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CONTENTS.

MEETING AT CHICAGO, JANUARY 9, 1920

MEETING AT NEW YORK, FEBRUARY 18, 19, 20, 1920

MEETING AT PITTSBURGH, MARCH 12, 1920

MEETING AT BOSTON, APRIL 9, 1920

MEETING AT WHITE SULPHUR SPRINGS, W. VA., JUNE 29—JULY 2, 1920

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THE SERIES SYSTEM OF STREET LIGHTING DISTRIBUTION

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The series system of distribution has been used almost universally for street lighting since the first use of electric lamps.

Lamps, both arc and incandescent are very simple and more efficient when designed for series operation than are the multiple type. The maintenance of constant power at the lamp terminals where lamps are thinly scattered over a wide area is much easier with a series system of distribution than with any other system. The burning of all street lamps in the city for certain specified hours makes it desirable to turn the whole circuit on and off from a certain point so that such a system whether series or multiple cannot be used to distribute power for other purposes. As therefore, a special system is necessary for the street lamps, it has usually been made of the series type for the above reasons.

The special apparatus required to operate a series system from constant potential is very simple and inexpensive. The constant-current moving-coil transformer is the main factor in the maintenance of the necessary constant current. The film cut-out socket for the lamp enables the continuity of the circuit to be maintained when the incandescent lamp breaks or is removed. Both are very simple and reliable even with unskilled operators.

Any system of street lighting of sufficient merit to supersede the series system must depend primarily on the development of simple, reliable and inexpensive control apparatus for the individual lamps to enable them to be operated on the existing multiple distribution circuits.

A SERIES system of lighting distribution is one in which all the current in the circuit passes through each individual lamp in turn. The power as measured in watts taken by each individual lamp is determined by the resistance drop across that lamp, the current in all the lamps being the same. By properly regulating this current at the station, the wattage and illumination is maintained at a constant value, regardless of the pressure drop in the conducting wires of the system. Special short-circuiting devices are employed to maintain the continuity of the circuit when the lamps burn out and to maintain a constant current in the circuit with the various conditions of load.

Such series systems are employed on a large scale to operate various types of lamps for street lighting. Conspicuous examples are the Mazda lamps in Chicago, the flame carbon arc lamps in Chicago and Indianapolis, the metallic flame or luminous arc lamps in Detroit, Pittsburgh and St. Louis as well as the old carbon arc lamps, both open and enclosed, which these lamps replaced. Where a suitable current and frequency are used the same circuit may operate both arc and incandescent lamps without difficulty.

As distinguished from the series system the multiple system of distribution requires lamps of the same voltage as the circuit rating, each of which has a resistance permitting the desired current to be taken. A simple switch at each lamp disconnects it individually from the circuit. As practically all factory and residence lighting is so operated approximately 95 per cent of the incandescent lamps used in this country are on multiple systems.

Other systems combine the principles of the series system with that of the multiple system by connecting a number of lamps in series and operating this series of lamps on a multiple circuit of higher voltage than is suitable for single lamps. Street cars thus use five of the substantial 110-volt lamps in series on their 550-volt circuits. However when one lamp breaks a filament, the other four will not operate as the circuit is then open. In some street lighting systems, notably at Milwaukee, series street lamps are connected in series and operated on constant potential in a similar manner. In such cases alternating current is employed and special devices are provided to maintain the continuity of the circuit when individual lamps go out.

EXTENT OF USE

Approximately 1,000,000, street lamps in the United States are supplied by series distribution circuits. This constitutes practically the entire street lighting distribution system of a large majority of the operating companies. Although requiring but a small proportion of the total kv-a. generating capacity, the system is of great importance to operating companies, because of the conspicuous position and essential duty of each small unit. It is especially notable that in their street lighting work, many operating companies render service to every one of the inhabitants whereas only 20 per cent or less, of the people may be making use of electricity in their homes. Yet, its total actual cost

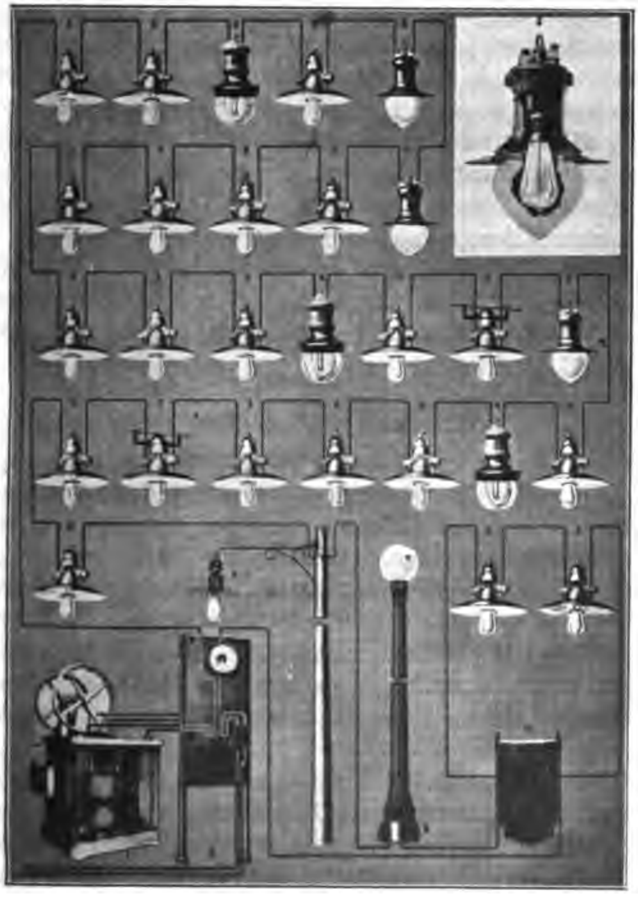

FIG. 1—STREET LIGHTING APPARATUS AS CONNECTED TO A CONSTANT-CURRENT MOVING COIL REGULATOR

may be only 1 or 2 per cent of the total city budget and seldom over one dollar ($1.00) per year, per inhabitant.

DEVELOPMENT

Although a great many other systems involving substitutes for the constant-current generator or moving-coil regulator have been tried out on a limited scale since the first introduction of the electric lamp some forty years ago, the series system is of as great relative importance as ever in street lighting, and is changed only slightly from the original concept brought out with the development of the first arc lamp by Mr. Charles F. Brush. This is an indication of the simplicity and fundamental soundness of the scheme.

The change from direct current to alternating current involved practically no change other than the lamps themselves, and the station equipment. The lines were seldom involved.

ADVANTAGE OF A SERIES SYSTEM

Street lamps constitute a comparatively small load in kv-a. which must be operated at various spacings over a wide area. The line cost per kv-a. of installation is relatively high. This is offset to a certain extent by a load factor of approximately 45 per cent where a 4000-hr. per year schedule is maintained. Furthermore, special precautions must be taken to maintain constant power at the lamps, regardless of the wide distribution. The series system is particularly adaptable to this service.

Consider that power is supplied to a 500-watt lamp every 500 feet along a street. A conductor smaller than No. 8 gage, is seldom used, as it would be too weak to be dependable, from a mechanical standpoint. From current-carrying consideration, a No. 14 wire would be ample although the watts lost in transmission, would be approximately four times as high as in a typical street lighting system.

If 110-volt multiple distribution is attempted, even with No. 8 conductor, not over two such lamps, the most distant being 1000 feet away, can be successfully operated, because of the poor regulation resulting from the voltage drop in the conductor. On a similar series circuit, 100 or more of such lamps, or a much larger number of smaller lamps, with as much as 40 miles of line wire, are readily operated.

Under such conditions there would not be over 50 kw. in load per square mile of territory served, and where the lamps are small, or spaced even further apart due to the presence of gas lamps in many locations or due to a scattered population, this load may be of the order of 5 kw. per square mile and 1 kw. per mile of single wire.

The low current in the series circuit prevents excessive drop in the line even with No. 8 conductor. The regulator compensates perfectly for this drop. The voltage of a single circuit is thus the sum of the voltages of the lamps on this circuit, and the line drop. It is evident that the long high-voltage circuits have the great advantage of less apparatus per kilowatt of load because fewer circuits will be required. Also a lower percentage of the line is used in simply running back to the station for purposes of control.

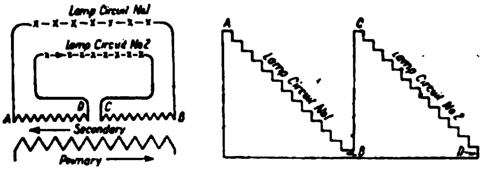

Fig. 2—Regulator with Two Interconnected Secondary Circuits and Development of the Diagram Showing the Voltage to Ground in the Various Parts of the Circuit.

The size of such a circuit may be limited by any one of several factors.

1. Voltage of the circuit to ground.
2. Number of lamps to operate on one circuit.
3. Insulation of lines and equipment.

Series lighting circuits on poles are subject to many grounds due to trees, lines breaking, crosses with other circuits and other causes. If an excessive voltage is supplied to any circuit an accidental ground at any point on the circuit causes a heavy strain on the insulation at other points because the drop across the large number of lamps is so great. This drop can be greatly reduced by cutting the circuit in parts and connecting a portion only of the generator or regulator winding between these sections. The whole combination consisting of from two to four sections is thus connected in series as shown in Fig. 2 and takes the same current throughout the circuits. Such a system has been used in the d-c. arc generators for many years and also in the constant-current regulating transformers.

Grounds must be promptly removed from such a system. Where double grounds occur, one on each of two circuits, they may, if of low resistance, both be placed near enough the station (Fig. 2 at *B* and *C*) to practically short-circuit one section of the regulator winding and the adjacent lamps, thus causing the current in that part of the circuit to increase to nearly 100 per cent of normal, while the remaining lamps would get practically no current. This double ground rarely occurs with good line construction, maintenance and methodical testing of the circuits. Equality of current in the two circuits may be insured by a relay so connected as to disconnect the regulator or circuits if current in the two circuits becomes unbalanced. In general the merits of such a system of interconnection are so great that it is in very general use, particularly where the voltage of a circuit would otherwise exceed 5000.

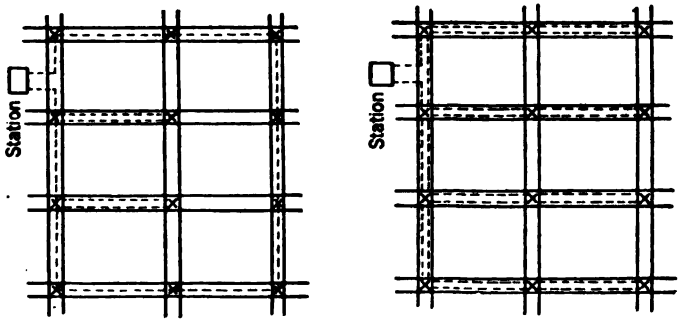

Fig. 3—Diagram of a Series Circuit on Streets Showing the Saving of Wire that May be Made by Using a Single-Wire Circuit

In many cases one circuit is used for the all night lamps of a white way system and the others for the half night lamps. The high light load efficiency of the regulating transformer makes this arrangement very suitable.

The number of lamps on a circuit is influenced largely by the density of the lighting units and their size. In extreme cases 500 are found on a single circuit where there is a wide area of suburban territory to be lighted only by low candle power lamps. The numerous lamp loops of such circuits greatly increase the chances of open-circuiting the whole system. To reduce the consequences of this the wiring should be arranged in loops so that cut-outs can be installed which automatically short-circuit and disconnect a broken loop (Fig. 3 shows typical loops).

The cost of insulating lines and equipment rises rapidly with the higher voltages. This fact must be considered and balanced with the cost of additional circuits necessary to maintain low voltages. On overhead lines some systems successfully operate arc lamps up to 8000 volts. Underground circuits are seldom operated at more than 5000 volts as any breakdown in their insulation is much more serious from the standpoint of repair.

It is of great advantage to be able, in a series circuit, to operate the lamps on one street from one wire of the circuit alone without it being necessary to have the return circuit on the same street (Fig. 3). This may save as much as 50 per cent of the total wire required, and as many street lighting circuits are on pole lines already necessary for residence lighting such a use of a single wire system means the saving of a very considerable portion of the total expense. Little or no additional wiring in such single-wire circuits is necessary to create loops by which large sections can be isolated either automatically as previously described or by means of a jumper. The automatic current regulation of the constant-current moving-coil transformer makes this perfectly feasible even though 90 per cent or more of the lamps be so cut out of circuit.

Line reactance with such small currents involved may usually be ignored as its only effect is to slightly reduce the load carrying capacity of the regulator. Variations of the lamps themselves from their rated voltage have a much greater effect on the total regulator capacity. For overhead lines this reactance drop at 60 cycles in single-wire circuits is approximately 50 per cent and in return-wire circuits, 25 per cent respectively of the resistance drop.[1]

CONTROL

The operation of the street lighting lamps from dark until dawn or over some other fixed period makes a special condition of control not required for any other type of service. It is of great advantage from the standpoint of time and labor to be able to control all these lamps from a single point. Such has generally been the practise, thus making it impracticable to use the street lighting circuits and lines for any other service. Therefore, where a separate circuit is run for street lighting the series system has many advantages from the standpoint of installation costs and regulation over any other system.

LAMP CHARACTERISTICS

In general it has been well worth while to make special lamps for series operation on account of the higher electrical efficiency or lumens per watt input.

In direct-current series arc lamps the wasteful sustaining resistance necessary to the multiple arc is omitted and the constant-current regulating device itself sustains the arc without material loss of power. In consequence of this difference, the direct-current arc lamp electrical efficiency is approximately 70 per cent for the multiple and 95 per cent for the series lamp. Similar figures for an alternating-current system of enclosed carbon arc lamps so widely used at one time, were 90 per cent and 95 per cent respectively. The smaller apparent difference is, of course, due to the use of a reactance coil, instead of a resistance in the multiple arc lamp for sustaining effect.

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[1] Formulas in Pender's handbook or in Bureau of Standards, Vol. 4, No. 2, page 317.

The series system has been retained for use with Mazda lamps, as the efficiency of these lamps when made for a series system is much better than that of multiple lamps, being, in the case of the 100-c.p., 6.6-amperes series lamp, some 20 per cent higher than a 110-volt lamp of such a candle power.

Direct-current arc lamps were used on the early series circuits because of the greater steadiness and efficiency of the arc on direct current than on alternating current. The alternating-current enclosed arc although much less efficient became very popular when alternating-current systems of generation and distribution became common because a series alternating-current circuit could be derived, from the usual 2300-volt constant-potential bus by a simple constant-current regulating transformer. This transformer had only a fraction of the first cost, maintainence cost or floor space of the motor generator or rectifier set necessary to operate a direct-current series circuit from the alternating-current constant potential bus. Yet the improvement, from the illumination standpoint, over all predecessors of the metallic flame, or luminous arc lamp introduced about 1907, was such that many of the direct-current series circuits necessary for their operation have since been installed.

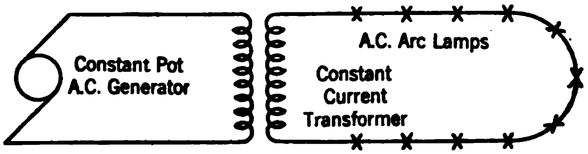

FIG. 4—CONNECTIONS OF A SERIES CIRCUIT TO A CONSTANT-POTENTIAL SOURCE BY MEANS OF A CONSTANT-CURRENT TRANSFORMER

Where incandescent lamps only are used there is obviously no need for direct current and it is well worth the slight expense to change the regulating equipment from direct to alternating current when direct-current series arc lamps are replaced by incandescent lamps. Such a change usually only involves the replacement of the station apparatus by a constant-current regulating transformer. An alternating-current circuit enables the use of the 20-ampere lamps from auto-transformers where high candle power lamps are desired. Safety coils can also be used on alternating current enabling the use of series lamps on fire alarm brackets, traffic posts and bridges in a way practically impossible on a high-voltage direct-current circuit with any degree of safety.

Frequency has had but slight effect in the development or use of series circuits. Apparatus has been available for all commercial frequencies except that arc lamps were never satisfactory on 25-cycle circuits.

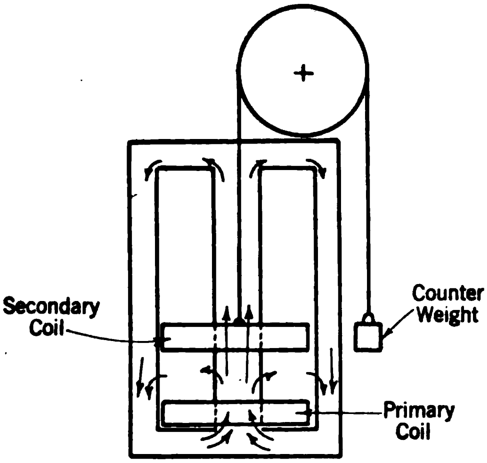

FIG. 5—FLUX RELATIONS IN A MOVING-COIL REGULATING TRANSFORMER SHOWING THE LEAKAGE OF FLUX BETWEEN THE STATIONARY AND THE COUNTERBALANCED MOVING-COIL

APPARATUS

In the simplest and most commonly used series systems, a special constant direct-current generator or a moving-coil transformer for alternating current is required at the station to control the circuit and to maintain the correct current at the lamp.

In the old direct-current arc lighting machine, special designs utilized the reactance of the armature to prevent excess current in the lamp circuits, and used devices for shifting the brushes to obtain the necessary voltage up to the limits of the machine. Some of these regulators were extremely accurate and were sensitive enough to operate the high efficiency incandescent lamps now commonly used. The constant-current regulator commonly used for alternating current depends upon the electrical repulsion existing between the primary and secondary coils of a transformer under load to produce and maintain a constant current in the secondary or lamp circuit. Such a regulator is of great value because it can automatically maintain a fixed secondary current through any number of lamps from one up to its maximum capacity or with a wide fluctuation

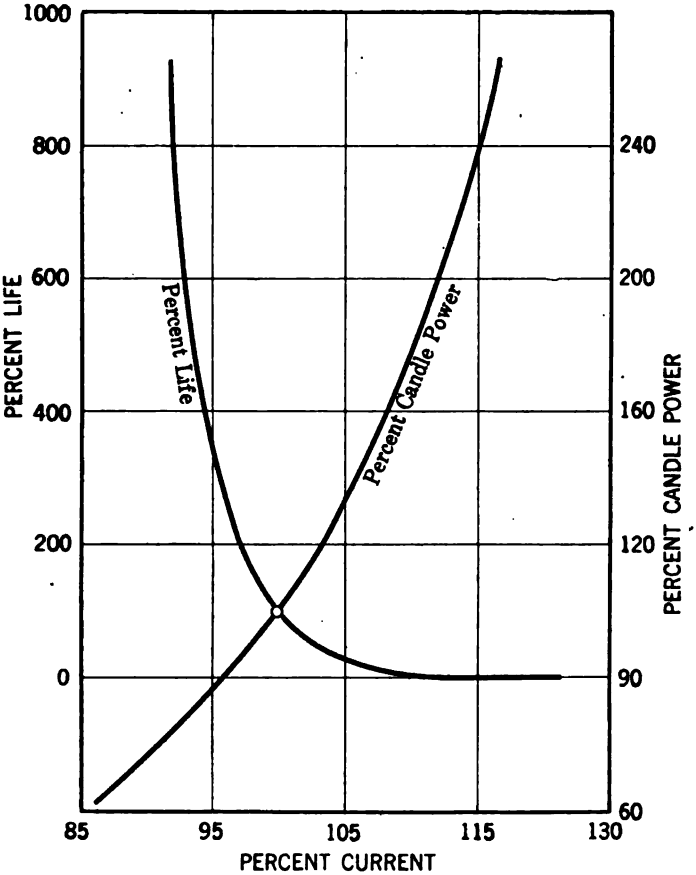

FIG. 6—CURVES SHOWING THE CHANGE IN CANDLE POWER AND IN LAMP-LIFE WITH A CHANGE OF CURRENT IN A TUNGSTEN FILAMENT

of the voltage on its supply circuit. When it is realized that an increase of 3 per cent in the current of the lamp circuit cuts the life of an incandescent lamp in half and thus doubles the renewal cost, it can readily be appreciated that such a regulator is very cheap protection for the lamps as compared with any system that does not compensate automatically for all variations. To get a more definite idea of this relation it should be remembered that in extreme cases such as certain suburban districts using a great number of low candle power lamps the value of the lamps on the circuit is equal to that of the regulator controlling them. Under normal conditions these lamps are renewed about three times per year. Accurate and reliable regulation is therefore necessary to keep these lamps up to rated candle power without getting at times a current sufficient to cause premature burnouts and increased expense in lamp renewals as well as in deductions for lamp outage. Renewals of four times per year would in this case require excess lamps equal the value of the regulator. The

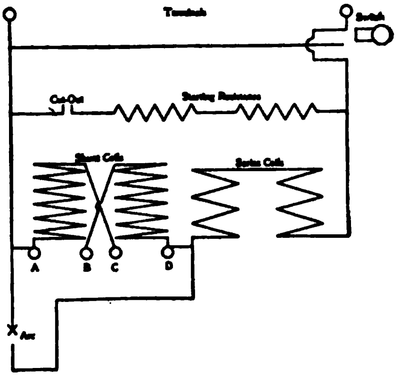

FIG. 7—SCHEMATIC DIAGRAM OF CIRCUITS IN A SERIES ARC LAMP SHOWING THREE PARALLEL CIRCUITS:

(1) Main circuit through the series coil and the arc.

(2) Shunt circuit to pull the carbons together when the voltage across the arc increases.

(3) Starting circuit consisting of resistance in series with a switch to cut it out when the arc burns.

The drop across the starting resistance through which the current first flows causes current to flow through the arc and series coil This coil pulls the carbons apart opening the starting circuit The shunt coil prevents the arc voltage becoming excessive and can close the switch, if necessary, in the starting circuit thus maintaing a safe current path through the lamp.

moving coil regulator is the only device which automatically compensates accurately for short circuits and double grounds on the series line or for voltage variation of the constant potential supply.

Such regulators are so substantial and reliable that small sizes have been successfully mounted on poles in transformer cases and operated by a time switch. Wherever possible however, they should be installed in a station so that the line can get more frequent and careful testing and an ammeter can be continuously kept in the lamp circuit.

In every series circuit the devices used on it must have a means of by-passing the current when the device itself is inoperative to prevent the opening of the whole circuit.

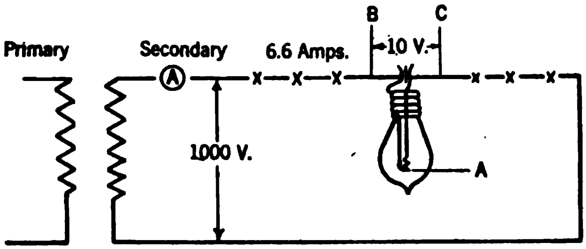

FIG. 8—THE FILM CUT-OUT SOCKET FOR INCANDESCENT LAMPS

When lamp filament *A* breaks, the voltage *BC* rises from that of the lamp to that of an open-circuited regulator causing the film to break down and short-circuit the lamp.

The arc lamp has a mechanism arranged as in Fig. 7 so that a shunt coil connected across the arc causes a switch to be closed and the arc short-circuited when the drop across it becomes excessive. Another coil in series with the arc and line acts to give the correct spacing of the carbons or electrodes.

For the incandescent lamp a simple form of switch is incorporated in the socket. An insulating film is placed between the two sides of this switch. This film has normally only to withstand the drop across the lamp filament, *A* (Fig. 8) of 10 volts. However, if the lamp burns out the filament itself gives the effect of an infinite resistance, and the full generated voltage of the regulator or generator is impressed upon the insulating film, causing it to puncture and short-circuit the lamp filament. In a moving coil regulator this maximum voltage is approximately 25 per cent more than the full-load voltage. A switch in the socket operated by the insertion of the lamp in this socket short-circuits this film when the lamp is removed.

Such film cut-out sockets are very simple and inexpensive, and enable men with little training easily to renew the lamps, even under trying conditions of lamp location and bad weather.

In general, devices other than lamps have seldom been used to any great extent on street series circuits. On account of the dark until daylight schedule it is often desirable to operate lamps indoors and around places where the high voltage possible on a series circuit cannot be tolerated. Underground circuits are often included in this status on account of the great expense of high-voltage cable. In such cases a special series transformer or safety coil is used. This safety coil can supply a small wattage at constant current and low voltage on the secondary circuit and yet be insulated for an extremely high voltage on the main series circuit.

Instruments or meters of a special nature are seldom required for series circuits. A high grade ammeter should be in series with the lamps for upon it depends the accuracy of the adjustment of the regulator and the life and candle power of all the lamps. The possible accuracy and permanence of the adjustment of the moving-coil regulator is well comparable to that of the ammeter, and little difficulty is experienced in its operation. Municipalities sometimes require a chart of the current in the series circuit and maintain curve drawing ammeters for this purpose. In other cities a jack on a pole allowing the insertion in the circuit of a plug with ammeter fulfills all requirements.

MAINTENANCE

Maintenance and testing of series circuits is special only in that checks through the day are advisable in order to repair before dark any damage that may have occurred during the hours of non use. Troubles usually show in the form of grounds and open circuits. Simple electrical tests locate troubles, of this nature. They must be promptly removed to avoid damage to the lamps or interruption of service during the dark hours.

SUMMARY

The advantage of the series systems of street lighting distribution consist of (1) effectiveness; (2) simplicity of control; and (3) high electrical efficiency of lines and lamps.

The disadvantages are (1) high cost of line per dollar of income; (2) high voltage; (3) special lamps.

The ideal system ultimately should be one to operate a highly efficient standard lamp from the existing multiple distribution lines necessary for residences and factories. Complicated special apparatus and special wiring, other than to the lamps is thereby eliminated. Reliable and inexpensive control devices must be available to turn the lamps on and off. Although much time and expense have been spent on such development ever since the lighting industry started, none has ever been sufficiently perfected to be worthy of generally replacing the simple series system.

DISCUSSION ON "THE SERIES SYSTEM OF STREET LIGHTING DISTRIBUTION" (HURLEY), CHICAGO, ILL., JANUARY 9, 1920.

A. E. Betts: I would like to find out what success different people have met with in the use of the refractors on such lamps, and also what effect it has had on illumination, after they have been cleaned.

E. Sweitzer: I am of the opinion that a refractor does materially benefit the distribution of illumination. It is found with the ordinary diffusing globe the natural angle of distribution is around 45 deg. below the horizontal. In the use of refractors the maximum angle is raised to about 10 or 15 deg., and consequently you get a very much wider distribution. I think fundamentally that is the basic reason for using refractors. I venture to say that there are very few installations having high candle-power where the refractors are not being used. Where large units are used I think it is of decided advantage. As far as maintenance cost is concerned, it is not excessively high. They are made quite heavily and with ordinary service, breakage should be very light. The operating records of central station companies who are using street lighting units with refractors will well bear out my contention.

A. E. Betts: The idea is that after a refractor has been in use for a short period of time it becomes very dirty and unless you institute a system of periodic cleaning, the lighting will be very materially reduced and the refractor of no benefit.

E. Sweitzer: That is true. From a purely commercial point of view I want to say that both central station companies and municipalities have gotten to the point of seriously abusing street lighting systems such as are now in operation since Mazda Lamps have become as prevalent as they are. Heretofore when they used arc lamps they had to get around and trim them at certain periods. A lamp had to be trimmed every hundred hours or 150 hours, and at that time the lamps were naturally cleaned, but when you come to Mazda Lamps the operating company seems to think "Well, here is something that will last a thousand hours" and while the contracting company calls for the lamps to be maintained in proper form, in a great many instances they are not given near the attention that the arc lamps were. If the refractor is permitted to get dirty naturally it absorbs some light, but if they are given reasonable attention, possibly, I would say six to eight times a year, there would be a very slight loss of light. Does that answer your question?

A. E. Betts: Yes, it does, only our experience is a little bit to the contrary. We find that if they are allowed to go even six weeks the amount of dirt accumulation on the refractor very materially reduces the light.

E. Sweitzer: Isn't it true that it is due very largely to the atmospheric conditions? Taking it as a whole I don't believe they should require such frequent attention.

A. E. Betts: We have not made any tests on the lamps, although we intend to do so, but it is evident that the light is very materially reduced.

F. F. Fowle: Inductive interference from series distribution systems is due fundamentally to two causes. One is the character of the current in the circuit and the other is the nature of the layout of the line circuit itself. In the early days with the use of old T. H. and Brush arc machines inductive interferences with telephone systems from series distribution, using generators of this type, were very severe, more severe than is encountered ordinarily today. That was due in large part to the fact that the wave form produced by those machines was very irregular and in the case of both machines there was a very peaked wave of electromotive force which arose instantaneously to very high and abrupt values and of course produced a very serious disturbance. The current was a constant in the sense that it had a constant average or effective value, but the instantaneous value even though it was so-called direct current, was anything but constant. With the passing out of direct-current series lighting and the introduction of the alternating-current system, of course, we come to the modern conditions. Series circuits are, generally speaking, of three types, the so-called open-loop series circuit, in which the line wire is led from lamp to lamp by the most available direct route without regard to anything else, and in that type of circuit the configuration of it is generally an irregular open loop. The second type is the closed loop circuit, in which the line wires are kept parallel so that while it is a series circuit it is a parallel two-wire series circuit in which the return wire is always kept on the adjacent pin, or at least on the same pole line with its mate.

The third type, so-called, is a combination of the two or a mixed loop circuit. The open loop circuit gives trouble of course from inductive interference if it is in the same neighborhood with telephone circuits, particularly circuits of open wire. The trouble with telephone circuits in cable is considerably diminished. The electrostatic induction is entirely removed by placing the telephone circuits in a cable, within a grounded metal sheet, but magnetic induction is affected but slightly. A practically sure and efficient remedy for inductive interference in series distribution can be obtained by using the closed loop. In that case the inductive interference is usually less than it would be from the ordinary multiple distribution system with the same spacing of conductors. Of course, its current is limited in value; the current is very frequently less than would be flowing in a multiple distribution system,—at least when the load is of fair proportions. There are no very effective means, from the telephone standpoint, to combat induction from the open loop construction in ordinary city distribution. It is usually impractical to transpose against it, because the points of change or discontinuity in the lighting circuit occur with such frequency that any attempt to put transpositions in the telephone circuit is impractical for the reason that the poles are not close enough together or they don't come close enough to the theoretical transposition point, and it would in any case make a badly cut up and disarranged telephone distribution system, that is, according to the open wire.

It is also of some advantage to place the lamps alternately on each side of the series circuit. Where the two wires of the series circuit leave the station, let us say the first lamp is in wire No. 1 and the next in No. 2, the third in No. 1, the fourth in No. 2, and so on. That means the point of zero potential or the neutral point in the series circuit will come at the far end and the drop of potential will be uniform in the two wires, so that if taking maximum values of one wire at the station at some particular instant as 1000 volts positive, the opposite end is 1000 volts negative, and at the far end of the loop at that same instant the potential is zero.

I would like to say a few words about the use of line conductors for series distribution. Some years ago I had an opportunity to make an experimental installation of copper plated steel lined wire in a small series distribution system. The lamps were closed carbon lamps operating on about 5 amperes, and there were 19 or 20 lamps in the circuit. It was operated from a 2300-volt bus through a regulating reactance. This circuit was carefully tested in normal operation for current, voltage, energy input, power factor, and then in the day time the lamps were all strapped out with a jumper at the terminals of each lamp. We then measured the true continuous current resistance of the lamp circuit and we also put a normal operating current around the line circuit, with the lamps cut out, and measured the impedance, effective resistance, and effective reactance of the line circuit and power factor. After these tests were completed, we took down the line wire, which was No. 6 medium hard drawn triple weather-proof copper, and on the same pin positions and precisely the same positions throughout the entire circuit we strung up some No. 8 hard drawn triple weather-proof copper cased steel. Then we repeated the whole series of tests and I can give you a few of the results that we obtained. We found that the No. 6 annealed copper had a breaking load of an average of 750 lb., a potential strength of approximately 38,000 lb. per sq. in.; No. 8 hard drawn, an average breaking load of 1200 lb. approximately, and about 97,000 lb. per sq. in. A test on the No. 6 copper circuit with the lamps cut out showed that it had a true resistance of about 9.05 ohms and the effective a-c. resistance, about 9.25 ohms. The ratio of the effective a-c. resistance to the true d-c. resistance was about 1.02 per cent. That is only about 2 per cent skin effect in that copper wire at sixty cycles. The true resistance of the copper clad steel circuit was practically four times as much, and the skin effect factor was practically the same as it was with the copper wire, that is, between two and three per cent. Taking into account the actual diameter of the wire and the temperature, we found that the conductivity of the copper wire was 97 per cent.

Another interesting result was the calculations on the measurements of the effective reactance of the copper circuit. This was a mixed loop circuit but more of the open loop type than of the closed loop. It was found that the total reactance of the mixed loop circuit, as calculated from the measurements, was the same as though it had been a parallel loop circuit from end to end, with a spacing of 11.5 ft. between the two wires. The measurements with the lamps in circuit showed that the power factor at the bus, which would include, of course, the power factor of the line and the power factor of the reactance combined, was, for the No. 6 copper circuit about 62 per cent, and for the No. 8 copper clad steel circuit about 66 per cent. The power factor of the line circuit with the lamps in it was only about 92 per cent for the No. 6 copper, and the same figure for the copper clad steel. That is to say, that shows that there was no particular additional reactive effect because of the steel core in the copper clad steel wire. The main effect of it was increased line resistance. The average watts lost in the line in this particular case was about 230 watts for the copper circuit and about a thousand watts for the copper clad steel circuit, that is, roughly, in the ratio of four to one.

In the matter of structural advantages and economies I gave you the figures on the breaking load of the two wires. Of course the No. 8 copper clad steel is much the stronger of the two and naturally would be the more reliable conductor, from a structural standpoint. However, its disadvantage is having about four times the resistance of the copper, and the increased energy loss.

Without going into details, taking the cost of copper wire as a basis, which at that time was in the neighborhood of sixteen cents, and assuming a schedule of 3000 hr. per annum of operation, it worked out that the turning point in economy, as between the two wires, was when the energy cost about 0.55 of a cent per kw-hr. If the cost of energy was more than that there was economy in using copper clad steel, but if the cost of energy was less than that, there was no economy in doing that. There was, of course, considerable increase for reliability, and the inference would be there was somewhat less maintenance cost. That circuit has been in service for some years and as far as I know it has never given any trouble.

G. N. Chamberlin (read by C. E. Skinner): In setting forth the advantages of the series system of distribution, I think Mr. Hurley has been more than fair in giving what he considers the disadvantages.

HIGH VOLTAGE

Modern arc lamps cannot be used to advantage on multiple circuits and series incandescent circuits are as a rule I believe not above 5000 volts. It is doubtful if this voltage, with the circuit wires widely separated and the flow of current restricted to practically normal, is not safer than 2200-volt lines carried, as they usually are, closely spaced and backed by unlimited current, in case of short circuit or double grounds.

SPECIAL LAMPS

It is well known that losses from theft where multiple lamps are used for street lighting are very large and even series lamps are at times brought in for free renewal.

Several central stations are now using an auto transformer, so that the low-voltage, high-current, large base lamps can be used on multiple circuits for no other purpose than to eliminate the loss from theft.

Series lamps have greater intrinsic brilliancy or a more compact filament. This allows of the more effective use of scientifically designed auxiliaries (reflectors, globes and refractors.)

It is characteristic of the incandescent lamp filament that its diameter decreases during life, due to evaporation. The decreased diameter results in increased resistance. With a constant-current supply the wattage is increased and the candle power maintained, while on any form of multiple or mixed circuit there must be a continual changing of the line potential in order to maintain contant candle-power and candle-power is what the central station sells and wishes to maintain as nearly constant as possible.

The speaker fully realizes the advantage to the manufacturer, distributor and central station of having the fewest possible types of lamps, but he believes that the series street lighting lamp is fully justified.

There is soon to be installed at Saratoga, N. Y., an ornamental duoflux unit consisting of 1000 candle-power and 250 candle-power lamp in one globe. A small evacuated bulb containing mercury and suitable terminals, tilts one way and then the other by successive throwing on of the circuit. The 1000 candle-power lamp will be used until midnight and the 250 candle-power from midnight until daylight. Simple switching apparatus can be made use of where a separate circuit is controllable from the station.

F. W. Parker (read by C. E. Skinner): In Mr. Hurley's paper under "apparatus" the statement is made "The moving coil regulator is the only device which automatically compensates accurately for short circuits and double grounds on the series line or for voltage variation of the constant potential supply." He further states that "The constant-current regulator depends upon the electrical repulsion existing between the primary and secondary coils under load to produce and maintain a constant current in the secondary or lamp circuit. Such a regulator is of great value because it can automatically maintain a fixed secondary current through any number of lamps from one up to its maximum capacity."

Is it not a fact that the repulsion action depends upon the strength of the magnetic field set up by the current in the primary and secondary coils of the regulator? The magnetic fields being proportional to the value of the current. Under short-circuit conditions coming on a part of the series circuit, it must necessarily result in a greater current, hence the repulsion action lags behind the current thereby introducing a time element during which the lamps remaining in the circuit are subjected to abnormal current.

What would be the value of this current with a short of fifty, seventy-five or eighty per cent, and under said conditions would the remaining lamps in the series circuit be injured or destroyed?

Regulators with stationary coils built with high inherent reaction, and operating upon the magnetic leakage principle are used for the control of series incandescent lamp circuits. With this type of regulator the time element mentioned above is entirely removed and it is possible to get any range of regulation desired by adjusting the inherent reaction. Is such equipment now available, and when built as described above, will it give full protection to series burning lamps when operating from a constant potential supply?

F. F. Fowle: The example is shown at the right of Fig. 3 in Mr. Hurley's paper. The example of the open loop circuit is shown at the left in Fig. 3.

E. N. Lake: I don't understand that the previous speaker intended to recommend a special copper clad wire for distribution, but if that was his intention I want to say that the use of copper clad wire may, under certain climatic conditions, prove to be very unsatisfactory. I refer particularly to locations where the lines may be close to salt water.

F. F. Fowle: I would like to ask Mr. Lake if he has reference to the bare wire or to the weather-proofed wire?

E. N. Lake: The bare wire.

F. F. Fowle: I will say, in general, that the corrosion of copper clad steel wire with regard to its life, would be the same as that of bare steel. It sometimes happens as you say, near salt water, that it corrodes rapidly, but I know that so far as recommending it is concerned there are a great many thousands of miles of that wire that are used both bare and insulated, and it is giving on the whole, very satisfactory service. I can see no reason why you shouldn't recommend it where the economics of the situation warrant.

E. N. Lake: To take an example, would you recommend it for use across salt marshes?

F. F. Fowle: Yes.

E. N. Lake: In the bare condition?

F. F. Fowle: Yes.

F. A. Vaughn: I would like to ask Mr. Lake where the corrosion occurs. I would get from inference, that it occurred on the steel, and with reference to your own statement it would seem to me it would make no difference, if the copper coating was perfect, whether there were steel wire inside or not. I should like to ask Mr. Lake if the salt water, or the salt air, did get in to the steel through the copper coating.

E. N. Lake: The particular installation that I refer to was along the border of salt water and it became necessary to replace the wire with solid copper wire. It was not possible, from the brief examination that was made, to determine whether or not it was defective wire. The assumption would be, of course, that it was defective in manufacture, but the fact is that, due to the processes of manufacture, you cannot always depend upon a uniform coating of copper over the steel core.

F. F. Fowle: Perhaps I can explain a little about the manufacture which will make it clear why corrosion sometimes occurs, and why the thin places sometimes occur. The copper clad steel wire has been made in this country by three different processes. It was first made by a company that has since gone out of business. It is now made by the Griffith and Roth processes. At one time the company which manufactured it under the Monarch process started by using steel rounds and coated them with copper. Then for some reason or other they abandoned that process and began using square steel bars and coated them with copper. That undoubtedly had a bad effect. When steel rods were used and coated with copper the steel was well centered in the copper coating, and it drew down, that is, it rolled down very uniformly in the hot rolling, and then drew down very uniformly in the cold drawing, except in one or two cases, but with the square sections there was obviously a thinner coating of copper at the four tips of the squares than there would be at the middle of the four sides, or between the tips of the squares, and therefore bare steel was more likely to occur at these points, and it did occur more frequently. While it was made at that time, with the square steel I think it gave some trouble. As far as I know at this time the two companies that are making it are both using round steel.

As to the character of the copper on the outside of the steel, all the researches that I have made on it have never indicated that the copper was anything but good metallic copper, unless the copper was oxidized. I have analyzed cases where we found that the wire had low conductivity and where it was determined upon getting the relative area of the copper and steel that the conductivity should have been normal. Upon an analysis being made of the copper it was found that the low conductivity was caused from within the copper itself. Copper is exceedingly sensitive to conductivity. A few tenths of a per cent of impurity in copper will effect the conductivity immensely, so that it is necessary to be careful to get the purest grade of copper on the copper clad steel, as it is to get it in the ordinary copper wire. I have known of many cases of premature corrosion of copper clad steel that didn't occur as a result of the steel showing through the copper, which really accounts for corrosion in many cases.

E. N. Lake: It would be very interesting to know whether there is any considerable quantity of it over salt marshes. I think it would be well to make a further investigation.

F. F. Fowle: I can't say specifically as to whether it is installed over salt marshes, although a great deal of it is in use in this country for railroad signal circuits. I think that, however, is made of weather proof wire.

Mr. Cameron: It is true that when a short circuit involving an appreciable amount of the street lighting circuit occurs, there is a time lag before the transformer accommodates itself to the new conditions. That interval isn't a matter of hours or minutes; it is of seconds or fractions of seconds, and one feature which helps even to reduce that interval is the fact that the action of the tungsten filament to increase this current is not instantaneous, either. The lamp comes up to heat after the current has increased. There is a time lag there also, which, for all practical purposes, eliminates the "leisureness" of the transformer in meeting the new conditions.

As to the second question, if it is so important that even intervals of that instantaneous nature are worthy of comment, how much more necessary is it to have a device which meets that condition automatically and instantaneously, rather than with a device where it is necessary to manually change taps after you have become acquainted with the conditions on the circuit, which is not a matter of seconds or fractions of seconds, but perhaps minutes or hours, in a division where the current may change ten per cent, which is one quarter the life of the lamp, before the next tap brings you a better condition. That is, your transformer, without regard to steps, automatically and instantaneously does it.

C. H. Shepherd: The Commissioners of Lincoln Park operate a street and park lighting system of some 1700 street series lamps which are supplied with power from 18—7½—ampere series circuits with voltages ranging from 2000 to 7000. Each circuit has its individual transformer, regulator, oil switch, control panel and instruments, the power for the entire system being derived from a 12,000-volt 3-phase power line from The Sanitary District of Chicago.

From the standpoint of the lay-out engineer these circuits divide themselves into three main classes, namely, open loop, closed loop and combination.

In order to properly segregate defective portions of circuits, pot heads are installed at various points to facilitate not only this operation, but the locating of trouble and maintenance of continuity of service as well. At the feeding points of all loops, disconnecting pot heads are installed with triple braided jumpers so that any loop may be disconnected, and shorted out of circuit in case of trouble. At the ends of all loops, pot heads are installed with ground connections arranged so as to be made immediately available in case of necessity, and where two circuits run parallel on the same boulevard it is thereby made an easy matter to select for service the two good sides of these circuits or loops in case of the development of a ground or open on one side of each. The pot heads installed at the ends of the circuits are also available for rotation testing and for opening any circuit for capacity tests. Pot heads are also installed at the ends of feeder lines on the energy side of the first lamp on each circuit and the last lamp respectively, thereby allowing the feed line to be disconnected and closed across for the purpose of making a bridge test.

In cases where it is necessary to interrupt the continuity of the cable sheath in such places where pot heads, triple braided lamp legs, or similar devices are installed, the cable sheath is bonded across the break and pot head sleeves, belled at the triple braided end and wiped into a sealed joint, 18 in. back of the bell, are installed, thus effectually taking care of the accumulation of static at these points. In cases where by polarity tests the cable is shown to be positive to ground ground rods are installed and bonded to the cable sheath but in places where the cable shows negative to ground the cable is either insulated, or no action is taken at all where the process of insulating would be too difficult.

In checking the rotation of lamps on circuits, a ground is placed at the center of the loop in question, and another ground placed on the right or left respectively of the circuit at the switchboard. This connection cuts out the lamps on the grounded leg allowing the lamps on the ungrounded leg to burn, with the feeder transformer at half-tap making it a very easy matter to check up a circuit diagram by merely running along the circuit in a machine and checking off the lamps on the right or left according to whether they burn or not.

On long closed loops sometimes involving an entire circuit, the lamps are connected on the step by step system, or electrically staggered on the line cables. This method cutting down the greatest length of cable between lamps, to two-lamp spans. Obviously this construction facilitates the locating of trouble and reduces the necessary cuts to a minimum. The rotation of lamps is checked after each change by the method outlined above.

Series circuits on this system are always treated as condensers, the conductor forming one plate and the grounded cable sheath, the other. In order to determine whether or not these circuits should be operated grounded or ungrounded, potential gradient tests were made covering each circuit in question. On ungrounded circuits the potential gradient is almost uniform due to the fact that there is no ground on the circuit, allowing the zero potential point to shift position to compensate for the changes in reactance on the circuit due to lamp burn-outs, regulator movements, etc. and the prevailing electrostatic conditions throughout the entire length of the circuit, including the feeder cable. With the apparent electrical center of the circuit grounded the potential gradient is unbalanced and shows a lack of uniformity due to the fact that the zero potential point is fixed by being grounded and the former flexibility and freedom for adjustment of the ungrounded circuit in response to changes of electrostatic and electromagnetic conditions is thereby seriously hampered. Such unbalance due to action of the regulator and to changes in electrostatic capacity caused by variations in the specific inductive capacity of the various dielectrics used in the cable insulation along the circuits are very apparent in making such a test.

After investigation of the potential gradient situation obtaining on this system, it was apparent that the advantages on an ungrounded system outweighed those inherent on a grounded system and it was therefore decided to operate the system ungrounded as it is a very easy matter to place grounds at the salient points of the circuit whenever necessary.

In the early days of this system the series circuits carried 7½-ampere enclosed-carbon arc lamps with an efficiency no better than the average for these units. Some five or six years ago the arc lamps were eliminated and a compensator construction substituted on all existing standards, each compensator supplying power to a 15-ampere 400-candle power type C lamp. As extensions to the system were being rapidly made, it was decided after certain experiments to feed lamps on all new construction from the secondary side of suitable series-multiple transformers and after a series of experiments and tests had been conducted with this end in view, a standard transformer was adopted and installed from that time on. These transformers have a partly open core—the primary and secondary windings being insulated by a dielectric having a strength of 20,000 volts, 60 cycle. The line and lamp connections are made with tinned brass wiping sleeves which are fitted with suitable lead sleeves soldered on, wiped to the cable sheath, properly insulated, filled, refilled and sealed, thus establishing a bell at each terminal and a 100 per cent cable sheath bond. This construction operates as well submerged as dry, no static trouble being experienced, as the secondaries are not grounded, but are carried in 600-volt duplex rubber and lead cable direct to the lamp.

The safety island light system is fed by transformers of similar construction, but of a different suitable characteristic and capacity, each transformer having as a secondary load five standard 56-watt railway type B lamps.

The capacity of the type C transformers is approximately 225 watts with a regulation within one-half of 1 per cent of normal full-load current and a rise in pressure from full load to open secondary of not more than 200 per cent of normal, the power consumption expressed as a figure including line loss, averaging about 272 watts per unit. The island light transformers on the other hand have a capacity of approximately 220 watts with a very low power factor, and a rise in pressure from full load to open secondary of not more than 50 per cent normal full-load pressure.

By the elimination of the arc lamps and substitution of the devices mentioned above, each circuit has been made metallically solid, allowing a number of advantages which did not exist previous to the time mentioned. At the present time three continuity devices are in service, *viz.*: film cut-outs, compensators, and series-multiple transformers.

Some two years ago a testing ground bus with suitable quick connecting devices was installed on the main series circuit switchboard allowing certain and rapid trouble shooting by means of using the lamps themselves as indicators. This process is extremely simple, it being only necessary in the case of a ground on the exterior circuit, to connect to the ground bus the side of the circuit nearest to the trouble as determined by a current test. The circuit is then cut in at half tap, the lamps on the grounded side naturally being cut out and a quick trip made to the last lamp out and the first lamp burning. This point being reached it is quickly checked by a reversal of the ground at the switchboard, the defective cable being then narrowed down to two lamp spans, after which the trouble is quickly located at the exact point by a visual test, magneto or bridge. In the winter of 1918 we had 161 cases of trouble and the average time of location was 17 minutes. The cost of this ground bus was about $150 and it paid for itself many times over from the first month of use.

As stated above, any feeder lines may be disconnected and shorted out at the feeder point leaving this line available in case of trouble for a Murray loop, or Varley loop test, or both. Open circuits on similar lines may be easily located by disconnecting sections, by the use of a capacity bridge, or by breaking down the insulation at the open point by means of high tension to ground.

As an adjunct to the above, a progressive total of cable lengths from right to left and from left to right on each series circuit is being prepared which we anticipate will greatly facilitate the locating of points at any given distance from the source of supply of any circuit, when such distance is indicated by means of a bridge or loop test.

W. A. Del Mar: Some open-circuit series-arc lighting systems have been installed. A pessimistic theorist might say that they would be objectionable from the point of view of inductive losses, excessive impedance, etc. I would like to know whether any engineers, who are familiar with such systems, are encountering any difficulties in operation.

Due to the small current used in series arc lighting, usually less than ten amperes, this objection is probably not important. Nevertheless, it would be very interesting to hear of some actual results in practical operation.

Henry Nixon: It is a fact that we have noted some differences in the current value at the phase end and the ground end of a long circuit. It hasn't been particularly serious, except that in the longer lines it has been necessary to install reactance in series at the point where the return circuit starts. That difference in the amperage between the phase and the grounded ends of the circuit I believe has never been more than about 0.4 of an ampere, but on some of our circuits we operate at a higher amperage than the rated amperage for the lamp. Mr. Synder, our operating engineer, was talking to me about it the other day and in the course of our discussion he drew my attention to the fact that we were now getting a longer life out of the four ampere lamps used on such circuits than we were out of similarly designed lamps.

It was our idea, from the few figures we had at hand, that the average amperage at which the lamps were operating—I don't know whether I make that clear or not—the average at which the average lamp was operating would be slightly less than the rated amperage of the lamp, giving a longer life to that lamp. So far as cable trouble is concerned, the mechanical troubles and grounds, etc., we consider them negligible. The only trouble we have is due to citizens driving parkway stakes through the cable and cutting it in two. This cable has been in service since 1913 and very few cases of trouble have come up except from that cause. I don't know whether Mr. Snyder has any figures here or not.

Mr. Snyder: I have not.

F. A. Vaughn: May I add that the Milwaukee system, I think, is the type of system that Mr. Del Mar speaks of, and while the system is not entirely installed, there are hundreds of thousands of feet of cable already in and operating, and this system is largely of the open-loop circuit type. The cable is broken up into isolated sections very often, and so far there has been no noticeable or appreciable effect on the system as it is operating at the present time. I don't know of any cases of trouble that have arisen from this cause, and while I think there are some of the engineers of the Bureau of Illumination Service here, and they have not called my attention to troubles of that kind, if I am incorrect, I hope they will correct me at this time, but it is a fact that there has been no noticeable effect from this theoretical thing that Mr. Del Mar speaks of.

C. E. Skinner: On the question of steel tape cable for series distribution, when it first came up there were quite a number of us that thought it was doubtful whether the inductive effect of the steel tape would permit its use, but a few tests on that point dispelled our fears and it is used extensively now. About 1912 there was quite a lot of it put in under my direction at Champaign, Ill. As far as I know they have had no mechanical troubles due to the breaking through of the steel tape sheet, and no electrical troubles to speak of outside of defective joints which were defective when they were laid.

F. F. Fowle: Talking about the inductiveness of the steel tape cable, which is quite comparable with the case of steel cables for ordinary power conductors, research has proved that it is a very interesting fact that whereas some of the grades of steel, that is, mild steel, are highly inductive, that a steel that contains as much carbon as the Sells-Martin is practically non-inductive. That is, measured by the skin effect, it is almost negligible. I made a series of tests on some steel cables of all grades up to the high strength strand. The high strength strand was merely a trade name, but it is really a steel which has something like sixty or seventy points of carbon, and the skin effect, even with a 3/8-in., seven-wire strand made of that grade of steel is on the order of only one or two per cent, whereas the strength of the same size of strand made of ordinary wire strands, the skin effect is very marked indeed, so that, by analogy, the magnetic circuit that would be created around a steel tape cable if the proper grade of steel is used, would be practically negligible, but if made of mild soft steel there might be a noticeable effect.

C. E. Skinner: There are tests on this that are a matter of record, and can be found with a little research. One test in particular on a certain open loop, a street lighting circuit, a No. 6 steel tape lead covered cable, the voltage drop over that was only a little over double the ohmic drop.

Has Mr. Vaughn had any experience with the use of straight lead cables without the use of tape. It seems that the tape is put on merely as a protection against accidents, as I understand it.

F. A. Vaughn: In the older Milwaukee system practically all is lead sheet cable without the steel tape. Of course that doesn't bring into account, however, the magnetic effect that Mr. Del Mar speaks of, if that is what you have reference to, because the steel or iron tape is missing. Of course, the effect from the lead sheet is present on the high-voltage a-c. circuits.

C. E. Skinner: There are some lead-covered cables being used without the steel tape, buried directly in the ground.

Member: I would like to ask Mr. Shepherd if he has had any experience with the steel line transformers. That is one point that has not been brought up, and I believe it would be interesting to hear something about those transformers, the "pull" type regulating transformers with the moving coil.

J. E. Royer (by letter): I will give you some idea of what is used in Spokane.

About a year ago all the overhead lights were changed from the 7.5-ampere series arc lamps to the 6.6-ampere Westinghouse Luxolite fixture with the incandescent lamp. The maintenance with the new system is about the same as under the old series arcs, but as the incandescent lamps take less voltage, it permitted putting more lamps per circuit on the same station equipment and in so doing spare transformer capacity was gained. Out of 1500 lamps in circuit at the beginning, 1558 replacements have been made during the year giving about 2000 hours as the average life per lamp, although in a few cases lamps have gone as high as 4500 hours burning. The causes for renewals are many, the greater per cent being breakage, high voltage (crosses) and faulty manufacture. The voltage of the primary circuit ranges from 5000 to 7000 volts while with the old series arcs in some circuits it was as high as 10,000 volts.

On sidewalk lighting the 6.6-ampere luminous d-c. arcs are used on constant current transformers with rectifier tubes, in most cases four tubes being used, two tubes in parallel on each side but only one tube per side taking the load. We have had quite a little trouble with tubes during the past year in the matter of hours life. The later tubes do not seem to have the good qualities the earlier ones had. I hope something in the way of care and treatment of tubes will come up in the discussion as we have tried drying, washing and rest, but as I said before, the life of the later tubes does not come up to what we used to get.

In the matter of testing circuits in daylight hours, we test with 220 d-c. every three hours for open or ground.

W. P. Hurley: The moving coil regulator has an inherent reactance of approximately 40 per cent when the coils are in the full-load position. This value increases to 100 per cent depending on the load which in turn determines the position of the coils. Tests with an oscillograph have shown that such a regulator has under full-load and short-circuit conditions, enough reactance to amply protect the series lamps remaining in the circuit. The current returns to approximately normal value within less than one second. These regulators are, in many installations, started by time switches without separating the coils or injuring the lamps. The maximum current in the lamp is approximately three times the normal current when the regulators are so started.

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MULTIPLE SYSTEMS OF DISTRIBUTION FOR STREET LIGHTING

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The wider use of multiple lamps supplied from standard secondary distribution systems is considered as a step in simplifying operating problems and reducing the disporportionate cost which goes to cover fixed charges on special equipment and additional lines for street lighting only. The advantages in simplicity and flexibility of multiple connected lamps are discussed with particular reference to the frequent changes and extensions of street lighting service required in growing cities. The more general adoption of multiple street lighting is stated to be contingent upon fuller standardization of suitable methods of control applicable generally to existing electrical power distribution systems. Different devices in use or proposed for control of multiple street lamps are briefly described and the characteristics desirable in such apparatus are outlined. Attention is directed to the small differences in efficiency of present multiple and series incandescent lamps.

THE problem of securing street lighting most economically from multiple connections is one whose possibilities are far from exhausted, for multiple street lighting circuits and suitable control devices have not as yet progressed to the same degree of standardization as those of the series type. In the past, each individual case has been worked out, as far as possible, by adapting existing apparatus to meet the requirements of the particular situation. The purpose of the present paper is to review the systems now in use with their methods of control, and to indicate some of the more promising avenues of development.

EXTENT OF USE

In European cities, multiple street lighting systems are practically as common for both arc and incandescent lamps as are series systems in this country. Even here, where series circuits predominate, there are, notwithstanding, a large number of straight multiple systems in operation. It is true that they are largely confined to cities where low-tension current only can be used. Of these cities, New York, with many thousands of such lamps, is the most prominent example.

Fig. 1—Typical Overhead Series Arc Lamp Post Showing Insulating Hanger

Fig. 2—Simplified Type of Fixture Designed for Multiple Incandescent Street Lamp

It is worthy of note, however, that in cities nominally lighted from series circuits, considerable numbers of street lamps will be found operated from the multiple distribution system.

Advantages

The following advantages which contributed to the standardization of multiple distribution for all other types of electric service apply also to street lighting.

1. Widest choice in number and sizes of lamps to be connected to circuit.
2. Ease of adding more lamps or lamps of higher wattage.
3. Simplicity and low cost of fixtures and accessories.
4. Safety.

The matter of flexibility is of immediate importance in the problem of designing street lighting for American cities and towns. There is not only steady growth in population and area covered, but there are also unexpected demands from time to time for increased street lighting service in given areas. The result, with series circuits of units of 50 to 100 lamps included on a single line, is that there must be an almost continual process of rearrangement and redivision of lamps among different circuits. In many cases service can only be provided after considerable delay and at relatively high cost, because existing circuits are so completely loaded that the addition of a few lamps makes necessary an extended revision of the system. No such limitations attend when the street lamps are operated from the general multiple distribution system.

An additional advantage of multiple street lighting is that, as mentioned above, the multiple circuit is now used exclusively by central stations for every other service. Were it used generally in street lighting as well, the reduction in types of apparatus required and the resulting simplicity of operation would benefit both the electrical manufacturer and the central station. At the present time, when a new real estate sub-division or allotment is opened, it is common practise for the lighting company to run a set of 2300-volt feeders to cover the plot, and to furnish service to houses as they are built. At the same time, a series transmission line is extended to furnish the necessary street lighting for the sub-division, and, in anticipation of increased requirements for service, both supply circuits are installed with a large excess capacity. The principal reasons for this duplication of service have been:

1. The arc lamp is inherently a constant current device and gives its best operation and greatest efficiency on series rather than on multiple circuits.

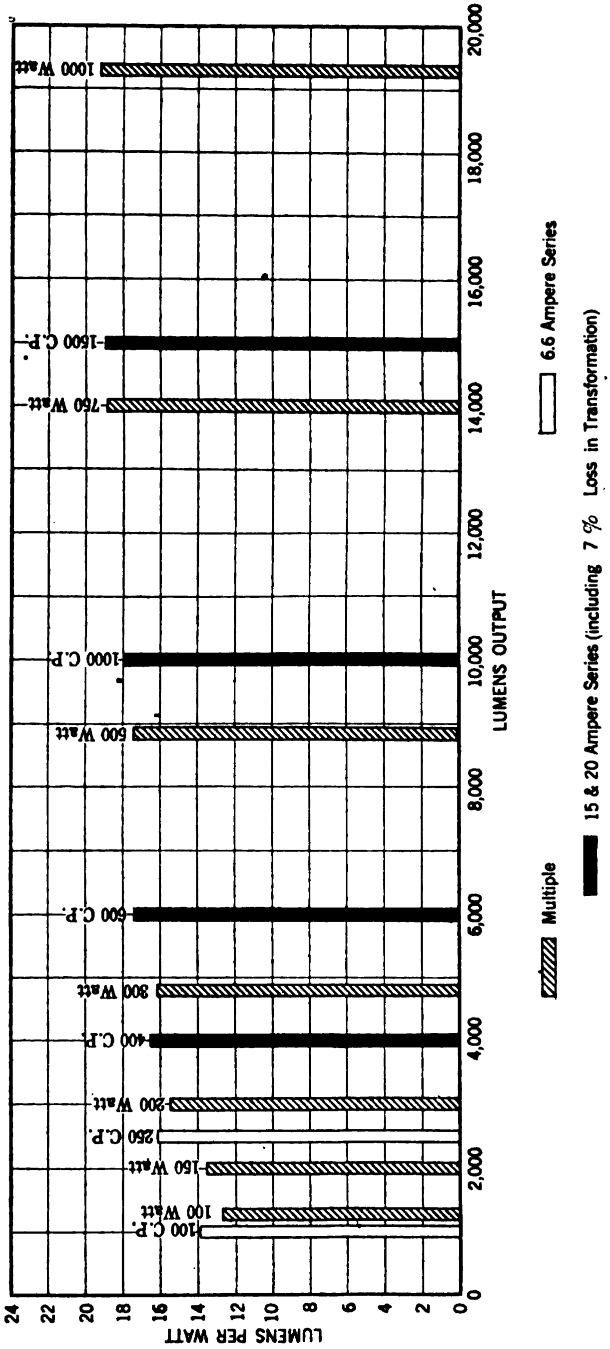

Fig. 3—Comparative Efficiency of Series and Multiple Incandescent Lamps

2. The series circuit is the simplest and most efficient method of supplying energy to comparatively small units scattered over wide areas; in many cases electric street lighting ante-dated the general use of electricity in residences by a considerable period.

3. A separate system of distribution has furnished a convenient means of automatically lighting and extinguishing street lamps from the central station.

There is an insistent demand for the extension of electric street lighting service in territory not now served by series lines, as for example, in Ohio cities, where in the past a considerable portion of the lighting has been furnished by natural gas, the supply of which is being rapidly depleted. In such territory the residences are, for the most part, furnished with current from overhead circuits along the rear lot lines, and the introduction of a pole along the street, to carry the series circuit has often met with a decided protest from the property holders. On the other hand, the cost of underground construction, with ducts running parallel to the curb, is almost prohibitive, especially in view of the increasing frequency with which concrete driveways now cross the parking in residence districts. High-tension underground lines passing through private property from the rear lot lines to the street, also involves expensive construction if safety is to be assured. Taken altogether, the conditions point toward the use of low-tension multiple circuits from the pole lines in the rear to the posts on the street, brought out either overhead or underground, as occasion may require. Under these circumstances the underground construction may be in the form of a relatively inexpensive lightly-insulated sheathed conductor protected simply by fuses at the transformer. Such a conductor is very much less expensive than that designed for high-tension work, and it can be installed without opening a wide trench across the lawn. If such a conductor is accidentally severed no serious harm can result, and the loss in itself is small.

CONTROL

Today if incandescent street lamps were to be operated throughout the twenty-four hours instead of at night only, there would be no discussion as to the desirability of connecting them to the existing multiple distribution system, but the obvious difficulty in this practise lies in obtaining a thoroughly satisfactory method for turning the lamps on and off. Of course, wherever the load is sufficiently concentrated to justify a separate circuit from the substation, as in the case of many so-called White Way installations in the business streets of cities, the lamps may be switched at the substation in the same way as in the case of series circuits. Ususally, however, to bring the circuits back to the sub-station entails a prohibitive expense in copper and

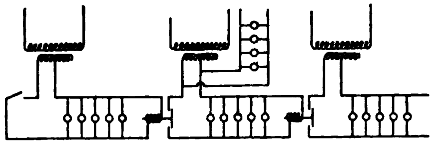

Fig. 4—Cascade Control of Multiple Street Lamps

it is necessary to supply the lamps from several different transformers or feeders located nearer the lamps. For example, in the case of residence districts, it would be particularly desirable if the street lamps could be supplied from the same circuits located in each block, which now provide service for the residential consumers, as in Fig. 8.

The simplest plan is of course to have the lamps turned on and off by hand through the agency of a patrol messenger. Where labor is cheap, as in some of the European and Asiatic countries, this has been found a satisfactory and economical solution. In this country there are many cities where the White Way lighting is turned on and off by the policemen on duty. However, in view of the higher cost of labor in the United States and the extended distances between lamps, hand control is less desirable. Furthermore, the fact that gas companies are forced to rely upon manual control, and the difficulties which they have experienced, have served to deter electric lighting companies from resorting to it, excepting where there was no practical alternative in the way of a mechanical or electrical control.

Most of the efforts toward the development of remote control of multiple street lamps on 110-220-volt circuits can be grouped under three general types of systems,—cascading, relay switches operated from a pilot wire, and automatic clock or impulse-operated relay switches without separate control circuit. Diagrams illustrating these three methods of remote control are shown in Figs. 4, 5 and 6.

In the cascade method the section of lamps nearest the sub-station or control point are switched on and off by hand or a clock control switch. Energizing of this section serves to close a relay operating the switch for a second section. At the end of the second section is a similar relay which, being energized, closes the circuit for the third section of lamps, and so on. Opening the switch on the first section of lamps serves to de-energize the other sections of lamps in the same order. It is obvious that a method of this kind can be used for either alternating- or direct-current systems, or even a combination, and the load may be carried from as many separate transformers or sets of feeders as desired.

In the cascade system a failure of current supply in any section will turn out the lamps in succeeding sections of that cascade. However, this difficulty can be minimized by provision for hand operation of the switches in case of failure of one of the units.

The so-called pilot wire control utilizes a somewhat different principle for switching the lamps on and off. In this method, as will be seen from Fig. 5, lamps of the system are grouped in sections of suitable size to be controlled by a single switch and the separate switches are operated from a pilot wire which is energized from the sub-station or control point. The pilot wire can obviously be made of low current carrying capacity since it does not carry the lamp current but simply serves to energize the switch relays. Switch relays of several different models have been developed in connection with this pilot wire control. The simplest type consists of a solenoid-operated switch, which is closed or opened, depending on whether the current is on or off the control wire. Preferably, switches of this type should be closed when the pilot wire is de-energized, so that if the pilot circuit is accidentally opened the lamps will all be lighted, even though the break should occur during the daytime and therefore result in some waste of current.

Another type includes a mechanism with on and off positions. With this, the first time the control wire is energized the lamps are lighted. The next time the lamps are turned off, and so on. Continuous application of current to the control wire is not necessary with this form, but there exists a possibility of lamps being left turned off when it was desired to have them on, or vice versa, due to the switches getting out of step. A pilot or signal switch at the control station has been suggested to obviate this uncertainty. Also, one manufacturer has provided for convenient hand setting of individual switches which might drop out of step.

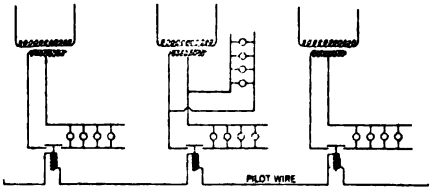

Fig. 5—Pilot Wire Relay Switch Control

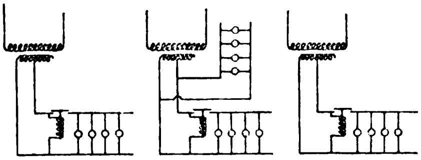

Fig. 6—Control by Automatic Clock or Impulse Operated Relay Switches Without Separate Control Circuit

In an endeavor to avoid the objections raised against the impulse-operated switches of the type just described other forms have been developed which operate from impulses of different strength. One of this type is illustrated in Fig. 7. The solenoid has two plungers so adjusted that a certain current will lift one and close the circuit. The impression of double this voltage on the control circuit lifts the second plunger which releases the catch and opens the lamp circuit. Such an arrangement using different impulses for the on and off position, has the advantage of avoiding the possibility of switches falling out of step. For instance, when turning on the lamps the control circuit can be given several impulses if desired without fear of leaving the switches in the wrong position.

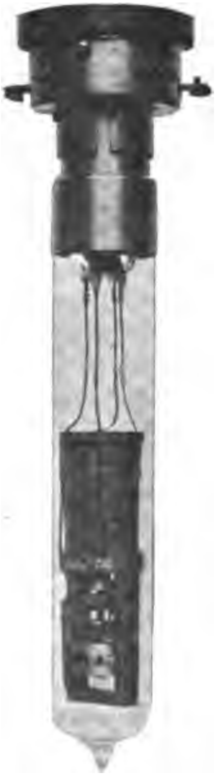

FIG. 7—SMALL RELAY SWITCH OPERATED IN INERT GAS

In order to reduce the wattage required to operate the switch to the lowest possible value and to be able to carry a large number on a single control circuit, the parts are made light in weight. Corrosion and the resultant failure of switches is avoided by sealing the entire mechanism in a glass bulb filled with an inert gas.

The capacity of these small switches is limited, but where a heavy current is to be handled they can be used as relays to operate a circuit breaker.

The pilot wires for operating the types of switches described may connect the switches in series. A greater flexibility in making extensions results if the switches can be operated in parallel, and in this case the neutral or grounded side of the distribution system may be used as one side of the control circuit and only one wire need be run to each switch. With a switch which operates on an impulse transmitted at low voltage, a relatively inexpensive galvanized iron wire such as is used in telegraph circuits, will frequently prove satisfactory.

Since the installation of a pilot or control circuit involves a certain amount of inconvenience and expense, considerable attention has already been directed toward the development of a remote-control switch system which would not require extra wiring beyond the necessary extension of the circuit from the feeders to the lamps. One of the earliest devices for accomplishing this result was the clock or time switch. Early designs were often unreliable in operation, and therefore quite unsatisfactory; however, there are available at present, time switches which withstand the severe conditions to which they are subjected. Again, time clock switches may be had which are electrically wound and which may be set to change the time of switching the lamps automatically, according to a prearranged schedule of burning. Experience has indicated that clocks require a considerable amount of attention and regulation to keep them operating and in close agreement with each other. This factor, together with the question of initial cost, has hindered the more extended use of these devices.

A tiny synchronous motor-operated clock switch requiring an almost negligible wattage, is also available for remote control switching on alternating-current circuits. This switch would find its best application in systems whose average frequency is well maintained and where there are but few interruptions of service.

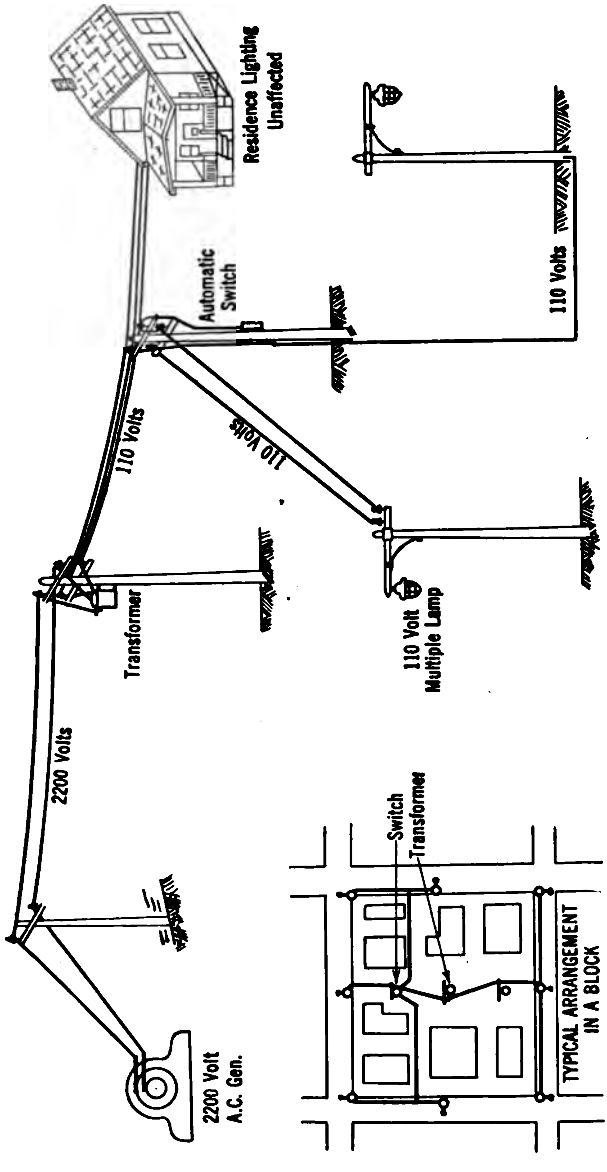

FIG. 8—PLAN OF OPERATING STREET LAMPS FROM SECONDARIES OF A HOUSE LIGHTING SYSTEM

Otherwise the switches are liable to require a considerable amount of resetting.

Quite a different and a more satisfying solution of the problem of providing an economical and satisfactory remote control for multiple street lamps would appear to lie in the development of a system of switch controls operated without pilot wires and without clock mechanisms, but so arranged as to be actuated at will from the power station or control point.

One remote control switch now on the market is operated by "winking", or quickly opening and closing the supply circuit. Where the other load carried on the same distribution system is of such character that a momentary interruption of power does not interfere with its operation, this method will work out successfully.

Also, experimental apparatus has been built showing the practicability of operating switch relays by varying the frequency of the alternating-current supply a few cycles above and below normal. The opening and closing of the relay circuit may be controlled simply by the vibration of metal reeds similar to those in the familiar frequency meter, tuned to the proper frequencies. An arrangement of this kind would appear to be practical, at least in smaller communities where there are no serious difficulties attendant upon momentarily changing the speed of the generators.

Different experimenters have proposed also the use of relay switches receiving impulses from wireless waves, though so far as is known, the opportunities of development in this field have not been exploited.

Operation of the switch relays by means of selenium cells has been suggested. The operation of such switches would, however, be entirely dependent on the amount of daylight, and would not necessarily control the lamps according to the fixed hours of burning involved in many street lighting contracts. Furthermore, it would not be possible to turn the system on and off for test or other purposes, except by the additional provision of hand-operated switches.

The development of a phantom-circuit-operated control switch has been described elsewhere[1], and with further development this plan would appear to have promising possibilities. At present, however, the expense of the apparatus is too great to permit of an individual control for each lamp.

TABLE I.

COMPARATIVE EFFICIENCIES OF GAS-FILLED INCANDESCENT UNITS.

MULTIPLE LAMPS

Watts	Lumens	Lumens per watt
100	1,260	12.57
150	2,040	13.66
200	3,100	15.51
300	4,840	16.11
500	8,750	17.45
750	13,900	18.48
1,000	19,300	19.33

SERIES LAMPS

Amperes	Nominal candle power	Approx. watts	Approx. lumens.	Lumens per watt.
6.6	100	72	1,000	13.96
6.6	250	155	2,500	16.11
15.	400	240*	4,000	16.65*
20.	600	344*	6,000	17.45*
20.	1,000	556*	10,000	18.00*
20.	1,500	809*	15,000	18.55*

*Includes 7 per cent allowance for loss in transformation.

It seems reasonable to believe, however, that apparatus operated without a pilot wire should soon be developed by means of which street lamps can be lighted and extinguished at will, in a manner which is not open to any of the foregoing objections. At the same time it is evident that such a system, to be successful, must not involve a high cost per unit installed; also, it must not require the addition of any considerable amount of equipment on the 2200 lines; finally, its mode of operation, particularly if it involves the use of high frequency, must be such that there is no possibility of

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[1] *G. E. Review, Vol. XX, No. 2, Feb. 1917.*

injury to transformer insulation or to any apparatus forming a part of the present standard distribution system.

LAMP CHARACTERISTICS

The serious loss in light output when incandescent lamps are operated at less than rated voltage, is shown graphically in Fig. 9. Because of this characteristic it is essential with multiple systems that lamps be used whose ratings correspond closely to the voltage measured at the socket. In most cases with series systems, the operation of all lamps at proper efficiency is assured if the current is maintained at the correct value at the station. In some multiple installations a large number of lamps may be carried on comparatively long branches, so that there exists a considerable difference in voltage between sockets near the feeders and at the far end. This difference in voltage may become so great that the expedient of using lamps of two or even three different voltages is sometimes followed. Under these circumstances every lamp post must be marked with a symbol indicating the proper

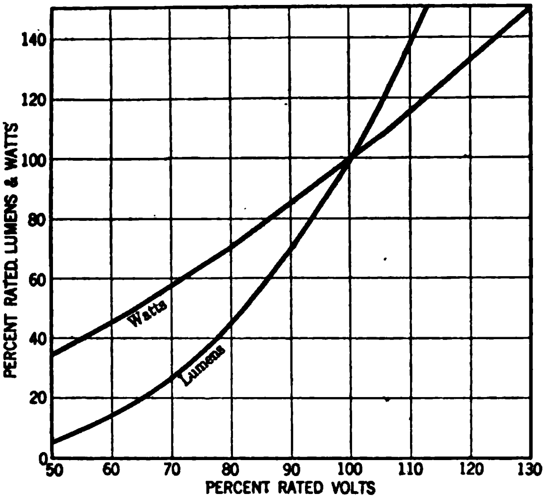

FIG. 9—VOLTAGE, CANDLE POWER AND WATTAGE CHARACTERISTICS OF MULTIPLE TUNGSTEN FILAMENT LAMPS

voltage of lamp to be used in the replacement of burnouts. In spite of this precaution, however, confusion of lamp voltages is very likely to arise unless the maintenance of the system is supervised with unusual care. A plan which is usually preferable is to provide for a very small drop between the street lamp and the supply circuit by running short branches and operating from one to three lamps on a branch. Table I shows the efficiency of the various sizes of multiple and series lamps computed in lumens per watt. For the 15-and 20-ampere units an allowance of 7 per cent has been made to cover loss in transformation. In Fig. 3 these data have been plotted graphically so that the comparative efficiency of series and multiple lamps of equivalent output may be readily seen. In the case of those series lamps which are available both in 6.6 and higher amperages, the more efficient type has been shown. The very slight inferiority of multiple lamps from the standpoint of efficiency is worthy of note.

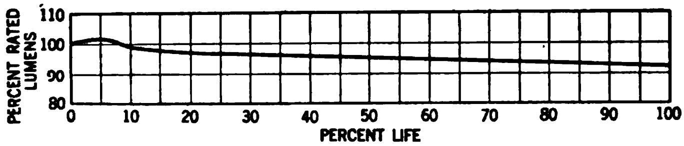

Fig. 10—Typical Life-Candle Power Performance of Multiple Tungsten Filament Lamps

The depreciation in candle power or falling off in light output during life, is comparatively slight for the modern types of gas-filled incandescent lamps. The curve of Fig. 10, based on life tests of a large number of lamps of the sizes customary in street lighting, illustrates the performance which may be expected.

Conclusion

A fundamental reason for the increasing interest in multiple systems for street lighting is the growing realization that with present systems but a small part of the total cost of operation per lamp per year (usually not more than one-third) is to cover the actual cost of light; *i. e.*, cost for current and lamp renewals, or electrodes; the major portion consists of fixed charges on special equipment, floor space rental in substations, and proportionate charges for the use of pole lines and underground ducts. Furthermore, these latter charges are abnormally high per kv-a. owing to the small load carried on the average series circuit. Twenty-three hundred and 11,000-volt multiple circuits may be loaded at 400 to 4000 kv-a., while a 6.6-ampere circuit occupying an adjoining duct, is usually considered fully loaded at 20-30 kv-a.

A DISCUSSION ON "MULTIPLE SYSTEMS OF DISTRIBUTION FOR STREET LIGHTING" (HARRISON), CHICAGO, ILL., JANUARY 9, 1920.

N. B. Hinson: I would like to ask Mr. Harrison why it is the switch is put on the secondary circuit rather than on the primary side of the transformer.

Ward Harrison: The reason we have shown the control in each case on the secondary side is that we feel in any system of multiple distribution it is desirable to be able to utilize the transformer already installed for house lighting circuits, as shown in Fig. 8 of the paper. Again, any system of switching has the possibility of getting out of order, and also the possibility of being grounded. If you can keep these switches off the 2200-volt line and install them on the 110, a ground or short circuit has no more serious effect on the system than if it were in your own house.

C. E. Skinner: The fact that the transformer would usually be there for other purposes would answer that question in most cases. The street lighting would simply be a part of the whole general scheme of multiple distribution. Where Mr. Harrison says that the question of control is one of the governing features he hit the nail squarely on the head, were it not for the limited distances that multiple circuits can be operated from the source of supply, the transformer or whatever it may be, the question of control would not be difficult, but on account of the fact that if you are going to use street lighting on a multiple plan you can only economically supply a very limited number of lamps from one center, means that you have to have a lot of control points.

N. B. Hinson: A list of the different types of street lighting systems of the Southern California Edison Company, Southern Division, will give some idea of the kinds of circuits used and tendency to adopt series circuits as standard.

There are seven cities with d-c. series arcs, three with a-c. series arcs, and two with both a-c. and d-c. series arcs. Six cities have multiple incandescent over head systems, only one of these being a very large city. There the lamps are controlled by a special primary circuit which feeds the street lighting transformers. Seven cities have series incandescent overhead systems, most of which are fed from constant-current transformers in the substations. A few are controlled by outdoor type constant-current transformers mounted on poles and operated by time switches. Eight cities have multiple lamps in ornamental posts only, and six series lamps in ornamental posts only. Some of the above cities which have overhead circuits also have ornamental. Twenty-seven of these systems are multiple lights, and twenty-two series. All of the multiple being old installations, the series are newer systems.

All of the series ornamental systems except one, are cut up into smaller systems so as to limit the voltage on each individual circuit to a value of less than 600 volts; only one system being operated at high voltage. This has been done as it is not considered safe to carry the high voltage of the series circuit direct to the post. Most of the multiple ornamental systems are operated from a special primary system which feeds the street lighting transformers.

The reason for the almost universal use of series circuits is due to the lower first cost of the system, either overhead or underground for ornamental posts, the reduced maintenance due to the longer life of lamps, and the increased efficiency of the series lamp, particularly on the smaller size units, which size predominates.

A brief description of a city which is lighted by ornamental posts only and which were installed quite recently, may be of interest.

Due to the high cost of material and the special requirements of a particular installation, a new type of system has been developed, which we have christened "Conduit Return". It has proven very satisfactory for other installations than the one for which it was designed. A brief description of the original conduit return system may be of interest.

The city in question has three distinct sections, long blocks and very irregular streets. It was desired to operate the lamps on a double schedule, that is, all of the lamps were to burn until 12 o'clock and then about half were to be extinguished and the remainder to burn all night. This means two circuits, and it was agreed that one side of the street should be the midnight and the other the all night. This simplifies the wiring, but the sections were so long in some cases a complete loop would run the voltage higher than that at which the rubber covered wire could be operated, and in other sections no complete loops could be made because of the streets, and in all of the sections there was an odd number of streets; also all the pole lines are in alleys and in one section the whole territory is underground so that the possible feed points were limited.

With the conduit return system these features were not difficulties at all. Each side of the street was a circuit to itself, the current flowing out on the wire through all the lamps and returning on the conduit to the transformer. The conduit was bonded together in the base of each post and special care taken to see that the joints were made so as to offer as little resistance as possible. The system has practically no reactance, the voltage drop and energy loss in the circuit are less than with a copper return circuit. Extensions are very easily made, trouble easy to locate and the first cost is very much less. The conduit was ½ in. galvanized with No. 8 D. B. R. C. wire.

At the feed points S. L. transfomers are located with a protective device to limit the voltage in case of an open circuit. These are fed from the substation by four overhead high-voltage series circuits which are regulated by four constant current transformers in the substation, two of which control the midnight circuit, and two the all-night. In this particular case it was more satisfactory to run out high-voltage circuits with S. L. transformers than to use R. O. transformers and a control primary.

J. M. Humiston: I want to give a little history of a municipally owned plant that I am familiar with, more for the sake of throwing some light on the discussion and pointing out some absurdities rather than giving any information to this meeting. This municipal plant originally served an area of about two-thirds of a square mile and was a Nernst lamp system operating at 220 volts. As the Nernst lamp went out of use the lamps were replaced with carbon filament lamps burning at 220 volts. Later these lamps were replaced, first by a 100-watt tungsten 220-volt lamp, and now they are gradually being replaced by 150-watt tungsten lamps, none of these lamps being of the nitrogen filled type. No effort has been made by means of refractor globes to improve the illumination. I imagine that the 220-volt pressure lamp has been one reason why none of these lamps have been stolen, because I have heard no complaints of thefts. About 1907, some five years after this Nernst lamp was put in operation, another part of the town was supplied with about 50 arc lights on a series circuit. About five years ago these were changed to the series incandescent type. At the present time there is one circuit consisting of 32 tungsten 350-watt series lamps burning on a circuit with a regulator, the circuit being supplied by 4000 volts at the bus bar, and another similar circuit with 83 of these same lamps. The 83 lamps, of course, are burning at approximately half the candle-power that they would have if the circuits were rearranged in such a fashion as to put the proper number of lamps on a circuit. These two systems are supplemented in part of the town by a post lighting system, the posts being about 100 feet apart. Some trouble has been experienced through these post lamps being damaged by autos. Four or five posts have broken off in this way. The original multiple system, using the 150-watt lamps has lights spaced at an average of 200-ft. centers, and that part of the town is the best lighted of any district that I know of around Chicago, with the exception of the boulevards and places where special effort has been made.

About 3 years ago I suggested changing this multiple system to a series system, but I found that there was opposition from the force that was maintaining the system because they didn't want to take the trouble to trim the trees, and they had had almost no·trouble from tree contacts against these 220-volt wires, and where the series system was in force they had had a great deal of trouble. In spite of the fact that the series system was in a district with comparatively few trees, and the multiple system was in a district with a great many trees. After surveying this I finally concluded to drop the matter of changing this to a series system, and now I am rather favorable toward maintaining the multiple system as it stands. The secondaries of this multiple system are entirely separated from any commercial distribution. There are five or six transformers located at proper intervals which serve the secondaries of this multiple system, the lamps being turned on, by the operator at the station charging the primaries that feed these transformers.

C. H. Shepherd: I would like to ask Mr. Harrison as to how the economies of multiple street lighting would be affected by the question of metering; whether the current would be metered or not, and if not, why not?

Ward Harrison: I think in general that it would be uneconomical to meter the current to each lamp, but rather to measure at reasonable intervals, if necessary, the wattage taken by each lamp, or by a predetermined group of lamps; although I don't see why this is any more necessary than to measure the current taken in the series circuits at the present time. In general, these lamps should be regarded as flat rate customers such as electric signs. There would be no objection, however, to treating the lamps as regular residence consumers with a wattmeter back on the pole, which could be read by the same man who reads the meters in the houses.

F. A. Vaughn: In connection with that point—just incidentally—it might be of interest to record that in Milwaukee during the transitory period from the old series system to the new series system an arrangement was made with the public utility to supply lamps on these short individual multiple circuits at a given rate per lamp, based upon the wattage rating of the lamp, and those lamps were temporarily burned for a number of months, many of them on that rate schedule. This was a flat rate schedule based on the rating of the multiple lamp. I wish that Mr. Harrison had mentioned in connection with the manual turning on and off of the lamps, that it is not so much the cheapness of the labor as the undependability of the labor that is a factor acting against adopting that as a good system. It seems to me that under the present-day circumstances undependability of the labor which you can get for that manual extinguishing and turning-on of the lamps is absolutely against its feasibility.

H. Goodwin, Jr.: Mr. Harrison has given us a summary of many methods of turning off and on the lamps, and he has mentioned the selenium cell. It might be interesting to add to this collection of methods a method which is in practical use for turning off and on gas buoys at sea and on lonely rocks or headlands. This daylight switch has been in satisfactory operation for many years. It is a Swedish invention but is in use in quite a number of lights in this country, particularly in Alaska and remote sections of the coast. This device consists of a steel cylinder covered with lampblack. There is a frame on top of this cylinder to which are attached steel posts of just the same material as the steel cylinder. There are about six or eight of those posts. They are covered with gold leaf. They reflect the light and the cylinder absorbs the light. Therefore, during daylight one expands and the others remain at the temperature of the air. There is a yoke around the bottom. Now if you pivot a lever you can get a considerable motion on the end of the lever, due to the light falling on the cylinder, and that is actually made to open and close a gas valve, which turns the gas buoys off during the day. This apparatus can be made so sensitive, and the adjustment so fine, that it could be set in this room with the lights turned on and yet if you hold a match within one foot of it, that would give enough additional light to turn the lights off. A satisfactory daylight switch has been developed. It is slightly expensive at the present time, but if sufficient demand is created, they could undoubtedly be produced in quantities that would allow them to be sold at a reasonable price.

Mr. Harrison also mentioned the pulling in of cable from rear lots and along parkways, etc. I don't know if you are all familiar with the method that has been used in Philadelphia to a considerable extent. The apparatus consists of a cable claw with two channel irons set parallel, there being a roller on each end, a straight plow share and a bulb on the lower end. You dig a hole in the ground, set this down into it, shackle the cable to the rear end and by means of an electric truck or a gasoline winch set up sixty or one hundred, or one thousand feet ahead, just pull that along 18 inches below the surface. The rear roller coming along closes the slot so you can hardly find it after the plow has passed. In that way they can pull in many thousand feet of cable in parkways, etc. in a single day.

F. A. Vaughn: I wonder if I may add, that I think the difficulty of laying the parkway cable under the driveways would be largely minimized if the driving-method was used. The ratio that Mr. Harrison gave as to the cost of crossing the street and crossing driveways, as compared with a block of cable, I think would be materially changed if a "jacking" system which we use in Milwaukee were utilized. I believe this is used here in Chicago—some modification of it, at least—for driving a pipe under the pavement or under the driveway. That is particularly easy on all the narrower driveways and is perfectly feasible even on asphalt covered streets.

Apropos of the control of the circuits I should like to record that in Milwaukee on the multiple side of the system, that is, on the primary transformers that feed the series circuits, we use time-switches and are having very good success with them. In order to show the magnitude of their present use, I might say that the Milwaukee system is about 40 per cent complete and ultimately will have between nine and ten thousand units, so we have had a pretty fair chance to try out the control by the time-switch method.

F. H. Bernhard: I wonder if Mr. Harrison could throw any light on the subject as to why this system is used so extensively in Europe and also some of the results we have compared with our system here.

Ward Harrsion: I am not familiar with all the causes underlying the greater use of the multiple system in Europe, but one reason is the fact that lamps are spaced much closer in a great many of the foreign cities than we have been in the habit of spacing them in this country. The line drop in the multiple system of distribution in these cities would consequently be less objectionable than here where there is a wider spacing of units; also, the fact that labor is cheaper there, or at least that it has been, affords an inexpensive method of turning the lights on and off.

Mr. Peaslee: The use in Europe is very common of what they call a "Pensioner", or a man that draws a pension from the government. I know this is the case in the lines in Paris and any time that any of the companies over there are incorporated they have some sort of a clause that gives them a government privilege of using men who are drawing war pensions, and men who are paid a pension in a great many of those cases are used, for a very slight sum, to go around and handle the switches, both in the gas lighting and electric lighting. It costs the company probably three or four cents a day extra to get a man to handle three or four blocks of that switching. And the reliability of that labor is really remarkable to anyone who has had experience with American labor, and that has been a very important feature of the continuance of it, and also of the continuance of the multiple systems in that part of the country.

Mr. Cameron: Isn't it a fundamental fact also that in most European cities, particularly of the smaller class, the streets are so narrow that the street in most cases provides also a sidewalk. The wires are usually fastened to the buildings, and must necessarily, from the standpoint of safety, be of low voltage.

George Nixon: As stated in Mr. Harrison's paper there yet remains much to be done in perfecting the multiple system of street lighting. In time, however, there may be developed specially for this purpose reliable remote control switches and automatic time switches and manufactured in quantities sufficiently large to be sold at a low enough price to allow their practical use to become quite common.

Among the practical applications of the multiple lights for street lighting not particularly mentioned by Mr. Harrison, is the lighting of small villages and towns from service off of a transmission line.

Often the amount of business to be obtained in a small village does not warrant the installation of a transformer unless all the business can be served from one transformer. On the same pole with the transformer is installed the time switch which automatically turns the street lights on each evening and off each morning. One side of the street lights would be connected to a special secondary wire and the other side to one of the secondary wires serving the residences.

The time switch is wound up once a week by the patrolman in that section of the line.

Since the swift automobile and auto trucks have come into such extensive use it has become more necessary than ever that these small villages be well lighted.

Presented at the Chicago Meeting of the American Institute of Electrical Engineers. January 9, 1920.

CONSTANT POTENTIAL SERIES LIGHTING

BY CHAS. P. STEINMETZ

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UNTIL recent years, street lighting in the United States has been almost exclusively by the high-voltage constant-current series system, either direct current or alternating current, and low-voltage constant-potential multiple systems have been used to a very limited extent only. This was for two reasons, one social-political, the other electrical.

To the democratic character of the country, only such form of lighting appeared suitable, as was applicable alike to the scattered suburbs and to the more densely populated center of the city. Thus high voltage had to be used, either a series system, or alternating current with individual transformers. The latter was excluded at first by the absence of a satisfactory alternating-current arc lamp and by the high cost and low efficiency of the small individual transformers. Thus American street lighting developed on the high-voltage series system, and much later only, with the gradual growth of civic pride of the city population, special lighting features were developed such as "White Way Lighting." The alternating-current arc lamp largely replaced the direct-current arc lamp; the small transformers became more efficient and less expensive; but still more efficient and economical was the constant-current transformers operating a whole circuit of alternating arc lamps, so that the conditions which led to the use of the high-voltage series system did not materially change.

The other reason was the electrical characteristic of the arc, which was the only illuminant of sufficient intensity and efficiency for street lighting. The arc is unsteady on constant potential, and requires a constant current or at least a circuit in which the voltage drops greatly with the increase of current. Thus, to operate an arc lamp on constant-voltage supply, a large series resistance or reactance is necessary, which makes it inefficient and, with alternating current, greatly lowers the power factor, and even then, the light is not as steady as when operating on a constant-current circuit.

As seen, of these two conditions, the first one requires high voltage, whether constant potential or constant current; the second one required constant current, which economically means series connection and high voltage. The first one still applies with practically the same force as before, but the second one has entirely changed, and thus makes a reconsideration and revision of our views on street lighting desirable.

With the exception of the luminous arc, which by its high efficiency and superior quality of light—perfect whiteness—holds its own, especially in high intensity and decorative lighting, such as "white way lighting," exposition lighting, etc., the arc lamp has practically disappeared from street lighting, and the gas-filled mazda lamp has taken its place. With the disappearance of the arc lamp however, disappeared the necessity of the constant-current circuit. As an incandescent lamp, the gas-filled mazda lamp is a dead resistance, equally stable on constant potential as on constant current, and while the necessity of using high voltage still remains—especially in American cities, which as a rule cover a far larger territory with their street-lighting districts, than European cities of the same population—the need of converting from constant potential to constant current has ceased, and the high-voltage constant-potential street-lighting systems with gas-filled mazda lamps, thus have become possible and are increasing in frequency, either as high-voltage constant-potential series systems, or as high-voltage alternating-current multiple systems with individual transformers.

Unlike the carbon-filament incandescent lamp of old, which has a negative temperature coefficient of resistance, the metal-filament mazda lamp has a positive temperature coefficient, that is with a change of circuit conditions, in the mazda lamp the voltage varies more than the current, while in the carbon-filament lamp the current varied more than the voltage. Therefore, in a constant-potential circuit, the mazda lamp,—other things being equal—will have a better life than in a constant-current circuit having the same percentage fluctuation of current, as the constant-potential circuit has fluctuations of voltage, while the reverse was the case with the old carbon-filament lamp. Constant potential thus is somewhat preferable for the mazda lamp, that is to give the same life of the mazda lamp, the regulation of the constant-current system must be closer than that of the constant-potential system.

The regulation of the available constant-current systems, such as given by the moving-coil constant-current transformer, reactor or similar devices, is amply close for the successful operation of mazda lamps. It is true that the incandescent lamp is much more sensitive to current variations than the arc lamp was, and while arc lamps are not affected by momentary current variations of 50 to 100 per cent, such as may occur in case of a surging of the circuit due to instability of the arc lamps, swinging grounds, etc., such current variations may be disastrous to the incandescent lamp.

However, in the constant-current mazda lamp circuit if reasonable care is taken of the insulation, the danger of surging hardly exists, and it is only where mazda lamps are operated in series with arc lamps, and the circuit is overloaded and the arc lamps allowed to get out of proper adjustment, that an impairment of the life of the mazda lamp, due to current surges, may occur.

As regards the regulation of constant-potential street-lighting systems, whether high-voltage series, or multiple systems with individual transformers, the problem of regulation of the voltage supply is eliminated by the fact that domestic lighting constitutes a large part of the load of the electric supply system, and the voltage regulation satisfactory for domestic lighting is equally satisfactory for constant-potential—series or multiple—street lighting. That is, constant-potential street lighting is operated from the same supply circuits and in the same manner as domestic lighting is.

Necessary in all series systems is a shunt protective device, that is, a device keeping the circuit closed in case one of the lamps open-circuits. Otherwise either all the lamps would go out or, if the circuit voltage is high enough, arcing occurs and causes more or less damage. The simplest form of shunt protective device is the film cut-out, used very largely in constant-current incandescent lamp circuits. Usually however, one of two other functions are combined with the shunt protective device; to automatically restore operation and to maintain the regulation of the circuit. In incandescent lamp circuits, the former is not necessary, as an open circuiting of the lamp means a burn-out, the lamp then has to be replaced, and when doing so, the film cut-out is renewed. In arc lamps however, the arc not infrequently breaks momentarily, and if then the film cut-out short-circuited the lamp, it would be put out of service. Therefore, in arc circuits, as a part of the arc lamp, a shunt magnet is used closing the circuit through a starting resistance, the latter giving sufficient voltage drop to allow the arc to start again when the electrodes come together.

In constant-potential series circuits, the operation of the film cut-out would reduce the circuit resistance, thereby increase the circuit current and impair the regulation and the life of the lamps. Thus the shunt protective device when operating, must insert a resistance or reactance into the circuit, such as maintains the total circuit impedance unchanged. For this purpose, the same device may be used as in arc lamps, that is, shunt magnet and starting resistance, making the latter equal to the lamp resistance. Such is used in the series operation of a number of arc lamps in direct-current circuits. Or a reactance may be used instead of the resistance, as in the series operation of two flame carbon arc lamps on constant-potential 110-volt circuits.

The objection to this device is the use of moving parts and contacts, and as series incandescent circuits are almost invariably alternating, as shunt protective device maintaining the circuit regulation, a shunted reactance is used, which remains continuously in circuit, and when the lamp is in normal operation, by-paths only a negligible current, operating below saturation; but when the lamp burns out, the shunt reactance magnetically saturates and thereby drops in reactance to the value required to maintain the regulation. A constant shunt reactance cannot give regulation, since with the lamp burned out, but all other parts of the circuit the same, the total circuit impedance necessarily is higher, thus the circuit current lower, and the reactance in this case thus must decrease, to maintain the same circuit impedance. However, with the lamp burned out, the voltage drop across the shunt reactance is much higher than with the lamp in operation, since in the former case, the reactance voltage combines in approximate quadrature with the remaining circuit voltage.

The requirements of such a regulating shunt reactance thus are: (1) with the lamp in operation, to consume as little current and power as possible. (2) When the lamp opens, to drop by saturation to such a value of reactance as to give the same total circuit impedance as before, and thus maintain the circuit regulation. (3) To maintain the circuit regulation over as wide a range as possible, up to at least 10 to 20 per cent of burned-out lamps, so as to give ample time for renewal. These requirements can be fulfilled by proper design in a very perfect manner.[1]

Such shunt reactance may be, and usually is combined with other features. It may be used as an autotransformer, so as to operate lamps of different current consumption in the same circuit or to operate a larger number of lamps in one circuit, without excessive voltage, by having the circuit current larger than the lamp current. Or it may be used as a transformer, with sufficient insulation between primary and second-

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Regarding the mathematics hereof, see "Theory and Calculations of Electric Circuits." Chapter XV.

ary so as to make the secondary, that is, the lamp, a safe low-voltage circuit, especially when locating the transformer away from the lamp, and grounding it. However, in this case care must be taken in the design of the transformer to guard against dangerous peak voltage. Assuming a 6.6-ampere 300-watt lamp. The normal lamp voltage, and thus transformer secondary voltage, then is 45.5 volts. At open circuit, that is, with the lamp burned out, the secondary voltage then would be of the magnitude of 100 volts effective. However, the voltage wave may be very greatly distorted by saturation, having high voltage peaks—at the moment of magnetic reversal—reaching into dangerous values.

The foremost disadvantage of the constant-potential series system is its lesser flexibility. From a constant-current transformer any number of lamps can be operated, with equally good regulation, from full load down to a fraction of it—though it is economically undesirable to operate a constant-current transformer much below full load, as with decreasing load, the power factor rapidly decreases. In a constant-potential series system, however, the number of lamps is given by the circuit voltage. Thus from a 2300-volt constant-potential supply could be operated $\frac{2300}{45} = 51$ lamps of 300-watts 45-volts, neither more nor less. This limitation is overcome by the use of transformer taps and idle regulating reactors. Thus considering a 100-lamp circuit, operated from an ordinary constant-potential transformer with 4500-volt secondary; giving the transformer secondary one 4 per cent and two 8 per cent taps, and using up to 3 idle reactors in series to the circuit, would give a 20 per cent range of load, which probably would be sufficient for most purposes.

A further disadvantage of the constant-potential system is its sensitivity to grounds. If two dead grounds occur in the same circuit, and thereby short-circuit a part of the circuit, the current in the remaining part of the circuit is increased, and if the short-circuited part of the circuit is considerable, all the lamps in the remaining part of the circuit may be burned out. How serious this is, depends on the quality of the circuit. However, with any reasonably well built and operated circuit, the probability of two dead grounds in the same circuit should be so remote as hardly to require much consideration.

On the other hand the great advantage of the constant-potential series system is its high power factor, of 95 to 99 per cent, the better efficiency, and especially, that it requires no station apparatus such as constant-current transformers, but can, and would be operated from a transformer on a pole in the distribution system, just like domestic lighting, and the mass of lines or cables running back to the stations, which are characteristic of the usual constant-current systems, due to the limited power per circuit, thus is eliminated.

In such constant-potential series lighting system, all the street-lighting circuits may be operated from one single feeder or a few feeders, which are connected or disconnected from the station at the proper time. While an advantage over the use of a separate feeder from the station for each street-lighting circuit, this is not as simple and convenient as connecting the constant-potential transformers on the poles in the street-lighting districts, which operate the street lighting, to the domestic-lighting feeders in their respective territory. This latter however requires some means of connecting and disconnecting the street-lighting transformers at the proper time of starting or stopping the street lights, since the domestic-lighting feeders are continuously alive.

Such may be done by manual operation of the switches, though such is rarely satisfactory.

Or it may be done by cascade operation, that is, having the street-lighting circuits overlap, and each start the next one, the first being started from the station. This may introduce some complications and limitations of the location of the transformers, and has the disadvantage that, if one switch fails, all the circuits beyond it are out of service.

	I Constant-current series system	II Constant-potential series system	III Constant-potential multiple system
(1) Type of lamp.	Arc lamp or incandescent lamp.	Incandescent lamps only.	Incandescent lamps only.
(2) Station apparatus.	Constant-current transformer or reactor, etc.	No station apparatus.	No station apparatus.
(3) Number of station circuits.	A separate circuit for every lighting circuit.	No special circuits from station.	No special circuits from station.
(4) Number of wires per circuit.	Single-wire circuit.	Single-wire circuit.	Two-wire circuit.
(5) Accessories at lamp.	Film cut-out, or autotransformer or series transformer.	Auto - transformer or series transformer.	Constant-potential transformer.
(6) Size of circuits.	Limited number of lamps per circuit.	Limited number of lamps per circuit.	Unlimited number of lamps per circuit.
(7) Flexibility.	Great flexibility in number of lamps.	Lesser flexibility.	Great flexibility in number of lamps.
(8) Power factor.	80 to 85 per cent with incandescent lamps. 70 to 75 per cent with arc lamps.	95 to 99 per cent.	Practically unity.
(9) Safety.	High voltage at lamp except when using transformer.	High voltage at lamp except when using transformer.	Low voltage at lamp, due to transformer

Or time switches may be used, that is, switches operated by a clock located on the pole at the transformer. Such clocks however, exposed to weather, are not always as reliable as desirable.

Or a pilot circuit may be used, that is, a low-voltage direct-current circuit carried to all the switches. This means an additional circuit.

Another method, which in many cases is very promising, is the use of the phantom switch, that is, a switch operated with direct current by a polarized relay, so that one direction of the direct current closes it, the opposite direction opens it, with the direct current sent to the switch over a phantom circuit, such as are extensively used in telephony. That is, a direct current is sent through a balanced reactance in the station, into the neutral of the alternating-current distribution system, and at the switch passes from the alternating neutral through another balanced reactance to the relay which operates the switch. Such arrangement permits electrical long-distance operation without any additional circuit. It is not applicable in circuits with grounded neutral. However, in primary distribution circuits a grounded neutral is hardly ever used.

Comparing then the three types of high-voltage street-lighting systems; the constant-current series system, the constant-potential series system, and the constant-potential multiple system: (See page 64)

NOTES

(2) For I, constant-current pole transformers have been designed, without moving parts, and are in successful use to some extent, especially for smaller circuits; but usually are either lower in power factor or inferior in regulation.

Theoretically, II may be operated directly from the primary distribution feeders, but usually it will be preferable to interpose constant-potential transformers on a pole in the lighting district, so as to reduce the trouble from grounds.

(3) In II or III, provision must however be made for starting and stopping the circuit, as discussed in the preceding.

(5) The film cut-out, as the simplest shunt protective device, is applicable only in I.

To secure low voltages at the lamp, so as to make it safe even when the circuit is alive, requires transformers in I as well as II.

However, these transformers are series transformers, that is, with a moderate number of primary turns, while in III, the transformer has to step down from the total circuit voltage to the lamp voltage, requiring many turns of fine wire in the primary, and thus is more expensive to build and insulate.

In III, several lamps may be operated from the same transformer, if near together, but this then means a two-wire secondary circuit in addition to the primary circuit. The same could also be done in I or II.

(6) Both series circuits are limited in the number of lamps per circuit, but this limitation, while serious in a constant-current circuit which has to go back to the station, is of no moment in the constant-potential circuit, as the latter does not need to run back to the station but is fed from the transformer on the pole in the lighting district, and such transformer is of sufficient size to be efficient and economical.

In addition to paper by Dr. Steinmetz a description of Chicago's lighting system was presented by Mr. Henry Nixon, as follows:

I have been asked to give a description of Chicago's new group lighting system. In order that you may know the general type of distribution system to which these group circuits have been added, I think it might be well to cover briefly our transmission and station equipment.

The energy used is three-phase, sixty-cycle, transmitted from the main substation to the thirteen municipal street lighting substations by three-conductor, 12,000-volt cables installed in municipal duct lines. The apparatus in all of these substations is similar, differing only in manufacture and in location in the station, except for three of our stations in which synchronous motors are operated for the benefit of the transmission system. The substation transformation is from 12,000 to 5050 volts, the secondary bus being of the three-wire, grounded neutral type. The path of the circuit is from the phase side of this bus, through an oil switch, ammeter and reactance type of regulator, out to the line, the return coming into the station and terminating on the grounded neutral bus.

The circuits are carried out of the station in sets of four, in 8-conductor, 5000-volt, lead-covered cables to a point where the first distribution to circuits is desired. At that point it branches usually into a 4-conductor and four single-conductor cables, the 4-conductor cable being carried on to a second point of distribution where it branches into four single conductors. From these branch points, the circuit construction varies according to the types of lamps and the local conditions to be met.

Chicago's street lighting system consists of some 50,000 incandescent lamps operated on four general classes of circuit construction. About one-half of the system is of the 600 candle power type of lamp mounted at a height of twenty-two feet. About eighty per cent of these 600 candle power lamps are operated on aerial circuits, using the No. 6 hard-drawn, weatherproof copper wire, and are fed out of the multi-conductor cables by means of single-conductor cables and potheads in the usual way. The remaining twenty per cent of these 600 candle power lamps are operated on underground circuits of the usual type, using 5000-volt, single-conductor, lead-covered cable in the street in ducts, and in some cases utilizing the area ways under the sidewalks.

In about 1913 the street lighting in the better residential districts was changed from gas to electric by the installation of 100 candle power, 4-ampere lamps on the existing gas posts, utilizing an adapter to fit the lamp socket to the post. Five thousand volt, steel-armored cable was installed in the parkway about eighteen inches below the surface. The circuits were quite long, averaging about 250 lamps to the circuit, and the result was that a large number of lamps were put out of service when the circuit was opened, due to the breaking of a post, the cutting of the cable, or other accidents. These circuits were also fed from the multi-conductor cables as explained before. This method of lighting residential streets proved very satisfactory in every regard except that it was open to the serious objection of being dangerous to both the public and the department employes. The posts were only 8 ft., 6 in. high, with a 14-in. ball globe surrounding the lamp and socket. When one of these globes was broken or removed, the binding post of the socket was exposed, and the potential of that binding post being very high, in some instances 5000 volts to ground, it was very hazardous for our men to work upon it, and also dangerous to the public. There are not many instances on record where citizens have been hurt in that way, but of course it only takes a few such accidents to cause you to look about for a safer system. Especially is this true, because when posts are knocked down, quite often the circuit would not be broken. We had one instance of this kind where a policeman attempting to remove such a pole was killed. Boys trying to steal the lamps from the sockets have also been injured, and some deaths have been caused in this manner.

In looking around for a method which would prove safer than the straight series system, two methods of circuit construction presented themselves, the individual transformer method and the group lighting system.

The group lighting system consists of several small series circuits, each operated from a series transformer, the primaries of which are energized by a 5000-volt main supply circuit, which in turn, in our system, is fed from the multi-conductor cable. This system is somewhat cheaper to install than the individual transformer system, it gives the same safety to the public, and is slightly more efficient in operation than the transformer method, and hence it was decided upon for installation in Chicago.

A 6.6-ampere, 72-watt, 100 candle power incandescent lamp is used, mounted upon a cast iron post 10 ft., 6 in. to the center of the light source. This gives an even better illumination than the original 100 candle power installation explained before. These lamps are spaced 150 ft. apart, staggered on opposite sides of the street, giving one lamp for each 75 lineal feet of street. From twenty to thirty of these units are operated in each group, two sizes of transformers being used according to the number of lamps in the group.

The group circuit construction consists of 600-volt, single-conductor, lead-covered jute-covered cable buried in the parkway about 18 in. below the surface. In answer to a question asked at the meeting this afternoon, I might say that we have had a larger number of cases of grounds and opens on this cable than we have experienced on the steel-taped cable installed in a similar manner. We attribute this to the fact that there has been a great impetus to building recently, and that each place where a new building is put up, in digging for the installation of water, gas, etc., the cable is quite apt to become severed. This 600-volt cable is very small and being covered with jute, it gives every appearance of being the small root of a tree, and without thinking the men cut it in two. Also in the installation of parkway fences, our cables quite often are damaged by having stakes driven through them, and it is the opinion of the repairmen that in many cases the steel taped cable would not have been cut where the jute covered cable was.

Only a few lamps being operated in each group, makes it possible to install three, four and even five of these series-series transformers in one manhole. The transformer used is a 6.6 to 6.6 ampere series transformer, having a secondary operating voltage of 300 to 450 volts. From twelve to sixteen of these transformers have their primaries connected in series by means of 500-volt, steel armored cable. The terminals of the transformer are connected to the lead sheath of the cable by means of a wiped joint, similar to the method used in the system described by Mr. Vaughn. This confronted us with the difficulty of having no test point in the primary circuit in the field, and, therefore, we considered it advantageous to install at the termination of the multi-conductor cable a disconnecting pothead by means of which the circuit can be opened and tested in the field.

Individual lamp failures are taken care of by the standard film cut-out socket, and this film puncturing at about 450 volts is practically instantaneous in its operation. The open-circuit voltage of the secondary of the transformer is very high, and would bring us back to the dangerous conditions of the straight series system, if it were allowed to operate at that voltage for any length of time. It is, therefore, necessary to provide some means for short-circuiting the transformer secondary when an open circuit occurs, due to a broken post or a cut cable. This is taken care of in our system by means of a simple film held between two clips, which clips are connected direct across the secondary terminals of the transformer. When the secondary voltage rises beyond the normal value, due to an open in the line, this film will puncture and short-circuit the secondary.

We find that a considerable number of our lamp filaments failed after the circuit is shut down in the morning, due apparently to the cooling of the filament. It, therefore, was necessary to provide that the puncturing voltage of the short-circuiting film be considerably higher than that of the film in the lamp socket. Otherwise if two or more of the lamp filaments had failed after the circuit was shut down in the morning, the film in the transformer would puncture before those in the lamp sockets, and the entire group would be out of service that night. In our system we have installed this secondary short-circuiting device as an integral part of the transformer in order to save the cost of a separate piece of apparatus and the extra cost of labor for installing the same. Our experience has indicated that it would be very desirable if we could develop an automatic circuit restorer which would short-circuit the secondary whenever an open occurred in the group circuit, but which would remove that short circuit as soon as conditions in the exterior circuit had been restored to normal. An instance of the desirability of such a device is when a patrolman in replacing a burned out lamp is careless in his methods; or if the spring on the socket short-circuiting switch has become weak, the film in the transformer punctures due to the momentary open circuit. When this occurs, it is necessary to go to the manhole, open it up, take the top off the transformer, and change the film, and in our system it is even necessary that another class of labor be called upon to do this work, with the result that that group is out of service for a considerable length of time. Up to the present time we have not been able to design anything which would operate satisfactorily, under the severe conditions existing in our manholes, at a cost which would not be prohibitive. So far the cheapest design which has been made up would cost approximately $14.00 to manufacture, and since this is about one-third of the cost of the transformer, we do not feel that its installation would be justified.

As a further measure of safety, the middle point of the transformer secondary is grounded. This brought up a rather interesting point—that the lamps adjacent to the transformer might be seriously damaged if an accidental ground should occur near the transformer. In order to remove this difficulty, the transformer was then designed in two parts having separate primaries, secondaries and cores, in order that the flux linking each primary and secondary might have no effect on that linking with the other primary and secondary. This practically makes two transformers in a single case. This transformer in operation with the middle point grounded is still open to the objection that an accidental ground in the exterior circuit may cause one-half of the transformer to be loaded more heavily than the other half, in just so much as the accidental ground is off the center of the circuit. This, however, has never worked any hardship on the transformer, so far as we know, inasmuch as it is immediately apparent in the reduced candle power of some of the lamps, and the ordinary patrolman will immediately notice the difference and report the trouble to the repairman who removes the ground. These grounds are practically all due to some exterior cause, as I have stated before, and are not due to any trouble in the system itself. This is indicated by reports we have of such cases, where 76 out of 77 such grounds were from exterior causes. These reports cover the operation of 275 groups for one year. Reports on transformer trouble show that 56 out of 59 such cases have been due to leaks in the transformer case. The original transformers were oil-filled, and immediately any water gets into the case, the oil will of course seep out quickly resulting in a burn out. Many of these leaky cases have been due to a patrolman carelessly replacing the cover of the short-circuiting device. Because of these troubles we are at the present time filling each transformer which fails with compound in place of the oil, and on our new contracts we are specifying compound-filled transformers to further reduce this trouble, inasmuch as these transformers operate in manholes where quite frequently the water entirely covers them. This is due to the fact that in some of the outlying territories the sewers are so near to the surface of the ground that it is impossible to give proper manhole drainage.

Except for the change from oil-filled to compound-filled transformers, and the desirability of securing an automatic circuit restorer in place of our present short-circuiting film, the system as originally designed is working satisfactorily in every respect.

DISCUSSION ON "CONSTANT POTENTIAL SERIES DISTRIBUTION FOR STREET LIGHTING" (STEINMETZ), CHICAGO, ILL., JANUARY 9, 1920.

J. R. Cravath: In considering the new systems which we have before us from a practical field one of the first things that is apparent is that there has been a tendency for some time past as a result of the development of the gas filled tungsten lamp to more improved street lighting and more underground work in residence districts. That, in turn, has led to a serious consideration and demand for lower voltage on the actual lighting circuits, I mean the circuits which distribute the energy to the lamps. At the same time the advantages of high-voltage transmission are just as great as ever. In fact, they are more necessary than ever with our large generating plants located a long distance from the point of distribution.

Now the reasons for this tendency to a lower lamp voltage are several. One is that it means cables made for lower insulation and consequently less expensive. Another is that it is undesirable to have high voltages in wires leading up lamp posts. That is, it is much cheaper and safer to have low voltage. It doesn't require high-voltage socket arrangements, and to a large extent does away with the necessity of expensive pot heads in the base of the post. Then there is a question of public safety. I never have heard of very many accidents from series circuits in underground lamp posts, but at the same time one must recognize that frequently they are on concrete bases which may be fairly well insulated from the earth, and in that case there is some possibility of danger to public or to employees.

Another advantage of the low-voltage lamp circuit is that fewer lamps are affected in case of trouble on that circuit. All those reasons have led, as I say, to a tendency to trying to maintain high-voltage transmission and high-voltage distribution up to a certain point near the lamps and from then on have low-voltage at the lamps. There are several variations of the way that has been carried out. One is to have the transformers directly at the lamp posts, as has been spoken of before by Dr. Steinmetz and others, or reactance coils at the lamps.

Another plan, one that has been used here in Chicago recently, and one of the most interesting ones we have to consider tonight, is a series transmission through high-voltage series circuits feeding a small series transformer of relatively low voltage, these series transformers being located in vaults and feeding a group of underground lamps at a comparatively low voltage.

F. A. Vaughn: The Milwaukee street-lighting system was designed about 1914 or 1915, and, with the war coming on at that time, there were certain stoppages in the construction, which set the work back somewhat in respect to the time of completion. It is now about 40 per cent or perhaps 50 per cent completed, and will contain about nine to ten thousand units when complete. The system is operated from constant-potential circuits, that is, from the ordinary 2300-volt alternating-current distribution system, as used by the public utility company for distributing to the houses and factories on the ordinary constant-potential basis. From such feeders are tapped transformers of approximately one-to-one ratio, with taps on the primary and on the secondary, very much as referred to by Dr. Steinmetz, to give a manual adjustment of voltages to the correct constant-potential for the given series-circuit with its particular number of lamps, and their types and sizes. Every complete circuit is grounded in the center, and the lamps are actually fed by series-loops with individual lamp-transformers, or, in certain places, a transformer, as Mr. Cravath has just said, may feed a group of lamps, or, in the case of the double-bracket units, may feed two lamps wired in the three-wire fashion on the secondary of the series-multiple transformers, that is, a three-wire cable runs up the pole from the transformers, each of the transformers is buried in the ground at the base of the pole. The apparatus at the substations consists of means of turning the circuit off and on, in addition to the transformers with the approximately one-to-one ratio of transformation, and induction-type inverse-current relays for the protection of the series circuits against grounds or short circuits, watt-hour meters, and perhaps a potential transformer or a current transformer or a jack from which readings can be taken of the potential and current of the system.

The various types of units on any given series-loop may be grouped promiscuously, incandescent, gas-filled tungsten-filament lamps of any size from 100 candle-power to 1000 candle-power, or multiple units of two or more of these lamps on one post.

Transformer stations are sometimes placed overhead (Fig. 1). What might be termed a more or less temporary type of substation, or a substation which could be used in the suburbs where municipal buildings were not available, or inside types not feasible. Several circuits are grouped into these substations. The substations are placed at the junctions—or, in the case of a group of four, five, six or seven circuits, they are placed at the point where their corners intersect. Thereby, a very few time-switches, which are used for the turning on and off of the lights in this particular case, are utilized—one time switch turning on a group or a multiple number of circuits, if desired.

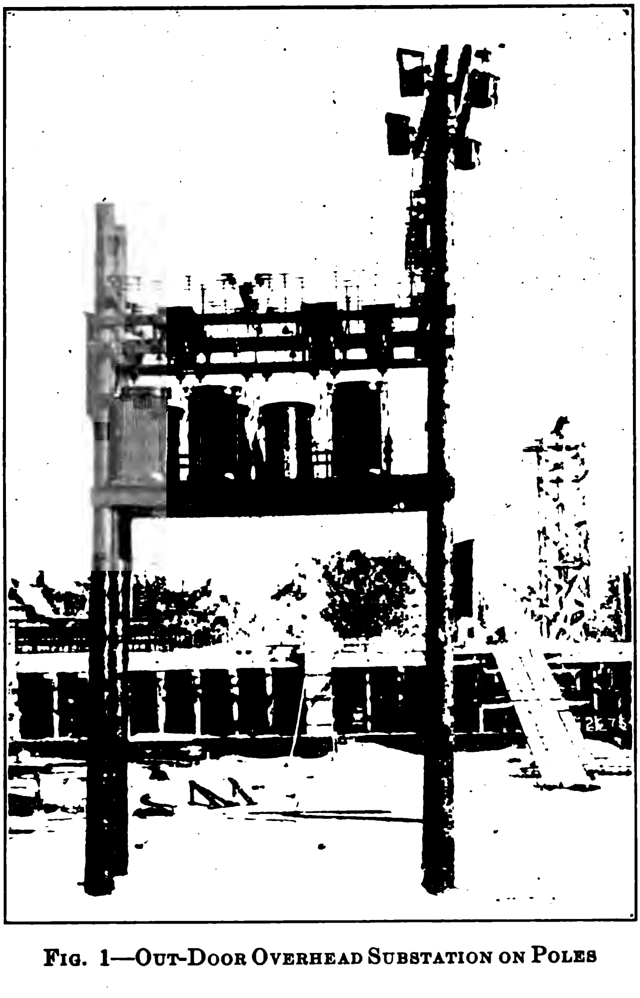

FIG. 1—OUT-DOOR OVERHEAD SUBSTATION ON POLES

In contradistinction to the above particular type of substation, an inside substation is also used with the usual switches and relays, watt-hour meters, etc.

(Fig. 2). This type is located in municipal buildings like the city-hall, the fire-station, school-houses or any other available place where floor space can be secured, and an adequate fire-proof compartment is built for the purpose.

I presume one of the most interesting detail parts of a system of this kind is the type of so-called series-multiple transformer which is used at the base of the poles for the purpose of supplying the lamps in the series-loop with constant current, and eliminating the high voltage of the series-loop from the lamp itself and from the lamp-post. In order to anticipate the possibility that a change in the desired intensity of illumination on any street might be desired, either due to increased traffic, change of type of business, or buildings on the street, or by demand from the merchants or occupants of the streets, the series-multiple transformers for this system were designed with a view to making their over-load and under-load, as well as the full-load characteristics conform to the possible requirement. Therefore Size I transformer was so designed that its 100 per cent load efficiency point falls between the load of a 100 candle-power lamp and that of two 80 candle-power lamps in series. It, however, was also designed to operate well with the 250 candle-power lamp. So, also, the Size II transformer was built for both the 250 candle-

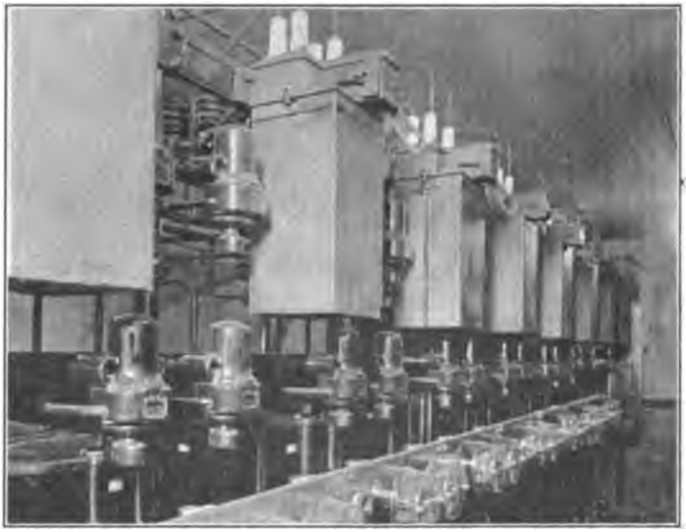

FIG. 2—SUBSTATION LOCATED IN BASEMENT OF AUDITORIUM

power, 6.6-ampere lamp and the 400 candle-power 6.6-ampere lamp; Size III, for the 400 candle-power, 6.6-ampere lamp and the 600 candle-power, 6.6-ampere lamp; Size X, for the 600 candle-power, 20-ampere lamp lamp and the 1000 candle-power, 20-ampere lamp; and Size XX for the 1000 candle-power, 20-ampere lamp; and an over-load range up to the equivalent of a 1700 candle-power, 20-ampere lamp.

Fig. 3 is a chart showing the per cent load on such series-multiple transformers and regulation of same for various lamp-loads. The left hand half of the chart shows the per cent load on the transformer of

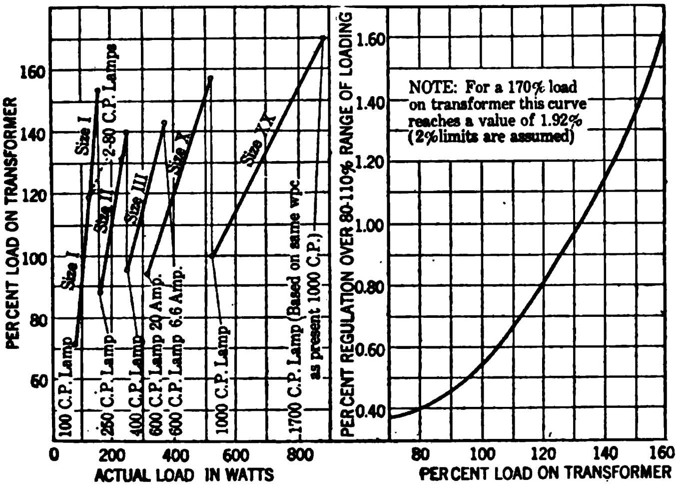

Fig. 3—Chart Showing Per Cent Load on Series Multiple Transformers and Regulation of Same for Various Lamp Loads

any given size and the actual load in watts with any given candle-power rate of lamp for which that transformer was designed. For instance, Size II transformer will operate at approximately 88.5 per cent of full load with a 250 candle-power lamp in the socket and at approximately 140 per cent load with a 400 candle-power lamp in the socket, and so on through the other sizes.

I might say that while an anticipation of the change in size of lamps was a matter of foresight in the days when the transformers were designed, the war brought on the fuel conservation problem and that brought on a change in the size of lamp very much more quickly than was anticipated, so that during the past two or three years these characteristics really came into play much more widely and much more quickly than was anticipated. Preparedness was again strongly justified.

The right-hand half of the chart is used as follows: Suppose we select the 400 candle-power lamp, to be used on Size II transformer. We find that this will be a 140 per cent load on the transformer. Now, passing to the right-hand half of the chart, and reading vertically above 140 per cent on the curve, we find that the per cent regulation over 80-110 per cent range of loading with 400 candle-power lamps in the socket will be 1.14 per cent. This means that even when using the size larger lamp than that for which the transformer is normally rated, and allowing these lamps to vary in their individual wattage consuming characteristics over a range of 80-110 per cent of normal, the per cent regulation of this transformer will be within 1.14 per cent.

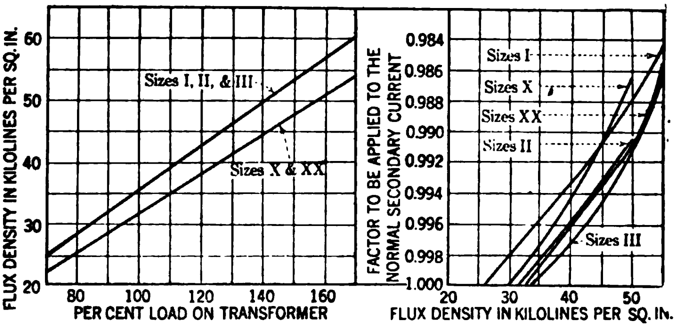

FIG. 4—CHART SHOWING FLUX DENSITY AND CURRENT OF THE SERIES-MULTIPLE TRANSFORMERS FOR VARIOUS DEGREES OF LOADING

Fig. 4 shows a chart giving the flux density and secondary current of the series-multiple lamp transformers for various degrees of loading. From this chart we may read that with 140 per cent load on the transformer, of either Size I, II or III, the flux density in kilolines per square inch is approximately 50. With this flux density of 50 kilolines per square inch, through the medium of the right-hand curve for Size II, we read a factor of 0.9904. This factor, when multiplied by the normal secondary current of 6.6 amperes, for Size II transformer, will give the actual secondary current of 6.53664 amperes. This is the current that will actually flow through the filament of the 400 candle-power lamp, when burned on the Size II, or 250 candle-power transformer.

After completing the electrical and mechanical design of the coils and deciding upon the actual requirements, as shown by the charts above, as well as the method of testing the manufactured product for compliance with these requirements, the results actually produced by the coils put out by the factory were recorded on "shot-gun" diagrams (which are really not, technically speaking, "shot-gun" diagrams), showing the factory test data for individual transformers.

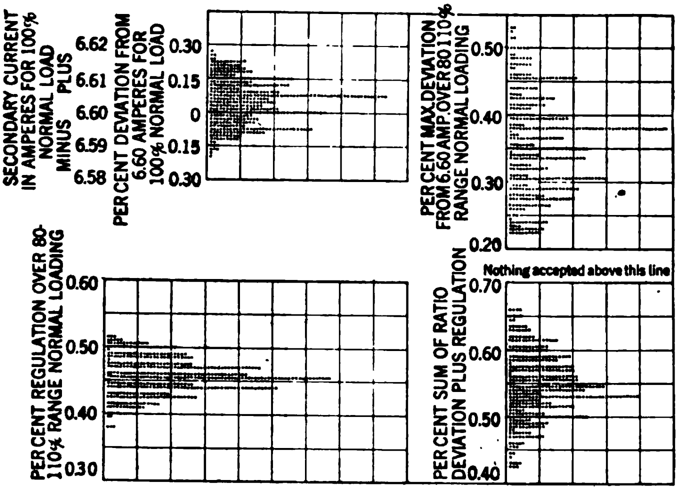

Fig. 5—"Shot-gun" Diagram Showing Factory Test Data of Series-Multiple Transformer, Size II

In explanation of these "shot-gun" diagrams, let a Size II transformer be selected. Fig. 5 contains the data for this size. Each dot on the diagram represents a transformer. The compliance of the factory's output, as determined by actual test at the factory, is indicated by the relative positions of these dots in the four groups in this diagram.

Since, according to the table on Fig. 4 the normal secondary current for this size transformer with 250 candle-power lamp is 6.6 amperes, all transformers which are perfect as far as ratio is concerned, will be grouped along a horizontal line indicated by 6.6 amperes, all transformers having any other ratios will be grouped at positions relatively above or below this line, according to their per cent deviation from 6.6 amperes, with 100 per cent normal load.

For the group of transformers shown on Fig. 5 the success of the factory output is indicated. Observe that this scale is very open and the fluctuations, therefore, are really very small, as indicated by the figures representing the per cent deviation from 6.6 amperes.

The lower left-hand group represents the variation in the actual transformers produced by the factory, as expressed in per cent regulation over the 80 to 110 per cent range of normal loading. This is the range named in the specifications which was so specified in anticipation of possible variations in the electrical characteristics of individual lamps.

The upper right-hand group of points gives the per cent of maximum deviation from 6.6 amperes over the 80 to 110 per cent range of normal loading. This maximum deviation is thus recorded as distinguished from the actual per cent regulation shown in the lower left-hand group of points, which latter gives the total range of regulation.

FIG. 6—INDIVIDUAL LAMP-TRANSFORMER AT BOTTOM OF STREET LIGHT POST, STEEL CASE TYPE

In the lower right-hand group of points is plotted the per cent of the ratio deviation plus the regulation, which it was assumed would represent the worst condition that could be anticipated from such a transformer operating with the regular factory production of 250 candle-power lamps.

A maximum limit of this last variation was set at 0.70 per cent, and nothing was accepted above this line in any size of transformer.

These transformers, as indicated earlier in my remarks—or at least the first ones installed in Milwaukee—were steel-cased transformers, connected with steel-taped cable laid in the ground, and they were buried at the base of the transformer in concrete cups and then covered with asphaltum to make them electrolysis-proof, and then covered up. Fig. 6 shows a typical installation of that type of case.

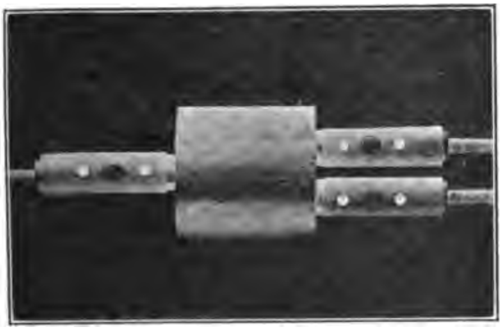

FIG. 7—BITUMINIZED FIBER TRANSFORMER CASE SPECIALLY DESIGNED FOR MILWAUKEE STREET-LIGHTING SYSTEM

Then another case was utilized, namely, that of bituminized fiber, composed of large and small bituminized fiber ducts, in which the whole thing was buried in the ground without further protection (Fig. 7).

Then wooden pump-log, or similar type of cylindrical wooden cases were also used and installed as shown in Fig. 8. This shows a double transformer used for a two-unit equipment, and it is really two transformers in one case. This is the type of double-bracket equipment.

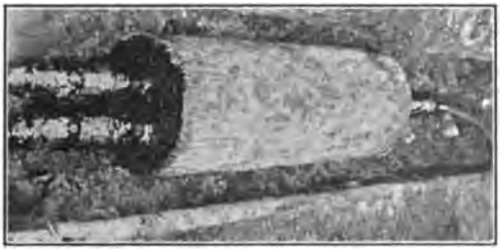

FIG. 8—INDIVIDUAL LAMP TRANSFORMER AT BOTTOM OF STREET LIGHT POST. WOODEN PUMP-LOG CASE TYPE

Fig. 9 indicates perhaps the most interesting type of unit that we have in Milwaukee, namely, the

30-ft. suspension-height hollow-concrete pole with two 1000 candle-power lamps, and the three-wire system on the secondary of the foregoing series-multiple transformer buried at the base of the pole, fed by under-ground steel-taped cable.

FIG. 9—30-FT. CONCRETE ROUND POST, DOUBLE BRACKET STREET-LIGHT UNIT

On certain of the streets in Milwaukee there is utilized the 30-ft. double-bracket unit spaced about 180 ft. apart, and on both sides of a wide boulevard. A practically uniform illumination of approximately one-half foot-candle is obtained by such installation—four units per block. Fig. 10 indicates a less novel type of mounting, that is, the 15-ft. mounting-height short-post type of unit with a single equipment of

250 candle-power lamps with the harp-type fixture, all of course equipped with the special Milwaukee-bowl-type refractor for the purpose of light distribution. These units are opposite and spaced about six times the mounting height.

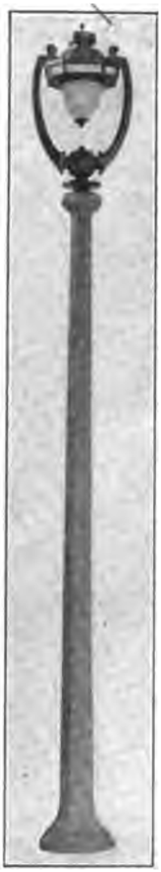

FIG. 10—15-FT. MOUNTING HEIGHT SHORT-POST TYPE OF UNIT, WITH A SINGLE EQUIPMENT OF 250-CANDLE-POWER LAMP WITH THE HARP-TYPE FIXTURE

Speaking particularly of the lighting of the country roads, I thought it might be apropos to show the idea in Milwaukee County, where we have 150 or more miles of concrete roads, and Fig. 11 is the way some of it is lighted. It shows how by a staggered system, 180 ft. apart, 400 candle-power lamps, a very-close-to-uniform illumination is obtained on those concrete roads in a comparatively simple manner—22½-ft. mounting-height units; with refractors for distribution of the light. The uniformity there was, to the human eye, practically constant, that is, it was practically uniform distribution.

D. W. Roper: The Commonwealth Edison Company have very few arc lamps. Anything that we have is somewhat old-fashioned nowdays, due to the fact,—first, that we have only a year to year contract with the City for a comparatively small number of lamps, and secondly, to the fact that we had anticipated they would take over these circuits at the present time, and they have actually taken over quite a few of them within the last few months. I don't think that there is anything that the Commonwealth

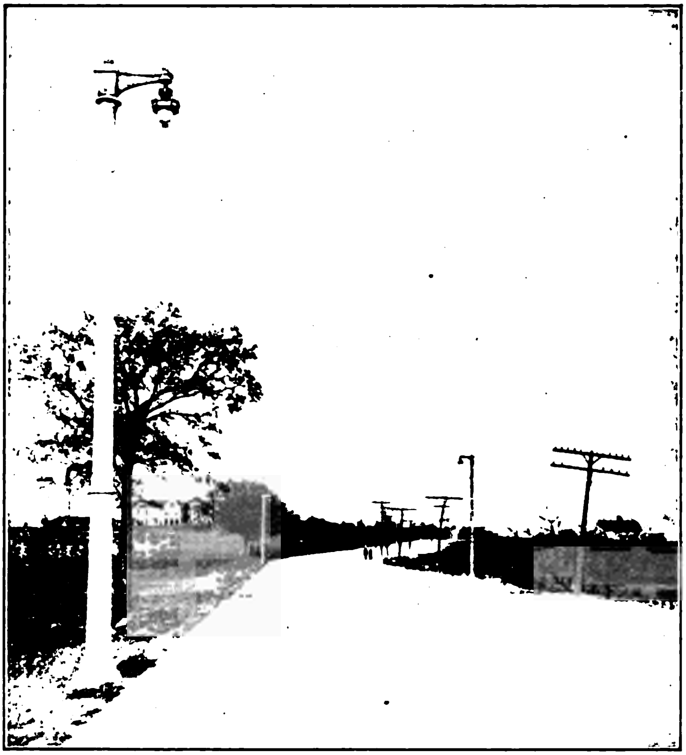

Fig. 11—Milwaukee Type of Installation Used on State and County Highways

Edison has that would qualify with some of these new combinations of series multiple circuits.

Ward Harrison: As I listened to Dr. Steinmetz it became evident that the system which he has described would also allow of the installation of an inexpensive underground construction between the lighting unit and an individual lamp transformer located on a pole at some convenient point. Under such circumstances, however, it probably would be inadvisable to install high-current lamps because the line drop 15 or 20 amperes would run up too fast with any ordinary size wire.

With the low voltage for the underground portion of the circuit, not only is the expense for cable reduced, but it is possible also simply to run out to the curb from the rear lot line at any point where it is desired to place a lamp. As the cable then runs practically parallel to all trenches which might be dug for gas, a sewer, a drive-way, or any other service, there is, therefore, much less likelihood of encountering and possibly cutting through it than where the line runs along the curb. However, good judgment must be exercised in locating such a circuit. If the cable is placed exactly on the property line between two lots it is almost certain to be disturbed if at some future time a fence is erected. On the other hand, if the cable is placed, say, one foot inside of the boundary, and put down approximately a foot below grade, it has a pretty safe resting place; in the suburbs a building is seldom placed to within a foot of the line, and again, very few amateur gardeners are really ambitious enough to drive a spade more than six or eight inches into the ground.

On the constant potential series system, while you can lay out the installations to give the correct number of lamps per transformer, it has been our experience that in nine out of ten of the cases where we have been asked to investigate unsatisfactory lamp performance on such circuits, we have found either the wattage of the lamps or the number of lamps on the circuit changed without making proper adjustment of the transformer taps. Of course, in a series circuit with a constant-current regulator this situation is automatically taken care of.

It seems that the choice between a constant-potential series circuit and the constant-potential multiple circuit will rest quite largely on how simple and inexpensive a method is devised for turning the lamps on and off in the latter system. If a device could be developed that operated without additional wires and determined that the series system should be selected. The fact that the County would own and maintain the lighting system practically eliminated the arc lamp because of the fact that the County would not have a regular arc man to maintain these lamps. In order not to have to duplicate transformer capacity by supplying constant-potential transformers on 11,000 to 2300 volts, and a constant-current from 2300 volts to the constant-current secondary, it was finally decided to have special constant-potential transformers built with 11,000-volt primaries and approximately 1500-volt secondaries so loaded as to deliver a current of 6.6 amperes and provided with secondary potential taps so as to allow a reasonable amount of regulation. Briefly the reasons for adopting in this case the constant series system were as follows: The line was entirely too long for satisfactory multiple distribution. There was no suitable location for a station type constant-current transformer equipment, the use of standard constant-current transformers would require a duplication of transformer capacity because of the constant-potential transformers necessary to reduce the standard primary voltage of the regular constant-current equipment and by the use of constant-potential series arrangement for the circuit a constant-potential transformer designed for 11,000-volt primary and the necessary secondary voltage to supply 6.6 amperes to the load could be more readily secured. In order that the current might be kept constant on the system under lamp failures at various posts, auto-transformers were used at each lamp outlet and these transformers were of such design that the primary current value through the transformer might increase to 150 per cent of normal value before the secondary current reached the burnout value of the lamp. This installation has been in operation now for over two years and given excellent satisfaction. The installation consists of 67—400 candle power units and 10—250 candle power units, and the breakage for the year 1919 consisted of 12 outside globes and the replacement of 24—400 candle power lamps. One day's time was charged against trouble work on the line due to a loose connection on one of the auto-transformers. This represents the entire expense in maintaining the system for one year. The man in charge of the installation reports that the breakage of globes and lamps was due primarily to malicious shooting. One-half of the installation burns from dusk to midnight and the other half from dusk to daylight, or approximately 4000 hours per annum.

The lights on the draw span and the single span connecting the draw span to the Vancouver, Washington, side of the river, are lighted by multiple lamps connected to the distribution system in Vancouver, Washington. These circuits are controlled by time switches and I wish to call attention to the fact that on the series system these switches have operated with very little trouble whatever, and have proven very satisfactory.

There are many cases, undoubtedly, in which a constant-potential series system has very decided advantages over any other, although the development of the pole type, constant-current transformer with moving coil in the comparatively wide range of sizes which are offered at the present time eliminates some of this advantage of the constant-potential series relation. This new type of transformer, together with a reliable time switch, which it is hoped may be developed in the near future, will undoubtedly prove the solution of a great deal of the outlying distribution for street lighting.

Presented at the Midwinter Convention of the American Institute of Electrical Engineers, New York, February 18, 1920.

ESSENTIAL STATISTICS FOR GENERAL COMPARISON OF STEAM POWER PLANT PERFORMANCE

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This paper briefly outlines a method for preparing statistical reports relating to power generation in steam power plants, whereby fairly close comparisons can be made of the efficiencies between different plants, without going into a detailed study of the thermal characteristics of the plants, or the intricate subject of power costs.

The essential items of steam power plant performance that should be recorded and a uniform method of expressing them, are given in tabular form, followed by an illustration demonstrating the advantage of the proposed method.

THE principal difficulty with operating statistics of steam power plants as published today by various State and Governmental bodies is that they are wholly inadequate for the purpose of determining the efficiency of a plant or making general comparisons between different plants. In practically every case the bare figure of coal consumption per kilowatt-hour is given without any reference whatever to the quality of coal and without a clear understanding as to the character of the load on the station. When load factors are given, the figures are often misleading because they are computed upon different bases. No uniformity exists in reporting the output of the power plant, the gross output being used in some cases and the net output in others. Because of these facts the reports as at present published are limited in their use, principally serving to show the total amount of power generated and the total quantity of coal consumed.

The object of this paper is to point out the essential items of steam power plant performance which should be recorded in statistical reports, and to suggest a uniform method of expressing them for the purpose of making a general comparison between different plants, without going into an analysis of the thermal characteristics or cost of generation.

It must be conceded that the proper criterion for comparing power plants is the commercial efficiency, that is, the total cost per kilowatt-hour sent out from the a-c. power station bus, which is the ultimate test of design and operation. To make such a comparison, a uniform system of accounting would have to be adopted, and suitable correction factors would have to be established whereby the operating results and conditions can be reduced to a more or less common basis, all of which is beyond the scope of general statistical reports. For this reason, the discussion of costs in this paper will be limited to that of coal which is the largest single item in the cost of generation.

What is said here refers particularly to steam power plants using coal as fuel and in which all the output is in the form of electrical energy, but the principles relate in their broad application to steam power plants using fuel other than coal and where both steam and electrical energy are supplied.

For the purpose of obtaining a better knowledge of what a plant is doing and for judging the relative merits of different plants, it is recommended that the following items be recorded in statistical reports relating to power generation in steam power plants, and that they be expressed in a uniform way as indicated. These items reflect the influence of station design and arrangement of apparatus, method of operation and management, load, quality and kind of fuel used, and at the same time indicate the character of the load imposed upon the station. For convenience the items are divided into two groups, coal characteristics and load characteristics.

I. *Coal Characteristics of the Plant*

(1) Average B.t.u. supplied to plant per kw-hr. net output.
(2) Thermal efficiency of plant in per cent.
(3) Average B.t.u. per dollar, coal as received (moist basis).
(4) Coal factor or pounds of coal per kw-hr. net output (moist basis).

(5) Average B.t.u. per pound of coal as received (moist basis).
(6) Cost of coal per ton (2240 pound) delivered alongside the plant.
(7) Kind and character of coal.
(8) Cost of coal in cents per kw-hr. net output.

II. *Load Characteristics of the Plant*

(9) Average daily load factor of load
(10) Maximum load for the year.
(11) Yearly load factor of load.

Interval of maximum load 15 minutes or 1 hour depending upon character of load.

(12) Kilowatt-hours net output for the year—(kilowatt-hours sent out from the a-c. bus).
(13) Installed rated capacity, that is, the aggregate maximum continuous rating of the generators in kilowatts.
(14) Average kilowatt-hours net output per kilowatt installed rated capacity.

Items 1, 2, 3, 8, 11 and 14 can readily be computed from items 4, 5, 6, 10, 12 and 13.

Coal Characteristics. The fundamental basis of determining and comparing steam power plant efficiencies as to coal consumption should be the average B.t.u. supplied per kilowatt-hour net output at the power station bus. This will give a basis for discussing and comparing plant efficiencies upon which there can be no misunderstanding. With this factor and a knowledge of the character of load, we have a very good criterion by which to judge the merits of a plant both as to design and operation. The average B.t.u. supplied to the plant per kilowatt-hour net output for the year is determined by dividing the kilowatt-hour net output for the year into the product of the total pounds of coal used during the year by the average B.t.u. per pound of coal as received, (moist basis). This measure of efficiency can be expressed in thermal efficiency or what is generally termed over-all plant efficiency, by dividing it into 3415.

Before a very close comparison can be made between two plants each of which uses a different grade of coal, the coal characteristics of both plants will have to be placed on the basis of coal having the same quality. This involves the effect of quality of coal on the boiler capacity and efficiency which must be taken into account if the object sought is to make an analysis of the thermal characteristics of the plants. However, since the purpose of this paper is to deal with general comparisons which can be made from statistical records without going into a detailed study of the plants, it is proposed to make fairly close comparisons which correct for the difference in the B.t.u. value of the coal itself, but which neglect the effect on the boiler capacity and efficiency. Notwithstanding this inaccuracy which will be relatively small, even with a large difference in the quality of coal, the comparisons as outlined are much more useful than any that can possibly be made from the statistical information published today.

Two plants can be directly compared as outlined above on the basis of the average B.t.u. supplied per kilowatt-hour net output, because both the coal factor and quality of coal are taken into account. A comparison however, cannot be made between the coal factors as given without first correcting for the difference in the B.t.u. value of the coal used. This can be done by multiplying the coal factor of either plant by the ratio of the B.t.u. value of the coal used by that plant, to the B.t.u. value of the coal used by the other plant, thus placing the coal factors on the same basis in so far as the B.t.u. value of the coal itself is concerned.

Because of the lack of laboratory facilities it may not be possible for some small plants to determine the average B.t.u. value of all the coal consumed during the year. In such cases the results of periodic tests should be recorded with an explanatory foot-note, as they will serve to give a general idea of the conditions under which the plant is being operated. There is nothing that upsets the boiler room force so much as changing the grade of coal without notice, since every change necessitates a readjustment of methods of firing. With a knowledge of just what kind of coal is being supplied, the cause of the change in efficiency can more readily be traced.

Generally speaking, the highest thermal efficiency is obtained when burning the highest quality of coal. On the other hand, a low grade of coal purchased at a low price may result in a better commercial efficiency. The B.t.u. per kilowatt-hour net output together with the B.t.u. per dollar will, therefore, give an index of the commercial efficiency of the plant, since coal is by far the largest item in the cost of power production.

Where mixed fuel is used, the relative amount of anthracite and bituminous coal should be recorded, together with the average B.t.u. value of each.

Load Characteristics. The character of load carried has a marked influence on plant efficiency, a fluctuating load and a load with sharp peaks and low load factor require more coal than an equally steady load or a load with a high load factor. In comparing plant performance therefore, it is essential to know the load conditions. These are best indicated by the ratio of the average load to the maximum load for the same period, which is known as the load factor of load and is usually expressed in percentage. Industrial and railway loads are less affected by seasons and daily weather conditions than lighting loads. For this reason it is recommended that the interval for the maximum load be fifteen (15) minutes for stations supplying an appreciable lighting load, and one (1) hour for all other stations.

Much confusion is caused throughout the electrical industry, due to the fact that a uniform method has not been used in determining the load factor. For the purpose of comparing the load conditions of various generating stations, the daily load factor seems to be most suited, and it should be the ratio of average net output during 24 hours to the net maximum load, multiplied by 100 to express in percentage. The load factor should be computed on the basis of 24 hours for the reason that it represents the relation between the actual and possible hours use of the maximum load. In comparing power station statistics which are usually on a yearly basis, the average daily load factor should be used because it is more representative of the operating conditions in the plant itself than a load factor based on the maximum of the month or year.

The fundamental basis of determining and comparing those items of power plant performance that are expressed in terms of unit output of the plant, should be the kilowatt-hour net output. This will eliminate any difference in power consumption of auxiliaries or power used for other purposes in the plant. The result will be that all the factors will be on the basis of the character of load imposed upon the plant.

The plant factor which is the ratio of the average hourly load for the year to the rated capacity of the plant is a measure of the utilization of the installed capacity. This is obtained by dividing 8760 into Item 14.

For the purpose of illustrating the advantage of the proposed over the present method for preparing statistical reports relating to power generation in steam power plants, the following comparison is made between a few of the items of two plants, A and B, which have similar load characteristics, but which use coal differing in quality and price.

A comparison of Items 1 and 2 shows that on the basis of plant *A*, plant *B* has a 12.5 per cent lower plant efficiency and requires 14 per cent more B.t.u. per kilowatt-hour net output at the station bus. If the effect on boiler capacity and efficiency due to quality of coal are neglected, the lower efficiency for plant *B* can be attributed principally to a difference either in station design or method of operation or both since the load characteristics are practically the same.

The equivalent coal factor of plant *B* based on coal having 14,000 B.t.u. per pound, is 1.83 pound or 0.23 pounds more per kw-hr. net output than that required for plant *A*. This means that for the same output and with coal having the same B.t.u. value, plant *B* consumes approximately 14 per cent more coal than plant *A*.

With the present method of recording power plant statistics, Item 4 showing the coal factors would be given, but Items 1, 2, 3, and 5 would not. In this case it would be impossible to tell whether or not any part of the difference in the coal factors was due to the quality of coal. If the relative merits of the two plants were judged from the coal factors alone. the comparison would be misleading for the reason that while the coal

		Plant A	Plant B
I. *Coal Characteristics of the Plant*			
1. Average B.t.u. supplied to plant per kw-hr. net output		22,400	25,600
2. Thermal efficiency of plant in per cent		15.24	13.34
3. Average B.t.u. per dollar, coal as received (moist basis)		5,226,666	5,734,400
4. Coal factor, or pounds of coal per kw-hr. net output (moist basis)		1.60	2.00
5. Average B.t.u. per pound coal, as received (moist basis)		14,000	12,800
6. Cost of coal per ton (2240 lb.) delivered alongside plant		6.00	5.00
7. Kind of coal		Bituminous	Bituminous
8. Cost of coal in cents per kw-hr. net output		0.428	0.446
II. *Load Characteristics of the Plant*			
9. Average daily load factor of load.	Interval of maximum load one hour	50	54
10. Maximum load for the year.		90,000	80,000
11. Yearly load factor of load.		40	38
12. Kw-hr. net output for the year.—(kw-hr. sent out from the a-c. bus)		315,460,000	217,248,000
13. Installed rated capacity, that is, the aggregate maximum continuous rating of the generators in kw		125,000	100,000
14. Average kw-hr. net output per kw. installed rated capacity		2,525	2,173

factor for plant "*B*" is 25 per cent higher than that for plant "*A*", the B.t.u. supplied per kw-hr. net output is only 14 per cent higher. This very clearly brings out the fact that a general comparison cannot be made between the coal factors of two plants without a knowledge of the relative B.t.u. value of the coals used.

Other interesting comparisons can be made from the above tabulations, one of which is the cost of coal per unit of output. The figures given in item 8 show that the cost of coal per kw-hr. net output for plant "*B*" is approximately 4.2 per cent higher than that for plant "*A*".

The above discussion demonstrates the need of a uniform standard method of recording the fundamental factors of steam power station performance in statistical reports, and this paper is presented with the hope that some action will be taken to adopt such a method.

DISCUSSION ON "ESSENTIAL STATISTICS FOR GENERAL COMPARISON OF STEAM POWER PLANT PERFORMANCE" (GORSUCH) NEW YORK, N. Y., FEBRUARY 18, 1920.

E. J. Cheney: I want to give you an idea that I have received as an employee of the Public Service Commission. Instead of objecting because the statistics that we publish are not sufficiently complete and understandable, we get, I suppose, 100 times as many complaints from the companies that make up these statistics, because we ask them for so much stuff, that they do not have time to do anything else than get it ready for us. We would be very glad to improve the reports, but do not like to push the matter too far.

If we started out to get good information—we try to have some now—and go to the logical conclusion, I do not know just where we would end. For instance, a man who is interested in meters, might like to have our report so arranged, and the data so set up by all the companies, that he could tell for each company how much it cost per meter to maintain them, how much per meter it cost to read them, how much per meter it cost to bill the consumers, how much to collect, etc. The distribution man might like to have data which would show the annual cost of pole maintenance, and he would like to get an idea of pole life in the case of all the other companies, and we could go on to an indefinite extent.

I cannot say that these things are not as important as generation, although comparative figures are perhaps more important for generation. I am heartily in favor of the idea of getting statistics which can be interpreted and compared, and I do think we probably can improve our report forms, so that the data we get will be more useful, without making them much more burdensome. Concrete and well thought out suggestions like those in Mr. Gorsuch's paper are most welcome.

W. S. Gorsuch: In regard to the first part of Mr. Cheney's remarks, I wish to say that in preparing data for different reports, the companies are at times put to additional work because in some instances the questions asked lack clearness and uniformity. The result of this is to give out the wrong information which of course will be misleading when comparisons are made.

If the regulating bodies would formulate their questions along the lines suggested in the paper, I believe that the companies would gladly furnish the data, for the reason that such information would be useful for comparative purposes not only to the regulating bodies, but to the companies themselves.

Presented at the Midwinter Convention of the American Institute of Electrical Engineers, New York, February 19, 1920.

ECONOMICAL SUPPLY OF ELECTRIC POWER
For the Industries and the Railroads of the Northeast Atlantic Seaboard

BY W. S. MURRAY
Consulting Engineer, New York City

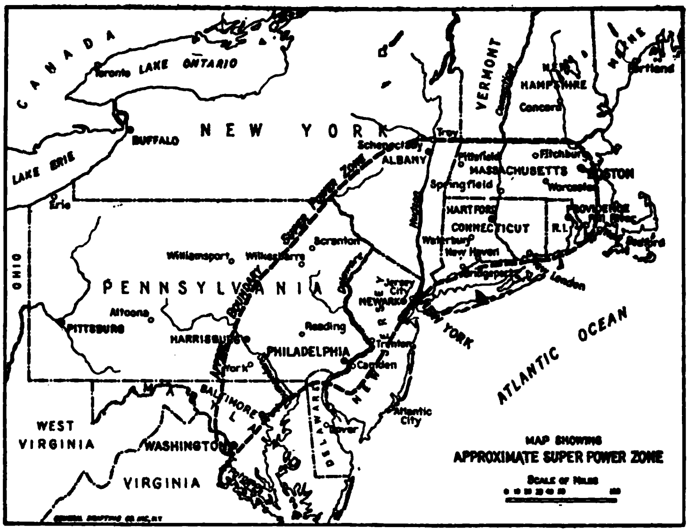
CONTRIBUTIONS TO SYMPOSIUM BY:

(1) W. Le R. EMMET, Consulting Engineer, General Electric Co.
(2) J. F. JOHNSON, Engineer Turbine Department, Westinghouse Machine Co.
(3) H. G. REIST, A-C. Engineering Dept., General Electric Co.
(4) F. D. NEWBURY, Manager, Power Engineering Dept., Westinghouse Elec. & Mfg. Co.
(5) W. B. POTTER, Chief Engineer, Railway and Traction Dept., General Electric Co.
(6) PHILIP TORCHIO, Chief Electrical Engineer, New York Edison Co.
(7) PERCY H. THOMAS, Consulting Electrical Engineer, New York.
(8) W. D. A. PEASLEE, Electrical Engineer, Jeffry-Dewitt Insulator Co.
(9) A. O. AUSTIN, Chief Engineer, Ohio Insulator Co.

THE super-power plan, briefly summarized, provides a means by which a present estimated machine capacity of 17,000,000 horse power—divided 10,000,000 for industrial purposes and 7,000,000 for the railroads—in a region between Boston and Washington and inland from the coast 100 to 150 miles, now operating with a load factor not exceeding 15 per cent, can be lifted to a load factor of greater than 50 per cent and possibly to 60 per cent, and a means by which, conservatively speaking, one ton of coal will do the work of two, and the railroads within the above zone, and those carrying coal into that zone will be relieved of transporting one-half the amount of coal required for power and lighting purposes. In short, the value of machine capacity from a utilization standpoint will be increased three to fourfold, and coal resources for the purposes named conserved twofold.

This means that a present plant capacity of 17,000,000 horse power can be replaced by one of not greater than 5,500,000 horse power, and that not less than 30,000,000 tons of coal per annum can be saved, which at $5.00 a ton will represent $150,000,000 a year.

Besides the above savings, two great departments of economy will be created; one applying both to the railroads and the industries, in the reduced cost of maintenance of machinery, and the other applying to the railroads alone, in the reduction of train miles, by virtue of this plan permitting consolidation of trains with resultant savings in train miles. It is estimated that these latter economies will effect a saving of another $150,000,000 annually thus making a total saving of $300,000,000.



MAP SHOWING APPROXIMATE SUPER POWER ZONE

The above are the direct savings as estimated from data collected from actual past operations of the specific order contemplated to be put into force in the zone under consideration.

The railroad situation in this country is too well known to hold any brief upon it in this paper. Not secondary to the concern of those private interests from whom the money has come to build and maintain them is that, now, of the people to whom their service means so much. Forgetting for the moment the much-mooted question as to who will own and control them, we cannot afford however, from a national standpoint, irrespective of ownership, operation or control, to overlook any means, large or small, which will be conducive toward reducing expense keeping the standard of American transportation upon its past high plane, and which will operate to hold the roads solvent.

This plan offers immediate relief from the present intolerable congestion by automatically increasing rail capacity without increasing track mileage. It also operates to reduce power equipment investment to a minimum. By the creation of an overhead common carrier system of power the present cargo space now required for industrial coal will be cut in half; train equipment in all classes will have its service factor doubled, and the present steam power equipment, replaced by electrical equipment, can be transferred to other divisions, where it is so vitally needed.

The super-power plan has lately been given much space in the pages of the technical and public press, and except to say that it contemplates the general application of electricity wherever economically possible in the factories and on the railroads, in the zone described, it will not be necessary to add length to this paper by a more detailed description. It will doubtless engage your interest however, to know the following facts:

1. The plan has the unanimous endorsement of Engineering Council representing the American Institutes of Electrical, Mechanical, Civil and Mining Engineers.

2. It has been presented to and unanimously endorsed by the

a. Western Massachusetts Engineering Association
b. Bridgeport Chamber of Commerce
c. Connecticut Chamber of Commerce

3. It is now in Committee with the U. S. Chamber of Commerce, and so far as I know has found only favor in their considerations for the amount of time they have been able to give it among their many other deliberations.

4. Among the many prominent men with whom I have had opportunity to discuss it, including U. S. Senators and Representatives, Engineers, Central Station Operators and Railroad Officials, and Financiers, all have expressed the keenest interest in it.

5. An appropriation of $250,000 forms a part of the sundry civil bill to provide the money necessary to an investigation, under the direction of the Department of the Interior. This investigation will be followed by a report allocating the losses incident to labor and material by virtue of the present inefficient form of power production and distribution with recommended procedure, to cause their elimination.

While satisfied in my own mind that the true economy of power production, and its distribution in this great industrial and railroad field lies in an organized power policy, it is far more important that we should hear the opinions of those men to whom is largely due the great advances of electricity's application. Therefore, the following letter addressed to Messrs. Emmet, Johnston, Reist, Newbury, Porter, Storer, Torchio and Thomas will explain itself:

(I would add that the address referred to in this letter is one which was delivered to the Connecticut Chamber of Commerce and is reprinted in the JOURNAL of the A. I. E. E. for January 1920).

> It has been said that the clash of rival systems has delayed the electrification of railroads. This, in my opinion, is true; but I question whether such a result, after all, has not been beneficial.
>
> Fifteen years has been the period over which we have had opportunity to analyze the theory and practise of railway electrification.
>
> Had we known fifteen years ago what we know today about electrification, and therefore could have decided the proper system to apply to each case, the financial stress under which the railroads were laboring would have prevented electrification.
>
> Today the industrial demand for power has grown so prodigiously that faced with the old methods of power production which we have been perpetuating, we find ourselves faced with two

factors, greatly militating against securing and maintaining supremacy in world's trade. Viz.: (1) throttled production, and (2) waste of the nation's natural resources.

The past fifteen years of education have opened the eyes of the "opposing camps" to certain fundamental factors which will now permit a prompt unanimity of opinion regarding what system applies. The railroad managers have in the past justly taken the ground that since we could not make up our minds which system was correct, they would wait until we could. But, as stated before, I believe that finance and not system has been the real cause of lack of action on the part of the railroads to electrify and that the delay has been truly beneficial since we can now stand on more solid ground for decision.

I am enclosing copy of an address made before the Connecticut Chamber of Commerce, November 19th, 1919.

While the super-power line as described in that address is an important adjunct to the plan proposed, the central idea is the indorsement of central station expansion through means of which national production may be increased and the conservation of our natural resources made secure.

As the major demand for power in the territory between Boston and Washington is for industrial purposes, I believe that I will be supported in the following conclusion:

(a) That prime generating machinery should be three-phase.

(b) That the power to be applied to railroads should be through synchronous motor generators with the synchronous motor fields designed to relieve to a maximum degree the direct-current excitation of the prime generating machinery.

(c) That the question of whether train propulsion be by direct or by alternating current should be settled purely upon the economic relations existing between the generating side of the motor-generator substations and the driving wheels of the motive power. In saying this I realize that this robs the single-phase system of the argument to which it has justly laid claim by virtue of the higher efficiency of static versus rotative substations.

Railroad electrifications, in the past, have been individual to themselves. Today I believe that you will agree that they will form only a part of a whole and indeed be a smaller part, the industrial load being the greater—both receiving power from the same source.

I would like to be first under the above conditions in recognizing two serious limitations to single-phase generation.

(a) Size of generators.

(b) Low power-factor regulation.

The Paper's Committee have assigned the 18th of February during the Midwinter Convention in New York as the date for the presentation of a composite paper bearing upon the subject as presented in the accompanying pamphlet.

It is respectfully requested that you, as expert authorities in the design and application of the apparatus in your respective

> **fields, keeping in mind the central idea of zone application of power, contribute your ideas, and while doing so, eliminate all thought of the past battle of systems.**
>
> **I further believe that you will agree that the hour has struck for a united front and that such a paper will help the advance of electricity's rightful use in the industrial and railway field.**

Since writing this letter contributions have been asked from Messrs. Peaslee and Austin, and their valuable opinions touching upon line and insulation form a part of this presentation.

To write a symposium of the contributions forming the principal part of this paper is at once both a delightful and easy task for there treads through such an effort the supporting element of unanimity of opinion. I shall not undertake to comment individually upon these authors' opinions, for that would be but a discussion of their papers. I will rather treat the subject as a whole, shaping its individual parts from the contributed opinions of the authors, using some cement of my own for coherence, and thus, I trust, arrive at a true composite summary.

I am immediately struck by what might be termed the composite voice of these authors. It is as though they say in unison, "We are ready and the conditions are ready to put this thing together." It is opportune here to quote from the address delivered by Past President Charles F. Scott before the New England Section of the National Electric Light Association, September 24, 1919: "The water powers which Nature has bestowed upon New England will be wastefully used until our best engineering abilities combine them into one great regional power system." Again, "The goal is the super-power system, the interconnection of hydraulic and steam stations—a single comprehensive power system, and the universal use of electric power"; and again, "A still larger view shows the power problem from New York to Washington to be closely related to that of New England. High-voltage transmission makes the region from Portland and Boston on the North to Baltimore and Washington on the South a single electrical area. The wide and more varied the field, the more favorable the diversity factor. Power plants at water powers, at coal mines and on tide-water at the cities, combined into one system insure the highest economy in power production."

Professor Scott shows by undeniable records of past growth of central stations, that if the present rate of increase is maintained in New England, its power requirement, now 1,500,000 kw. will be 5,500,000 kw. in ten years. What is true of New England is true throughout all the centralized industrial and railroad districts of the United States. The super-power system now so urgently needed here in the northeast will find demand elsewhere and related in time of application to the industrial and railroad density of the regions to be considered.

We have indeed come to the realization that electricity solved the problems of high-speed economic production and has become the agent of a tremendously diversified utilitarianism. I heard a representative of one of the larger electrical companies say he had just closed an order for a million motors for electrical washers. Think of this as a million kilowatt, and then think also of the diversity factor of such a load! This is but one illustration. The electric range, the toaster, curling iron—all these little implements depending upon electricity, integrate great power loads upon central stations. The amount of power a one-kw. washer takes in an hour would move a 40-ton freight car half a mile. Insignificant as the individual power application may be, their combination makes the central force to drive them immense.

Mr. Emmet more than confirms the statements made to the Connecticut Chamber of Commerce relative to the great economy of electric over steam locomotives and draws deserved attention to the more reliable operating characteristics of the electric engine over that of the steam during the winter months. Traction requirements in such season "go up" and the electric locomotive's capacity automatically "goes up" to meet them, while the steam locomotive's capacities when they are most needed "go down." Who has not had the experience when upon a steam train during bitter cold weather, of having his question, "What's the matter?" answered by the conductor, "We can't make steam."

Mr. Emmet's contribution, though brief, not only brings out the fact that individual turbine sizes are now available for zone power application, and his closing paragraph, while summating important conditions to be satisfied, points to his belief that real results can be accomplished by cooperating in the art of power production—this to my mind is the crux of the situation, for if plant *A* is inter-connected with plant *B*, then the customers of district *A* and those of district *B* both can enjoy the fruits of economical power production, as upon those joint districts could be impressed the voltage from selected machines of high efficiency synchronized upon a common transmission bus.

In comparing the economy of present power production throughout the Boston-Washington zone, with the proposed super-power system, I have stated elsewhere that the former will average at least, if not more than, 40 lb. of steam per kw-hour, and that the average consumption per kw-hour for the super-power line will not be greater than 15 lb. In this connection it is most interesting to note Mr. Johnson's figures of ten, or even less than ten pounds of steam per kw-hour.

As Mr. Johnson has pointed out, "Constant output at as nearly as possible full normal capacity is the first requisite to high efficiency or its equivalent, low cost of production." This is but another way of saying that there can be selected from the inter-connected plants of those now operating and those to be built, the high efficiency units, their size and location related to required distribution, to supply the base load, with the governors of each machine set to contribute its power at highest efficiency.

It is indeed encouraging to find no qualifications in Mr. Johnson's viewpoint regarding the super-power system, but rather a reference to the specific nature of the apparatus necessary to produce satisfactory results, and especially his reference to the adaptability of the multi-cylinder type. It is quite clear that Mr. Johnson's shop specifications are ready and this department of the super-power system only awaits orders to construct. Mr. Emmet and Mr. Johnson have given us the assurances regarding the steam element in the super-power generator. What do Mr. Reist and Mr. Newbury say regarding the other half of the turbo-generators? Mr. Reist says: "So many generators of from 30,000 to 50,000 kv-a. are at present in operation, giving satisfactory results, that there need be no hesitancy in considering generators such as those now used or larger."

Mr. Newbury says: "The generating element in the proposed Boston-Washington power supply system does not involve anything new or untried. The individual power stations need not, and probably would not, be any larger than stations now in operation, or under construction: Certainly any probable station involved in carrying out this super-power development could be designed with steam and electric generating units of size now available."

Mr. Reist's remarks regarding the standardization of potential, methods of insulation and the closed cycle of ventilation are indeed illuminating and all point to that reliability of construction so necessary to continuity of service. It is surely a step in the right direction, and a mark of determination to save thermal losses when, as Mr. Reist points out, the generator itself is made to function as a feed water heater. It may be said indeed that this will be a clean and efficient way to accomplish generator ventilation.

Mr. Reist has opened up an interesting suggestion regarding the consideration of fifty cycles as a standard for world's frequency. Personally, I cannot offer a friendly port to this thought. I believe our generating and transformer investment in the two generally accepted standards of frequency *viz.* 25 and 60 cycles is of an order too large to change now. Certainly, generating equipment of 40 and 62½ cycles should be discouraged. In Mr. Thomas' presentation of the transmission and distribution problem for the super-power zone much valuable suggestion is offered to confirm the standard for prime generating power at 60 cycles.

I think Mr. Newbury has pointed out a fact, not to be denied, in saying that the "super-power scheme will have to be developed by building up local central stations. Its growth must be outward from these centers; but I believe he would agree in the importance of inter-connection of plants through high-voltage transmissions to secure not only diversity factor of load, but *economic* generation and transfer of power from one district to another. The operator who has tried to secure for his company the maximum number of kw-hours per annum from the minimum number of tons of coal burned, has indeed, many times been burdened with the thought of the low efficiency of some of the turbines in his plant that he has had to run on base load when a neighboring plant has had a machine lying idle whose efficiency was the equal of his best machine. Diversity factor though important pales in value when compared to this. That is one of the operating conditions I infer Mr. Emmet had in mind when he made reference to cooperation. I can easily conceive of a tie line between two plants without a single kilowatt of additional capacity paying a handsome return on the investment necessary.

Mr. Potter's able contribution on the traction side of the problem needs but a small amount of cement to bind it to this symposium. He has furnished the cement along with his paper. In short, he tells us that every form of railroad service, passenger, freight or switching can be performed and better performed, by the electric locomotive. The record of electric motive power now in service doing terminal, level and mountain service, and the mileage electrified, bespeaks the practicability of electric traction in the zone under contemplation. Table II is indeed a consolidation of interesting facts and I am pleased to note that he arrives at 14,000,000 tons of coal as the amount to be saved by electric traction over-steam which is 2,000,000 tons greater than the figures I have named in an approximate preliminary study of the situation. On the other hand, I think Mr. Potter has been conservative on his estimate regarding the maintenance figures as I believe the savings here will be *much* more rather than "more" in the amount he has named.

An illness has robbed us of Mr. Storer's direct contribution to this paper, but we find his ample support in the traction field by the recent remarks he made before the Western Railroad Club, December 15, 1919. A copy of these remarks is before me. In them he refers to President Townley's paper, read before the Boston Branch of the A. I. E. E. Mr. Townley's paper was epoch-making in its insistance that the railroad man and the electrical man view the railroad problem *together* and not *apart.* Fifteen years of association between the railroad and the electrical man has brought this amalgamation. Mr. C. L. Bardo, General Manager of the New Haven road was one of the first to recognize this necessity. Mr. Bardo recognized that the requirements necessary to efficient operation by electric or steam locomotive were equal—*but opposite!* He observed that an electric engine would haul twice the tonnage for which it was designed and then apparently without rhyme or reason go out of business three weeks later. Whereas, he noted later, if the designed service duty of the electric engine was strictly adhered to it would do its work on half the coal and half the maintenance, and would double or treble its mileage per engine failure.

Quoting from Mr. Storer's paper, "Actually, electricity has been tried in every known kind of railroad service from street railways to high-speed passenger locomotives; from mining locomotives to the heaviest freight locomotives for mountain grade and tunnel service and from switching in the great classification yards to fast freight trains on trans-continental lines. In every case electricity has shown its ability to do its work as well as, and in most cases, it has done better than the steam locomotive has done and would do it."

Mr. Storer closes his paper by saying: "The big question of capacity, fuel conservation, shortage of labor and the improvement of terminals, will exert all the pressure toward electrification that the railways can stand"; all of which points to the necessity of the super-power system which should be ready to contribute its economies to such an end.

High tribute is due Mr. Torchio for his able and intensely interesting contribution regarding the relative value of water versus steam power development. Looking at his study of the situation from conservation standpoint, brings to mind certain remarks made by Mr. Frederick Darlington, whose able services rendered the War Industries Board during the war are well known and appreciated throughout the country. Mr. Darlington said:

"In the central power station business so much of the cost of service goes to pay for invested capital that it is most essential that the interest rates should be kept low which can be done only by making the investment safe, that is; by so regulating rates that principal and interest will be secured. From a national point of view, as effecting the general industrial efficiency of our manufacturing and power-consuming districts, it is relatively unimportant whether the cost of power is one mill or two mills per unit more or less than some established rate, whereas it is of vital importance for the conservation of resources, for economy of production and for general industrial efficiency that the bulk of the power used should be made by central systems as against isolated plants; therefore let us try to get our law-makers and public executives, national, state and municipal, to take the Government point of view, in other words, to think in terms of the war which are also terms of peace from a Government standpoint, and uniformly and rationally to encourage central power development, provide for a just return to capital in electric power business and grant monopolies under regulations that will foster co-ordination and interstate operations."

The point made by Mr. Torchio regarding the decreasing advantages of water powers on rivers without regulative storage, in proportion to their seasonal variation in flow and especially where their energies can be applied only to independent systems carrying low load factor, has been one, and rightly so, to discourage their development.

It is right here where care must now be used to differentiate between the past small and confined conditions as compared with the present large and unconfined conditions. Where, in the past, rivers whose energies were to be applied to a confined system, and whose load curve for one day could be superimposed upon that of the following with near coincidence, were developed for one second-foot per square mile of drainage area, these same rivers, today, are being developed for two, three and even four second-feet per square mile of drainage area, and we have arrived at this condition because of the tremendous progress made in the inter-connections of systems through the means of which extra load capacity and diversity factor are offered. Today water power development is looking quite as much, if not more, for kilowatt-hours as it is for kilowatts, and therefore the more the energy requirement and diversity factor in inter-connected systems, the greater is the opportunity for unregulated streams to pour their kilowatt-hours into such a composite reservoir of power. This is only saying in another way what Mr. Torchio has already said.

Mr. Torchio's table consolidating a "Review of supply and consumption of coal by classes and territories, and the relation which the potential water powers bear to the amount of coal used in the United States," brings out and confirms my statement that in the east we can never expect a greater contribution from water than 10 per cent of the total power requirement, and it is to the economy of steam produced power that we must earnestly address ourselves. On the other hand, we must be as assiduous in our effort to avail ourselves to a maximum degree of the little that nature has bestowed upon us.

Of great interest comes the check by Mr. Torchio on Mr. Potter's figures of coal to be saved by the electrical operation of the railroads. As stated before, the computed horse power of locomotives in the zone under consideration is 7,000,000. Mr. Torchio shows a total saving of 94,000,000 tons, should the 50,000,000 horse power of steam locomotives in this country be turned into electrical operation. Seven-fiftieths of 94,000,000 would be 13,160,000—a figure lying between my own estimate and that of Mr. Potter's. Again, on the industrial side, Mr. Torchio shows a saving of 61,700,000 tons saved if the present plants in the country, aggregating 25,250,000 horse power, were operated from the lines of high-efficiency central stations. As the computed horse power of the zone under contemplation calls for an aggregate capacity of 10,000,000 horse power, this would accordingly indicate a saving of 10/25 of 61,700,000 or 24,500,000 tons, which added to the above figure, for the railroads, of 13,160,000 tons, makes a total saving of approximately 38,000,000 tons. Both Professor Breckenridge of Yale, Chairman of the Fuel Conservation Committee, Engineering Council, and Director George Otis Smith, head of the department of Geological Survey, have accused me of being too conservative on these savings, and I am glad to see them showing more than less.

We have discussed the generation and utilization of power. The link between its generation and utilization is transmission and distribution. Mr. Percy H. Thomas has contributed a most valuable and illuminating discussion upon this very vital factor of consideration, and it forms a most important part of this presentation. Again we see the objective brought out by Mr. Thomas in that the great base load of the super-power system must be carried by the large, high-economy steam plants supplemented by water power; peak loads and regulative features of the system being taken care of in the main, by "present large generating stations."

Mr. Thomas's comprehensive grasp of the transmission and distribution problem of the super-power zone leaves me with but one recommendation, and that is, your most careful perusal of his paper. The functioning of capacity against inductance in the respective square relation between voltage and current must commend itself to you in the matter of maintaining unity power factor for transmission. Here we see transmission economy maintained and automatic protection against the evil effect of short circuit. A point of almost fascinating interest is the one brought out by Mr. Thomas in the capacity effect on the main line between Boston and Washington to preserve continuity of high power factor and the ability to transfer power north or south without undue loss. The main line will be a veritable static condenser to the whole system, and with such excellent distribution of generating centers, magnetization will provide means of stabilizing and maintaining a constant voltage throughout the whole system.

The simplicity of construction and reduction to a minimum of the switch gear applicable to the 220,000-volt primary transmission, making use to the maximum of present equipment installed, robs the proposed new generating equipment and super lines of much complication and cost.

Mr. Thomas touches on the cost of super-power equipment to the extent of 900,000 kw., this inclusive of generating and transmission equipment up to 700 miles, and names a figure of $150,000,000. Twice that capacity, in combination with the present large plants, would go a long ways toward handling the present industrial and railroad loads in the zone named, Mr. Potter's figures for the railroad requirements being only 750,000 kw.

Mr. Austin's and Mr. Peaslee's contributions regarding the characteristics of line and insulation for high-voltage transmission carry conviction. Particularly interesting is Mr. Peaslee's appeal for ruggedness in the insulator, and Mr. Austin's appeal for simplicity of construction throughout, again touching on many points brought out in Mr. Thomas's paper.

Their papers may be interpreted briefly by saying that in the department of insulation, which is second to none in importance to maintain both economy and continuity of operation, the problem is already solved. Indeed, I think that it can be well said that so great are the returns portended by such a proposed system of organized power policy, that like the great bridges built to sustain a high density of transportation, so may these trunk and tie transmission lines be built to carry their great electrical loads. Their very largeness will bring unusual factors of safety both electrical and mechanical.

With these savings in sight, and with the means at hand to effect them, these means being but the placing in operation of intensive practises long tried out and demonstrated as sound in principle—can this nation afford to be classed as one drunk with the wealth of her natural resources, and rolling in a criminal debauch of their treasure? Is it to be the Americanism of the future to permit this intolerable record of waste to go on?

It is hardly within the province of this paper, should the investigation proposed be made and confirm these projected results, to make suggestions regarding the methods of procedure looking toward the financing and construction of the regional plant required. The following points however, are significant:

1. Conservation in fuel, labor and materials reduced to money equivalents represent approximate savings of $300,000,000 per annum.

2. The highest representative engineering talent in the country agrees that the plan is entirely practical.

3. The securities which represent an investment in such a property (said property to be confined to the features of generation and inter-connecting transmissions) whether underwritten by the

a. States, or

b. Federal Government, or

c. Private Interests, or

d. Private Interests guaranteed by the states or Government will carry a low interest by virtue of their great national importance and financing should not be difficult.

4. Our northeast Atlantic seaboard is the natural finishing shop of American industry, populated with skilled labor to which should be available a cheap and reliable power and by it will be secured high-speed production for shipment in American bottoms (our Merchant Marine) to maintain our supremacy in world's trade.

5. Industrial concerns are far behind in their power requirements and a delay in a supply of electrical power to them will force them to resort to and perpetuate the present wasteful method of supplying it by plants built by themselves. The investment necessary

to their motor requirements will be gladly borne by them and financed immediately.

6. The railroads in this zone are ready for electrification as only by it will capacity be increased: first by the relief from congestion of coal traffic, and second by higher speed and tractive efforts offered in the use of electrical locomotives, thus obviating the immediate necessity of disproportionate increase in track mileage and equipment. The one alternative or the other must be chosen and the right one is apparent.

The following is a summation of approximate investment cost and return for the super-power zone:

SUPER-POWER SYSTEM

(a) New machine capacity 2,700,000 kw.
High-tension transmission mileage 2100
(b) Unit cost per kw. inclusive of transmission lines, $164
(c) Total cost super power system
2,700,000 × $164 = $442,800,000

RAILROADS

(a) Present steam horse power 7,000,000
(b) On account increased speed, traction and machine factors 100 per cent steam horse power can be replaced by 80 per cent electrical horse power
7,000,000 × 80 per cent = 5,600,000 h.p.
(c) Cost of electrical locomotives per horse power = $80(high)
5,600,000 × 80 = $448,000,000
(d) 30,000 miles (single track) to be electrified
Cost per mile $7,500
30,000 × 7,500 = $225,000,000
(e) Total cost railroads $673,000,000

INDUSTRIAL PLANTS

(a) Total horse power 10,000,000 of which 5,000,000 to be changed to electrical drive.	
(b) Cost per horse power inclusive of transformers and local distributions $25	
(c) Total cost industrial plants 5,000,000 × $25 =	125,000,000
Total zone investment	$1,240,800,000
Total savings per annum	300,000,000
Average return on investment	24 per cent

1. W. L. R. EMMET

LARGE STEAM TURBINES

In discussing the broad expediency of power distribution from central steam stations, it is not necessary to consider the relative merits of turbines of different types. The value of the product of such machines is so great in proportion to their cost that wide variations of efficiency cannot be tolerated, and it may be said that all large turbines are good as compared with other apparatus for obtaining power from fuel. In large turbine units, a variation of 10 per cent in efficiency might justify the scrapping of one machine and the purchase of another, unless the inferior machine could, as is usually the case, be used as a reserve or peak load unit.

The best steam turbine station equipments, operating under favorable conditions, can deliver a horse power hour in the form of electricity with an expenditure of one pound of coal, while four pounds are required to deliver a horse power hour to the drawbar of a good locomotive. The locomotive is further subject to many disadvantages, its fuel supply must be delivered and stored on a relatively small scale in many inconvenient places, and its efficiency is greatly affected by conditions of temperature. Its water supply is also a matter of much trouble and expense. While the comparison of efficiency between the large power station and the smaller engine or turbine equipment used in small stations and isolated power plants is less striking than that with the locomotives, it is nevertheless highly unfavorable to the small plant, and this is fully demonstrated by the rapid growth in the use of central station power in industrial plants large and small. The important requirements for economical power production in turbine stations are: large and continuous demand, facilities for economical purchase, handling and storage of fuel, and an ample supply of condensing water.

Power delivered near centers of demand where load factor is high has a much higher value than that produced in remote places, that is, it pays to transmit power long distances for occasional or irregular demand, while for large and continuous demand it is profitable to produce it locally.

Our large rivers with their fertile valleys have naturally focused our centers of population and of transportation facilities. They afford the requisites for economical power production, and the lines of railways correspond generally to the natural arteries for its distribution. To accomplish ideal results in power production for general uses over large areas will require broad cooperation, and dependable financial conditions under such franchises as will give reasonable security against political attacks, and which will give scope for the many progressive individual activities which are gradually developing the electrical art and educating the public to its uses.

2. J. F. JOHNSON

LARGE STEAM TURBINES

The economy benefits resulting from the proposed consolidation and expansion of electric power production, distribution, and application in a portion of the New England and middle Atlantic section, properly organized and managed, and amply financed, would undoubtedly be very great. Problems involved would require the services of our greatest engineers of finance, organization, distribution, and production, but the writer can see no reason why these should not be satisfactorily and profitably solved.

The problem affecting the steam turbine designer is only a very small portion of the general problem of production. With an electric power generating station, as with a manufacturing establishment, constant output at as nearly as possible full normal capacity is the first requisite to high efficiency or its equivalent, low cost of production.

Studies of the load curves of the various districts served will, no doubt, be made to determine the probable load factor obtainable by the proposed consolidation and expansion.

The greatest amount of saving in cost of production of power is to be made by the discontinuance of the many small and medium sized generating stations, many of which contain apparatus which is either obsolete, incomplete, or uneconomically arranged, and replacing them with large capacity stations designed for maximum reliability and efficiency. This would include the development of available water powers to the maximum degree feasible, and the addition of such steam power stations as would be necessary. Against this saving would, of course, have to be charged the costs and losses incident to the increased distribution system required.

The design of the steam generating stations, such as would be required, should probably not differ materially from those being employed for our largest and most modern stations now building, and their efficiency would not differ materially from those now obtainable except as affected by the higher load factor under which they would operate.

Reliability would, of course, have to be the first aim of the designer, and this requisite would preclude the adoption of experimental apparatus or conditions widely removed from those now used and known to produce satisfactory results.

Generating stations of from 200,000 to 300,000 kw. capacity, employing generating units of from 50,000 to 75,000 kw. capacity, each operating on steam conditions of 300 lb. pressure, 200 deg. fahr. superheat, and 29 in. vacuum referred to a barometer of 30 in., would involve no difficulties of design, construction, nor operation, and from such stations a steam consumption rate of 10 lb. or less per kw-hr. and total station rate of less than 1½ lb. of good quality coal per kw-hr. output should be obtained.

Steam turbines of the multi-cylinder type, similar to those now in use and under construction would possess advantages because of their reliability, efficiency, and flexibility, and also because of their adaptability to other operating conditions, should changes become advisable by reason of further experience or development. Only the high pressure element of a multi-cylinder unit would require alteration to meet such a change of conditions.

3. H. G. REIST

LARGE A-C. GENERATORS

The following brief discussion has reference to generators needed in a proposed super-power zone extending through the southern part of New England and the southeastern part of New York and Pennsylvania to Washington, D. C. In this area there would naturally be a number of power houses located at points where condensing water was available and where coal could be delivered with the least haulage by rail. Probably most of these power houses would be located on tide-water; others, in the vicinity of the coal mines.

The size of the generating units for such a project will undoubtedly be determined largely by the output required from individual power houses. Probably there would be at least five generating units in each installation, so that repairs and inspection of the units could be made without much reduction in output at any point. Any reduction in output would have to be supplied from neighboring power houses and the transmission of large masses of power should naturally be avoided to limit the line losses. Nor should there be an excessive number of machines, as this would increase the cost of attendance and installation. A second consideration in determining the size of the units is the economical maximum size of the steam turbine. This subject was discussed at length at a recent meeting of the A. I. E. E. and we may assume that three-phase generators can readily be built for the limiting sizes of the steam units suggested at that time. It seems probable that whatever frequency is selected for this system of distribution, turbines and generators can be supplied to meet the most desirable number of units to be placed in each installation. So many generators of from 30,000 to 50,000 kv-a. are at present in operation, giving satisfactory service, that there need be no hesitancy in considering generators as large as those now in use, or larger.

The potential of these generators should be within the limits of our experience, that is, not above 13,200 volts and preferably lower, if this does not cause inconvenience in the lines leading from the generators to the step-up transformers. The efficiency of large modern generating units is very high. The losses may be expected to be less than 2 per cent of the output of the machine, but even with these small losses, careful attention must be paid to the design of the machine to get satisfactory ventilation so as to keep all parts of the machine reasonably cool. While it is permissible to allow a high temperature on the rotating element, with normal maximum potential of 125 or 250 volts in the windings it is better to restrict the heating in the stationary element to a temperature which will not impair the materials used for insulating the windings. It is recognized that nearly all our flexible insulating materials, such as, paper, cloth, fibre and varnishes, deteriorate at a temperature slightly above 100 deg. cent., with greater or less rapidity. The well-known exception to this rule is mica, but an insulation of mica as applied in electrical machinery, consists of a multitude of thin flakes held together by shellac, or other varnishes which also seal the interstices between the flakes. The electrical resistance of all such varnishes decreases with increase in temperature. With some of these materials, the resistance decreases rapidly at temperatures as low as 75 to 80 deg. cent. Other varnishes, especially some that have been developed in recent years, retain their resistance at considerably higher temperatures. When the subject is considered from all standpoints, it seems undesirable to operate such important machinery at a higher temperature than is allowed by "Class A" insulation in the American Institute Rules. Nor is there much objection to insisting on low temperature on such large machines, since the increase in size and cost of cool machines is negligible as compared with those operating at higher temperatures. Moreover, the mechanical construction should be such as has been tested and found to be reliable. The machines should be designed to have minimum losses and an efficient system of ventilation.

Big generators are cooled by circulating a very large amount of air through the various ducts provided for this purpose in the machine. Any soot or dirt in the air, and the air always carries some of this material, is liable to be deposited on the surfaces of the ventilating ducts and on the exposed surfaces of the coils, thus greatly reducing the efficiency of these surfaces in transmitting the heat to the circulating air. To avoid this deterioration, the air is generally passed through air-cleaning apparatus which removes about 98 per cent of the dirt contents, but even with this cleaning, a considerable amount of dirt will accumulate. An improved method of cooling which has been adopted in a few cases and which has excited the interest of engineers in many other installations, is to enclose the air used in cooling the generator completely, and to re-circulate it, thus not taking in any new air. The circulating air may be cooled by the familiar spray system, but in many cases it is preferable to cool it by a system of radiators, similar to those used on automobiles, only they function to cool the air passing through, instead of cooling the water, as in a motor car. Under certain conditions, the water from the condensed steam may be used as a cooling medium in these radiators. This offers a method of returning part of the heat to the boilers, thus saving a small amount of heat which ordinarily is lost. In this way there is no opportunity for dirt to get into the system.

A closed system of ventilation offers some advantage in extinguishing internal fires which sometimes occur, since it will be possible to retain gases or vapors, which may be used in putting out the fire. It is difficult to do this with the ordinary system of ventilation, since the rapid circulation of the air, of which there is a continued new supply, blows gases out of the machine very rapidly even if enclosing dampers are provided.

On account of the great capacity of the lines in such a large system, there may be some advantage in the use of induction generators. Such machines cannot readily be built in as large sizes as synchronous generators, for the following reasons:

1st. Such machines, having smaller airgaps, are more difficult to ventilate than standard synchronous generators;

2nd. Magnetizing current must be supplied through the armature winding. Therefore, larger current is required in the windings which have to be insulated against the potential of the machine;

3rd. The rotor must be at least partly built of laminated iron, making it impossible to design as stiff a structure as otherwise, thus reducing the critical speed.

Roughly, the size of an induction generator would probably be from 25 per cent to 30 per cent larger than a synchronous generator and the cost of the machine in proportion. Since the present limit in the design of steam turbine generator sets is due to the turbines and not to the generator, except possibly at 3600 rev. per min., I believe that it is possible to construct induction generators in sizes comparable with the most efficient operation of the turbine, that is, perhaps 20,000 to 25,000 kw. at 1800 revolutions, and 40,000 kw. at 1200 revolutions, for 60-cycle machines. We might assume that the same conditions hold in connection with 25-cycle generators. The efficiency of induction generators would probably be about 1 per cent lower than of synchronous machines.

In the territory to be supplied by the super-power system, power is at present being distributed at a number of frequencies, such as, 25, 40, 60, 62½. I will not attempt to explain the reason for these various frequencies, but no doubt all of them were, or seemed, justified when they were adopted. The present tendency is to standardize 60 cycles, rather than 25 cycles, since the latter cannot be used for lighting and higher frequency gives greater flexibility to the speed of industrial motors. Synchronous converters and motor-generators can more readily be operated from 60 cycles for transforming to direct current, than from 25 cycles, since there is greater choice in the number of poles of the motors. The use of 60 cycles will be a handicap on the transmission line, but probably the line will be sectionalized and very long transmission of power will be the exception, rather than the rule, so that it is possible that these inconveniences may be overcome. Since much machinery has to be replaced, and a large amount of new machinery added, and since the transmission conditions would be better at a frequency lower than 60 cycles, I think it is well to give consideration to the use of 50 cycles on such a project. This frequency is entirely satisfactory for lighting, for commercial motors of all sizes, and for almost all other work, and it would probably be found that ultimately its adoption would be a great advantage to this country commercially, since we should then have what is going to be the world frequency outside of the United States. It might be well to consider the adoption of this frequency now, rather than to let the matter drift and suffer the inconveniences of the continued use of a frequency different from the rest of the world, and ultimately to find it desirable to change our standard at much greater inconvenience and expense than if it were done now.

4. F. D. NEWBURY

LARGE A-C. GENERATORS

I have given this subject, as presented in Mr. Murray's several letters and in his paper printed in the January A. I. E. E. JOURNAL, careful consideration. I have not been able to give this matter the time it deserves, but trust the following notes will give my point of view, in regard to the generating end of this subject.

The generating element in the proposed Boston-Washington power supply system does not involve anything new or untried. The individual power stations need not, and probably would not, be any larger than stations now in operation, or under construction. Certainly, any probable station involved in carrying out this super-power development could be designed with steam and electric generating units of a size now available.

Suppose we assume a station of 300,000 kw. to 500,000 kw. total capacity. We have available single-shaft generating units up to 40,000 kw., with generators of 50,000 kv-a. capacity. The preferred speed for such a unit would be 1200 rev. per min., 60 cycles. Eight to twelve such units would give the assumed total station capacity.

There has also been developed and built triple-shaft cross-compound units of 60,000 kw. Five to eight such units would constitute as large a station as has been suggested. This triple-shaft unit consists of three 20,000 kw. turbine cylinders and generators. There is one high pressure element at 1800 rev. per min. and two low pressure elements at 1200 rev. per min.

All of the generating units referred to are actually conservative in size, and by no means represent the largest units of the speeds in question that could be designed. Information concerning this latter point is given in the recent papers by Eskil Berg and J. L. Johnson, on Steam Turbines, and the writer on the subject of Turbo Generators.

As regards the general subject of the single source of power supply within the zone outlined, I believe that the scheme will have to be developed by building up local central stations. It would seem that the greater part of the gain in economy can be secured if all the power requirements of the given local territory, such as Boston or New York, or Philadelphia, or Baltimore, were supplied from local central station systems. The problem of connecting up such local systems in one super-power scheme and interchanging power in both directions from each local system presents a problem of considerable engineering difficulty, and questionable economic advantage in the present state of the art, and in the present state of central station development. There remains so much to be done in developing each local territory that it would seem wisest to carry this on before attempting linking up several local power systems.

5. W. B. POTTER

HEAVY TRACTION LOCOMOTIVES

The suggested system of interconnected economic power generation and distribution throughout the proposed super-power zone should adequately and advantageously provide for the electric operation of the railways within this zone, as well as for the power required for industrial and other purposes. The electrification of these railways would not only insure a substantial reduction in the amount of coal otherwise consumed by the steam locomotives, but would also materially reduce the cost of maintaining the motive power units. Electrification would also provide a more reliable service for all classes of traffic and be a welcome improvement to the traveler as passenger trains would be less frequently late, especially during the winter. The colder the weather the greater is the reserve power of the electric locomotive, which is a much better characteristic than that of the steam locomotive whose power under similar conditions is correspondingly diminished.

There are numerous illustrations of electric operation, which are comparable to the service within the zone under consideration, as well as many other examples of railway electrification throughout the country and abroad, which afford conclusive evidence as to the successful operation of railways with electric power. In fact, a large number of railway electrifications are already embraced within the limits of the proposed zone, and while they do not represent a large proportion of the total mileage, their traffic statistics are available and can readily be studied as a basis for determining the demands of the whole area. A tabulation of these electrifications shows that in this area there are already 380 miles of electric route, embracing 1450 miles of single track and operating 230 electric locomotives and about 1000 motor cars for multiple unit suburban service. A table showing the data of the various roads embraced in this statement, is presented herewith.

TABLE I

Steam Railroad Electrification in the Super-Power Zone

Railroads	Date of electrification	Route Miles	Total mi. of track	No. of locos.	Motor Cars
Balt. & Ohio R. R.	1895	3.6	8.	9	0
L. I. R. R.	1905	88.63	218.	0	477
N. Y. C. & H. R. R. R.	1906	54.00	258.	73	221
W. J. & Seashore R. R.	1906	74.60	150.26	0	109
N. Y. N. H. & H. R. R.	1907	81.63	527.49	106	27
Penn. R. R. (New York)	1910	18.73	97.49.	33	8
Boston & Maine R. R.	1911	7.97	21.50	7	0
N. Y. West & Boston	1912	18.23	54.41	1	40
Penn. R. R. (Phila.)	1915	30.5	116.3	0	115
		377.89	1451.45	229	997

In order to obtain a general picture of the railroad traffic which would be affected by the power supply of the super-power zone, we have made a study of the traffic conditions of the territory covered by the zone. In making this study, we have taken data from the Operating Reports of the United States Railroad Administration, extending over the months of 1919. for which comparable figures are available.

The reports of the Railroad Administration do not give separate traffic statistics of the various divisions of the roads embraced in their report, and there is necessarily some uncertainty in estimating the portion of each road and the traffic which would be embraced in the zone. We have tabulated the mileage of those divisions of each road which would presumably be included in the proposed super-power zone, thus determining the percentage of the total mileage of that road lying within the zone. This percentage, or ratio, we have applied to all other data of the road in order to determine the traffic within the zone. This factor, therefore, determines the number of locomotives, the amount of traffic which would be handled electrically instead of by steam, and the tonnage of coal which would be replaced by electric power. In view of these assumptions as to probable area of the super-power zone and the amount of included traffic, the estimate as given can only be an approximation.

As the detailed figures obtained from the Operating Reports do not apply to switching service, we have added 20 per cent to the mileage and tonnage to cover switching service, and as the power requirements per ton-mile for switching are approximately double those for main line service, we have added 40 per cent to cover the coal consumed in the switching.

On this basis we estimate that the railroad traffic in the region covered by the zone can be represented, approximately, by the following table:

TABLE II

Railroad Traffic in the Super-Power Zone (Passenger, Freight and Switching)

Miles of route	12,000
Miles of single track	30,000
Locomotives in service	8,100
Locomotive miles annually	185,000,000
Gross ton miles annually, including main line and switching movements of passenger trains, freight trains and locomotives	170,000,000,000
Tons of coal consumed annually	21,000,000

Considering railway electrification broadly throughout the whole country and including only those lines which handle freight and passenger service with electric locomotives, we find there are about 700 electric locomotives operating over 5000 miles of route.

There have been some data published on the results of heavy electrification. The papers to which we would particularly refer are,—

Murray—*Electrification Analyzed and its Practical Application to Trunk Line Roads.* TRANS. A. I. E. E., Vol. XXX, 1911.

Cox—*Electrical Operation of the Butte, Anaconda & Pacific Rwy.* TRANS. A. I. E. E., Vol. XXXIII, Sept. 1914.

Beeuwkes—"Operating Results from the Electrification of the Trunk Line of the C. M. & St. P. Ry." New York Railroad Club—March 16, 1917.

From data available, it would appear that the ton-miles moved by 6½ pounds of coal in a steam locomotive is approximately equal to that which can be moved by one kilowatt-hour delivered from the power station. Applying this ratio to the last item in Table No. 2, the electric energy required to handle the traffic now handled by the 21,000,000 tons of coal to steam locomotives would be approximately 6,500,000,000 kw-hr.

If we assume 40 watt-hours per ton-mile at the power station, which checks fairly with the records of a mixed service of main line and switching, the total energy for moving the assumed traffic of 170,000,000,000 ton-miles, would be approximately 6,800,000,000 kw-hr.

The actual requirements would, however, be something less. It has been estimated that of all the tonnage moving over the railroad, approximately 12 per cent is taken up with the movement of railroad coal to points of distribution, including a second movement of the same coal in the locomotive tenders. Making an allowance for railroad coal that would still be required, a reduction of 10 per cent would seem a fair estimate. This would correspondingly reduce the yearly power requirements to about 6,000,000,000 kw-hr.

On the basis of probable load factor, this would call for about 1,250,000-kw. of power station equipment.

CONCLUSIONS

The conclusions to which this discussion point may be summarized as follows:

1. Of the whole mileage included in the zone, a not very large proportion has been electrified, but main line electrifications now in operation are of sufficient extent and carry tonnage of a character to present data which can be fairly applied to the traffic of the whole district.

2. The traffic within the zone now handled by steam locomotives, if handled electrically, would require an average output of less than 750,000 kw. and if produced entirely by coal-burning electric power stations, would reduce the coal requirement for transportation purposes from 21 million tons to 7 million tons annually.

3. As a certain proportion of the electric power will be produced from hydraulic power stations, this coal requirement will be reduced in proportion as advantage is taken of hydraulic operation.

4. The reduction in cost of maintaining the motive power units would be a large amount which estimated from the locomotive mileage would be in the order of $15,000,000 or more, annually.

6. PHILIP TORCHIO

ROLE OF WATER POWER AND COAL IN SUPER-POWER SYSTEMS

Summary. A review of the resources and consumption of coal and the availability of potential water powers in the United States shows that:

1. The *Western States* (Mountain and Pacific) have resources in both coal and water powers to meet indefinitely all their heat and power requirements from either source of supply; the potential water powers alone being large enough to supply over six times all heat and power requirements of 1915.

2. The *other states*, with corresponding heat and power requirements 40 times greater than the Western States, have actually smaller resources in coal and potential water powers, the latter capable of supplying only 8 per cent of their total heat and power requirements. It follows that these states must indefinitely, so far as present human knowledge can foresee, depend upon the use of coal to supply the great bulk of these needs.

3. These states, comparatively so deficient in water powers, consumed in 1915, 528,000,000 tons of coal, about one-half for generating power and one-half for generating heat. They obtained an average efficiency from coal of 5 per cent for the power and 50 per cent for the heat. With present methods of generating power in large, modern central stations the efficiency from coal has been raised to 19 per cent. If all the power had been so generated, the coal saving would have amounted to 185,000,000 tons and the railroads relieved of two thirds of this kind of freight.

4. As the investment cost of a steam plant is only about one-third or one-quarter that of a waterpower development with its transmission connections and steam reserve, and as all other items of investment for the electrification of industrial plants would be the

same with power generated either by steam or waterfalls, it follows that every dollar invested in modern steam plants will make available several times more power for the industries than a dollar invested in hydroelectric plants.

5. The offset for the greater overhead charges of hydroelectric power is the saving in coal. The value of this saving will be more or less, according to whether the power is used more or fewer hours every day of the year, and according to the unit price of coal.

Under American conditions, the value of coal saving derived from the use of water power is usually smaller than the difference in overhead charges between steam plants and hydro-plants, except in cases where the power is used continuously every hour of the year, or where the development is connected to a large steam power system which will absorb the entire possible output.

6. With certain industries like electro-chemical processes and therefining of metals, using immense amounts of powercontinuously, the net cost of hydroelectric power may be considerably less than steam power. There is a great advantage to the whole Nation of devoting as much as possible of the economical water powers, particularly those which are continuous or nearly so and those without regulatory daily storage to intensify the development of these industries which can prosper and render their full benefit to the Nation only under conditions of mass production and most favorable cost of power.

7. In considering the technical and practical features of the special problem affecting all the states (exclusive of the Western States) having insufficient potential water power to meet their present and larger future demands, emphasis should be laid on the importance of concentrating the national efforts to secure the largest possible coal savings in co-ordinating modern methods of power generation by steam. Heavy losses would follow a policy of handicapping these developments by any method of water power development which would involve diverting from them the few but most economical large commercial loads,

and duplication of plant and substitution of water power, and relegating the steam plants to supply the larger in aggregate but less desirable loads of small commercial and domestic services, with poor utilization of the apparatus on account of low average use of power each day during the year.

Where water powers are used as auxiliaries to general power distribution systems, the most economical conditions for both steam and hydroelectric generation can be fulfilled only where a relatively small proportion of the installed generating capacity is hydroelectric.

8. These conclusions would not apply to the Mountain and Pacific States, where the potential water powers are enormous and of cheaper development and can be depended upon to furnish all local power requirements to a very distant future.

THE UTILIZATION OF WATER POWERS AS A MEASURE OF FUEL CONSERVATION

The harassing experiences of coal famines have brought forth repeatedly pressure that immediate efforts be directed to the development of the potential water powers of the country. While from the general standpoint of conserving the national coal resources the proposition deserves the most favorable consideration, it is questionable whether quicker and greater results could not be obtained by concentrating the effort upon improving and changing wasteful methods of utilizing coal.

This we shall discuss under two main headings, one surveying the value of "Potential Water Powers as Sources of Saving Fuel;" the other reviewing the "Technical and Practical Features Attending the Utilization of Water Powers;" so that with the aid of the first survey we may arrive at a clear understanding of the logical lines of development and application of potential water powers for the best interests of the Nation.

1. *Potential Water Powers as Sources of Saving Coal.*

In order to appreciate the situation with respect to utilization of waterpower, it may not be amiss to give a bird's-eye view of the supply and consumption of coal by classes and territories and the relation which the potential water powersbear to the amount of coal used in the United States.

In Table I are given, in detail, for the year 1915, the amount of coal consumed under the two main classifications of coal used for:

Primary power	267,000,000	tons:
Heating	275,000,000	"
Total	542,000,000	"

On the basis of estimates made by the United States Geological Survey, and assuming that only about 60 per cent of the total supply will be recoverable, we find that the present supply of coal in the United States, exclusive of Alaska, if consumed at the rate of the 1915 yearly consumption, will last *four thousand years.* The supply in the Western States amounts to about two-thirds of the total of the United States, exclusive of Alaska. The supply in the remaining states, if consumed at the 1915 rate of about 500,000,000 tons per year, would last *one thousand years.* The greatest shortage is in the supply of anthracite, which, if consumed by the anthracite using states at the rate for 1915, would last only *one hundred years.* In any case, therefore, the saving of coal during one generation is not of vital importance.

This disposes of the immediate urgency of considering the conservation of coal per se, at least for the time being, while more pressing subjects demand attention. The vital importance in conserving railroad facilities, need not be considered provided similar coal savings are made by one means or another.

The next point to visualize is the relation between the national coal requirements and the extent to which they could be supplied by the utilization of water powers in place of coal. We find, using Table I, that the 542,000,000 tons of coal produce a total energy figured in terms of million British thermal units of 3,908,320,000 units,

121,620,000 in the Western States, and
3,786,700,000 in all other states.

ATER POWERS BEAR TO THE AMOUNT OF COAL USED IN THE UNITED STATES

Total energy utilized (In millions b.t.u.)			Per cent efficiency	Possible efficiency with modern improvements applied	Difference —Tons of Coal— which could be saved if its value balances carrying charges of extra investment required
				Per cent	
153,000,000			4.99	18.98	94,000,000
40,800,000					
43,300,000			9.45	18.98	9,000.000
			....	....	....
119,200,000			5.66	18.98	61,700,000
30,600,000			....	....	....
24,450,000			5.66	18.98	12,600,000
25,470,000			10.60	18.98	4,400,000
6,500,000			5.15	18.98	3,800,000
369,420,000	Western states	9,420,000	5.75	18.98	185,500,000
	Other states	360,000,000			
73,900,000	Western states	22,900,000	....	....	....
	Other states	51,000,000			
443,320,000	Western states	32,320,000	....	18.98	185,500,000
	Other states	411,000,000			
297,000,000			60	70	12,800,000
995,000,000			35	60	49,000,000
657,000,000			65	85 + By-products	10,000,000 + By-products
408,000,000			85	85	0
108,000,000			85	85 + By-products	0 + By-products
465,000,000	Western states	89,300,000	52.5		71,800,000 + By-products
	Other states	3,375,700,000			
.834,420,000	Western states	98,720,000		....	257,300,000 + By-products
	Other states	3,735,700.000			
73,900,000	Western States	22,900,000		....	
	Other states	51,000,000			
.908,320,000	Western states	121,620,000		....	
	Other States	3,786,700,000			
				....	
746,000,000					
300,000,000					
.046,000,000					

If all the water powers of the country were utilized they could replace 1,046,000,000 or 27 per cent of the above stated units, of which are available
746,000,000 in the Western States
= 614 per cent of total energy used, and
300,000,000 in all other states
= 8 per cent of total energy used,

The water powers already developed represent 73,900,000 units of which
22,900,000 in the Western States
= 19 per cent of total energy used, and
51,000,000 in all other states
= 1.4 per cent of total energy used,
while the Western States possess large waterpower resources to furnish several times their total power and heat requirements. All other states, are inadequately supplied with water powers. Considering the present commercial limitations of distances for electric power transmission, they could not receive assistance from the excess water powers of the Western States. Their own water powers, even if fully developed, could not replace more than 8 per cent of the energy required. As about one-sixth of their waterpower is already utilized, the *balance would give only about 6 per cent of their power and heat requirements.* From this arises the necessity of an enlightened national policy encouraging and protecting the efforts of such industries as could be instrumental in realizing savings of coal with all other incidental economies.

While the resources of these states are very great, the coal consumption has been steadily increasing from year to year at the rate of about 10 per cent, and if this increase continues, even the present immense reserve would be exhausted in a small fraction of the 1000 years, without large water power resources to fall back upon in a future not distant as measured by the life of nations.

Coal is used to produce either *primary power* or *heating.*

Referring to Table I we find that all of the coal used in the United Stated approximately *one-half* is for producing power and *one-half* for producing heat.

We find further that in producing power only about 5 per cent of the total energy in the coal is utilized, while in producing heat about 50 per cent is utilized. (These figures do not take into account additional losses met in the transmission and utilization of power or heat, as the case may be).

It is evident from these figures showing the low economy of coal for producing primary power that in this field the utilization of water powers would produce the greatest savings in coal.

The economic value of water powers in conserving coal resources is therefore manifest, but *one of the questions to be considered* is whether at the present time quicker and greater results could not in most instances be obtained by the alternative proposition of securing equivalent or even greater economies through improvements in present methods of generating power or heat from coal.

Analyzing the classified means of power production by services, we find that central stations and electric railways realize the highest efficiency from coal, while all other classes of steam power producers obtain from coal about one-half as much as the central stations. Furthermore, with modern equipment the central stations could obtain nearly twice as much work per pound of coal as the present average from their existing equipments. By supplying with electric power all railroads and industries using steam power this could be done, if generated by modern equipment in central stations, for about one-third the present consumption of coal. The total saving would represent 185,500,000 tons of coal, and the railroads would be relieved of two-thirds of this kind of freight.

To realize these coal economies, it would be necessary to make investments for new steam or hydraulic generating equipment to replace the discarded existing steam locomotives on railroads and steam engines in private plants. In all other cases, with the exception of 11,000,000 horse power in existing private plants which already generate electricity for their operation, it would be necessary to introduce a universal electrification of services, substituting electric motors for mechanical drive in plants and electric locomotives on railroads.

In ultimate analysis, each individual case would have to be studied on its economic merits, but, aside from many technical advantages which the electric operation presents over the operation by steam, which advantages may be of greater importance than the mere saving of coal, in general, for a rough approximation, one may assume that a proposition for electrifying an existing steam-driven plant or a steam railroad will be economically advantageous or not according to whether or not the *fuel saved* will *balance the carrying charges of the cost of electrification.* It will appear, therefore, that in general the unit price of fuel will be the fundamental determining factor of the solution. As an illustration, it has been found that in Italy, where the coal price before the war was ten dollars per ton delivered to the locomotive, the saving from electrification of two-track railroads, with electricity at 1 cent per kw-hr., balanced the extra carrying charges of electrification when the roads consumed 700 tons of coal per mile—equivalent to $7000 cost of coal per mile of road.

I repeat that there are many other considerations to take into account in any specific problem, but the broad fundamental principle holds that the unit price of fuel will determine the net economy of transformation from one power service to another of superior fuel economy. While, therefore, by electrification of railroads and manufacturing plants and substitution of modern efficient apparatus in central stations, we could today, reduce the yearly coal consumption for primary power from 267,000,000 tons to 82,000,000—*a saving of* 185,000,000 *tons*—it is doubtful whether the coal saving would balance the carrying charges for the extra investment required.

In different cases the economic advantages would often times be lacking. This would apply particularly to many railroads, but, on the other hand, the centralization of supply for manufacturers and isolated plants, mines, quarries, etc., would in the majority of cases come within economic limits, and the major part of the indicated possible savings for these services of 74,300,000 tons could be realized in a relatively short time. To accomplish this most economically the central station power undertakings already covering the country with a network of transmitting lines, distributing stations and service lines:

1. Should further centralize their production and increase the interconnections between their own and neighboring generating plants, and possibly locate new plants in the coal fields or at by-product gas and coke works; and
2. Should install modern generating apparatus to obtain the highest coal efficiency, and operate this new apparatus more or less continuously, reserving the existing less economical equipment for operation during occasional short periods of high demand for power, such as the evening hours of the winter months. In this manner the added modern equipment would not only serve to carry the new load of the industries transferred to central station service, but would also make a *large saving of coal for the existing load of the central stations.*

It is necessary that we fully realize the importance of this double saving from the addition of modern generating apparatus to existing steam central stations and also the fact that if hydraulic power be so utilized as to appropriate the most economical portion of the steam central station load, the operating costs of the remaining load on these stations, which will usually be required to supplement the hydraulic power, would increase perhaps more than the saving due to the lower cost of the hydraulic power. In addition, the new apparatus for a steam station could be located nearer to the customers who are to use the power, thereby saving the cost and losses of long transmission lines, and the safety of operation and reliability of the electric service would be incomparably greater under all conditions of weather or season or possibility of inimical interference with the lines.

In the foregoing we established the facts that:

All potential water powers are insufficient to provide the total heat and power requirements of the Nation;

The shortage is particularly striking in the case of the states east of the Mountain and Pacific States, hence

The railroads and industries of these Eastern States must always depend largely for their power upon coal;

If, in these states, steam central stations are confined mainly to supplementing hydraulic power from extensive hydro-electric

systems, their power costs would increase, offsetting the savings from hydraulic power;

If, on the contrary, central station systems, unified on a large scale, a scale commensurate with that which would prevail in the case of hydroelectric systems, were developed with modern apparatus and correlated with new plants at coal fields, and by-product gas and coke works, etc., the savings in coal would in a few years fully equal the savings from the total potential water powers in the states east of the Mountain and Pacific States. Incidentally, these savings could be realized with a considerably smaller outlay of capital and within a shorter time than that required for the hydraulic development. Also, this program would avoid duplication of plants and conserve an important industry, already splendidly organized with plants and trained men, all confirmed in the enthusiastic faith that their work is the most material and influential factor in the upbuilding of the efficiency of the Nation.

In this respect, after all is said and done, one must admit that the men who are accomplishing the most in saving coal are the central station companies with their power engineers and the manufacturing companies with their motor salesmen, who are achieving the real results by substituting the wasteful small plants operating at 5 and 10 lb. of coal per horse power-hour with the central station supply operating at a coal economy to save over two-thirds of the coal for the same service. It may not be amiss to point out that the merits of waterpower or steam power are of only relative importance to a public utility operating under governmental regulation. Once the prices of service are regulated upon the basis of a reasonable return on investment values, it will be found that the relative savings from hydraulic power over steam power are of very small order as affecting the total cost of service to a customer. These small differences, with a unified system of steam-power generation as suggested, would entirely disappear for practically all domestic and manufacturing classes of service. The only possible exception would be for those rare instances where the customer makes use of a constant power almost continuously throughout the year.

This leads us to the study of the relative value of waterpower and coal power according to conditions of use and applications. We shall find, for instance, that for certain products which the Nation needs the producer must make use of cheap, continuous power, as these necessities could not be commercially obtained in competition with the same products obtained by the use of coal or from importation from other countries, except by power at an extremely low cost. These products, necessary to the Nation, consume in bulk enormous amounts of energy continuously throughout the year, and to these necessities it is obvious that hydraulic power could be applied at greater advantage than to almost any other application which does not make a similar large and continuous use of the available waterpower. These technical and practical points are covered in the following second part of this presentation.

II *Technical and Practical Features Attending the Utilization of Water Powers*

In normal times the exploitation of a water power is justifiable only when the expected economic saving equals or exceeds the results that could be obtained from steam power. The saving must also be predicated upon the existence of an assured market for the sale of the hydraulic power developed. In a few instances, hydraulic power has been secured as incidental to improvements in navigable rivers. It is impossible to generalize and give the value resulting from such double utilization, but it may be said that none of these improvements has been economically justifiable purely from the value of the hydroelectric power developed.

In general, in the study of a water power development one must consider, besides the costs of the development,

The amount of water flow and its variation at different seasons, and

The maximum possible seasonal and daily utilization of the water powers by the industries to be served.

With the exception of a few rivers, like those fed from the Great Lakes, the water flow and consequently the power available for most of the potential water powers in the United States east of the Western States is very variable and without facilities for building in the mountain regions suitable reservoirs to impound the water for the low periods. This is practicable only in situations like those existing in the West. On this account, it has been found necessary to install in conjunction with water powers, large auxiliary steam plants to supplement the deficiency of the water power at times of low water flow, except in instances where water powers are used as auxiliaries to steam systems of relatively large capacity.

The second point under consideration, that of daily utilization of the power by the industries served, is very important and is often overlooked by men not familiar with the operation of power plants. Only certain special industries, like electrochemical industries producing aluminum, carborundum, special steels, caustic soda, nitrate, calcium carbide, etc., can use available power continuously every hour of the day and every day of the year. The average power user, on the contrary, utilizes the maximum power for, say, eight hours a day only, and more or less irregularly even for that period, with no use on Sundays, holidays, etc. This characteristic of individual users makes the utilization of the maximum available power very low. By combining a great number of users, all supplied from one power plant, the conditions are ameliorated in respect to the utilization of the maximum power demand upon the central plant, as the resultant maximum demand is considerably less than the sum of the individual demands of every user. This results from the fact that all users do not reach their maximum power demand at the same time. But, notwithstanding this averaging of individual demands upon the central plant, the net utilization of the resultant maximum demand is still about eight hours a day or 2920 hours out of the possible 8760 hours in a year. In the case of water power plants without regulatory daily storage: the unused power between 2920 and 8760 hours in the year is entirely wasted.

Without further emphasizing this point of the poor utilization of the maximum power in communities, it appears that *where continuous water power is available*, the greatest usefulness is derived by applying it first to special industries and processes which require large amounts of electric power and can be operated continuously, or nearly 8760 hours each year, so that the waste through unused power is reduced to a minimum. If that is not possible, then the best means of utilizing the water power to the greatest advantage is to have it exploited by large power undertakings which can either so arrange their operations that they can utilize all the water power for carrying that portion of their load which is nearly constant throughout the year, leaving the heavy season peak loads to be supplied by their reserve steam stations, or can otherwise distribute the electric power over large territories, including cities, mining fields, agricultural sections requiring electric pumping for irrigation and other uses, so that the resultant utilization of the water power distributed over different places and periods of day or season is high, and the waste of unused available waterpower is reduced. This method of exploitation is economically feasible only where the price of coal is relatively high.

In the case of water powers with variable flow, presenting large variations between minimum and maximum available water power, the necessity for extensive operation of auxiliary steam plants greatly reduces the economic value of the hydro development as an independent source of power, a condition which exists with most potential water powers of rivers east of the Pacific States, where impounding of water is usually impracticable. Economic utilization of such water powers is frequently contingent upon their use in connection with steam power systems so large that the water power is a relatively small proportion of the total.

The economic value of a potential water power is calculable by comparing the investment and operating costs of the water development, plus the steam plant reserve when required, with the cost of an equivalent steam plant or plants. If the power is not to be utilized *in situ* and must be transmitted over long-distance transmission lines, the cost of these lines and the cost of their maintenance and operation must be added to the cost of water power.

To visualize the relative values of water powers under different conditions of water flow and utilization of power, we may make a few comparisons to approximately cover conditions existing in different localities. As costs figures for present prices are lacking I have used pre war prices.

To arrive at a basis for comparing costs, we have reviewed several existing hydroelectric developments and found that the costs per *kilowatt maximum* of power development were as follows:

One case—continuous power = $167
One case— " " = $198
One case—non-continuous power = $188
One case— " " = $238
One case— " " = $222

The costs of these developments would be materially higher under present prices of labor and materials, but I shall neglect this feature and assume a cost of $180 per kw. maximum—a figure smaller than the average of the above five costs—and also neglect the extra cost for transmission lines where required. We shall also assume the low rate of 8⅓ per cent for interest, taxes and up-keep.

On the other hand, for the cost of steam plants, I shall assume $60 per kw. maximum—as the corresponding cost before the war—and the rate of 11 per cent for interest, taxes and up-keep.

The estimates for six illustrations are based in each case on the development of a 100,000-kw. plant. In the several cases the plant investments were:

Hydro without steam auxiliary.......	$18,000,000
Hydro with 50 per cent steam auxiliary	21,000,000
Hydro with 100 per cent steam auxiliary	24,000,000
All steam.........................	6,000,000

These figures clearly illustrate the point that for the same amount of power the hydroelectric development requires several times the capital outlay of an equivalent steam plant.

From the detailed estimates of power costs I have abstracted the results in the following tables of comparison.

Before proceeding with the presentation of these comparisons, it may perhaps be well to caution the reader that the figures apply to the *increment costs* of securing power from new plants under the conditions given in the assumptions, and would not apply, for instance, to the cost of power from the average steam station operating apparatus considerably more expensive and less efficient than large turbo units of modern design.

It may also not be amiss to state here that the steam central station industry has made almost revolutionary progress in raising the efficiency to a maximum of about 19 per cent of the energy in coal, while older plant and the average isolated plant of today utilize less than 6 per cent. It is also reasonable to expect that, with time, still further progress will be made.

On the other hand, in the case of water powers, hydraulic plants have started with, say, 80 per cent recovery of the potential energy, and efficiencies of even 90 per cent have been reached so that no material progress may be expected in increasing the efficiency of hydraulic plants.

As a corollary to these considerations, it follows that with the free play of economic laws in industry, everyone must recognize the latent weakness of extensive water power development, except under the most favorable conditions of cost of development, marketability of product and liberal terms of water grant.

The following tables give comparisons of the *relative values* of hydroelectric and steam power for a number of specific conditions of service which cover all possible ranges of conditions that may exist for different situations of power production from very large plants equipped with modern apparatus.

I again note that the costs given are only theoretical relative increment costs based on pre-war prices of plant installation and operating labor. Actual costs were higher before the war and would be considerably higher at present both for hydroelectric as for steam power, but relatively higher for hydroelectric on account of greater investment charges.

I. WATER POWER CONTINUOUS—33⅓ PER CENT

Average Yearly Use of Kw. Maximum demand = 2920 Hours—Auxiliary Steam Plant 50 Per Cent of Capacity of Hydro Plant.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydroelectric	0.564 cent	0.564 cent	0.564 cent	0.564 cent
Hydro with 50 per cent steam	0.701	0.711	0.731	0.751
All steam	0.393	0.460	0.593	0.727

II. WATER POWER CONTINUOUS—66⅔ PER CENT

Average Yearly Use of Kw. Maximum Demand—5840 Hours—Auxiliary Steam Plant 50 Per Cent of Capacity of Hydro Plant. Similar estimates Have Been Made, from Which We Abstract the Results Comparing the Yearly Unit Costs and Cents Per Kilowatt Hour for the Different Sources of Power.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro-electric	0.297 cent	0.297 cent	0.297 cent	0.297 cent
Hydro with 50 per cent steam	0.367	0.372	0.382	0.392
All steam	0.250	0.316	0.450	0.584

III. WATER POWER CONTINUOUS—91 PER CENT

Average Yearly Use of Kw. Maximum Demand = 8000 Hours. In this Case, as the Power is Practically Used Continuously, We Assume that, when Transmitted to Long Distances, it Will Require a 100 Per Cent Reserve Steam Station—Instead of the 50 Per Cent Assumed in the Two Previous Cases.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro-electric	0.218 cent	0.218 cent	0.218 cent	0.218 cent
Hydro with 100 per cent steam	0.298	0.302	0.322	0.329
All steam	0.199	0.265	0.398	0.530

In the case of water power with variable flow, a condition prevailing on almost all rivers except those fed from the Great Lakes, we find that it is necessary to install a 100 per cent steam plant auxiliary for carrying the deficiency of low water flow and as a reserve against interruptions of transmission lines if the water power is utilized at considerable distance from the power plant.

The amount of energy output required from the auxiliary steam station depends upon the load factor of the system and the period of drought on the hydro plant. For average conditions we obtain the following comparative results for similar conditions of service as figured in the case of *a constant water flow.*

IV. WATER POWER VARIABLE FLOW—33⅓ PER CENT

Average Yearly Use of Kw. Maximum Demand = 2920 Hours (2190 W.P —730 Steam)—100 Per Cent Reserve Steam Station.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro with 100 per cent steam................	0.825 cent	0.841 cent	0.876 cent	0 910 cent
All steam...............	0.393	0.460	0.594	0.726

V.—WATER POWER VARIABLE FLOW—66⅔ PER CENT.

Average Yearly Use of Kw. Maximum Demand = 5840 Hours (4380 W. P.—1460 Steam)—100 Per Cent Reserve Steam Station.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro with 100 per cent steam................	0.448 cent	0.465 cent	0.499 cent	0.532 cent
All steam...............	0.250	0.317	0.450	0.585

VI—WATER POWER VARIABLE FLOW—91 PER CENT

Average Yearly Use of Kw. Maximum Demand = 8000 Hours (6000 W. P. —2000 Steam)—100 Per Cent Reserve Steam Station.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro with 100 per cent steam................	0.341 cent	0.356 cent	0.386 cent	0.416 cent
All steam	0.199	0.265	0.399	0.530

Under present prices these estimated costs would be larger. This would influence especially the overhead charges for plants, which would make the relative increases larger for hydroelectric than for steam power. It appears from these comparisons that hydroelectric power offers material advantages only when it is continuous and is so used, but that, on the other hand, the cost of steam power is less than that of hydroelectric power for ordinary conditions of service, load factors and price of coal. For such conditions it may also be noted that even relatively large differences in unit cost are of a trifling amount in comparison with the total cost of service including all other items, common both to steam as well as to hydroelectric power, covering costs of transmission, distribution, metering, billing, etc.

Recognizing these economic conditions, it becomes evident that in shaping the policies of utilization of potential water powers to the best end of preserving investment and conserving fuel resources, hydroelectric developments yielding relatively large amounts of continuous power should be co-ordinated primarily to supply energy for the production of products which require large amounts of continuous power such as can best be generated from water powers. Following this policy, it is conceivable to see large centers of electrochemical industries developed in a not distant future to produce nitrates for chemicals, munitions and agricultural fertilizers, caustic soda, electrolytic copper, electrolytic zinc, aluminum, calcium carbide, carborundum, graphite, etc. As for other applications, while today it may appear visionary to state that eventually no steel will be produced without passing through the electrical refining process, some steel men are firmly convinced that electrical steel will ultimately become standard, because of the saving that all users of such steel will be able to make in reduction of material owing to its great elastic limit and its uniformity.

Some of these industries are in their infancy; others are now either consuming great quantities of coal or contending against great odds in their development because of shortage in cheap electric power within commercial distance of the raw products.

By concentrating waterpower developments into these intensified fields of electrochemical and refining products, there would result the attendant economies of mass production which are an absolute necessity to enable any American industry to flourish or meet foreign competition with its cheaper labor. If, for instance, the utilization of the major part of the Niagara Falls power could be arranged for between the United States and Canadian Governments, it would not be a great stretch of imagination to forsee that center become the Pittsburgh of electrochemistry and electro metallurgy with railroads and waterways reaching out to the most inland points and to the oceans.

Developments on rivers with irregular flow can in general only be used with full economy when employed as auxiliaries to or in connection with steam power systems of relatively much larger capacities.

In the mountain and Pacific States, where the potential water powers are enormous, the development of these resources, to save the wasteful use of fuel oil, should be fostered on the widest possible scale.

7. PERCY H. THOMAS

The Super-Power Plant

The following discussion is intended to cover the chief factors underlying the economy and operative feasibility of the proposed scheme for interconnecting the electric power systems of the district of the country lying along the north Atlantic seaboard and putting them in reach of the cheapest sources of steam and water power. With the interconnection is associated the electrification of the principal railways. The existing power companies in the district and the railroads now electrified or which it is proposed to electrify constitute the principal power consumers and distributors and must form the nucleus of the system.

The most fundamental object and the most important advantages of the project from the community's point of view, are the conservation of coal and the relief of the railroads from the burden of hauling coal for their own use and for power supply, which coal haulage constitutes more than one-third of the total traffic. Other advantages are mutual support and interchange of power among power companies, more favorable diversity factor and cheaper generation of power.

To conserve fuel most effectively requires both the development of as much water power as may be economically justified and the burning of coal in the most economical manner, as well as the use of low grades of coal. To relieve the railroads of the burden of freight, requires the burning of coal as near the mines as feasible. Furthermore, economical generation of steam power requires use of large power houses. To meet all these conditions requires long transmission and a very great premium is thus put on low priced and economical transmission lines. In fact, operative practicability and low cost of transmission are the essential features of this project. This means the simplest practicable layout and the greatest possible power capacity.

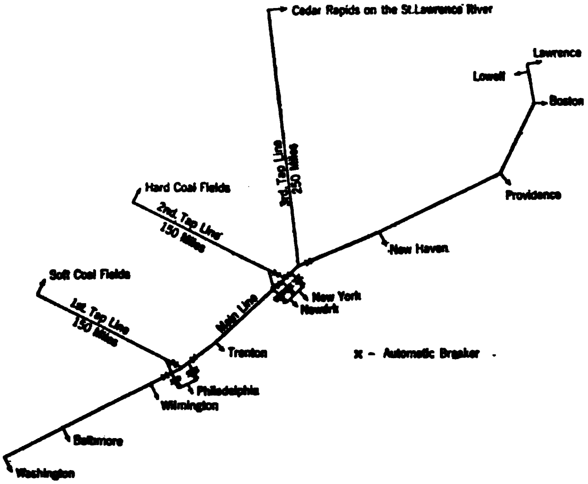


FIG. 1

The system shown in outline in Fig. 1 is suggested as one solution of this problem and as a suitable basis for discussion.

This system consists of a main 250,000-volt line connecting Washington or Baltimore with Boston, via Wilmington, Philadelphia, Newark, New York, New Haven and Providence. This line is fed from a group of large stations at the nearest soft coal fields—perhaps on the upper branches of the Susquehanna, but in any case where condensing water can be obtained and from a second group at the hard coal fields, probably supplemented by the Susquehanna and Delaware River water powers. Each group of such powers would feed through a tap transmission line. If a large plant or plants on the St. Lawrence River should turn out to be feasible, another tap line could be run to connect with the main line probably where this line crosses the Hudson River. The length of this line would be about 250 miles, this distance being taken to a junction point with the main line. The total distance to New York probably will be 300 miles or somewhat over.

I would like to call attention to the fact that I have not fixed upon any definite locations for actual coal mine power houses, but have made assumptions as to the probable minimum distances involved. The actual selection of a site or group of sites might add materially to these distances. The only effect, however, of an increase of 25 per cent to 35 per cent in the transmission distance would be to increase the line losses approximately proportionally to the distance and the cost in proportion to the added length. It would have very little effect in the final conclusions. Since the exact location of a site must be determined by suitable cooling water it will obviously be necessary to make use of a certain amount of railroad haulage of coal. This would be necessary in any case since coal must be collected from a number of mines. This is not objectionable if the coal does not have to be hauled long distances nor over main trunk lines. The west branch of the Susquehanna around Williamsport and Lockhaven is one possible site for a large plant. This plant would be fed from the Clearfield coal district. As an alternative, a power station might be located on the Potomac River somewhat below the Maryland coal fields and the transmission from there to Baltimore would be less than 150 miles. The latter alternative would probably work out to about the equivalent of the other as to cost and as to operating quality.

These two or three generating groups would supply the base load or the maximum continuous 24-hour power for all the territory fed by the main line—amounting to perhaps one-third of the peak load. To carry out this idea with the assumed conditions, the single-circuit tap lines should have the maximum possible capacity, which is here taken as 350,000 to 400,000 kw. per circuit. To secure such a capacity at 250,000 volts, requires a number of innovations, which will be discussed later.

It will not be desirable for consumers except the very largest to be connected directly to the 250,000 volt lines on account of complications and the expense. Local consumers in any district can be supplied from the nearest large distributing company. Small water powers or groups of neighboring powers up to 25,000 to 50,000 kw. would be connected with the nearest distributing system without connection to the 250,000-volt system.

As the load in the district grows, additional 250,000-volt circuits can be added from suitable generating sites—probably all more or less widely separated—preferably single-circuit lines.

It will be desirable to make use of and depend upon the present large generating stations to supply all peak and breakdown service. Voltage would be independently controlled at the principal points of the system, as for example, Philadelphia, Baltimore, New York, Boston, as will be later explained.

For example, Philadelphia would receive the first tap line and control its voltage and feed the surplus up and down the main line. Similarly with Newark and New York for the second tap line, and also for the third line to the St. Lawrence if this should be later added.

General Features. The new generating plants should be designed for maximum simplicity and low cost. They should be based upon full-load operation at 90 to 100 per cent load factor. Turbines and generators should be designed for best efficiency at this load. Boilers should be built for high rating and long continuous runs—with distilled water makeup. Very likely coal should be burned in powdered form and the largest units should be used.

The highest steam pressure, probably 500 lb., with at least 200 deg. superheat and economizers with air heaters for preheating air for combustion from exhaust gases and the heat in the generator cooling air should be used. A house turbine operating condensing with the condensate of the main units for cooling water would be used having as large a capacity as can be handled with the amount of condensate available. In addition, atmospheric exhaust auxiliaries would be used to the maximum extent permissible without too high a feed water temperature.

Generators would be synchronous type with governors for controlling the field current, the governors being set for a definite load and the fields controlled automatically to keep a definite power factor—about 98 per cent to 99 per cent lagging and also to take a certain definite amount of charging current at no-load. The reason for this will be explained later.

There would be great advantage in the use of non-synchronous or induction type generators, but their bad power factor and high cost are too much of a handicap to be overcome, unless the characteristics of such generators can be made much more favorable in large sizes than in present sizes; which unfortunately does not seem to be the case.

Large power houses must be located near large and reliable supplies of cooling water—400,000 kw. would require approximately 1500 second-feet flow—or a large lake whose waters could be used over and over, cooling meanwhile.

If a large enough power plant could not be gotten at one point, several closely adjacent plants might be used, all feeding at perhaps 15,000 volts to some central point for stepping up.

High-tension transformers would be connected directly to the line—with opportunity of uncoupling the line dead—but no standard disconnect switches. Something in the nature of a very long enclosed fuse might be introduced for short-circuit protection.

Synchronizing could be done on either end, but on the low-tension side, preferably in the new power houses, however.

No means for automatic circuit opening would be provided, except at the six points (7 points if the third tap line be supplied) shown in Fig. 1. At points where one line connects with another, appropriate towers should be placed close together and facilities provided so that the lines could be cut apart in case of emergency.

The transmission lines proposed have a capacity power something like three or four times as large for a given voltage as heretofore proposed. They have the further advantageous property that they transmit normal power with a very small loss of energy or voltage and at the same time will permit only a relatively small current to flow in short circuit. This is a rather remarkable and very useful property. For example, either 150 mile tap line will permit only about three or four times full load with one end short-circuited.

This favorable design of line is obtained only by careful proportioning and if the load on the line be properly chosen. The load is so taken that the charging current of the line neutralizes the lagging effect of the load current in the reactance of the line.* The proper load for this balance can be made to approximate the desired load of 300,000 to 400,000 kw. by using such construction as to give an abnormally large line capacity and a correspondingly small line reactance. The use of aluminum and the dividing of the line conductor into several separated cables or parts as described hereinafter, sometimes called the "Split Conductor," have a very marked effect to increase capacity and reduce inductance.† This construction also helps greatly in reducing corona and skin effect. Obviously if load and line constants are so chosen that the inductive effect of the line current is balanced by the capacity current at normal voltage, this balance will be lost when a short circuit occurs and the charging current largely disappears. As the charging kv-a. go down as the square of the voltage, and the inductive effect goes up as the square of the current, the

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*See *Output and Regulation in Long Distance Lines*, by Percy H. Thomas. TRANSACTIONS A. I. E. E., 1909.

†TRANSACTIONS A. I. E. E., 1909, loc. cit.

capacity effect will be negligible on short-circuit conditions, leaving the reactance drop the controlling factor. This explains the favorable behavior of this line to limit short circuits above stated.

The layout of Fig. 1 is favorable, since the general effect of the long interconnecting or main line is to increase the system power factor. Power in large quantities is not regularly transmitted in any one direction over the whole length of the main line but blocks of power will pass in alternate directions in adjacent parts and the drop voltage along this line is practicably negligible. For this reason the capacity will not be neutralized in the main line by the effect of the load current in the line inductance but will be available for power factor correction.

It has just been stated that the drop between any two neighboring large stations on the main line will be very small. This is not only desirable but necessary for when power is to be quickly supplied in reverse direction from normal flow, as for example, on a sudden call for help, this reversal of power could hardly be accomplished practically, if there were a large line drop, since the voltage delivered to the station ordinarily sending out power would be too low to be useful when the direction of power was reversed.

But with a line of very low ohmic drop, like the lines here proposed, transfer of energy backward or forward on the same line may be easily accomplished, for by manipulating the field strengths of the generators on both ends of the line, out of phase or circulating current between the generators can be made to counteract the ohmic drop of the load current. This increases line losses somewhat but line loss would be unimportant at time of emergency and would be small in amount in any case. As a net result of these considerations power can be sent in either direction between two points without changing the voltage at either end, the necessary flow of power being caused by suitable change in the power factor of the passing current.

OPERATION

As already stated, under normal conditions the new power houses would send current into the main lines in a constant rate 24 hours a day and no changes would be made for load or seasonal conditions. In case of accidents to line,—which should be very rare,—it is proposed that a certain amount of automatic protection would be provided. The transformers at the station which receives power from one tap line are divided into two parts, one connected to the tap line, the other to the adjacent main line. In case of trouble on the tap line, the tap line circuit breaker would open leaving the connection between sections only through the banks of transformers, stepping down through one and up through the other, and then the main line transformers or the tap transformers would be automatically cut off on the low-tension side as might be called for, leaving the good line operating. The bad section of the system having been cut out, the good parts of the plant could be operated without the bad part. This would be accomplished by relays.

In testing the insulators and making changes of insulators, it is assumed that this work will be done alive by the method used by Johnson in Georgia. While this sounds chimerical to a person not familiar with what has already been accomplished, I believe it to be entirely possible and safe.

Starting Up. Starting could be readily accomplished on account of the subdivision of the high-voltage lines. Philadelphia will excite the section of the line to Newark by the low-tension switches through one set of transformers. This is too short in length for the charging current to be important and will furthermore merely relieve the lagging power factor of the Philadelphia system without increasing load or current. New York can then synchronize with Philadelphia on the low side. Philadelphia will then excite first tap line on its second bank of transformers. This will call for no increase of current at Philadelphia as before on account of the great excess of load lagging current. By connecting the power house transformers at the coal mines before exciting the line, no synchronizing on high voltage would be necessary. Newark or New York would similarly excite the second hard coal tap line (and the third if installed). The line in Baltimore may then be energized at Philadelphia and Baltimore synchronized on the low-tension side. Similarly with the line to Boston, and the system is operating ready for load. The two tap lines which so far are connected only by step up transformers would be connected to the main line at leisure by the high-tension circuit breakers, the system being already synchronized.

The switches or breakers shown in Fig. 1 can be used for automatic cutting apart of the sections of the high-tension system as already explained. On account of the characteristics of the transmission lines as here laid out, these coal mine stations will a dd little tothe short-circuit currents; and one center like New York will supply little short-circuit current to another as Philadelphia.

Spare Lines. This scheme of Fig. 1 offers no spare lines. It is not believed that the expense of a spare line is warranted. It will be noticed that no accident at any one point of the line can cut out more than a small part of the system except momentarily—nor more than one of the mine power plants. If the system grew it would be possible to add other tap lines at suitable points, which would reduce the power shortage due to any failure without providing any idle lines. It would probably be more economical for the community as a whole to make the best shift possible to a certain extent in case of a serious line failure than to pay the charges and endure the disadvantage of the complication of spare lines and additional switching facilities.

Transmission Line. The conductor has been chosen as consisting of three 600,000-cir. mil. aluminum with steel cores. These would be placed in a horizontal plane, 18 to 24 in. apart each hanging from two strings of insulators suspended at an angle, as shown in Fig. 2. This would reduce side swinging and help clearances. Furthermore, if one insulator string should be broken the other would hold up the conductors.

The principal strain on the towers, according to the usual test specification, is due to a broken conductor causing torsion. To reduce this requirement and so lighten the towers as far as possible—suspension towers should be arranged with the conductors free to slide in the hanger, or held only by a moderate grip, having a maximum positive hold. The conductor will tend normally to equalize longitudinal stresses in the adjacent spans automatically and not to develop unevenness. The one danger is the uneven collecting or falling of ice and sleet. A moderate grip on the hangers such as would be justified by the strength necessary in the tower for other requirements would be sufficient to prevent any permanent slipping except in case of a conductor broken in all three cables.

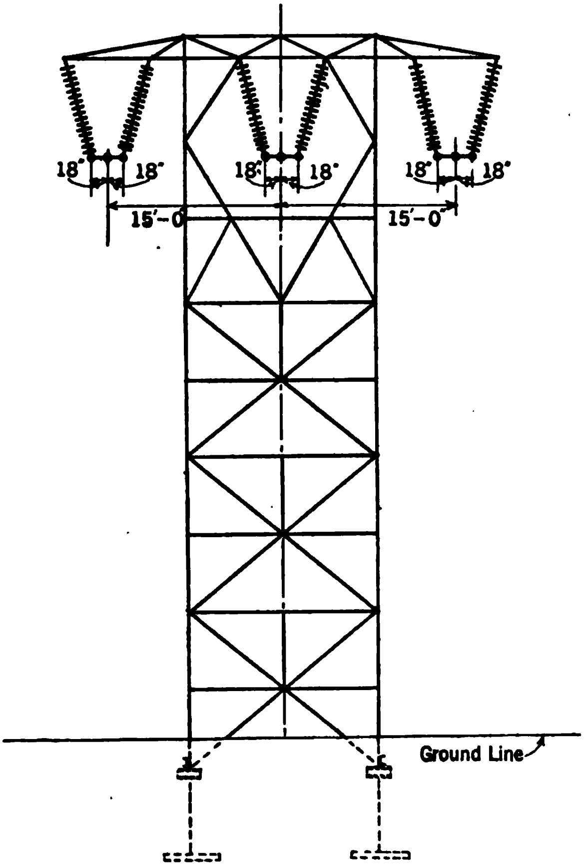

Fig. 2

In this case the conductor will slip in one or more hangers and drop on the ground. The drag on the ground will prevent the line slipping in many towers. Repair in any case will be a big undertaking. The breaking of a single cable of the three of any phase would cause an electrical ground but no slippage.

The assumptions here made are confessedly a departure from standard practises of the present and mature discussion may or may not prove them warranted. The results accomplished will be somewhat less installation cost, with, I believe, no material increase in the danger of line interruption, this being on account of the special hanger or cable clamp assumed.

For most economical construction all ground wires should be omitted. Whether the installation of ground wires would eliminate enough trouble to warrant the expense and complication introduced is a debatable point. It is probable that indirect or induced surges will not be sufficiently severe to affect the lines, since the lightning effects do not depend primarily on the line voltage and since disturbances of this nature on our highest voltage lines seem comparatively rare. Direct strokes of magnitude will not be turned aside by ground wires and the most that can be expected of ground wires would be the elimination of certain disturbances of intermediate severity.

The 150-mile tap lines as shown when delivering 300,000 kw. at 250,000 volts and 99 per cent power factor lagging will have an energy loss of 3½ per cent and a drop of about 5 per cent and the generator power factor unity. It is essential that calculation of this line be made by an accurate formula, such as the hyperbolic function method, taking account of distributed inductance and capacity. Formulas based on localized capacity will give misleading results. The critical condition to cause this failure of the approximate formulas is the one here existing, *viz.*, that the leading and lagging effects of capacity and inductance balance, leaving substantially ohmic losses. With the reactance seven times the resistance the importance of this consideration is evident.

With 400,000 kw. delivered, the energy loss would be 5 or 6 per cent and the line drop about 9 per cent. With 300,000 kw. and a lower power factor the efficiency would go down and the line drop up, but a considerably lower power factor would not give unduly unfavorable conditions. It is here assumed that the existing generating apparatus when coupled up with these lines will be able to supply the necessary out-of-phase current to permit a high power factor in the delivered line load. If not, synchronous condensers can be used as is now customary. With a lightly loaded line the automatic power factor control on the coal mine generators already described will secure a condition such that there will be almost no voltage change between light load and full load at the power house. Since voltage is established at the receiving end there will be no voltage variation there.

The essence of this arrangement is that the coal mine generators shall be set to take automatically a definite power factor at and near full load and yet will take automatically a certain amount of charging current at or near no load. These adjustments should be so made that with full load a slightly lagging power factor shall be maintained automatically at the receiving end of the tap lines. With the voltage regulating apparatus of the distributing system at the receiving end in entire control of the delivered voltage the system will maintain itself in proper adjustment to give the efficient line conditions required—that is approximately unity power factor in the line.

Costs. It having been established in the general terms that the operating characteristics of the super system as proposed are satisfactory and in many ways unexpectedly favorable, the questions of capital and operating costs remain.*

Capital Cost. It may be assumed that a 400,000-kw. power station may be built, under reasonably favorable price conditions for $100 a kw. including overhead necessary for design and construction; that the

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*In this present discussion very free use has been made of the excellent paper, *Problems of 220-Kv. Power Transmission* by A. E. Silver, A. I. E. E., 1919.

transmission lines as proposed, *viz.*, two 150-mile 250,000-volt, 400,000 kw. tap lines and 100 miles of similar capacity line for the central part of the main connecting line, and 250 miles of 200,000-kw. line, can be built at an average cost of $47½ a kw. generated, including overhead. Also that the necessary 250,000-volt step-up and step-down transformers with 250,000-volt switches could be installed for $15.00 a kw., giving the following table:

	Per kw.	
2-400,000 kw. power houses	$100.00	
700 miles 250,000-volt transmission lines	47.50	
250,000-volt transformers and switches	15.00	
Miscellaneous	1.50	$164.00
Total for 900,000 kw., allowing 12½ per cent spare		$150,000,000.00

Taking into account the added cost of real estate, the congestion of fuel handling in the large cities and the complication of handling large blocks of power underground from city stations, a fair estimate of the cost of installing plants to generate this same amount of power at the several large centers, supposed to be supplied, would be about $125.00 per kw., leaving an excess capital cost on account of the transmission scheme of $40.00 per kw., a very small amount for the benefits offered. The value of the 800,000 kw. delivered to the large distributing companies should be in the neighborhood of $20,000,000 to $25,000,000 a year, with present or slightly better conditions.

These cost figures are fairly liberally estimated on the basis of routine construction, probably 10 to 20 per cent should be added to cover additional cost due to the unusual character of much of the work.

Comparing the cost of transmitting the power over transmission lines with the cost of hauling coal by rail, we have to compare the freight on five tons of coal (a sufficient quantity to produce a kilowatt for a year) against the cost of 10 per cent additional coal (required to cover losses in transformers and transmission) plus $3.50 fixed charges on excess capital cost of one kilowatt for the transmission system. That is with coal at $1.00 per ton at the soft coal power station, five and one-half dollars worth would be required for each kilowatt-year. Adding $3.50 excess or fixed charge, gives $9.00. For coal hauling to be on an even basis, coal freight would be 80 cents a short ton. Most of this is on account of interest on the excess investment. This, however, charges the transmission system with 700 miles of line while the actual distance of transmission is only 2 × 150 miles. If the cost of 150 miles, the direct transmission only, is considered, the capital cost may be taken as 12½ per cent less, giving a fixed charge of $2.00 a kw. and a corresponding freight rate of 50 cents a ton. All the above figures are in short tons.

I wish to say that all the above dimensions and estimates are based on the assumption, that the interconnecting system will be characterized by extreme simplicity and that very little so-called flexibility will be provided. This is necessary to reduce costs and to simplify operation and is justified by the completely equipped plants already operating in the inter-connected district.

These approximate figures give an idea of the actual and relative cost and of some of the results that might be obtained by interconnecting the district and supplying from the coal fields the base load for two or three million kilowatt of maximum load now existing. With the future growth of the load and the electrification of the railroads, the St. Lawrence development, or additional coal mine plants, might well add another half million kilowatts in one or two circuits, on a basis more or less on a par with the above, except that if the new power should be hydraulic, it would probably call for a somewhat larger capital expenditure and would show a lower operating cost.

Attention is again called to the fact that the present discussion is based on a hypothetical typical case and not on an actual group of definitely located power houses. Variations of 25 to 50 miles in the length of transmission will, however, make little difference in the net result.

It is to be hoped that as the cost of generating electric power is reduced, partly by the development of such large scale systems as here discussed, partly by the natural development of the art, and partly also by the freer use of local distribution mains, that the use of electric power for heating and cooking and small power will be enormously increased, which will warrant the building of the largest systems, becoming a great source of comfort to the community.

So far no account has been taken of the benefit from the inter-connecting system here under discussion in its opportunity to take advantage of diversity factor in general power consumption nor of its very great advantage in raising the load factor of a railway load, which is at best very low. The relay and breakdown service of the inter-connecting line would be of great benefit. Another advantage accruing is the ability of the system to apply a surplus of power existing in any one system, either temporary or permanent, to any load that may be available. This ability is of much importance in a growing district. Furthermore, the ability to locate very large consumers of power at the most favorable location would be of importance. For example, a large electric furnace plant can now only be placed near some large system, without installing a special power plant. Other similar advantages of the inter-connected system exist, but enough has been said to warrant a careful study of its advantages, disadvantages, possibilities and limitations.

8. W. D. PEASLEE

HIGH-TENSION INSULATORS

The utilization of 220,000 volts or higher for the transmission of power is not a problem that holds any terrors for us as insulator manufacturers.

Due to the size of the cable necessary on such aline and to the fact that it seems rational to design theline for more severe storm loading conditions than standard for an ordinary line, the standard disk suspension insulator as at present manufactured is not entirely satisfactory from a standpoint of mechanical strength.

An insulator for 220,000 volts will have to stand very severe operating conditions as to mechanical stresses and dielectric flux concentrations, and it is very important that certain features of insulator design be followed carefully in any insulator presented for this service. Due to the large size conductors necessary and the spans, the mechanical stress will be considerably in excess of that met with in high-voltage lines now in operation.

The ratio of puncture to flashover in any unit is a very important feature of its design and one which unfortunately has been neglected in most conventional designs of insulators. Due to the effect of transients added to the normal frequency voltage of the line and their time lag or impulse ratio, it is possible to stress the end unit of a string to very high values, and for that reason its actual puncture resisting power should be very high. The puncturing voltage of a disk insulator for such service should never be less than 2½ times the dry flashover voltage of the unit.

In order to reduce corona formation as much as possible, the unit should be designed to reduce the flux concentration and in such a way that the corona forming voltage will be considerably above the operating voltage impressed upon the unit.

For an insulator for service of this kind strength is an important factor. The lines are very important and the ability of the insulator to withstand power arc shock and other events, is of the greatest importance to the continuity of service of the line. We have made a very careful study of the requirements of this insulator.

We are at work at the present time on a disk insulator to meet this requirement. It will be a disk insulator probably of larger diameter than the present design, with a mechanical strength sufficient to meet the strains imposed by the larger loadings necessary in such a line; dry flashover in the neighborhood of 125,000 volts. It will be designed in accordance with the correct theory of surface design between the air and porcelain, and with this design the corona-forming voltage will be very high. The approximate puncture voltage per disk will be in the neighborhood of 300,000 volts, thus securing the high ratio of puncture to dry flashover now recognized as a vital point in correct insulator design.

From these characteristics you will see that this disk will be very well suited for this particular work and our research work has progressed far enough so that we feel confident that by the time such a line is an active proposition, we will be able to undertake the insulation problem. Not more than eight to ten units will be required for the 220,000-volt grounded *Y* line and this will give a string of insulators from 65 to 80 in. in length.

9. A. O. AUSTIN

HIGH-TENSION TRANSMISSION SYSTEMS

It would seem that the super-power project such as Mr. W. S. Murray has outlined, must become more and more of a necessity with each succeeding year. The project would necessarily change in character in time, but the sooner it is started, the greater the saving that would be effected, both as to the economy of operation and the saving of natural resources. The very magnitude of a system of this kind tends to postpone a start on the project, even though the possible economies increase with the size and completeness of the system.

The project must necessarily depend upon an efficient transmission system, and as the cost of the connecting transmission line will be an important factor, it is necessary that we consider the state of the art with a view of adopting a transmission system which will show a maximum economy. Anything effecting the cost of the transmission system should be given careful consideration, for the greater the economy shown the more quickly will a system of this kind be put in operation. Where a project of this magnitude is involved it would seem that a departure from the usual practise is well warranted, particularly where an increase in reliability and a decrease in cost can be effected.

It is possible that at the start, a large part of the system could be connected to advantage with a transmission line not exceeding 110 kv. It is probable, however, that power will be transmitted from large hydroelectric or steam plants located at a considerable distance. The increasing cost of coal will make it highly advisable that the future additional generating equipment be located where the greatest economy can be effected, rather than split up into a lot of smaller, less efficient stations.

The adoption of a high line voltage for the main trunk system would show a saving which might hasten the project several years, consequently it is important that we consider various factors which may be taken advantage of, to increase the efficiency or lower the cost of the line.

The real problem on a large transmission system operating at a high voltage, has usually been that of regulation, and it would seem that the present project is no exception to the general rule. It would seem that the future demands for power in the zone covered by the project would permit the installation of equipment without additional cost, which would give the necessary regulation to insure the success of the system All that would be necessary would be to form a comprehensive plan and install future equipment so as to make the scheme effective.

A transmission line operating at 220 kv. or slightly above will involve large conductors, which would give the system a high degree of mechanical reliability not afforded in the smaller systems. In addition the insulation which would be necessary for the line and equipment, would make the system free from lightning trouble. These two factors would be important in eliminating the necessity for stand-by plants, and should hasten the adoption of the project. The large amount of power would permit of apparatus of a high degree of strength and reliability, so that shut-downs from failure of same would be negligible.

A careful analysis of operating systems would probably indicate that the greatest danger to a system of this kind would be that due to suddenly dropping a large portion of the connected load. This hazard could probably be entirely eliminated by tieing in the main transformers connected to the system, so that they could not be disconnected by the operator unless the voltage of the system was lowered or a defective unit alone be disconnected. Since apparatus would probably be disconnected only in case of failure, a shunted fuse or other inexpensive device could be used to disconnect the defective unit. This would eliminate a considerable cost for oil switches and the freedom from lightning would probably permit the elimination of lightning arresters. This would effect a considerable saving and might advance the installation of the completed project several years.

It is highly desirable to make the main system simple to give the highest degree of reliability and it would seem that in carrying this out that economies could be effected which would more than off-set the increased cost for the higher voltage transformers necessary to permit an economic transmission system for supplying power for a considerable distance. The large conductors necessary for a system such as this would require heavy working stresses for the insulators. The insulator art, however, is such that high ultimate insulators with longer life can be produced at a much lower price than they could previously. It is now possible to procure insulators such that a single string will carry the line and at a cost probably not more than 50 to 60 per cent of that required for a 110-kv. line. From this it will be seen that the cost of installation for a 220-kv. project would probably be less than the cost of some of the 150-kv. lines previously installed. The transmission art has advanced rapidly and many economies could be effected by carrying out a comprehensive scheme, which are not possible in many of the existing systems.

Large additional amounts of power will be required in the district effected by the project and much could be saved by adopting a plan as soon as possible. It would seem that there is no new or untried principle involved, and that by making a survey it would be possible to show the economies which would be effected not only for the present, but for some years to come. It takes considerable time to put large projects into operation, hence it is important that we do not confine our attention entirely to the immediate economies, but look more to the future.

NOTE: For the complete discussion of this paper see JOURNAL of A. I. E. E., August, 1920, page 766.

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PRINTING TELEGRAPH SYSTEMS

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Abstract of Paper

This paper describes the Creed, Murray Automatic, Siemens & Halske, Baudot and American Multiplex Printing Telegraph Systems, and their methods of operation. A discussion of the operating features of the systems described is dealt with under the following headings:—

Accuracy Under this heading it is shown that printing systems are more accurate than Morse working and that the printing system which permits of sufficient time for checking of messages is likely to be the most accurate.

Speed of Service. It is shown wherein the multiplex systems are superior to the high-speed systems.

Operator Output. Under this heading reference is made to actual results, both at home and abroad.

Maintenance. Outlines method proposed for detecting incipient faults in apparatus and securing uninterrupted service.

Line Economies. The advantages of using the five-unit code are discussed.

Flexibility. Examples are given of the flexibility of multiplex systems in linking up a number of cities, by means of forking repeaters, thereby resulting in either line or operator economies.

THE object of this paper is to describe briefly those printing telegraph systems in use today which are designed for handling traffic at over 100 words per minute, and to discuss their operating features. This minimum speed limitation has been chosen as it effectively divides the various printing systems into two classes as regards traffic carrying capacity, each of which classes is of sufficient interest to warrant treatment in a separate paper. Practically all the printing telegraph systems not considered in this paper have a carrying capacity of from 50 to 70 words per minute, and are consequently in a class by themselves. The systems capable of carrying heavy traffic loads may be divided into two classes.

(1) High-speed systems.
(2) Multiplex systems.

By a high-speed system is meant a system in which the sending and recording machines operate at the same speed as the signals are transmitted.

By multiplex, is meant a system in which a plurality of sending machines and a plurality of receiving machines are directly connected in the telegraph circuit and individually operate at a lower speed than that at which the signals are being transmitted.

The systems to be discussed are as follows:

High-Speed Systems. Creed (continental Morse code) tape transmission. Signals received on perforated tape which is fed through a printer. Messages are printed on a tape.

Murray automatic (Baudot or five-unit code) tape transmission. Signals received on a perforated tape which is fed through a printer. Messages are printed in page form.

Siemens and Halske (Baudot or five-unit code) tape transmission. Signals received in printed form on a tape.

Multiplex Systems. Baudot (Baudot or five-unit code) direct keyboard transmission. Signals received in printed form on a tape.

American Multiplex (Baudot or five-unit code) tape transmission. Signals received as printed message in page form.

Several years ago there was a diversity of opinion as to the relative merits of high-speed and multiplex systems, and the advantages of the five-unit code over other telegraph codes were not generally appreciated. The tape-transmitting, page-printing multiplex system using a five-unit code, however, has demonstrated its superiority over all others and one object of this paper is to indicate the reasons for this result.

Perhaps the most elementary form of electric signaling is that in daily use by most executives, where one depression of a push button summons a messenger boy, two depressions means stenographer, and so on, a bell or buzzer being the telegraph instrument of call. To send the 26 letters of the alphabet in this fashion, where one beat represents *a*, two beats *b*, three beats *c*, and so on, would, obviously, be a tedious process. By arranging that some beats are short and some long and forming combinations of these short and long beats, a code can be built up. This is the basis of the Morse code. For actual telegraph purposes the push button is replaced by a suitable key and the bell or buzzer, by a sounder. Fig. 1A.

The sounder is an instrument which requires a considerable amount of energy to actuate it. Consequently, when a line exceeds a few miles in length it becomes economical to introduce for the reception of the signals, a more sensitive instrument than a sounder, namely, a relay, and to cause the sounder to be actuated locally as shown in Fig. 1B. The relay alone is unsuitable because the noise produced is not sufficiently loud to be heard with distinctness at all times. With such a system, telegraph signalling can be carried on over several hundred miles provided the insulation of the line is good. Under wet weather conditions, however, the strength of the operating current at the receiving end of the line is reduced and the relay must be re-adjusted.

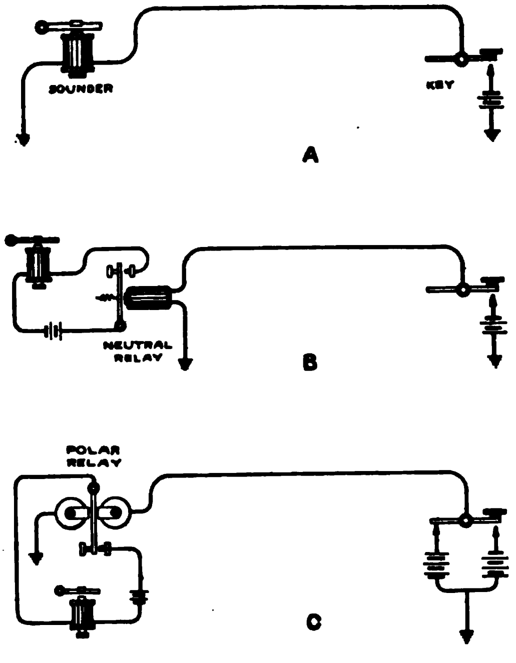


FIG. 1

If, instead of a relay of the simple "neutral" type, and a single-pole battery, there be used a polarized relay and a two-pole battery, Fig. 1C, the need for readjustment of the relay to meet variable insulation conditions is eliminated. The armature of the polar relay is acted upon in both directions. An impulse from the negative or "marking" battery causes the tongue of the polarized relay to make contact with its right hand or marking stop, whilst an impulse from the positive battery reverses its movement.

The positive or "spacing" battery takes the place of the retractile spring in the neutral or non-polar relay. As the current from both batteries is affected equally by variations in the line insulation, this is the equivalent of automatically varying the strength of the tension spring of a neutral relay to the exact degree of adjustment necessary.

Fig. 2 represents oscillograph records of signals transmitted over an artificial line simulating a 400-mile open-wire circuit. *A* shows the signals repeated by a polar relay, and *B* the signals repeated by a neutral relay when the insulation of the line was perfect. *C* and *D* show the repeated signals by the polar and neutral relays respectively, when the insulation resistance of the line is approximately one quarter of its conductor resistance. No readjustment of the relays was made.

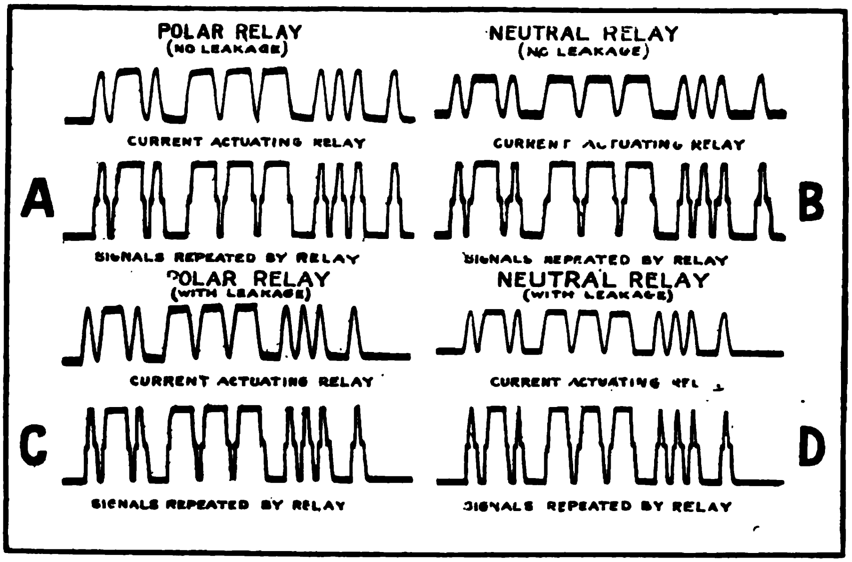


Fig. 2

It will be seen that the repeated signals in the case of the polar relay are the same under the two conditions, whilst in the case of the neutral relay reduced current has produced "pin-pointed" dots. The superiority of the double current or polar method is evident.

A simple circuit of this type, when operated by hand Morse, provides for the transmission of one message at a time at a speed of approximately 30 words per minute. The next advance is to increase the carrying capacity of the circuit by transmitting two messages simultaneously—one in each direction. This was achieved by the invention of the duplex system.

By providing a polar relay having two equal windings, and an assembly of resistances and condensers to simulate the real line, at each end of the circuit, as shown in Fig. 3, signals may be transmitted from station *X* and repeated by the relay at station *Y* and signals from *Y* repeated by the relay at station *X* without interference with each other. The outgoing currents at each station divides equally at the common terminal of the home relay, the paths via the line *L* and the artificial line *A L* having equal impedances. Passing through the two balanced windings of the home station relay in this manner, they produce no effect on it. When the current reaches the distant station, it passes through the line side winding of the relay *R* and then divides between the two paths to ground, one via the transmitting key and battery, the other through the artificial line *AL*. That part of the current through the artificial line winding of the relay passes in the direction to assist the current through the line winding. The relay, therefore, is actuated in accordance with the polarity and duration of the incoming signals.

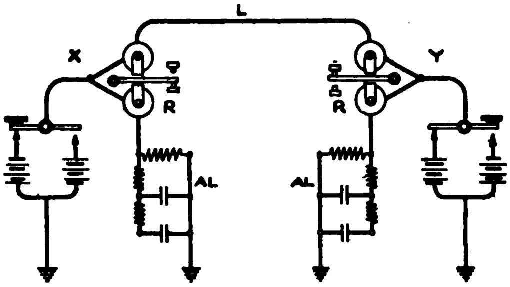


Fig. 3

Except in the case of the longest duplex lines, it is possible to transmit signals at a speed much in excess of the fastest hand operation. In order, therefore, that a line shall be made to carry an increased load, an automatic transmitter for sending and a high-speed tape-recording instrument for reception, are used. Wheatstone designed such instruments in 1858 and these have undergone improvements from time to time, until today, they are capable of operation at a speed of over 400 words per minute. That is to say, the limit of speed of the apparatus is over 400 words per minute. The complete Wheatstone system comprises a perforator, an automatic transmitter and a tape recorder. The original type of perforator has three keys which are struck by small mallets held in the hand. The left hand or dot key perforates two signal holes and a center feed hole in vertical alignment across the tape. The middle or space key perforates a center feed hole, and the right hand or dash key perforates four holes, two being signal holes and two being feed holes. This type of perforator is rapidly being replaced, however, by modern keyboard perforators of which the Kleinschmidt, is probably the best known.

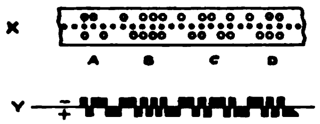


Fig. 4

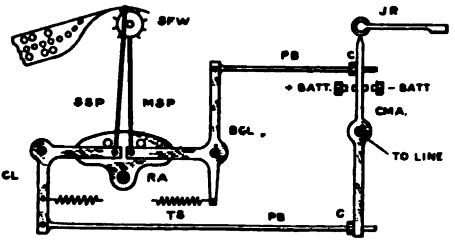


Fig. 5

Fig. 4x shows a piece of tape with the letters *A*, *B*, *C*, *D* perforated.

The automatic transmitter for operation in conjunction with this tape, (Fig. 5) has two vertical selecting pins, each pivoted to a bell crank lever *B C L*. The two bell crank levers control the movements of the contact-making arm *C M A* by means of the push bars, each of which has a collar or flange *C*. Each push bar extends beyond the collar, through a hole in the contact lever which acts as a guide or sliding support for that end of the push bar. When pushed beyond the dead center of the jockey roller, *J R*, the jockey roller ensures firm contact being made and also retains the contact lever in the position placed whilst one pushbar is being withdrawn and the other advanced.

The upward movement of the selecting pins *S S P* and *M S P* is controlled by the rocking arm *R A* and by the perforations in the tape. Power is transmitted to the rocking *R A* and the sprocket feed wheel *S F W* from an electric motor and these units of the driving mechanism are so geared that the marking selecting pin *M S P* rises so as to strike the tape exactly opposite a feed hole. When no signal hole occurs the selecting pin is not free to rise, consequently, the contact-making arm remains in the spacing position.

When a signal hole does occur the "marking" selector pin *M S P* passes through it, and the push bar is forced to the right by the tension spring *T S*. As the feed wheel rotates, moving the tape along, the rocking arm releases the spacing selecting pin *S S P* and withdraws the marking selector pin *M S P*. The upper end of the spacing selector pin is so guided as to engage with the signal hole in the lower row on the tape. When a signal hole occurs, the spacing selector pin passes through it and the lower push bar *P B* moves to the right causing the lever *C M A* to make contact with the positive battery contact point.

In the case of a dash or a space, both selector pins are prevented from rising once owing to the absence of signal holes. During these periods, the contact-making arm remains at rest.

During a dash it rests against the negative battery contact and during a space against the positive battery contact. The signals passing to the line are shown at *Y* in Fig. 4.

For a speed of 120 words per minute each selecting pin is freed to rise through the perforations in the paper tape at the rate of 54 times per second, the rest of the mechanism operating at a like rate. Obviously, the adjustments in such a machine must be extremely accurate.

This method of transmission is used in the Creed printing system which was designed to supplement the Wheatstone system and thereby to effect labor economies and reduction of delay at the receiving end, two features in which the Wheatstone system compares unfavorably with printing systems.

CREED SYSTEM

This system has been in commercial use in England and several other countries for the last 12 years. Actually it may be described as a development of the Wheatstone system. So far as the sending station is concerned there is no change from the apparatus and method of operation used in the Wheatstone. The messages are first handled by operators at keyboard perforators who prepare them in the form of perforated tapes. These operators are generally seated close to the transmitter, and as they complete each length of tape containing two or three messages, pass it forward to an operator who feeds the tape into the Wheatstone transmitter.

At the receiving station the incoming signals are made to actuate a "receiving perforator" which reproduces a perforated tape identical in all respects to that which is fed into the Wheatstone transmitter at the sending station. This perforated tape is then fed into an automatic printer which prints the messages in bold Roman characters on a paper tape.

The general practise is to have the receiving perforator and printer mounted conveniently near each other so that the perforated tape is fed into the printer with minumum delay.

As the printed tape comes from the printer it is fed through a small gumming device, or if gummed tape be used, over a moistening roller, thence to the gumming operator. The duty of this operator is to paste the tape neatly on the standard telegraph blanks. Where the speed is low, such as on circuits feeding or fed by a submarine cable this operator checks the messages as well, but on circuits operated at 100 words per minute the gumming operator is able to perform only the work of pasting; the checking being done by a second operator. This system is an improvement over the Wheatstone system in that it eliminates the laborious process of tape translation.

Receiving Perforator. Fig. 6 shows in diagrammatic form the construction of the Creed receiving perforator which reproduces a perforated tape identical with that fed into a Wheatstone transmitter at the distant end of the line.

A represents a polarized relay of the British Post Office type. No electrical contacts are used, and in place of the contact-making tongue, a light tongue *R T*, having a small slide valve *S V*, at its free end, is provided. Movements of this valve *S V* control a supply of compressed air to the small relay engine *R E*, causing its piston to move from side to side. The rod of this piston carries two circular disks which act as valves controlling the air inlets to a larger double-acting piston (Fig. 7). This larger piston has a rod projecting from

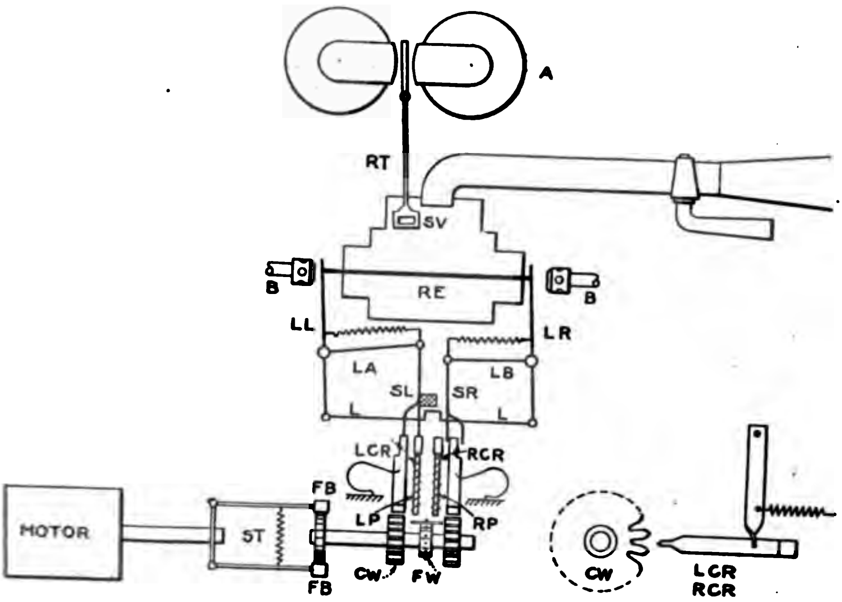


Fig. 6

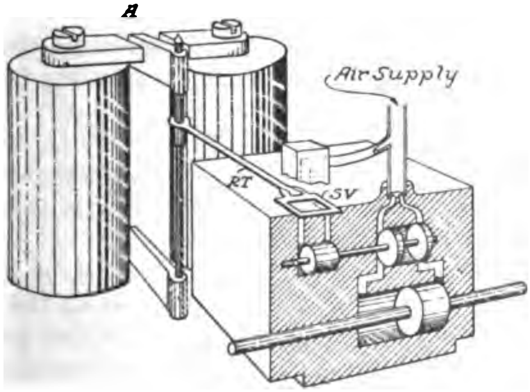


Fig. 7

each side of it, the rods projecting through the cylinder cover, and acting upon the bell crank levers *L L* and *L R*, (Fig. 6). The adjustable buffers *B* regulate the travel of the piston and piston valves. The arms *L A* and *L B* transmit the movement to the striker arms *S L* and *S R*, the free ends of which are bifurcated for the purpose of thrusting against the punches *L P* and *R P* and corrector rods *L C R* and *R C R*. The punches and corrector rods are mounted and guided in a separate block which also carries the die-plates, feed wheel, and corrector wheels. The ends of the corrector rods have flattened pointed ends which enter between the teeth of the corrector wheels. The punches and correctors are the parts most subject to wear and this arrangement provides for the quick replacement of the punch block, there being generally two such blocks provided with each machine. Springs are provided for restoring the punches and corrector rods as shown in Fig. 6.

The paper tape which is previously center-holed is fed up between the die-plates past the punches and engages with the small sprockets of the feed wheel *F W*. The feed wheel and correcting wheels *C W* are firmly mounted on the same spindle which is rotated by an electric motor. The corrector rods move in advance of the punches and hold the corrector wheels and the feed wheel in the correct position so that the perforations made by the punches shall be in correct vertical alignment with the feed holes. This ensures that the perforated tape will properly actuate the printer. As the tape is momentarily stopped at the instant of perforating, a friction drive is introduced between the motor and the feedwheel spindle. The exact degree of tension is secured by regulating the position of the spring *S T* along the two rods carrying the friction clocks *F B*.

Connecting the ends of the bell crank levers *L L* and *L R*, is a link *L* on which is a tappet piece which pushes on the striker bars laterally, and causes them to slide clear of the corrector rods and punches at the right moment.

As the two signal holes forming a dot must be in vertical alignment with their corresponding feed hole on the tape and a small interval of time elapses between the two perforating operations, during which time the tape has traveled a little, the right-hand correcting wheel and the right-hand punch are given a lead equal to one-half tooth of the corrector wheel.

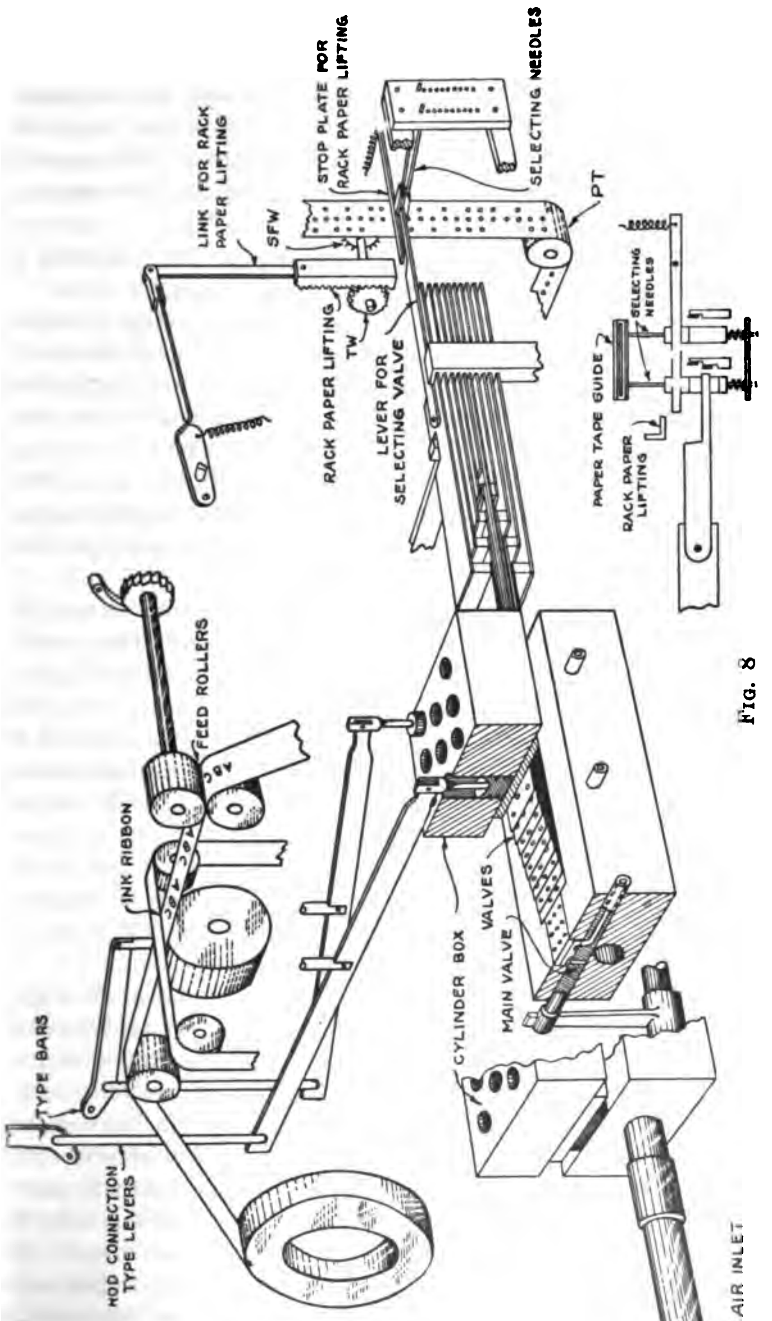

TYPE BARS
ROD CONNECTION TYPE LEVERS
INK RIBBON
ABC
FEED ROLLERS
LINK FOR RACK PAPER LIFTING
SFW
STOP PLATE FOR RACK PAPER LIFTING
SELECTING NEEDLES
PT
RACK PAPER LIFTING
TW
LEVER FOR SELECTING VALVE
SELECTING NEEDLES
PAPER TAPE GUIDE
RACK PAPER LIFTING
CYLINDER BOX
VALVES
MAIN VALVE
AIR INLET

FIG. 8

Air pressure at approximately 30 lb. per square inch is required to operate the air engine.

The speed at which the paper tape is fed forward may vary within fairly wide limits as the correcting rods will advance or retard it at every signal. On one occasion the writer has witnessed the machine perforating correctly at 140 words per minute while the tape speed was adjusted for 100 words per minute.

When receiving reversals at 120 words per minute, this machine perforates 108 holes in succession per second.

Printer. The perforated tape is next fed through the Creed printer, which reproduces the message in Roman characters on a paper tape, suitable for pasting to a telegraph form. The received perforated tape can be run through the printer several times and, in addition, it can be utilized to re-transmit the signals to any other line or lines equipped with Wheatstone apparatus. Pneumatic pressure is used for the printing and a small electric motor supplies the necessary power to drive the camshaft.

The perforated tape is fed forward, letter by letter, in a guideway, in front of a series of ten pairs of selecting needles, one needle of each pair being associated with one of a series of ten slide valves. These valve plates, made of thin sheet steel, can be made to occupy one of two positions, selected and non-selected, so that a number of different combinations can be set up. Each valve plate has a number of holes or perforations in it and when a combination has been set up, a passage exists through the ten valve plates to one of the small cylinders in the cylinder box. Compressed air can thus be admitted forcing up the small piston which acts on the end of a lever connected to one of the type bars.

A diagrammatic sketch of the printer is shown in Fig. 8. The perforated tape *P T* is fed by means of a sprocket feed wheel *S F W* between two vertical guide plates which have ten pairs of holes positioned to permit of the entry of the ten pairs of selecting needles. Should the perforated tape have a continuous series of dots all the selecting needles would have entry. Particular combinations of perforation in the tape will permit only certain of the selecting needles to enter, and it is by this means, that the spacing and letter selections are made. The feed wheel is mounted on the same spindle as the toothed wheel *T W* which is rotated by the vertical feed rack actuated by a cam on the main shaft. An examination of the perforated tape, Fig. 4, will show that between each letter combination there is a clear blank space which would block the entrance of one pair of selecting needles in the same horizontal plane also that the lower row of signal holes alone is sufficient to differentiate between the various letters, figures and punctuation marks; provided that the spacing between the letters is recognized. These are the two underlying features upon which the selecting mechanism of the printer is designed.

If a space occurs in the tape neither of the pair of selecting needles opposite this position will find entry through the guide plates, and will consequently permit the space lever which they control to remain in position to block the downward movement of the feed rack. If a space position occurs opposite, say, the sixth pair of needles, the rack will move down to that position so that on its upward movement it will cause the toothed wheel and feed wheel to rotate and carry forward the tape an equal distance. If the space occurs opposite the second pair of needles, which would be the case for the letter *e*, then the tape is fed upwards two space holes.

If one or both of the needles of a horizontal pair finds entry through perforations in the tape, the space lever is moved clear of the feed rack.

The valve selecting levers are controlled by the selecting needles which engage with the lower row of signal perforations only. When one of these levers is pushed forward by a selecting needle, it is in position to be moved laterally by the rack which has a lateral motion imparted to it at the right moment, by a cam.

Although selecting needles below a space position enter the signal holes of the next or part of the next letter combination on the tape, and carry with them the valve selecting levers, these levers are not acted upon by the rack because it does not descend so far.

Consequently, only those selected levers above the space position are moved to the left and they carry with them the slide valves to which they are fixed.

As soon as the slide valves have been moved they are compressed between the two stout plates forming the slide-valve chamber. The main air valve is then opened by cam action, admitting compressed air to the slide-valve chamber, whence it finds passage through the ten valve plates by the one hole made available by the particular valve setting, and forces up the small piston which actuates the corresponding type lever.

The slide-valve chamber is then opened, and the valves are returned to normal position. Meanwhile the paper-lifting rack is drawn up by cam action and in its upward moment it engages the toothed wheel *T W* so that the paper tape is drawn up the proper distance for the selection of the next letter. The path of movement of the rack forms a narrow rectangle, the length of which is determined by the length of space occupied on the tape by the letter combination printed.

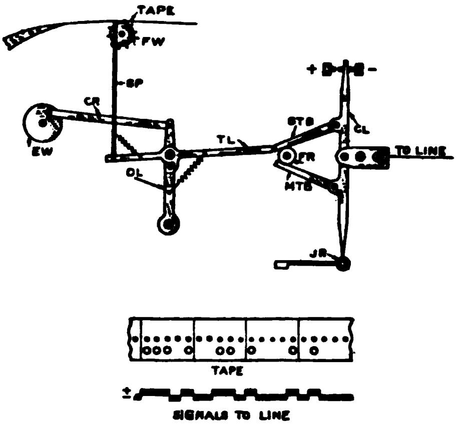


Fig. 9

It will be seen that the differential feed of the tape involves a selecting mechanism which is not required in the case of systems using an equal unit code, which are described later.

This printer has been operated in laboratory tests up to 130 words per minute, but for commercial use it is found best to work it at a speed not exceeding 100 words per minute.

The air pressure required is the same as for the perforator.

Murray Automatic System

The Murray Automatic System is not widely used, but where it has been installed it has rendered good service.

At the transmitting station the method of operation closely resembles the Creed. A number of operators perpare the messages on paper tapes by means of keyboard perforators. This tape has feed holes identical with those on the Wheatstone tape, but with signal holes in alignment with the feed holes, on the lower side only. The perforated tapes are passed to the transmitter operator who feeds them into the transmitter.

At the receiving station the messages are received on a receiving perforator which produces a perforated tape identical with that fed into the transmitter at the sending station. This tape is then passed into an automatic printer which prints the messages in page form ready for delivery. In addition to the advantage of the shorter code used this system eliminates the gumming process as used in the Creed.

Fig. 9 shows the appearance of Murray automatic tape. In this case, every letter and symbol occupy the same length of tape (one-half inch) so that a differential tape feed feature is eliminated in the perforator, the tape being fed forward through the perforator letter by letter. Only five punches are needed, all combinations being formed by various groupings of these.

The vertical marks on the tape after every fifth hole are for the purpose of enabling the operators to read the signals.

The transmitter is driven by a phonic wheel motor which is controlled by a vibrating reed. The unloaded reed is designed to vibrate at a certain speed and by loading it with weights, which can be fixed at any position along the reed, the speed can be reduced. The speed of the transmitter can be adjusted by this means over a considerable range.

In the Murray automatic transmitter (Fig. 9) only one selecting pin *S P* is needed. The tape feed wheel *F W* is coupled direct to the phonic wheel motor whilst the eccentric wheel *E W* is geared to rotate at ten times the speed of the feed wheel, that is, once for each sprocket of the feed wheel. For each rotation of the eccentric wheel *E W*, the oscillating lever *O L* makes one complete movement, carrying with it the thrust lever *T L*. If there be no signal hole in the tape, the selecting pin *S P* does not rise so that the thrust lever *T L* strikes the upper or spacing thrust bar *S T B* and carries it past the dead center point of the jockey roller *J R*. The contact lever *C L*, therefore, makes contact with the right-hand battery contact point.

At the next step of the feed wheel *F W* the eccentric wheel *E W* makes another complete revolution and the thrust lever *T L* again moves to the right. If no signal hole exists movement of the thrust lever *T L* will be identical with its previous movement. The contact lever, however, remains in the position set by the previous thrust, due to the jockey roller *J R*.

If the next step of the feed wheel brings a signal hole in line with the selecting pin *S P* the thrust lever *T L* will be tilted downward and will strike the lower or marking thrust bar *M T B* and cause the contact lever to make contact with the other pole of the battery.

With this transmitter operating at a speed of 120 words per minute the pin selecting mechanism is required to operate at the rate of 60 times per second, that is, the eccentric wheel *E W* makes 60 revolutions and the thrust lever oscillates 60 times per second.

Murray Automatic Receiving Perforator. The circuit arrangements of the Murray receiving perforator are shown in Fig. 10.

The vibrating reed *V R* is provided with a weight *W* fixed at some point along the reed so that the rate of vibration of the reed will be slightly faster than twice the frequency of the incoming signals. The reed is kept in constant vibration by means of the local battery *L B*, the circuit being via the contact *V*, the vibrating magnet and the back or front contact of the governing relay. The amplitude of vibration is limited by the buffer springs *B S*. If the amount of energy acting upon the reed be increased, the rate of vibration increases. Conversely, if the amount of energy be reduced, the rate of vibration is reduced. The punching and governing relays are connected in series from the *M* or marking contact of the line relay. The punching magnet is operated when the tongue of the punching relay is pulled up and the contact *P* on the reed is closed. Consequently, if the tongue of the line relay remains in the marking position the punching magnet will be operated each time the vibration of the reed closes the contact *P*. The spacing magnet is operated, when the *S* contact of the reed is closed. If no signals are passing the spacing magnet continues to feed forward the tape, but after about a foot of blank tape has passed an automatic switch *A S*—a small mechanical device—opens the circuit of the spacing magnet *S M* and the tape is stopped. The first marking signal closes this switch and it remains closed so long as signals are passing. The actuation of the spacing magnet operates the escapement mechanism on the motor drive shaft *M D S*. It will be obvious that the punching and spacing magnets

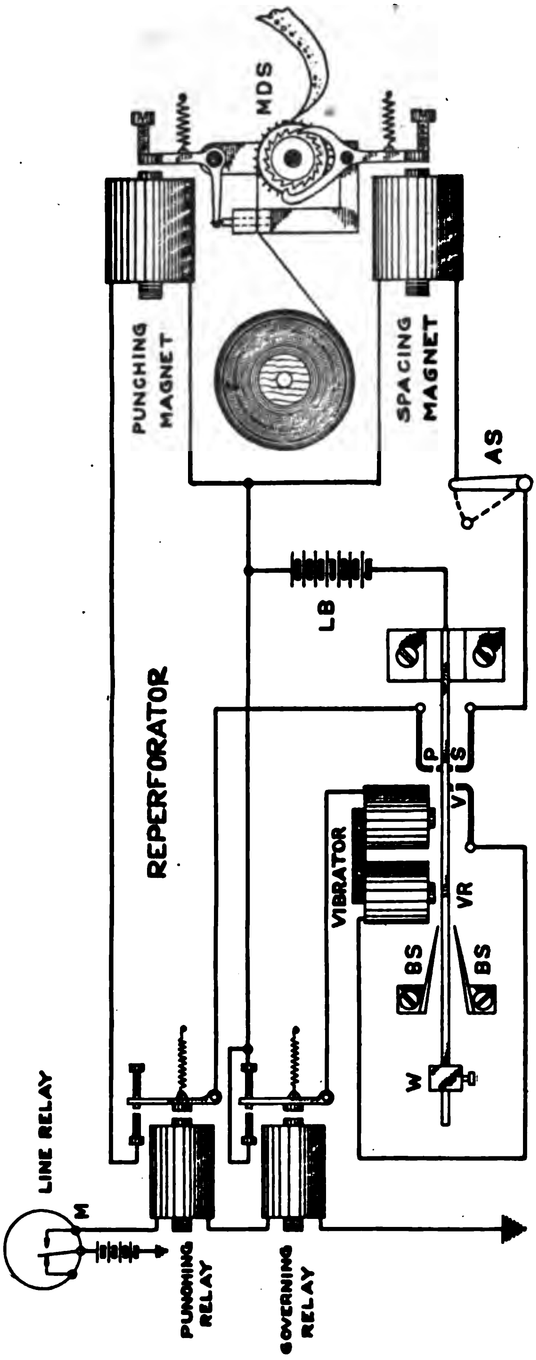

MDS
PUNCHING MAGNET
SPACING MAGNET
AS
LB
REPERFORATOR
P
S
V
VIBRATOR
VR
BS
BS
W
LINE RELAY
M
PUNCHING RELAY
GOVERNING RELAY

Fig. 10

work alternately, the tape being fed forward when the punch has been withdrawn.

Speed Control of Reed. The frequency of the reed is adjusted to be about 2 per cent or 3 per cent faster than twice that of the incoming signals and this frequency will be maintained so long as the tongue of the governing relay is stationary or moves from one contact to the other at the same instant that the *V* contact on the reed is closed.

Actually, the incoming signals cause the governing relay to be operated slightly out of phase with the closing of the contact *V* on the reed, so that the amount of energy passing through the vibrating magnet is reduced until the contacts are again simultaneous. Consequently, the frequency of the reed is reduced momentarily. When signals are passing, speed correction of the reed by this means may take place about two or three times per letter signals if such be necessary, but generally the speed of the reed is such that one correction per six or seven letters is sufficient to maintain unison.

Printer. The latest type of Murray high speed printer comprises six horizontal slotted combs, five of which have a selecting needle fitted on one end for engaging with perforations in the tape. The sixth comb has no selective needle. Mounted in front of the slotted combs is a series of vertical selecting bars. A small electric motor drives the main shaft which extends across the whole length of the base and all movements are controlled by a series of cams mounted on this shaft.

The tape is fed round a wheel which acts both as a feed-wheel and a guide for the selecting needles.

The type bars are only 2 inches in length and in order to have them all strike at the same position on the platen they are mounted in a circle. Connecting rods are led down from the typebars to a series of hooks one of which is positioned in front of each vertical selecting bar.

The tape wheel is advanced against the selecting needles so that those which find a blank in the tape are pushed to the left leaving the selected needles with their slotted combs in place. The next movement in the cycle of operation is the withdrawal of the restoring bar permitting all vertical bars to move forward, but only one vertical bar finds a free path to move, the others being stopped by the blank portions of the slotted combs. The bar so selected, pushes forward the hook immediately in front of it, placing it in the path of the striker bar. As soon as the striker bar comes down, and engages with the hook the restoring bar pushes back the vertical selecting bar so that the setting up of the next letter combination may be proceeded with. This overlapping of the cycle of operation is a distinct advantage permitting a slower operation of moving parts and consequently less wear and tear.

With five slotted combs only 31 combinations can be made, but by adding a sixth comb, which can occupy one of two positions, the number of selections can be doubled. Only 56 selections, however, are required. The sixth bar is moved when the "figure shift" signal occurs in the tape. The vertical bar selected by this signal combination causes the sixth slotted comb to slide in the same way as do the five selecting combs and in this position all the vertical selector bars associated with lower case characters are rendered non-selectable whilst the vertical bars associated with the upper case characters may now be selected. The "letter shift" signal restores the sixth slotted comb to normal.

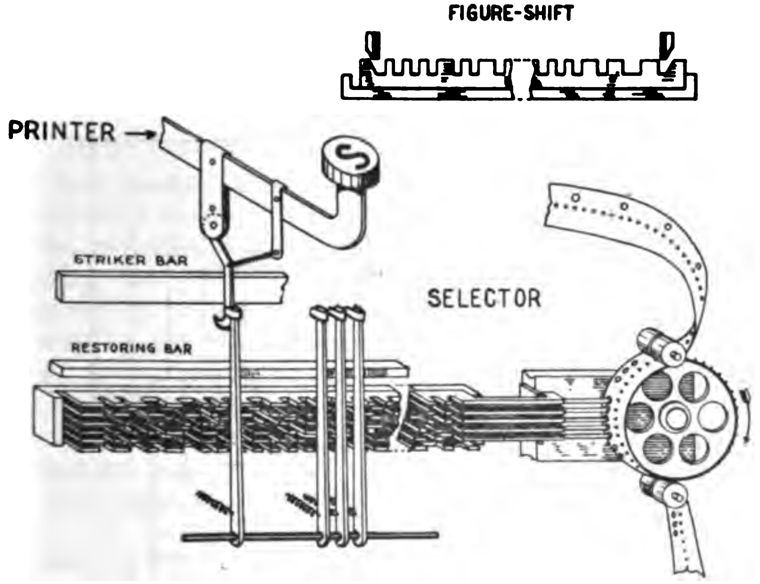


Fig. 11

Fig. 11 shows the selecting mechanism of the old Murray automatic printer, which does not differ fundamentally from that of the new high-speed printer. In the new machine, however, typewriter keys are not provided.

SIEMENS & HALSKE HIGH SPEED SYSTEM

The Siemens and Halske is another type of high-speed system. At the sending end the operations are practically identical with those of the Creed and Murray Automatic in having several perforating operators and one transmitting operator. At the receiving station the perforated tape is eliminated, the messages being printed on a tape direct from the line signals. This tape is pasted on the message blanks.

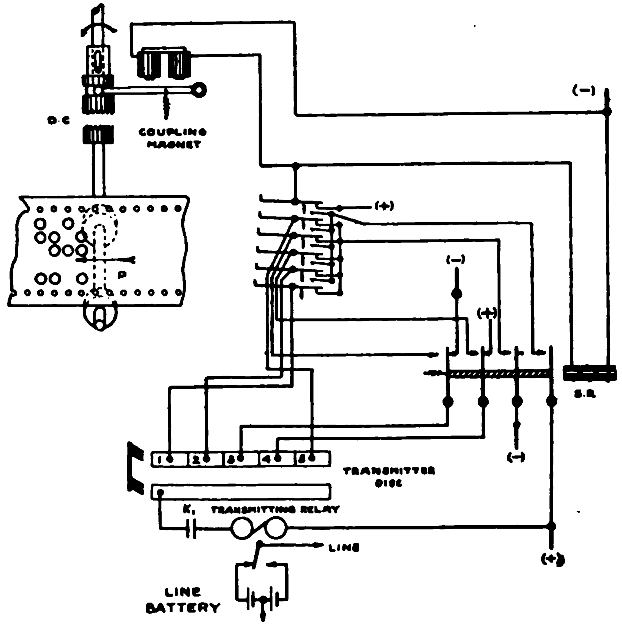


FIG. 12

This system shows a further development in that the intermediate process of perforating a tape at the receiving station is eliminated, but the gumming process is continued.

Fig. 12 shows the high-speed transmitter, used in the Siemens and Halske printer system. The tape in this system is perforated on a keyboard perforator. The five-unit code is used, so that a maximum of five perforations is required per letter combination. The tape, however, differs from the Murray Automatic tape in this respect, the signal perforations are made across the tape instead of along it, so that a space of only 0.1 in. per letter is required, a wider tape being used.

With this form of perforated tape the transmitting mechanism must include means for securing the proper sequence of signals. This is achieved by using a rotating distributor which consists of two metal rings over which contact brushes pass at the rate of one revolution per letter. One ring is solid, the other is divided into five equal segments.

Fig. 12 shows the general arrangement of the transmitting circuit. The perforated tape has two rows of feed holes for ensuring its accurate movement over the six contact fingers which are located below the tape. When blank tape is being fed through the transmitter, the centrally pivoted fingers are held down by the tape so that they rest against their upper contact stops.

When no tape is in the transmitters all fingers are free to rise, and the sixth or top finger then closes a circuit through the coupling magnet and the switching relay *S R*. Actuation of the coupling magnet withdraws the clutch *D C*, thereby stopping the transmitter. The function of the switching relay will be referred to later.

In rising, the five selecting fingers which engage with the signal perforation in the tape, make contact with their lower stops. The upper contact stops are connected to the positive pole of the local transmitting battery and the lower stops to its negative pole. The five selecting fingers are connected to the five segments of the transmitting ring. The selecting fingers are not in lateral alignment; the second finger is 0.020 in. to the left of No. 1, No. 3 is 0.020 in. to the left of No. 2 and so on, so that as the tape is drawn over them they engage with the perforations in sequence and No. 1 engages immediately after No. 5 has engaged with a perforation of the preceding lateral group of perforations. The distributor brushes are so geared that at the moment finger No. 1 engages with a perforation the brushes start to sweep over segment 1; when finger No. 2 engages with a perforation the brushes start to sweep over segment 2 and so on. The solid ring which is in connection with the segmented ring through the rotating brushes is connected through a condenser to the transmitting relay. Consequently, when a perforated tape is inserted and the selecting fingers are set up in different combinations,positive and negative impulses will pass through the transmitting relay and actuate it.

When no tape is in the transmitter the transmitting relay would remain at rest, sending out either a steady positive or negative current. As the system is a synchronous one and the maintenance of synchronism between the two ends of the line is dependent upon the reception of signals, provision is made for sending a signal once per revolution in the following manner:

The switching relay is actuated when there is no tape in the transmitter, as has already been described. The actuation of this switch removes the normal battery connection to the selecting fingers and connects negative battery direct to segment 3 and positive battery to segment 4 of the distributor.

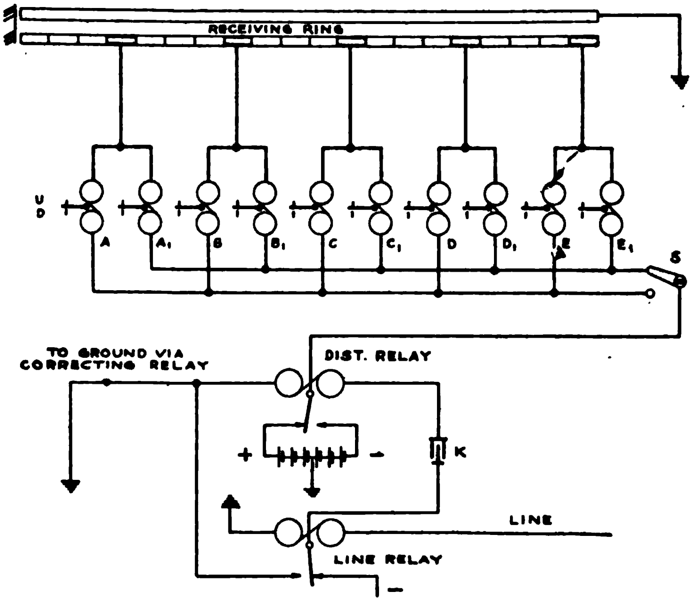


Fig. 13

Consequently, the transmitting relay will send out a reversed pulse each time the brush sweeps over segment 3.

Receiving. The receiving apparatus of the Siemens and Halske system comprises a distributor with rotating brushes which make one revolution per letter.

One noteworthy feature of the system is the use of storing relays which provides an overlap arrangement.

Fig. 13 shows the line and local relays with the receiving ring of the distributor and the storing relays. The storing relays are polarized and are arranged in pairs as shown. The switch *S* is reversed mechanically at each revolution so that relays *a*, *b*, *c*, *d*, and *e* are connected in sequence to the tongue of the distributor relay for one revolution and relays *a′*, *b′*, *c′* *d′* and *e′* for the next.

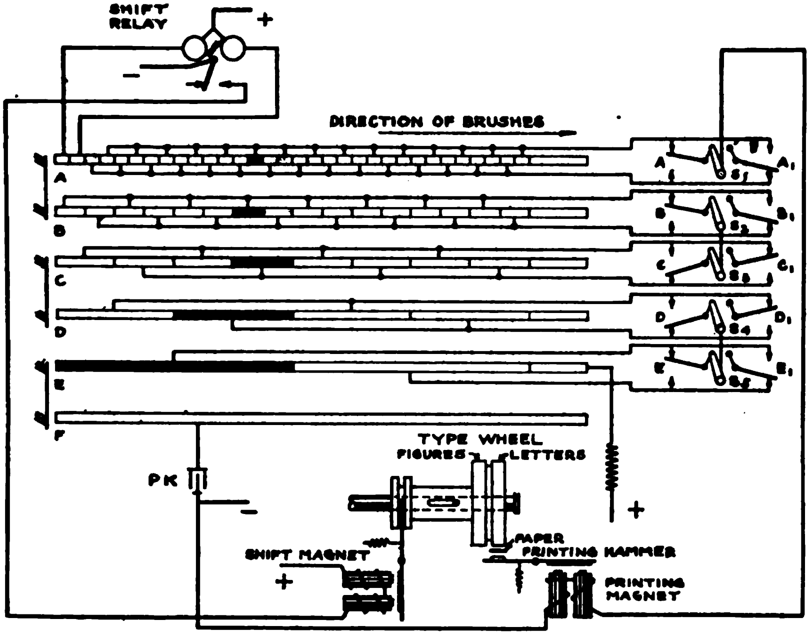


Fig. 14

The distributor relay is actuated from the tongue of the line relay through the condenser *K* and follows the movements of the line relay. During the reception of signals forming one letter combination the brush makes one complete revolution. Assuming the armature of the relays move to the "up" or *U* position when a positive impulse passes, and to the "down" or *D* position when a negative impulse passes, then a letter combination, say + + − − +, would cause the armature of relays *a b* and *e* to move to the up position and the tongues of relays *c* and *d* to the down position. Being polar relays the tongues remain in the position set until reversed by an impulse of opposite polarity.

At the next revolution the next combination would be set up on relays, *a*′, *b*′, *c*′, *d*′ and *e*′. Meanwhile, the set-up on the relays *a*, *b*, *c*, *d*, and *e* is used to effect the selection and printing of the previous letter.

Fig. 14 shows the translator portion of the distributor. It comprises six segmented rings. These are connected to the contacts of the storing relays shown in the preceding figure. The tongues of each pair of relays are connected to switches S_1 S_2 S_3 S_4 and S_5, which are reversed once per revolution, the reversal taking place whilst the brushes sweep over the short blank portion of the rings.

The brushes are mounted on the same shaft as the typewheel and the letters are arranged around the periphery of the typewheel to accord with the code used.

As the brushes sweep over the blank portions of the rings, the condenser *PK* is charged by being connected across the + and − poles of the battery. The operation of the translator will readily be understood by following the selection of one letter. Assume that the switches are in the position shown and that the tongues of relays *a*, *b*, and *e* are "up" and those of *c* and *d* are in the "down" position as shown in the figure. The segments which are involved in this particular selection are shown in black. It will be seen that when the brushes reach segment 14 of the first ring a circuit is provided for the discharge of the condenser *PK* through the printing magnet. This magnet is actuated, causing the paper tape to be impressed against the typewheel.

The typewheel carries two series of characters, one being letters, the other figures and the usual punctuation marks. In the normal position the letter series is in position for printing and by sliding the wheel along its shaft the figure series is brought into the printing position.

The positioning of the typewheel for printing letters or figures is effected by a shift magnet which is controlled by a differentially wound polar relay. Two of the 32 segments of the first ring are connected to the windings of this polar relay so that the position of its tongue is determined by the "figure shift" and "letter shift" signals which are transmitted over the line.

Maintenance of Synchronism. Between each of the five short receiving segments of the receiving ring, there are three correcting segments as shown in Fig. 15. As the signals are being received by the line relay, the brush which sweeps over this ring should be on one of the *C* segments, at the moments when the line relay tongue moves from one contact to the other. Under this condition, the center portion of each signal impulse is utilized for the actuation of the storing relays. If the brush is on an *R* correcting segment, it is in advance of its correct position and must be retarded, if on an *A* segment it means that the motor has been running slow and must be accelerated.

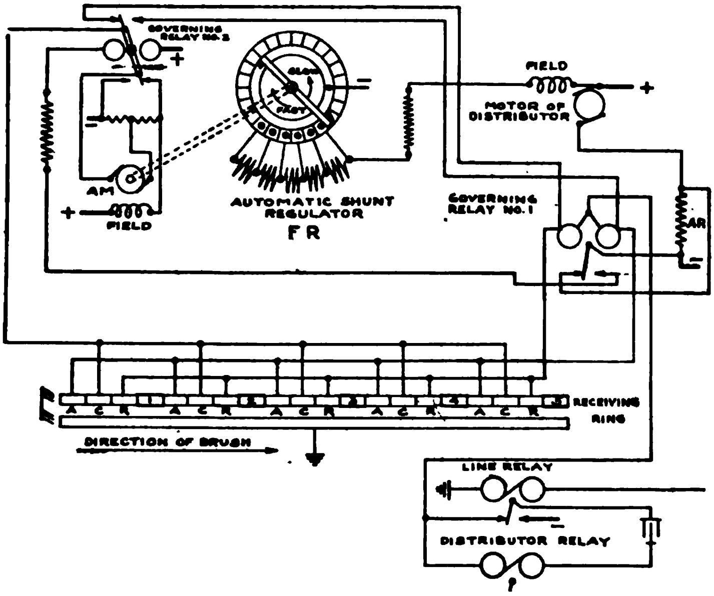


Fig. 15

The speed of the motor which carries the brushes is governed by a resistance *A R* in series with its armature and an automatically adjusted resistance *F R* in series with its field winding. If the motors at the two stations are running at approximately the same speed, the intermittent shunting of the resistance *AR* is sufficient to maintain synchronism. The field resistance *F R* is used to control wider speed variations.

When the two motors are running in synchronism the operation of the correcting circuit is as follows:

Assuming the armature of the line relay just makes contact with its right-hand contact when the brush reaches the first *C* segment of the receiving ring a negative current impulse will pass through the distributor relay to the mid point of the windings of the No. 1 governing relay, through the left-hand winding of this relay to the left-hand contact and upper tongue of the No. 2 governing relay via contact *C* to ground. The armature of No. 1 governing relay will be moved to the right, thereby closing a circuit from the negative pole of the battery through the No. 2 governing relay and also inserting the resistance *A R* in series with the armature winding. The position of the No. 2 governing relay armature is thereby reversed, so that when the brush reaches the second *C* segment the circuit from the negative pole will be through the distributor relay to the mid point of the windings of No. 1 governing relay, through the right-hand winding of this relay to the right-hand contact and upper tongue of No. 2 governing relay, to ground via the second *C* segment. The No. 1 governing relay is reversed, the resistance *A R* is again short-circuited and the armature of No.2 governing relay which is normally biassed is restored to the position shown in Fig. 15.

So long as the brush is on a *C* segment, at the moments when the line relay armature is just making contact with the negative battery the armatures of the two governing relays will move from one contact to the other. The resistance *A R* will be cut in and out at each movement of the armature of No. 1 governing relay. The lower tongue of No. 2 relay, which is insulated from the upper tongue, controls the direction of the current through the armature of the small auxiliary motor *A M*, so that if this lower tongue moves rapidly backwards and forwards the armature of the motor merely oscillates. If, however, the power connections to this motor remain undisturbed for a few seconds, the armature commences to rotate and cause a change in the resistance *F R* which is in series with the field of the larger motor.

Should the brush be on an *R* segment, the current impulse through the condenser and the distributor relay will be via the left-hand winding of the No. 1 governing relay to the *R* segment, across the brush to ground. The tongue of the No. 1 governing relay will move to the right, opening the shunt across the resistance *A R* and thereby producing a slight reduction in the speed of the motor.

If the brush is on an *R* segment, when the next correcting impulse is generated the two governing relays will remain undisturbed. Consequently, the resistance *A R* remains in circuit. Should this condition continue for several successive revolutions of the brush, the auxiliary motor will have time to rotate and to aid in retarding the speed of the motor by cutting out resistance in series with the field.

In the event of the brush being on *A* segments, when correcting impulses are generated, the No. 1 governing relay armature will move to the left-hand contact and shunt the resistance *A R*, whilst the position of the No. 2 governing relay will reverse the direction of rotation of the auxiliary motor should the lagging of the brush continue.

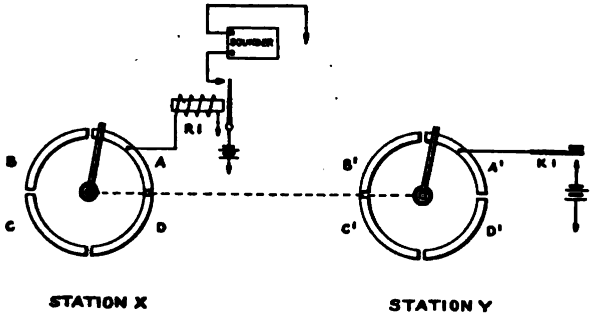


FIG. 16

The auxiliary motor shaft is connected to the adjustable rheostat arm by means of a worm reduction gearing.

This system is operated under duplex conditions. Although provision is made for securing a maximum speed of about 160 words per minute, the normal working speed is about 120 words per minute in each direction.

MULTIPLEX SYSTEM

Multiplex printing telegraph systems are all based on the Baudot system which was invented in 1875 by Emile Baudot, a Frenchman.

The principle of the multiplex system is illustrated by Fig. 16. If two metal rings be each divided into four segments of uniform length, and contact brushes, connected by a wire, arranged to rotate in synchronism, then segment *A* at *X* will be in electrical connection with segment A^1 at station *Y*, during one quarter of each revolution of the brushes; segment *B* will be connected to B^1 for a like interval of time and so on.

Communication by automatic Morse could be carried on, but it is obvious that the speed of rotation would have to be such as to permit of the longest Morse signal combination being transmitted from each segment. There would consequently be considerable loss of time when sending the short length signal combinations. Transmission of one word per segment would be equally unsatisfactory as words vary in length. Obviously, if there is to be no lost time, each signal combination representing a letter or symbol should occupy the same length of time in transmission. In order to obtain sufficient signal combinations to represent the letters of the alphabet, it is necessary to divide this time interval into five units, and arrange that during each time unit a selecting or non-selecting impulse shall be transmitted over the line. This arrangement will provide 32 possible combinations. By adopting "upper case" and "lower case" characters, as is done in typewriter practise, and arranging that two of the signal combinations shall automatically shift the printing mechanism, so as to print "upper case" or "lower case" characters, the number of selectable characters is practically doubled.

In completing this simple arrangement two things are essential:

(1) Means for keeping the two rotating brushes in synchronism.

(2) Means to change the keyboard set-up so as to send a fresh letter combination at each revolution.

Synchronism may be maintained in the following manner:

Assuming 20 segments are to be used for actual signalling, two extra segments are provided at each end. In Fig. 17, these extra segments are numbered 21 and 22. Segment 21 at the sending station is connected to a battery. Segment 22 at the receiving station is connected to a relay.

The brushes are driven by electric motors, the speeds of which are so adjusted that the brush at station *X* rotates slightly faster than that at station *Y*, say about ¾ deg. or 1 deg. per revolution. When the brushes are started off in synchronism, they will sweep over like numbered segments at the same moment. The brush at station *X*, however, will gradually gain so that whilst the *Y* brush is still on segment 21, brush at *X* will be on segment 22. When this condition exists, an impulse will flow from the correcting battery at *Y* connected to segment 21 through the correcting relay at *X*. If it be arranged that the actuation of this relay shall slow down momentarily the speed of the motor at *X* or else by mechanical means retard the brush, the two brushes will again be in synchronism. By this arrangement the duration of the braking effect can be made proportional to the amount of retarding required.

With the arrangement of segmented rings, as shown in Fig. 17, it will be obvious that, as the brush at the corrected station must be slightly out of phase with the incoming correcting impulses, before correction can take place, it will be out of phase with the incoming signal impulses. Consequently, an impulse which should flow say, via segment 17 to the selecting relay connected thereto, may overlap and flow via segment 18 to another selecting relay. The overlapping portion might be sufficient to actuate this selecting relay and thus produce defective selection. This is obviated by using small segments for the receiving ring so that only the middle portion of each signal impulse is utilized to actuate the selecting relays. Fig. 18 shows receiving ring segments of half the length of the sending segments; *a* represents signals in synchronism in which case the middle half of each signal impulse is picked out for actuation of the selecting relays, *b* represents the case where the brushes are ahead of their correct position, and *c* represents the reverse condition. It will be seen that only approximate synchronism is necessary to ensure that the impulses shall be switched to the correct selecting relays.

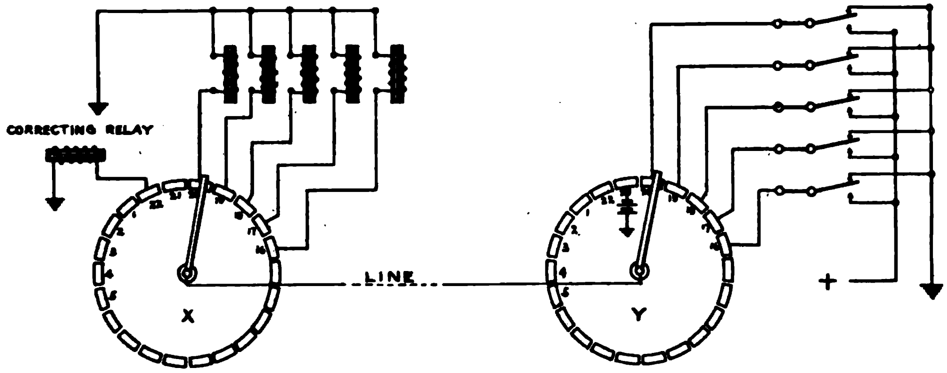


FIG. 17

This feature of utilizing only the middle portion of each signal impulse is advantageous also in permitting the system to be operated with signals which are considerably distorted. Such distortion may be caused by the electrical characteristics of the line, or by extraneous interference such as induction from other circuits.

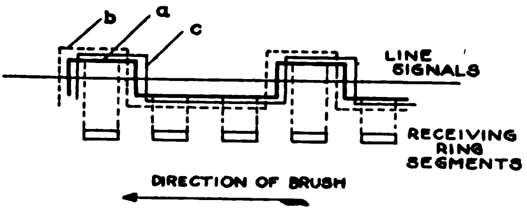


Fig. 18

Baudot Multiplex

The Baudot Multiplex System was invented in 1875 by Emile Baudot, a telegraph operator in the French service. Despite the fact that it was put into commercial operation over 40 years ago it held the field against all competitors until recent years.

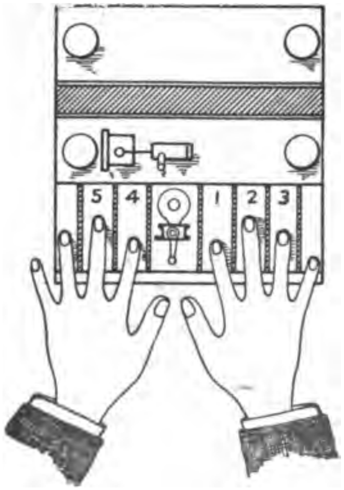


Fig. 19

Next to the conception of the multiplex principle as embodied in the Baudot, perhaps the most important development was the doubling of its carrying capacity by adapting it for duplex operation, which was successfully accomplished in 1910 by Mr. A. C. Booth of the British Service.

Other systems, closely resembling the duplex Baudot, and incorporating features borrowed from the typewriter have since been developed, and these are likely to supplant the Baudot in the future. At the present time a large precentage of the telegraph traffic in France is carried by the Baudot system.

Transmitter. Transmission on the Baudot system is effected by means of small keyboards each having five keys similar to piano keys, four such keyboards (Fig. 19) being connected to a four channel or quadruple set. As the keys are depressed they are automatically locked by small hooks and remain in the depressed position until the brush has swept over the five segments of the sending ring connected to the particular transmitter. An electromagnet mounted in the keyboard is then energized by an impulse through a local ring on the distributor, and releases the hooks, at the same time producing an audible click in the transmitter. This gives a signal,

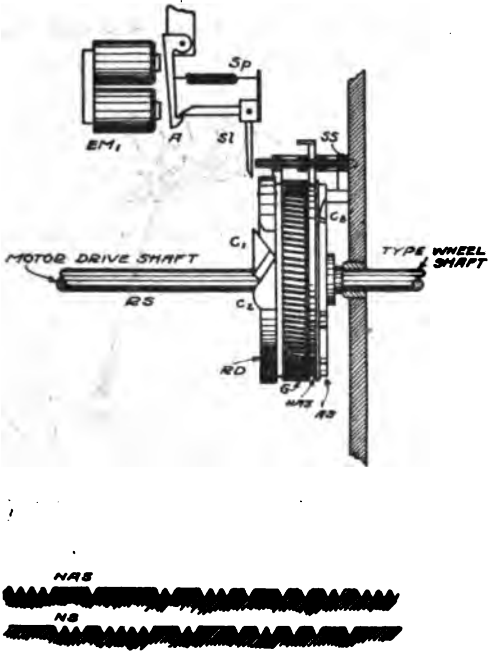


FIG. 20

termed the Cadence signal, to the operator who immediately depresses one or more of the keys setting up the next letter combination.

Printer. There are several types of Baudot printer, all of which are entirely mechanical except the initial actuation of the selecting magnets. In this paper it is proposed to describe briefly the salient features of the "Carpentier" type.

In Fig. 20 $E\,M_1$ represents one of the five selecting magnets connected to the receiving segments on the distributor. These five selecting magnets are mounted close together so that their armatures A and selecting levers $S\,L$ are grouped together and adjacent to their respective sliding selectors one of which $S\,S$ is shown in the figure.

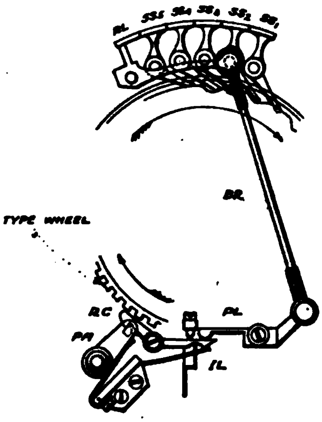


Fig. 21

When an electromagnet is actuated its armature A is pulled forward releasing the selecting lever $S\,L$. As the rotating disk $R\,D$ moves round a selecting lever which has been selected is caught by the cam C_1, and thereby is caused to press against its corresponding sliding selector $S\,S$ forcing the toe piece of the sliding selector into the channel marker $A\,S$. A selector lever which has not been actuated remains on the left side of the cam C_1, as it passes, so that its sliding selector remains with its toe piece in the channel marker $N\,A\,S$.

Immediately a selecting lever has performed its function of causing the sliding selector to be moved it is restored to normal by the cam C^2 and its upper arm is caught by the armature A.

The rotating shaft driven by an electric motor carries the disk *R D*, the gear *G*, the disks *N A S* and *A S* and the typewheel. The disks *N A S* and *A S* are notched as shown in Fig. 20. A front view of the sliding selectors is shown in Fig. 21. Normally these sliding selectors rest with their heads abutting against each other, irrespective of whether they have been selected or not. As the disks rotate notches will pass under their toe pieces, but no movement will take place until a notch is under each toe piece. All toe pieces will then drop into the notches simultaneously, causing the heads to move together towards the right. As the disks continue to rotate the toe pieces will be sharply thrown up by an unnotched portion of the periphery, causing the heads to overshoot and strike sharply against the releasing lever *R L* (Fig. 21). Actuation of this lever causes the releasing bar *B R* to pull up the pedal lever *P L* which strikes the intermediate lever *I L*, and removes the retaining pawl from the printer arm *P A*. The printing mechanism is thereby released, the paper tape is impressed against the typewheel, and the desired character printed.

The typewheel has characters on 31/40ths of its periphery, and whilst the blank portion of 9/40ths passes in front of the printing position the printing arc is restored to normal.

Letters of the alphabet alternate with figures and symbols. When it is desired to print a figure or symbol the "figure shift" signal combination causes the typewheel to be displaced from normal equal to the angular distance between a figure and a letter, so that on receipt of the next signal combination a figure shall be opposite the paper tape instead of a letter. The typewheel is restored to normal by the "letter shift" signal combination.

The speed of rotation of the typewheel must correspond approximately to that of the distributor brushes, and provision for maintaining approximate synchronism between these instruments must be made.

The Baudot system is generally operated at 180 revolutions of the distributor per minute so that an operator sets up letter combinations at the rate of three per second. Assuming six letters per word, this is equal to a speed of 30 words per minute per transmitter or 120 words per minute for the four transmitters. The system may be operated duplex, giving a total carrying capacity of 240 words per minute. One circuit has been in operation for several years between London and Birmingham, giving six channels in each direction—a total carrying capacity of 360 words per minute.

Synchronism. In the Baudot system the brush holder is connected to the motor shaft through gears, and a mechanical device actuated by the correcting relay causes one of the intermediate gears to be rotated one tooth which has the effect of stepping back the brushes with respect to the driving shaft.

MULTIPLEX WITH TAPE TRANSMISSION AND PAGE PRINTING

There are two multiplex systems of this type—the Murray and that developed in the research laboratories of the Bell System in collaboration with the engineers of the Western Union Telegraph Company. The Murray system was the forerunner in showing the advantages to be gained by the use of keyboard perforators instead of direct transmission, as in the Baudot system, and in the use of a page printer, which turns out finished messages ready for delivery, thereby eliminating the gumming process.

A two-arm Murray multiplex installation has been in commercial use in England for about ten years. The war, however, prevented the manufacture of further installations; otherwise, it is probable that complete quadruple duplex installations of this system would have been in service by this time.

The multiplex system designed in this country resembles the Murray as regards the use of perforators, page printers, and rotating distributors, but differs from it in many important details. What will be said in this paper concerning the operation advantages of the American multiplex applies generally to the Murray multiplex.

The American multiplex system, like the Murray system, is based fundamentally on the duplex Baudot system, and was developed primarily to meet the needs of the Western Union Telegraph Company. The further development of the American system, providing greater flexibility by means of forking and extending the individual arm or channel, has been developed in the research and development laboratories of the Bell System.

This system differs from the Baudot system in the following respects:

Messages are first prepared on keyboard perforators, instead of being transmitted direct from five key transmitters. No special signals are transmitted over the line for the purpose of maintaining synchronism. Messages are printed in page form instead of on tape, thereby eliminating the gumming operation. For a complete quadruple duplex equipment, that is, one which provides for the simultaneous transmission of eight messages—four in each direction—there is required at each end of the line:

Fig. 22

Distributor with driving fork and impulse motor

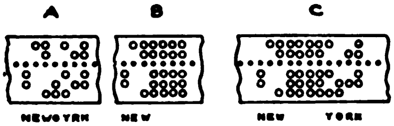


Fig. 23

Four transmitters
Four keyboard perforators
Four automatic control units
Four printers
Line and local relays, and duplex balancing equipment.

Messages are first prepared as perforations on a tape, by means of a keyboard perforator as shown in Fig. 22. The perforations for each character are made across the tape and occupy a space of 0.1 in. per letter character. Hence a 30-word message occupies a length of only 18 inches of tape.

Perforator. The perforator comprises three rows of keys arranged according to standard typewriter practise, which control the selection of the punches. The depression of a key, in addition to effecting the selection of a group of punches, causes a circuit to be closed through an electromagnet whose armature drives the selector punches through the paper tape.

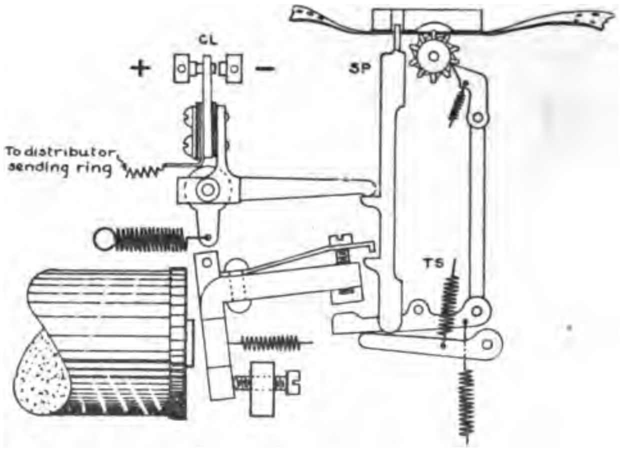


Fig. 24

Provision is made for back spacing the tape, so that in the event of an error having been made, and observed by the operator, she may cancel it in such manner that it does not appear on the printed record at the distant end. For instance, if the word New York is to be perforated, and in error, the *O* key is depressed before the *Y* key as shown at *A* Fig. 23, the tape would be back spaced and all letters back to and including the *Y* would be cancelled as shown at *B*. Then the word would be continued commencing at the *Y* as shown at *C*. This feature is referred to as the "rub-out" and permits the operator to erase errors from the tape which prevents them from appearing in the printed message.

Transmitter. The transmitter, Fig. 24, has five contact levers, *C L*, insulated from each other and controlled by five selecting pins, *S P*. The contact levers normally rest against a busbar connected to one pole of the line battery. These contact levers are connected to five consecutive segments on the sending ring of the distributor. When a selecting pin rises through a signal hole in the perforated tape, its contact lever moves over and makes contact with another busbar connected to the other pole of the line battery.

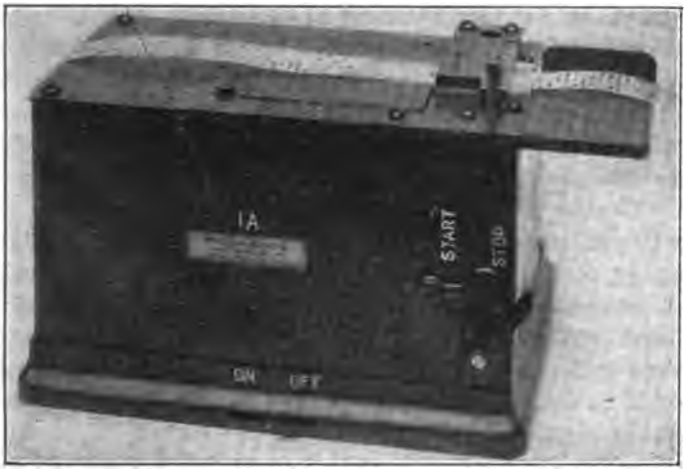


Fig. 25

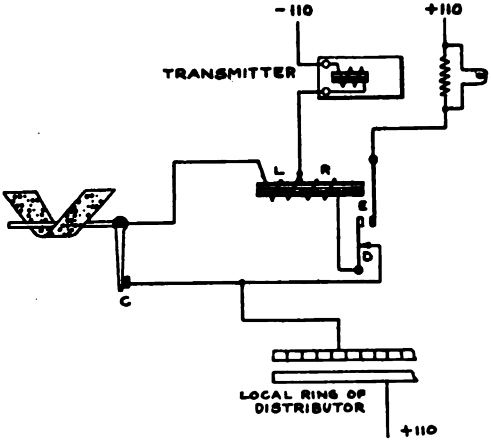


Fig. 26

Normally each selecting pin tends to move upwards due to a tension spring, *T S*. Only those pins which find a signal hole in the tape, however, are free to rise, the others being held down by the blank section of the tape.

The contact levers associated with those pins which rise through the signal holes move over to the opposite busbar. Consequently, the five segments of the distributor connected to the transmitter are placed in contact with one or other of the two poles of the line battery according to the perforations in the tape. As soon as the brush on the distributor has swept over these five segments, a current impulse which may be termed the "stepping pulse" is made to pass through the electromagnet which depresses all the selecting pins and by means of a ratchet and tape feed wheel similar to that in the perforator, causes the tape to be stepped forward ready for the next letter combination to be set up. Fig. 25 shows the transmitter with covers in place.

Auto Stop. Although a skilled operator has no difficulty in maintaining an ample length of perforated tape between the perforator and the transmitter, conditions arise in normal operation when a perforating operator may have to suspend work temporarily. The transmitter, however, continues to feed forward the perforated tape and were no provision made, the tape would become torn as soon as the slack was taken up between the perforator and the transmitter. This is taken care of by a device, termed the "auto-stop," which consists of a light lever and a differentially wound relay connected as shown in Fig. 26. The lever is so placed between the perforator and the transmitter that the slack perforated tape sags beneath it, but does not touch it. The weight of the long arm of the lever ensures the contact at *C* being closed. The stepping pulse from the local ring of the distributor to the transmitter magnet has two paths, one via contact *C*, the other via contact *D*. Consequently, the current divides equally between the differential windings *L* and *R*, of the relay, which remains inoperative. Whenever the sag in the tape is taken up sufficiently to carry with it the contact lever, the contact at *C* is opened, so that the next stepping inpulse from the local ring of the distributor passes through one winding only of the differential relay and actuates the relay. Its armature causes contact at *E* to be made and that at *D* to be broken. The relay is therefore locked up and the magnet of the transmitter is energized. So long as this condition is maintained the selecting pins of the transmitter are held in the position for transmitting the "blank" signal which has no effect whatever upon the printer at the distant station.

As soon as the operator commences perforating and the slack tape allows the contact lever to drop, the first "stepping

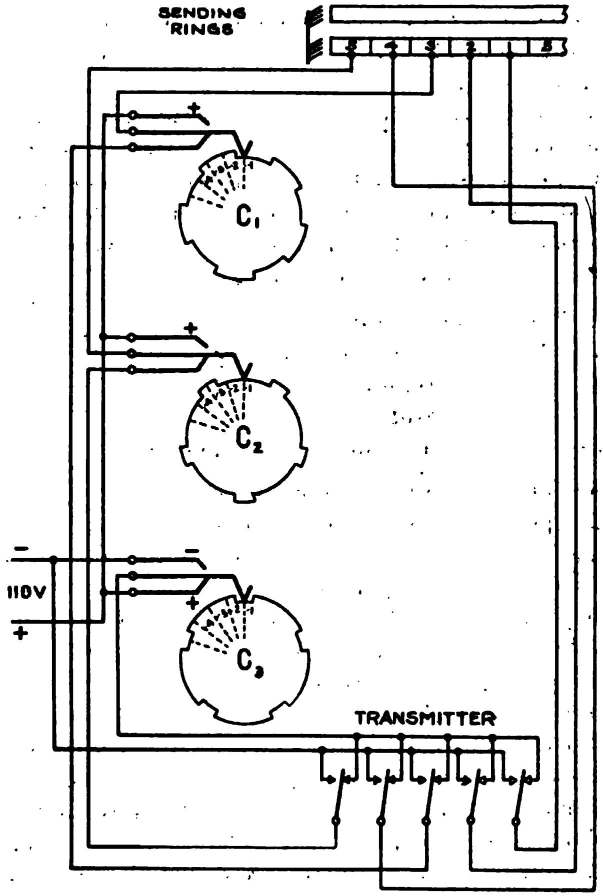


FIG. 27

pulse" will cause the unlocking of the differential relay and normal transmitting conditions to be restored.

Auto Control. While not an essential element of the system, the auto control is a desirable adjunct. In order to save time, it is advantageous to be able to signal the distant station operator without having to wait until a message in course of transmission has been completed, and it is essential that such signals shall not involve mutilation of the printed record of such message.

This is secured by means of the "auto control" device which is connected in the circuit between the transmitter and the sending ring of the distributor. The "auto stop" feature is incorporated in this device, the circuits being inter-connected, but for the sake of clearness the auto control circuit alone is shown in Fig. 27.

The auto control, therefore, is a device for automatically stopping the transmission of a message and for sending over the line the necessary signals to ring a bell mounted on the printer, at the distant station. Provision is made for ringing the bell once, twice, up to five times in succession. A code of signals can be used. For example:

1 ring may mean "stop"
2 rings " " "rerun last message"
3 rings " " "repunch " " "
4 rings " " "start"
5 rings " " "attendant" or "supervisor."

The auto control consists of a starting sector or handle with five finger positions, connected to a small spring motor, three cams rigidly mounted on a spindle, acting upon three separate groups of contact springs and an escapement wheel controlled by an electromagnet as shown by Fig. 28. The starting handle is actuated by inserting a finger in one of the five finger positions and pulling it forward to its starting position which is determined by a finger stop. The spring motor is wound up by this forward movement and when the finger is removed, the motor causes the spindle carrying the three cams to be rotated, at the same time restoring the handle back to normal position. The "stepping pulses" which normally actuate the electromagnet of the transmitter are diverted so as to actuate the electromagnet controlling the escapement wheel of the auto control device so that the cams are rotated one step for each "stepping pulse."

In order that the circuits may be readily traced, dotted lines have been drawn to show the position of the middle springs on the cams at each step. The normal position of the springs is shown in Fig. 27.

The first step, that is, to position marked No. 2; causes the middle spring of cams 1 and 3 to be raised. The polarity of the right hand contacts on the transmitter is changed from

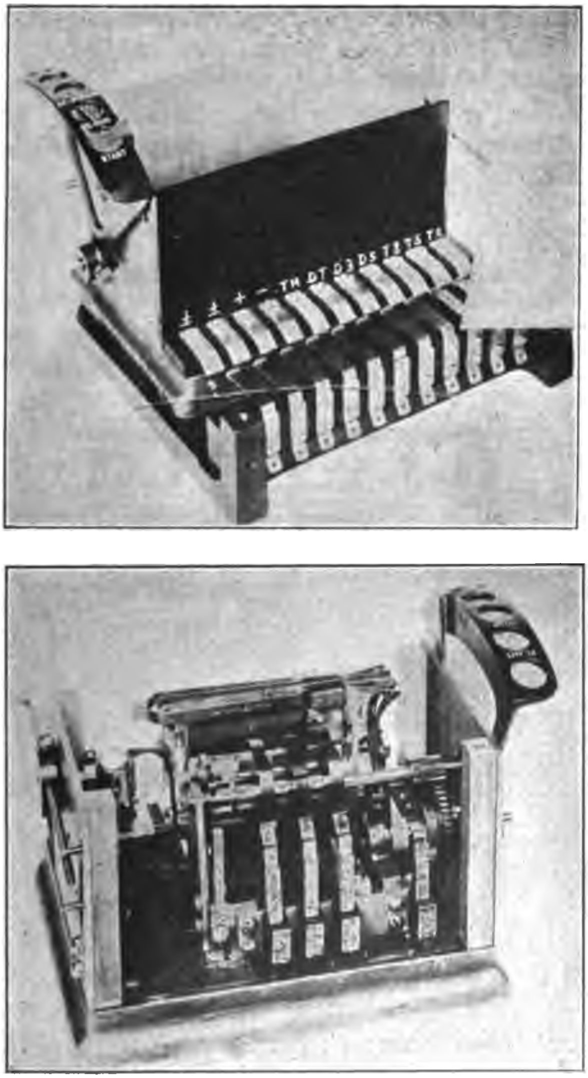


Fig. 28

positive to negative by the action of cam 3 whilst the actuation of the spring on cam C1 causes segment 3 on the sending ring to be connected to the positive line battery, irrespective of the polarity applied to the third contact lever of the transmitter. Consequently, the five segments will send – – + – –, which is the "figure shift" signal. This will actuate the printer at the distant station and adapt it for "upper case" operation. The next step forward of the cams will cause all springs to be raised. This will reverse the polarity applied to the fifth segment so that the next signal transmitted will be – – + – +, which in the "upper case" is the combination for "signal" or "bell." The effect of this will be to cause the bell to ring at the distant station printer.

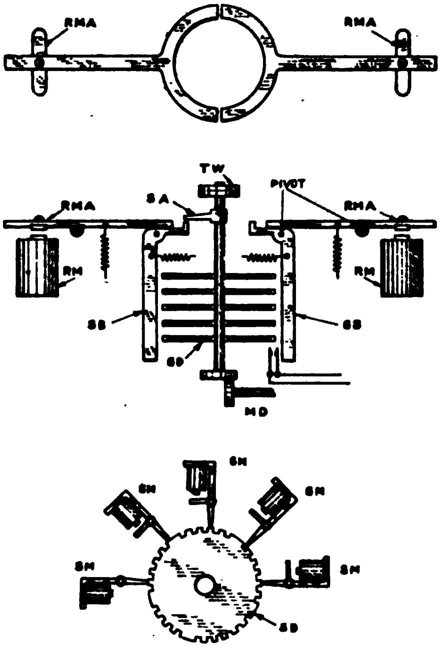


Fig. 29

At the third step of the cams, the middle spring of cam 3 only will be raised, so that negative voltage is applied to all segments. This is the "letter shift" signal and causes the typewheel of the printer to be restored to normal. The fourth step restores the circuit conditions to normal ready for transmitting direct from the transmitter.

This cycle of operation is repeated one, two, three or fou times, according to the setting of the handle. None of the signals transmitted by the auto control device actuates the spacing or lateral movement of the paper carriage so that the auto control may be operated whilst the printer is in

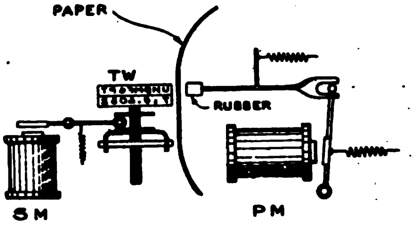


Fig. 30

the middle of a word. When the auto control signals have been sent, the normal transmission of the message continues, and the printer completes the unfinished word.

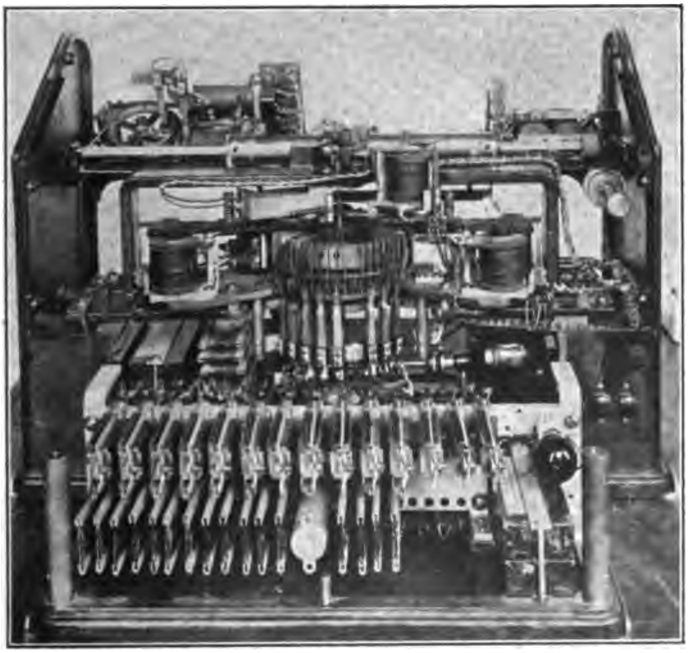


Fig. 31

Printer. The printer is of the movable paper carriage type, and all operations such as causing the paper carriage to be returned for the printing of a new line and moving the paper upwards equal to the distance between lines, are performed automatically under control of signals from the sending station.

FIG. 32

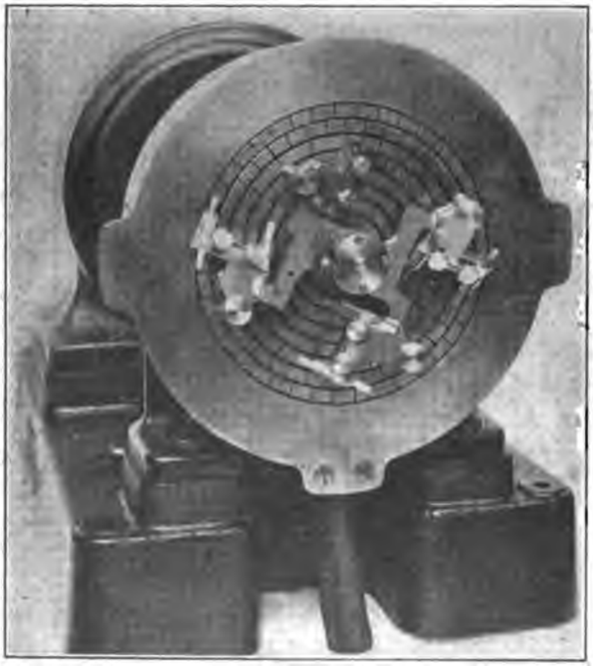


FIG. 33

A typewheel *TW*, Fig. 29, mounted on a vertical shaft, is made to rotate by means of a small electric motor. Projecting from the typewheel shaft is a short arm *S A*, the free end of which just clears the ends of the selector bars *S B*. When stopped by a raised selector bar, the typewheel is so positioned as to print the particular letter associated with that selector bar. Normally, the selector bars are held down by the semicircular ends of the armature levers *R M A* of two release magnets *RM*, Fig. 29.

The five horizontal selector disks are mounted around a hollow drum or shaft enclosing the typeshaft. Each disk has notches so arranged around its periphery, that when one of the disks is moved through a small angle, a notch on each of the disks will be in vertical alignment so as to permit the vertical leg of one selector bar to move inwards. If two disks are moved, the alignment of notches will occur opposite another selector bar, if three disks are moved the alignment will take place opposite still another selector bar and so on. With five selecting disks, 32 possible selections are available. There are 31 selecting bars, each one of which is associated with a character to be printed, or contacts to be closed to cause one of the several functions which the printer has to perform. One selection is not used, since it is desirable to have one combination which will cause no operation in the printer. The selector bar on the right, Fig. 29, is shown as a function or "stunt" bar. These functions are "space," "carriage return," "line feed," "figure shift," "letter shift" and "bell."

The typewheel is mounted on the typeshaft in such a manner that it may be raised so as to bring a second row of characters opposite the printer hammer. This operation is performed by means of the shift magnet *SM*, as shown in Fig. 30. When the "figure shift" signal is received, a "stunt" bar is selected and closes the contacts of the shift magnet *SM*. When the "lettershift" signal is received, another "stunt" bar is selected, which opens the circuit of the "shift magnet" and the typewheel is brought down to its normal position.

The paper on which the messages are printed is fed round the platen of the paper carriage which slides laterally on a square shaft. Fig. 31 shows the printer with all covers removed. Fig. 32 shows an operating table with all equipment in position.

Distributors. The distributor face plate consists of eight rings, four being solid and four segmented, as shown in Fig. 33. The rings are associated in pairs, one segmented and the other solid. The two brushes which sweep over the two rings of a pair are in electrical contact with each other so that the solid ring is connected in sequence to all the segments of its associated ring, once per revolution. The outer or receiving ring has 40 segments and is associated with the fifth ring. The second or correcting ring (40 segments) is associated with the sixth ring. No. 3 is the sending ring (20 segments) and is associated with No. 7. The innermost segmented ring (20 segments) is termed the "local" and is associated with the innermost solid ring. The brushes are made from thin copper wire and make contact on one segment only at a time. They are mounted on a balanced brush holder which is carried by a spindle projecting through the center of the face plate.

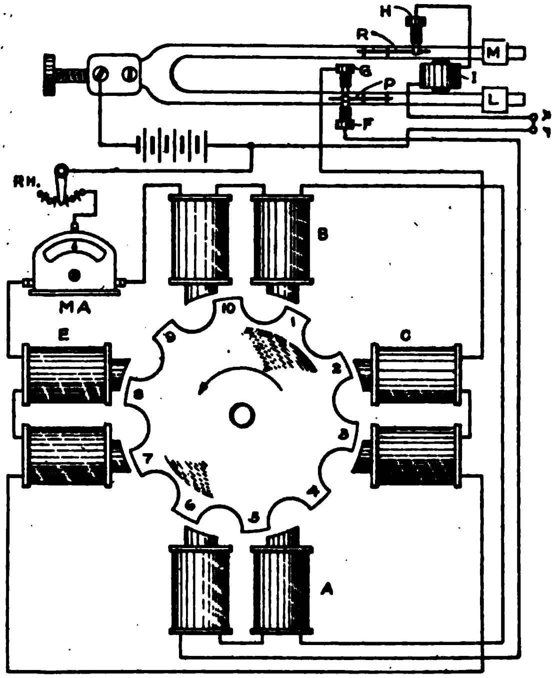


FIG. 34

Two types of distributors have been designed, one in which the brush holder spindle is directly connected to the shaft of the driving impulse motor and the other in which intermediate gearing is introduced between these parts, for a purpose which will be described later.

The La Cour or phonic wheel motor together with the electrical driving fork is shown in schematic form in Fig. 34.

The fork carries two contact springs *R* and *P* in electric contact with the fork itself. Mounted below the tines of the fork is an electromagnet *I* for causing the fork to vibrate. When the tines are not loaded with weights *M L* the fork vibrates at the rate of approximately 55 cycles per second. If a slower rate is desired, weights are fixed on the tines as shown in the figure. The rate of vibration can be varied between 25 and 55 cycles by this means. Provision is made for effecting small changes in speed, by means of additional weights near the base of the fork, which are so mounted as to be slidable along the tines by an adjusting screw operated by a handle or knob at the end of the fork. This adjustment can be made whilst the fork is in vibration. A photograph of the fork is shown in Fig. 35. In the case of a quadruple or a four arm installation, the speed of the fork in cycles per second corresponds to the speed of operation of each channel in words per minute, so that if the fork vibrates at 50 cycles per second each transmitter operates at a speed of 50 words per minute.

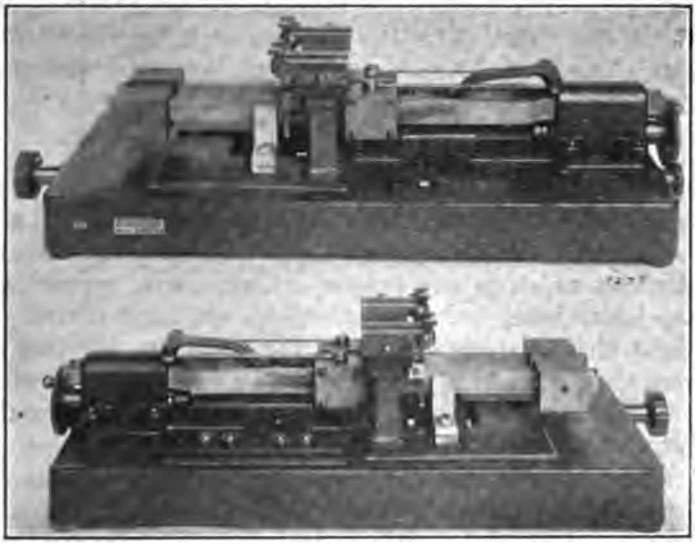


Fig. 35

The phonic wheel motor has an armature so shaped as to form ten teeth or polefaces, equally spaced, around the periphery. Mounted 90 deg. apart are four electromagnets with polefaces shaped so as to be concentric with the edge of the polefaces or teeth on the armature.

The electromagnets diametrically opposite are connected so that *A* and *B* (See Fig. 34) are energized together and *C* and *E* together. Consequently, when the fork is vibrating, *A* and *B* will be energized alternately with *C* and *E*. The number of the polefaces on the armature is such that the armature makes one complete revolution during ten complete vibrations of the fork. The differential milliammeter *MA* shown in the figure is for the purpose of indicating whether the adjustment of the fork contacts in the motor circuits is correct.

SYNCHRONISM

Mechanical Correction. As previously stated two types of distributors are used. They differ only in so far as one is provided with a mechanical correcting or synchronizing mechanism which is not present in the other, synchronism in the latter being secured entirely by electrical means. In the case of mechanical correction the correcting impulses are generated by a reversal of current in one direction only, whilst in the electrical correcting method the correcting impulses are generated by current reversals in both directions.

A schematic diagram of the circuit for mechanical correction is shown in Fig. 36. The distributor motor at the corrected or speed controlled station is arranged to run slightly faster than the motor at the correcting or speed controlling station. If reversals of current in the line signals take place when the brushes which sweep over the correcting rings at the controlled station are on a dead or unconnected segment, as at the point indicated by the arrow marked *X*, no correction takes place. But the brushes at the speed controlled station will gain over those at the speed controlling station so that after one or two revolutions the brushes at the former station will be leading, and be on a "live" segment at the moment when the corrector relay tongue which follows the movements of the line relay, moves to its *M* contact.

Before the "break" relay *B R* has had time to pull up its armature, an impulse passes via the back contact of *B R* to the solid correcting ring, across the brushes, the "live" segment, switch *S* through the correction relay *C R* to the −110 *V*. The corrector relay is actuated and is locked up due to current from the +110 *V* flowing via the armature and front contact of *C R* and the back contact and lever on the stepping pawl *P*. A path is also provided from the +110 through the corrector

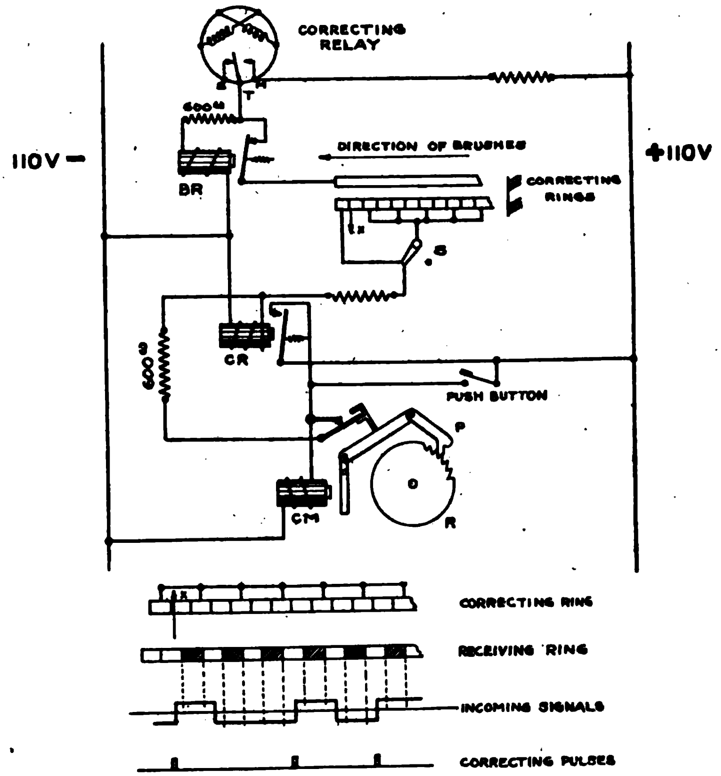


Fig. 36

magnet *C M* which is energized. Actuation of the armature of *C M* causes the pawl *P* to step the ratchet wheel one tooth. As it completes its stroke the pawl lever causes the circuit from the +110 to be opened so that the corrector relay *C R* and the corrector magnet *C M* are restored to normal. The stepping of the ratchet wheel has the effect of momentarily stopping or retarding the movement of the brushes so that they are again in sufficiently close phase relationship with the incoming signals to insure the correct distribution of the electing impulses to the printer selecting relays.

The mechanical feature of the correcting device is shown in Fig. 37. The motor shaft carries with it the frame *F* on which is mounted the corrector magnet, the ratchet wheel *A*, gear wheel *B*, the pawl and jockey roller.

A and *B* are fixed on the same spindle and are prevented from rotating by the jockey. Consequently, when the motor shaft is rotated, gear *B* transmits motion to the brush holder shaft in the direction indicated by the arrow marked *N*. When a correcting impulse actuates the corrector magnet, the pawl steps the ratchet wheel *A* one tooth in a counter-clockwise direction, so that *B* advances one tooth around *C* in the direction marked *N*, which is the equivalent of stepping the gear *C* in the direction marked *R*. The resultant effect is that the brush holder shaft is momentarily retarded.

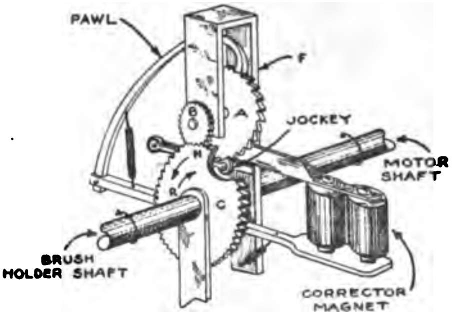


FIG. 37

Phase Finding. In order to bring the brushes at the two stations into proper phase relationship, *i. e.* into synchronism, a switch is provided at the speed-controlling station for sending a long negative pulse from sending segments 1 to 19 and a positive pulse from sending segment 20. The switch marked *S*, Fig. 36, at the speed-controlled station is thrown to the right, disconnecting all the "live" segments but one. Consequently, only one correcting pulse will be generated per revolution of the brushes and it will be inoperative until the brushes at the corrected station overtake the incoming signals, at the position marked *X*.

Electrical Correction. Fig. 38 shows in schematic form the circuit arrangements for electrical correcting. A distributor arranged for electrical correction has the impulse motor directly connected to the brush holder and correction is effected by altering the speed of the fork which determines the speed of the motor. When the resistance *R* is in series with the electromagnet of the driving fork at the speed-corrected station, the speed of the fork is slightly faster than that of the fork at the speed-controlling station, and when *R* is short-circuited the speed of the fork is slightly lower than that of the fork at the other station. The function of the

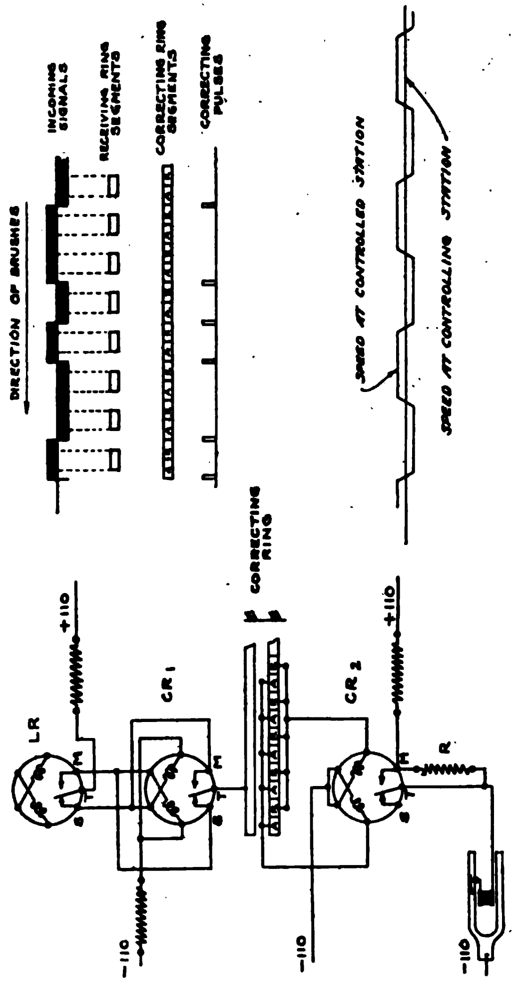


FIG. 38

either for delivery or for retransmission over another circuit. Messages for delivery by messenger have to undergo further handling. As such further handling, however, is not directly associated with or affected by the system of transmission over the wires, time occupied in this connection is not included.

Reviewing the methods of operation of the various systems under consideration, it will be shown wherein time losses, which occur in the high-speed systems, are avoided in the multiplex.

At the Sending End. In the high-speed systems messages are first prepared by perforating operators, seated conveniently near to the transmitting operator who feeds the perforated tapes into a high-speed transmitter. In order that as little time as possible be lost by the perforating operators in passing the message forms with the corresponding tapes to the transmitting operator, it is the usual practise to perforate two or three messages on each tape. If such tapes are immediately fed into the transmitter, the first message has already undergone a delay equal to the time taken to perforate all three messages and the second message has undergone a delay equal to the time taken to perforate the second and third messages. The transmitting operator, however, has no time to sort out the tapes from the various operators so as to transmit messages in their exact order of priority. It usually happens, therefore, that a certain proportion of the messages undergo further delay before actual transmission takes place. It may be taken that the average delay in the case of high-speed systems is not less than three minutes.

With a multiplex system transmission over the lines takes place almost immediately the keys are actuated. In other words a multiplex provides practically direct connection between operator and operator, as is the case with Morse working.

At the Receiving End. There is not much difference between the high-speed and multiplex systems as regards speed of service, a slight advantage, however, resting with the multiplex.

Unfortunately, telegraph traffic does not glide along as smoothly as has been outlined above. Errors creep in and must be corrected. With the high-speed systems, a correction operator is required at each end of the circuit. Messages dealing with the correction of errors pass between these operators and although given priority over other traffic several minutes elapse in their transmission in each direction. Therefore, before an inquiry regarding a message can be received, a considerable number of other messages will have been transmitted, and unless these messages have been arranged in numerical order, some time is needed to trace the required message. Further delay will occur in the transmission of the answering message so that the total delay to a message in which a correction is required may amount to many minutes.

With the multiplex systems the mode of working admits of such corrections being asked and obtained almost instantly.

In a paper read before the Post Office Telephone & Telegraph Society, London, on November 24th, 1913, Mr. John Newlands, Controller of the Central Telegraph Office, states:

> There is no doubt that all existing systems of telegraphy which involve the preliminary punching of each message tend to create delay, because even when the punching operation is completed, the message is not yet on its way, the process of transmission through the transmitter has still to be carried out, and when there are two messages on each slip, and each separate slip must perforce await its turn—there is always an appreciable loss of time, which does not take place, for example, on the Baudot System nor even in Morse Keying.

It is interesting to note that this advantage of multiplex operation has been recognized in England and an effort has been made to adapt the Wheatstone System to the multiplex principle, so far as transmission is concerned. The arrangement is as follows:

Beside each perforating operator is a motor-driven Wheatstone transmitter. These transmitters are connected to an electromagnetic rotary switch which is arranged to switch the line to each transmitter in sequence. The switch is actuated by a signal from the transmitter to which it is connected, each actuation transferring the connection to the next transmitter. When perforating operator No. 1 completes a message, he depresses a special key on the perforator which perforates a hole in the tape, not in alignment with the signal holes. As this hole passes through the transmitter, it causes actuation of the switch, which stops the motor of the No. 1 position transmitter and starts the motor of the No. 2 position transmitter, at the same time switching the line from No. 1 to the No. 2 transmitter. Similar operation at the end of the message transmitted by the No. 2 transmitter switches the line to the No. 3 transmitter and so on in rotation.

It is doubtful if such a scheme will prove satisfactory, however. Messages are not of uniform length, nor do operators perforate at exactly the same speed. It effects the elimination of the transmitting operator, but it is difficult to see wherein much improvement can be secured towards providing a quicker speed of service. The use of two or more high-speed transmitters which are operated intermittently involves added complexity and cost to the sending end apparatus.

ACCURACY OF PRINTING TELEGRAPHS

Whatever may be the system of telegraphy employed, the standard of accuracy is dependent upon the care with which the messages are checked. In Morse working, the receiving operator is expected to check the accuracy of a message whilst performing the mental operation of translating the dots and dashes into letters and the physical operation of recording them on a typewriter. Actually three functions are performed simultaneously:

1. Translating telegraphic signals into letters, mentally.
2. Operating a typewriter, selecting correct keys.
3. Scrutinizing the typed copy to see that it contains no errors.

Although it has been claimed that the two first operations are performed entirely automatically by skilled operators, such is not the case except in rare exceptions. If a function is being performed automatically, that is, requiring no attention of mind, then the mind can be employed on some other function. It is only necessary to request a Morse operator to read some message other than that he is in the act of receiving, or to engage him in conversation to show how little attention he is able to give it, without interfering with his main task. Psychological authorities agree that if the mind be engaged in two or more independent actions the principal action is hampered by the side issues.

Apart from its being a slower process than Morse sounder working, the translation of telegraph signals from a tape as in the Wheatstone system involves a division of attention between the tape and the sheet on which the message is being written or typed with the result that the liability to making errors is increased. When continued for a long period it also tends to produce eye strain.

A page-printing system necessitates only the checking of the messages, and consequently that work can be given un-divided attention by the operator, with the result that it is done better.

It may be of interest to mention that a European telegraph administration, desirous of securing data in regard to undetected telegraphic errors, caused a careful examination to be made of a large number of messages which were handled by different telegraph systems. This examination showed that the number of undetected errors per 100 messages was as follows:

Morse tape recording	0.11
Morse sounder	0.09
Hughes printer	0.04

It is understood that the introduction of the multiplex printing system in the Western Union Service has resulted in a proportionate improvement as regards accuracy, closely approximating these figures.

When it is realized that telegraphic errors oftentimes result in financial loss, as well as involving considerable expense in official correspondence and investigation, the greater accuracy secured by a printing system must be regarded as an important factor in its favor.

OUTPUT

The introduction of printing telegraphs in place of the Morse has not resulted in so great a percentage gain in operator output as has been achieved by the use of automatic machines in other fields of activity, nor can they ever be expected to do so. The human factor cannot be eliminated entirely; every message must be handled—letter by letter—by the sending operator, and be checked at the receiving end by the receiving operator.

The keyboard perforator, however, has been responsible for a considerable increase in speed of working at the sending end, whilst at the same time its operation involves less mental and physical fatigue on the part of the operator.

At the receiving end of a printing system, the operating duties are less exacting than those of the Morse operator. Although messages must be handled without delay the nature of the work permits of a freedom to the operator which cannot possibly be felt by Morse operators. Consequently, the operators are under less mental strain and can handle a larger volume of business.

It has been found that, in general, printing systems have shown some improvement over the Morse and Wheatstone systems, as regards labor economies.

The following statement, extracted from Mr. John Newlands' paper previously referred to, gives the comparative results of tests of various systems of telegraphy in England. As British messages average 24 words, whilst American messages average 30 words, I have equated the figures to American messages, as shown below:

System	Average per Operator Hour	
	British Messages	American Messages
Baudot Multiplex	43	34.4
Wheatstone with Creed	35	28
Morse Sounder	31	24.8
Wheatstone	26	20.8

The Siemens & Halske and American Multiplex Systems have been installed in the London office since Mr. Newlands' paper was written. No results have yet been published concerning these systems.

In the foregoing statement are shown the four systems of telegraphy by means of which probably 95 per cent of the British telegraph traffic is carried.

The multiplex Baudot shows an output per operator of 23 per cent greater than the high-speed system—the Creed, despite the fact that the Baudot sending operators are limited to a maximum speed of 30 words per minute. Assuming an increase in output pro-rata with the increase in speed resulting from the use of the keyboard perforators and automatic transmitters, say 45 words per minute, the output per operator would be 64 British messages or 51.6 American messages per hour.

All these foregoing figures refer to team work on one or more circuits.

In the 1919 April issue of the "Western Union News," some interesting records of work done on keyboard perforating machines by female operators are given. From 100 to 130 messages per hour have been perforated by a number of operators for short periods, up to eight hours. For periods covering from one month to one year, the hourly average range between 60 to 79 messages.

In one particular case a female operator maintained an average of 74 messages per hour for a period of eight hours during the first month after graduation from the training school.

In the same magazine is given data pertaining to record performances of Morse operators which compare very favorably with the work of the keyboard perforator operators. In order to achieve such results, however, it is necessary that the operators at both ends of the circuit shall be experts whereas in automatic operation the transmitting operator output is not limited by the skill of the receiving operator.

MAINTENANCE

Available information regarding the cost of maintaining installations of the various systems described is unfortunately too meagre to be of value in making direct comparisons.

Printing telegraph machines have not yet reached that stage of perfection where they never get out of adjustment and never have broken parts. Consequently, provision should be made for the removal of faults due to such causes as promptly as possible. What is even more important, however, is the provision of means for the detection of incipient faults before they have developed sufficiently to interfere with operation and for the quick restoration of service when it has been interrupted by an apparatus fault. In this respect, the multiplex systems are superior to the high-speed systems. For a quadruple duplex installation, it is necessary to provide only one spare instrument of each kind, except the distributor. In the case of the American multiplex system, such spare apparatus is mounted on a test table, which is provided with a small local distributor so that the machines may be tested in operation under conditions even more severe than in actual service. Spring clips are used for effecting the electrical connections on the operating and test tables and any machine can be removed and another placed in its stead in a few seconds. If the test table is located in close proximity to the operating tables, interruption to the flow of traffic due to machine trouble can therefore be almost entirely eliminated. A fault in any of the machines, except the distributor, will stop the flow of traffic in one direction over one channel only. Faults in the distributor are of rare occurrence. Generally a spare distribution face is available, and can be substituted in a few minutes.

In the case of high-speed systems an apparatus fault in the transmitting or receiving apparatus causes total suspension of traffic flow in one direction, and generally involves temporary interruption to the traffic in the other direction whilst the distant station is being advised of the trouble.

The percentage of spare apparatus must obviously be greater than in the case of the multiplex systems, except where four or more high-speed installations are in service in one office, and in such cases it is practically impossible to effect apparatus changes so quickly as can be done with multiplex machines. Where a complete spare set of machines is provided, it is possible, of course, to switch over to the spare set without much delay.

One important factor in connection with the maintenance of apparatus is the periodical inspection and overhaul. Such inspections reveal incipient faults. The test table and local distributor of the American multiplex system are particularly valuable for this purpose, as the more severe conditions under which the machines can be tested will reveal small defects or faulty adjustments before such defects make themselves felt under normal working conditions.

For example, if a printer operates at 65 words per minute during one inspection test, and only 60 words per minute at the next, it is an indication that some slight fault exists, which can be detected and remedied before it affects the machine at normal working speed of 50 words per minute.

Provided a properly organized system of periodical cleaning and testing is introduced, it is possible to maintain a service practically free from interruption due to apparatus troubles.

The most satisfactory method of maintaining such apparatus in correct adjustment is to specify spring tensions, distance of movement of relay armatures, airgaps etc. and to use suitable gauges for indicating whether the adjustments of the various units are within the specified limits.

LINE ECONOMIES

In the synchronous type of printing telegraph systems no spacing between signal combinations is required and consequently the code is considerably shortened as compared with the Morse where the spacing between letters is equal to three dot lengths. From studies which have been made it has been found that the average signal combination in the American Morse code is equal to eight dots, and that in the continental

Morse code nine dots. In the synchronous systems the signal combinations are all five-dot lengths. As the dot is the unit of length these codes are generally referred to as eight, nine and five-unit codes. The advantage of a shorter code can perhaps best be illustrated by example. Assuming that over a particular telegraph circuit a maximum transmission speed of 45 cycles per second can be obtained. This speed of transmission will provide a carrying capacity of 100 words per minute for a nine-unit code, 112.5 words per minute for an eight-unit code and 180 words per minute for a five-unit code. Should the traffic load be such that 100 words per

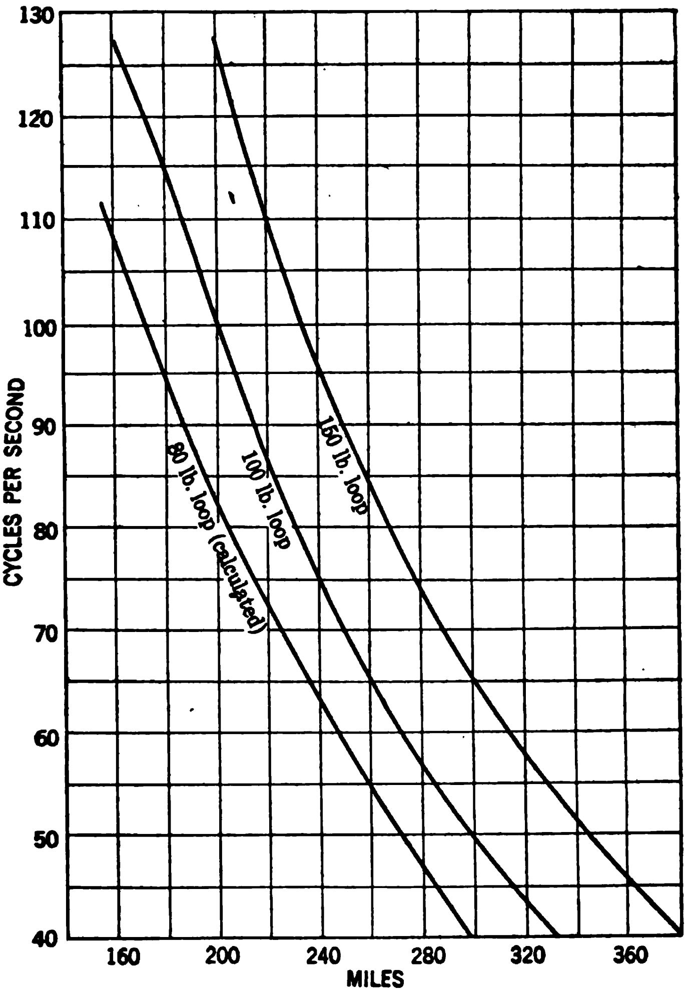
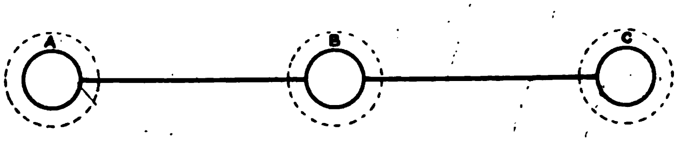
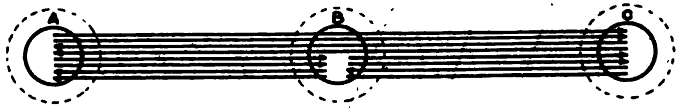


FIG. 39—METALLIC LOOP CONDUCTORS
Curves showing relation between conductor weight, mileage and speed of transmission.

minute will be sufficient to carry it then the transmission speed of the five-unit code system may be reduced to 25 cycles per second. This reduction in speed will permit of operation under adverse weather conditions which tend to disturb the duplex balance, and which would necessitate a reduced speed of working on systems using an eight or nine-unit code. The author has no data as regards actual transmission speeds over aerial lines of different lengths, but in Fig. 39 are shown the results of speed tests of underground cable circuits, which were made in England several years ago.



Wire Facilities



Traffic Carrying Facilities

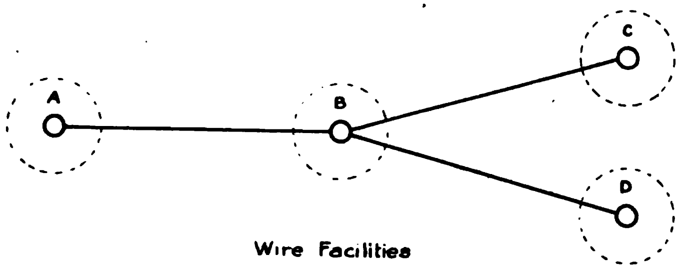
Fig. 40

It will be seen that over a metallic return circuit of 270 miles in length having copper conductors each weighing 150 lb. per mile, a transmission speed of about 80 cycles per second is secured. This would provide a carrying capacity of 177 words per minute with a system using a nine-unit code. With a system using a five-unit code an equivalent carrying capacity in words per minute can be secured by a transmission speed of 44 cycles per second, and with the same gauge of conductors this speed can be obtained over a circuit of approximately 364 miles in length, or if 100-lb. conductors be used, over a circuit of 318 miles in length.

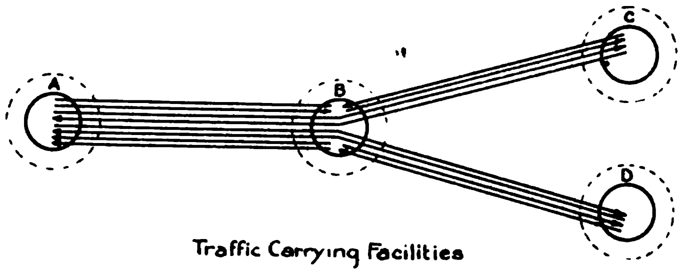
From the speed test figures it has been calculated that a transmission speed of 44 cycles per second could be secured over a 270-mile metallic circuit having conductors of about 75 or 80 lb. weight per mile. Assuming the wire to cost 20 cents per lb., there would be a capital saving in copper of $7560 on a 270-mile circuit, by using a five-unit code instead of a nine-unit code.

Flexibility. An outstanding feature of the multiplex system is the flexibility which it provides in linking up a number of offices with a minimum of line plant. Fig. 40 shows two quadruple duplex circuits radiating from a repeater at station *B*, and providing the following communications:

2	Channels	*A* to *B*	2	Channels	*B* to *C*
2	"	*A* to *C*	2	"	*C* to *B*
2	"	*C* to *A*	2	"	*B* to *A*



Wire Facilities



Traffic Carrying Facilities

FIG. 41

Another arrangement is shown in Fig. 41 which represents a quadruple duplex circuit between *A* and *B* linked up with two double duplex circuits between *B* and *C* and *D*.

This arrangement will provide direct communication between all offices as indicated:

2	Channels	*A* to *B*	2	Channels	*B* to *A*	1	Channel	*B* to *C*
1	"	*A* to *C*	1	"	*C* to *A*	1	"	*C* to *B*
1	"	*A* to *D*	1	"	*D* to *A*	1	"	*B* to *D*
						1	"	*D* to *B*

A third arrangement is that shown in Fig. 42 in which the four arms of a quadruple are shown extended to four duplex printer circuits radiating from one terminal station.

By this means direct communication can be provided between station *A* and four small stations *C*, *D*, *E* and *F*, neither of which perhaps may have sufficient traffic to warrant a direct wire to station *A*.

The duplex printer referred to is a two-channel system using practically the same apparatus as the multiplex, and a small distributor in which the brushes rotate once per letter combination transmitted. No synchronising arrangements are required, and as the brushes are momentarily stopped when they reach the normal or rest position the system has been appropriately termed "The start-stop system."

It will be obvious that by the combination of quadruple or double-duplex multiplex and "start stop" installations it should be possible to effect considerable economies.

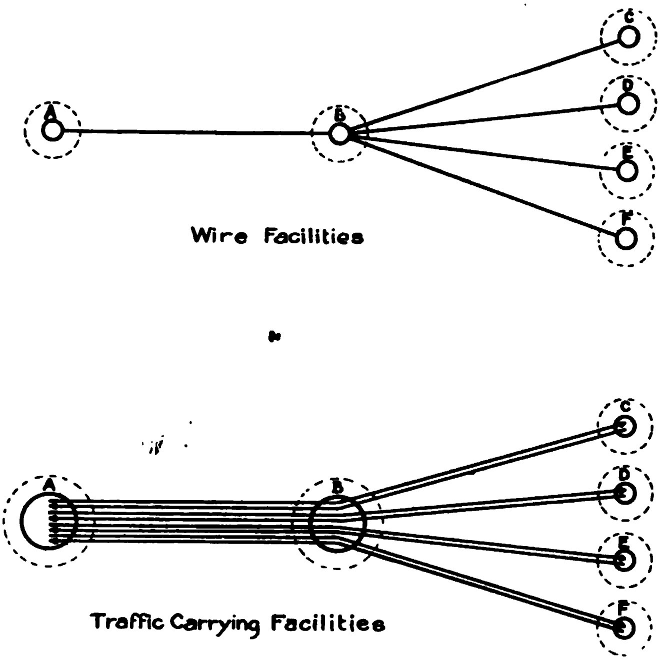


Fig. 42

In addition to the financial savings such circuit combinations help to accelerate the service by reducing the number of times a message is handled, and this in turn tends towards a higher standard of accuracy.

We cannot anticipate the transmission of all telegrams by connecting the originating office with the office of destination as is done in telephone practise but the multiplex in conjunction with the "start stop" will materially help towards developments along such lines.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 19, 1920.

MAXIMUM OUTPUT NETWORKS FOR TELEPHONE SUBSTATION AND REPEATER CIRCUITS

BY GEORGE A. CAMPBELL AND RONALD M. FOSTER

ABSTRACT OF PAPER

Ideal telephone substation and repeater circuits are shown to present output and input requirements which can be met by a type of circuit containing four resistances, each of which has maximum output; maximum output is found to involve biconjugacy, *i.e.*, the four resistances fall into two pairs without active electrical connection between the two resistances of either pair; all circuits meeting these requirements with the minimum number of elements are enumerated; the necessary formulas for proportioning these circuits and also circuits having superflous elements are derived. The discussion of the properties of these four-resistance circuits, to which a consideration of three-resistance circuits is added, is a contribution to the important problem of the properties of the most general circuit containing ideal transformers.

1. INTRODUCTION

ALL networks having the minimum number of elements which can be used to connect four resistances so that an electromotive force, placed successively in each of the resistances, will have the maximum possible output, $E^2/4R$, into the remaining three resistances, and also some circuits with superfluous windings, are enumerated and briefly discussed in this paper. These networks are called "maximum output networks," a term which will naturally be used to include also all networks connecting any number of resistances when a given source of energy in any resistance has its maximum output into the remaining resistances. The term "maximum output circuit" is also employed, and while no hard and fast distinction is made, it is usually used to include the resistances together with the connecting network. The maximum output circuit, consisting of two resistances connected by a transformer having the turns in the two windings in the same ratio as the square roots of the resistances, is well known. With three or any odd number of resistances it can be shown that a maximum output circuit is impossible.

The outstanding property of four-resistance maximum output circuits is that the resistances are conjugate in pairs; the circuits are said to be biconjugate. Thus if (A, B) is one conjugate pair of resistances and (C, D) the other conjugate pair of resistances an electromotive force in A, B, C, D will produce no current in B, A, D, C, respectively. This remarkable property is, as we shall see, the precise condition which is especially called for in telephone substation and repeater circuits. The biconjugacy condition from a practical standpoint may be more important than that of maximum output, but the latter has been chosen as the term for designating this class of circuits since maximum output involves biconjugacy, whereas biconjugacy does not necessarily involve maximum output.

The maximum output and conjugate properties may be illustrated quite simply by taking a Wheatstone bridge having the same total resistance in each of the six branches. An electromotive force in any branch will then have its maximum output into the remaining branches, divided equally between the four adjacent branches, the non-adjacent branch receiving nothing and thus being said to be conjugate. The circuit is thus a maximum output circuit with six resistances and three pairs of conjugate branches.

It is found that the minimum number of elements which the network may contain, provided no unnecessary restrictions are permitted between the input ratios, as defined in Section 4, and the ratios between the four resistances, is six, which may be six windings, three on each of two transformers. The enumeration of the distinct networks which can be formed with two three-winding transformers shows a total of 57,340, all of which have the same theoretical possibilities on the assumption that ideal transformers are employed. In practical applications it is usually allowable to employ a network which imposes one or more restrictions between the input ratios and the resistance ratios. A complete enumeration of such networks shows that there are 23,664 distinct networks involving a three-winding transformer and a two-winding transformer; 2,361 distinct networks made up of two two-winding transformers; 136 networks employing but one three-winding transformer; and 38 distinct networks made up of but one two-winding transformer. The complete list of possible networks thus totals 83,539; and no other networks are possible which do not include a greater number of elements for accomplishing the same end.

Each of these distinct networks presents the possibility of from one to eight distinct proportionings, all of which have the same energy inputs but different combinations of current directions in the branches. The resulting total number of distinct cases is over half a million distributed as shown in Table I.

TABLE I

Transformers	Windings	Restrictions	Distinct networks	Total cases
2	3 + 3	0	57,340	446,234
2	3 + 2	1	23,664	124,536
2	2 + 2	2	2,361	6,596
1	3	2	136	316
1	2	3	38	40
			83,539	577,722

2. Substation Circuits

The telephone substation circuit consists primarily of a transmitter and a receiver with a network for connecting them to a line. The primary requirements are—

1. Sending efficiency the greatest possible consistent with the receiving efficiency.
2. Receiving efficiency such that line noise will interfere as little as possible with telephone conversation.
3. Elimination of side-tone (the disturbance produced in the talker's own receiver).
4. No restrictive limitations as regards the impedances of the transmitter, receiver, and line.

It will be assumed in this paper that any given transmitter has a fixed resistance and when spoken into generates an electromotive force the magnitude of which is independent of the circuit to which the transmitter is connected. A given receiver is also supposed to have a fixed resistance independent of the magnitude or frequency of the current passing through it and its reactance is assumed to be annulled by a suitable impedance associated with the receiver. Similarly, the line reactance will be assumed compensated, leaving a fixed resistance containing a fixed electromotive force arising from the line noise. These fixed resistances of the transmitter, receiver, and line will be designated by T, R, and L.

The substation conditions may now be put in the following mathematical form: Three arbitrarily chosen resistances T, R, L are to be so connected by a network that L will have the maximum possible input into T and R;* the ratio between these two inputs to have an arbitrary value y as found experimentally to be the best for reducing line interference; and the input between T and R to be zero. These conditions are identically those stated in proposition 6 of Appendix I, and from that proposition it follows that the ideal substation circuit must have at least one auxiliary resistance X and if there is but one, making a total of four resistances, they are connected by a maximum output network.

In the engineering design of substation circuits a large number of practical requirements must be met; such as transmitter-current supply, signaling to and by the subscriber, proper allowance for the departure of the line, transmitter, and receiver impedances from the fixed reactanceless resistances assumed in this elementary mathematical treatment, the allowance for the deviations between actual and ideal transformers, and the question of costs. These practical requirements assume different forms, depending upon the particular use to which the substation circuit is to be put, and the best answer can be reached only on the basis of a detailed study of a large amount of experimental data. This side of the problem will not be entered into in this paper, since it is a large problem and does not form a part of a purely mathematical discussion of the available circuits. This paper is limited to the problem of discussing, as fully as may be, all of the possible ideal circuits which should form the starting point for the practical discussion of substation circuits.

3. Repeater Circuits

One of the applications of telephone repeaters is for two-way amplification of telephone conversations on long lines, the repeater being placed near the midpoint of the line. When the person at the east end of the line is talking, the speech wave passes from the line marked E, Fig. 1, to the receiving element R of the telephone repeater, which produces an amplified effect in the transmitting element T which is passed along to the line marked W. When the person at the western end

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*The restatement in this form of condition 1 is possible by virtue of the reciprocal property of the input ratio as explained in section 6 of Appendix I.

of the connection is talking, the path taken by the transmission is not reversed in every particular, since it cannot be reversed through the repeater, but must traverse the repeater from the receiving to the transmitting element in order to be amplified. Its course is thus from W to R to T to E. To make the system as efficient as possible the transfer of energy between E and R, T and E must each be as large as possible, and since the input ratio is a reciprocal property, the combined input from E into R and T must be a maximum. Similarly, the input from W into R and T must be a maximum. Moreover, none of these four inputs can be zero, otherwise there would be no transmission in at least one direction. These conditions are identical with the requirements specified in proposition 7 of Appendix I; the network required for connecting the four elements E, W, R, T is therefore a maximum output network for use with four resistances. The necessary biconjugacy of such networks satisfies two additional requirements for repeater circuits. There is no direct transmission between E and W, which might happen to be superposed on the amplified output in such a way as to produce a certain amount of confusion. Finally, and most important of all, the circuit is non-singing, that is, there is no feeding back of energy from T into R through the network and connected lines E and W, and thus the presence of the source of energy presented by the repeater does not lead to the automatic production of an alternating current which might interfere with the easy reception of speech in case it fell within the audibility range.

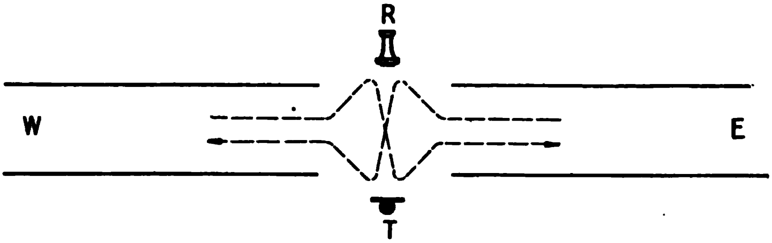

Fig. 1

Repeater networks must not unbalance the lines to which they are connected, and they present, of course, a large number of other practical conditions. These will not be taken up here, but this paper will, however, contribute towards meeting them by discussing the ideal circuits satisfying the above fundamental conditions, without imposing any restrictions upon the magnitudes of the four resistances presented by the two lines and the receiving and transmitting elements of the amplifier.

4. General Properties of Maximum Output Circuits

This investigation of maximum output networks was actually carried out by first determining, some years ago, the possibilities of one two-winding transformer, then of one three-winding transformer, and showing that no further increase in the number of windings on a single transformer would change the restrictions which were imposed on the input ratios and the resistances. When it came to extending the investigation to include all cases where no unnecessary restrictions should be imposed, it became highly desirable, on account of the large number of circuits which must be included, to find a general circuit including all cases, so that the properties of individual circuits could be deduced as special cases. Upon looking at the problem from this viewpoint it was found that the comparatively simple circuit of Fig. 4 presented the general problem; it consists merely of four separate closed loops, each containing one of the four resistances and one winding from each of two four-winding transformers. To distinguish this circuit from others it has been called the master circuit.

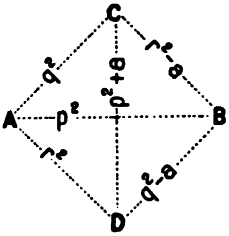

Fig. 2

Certain properties which had been noted in the earlier circuits were found to hold for the master circuit, and the question was raised as to whether they were not fundamental and deducible directly from the conservation of energy. This proved to be the fact, and these general properties will now be considered before proceeding to the master and other concrete circuits.

We assume that one of the four resistances is the source of activity, and that it contains an invariable sinusoidal electromotive force of amplitude E, and has a fixed effective resistance R. Its maximum possible output or external activity has then the definite value $E^2/4R$. In general, the actual output will be only a part of this, and the ratio of this part to the possible maximum is called the output ratio. Each of the other resistances in the circuit will absorb a certain amount of the energy and the ratio of this energy to the maximum possible output $E^2/4R$ is called the corresponding input ratio. The input ratios are positive and not greater than unity, and the sum of all the input ratios for a given impedance paired with every other impedance in the circuit cannot exceed unity.

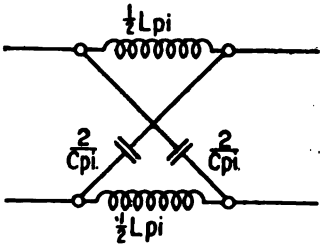

Fig. 3

In case the input ratio between two resistances, neither of which vanishes, is zero, the two branches containing the resistances are said to be conjugate. The input ratio possesses the reciprocal property, being unchanged when the electromotive force is transferred so that the driving and driven points change places, the passive network remaining unchanged.

A circuit containing four resistances presents thus only six distinct input ratios between pairs of resistances. These six input ratios are not entirely independent but must satisfy conditions imposed by the conservation of energy as is shown in Appendix II. These conditions assume very simple form when the circuit is a maximum output circuit: the input ratios for any two mutually exclusive pairs of resistances are equal, so that there are but three distinct input ratios; the sum of the three input ratios is equal to unity. One of the three input ratios must be equal to zero and thus a maximum output circuit has two pairs of conjugate resistances. Such a circuit is said to be biconjugate, and the fact that biconjugacy accompanies the maximum output of four resistances is the outstanding characteristic of these circuits. The formal proof of biconjugacy is given in Appendix I in connection with formula (2) which is the mathematical expression of biconjugacy.

There is also a restriction as to the direction of current flow which is given by formula (5) which assumes the simple form $q\, q_1\, r\, r_1 < O$ of (7) if all driving-driven point impedances of the network are pure resistances. This means that three of the four quantities q, q_1, r, r_1 may have their signs arbitrarily determined, but the sign of the fourth must be chosen so that the product of all four will be negative. Physically, this may be explained as follows: Insert a positive electromotive force in resistance A, and note the direction of the current flow which is produced in resistance C; now transfer the electromotive force to C making its positive direction correspond with the direction of current flow, and note the direction of the current produced in resistance B. Proceed in the same way to transfer the electromotive force to B, and then to D; it will be found that the current which flows in A will have its direction opposed to the electromotive force as originally placed in A.

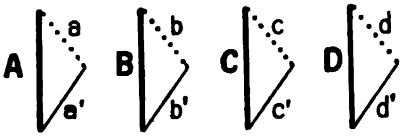

Fig. 4

The three arbitrary sign choices give eight sign combinations, and thus there are for every maximum output circuit eight proportionings which leave the distribution of energy unchanged, but alter the relative direction of current flow in the four resistances. Where the resistances occur in two, three, or four essentially disconnected portions of the circuit the essentially different sign combinations are reduced from eight to four, two, and one, respectively. Thus with the master circuit of Fig. 4, which consists of four entirely distinct loops, the positive direction in each branch is unrelated to the positive direction in the other branches, and the eight sign combinations are not essentially distinct. Because of the alternative sign combinations which are always involved, irrespective of whether they lead to circuits which are practically distinct or not, the mathematical formula for the proportioning of the transformers is complicated when expressed in terms of the input ratios and involves radicals, as is seen by reference to (**30**).

A maximum output circuit with four resistances which does not impose unnecessary restrictions must have at least four degrees of freedom, as is stated in proposition 5 of Appendix I. This is the starting point for the enumeration of all possible circuits employing the minimum number of elements. By permitting two or three restrictions a three-degree circuit may be employed in place of a four-degree circuit, but it is not possible to have a four-resistance maximum output circuit of two degrees of freedom without allowing the resistances to separate into two pairs which do not interact, and such circuits are thus of no value in connection with the applications which we have under consideration.

A circuit containing only four resistances must be a maximum output circuit if three of its resistances have maximum output, or if two have maximum output into the other two, or if one has maximum output into two which are mutually conjugate, as is proved in propositions 6, 7, 8 of Appendix I. These propositions bring the substation set and the repeater circuit within this class of circuit, as was stated above.

Maximum output circuits having more than four resistances present questions of considerable complexity, and a discussion of them is not included in this paper since they are not required for the particular applications under consideration.

5. THE MASTER CIRCUIT

The mathematical treatment of the master circuit is contained in Appendix III, where a rule is given for proportioning the winding turns when the magnitudes of the four resistances, the three input ratios, and the current directions are assigned. The procedure is to calculate the values of P, P_1, Q, Q_1, R, R_1 by formulas (**30**), (**31**), or (**32**), and then use any two of the ratios (**29**), or any linear combinations of the four ratios (**29**) which give distinct proportions. Thus, all eight windings may be retained, or one winding may be dropped from each of the two transformers; but at least one winding must remain in series with each of the four resistances.

In practical applications, it may be necessary to consider the effect of either accidental or intentional variations in the magnitudes of the resistances, or of the turns, which will cause the circuit to depart from exact conformity with the maximum output and biconjugate conditions assumed as the basis of the above rule. Accordingly, a generalized rule is given in formula (**41**) which may be employed for discussing the behavior of the master circuit under any conditions.

In the master circuit we have what is probably the highest possible degree of symmetry among the resistances and windings. This may be of importance in practical applications where the electrical balance of the circuit calls for symmetry, or where a symmetry of the circuit will reduce the effect of the departure of practical transformers from ideal operation.

6. UNRESTRICTED MAXIMUM OUTPUT NETWORKS EMPLOYING THE MINIMUM NUMBER OF WINDINGS

The master circuit with two windings eliminated, is an unrestricted maximum output circuit employing the minimum number of windings; permutation of the resistances gives a total of only six distinct circuits. That permutation of the resistances in the remaining fifteen four-degree geometrical circuits should increase this total to 57,340, or 446,234 cases if attention is paid to the direction of current flow, is one more striking illustration of the large number of possible combinations presented by even a few elements.

All of the distinct master circuits can be made up by the use of two ideal transformers each having six windings with turns in the proportion $P : P_1 : Q : Q_1 : R : R_1$ as is shown by the rule (**29**). Two 18-winding ideal transformers, having the same six winding turns in triplicate, are sufficient to give all of the 446,234 cases, as is shown by the formula (**50**) employed in stating the rule in Appendix IV for calculating the winding turns for all of these cases. Each winding may involve from one to six of the windings connected, either aiding or opposing, but it never requires two windings such as P, and so each set suffices for a winding in every possible case.

The complete list of these circuits is presented by Table V in the form of 16 charts, one for each of the four-degree circuits of Fig. 5. The interpretation of the chart will be understood by considering the meaning of the insert in the chart for circuit **M**, the first line of which reads $M\,2463\,AC\,56.12 = 672$. The first group of four figures comprises the four branches

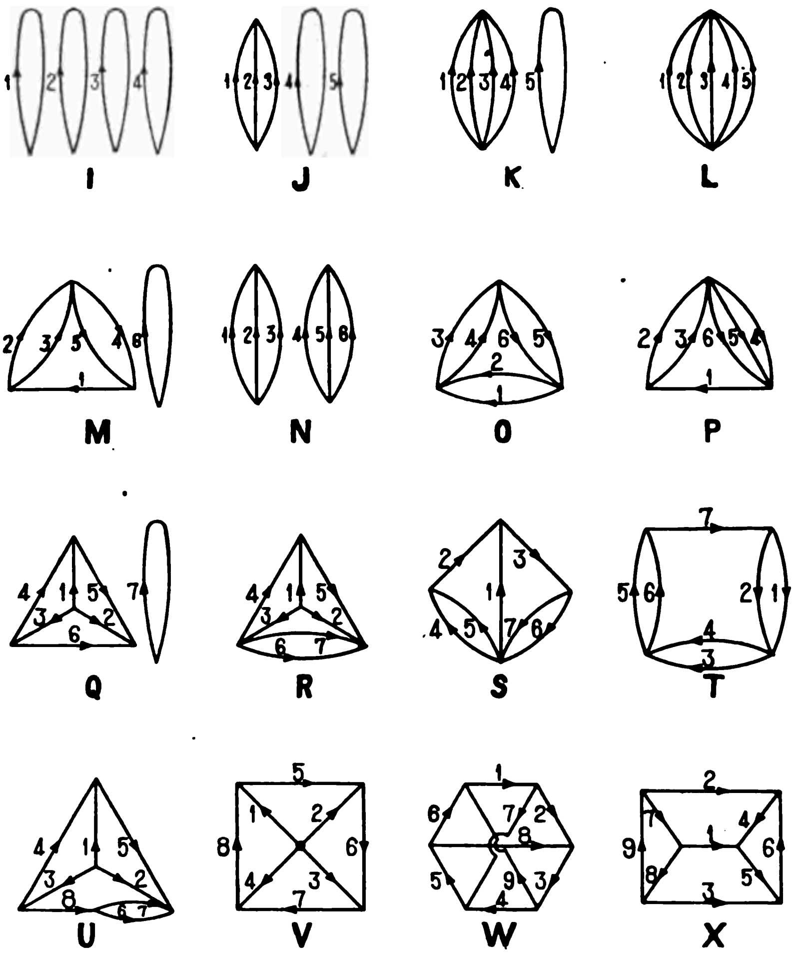

FIG. 5—GEOMETRICAL CIRCUITS WITH FOUR DEGREES OF FREEDOM

in which the four resistances are placed; A is a reference letter used in the body of a chart to show every distribution of transformer elements with which this resistance distribution occurs (for example the first entry of A shows that the two transformers may have their windings in branches **1**, **2**, **5** and

1, 2, 6); the next part of the insert is 56, which is the number of times the reference letter A, or C which includes A in combination with other reference letters, occurs in the chart; 12 is the number of permutations of the four resistances, considered non-interchangeable, among the four branches, 2, 4, 6, 3 which give distinct geometrical circuits; 672, the product of 56 and 12, is the total number of distinct circuits for this resistance grouping in the assigned branches. The second line shows 780 cases for the only other possible distinct group of branches for the resistances, and the total of all distinct circuits obtainable with the geometrical circuit **M** is given as 1452.

The method of making these charts is sufficiently evident. It is necessary to make the distribution of resistances such that they enjoy four degrees of freedom; to count the distinct permutations of resistances in the geometrical circuit; to make each transformer enjoy three degrees of freedom, and the two transformers taken together four degrees of freedom; to have at least one element in every branch of the circuit; and finally to apply the test for distinctness of circuits at each step. Crosses in the charts indicate the failure of the six windings to utilize the required number of degrees of freedom.

Table II under the heading "Windings 3+3" gives the total number of distinct circuits for each of the fifty-five distributions of resistances in four-degree circuits, classified according to the maximum number of circuits which differ only in permutations of resistances. These permutations depend upon both the asymmetry of the fundamental geometrical circuit and the asymmetry introduced by the transformer windings. In Table V the listed permutations are based entirely upon the asymmetries of the geometrical circuit; the asymmetries of the windings are taken care of in the body of the charts which makes this enumeration agree with that of Table II.

Typical cases which will serve as examples illustrating the winding ratios required, are shown by Table III which contains, among others, all symmetrical cases having less than 12 permutations. The first line, for example, shows that taking the four-degree circuit L and placing the resistances and windings in the branches whose numbers appear immediately below the letters designating these resistances and windings, *viz.*, **1234**, **125**, **345**, we have an unrestricted maximum output circuit provided the windings on one transformer are in the ratio $(P_1 + Q - R)$: $(P_1 - Q_1 + R_1)$: P_1 and on the other

TABLE II

Enumeration of Distinct Maximum Output Networks. With Resistance Permutations.

Four degree circuit . 5	Resistances in branches	Windings in branches (part)	Windings and Permutations																		
			Windings 3 + 3					Windings 3 + 2				Windings 2 + 2				Windings 3 + 1			Windings 2 + 1		
			3	4	6	12	24	4	6	12	24	3	6	12	24	4	12	24	3	12	24
I	1234	1234			1																
J	4125	3415				4	4			1	2										
K	1253	4512				5	3	1		3	2						1				
L	1234	5123	1			3		1		2						1					
M	2463	1526				4	26			2	15				1						
	1	3561,2				7	29			1	16			1	2						
N	1245	3614	1			6	7			2	4			1							
O	1235	4612,3			2	11	19			5	17			2	3		1	2			1
P	2451	3645,2,1				11	41			6	28				6						
	3	1624			1	7	18			4	11				1		1				
	6	1345				8	9	1		7	6				1						
Q	4572	6137				7	31			5	14			1	1						
	6	12374				2	9			3	2			1							
R	6732	45167,3				5	21		1	2	9			1	1						
	31	4526				10	53			3	30				3						
	35	4216			1	10	22		1	4	14			1	1						
	6324	7516,3,2					131				76				13		1	1		1	
	1	7524,1				12	63			6	31			3	4						
	11	7426,5,1				14	65			8	36			3	4		1		1		
S	4623	1574,2				5	67			1	34			1	4						
	57	12346				4	8			1	3										
	27	1356				7	52			2	25				2						
	21	3571,2,4.					127				67				11						
	51	2374				7	49			2	27				2						
T	1352	7461				8	22			1	18				1						
	7	2467,1		1		10	18			1	11			1	1						
U	6732	45186			1	4	14			3	3										

TABLE II (*Continued*)

Enumeration of Distinct Maximum Output Networks. With Resistance Permutations.

Four degree circuit Fig. 5	Resistances in branches	Windings in branches (part)	Windings and Permutations																		
			Windings 3 + 3					Windings 3 + 2				Windings 2 + 2				Windings 3 + 1			Windings 2 + 1		
			3	4	6	12	24	4	6	12	24	3	6	12	24	4	12	24	3	12	24
	41	53286				2	31				8										
	42	58186			1	5	15			3	3										
	6832	45176,8,3				7	46			6	12			1	1						
	42	5317				13	49			8	18			1	1						
	41	5327					111				44				3						
	6841	52874,1				4	40			3	9			1	1						
	15	4287				10	48			9	18			1	1						
	42	5187					107				45				3						
V	5123	46785,1,2					86				25				3						
	34	2678					108				47				3						
	5614	2378					112				46				3						
	17	2348					117				45				3						
	13	24785,1				2	49				16			1	1						
	5712	34685,1				9	44			2	12			3							
	13	2468				9	52				23			3							
	5678	12345			2	6	11			2	3	1	1								
W	1234	56789				2	22			1	4										
	5	46789				7	22			2	4										
	9	456782				2	3														
X	1247	356891,4				2	7														
	48	35679					47				9										
	69	34578				7	22			2	4										
	68	34579				3	24			2	4										
	1697	23458					50				10										
	47	23589					45				9										
	48	23579					46				9										
	4786	12359				2	21			1	4										
	9	123567,8					12														
Total.			2	1	9	263	2,255	3	2	116	927	1	1	27	85	1	5	3	1	1	1

ABLE III

at Circuits with Minimum

Cu Fig.	Ohing Ratios x′	Permutations of resistances	Sign combinations	Total cases
L	$P - Q + R_1$	3	8	24
	P	3	4	12
T	$- P - Q - R$	4	8	32
I	Q	6	1	6
O	$- P_1 + Q$	6	8	48
O	$- P - Q + Q_1 - R +$	6	8	48
P	$P - Q - R_1$	6	8	48
R	$- P_1$	6	8	48
U	$- Q + R_1$	6	8	48
U	$- Q + R_1$	6	8	48
V	P	6	8	48
V	$- Q_1 - R_1$	6	8	48
	a this table)........................	57,340		446,234
L	$P + Q_1 - R$	4	6	24
K	Q_1	4	3	12
P	$- P_1 + Q - R$	4	6	24
R	$- Q + R_1$	6	7–8	44
R	$P + Q - R_1$	6	7–8	44
	on not in this table)................	23,664		124,536
V	P 57, 368)	3	4	12
V	P	6	4	24
	ons not in this table)................	2,361		6,596
L (	I	4	4	16
K (	II	12	1	12
O (	II	12	2	24
P (	I	12	4	48
R (	II	12	2	24
R (	I	12	4	48
O (	II	24	2	48
O (	II	24	2	48
R (	II	24	2	48
	....	136		316
R (	V 56)	3^9	2	4
R (	III	12	1	12
O (	III	24	1	24
	....	38		40
	....	83,539		577,722

1 ɔy the infinite impedance
? branches.

2 mn of Fig. 7 are include
luded in the total.

3

4

5

6

[CAMPBELL & FOSTER]

transformer in the ratio $(P - Q + R_1) : (P + Q_1 - R) : P$. Moreover this circuit is symmetrical with respect to branches 1 and 2 which may be interchanged without giving a distinct circuit; similarly with respect to branches 3 and 4; and finally there is a third symmetry which does not alter the circuit, if branches 1 and 2 are changed with branches 3 and 4, respectively. This leaves three distinct permutations for the resistances, and since circuit L has eight sign combinations for the currents, the total number of different cases is 24.

In place of two transformers each having three windings, we might employ one transformer having a two-mesh core and six windings, since the two types of transformers are shown by Appendix VI to be exactly equivalent when they may be regarded as ideal transformers. If two windings, one from each of the two transformers, are in series, the two-mesh transformer may be arranged to replace them by one winding; a total of five windings will then be sufficient. With actual transformers there are practical differences between single-mesh core and multi-mesh core types, which has led to a proposal that multi-mesh core transformers be used, the original suggestion having been made by Mr. K. S. Johnson.

7. MAXIMUM OUTPUT CIRCUITS EMPLOYING ONE TRANSFORMER

If certain relations hold between the resistances and the input ratios, one or two of the windings of one or both of the three-winding transformers may require only zero turns, when certain circuit connections are employed; these special cases have been included in Tables I, II, and III, and the restrictions or relations imposed are given by Table III. The more important, of the special cases are those in which one of the two transformers may be entirely eliminated; this happens when two of the three windings vanish, leaving the other winding to open effectively the branch in which it occurs on account of the infinite impedance it introduces, and thereby reduce the degree of the circuit from four to three.

All of the 174 distinct maximum output circuits with four resistances and one transformer having no superfluous windings are shown in Fig. 7*, distinct symbols being employed to represent the transmitter T, the receiver R, the line L, auxiliary

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*The correctness of this enumeration of the 174 circuits has been independently checked by Mr. F. W. McKown.

into the transmitter and receiver, respectively. These limitations are also given in graphical form by Fig. 8 where the two resistance ratios are plotted for each of the circuits for both $y = 1$ and $y = 1.5$. The curves occurring on this drawing are marked with reference letters *A*, *B*. . . . *M*, *M'*, *N*, *N'* which also occur on Fig. 7, by means of which the curve applying to any one of the 174 circuits may be readily found. It will be noted that a few of the curves have more than one branch for a given value of *y*, *e. g.*, *A*, *N*, *N'*, *M*, *M'*, the last of which falls off the drawing.

Fig. 8 shows, for example, that if $(T/L = 1,\ R/L = 2)$ no one-transformer circuit can be adapted to give maximum output with any value of y in the range from 1 to 1.5. If, however, $(T/L = 2,\ R/L = 2,\ y = 1)$ all of the circuits bearing the reference letters *A*, *B*, *E*, *F*, and *G* may be employed; these are circuits *I*, 1-28; *II*, 1-54; *III*, 1-3; *IV*, 13-24 of Fig. 7, a total of 97 distinct circuits from which to choose in making practical application.

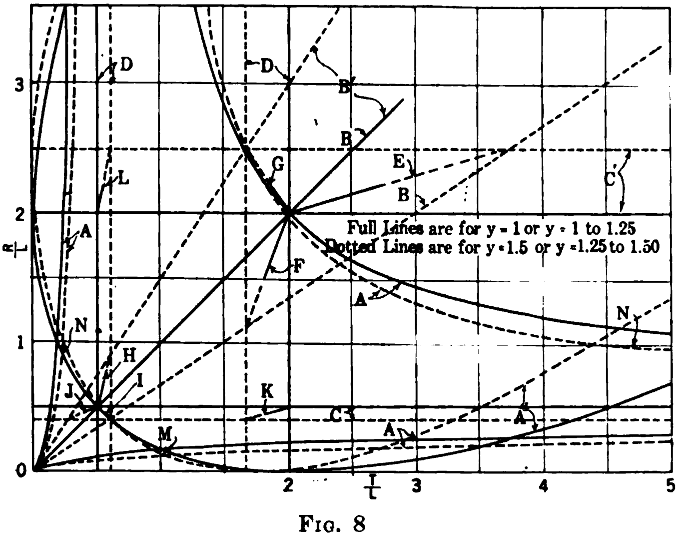
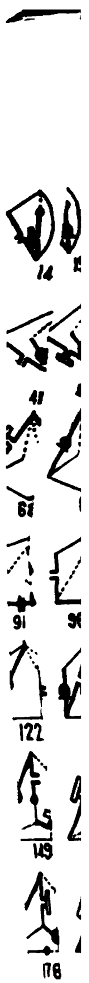
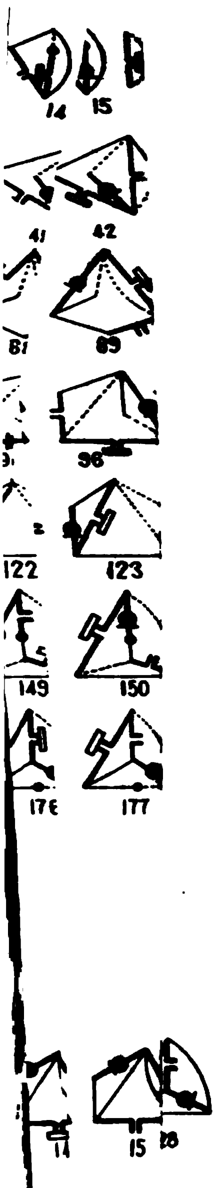


FIG. 8

8. MAXIMUM OUTPUT CIRCUITS WITH SUPERFLUOUS WINDINGS

The enumeration of maximum output circuits, with the minimum number of windings, has been carried through for





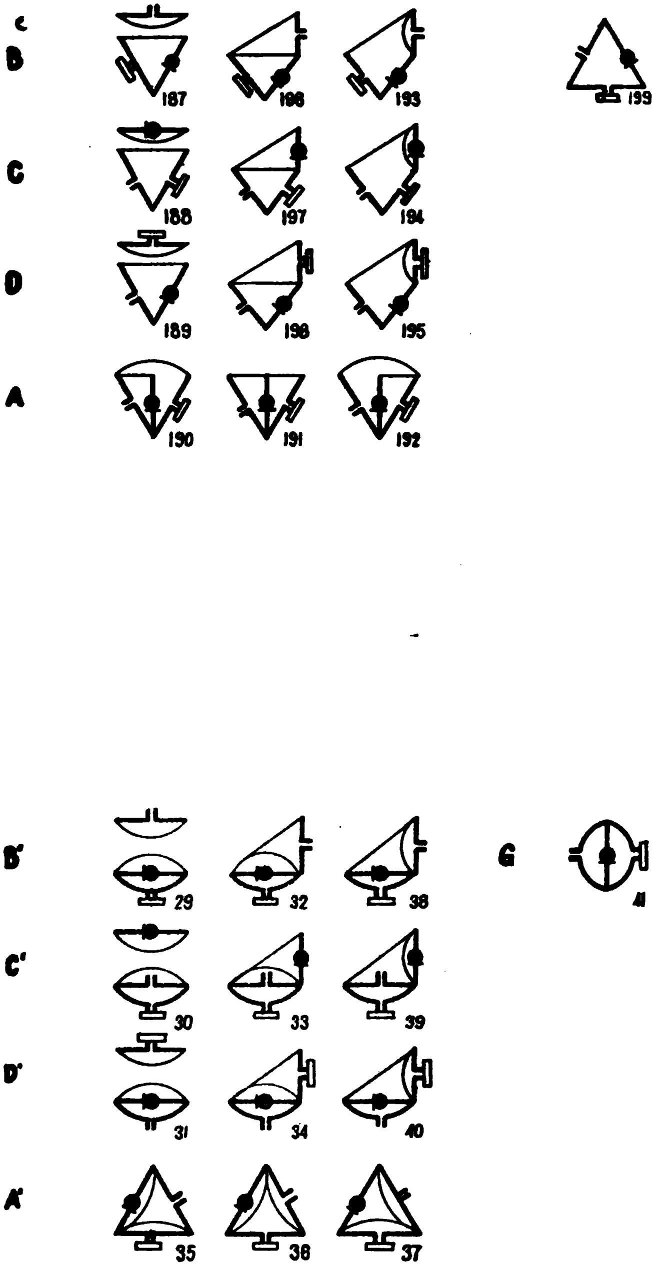


FIG. SUBSTA

[CAMPBELL & FOSTER]

the purpose of facilitating the selection of the simplest possible circuit for practical applications. In many cases, however, conditions will be presented which cannot be adequately met by the use of just these circuits, and it is necessary to turn to others which have a somewhat larger number of windings. When no limit is placed on the number of windings, there is also no limit to the number of possible maximum output circuits.

A group of circuits which seem to present certain broad practical possibilities are the fifty-five four-degree circuits with four resistances of Table II, when employed with a winding in each branch from each of two transformers. This gives from two to twelve extra windings and permits the introduction of additional symmetries. The circuit having two superfluous windings is the master circuit which is important on account of the high degree of symmetry it presents.

Maximum output circuits may be obtained from any geometrical circuit with four or more degrees of freedom by inserting four resistances in four branches which utilize four degrees of freedom, and then proportioning two or more ideal transformers so as to meet the master circuit requirements for the desired choice of conjugate pairs of resistances, input ratios, and directions of current flow. A possible way of proportioning the transformers is given in Appendix IV.

9. CIRCUITS COMPOSED OF THREE RESISTANCES AND IDEAL TRANSFORMERS

The ordinary telephone substation employs a transformer, or induction coil, to connect the three resistance elements, transmitter, receiver, and line, and includes no balancing resistance. In order that a comparison of these circuits with the maximum output circuits may readily be made by any one interested in substation circuits, these circuits will be discussed and all possible cases involving the minimum number of elements will be listed.

Appendix V gives the mathematical discussion from which it appears that there are two entirely distinct types of circuit, for one of which the plus sign of (54) holds, and for the other the minus sign. These will be called the series and parallel types of circuit, respectively, since the two types have the directions of current flow corresponding to the simple series and parallel connection of three resistances. This distinction was first pointed out, and the terminology adopted by, Mr. K. S. Johnson, who made a systematic investigation and enumeration of the 54 circuits obtainable with three resistances and not more than one transformer. While the two types of circuit differ as regards the directions of current flow, the energy relations which depend upon the squares of p, q, r are the same for both types. One resistance may be made to have its maximum output divided arbitrarily between the other two resistances; but it is not possible to have maximum output from all three resistances, or from two resistances, unless the third resistance is effectively disconnected from the system.

The enumeration of all circuits containing the minimum number of elements which impose no restrictions, in addition to those which are inherent between the input ratios and the resistance ratios, has been made in the same matter as for the four-resistance maximum output circuits. The following totals were found:

TABLE VII

Type	Transformers	Windings	Restrictions	Distinct circuits	Total cases
Series................	2	2+2	0	186	681
Parallel..............	1	3	0	28	97
Series................	1	2	1	12	24
Parallel..............	1	2	1	12	24
Series................	0	0	2	1	1
Parallel..............	0	0	2	1	1
				240	828

All 240 distinct circuits are presented by Fig. 9 where the three resistances are drawn in heavy lines, the windings of one transformer in light lines and of the other transformer dotted. On Fig. 9 series type circuits appear above, and parallel type circuits below; all circuits imposing no restrictions occur in the left blocks, those imposing one restriction in the middle blocks, and those imposing two restrictions in the right. Several different restrictions are involved which are designated by the letters A, A', B, B', C, C', D, D', G, I placed at the left of each row of circuits to which they apply. The same letters are found on Fig. 8 where the restrictions between the values of R/L, T/L, and $y = p^2/q^2$ are shown graphically, on the assumption that $p^2 + q^2 = 1$ so that the line L has maximum output.

All the information necessary for calculating the turn-ratios of the transformer windings, to give maximum output of any one of the three resistances divided in any desired proportion between the other two resistances and with any possible directions of current flow, is given in Table VI. This table is similar to Table III and does not require special explanation. Since Table VI gives the complete rules for proportioning each of the circuits, it is not necessary to go back to the formulas of Appendix V in making the calculation for a particular circuit.

Without the restriction to the minimum number of elements there is no limit to the total number of possible circuits. Any circuit with three or more degrees of freedom, having the three resistance elements in branches utilizing three degrees of freedom, and a winding from each of one (two) or more ideal transformers in every branch, may be proportioned to be an unrestricted parallel (series) type circuit. Every such circuit is equivalent to one of the 214 unrestricted circuits shown on Fig. 9, so far as the electrical behavior in the resistance elements is concerned, as is readily seen from the properties of ideal transformers stated in Appendix VI.

Two two-winding transformers so connected into a circuit that one, and only one, closed path is formed by windings, are equivalent to a three-winding transformer, and for some purposes may be regarded as of the same order of complexity. Accordingly it may be noted that there are 186 unrestricted parallel type circuits each employing two two-winding transformers, equivalent in electrical behavior to the 28 unrestricted parallel type circuits using a three-winding transformer shown on Fig. 9.

APPENDIX I

GENERAL PROPERTIES OF MAXIMUM OUTPUT NETWORKS AND CIRCUITS

Without going into details with respect to the network to be employed, a number of important properties of maximum output networks and circuits can be deduced from the conservation of energy and other fundamental considerations. These properties will first be stated as eleven propositions and will then be proved.

1. The network connecting the four resistances can contain

no resistances, or other sources of dissipation, and must thus be made up entirely of reactances.

2. Between the six pairs of the four resistances ($A, B; C, D$) ($A, C; B, D$) ($A, D; B, C$) there are but three input ratios p^2, q^2, r^2; their total is equal to unity, and at least one of the three must vanish, *i. e.*,

$$p^2 + q^2 + r^2 = 1 \tag{1}$$

$$p\,q\,r = 0 \tag{2}$$

3. The driving point impedances S_{11}, S_{22}, S_{33}, S_{44} and driving-driven point impedances S_{12}, S_{13}, etc., or one-point and two-point impedances between points 1, 2, 3, 4 located in the four resistances A, B, C, D have the following magnitudes:

$$\left.\begin{aligned} S_{11} &= 2\,A, \quad S_{22} = 2\,B \\ S_{33} &= 2\,C, \quad S_{44} = 2\,D \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} |\,S_{12}\,| &= \frac{2\,|\,\sqrt{A\,B}\,|}{|\,p\,|}, \quad |\,S_{34}\,| = \frac{2\,|\,\sqrt{C\,D}\,|}{|\,p\,|} \\ |\,S_{13}\,| &= \frac{2\,|\,\sqrt{A\,C}\,|}{|\,q\,|}, \quad |\,S_{24}\,| = \frac{2\,|\,\sqrt{B\,D}\,|}{|\,q\,|} \\ |\,S_{23}\,| &= \frac{2\,|\,\sqrt{B\,C}\,|}{|\,r\,|}, \quad |\,S_{14}\,| = \frac{2\,|\,\sqrt{A\,D}\,|}{|\,r\,|} \end{aligned}\right\} \tag{4}$$

4. The phase angles of the two-point impedances are subject to one condition, provided four of these impedances are finite, *viz.*, the angles of the products of non-adjacent finite two-point impedances differ by 180 degrees, *e. g.*,

$$a\,r\,g\,(S_{13}\,S_{24}) = a\,r\,g\,(-\,S_{14}\,S_{23}) \tag{5}$$

a condition which is also contained in the relation

$$\frac{S_{13}\,S_{23}'}{S_{14}\,S_{24}'} = -\,\frac{C}{D}\,,\ \text{where } S_{23}' = \text{conjugate of } S_{23} \tag{6}$$

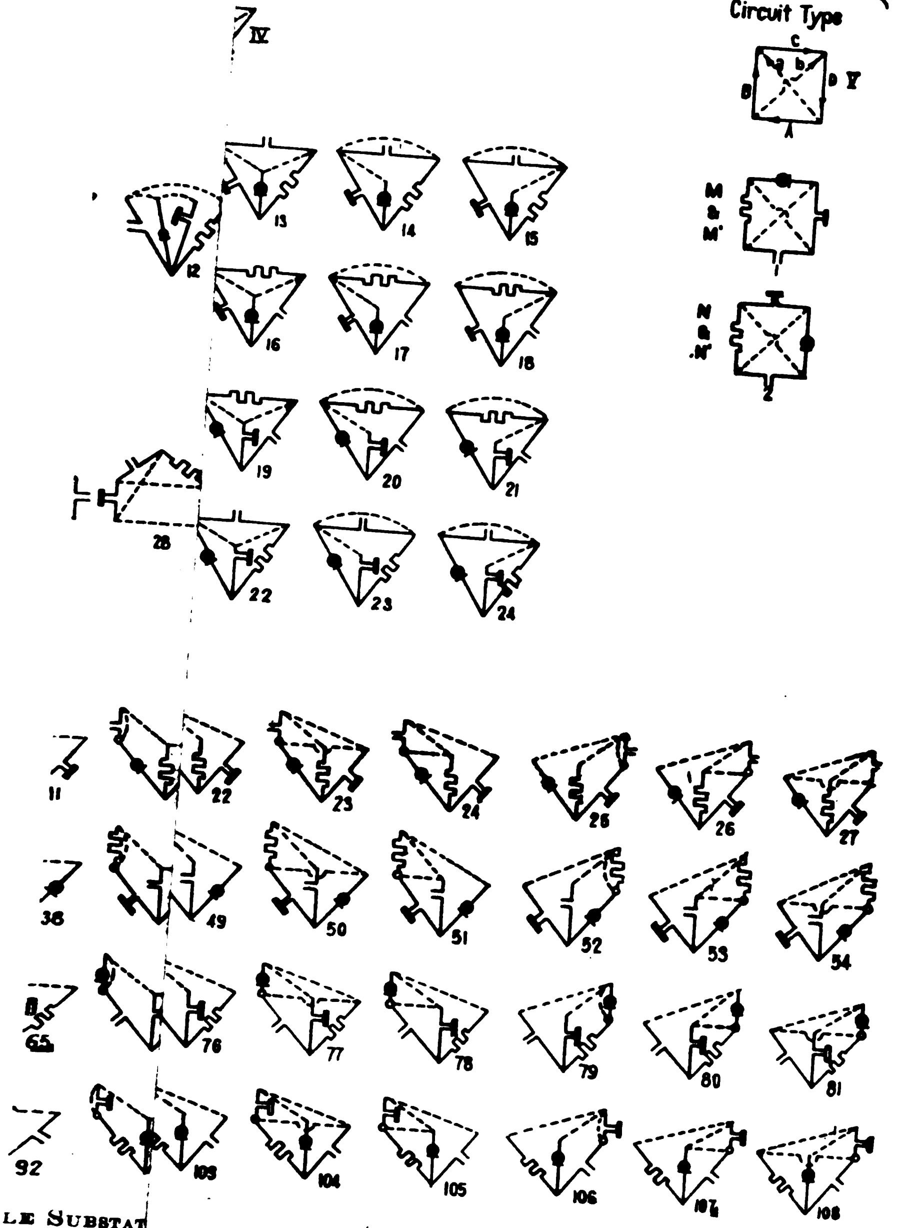
If all S's are real and $p^2 = 0$ we may conveniently use the expressions

$$\left.\begin{aligned} S_{13} &= \frac{2}{q}\left|\sqrt{A\,C}\right|, \quad S_{24} = \frac{2}{q_1}\left|\sqrt{B\,D}\right| \\ S_{14} &= \frac{2}{r}\left|\sqrt{A\,D}\right|, \quad S_{23} = \frac{2_1}{r}\left|\sqrt{B\,C}\right| \\ &q^2 + r^2 = 1, \quad q_1 = \pm\,q, \quad r_1 = \mp\,r \\ &q\,q_1\;r\,r_1 < 0. \end{aligned}\right\} \tag{7}$$

where q, q_1, r, r_1 are real, either positive or negative.

sformer

ps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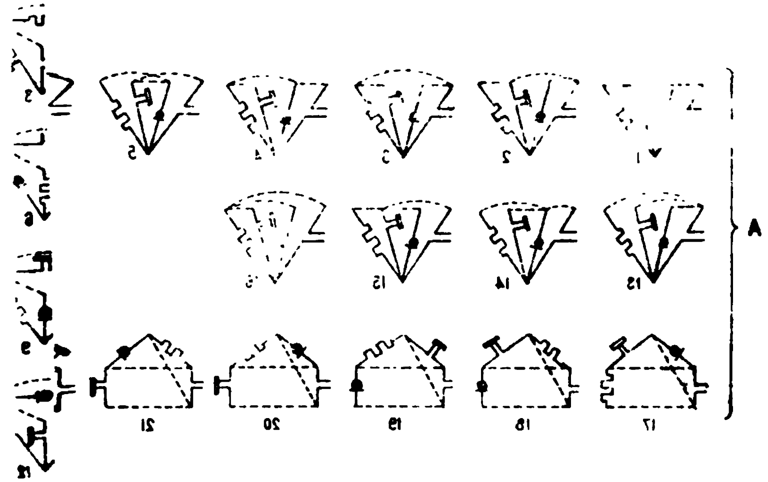

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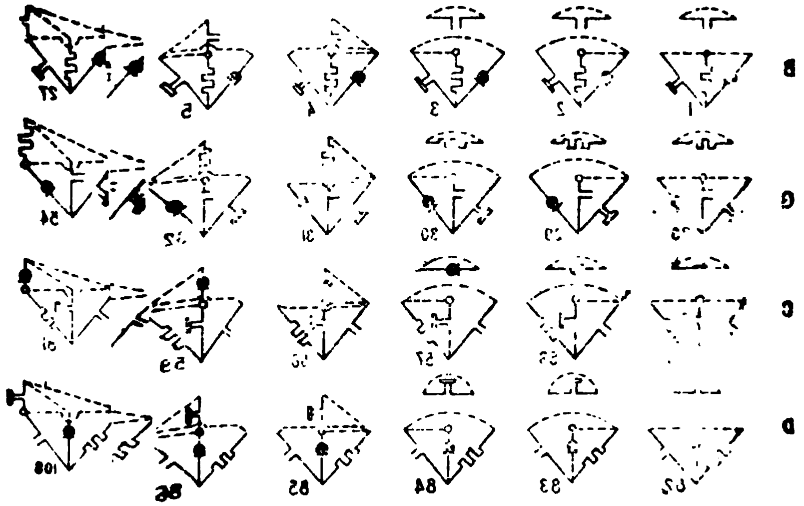

5. A maximum output circuit which imposes no restriction upon the values of the four resistances, and only the necessary restrictions (1) and (2) between the input ratios, must have at least four degrees of freedom; for given resistances and given input ratios the two effective degrees of freedom remaining in a four-degree circuit containing two ideal transformers are sufficient for maximum output.

6. If a circuit contains three resistances, one of which has its maximum output into the other two, (and not into one alone), these two being mutually conjugate, the circuit must contain at least one additional resistance, and if these four resistances are the only resistances, the connecting network is a maximum output network.

7. If two resistances each have their maximum output into the same two resistances, the circuit can include no resistance except these four, and the connecting network is a maximum output network.

8. If a circuit contains but four resistances, of which three have maximum output, then the fourth resistance must also have maximum output.

9. In case all two-point impedances are real, increasing the value of all four resistances in the same ratio leaves the maximum output property unaltered; increasing the value of but two resistances which are conjugate in the same ratio w destroys the maximum output property, but all one-point and two-point impedances are increased by the common ratio $\frac{(w+1)}{2}$; increasing the values of two conjugate resistances (A, B) in the ratios (v, w), respectively, changes the value of p_1 from zero to $\frac{2\,q\,r\,(w-v)}{(w+1)\,(v+1)}$.

10. A maximum output network made up entirely of ideal transformers will have real two-point impedances, and will hold for all frequencies.

11. The two-point impedances of a given maximum output circuit may all be made real at a given frequency, or be given any assigned phase angles consistent with the phase conditions (5) by inserting sections of ideal artificial crossed-capacity lines between the resistances and the network. The artificial

sections leave the maximum output property standing for all frequencies if it held for the original network.

Proofs. It is known that a given electromotive force E, in a given resistance A, cannot have a greater output than $\frac{E^2}{4A}$ into the circuit outside of A, and that it will have this output into any resistance B, provided the two resistances are connected by an ideal transformer of suitable ratio. Since the total possible output from a resistance can be transferred to any second resistance, it follows that the maximum output network, as defined above, cannot itself absorb any of the energy, and in consequence cannot contain any resistances or any other dissipative sinks; this is proposition 1.

Proposition 3 giving the magnitude of the S's follows from the definitions of p, q, r and the S's. The values given in (7) follow by condition (5) as the most general possible if the S's are real.

In Fig. 2 let p^2, q^2, r^2 be the input ratios for the energy received in B, C, D, respectively, from an electromotive force in A, and therefore $p^2 + q^2 + r^2 = 1$, since by assumption the total output into B, C, D is the maximum possible. These input ratios enjoy the reciprocal property, that is, if the electromotive force is transferred from A to B, the amount of energy received in A will still be a p^2 part of the maximum which any network could transfer from B to A. With this reciprocal property recognized it can now be shown, as a condition imposed by energy conservation, that the remaining three input ratios of the maximum output circuit must also be p^2, q^2, r^2. For suppose that, instead of p^2 between C and D, we had $p^2 + a$; then to make the total output of D and C each equal to unity we must have $q^2 - a$ between D and B and $r^2 - a$ between C and B. But B must also have its maximum output and this requires $a = 0$.

The conservation of energy will be satisfied by virtue of the preceding conditions for an electromotive force in any one of the four resistances, but it must also be satisfied when two independent electromotive forces are simultaneously active in different resistances. For this the conditions are given by (18) with $t_{11} = t_{22} = t_{33} = t_{44} = 1$ since each of the four resistances is to have maximum output. Rewrite the first three of the six conditions (18) as follows:

$$\left.\begin{aligned} t_{13}\,t_{23}' &= -\,t_{14}\,t_{24}' \\ t_{12}'\,t_{23} &= -\,t_{14}'\,t_{34} \\ t_{12}\,t_{13}' &= -\,t_{24}\,t_{34}' \end{aligned}\right\} \qquad (8)$$

and their product is

$$|\,t_{12}\,t_{13}\,t_{23}\,|^2 = -\,|\,t_{34}\,t_{24}\,t_{14}\,|^2$$

which requires that at least one t on each side vanish, and reference to the set of three equations shows that the vanishing pairs must be t_{12}, t_{34}; t_{13}, t_{24}; or t_{23}, t_{14}; thus p, q, or r must vanish for they are of the same magnitude as the t's. This was expressed above by the condition $p\,q\,r = 0$. Proposition 2 with its noteworthy biconjugacy condition is thus proved as a necessary consequence of the conservation of energy.

All but two of the six conditions (18) contain any one of the pairs t_{12}, t_{34}; t_{13}, t_{24}; or t_{23}, t_{14}, and thus all but two are satisfied by the vanishing of any one pair. The remaining two conditions would be satisfied by a second pair of vanishing t's, but we shall assume that this is not the case since the circuit would then fall into two electrically unconnected parts, each containing a pair of resistances, and this arrangement has no interest for us. Let the one and only vanishing pair of t's be t_{12}, t_{34}. The remaining two conditions are, if written in terms of the S's,

$$\frac{S_{13}\,S_{23}'}{S_{14}\,S_{24}'} = -\,\frac{C}{D}$$

$$\frac{S_{31}\,S_{41}'}{S_{32}\,S_{42}'} = -\,\frac{A}{B}$$

both of which are consistent with the necessary magnitudes of the S's as given in (4) and both of which require the angular condition (5).

Proposition 5 may be proved as follows: In case it is p that is zero and all two-point impedances are real, and there is but one electromotive force, which is located in one of the four resistances, the currents in the four resistances must stand in the proportion

$$\left.\begin{array}{lccccccc} & |\sqrt{CD}| & : & 0 & : & q\,|\sqrt{AD}| & : & r\,|\sqrt{AC}| \\ \text{or} & 0 & : & |\sqrt{CD}| & : & r_1\,|\sqrt{BD}| & : & q_1\,|\sqrt{BC}| \\ \text{or}\ q\,|\sqrt{BC}| & & : & r_1\,|\sqrt{AC}| & : & |\sqrt{AB}| & : & 0 \\ \text{or}\ r\,|\sqrt{BD}| & & : & q_1\,|\sqrt{AD}| & : & 0 & : & |\sqrt{AB}| \end{array}\right\} \quad (9)$$

according as the electromotive force is placed in A, B, C, D, respectively, which follows from the values for the one-point and two-point impedances given in (3) and (7). These proportions are not completely independent, for the sum of the first three multiplied by $q\,|\sqrt{B}|$, $r_1\,|\sqrt{A}|$, $-\,|\sqrt{D}|$, respectively, vanishes, with similar linear relations for any three of the four proportions. The circuit taken as a whole has, therefore, but two effective degrees of freedom. The linear relations between the proportions are, however, not independent of q, r, A, B, C, D, and hence the circuit must have inherently at least four degrees of freedom; the effective degrees of freedom may be reduced to two by ideal transformers or other means which impose fixed relations between the currents in different branches. The number of independent transformers, could not be more than two, since each transformer imposes one linear relation among the currents. It is shown in Appendices III and IV that any circuit having four degrees of freedom may be made a maximum output circuit by properly distributing the resistances and the six windings of two suitably proportioned three-winding transformers among the branches, which completes the proof of proposition 5.

To prove proposition 6 let q^2, r^2, $p^2 = 0$ be the input ratios between A, C; A, D; and C, D, where $q^2 + r^2 = 1$ and neither q nor r vanishes. Assume that C does not have its maximum output and insert reactance and an ideal transformer between C and the network, and so proportion them that the output of C shall be a maximum, which must be possible since it has initially an input into A, and it is necessary only to annul the reactance of the remainder of the system and step its impedance up or down so as to make it equal to the resistance C. All inputs from C will be increased in the same ratio and thus q^2 must be increased; the input from A to D will not be changed since D is conjugate to C; the total input from A into all resistances other than C and D may be increased, but cannot be decreased, since it is zero to start with. The total output from A has thus been increased; but this is impossible since A was assumed at the start to have its maximum output. It follows that C also had originally its maximum output. Similarly for D. But by assumption neither q^2 nor r^2 is equal to unity, and therefore there must be at least a fourth resistance in the circuit to receive the output from C and D which is not absorbed by A. If we assume that there is but one additional resistance B, the inputs of C and D into this resistance must be r^2 and q^2, and this ensures that the entire circuit possesses the maximum output property.

To prove proposition 7 let q^2, r^2, $q^2 + a$, $r^2 - a$ be the input ratios between the resistance pairs A, C; A, D; B, D; B, C, respectively, where $q^2 + r^2 = 1$. These are the most general relations which meet the conditions of the proposition and they make the output of C and D equal to $q^2 + r^2 \pm a +$ (input into other resistances than A and B). Since no output can be either greater than unity, or negative, a must vanish, and there can as well be no input into any resistances other than A and B. The circuit is therefore a maximum output four-resistance circuit.

Proposition 8 is proved by substituting $t_{11} = t_{22} = t_{33} = 1$ in (18), which reduces the first three of the conditions to the form (8) used above, and making one of these substitutions in the remaining three conditions which shows that $t_{44} = 1$, and therefore the fourth resistance also has maximum output.

To prove proposition 9 note that multiplying the magnitudes of two conjugate resistances, say C, D, by the factor w, multiplies the associated one- and two-point impedances S_{33}, S_{44}, S_{31}, S_{32}, S_{41}, S_{42}, S_{34} (equals infinity) by the common factor $\frac{(w+1)}{2}$ since the outgoing current for given electromotive forces in these resistances is cut down in this ratio. To determine the new values of the three remaining impedances S_{11}, S_{22}, S_{12}, substitute the values of the seven impedances just found in (16) and (18) the first three of which then give $S_{11} = A\,(w + 1)$, $S_{22} = B\,(w + 1)$, $S_{12} =$ infinite. Thus all of the one- and two-point impedances have been multiplied by the same factor $\frac{(w+1)}{2}$ as stated in the second clause of the proposition; one consequence is that the biconjugate relation is left unchanged. Now increase resistances A, B to $A\,w$, $B\,w$ which changes the one-point impedances at these resistances to $2\,A\,w$, $2\,B\,w$ and leaves them conjugate; since they have maximum output the circuit is now a maximum output circuit by proposition 7. Multiplying the magnitudes of two conjugate resistances A and B by the factors v and w, respectively, multiplies the associated one- and two-point impedances S_{11}, S_{31}, S_{41} and S_{22}, S_{32}, S_{42} by the common factors $\frac{(v+1)}{2}$ and $\frac{(w+1)}{2}$, respectively, S_{12} remaining infinite. Substituting these values in (16) and (18) we find that

$$t_{34} = p_1 = \frac{2\,q\,r\,(w-v)}{(w+1)\,(v+1)}\,.$$

No formal proof of proposition 10 is necessary at this point since networks composed of ideal transformers are investigated in considerable detail in Appendices III and IV.

Proposition 11 follows from the fact that the ideal, artificial crossed-capacity line, Fig. 3, has a pure resistance iterative impedance $\sqrt{L/C}$ which is independent of frequency and may be given any phase angle $2\tan^{-1} 1/2\,p\sqrt{L\,C}$ per section. Three artificial line sections are sufficient, but four, one for each resistance, may be used to give symmetrical expressions for all two-point impedances. The iterative impedance of any section of line is equal to the square root of the product of the open and closed circuit impedances. The phase difference, that is, the lag of the current at the far end behind the current at the transmitting end, is the imaginary part of the propagation constant, which is equal to the anti-hyperbolic tangent of the square root of the quotient of the closed-circuit impedance divided by the open-circuit impedance. For the artificial line section of Fig .3 the open- and closed-circuit impedances are

$$\frac{1}{2}\left(\frac{1}{2}\,L\,p\,i + \frac{2}{C\,p\,i}\right) \text{ and } \frac{2\,L}{C\left(\frac{1}{2}\,L\,p\,i + \frac{2}{C\,p\,i}\right)} \tag{10}$$

respectively. Using these we obtain the values stated above for the iterative impedance and phase angle.

APPENDIX II

CONSERVATION OF ENERGY

In any network made up of resistances, self inductances, mutual inductances, capacities (R_q, L_q, M_{qr}, C_q) with instantaneous impressed electromotive forces (e_q) and currents (i_q, i_r) the conservation of energy demands

$$\sum e_q i_q = \sum R_q i_q^2 + d/dt \left\{ \sum \frac{1}{2} L_q i_q^2 + \sum M_{qr} i_q i_r + \sum \frac{1}{2C_q} \left(\int i_q dt \right)^2 \right\} \qquad (11)$$

and in the special case of steady cisoidal oscillations with impressed electromotive forces E_q cis ($p\,t$), currents I_q cis ($p\,t$), self impedances Z_q, mutual impedances Z_{qr}, this condition becomes, after dropping the common factor cis ($2\,p\,t$),

$$\Sigma E_q I_q = \Sigma Z_q I_q^2 + 2\,\Sigma Z_{qr} I_q I_r \qquad (12)$$

By introducing one- and two-point impedances, the electromotive forces and currents may be eliminated and two sets of conditions of the following forms obtained:

$$\frac{1}{S_{jk}} = \sum Z_q \frac{1}{S_{jq} S_{kq}} + \sum Z_{qr}' \left\{ \frac{1}{S_{jq} S_{kr}} + \frac{1}{S_{jr} S_{kq}} \right\} \qquad (13)$$

$$\frac{1}{2} \left(\frac{1}{S_{jk}} + \frac{1}{S_{jk}'} \right) = \sum \frac{R_q}{S_{jq} S_{kq}'} \qquad (14)$$

or $$t_{jk} + t_{jk}' = \Sigma t_{jq} t_{kq}' \qquad (15)$$

where $$t_{jk} = \frac{2 \,|\sqrt{R_j R_k}\,|}{S_{jk}} \qquad (16)$$

and a prime accent indicates "conjugate of."

With a total of four resistances, there are ten relations of the form (15) which it is convenient to have written out in full:

$$\left.\begin{aligned}
|t_{11} - 1|^2 + |t_{12}|^2 + |t_{13}|^2 + |t_{14}|^2 &= 1 \\
|t_{12}|^2 + |t_{22} - 1|^2 + |t_{23}|^2 + |t_{24}|^2 &= 1 \\
|t_{13}|^2 + |t_{23}|^2 + |t_{33} - 1|^2 + |t_{34}|^2 &= 1 \\
|t_{14}|^2 + |t_{24}|^2 + |t_{34}|^2 + |t_{44} - 1|^2 &= 1
\end{aligned}\right\} \qquad (17)$$

$$\left.\begin{aligned}
(t_{11}-1)\,t_{12}' + t_{12}\,(t_{22}'-1) + t_{13}\,t_{23}' + t_{14}\,t_{24}' &= 0\\
(t_{11}-1)\,t_{13}' + t_{12}\,t_{23}' + t_{13}\,(t_{33}'-1) + t_{14}\,t_{34}' &= 0\\
t_{12}\,t_{13}' + (t_{22}-1)\,t_{23}' + t_{23}\,(t_{33}'-1) + t_{24}\,t_{34}' &= 0\\
(t_{11}-1)\,t_{14}' + t_{12}\,t_{24}' + t_{13}\,t_{34}' + t_{14}\,(t_{44}'-1) &= 0\\
t_{12}\,t_{14}' + (t_{22}-1)\,t_{24}' + t_{23}\,t_{34}' + t_{24}\,(t_{44}'-1) &= 0\\
t_{13}\,t_{14}' + t_{23}\,t_{24}' + (t_{33}-1)\,t_{34}' + t_{34}\,(t_{44}'-1) &= 0
\end{aligned}\right\} \tag{18}$$

In case all t's are real, and we write p, p_1, q, q_1, r, r_1 for t_{12}, t_{34}, t_{13}, t_{24}, t_{14}, t_{23} and u_j for $(t_{jj}-1)$ then (17) and (18) may be written

$$\left.\begin{aligned}
u_1^2 + p^2 + q^2 + r^2 &= 1\\
u_2^2 + p^2 + q_1^2 + r_1^2 &= 1\\
u_3^2 + p_1^2 + q^2 + r_1^2 &= 1\\
u_4^2 + p_1^2 + q_1^2 + r^2 &= 1
\end{aligned}\right\} \tag{19}$$

$$\left.\begin{aligned}
p\,(u_1+u_2) + (q\,r_1 + r\,q_1) &= 0\\
q\,(u_1+u_3) + (p\,r_1 + r\,p_1) &= 0\\
r_1\,(u_2+u_3) + (p\,q + p_1\,q_1) &= 0\\
r\,(u_1+u_4) + (p\,q_1 + p_1\,q) &= 0\\
q_1\,(u_2+u_4) + (p\,r + p_1\,r_1) &= 0\\
p_1\,(u_3+u_4) + (q\,r + q_1\,r_1) &= 0
\end{aligned}\right\} \tag{20}$$

Excluding the cases for which

$$p\,p_1 = q\,q_1 = r\,r_1 \tag{21}$$

which are series, parallel, and other connections of the four resistances that cannot give a maximum output circuit, we find that in all remaining cases the following, of which the first three are independent, are among the consequences of (19) and (20)

$$t_{11} + t_{22} + t_{33} + t_{44} = 4 \tag{22}$$

$$p\,q\,r + p\,q_1\,r_1 + p_1\,q\,r_1 + p_1\,q_1\,r = 0 \tag{23}$$

$$p^2 + p_1^2 + q^2 + q_1^2 + r^2 + r_1^2 = 2 - \frac{(q_1^2 - q^2)(r_1^2 - r^2)}{2(p_1^2 - p^2)} - \frac{(r_1^2 - r^2)(p_1^2 - p^2)}{2(q_1^2 - q^2)} - \frac{(p_1^2 - p^2)(q_1^2 - q^2)}{2(r_1^2 - r^2)} \tag{24}$$

$$= 2 + \frac{p^2 + p_1^2}{2}\left(\frac{q\,q_1}{r\,r_1} + \frac{r\,r_1}{q\,q_1}\right) + \frac{q^2 + q_1^2}{2}\left(\frac{r\,r_1}{p\,p_1} + \frac{p\,p_1}{r\,r_1}\right) + \frac{r^2 + r_1^2}{2}\left(\frac{p\,p_1}{q\,q_1} + \frac{q\,q_1}{p\,p_1}\right) \tag{25}$$

$$p^2 + q^2 + r^2 = 1 - \frac{[\,\Sigma\,(p_1^2 - p^2)(q_1^2 - q^2)\,]^2}{4(p_1^2 - p^2)(q_1^2 - q^2)(r_1^2 - r^2)} \tag{26}$$

$$[r^2 (p^2 + p_1^2 + q^2 + q_1^2)(p\,q_1 + p_1\,q)^2]^2 = 4\,r^4 (p\,p_1 - q\,q_1)^2 + 4\,r^2 (p\,q_1 + p_1\,q)^2 \tag{27}$$

$$2\,p^2 + 2\,q^2 + r^2 + r_1^2 = 2 - \frac{p^2 (r_1 + r)^2}{2\,q^2} - \frac{q^2 (r_1 - r)^2}{2\,p^2}$$

if $p_1 = p,\ q_1 = -q$ (28)

The proof of (14) may be made by inserting in branches j, k the direct and retrograde electromotive forces $e_j = E_j \operatorname{cis}(p\,t)$, $e_k = E_k' \operatorname{cis}(-p\,t)$ which together produce in any branch q the current

$$i_q = \frac{E_j}{S_{jq}} \operatorname{cis}(p\,t) + \frac{E_k'}{S_{kq}'} \operatorname{cis}(-p\,t)$$

Substituting these values for e, i in (11) gives a homogeneous second degree equation in the E's and since the E's are independent, the coefficients of E_j^2 $E_k'^2$ and E_j E_k' must vanish individually. The coefficient of the product $E_j\,E_k'$ is independent of t, since $\operatorname{cis}(p\,t)\operatorname{cis}(-p\,t) = 1$, consequently that part which involves differentiation with respect to t contributes nothing, the other terms give (14), or (15) after multiplication by $4\,|\sqrt{R_j R_k}\,|$. Similarly, condition (13) is found by substituting two direct electromotive forces E_j, E_k with the resulting currents $I_q = E_j/S_{jq} + E_k/S_{kq}$ in (12) and equating the coefficient of $E_j\,E_k$ to zero.

Multiply equations (20) by p_1, q_1, r, r_1, q, p and add in pairs giving the three equations:

$$\begin{aligned} pqr + pq_1r_1 + p_1qr_1 + p_1q_1r &= pp_1\,(u_1 + u_2 + u_3 + u_4) \\ &= qq_1\,(u_1 + u_2 + u_3 + u_4) \\ &= rr_1\,(u_1 + u_2 + u_3 + u_4) \end{aligned}$$

whence either (21) or both (22) and (23) must hold. Assuming the latter alternative, taking the sum of (19) and reducing by means of (22) we find

$$\begin{aligned} &2\,(p^2 + p_1^2 + q^2 + q_1^2 + r^2 + r_1^2) \\ &\quad = 4 + (u_1 + u_2)\,(u_3 + u_4) + (u_1 + u_3)\,(u_2 + u_4) \\ &\qquad + (u_1 + u_4)\,(u_2 + u_3) \end{aligned}$$

and substituting for each term in parentheses its value as given by (20) we obtain (25). Combining terms in (25) which have the same denominator and reducing by (23) we may obtain (24), or by a rearrangement of terms (26); (27) is obtained by eliminating r_1 between (23) and (24).

If $p_1 = \pm p \neq 0$ then (23) may be written

$$p\,(q \pm q_1)\,(r \pm r_1) = 0$$

which requires that either $q_1 = \mp q$ or $r_1 = \mp r$, or in other words if one of the three pairs p, p_1; q, q_1; r, r_1 have the same absolute values then another pair must have the same absolute value and the two pairs will have opposite sign relations.

That condition (21) does not include any circuits having maximum output possibilities is a consequence of proposition 2 above which demands that of the three products $p\,p_1$, $q\,q_1$, rr_1 at least one shall be finite and at least one shall be zero.

A more general discussion of the conditions imposed by the conservation of energy will be presented in another paper.

APPENDIX III

DISCUSSION OF THE MASTER CIRCUIT

Rule: The winding turn ratios $a : b : c : d$ of each transformer of the master maximum output circuit must be equal to the sum, after multiplication by arbitrary parameters α, β, γ, δ of any two or more of the four ratios

$$\left.\begin{array}{cccc} 0 : & P : & Q & : R \\ -P : & 0 : & R_1 & : Q_1 \\ -Q : & -R_1 : & 0 & : P_1 \\ -R : & -Q_1 : & -P_1 & : 0 \end{array}\right\} \qquad (29)$$

where P, P_1, Q, Q_1, R, R_1 *are given by*

$$\left.\begin{matrix} |\sqrt{CD}|\,P & : |\sqrt{BD}|\,Q & : |\sqrt{BC}|\,R & : \\ |\sqrt{AB}|\,P_1 & : |\sqrt{AC}|\,Q_1 & : |\sqrt{AD}|\,R_1 & \end{matrix}\right\}$$

$$= \left\{\begin{matrix} q_1 : q\,r & : -\,q^2 : \\ q \;: q_1\,r & : q\,q_1 \end{matrix}\right\}, \text{ if } p = 0 \qquad (30)$$

$$= \left\{\begin{matrix} r^2 & : -\,r_1 : & -\,p\,r \;: \\ -\,r\,r_1 : & r \;: & p\,r_1 \end{matrix}\right\}, \text{ if } q = 0 \qquad (31)$$

$$= \left\{\begin{matrix} q^2 & : -\,p\,q & : -\,q_1 : \\ q\,q_1 : & p\,q_1 : & q \end{matrix}\right\}, \text{ if } r = 0 \qquad (32)$$

and A, B, C, D *are the resistances*

$p^2 = p_1^2$, $q^2 = q_1^2$, $r^2 = r_1^2$ *are the input ratios and are subject to the further conditions—*

$p^2 + q^2 + r^2 = 1$

$p\,q\,r = 0$

$p\,q = -\,p_1\,q_1$, $qr = -\,q_1\,r_1$, $r\,p = -\,r_1\,p_1$ *which restrict the current directions. The parameters* α, β, γ, δ *and* α', β', γ', δ' *for the two transformers must be such that corresponding turn ratios are not identical; distinct sets of parameters not being necessarily sufficient since but two of the four ratios* (29) *which they multiply are distinct.*

Proof: For the case $p = 0$ the current proportions (9) must all satisfy the ampere-turn condition for each of the two ideal transformers which gives two and only two of the three conditions required to fix the turn ratios $a : b : c : d$ for a transformer since but two of these proportions are independent. The third condition is optional and it is convenient to employ one of the same form as the other two but with arbitrary parameters α, β, γ, δ :

$$a\,\alpha + b\,\beta + c\,\gamma + d\,\delta = 0 \qquad (33)$$

and if use is made of the last two proportions of (9) the two ampere-turn conditions are

$$\begin{aligned} a\,q\,|\sqrt{BC}| + b\,r_1\,|\sqrt{AC}| + c\,|\sqrt{AB}| & & = 0 \\ a\,r\,|\sqrt{BD}| + b\,q_1\,|\sqrt{AD}| & + d\,|\sqrt{AB}| & = 0 \end{aligned}$$

Solving these three equations we find that $a : b : c : d$ is equal to the sum after multiplying by α, β, γ, δ of the four proportions:

$$\begin{array}{lllllll}
0 & : & |\sqrt{A\,B}| & : & r_1\,|\sqrt{A\,C}| & : & -q_1\,|\sqrt{A\,D}| \\
|\sqrt{A\,B}| & : & 0 & : & q\,|\sqrt{B\,C}| & : & r\,|\sqrt{B\,D}| \\
r_1\,|\sqrt{A\,C}| & : & -q\,|\sqrt{B\,C}| & : & 0 & : & (q\,q_1 - r\,r_1)\,|\sqrt{C\,D}| \\
q_1\,|\sqrt{A\,D}| & : & -r\,|\sqrt{B\,D}| & : & -(q\,q_1 - r\,r_1)\,|\sqrt{C\,D}| & : & 0
\end{array}$$

respectively, which is one of the numerous forms in which the above rule (**29**) might be written as is proved by multiplying this form by q_1 and reducing by means of $q\,r = -q_1\,r_1$, $q^2 = q_1^2$, $q^2 + r^2 = 1$.

The four proportions of (**29**) are given for the sake of symmetry, but the solution derived from any two, say the first two:

$$\left.\begin{array}{l}
a : b : c : d \\
\qquad = -\beta\,P : \alpha\,P : \alpha\,Q + \beta\,R_1 : \alpha\,R + \beta\,Q_1 \\
a' : b' : c' : d' \\
\qquad = -\beta'\,P : \alpha'\,P : \alpha'\,Q + \beta'\,R_1 : \alpha'\,R + \beta'\,Q_1
\end{array}\right\} \qquad (34)$$

is perfectly general.

The three formulas (**30**), (**31**), (**32**) may be combined into a single formula in various ways, for example:

$$\left.\begin{array}{l}
|\sqrt{C\,D}|\,P : |\sqrt{B\,D}|\,Q : |\sqrt{B\,C}|\,R : \\
|\sqrt{A\,B}|\,P_1 : |\sqrt{A\,C}|\,Q_1 : |\sqrt{A\,D}|\,R_1
\end{array}\right\}$$

$$= \left\{\begin{array}{l}
\sqrt{1-p^2} : \sqrt{1-q^2} : \sqrt{1-r^2} : \\
\sqrt{1-p^2} : \sqrt{1-q^2} : \sqrt{1-r^2}
\end{array}\right\}$$

$$= \left\{\begin{array}{lll}
1 - p^2 & : p_1\,q_1 - r_1 & : p_1\,r_1 - q_1 : \\
q\,q_1 - r\,r_1 & : r - p_1\,q & : q - p_1\,r
\end{array}\right\}$$

The first formula fails to give the signs and neither formula applies to the circuit except when proportioned for maximum output of all four resistances; a general formula is given below.

The necessary and sufficient conditions in order that the master circuit may be a maximum output circuit are:

$$\left.\begin{array}{l}
A\,B\,[c\,d]^2 = C\,D\,[a\,b]^2 \\
A\,C\,[b\,d]^2 = B\,D\,[a\,c]^2 \\
A\,D\,[b\,c]^2 = B\,C\,[a\,d]^2
\end{array}\right\} \qquad (35)$$

and, since all factors must be positive, this may be put in the equivalent form

$$A^2 : B^2 : C^2 : D^2 = [a\,b]^2\,[a\,c]^2\,[a\,d]^2 : [a\,b]^2\,[b\,c]^2\,[b\,d]^2 \\ : [a\,c]^2\,[b\,c]^2\,[c\,d]^2 : [a\,d]^2\,[b\,d]^2\,[c\,d]^2 \qquad (36)$$

The six quantities $[a\,b]$, $[a\,c]$, $[a\,d]$, $[b\,c]$, $[b\,d]$, $[c\,d]$ are abbreviations for $(a\,b' - a'b)$, $(a\,c' - a'c)$, etc., and satisfy the identity

$$[a\,b]\,[c\,d] - [a\,c]\,[b\,d] + [a\,d]\,[b\,c] = 0 \qquad (37)$$

but are otherwise unrestricted as is readily seen by making $d = a' = 0$, $b' = 1$.

Proof: To establish (35) it is sufficient to show that these conditions are necessary and sufficient to make the one-point impedances at A, B, C, D equal to $2\,A$, $2\,B$, $2\,C$, $2\,D$ as required by (3). The determinant* of the network and its principal cofactor are:—

$$\Delta = \begin{vmatrix} A & 0 & 0 & 0 & a & a' \\ 0 & B & 0 & 0 & b & b' \\ 0 & 0 & C & 0 & c & c' \\ 0 & 0 & 0 & D & d & d' \\ a & b & c & d & 0 & 0 \\ a' & b' & c' & d' & 0 & 0 \end{vmatrix} \qquad (38)$$

$$= \Sigma\,A\,B\,[c\,d]^2$$
$$= A\,B\,[c\,d]^2 + C\,D\,[a\,b]^2 + A\,C\,[b\,d]^2 + B\,D\,[a\,c]^2 \\ + A\,D\,[b\,c]^2 + B\,C\,[a\,d]^2 \qquad (39)$$

$$\Delta_{11} = B\,[c\,d]^2 + C\,[b\,d]^2 + D\,[b\,c]^2$$
$$= \frac{(\text{terms of } \Delta \text{ containing } A)}{A} \qquad (40)$$

The one-point impedances at A and B are

$$S_{11} = \Delta / \Delta_{11} = 2\,A \text{ by } (3)$$
$$S_{22} = \Delta / \Delta_{22} = 2\,B$$

---

*Each ideal transformer introduced into a network adds a border to the network determinant; each term in the border equals the number of turns of the ideal transformer in the circuit corresponding to the row or column in which the term occurs, as is proved in Appendix VI.

and require $\Delta - A\ \Delta_{11} - B\ \Delta_{22} = 0$

or $A\,B\,[c\,d]^2 = C\,D\,[a\,b]^2$

and by symmetry the other two conditions of the set (35) follow; from these three relations (36) is obtained.

Generalized Rule: *Rule* (29) *for the winding turn ratios of the master maximum output circuit may be generalized to permit any assigned values of* p, p_1, q, q_1, r, r_1 *which are real and satisfy the fundamental conditions* (23) *and* (24) *by replacing the right side of* (30, 31, 32) *by*

$$\left\{\begin{array}{l} r\,(q + q_1 v) : r\,(p_1 v - p) : - v\,(p\,q_1 + p_1\,q) : \\ r\,(q\,v + q_1) : r\,(p\,v - p_1) : - (p\,q_1 + p_1\,q) \end{array}\right\} \qquad (41)$$

where v

$$= \frac{r^2\,(p^2 + p_1^2 + q^2 + q_1^2) + (p\,q_1 + p_1\,q)^2 + 2\,r\,(p\,q_1 + p_1\,q)}{2\,r^2\,(p\,p_1 - q\,q_1)}$$

Proof: From (7) by symmetry

$$p = \frac{2\,|\sqrt{A\,B}|}{S_{12}} = \frac{2\,|\sqrt{A\,B}|\,\Delta_{12}}{\Delta}$$

$$= -\frac{2\,|\sqrt{A\,B}|}{\Delta}\Big[C\,[a\,d]\,[b\,d] + D\,[a\,c]\,[b\,c]\Big]$$

where Δ is the determinant (38) and Δ_{12} is the cofactor of the element in the second column of the first row. To abbreviate set

$$\left.\begin{array}{l} L = [a\,b]\,|\sqrt{C\,D}|\,,\ M = [a\,c]\,|\sqrt{B\,D}|\,, \\ \qquad\qquad\qquad\qquad N = [a\,d]\,|\sqrt{B\,C}| \\ L_1 = [c\,d]\,|\sqrt{A\,B}|\,,\ M_1 = [b\,d]\,|\sqrt{A\,C}|\,, \\ \qquad\qquad\qquad\qquad N_1 = [b\,c]\,|\sqrt{A\,D}| \end{array}\right\} \qquad (42)$$

which by virtue of (37) must satisfy the identity

$$L\,L_1 - M\,M_1 + N\,N_1 = 0 \qquad (43)$$

By (39)

$$\Delta = L^2 + L_1^2 + M^2 + M_1^2 + N^2 + N_1^2 \qquad (44)$$

and from the cofactors we derive:

$$\left.\begin{aligned}
p &= -\frac{2}{\Delta}(M N_1 + M_1 N)\\
p_1 &= -\frac{2}{\Delta}(M N + M_1 N_1)\\
q &= -\frac{2}{\Delta}(N L_1 - N_1 L)\\
q_1 &= -\frac{2}{\Delta}(N L - N_1 L_1)\\
r &= -\frac{2}{\Delta}(- L M_1 - L_1 M)\\
r_1 &= -\frac{2}{\Delta}(L M + L_1 M_1)
\end{aligned}\right\} \qquad (45)$$

Let $N = v N_1$ and we find that the ratios $L : M : N : L_1 : M_1 : N_1$ have the values given by the right side of (41). The expression for v is determined by the condition that the value (44) for Δ must be such as to satisfy any one of the equations (45). Substituting we find that this requires

$$(F - G) v^2 + 4 E v - (F + G) = 0 \qquad (46)$$

where $E = r^2 (p p_1 - q q_1)$
$F = 2 r (p q_1 + p_1 q)$
$G = r^2 (p^2 + p_1^2 + q^2 + q_1^2) + (p q_1 + p_1 q)^2$
$G^2 = 4 E^2 + F^2$ by (27)

Equation (46) proves to be essentially a perfect square, giving

$$v = \frac{2E}{G - F} = \frac{G + F}{2E}$$

which upon substituting the values of E, F, G gives the value of v stated in the above rule. This value of v satisfies (43) which is, in the present notation,

$$E v^2 - G v + E = 0 \qquad (47)$$

By substituting in succession $a = 0$, $b = 0$, $c = 0$, $d = 0$ in (42) and comparing with (29), we find that $L : M : N : L_1 : M_1 : N_1$ is the same as the left side of (30) which completes the proof of the generalized rule (41).

From (41) the special cases (30, 31) may be derived by first making $p = \pm p_1, q = \pm q_1, r = \mp r_1$, dividing through by the common factor $2\,p\,q$ and then setting $p = 0$ or $q = 0$. The ratios (41) become indeterminate if $r = 0$ or $p = \pm p_1$, $q = \mp q_1$ but on evaluating by means of (23) we obtain (32). As a practical rule for proportioning the maximum output circuit (30, 31, 32) is much simpler than (41) and has, therefore, been presented as the working rule.

The behavior of the circuit of Fig. 4 has been shown to depend solely upon the six quantities (42) but since the ratio of these quantities only is involved in (45) and also in the expressions for t_{11}, t_{22}, t_{33}, t_{44} which correspond to the one-point impedances and since (43) must hold, there are but four independent parameters involved. This is also the number of independent quantities among p, p_1, q, q_1, r, r_1 in view of conditions (23), (24). The ratios of the turns of two three-winding transformers is also four. The number of independents is thus the same number four in each of the three aspects of the circuit, as it should be.

APPENDIX IV

GENERAL RULES FOR PROPORTIONING IDEAL TRANSFORMERS IN MAXIMUM OUTPUT CIRCUITS

In Appendix III a general rule for proportioning the winding turns for the master circuit has been derived; this rule may now be extended to all other maximum output circuits, by observing that whatever the circuit connections, the operation of the circuit is determined by the ratio of winding turns connecting every two pairs of resistance terminals, as explained in Appendix VI, where it is shown how ideal transformers may be substituted for each other, without changing their effect in the circuit.

Take any geometrical circuit having at least four degrees of freedom. Pick out four branches opening four separate degrees of freedom. Insert in these branches the four resistances and any set of windings on two transformer cores which meet the requirements (29-32) of the master circuit. Add, quite arbitrarily, any number of sets of transformer windings, each set comprising a winding in every branch connected to a selected branch-point, the number of turns counted outward from the branch-point being the same in each winding of the set and the windings all being on one core which may or may not carry other sets of windings including one of the sets corresponding to the master circuit. The transformers may be combined into multi-mesh core transformers, or separated into two-winding transformers as explained in Appendix VI. If a transformer winding is not placed in every branch of the geometrical circuit that branch must be short-circuited.

That this procedure always gives a maximum output circuit follows from the fact that the added sets of windings each give an algebraic total of zero turns in every closed path, and are thus without effect; with these windings short-circuited, the circuit becomes a master circuit, with an added network of short-circuits which have no effect, and may be eliminated.

If the circuit is of the fourth degree, it has been shown that there are 57,340 distinct unrestricted circuits employing two three-winding transformers. The proportioning of these windings might be carried out by means of the above directions, but it is convenient to have a more specific rule such as the following:—

The circuit, the values of the four resistances A, B, C, D, and the values of p, q, r are supposed given, and the values of P, P_1, Q, Q_1, R, R_1 determined by (30, 31, or 32). Select one of the transformers with windings x, y, z; trace the four paths in the circuit which contain no winding of this transformer, no winding but x, no winding but y, and no winding but z, respectively. Assign positive directions arbitrarily for the first path and concurrently with that of the branch containing the winding in the other paths. Form the array

$$\begin{matrix} A_0 & B_0 & C_0 & D_0 \\ A_x & B_x & C_x & D_x \\ A_y & B_y & C_y & D_y \\ A_z & B_z & C_z & D_z \end{matrix} \tag{48}$$

where the element of the array in the ith row and kth column is $+1$, -1, or 0, according as the ith path traverses the kth resistance in the positive direction, the negative direction or not at all. Make the turns in winding x equal to the sum of all the elements in the following array after multiplying each by the corresponding elements in the border row and column which are taken from (48)

$$\begin{array}{cccc|c} 0 & P & Q & R & A_0 \\ -P & 0 & R_1 & Q_1 & B_0 \\ -Q & -R_1 & 0 & P_1 & C_0 \\ -R & -Q_1 & -P_1 & 0 & D_0 \\ \hline A_x & B_x & C_x & D_x & \end{array} \tag{49}$$

or, in expanded form,

$$\left.\begin{array}{l} (A_0 B_x - A_x B_0)\, P + (A_0 C_x - A_x C_0)\, Q + (A_0 D_x - A_x D_0)\, R \\ \quad + (C_0 D_x - C_x D_0)\, P_1 + (B_0 D_x - B_x D_0)\, Q_1 \\ \qquad\qquad\qquad + (B_0 C_x - B_x C_0)\, R_1 \end{array}\right\} \tag{50}$$

This rule may be proved by adding the four proportions of (29) after multiplying them by A_0, B_0, C_0, D_0, respectively, and putting the resulting number of turns with signs reversed directly in series with the four resistances, when the four paths considered above may be shown to have each a total of zero turns; the added windings and the windings resulting from the rule are therefore mutually equivalent.

It may be shown that the expressions of the form $(A_0 B_x - A_x B_0)$ do not assume the values ± 2 since the two terms cancel if both paths contain both resistances which require both to be in the common part of the two paths.

To illustrate the use of this rule, take the circuit which is listed on the eighth line of Table III, *viz.*, circuit **R** with resistances in branches 6, 7, 3, 5 and the two sets of windings in branches 1, 2, 4 and 1, 6, 7. The arrays (48) for the two transformers are found by inspection to be

$$\begin{array}{cccc} -1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 1 \\ 0 & -1 & -1 & 0 \\ -1 & 0 & 0 & 1 \end{array} \qquad\qquad \begin{array}{cccc} 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}$$

and substituting these quantities in formula (50) for each of the windings gives

$$x : y : z = (P + Q + Q_1 - R - R_1) : (P + Q - R_1) \\ : (P + Q_1 - R)$$

$$x' : y' : z' = (-P_1) : (P_1 + Q + R) : (P_1 + Q_1 + R_1)$$

as listed in Table III.

APPENDIX V

CIRCUITS WITH THREE RESISTANCES

The circuits in question have real one- and two-point impedances; (**24-27**) all apply if $p_1 = q_1 = r = 0$ so as to separate the fourth resistance completely from the other three, and if the subscript from r_1 which is no longer needed is dropped (**24**) becomes:

$$\left.\begin{aligned} & p^2 q^2 + q^2 r^2 + r^2 p^2 \pm 2\, p\, q\, r = 0 \quad \text{or} \\ & \frac{r}{p\, q} = \frac{\sqrt{1-(p^2+q^2)} \mp 1}{(p^2+q^2)} \end{aligned}\right\} \tag{53}$$

$$= \mp 1, \text{ if } (p^2+q^2) = 1 \tag{54}$$

$$= \mp \left[\frac{1}{2} + \frac{p^2+q^2}{8} \right] \text{or} \mp \left[\frac{2}{p^2+q^2} - \frac{1}{2} \right]$$

$$\text{if } (p^2+q^2) \text{ is small} \tag{55}$$

In these forms the condition gives the information which is needed in considering side-tone reduction since the energy involved in the side-tone and the energy which reaches the receiver at the listening end of the line (transmission attenuation being neglected) are in the ratio $r^2 : p^2 q^2$ if L, T, R are associated with A, B, C. For a given value of $(p^2 + q^2)$, the total output of A, there are two possible values for the ratio $r^2 : p^2 q^2$; we are interested primarily in and will confine our attention to the smaller value. For very small values of the total output $(p^2 + q^2)$, the value of the ratio is shown by (**55**) to be 1 : 4, which is also its minimum possible value; the ratio increases continuously with increasing values for $(p^2 + q^2)$, becoming equal to 1 : 1 when $(p^2 + q^2)$ attains its maximum value, unity, as is shown by (**54**). The amount of energy then appearing as side-tone is equal to the amount of energy delivered to the receiver at the listening substation (the effect of line attenuation being ignored). This is not the nearest approach to side-tone elimination, for, as we saw above by making $(p^2 + q^2)$ small, the amount of energy appearing as side-tone can be reduced to one quarter of this amount, but this improvement is not enough to be practically satisfactory, and it is obtained only at a serious sacrifice in the absolute value of the volume of receiving at the listening station. The mathematical work follows that for four-resistance circuits, so only an outline and the results are given below:

Series Type Circuit. Circuit D, Fig. 6, symmetrical in $A, B, C, a, b, c, a', b', c', r, q, p$.

$$p^2 q^2 + q^2 r^2 + r^2 p^2 = 2\,p\,q\,r \tag{56}$$

$$\Delta = \begin{vmatrix} A & 0 & 0 & a & a' \\ 0 & B & 0 & b & b' \\ 0 & 0 & C & c & c' \\ a & b & c & 0 & 0 \\ a' & b' & c' & 0 & 0 \end{vmatrix}$$

$$= \Sigma A\, [b\, c]^2 \tag{57}$$

$$\Delta_{11} = [b\, c]^2 \tag{58}$$

$$\Delta_{12} = -\, [a\, c]\, [b\, c] \tag{59}$$

$$\left.\begin{aligned} p &= -\ 2/\Delta \mid \sqrt{A\,B} \mid [a\,c]\ [b\,c] \\ q &= +\,2/\Delta \mid \sqrt{A\,C} \mid [a\,b]\ [b\,c] \\ r &= -\ 2/\Delta \mid \sqrt{B\,C} \mid [a\,b]\ [a\,c] \end{aligned}\right\} \tag{60}$$

Hence

$$[b\,c] : [c\,a] : [a\,b] = p\,q \mid \sqrt{B\,C} \mid : r\,p \mid \sqrt{A\,C} \mid : q\,r \mid \sqrt{A\,B} \mid \tag{61}$$

and the special two-winding cases are:

$$\begin{aligned} a : b : c = \quad & 0 \quad : \quad q \mid \sqrt{B} \mid : -\, p \mid \sqrt{C} \mid \\ & -\, r \mid \sqrt{A} \mid : \quad 0 \quad : \quad p \mid \sqrt{C} \mid \\ & r \mid \sqrt{A} \mid : -\, q \mid \sqrt{B} \mid : \quad 0 \end{aligned} \tag{62}$$

of which but two are independent. For maximum output of A, $(p^2 + q^2) = 1$ and (56), (57) become

$$p\,q = r \tag{63}$$

$$\Delta \;= 2\,A\,[b\,c]^2$$

Parallel Type Circuit. Circuit D, Fig. 6, symmetrical in $A, B, C, a, b, c, r, q, p$.

$$p^2 q^2 + q^2 r^2 + r^2 p^2 = -\,2\,p\,q\,r \tag{64}$$

$$\Delta \;= \begin{vmatrix} A & 0 & 0 & a \\ 0 & B & 0 & b \\ 0 & 0 & C & c \\ a & b & c & 0 \end{vmatrix}$$

$$= -\,\Sigma\,A\,B\,c^2 \tag{65}$$

$$\Delta_{11} = -\,B\,c^2 - C\,b^2 \tag{66}$$

$$\Delta_{12} = C\,a\,b \tag{67}$$

$$\left.\begin{aligned} p &= 2/\Delta\,|\sqrt{A\,B}|\,C\,a\,b \\ q &= 2/\Delta\,|\sqrt{A\,C}|\,B\,a\,c \\ r &= 2/\Delta\,|\sqrt{B\,C}|\,A\,b\,c \end{aligned}\right\} \tag{68}$$

$$a : b : c = \frac{|\sqrt{A}|}{r} : \frac{|\sqrt{B}|}{q} : \frac{|\sqrt{C}|}{p} \tag{69}$$

For maximum output of A, $(p^2 + q^2) = 1$, which makes (64) and (65) assume the forms

$$p\,q = -\,r \tag{70}$$

$$\Delta \;= -\,2\,B\,C\,a^2 \tag{71}$$

APPENDIX

IDEAL TRANSFORMERS

Circuits containing ideal transformers may be solved for cisoidal oscillations by using the ampere-turn and volt-per-turn properties of ideal transformers, or by solving first for ordinary transformers, and then evaluating the expressions on passing to the ideal transformer limit, but in general discussions, such as that of the master circuit, it is more convenient to employ the following symmetrical bordered determinant:—

$$\begin{vmatrix} Z_{11} & Z_{12} & \cdot & \cdot & Z_{1n} & N_{11} & \cdot & \cdot & N_{1m} \\ Z_{21} & Z_{22} & \cdot & \cdot & Z_{2n} & N_{21} & \cdot & \cdot & N_{2m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{n1} & Z_{n2} & \cdot & \cdot & Z_{nn} & N_{n1} & \cdot & \cdot & N_{nm} \\ N_{11} & N_{21} & \cdot & \cdot & N_{n1} & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ N_{1m} & N_{2m} & \cdot & \cdot & N_{nm} & 0 & \cdot & \cdot & 0 \end{vmatrix} \qquad (72)$$

where Z_{jk} is the mutual impedance between closed paths j, k with all transformer windings short-circuited and N_{jk} is the number of turns in the closed path j on the core k. The bordered determinant is used in exactly the same way as the ordinary network determinant for determining impedances, currents and voltages.

For transformers which closely approximate to the ideal it is convenient to use the following amplified form of (72):—

$$\begin{vmatrix} Z_{11} & Z_{12} & \cdot & \cdot & Z_{1n} & N_{11} & \cdot & \cdot & N_{1m} \\ Z_{21} & Z_{22} & \cdot & \cdot & Z_{2n} & N_{21} & \cdot & \cdot & N_{m2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{n1} & Z_{n2} & \cdot & \cdot & Z_{nn} & N_{n1} & \cdot & \cdot & N_{nm} \\ N_{11} & N_{21} & \cdot & \cdot & N_{n1} & Y_{11} & \cdot & \cdot & Y_{1m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ N_{1m} & N_{2m} & \cdot & \cdot & N_{nm} & Y_{m1} & \cdot & \cdot & Y_{mm} \end{vmatrix} \qquad (73)$$

where $Y_{jk} = i\, \mathcal{R}_{jk}\, h^{-1}\, p^{-1} = i\, 0.7958\, \mathcal{R}_{jk}\, p^{-1}$ has the dimensions of an admittance; $\mathcal{R}_{jk}$ is the mutual reluctance between magnetic meshes j, k; h is the factor for converting ampere-turns into gilberts.

This determinant is obtained by writing Kirchhoff's electromotive force equations for the n independent closed electrical paths, and the corresponding magnetomotive force equations for the m independent closed magnetic meshes, which are of the forms:

$$\left.\begin{aligned} Z_{11} I_1 + Z_{12} I_2 \ldots + Z_{1n} I_n + N_{11} \dot{\phi}_1 \ldots \\ + N_{1m} \dot{\phi}_m = E_1 \\ - h N_{11} I_1 - h N_{21} I_2 \ldots - h N_{n1} I_n + \mathcal{R}_{11} \phi_1 \ldots \\ + \mathcal{R}_{1m} \phi_m = 0 \end{aligned}\right\} \qquad (74)$$

Divide the magnetomotive force equations by $-h$ and for the magnetic fluxes substitute their time rates, using the relation for cisoidal oscillations $\dot{\phi}_j = i p \phi_j$. The complete set of $(n + m)$ equations are now linear in the currents $I_1, \ldots I_n$ and the time rates of flux $\dot{\phi}, \ldots \dot{\phi}_m$ and (73) is the determinant of the set of equations.

The discussion of (73) would include the application of complex quantities so as to include transformers having any self and mutual impedances, due to winding and core losses, but this falls outside the scope of this appendix.

Returning to (72), which is derived from (73) by making all Y's zero, we will briefly indicate the results which follow from applying the theory of linear transformations to this determinant, and then state, without formal proof, a number of theorems which apply to ideal transformers in general, finding a particular application in the case of the maximum output circuits.

Multiplying both the $(n + 1)$st column and row of (72) by a_{11} ($\neq 0$) and adding to them a_{12} times the $(n + 2)$nd column and row, respectively, leaves the impedances, currents, and voltages unchanged, since the determinant and the minors involved all remain unchanged except for the same factor a_{11}^2. Generalizing this we find that each of the m bordering columns and rows of (72) may be replaced by linear combination of these columns and rows, respectively, without changing the impedances, currents, and voltages, provided the determinant of the transformation

$$\begin{vmatrix} a_{11} & . & . & a_{1m} \\ . & . & . & . \\ a_{m1} & . & . & a_{mm} \end{vmatrix} \tag{75}$$

does not vanish; the transformation is then called non-singular. There are thus ∞^{m^2} changes which may be made in m ideal transformers without any resulting modification in the electrical behavior of the network.

By subtracting bordering rows and columns one from another, after multiplication by suitable constants, it is evidently possible to reduce one or more of the windings to zero turns, unless there is but one linearly independent transformer. This simplifies the set of transformers and thus it is of importance to know how many windings may be reduced to short circuits in this way. It may be shown that $(m - 1)$ windings on each of the m transformers may be reduced to short circuits, provided the m transformers are linearly independent. In this case at least one of the m-rowed determinants of the transformer windings will not vanish, and by transposing paths this may be made the determinant

$$D = \begin{vmatrix} N_{11} & . & N_{1m} \\ . & . & . \\ N_{m1} & . & N_{mm} \end{vmatrix}$$

which will be transformed to

$$D' = \begin{vmatrix} 1 & 0 & . & . & . & 0 \\ 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & 1 \end{vmatrix}$$

by means of the linear transformation (75) if the parameters have the values

$$a_{ij} = D_{ji}/D \tag{76}$$

where D_{ji} is the cofactor of the element N_{ji} in the determinant

D. Each transformer will have, after this transformation, $w = (n + 1 - m)$ windings where n, m are the number of degrees of freedom of the geometrical circuit and the number of independent transformers. But each independent transformer introduces an independent ampere-turn condition between the currents; the number of effective degrees of freedom is thereby reduced to $e = n - m$. Thus if n, e are given the number of transformers and the necessary windings on each are $m = n - e$, $w = e + 1$. For the unrestricted maximum output circuit $n = 4$, $e = 2$ which makes $m = 2$, $w = 3$; in other words, two ideal transformers each with three windings are required.

IDEAL TRANSFORMER THEOREMS

1. An ideal transformer is an electrical device which may be used to transfer energy, but itself can neither absorb, store, nor supply energy. Its action may be expressed in terms of "winding turns" upon one or more magnetic core meshes. A single-mesh magnetic core is assumed in theorems 2–14.

2. For an ideal transformer the total ampere-turns equal zero and the voltage drop is the same for every winding turn; therefore, n windings impose a total of n distinct conditions, *viz.*, the total ampere-turns equal zero and the potential differences between terminals are proportional to the algebraic number of turns connecting the terminals.

3. An ideal transformer may be replaced by a fictitious closed loop, having self impedance equal to zero, and mutual impedance with each real branch of the circuit equal to the algebraic number of turns contained in this branch.

4. Introducing an ideal transformer into a network adds a border to the network determinant, each term of which equals the number of winding turns in the closed path corresponding to the row, or column, in which the term occurs.

5. Each independent ideal transformer reduces by one the effective degrees of freedom of the network; the greatest number of independent transformers which can be introduced into an n-degree circuit without effectively opening every closed path is thus $(n - 1)$. The number of transformers and the necessary windings on each are $m = n - e$, $w = e + 1$ if n, e are to be the geometrical and the effective degrees of freedom of the network.

6. With m independent ideal transformers connected into

any portion of a network, presenting n degrees of freedom, the general case demands but $(n + 1 - m)$ windings on each of the transformers.

7. Two sets of ideal transformers which are mutually derivable, the one from the other, by a non-singular linear transformation, are electrically equivalent.

8. The electrical effect of a set of ideal transformers will not be altered by adding one or more transformers, each of which is a linear combination of the original set of transformers, or by removing one or more transformers, each of which is a linear combination of the reduced set of transformers. In every case the determinant (72) to be used for determining the impedances is one for which all linearly dependent transformers have been eliminated; the value of the determinant is otherwise zero and the mathematical expression for the impedances would require evaluation.

(9) An ideal transformer with n windings, none of which has zero turns, may be replaced by $(n - 1)$ transformers each with two windings in any way which leaves unchanged the ratio of the number of turns which are upon any core and directly connect any two pairs of points of the network.

10. Short-circuiting one winding of an ideal transformer, for which the number of turns is not zero, short-circuits all windings.

11. Each winding of a transformer v may be short-circuited provided one or more of its windings, which do not have an algebraic total of zero turns, form part of a closed path p completed by windings on other transformers, each of which contributes a total of zero turns to the closed path, or is made to do so by the linear transformation of adding to the transformer t the winding turns of transformer v, after multiplication by $(-N_{tp}/N_{vp})$, which is the ratio, with sign reversed, of the total algebraic turns in the closed path p on transformers t and v.

12. Any closed path, made up of windings from ideal transformers, may be opened by omitting any one branch of this closed path, provided every transformer contributes an algebraic total of zero turns to the closed path.

13. One or more windings of an ideal transformer, which can be cut once only by a closed surface, that cuts no other part of the electrical system, may all be increased or decreased by the same number of turns counted outward through the surface;

in this way any one of these windings may be reduced to zero turns, *i. e.*, to a short circuit.

14. A two-winding ideal transformer connected between an element having impedance and the point of measurement, multiplies impedances, electromotive forces, and currents by the factors a^2, $-a$, and $-a^{-1}$, respectively, where $a = N_1/N_2$ is the ratio of the number of turns on the measuring side N_1 to the number of turns on the element side N_2 each counted in the assumed positive direction for the closed path.

15. An ideal transformer with an n-mesh magnetic core, may be replaced by n ideal transformers with single-mesh magnetic cores; the above theorems, which have been intentionally phrased to apply specifically to single-mesh core ideal transformers, may thus be applied to multi-mesh core transformers with slight modifications.

APPENDIX VII

GEOMETRICAL CIRCUITS WITH FOUR DEGREES OF FREEDOM

By distinct electrical circuits we mean those which may have different internal electrical properties, demanding in the first instance, different conditions of continuity. For the purposes of this paper it is necessary to determine the distinct geometrical circuits having four degrees of freedom.

For geometrical circuits, the geometrical condition of distinctness may be put in the following concrete form: Assume that wire or rubber thread models have been made of two circuits and (a) that the models may be translated and rotated, (b) that the wires may be bent or stretched, (c) that wires or groups of wires, which have a series connection may be interchanged, and (d) that any two disconnected portions may be connected at a single point. Then the circuits are not distinct if the wire models may be brought into geometrical coincidence by these four transformations; otherwise they are distinct. After the geometrical circuits have been replaced by physical electrical circuits, it is to be understood that what is required is in every case the coincidence of interchangeable elements.

Geometrical circuits having four degrees of freedom must have not less than four, nor more than nine, branches, provided branches which are in series, and must thus carry the same current, are excluded from the count. The lower limit is evident; the upper limit is proved by noting that in any connected portion of the circuit having more than one branch-point, at least three branches terminate at every branch-point, and for any circuit whatever $p = b - f + s$ where f, b, p, and s are the number of degrees of freedom, of branches, of branch-points, and of separate portions of the circuit, respectively. Set $f = 4$, since we are dealing with a four-degree circuit and $s = 1$, corresponding to a singly connected circuit, as we wish the largest value of b for any given p. In this case, $2b \geqq 3p$, since each branch has two ends, and each branch-point has connected to it at least three of these ends. It follows that $b \leqq 9$.

By trial, it is readily found that **W** and **X** of Fig. **5**, are the only possible nine-branch circuits in which three branches terminate at each of the required six branch-points. Circuits having eight branches are now found by allowing first one and then another branch of **W** and **X** to shorten and vanish. In **X** it is sufficient to consider the vanishing of branches **1** and **9** since all other branches are by symmetry equivalent to one or the other of these two. In **W** there is complete symmetry and it is sufficient to consider the vanishing of branch **1**. With the understanding that **X** − **1** = **V** means that circuit **X** after the vanishing of branch **1** becomes equivalent to circuit **V** (ignoring the numbering and the positive direction of branches) the deduction of all of the sixteen four-degree circuits is shown by the following table:

$X - 1 = V,$
$X - 9 = U,$
$W - 1 = V,$
$V - 1 = S,$
$V - 5 = R,$
$U - 1 = T,$
$U - 3 = S,$
$U - 6 = Q,$
$U - 8 = R,$
$T - 1 = M,$
$T - 7 = O,$
$S - 1 = N,$
$S - 2 = P,$
$S - 4 = M,$
$R - 1 = O,$
$R - 2 = P,$
$R - 6 = M,$
$Q - 1 = M,$
$P - 1 = L,$
$P - 2 = K,$
$P - 4 = J,$
$O - 1 = K,$
$N - 1 = J,$
$M - 1 = K,$
$M - 2 = J,$
$L - 1 = I$
$K - 1 = I$
$J - 1 = I$

It is thus shown that the only possible four-degree circuits are the sixteen shown by Fig. 5, where each branch is numbered for convenience in reference and has an assigned positive direction. Fig. 6 gives all circuits having one, two, and three degrees of freedom.

DISCUSSION ON "PRINTING TELEGRAPH SYSTEMS" (BELL), AND "MAXIMUM OUTPUT NETWORKS FOR TELEPHONE SUBSTATION AND REPEATER CIRCUITS" (CAMPBELL AND FOSTER), NEW YORK, N. Y., FEBRUARY 19, 1920.

General George O. Squier: I believe that the printing telegraph is the thing, and coming to be more and more the method of the future, and I find that is the opinion in Europe where I have recently made a visit of inspection.

The printing telegraph was used a very great deal in the war and we early found that the only hope of handling the large amount of traffic was to get some sort of a hopper that we could pour it into, ad lib., and get it out at the other end. The volume of traffic in operating a large army in France is unbelievable, and luckily we were able to take over this very same system to Europe and put it into operation. This same system described today, was in extensive operation. In my inspection I saw poured into this hopper messages between the headquarters of General Pershing at Chaumont and the great business end of the army down at Tours. And without that method of carrying the vast traffic, I am perfectly certain the whole machine would have collapsed. It was absolutely necessary to have a perfect system of communication at all times, and that solved it. We also put this system in between London and Paris, and Tours and Chaumont, so that you could get a page printed record immediately out in France. If the war had gone on, we proposed to have that all over between the different headquarters.

So, the printing telegraph really went to the battlefield for the first time, and we got away with it, eight messages on the line. It proved for the first time that this sort of technical apparatus could go to the battlefield. That was encouraging and made us think that this apparatus as well as other can be operated by the ordinary soldier.

In regard to Little Silver, the Government Laboratory for radio research work. There we have large research laboratories, and are carrying out various forms of signalling, of all sorts, and of all the different methods of signalling that we have to consider in the army, I think it is safe to say that the method of signalling I am now employing is probably the best, that is, the human voice.

That is the reason why the telephone is so superior to all other methods and we can afford therefore, in any case, to exert every technical effort to extend the range of the human voice. Every one, by ordinary conversation of course, is an expert signal man. You do not have to teach anybody to operate this puncher or to learn the Morse code, but you make a signal man at your mother's knee. However, that appeals to the ear only, and we must have records. So that the eye method must come in and of those, the typewriter, telegraph printing system is the best. All these other things are substitutes. We learn to speak directly and that is best from the oral standpoint. We learn to read and that is best from the optical standpoint; those two methods go hand in hand, and they have each forged to the front. That is why the printing system is ahead of any other system.

Now that science has advanced to such a state that we really can produce a large number of channels, reliable channels, along one conductor, we are within a measureable distance, I believe, of extending the telegraph system, the eye system, and beginning to approach the method that we now have for telephoning. The fundamental difference between the operation of the telephone and telegraph is that in the telephone each operator is exclusively connected with the other man, and has the line all to himself. In the telegraph you have to write something out then hand it in to somebody else, or have it sent or phoned. In that respect the telephone has gone much further than the telegraph, and I believe we are now within a measurable distance, of having both systems in the house, so that if you call an individual residence, if the line is busy, you need not be told that, but just leave the message. That would be ideal.

It is not possible now due to wire difficulty. We have already the Telautograph. That is perhaps useful in its way. I look for something not so far away due to the fact that we can have so many independent channels.

General Edgar Russel: I can speak of the multiplex telegraph with positive affection. It was such a good friend to us at the time of our severe need, and it helped us out in such a marked way, when difficulties seemed to be mounting beyond the possibility of our meeting them.

I have been greatly interested in Mr. Bell's paper today, and I join with General Squier in hoping we may hear more of the engineering considerations that brought out this remarkable instrument. I was very much pleased to get some statements about comparative traffic possibilities. I might have gained the idea from what I saw of the operation of the multiplex in France, that its capacity was unlimited already, and that it needed no further improvement. I feel that we owe a tremendous debt to the engineers that developed this remarkable telegraph system that served an invaluable purpose when our needs in the A. E. F. were so severe.

At the time when the telegraph business in France was growing beyond all bounds, and the mounting traffic curve seemed as if it were going to fall over backwards, these instruments we succeeded in getting over in the midst of vast tonnage and personnel difficulties, rescued us from a very difficult situation. After these instruments were put in operation our very greatest difficulty, the lack of Morse operators, became much less acute. When the multiplex was established between our headquarters at Tours and

Paris and Chaumont, traffic difficulties immediately gave way, and our concern about insufficient telegraph service between those main points practically vanished.

The possibility of putting, as we did by composite devices on the telephone lines, a six-way multiplex transmission was, I believe, first demonstrated in France. I do not understand that it has been the practise in this country to composite for six-way transmission. The system succeeded wonderfully. When the American cable was laid across the English channel, we operated it with this six-way system between our headquarters in Chaumont, France, and London.

Our multiplex telegraph system in France was a standing wonder to our French friends. As stated by the lecturer, the Baudot printer system is carrying the bulk of traffic over there, and of course they were well acquainted with the old types of tape printers. Although a few of the French experts connected with their telegraph system had seen the American multiplex, most of them would stand and watch with keen interest this page printer in operation with its fascinating automatic shifting device at the end of each line.

When I went to London, I found in the Great British Post Office Central four or five multiplex instruments in operation, and I was told by those in charge of the systems there, that whereas they recognized its many excellent points, they had a very elaborate set of reasons why the Baudot tape printer system was better. I confess I was unable to answer many of their arguments, for today I got for the first time a clear understanding of the reasons why the American system of multiplex is far superior to the Baudot and kindred systems.

I think that the speedy application of the American multiplex to our needs in France was the outstanding feature of our telegraphic accomplishments. The way in which we arrived at our decision to adopt the multiplex may be of interest. Before I started to France with General Pershing on May 28th, 1917, I had only a few days in which to decide. But I had been in consultation with General Squier at Washington, who had assured me he was going to support us with every possible assistance that could be secured in this country, and when I came to New York and saw Colonel Carty, and some of his associates in the great companies controlling the supply of personnel and material, I realized that with such support as that, I would be justified in recommending that we go ahead with the installation of the modern multiplex and the modern telephone repeater systems.

As you understand, we were far from our base, in a strange country, and where we could not get spare parts or the facilities for repair. Besides there were tonnage difficulties and difficulties of manufacture in this country owing to the conflicting activities of war. But I now feel as I look back on that critical time, when we had only a day or two to decide, what a serious mistake we would have made, if we had assumed the conditions of war would not permit us to avail ourselves of this wonderful modern invention. In the midst of the enormously growing demands upon us we could say to officers "Gentlemen, bring us all the telegrams you want. You do not need to limit your telegraphic correspondence in any degree."

Those who have been over in France know of the delays we had in getting mail; in fact, the difficulties in the mail service seemed insuperable, but by the free use of the telegraph service, which this instrument gave us, we were enabled to transmit promptly this vast volume of messages, which the mail service was inadequate to handle expeditiously.

C. E. Davies: In Canada we are using automatic telegraphs to handle approximately 65 per cent of our 24-hour load. I can only add my evidence to what has been said in the paper as to the accuracy compared with Morse. Our records, dealing with complaint cases from customers, actual complaints of errors with the automatic, show approximately a ratio of 3 to 7, with Morse. It is the intention of the company to extend the automatic principle throughout, as soon as the equipment is available.

I can remember twenty years ago, as a Morse operator, the amused tolerance with which we looked on anything in the nature of a suggestion that we could handle business by automatics. That feeling was gradually wiped out through improvements in the automatic system until today it is well recognized that the automatics will never be replaced by the Morse system.

The problems that we have to face in Canada are somewhat different than in the United States, inasmuch as the transcontinental lines are from east to west and cover great distances. We have been enabled to operate circuits which have been affected with power induction satisfactorily, but as yet we have not been able to find any solution to a difficulty presented by frequent troubles due to the Aurora Borealis. Some time ago I suggested to our chairman it might be a matter of interest to the engineers of the A. I. E. E. to make an investigation as to these disturbances. I do not know whether anything has been accomplished along that line or not. The Canadian lines are interrupted considerably more, I believe, than the American lines.

The disturbances seem to be principally confined to circuits running east and west, and as we are now installing triple channel multiplex circuits from Montreal and Toronto to Winnipeg, and Vancouver, that is going to be one of our principal problems, how to operate during the periods of interruption due to the Aurora Borealis.

H. A. Emmons: I have been greatly interested in Mr. Bell's paper, primarily in connection with outputs. What he has said in comparing the various methods checks our own experience in the Western Union Service. We find that our accuracy corresponds very closely to what the British figures show, and that our output is going up steadily.

We started in 1915, installing multiplex apparatus on a wide or rather on a broad plan of equipping all trunk circuits throughout the United States and Canadian connections with printing telegraph apparatus of multiplex type. Today the trunk circuit traffic handled by printing telegraphs amounts to about 38,000,000 messages a month in round figures. That is about 70 per cent of the trunk circuit traffic that the Western Union Company handles. Those are not originating messages; many messages are handled more than once due to difficulties of reaching various isolated places, many of which are small places.

It may interest the engineers to know that the telegraph company operates a New York-San Francisco direct multiplex circuit which is about 3200 miles long, and I received a report the other morning, stating that the operators on this circuit have run 90 messages an hour. Each operator had run 90 messages an hour for the 12 o'clock to 8 a. m. tour. That is a most remarkable output on a circuit that is 3200 miles long. It is an equivalent of 180 messages an hour, in both directions, between New York and San Francisco. We do not expect to get that during day hours. There are too many interruptions over such a great mileage of wire.

One of the most remarkable things in regard to the multiplex is its freedom from interruption. Figures which I have been getting together largely indicate that the maintenance is of a high order; the men handling the apparatus appear to become so familiar with it that interruptions over any extended period are very few. It is most gratifying to the engineers who have fathered this device to find that this is true. We had a great many misgivings initially that the personnel detailed to the maintenance of the apparatus might not measure up to the high standard that would be required. I am happy to say that it is the exception rather than the rule that these men fall down on their jobs.

It shows what men can do if they specialize, and these men are specialists. If they were required to do many things besides maintain the apparatus, it would not be possible. As we all know, this is an age of specialists, and I feel sure that if printing telegraphs are to be a success in any company, the men that handle the apparatus and the operators that operate it, must be specialists. We must not expect that they will be all around men. Some of them will be, many of them will be, but the average man, making the personnel of a telegraph organization must be a specialist when it comes to printing telegraph apparatus.

General Squiers spoke of the personnel that went overseas.

Here in America the printing telegraphs are operated almost exclusively by young women. When the war came on we knew that there was going to be a great need for telegraph operators overseas. There were few multiplex operators who were men, most of them were women, and the army had no provision at that time for handling women. We learned very early that multiplex operators were going to be required overseas. Many of them volunteered, they got into various organizations outside of the signal corps, and it was with some difficulty that they were transferred to the signal corps.

I think that was accomplished, however, and the first lot of young men that went overseas, if my recollection is correct, more than 25 per cent of them were thoroughly experienced operators, attendants, and test-board men, making up the necessary personnel for operating that overseas multiplex, and I am exceedingly glad to learn that they came through under the difficult war conditions which they met.

P. M. Rainey: One difficulty which the telegraph engineer experiences in his development work is that of securing complete information to enable him to determine the requirements of the field. By this I mean information which will enable him to anticipate as many as possible of the problems which are likely to arise in the field. As I see it the greatest success will result from the close coöperation of those who are doing this laboratory or development work, and those who best know conditions in the field.

J. P. Edwards: I have noticed only one item of statistics that I consider out of line insofar as the Western Union Traffic is concerned, and that is the statement of the Chairman that 80 per cent is handled by automatics. That of course you found corrected in Mr. Emmons' statement. Approximately 47 per cent of the total traffic handled is handled automatically, principally by the multiplex system. The facility with which the traffic has been handled for a number of years on account of its growth could not have been accomplished except with the development of the multiplex in the Western Union, because generally speaking we are doing business today on the same wire plant that we had five years ago, and approximately the same wire plant we had ten years ago.

The Morse operators have been quite concerned for a number of years that the automatic was displacing them, and Mr. Davies has pointed out his experience of the concern of the men with whom he has been associated for years over the approaching disappearance of their occupation. As a matter of fact, the Western Union is in need of more Morse operators today than ever in the history of the company.

E. R. Shute: In training the staff for the operation of the printing telegraph system, we have established schools in the principal cities where we teach the students the touch system of keyboard manipulation and our operating practises.

The students qualify and handle approximately the capacity of the circuits, as operated by the Western Union Telegraph Company, in about three months. After reaching this stage, that is, graduating from the training school, they are given further instructive supervision for from three to six months, when they become fully qualified printing telegraph operators.

There is nothing that I think of that I can add to the paper presented by Mr. Bell. The large number of engineers who do not know that there is a printing telegraph system in extensive use is surprising to one who knows of the work done in this line and I think we should have more papers on this subject in the future.

R. E. Chetwood: There has not been very much change in line construction, on account of the use of automatic apparatus. Of course, the larger output per wire, due to the multiplex has made it desirable that we have more reliable line construction, and more reliable wires than we did previously. When a wire is handling from 250 to 400 messages an hour, you can afford more expensive and stronger construction.

The result of that has been that in our line construction we have strengthened our pole lines considerably; we have also replaced some of the smaller sized iron wires when they reached their normal life with heavier iron wires. We are using a great deal of No. 4 and No. 6 B. W. G. iron wire, where previously, No. 8 and No. 9 B. W. G. wires were used. That gives us greater continuity of service and the wires have not been subject to the troubles which wires of less strength receive in winter, fall and spring, due to the combined effect of wind and ice.

In general, as Mr. Edwards pointed out, the wire mileage has not increased and that is due entirely to the use of the multiplex apparatus. There was a tremendous increase in telegraph business during the war. It came at a time when we could not have obtained new wire to string on the lines, to handle the increased business, and it was the multiplex and other printing telegraph systems that allowed us to handle the tremendous increase in business, without increasing the wire plant.

George R. Benjamin: Regarding the Aurora Borealis and its effect on transmission we have arranged to operate the circuits metallically. Ordinarily the telegraph circuits are operated with a grounded return. To take care of such time as we are seriously affected on the more important circuits, we have arranged to connect them up and operate them metallically.

The Western Union service is using, and other companies are using a considerable number of Morkrum sets, stop and start method, and I regret very much Mr. Bell did not say something about the Morkrum apparatus. As regards the multiplex, it is really a development of the Baudot but there ar many important changes, as Mr. Bell suggested. The changes are all intended to increase the efficiency of multiplex operation. The Baudot system is limited as to speed, the speed ranges being between 30 and 40 words per channel. In some cases it is 30 words and in exceptional cases they have arranged to operate at 40. The use of perforated tape for transmission increases the efficiency because it does not admit ordinarily of any stoppages or periods of wait. The method of synchronism saves in line time, about fifteen or twenty-five per cent; that is, with a lower line frequency, it is possible to get the same output as the Baudot operating at say, 30 or 40 words a minute. The page printer was devised primarily to meet American requirements. It is possible that tape printing should be taken into consideration. The page printer of course is very much more complex, and it requires more attention and has many more parts. It also increases the maintenance. The American people have not been accustomed to tape printing and probably will not take to it very kindly at first, but I think eventually they will go into it and turn out a good job, and they will be satisfied with the good service.

Lloyd Espenschied: The phase of Mr. Campbell's paper which is probably of most immediate interest to the engineer is that of the conjugacy obtaining between the terminals of the network. This conjugacy is the property of mutual balance which is familiar to the communication engineer as employed in separating the oppositely directed channels of two-way transmission circuits.

His work should find application in all branches of communication, for example, to direct and alternating-current telegraphy, to telephony, to high-frequency currents transmitted by wires or by radio, and so on. To expand Dr. Campbell's paper into one which would feel more familiar to the engineer, would require approaching it from the engneering or application aspect. There is here a fertile field and I trust that somebody will find it within their province to pursue the subject further by taking up particular problems, in coordination with the present paper, such as that of substation circuits to which the paper so directly applies. Reference is made to the telephone repeater but the repeater as commonly practiced today makes use of only a very limited form of Dr. Campell's circuit arrangements.

O. B. Blackwell: It will be of interest in connection with Dr. Campbell's paper to point out that in telephone and telegraph transmission we have a class of problems which do not occur in power transmission. These problems arise from the fact that a telephone or telegraph circuit is used in general not only for transmitting messages going, let us say, from east to west, but also messages going from west to east. At certain points in the circuit such as at the subscribers' sets and at repeater points, it becomes desirable to branch the circuit into two parts or "channels" one of which carries transmission east to west, and the other of which carries transmission west to east. At these points it becomes very important that transmission going in one direction shall cause as small currents as possible in the channel arranged for currents going in the opposite direction. It is these factors that make Dr. Campbell's work of the greatest value to us in that he discusses broadly the problems which arise in thus dividing a telephone or telegraph circuit into an east-going and west-going channel, with maximum efficiency and with minimum mutual interference between the two channels.

I think perhaps the paper does not sufficiently emphasize one fact. Dr. Campbell in his discussion refers to his four elements as resistances. This does not mean, of course, that the application of this work is limited to those cases in which the elements are actually pure resistances. In the practical cases in which we apply it the elements have both resistances and reactances.

John H. Cuntz: A word as to the training of multiplex operators for service overseas might be of interest. When the first lot of multiplex men were sent over, as Mr. Emmons has told you, it was very soon found that it would be extremely difficult to obtain trained multiplex men in this country; in fact, it was absolutely impossible to spare any more. So it devolved upon the Signal Corps to train the necessary men.

Up at the College of the City of New York we already had a Signal Corps school, where men were being trained in radio and general Signal Corps work, and it was decided to change that over, a part of it at least, into a multiplex school. The order came through and the multiplex school was started immediately, and we realized from the beginning that in order to get men ready as soon as possible, it was best not to take radio men, or men skilled in electrical work, but to train touch typists as multiplex operators. We got hold of these touch typists as fast as possible, and we trained them and sent them over within the required time.

Orders subsequently came through for more and more men, and the supply of touch typists gave out, so we had to get rather raw material and train them as well as possible. At the same time, the Western Union Telegraph Company coöperated with us in the most remarkable and able manner, and the whole country was scoured for possible multiplex men. We secured them from the Pacific Coast, from Florida and from everywhere in between. A great many multiplex men unfortunately had already been transferred to other branches of the service, but we got them from the infantry, and artillery and various other branches; and they were concentrated at the City College and sent overseas in quick time.

By the fall of 1918, we had already sent over several hundred, and as it was contemplated to enlarge the multiplex system on the other side, plans were already under way for training several thousand, which would have gone through if the armistice had not come along just then. As far as we can hear, the multiplex men we sent over, after a little necessary further training, acquitted themselves very favorably, and did particularly well.

John H. Bell: The only remarks which seem to call for any attention are those by Mr. Benjamin. He mentioned that I do not refer to the Morkrum System. I do by inference on the first page,—"practically all the printing telegraph systems not considered in this paper have a carrying capacity of from 50 to 70 words per minute, and consequently are in a class by themselves." It is for that reason I did not include the Morkrum System. If I had included the Morkrum, I would have had to include a description of the Western Electric start-stop, the Hughes, and the Harrison, and that would have made the paper very much too lengthy.

G. A. Campbell: No questions have been raised which need any word from me. Of course, the paper, as it is presented, is a mathematical one, and no attempt has been made here to go into the physical problem, which I hope, will be presented by some one else at another time.

Presented at the 8th Midwinter Convention of the
American Institute of Electrical Engineers,
New York, February 19, 1920.

A METHOD FOR SEPARATING NO-LOAD LOSSES IN ELECTRICAL MACHINERY

BY CARL J. FECHHEIMER
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The method proposed makes use of idle operation of the machine as a motor, the voltage being varied, and speed kept constant. After deducting the armature $I^2 R$ losses from the watts input, the remaining watts are plotted against the voltage. A formula is derived based upon the assumption that the watts are equal to constant windage and friction loss plus core loss which latter varies as a constant power of the voltage. In applying the method, tangents to the curve are drawn at two points, and from the slopes of these tangents, the voltages and watts at these points, the exponent of the core loss curve, the core loss, and the windage and friction may be calculated with the use of the equations derived. An example is given of the close agreement with the test curve in the case of an induction motor; and other examples are cited of close agreement of the core loss and windage and friction losses with the losses measured by means of the usual belted method. The fact is pointed out that in some machines more accurate results are obtainable by means of the proposed method than with the usual belted method.

THE usual method of measuring iron, and friction and windage losses in direct-current and synchronous alternating-current machinery consists in driving by belt from a small direct-current motor and measuring the power input. This method has three disadvantages: (a) The losses in the belt, and the losses in the driving motor are somewhat uncertain, especially the former; (b) the driving motor belt, and possible other auxiliaries require extra floor space, and necessitate equipment which may not be available or difficult of application; and (c) in driving some machines, especially turbo-generators with large stored energy in rotating parts, it is difficult to average with accuracy the current input to the driving motor.[1]

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[1] The author believes that the belt friction loss may vary while taking observations when driving large turbo generators, which may introduce an additional error.

The method outlined below may at times necessitate equipment which is not available, or it may require more changes than would warrant its use.

The method is one that has been used frequently. We believe, however, that the method of separation of iron from friction and windage losses is new. It consists in operating the machine idle as a motor, at constant speed, with various voltages impressed across the armature, the watts input to the armature being measured. In the case of a d-c. machine, the excitation and voltages are simultaneously changed so as to maintain constant speed. With synchronous a-c. machines, it is desirable to maintain approximately unity power factor by altering the excitation with the voltage, although slight departure from minimum current will not introduce appreciable errors. With induction machines, the method is the usual idle running saturation curve—the voltage being varied and frequency kept constant. It is advisable to employ integrating instruments for the power, although in some cases indicating instruments will give satisfactory results.

In d-c. and a-c. synchronous machines, the armature $I^2 R$ loss is usually negligible, but in induction machines, this loss is, in general, of appreciable magnitude, and should, therefore, be deducted from the power input.

If we plot volts as abscissas, and power input to the armature[2] (or primary, if an induction motor) as ordinates, the curve will be like that shown in Fig. 1. It is well known that, in many machines, if the laminated iron subject to cyclic changes in flux densities is worked above the knee[3] of the *B-H* curve, there is stray flux produced which may, by its rate of change, induce

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[2] It is understood that if the $I^2 R$ losses in the armature or primary are appreciable, they must be deducted from the power input.

[3] We appreciate that "knee" of curve is liable to be misleading. It appears to us, that with ordinary iron, the departure referred to begins around 12,000 lines per sq. cm. in the core, and 16,500 lines per sq. cm. in the teeth. In giving the method of separation to our Testing Department we have asked them to select the high point at approximately 90 per cent of normal voltage.

electromotive forces in parts external to the laminated iron structure, and such differences of potential may cause currents to circulate, and thereby give rise to by no means negligible losses. If this condition obtains, the total so-called iron losses augment at a higher rate than ordinarily and we shall, therefore, consider only that part of the loss curve below the point at which the high loss gradient begins. We may then write the equation of the curve:

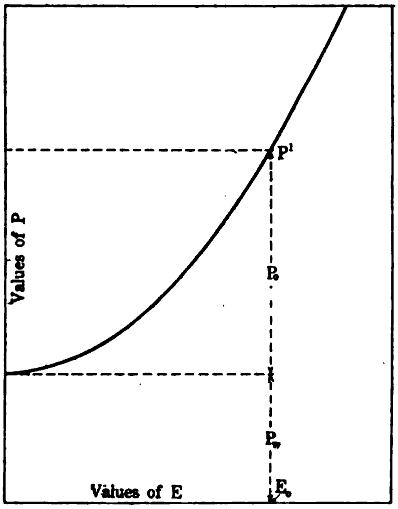

FIG. 1

$$P = P_w + P_0 \, (E/E_0)^n \qquad (1)$$

Where E = any voltage,

E_0 = voltage below "knee" of B-H curve,

n = constant exponent which indicates the rate at which iron loss increases with voltage,

P_0 = iron loss at voltage E_0,

P_w = friction and windage loss (constant at constant speed),

P = total loss = sum of friction and windage and iron losses.

Or, transposing P_w and taking natural logarithms,

$$\log_e (P - P_w) = \log_e (P_0/E_0)^n + n \log_e E \qquad (2)$$

Differentiating with respect to E, and remembering that $(P_0/E_0)^n$ is a constant,

$$\frac{1}{(P-P_w)}\frac{dp}{dE}=\frac{n}{E} \qquad (3)$$

Or, solving, $P_w = P - \frac{E}{n}\left(\frac{dp}{dE}\right)$ (4)

Select two points[4] on the curve;

For the first:

$$P = P_1 \quad E = E_1 \quad \text{and} \quad \left(\frac{dp}{dE}\right) = \left(\frac{dp}{dE}\right)_1;$$

For the second:

$$P = P_2 \quad E = E_2 \quad \text{and} \quad \left(\frac{dp}{dE}\right) = \left(\frac{dp}{dE}\right)_2$$

Then,

$$P_w = P_1 - E_1/n\left(\frac{dp}{dE}\right)_1 = P_2 - E_2/n\left(\frac{dp}{dE}\right)_2 \qquad (5)$$

Whence

$$n = \frac{E_2\left(\frac{dp}{dE}\right)_2 - E_1\left(\frac{dp}{dE}\right)_1}{P_2 - P_1} \qquad (6)$$

In this equation, all quantities in right-hand member may be determined from the curve; $\left(\frac{dp}{dE}\right)_1$ is the slope of curve at point (1) determined by drawing tangent to curve at that point, etc.

Having found n, we may substitute in (5), and evaluate P_w; or we may solve for P_w without solving for n, thus:

$$P_w = \frac{E_2 P_1\left(\frac{dp}{dE}\right)_2 - E_1 P_2\left(\frac{dp}{dE}\right)_1}{E_2\left(\frac{dp}{dE}\right)_2 - E_1\left(\frac{dp}{dE}\right)_1} \qquad (7)$$

Finally, if P' = power input at normal voltage, E_0

$$P_0 = P' - P_w \qquad (8)$$

We now have all of the constants in equation (1), and the curve may readily be checked by calculating a number of points.

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[4] See previous foot-note in regard to "knee" of curve for selection of higher point.

We give below data from a typical test curve of a three-phase induction motor; the equivalent single-phase resistance (1.5 × resistance between terminals) = 0.0438 ohm.

TABLE I

Volts	Amperes	Watts input	I^2R	Watts-I^2R
220	104	3640	475	3165
200	93	3210	380	2830
175	80	2730	280	2450
150	68	2320	202	2118
125	57.5	1940	145	1795
100	47	1640	97	1543
75	36.5	1410	59	1351

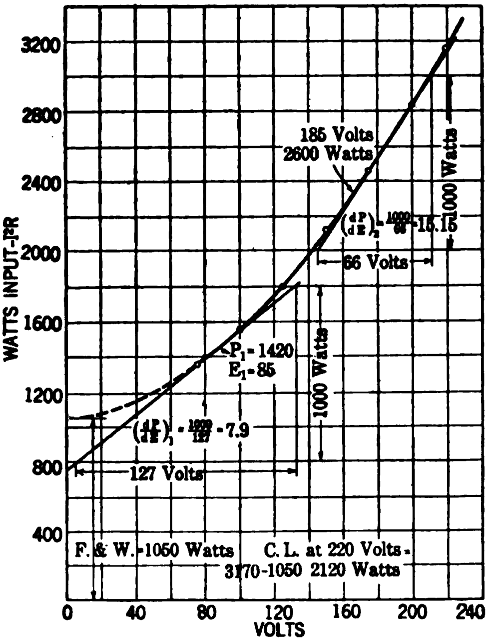

FIG. 2

We shall plot (Watts $- I^2R$) against volts as in Fig. 2. If we draw tangent to curve at point $E_2 = 185$ volts, we obtain $\left(\frac{dp}{dE}\right)_2 = 15.15$, and $P_2 = 2600$ watts. Similarly, for point 1, we take $E_1 = 85$ volts, $P_1 = 1420$ watts, $\left(\frac{dp}{dE}\right)_1 = 7.9$.

Substituting in equation (6)

$$n = \frac{185 \times 15.15 - 85 \times 7.9}{2600 - 1420} = 1.81,$$ the exponent for iron loss.

Then from equation (5),

$$P_w = 2600 - \frac{185}{1.81} \times 15.15 = 1050$$ watts friction and windage.

From equation (8), $P_0 = 3170 - 1050 = 2120$ watts iron loss at 220 volts. Substituting in equation (1),

$$P = 1050 + 2120 \left(\frac{E}{220}\right)^{1.81}$$ is the equation of the curve.[5]

We now give the values of watts $- I^2 R$ as determined from test (Table I) and those calculated from the equation:

TABLE II

Volts	Watts-I^2R	
	Test	Calculated
220	3165	3170
200	2830	2830
175	2450	2450
150	2118	2092
125	1795	1812
100	1543	1558
75	1351	1350
0	....	1050

It will be found that Fig. 3 is useful in evaluating $(E/E_0)^n$ or $(E_0/E)^n$. Plotting K as abscissas and K^n as ordinates on logarithmic paper, gives a straight line which may also be useful.

We give, as examples of check between the usual belted method and the method proposed, two synchronous machines on which losses were measured both ways. (See following table.)

It is interesting to note that slight inaccuracies in drawing the tangents to the curves at the two points do not introduce great errors. For example, in Fig. 2, the tangent at point 2 can be drawn but little, if any, different from that indicated; but at point 1, we might,

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[5] In this case, the iron is worked below "saturation" at normal voltage, so that we have taken E_0 to correspond to normal voltage of 220.

as extreme with not careful work, draw the tangent so that its slope $\left(\frac{dp}{dE}\right)$ is 8.3 instead of 7.9. This gives for the exponent "*n*" 1.774 instead of 1.81 and for P_w 1020 watts instead of 1050 and for P_0 2145 instead of 2120. The difference in friction and windage is thus 3 per cent and in core loss 1.2 per cent. Although this example does not so indicate, the exponent "*n*" usually changes by a larger percentage with slight changes in tangents than does the friction and windage or core loss. Also in the example, the exponent is less than 2,

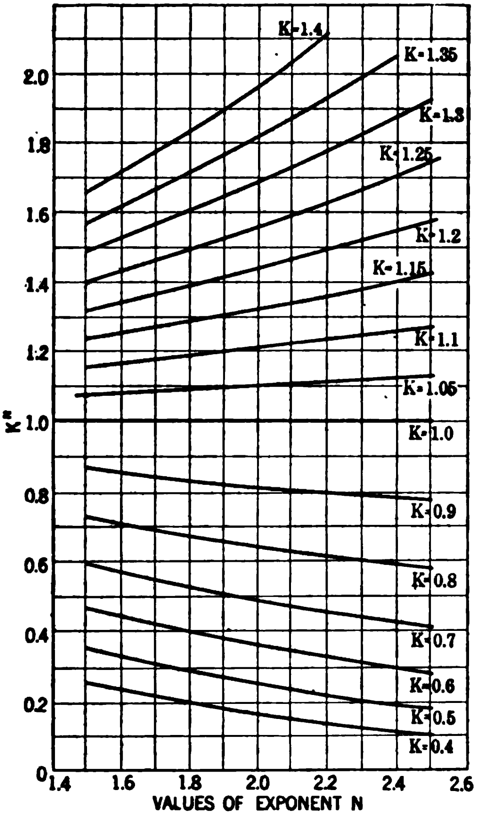

Fig. 3

whereas in those synchronous machines which we have investigated, the exponent is slightly above 2.[6]

Machine rating				Kw. core loss at nor. voltage		Kw. friction and wind		
Kv-a.	Volts	R.P.M.	Ph.	Belt	Proposed method	Belt	Proposed method	Remarks
1340	2200	900	3	16.1	16.4	28.	27.6	Part of m-g. set
10,000	4160	1800	3	129.5	124*	143.5	144	Turbo generator

*Machine was warm when making this test, which lowered the iron loss slightly. Machine was started cold for belted test.

In using this method of separation of no load losses, we suggest the advisability of adopting a large scale, especially for the power. For example, if in Fig. 2, had we selected the zero of ordinates to be 1000, we could have doubled the scale and still have plotted the curve on standard cross section paper.

We have obtained quite reliable results with synchronous machines by allowing the integrating wattmeter to run three minutes for each reading. Much depends upon the steadiness of voltage and frequency. It is doubtful whether integrating instruments can be employed to equal advantage with direct current, owing to the fact that such instruments are less reliable than for alternating current.

We have used the method successfully with a 13,500-kv-a. turbo generator, the normal speed of which was 2185 rev. per min. In that case, it was found difficult to obtain consistent data with the ordinary belt method, owing probably, in part, to change in belt loss. We have not experienced difficulty in starting polyphase synchronous machines, having simultaneously brought

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[6] Several years ago, the writer observed when plotting on logarithmic paper, core loss curves of synchronous machines (taken by belted method), that the exponent was greater than 2, even below those densities which correspond to beginning of saturation. It would be interesting to have explanations from Institute members.

up to speed the turbo generator and the machine supplying power to it.

While the foregoing application is outlined for separation of iron loss from friction and windage loss, and determination of the equation of the iron loss curve below saturation, it is obvious that it may be applied to any problem in which the general equation is similar in form.

DISCUSSION ON "A METHOD ON SEPARATING NO-LOAD LOSSES IN ELECTRICAL MACHINERY", (FECHHEIMER), NEW YORK, N. Y., FEBRUARY 19, 1920.

V. Karapetoff: Equation (1) in Mr. Fechheimer's paper is of the form that has interested me for a number of years. Now and then a problem arises in which a quantity varies (or may be assumed to vary) according to the *n*th power of an independent variable, plus a constant term. The expression of the no-load losses in an electrical machine is one such case, and unfortunately the logarithmic paper cannot be used for the determination of the exponent n, because of the presence of the other constant term in the equation.

The separation of the core loss from the friction by the method described is not new in principle, although I do not recall having seen the use of a fractional exponent in just this way. In my "Experimental Electrical Engineering", in the chapter entitled "Direct-Current Machinery—Efficiency and Losses", I give a similar formula, with the one exception that the exponent n is assumed to be equal to 2. This is nearly correct for machines working at a slightly higher degree of saturation than that selected for the numerical example in the paper under discussion. In many cases it will be simpler to obtain an approximate value of the friction losses using the exponent 2, and then obtain more accurate values of P_w and n by a few trials, using logarithmic paper. This possibility does not detract, of course, from the ingenuity of the method as used by Mr. Fechheimer in application to large machines.

The introduction of the quantities P_0 and E_0 does not seem to be necessary, since they do not appear in the final expressions (6) and (7). It may be preferable to write (1) in the form

$$P = P_w + K E^n \qquad \text{(a)}$$

The application of this formula is limited to the range of flux values for which n is approximately constant; this statement takes the place of P_0 and E_0.

In the place of expression (6) I wish to suggest a different formula. Taking a derivative of (a) with respect to E, and introducing for the sake of brevity

$$p = d\,P/d\,E \qquad \text{(b)}$$

we obtain

$$p = k\;n\;E^{n-1} \qquad \text{(c)}$$

Writing this expression for two points, 1 and 2, on the curve in Fig. 2, we get

$$\frac{p_2}{p_1} = \left(\frac{E_2}{E_1}\right)^{n-1} \qquad \text{(d)}$$

$$n - 1 = \frac{\log\left(\frac{p_2}{p^1}\right)}{\log\left(\frac{E_2}{E^1}\right)} \qquad \text{(e)}$$

Whether formula (6) or eq. (e) is used it is advisable to compute n for more than one pair of points, in order to see if it remains approximately constant.

In order to increase the accuracy of the method, it may be advisable to compute the friction loss P_w from the data for several points on the curve, and to take the average value. This may be done as follows: Multiplying both sides of equation (c) by E we get

$$p\,E = n\,K\,E^n \qquad \text{(f)}$$

Substituting the value of $k\,E^n$ derived from this expression in to equation (a), we obtain

$$P = P_w + \frac{p\,E}{n} \qquad \text{(g)}$$

Let (g) be written for several points, say, on the curve. Adding these expressions term by term we get

$$\sum P = q\,P_w + \frac{1}{n}\sum p\,E \qquad \text{(h)}$$

$$P_w = \frac{1}{q}\left(\sum P - \frac{1}{n}\sum p\,E\right) \qquad \text{(i)}$$

The value of P_w so computed will represent the true average value for the whole curve and for the average value of n.

A much closer approximation to the real shape of the experimental $P - E$ curve is obtained by the addition of a quadratic term to expression (1), thus:

$$P = P_w + a\,E^2 + k\,E^n \qquad \text{(j)}$$

The addition of this term, which represents the eddy current loss, and possibly some of the hysteresis loss, makes it possible to carry the analysis to much higher degrees of saturation than is permissible with the less accurate formula (1). Moreover formula (j) permits a simultaneous use of four points on the $P - E$ curve instead of two, as in Mr. Fechheimer's method. 'To determine the unknown coefficients P_w, a, k, and n, we differentiate equation (j) with respect to E and multiply the result by E. This gives

$$E\,p = 2a\,E^2 + n\,k\,E^n \qquad \text{(k)}$$

Eliminating $k\,E^n$ between (j) and (k), as before, we obtain

$$n\,P + E^2 a\,(2-n) - n\,P_w = E\,p \qquad \text{(l)}$$

We now take two points, 1 and 2, on the given curve, write equation (l) for both, and subtract one expression from the other to eliminate $n\,P_w$. The result is

$$n\,(P_2 - P_1) + a\,(2-n)\,(E_2^2 - E_1^2) = E_2\,p_2 - E_1\,p_1 \qquad \text{(m)}$$

A similar equation is then written for two other points on the curve, say 3 and 4. These two equations may then be solved together for n and $a\,(2-n)$, considering the latter expression as one variable. Knowing a and $n\,P_w$ may be computed from equation (l). Finally k may be determined from equation (j), should this quantity be deemed to be of interest.

While the actual computations are more involved than with the simpler formula (l), the resultant curve passes through four points on the given curve, instead of two only. This resultant curve will also probably fit the given curve so closely that check computations for per cent error will be unnecessary in most cases.

In an exceptionally important case involving a dispute, or with a large number of experimental data of comparatively small accuracy, it may be advisable to use the method of least squares, so as to obtain not one of the possible solutions but *the most probable curve* and *the probable error*. Since one of the unknown quantities, n, is an exponent, an expansion according to Taylor's theorem is necessary. A similar case in thermodynamics has been treated in detail.*

In some cases, especially with large machines, the *retardation method* † for determining the friction loss may prove preferable to the one described in the paper under discussion; or else the two methods may be used to check each other. The machine under test is brought to a speed say 10 per cent above that at which the friction is to be measured. Then the power is shut off and the machine is allowed to slow down to a speed about 10 per cent below the desired speed. While the machine is slowing down, a speed-time curve is taken, using an electric tachometer; or else a decelera-

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*V. Karapetoff, "Engineering Applications of Higher Mathematics", Part III, p.62.

†V. Karapetoff, "Experimental Electrical Engineering", Vol.I Cap. on "D-C. Machinery—Efficiency and Losses".

tion-time curve may be taken using a magneto in series with a galvanometer and a condenser. This test is performed with the field current on, and then again with the field circuit open. The field current must have exactly the same value as during a run at the rated speed at which the total loss is determined with a wattmeter, as in the test described in Mr. Fechheimer's paper. The slopes of the two retardation curves will give the relative proportion of the friction in the total stray loss, and knowing the numerical value of this loss from the wattmeter reading, the amount of friction loss is readily computed.

Thus, with a simple additional test the hypothesis such as equation (1) becomes unnecessary, and the computations are reduced to dividing a wattmeter reading in a known ratio. The retardation test has been further improved by Professor Robertson* who has described a very accurate stroboscopic method for speed-time measurement, and by D. Kapp† who has shown how to vary the retardation rate at will by keeping the machine connected to the line.

I believe that Mr. Fechheimer is wrong in his statement that the retardation method requires the determination of the moment of inertia of the revolving part, and I do not think any of us would care to use this method if it were based on the accuracy of the calculation of the moment of inertia. There are at least four methods whereby the moment of inertia can be eliminated out of the computations, one method is to put a brake on the shaft as an additional known friction loss, and to perform one run with the brake and one without. From the slopes of the two curves the moment of inertia may be eliminated.

Another method is to change the unknown moment of inertia by adding a mass of a known moment of inertia, and again to make two runs, one with the additional mass, and one without it. There is also a purely electrical method proposed not long ago by Dr. Kapp (see reference above), in which he allows the machine once to retard and once to accelerate, while the machine is connected to the source of power through a large resist-

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*Professor D. Robertson, "Separation of the No-Load Stray Losses in a Continuous-Current Machine by Stroboscopic Running-Down Methods", *The Journal of the Institution of Electrical Engineers* (British) Vol. 53, 1915, pp. 308 to 328.

†Dr. Gisbert Kapp, "Experimental Determination of the Moment of Inertia of a Continuous Current Armature", *The Journal of the Institution of Electrical Engineers*, (British) Vol. 44, 1910, pp 248 to 254.

ance. This also gives two equations, from which the moment of inertia is eliminated. Perhaps the simplest method for eliminating the moment of inertia is to perform one run by the retardation method, and another at a constant speed, taking one wattmeter reading. From the two runs the moment of inertia may be eliminated and this is probably the method most used.

In the particular class of machines with which Mr. Fechheimer is concerned, it seems to me that the retardation method is especially proper, and particularly with the refinements that have been brought into the art within the last few years. One of the refinements is the use of very accurate measuring devices for determining instantaneous speeds. Another refinement is the use of Dr. Robertson's stroboscopic method referred to above.

William F. Dawson: Evidently the main object of Mr. Fechheimer's paper was to bring out the advantage of this method of separating no-load losses. I may say for the sake of historical accuracy, that the method is not new. I have known about it for twelve or fifteen years and it was in use before that. The method is very convenient and is highly to be commended. Anybody who has ever had to do with using belts for getting core losses must realize their limitations. Unless they are specially prepared and glazed joints take the place of lacings there are many difficulties, and for turbo-generators, it is practically impossible to curve any belt around a small shaft or available pulleys so as to drive the load evenly and uniformly.

My own experience with respect to testing turbo-alternators began eight or nine years ago. With the then available methods, it was difficult to get core losses. We could take them by retardation, but as Mr. Fechheimer says, they were not satisfactory. It is possible, as Dr. Karapetoff says, to take them without knowing the moment of inertia. At that time however, I could not be sure of the speed. Tachometers were not satisfactory, and counting machines, while suggested, were not available. Therefore we adopted the method described by Mr. Fechheimer. The turbo alternators, with the turbine wheels connected to them, were driven as synchronous motors, and by varying the electromotive forces supplied by the generator, and taking a full curve for each voltage reading, we obtained the minimum input or unity power factor.

As a rule, we found that while we got very excellent readings in the upper part of the curve, we could not get steady readings much lower than one-third the normal voltage and so had to extrapolate the curves but the results were very satisfactory. I never thought that the exponent was constant, and I have run much above the saturation and found exponents as high as five. I think—I do not know —that the explanation is as Prof. Karapetoff has pointed out. Core losses of one group of materials are superimposed upon the core losses of another group of material. Judging by the saturation curve of the machine, one may be below the knee, but there are a number of elements which enter into the saturation, the saturation of the armature core itself, which is the principal thing, and saturation of the teeth which will often saturate more quickly than any other part of the magnetic circuit. The magnetic length is short and it has small effect upon the saturation curve, but a large effect on the core loss. Machines with large air gaps straighten out the saturation curve enormously. In some machines the focault current losses are generated by magnetic potential difference across the slot and these introduce a new element, depending on the width of the conductor.

There is one interesting thing about this scheme. One can test the turbo-alternator, and separate these no-load losses, even with the turbine wheels on. But one must be cautious, because in running a turbo-alternator either as a synchronous motor or synchronous condenser, without steam passing through the turbine, the wheels and buckets may become so hot as to damage them. For this reason it is a standard practise with us to test with the wheels removed, or else to lead low pressure steam into the turbine, in such a way, as not to cause any motoriug.

Those interested in the steam end of the set may at the same time measure the turbine rotation losses, at various degrees of vacuum. Do not try to maintain such a high vacuum that it cannot be maintained steadily. We found we could do very well with a vacuum of 27 or 28 inches (2 inches to 3 inches absolute).

There is another method of separating these no-load losses. I will not go into that method now but the article I have prepared will appear in the February number of the *General Electric Review*.

W. I. Slichter: The discussion of the variation of the exponent *B*, which causes variation in core loss, is most interesting. We have not only the cause of this variation, which we find in simple samples of iron; but we have this variation which has been alluded to as the flux seeking different paths and causing additional eddy currents in the solid masses. There is also a third cause, which I should contribute to this, that as we go up in saturation, the wave shape in our flux will be shaped, and the relation which we figure between voltage and flux density at one point will not be the same at another. The matter, therefore, must be handled with great care by one not experienced with it.

P. L. Alge (by letter): Mr. Fechheimer's method of extrapolating an incomplete curve by assuming it to be of a simple logarithmic character has proved useful in many fields of experimental science, notably that of ballistics, and his application of it to the separation of no load losses is very appropriate.

Mr. C. MacMillan, has proposed a somewhat similar method, which we use to arrive at the same end. This method consists in plotting the excitation watts against voltage squared, thus bringing the lower experimental points much nearer to the axis of zero voltage, and drawing a representative straight line, or at worst a very flat curve, through the experimental points by eye. If the core loss did obey a square law, the points would fall exactly on a straight line and this line would be very accurately determined. In practise, the law is so nearly a true square law that the points over the middle range ot voltage do tall nearly on a straight line and it is not difficult to choose a representative line. The method amounts to assuming the value of n to be 2 and taking a mean value of the coefficient of V^n. It has the advantages of requiring no calculation except squaring the voltage, of throwing the excitation curve so near to the axis of zero volts as to require but a very short extrapolation, and of giving the curve to be drawn the simplest possible shape.

The chief difficulty we have met with in using this method has been that the experimental points frequently give indications of obeying two quite different laws of variation in different parts of the curve, thus making any extrapolation inaccurate. (Both parts are frequently straight lines when watts are plotted against volts squared). However, this difficulty applies to any method of extrapolation and it can only be said that orderly methods of extrapolation like Mr. Fechheimer's and Mr. Macmillan's, will give much better results than the usual hit or miss method of drawing the watts-volts curve by eye.

The question raised by Mr. Fechheimer, as to the causes of variations of core loss with voltage at a higher rate than the square is a very interesting one. I believe there are four principal reasons which may cause n to be greater than 2:

1. Lessening of the effective area of flux path with

increase in density. As the flux density rises, saturation sets in in some places notably at the tooth corners and at unannealed edges of the punchings, so that the flux is crowded into a smaller area of iron the higher the density is. This lessening of area of path causes the flux density to rise more rapidly than the voltage, thus causing the losses to go up faster than the square of the voltage.

2. Increasing distortion of wave form with increase in density. The wave form of flux variation at any particular point of the iron is not usually sinusoidal, even though the total flux varies sinusoidally: since the distribution of flux between available parallel paths changes markedly with increase of density. Thus eddy currents of triple and higher frequencies exist in the iron even at low densities and with sinusoidal variation of total flux. These higher frequency eddy losses may increase at a considerably faster rate than the square of the voltage.

3. Lessening of effective resistance of the eddy current paths with increase in voltage. Any eddy current losses caused by imperfect insulation between stampings may be expected to rise at a higher rate than the square of the voltage since the contact resistance between stampings will decrease as the eddy currents increase.

4. Crowding of flux into iron outside the laminations. As Mr. Fechheimer, has observed, at high densities some flux will be crowded into the solid iron frame or other magnetic parts contiguous to the laminations. These parts will have a much higher specific loss than the laminations, so that after they begin to receive flux the total losses will rise quite rapidly.

C. J. Fechheimer: The trend of much of the discussion is in regard to the variation of the exponent "n", which I took to be constant. We must remember that in commercial testing, there is no desire to secure laboratory precision. I have plotted core loss curves of a large number of machines on logarithmic paper, and the curves were always a straight line indicating the constancy of the core loss exponent "n", provided there was no saturation in any parts of the magnetic circuit undergoing cyclic change. I do not mean to define just what the point of the beginning of saturation is other than as stated in the third footnote in the paper. The shape of the no-load saturation curve is not necessarily a means for determining the point of departure, as the saturation may be in the parts undergoing cyclic change, such as the armature core, or it may be in the poles in which the flux is constant. Inasmuch as the tests are usually made in the testing department of the manufacturing company the designer could indicate to the testing department what is the highest point on the curve to which the tangent might be drawn. In all cases in which the method has been used, the curves calculated by means of equation (1) after "*n*" and "*P w*" had been evaluated by means of equations (6) and (4) were almost in exact coincidence with the test curves. I therefore, do not believe it would be wise to ask our testing departments to employ the more tedious formulas which Prof. Karapetoff has so nicely worked out.

As I understand Mr. Dawson's statement in regard to "foucalt current losses generated by magnetic potential difference across the slot", such losses at no-load are either due to saturation of the teeth to which I have just referred, or arise from some of the main flux entering the slot; in the latter case, these losses are proportional to the square of the voltage and are included as part of the so-called "iron" loss, regardless of the method of measurement.

Prof. Slichter's reference to the change in wave shape of flux at different points on the saturation curve, I understand to be a change in flux wave shape in the air gap. That is modified if the teeth are saturated, and so long as we keep below high tooth densities, no error is introduced.

I agree with Mr. Dawson that the exponent "*n*" may be as high as five, well above saturation. The losses increase at a very high rate, once the flux does not remain in those parts to which it is supposed to be confined.

In abstracting the paper, I made mention of the retardation method of measuring no-load losses, and pointed out its disadvantages, difficulty in securing accurate measurement of speed, and in calculating the moment of inertia. Prof. Karapetoff tells us of refined methods of taking speed-time curves and of means of evaluating the moment of inertia experimentally. Such improvements undoubtedly have advantages, especially if employed in laboratories; but our commercial testing departments, which today are loaded with more instruments than they would like to have, would very reluctantly introduce such devices as Prof. Robertson's stroboscope or additional brake (the friction loss in which is probably subject to variation). I am not entirely familiar with some of the methods of measuring moment of inertia about which Prof. Karapetoff tells us, and I believe that he has given us a number of suggestions which may prove helpful.

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New York, February 19, 1920.*

INHERENT REGULATION OF CONTINUOUS CURRENT CIRCUITS

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This paper discusses the voltage changes, inherent in d-c. circuits, upon change of load. These variations are independent of $i\,r$ drops or speed of prime movers. A simple means of mitigating their effect is given.

INTRODUCTION

IN developing a device to measure time to one millionth of a second we were at one time confronted with the question, how is the voltage of a "flat" compounded generator affected by an increase or decrease of load? We were not interested in what happened as indicated by a voltmeter, (as the generator was carefully compounded for the range of load under consideration), but what happened at the instant the load was changed. We were surprised at the extent of the voltage drop, indicated by the oscillograph, when the load was increased. The very interesting oscillograms and data resulting from further investigation of this drop form the basis of this paper.

The principles governing this drop are set forth in elementary text books on engineering, and while simple in this form their application to a network of conductors supplied with current by a generator is difficult to follow, and when mathematically stated lead to very complex equations that can be comprehended only by a painstaking study of the problem.

The result is that the busy engineer with insufficient time to concentrate upon this subject frequently encounters, in practise, manifestations of this drop without realizing the cause or recognizing the principle.

We find very little written on the subject; a suggestion of it here and there in articles referring to transients, and with the exception of reference made to it in Lamme's paper[1] on commutation, no comprehensive statement of the case has come to our notice.

The simple explanation of this dip in voltage given in Lamme's paper follows verbatum: "Assume, for instance, a 100-volt generator supplying a load of 100 amperes—that is, with one ohm resistance in circuit. The drop across the resistance is, of course, 100 volts. Now, assume that a resistance of one ohm is thrown in parallel across the circuit. The resultant resistance in circuit is then one-half ohm. However, at the *first instant* of closing the circuit through the second resistance, the total current in the circuit is only 100 amperes, and therefore the line voltage at the first instant momentarily must drop to 50 volts. However, the e. m. f. generated in the machine is 100 volts, and the discrepancy of 50 volts between the generated and the line volts results in a very rapid rise in the generator current to 200 amperes. If the current rise could be instantaneous, the voltage dip would be represented diagrammatically by a line only; that is, no time element would be involved. However, *the current cannot rise instantaneously in any machine,* due to its self-induction, and therefore, the voltage dip is not of zero duration, but has a more or less time interval. The current rises according to an exponential law, which could be calculated for any given machine if all the necessary constants were known. However, such a great number of conditions enter into this that it is usually impracticable to predetermine the rate of current rise in designing a machine; and it would not change the fundamental conditions if the rate could be predetermined, as will be shown later."

Lamme points out that, "If a given load is thrown on a machine, the dips will be relatively less the higher the load the machine is carrying. Also, if the *same percentage* of load is thrown on each time, then the dips should be practically the same, regardless of the

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[1] B. G. Lamme, "Physical Limitations in D-C. Commutating Machinery." TRANS. A. I. E. E. 1915, Part II;

load the machine is already carrying." He also gives a table of test results to substantiate this taken from oscillograph curves which were considered too faint to be reproduced in the paper.

In our investigation of the "dip" we were able to get very good oscillograms though the phenomenon is a difficult one for the oscillograph to record. It is the purpose of this paper to extend the evidence by oscillograph records and plots drawn up from a mathematical consideration of a range of ideal circuits, which give at a glance a better understanding of the relative importance of various conditions than would be disclosed in pondering over a word picture.

LOAD INCREMENTS

Several types of generators were examined and the "dip" found in all in accordance with the theory within the limitations of the oscillograph.

The following oscillograms were obtained from the same generator under various conditions of load and may be considered to represent fairly the performance of any continuous-current generator, making due allowance for generator constants.

The performance of any continuous-current generator in this regard can be more easily understood by considering what happens when

A non-inductive load is added to a generator carrying no load.........................(Case one);

An inductive load is added to a generator carrying no load.........................(Case two);

A non-inductive load is added to a generator carrying a non-inductive load.............(Case three);

An inductive load is added to a generator carrying a non-inductive load..............(Case four);

A non-inductive load is added to a generator carrying an inductive load..................(Case five);

An inductive load is added to a generator carrying an inductive load..................(Case six).

CASE ONE

If an ideal non-inductive resistance is thrown on the circuit of a separately excited generator carrying no load, there will be an instant demand for current of value $\frac{E}{r_1}$ where E is the voltage at the terminals to which load is applied and r_1 is the resistance of the applied load.

Owing to the inductance of armature and leads, current cannot be supplied instantly, and as no current was flowing before load was applied and did flow afterward, obviously it must have increased from zero value. As $E = IR$ the voltage must drop to zero at the instant the load is applied, and increase along a simple exponential curve depending only upon the time constant of the circuit, to the value equal to the generated voltage minus the internal IR drop.

The drop to zero is therefore, fundamental and is independent of the type of the generator, its size, its generated voltage, or the value of the applied load. These variables influence only the time of recovery to normal voltage and the form of the recovery curve.

Considering the inductance of the circuit to be confined to the armature: The instant demand for current gives rise to an e. m. f. of self-induction which opposes any increase of current (at that instant zero) and with respect to the generated voltage is therefore a counter e. m. f. The value of this counter e. m. f. with respect to the value of the generated voltage at any instant during the transient, is just sufficient to allow current to flow into the circuit at a rate such that the e. m. f. of self-induction ($L \frac{di}{dt}$) at any instant added to the IR drop at that instant equals the generated voltage. Therefore, at the instant when load was applied and the current was zero the counter e.m.f. of self-induction just equaled the generated voltage in value, consequently the voltage of the circuit reduces to zero.

If, as in all practical cases, part of the inductance is in the leads of the external circuit, the voltage at the generator terminals should drop instantaneously to a value approaching zero as the inductance of the external circuit approaches zero.

Capacity effects greatly complicate matters and their consideration precludes a simple presentation of the phenomena. This is particularly so in the case of underground cable distribution where we find distributed capacity and inductance to a considerable extent. However, in the case of isolated plants, overhead distribution, and branches of underground systems shut off, as it were, by the inductance of protection devices, instruments, meters, etc., the available capacity is, in general, so small as compared to the instant demand for energy that capacity effects have been neglected. Those who wish to consider capacity effects in connection with this phenomenon can refer to such standard works as "Transient Phenomena" by C. P. Steinmetz, and "Propagation of Electric Waves Along Telegraph Conductors" by J. A. Fleming.

The following conclusions and equations assume lumped constants that remain constant during the time necessary for the readjustments to take place, and are applicable also to battery circuits where there is a series inductance that would correspond to the inductance of the generator armature, as for instance, the series coils of a circuit breaker or watt-hour meter.

As previously pointed out, applying a non-inductive load to an unloaded generator results, at the generator terminals, in an instantaneous drop of voltage to zero. From this point the voltage returns to normal value according to an exponential curve.

$$e = E\,(1 - \epsilon^{-\alpha_1 t}) \qquad (1)$$

where

e is voltage at any instant
E = normal voltage
t = time since application of load.

$$\alpha_1 = \frac{r}{L_a}$$

where

r = complete resistance of circuit.
L_a = inductance of armature.

Equation 1 can be developed from

$$e = ir_1 \qquad (2)$$

where i is instantaneous value of current
r_1 = resistance of applied load.
and

$$i = I\,(1 - \epsilon^{-\alpha_1 t}) \tag{3}$$

where $$I = \frac{E}{r_1}$$

At $t = 0$, that is at the instant of closing the switch $\epsilon^{-\alpha t}$ reduces to 1, and equation 3 becomes

$$i_{(t=o)} = 0 \tag{4}$$

which substituted in (2) reduces (2) to

$$e_{(t=o)} = 0$$

From the foregoing it will be seen that the voltage drop is independent of the value of the resistance applied, also that the velocity of return to normal is a function of the resistance, and an inverse function of the inductance.

The natural question as to the whereabouts of the voltage generated by the rotating armature of the machine, which is dependent only upon the speed of rotation and the strength of field, can be answered by a study of the reactions of the armature itself, the instant of closing the circuit.

Because the armature is inductive it resists any attempt to change the value of current flowing in it by a counter e. m. f. = e_c

$$e_c = L_a \frac{di}{dt} \tag{5}$$

which by differentiating (3) with respect to t, and substituting in (5) becomes

$$e_c = rI\,\epsilon^{-\alpha_1 t} \tag{6}$$

$$= E\,\epsilon^{-\alpha_1 t} \tag{7}$$

which at $t = o$ becomes

$$\underset{t=o}{e_c} = E$$

From 7, 3, and 2 it is seen that the sum of the external ir_1 voltage and the armature reactance voltage neglecting armature resistance, is always equal to the generated voltage, that is

$$ir_1 + L_a \frac{di}{dt} = E$$

Fig. 1 is a plot from calculated values and illustrates how a change in the value of the applied non-inductive load affects the shape of the voltage recovery curve and the time of recovery. The solid lines represent current increase as well as generator e. m. f. The broken lines represent the decrease in counter e. m. f. as the current approaches its steady value. The sum of the generator voltage and the counter e. m. f. at any instant during the transient equals 100 per cent of the generator voltage when the transient terminates.

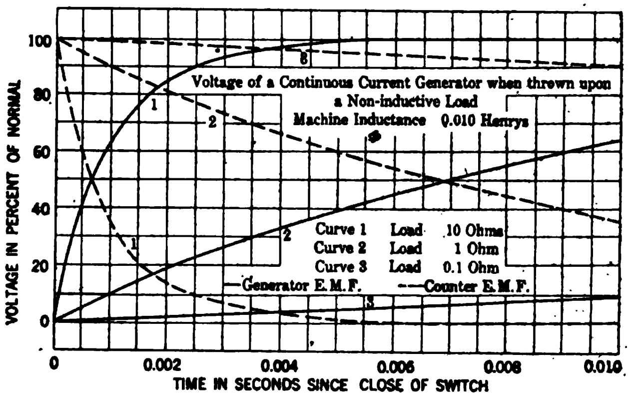

FIG. 1

Some oscillograms from actual tests are shown in Fig. 2. None of these show the voltage drop completely to zero, as should be expected, for a simple calculation is sufficient to show that in every case, the rate of rise of voltage was such that, assuming a natural period of 4000 cycles per second for the oscillograph vibrator, the voltage would be sufficiently great to give measurable deflections before the vibrator could return to zero. The table of test results given in Lamme's paper, taken from oscillograph curves, does not indicate sufficient drop in cases where the added load was a large part of the combined load, particularly where a load of 417 amperes was added to a 1200-volt

FIG. 2

generator carrying no load. The volt "dip" from 1200 to 500, there recorded, we believe due to the limitations of the oscillograph (mentioned by Lamme), should have been substantially to zero volts.

The oscillograms show clearly the variation in the rate of recovery and time of recovery with varying demand upon the generator. The distance between each peak represents an elapsed time of approximately 0.001 second. Some of the detail in the original oscillograms has disappeared due to the several photographic processes necessary to reduce them to print. None have been retouched for any purpose.

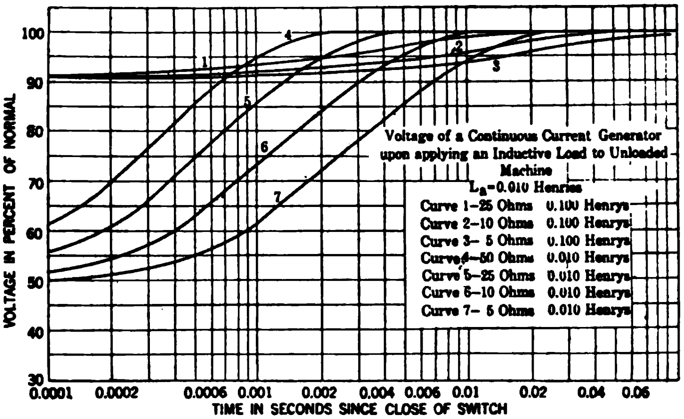

FIG. 3

CASE TWO

An inductive load thrown upon an unloaded machine.

Under these conditions the voltage can never drop to zero, since

$$e = ir + \frac{di}{dt} L_1 \tag{9}$$

where L_1 is inductance of load.

As in equation (3) the current will rise according to an exponential law,

$$i = I(1 - \epsilon^{-\alpha_2 t}) \tag{10}$$

where $$\alpha_2 = \frac{r_1}{L_a + L_1}$$

differentiating we get

$$\frac{di}{dt} = \frac{r_1 I}{L_a + L_1}\,\epsilon^{-\alpha t} \qquad (11)$$

$$= \frac{E}{L_a + L_1}\,\epsilon^{-\alpha t} \qquad (12)$$

substituting (10) and (12) in 9 we get

$$e = E\,(1 - \epsilon^{\alpha t}) + \frac{L_1 E}{L_a + L_1}\,\epsilon^{-\alpha t} \qquad (13)$$

which at $t = o$ becomes

$$e_{t=o} = \frac{L_1}{L_a + L_1}\,E \qquad (14)$$

That is, at the instant of closing the circuit the voltage of the machine divides in the ratio of the inductance of the external and internal circuits, and there is an instantaneous drop to this value. The instantaneous voltage drop depends upon the ratio of the inductances, and is independent of their absolute value, or of the resistance of the circuit. The velocity of return to normal voltage, however, depends, upon the resistance, since at the instant of closing the switch

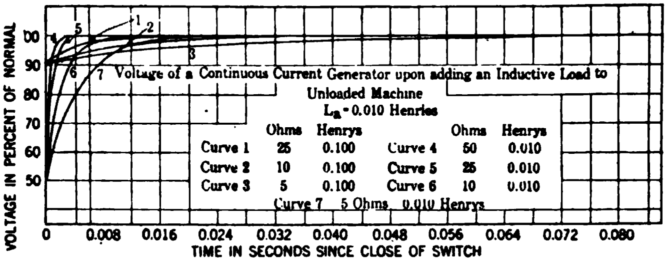

FIG. 4

$$\frac{dt}{dt} = \frac{rI}{L_t}$$

where $L_t = L_a + L_1$ (15)

Fig. 3 is a plot of calculated values on semilogarithmic paper and shows how a change in the value of resistance and inductance of applied load affects the extent of voltage drop and the rate of rise to normal.

Fig. 4 gives the same data plotted on cross section paper.

Oscillograms from tests under several conditions of load are shown in Fig. 5.

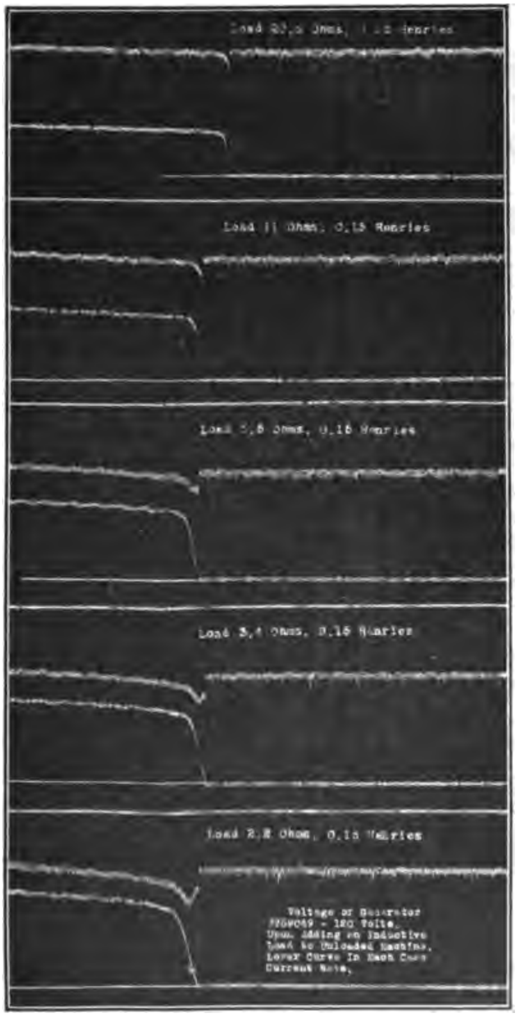

Fig. 5

Case Three

A non-inductive load added to a machine carrying a non-inductive load.

In this case there is an instantaneous drop of voltage to a value that is equal to the combined resistance of the circuits, times the current furnished by the machine at the instant of closing the switch,—that is

$$e_{t=o} = \frac{I_1 (r_1 + r_2)}{r_1 r_2} \qquad (16)$$

where

r_1 = steady load.
r_2 = added load.

$$I_1 = \frac{E}{r_1}$$

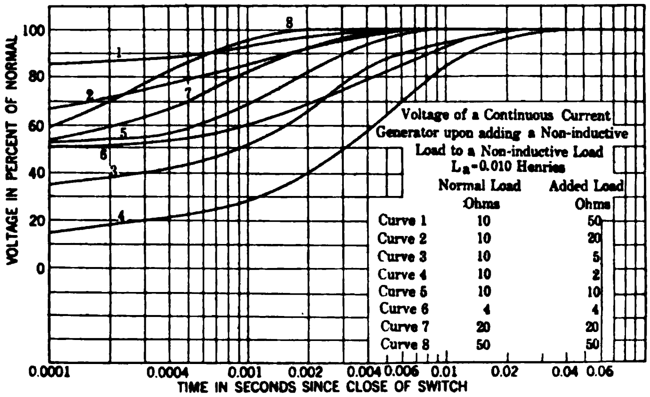

FIG. 6

The current builds up along an exponential law.

$$i = I_1 + I_2 (1 - \epsilon^{-\alpha_s t}) \qquad (17)$$

where

$$I_2 = \frac{E}{r_2}$$

$$\alpha_s = \frac{(r_1 + r_2)}{r_1 r_2 L_a}$$

and the voltage is

$$e = iR \qquad (18)$$

where

$$R = \frac{r_1 + r_2}{r_1 r_2}$$

$$e = I_1 R + R I_2 (1 - \epsilon^{-\alpha_s t}) \qquad (19)$$

Fig. 6 is a plot of calculated values to logarithmic abscissas for several values of steady load and added load.

Fig. 7 gives the same data plotted to rectilinear coordinates.

Oscillograms from tests under several conditions of load are shown in Fig. 8.

Oscillograms for similar values of load added to a generator under no load (case one) are shown for comparison.

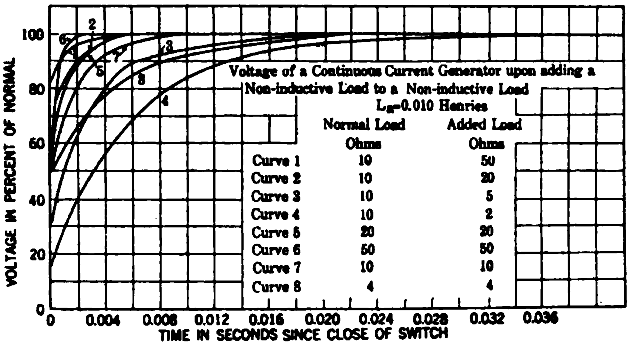

FIG. 7

CASE FOUR

The machine carrying a non-inductive load, and an inductive load added.

In this case there is no instantaneous drop of voltage but the voltage falls on an exponential curve, and rises on a similar curve of smaller exponent value. The voltage across the two branches is alike at every instant and is

$$e = i_1 r_1 = i_2 r_2 + \frac{di_2 L_2}{dt} \tag{20}$$

where i_1 is the instantaneous value of current in branch 1, i_2 the instantaneous value of current in branch 2. L_2 = the inductance of branch 2.

$$i_1 = I_1 + K\epsilon^{-\alpha_3 t} - K\epsilon^{-\alpha_4 t} \tag{21}$$

where

$$K = (r_2 - \alpha_4 L_2)B$$

where

$$B = \frac{\alpha_5 I_2 r_2 + E}{r_1 L_2(\alpha_5 - \alpha_4)}$$

where

$$\alpha_4 = \frac{S - q_1}{2 L_2 L_a}$$

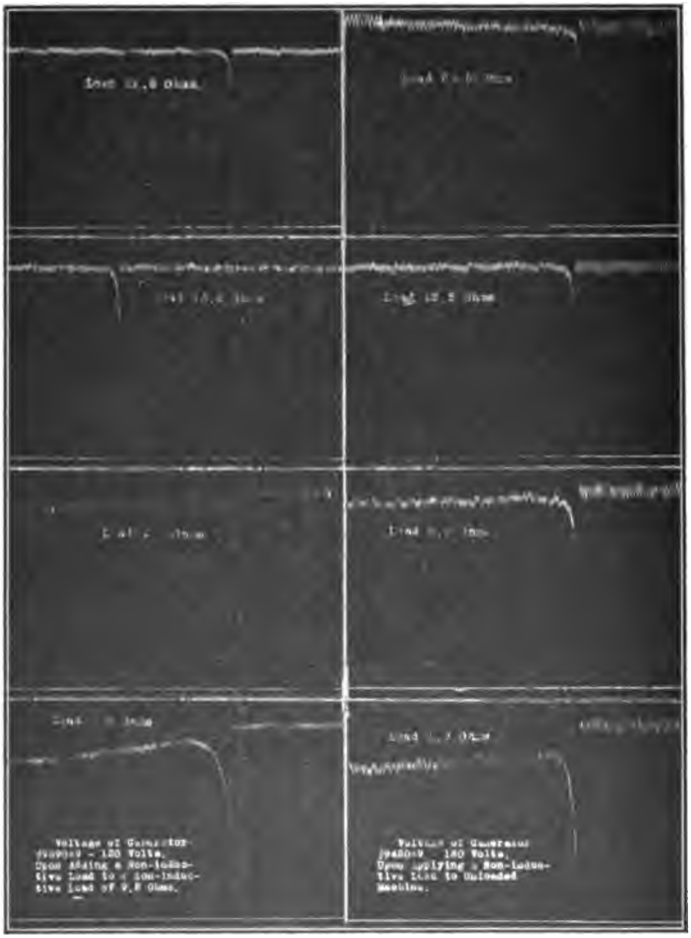

FIG. 8

and

$$\alpha_5 = \frac{S + q}{2 L_a L_2}$$

where

$$q_1 = \sqrt{S - 4 r_1 r_2 L_a L_2}$$

where

$$S = L_a (r_1 + r_2) + L_2 r_1$$

At $t = o$ (21) of course reduces to

$i_{1_{t=o}} = I_1$ and the voltage becomes (22)

$$e_{t=o} = I_1 r_1 = E \tag{23}$$

Fig. 9 is a plot of calculated values for several conditions of steady and added load.

Oscillograms from tests under several conditions of load are shown in Fig. 10 in comparison with similar values of non-inductive load added to a generator carrying non-inductive load. (Case three).

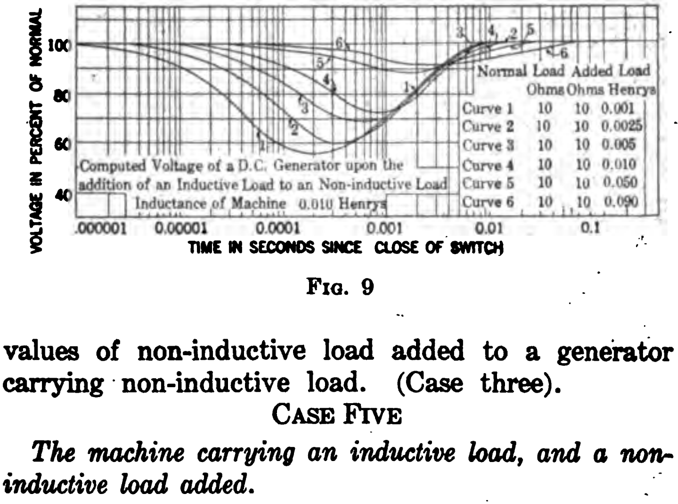

FIG. 9

CASE FIVE

The machine carrying an inductive load, and a non-inductive load added.

In this case there is an instantaneous drop to zero and a building up from there on an exponential curve.

$$e = i_1 r_1 + L_1 \frac{di_1}{dt} = i_2 r_2 \tag{24}$$

where

$$i_2 = I_2 + K_2 \epsilon^{-\alpha_6 t} + K_3 \epsilon^{-\alpha_7 t} \tag{25}$$

where

$$\alpha_6 = \frac{S_2 - q_2}{2 L_a L_1}$$

$$\alpha_7 = \frac{S_2 + q^2}{2\,L_a\,L_1}$$

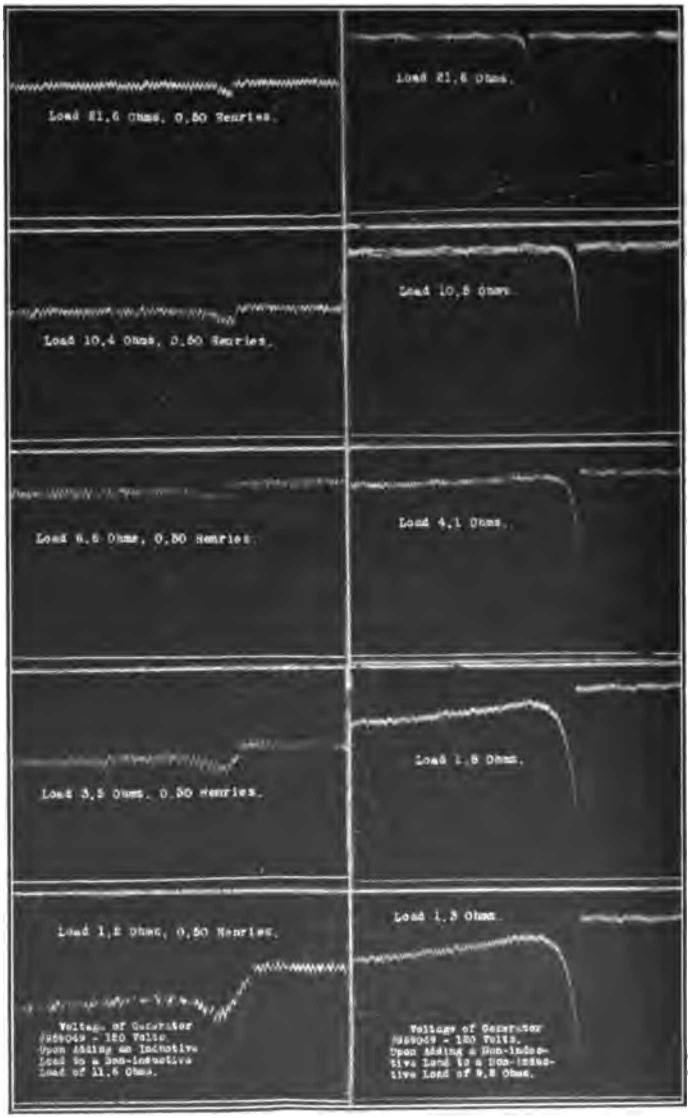

FIG. 10

$$S_2 = L_a\,(r_1 + r_2) + L_1\,r_2$$

and

$$q_2 = \sqrt{S^2 - 4\,r_1\,r_2\,L_a\,L_1}$$
$$K_2 = B_2\,(r_1 - \alpha_6\,L_1)$$
$$K_3 = (I_2 + K_2)$$

where

$$B_2 = \frac{I_2}{L_1\,(\alpha_6 - \alpha_7)}$$

Substituting (25) in (24) we get

$$e = r_2\,(I_2 + K_2\,\epsilon^{-\alpha_6 t} + K_3\,\epsilon^{-\alpha_7 t}) \qquad (26)$$

which on equating $t = o$ becomes

$$e_{t=o} = 0 \qquad (27)$$

since in (26) both K_2 and K_3 are negative numbers and $-K_2 - K_3 = I_2$.

Fig. 10-A is a calculated plot of several values of load added to a 10-ohm load of variable inductance.

Oscillograms from tests under several conditions of load are shown in Fig. 11 in comparison with similar values of non-inductive load. (Case three.)

CASE SIX

An inductive load added to an inductive load.

There is in this case an instantaneous drop in voltage, but never to zero. The instantaneous voltage is approximately equal to the voltage of the machine, times the ratio of the added inductance to the sum of the added load and machine inductances. The voltage builds up from this value along an exponential curve of four terms. The current builds up from zero according to an exponential curve of two terms.

$$e_1 = e_2 = i_1\,r_1 + L_1\,\frac{di_1}{dt} = i_2\,r_2 + L_2\,\frac{di_2}{dt} \qquad (28)$$

$$i_1 = I_1 + K_4\,\epsilon^{\alpha_9 t} - K_4\,\epsilon^{-\alpha_9 t} \qquad (29)$$

where

$$K_4 = B_3\,(r_2 - \alpha_9\,L_2)$$

$$B_3 = \frac{-\,I_2\,(r_2 - \alpha_8\,L_2)}{(r_1\,L_2 - r_2\,L_1)\,(\alpha_9 - \alpha_8)}$$

$$\alpha_8 = \frac{S_3 - q_3}{2\,L_s}$$

$$q_3 = \sqrt{S_3^2 - 4\,L_3\,r_1\,r_2}$$
$$S_3 = L_a\,(r_1 + r_2) + L_1\,r_2 + L_2\,r_1$$
$$L_3 = L_1\,L_2 + L_a\,L_1 + L_a\,L_2$$

Differentiating (29) we get

$$\frac{di_1}{dt} = -\,\alpha_8\,K_4\,\epsilon^{-\alpha_8 t} + \alpha_9\,K_4\,\epsilon^{-\alpha_9 t} \qquad (30)$$

Substituting (29) and (30) in (28) reduces it to

$$e_1 = r_1\,\{\,I_1 - K_4\,[\epsilon^{-\alpha_8 t} - \epsilon^{-\alpha_9 t}]\,\}$$
$$+ L_1\,K_4\,(\alpha_8\,\epsilon^{-\alpha_8 t} - \alpha_9\,\epsilon^{-\alpha_9 t}) \qquad (31)$$

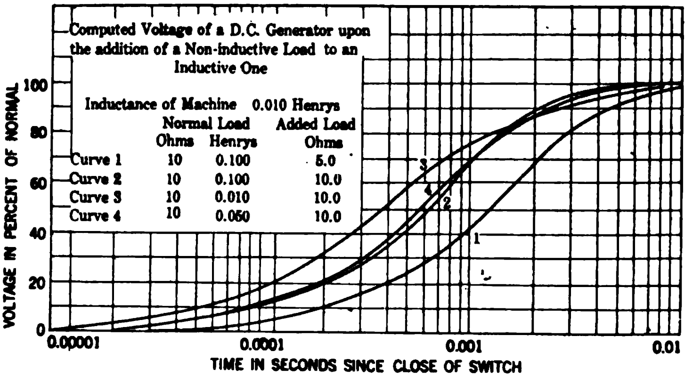

FIG. 10-A

This at $t = 0$ becomes

$$e_{1\,t=0} = r_1\,I_1 + L_1\,K_4\,(\alpha_8 - \alpha_9) \qquad (32)$$
$$= E + L_1\,K_4\,(\alpha_8 - \alpha_9) \qquad (33)$$

The second term of this expression is always a negative quantity since α_9 is always greater than α_8.

Fig. 12 is a plot of calculated values for several conditions of steady and added load.

Oscillograms from tests under several conditions of load are shown in Fig. 13 in comparison with similar values of inductive load added to a generator carrying non-inductive load. (Case four).

LOAD DECREMENTS

A load removed from the circuit produces effects opposite to those when load is added, namely, a voltage rise instead of dip.

It is well-nigh impossible to express mathematically such cases or analyze them experimentally, due to our inability to assign proper value to the increasing resistance of the arc that forms when the load is removed.

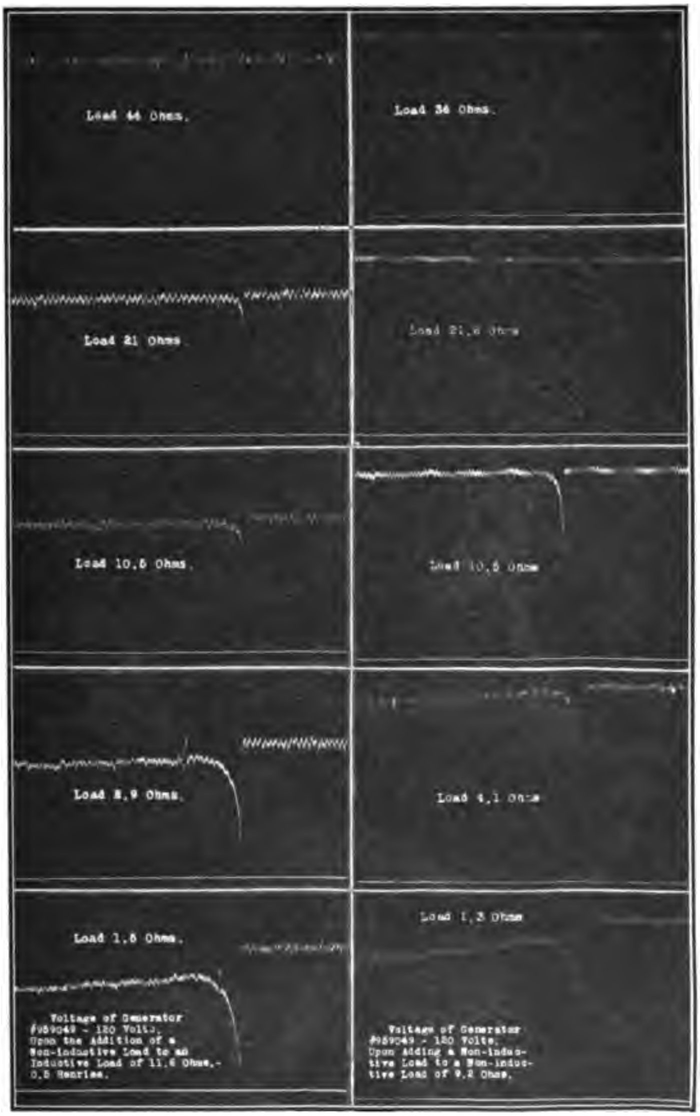

FIG. 11

CONCLUSION

The foregoing equations can of course be at best but first approximations especially at values well along the recovery curve. No allowance has been attempted for the variation of permeability with flux density, or for the individual characteristics and reactions of machines themselves. However, the equations will be found valuable and reliable for the computation of voltages near the start of the transient, which should make them of value in the determinations of those conditions that would cause flicker of lamps upon the loading of a generator or circuit.

The principles and equations already laid down lead to some rather unusual conclusions regarding the means of improving the instantaneous regulation of circuits where flickering of lamps is objectionable. The voltage changes that have been discussed are inherent in such circuits, and in general, in large measure are independent of the cross section of feeder or service lines. No matter how large the cross section there will be a voltage dip that is inherent in the circuit, and if the inductance of the series circuit is large enough the application of a load will cause flicker. Being dependent upon the inductance of the circuit for their duration, attempts at their elimination by the compounding of

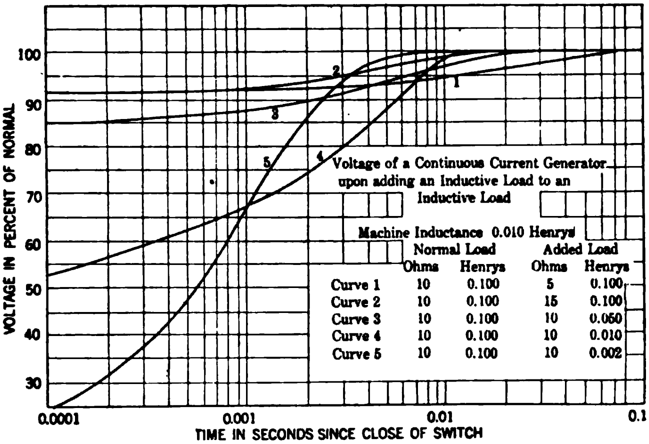

FIG. 12

machines are obviously steps in the wrong direction. Better instantaneous regulation will be obtained from

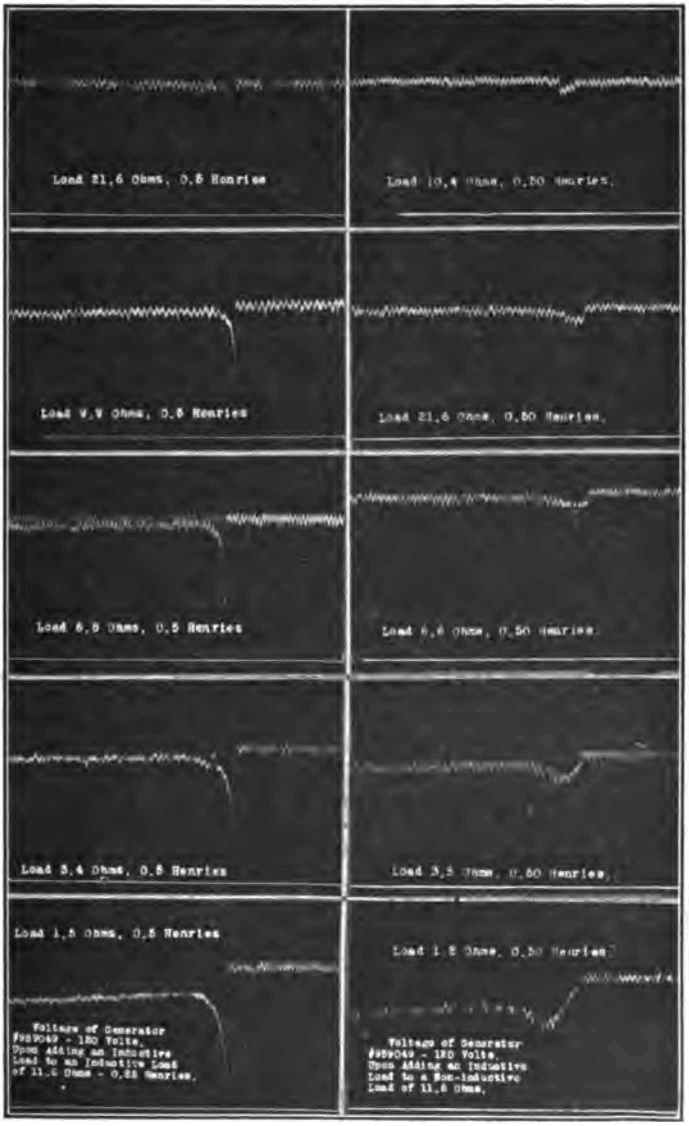

FIG. 13

shunt machines, than from compounded ones due to the lesser inductance of the former. Not only does compounding lead to a greater duration of transient conditions but the extra windings have no effect on the generated voltage of the machine until the current has risen well along the recovery curve.

The disturbances of large loads applied to lines can be mitigated by the insertion of suitable values of inductance coils of low resistance in series with the applied loads. In general the larger the value of the inductance applied the smaller will be the effect of a given load on the rest of the line. The fundamental principles involved in this application were discussed under the second case of load increments. It was there shown that the greater the ratio of the applied inductance to the machine inductance, the less will be the instantaneous drop. The other cases, that of inductance added to loads already on the lines, follow this same principle though the analytical treatment of them is not so simple.

DISCUSSION ON "INHERENT REGULATION OF D-C. CIRCUITS" (ELLIS AND ST. CLAIR), NEW YORK, N. Y., FEBRUARY 19, 1920.

V. Karapetoff: I should like to suggest to the authors of the paper that they mark their specific cases more distinctly. For example, when an inductance is added to the resistance, it is understood from the text that the inductance is put in series with the resistance. In another case they add a resistance and an inductance in parallel, and in one of the cases the machine is supposed to be carrying an inductance load. I do not know how a direct-current machine can carry a purely inductive load; perhaps it may, but certainly the paper needs a more explicit marking of the cases.

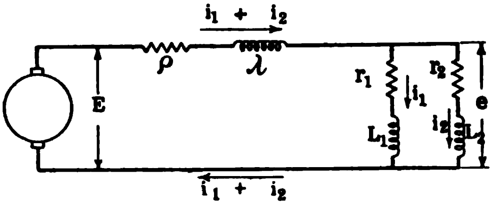

FIG. 1

Some years ago I read a paper on the concentric method in education in which I advocated going from the simple to the more complicated and from the concrete to the abstract. Now, this paper is written according to the concentric method, beginning with the simplest case and going to complicated cases. It may therefore seem inconsistent on my part to criticise a paper for being so written. In a professional paper I would rather see a more general case presented, first—I mean a case that admits of mathematical treatment without undue complication, and from which other cases may be derived as specific examples. Otherwise we can go on multiplying these cases, and yet not get a bird's eye view of the whole situation and of the possibilities of the method itself. I should like to show that all these cases and many others are comprised in one, which permits of a very simple integration.

Fig. 1. shows a d-c. generator loaded on two branch loads in parallel. Each load consists of a resistance r and of some inductance L in series with it. The resistance ρ and the inductance λ of the armature and of the leads to the load are concentrated

in the line, thus giving an ideal armature with a constant induced e. m. f., E. The change in the armature reaction, due to the sudden change in the load, is disregarded as it is in the original paper.

At the instant $t = o$ the original values of the three resistances and of the three inductances are *suddenly* changed, and the values shown in the figure are the *new* values. The values of inductance previous to $t = o$ are of no interest since the currents then are steady; instead of giving the values of the resistances previous to the change, we shall assume that the steady currents up to the time $t = o$ are known.

After the instant $t = o$ the transient currents in the circuit are expressed by the simultaneous differential equations:

$$e = i_1 r_1 + L_1 \frac{d i_1}{d t} \tag{1}$$

$$e = i_2 r_2 + L_2 \frac{d i_2}{d t} \tag{2}$$

$$E = \rho (i_1 + i_2) + \lambda \frac{d}{d t} (i_1 + i_2) + e \tag{3}$$

where e is the transient load voltage. The result of the elimination of any two out of the three unknown functions, *viz.*, i_1, i_2, and e, leads to a linear differential equation of the second order. Hence, the solution is of the well-known form

$$i_1 = I_1 - \frac{\alpha}{r_1 - m L_1} \epsilon^{-mt} - \frac{\beta}{r_1 - n L_1} \epsilon^{-nt} \tag{4}$$

$$i_2 = I_2 - \frac{\alpha}{r_2 - m L_2} \epsilon^{-mt} - \frac{\beta}{r_2 - n L_2} \epsilon^{-nt} \tag{5}$$

In these equations I_1 and I_2 are the *final* values of the currents when $t = \infty$, and α and β are the constants of integration to be determined from the initial currents at $t = o$. The decrements (or the increments) m and n, according to which the transient currents vary from their initial to the final values, are functions of the six given constants of the circuit.

Substituting the values of i_1 and i_2 from (4) and (5) into (1), (2), and (3), and equating to zero the coefficients of ϵ^{-mt} and ϵ^{-nt}, since the results must be identically true at any instant of time after $t = o$, we get

$$\frac{1}{r_1 - m\,L_1} + \frac{1}{r_2 - m\,L_2} + \frac{1}{\rho - m\,\lambda} = 0 \tag{6}$$

and a similar expression for n. Since the expressions for m and n are identical, and since equation (6) is a quadratic with respect to m, we conclude that m and n are the two roots of equation (6) which thus can be readily computed. Both roots of equation (6) are always real.

Let the steady currents in the two load branches, at the time preceding the sudden change, be ρ_1 and ρ_2 respectively. Because an electric current possesses electromagnetic inertia and cannot be changed suddenly the values of the transient currents at the instant immediately following $t = 0$ are also equal to ρ_1 and ρ_2. Substituting these values into eqs. (4) and (5) when $t = 0$, we get

$$\rho_1 = I_1 - \frac{\alpha}{r_1 - m\,L_1} - \frac{\beta}{r_1 - n\,L_1} \tag{7}$$

$$\rho_2 = I_2 - \frac{\alpha}{r_2 - m\,L_2} - \frac{\beta}{r_2 - n\,L_2} \tag{8}$$

From these expressions the values of α and β can be easily computed.

Thus, the whole problem is reduced to the following four simple steps:

(1) Compute the initial steady currents ρ_1 and ρ_2 by Kirchoff's laws;

(2) Compute the final currents I_1 and I_2 at $t = \infty$ by Kirchoff's laws;

(3) Solve the quadratic equation (6) for m, and take the two roots of it as the values of m and n.

(4) Solve the simultaneous linear equations (7) and (8) for α and β.

The transient currents are then given by the equations (4) and (5) and the variable voltage across the load is

$$e = E_0 - \alpha\,\epsilon^{-mt} - \beta\,\epsilon^{-nt} \tag{9}$$

where

$$E_0 = I_1\,r_1 = I_2\,r_2 \tag{10}$$

is the final load voltage at $t = \infty$.

H. R. Summerhayes: The work of Messrs. Ellis and St. Clair has a very practical application in that it refers to the phenomenon observed when elevator loads are thrown on to d-c. generators, and the flicker in lights, of course, is observed when the generators are relatively of small size. There is a dip in the voltage, when the elevator load is thrown on, observable even when d-c. service is obtained from the largest central stations, with capacity of a million amperes, more or less, and that is due to the self-inductance of the whole circuit, which is relatively very low, so that the dip has very little effect on the lamps. But one reason that an elevator load is so particularly objectionable in small plants, as far as lighting is concerned, is that you not only add the motor load, that is, the inductive load, but in many instances, to secure a better control, a non-inductive resistance is thrown in shunt across the circuit at the same time as the elevator motor in series with its starting resistance, and that may account for the rather large dips that are observed.

The result of many investigations has been summarized in what you might call a rule of thumb formula that with a d-c. generator of any size, you can throw on about one-sixth load without objectionable flicker being caused on the light supplied from that generator. That is a rough rule, of course, but it appears to hold for many sizes and types of generators, but if you try to throw on more than one-sixth load, the flicker in the lights is objectionable, and has to be met by such expedients as inducing inductance in the motor circuit or by other methods.

L. W. Thompson: A second method to overcome the dip that occurs in d-c. generators is to inductively couple the power and light circuits, through a one to one ratio transformer, so that at each sudden application of power load, a compensating pulse of energy is added to the lighting circuit at the instant the drop occurs.

Good results have been obtained from this method and it is readily applicable, for it is only necessary to add the equivalent of a 15 kw., 60-cycle lighting transformer for every 75-kw. generator capacity.

B. W. St. Clair: The general equations for transients are, of course, well known. To reduce them to fit specific cases requires more time than many engineers can afford and also demands mathematical transformations that are difficult for many practical engineers to follow, though they are simple enough to mathematically inclined engineers and physicists. The simpler cases require very simple equations, quite different in appearance from the general equations. It is much easier to substitute numerical values into these simplified equations for these cases than it is to substitute in the general equation. It is for this reason that the paper is written as it is. The paper does not attempt to cover all possible cases, but it does cover some of the more usual ones.

Most of the oscillograms reproduced in the paper do not show the voltage drop completely to zero in those cases where the theory carries it to zero. This is not due to a breakdown of the theory, but to limitations of the oscillograph. We made numerous tests of the effect of the damping of the oscillograph vibrator upon the faithfulness of the records of these extremely rapid transients. The value of damping used for these records was one that gave quite accurate results in most cases after the return from zero voltage was 25 per cent completed. A few simple computations are sufficient to show that it is impossible in most cases for the vibrator to reach zero where the change from full voltage to zero was instantaneous before the voltage recovery was well along on its cycle.

I am afraid that Dr. Karapetoff has misread those portions of the paper where the six specific cases are described. There is no case where the machine is supposed to be carrying a pure inductance. What is meant by an "inductive load" is, of course, one that has finite resistance. This is the usual meaning in engineering literature for the term. The curves and oscillograms for these inductive cases give the resistance and the inductance of the oncoming load.

Our case 1—the most simple one possible corresponds to a load of lamps added to a machine otherwise unloaded. Case 2—to a motor load in place of a lamp load added to an unloaded machine. Case 3—is similar to a lamp load added to a lamp load and this is one that may cause much trouble from flicker in office buildings where isolated plants are used. Case 4 corresponds to a motor load added to an existing lamp load. This is a case that has been the cause of numerous complaints and investigations in this city where elevators and lights are operated from the same generator. The fifth case is that of lamps added to a motor load and the last that of a motor load added to a motor load. This is Dr. Karapetoff's general case. A comparison of its equations and those of some of the earlier cases will show the desirability of using the specific rather than the general equations for numerical computation.

To use the general equation requires first of all the solution of at least two and generally more simultaneous equations before the main equations can be attacked.

The impulse transformer described by Mr. Thompson is a very ingenious device. It is an attempt to add to the lamp circuit another transient that will be of opposite polarity to the first so that if exactly the counterpart of the first it will neutralize the first one. It is, however, impossible to make this correction exact. The transients change with the lamp load, the starting currents required by the elevator, etc. It can be looked upon as a special case 6. There is always a drop of voltage in the lamp circuit. There is also added to that circuit a sudden rise of voltage. However, as the two transients generally have very different duration times, the correction is never complete. The voltage will change. Whether this change will cause perceptible flicker depends upon the rate of change, the size of the lamps, etc. The resulting flicker will be lessened in general if in addition to the transformer a large inductance of low resistance is added to the elevator circuit.

Philip Torchio: How would this be complicated if you took into account the temperature coefficient of lamp filaments?

B. W. St. Clair: This is something we tried to work out but found it a difficult thing to do. Dr. Langmuir has worked out the change in temperature and candle power as functions of filament sizes for sinusoidal voltages but not for impulse voltages. It is difficult to get an expression for the rate of cooling of the filament during these voltage shifts.

Presented at the 8th Midwinter Convention of the
American Institute of Electrical Engineers,
New York, February 19, 1920.

THE MEASUREMENT OF PROJECTILE VELOCITIES

BY PAUL E. KLOPSTEG
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AND
ALFRED L. LOOMIS
Formerly Major, Ordnance Dept., U. S. A.

The paper discusses the requirements imposed by proving ground practise upon a chronograph which is intended for general ammunition testing. Instruments of the standard pre-war pattern were entirely inadequate in number for testing the immense quantities of ammunition contracted for by the Government during the war. The development work undertaken to cope with the whole problem of velocity testing was first directed towards devising methods for speeding up measurements with the existing instruments. Conditions made it necessary to lay aside this work and concentrate upon the development of a totally new device for obtaining the results both speedily and accurately. The instrument which was developed, and adopted as a standard ordnance chronograph, is designated "Aberdeen chronograph." An account of its development is given. It is an assembly of many standard parts, with a few necessary special ones. Rapid production of the instruments was thereby made possible. The Aberdeen chronograph, and the procedure in determining velocities by this means, are described in detail. For the sake of comparison, the Boulengé chronograph is briefly described. Comparative results as to speed and accuracy of measurement are given.

INTRODUCTORY

ONE of the very important measurements which must be made at an army proving ground is that of projectile velocity. Not only is it essential that such measurements be made, in connection with ammunition testing and development, but it is equally essential that the results be reliable. Data of the greatest accuracy are needed in certain problems of experimental development. Velocity measurements for the computation of range tables must be as free from error as possible. In making powder tests, seven "uniformity" rounds are fired; the measured velocities, for acceptance of the powder, must agree within narrow limits. Rejection of a lot usually means the rejection of thousands of dollars' worth of material.

The difficulty of making these speed tests is increased by the fact that the portion of the flight of a projectile with which we are directly concerned, is an extremely transient phenomenon. The interval during which the shell lends itself to the measurement is of the order of hundredths of seconds. This fact adds a third difficult requirement to the two above mentioned, namely, certainty of obtaining the record when the projectile passes. Failure to do so necessitates the firing of an extra round, sometimes at a cost of hundreds of dollars. During the emergency there were several additional requirements. The apparatus had to fulfill the demand of giving good results day after day, under all conditions of weather, and this in the hands of operators without technical training.

It was recognized in the early summer of 1917 that when deliveries of the immense quantities of ammunition contracted for by the government would commence, the matter of making the necessary velocity tests quickly and accurately would prove an embarrassing problem. The existing chronographs, of the Boulengé pattern, were limited in number, and manufacture of enough of these instruments in time to meet the situation was, because of their mechanical complexity, out of the question. In order to represent adequately what the problems were and how they were met, we shall first make brief mention of some of the characteristics of the Boulengé chronograph. A fuller description is contained in War Department Circular No. 1682, and in the 5th edition of Foster's Handbook, page 1128.

In all the various practical instruments devised for velocity determinations, the measurement is that of the time interval required by the projectile to traverse a measured section of its path. With the Boulengé chronograph, this section is included between two wire screens, the breaking of which by the projectile actuates the recording mechanism. The latter utilizes two electromagnets, each in series with one of the respective screens. These magnets, by virtue of the current through them so long as the screens are intact, support, respectively, a rod to receive the record, and a weight, to release a marking "knife" by falling upon a trigger. The rod and weight are intended to fall freely under the action of gravity. The origin of readings on the rod is the mark made by the knife when the rod is suspended. A simultaneous break of circuits of the two magnets, produced by a device called a disjunctor, produces a second mark, somewhat higher up, which represents the time required by the instrument to function. When the projectile is fired through the screens, it breaks, first, the circuit of the magnet supporting the rod, then that of the circuit supporting the weight, thus making a third mark, still higher up on the rod. Measurement of the distances from the origin of readings to the two other marks mentioned gives the data from which the time interval may be turned up in tables. From the interval so found, the velocity is determined by the usual formula.

For the sake of a later comparison, statements should be made as to the accuracy, and as to the best conditions for precision of measurement with the Boulengé chronograph. Cranz in his "Lehrbuch der Ballistik"—which is probably the most authoritative work on ballistics yet produced—states that a given Boulengé chronograph is capable of repeating its reading of a certain time interval to 0.0001 second; but that, because of certain constant errors, which are very difficult to avoid or to determine with certainty, the absolute value of the time interval as measured by the instrument is, at best, reliable to 0.001 second. This statement is fully borne out by experience. The interval most favorable for precision of measurement being about 0.11 to 0.12 second, the distance between screens should be variable so that the distance best suited to a particular gun may be used. With the 155 mm. gun, for instance, this distance is 200 feet; on account of the blast, the first screen is placed about 100 feet from the muzzle. As a result, the second screen must be very high, even with moderate elevations of the gun. Thus, with an elevation of but thirty degrees, the second screen must be about 175 feet above the level of the gun. It is needless to point out, perhaps, that a movable screen support which can rigidly hold a screen at a height of 175 feet, has not yet been built. Even were this possible, the time consumed in hauling down a screen from a height of 175 feet, repairing or replacing it, and hauling it up again, would be so considerable as to render rapidity of testing quite impossible. The immediate remedy may be sought in a greatly shortened screen distance. In this case, of course, a more accurate recording instrument would be desirable.

In addition to the undesirable feature of requiring so much time for replacement, the wire screens have another serious drawback, which has repeatedly been mentioned in the writings of the Germans, the French and others upon the subject, and which thrusts itself upon the attention of anyone making a careful study of them. This is the very considerable uncertainty as to whether the recorded interval actually and accurately corresponds to the interval taken by the projectile to travel the measured distance between screens. The uncertainty is due in part to persistence of contact of the wires with the projectile, even after they are broken; in part also to the fact that the instant of break depends somewhat upon whether a wire is squarely hit by the tip of the projectile, or whether the tip strikes through the screen between neighboring wires. It happens occasionally—in spite of the fact that the wires are ordinarily spaced about one-third of a caliber apart—that the projectile slips through the screen without breaking a wire.

With the idea of eliminating the wire screens as the signaling end of the Boulengé arrangement, one of the writers in the summer of 1917 presented to the Sandy Hook Proving Ground a plan for "wireless" screens which had occurred to him some months before, in connection with other work upon the measurement of short time intervals upon which he had been engaged. Essentially, a screen of the type proposed was to be a mutual inductance coil of sufficiently large diameter to permit firing a projectile through it without great risk of injuring the windings. If a steady magnetic field is set up within such a coil by a current through one of the windings, it follows that any sudden disturbance of the field—by the passage of a projectile of magnetic material, for example—will produce a corresponding momentary electromotive force in the other winding of the coil. Preliminary tests showed the supposition to be correct.

For the purpose of developing this idea into a routine method, and to work upon other related problems, one of the present writers entered upon work at the Sandy Hook Proving Ground as Electrical Engineer in the Ordnance Department. The preliminary work was carried on at the Bureau of Standards, where facilities were kindly offered, and where much splendid cooperation was given, particularly by Dr. P. G. Agnew and Mr. W. H. Stannard. At the same time other possible solutions of the problem of speeding up the tests were being considered, and apparatus was being built. Frequent trips were made to Sandy Hook for field tests, which had to be made under very adverse conditions on Saturday afternoons and Sundays, so as not to interfere with routine firing.

The inductor scheme as partially developed was put to actual test, with very promising results; the Boulengé chronograph used with two such coils to all appearances functioned in the same manner as though the projectile had broken the usual type of wire screens. Unfortunately the work had to be discontinued before the development could be considered through the experimental stage, because of the removal of the proving ground from Sandy Hook to Aberdeen, Md. Here experiments with the coils became prospectively impossible for some months, and the work was concentrated upon another device which seemed to hold considerable promise, and with which tests could more readily be made. This device had been constructed at the Bureau of Standards for the purpose of recording signals produced by the "bow wave" of the projectile. The instrument which was finally evolved was called the Aberdeen chronograph. Work upon the inductor screens was not resumed because the newly developed instruments met all the needs which necessitated the development work, and their rapid production, as well as other problems occupied attention until the close of the war. It has been reported recently, however, that the inductor method is being applied with success in England.

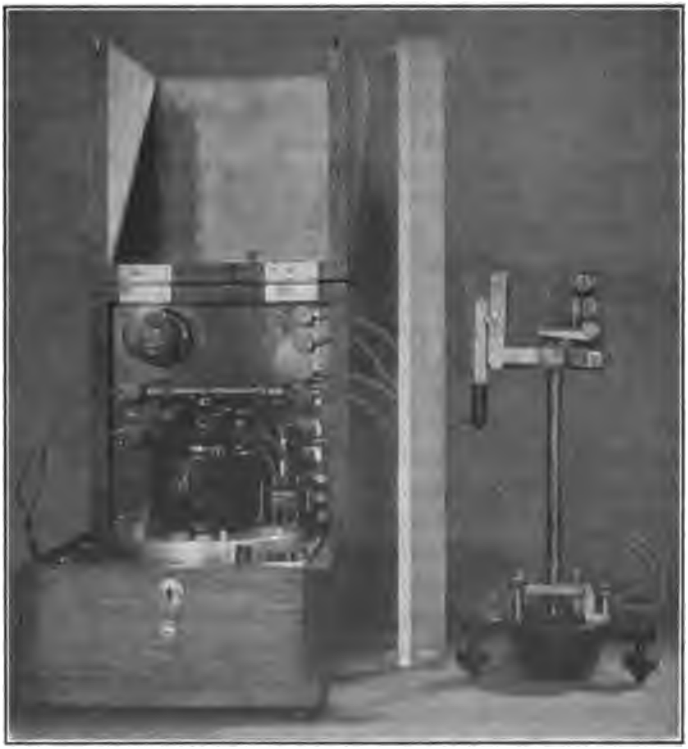

FIG. 1—ABERDEEN CHRONOGRAPH, FALL APPARATUS AND DIRECT-READING VELOCITY SCALE. INSTRUMENTS IN OPERATING POSITION.

DEVELOPMENT OF THE ABERDEEN CHRONOGRAPH

The device mentioned in the preceding paragraph consisted of a Leeds & Northrup governed motor, on the vertical shaft of which was mounted a flywheel with a smooth rim. A strip of paper could be fastened upon the rim to receive the records produced by the projectile when it actuated the signaling device. Up to this point the instrument differed from previously described ballistic chronographs only in that a constant speed motor was used. All such chronographs, however, utilized the breaking of wires for the signals, the interruption of the current producing a spark from a point in close proximity to the rapidly revolving drum. In this case, the recording system was designed for open-circuit operation, *i.e.*, the spark was to occur when a circuit was closed by the projectile. In connection with such circuits, it had been planned to use automatic contacts which should close when the bow wave of the projectile impinged upon them. The task of constructing such switches so that they would operate unfailingly and under all conditions proved formidable. The greatest possible speed in getting something usable remained imperative, and the urgency became greater as time passed.

It then occurred to one of us to try a new form of screen, or better, a form of contact target, in which contact should be established mechanically by the tip of the projectile. This target in its simplest form consists of a pair of metal sheets, of suitable thickness, separated by an insulating layer. Experiments immediately demonstrated the success of such an arrangement. Thus, the recording mechanism arranged for "make" circuit signals, together with the contact targets, became a most promising basis for further work of development.

On March 2, 1918, a modified design of the Mark I instrument* was taken to the Leeds & Northrup Company, and on March 15th three of the new Mark II instruments had been completed and were ready for operation at Aberdeen. These three instruments were used daily in routine measurements, in a shanty of rough boards on the gun platform, and so successful were they that it was decided to proceed immediately with the construction of the instruments in quantity. A new design, Mark III, including valuable suggestions from the manufacturers, was worked out which, with the exception of minor details, became standard. In

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*Mark I, Mark II, etc., is the War Department designation of successively developed types of ordinance material.

these, of course, the faults revealed by experience with the Mark II instruments were eliminated. Much experimenting in the laboratory and under service conditions was carried on in the meantime on both the electrical and mechanical components of the apparatus. Several further improvements were made, and from this point the production of Mark IV Aberdeen chronographs in lots of 25 proceeded without delay.

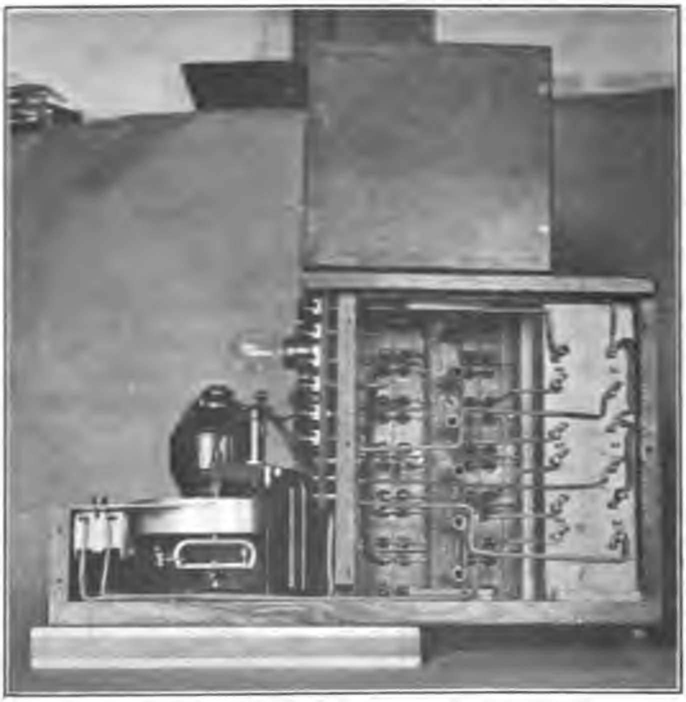

FIG. 2—ABERDEEN CHRONOGRAPH, SIDE VIEW, WITH CASE OPENED FOR INSPECTION OF PARTS

An important factor in the success of the Aberdeen chronograph in the emergency, aside from questions of its excellence as an instrument, was the fact that from the time production began until the last instrument needed had been delivered, the average rate of production per working day was one complete instrument. This became possible, even with manufacturing facilities already overtaxed, because the design included many standard parts.

DESCRIPTION OF MARK IV ABERDEEN CHRONOGRAPH

Fig. 1 shows the chronograph ready for operating, and Fig. 2 is a side elevation, with the case open for inspection of the circuits. The size of the apparatus may be judged by comparison with the 12 inch scale, shown in the latter view. In Fig. 3 the case of the chronograph is shown closed, with the carrying handle attached.

FIG. 3—CHRONOGRAPH IN CLOSED CASE WITH CARRYING HANDLE

The mechanical part of the device is seen by reference to the photographs to consist of a shallow cylindrical drum of aluminum, mounted on the vertical shaft of a small (110-volt direct-current) motor. On the lower end of the same shaft is carried the constant speed governor, upon which depends, in large measure, the accuracy of the instrument. It is a development of the governor used in the Leeds & Northrup temperature recorder. The function of the governor is to control the power input into the motor. Its *modus operandi* may be understood by reference to Fig. 6. A thorough analysis of the device resulted in the design illustrated, which is capable of holding the speed so constant that the mean deviation from average speed, as determined in repeated tests over a fixed 0.2 second interval, was less than 1/10 per cent. Another way of representing this fact is in showing the results of a series of observations. Taking 100 measurements of a 0.2 second interval, 62 were within 0.04 per cent of average speed; 70 within 0.05 per cent; 77 within 0.06 per cent; 95 within 0.07 per cent; 97 within 0.08 per cent; and 100 within 0.09 per cent. In these measurements, therefore, it is safe to assume a maximum probable error of ±0.1 per cent. A speed of 25 rev. per sec. is thus maintained; the method for checking this value will be described later.

A specially prepared strip of paper receives the record marks. The paper, which is blue, has a thin coating of white paraffin on one side. When a spark passes through the paper, it not only perforates the latter, but also melts a tiny bit of the paraffin, leaving a bright blue, easily distinguishable spot. The width of the strip is equal to the depth of the cylindrical drum, and its length to the inner circumference of the drum. It is held in position very firmly against the smooth inner surface of the drum, on account of the rapid rotation of the latter. By this arrangement, much saving of time is effected, since the record strip may be easily inserted in the drum, and, with a little practise, removed while the motor is running at full speed. With the speed mentioned, and an inner circumference of the drum of 500 mm., the paper strip is given a linear speed of 12,500 mm. per second. Thus a very open and uniform time scale is secured. Three spark points (Fig. 6), in the same vertical line, are held within 1 mm. of the record strip by an insulating block. A spring latch holds the points either in the recording position or at some distance from the paper.

Reference to Fig. 6 shows that one of the electrical spark-recording units consists essentially of a condenser, and the primary of a transformer, both in a circuit with the contact target. Up to the instant of firing, the condenser is kept charged through high resistances, from the supply line which furnishes the power for the motor. The supply line of 110- or 220-volts direct current, is connected at *AB*. When the tip of the projectile strikes the target, contact is established between the plates, and a discharge passes through the transformer resulting in a spark from the corresponding recording point. The main condenser has a capacity of 10 micro-farads. Later work showed that an auxiliary condenser of 6 microfarads, connected in parallel with the plates of the contact target, greatly intensified the spark. The complete circuits are shown in the diagram. Three recording units were provided, as shown. Normally two only were used, the third being for emergency, or for use in cases where average velocity values over two adjacent sec-

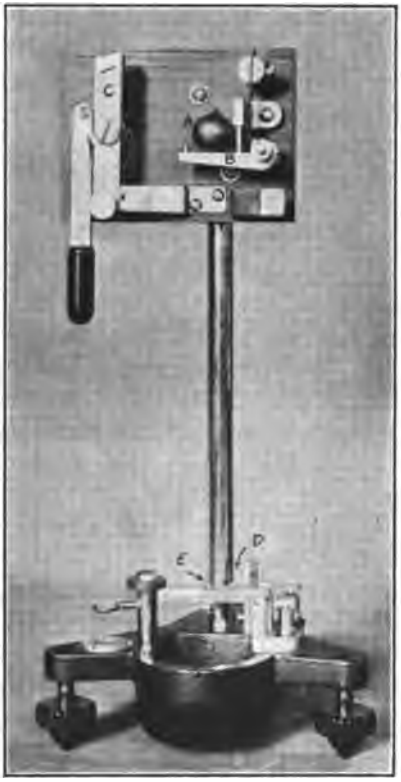

FIG. 4—FALL APPARATUS FOR VERIFYING ACCURACY AND CONSTANCY OF MOTOR SPEED

tions of the trajectory were to be determined on the same instrument.

A ready means for verifying the speed of the rotating drum is essential, to give the operator confidence in the performance of his instrument if for no other purpose. If we could, in some manner, produce two sparks, separated by an accurately-known short interval of time, we should have a ready means for checking the accuracy and constancy of the motor speed. A device which could be utilized for this purpose had previously been worked out in connection with psychological time measurements.* This is a "fall apparatus," the modified form of which for use with this chronograph is seen in Fig. 1, and shown in detail in Fig. 4. A ¾-inch steel ball *A* rests upon a spring-supported horizontal lever *B*. When the counterpoised hammer *C* which is under spring tension is released, it strikes the end of the lever a smart blow, knocking it clear of the ball. At the instant of striking, the contact between the hammer and lever produces a spark from, say, the uppermost spark point in the drum. Simultaneously the ball begins to fall. When it reaches the bare wire *D*, held on an insulated post, it makes a wiping contact with the latter, and then touches the tungsten plate *E*, between which and the wire it closes the second circuit. At the instant contact is made at *E*, a second spark jumps, say from the next lower spark point. Obviously we may adjust with great exactness the distance between the lowermost point on the ball, when it is in position on the upper lever, and the plate *E*, so that the time of fall between these two points is 0.2 second. This corresponds to five revolutions of the drum. If the speed of the latter is correct, two marks will be found upon the strip, one directly above the other. With slightly higher or lower speed than the correct value, the two marks will be displaced with reference to each other. From the amount of this displacement the constant error in speed becomes easily known. Thus, when the fall apparatus is kept permanently connected as shown in Fig. 1, it is a simple matter at any time dur-

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*Klopsteg, *Journal of Experimental Psychology*, II, 253, 1917.

ing the course of the day to calibrate the chronograph. In practise, the speed remained so constant in a day's run that calibrations were made only two or three times during the day.

Fifty feet was chosen as the standard distance between targets for all velocities from 500 to 2500 feet per second. With a fixed distance between the contact targets, and fixed speed and dimensions of the drum, there is a one-to-one correspondence between distance between record marks and velocity. Thus it becomes possible to construct a direct-reading scale for velocities which is suitable for all the values between the limits mentioned. The scale was so designed that, when the shot was fired, regardless of the relative positions between the spark points and the abutting ends of the record strip, the velocity readings could be made without possibility of mistake. The scale is shown in Fig. 1, and again in the photograph of the accessories case (Fig. 5). This picture also shows the fall apparatus taken apart and in the case.

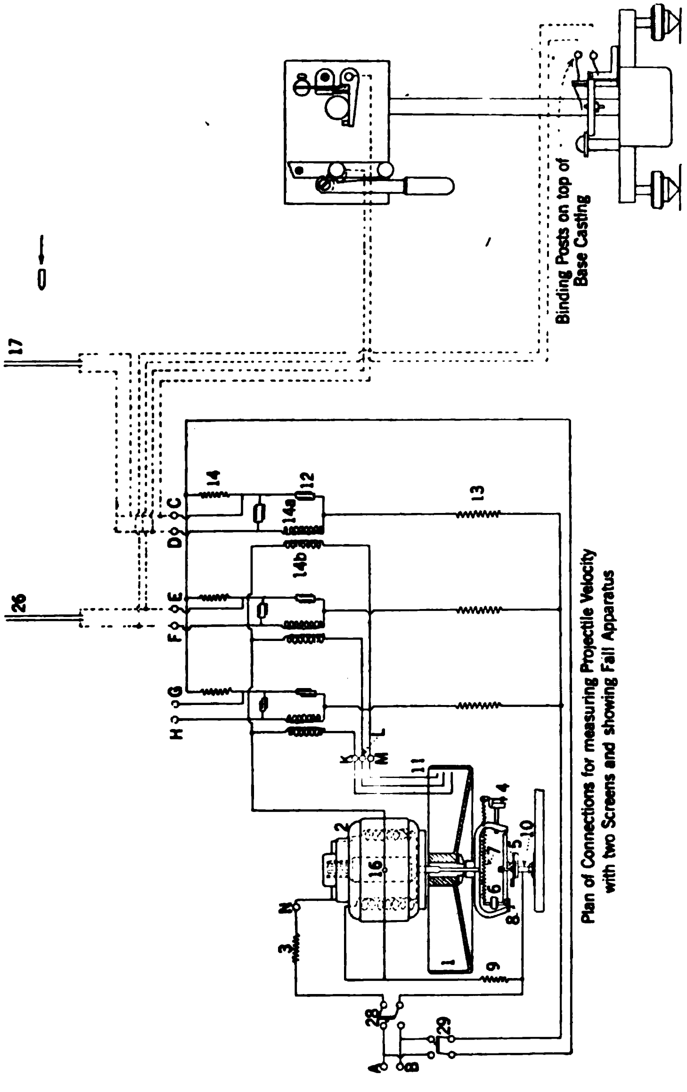
In its contact target arrangement, already described, the Aberdeen chronograph is unique among the ballistic chronographs. The fact that the recording spark occurs when the tip of the projectile establishes contact between the metal plates has the consequence that the otherwise great uncertainty in the distance between the points at which the signals occurred is practically eliminated. It may here be repeated that such uncertainty is probably the chief point of weakness, so far as accuracy is concerned, in the ordinary wire screens used with the Boulengé chronograph. That such errors are truly minimized with the contact targets is shown by the fact that the distance between targets could be reduced to 25 or 30 feet without seriously affecting the agreement among several instruments measuring the same time interval.

PROCEDURE IN VELOCITY MEASUREMENTS

The application of the Mark IV chronographs to velocity measurements in such a way as to secure the data with minimum loss of time was in itself a development problem. Three instruments were used with the same pair of contact targets, and the observations were independently made and entered in the data book by three respective observers. The averages were then entered in a fourth book, together with the necessary accompanying data as to the gun, kind of projectile, etc.

The contact targets were made up beforehand in quantity. For the three-inch guns they consisted of lead foil strips about 3-in. by 12-in. which were attached with hot paraffin to the opposite sides of a sheet of paraffined building paper 5-in. by 17-in. Much of the routine firing was done with constant elevation of the guns, so that permanent supports for the screens could be used. These supports consisted of uprights, each of which had two cross-arms at the proper height, extending out on one side. The lower cross-arm was provided with a spring clamp, on the insulated inner surfaces of the jaws of which there were contact buttons which made connections with the lead foil sheets. The upper arm also carried a clamp which supported the contact target in a vertical position, and between the two clamps the target was held with sufficient firm-

FIG. 5—ACCESSORIES CASE WITH NECESSARY AUXILIARY APPARATUS



Plan of Connections for measuring Projectile Velocity with two Screens and showing Fall Apparatus

FIG. 6

ness to prevent its being moved appreciably by the wind. From the contact buttons on the lower clamp, wires were run in conduit to the instrument building, just behind the gun platform.

In undertaking a series of measurements, the motors are started running, and their constancy and accuracy of speed verified by a test with the fall apparatus. In the meantime, the gun is made ready and the contact targets put in place. The strips on which the "check records" are made are removed, and clean strips spun into the drum. The instruments may then continue to run *ad libitum* until the shot is fired. The motor switch button is then turned off, the strip with the velocity record removed and a new one inserted. (All strips are marked beforehand for proper identification). With the direct reading velocity scale, the velocity is read directly from the strip, and entered in the record book. The proper correction figure for "reduction to muzzle"* and for motor speed is added, and the corrected reading put down. All this is about a half minute's work. In the meantime, the detail of men outside has replaced the screens and reloaded the piece, and the next shot may be fired. In this manner velocities could be measured at an average rate of one round per forty-five seconds, or from three to five times as rapidly as by the older method. The advantage of speed of measurement is indicated by the fact that, although the Aberdeen Proving Ground was not established until January, 1918, and the new chronographs were not put in regular service until June, seventy per cent of all velocity measurements made at the Aberdeen Proving Ground before the cessation of hostilities, were made with Aberdeen chronographs.

A comparison of the two types of chronographs, Boulengé and Aberdeen, brings out clearly the fact that the former is distinctly a laboratory instrument, requiring a very stable foundation at a minimum dis-

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*By "reduction to muzzle" is meant the process of transforming the observed velocity to that which obtained at the muzzle, on the assumption that the projectile experienced uniform retardation from the instant it left the gun.

tance of several hundred yards from the gun platform, while he latter is a field instrument, which may be and is being used immediately behind the gun platform; or it may be set upon the ground as near the gun as may seem desirable. Proximity to the gun does not affect the records. The facts previously mentioned with reference to the two types of instruments may already have suggested that, while it takes skilled operators, who are thoroughly familiar with the sources of error in the Boulengé and how to avoid them, to get acceptable results, the Aberdeen chronograph has from the outset been successfully used by operators who had had no previous training; they were enlisted men, taken from the ranks.

Many competitive tests as to accuracy were made during the process of development, measuring the velocity of the same projectile with a set of three Boulengé and a set of three Aberdeen chronographs. Average dispersions* for twenty rounds were computed for many series of firings. Invariably the average dispersion with the Boulengé chronograph was from two to four times as great as with the Aberdeen. In routine firings, the three Aberdeen chronographs agreed within one or two feet per second on a velocity of 1700 feet per second, and only rarely did the maximum dispersion at this value become as great as 5 feet per second.

One source of gratification is the fact that in spite of the "momentum of custom"—a phrase borrowed from a recent article upon the subject—the Aberdeen chronograph was one of the few devices developed under the pressure of necessity, which was fortunate enough to find a place in actual service of the government, and to play an important part in increasing the efficiency of the immense program of preparation which was so well under way when the armistice put an end to the hostilities.

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*By dispersion is meant the maximum difference between any two of the three instruments used. The average dispersion is the sum of the dispersions divided by the number of rounds.

DISCUSSION ON "THE MEASUREMENT OF PROJECTILE VELOCITIES" (KLOPSTEG AND LOOMIS), NEW YORK, N. Y., FEBRUARY 19, 1920.

E. E. F. Creighton: There are some questions that come up that possibly may not be quite in line with the paper, but perhaps Mr. Klopsteg would be willing to answer them. First, in regard to the velocity of the projectile inside the gun—are those instruments adapted to such measurements? Since in all photographs we have seen the projectiles are ahead of the gases, why is it necessary to move the screen out about 100 feet to get rid of the effect of the gases?

Another question is in regard to the bow wave—can the bow wave break the screen wires, and does the factor of wire drawing enter into the problem? If the projectile can pass between wires that are less than the caliber, that means the wires are slipped aside without breaking. Would it not be quite possible to have considerable wire drawing without breaking the wire at the time the projectile passed through.

Regarding the insulation between the two plates—to me it is an entirely new way of making contact. If a paper was placed between the two plates, how could the contact be made permanent? Would not the insulation remain between the two plates?

Regarding the absolute time—it must be evident to all of those who handle instruments that the relative time of making contact would be extremely accurate, but there enters into the time between the contact and the spark a certain time element in the inductance of the circuit.

Clayton H. Sharp: Once in the course of business a problem came to me to determine the relative velocities of flight of different kinds of golf balls. The question was the effect of the character of the rubber core in the golf ball. This introduced some of the problems of projectile flight. In the first place, of course, it was necessary to hit the balls a uniform blow. An attempt was made to do that by taking a d-c. motor and putting the equivalent of a golf club on the face of a pulley on the end of the motor. The club was arranged to slide longitudinally in guides and had two positions corresponding to slightly different circles of swing, and was pulled by a spring from the "in" position to the "out" position. In the "in" position the club cleared the golf ball which was set up on a tee. With the motor in operation a trigger could be tripped allowing the club to go to the "out" position and to strike the ball. The strength of the blow depended on the linear velocity of the head of the club which could be computed from the radius and the rev. per min. of the motor.

The ball passed through two wire screens quite close together. In order to measure the time we took a synchronous motor and put a wooden pulley on it, and put a circular chart on the end of the pulley. An electromagnet connected up with the screens and actuating a stylus on the chart left a record of the times of passage through the screens. It had to be done indoors, with a limited space, and did not succeed quite as well as we might have wished, but at least it was the application of the principle of constantly driven motor to the measurement of very short time intervals.

Roy Kegerreis: About the blast reaching the screen before the projectile; the problem of the Helmholtz sound resonator is somewhat the same. If one calculates anything about that it is necessary to calculate the magnitude and direction of the velocity of each particle of air at every point, both within and without the hollow spherical resonator. That problem has been solved and leads to a pretty complicated sort of mathematics. Somewhat the same thing happens in the case of the gun.

A rough concept of the action of the blast before the gun may be had by imagining a row of blocks set up rather near to each other. If the one at the end were pushed, the next would soon move, and the next, and so on until a large number of blocks would be pushed ahead, the number of moving blocks being ever on the increase.

The force which the gases from the gun exert may be compared to the force which pushes the first block and the moving blocks may be compared to the air particles before the gun. This shows why the actual front of the compression wave may be ahead of that shown on a photograph, since the compressed air before the products of combustion from the gun is so nearly transparent.

P. E. Klopsteg: We made no tests to determine projectile velocities inside the gun. The primary effort in this development work was to secure a device which would quickly and accurately measure the external velocity, and the work was carried on constantly with that in view. It was never considered feasible to adapt this device to measuring the extremely short intervals involved in the velocities inside the bore.

The Bureau of Standards, however, has been carrying on very extensive and elaborate investigations for the Navy Department, using an oscillograph driven at very high speed, something like ten meters a second, for making determinations of that kind.

The effect of the blast is merely this. Although in the photograph the projectile is always shown ahead of the blast, it has been shown very clearly that at the instant the projectile leaves the muzzle, some of the blast does get ahead of the projectile for a short distance, and that the projectile is then actually accelerated by the pressure of the blast behind it, for a distance of some few feet after it leaves the bore of the gun. However that may be, the blast from the large gun does continue along after the projectile has left, and if the first screen was not placed far enough away, the blast might do damage to the screen support rather than the wires.

As to the effect of the bow wave on the breaking of wires, I am unable to answer the question, and to the question as to whether there is wire drawing, I am unable to give a definite answer.

The effect of the projectile going through the screen without breaking the wires was observed in work on the 37 mm. gun, which required a screen made of fine wires very close together. These wires were woven back and forth between two pieces of paper glued together, and it was just a curious happening that in one or two cases the projectile got through one of the two screens without breaking the wires.

The insulation between the two plates need not be very high. As I attempted to show in the diagram of connections, the condenser is permanently charged from the line, and since the plates are in parallel with the condenser, the plates are also permanently charged from the line, through a high resistance. The passage of the spark requires only a momentary, not a permanent contact. This makes possible the arrangement used in the fall apparatus, such that when the hammer comes around and hits the lever a blow there is only a momentary contact, causing the spark to jump. With the contact targets, in most cases it is found that there is no contact between the perforated plates after the projectile has passed.

As to relative and absolute time, the accuracy of the measurements for the Aberdeen chronograph depended on the accuracy of the fall device. If we want to measure to accuracies of a few hundredths of a per cent, rather than tenths of a per cent, then we must know accurately the acceleration due to gravity and make allowance for it. In the use of the fall apparatus, as nearly as we could determine with all the experimental facilities we had, the same interval, within 0.00005 second, was required for the ball to fall a given distance from the upper to the lower contact point, and operate the contacts.

As regards the time lag between the passage of the spark and the contact at the projectile, the accuracy of the arrangement is assured by the similarity of the two circuits. One may assume, I think, that if a lag takes place at the first screen, there will be an equal lag at the other. That was assumed all through the measurements, and the results indicated that the measurements were giving satisfactory values.

C. H. Sharp: Could not the drum speeds be accurately calibrated by making it in the form of siren, and blowing through it, or matching it up against a tuning fork?

P. E. Klopsteg: Yes, it could. Again, the reason for not going to a more complicated arrangement for this purpose was that it was necessary in many cases to use the apparatus in the field, and laboratory devices of that kind could not be handled by unskilled men. We had considered at one time driving the drum with a shunt-wound d-c. motor, so wound and supplied with a source of current such that it would run at approximately normal speed, and then to mount the rotor of a synchronous motor on the same shaft. The auxiliary synchronous motor could be driven by means of a tuning fork, thus regulating the speed very precisely. That was given up as a complicated and unnecessary labor. The device used was found in practice to give all the accuracy desired.

C. H. Sharp: Regarding the Boulengé chronograph, the thought strikes me, at least, of its being a cumbersome and awkward and indefensible device. Why is it that it has been so generally accepted in ballistics?

P. E. Klopsteg: That is difficult to answer. One reason is that all military men grew up with the Boulengé chronograph, and it is a case of "momentum of custom," as I have indicated in the paper, which keeps it going. There is presumably a psychological reason for it—a false assurance, perhaps, brought about by the knowledge that when the rods hang, the two circuits are intact, and when the rods fall, the circuits are broken. The fact is too often overlooked that the Boulengé chronograph, although it is an instrument which when properly installed will seldom fail to give a record of some kind, it is nevertheless a delicate measuring instrument, the successful operation of which requires not only great care but considerable manipulative skill and a knowledge of all the possible sources of error. Judging from publications prior to 1915, it appears that the German authorities have conflicting views on the reliability of the Boulengé chronograph.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 20, 1920.

A NEW FORM OF VIBRATION GALVANOMETER

BY P. G. AGNEW
Secretary, American Engineering Standards Committee

Vibration galvanometers are very useful in null measurements, but have not been much used in industrial laboratories on account of their being sensitive to external vibrations and requiring delicate adjustments. The present instrument, which has a sensitivity higher than other forms of the moving-iron type, but less than that of the most sensitive forms of the moving-coil type, has the advantages of sturdiness, quick responsiveness, and freedom from the effects of external vibration. It consists essentially of a fine steel wire, mounted on one pole of a permanent magnet, and so arranged that the free end of the wire may vibrate between the poles of an electromagnet through which the current to be detected passes.

THE vibration galvanometer is a very useful instrument in alternating-current measurements where null methods can be used as, for example, in an almost endless variety of bridge measurements, in various applications of the alternating-current potentiometer, and in testing instrument transformers. As in the case of direct-current galvanometers, there are two general types of vibration galvanometers, the movving-coil type and the moving-iron type. In each type the moving element is mechanically tuned so that its natural period is the same as that of the alternating electromagnetic forces produced by the current to be detected, thus using the principle of resonance to produce a relatively large motion for a very small current.[1]

The reading is usually made by observing the image of an electric lamp filament reflected in a very small mirror attached to the moving system, by means of a

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[1] For a general discussion of vibration galvanometers, see: Laws' Electrical Measurements, 1917, p. 434; F. Wenner, TRANS. Am. Inst. Elec. Engineers, 31, p. 1243; F. Wenner, *Bull. Bureau of Standards*, 6, p. 347, 1910; A. Campbell, *Proc. Physical Society of London*, 26, p. 120, 1914.

telescope, or by a projection upon a screen. When current passes through the instrument the vibration of the moving element causes the image of the filament to appear to broaden out into a band.

It is evident that a vibration galvanometer of either type is simply a specialized form of synchronous motor, the whole mechanical output of which is used in overcoming air friction and elastic hysteresis.

The vibration galvanometer has been used but little in industrial laboratories, its principal use being in physical laboratories, and principally in precision work. The chief reasons for this limited use are that it is easily disturbed by external mechanical vibrations, and that delicate adjustments are necessary. In the present form of instrument these difficulties are greatly reduced. It is not, however, as sensitive as some forms of the moving-coil type.

Principle of Operation. The present instrument is of the moving-iron type. It consists essentially of a fine steel wire, mounted on one pole of a permanent magnet, and so arranged that the free end of the wire may vibrate between the poles of an electromagnet through which the current to be detected passes. If an unmagnetized steel wire, *W*, is held near the pole of an electromagnet, as in Fig. 1, the end of the wire will be pulled toward the pole of the electromagnet during each half wave of the current flowing in the winding of the electromagnet. That is, the wire will vibrate with twice the frequency of the current. If the wire be magnetized by mounting it on the pole of a permanent magnet, the free end of the wire will be alternately attracted and repelled by the alternating flux of the electromagnet. That is, the wire will vibrate with the same frequency as that of the current. But what is of more importance, the alternating mechanical pull will be very much greater than with an unpolarized wire, since the total flux from the wire is much greater.

The permanent magnet plays the same role in increasing the motion of the wire that the permanent magnet in a telephone receiver does in increasing the motion of the diaphragm. The total pull varies as B^2, and the change in the pull varies as the change in

B^2 resulting from the alternating current. Hence if a given small alternating current produces a change in flux, ΔB, the alternating mechanical pull is proportional to $B \Delta B$, instead of to $\overline{\Delta B^2}$, as it would be if the wire were not polarized by the permanent magnet.

Fig. 2 shows one of the first arrangements experimented with. A small electromagnet is mounted on the inside pole face of the permanent magnet, and the fine wire vibrator on the opposite pole face. The operation is readily seen. During one half-cycle the flux from the pole tip A is slightly strengthened, and that through B is weakened. Hence the vibrator moves toward A. During the next half-cycle the conditions are reversed and the vibrator moves toward B, and so on.

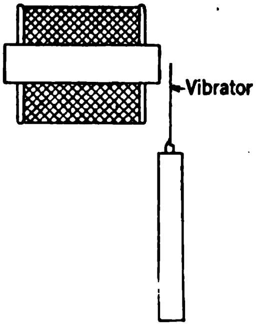

FIG. 1

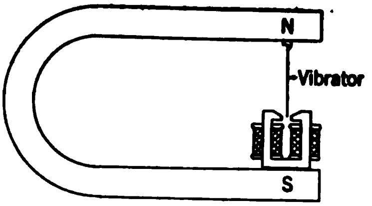

FIG. 2

Considering the arrangement as a motor, it is easy to see how the back electromotive force is generated. If the vibrator is moved back and forth by mechanical means, magnetic lines of force from the end of the vibrator move back and forth between A and B, alternately increasing and decreasing the flux in each, and thus generating an alternating electromotive force in the winding.

The vibrator, which is, of course, mechanically tuned so that its period is the same as that of the current to be detected, is observed by a microscope mounted perpendicular to the plane of the paper and focused on the end of the vibrator. In this way a much smaller motion can be detected than is possible with the unaided eye.

During a set of experiments undertaken in order to determine optima of conditions, such as size and shape of electromagnet, shape of pole tips, length of gap, and distance between end of vibrator and pole tips, an interesting observation was made. As a limiting case, the air gap of the electromagnet was reduced to zero by using a closed core as shown in Fig. 3. Even under this condition the device was nearly a tenth as sensitive as with the best arrangement that could be obtained with the same core after it had been sawed so as to form an adjustable gap.

Arrangement for Greatest Sensitivity. Of the many arrangements of parts that have been tried, that which gives the greatest sensitiveness and general convenience of working is shown in Fig. 4.

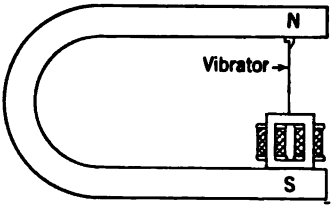

FIG. 3

The electromagnet is placed outside the gap of the permanent magnet, but in a position in which an appreciable amount of flux from the latter passes through the cores of the former. It is to be noted that in principle the arrangement is identical with the earlier one described above, the most important difference in the details of the arrangement being the weaker field to which the core is subjected. The two principal advantages of putting the electromagnet outside are: first, other things being equal, the instrument will be most sensitive when the core is working at the point of maximum differential permeability, and the point of maximum differential permeability occurs at very low values of B, (ideally, at $B = O$); second, with low frequency vibrators, which must be of very small diameter, the effect of the strong magnetic field upon the frequency of the vibrator introduces practical difficulties in tuning, since the magnetic forces combine with the elastic force to change the total restoring force, thus changing the frequency. The relative positions of the permanent magnet and the electromagnet were varied systematically, but, as would be expected, the one shown gives the best results.

A high differential permeability is necessary, as just mentioned, for sensitiveness. This means a high initial permeability in the core. Decidedly the best results have been obtained with cores made of good sheet silicon transformer steel. Results seemed to indicate that the sensitiveness varies as the square of the initial permeability instead of as the first power, as was expected, but sufficiently accurate data was not obtained to make certain on this point.

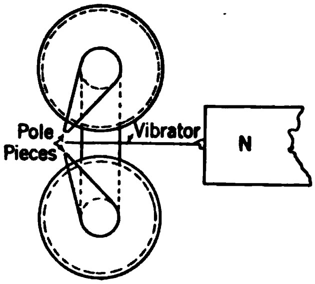

Fig. 4

If it is attempted to increase the sensitiveness by bringing the end of the vibrator close to the pole tips, a condition of instability is reached, the vibrator being pulled over to one or the other of the pole tips. Other practical difficulties arise, such as changes in the tuning of the vibrator. In general, the shorter the gap, the closer can the end of the vibrator be placed to the pole tips without such difficulties being encountered. Considerable changes may be made in length of gap, shape of pole tips, etc., without greatly affecting the maximum working sensitiveness attainable by the various adjustments.

Generally satisfactory results are obtained with the pole tips brought down in the form of truncated pyramids, the faces being about 2 by 0.5 mm., the short edge being parallel with the plane of vibration of the vibrator, a gap of about 1.5 mm., and the end of the vibrator about 1.5 mm. from the pole tips. An improvement by a factor of 1.5 to 2 may be obtained by setting the pole tips at an angle, as the field is intensified by such an arrangement, (See Fig. 4).

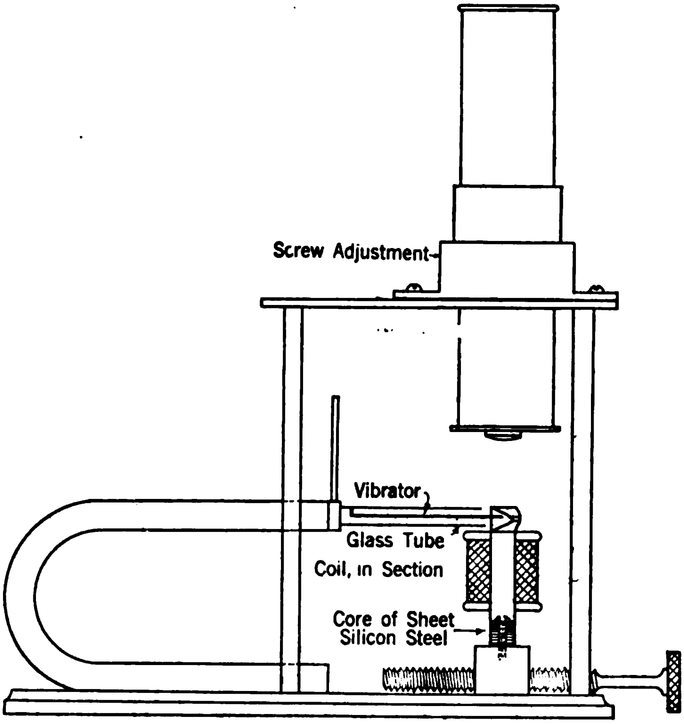

Fig. 5

The Vibrator. The vibrator is mounted on a small base of soft steel, as shown in Fig. 6, and is held in place magnetically, by merely placing the base on the face of the magnet. For convenience in changing vibrators for different frequencies, an aluminum wire is inserted in the base as a handle. For a 60-cycle vibrator a 0.1-mm. wire is convenient, and with this diameter a length of about 33 mm. is required. A 0.04-mm. wire of the same length has a frequency of about

25 cycles. For soft steel wires, the following formula is sufficiently accurate for design purposes:

$$\text{Frequency} = 65{,}000\,\frac{\text{diameter}}{(\text{length})^2}$$

where the dimensions are in centimeters.[2]
For piano wire the constant is higher, about 70,000.

At first it was hoped that such a vibrator might be permanent enough in its frequency to serve the purpose of a frequency meter as well as that of a galvanometer, but it is not feasible, since the actual resonance frequency depends to a small extent upon the position of the vibrator in the magnetic field. Advantage may be taken of this fact, however, to provide a fine adjustment for tuning the vibrator to the exact frequency desired. If an auxiliary small magnet, or better, a pair of such magnets, is placed near the end of the vibrator, the frequency may be changed several per cent, by changing the position of the auxiliary magnet. While it does not readily provide as large a range of adjustment, a more convenient method is to move an iron rod toward or away from one pole of the permanent magnet by a screw motion, as shown in Fig. 5.

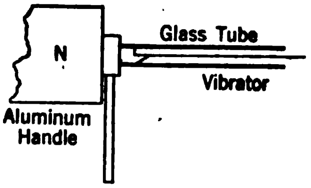

Fig. 6

If the vibrator is polished, and illuminated by a horizontal beam of light, a sharp line of light may be obtained in the microscope, very similar in appearance to the image of an incandescent filament of moderate brilliancy when viewed in a telescope. Satisfactory readings may also be made by viewing the wire in the ordinary way. In comparing sensitiveness it is convenient to use a vibration just sufficient to make the vibrator appear of double diameter.

It is well to keep the vibrator covered with a very thin film of oil to prevent corrosion. The effect of such a film on the frequency is too small to be at all inconvenient. For many purposes it is convenient to shield the vibrator with a glass tube, as in Fig. 6.

Performance. With a magnifying power of 50 to 100, which has been found satisfactory under working conditions, a motion of the vibrator of five microns is easily visible. With a one-ohm winding an electromotive force of three microvolts can be detected. The construction is such that high resistance coils may easily be wound for high current sensitivity. With a 270-ohm winding the sensitivity is such that a current of 0.05 microampere can readily be detected.

The chief advantages of the instrument are its sturdiness, the ease of adjustment, its quick responsiveness, and its freedom from the effects of external vibration. In the last characteristic the instrument is an order of magnitude better than any other form of vibration galvanometer with which the author is familiar. Both the freedom from external vibration and the quick responsiveness are due to the relatively large damping by air friction, and the extremely small mass of the moving element. Although the vibration galvanometer depends upon the principle of resonance, an appreciable amount of damping is necessary to give a reasonably quick response to changes in circuit conditions, and the smaller the mass of the moving element, the smaller the amount of energy necessary for a given amplitude, and the more quickly will the requisite amount of energy be supplied from the circuit.

The "resonance range" is about one per cent. That is to say, if the frequency of the current is one per cent above or below the frequency of resonance, the amplitude of vibration will be half as great as at resonance.

The efficiency of the present instrument as a motor is very low, the back electromotive force being only a

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2. This formula is in the form given by Rayleigh, (Theory of Sound, Vol. 1, art. 171), for the design of the rectangular prong of a tuning fork,

$$N = 84{,}590\ t/l^2$$

where N is the frequency, t the thickness, and l the length.

few per cent of the applied electromotive force. If it should be found possible to increase the electrical efficiency of the device, as a motor, to 50 per cent, the sensitivity would be increased in like ratio, and the instrument would be able to do the work of the most sensitive form of the moving-coil type. Wenner has shown that it is possible to develop an electrical efficiency of over 97 per cent in a vibration galvanometer of the moving-coil type.[3]

The instrument, in various stages of development, has been used in routine testing at the Bureau of Standards for more than three years. While many improvements have been made in detail, a much-to-be-desired radical increase in electrical efficiency has not been accomplished. One possibility is the use of a hardened steel vibrator, permanently magnetized, but a preliminary substitute experiment, in which a soft iron vibrator was surrounded by a magnetizing coil, did not show encouraging results. No attempt has been made to put the vibrator in a vacuum so as to reduce the air friction. It would be necessary to use a fairly high vacuum as the resistance of the air is nearly independent of the pressure down to a pressure of about one mm. of mercury. The sharpness of tuning necessary would increase as the sensitiveness increased, as would also the response to external mechanical disturbances, especially disturbances synchronous with the current. A very promising line of attack which has not been tried is the use of a vibrator made of vacuum process iron, to secure, if possible, a much higher flux in the vibrator.

I am indebted to Mr. J. B. Dempsey for a large part of the experimental work, and for valuable suggestions.

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[3] Wenner, *Bull. of the Bureau of Standards*, 6, p. 364, 1910.

DISCUSSION ON "A NEW FORM OF VIBRATION GALVANOMETER" (AGNEW), NEW YORK, N. Y., FEBRUARY 20, 1920.

John B. Whitehead: To one who has worked with the older forms of vibration galvanometer, involving the string suspension, it is obvious that this instrument presents many advantages. Simplicity of operation is at once conspicuous. I want to ask Dr. Agnew if he will not say a word on the adjustment of the instrument to different values of frequency. The suggestion is made here that with a given standard frequency, such as 60 or 25, vibrators are readily designed, but for frequencies which differ by several per cent from these, it would be necessary to include some provision for accurate tuning.

I have recently had occasion to measure alternating currents of very small values, and have found very great assistance indeed in the use of d-c. galvanometers in combination with hot cathode tubes, or kenotrons. These tubes rectify completely at all frequencies, and the use of the most sensitive forms of d'Arsonval instrument is thus possible. I suspect, from the figures given in the paper, that the sensitivity reached with the vibration galvanometer may not be reached with the d'Arsonval instrument, and if so, it will still be necessary to go to the vibration galvanometer in order to get these extreme limits of sensitivity. However, I want to call attention to the very great ease with which one may measure small alternating currents using kenotrons and galvanometer.

Clayton H. Sharp: I think it should be pointed out once more that the great advantage of this instrument is that it makes, apparently, the vibration galvanometer, an instrument available for many classes of usage to which vibration galvanometers as hitherto constructed have not been applicable. The ordinary type of vibration galvanometer as we have known it in the past has been practically unusable in a building which was subject to vibrations having a periodicity somewhat similar to the natural period of the instrument. If Dr. Agnew has succeeded, as I take it he has in producing an instrument which is not so affected by building vibrations while still retaining a satisfactory degree of sensibility electrically, he has advanced the art a very material step.

W. H. Pratt: Some six or eight years ago I had occasion to determine the moment of inertia of the moving system of an air damping instrument. After giving the procedure sufficient consideration, I attempted to determine the time of vibration of this moving system on the conditions of no damping, by conducting the experiment in a vacuum tank. I immediately discovered what has been pointed out in the paper, that the damping is reduced by a very small amount, until the condition of high vacuum is attained.

P. E. Klopsteg: Another modification of the instrument suggests itself, and that is the use of a tungsten filament mounted, possibly with an elastic mounting, in an exhausted glass tube, to pass the direct current through the filament so as to heat it, and to use the same form of electro-magnet for the alternating current, and observe the heated filament with the microscope. This would be an arrangement somewhat similar to the Einthoven string galvanometer, except that the filament is made incandescent with direct current, and the alternating current passed through the magnet windings. Tuning might be accomplished by using a magnet or pair of magnets to change the tension of the filament. Of course, the process might be reversed, presumably, and the regular Einthoven arrangement used, that is, to use the direct current in the magnet and send the alternating current through the filament. With this arrangement the filament would require outside illumination. The other plan seems more worth trying, namely, to heat the filament with direct current, and allow the alternating flux, due to the current being detected, to vibrate the filament and observe the incandescent filament with a microscope.

P. G. Agnew: Replying to Professor Whitehead's question in regard to the method of tuning, the magnetic control described in the paper gives a frequency adjustment up to 5 per cent. For more than 5 per cent, the vibrator is changed. In practise the change is a simple matter. I usually keep a few pieces of wire hung up in the laboratory, oiled, so as to avoid corrosion, and when it is necessary to use a different frequency, say 30 cycles, one mounts a piece of the wire, and clips it off to the calculated length. If the diameter of the wire is known, the formula gives the length to a few per cent, and so only one or two trials are necessary to get a satisfactory adjustment. The mounting and adjustment of a vibrator can easily be made in ten or fifteen minutes.

The forms suggested by Dr. Klopsteg would actually be entirely new forms of the vibration galvanometer, rather than mere modifications of the present form. The development of Dr. Klopsteg's ideas would certainly constitute an interesting and a valuable contribution to the subject of electrical measurements.

Dr. Sharp has very clearly pointed out the most important advantage of the instrument. At first sight one would expect the present instrument to be affected much more than it is by external vibration, in fact, much more than is the coil type. In an axially mounted coil we would expect, as a first approximation, that the instrument would be independent of external vibration, which could be effective only through accidental variation from axial symmetry, while in the present form the vibrator would be expected to respond to external vibration. Experience shows, however, that the present form is much less affected than is the coil type by external vibration. The principal reason for this is to be found in the extremely small mass of the vibrator, coupled with the damping. Only a minute amount of energy is involved in the vibration and this is quickly dissipated in the damping.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 20, 1920.

A PRECISION GALVANOMETRIC INSTRUMENT FOR MEASURING THERMOELECTRIC E. M. FS.

BY T. R. HARRISON AND PAUL D. FOOTE
Both of the U. S. Bureau of Standards

A new principle has been developed whereby an ordinary millivoltmeter may be converted into an instrument in which the usual errors arising from a variable line resistance are entirely eliminated. The instrument measures true e. m. f. in a simple circuit or if connected across a resistance or network through which a current flows it indicates the potential drop which would have existed had the instrument not been connected. In this respect it functions as a potentiometer, yet it does not operate on the potentiometric principle since it does not require a standard cell (or the equivalent) or an auxiliary battery, the only e. m. f. employed in the adjustment being that of the source measured. No loss in precision of setting results; in fact the adjustment may be readily made to 10 times the scale accuracy. Various wiring diagrams are shown and methods are discussed for constructing instruments of zero temperature coefficient and properly damped. A new deflection potentiometer is described which offers (considerable advantage over the ordinary type for small e. m. fs. in a circuit of variable resistance.

THE maximum e.m.f. developed by a rare-metal thermo-couple is about 15 to 18 millivolts, and that developed by a base-metal couple about 50 to 80 millivolts. To measure such small e.m.fs. accurately by a galvanometric method requires the use of specially constructed millivoltmeters. A millivoltmeter, although graduated to read e.m.f. is fundamentally a current-measuring device. The deflection of the pointer is approximately proportional to the current flowing through the instrument.

If e is the e.m.f. developed by the couple, R_g the resistance of the galvanometer, and R the resistance of the lead wires and couple, the current flowing $= \frac{e}{(R_g + R)}$ · Hence in order that a certain deflection correspond to a definite e.m.f., the scale of the instrument must be graduated for a preassigned value of $R_g + R$. Any change in the resistance R from this standard value causes an error in the observed reading of the e.m.f. Such variations may arise from bad contacts, deterioration of the couple, change in depth of immersion, temperature coefficient of lead wires and couple, etc.

In order to minimize the effects of these variations the resistance R_g of the galvanometer is made large compared to that of the line and couple R. This latter resistance in general practise may vary from 1 to 15 ohms. Hence the resistance R_g must be very great in order that changes in R be negligible. The construction of a millivoltmeter having the combined features of high resistance and robustness sufficient to resist the mechanical shocks to which it is necessarily subjected in any industrial plant is an exceedingly difficult problem. The highest resistance ever employed is about 1200 ohms, and instruments are manufactured having a resistance of only 10 ohms. All questions considered, the most satisfactory pivot instruments from the pyrometric standpoint have a resistance of about 5 to 10 ohms per millivolt.

If an instrument having a resistance of 100 ohms is calibrated to read correctly for a line and couple resistance of 2 ohms, and if the resistance of the latter for one of the various reasons mentioned above changes from 2 to 5 ohms, the indicator will read in error by 3 per cent or 30 deg. at 1000 deg. cent. With a 10-ohm indicator the error would be about 20 per cent, or 200 deg. at 1000 deg. cent. Such errors are of serious importance, and are not easily detected unless measurements of the line resistance are frequently made. These measurements require the use of an auxiliary instrument such as a Wheatstone bridge, and then corrections must be applied to the observed reading of the millivoltmeter to take account of the variation in line resistance from the value for which the indicator was standardized.

In the following discussion an instrument is described in which provision is made whereby the total resistance of the circuit may be easily adjusted to the preassigned value for which the scale of the instrument is graduated without requiring the use of any auxiliary instrument or a source of e.m.f. other than that of the couple being measured. Although the instrument was designed primarily for pyrometric purposes, its usefulness is not confined to this field alone.

DESCRIPTION OF COMPENSATED INSTRUMENT

Fig. 1 illustrates the apparatus in a general form. When the instrument is used as a simple temperature indicator, the resistance of the circuit through which the current from the thermocouple flows is made up as follows: The resistance *T* of the thermocouple, resistance *L* of the line or lead wires, a rheostat of resistance r_x which may be adjusted thus altering the resistance of the circuit, a resistance r_2, a resistance r_3 including the resistance of the moving element, r_3 being shunted by resistance r_4, r_5, and r_6 connected in series. In addition to these parts of the circuit, arranged as shown in the figure, there is a resistance r_7 connected between points *C* and *K*, and a key *K* which when closed, short-circuits r_5, and connects r_4, r_6, and r_7 directly together through the key.

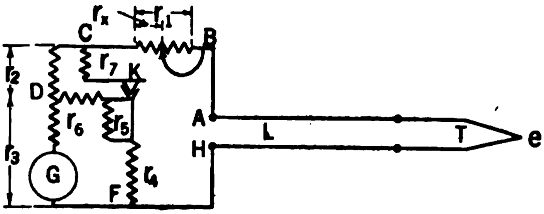

FIG. 1—GENERAL DIAGRAM OF APPARATUS FOR ELIMINATING ERRORS DUE TO UNKNOWN LINE RESISTANCE, FROM WHICH TWO SIMPLE TYPES ARE DERIVED

By adjusting one or more of the resistances until a definite relation is obtained between the deflections of the moving element *G* when the key is open and closed respectively, a definite relation will be established between different resistances of the system. The most convenient relation between the respective deflections is identity, and this relation will be used in the following discussion. In the general case here described, identical readings could be established by adjusting any of the resistances r_x, r_2, r_3, r_4, r_5, r_6, or r_7, but it will appear directly that one desirable method is to adjust r_x, for then the sum of r_x, L, and T is brought to a definite value. The instrument may be so designed that when $r_x + L + T$ = a constant $= r_1$, the total resistance of the circuit is the preassigned value for which the instrument was calibrated. The relation between the resistances may be deduced as follows:

Representing the current flowing through r_3 by i'_3 when the key is open and by i_3 when the key is closed, and the e.m.f. which the couple develops by e, when the key is open we have:

$$\frac{e}{i'_3} = r_1 + r_2 + r_3 + \frac{(r_1 + r_2)\, r_3}{r_4 + r_5 + r_6} \tag{1}$$

When the key is closed we have from Kirchhoff's laws the following seven equations:

$$\begin{aligned}
&i_1 - i_2 - i_7 = 0\\
&i_2 - i_6 - i_3 = 0\\
&i_3 + i_4 - i_1 = 0\\
&i_7 + i_6 - i_4 = 0\\
&i_2\, r_2 - e + i_1\, r_1 + i_3\, r_3 = 0\\
&i_2\, r_2 + i_6\, r_6 - i_7\, r_7 = 0\\
&i_3\, r_3 - i_4\, r_4 - i_6\, r_6 = 0
\end{aligned}$$

in which i with subscript represents the current through the resistance designated by the corresponding subscript.

Solving these equations we obtain:

$$\begin{aligned}
&\frac{e}{i_g}\left[r_4\,(1 + r_2/r_6) + r_7\,(1 + r_4/r_6)\right]\\
&\quad = r_7\left[\frac{(r_1 + r_2)\, r_3}{r_6} + (r_1 + r_2 + r_3)\,(1 + r_4/r_6)\right]\\
&\qquad + (r_1\, r_3 + r_1\, r_4 + r_3\, r_4)\,(1 + r_2/r_6) + r_2\,(r_1 + r_4)
\end{aligned} \tag{2}$$

If the deflection of the galvanometer is the same whether the key is open or closed, the current flowing through the galvanometer coil is equal in the two cases

($i'_3 = i_3$) and we may substitute for e/i_3 in (2) the value given for e/i'_3 in (1). Upon simplification of the equation thus obtained we have (3) which is the general conditional equation for the relation between the resistances necessary in order that the deflection of the galvanometer be not altered by closing the key K:

$$r_2 r_4 + \frac{r_3 (r_1 + r_2)(r_2 r_4 + r_4 r_6 - r_5 r_7)}{(r_2 + r_6)(r_4 + r_5 + r_6)}$$

$$= r_1 r_3 + \frac{r_2 r_6 (r_1 + r_4)}{r_2 + r_6} \qquad (3)$$

By assigning certain special values to some of the resistances, equation (3) may be very much simplified.

Making $r_5 = \infty$ (*i. e.* no connection directly across from one switch contact to the other) we obtain:

$$r_2 r_4 - \frac{r_3 r_7 (r_1 + r_2)}{r_2 + r_6} = r_1 r_3 + \frac{r_2 r_6 (r_1 + r_4)}{r_2 + r_6} \qquad (4)$$

If in addition to making $r_5 = \infty$ we make $r_7 = 0$ the equation reduces to the form:

$$r_2 r_4 = r_1 r_3 + \frac{r_2 r_6 (r_1 + r_4)}{r_2 + r_6} \qquad (5)$$

and finally, upon making $r_6 = 0$ in addition to making $r_5 = \infty$ and $r_7 = 0$, we have the important relation *viz.*:

$$r_2 r_4 = r_1 r_3 \quad \text{or} \quad r_1 = \frac{r_2 r_4}{r_3} \qquad (6)$$

This arrangement will be discussed fully below. If we impose the sole condition that $r_5 = 0$, equation (3) reduces to the form:

$$r_1 = \frac{r_2 r_4}{r_6} \qquad (7)$$

This is also a very simple form which possesses the advantage that it is independent of the value of r_7, and therefore of contact resistance at the key. For high sensitivity in adjusting r_1 to the proper value, r_7 should be practically zero and r_4 should be also low in order that a large current may flow through r_1 when the key is closed.

Fig. 2 shows a diagram of the instrument where $r_5 = \infty$ and $r_6 = r_7 = 0$ as discussed above. The circuit *C D G F* is an ordinary millivoltmeter or galvanometric indicator of resistance $r_2 + r_3$, the latter resistance including that of the moving element *G*. In series with this is an adjustable rheostat *C B*, the maximum value r_1 of which is chosen equal to the maximum value of the resistance of the line and couple likely to occur in practise. The galvanometer is calibrated to read correctly when the total resistance *R* of the circuit has the preassigned value $r_1 + r_2 + r_3$. If in the construction of the instrument $\frac{r_2 r_4}{r_3}$ is made equal to r_1, then when the rheostat *C B* is adjusted to a value r_x such that the deflection of the instrument is the same whether the key is open or closed, the sum of resistances $r_x + L + T$ must equal r_1 and the resistance $r_x + L + T + r_2 + r_3$ of the circuit with the key open must equal the preassigned value $R = r_1 + r_2 + r_3$ for which the scale of the instrument was graduated. Thus it follows, as will be shown more clearly later, that *the instrument measures true e.m.f. in a simple circuit, or if connected across a resistance through which a current flows it will indicate the potential drop which would have existed had the instrument not been connected. In this respect its action is similar to that of a potentiometer. The principle may be also applied to net work conductors, an extension of which leads to a serviceable deflection potentiometer.*

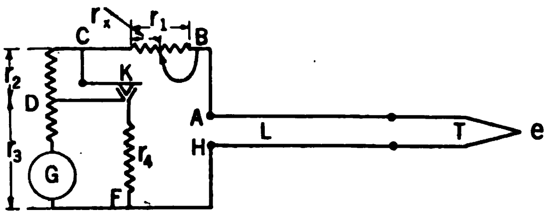

FIG. 2—OPEN SHUNT TYPE FOR HIGH SENSITIVITY, ESPECIALLY SUITABLE FOR THERMOCOUPLES

Equation (6) may be deduced in a simple manner as follows: If e represents the e.m.f. developed by the couple, e' the potential drop across $D\,F$ when the key is open, and e'' the potential drop across $D\,F$ when the key is closed, we obtain the equations:

$$e' = \frac{e\,r_3}{L + T + r_x + r_2 + r_3} \tag{8}$$

and

$$e'' = \frac{e\,r_3\,r_4}{(L + T + r_x)\,(r_3 + r_4) + r_3\,r_4} \tag{9}$$

If r_x is so adjusted that these two potential drops and hence the deflections of the indicator are the same, we have on equating (8) and (9):

$$L + T + r_x = \frac{r_2\,r_4}{r_3} = \text{a constant} = r_1 \tag{10}$$

by construction. Adjustment of the rheostat alters the readings both when the key is open and closed. A simple process for operating the instrument is accordingly as follows:

1. Read the instrument with the key open.
2. Close the key and adjust the rheostat $C\,B$ until the instrument reads approximately the same as in 1.
3. Repeat 1 and 2 if necessary.

MAGNIFICATION OF ERRORS

By making r_3/r_4 equal to from 5 to 10, the ease with which the proper setting can be obtained is greatly improved. That this is true will be seen from the following consideration.

If the instrument is calibrated to read correctly when the resistances are related according to the equation $r_1/r_2 = r_4/r_3$ and if the rheostat $C\,B$ is out of adjustment by an amount $\delta\,r_1$, the instrument will read in fractional error, when the key is open, by an amount:

$$\frac{\delta\,e'}{e'} = -\frac{\delta\,r_1}{r_1 + r_2 + r_3}$$

and when the key is closed by an amount:

$$\frac{\delta e''}{e''} = - \frac{\delta r_1}{r_1 + \dfrac{r_3 r_4}{r_3 + r_4}} \qquad \text{whence}$$

$$\frac{\delta e'}{\delta e''} = \frac{r_1 r_3 + r_1 r_4 + r_3 r_4}{(r_3 + r_4)(r_1 + r_2 + r_3)} \tag{11}$$

If $r_1 = r_2$ and $r_3 = r_4$, the above expression simplifies to $\frac{\delta e'}{\delta e''} = \frac{1}{2}$ or $\delta e'' - \delta e' = \delta e'$. Therefore when $r_3/r_4 = 1$ the difference between the readings will equal the error in the reading when the key is open. If however, r_2/r_1 and r_3/r_4 are made large, say equal to 10, we find $\frac{\delta e'}{\delta e''} = \frac{1}{11}$ or $\delta e'' - \delta e' = 10 \, \delta e'$. Thus if $r_3/r_4 = 10$, the error when the key is open is only 1/11 of the error when the key is closed, or 1/10 of the difference between the readings when the key is open and closed respectively.

In the process for operating the instrument described it is rarely necessary to make a second adjustment when the resistances r_3/r_4 are in as high a ratio as 10. In position 1, the instrument functions as an ordinary galvanometer. The single setting in position 2 reduces the error in the ordinary galvanometer to 1/11 of itself, which is usually sufficient.

The adjustment for the proper external resistance, if desired, can be made with 10 times the precision necessary. Thus, if the galvanometer can be read to 1/10 of a scale division, the line resistance can be set for an error of only 1/100 of a scale division, which is at least 10 times the accuracy possible with any indicating instrument. Hence the factor of variable line resistance which may give rise to very serious errors is easily and accurately controlled by a simple mechanical adjustment.

MULTIPLE INSTALLATION

The device is readily applicable to multiple installations of different line resistances. For multiple point recorders and indicators as many resistances *C B* may be employed as there are couples. These may be inexpensive rheostats having a resistance of approximately 15 ohms each, located in each couple line between the couple and the selective switch. These rheostats may be adjusted in the manner described whenever convenient or necessary. The fact that it is not essential to have the extra resistances adjusted to precisely 15 ohms is of great advantage from the constructional point of view. The instrument may be calibrated for exactly 15 ohms external resistance. If the adjustable resistances accordingly have values of from 16 to 17 ohms, from 1 to 2 ohms more are cut out of the circuit in the process of obtaining a setting than would be the case if resistances of precisely 15 ohms were employed. In either case, however, the final e.m.f. reading is correct.

The following illustrates a suitable proportioning of resistances for a 300-ohm indicator with $r_3/r_4 = 10$:

$r_1 = 15$ ohms.
$r_2 = 150$ ohms.
$r_3 = 135$ ohms.
$r_4 = 135/10$ ohms.
$r_1 + r_2 + r_3 = 300$ ohms.

PROPER DAMPING

In designing an instrument of the type discussed above, care must be taken to avoid excessive over damping when the key is closed. The resistances effective in damping are $R = r_1 + r_2 + r_3$ when the key is open and $R_d = r_3 + \frac{r_1 r_4}{(r_1 + r_4)}$ when the key is closed. It is evident that R_d is less than R. If we let $K = r_2/r_1 = r_3/r_4$ the following two relations are obtained:

$$r_1 = \frac{R - R_d}{K + \frac{R - R_d}{R}} \qquad \text{(a)}$$

$$r_3 = r_1 K \frac{R_d}{R - R_d} \qquad \text{(b)}$$

For the simple indicator R is known as soon as the scale range is chosen. Suitable values may be selected for r_1 and K and equation (a) solved for R_d. If the simple indicator is not seriously overdamped when its total resistance is R_d, this value of R_d is satisfactory, and r_3 may be computed from equation (b). The values of r_2 and r_4 follow from K.

If, however, the simple instrument is too sluggish when its total resistance is R_d, the minimum allowable value of R_d may be determined experimentally, and this value, substituted in equation (a), determines r_1 with proper choice of K; whence r_2 and r_3.

TEMPERATURE COEFFICIENT

In the method so far described, no account has been taken of the temperature coefficient of the instrument. An increase in the temperature of the instrument causes a slight weakening of the springs which tends to allow a given current flowing through the instrument to produce a greater deflection[1]. This tendency is opposed by the effect of a slight decrease in the magnet strength. The effect of the weakening of the springs is almost always in excess of the effect due to the decrease in magnet strength. Thus an instrument becomes more sensitive to current as its temperature rises. In order that a given deflection shall continue to indicate the e.m.f. marked on the scale, the current which this e.m.f. causes to flow through the instrument must decrease with increasing temperature. This requires that the resistance of the circuit increase slightly with increase in temperature. The construction of the instrument is such that its resistance really does so increase. Nearly all of the moving coils used in millivoltmeters are made of copper, the resistance of which increases about 0.4 per cent of its initial value per degree rise in temperature. The majority of instruments require that from 1/13 to 1/40 of the resistance of the galvanometer be composed of copper in order to neutralize the excess of the effect of weakening of springs over decrease in magnet strength and thus give the instrument a zero temperature coefficient.

For an instrument which is constructed with the proper proportion of copper in the circuit, the value of

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[1] See Bureau of Standards Circular No. 20.

r_1 should be the same whatever the temperature of the instrument. This requires that the value of $\frac{r_2 r_4}{r_3}$ also must not vary with temperature. The resistance of r_2 which is of manganin, is practically independent of temperature, but r_3 is composed partly of copper, since it includes the moving element. Therefore in order that r_4/r_3 shall remain constant with changing temperature, r_4 must be composed of the same proportion of copper and manganin as r_3.

Any greater proportion of copper in the instrument than that required as stated above will result in too great a rate of increase of resistance with increase in temperature, and hence the instrument will read too low when its temperature is higher than that at which it was calibrated. Instruments are often made with a greater proportion of copper than is thus required in order to secure a more substantial design suitable for industrial practise and to avoid other difficulties which arise in the construction. One method of making such an instrument read correctly at different room temperatures would be to reduce manually the resistance of the circuit by an amount equal to the increase in resistance of this extra amount of copper in the circuit. The instrument described in the present paper may be arranged so that this compensation is included in the adjustment for variable line resistance without any extra constructional features or manipulation. This is done by allowing the resistance r_4 to have a greater proportion of manganin to copper than r_3. The necessary proportion is found as follows:

Let Q = temperature coefficient of the simple indicator.
P = temperature coefficient of the compensated indicator.
η = temperature coefficient of simple indicator contributed by the copper.
ν = temperature coefficient of simple indicator contributed by springs and magnet.
α = temperature coefficient of r_3.
β = temperature coefficient of r_4.

η' = temperature coefficient of compensated indicator contributed by all changes in resistance.

The total temperature coefficient of an indicator is made up of the two parts, the temperature rate of change per ohm of resistance and the combined coefficient of the springs and magnet. Hence for the simple indicator we obtain:

$$Q = \eta + \nu = \frac{\frac{d}{dt}(r_1 + r_2 + r_3)}{r_1 + r_2 + r_3} + \nu$$

$$= \frac{\frac{d\,r_3}{dt}}{r_1 + r_2 + r_3} + \nu \qquad (12)$$

since r_1 and r_2 are constants. For the compensated indicator, however, $r_1 = \frac{r_2 r_4}{r_3}$ is a function of the temperature since both r_3 and r_4 depend upon the temperature. Hence:

$$P = \eta' + \nu = \frac{\frac{d}{dt}(r_1 + r_2 + r_3)}{r_1 + r_2 + r_3} + \nu$$

$$= \frac{\frac{d\,r_1}{dt} + \frac{d\,r_3}{dt}}{r_1 + r_2 + r_3} + \nu$$

$$= \frac{r_1\left(\frac{1}{r_4}\frac{d\,r_4}{dt} - \frac{1}{r_3}\frac{d\,r_3}{dt}\right) + \frac{d\,r_3}{dt}}{r_1 + r_2 + r_3} + \nu \qquad (13)$$

since $$\frac{d\,r_1}{dt} = \frac{r_2 r_4}{r_3}\left(\frac{1}{r_4}\frac{d\,r_4}{dt} - \frac{1}{r_3}\frac{d\,r_3}{dt}\right)$$

$$= r_1\left(\frac{1}{r_4}\frac{d\,r_4}{dt} - \frac{1}{r_3}\frac{d\,r_3}{dt}\right)$$

If in equations (12) and (13) we substitute

$$\alpha = \frac{1}{r_3}\frac{d\,r_3}{dt} \quad \text{and} \quad \beta = \frac{1}{r_4}\frac{d\,r_4}{dt}$$

we obtain the following:

$$Q = \frac{r_3\,\alpha}{r_1 + r_2 + r_3} + \nu \tag{12)'}$$

$$P = \frac{r_1\,(\beta - \alpha) + r_3\,\alpha}{r_1 + r_2 + r_3} + \nu \tag{13)'}$$

Whence:

$$P = Q + \frac{r_1\,(\beta - \alpha)}{r_1 + r_2 + r_3} = \text{temperature coefficient of compensated indicator.} \quad \textbf{(14)}$$

In order to obtain the condition for an instrument of zero temperature coefficient put $P = 0$ in the above equation, whence:

$$\frac{r_1\,(\alpha - \beta)}{r_1 + r_2 + r_3} = Q \quad \text{(condition for zero temperature coefficient)} \quad \textbf{(15)}$$

The coefficients α and β are due to the copper content in the resistances r_3 and r_4. If we assume the temperature coefficient of copper to be 0.004 we obtain:

$$\alpha = \frac{\text{per cent of copper in } r_3}{25{,}000}$$

$$\beta = \frac{\text{per cent of copper in } r_4}{25{,}000}$$

On substituting these values of α and β in equation (**15**) the following equation is obtained for the condition that the compensated indicator have a zero temperature coefficient:

(per cent of copper in r_3) − (per cent of copper in r_4)

$$= 25{,}000\,\frac{r_1 + r_2 + r_3}{r_1}\,Q \quad \textbf{(16)}$$

If the simple indicator has been so designed that its temperature coefficient is zero, *i. e.* $Q = 0$, equation (**16**) shows that the copper to manganin ratio should be the same in the two resistances r_3 and r_4.

If the simple indicator has a too high copper content so that its temperature coefficient for voltage is positive, *i. e.* $Q > 0$, the experimentally determined value of Q

for the simple indicator and the known copper ratio in r_3 may be substituted in equation (**16**) and the proper copper ratio for the shunt r_4 may be readily computed.

Perfect compensation cannot be obtained if $\frac{r_1}{(r_1 + r_2 + r_3)}$ is small and Q is large. The greatest degree of compensation is then secured when r_4 is made entirely of manganin, *i. e.* $\beta = 0$. Hence on making $\beta = 0$, from equation (**15**), we obtain as a condition for the possibility of perfect compensation that:

$$Q \leq \frac{r_1 \alpha}{r_1 + r_2 + r_3} \tag{17}$$

Many simple indicators have a temperature coefficient Q which is larger than the maximum value expressed by equation (**17**). In this case the shunt r_4 should be constructed entirely of manganin and the relation between the temperature coefficient of the compensated indicator, P, and that of the simple indicator, Q, readily derived from equations (**12**) and (**13**), is as follows:

$$P = Q\left(1 - 0.004 \frac{r_1}{r_3 Q} \cdot \frac{\text{ohms of copper in } r_3}{r_1 + r_2 + r_3}\right) \tag{18}$$

or

$$P = (\eta + \nu)\left(1 - \frac{r_1}{r_3} \frac{\eta}{\eta + \nu}\right) \tag{19}$$

Equation (**19**) is useful for computing directly the value of P on the assumption of a definite value for ν. Usually ν is approximately equal to $- 0.0002$. We will consider the application of the above equations to the instrument already described.

$r_1 = 15$ ohms.
$r_2 = 150$ "
$r_3 = 135$ "
$r_4 = 13.5$ "
$r_1 + r_2 + r_3 = 300$ "
Let $r_3 = 50$ ohms copper + 85 ohms manganin.
$Q = + 0.00047$ (assumed to be determined by experiment).

On applying the test given by equation (**17**) it is found that $Q > \frac{r_1 \alpha}{(r_1 + r_2 + r_3)}$. Hence perfect compensation is impossible for the above instrument. By making the shunt entirely of manganin, we obtain on substituting the proper values in equation (**18**):

$$P = (0.00047)\,(0.84) = 0.00040$$

Thus the compensated indicator has a temperature coefficient 16 per cent smaller than that of the simple indicator. The same conclusion follows also from equation (**19**).

USE OF THE COMPENSATED MILLIVOLTMETER FOR MEASURING POTENTIAL DROP

It is at times desirable to measure a small potential drop across a resistance through which a current is flowing. When an ordinary millivoltmeter is employed for this purpose, the potential drop is altered to some lower value which is indicated by the instrument, provided it is graduated to read potential difference at its terminals or at the ends of calibrated lead wires. A similar reading may be obtained with the instrument here described if the rheostat *C B* is set at a marked position such that $r_x = r_1$, since with this adjustment the instrument measures potential drop across its terminals.

Often however, the measurement actually desired is the potential drop which existed across the resistance without the millivoltmeter being connected. This value which may be obtained potentiometrically is also readily measured by the compensated indicator, as will be seen in the following section. When the rheostat *C B* is so adjusted that the deflection of the instrument is the same with the key open or closed, the reading is the potential drop which would exist were the instrument disconnected. One method employed for compensating for irreproducibility of the temperature-e.m.f. relation of different thermocouples of a given type is by the use of shunt and series resistance in the head of the couple.[2]

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[2] See Foote, Harrison and Fairchild, "Thermoelectric Pyrometry," Pyrometer Symposium, Chicago, 1919.

An ordinary millivoltmeter reads the potential drop existing across the shunt when the instrument is connected. This value depends upon the resistance of the indicator. The reduction in the observed e.m.f. of the couple (potential drop across the shunt) is greater the lower the resistance of the meter. With the compensated indicator however, the ratio of the shunt and series resistances may be definitely adjusted and the instrument, regardless of its resistance, will always indicate, similarly to a potentiometer, that proportion of the e.m.f. e developed by the couple which is represented by the expression

$$e \cdot \frac{\text{shunt resistance}}{(\text{shunt resistance} + \text{series resistance})} \cdot$$

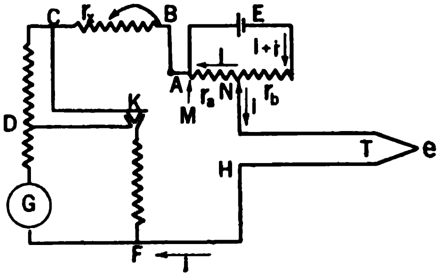

FIG. 3—IN CONJUNCTION WITH ANY POTENTIOMETER OF LOW RESISTANCE THE INSTRUMENT PRODUCES A PERFECTLY COMPENSATED DEFLECTION POTENTIOMETER

DEFLECTION POTENTIOMETER

The compensated indicator may be used as a galvanometer in conjunction with an ordinary potentiometer of low resistance forming a perfectly compensated deflection potentiometer, the compensation being effected by the adjustment of r_x in the manner already described.

This is shown in the following discussion, a special case of which resolves itself to the condition above mentioned in which the instrument may be used to measure the potential drop, which existed before the instrument was connected, over a resistance carrying a current.

Fig. 3 illustrates the use of the instrument as a deflection potentiometer. The potentiometer circuit consists of the battery E and the resistances r_a and r_b the latter including the internal resistance of the battery. When the indicator or galvanometer circuit is broken, for example at A, the potential drop e_1, across r_a is given by equation (20):

$$e_1 = \frac{r_a}{r_a + r_b} E \tag{20}$$

Upon connecting the instrument to the potentiometer so that the e.m.f. e of the thermocouple (or any source of e.m.f.) T opposes e_1, the currents flowing through r_a and r_b are unequal and different from the original value. Assume that the currents now flowing are as indicated in the figure, I being the current in r_a, i the current in the indicator or galvanometer circuit, and $I + i$ the current in r_b. Since the sum of e.m.fs. and potential drops around any closed circuit is zero, the following two equations may be written:

$$E = (r_a + r_b)\, I + r_b\, i \tag{21}$$

$$e = r_a I - R\, i \tag{22}$$

in which R equals the resistance of the circuit from M, through $A\ B\ C\ D\ F\ H$ etc. back to N. Substituting the value of I from equation (21) in equation (22) and subtracting this value of e from the value of e_1, given by equation (20) we obtain:

$$e_1 - e = i\left(R + \frac{r_a r_b}{r_a + r_b}\right). \tag{23}$$

This is the equation of a deflection potentiometer.[3]

If now the resistance r_x is so adjusted that the reading of the instrument is the same whether the key K is closed or open, we have the following relations: Representing the values of i with the key open and closed respectively by i' and i'', we have for the potential drops across $D\ F$ in the two cases, $i'\, r_3$ and $i'' \dfrac{r_3 r_4}{r_3 + r_4}$.

Equating these we obtain:

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[3] For general theory and design of deflection potentiometers see Brooks, Bull. Bu. Stds. Vol. 8, 1911, p. 395 (Sci. paper 172) and p. 419 (sci. paper 173).

$$i'' = i' \frac{r_3 + r_4}{r_4} \tag{24}$$

the condition necessary for equal deflections.

We also have the values R' and R'' for R with the key open and closed respectively.

$$R' = r_x + L + T + r_2 + r_3 \tag{25}$$

$$R'' = \frac{(r_x + L + T)\,(r_3 + r_4) + r_3 r_4}{r_3 + r_4} \tag{26}$$

From equation (23) it follows that:

$$e_1 - e = i'\left(R' + \frac{r_a r_b}{r_a + r_b}\right) = i''\left(R'' + \frac{r_a r_b}{r_a + r_b}\right)$$

since both e_1 and e are independent of R, e being the e.m.f. of the couple and e_1 being defined by equation (20). If the values of R' and R'' from equations (25) and (26) and the value of i'' in terms of i' from equation (24) are substituted in the above equation, we obtain:

$$\frac{r_2 r_4}{r_3} = r_x + L + T + \frac{r_a r_b}{r_a + r_b} \tag{27}$$

But by the construction of the instrument $r_1 = \dfrac{r_2 r_4}{r_3}$.

Hence from equation (25), $R' = \dfrac{r_1 + r_2 + r_3 - r_a r_b}{(r_a + r_b)}$.

If this value of $R = R'$ is substituted in equation (23) we obtain:

$$e_1 - e = i\,(r_1 + r_2 + r_3) \tag{28}$$

The compensated indicator is calibrated for a total resistance $r_1 + r_2 + r_3$ so that the current i flowing through the instrument produces a deflection which indicates directly the difference between the e.m.f. e of the couple and the potential drop e_1 existing across r_a with the instrument disconnected. The potentiometer is graduated to read the value of e_1. *Thus when the compensated indicator is used as the galvanometer of an ordinary potentiometer, a deflection potentiometer is obtained which requires no compensation other than that provided by the indicator.*

Evidently the instrument has the disadvantage for continuous use as a deflection potentiometer that a new adjustment is required whenever the dial setting of the potentiometer is changed enough to cause a material difference in the value of $\frac{r_a r_b}{(r_a + r_b)}$. In any form of a deflection potentiometer however, changes in the resistance of the couple and leads, unless compensated for, will introduce errors in the galvanometer readings. The ordinary deflection potentiometer may be provided with a rheostat, in series with the couple and galvanometer, which is adjusted until the full scale range is equivalent to one step on the dial. The instrument here described compensates simultaneously for changes in both the couple resistance and potentiometer resistance. This adjustment is simpler than the adjustment for scale sensitivity of the ordinary deflection potentiometer, and hence if the resistance of the couple varies seriously, the new type of deflection potentiometer appears to be the more easily operated.

If in equation (28) we let $e = 0$, we obtain the condition which applies when the compensated indicator is connected across a resistance carrying a current. *It follows directly from the reasoning presented above that the instrument indicates the potential drop, e_1, which would exist if the instrument were not connected.*

CALIBRATION OF THE COMPENSATED MILLIVOLTMETER.

Another useful application of this feature may be employed in calibrating the scale of the indicator. The scheme of connections is the same as that shown in Fig. 3 except that there is no thermocouple T in the circuit. The instrument is connected directly, with the proper polarity to give positive deflections, to the e.m.f. terminals of a potentiometer of sufficiently low resistance that, for all settings, $\frac{r_a r_b}{(r_a + r_b)}$ plus the resistance L of any leads employed is not greater than r_1, and the galvanometer terminals of the potentiometer are short-circuited. When the resistance $C B$ is so adjusted that the deflection of the indicator is unaltered by depressing the key K, the reading of the potentiometer gives the value of the e.m.f. which should be indicated by the millivoltmeter. The other obvious method of calibrating the instrument in terms of the potential drop across its terminals when the resistance $r_x = r_1$ is not as simple experimentally, and the former method is applicable at any temperature when the shunt resistance r_4 is constructed with the proper ratio of manganin to copper (see equation 16) necessary to give a zero temperature coefficient. With the latter method however, the calibration should be made at the temperature for which a definite marked position of the rheostat $C\,B = r_1 = \frac{r_2 r_4}{r_3}$. This position can be a definite one at only one temperature for a compensated instrument, if the simple indicator has a temperature coefficient. However, the errors likely to arise from not considering this factor are usually of negligible importance.

USE OF COMPENSATED MILLIVOLTMETER FOR MEASUREMENT OF POTENTIAL DROP ACROSS A NETWORK

It follows from the above discussion that the compensated indicator may be used to measure the true potential difference between any two points of a complicated network existing were the instrument not connected. In this respect the instrument acts as a potentiometer. This is more clearly demonstrated as follows:

In Fig. 4 let r = resistance of network between M and N, excluding branch $M\,P\,N$.

R = resistance of branch $M\,P\,N$.

e_2 = potential drop across $M\,N$ for condition 1. Primed letters refer to condition 2, unprimed letters to condition 1.

The current i_{MN} which flows from M to N when an e.m.f. E is introduced in the branch $C\,D$ of the network is equal to the current i'_{CD} which flows from C to D when the same e.m.f. is introduced in the arm $M\,N$[4].

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[4] Jeans Math. Theory of Elec. and Mag. 3rd Ed. p. 327. See also Maxwell.

These two conditions are shown by (a) and (b) of Fig. 4.

$$i_{MN} = i'_{CD} \text{ by the above theorem.} \qquad (29)$$

$$i_{MN} = \frac{E}{R+r} \text{ from Ohm's law. Hence:}$$

$$i'_{CD} = \frac{E}{R+r} K \text{ where } K < 1 \text{ is a constant}$$

depending upon the values and the grouping of the separate resistances of the network. Also,

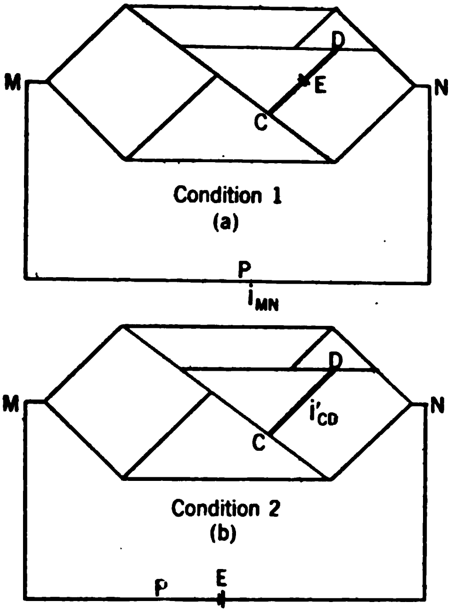


FIG. 4—SHOWING GENERAL DEFLECTION POTENTIOMETER PRINCIPLE, OR POTENTIOMETRIC NATURE OF READINGS BY THE COMPENSATED INSTRUMENT

$$i_{MN} = \frac{e_2}{R} = \frac{E}{R+r} K \text{ or} \qquad (30)$$

$$e_2 = \frac{E}{1 + r/R} K \qquad (31)$$

When $R = \infty$ let $e_2 = e_1$, whence from equation (31) $e_1 = E\,K$. Substituting this value of $K = e_1/E$ in equation (30) we obtain:

$$i_{MN} = \frac{e_1}{R + r} \tag{32}$$

Thus the current flowing in the branch R is equal to the potential drop across $M\,N$ with the branch circuit open, divided by the total resistance of the circuit.

Suppose the branch circuit to be the compensated indicator. The current flowing through the instrument is given by equation (32), and the two values R' and R'' of R (key open and closed) are expressed by equations (25) and (26). Hence:

$$\text{Key open } i' = \frac{e_1}{r_x + L + T + r_2 + r_3 + r}$$

$$\text{Key closed } i'' = \frac{e_1}{(r_x + L + T) + \dfrac{r_3 r_4}{r_3 + r_4} + r}$$

If r_x is adjusted for equal deflections with the key open and closed we may substitute these values of i' and i'' in equation (24) and obtain:

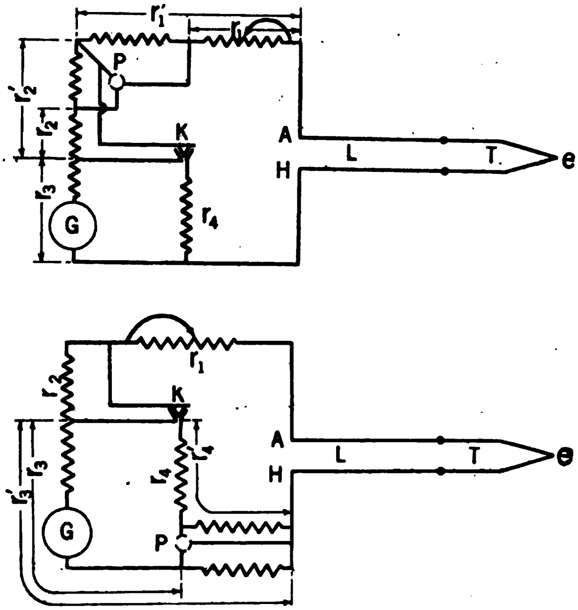
$$\frac{r_2 r_4}{r_3} = r_x + L + T + r = r_1 \text{ (33) as in equation (27)}$$

Thus with the key open $e_1 = i'\,(r_1 + r_2 + r_3)$. The instrument is calibrated for the total resistance $(r_1 + r_2 + r_3)$ so that the current i' produces a deflection which indicates directly the true potential difference e_1 existing between any two points of a network. If there are several e.m.fs. E_1, E_2, etc. in different arms of the network, it readily follows that the instrument still measures the true potential drop, on open external circuit, between any two points to which it is connected.

DOUBLE SCALE RANGE

Figs. 5 and 6 illustrate modifications of the simple design shown in Fig. 2 to permit the use of two different e.m.f scales. In Fig. 5 the resistances r_1 and r_2 are increased in the same ratio and by such an amount that the total resistance $r_1' + r_2' + r_3$ gives the desired sensitivity for the high range scale. Thus $r_1/r_2 =$

$r_1'/r_2' = r_4/r_3$ and the factor for the magnification of errors is the same with either range. With the plug in at *P*, $r_1' - r_1$ and $r_2' - r_2$ are short-circuited and the instrument operates on the low range scale. With the plug out r_1 is increased by the fixed resistance $r_1' - r_1$ and r_2 is changed to r_2', and the instrument operates on the high range scale.



Figs. 5 and 6—Double Range Instruments, Compensated in Either Range

In the design shown by Fig. 6 the resistances r_3 and r_4 are increased in a similar manner such that $r_1/r_2 = r_3/r_4 = r_3'/r_4'$.

USE OF AN UNADJUSTABLE VALUE FOR r_1

In the foregoing discussion adjustment of the resistance r_1, which consists in part of the line and couple, has been considered. As has already been pointed out, if any of the four resistances in Fig. 2 is adjusted to give equal galvanometer deflections with the key open and closed, the relation $r_1/r_2 = r_4/r_3$ will be established. In Fig. 7, r_3 is made equal to r_4, and its value is chosen such that the instrument reads correctly when the total resistance of the circuit equals $r_2 + r_3 + r_x$ where $r_2 + r_x$ is the total resistance of the rheostat. With this arrangement $r_1 = L + T$. When switch S is in position B', adjustment for equal deflections makes $r_1 = r_2$, (since $r_4 = r_3$). After this adjustment switch S is thrown into position B, and the resistance of the circuit, $L + T + r_x + r_3$ equals $r_2 + r_x + r_3$, the resistance for which the instrument reads correctly. The process of adjusting is to close the key K and read the instrument, open the key and adjust the rheostat until the instrument shows the same deflection, and then throw S into the opposite position. The method has the advantage that the reading with the key closed

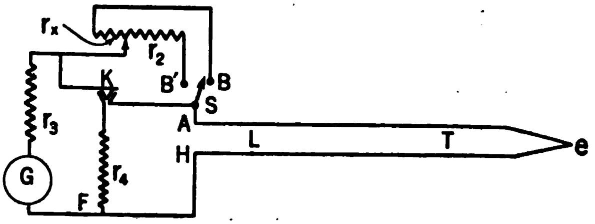


Fig. 7—Modification of Fig. 2

is independent of the setting of the rheostat; therefore, when adjusting the rheostat with the key open, the reading to which the instrument should be brought is accurately known. It is immaterial which way switch S is set during the adjustment, provided its position is changed after the adjustment has been made. Three disadvantages will be noted. (1) Since $r_3 = r_4$, there is no multiplication of errors in setting; (2) an extra switch S is required, and this switch must be operated each time an adjustment is made; (3) any contact resistance existing in the rheostat will alter the final resistance of the circuit by double this amount. Such contact resistances constitute no error in the instrument shown in Fig. 2.

Fig. 8 shows a design, somewhat similar to that illustrated by Fig. 7, which is modified to give a multiplication of errors on setting. Resistance r_4 is made equal to r_3/m, where m is any desirable factor for multiplication of errors. The total resistance $r_x + r_y$ of the upper rheostat is made equal to the maximum allowable line and couple resistance. The total resistance of the rheostat of which r_2 is part is made equal to $m\ (r_x + r_y)$. Adjustment for equal deflections with switch S at B' makes $r_2 = m\ (L + T)$, whence, since the two rheostats are adjusted simultaneously, $r_y = L + T$. This is removed from the circuit upon setting switch S in working position B. Resistance r_3 in Fig. 8 is made less than r_3 in Fig. 7, in order to compensate for a higher value of r_2. Resistance C is added to bring the total resistance of the circuit in the operating position up to the proper value.

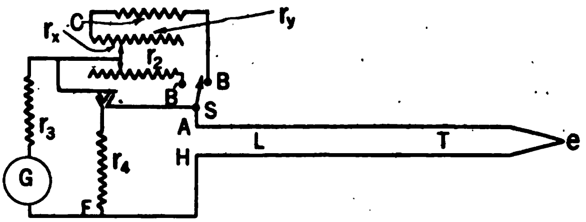


FIG. 8—MODIFICATION OF FIG. 7 ARRANGED FOR ACCURACY OF ADJUSTMENT

In Fig. 9 the resistance r_4 is adjustable. By construction r_2 is made equal to r_3. Switch S is closed and r_4 is adjusted until equal deflections of the galvanometer are established. This makes $r_4 = r_1 = L + T$. Opening switch S adds resistance r_x to the circuit, making the total resistance equal to $r_2 + r_3 + x$, the resistance for which the instrument is calibrated. With S closed and K open the reading is independent of the setting of the rheostat. This reading is noted. With switches K and S closed r_4 is adjusted until the reading is the same as that determined above. The factor m increases rapidly as resistance $L + T\ (= r_1)$ decreases. Its value is given by the expression $m = \dfrac{r_2 + r_3 + x}{2\,r_4\,(1 + r_4/r_3)}$

COMPENSATED MILLIVOLTMETER WITH $r_6 = 0$

The circuit for an instrument designed according to equation (7) in which $r_6 = 0$ is shown by Fig. 10. The instrument possesses many advantages, one of these being the complete elimination of objectionable effects of contact resistances. It is further possible to compensate for temperature coefficient by the well known Swinburne[5] method, to obtain any desirable magnification ratio for errors, and to secure an instrument which is satisfactorily damped either with the key open or closed. Where a desirable magnification ratio cannot be otherwise obtained a battery may be inserted between C and K, Fig. 10. The proportioning of the various resistances is illustrated by the following discussion:

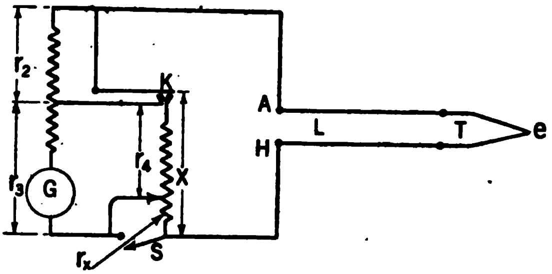


FIG. 9—ANOTHER MODIFICATION OF FIG. 2

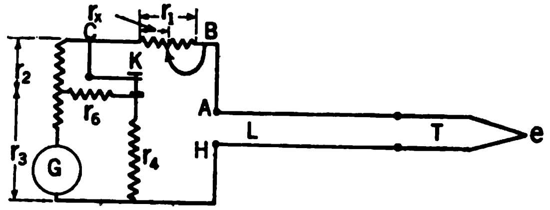


FIG. 10—CLOSED SHUNT TYPE OF COMPENSATING INDICATOR

Let i' = current through the galvanometer coil necessary to produce full scale deflection.

e' = maximum e.m.f. measured = highest scale reading.

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[5] Hallo and Land, Electric and magnetic measurements and measuring instruments, 1906, p. 260.

R_g = resistance of the moving copper coil.
m = magnification ratio for errors $\left(\frac{\text{key closed}}{\text{key open}}\right)$.
R_d = proper damping resistance.

Hence:

$$\frac{(r_3 + r_4 + r_6)\,(r_1 + r_2)}{r_4 + r_6} + r_3 = e'/i' = S$$

(Scale range) (**34**)

Equation (**34**) is the condition for scale range such that an e.m.f. e' produces a full scale deflection.

$$r_1/r_2 = r_4/r_6 \qquad \text{(Key adjustment)} \qquad (\mathbf{35})$$

Equation (**35**) is deduced as shown by equation (**7**) and expresses the condition that the deflection of the galvanometer is the same with the key K open or closed.

$$r_1 + r_2 = (r_4 + r_6)\,\frac{R_g}{r_3 - R_g} \qquad \text{(Temperature compensation)} \qquad (\mathbf{36})$$

Equation (**36**) imposes the condition for temperature compensation by the Swinburne method. The resistances r_4 and r_6 must be of copper.

$$\frac{1}{m} = \frac{\delta e'}{\delta e''} = \frac{{}_1 + r_4 - r_4\,\dfrac{r_4}{r_3 + r_4 + \dfrac{r_2\,r_6}{r_2 + r_6}}}{r_1 + r_2 + \dfrac{r_3\,(r_4 + r_6)}{r_3 + r_4 + r_6}}$$

(Errors) (**37**)

Equation (**37**) shows the relation between the error in a setting with the key open and with the key closed. This is derived in a manner similar to that employed in the discussion of Fig. 2. For this relation r_7 is roughly zero.

$$R_d = r_3 + \frac{(r_1 + r_2)\,(r_4 + r_6)}{r_1 + r_2 + r_4 + r_6} = r_3 + \frac{(r_1 + r_2)\,r_4}{r_1 + r_4}$$

(Damping) (**38**)

Equation (**38**) gives the total resistance of the galvanometer circuit with the key open. This should equal the proper damping resistance which is slightly greater than the critical damping resistance. It will be noted that when the resistance r_x is adjusted for identical deflections with the key open and closed the resistance effective in damping is the same with the key in either position, and hence if the instrument is properly damped with the key open, it will be properly damped with the key closed.

The above 5 equations serve to determine the 5 variables r_1, r_2, r_3, r_4, and r_6. However, the condition for proper damping, equation (38), need not be satisfied exactly. The behavior of an instrument usually allows considerable tolerance in the value of R_d. Hence in order to obtain wide limits for construction, the first four equations may be solved as indeterminants and the best values of the resistances consistent with a suitable damping resistance may be chosen for the final design. The third term in the numerator of the right hand member of equation (37) is usually small. In place of equation (37) we shall assume the following relation:

$$\frac{1}{K} = \frac{r_1 + r_4}{r_1 + r_2 + \dfrac{r_3\,(r_4 + r_6)}{r_3 + r_4 + r_6}} \tag{39}$$

The solution of equations (34), (35), (36), and (39) is as follows:

$$r_1 = \frac{R_g\,S}{K\,r_3\left[1 - \dfrac{r_3\,R_g}{(r_3 - R_g)\,(S - r_3)}\right]} \tag{40}$$

$$r_2 = S - r_1 - r_3 - \frac{r_3\,R_g}{r_3 - R_g} \tag{41}$$

$$r_4 = \frac{r_1\,(r_3 - R_g)}{R_g} \tag{42}$$

$$r_6 = \frac{r_2\,r_4}{r_1} \tag{43}$$

The following example illustrates the method of applying the above equations: It is desired to convert an ordinary millivoltmeter, scale range 0 – 15 millivolts into the above compensated instrument having a scale range 0 – 60 millivolts. The resistance of the simple indicator is 250 ohms of which the copper content R_s = 60 .ohms. The instrument if suitably designed is accordingly properly damped on R_d = 250 ohms. Since the scale range is to be increased by the factor 4, the value of $S = 4 \times 250 = 1000$ ohms. Let K = 10. This is closely equal to m the magnification ratio for sensibility to changes in r_1 with the key closed. For various assumed values of r_3 compute r_1 from equation (40). For corresponding values of r_3 and r_1 compute r_2 from equation(41). Similarly for equations (42) and (43).

The following table illustrates the results thus obtained:

Values of Resistances
for r_5 =0

r_3	r_1	r_2	r_4	r_6	$r_3 + \frac{(r_1 + r_2)(r_4 + r_6)}{r_1 + r_2 + r_4 + r_6}$	m
65	15.3	140	1.27	11.7	77	10.0
70	47	463	7.84	77.3	143	
80	55.5	626	18.5	208.5	250	10.2
90	53.5	676	26.7	338	333	
100	50	700	33.3	466	400	10.3
200	26.8	688	62.5	1605	700	
300	17.85	607	71.4	2428	800	10.7

Any of the above designs gives a millivoltmeter of scale range 0 – 60 millivolts, compensation for temperature coefficient by the Swinburne method and complete elimination of the error due to uncertain line drop. The values corresponding to r_3 = 80 furnish the proper damping, 250 ohms, characteristic of the particular instrument selected. If we substitute the values of the various resistances in equation (37) we obtain m = 10.2 which gives the ratio of sensitivities with the key closed and open.

If equations (40) to (43) do not furnish satisfactory values for the resistances with a suitable choice for K, equation (36) may be omitted and equations (34), (35) and (39) may be solved indeterminately similar to the manner above outlined. If the final choice of values for the various resistances results in an excessive temperature coefficient of the compensated instrument, this can be minimized by making r_4 and r_6 of unequal temperature coefficients, thus allowing the adjustment of r_1 to vary with the temperature as was done with the instrument illustrated by Fig. 2.[6]

The instrument on the right in Fig. 11 is an ordinary millivoltmeter. The other instrument is the compensated millivoltmeter constructed according to the wiring diagram shown in Fig. 2. It will be noted that the additional wiring requires no alteration in the size of the finished instrument.

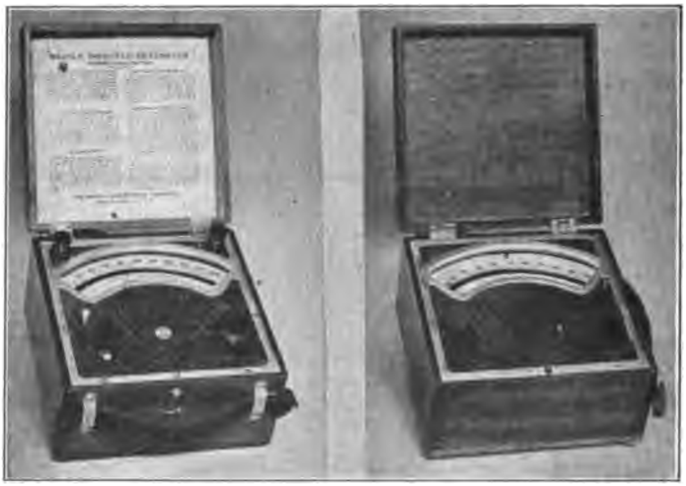

FIG. 11—COMPARISON OF INSTRUMENTS WITH AND WITHOUT COMPENSATING DEVICE. COMPENSATED INSTRUMENT SHOWN ON LEFT

SUMMARY

A new principle has been developed whereby an ordinary millivoltmeter may be converted into an

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[6] This method of treatment has recently been developed more fully and will appear in an early issue of the *Journal Wash. Acad. of Sci.* In this treatment the compensation for coefficient of springs and magnet is considered. This condition is more easily satisfied than is that required by eq. (36). This method is useful in making low range instruments.

instrument in which the usual errors arising from a variable line resistance are entirely eliminated. All of the several modifications described afford some simple means by which the total resistance of the galvanometer circuit, internal and external, is adjusted to a preassigned value for which the scale of the instrument is calibrated. In one form now manufactured the instrument consists of an adjustable resistance r_x in series with the moving coil and swamping resistance of the millivoltmeter. On depressing a key part, r_2, of the swamping resistance is short-circuited and the remaining part together with the moving coil, r_3, is shunted by a resistance r_4. The instrument is calibrated in terms of the potential drop across its terminals for a maximum value of $r_x = r_1$. In the construction the resistances are proportioned according to the relation $r_2 r_4 = r_1 r_3$. If the resistance r_x is so adjusted that the deflection of the pointer is unchanged by depressing the key it is shown that the total resistance of the circuit is that for which the instrument is calibrated, the sum of r_x and all external resistance being thus made equal to r_1. Hence it follows that the instrument measures true e.m.f. in a simple circuit, or if connected across a resistance or network through which a current flows it indicates the potential drop which would have existed had the instrument not been connected. In this respect it functions as a potentiometer, yet it does not operate on the potentiometric principle since it does not require a standard cell or an auxiliary battery, the only e.m.f. employed in the adjustment being that of source measured.

By constructing r_3/r_4 equal to from 5 to 10 it is possible to adjust r_x with 5 to 10 times the precison necessary. Thus if the galvanometer can be read to 1/10 of a scale division the line resistance may be adjusted with a precision equivalent to 1/100 of a scale division, which is at least 10 times the accuracy possible with any indicating instrument. This principle of magnification of errors greatly facilitates the proper adjustment of r_x. By varying the copper to manganin ratio in r_4 it is possible to produce a compensated instrument of zero temperature coefficient from a millivoltmeter having an excessive copper content.

The instrument may be used as a galvanometer with an ordinary potentiometer, forming a deflection potentiometer which requires no compensation other than that provided by the indicator. Such an instrument is especially serviceable in thermocouple work.

The compensated millivoltmeter may be used in multiple installations of thermocouples having different line resistances, as many resistances r_x being employed as there are couples. These may be inexpensive rheostats, one located in each line between the couple and selective switch, and still the accuracy of adjustment will be as high as though precision rheostats were employed.

Methods are described for constructing instruments of double scale range with compensation in both ranges.

Specifications are given for an instrument which compensates for line resistance, which has a zero temperature coefficient, and which is properly damped either with the key closed or open.

DISCUSSION ON "A PRECISION GALVANOMETRIC INSTRUMENT FOR MEASURING THERMOELECTRIC E. M. FS." (HARRISON AND FOOTE), NEW YORK, N.Y., FEBRUARY 20, 1920.

J. B. Whitehead: In the abstract the statement is made: "The instrument measures true e. m. f. in a simple circuit, or if connected across a resistance or network through which a current flows it indicates the potential drop which would have existed had the instrument not been connected;" which suggests that the instrument takes no current, or functions, as does the electrostatic voltmeter. This, however, cannot be true—hence my question. It is quite obvious, that even if limited to correction of drop in the leads, the principle has great value for refined measurements.

A. E. Kennelly: It is very important to be able to secure potentiometer apparatus, so compensated that no errors are produced by introducing it into the circuit it measures. The principle represented here is a very general application and it ought to prove very valuable to laboratory men.

W. D. A. Peaslee: In connection with instruments for measuring thermo-electric currents of this kind, I recently ran into an interesting phenomenon that may be of interest to you. In using some rather high temparature kilns in the manufacture of porcelain, we began to get, under certain conditions, very freaky readings on the thermocouples. They were used in certain parts of the kiln, some base metal couples, and in other parts rare metal couples. On the recommendation of some men who we thought knew more about the matter than we did, we were using switches, and installed indicating and recording instruments, and got some very peculiar readings. We got readings after the apparatus had been in operation a little while, that were the sum of the base metal and rare metal readings, and got a reading that would be the difference in these readings and then some that were some integral of the two. Then we would have readings for a while that did not seem to mean anything. We finally got into it, and I came to the conclusion that there was some sort of ionic conduction through the terrifically hot gases of the kiln that gave us a leakage combined with a dust leakage, which was in the switch. The manufacturers of these instruments stated that it was absolutely impossible. By the simple experiment of closing the dampers, in some of the gas chambers, we proved that in these high temperatures it was possible, and it was possible also in these high temperatures to get electrified drift of some kind that made your errors in these instruments of too great a magnitude to be permissible in that work.

We had a very interesting thing occur which the paper brings to mind. We have indicating instruments on the kilns by which the kiln operators do their firing, and they read a log from these indicating instruments. If the men happen to play a game of cards in the middle of the night, and neglect their duty, it results in rather serious consequences in the kiln, and everything is lovely in the morning, except you have lost $10,000 or $12,000 worth of porcelain. We rigged up a means in our research laboratory by which we could plug the recording instrument on to any indicating instrument at will, so that the men did not know what indicating instrument was on, but through some rather subterranean channels, I found the men immediately knew—we could not figure it out. When we plugged the recording instrument and indicating instrument in parallel, to a thermo-coupled circuit, whenever the man reads his indicating instrument, he gets a static kick on the indicating instrument every time he starts reading across. He has eighteen couples on which he presses a button to take his readings, and when he comes to the one to which the recording instrument is coupled he gets a kick, and he says—"The old man is watching me on that one," and we always get results on that instrument. I am anxious to get some system whereby we can get our recording instrument on the indicating instrument without their knowledge.

It really is a very delicate problem. In firing porcelain in tunnel kiln, you fire to very nice limits, and not only that, the time curve to be arrived at is very important and it is desirable that the firing process should be very strictly under control.

That is the problem on which I would like to keep some of the men interested in thermocouple indicators and recording instruments—to give us a system where we can put in 30 or 40 indicating and a few recording instruments, and be sure of our results, and also to be sure that the men we are keeping check on are not as well posted on what we are doing as we are.

E. M. Hewlett: If you put a dummy resistance near to the instrument itself, you might know these instruments at any time, without this kick, I should say.

W. D. A. Peaslee: I have tried that—I will admit that none of the manufacturers agreed with me, but I have become used to that, and it does not bother me so much anymore. I believe most of it, some sort of a charge, due to the friction of the paper roll in the recording instrument, giving you a static disturbance of some kind, because we have tried all sorts of arrangements, and resistances now manufactured, and have not been able to prevent a snappy kick when the button is pressed in. We have tried one thing which will be put in sometime during this week, in which we disconnect completely the recording instrument, and allow it to discharge itself, and after that is done connect the thermo-couple to the indicating instrument.

E. M. Hewlett: What I meant was, instead of letting the man pick out the particular instrument, you could contrive something that would not let him do that, something else that could go on at the same time.

W. D. A. Peaslee: That is what I was aiming to do, but they have me outguessed all the time.

Clayton H. Sharp: I have examined with a great deal of interest the diagrams which are given showing how an ordinary milli-ammeter may be converted into one of the compensated type. There are many times when one of these plans of connection might be used to advantage, and as I look this paper over, the Fig. 7 seems to offer a scheme whereby a very simple and ordinary milliammeter can be arranged so that it will work on this compensated principle. Of course, one cannot go ahead and convert any instruments if there are legal restrictions on the use of this plan, but inasmuch as the development has come from the National Bureau of Standards, I take it that that condition is fulfilled. I think, however, we ought to be warned if that is not true.

H. B. Brooks: In regard to Dr. Whitehead's question as to the way the chief advantage is reached. Of course, at the start one recognizes that network matters are not always evident by mere inspection—it is necessary to trust to Kirchhoff's laws, and rely on your mathematics, to a certain extent, but so far as I can I will illustrate the matter.

Suppose we have a source having zero internal resistance, for example, a storage cell, connected to a voltmeter which consists of a moving coil of relatively low resistance in series with a large resistance. Suppose further that we connect one terminal of a shunt resistance to the terminal of the storage cell which is connected directly to the moving coil of the instrument. Now if we connect the other end of the shunt resistance to the other pole of the storage cell, the voltmeter will show no change in reading. This is because our shunt resistance has gone on in such a way as to simply load the source a little more. If we connect the same shunt resistance in parallel with the moving coil, we have not appreciably changed the current through the series resistance but part goes through the shunt and the deflection is decreased.

Instead of connecting the end of the shunt circuit to the junction of the series resistance and the moving coil, we may connect it at some intermediate point on the series resistance. We will find it necessary to cut out some resistance from the unshunted part of the series resistor to maintain the deflection at the original value. When the proper relations exist for the instrument to measure true e. m. f., the additional current drawn from the source is so divided that the current which flows through the moving coil is exactly equal to the current before we applied the shunt. It is not evident what the proportions must be, but it is necessary to depend on the mathematics of the case.

J. B. Whitehead: Is more load taken from the source?

H. B. Brooks: It is; more current is supplied by the thermocouple when the key is depressed. In spite of this, the instrument measures the true e. m. f. of the couple.

J. B. Whitehead: It is suggested in the second sentence of the abstract that that is so.

H. B. Brooks: That is the way we get a magnification of the rheostat adjustment error. We have an adjustable rheostat which compensates for the resistance in the source. The instrument may be so designed that if the reading changes by one-tenth of a division upon closing the key, the error due to incorrect adjustment of the rheostat with the key open will be only from one-fifth to one-tenth of this amount.

Referring to the question regarding legal matters, the authors requested that any question of that sort that might be asked should be written into the minutes of the convention, and that the question be transmitted to them in order that they might reply in their own words.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 20, 1920.

NOTES ON THE SYNCHRONOUS COMMUTATOR

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In the use of the synchronous commutator in series connection as a suppressor, serious errors may arise due to relatively small amounts of capacity in the commutator and galvanometer circuits. The magnitude of these errors is studied for a number of different connections, and methods for eliminating them are pointed out. A number of wave forms are given, indicating the nature of the errors.

Used as a shunt suppressor the commutator is far more reliable and this method of connection is always to be preferred.

An appendix gives a theoretical analysis of two cases investigated and shows a close agreement with the experimental observations.

IN certain methods of measurement of the crest values of high alternating voltages it is necessary to measure accurately the average values of very small alternating currents. One of the commonest of these methods is that in which a condenser, in series with a low resistance, is placed across the voltage and the voltage drop over the resistance, due to the charging current, measured on an instrument of D'Arsonval type with high series resistance, through the use of either a synchronous commutator or rectifying vacuum tubes.

In these methods, if the commutator is used the instrument receives either a rectified alternating current, made up of both half waves, or a uni-directional pulsating current, made up of alternate half waves with the intermediate opposite half waves cut out either by opening the circuit (series commutator), or by short-circuiting the instrument (shunt commutator). When the commutator is used so as to eliminate alternate half waves, F. Bedell[1] has called it a *suppressor*. If the rectifying vacuum tube is used the current in the measuring instrument is always pulsating with alternate half waves cut out. In any of the above mentioned cases the instrument reads the average current in terms of its calibration by continuous current. For the cases in which the current is pulsating, the instrument reading is multiplied by two to obtain the average value of the condenser charging current, it being assumed that the active half wave of charging current is accurately reproduced in the instrument and that in the intervening half wave interruption no current flows in the instrument.[2]

Measurements of crest value by these methods are probably the best so far obtained, but apparently no special effort has been made to put them on a precision basis. When a sensitive galvanometer is used, the pulsating character of the current raises questions as to the influence of capacitance and reactance in the instrument circuit, and as to the validity of the factor 2 mentioned above. The observations recorded below show that under certain circuit conditions serious errors may arise.

THE APPARATUS

The commutator used in the experiments was six inches in diameter and had six brass segments ½-in. wide, insulated from each other by $^1/_{32}$-in. built-up mica, and from the hub by standard V-shaped commutator bushings. Two brass rings, 6-in. in diameter and ½-in. wide, were mounted one on each side of the central commutator segments and insulated from them by intermediate bakelite rings. Screws holding the outer brass rings in place, passed through the bakelite connecting the outside continuous rings to opposite sets of three alternate commutator segments. The commutator was mounted on the end of the shaft of the 6-pole generator. Four brushes of $^1/_{64}$-in. thick

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[1] F. Bedell, *Journal of Franklin Institute*, No. 176, 1913, p. 385.

[2] W. E. Sumpner, *Philosophical Magazine*, p. 155, January 1905.

spring bronze, stiffened with backings of steel spring, were carried on a graduated bakelite disk arranged for complete rotation about the axis and clamping in any position. The commutator was machine-constructed throughout. It was frequently tested for insulation and showed throughout a resistance of the order 10^3 megohms.

The 6-pole generator was single phase and had a capacity of 5 kw. at 60 cycles and 120 volts. It was driven by an adjustable-speed continuous-current shunt motor, permitting a range of frequency from 20 to 90 cycles. The armature was surface wound, giving a smooth wave approximating closely to sine shape.

Two D'Arsonval galvanometers were used, both of suspension type, read by telescope and scale. One, No. 24515, had a resistance of 115 ohms, a sensitivity of 40 megohms, an undamped period of 9.5 seconds, and a critical damping resistance of 560 ohms. The corresponding figures for the other galvanometer, No. 23518, were 1680 ohms, 1280 megohms, 22 seconds, and 3400 ohms. The sensitivities for the two instruments when critically damped therefore were 33 and 420 megohms, respectively.

COMMUTATOR AS SERIES SUPPRESSOR

Obviously the most rigid test of the circuit and instrument for pulsating current is to apply a continuous electromotive force, chopping it up into alternate intervals at full value and zero value by means of the series commutator, as indicated in full lines in Fig. 1. The complete interruption of the current and the sudden application of the full electromotive force accentuate any conditions tending to upset the perfect rectangular half wave alternating with a half period of complete interruption. The test of this performance is the ratio of the readings of the galvanometer with commutator at standstill and when running. This ratio should be 2. This arrangement uses only two brushes and will be spoken of as the series connection of the commutator.

Fig. 1 shows the commutator in series connection; that is, as a simple make and break device in the circuit containing a single dry cell E, resistance $R = 10^6$ ohms and $r = 10^4$ ohms and the galvanometer shunted with its critical damping resistance. Several different types of resistance were used for R; a series of carbon lightning arrester rods aggregating 200,000 ohms, a series of so-called "lavite" units aggregating 10^6 ohms, and a precision set of non-inductive, capacity-free, manganin units aggregating 10^6 ohms, in which the wire was wound in single layers on thin micanite plates, suspended on glass rods.

A number of observations were made at different speeds of the commutator C, and with S open and closed, corresponding to running and standstill conditions for C. In every case it was found that the ratio of galvanometer deflections for running and standstill was greater than 0.5. A typical set of observations is shown in Table I and plotted in Fig. 2 as curve No. 2. The speed, in terms of the frequency of the generator, is plotted as abscissa, and the ratio of running to standstill deflection of the galvanometer as ordinate.

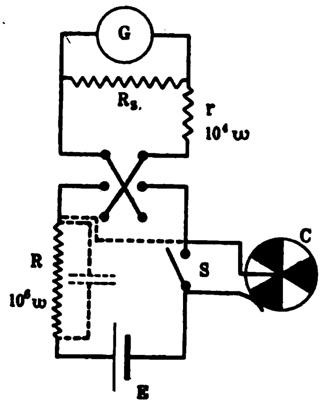

FIG. 1—COMMUTATOR CONNECTED AS SERIES SUPPRESSOR

The increase of the ratio of running to standstill deflections with the frequency obviously suggests the presence of capacity as a disturbing element. This was confirmed by a rearrangement of the apparatus and connections so as to reduce the capacity in circuit. In Fig. 2, curve No. 1, the resistance R was connected into the circuit over a pair of leads consisting of twisted lamp cord about 100-ft. long and having a capacity of about 0.005 microfarads. In curve No. 2, R^1 was a small compact resistance of the same

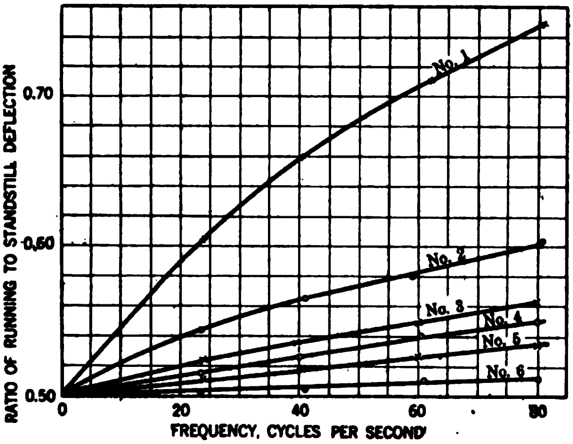

Fig. 2—Ratio of Running to Standstill Deflection for Various Speeds of Commutator. Commutator in Series.

No. 1—Leads for 10^6 ohm Resistance: 100 ft. Twisted Lamp Cord (Capacity about 0.005 microfarads). Leads for Commutator: 15 ft. Twisted Lamp Cord (Capacity about 0.0007 microfarads), Brush Narrower than Insulation between Segments.

No. 2—Leads for 10 ohm Resistance: Short Separated Wires, Leads for Commutator: Same as No. 1. Brush Wider than Insulation between Segments.

No. 3—Same as No. 1 except that Leads for 10^6 ohm Resistance Were Separated to Two 100 ft. Wires.

No. 4—Same as No. 2 except that Brush was narrower than Insulation between Segments.

No. 5—Same as No. 3 except that Commutator Leads Were also separated.

No. 6—Same as No. 4 except that Commutator Leads Were Separated

value as that pertaining to curve No. 1, but connected to the circuit with short leads. The remaining curves showing decreasing departure of the ratio of running to standstill values from 0.5, show the successive improvements resulting from shortening and separating the various connections of Fig. 1. It should be noticed, however, that under the very best conditions of short, well-separated connections, with the arrangement of Fig. 1, there is always an error in assuming a ratio of running to standstill deflections of 0.5 and this error increases with the frequency.

TABLE I

Speed cycles	Deflection of galvanometer						
	Running			Standstill			Running / Standstill
	left	right	mean	left	right	mean	
23.5	5.6	5.59	5.595	10.22	10.32	10.27	0.545
41	5.8	5.80	5.80	10.22	10.32	10.27	0.565
59	5.95	5.95	5.95	10.22	10.32	10.27	0.580
81	6.2	6.18	6.19	10.22	10.32	10.27	0.603

By means of the method of connection shown in Fig. 3, it was found that in the half wave in which the circuit is closed by *C*, the current is greater than the standstill value, and during the half wave in which the circuit is opened by *C* there is still some current in the positive direction. *B* is a second com-

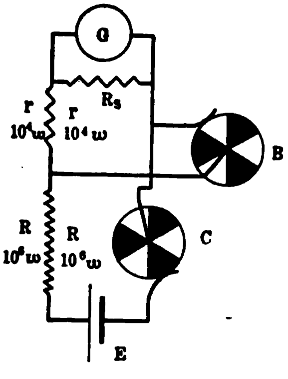

Fig. 3—Series and Shunt Suppressors in the Same Circuits

mutator on the same shaft with *C* and is connected in shunt with the galvanometer, as indicated. By proper setting of the brushes of *B* it is a simple matter to short-circuit the galvanometer for the periods of opening or closing by *C*, and thus read the current for the opposite half wave. For example, at standstill, with *B* open and *C* closed, the galvanometer reading was 9.9; when running and with *B* set for reading during closed *C* and open *C*, respectively, the readings were as follows:

TABLE II

Speed cycles	Closed C	Open C
23	5.78	1.1
41	6.15	1.38
60	6.5	1.7
77	6.7	....
80	....	1.9

Since in each case the galvanometer is receiving current only one-half the time, the readings must be multiplied by 2 to obtain the full values. They show therefore that during the half wave of closed *C* the current is greater than 9.9, and during the half wave of open *C* the current is greater than 0. Moreover the excess of current value increases with the frequency.

A further study of the influence of capacity with the connection of Fig. 1 was made by inserting additional capacity in shunt to the resistance *R*, between the commutator leads, and between commutator segments. In the upper part of Fig. 4 curves are plotted showing the variation of the ratio of running to standstill deflection with the frequency for series commutator and for different values of capacity connected in shunt to the resistance *R*. The lower half of Fig. 4 gives curves taken with the commutator in shunt connection. These curves will be referred to below. Similar curves are obtained when the capacity is connected between the leads to the commutator.

In Fig. 5 the two lower curves were taken at 60 cycles with various values of capacity connected between the opposite sets of segments of the commutator.

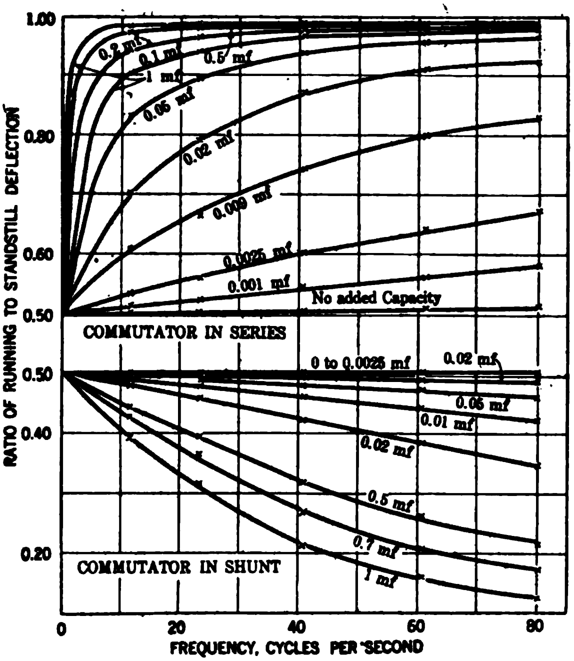

FIG. 4—INFLUENCE OF CAPACITY IN SHUNT TO 10^6 OHM RESISTANCE UPON RATIO OF RUNNING TO STANDSTILL DEFLECTION. BRUSH NARROWER THAN INSULATION BETWEEN SEGMENTS

This connection is made by connecting the capacity in parallel with the two outer collector rings of the commutator. Curve No. 2 is taken with a brush which is thicker than the insulation between segments and curve No. 3 is taken with a brush which is thinner than the insulation between segments. It is noteworthy that the use of the thick brush, short-circuiting as it does the capacity shunted around the commutator and thus discharging the capacity, is the cause of a wide error in the value of the ratio of running to standstill deflections. Curve No. 1 shows that the effect of adding capacity between commutator leads is similar to that of adding capacity between the terminals of *R*.

In view of the foregoing results, it appeared of interest to study the wave form of the current in the galvanometer. The current values being too small for the oscillograph, the method shown in Fig. 6 was adopted. *C* is the rectifying commutator already described arranged for series connection, that is, using only two brushes and operating as a half cycle make and break. Switching is also provided so that *C* may be arranged in shunt connection or cut out entirely. *D* is a metal disk also mounted on the generator shaft and equipped with three narrow insulating sectors set in its outer surface. Two brushes on *D* short-circuit the galvanometer, as indicated, during a complete cycle except for the brief interval when the short circuit is interrupted by one of the insulating segments, that is, once in each cycle of make and break. The particular instant in the cycle at which the galvanometer receives current is varied by rota-

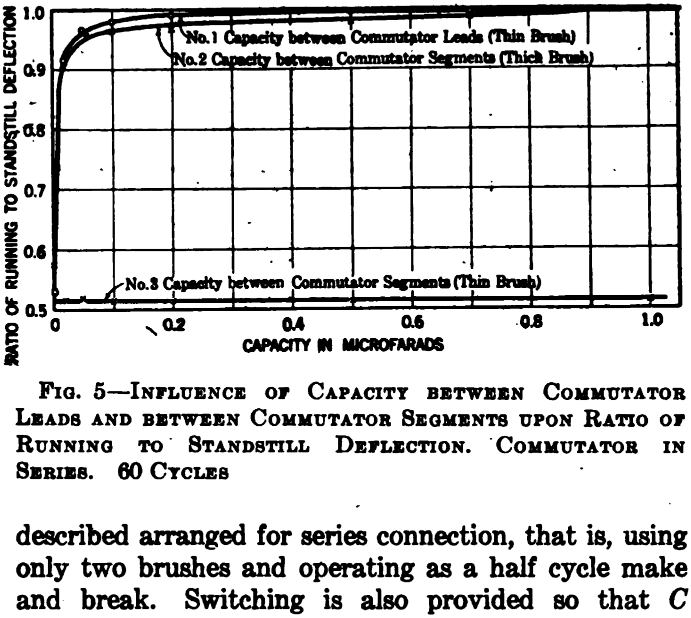

FIG. 5—INFLUENCE OF CAPACITY BETWEEN COMMUTATOR LEADS AND BETWEEN COMMUTATOR SEGMENTS UPON RATIO OF RUNNING TO STANDSTILL DEFLECTION. COMMUTATOR IN SERIES. 60 CYCLES

ting the brushes on *C*. The arrangement constitutes a point by point method for studying the current in the circuit of the commutator *C*.

Observations were taken with *C* both in series and in shunt. A typical set is given in Table 3 and plotted in Fig. 7. It was found that with *D* operating and *C* cut out completely, that is, with steady current in the battery circuit, the deflection of the galvanometer had a slow and irregular variation and usually different values for opposite positions of the reversing switch. These variations were due to thermal electromotive forces or other variable contact conditions at the brushes of *D*. In order to eliminate this trouble right and left readings of the steady current (*C* not in circuit) and of the instantaneous current (*C* in circuit) were taken in quick succession and their mean values used. The wave form is obtained by taking for each brush setting the ratio of the mean of the instantaneous readings to the mean of the steady readings.

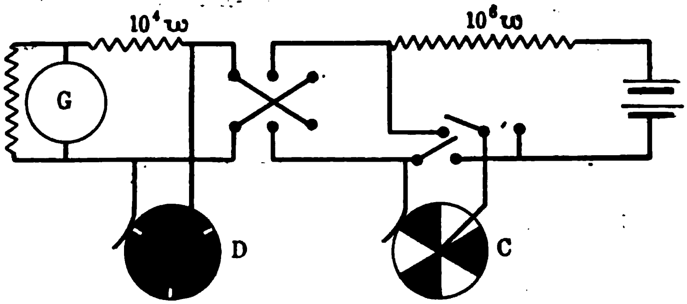

Fig. 6—Connections for Taking Wave Forms of Series and Shunt Suppressors

The more interesting results are plotted in the curves of Figs. 7, 8 and 9, showing several forms for both series and shunt connections of the commutator. In the case of Fig. 7 the frequency was 60 cycles and the 1-megohm resistance *R* and the commutator *C* had leads consisting of twisted lamp cord 100-ft. and 15-ft. long (capacities about 0.005 and 0.0007 microfarads) respectively, the brush was thinner than the width of the insulating segments in the commutator *C*. In Fig. 8 the conditions were the same at a frequency 24 cycles, and in the lower curve a brush thicker than the insulating commutator segments was used. In Fig. 9 short separated leads of minimum capacity were used to *R*, and the 15-ft. twisted leads to *C*.

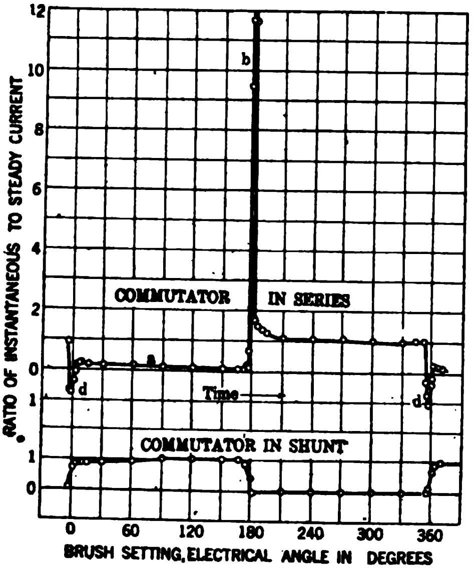

Fig. 7—Wave Form of Current Through Galvanometer 60 Cycles. Brush Narrower than Insulation between Segments. Leads for 10^6 Ohm Resistance: 100 ft. Twisted Lamp Cord (Capacity about 0.005 Microfarads. Leads for Commutator: 15 ft. Twisted Lamp Cord (Capacity about 0.0007 Microfarads)

As already indicated, the results shown in the foregoing figures are due to the capacity located in different parts of the connections. For example, in Fig. 7, during open circuit by the commutator, the capacity of the commutator itself charges as indicated at (*a*) and *C′* the capacity around the resistance *R* discharges through the resistance. At (*b*) on the instant of closing, the capacity *C′* charges, giving an excess current through the galvanometer which immediately tends to fall to the value fixed by the resistance *R*.

TABLE III

60 cycles, brush narrower than insulation between commutator segments, leads for 10^6 ohms resistance, twisted lamp cord 100 ft. long, (capacity about 0.005 microfarads), leads for commutator—twisted lamp cord 15 ft. long, (capacity about 0.0007 microfarads)

Electrical Angle degree	Commtutator in Series							Commutator in Shunt						
	Deflection for instantaneous current (*C* in circuit)			Deflection for steady current (*C* not in circuit)			Ratio: inst. st'dy	Deflection for instantaneous current (*C* in circuit)			Deflection for steady current (*C* not in circuit)			Ratio: inst. steady
	left	right	mean	left	right	mean		left	right	mean	left	right	mean	
-6	1.7	1	1.35				.97							
-3	-.6	-1.4	-1	1.80	1.00	1.40	-.72	.28	- .1	.09	1.22	.98	1.10	.08
-1.5	-.62	-1.88	-1.25	2.08	1.12	1.60	-.78							
0	.20	-.15	-.65				-.41	.88	.72	.80				.73
1.5	.5	-.88	-.19				-.12							
3	.9	-.47	.215				.14	.95	1.00	.975				.89
6	.85	-.32	.265	2.08	1.12	1.60	.17	.95	1.00	.975				.89
9	.92	-.30	.31				.19	.95	1.00	.975				.89
15	.63	-.02	.305				.19	.95	.92	.935				.85
30	.60	-.08	.26				.16	1.02	.92	.970				.89
60	.57	-.17	.2				.12	1.02	.92	.970	1.20	.99	1.095	.89
90	.48	-.22	.13				.08	1.10	1.00	1.05	1.12	.92	1.02	1.03
120	.57	-.32	.125	2.18	1.12	1.65	.08	1.10	1.00	1.05				1.03
150	.7	-.42	.14				.09	1.10	1.00	1.05				1.03
165	.7	-.47	.115				.07	1.10	1.00	1.05				1.03
171	.72	-.47	.125				.08							
174	.78	-.45	.165	2.18	1.12	1.65	.10	1.08	.92	1.00				.98
175.5	.97	1.09	1.03				.62							
177	16.2	15.6	15.9	2.22	1.12	1.67	9.52	.62	.21	.42	1.12	.92	1.02	.41
178.5	19.0	18.6	18.8				11.70							
180	18.6	17.5	18.05				11.63	.18	-.20	-.01				.01
181.5	2.9	2.0	2.45				1.63							
183	2.68	1.82	2.25				1.52							
186	2.42	1.55	1.99	1.78	1.05	1.415	1.40							
195	1.42	2.2	1.81				1.25							

At (*d*) if the brush is narrower than the insulating segment of the commutator, the counter-electromotive forces of the capacities *C* and *C′* are in series and therefore greater than the battery electromotive force *E*; hence, there is momentarily a reverse current after the circuit opens. Then as *C′* discharges through *R* the electromotive force falls and the battery charges the capacity of the commutator, resulting in

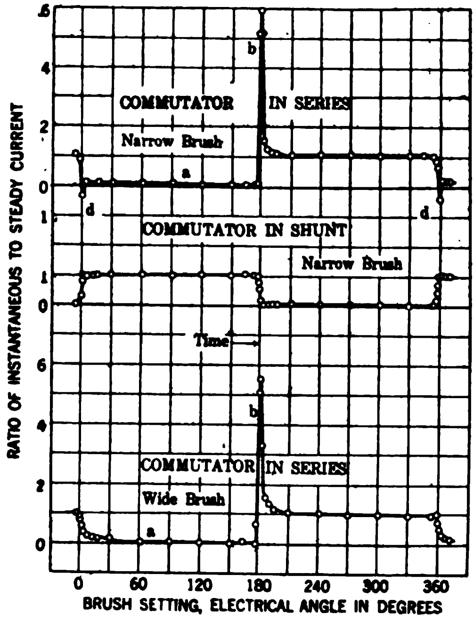

FIG. 8—WAVE FORM OF CURRENT THROUGH GALVANOMETER. 24 CYCLES. ALL LEADS SAME AS FIG. 7

a positive charging current for the interval (a). If the brush on *C* is wider than the insulating segment, the capacity of *C* is discharged through the brush and during the following interval of open circuit the charging current of this capacity is higher. (See Fig. 8, lower curve).

When the leads to the commutator contain capacity with either a wide or narrow brush, the capacity discharges through the commutator and not through the galvanometer at each interval of make, and therefore this capacity charges through the galvanometer during each interval of break; hence there is excess current through the galvanometer at each interval of break, resulting in an increase of the ratio of running to standstill conditions. (See curves of Figs. 5, 7, 8, 9).

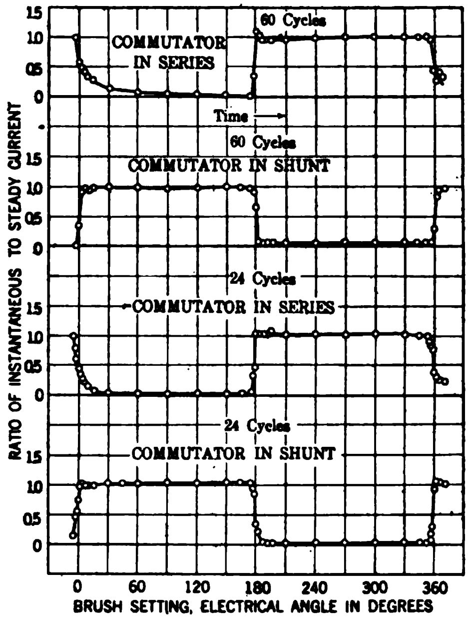

Fig. 9—Wave Form of Current through Galvanometer, Brush Narrower than Insulation between Segments. Leads for 10^6 Ohm Resistance: Short Separated Wires. Leads for Commutator: 15 ft. Twisted Lamp Cord (Capacity about 0.0007 Microfarads)

The effect of capacity between segments in the commutator itself is as follows: This capacity charges during open circuit. If a thick commutator brush is used, the capacity discharges through the brush at the instant the brush bridges the insulating commutator segment, and hence the capacity charges in each interval of open circuit, giving excess galvanometer current and increased ratio of running to standstill current. If, on the other hand, the commutator brush is narrower than the insulating segment, the capacity retains its charge except for loss due to leakage and hence there is no recharge and no excess galvanometer current after the first charging interval, consequently the ratio of running to standstill current in the galvanometer is independent of the value of the capacity and approximately equal to 0.5. (See curves of Fig. 5).

The apparent errors of the series connected commutator may be reduced to a minimum or avoided entirely in two ways: First, by any method of connection whereby the current in the resistance *R* is not interrupted. Two methods of accomplishing this are shown in Figs. 10 and 11. In Fig. 10, by using three brushes on the commutator during the period in which the galvanometer current is interrupted, the current in *R* is maintained at constant value by means of the resistance *R′* of value equal to that in the galvanometer circuit. A number of accurate observations with this method of connection at various values of frequency and capacity were made, and the ratio of running to standstill deflection of the galvanometer was very accurately 0.5 throughout, except for the case of large capacity connected between the points (*b*) and (*c*). In Fig. 11 the current in *R* is also constant and the galvanometer may be shunted around either of the resistances *r*. In a series of experiments with this connection, 120-volts continuous potential was used, and capacity up to 1 microfarad was shunted

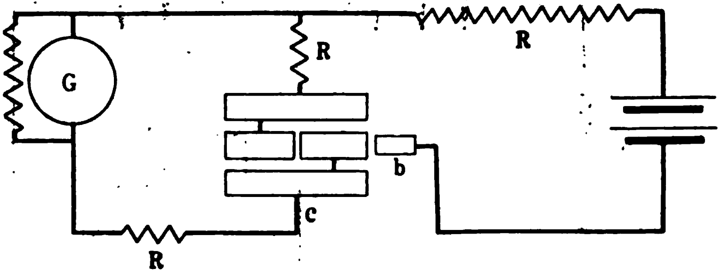

FIG. 10—CONNECTIONS OF SERIES SUPPRESSOR FOR ELIMINATING ERRORS DUE TO CAPACITY

reduce the charging current of any small capacity which may exist. For values of the ratio R/r greater than 100, the current pulsation is small and if the circuit has no capacity, the error is small (values of ratio R/r at the above frequencies being 0.502, 0.501, 0.501 and 0.500).

Capacity, however, soon introduces trouble. When located in the leads to the commutator it charges on open commutator and discharges on a closed commutator. Thus during the open period current is shunted from the galvanometer, lowering the ratio of running to standstill deflections. The phenomenon is the same for thin and thick commutator brushes. In a series of observations at 24 cycles, the ratio decreased from 0.504 to 0.298 in increasing the capacity between the leads from 0 to 1 microfarad.

When there is any capacity between the commutator segments, this capacity charges and stays charged without influence on the ratio of running to standstill deflections if the commutator brush is thinner than the insulation between segments. With a thick brush, however, the capacity discharges when the brush bridges the insulating segment and then charges in the position of open commutator, the performance then being as in the case of the foregoing paragraph. The capacity between commutator segments, as measured, was found to be 0.00126 microfarad. In a series of observations the addition of 0.02 microfarad between commutator segments changes the ratio of running to standstill values from 0.498 to 0.488 with still further lowering for increase of capacity.

When there is capacity in the leads to R on closed commutator, this capacity receives the full battery e.m.f. On open commutator at the instant of opening, the counter e.m.f. of the capacity is equal to that of the battery and no current flows in the galvanometer G. As the capacity discharges through R there is a delay in the rise of current to its full value, thus causing a lowering of the ratio of running to standstill deflections, and in greater amount the greater the capacity shunting *R*. (See Fig. 7, and curves in the lower half of Fig. 4).

PERFORMANCE ON ALTERNATING CURRENT

The errors arising from the presence of capacity in using the commutator with alternating current are smaller than those that have been described. This is to be expected since on commutation the alternating current tends to rise gradually from the zero value and not abruptly as in the case of the pulsating continuous current. In a number of observations taken with alternating current introduced by means of a transformer located in place of the battery of Fig. 1 and with the commutator connected in both series and shunt for reading alternate half waves and also for the reading of full rectification by means of the addition of two more brushes to the commutator in the usual manner, the ratios were found to be very accurately 0.5 under *properly selected* conditions. However, even with careful elimination of capacity

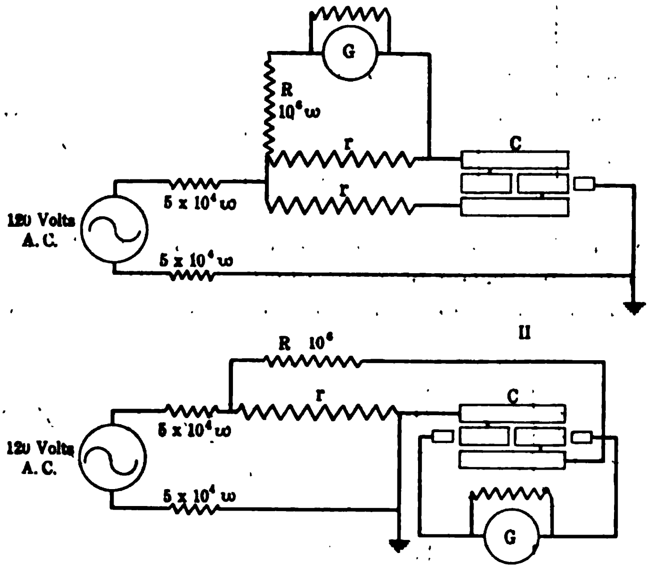

FIG. 13—SERIES COMMUTATOR. ELIMINATION OF ELECTROSTATIC UNBALANCING

TABLE IV

r ohm	Deflection of Galvanometer						
	Full Wave (Fig. 13, I)			Half Wave (Fig. 13, II)			Half wave / Full wave
	left	right	mean	left	right	mean	
1000	3.63	3.69	3.66	1.83	1.83	1.83	0.500
1000*	3.66	3.70	3.68	1.83	1.83	1.83	0.497
2000	7.30	7.38	7.34	3.63	3.68	3.655	0.499
2000*	7.30	7.32	7.31	3.63	3.63	3.63	0.497

from all connections considerable errors may be introduced by reason of unsymmetrical electrostatic relation of the various parts of the circuit. For example,

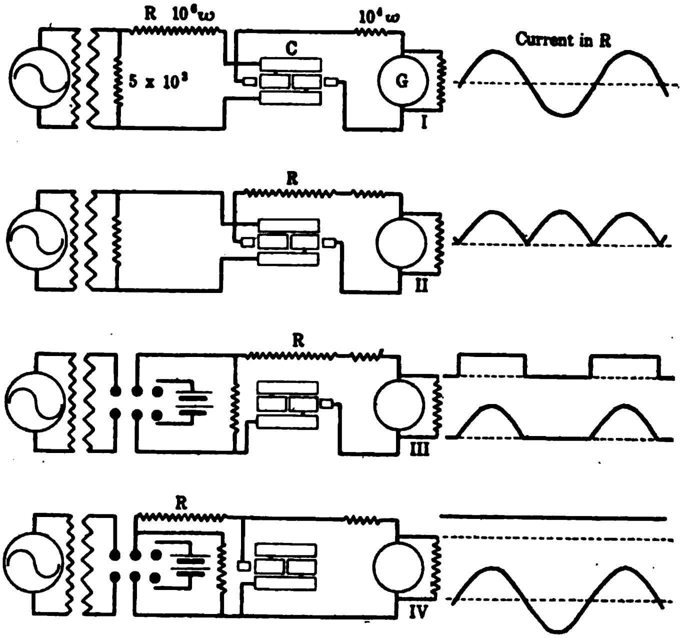

FIG. 14—SERIES AND SHUNT COMMUTATORS. COMPARISON OF VARIOUS TYPES OF PULSATING CURRENT IN SERIES RESISTANCE

TABLE V

Electrical angle degree	Deflection of Galvanometer											
	Connection I			Connection II			Connection III			Connection IV		
	left	right	mean	left	right	mean	left	right	mean	left	right	mean
-3	-.2	-.1	-.15	-.32	-3.	-.31	-.25	-.25	-.25	-.1	-.02	-.06
0	.32	.5	.41	.22	.28	.25	.02	.02	.02	.22	.28	.25
3	.88	1.1	.99	.78	.8	.79	.31	.31	.31	.48	.52	.5
9	1.99	2.1	2.05	1.8	1.85	1.83	.8	.82	.81	.98	1.08	1.03
15	3.0	3.12	3.06	2.88	2.9	2.89	1.32	1.38	1.35	1.5	1.6	1.55
30	5.37	5.5	5.44	5.21	5.28	5.25	2.5	2.58	2.54	2.7	2.78	2.74
45	7.37	7.52	7.45	7.22	7.32	7.27	3.51	3.59	3.55	3.67	3.78	3.73
60	8.8	9.1	8.95	8.8	8.9	8.35	4.31	4.39	4.35	4.4	4.5	4.45
75	9.7	9.98	9.84	9.69	9.81	9.75	4.8	4.88	4.84	4.88	5.00	4.89
81	9.9	10.08	9.99	9.82	10.0	9.91	4.9	4.95	4.93	4.92	5.1	5.01
87	9.98	10.1	10.04	9.92	10.07	10.0	4.95	5	4.98	4.95	5.1	5.03
90	9.98	10.1	10.04	9.95	10.05	10.0	4.97	5.01	4.99	4.95	5.1	5.03
93	9.95	10.07	10.01	9.9	10.07	10.04	4.93	5.	4.97	4.91	5.1	5.01
105	9.6	9.75	9.68	9.6	9.72	9.66	4.8	4.88	4.84	4.75	4.9	4.83
120	8.62	8.68	8.65	8.62	8.72	8.67	4.32	4.4	4.36	4.2	4.4	4.3
135	7.02	7.02	7.02	7.07	7.15	7.11	3.58	3.62	3.6	3.4	3.58	3.49
150	4.88	4.85	4.87	4.98	5.05	5.02	2.53	2.6	2.57	2.32	2.5	2.41
165	2.38	2.38	2.38	2.5	2.62	2.56	1.32	1.38	1.35	1.22	1.1	1.16
177	.28	.2	2.4	.38	.43	.41	2.7	.32	.30	.08	.18	.13
180	-.2	-.2	-.2	-.08	0	-.04	.09	.1	.10	-.2	-.09	-.15
183	-.8	-.97	-.89	-.7	-.68	-.69	-.3	-.2	-.25	-.5	-.4	-.45

introducing alternating e.m.f. in place of the battery in Fig. 11 for obtaining alternate half waves in the galvanometer, and then by a simple change of connections sending the complete rectified alternating current through G, the ratio of half wave to full wave deflections was found to be 0.511. When, however, the circuit was rearranged, as shown in Fig. 13, so as to divide the resistance R between the two halves of the circuit, the results of Table IV are obtained, showing that the ratio for half wave to full wave deflection of the galvanometer is very close to 0.5. In these circumstances capacity (up to one microfarad) may be shunted to r or R without affecting the ratio 0.5 of half wave to full wave. The position of the brushes was usually set for maximum deflection, although the method of setting for zero deflection and then shifting 90 electrical degrees was sometimes used.

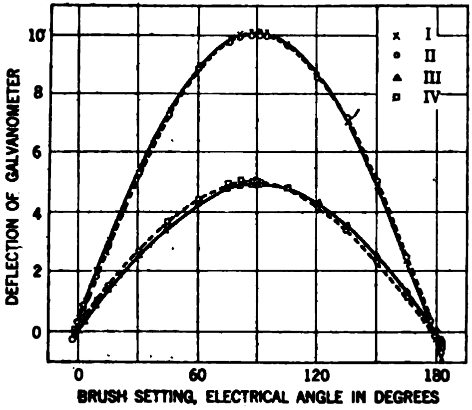

Fig. 15—Deflection of Galvanometer for Various Positions of Brushes, Connection Fig. 14. Capacity Eliminated as Far as Possible

For further study of the influence of capacity, the connections shown in Fig. 14 were used. Connection *I* gives complete rectification and normal alternating current through the resistance R. Connection *II* gives complete rectification with pulsating current through R. Connection *III* is the simple series connection for both R and the galvanometer and therefore has pulsating current in R with alternate half waves eliminated. Connection *IV* is ordinary shunt connection with normal alternating current in R. In all cases current is introduced into the circuit through a step-down transformer giving very low voltage in the galvanometer circuit.

Table V gives the readings taken with each of the methods of connections shown in Fig. 14, for various

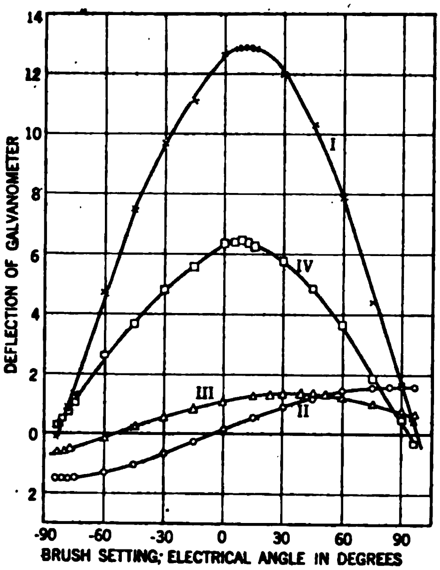

Fig. 16—Deflection of Galvanometer for Various Positions of Brushes, Connection Fig. 14. Resistance R Shunted by 0.02 Microfarad Capacity

positions of the commutator brushes. The results are plotted in Fig. 15. It is of interest to note that in this case in which capacity has been eliminated as far as possible, the ratio of the half wave to the full wave deflection is very closely equal to 0.5 for all positions of brushes, independent of the method of connection.

Fig. 16 shows the effect of shunting the resistance R with 0.02 microfarad. In this case the ratio of the maximum value for connection *IV* to the maximum value for connection *I* is 0.498, thus indicating the reliability of the shunt method of connection. On the other hand, the ratio of the maximum value for connection *III* to the maximum value for connection *I* is only 0.105, showing the unreliability of the series commutator connection when the circuit contains any capacity.

In order to compare the disturbance introduced in the series connection by the presence of capacity when using continuous and alternating currents, observations were taken with very small values of capacity by making the connections to *R* with two lengths of twisted lamp cord, introducing capacities of 0.002 and 0.006 microfarads respectively. The observations were taken at 60 cycles and the results are given in Table VI .The upper line gives the ratios of half wave to full wave for alternating current, as determined by connections *III* and *II*, respectively, and the lower line the ratio of running to standstill deflection using continuous current.

TABLE VI
INFLUENCE OF CAPACITY ON HALF WAVE READING

Capacity m.f.	0	0.002	0.006	0.02
A. C.	0.496	0.509	0.616	0.849
D. C.	0.51	0.602	0.741	0.907

It will be noticed, while that generally in the same direction, the disturbing effect of the capacity is less for alternating than for continuous current.

DISCUSSION

Small alternating currents may be measured by means of a synchronous commutator with a high sensitivity D'Arsonval galvanometer. The combination may be placed directly in the circuit, or perferably may be used in conjunction with high non-inductive series resistance to measure the voltage drop over a relatively low non-inductive resistance carrying the current to be measured.

The commutator may be connected in series with the galvanometer so as to open and close its circuit during alternating half waves, in shunt so as to short-circuit the galvanometer during alternate half waves, or it may be connected so as to completely rectify the alternating wave, thus making use of both half waves. In the two former methods of connection the commutator is spoken of as a suppressor.

In all methods of connection under steady conditions the galvanometer reads the average value of current passing through it in terms of its continuous current calibration. With either type of suppressor, since the instrument receives current during only one-half cycle, the value of the reading is multiplied by two in order to obtain the average value of the alternating current as based on the continuous current calibration.

Using the series suppressor, serious errors may arise due to relatively small values of capacity in the resistances and connections of the galvanometer and commutator circuit. Owing to the interruption of the circuit the capacity charges and discharges in such a way as to increase the instrument current during the closed half cycle and to raise it above zero value in the open half cycle. This source of error may be avoided if during the open half cycle the source of alternating current or electromotive force is kept closed by a second circuit equivalent to that of the galvanometer. This may be done by using the opposite segments of the commutator. (See Fig. 10).

The shunt suppressor is far more reliable and with careful elimination of capacity from the galvanometer circuit the errors may be reduced to negligible values.

If it is desired to use the series suppressor the circuit conditions may be tested by taking the ratio of the deflections due to the pulsating current, consisting of alternating half waves, and that due to the completely rectified alternating current by means of connections similar to those in Fig. 14. This ratio should be 0.5.

This test should be followed by one applying a continuous e.m.f. with the commutator connected as either a series or a shunt suppressor. The test lies in the ratio of the running to standstill deflections of the galvanometer. This ratio should be 0.5. For the simple series suppressor the value will be always higher than 0.5 unless an auxiliary circuit is used. With the shunt suppressor the value 0.5 may be very closely reached.

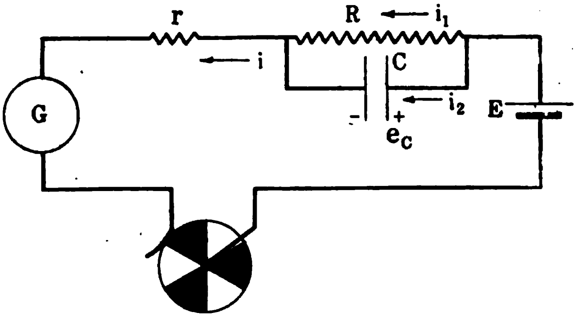

Fig. 17

In using rectifying vacuum tubes in place of a commutator, two tubes and two resistances must be placed in the alternating circuit. The voltage drop over either resistance may be used for measuring the current. In this case the e.m.f. applied to the galvanometer is the pulsating unidirectional e.m.f. due to alternate half waves. Since no commutator is used, the galvanometer circuit with its series resistance is closed at all times and the presence of capacity introduces no error.

Following is a summary of the conclusions from the tests:

(1) In the use of the synchronous commutator as a series suppressor, serious errors may arise due to relatively small amounts of capacity in the commutator and galvanometer circuits.

(2) Used as a shunt suppressor the commutator is far more reliable and this method of connection is always to be preferred.

(3) The galvanometer should be calibrated with continuous current with the commutator both at standstill and running as an ordinary make and break. If the ratio of these two readings is 2, the circuit conditions will introduce no error with alternating current.

(4) Several sources of error due to the commutator are eliminated by the use of vacuum tube rectifiers.

(5) A number of wave forms of alternating current of very low value are given.

APPENDIX

The following mathematical analysis of the case of the series suppressor as described in the paper gives results in close agreement with those observed.

I. Series suppressor in which the high resistance R is shunted by capacity C and continuous electromotive force E applied. No other capacity in circuit.

Referring to Fig. 17, let i, i_1 and i_2 be the instantaneous currents through galvanometer, R, and C (charging current) respectively, and e_c the instantaneous value of the voltage over C, and let their positive directions be those shown by arrows.

Consider the interval (half cycle) during which the circuit is closed by the commutator. Let us call this interval "closed half cycle". During this half cycle C is charged.

We have

$$i\,r + i_1 R = E \tag{1}$$

$$i_1 R = e_c = 1/C \int i_2\, dt \tag{2}$$

$$i_1 + i_2 = i \tag{3}$$

Eliminating i and i_2 from these equations we obtain

$$\frac{d\,i_1}{dt} + \frac{g}{C} i_1 = \frac{E}{C\,R\,r} \tag{4}$$

where $$g = \frac{R + r}{R\,r} = \frac{1}{R} + \frac{1}{r}$$

The solution of this differential equation is

$$i_1 = A\epsilon^{-\frac{g}{C}t} + \frac{E}{R + r} \tag{6}$$

where A is an integration constant to be determined by boundary conditions.

From (2) and (6)

$$e_c = A\,R\,\epsilon^{-\frac{s}{C}t} + E\,\frac{R}{R+r}$$

If we take the instant of the closing of the circuit as $t = 0$, $t = \frac{1}{f}$ at the instant of opening, f being the frequency, and the value of e_c at these two instants are respectively

$$|e_c|_{t=0} = A\,R + \frac{E}{R+r} \tag{7}$$

$$|e_c|_{t=\frac{1}{2f}} = A\,R\,\epsilon^{-\frac{s}{2fC}} + E\,\frac{R}{R+r} \tag{8}$$

Next consider the interval (half cycle) during which the circuit is open. Let us call this interval "Open half cycle."

During this half cycle the charge of C discharges through R and the equation of the circuit is

$$\frac{d\,e_c}{dt} + \frac{e_c}{C\,R} = 0 \tag{9}$$

the solution of this equation is

$$e_c = A'\,\epsilon^{-\frac{t}{CR}}$$

where A is an integration constant to be determined by boundary conditions.

If we take the instant of the opening as $t = 0$, the instant of the closing is $t = \frac{1}{2f}$ and the values of e_c at two instants are respectively

$$|e_c|_{t=0} = A' \tag{10}$$

$$|e_c|_{t=\frac{1}{2f}} = A'\,\epsilon^{-\frac{1}{2fCR}} \tag{11}$$

After the steady state of the circuit is established, the values of $|e_c|_{t=0}$ and $|e_c|_{t=\frac{1}{2f}}$ of the closed half cycle must be equal to the values of $|e_c|_{t=\frac{1}{2f}}$ and $|e_c|_{t=0}$ respectively of the closed half cycle, so that (10) and (11) must be identical with (8) and (7) respectively, *i. e.*,

$$A' = A\,R\,\epsilon^{-\frac{g}{2fC}} + E\,\frac{R}{R+r}$$

and $$A'\,\epsilon^{-\frac{1}{2fCR}} = A\,R + \frac{R}{R+r}R$$

Eliminating A' from these two equations, we have

$$A = -\frac{E}{R+r}\,\frac{1-\epsilon^{-\frac{1}{2fCR}}}{1-\epsilon^{-\frac{g+1/R}{2fC}}}$$

Putting this value of A into (6) and replacing g by $1/R + 1/r$ we have for the value of i_1 during the closed half cycle

$$i_1 = \frac{E}{R+r}\left\{1-\frac{1-\epsilon^{-\frac{1}{2fCR}}}{1-\epsilon^{-\frac{1}{2fC}(2/R+1/r)}}\,\epsilon^{-1/C\,(1/R+1/r)t}\right\} \tag{12}$$

From this equation and (1), we have for the closed half cycle

$$i = \frac{E}{R+r}\left[1+\frac{R}{r}\,\frac{1-\epsilon^{-\frac{1}{2fCR}}}{1-\epsilon^{-\frac{1}{2fC}(2/R+1/r)}}\right.$$
$$\left.\cdot\,\epsilon^{-1/C\,(1/R+1/r)\,t}\right] \tag{13}$$

If I be the value of steady current through galvanometer or the value of i when there is no interruption of circuit, then

$$I = \frac{E}{R+r}$$

Therefore the ratio of i and I or instantaneous current to steady current through galvanometer for any particular time t during the closed half cycle is

$$\frac{i}{I} = 1 + \frac{R}{r}\,\frac{1 - \epsilon^{-\frac{1}{2fCR}}}{1 - \epsilon^{-\frac{1}{2fC}(2/R + 1/r)}}\,\epsilon^{-1/C(1/R + 1/r)t}$$

$$= 1 + \alpha\,\frac{1 - \epsilon^{-\frac{1}{2fCR}}}{1 - \epsilon^{-\frac{1}{2fCR}(2+\alpha)}}\,\epsilon^{-\frac{1}{2fCR}(1+\alpha)2ft} \qquad (14)$$

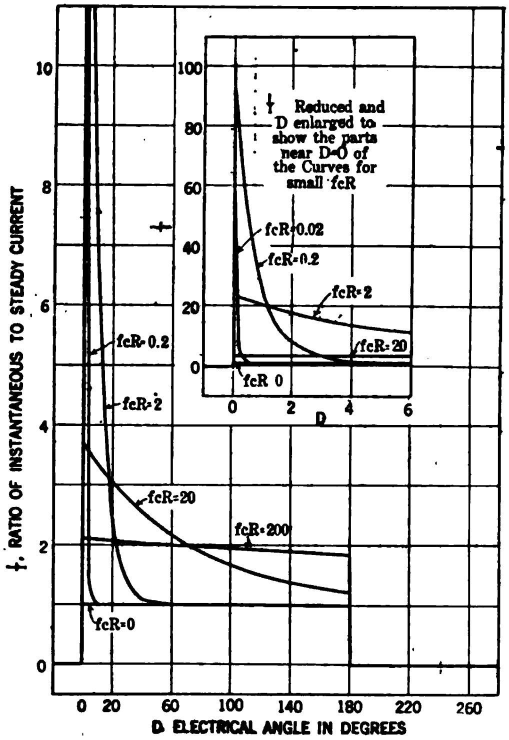

Fig. 18—Wave Form of Current through Galvanometer, R Shunted by C, Commutator in Series. Calculated by Equation (15) $\alpha = R/r = 100$.

where $\alpha = R/r$

If we denote the time by degrees of the electrical angle D,

$$t = \frac{1}{2f}\,\frac{D}{180}$$

Putting this value of t into (14) we have

$$\frac{i}{I} = 1 + \alpha \frac{1 - \epsilon^{-\frac{1}{2fCR}}}{1 - \epsilon^{-\frac{1}{2fCR}(2+\alpha)}} \epsilon^{-\frac{1}{2fCR}(1+\alpha)\frac{D}{180}} \qquad (15)$$

This equation gives the ratio of instantaneous to steady current through galvanometer for any particular position of brush during closed half cycle. The value of this ratio for the open half cycle is, of course, zero.

This equation shows that i/I for any particular value of D is function only of $f\,C\,R$ and α.

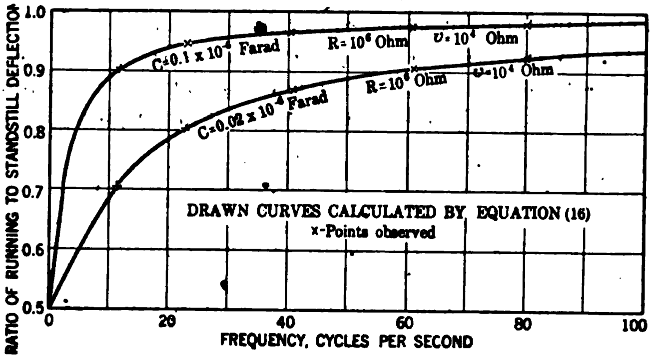

FIG. 19—CURVES BETWEEN FREQUENCY AND RATIO OF RUNNING TO STANDSTILL DEFLECTION. COMMUTATOR IN SERIES

In Fig. 18 curves between D and i/I computed from equation (15) for $f\,C\,R = 0.02, 0.2, 2, 20$ and 200 farad ohm sec.$^{-1}$ and $\alpha = 100$ are given. Corresponding curves as observed are given in Figs. 7 and 8.

The ratio of mean value of i during one cycle to steady current I, or the ratio of running to standstill deflection, S, is as follows:

$$S = \int_0^{1/2f} \frac{i\,dt}{I\,1/f}$$

$$= f \int_0^{1/2} \left[1 + \alpha \frac{1 - \epsilon^{-\frac{1}{2fCR}}}{1 - \epsilon^{-\frac{1}{2fCR}(2+\alpha)}} \epsilon^{-\frac{1}{2fCR}(1+\alpha)2ft} \right] dt$$

by (14)

$$\therefore S = \frac{1}{2}\,\frac{f\,C\,R}{1 + 1/\alpha}\,\frac{\left(1 - \epsilon^{-\frac{1}{2fCR}}\right)\left\{1 - \epsilon^{-\frac{1}{2fCR}(1+\alpha)}\right\}}{1 - \epsilon^{-\frac{1}{2fCR}(2+\alpha)}} \tag{16}$$

It is seen from equation (16) S is also a function only of $f\,C\,R$ and α.

Fig. 19 shows the close agreement between the observations and theory, the drawn curve shows the values calculated by formula (16) and the points marked are those taken from Fig. 4, the results of observation. The values of capacity are 0.02, and 0.2 microfarad which is so large that the capacity between commutator segments does not cause appreciable error.

II. A similar analysis shows that capacity between the leads to the commutator has very closely the same effect as capacity in shunt to R; see Figs. 4 and 5.

DISCUSSION ON "NOTES ON THE SYNCHRONOUS COMMUTATOR" (WHITEHEAD AND ISSHIKI) NEW YORK, N. Y., FEBRUARY 20, 1920.

P. G. Agnew: I have been reminded of an observation I made several years ago in connection with a curve tracer of the Rosa type. The device was quite similar to that of Prof. Whitehead, because it was always used as a mill instrument, and high capacity effects were found to be of importance. Unless the connections are correctly made, there are certain capacity effects induced which shift the whole curve, there is also a very small capacity effect, due to the brushes, as a commutator segment impinges a brush.

Another interesting illustration came up with Dr. Dorsey's work at the Bureau of Standards in the determination of the ratio of electrostatic unity to the importance of a very small capacity effect. He was using a spherical condenser, and of course a very small wire was used to establish connection inside of the sphere, but the effect in that small wire was slightly different. It had to be raised to make the connection; and the changing capacity, due to a small change in position, one millimeter, had to be taken account of and the numerical magnitude determined. It was a very troublesome job.

E. D. Doyle: Dr. Whitehead has pointed out the use of a commutator or suppressor for alternating current rather than alternating voltages. The work we have done at the laboratories has been rather different from this, in that we wanted to commutate low voltages, running down as low as 10 to 50 microvolts. We found that in such cases a commutator will not serve the purpose, in that for the lower values, the commutator seems to develop an infinite resistance. We have had to use butt contacts, rather than sliding contacts a synchronous motor rotating a cam which moves the contacts back and forth thereby effecting the reversals as required.

A. E. Kennelly: In the continuous current circuit, the current becomes steady after a certain time, usually a very short time, and then as a rule no capacitance effect is observed. With an a-c. circuit we all know that capacitance effects reveal themselves in a number of ways and must be taken into account, and if we do not take them into account, we get results which are ridiculous when interpreted, unless we make proper allowances.

This paper calls our attention to the fact that in the d-c. circuit when periodically opened and closed, the capacitance effect may be much more serious than we are apt to realize at first sight. In this case, the difficulties were overcome by reducing the circuit to the continuous-current type.

J. H. Morecroft: I think of one point in respect to Dr. Agnew's paper which is worth while emphasizing and is perceived at once, if one calculates the amount of power necessary to actuate this vibration galvanometer sufficiently to give a reading. If one takes Dr. Agnew's data, he will find that this instrument gives, at the frequency mentioned in the paper, (that is to say, 25 and 60 cycles) a readable deflection with 10^{-12} watt, one millionth of one millionth of a watt. Evidently this gives the experimenter an instrument, which, at the frequency noted, is comparably better than any other type of instrument available.

I have used the vibration galvanometer a great deal in laboratory work, especially at the low frequencies where the telephone was not usable, and have encountered the difficulty Dr. Sharp mentioned—every time a car runs up and down the street outside the laboratory, the galvanometer goes off the scale, and we have to wait until the cars stop running before we can make any determinations. That was with a very sensitive type of vibration galvanometer recently described by Dr. Wenner. I think this galvanometer described by Dr. Agnew is a very wonderful device and I hope soon to have one. There is one point which struck me both in Dr. Whitehead's paper and in his discussion of Dr. Agnew's paper, and that is the use of the vacuum tube in rectifying current in place of a commutator.

I think Dr. Whitehead is aware of the fact—he did not point it out—that an instrument of the type Dr. Agnew tells us about would give a deflection varying with the first power of the impressed voltage, that is, it gives a scheme of sensibility, which does not fall off too rapidly as the voltage goes down. If one tries to use the two electrode vacuum tube in place of the commutator, one will find at once that it has a very bad quality. The amount of rectified current which one can get with the tube varies as the square of the impressed voltage, and it will be appreciated that as the voltage goes down, the rectified part of the current, (which is the only thing the direct-current voltmeter has cognizance of) falls off more rapidly; hence, anyone trying to use a vacuum tube in place of the commutator should have that point in mind, that the sensitiveness of these rectifying tubes falls off very rapidly as the impressed voltage decreases.

This difficulty can be remedied to some extent, by impressing a polarizing voltage on the tube, so that there is a large direct current flowing through the tube all the time, but, of course, one must not have that current flowing through the galvanometer being used as detector, and there must be a balanced circuit put in, otherwise the device is no good, and that makes it somewhat difficult to use the vacuum tube, as a sensitive rectifier. It sounds nice to use a vacuum tube, having no moving contacts—it is an excellent thing to get away from moving contacts—still the vacuum tube does labor under some difficulties.

In our laboratory we noticed also an effect which one of the members, discussing Dr. Whitehead's paper, brought up, that has to do with the use of commutators on very low-voltage circuits. It seems that a contact having very few ohms resistance for ordinary currents may possibly open the circuit for a very low voltage. In the case of sliding metallic contacts in circuits excited perhaps with micro-volts, the contact actually seems to open. Thus, there is a limiting voltage beyond which the contact does show a very high resistance, and is comparatively useless. It will also be found that the rubbing of the two metals (brush and commutator) will itself give more than a micro-volt and hence, unless one is very careful the errors involved due to the rubbing of one metal against another are likely to be greater than the thing being measured.

J. B. Whitehead: I thank Prof. Morecroft for emphasizing the point I meant to bring out when I mentioned the use of the Fleming valve as a rectifier, namely, the difference in sensitivities, the vibration galvanometer having an evident advantage there, for the reasons which Prof. Morecroft has mentioned. There are however, many cases in which extreme sensitivity is not necessary, and in these cases the vacuum tube appears to have many advantages.

In connection with the second point that Prof. Morecroft has raised, namely, the difficulties of the commutator at low values of voltage, we encountered them to some extent, in taking our wave forms, and we have described in one of the paragraphs in connection with Fig. 6, just how the trouble, as we found it, was taken care of by taking rapid reverse readings, with commutator running on closed circuit, and on open circuit. It is necessary to take immediately successive observations in order to correct for conditions at the point of contact of the brush with the commutator.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 20, 1920.

OSCILLOGRAPHS AND THEIR TESTS

BY A. E. KENNELLY, R. N. HUNTER AND A. A. PRIOR

ABSTRACT OF PAPER

A method and technique for the testing and calibration of oscillographs is described, using an auxiliary vibrator or "oscillographmeter" for the production of Lissajous optical figures, whereby the resonant frequency f_0 of the tested oscillograph may be readily ascertained. From this and one other test, which is preferably a comparative calibration at 60 ~ and at the resonant frequency, the bluntness of resonance B of the oscillograph is determined.

From the two essential constants f_0 and B of an oscillograph, its indications at any assigned frequency can be corrected for the inertia of its vibratory system. At high frequencies, the correction may be relatively large.

A number of oscillographs have been tested for their f_0 and B. The principal results obtained are reported in the paper.

ALTHOUGH oscillographs have come into very general electrical engineering use, and are invaluable in many laboratory investigations, no methods for testing them seem to have been published. It is important to develop methods for the testing of oscillographs, because all those oscillographs which employ mechanical vibratory systems behave differently to alternating currents of different frequencies. If calibrated at say 60 ~, their calibration at other frequencies, and especially at high harmonic frequencies, will be different. If an oscillographic waveform is analyzed into its Fourier components, the apparent value of each component requires to be corrected, both as to amplitude and as to phase. In some cases, these corrections may be very large; while in others they may be insignificant; but in all cases the oscillographer should know, and be able to determine, the magnitude of the correction to be applied to the terms of his Fourier analysis, if only for the purpose of assuring himself that these corrections are too small to be worth taking into account, for those purposes to which his oscillographic records are to be applied. Strictly speaking, no oscillogram is complete, as a technical record, without an appended index of correction. It has already been pointed

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*Bibliography 7, 12.

out that these corrections are due*; but no technique has been published, so far as the authors are aware, for determining the range of frequency errors in any particular instrument. It is the object of this paper to supply a technique for the testing of oscillographs, as the outcome of laboratory researches conducted to that end for the last two years.

Before discussing the technique of oscillograph testing, it is desirable to consider certain preliminary and underlying principles relating to the operation of the mechanically vibrating oscillograph. These principles are by no means new†; but the forms in which they are here presented are believed to be distinctive, and relatively easy for the student to apprehend. They depend upon a close analogy between the mechanics of a simple vibratory system and the electromagnetics of a simple alternating-current system.

Vector Current Impedance of an L. R. S. Branch. Fig. 1 represents a simple branch circuit *A B*, between a pair of alternating-current mains *m m′*, which are maintained at a constant r.m.s. voltage, by means of voltmeter *V*. The branch contains a condenser of capacitance *C* farads, a resistance of *R* ohms and an inductance of *L* henrys. Any resistance associated with the inductance is supposed to be displaced into, and segregated with, the resistance *R*. The values of *C*, *L* and *R*, are assumed to remain fixed and unchanged.

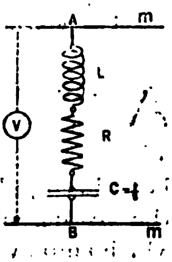

FIG. 1—SINGLE *LRS* BRANCH CIRCUIT

The a-c. generator connected to and supplying the mains *m m′*, is supposed to be able to supply, at will, any desired frequency, from the lowest to the highest, without altering the r.m.s. e.m.f. *E* volts. This e.m.f., while being varied in frequency from nearly zero to a very high value, remains simply sinusoidal throughout. In other words, the impressed e.m.f. is supposed to be pure, and devoid of harmonics. Then, at any impressed frequency *f* cycles per second, and impressed angular velocity $\omega = 2\pi f$ radians per second, the impedance of the branch circuit will be

$$Z = R + j\left(L\,\omega - \frac{1}{C\,\omega}\right) \qquad \text{ohms} \angle \quad (1)$$

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†Bibliography 2, 4, 5, 7a, 8, 12.

or, if $S = 1/C$, is the elastance of the condenser in darafs, this becomes

$$Z = R + j\,(L\,\omega - S/\omega) \qquad \text{ohms} \angle \quad (2)$$

the graph of this impedance, as is well known, is the straight line $A\,B\,C\,D\,E\,F$, Fig. 2, in the impedance plane, parallel to the Y axis, and distant R ohms therefrom. Commencing with frequency zero, the impedance $j\,L\,\omega$ of the reactor is indefinitely small, but the impedance $-\,j\,S/\omega$ of the condenser is infinite; so that the impedance Z then has the size of infinity, in the direction of $B\,A$ produced, and with a slope α of $-$ 90 deg. As the impressed angular velocity is increased, both the size and the slope of Z diminish; until, at a certain frequency, which may be denoted by ω_1, the size of the reactance $(L\,\omega - S/\omega)$ is negatively just equal to the size of the resistance R. The impedance will then be $O\,B$ Fig. 2, or

$$Z_1 = R - j\,R = R\,\sqrt{2} \diagdown 45 \qquad \text{ohms} \angle \quad (3)$$

the slope α being then $-$ 45.

Increasing the angular velocity up to the resonant value, which may be denoted by ω_0, the reactance $L\,\omega - S/\omega = 0$, and the impedance becomes the real value $O\,D = R$ ohms, of slope 0 deg.

Increasing the angular velocity up to a certain value ω_2, when the reactance has a size that is positive and just equal to R, the impedance reaches the vector value $O\,E$, and is:

$$Z_2 = R + j\,R \doteq R\,\sqrt{2} \angle 45. \qquad \text{ohms} \angle \quad (4)$$

The values of angular velocity ω_1 and ω_2, may be called the lower and upper *quadrantal angular velocities*, respectively. Similarly, Z_1 and Z_2, corresponding thereto, are the *quadrantal impedances*.

Finally, if the impressed angular velocity is increased towards infinity, the reactance of the condenser C becomes indefinitely small; but that of the inductance L indefinitely great; so that the impedance of the $C\,L\,R$ branch approaches infinity in size, and + 90 deg. in slope, and lies in the vector direction from O, towards $E\,F$ indefinitely produced.

The impedance diagram of Fig. 2 has been prepared for the case in which $R = 100$ ohms, $C = 0.5 \times 10^{-6}$ farad and $L = 5 \times 10^{-3}$ henry. Each unit on the scale of the diagram then represents 100 ohms. The resonant angular velocity ω_0 is thus 20,000 radians per second, corresponding to a frequency of $f_0 = 3183 \sim$. If we denote by u, the ratio of the impressed to resonant frequency, or

$$u = \omega / \omega_0 = f / f_0 \qquad \text{numeric} \qquad (5)$$

the small circles along the impedance locus, Fig. 2, are marked with their corresponding values of u, for this particular case, from $u = 0.2$ or $\omega = 4000$, to $u = 5$ or $\omega = 100{,}000$.

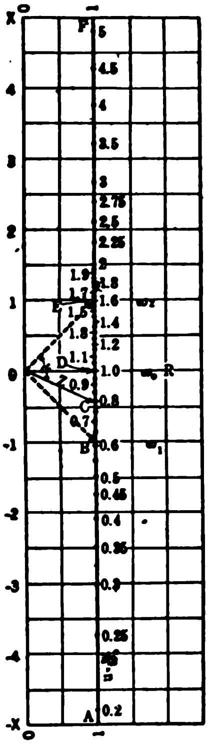

FIG. 2—RECTILINEAR GRAPH OF IMPEDANCE TO CURRENT FOR CASES OF $\left\{\begin{matrix} R = 10^{1}. & L = 5 \times 10^{-3}, & S = 2 \times 10^{6} \\ r = 10^{-2}, & m = 5 \times 10^{-7}, & s = 2 \times 10^{2} \end{matrix}\right\}$ $w_0 = 20{,}000$ $\Delta = 10{,}000$ $B = 0.5$

IN THE ELECTRIC CASE, UNIT LENGTH = 10^2 OHMS.

IN THE MECHANIC CASE, UNIT LENGTH = 10^{-2} MECHANIC ABSOHMS, OR DYNES-PERP. CM. PER RADIAN PER SEC.

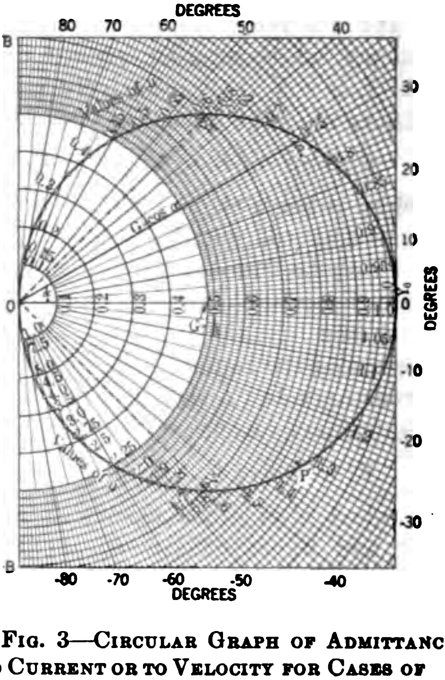

FIG. 3—CIRCULAR GRAPH OF ADMITTANCE TO CURRENT OR TO VELOCITY FOR CASES OF

$$\left\{\begin{matrix} R = 10^{2}, & L = 5 \times 10^{-3}, & S = 2 \times 10^{6} \\ r = 10^{-2}, & m = 5 \times 10^{-7}, & s = 2 \times 10^{2} \end{matrix}\right\}$$

$w_0 = 20{,}000$ $\Delta = 10{,}000$ $B = 0.5$

IN THE ELECTRIC CASE, UNIT LENGTH REPRESENTS 10^{-2} MHO.

IN THE MECHANIC CASE, UNIT LENGTH REPRESENTS 10^{2} MECHANIC ABMHO OR RADIANS PER SEC. PER UNIT V.M.T.

The lower quadrantal value of impedance $O\,B$, occurs at $u = 0.618$, or $\omega_1 = 12{,}361$ radians per second. The upper value $O\,E$, occurs at $u = 1.618$, or $\omega_2 = 32{,}361$. The range between these two quadrantal values of impressed angular velocity may be described, for convenience as the quadrantal range.

As will be subsequently discussed, the quadrantal range of u in each case corresponds to the sharpness of resonance. In this case, the quadrantal range of u is from 0.618 to 1.618, or amounts to 1.0.

If we form the graph of the admittance Y; of the branch $L\,R\,S$, by forming the successive planevector reciprocals of the impedance Z, we obtain the circle $O\,Y_1\,Y_0\,Y_2$, Fig. 3, around which advance is made clockwise, as the impressed frequency is increased. $O\,Y_1$ and $O\,Y_2$, are the quadrantal admittances at the lower and upper quadrantal angular velocities ω_1 and ω_2 respectively. Any point in the Z graph, Fig. 2, characterized by the slope $-\ \alpha$ deg., of the planevector $O\,P$, corresponds to a point P, in the Y graph Fig. 3, having for its planevector $O\,P$ the slope $+\ \alpha$ deg, and the size $G \cos \alpha$, where $G\ (= 1/R)$, is the conductance of R in mhos.

It is easy to show that the two quadrantal angular velocities ω_1 and ω_2, whose vector admittance slopes are 45 deg. plus and minus respectively, have, as their geometrical mean, the resonant angular velocity, or

$$\sqrt{\omega_1\,\omega_2} = \omega_0 \qquad \frac{\text{radians}}{\text{sec.}} \qquad (6)$$

Moreover, this relation is not confined to the quadrantal pair of angular velocities. Any pair ω_1' and ω_2', characterizing the vectors $O\,P$ and $O\,P'$ in Fig. 3, whose slopes are respectively $+\ \alpha$ deg. and $-\ \alpha$ deg., are subject to the relation expressed in (6), or have ω_0 as their geometric mean.

Vector Maximum Cyclic Current in a C L R Branch. It will be evident from a consideration of the vector admittance graph Fig. 3, that if a maximum cyclic constant voltage E is impressed, at standard phase, on the $C\,L\,R$ branch, at varied frequency, the vector graph of maximum cyclic current in the branch will be a circle, similar to that of Fig. 3, since

$$I = E\,Y \qquad \text{max. cyc. amperes} \angle \qquad (7)$$

where I is the maximum cyclic vector current. In fact, Fig. 3 may then be interpreted as a vector current graph, if the scale of the diagram be properly chosen. Consequently, as the frequency is increased from zero to infinity, the maximum cyclic current changes from 0 ∠ 90 deg. to 0 ⦢ 90 deg. amperes, through the resonant value $E\ G$ ∠ 0 deg.

Corresponding Mechanical Vibrating System. Fig. 4 represents the elements of a simple bifilar, oscillographic vibrator, in which two similar parallel fine wires or strips, $A\ B$ and $C\ D$, carry the same alternating current in a uniform powerful magnetic field. The direction of the field lies substantially in the plane $A\ B\ D\ C$. At the center of the rectangle $A\ B\ D\ C$, the small mirror m is fastened symmetrically. The two wires are supposed to be equally stressed in tension. The vibrator is immersed in a damping fluid, such as air, oil, or glycerine.

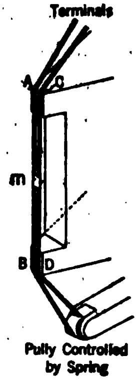

FIG. 4—OSCILLOGRAPH VIBRATOR

When a maximum cyclic current I absamperes passes through the vibrator, it produces a maximum cyclic vibromotive torque on the mirror

$$F = A\,I \quad \text{max. cyc. dyne-perp.-cm.} \angle \qquad (8)$$

This is a planevector alternating torque, proportional to I, and measurable in dynes acting perpendicularly to a radius arm of 1 cm., supposed to be erected perpendicularly to the plane of the mirror. The coefficient A is a real numeric, depending on the strength of the magnetic field, the length and the tension of the vibrator wires, etc. The phase of the torque will be the same as the phase of the current, if the wires are of non-magnetic material. The angular displacements produced in the vibrator mirror are assumed to be small; *i. e.*, of the order of a few degrees only.

The alternating vibromotive torque F (abbreviated v.m.t.), acting on the vibrator of Fig. 4, corresponds to the alternating e.m.f. E, acting on the $C\ L\ R$ branch of Fig. 1. F tends to produce an alternating angular velocity of the mirror, just as E tends to produce an alternating current in the $C\ L\ R$ branch. Just as E is opposed by an electric impedance Z, expressed as in (2), so F, is opposed by a mechanical impedance z, expressed by a similar equation

$$z = r + j\,(m\,\omega - s/\omega) \qquad \frac{\text{dynes perp. cm.}}{\text{radians per sec.}} \angle \quad (9)$$

Here r is the frictional resistance of the vibrator $A\ B\ C\ D$, Fig. 4, to alternating angular velocity, which causes energy to be dissipated as heat in the damping field. The quantity $(m\,\omega - s/\omega)$ is that mechanic resistance of the vibrator to alternating angular velocity, which causes energy to be shifted cyclically from the resilient wires to the inertia of the vibrating system, and which, therefore, occupies mechanical power in cyclic storage and release, but without dissipation. The quantity m corresponds to the inductance L of the branch circuit, and is the moment of inertia of the vibrating system, expressible in gm – cm², or what is equivalent thereto, in $\dfrac{\text{dynes-perp. cm.}}{(\text{radians per sec.})^2}$. The quantity s corresponds to the elastance S of the branch circuit, and is the elastic stiffness, or resistance to angular displacement, of the vibrator, expressible in dynes-perp.-cm. per radian. The complex quantity Z, is thus a mechanic impedance, analogous to electric resistance, and capable of being expressed in mechanic absohms $\angle$.

Vector Angular-Velocity Impedance. In view of the corresponding relations between an electric branch circuit, and a mechanic vibrator, the impedance graph of Fig. 2 may be regarded as representing the graph of impedance to angular velocity of the vibrator mirror in Fig. 4. At a very low impressed angular velocity, this impedance z commences at $\infty \measuredangle 90$ deg. mechanic absohms. At a very high impressed ω, z ends at $\infty \angle 90$ deg. mechanic absohms. At resonant angular velocity, ω_0, z passes through the value $r \angle 0$ deg.

Vector Angular-Velocity Admittance. In view of the same electric-mechanic analogy, the circular vector admittance graph to alternating current, Fig. 3, may also be regarded as a vector admittance graph to alternating torque. As the angular velocity impressed on the vibrator increases from zero to infinity, the admittance $y = 1/z$, varies from $0 \angle 90$ deg., through $g \angle 0$ deg., to $0 \measuredangle 90$ deg. mechanic abmhos, where $g = 1/r$, is the "mechanic conductance" of the system, expressible also in $\dfrac{\text{radians per sec.}}{(\text{dynes perp. cm.})}$.

The circular admittance graph of Fig. 3, corresponds not only to Fig. 2, by inversion; but also to either of the following cases: Electrically, $R = 100$ ohms, $G = 10^{-2}$ mho, $L = 5 \times 10^{-3}$ henry, $S = 2 \times 10^6$ darafs.

Mechanically, $r = 10^{-2} \dfrac{\text{dynes perp. cm.}}{\text{radian/sec.}}$,

$$g = 100 \frac{\text{radian/sec.}}{\text{dyne perp. cm.}}, \; m = 5 \times 10^{-7} \text{ gm.} - \text{cm.}^2,$$

$$s = 2 \times 10^2 \quad \frac{\text{dyne perp. cm.}}{\text{radian}}$$

In either case, the resonant angular velocity $\omega_0 = 20{,}000$ radians/sec.

In the electric case, unit size on the graph corresponds to 10^{-2} mho.

In the mechanic case, unit size corresponds to

$$10^2 \frac{\text{radian/sec.}}{\text{dyne perp. cm.}}.$$

The resonant range in u is from $u_1 = 0.618$, through 1.0 to $u_2 = 1.618$.

The size of the resonant admittance is G mhos electrically; or $g \dfrac{\text{radians/sec.}}{\text{dyne perp. cm.}}$ mechanically; or what may be described as "mechanic abmhos", since the mechanic units are all referred, for convenience, to the C. G. S. system.

The size in Fig. 3 of the admittance Y or y, at any slope α deg., is $G \cos \alpha$ electrically, or $g \cos \alpha$ mechanically.

We may express the maximum cyclic angular displacement of the vibrator mirror by Θ radians $\angle$. The instantaneous angular velocity will then be $\dfrac{d\theta}{dt}$ or $\dot{\theta}$ radians per sec. and the maximum cyclic value of the mirror's angular velocity may then be denoted by $\dfrac{d\Theta}{dt}$, or $\dot{\Theta}$ radians per sec. Here $\dot{\Theta}$ corresponds to I.

Folliowng the same analogy with reference to (6),

$$\dot{\Theta} = F y \qquad \text{max. cyc. radians per sec.} \angle \quad (9)$$

where the v.m.t. F is taken as of standard phase, or zero slope. Consequently, Fig. 3 may be interpreted as the graph of angular velocity produced by an exciting current I max. cy. absamperes in the vibrator, at varied angular velocity, the scale of the diagram being suitably chosen. As ω varies from zero through ω_1, ω_0, and ω_2, to infinity, $\dot{\Theta}$ varies from $0 \angle 90$ deg., through $F y_1 \angle 45$ deg., $F g \angle 0$ deg., $F y_2$ ⦢ 45 deg. to 0 ⦢ 90 deg., radians per sec. ∠. This means that at the angular velocity of resonance, the angular velocity of the mirror is greatest, and is in phase with the exciting alternating current. At a very low frequency, $\dot{\theta}$ is insignificantly small, but in leading quadrature to the exciting current. At a very high frequency, $\dot{\theta}$ is again very small, but in lagging quadrature to the exciting current. At the quadrantal frequencies f_1 and f_2, the size of the angular velocity will be 0.707, or $2^{-1/2}$ of that at resonance, and will be in semiquadrature with the exciting current.

Thus in Fig. 3, interpreted electrically at 10^{-2} mho per unit of length, with impressed frequency $\omega = 10{,}000$ radians per second, or $u = 0.5$, $Y = 0.555 \times 10^{-2} \angle 56.3$ deg.; so that 1 volt max cy. e.m.f at this frequency and standard phase applied to the $C\,L\,R$ circuit, of the constants chosen above, would establish a current of $0.555 \times 10^{-2} \angle 56.3$ deg. max. cy. ampere, or 5.55 max. cy. milliamperes, leading the e.m.f by 56.3 deg. If, however, the mechanic application of Fig. 3 is considered, with 100 mechanic abmhos to the unit of length, one max. cy dyne-perp-cm. applied to the vibrator, at $\omega = 10{,}000$ or $u = 0.5$, would establish a max. cy. angular velocity of $55.5 \angle 56.3$ deg. radians per sec. As the mirror crossed the zero point on its scale, it would be moving with the angular velocity of 55.5 radians per second, or 3180 degrees per second, and this zero would be crossed 56.3 deg. ahead in phase of the impressed v.m.t.

In the electric system, resonance occurs at

$$\omega_0 = \frac{1}{\sqrt{C\,L}} = \sqrt{S/L} \qquad \frac{\text{radians}}{\text{sec}} \tag{11}$$

In the mechanic system, resonance correspondingly occurs at

$$\omega_0 = \sqrt{s/m} \qquad \frac{\text{radians}}{\text{sec}} \tag{12}$$

Damping Constant of Oscillating Systems, Electric and Mechanic. If we energize either the condenser or the reactor or both these elements in Fig. 1, and removing, from the mains, the branch circuit $C\,L\,R$, close it on itself, the damping factor of its current, assuming no loss of power in either S or L is known to be

$$\Delta = \frac{R}{2\,L} \qquad \text{hyps per sec.} \tag{13}$$

This may be regarded as the hyperbolic angular velocity of decay in the circuit, expressible in hyperbolic radians per second. Similarly, if we energize either the wires, or the mass of the vibrator, or both these elements in Fig. 4, the damping factor of the angular velocity, assuming no loss of power in either s or m, is:

$$\Delta = \frac{r}{2\,m} \qquad \text{hyps per sec.} \qquad (14)$$

This is also the hyperbolic angular velocity of decay in vibration. This quantity Δ is very important in oscillographic theory. The reciprocal of Δ may be called the *oscillatory time constant* of either system, or

$$\tau_o = 1/\Delta = \frac{L}{(R/2)} = \frac{m}{(r/2)} \qquad \frac{\text{seconds}}{\text{hyp. radian}} \qquad (15)$$

This is equal to the time in which free oscillations fall to $1/\epsilon$ th in size. The damping factor may also be obtained from the quadrantal angular velocities, by the relation

$$\Delta = \frac{\omega_2 - \omega_1}{2} = \frac{f_2 - f_1}{\pi} \qquad \frac{\text{hyp}}{\text{sec.}} \qquad (16)$$

or from any angular velocity ω, at which the slope of the vector admittance is α deg., by the more general relation:

$$\Delta = \frac{\omega^2 - \omega_o^2}{2\,\omega \tan \alpha} \qquad \frac{\text{hyps.}}{\text{sec.}} \qquad (17)$$

In the case represented by Figs. 2 and 3, $\Delta = 10{,}000$ hyps. per sec.

Sharpness and Bluntness of Resonance. The sharpness of resonance of a circuit, or of a vibrator, may be defined as the ratio of the resonant angular velocity to the damping factor, or

$$\Lambda = \omega_o/\Delta = \frac{\omega_o}{\left(\dfrac{\omega_2 - \omega_1}{2}\right)} = \frac{2\,\omega_o\,\omega_1}{\omega_o^2 - \omega_1^2} = \frac{2\,\omega_o\,\omega_2}{\omega_2^2 - \omega_o^2}$$

$$= \frac{f_o}{\left(\dfrac{f_2 - f_1}{2}\right)} = \frac{2\,\pi f_o}{\Delta} = \frac{m\,\omega_o}{(r/2)} = \frac{s}{(r/2)\,\omega_o}$$

$$= \frac{\sqrt{m\,s}}{r/2} = \frac{\sqrt{L/C}}{R/2} \qquad \text{numeric} \qquad (18)$$

Very sharp resonance thus implies a large ratio of ω_o to the semi-quadrantal range $\frac{(\omega_2 - \omega_1)}{2}$. Resonance sharpnesses in designedly resonant electric circuits are commonly of the first order, (near 10^1) and occasionally exceed the second order (10^2). In vibrators, the magnitudes encountered are of similar orders. In air-damped vibration galvanometers, a resonant sharpness as high as the third order, (1000) may be met.

The reciprocal of the sharpness of resonance may be called B, the bluntness of resonance

$$B = 1/\Lambda = \Delta/\omega_0 = \frac{\left(\frac{\omega_2 - \omega_1}{2}\right)}{\omega_0} = \frac{\omega_o^2 - \omega_1^2}{2\,\omega_o\,\omega_1}$$

$$= \frac{\omega_2^2 - \omega_o^2}{2\,\omega_o\,\omega_2} = \frac{\left(\frac{f_2 - f_1}{2}\right)}{f_o} = \frac{\Delta}{2\,\pi\,f_o} = \frac{(r/2)}{m\,\omega_o}$$

$$= \frac{(r/2)\,\omega_o}{s} = \frac{(r/2)}{\sqrt{m\,s}} = \frac{R/2}{\sqrt{L/C}} = \frac{u_2 - u_1}{2} \quad \text{numeric} \qquad (19)$$

When the bluntness of a $C\,L\,R$ circuit, or of a vibrator, attains the value unity, the system is critically damped, or is just aperiodic. If it exceeds unity, the system is overdamped, or ultraperiodic. If the bluntness is less than unity, or the sharpness greater than unity, free oscillations can occur. With high sharpness, numerous oscillations accompany oscillatory decay. Air-damped oscillographic vibrators commonly have a sharpness Λ of 100 or more, or a bluntness B of about 0.010 or less. Oil damped vibrators may easily have a bluntness B greater than unity. These numerical coefficients Λ and B are important in oscillographic theory.*

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*The resonance sharpness thus defined is double that used in some publications. See Bibliography 11, 13, 17. As defined in (18), it may be distinquished as Λ_0, and has certain algebraic advantages over the lesser value $\Lambda_s = \frac{\omega_0}{2\,\Delta}$. In this paper Λ stands for Λ_0, and its reciprocal B for B_0. Λ_0 may be regarded as an oscillatory sharpness, depending on the oscillatory time constant $\tau_0 = \frac{2\,L}{R}$ seconds, and as distinguished from the sharpness of sustained alternation Λ_s, depending on the time constant of continuous-current application $\tau_s = L/R$ seconds. Hence $B_s = 2\,B_0$, $\Lambda_s = \Lambda_0/2$; or $B_0 = B_s/2$ and $\Lambda_0 = 2\,\Lambda_s$.

In the case represented in Figs. 2 and 3, $\Lambda = 2$, and $B = \frac{1}{2}$. This means, therefore, that in the case of those figures, the resistance to either electrical or mechanical motion is just half of that which would render the system aperiodic, or critically damped.

Rapidity of Change in Phase Angle α, of I or $\dot{\theta}$, with respect to ω. In sharply resonant systems, ($\Lambda > 1$), a small change in the impressed angular velocity will involve a rapid change in the phase angle α of current I, or angular velocity $\dot{\theta}$, Fig. 3, near resonance. In cases of very sharp resonance, or small damping, a change of a few radians per second in ω, near to ω_0, may carry the vector I, or $\dot{\theta}$, around the quadrantal range in slope (from $\alpha = +45$ deg. to $\alpha = -45$ deg.). On the other hand, in bluntly resonant systems ($B > 1$), it may require a relatively large change in ω to carry the vector I or $\dot{\theta}$ over the same range. In the neighborhood of the resonant angular velocity ω_0, we may notice that

$$\frac{d\,\alpha}{d\,\omega} = -1/\Delta = -\tau_0 \qquad \text{seconds} \quad (\mathbf{20})$$

so that the phase α changes near resonance by $-\tau_0$ radians per single radian per second in ω. By (**14**), this change is rapid in a system of large L and small R, or of large m and small r. With a large frictional resistance r, this rate of change in α becomes small.

Again, the quadrantal range in α is from α_1 to α_2, or -90 deg. or $-\pi/2$ radians (see Fig. 3). The corresponding change in angular velocity is $\omega_2 - \omega_1 = 2\,\Delta = 2/\tau_0$. Consequently, the average rate of change in α with respect to ω, over the whole quandrantal range is

$$\frac{\alpha_2 - \alpha_1}{\omega_2 - \omega_1} = \frac{-\pi/2}{(2/\tau_0)} = -(\pi/4)\,.\,\tau_0 = -0.7854\,\tau_0 \qquad \text{seconds} \quad (\mathbf{21})$$

or the average rate of change in α with respect to ω is 78.5 per cent, over the whole resonant range, of what it is at resonance, where the rate is a maximum. Above and below the quadrantal frequencies, at which the reactive factor

$$\frac{m\,\omega - s/\omega}{r} = \pm 1,$$

the rate of change in α with change in ω diminishes. At large reactive factors, it diminishes approximately as the inverse square of the reactive factor.

Impedance to Displacement. The foregoing discussion relates to the evaluation of the maximum cyclic current I in a $C\,L\,R$ branch, or to the maximum cyclic angular velocity $\dot{\theta}$ of a vibrator mirror. This maximum cyclic angular velocity occurs at the moment when the mirror is passing through its zero or central position. The oscillographer is only indirectly interested, however, in the angular velocity of his mirror. He is directly interested in the angular displacements, or elongations, of his mirror. In the electric-circuit analogy, the mirror displacement corresponds to the alternating displacement of electricity, or electric quantity, Q coulombs $\angle$. The quantity Q is the time integral of the current I. In the assumed condition of a simple harmonic or purely sinusoidal current, the quantity Q is also sinusoidal. It is in direct proportion both as to magnitude and phase with the voltage across the condenser, or is in lagging quadrature with the current. We are therefore led to study the impedance to displacement or "displacement impedance" of a branch or a vibrator, in order to evaluate θ, the maximum cyclic angular displacement of a vibrator mirror.

It is easily shown that the displacement impedance Z', for the simple alternating electric circuit, is

$$Z' = j\,\omega\,\{R + j\,(L\,\omega - S/\omega)\} = (S - L\,\omega^2) + j\,R\,\omega \qquad \frac{\text{volts}}{\text{coulombs}}\angle \qquad (22)$$

and similarly for the mechanical vibrator:

$$z' = j\,\omega\,\{r + j\,(m\,\omega - s/\omega)\} = (s - m\,\omega^2) + j\,r\,\omega \qquad \frac{\text{dyne perp. cm.}}{\text{radian}}\angle \qquad (23)$$

$$= s\,\{(1 - u^2) + j\,2\,B_0\,u\} = s\,\{(1 - u^2) + j\,B_s\,u\} \qquad \frac{\text{dyne perp. cm.}}{\text{radian}}\angle$$

As u varies from 0 to infinity, z' traces a parabola. When $B_0 = 0$, the parabola collapses into a straight line along the x axis. When $B = 8$ it nearly coincides with a straight line parallel to the Y axis. Consequently, the max. cyclic displacement of electricity in the branch circuit, and accumulated in the condenser, from an impressed max. cyclic e.m.f. E volts $\angle$ on the branch, will be

$$Q = E/Z' \qquad \text{max. cy. coulombs } \angle \qquad (24)$$

Similarly, the max. cyclic displacement of the mirror of the vibrator, under a max, cyclic v.m.t. F dynes-perp.- cm., will be:

$$\Theta = F/z' \qquad \text{max. cy. radians } \angle \qquad (25)$$

Fig. 5 is a graph of displacement impedance, for a case of $\Lambda = 2$, or $B = 0.5$. In such a case, the resistance r to motion is just equal to the surge resistance $\sqrt{m\,s}$, or

$$R = \sqrt{L/C} \qquad \text{ohms} \qquad (26)$$

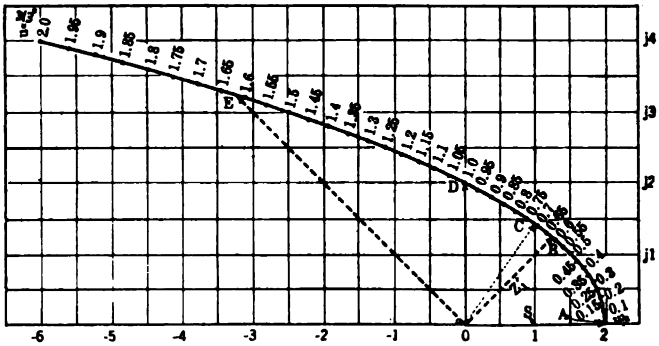

FIG. 5—GRAPH OF IMPEDANCE TO ELECTRIC OR MECHANIC DISPLACEMENT FOR CASES OF

$$\left\{\begin{matrix} R = 10^2, & L = 5 \times 10^{-3}, & S = 2 \times 10^6 \\ r = 10^{-3}, & m = 5 \times 10^{-7}, & s = 2 \times 10^2 \end{matrix}\right\}$$

$$w_0 = 20{,}000 \quad \Delta = 10{,}000 \quad B = 0.5$$

IN THE ELECTRIC CASE, UNIT LENGTH REPRESENTS 10^6 VOLTS PER COULOMB.

IN THE MECHANIC CASE, UNIT LENGTH REPRESENTS 10^2 DYNES-PERP. CM. PER RADIAN.

and the resistance is just 0.5 critical. In the current impedance graph of Fig. 2, similar relations between the ohmic and surge resistances were chosen; so that the graph of Fig. 5 may be derived from that of Fig. 2, according to (21), by applying to each vector, at angular velocity ω, the factor $j\ \omega$, which rotates the vector counter-clockwise through 1 quadrant and extends it in the numerical ratio ω, which ratio increases steadily as the frequency is increased. We thus rotate the diagram of Fig. 2 through + 90 deg., and apply the factor ω throughout.

It will be evident that at $\omega \doteq 0$, the initial displacement impedance is S or s, as the case may be, taken as $O\,A$ Fig. 5, along the axis of real quantities. At the lower quadrantal angular velocity ω_1, this impedance is $O\,B$, at a slope of 45 deg. At resonance, or ω_0, this impedance has reached $O\,D$, along the j axis. At ω_2, it has reached $O\,E$, at the slope of 135 deg., and thence, as the angular velocity is increased indefinitely, the displacement impedance tends to an infinite size, at slope 180 deg. It will be observed that the curve of displacement impedance is a parabola $A\,B\,C\,D\,E$ Fig. 5, with its axis along the line of real quantities.

The displacement impedance is a minimum at that point C in the curve, where the tangent is perpendicular to the radius vector.

Displacement Admittance. The reciprocal of displacement impedance, or "displacement admittance," is of still greater interest and practical importance from the oscillographic viewpoint. From (23), considering only the mechanical case, the displacement admittance is

$$y' = 1/z' = \frac{1}{(s - m\,\omega^2) + j\,r\,\omega} = \frac{1}{m\,(\omega_0^2 - \omega^2) + j\,r\,\omega}$$

$$= \frac{1}{m\,\{\,(\omega_0^2 - \omega^2) + j\,2\,\Delta\,\omega\,\}} \qquad \frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \tag{27}$$

$$= \frac{1}{s\,\{\,(1 - u^2) + j\,2\,B_0\,u\}} = \frac{1}{m\omega_0^2\,\{\,(1 - u^2) + j\,2\,B_0\,u)\}}$$

$$= \frac{y}{j\,\omega} = -\,j\,y/\omega \qquad \frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \tag{27a}$$

At $\omega = 0$, $y' = y_0' = 1/s\ \angle 0$ deg.

$$\frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \tag{28}$$

At $\omega = \omega_1, y' = y_1' = \dfrac{1}{\sqrt{2}\,r\,\omega_1}\ \overline{\diagdown}\ 45$ deg.

$$\frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \tag{29}$$

$$\text{At } \omega = \omega_0,\ y' = y_0' = -j\,\frac{1}{r\,\omega_0} = \frac{1}{r\,\omega_0}\ \overline{\diagdown}\ 90 \text{ deg.}$$

$$\frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \qquad (30)$$

$$\text{At } \omega = \omega_2,\ y' = y_2' = \frac{1}{\sqrt{2}\,r\,\omega_2}\ \overline{\diagdown}\ 135 \text{ deg.}$$

$$\frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \qquad (31)$$

$$\text{At } \omega = \infty,\ y' = y'_\infty = 0\ \overline{\diagdown}\ 180 \text{ deg.}$$

$$\frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \qquad (32)$$

Displacement Admittance Graph. Referring to (**27**a), and to Fig. 6, it will be seen that if we draw the circular graph of $\dot{\theta}$, $O\,B\,C\,D$, and at any particular value of ω, or $u\,\omega_0$, such as $\omega = 10{,}000$, draw the vector $O\,P$, of size 0.555×10^2 $\frac{\text{radians per sec.}}{\text{dyne perp. cm.}}$, at slope $\alpha = 56.3$ deg., we may then draw perpendicularly thereto the vector $O\,p$, and dividing 55.5 by ω obtain, to a correspondingly altered scale, the displacement admittance of 5.55×10^{-3} radian per dyne-perp.-cm., at a slope of β deg. $= -33.7$ deg. This means that if unit max. cy. v.m.t., or 1 dyne-perp.-cm., be applied to the vibrator of this case, at $\omega = 10{,}000$, the max. cy. displacement of the mirror will be 5.55×10^{-3} radians $= 0.32$ degree. This elongation, or max. cy. angular displacement, would occur 33.7 deg. in phase behind the corresponding elongation in torque or exciting current.

Proceeding in this manner, we might map out the entire displacement admittance* graph a, p, b, c, d, O. At or near zero frequency, denoted by ω_a, the displacement admittance would be the vector $O\,a$, in phase with the exciting current of the vibrator. At the lower quadrantal frequency ω_1, it would be the vector $O\,b$. At the upper quadrantal frequency, it would be $O\,d$. At resonance, it would be $O\,c$, in lagging quadrature with the excitation.

The maximum admittance would occur at ω_d, in this case at $u = 0.707$, or $\omega = 14{,}140$. This vector will vary in position

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*Bibliography 18, p. 505 and 511.

with different values of B. *It is to be noted, however, that the maximum admittance never occurs at resonance, although in sharp vibrators, it will lie close to resonance.*

If we consider Fig. 6, in terms of its electric representation, the scale being 10^{-2} mho. per unit length, then at $\omega = 10{,}000$ and $u = 0.5$, the electric displacement admittance would be $0.555 \times 10^{-6} \nwarrow 33.7$ deg. coulomb per max. cy. volt on the $C\,L\,R$ branch terminals.

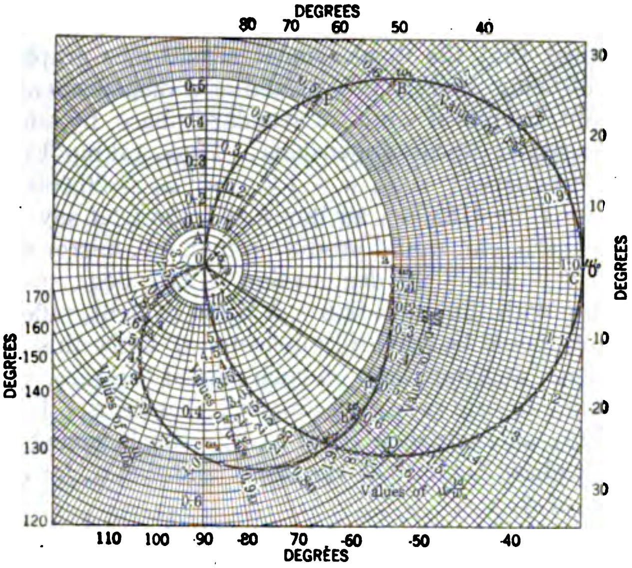

FIG. 6—GRAPH OF DISPLACEMENT ADMITTANCE FOR CASES OF

$$\begin{Bmatrix} R = 10^2, & L = 5 \times 10^{-3}, & S = 2 \times 10^6 \\ r = 10^{-2}, & m = 5 \times 10^{-7}, & S = 2 \times 10^2 \end{Bmatrix}$$

$w_0 = 20{,}000 \quad \Delta = 10{,}000 \quad B = 0.5$

FOR CIRCLE $A\ B\ C\ D$ { IN THE ELECTRIC CASE, UNIT LENGTH = 10^{-2} MHO. IN THE MECHANIC CASE, UNIT LENGTH = 10^{-2} MECHANIC ABMHOS, OR $10^2 \left(\frac{\text{RADIANS}}{\text{SEC.}}\right)$ PER UNIT V.M.T.

FOR GRAPH $a\ b\ c\ d$ { IN THE ELECTRIC CASE, UNIT LENGTH = 10^6 COULOMB PER VOLT. IN THE MECHANIC CASE, UNIT LENGTH = 10^{-8} RADIAN PER UNIT V.M.T.

It is also easy to show that the displacement admittance graph $a\ b\ c\ d$, Fig. 6, is obtainable, by direct vector inversion, from the displacement impedance graph $A\ B\ C\ D\ E$, of Fig. 5.

Displacement:

Since

$$\Theta = F\,y' \qquad \text{max. cy. radians } \angle \qquad (33)$$

and

$$Q = E\,Y' \qquad \text{max. cy. coulombs } \angle \qquad (34)$$

it is evident that the displacement admittance graph is also capable of being directly interpreted as a displacement graph, to constant impressed v.m.t. at standard phase, by adopting a suitable scale of linear dimensions. Thus, taking Fig. 6, at resonant frequency; *i. e.*, $u = 1.0$, and $\omega_o = 20{,}000$ radians per sec., or $f_o = 3183 \sim$, an impressed v.m.t. of say 10 max. cy. dynes-perp. cm. would produce a max. cy. mirror displacement of $\Theta = 0.05 \measuredangle 90$ deg.· radian $= 2.87$ deg., in lagging quadrature with the excitation. A beam of light, reflected by the mirror, would be cyclically deflected to twice this angle or ± 5.74 deg., at elongation, on each side of the zero, owing to the doubling optical reflecting property of mirrors. It may be noted that, at resonance ($u = 1$), the displacement is just 90 deg. in phase behind the excitation and v.m.t., for any and all values of B.

Formulas for Displacement Admittance. Remarking that the slope β deg. of the displacement admittance Y' or y', is always negative, lying between the values $-\ 0$ deg. and $-\ 180$ deg., we have

$$\omega = \sqrt{\omega_o^2 + \Delta^2 \cot^2 \beta} + \Delta \cot \beta$$

$$= \omega_o \left\{ \sqrt{1 + B^2 \cot^2 \beta} + B \cot \beta \right\} \qquad \frac{\text{radians}}{\text{sec.}} \qquad (35)$$

$$\text{or} \quad u = \omega / \omega_o = \sqrt{1 + B^2 \cot^2 \beta} + B \cot \beta \qquad \text{numeric} \qquad (36)$$

The polar equation of displacement admittance is

$$|\,y'\,| = \frac{-\sin \beta}{r\,\omega} = \frac{-\sin \beta}{r\,\omega_o \left\{ \sqrt{1 + B^2 \cot^2 \beta} + B \cot \beta \right\}} \qquad \text{numeric} \qquad (37)$$

$$= \frac{-\sin \beta}{2\,B\,s \left\{ \sqrt{1 + B^2 \cot^2 \beta} + B \cot \beta \right\}} \qquad \text{numeric} \qquad (37a)$$

The corresponding vector equation is

$$y' = \frac{-\sin\beta}{r\,\omega} \angle -\ \beta \text{ deg.}$$

$$= \frac{-\sin\beta \diagdown \beta \text{ deg.}}{2\,B\,s\ \{\sqrt{1 + B^2\cot^2\beta} + B\cot\beta\}}$$

$$= \frac{-\sin\beta \diagdown \beta \text{ deg.}}{2\,B\,s\,u} \qquad \frac{\text{mirror radians}}{\text{unit v.m.t.}} \angle \qquad (38)$$

The slope β, pertaining to any assigned value of ω, is

$$\beta = \tan^{-1}\left(\frac{2\,\Delta\,\omega}{\omega^2 - \omega_o^2}\right) \qquad \text{degrees} \qquad (39)$$

or $$\beta = \tan^{-1}\left(\frac{2\,B_o\,u}{u^2 - 1}\right) = \tan^{-1}\left(\frac{B_s\,u}{u^2 - 1}\right) \text{ degrees} \qquad (40)$$

In the testing of oscillographs, the calibration is ordinarily conducted at a relatively low frequency, say 60 ~, which, for practical purposes, may be regarded as of the same effect as zero frequency, or ω_s. By (**28**), the displacement admittance at ω_s is $1/s$, at zero slope. It is desired to find what ratio the admittance will bear at any impressed angular velocity ω, (or its ratio u), to that at the calibration frequency ω_s. From (**38**) and (**28**) we obtain

$$D = y'/y_s' = \frac{-\sin\beta \angle -\ \beta \text{ deg.}}{2\,B_o\,u} = \frac{-\sin\beta \diagdown \beta \text{ deg.}}{B_s\,u}$$

$$= \frac{-\sin\beta \diagdown \beta \text{ deg.}}{2\,B_o\,\{\sqrt{1 + B_o^2\cot^2\beta} + B_o\cot\beta\}}$$

$$\text{numeric} \angle \qquad (41)$$

This is an important formula. It gives the vector *deviation factor* D, for any oscillograph at any impressed ω, when the values of ω_o and B are known for the instrument. Thus, in the case represented by Fig. 6, where ω_o is 20,000, and $B = 0.5$, if the instrument is calibrated at 60 ~ $= \omega_s$, it may be required to know what will be the deviation factor at say $\omega = 5000$ or $u = 0.25$. Here, by (**40**),

$$\beta = \tan^{-1}\frac{0.25}{-0.9375} = \tan^{-1}(-0.2667) = -(14 \text{ deg. } 56 \text{ min.})$$

Then by (41)

$$D = y'/y'_s = \frac{-\sin(-14\text{ deg. }56\text{ min.})\ \overline{\diagdown}\ (14\text{ deg. }56\text{ min.})}{2 \times 0.5 \times 0.25}$$

$$= \frac{0.2577}{0.25}\ \overline{\diagdown}\ 14\text{ deg. }56\text{ min.} = 1.031\ \overline{\diagdown}\ 14\text{ deg. }56\text{ min.}$$

This means that the max. cy. displacement produced at 796 ~, or 5000 radians/sec. will be 3.1 per cent greater than that produced by the same impressed torque at 60 ~, and will lag 14 deg. 56 min. in phase behind the impressed torque; whereas at 60 ~, the phase lag of displacement would be insignificant in such a vibrator. The *correction factor* will be the reciprocal of the deviation factor, or in this case 0.969 $\angle$ 14 deg. 56 min., at 796 ~. The frequency of 796 ~ would be 13 times 61.23 or would be the thirteenth harmonic of 61.23 ~. The lag of mirror displacement to this harmonic would thus be nearly 15 deg. of the harmonic frequency. The position of the recorded 13th harmonic would thus be retarded by nearly 15 deg. of its own cycle, and the harmonic, as recorded photographically should be advanced through more than 14° of its own cycle, in order to place it in proper relation to the fundamental. Its recorded or apparent magnitude should also be reduced 3 per cent, in order to have the same scale of amplitude as the fundamental and calibrated frequency.

The following Table of important displacement frequencies may be convenient for reference.

Of the above 8 important angular velocities, Nos. 1, 2, 5 and 7 are always presented, at the successive negative slopes of 0 deg., 45 deg., 90 deg., and 135 deg. No. 3, the maximum admittant value of ω, is always to be found when $B < 1$, or $\Lambda > 1$. For values of B equal to, or greater than, unity, which we may call blunt cases, ω_d has no independent existence, because the greatest admittance is then found at $\omega = \omega_s = 0$. Angular velocity No. 4, ω_f, is always presented when $B < 1$; but in blunt cases, the system is too heavily damped to permit of free vibration. No. 6 is always presented, but its phase position varies with B. It is midway in angular velocity between ω_1 and ω_2; but is not midway between them in phase. No. 8, the upper angular velocity at which the displacement has the same size as at ω_s, is not found when $B > 0.707$. In

TABLE I.

PRINCIPAL IMPRESSED ANGULAR VELOCITIES OF A VIBRATORY SYSTEM.

	Angular Velocity radians/sec.	Displacement Admittance Mirror radians/dyne p. cm.	
1. Initial, (near zero),	ω_s	$y_s' = 1/s \angle 0^\circ = c \angle 0^\circ$	(42)
2. Lower quadrantal,	$\omega_1 = \sqrt{\omega_0^2 + \Delta^2} - \Delta = \omega_0 \{ \sqrt{1 + B^2} - B \}$	$y_1' = \frac{1}{\omega_1 r \sqrt{2}} \angle 45^\circ = \frac{g}{\omega_1 \sqrt{2}} \angle 45^\circ$	(43)
3. Max. admittant,	$\omega_d = \sqrt{\omega_0^2 - 2\Delta^2} = \omega_0 \sqrt{1 - 2B^2}$	$y_d' = \frac{1}{r\,\omega_f} \angle \tan^{-1} (\omega_d / \Delta)$	(44)
4. Free vibrational,	$\omega_f = \sqrt{\omega_0^2 - \Delta^2} = \omega_0 \sqrt{1 - B^2}$	$y_f' = \frac{1}{r \sqrt{\omega_f^2 + (\Delta/2)^2}} \angle \tan^{-1} \frac{\omega_f}{(\Delta/2)}$	(45)
5. Resonant,	$\omega_0 = \sqrt{\omega_0^2 - 0} = \omega_0 \sqrt{1}$	$y_0' = \frac{1}{r\,\omega_0} \angle 90^\circ = \frac{-j\,g}{\omega_0}$	(46)
6. Midquadrantal,	$\omega_{12} = \sqrt{\omega_0^2 + \Delta^2} = \omega_0 \sqrt{1 + B^2}$	$y_{12}' = \frac{1}{r \sqrt{\omega_0^2 + 5\,\Delta^2/4}} \angle \tan^{-1} \left\{ \frac{\omega_{12}}{(\Delta/2)} \right\}$	(47)
7. Higher quadrantal,	$\omega_2 = \sqrt{\omega_0^2 + \Delta^2} + \Delta = \omega_0 \{ \sqrt{1 + B^2} + B \}$	$y_2' = \frac{1}{r\,\omega_2 \sqrt{2}} \angle 135^\circ = \frac{g}{\omega_2 \sqrt{2}} \angle 135^\circ$	(48)
8. Duplicate initial,	$\omega_{s2} = \omega_d \sqrt{2} = \omega_0 \sqrt{2\,(1 - 2B^2)}$	$y_{s2}' = c \angle \tan^{-1} \left(\frac{2\,\Delta\,\omega_{s2}}{\omega_d^2 - 2\,\Delta^2} \right)$	(49)

sharp cases, it is always present. Its phase position, or slope, varies with B.

A displacement admittance graph presenting all of these 8 angular velocities is seen in Fig. 7, for the particular case of $\omega_0 = 25{,}000$, or $f_0 = 3979$ ∼, and $B = 0.225$. Here y_s' is taken as 1, corresponding to an oscillographic standard calibration at or near zero frequency. We find y_1, y_0 and y_2, all at their corresponding slopes of − 45 deg., − 90 deg. and − 135 deg., with the respective sizes of 1.9642, 2.22 and 1.2571, at the frequencies 3182, 3979 and 4973 ∼. The maximum admittance value y', at $\omega_d = 23{,}701$, or $f_d = 3{,}772$ ∼, is 2.2807, occurring at $\beta = -$ 76 deg. 38 min. 55 sec. The free vibrational value y_f', at $\omega_f = 24{,}359$, or $f_f = 3{,}877$ ∼, is 2.2657 ⦣ 83 deg. 24 min. 50 sec. The midquadrantal value y_{12}, at $\omega_{12} = 25{,}625$, or $f_{12} = 4078$ ∼, is 2.1551 ⦣ 96 deg. 15 min. 49 sec. Finally, the duplicate initial value of y', or y_{s2}', at $\omega = 33518$, or $f_{s2} = 5334$ ∼, is 1.0 ⦣ 142 deg. 53 min. 30 sec. In the oscillograph corresponding to this case, the deviation factor at $\omega = 4000$, or $f = 636$ ∼, would be 1.023 ⦣ 4 deg. 13 min. and at $\omega = 8000$, or $f = 1272$ ∼, it would be 1.101 ⦣ 9 deg. 7 min.

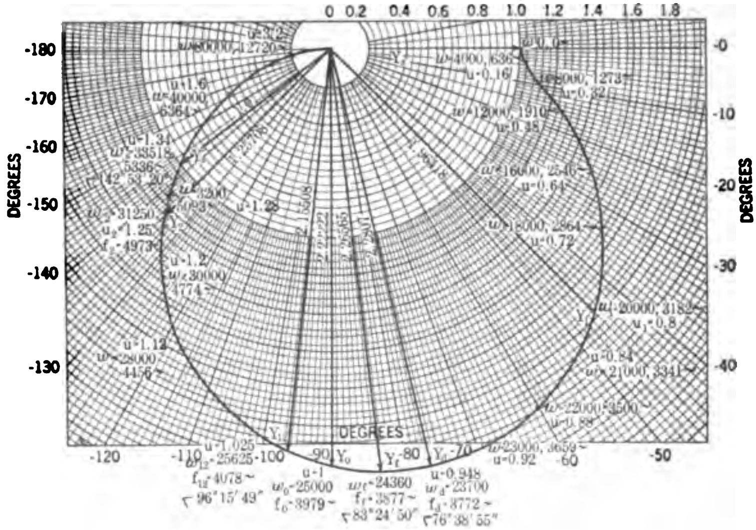

FIG. 7—VECTOR DISPLACEMENT ADMITTANCES

$w_0 = 25{,}000$ $\Delta = 5{,}625$ $B = 0.225$ $s = 10^6$ $m = 0.0016$ $r = 18$

Fig. 8 is a vector chart of the deviation factor D for any value of bluntness B between $B = 0.25$ and $B = 1.0$. Enter the chart with u, the ratio of the impressed angular velocity; to the known resonant angular velocity ω_o. The intersection of the u line and the B line will lie on a radius vector whose size and slope make the factor D.

Fig. 9 is a similar vector chart for finding D, when B lies between 1.0 and ∞. There is actually but little change in the form of the curve for values of B exceeding 4. The limiting curve is a semi-circle of unit diameter on the real axis.

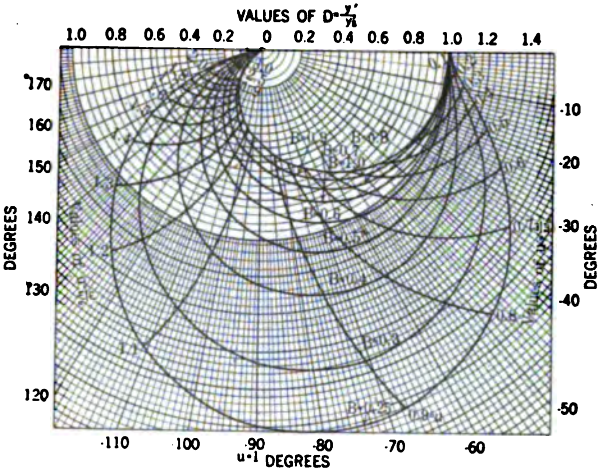

FIG. 8—PLANEVECTOR CHART OF THE DEVIATION FACTOR D FOR ANY OSCILLOGRAPH WHOSE BLUNTNESS LIES BETWEEN $B = 0.25$ AND $B = 1.0$, AND WHOSE RESONANT FREQUENCY IS GIVEN.

EXAMPLE: IF $B = 0.6$, AND $u = 0.3$, THEN $D = 1.022 \,/\overline{21.6\text{ DEG.}}$

Fig. 10 is a similar vector chart for D, when B lies between 0.05 and 0.25 inclusive. As B diminishes, the curve of D rapidly approximates to a circle of diameter $\frac{1}{2B} = \Lambda/2$, or radius $\frac{1}{4B}$, and having its center displaced approximately 0.25 unit to the right of the axis $-O\gamma$. At $B = 0$, this circle has an infinitely great diameter.

Constants of an Oscillograph Vibrator. It may be seen from an inspection of (**39**), (**40**) and (**41**), that the deviation factor of any oscillograph vibrator can be either computed or identified from the foregoing curve sheets, if we can ascertain its two essential working constants ω_o and Δ; or what is equivalent thereto, ω_o and B, or f_o and B. The tests of oscillographs thus direct themselves towards the determination of these two constants. If, however, not merely the deviation factor of a vibrator; but also its essential elements are required; then we must seek for the four constants A, m, r and s. This calls, as we shall see, for an additional test.

If we divide (**46**) by (**42**), we find:

$$y_s'/y_o' = j\,2\,B_o = 2\,B_o \angle 90 \text{ deg.} = B_s \angle 90 \text{ deg.} \qquad \text{numeric} \angle \qquad (\mathbf{50})$$

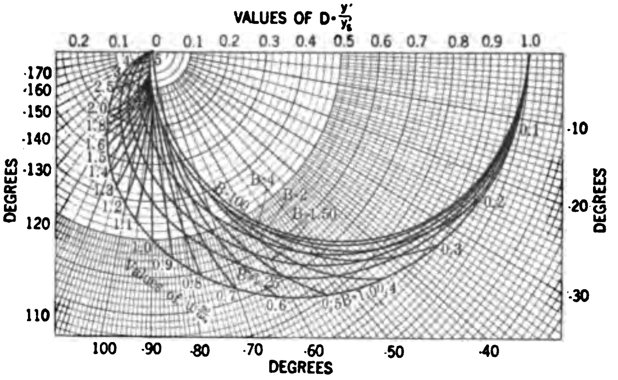

FIG. 9—PLANEVECTOR CHART OF DEVIATION FACTOR $D = y'/y'_s$ FOR ANY OSCILLOGRAPH WHOSE BLUNTNESS LIES BETWEEN $B = 1$ AND $B = \infty$, AND WHOSE RESONANT FREQUENCY IS GIVEN.

EXAMPLE: IF $u = 0.5$ AND $B = 2.0$, THEN $D = 0.468 \,/\, \overline{69.4 \text{ DEG.}}$

If, therefore, we can (1) identify the resonant frequency f_o, of a vibrator, and (2) compare the calibration of the vibrator at this frequency with that at zero frequency, we shall have; see Figs. 6 to 10:

$$\left| \frac{y_s'}{y'_o} \right| = 2\,B_o = B_s \qquad \text{numeric} \qquad (\mathbf{51})$$

The bluntness B_o of the vibrator will then be half the ratio of the max. cy. displacement at zero frequency to the max. cy. displacement at resonant frequency, equal current excitations and impressed torques being used in each test. If the vibrator is sharp, the same fact may be expressed by saying that *the sharpness Λ_o will be twice the ratio of the electric exciting*

current needed to produce the same max. cy. displacement, or deflection, at zero frequency, as will produce it at resonant frequency.

All that is necessary, therefore, for finding the deviation factor of a vibrator at any impressed frequency, is to identify its resonant frequency f_o, and then compare its calibration at that frequency with that at f_s, a low frequency, practically equivalent to zero.

An equivalent alternative formula is:

$$\Delta = \frac{\omega_o}{j^2} \cdot \frac{y_s'}{y_o'} = \pi f_o \frac{y_s'}{y_o'} \nwarrow 90 \text{ deg. } \frac{\text{hyps}}{\text{sec.}} \angle \qquad (52)$$

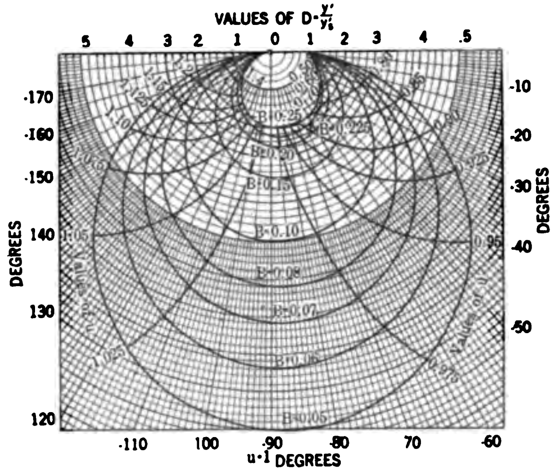

FIG. 10—PLANEVECTOR CHART OF THE DEVIATION FACTOR $D = y'/y'_s$ FOR ANY OSCILLOGRAPH WHOSE BLUNTNESS LIES BETWEEN 0.05 AND 0.25, AND WHOSE RESONANT FREQUENCY IS GIVEN.

EXAMPLE: IF $u = 0.9$, AND $B = 0.08$, $D = 4.17 \,/\,\overline{37.1 \text{ DEG.}}$

Oscillographmeter. In order to detect and identify the resonant frequency f_o of a vibrator, which, as above pointed out, is not the same as, and may even be remote from, the frequency f_d of maximum displacement, an oscillographmeter has been devised and constructed. It is shown separately in Fig. 11, and mounted in front of an oscillograph under test in Fig. 11-a. It consists of a simple permanent-magnet auxiliary vibrator, air-damped, and therefore of very sharp resonance. This vibrator is firmly set with its vibration axis horizontal, and with its mirror close to that of the tested oscillograph mirror, whose vibration axis is vertical. A strong beam of light is then directed first on the oscillograph mirror, thence on to the auxiliary mirror, and thence vertically upwards to a horizontal sheet of thin graduated paper; so that the path of the vibrating beam on this paper can be examined. The optical arrangement is indicated in Fig. 12.

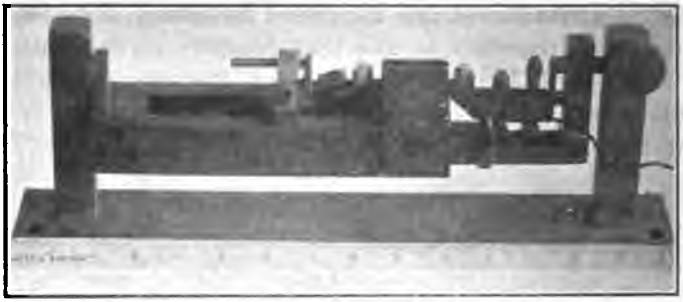

FIG. 11—OSCILLOGRAPHMETER OR AUXILIARY VIBRATOR. LENGTH OF MIRROR 2.3 MM. BREADTH 0.9 MM.

If the oscillograph and the auxiliary vibrator are operated from the same audio-frequency source of adjustable frequency; then they will both vibrate with that frequency. Because their vibration axes are set mutually perpendicular, the

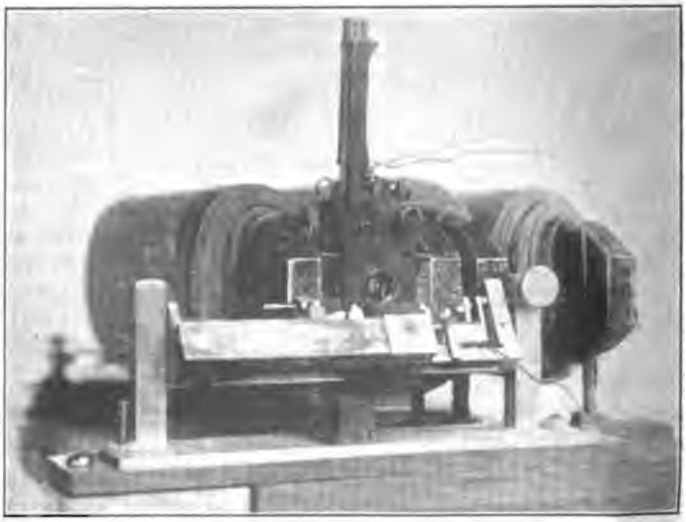

FIG. 11-A—OSCILLOGRAPHMETER APPLIED TO OSCILLOGRAPH FOR TEST OF LATTER.

vibratory path of the beam of light on the paper plane *F F*, Fig. 12, will be a Lissajous figure*. If the max· cy. displacements of the two mirrors are cophasal, the figure formed by the moving beam will be a straight line. If they are not cophasal, the figure will be an ellipse. If they are in quadrature, this ellipse will include its maximum area and its two perpendicular axes will be both equal and parallel to the two component displacements. If these two components are equal, the ellipse will, at this quadrature phase relation, become a circle.

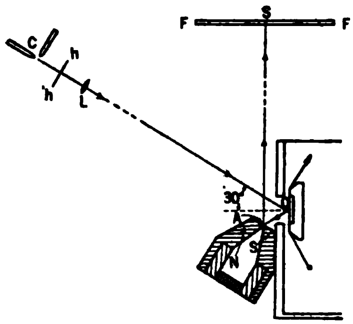

FIG. 12—OPTICAL SYSTEM OF OSCILLOGRAPHMETER *A* AND THE TESTED OSCILLOGRAPH *O*. LENGTH OF *A* MIRROR 2.3 MM. WIDTH 0.9 MM. DISTANCE *O–A* = 1.1 CM. DISTANCE *A–S* = 50 CM.

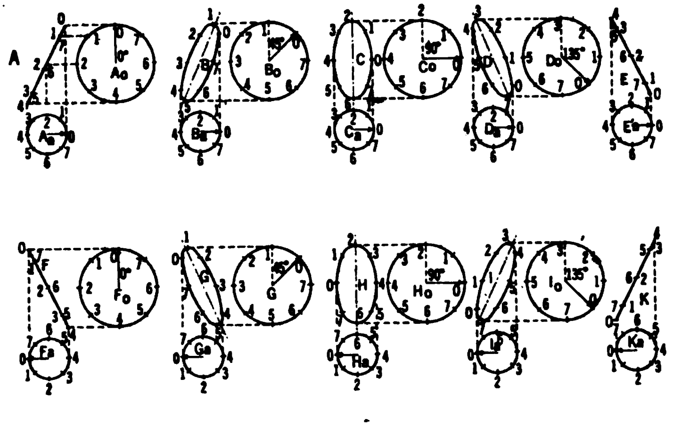

FIG. 13—LISSAJOUS FIGURES WITH COMPONENTS IN THE RATIO OF 2 TO 1 AND VARIOUS PHASE DIFFERENCES

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*Bibliography 1.

The development of Lissajous ellipses is outlined in Fig. 13, for the particular case in which the auxiliary vibrator amplitude is half of the oscillograph vibrator amplitude. Commencing at *A*, the auxiliary vibrator A_a, starts at standard phase, and the oscillograph A_o, set in mechanical quadrature therewith, also at standard phase. The two vectors then rotate with equal angular velocities. The path of the reflected beam will be straight line *A*; *i. e.*, an ellipse of negligibly small minor axis.

When the oscillograph vibrator is lagging 45 deg. in phase with respect to the auxiliary vibrator, the path of the beam of light on the plane of the receiving sheet, will be the ellipse *B*. Again, with the oscillograph vibrator 90 deg. behind C_a in phase, the path of the beam will be the upright ellipse *C*. With 135 deg. of lag, the ellipse will incline backwards as at *D*. With 180 deg. of lag, the ellipse will collapse into the straight line *E*.

The effect of a reversal of phase in either the oscillograph or the auxiliary vibrator, is shown at *F, G, H* and *K*. In each case it will be seen that, at quadrature, the ellipse stands upright. The direction of change in inclination of the ellipse is, however, relatively reversed in the two series.

In making the test of the oscillograph, we must first make sure that the resonant frequency of the auxiliary vibrator is not near to the resonant frequency of the oscillograph vibrator. If the two resonant frequencies should happen to approach too closely, the tension of the auxiliary vibrator must be mechanically adjusted, in either direction, so as to separate them. All that is required of the auxiliary vibrator is that it shall be sharp, and also out of tune with the tested oscillograph. The various constants ω_o and *B*, or ω_o and Δ, or *A, m, r* and *s*, of the auxiliary vibrator are of no concern, provided that its *B* is very small, and that its ω_o is not close to the ω_o of the oscillograph vibrator.

Connections for Test. The electrical connections for the test of an oscillograph by means of the oscillographmeter, or auxiliary vibrator, are indicated at *A*, Fig. 14. *S* is the source, or secondary coil of an oscillator, supplying, in parallel, both the oscillograph *O*, and the auxiliary vibrator *A*, each through an adjustable resistance. The two supply currents will thus have the same phase, and the two impressed v.m.ts. will be cophasal. The Lissajous figure, obtained from the optical combination of the two vibrators, will then be a straight line, as at *A*, Fig. 13, if the impressed frequency is low, and remote from resonance in either vibrator. The impressed frequency is now raised, until the resonant angular velocity ω_0, or resonant frequency f_0, for O, is approached. The Lissajous ellipse will now begin to standup. The impressed frequency is now carefully adjusted, until the Lissajous ellipse is upright, and the frequency then noted. If the two vibratory components are equal, the Lissajous ellipse, at resonance in O, will be a circle, as shown at A, Fig. 14. This, however, is an unsuitable adjustment, since it does not lend itself to precise observation. An ellipse of say 2 : 1, or 3 : 1, in major and minor diameters is preferable, as at C, Fig. 13.

If the optical adjustment is fine, so that the beam of light is sharp and thin, it may be more easy to observe the existence of the thin cophase straight line of A, E, F and K, in Fig. 13, than the verticality of an ellipse. This is a matter of personal choice with the observer. If he prefers the straight line, or collapsed ellipse, then the connections of Fig. 14 B, may be used. Here, the resistance in one of the two vibrator circuits, O as shown, is replaced by an adjustable condenser. This has the effect of advancing the phase of the exciting current in that branch by 90 deg., so that the two impressed v.m.ts. will be in quadrature. If the two vibrator displacement slopes are cophasal, the Lissajous figure will then be the circle, or upright ellipse. At displacement slope quadrature, on the contrary, the two actual displacements will be cophasal, and resonance in O will show itself by the Lissajous straight line of Fig. 14B.

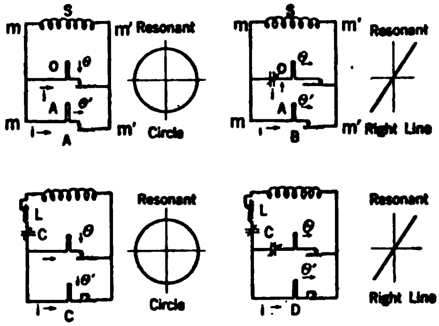

FIG. 14—METHOD OF TESTING OSCILLOGRAPH VIBRATORS BY MEANS OF OSCILLOGRAPHMETER

With the auxiliary vibrator air damped, and reasonably well constructed, its bluntness B will be very small, and its displacement slope will then be negligible, until within a few cycles per second of its resonant frequency. This means that the displacement slope of the auxiliary vibrator will either be substantially 0 deg., or substantially 180 deg., provided that the two vibrators are not closely coresonant.

It has been found that when a Vreeland oscillator is used as the source of impressed frequency, difficulties may arise owing to small parasitic harmonics or impurities in the derived current wave, which may give rise to false resonances, and erroneous results, in the behavior of the sharply resonant auxiliary vibrator. Much trouble was experienced in the first few months of this research, by such parasitic resonances. They can be eliminated, however, by electrically tuning the main testing circuit to the impressed frequency, as indicated in Fig. 14 at C and D. An adjustable inductor and condenser are so varied, that their combined reactance is negligibly small to the main impressed frequency from the source; while opposing a very considerable reactance either to parasitic frequencies, or harmonics. If this plan is used, care must be taken to make the proper adjustment of reactance at each change of impressed frequency. With the ordinary connections of a pliotron, or triode vacuum-tube oscillator, this difficulty has not been encountered.

Technique of Test. Having identified the resonant frequency f_o, or resonant angular velocity ω_o, of the oscillograph vibrator, with the aid of the oscillographmeter, and suitable Lissajous optical figure, the oscillograph is calibrated for its specific deflection, or maximum cyclic deflection per unit of r.m.s. exciting current, both at a low impressed frequency, f_s, say 60 ∼, and at f_o. In this part of the test, the oscillographmeter is either removed, or left out of circuit. As we have already seen in connection with (**51**), the ratio of these two calibrations, or specific deflections, is equal to the value of 2 B for the instrument. Having found ω_o and B, we are able to find the vector deviating factor, and its reciprocal, the correcting factor, for the instrument, at any or all impressed frequencies that may be used. Defining the specific deflection θ as the vector scale deflection of the oscillograph divided by the r.m.s. value of the exciting current in the oscillograph vibrator, we have, if θ_s is the specific deflection at low frequency, and to standard phase

$$D = \theta/ = \theta_s \frac{1 - u_s^2 + j\,2\,B_o u_s}{1 - u^2 + j\,2\,B_o u} = \frac{1 - u_s^2 + j\,B_s u_s}{1 - u^2 + j\,B_s u}$$

numeric ∠ (**53**)

Since the ratio ω_s/ω_o is usually a negligibly small real number, this may be written without serious error

$$D = \theta/\theta_s = \frac{1}{1 - u^2 + j\,2\,B_o u} = \frac{1}{1 - u^2 + j\,B_s u}$$

$$= \frac{-\sin\beta\angle-\beta}{2\,B_o u} = \frac{-\sin\beta\nwarrow\beta}{B_s u} \qquad \text{numeric} \angle \quad \textbf{(54)}$$

which agrees with (50) and (51), at resonance, when $u = 1$.

If we seek only the size of this ratio, and ignore the slope or phase deviation, we may write this in the form

$$|D| = \left|\frac{\theta}{\theta_s}\right| = \frac{1}{\sqrt{1 + 2\,u^2\,(2\,B_o^2 - 1) + u^4}} \qquad \text{numeric} \quad \textbf{(55)}$$

which is devoid of imaginary quantities. The correcting factor is then

$$K = 1/D \qquad \text{numeric} \quad \textbf{(56)}$$

When making specific deflection tests with the optical arrangement of Fig. 12, it has been found advantageous to insert a suitable lens L, with a fine cross hair on it, into the path of the beam. The positions of the hair can then be detected at each end of the vibrating beam of light on the graduated paper scale $F\,F$. When, however, Lissajous figures are being examined, the lens and crosshair are withdrawn.

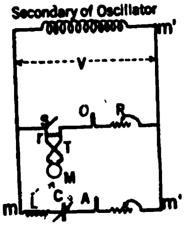

FIG. 15—METHOD OF DETERMINING QUADRANTAL POINTS

It should be remembered that the angular deflection of the beam of light reflected from the oscillograph's mirror is double the angular deflection of the mirror. Since the equations refer to the angular displacements of the mirror, this factor of 2 should be taken into account in interpreting the deflections.

Alternative Methods. Instead of determining B by the ratio of specific deflections θ_s and θ_o, it is possible, after finding ω_o, to find Δ, and hence B, by identifying optically one, or preferably both of the quadrantal angular velocities ω_1 and ω_2. The electrical connections for this are shown in Fig. 15. The plan is to introduce such equal resistance and reactance into the auxiliary branch circuit, as shall cause the exciting current and v.m.t., in one circuit, to differ in phase from that in the other by

45 deg., and so to enable quadrantal frequency to be identified optically from a Lissajous diagram. After observing ω_o, and then say ω_1, we can compute ω_2 by (6). Again, observing all three values ω_1, ω_o and ω_2, they should check by (6). We then find Δ by (**16**), and B by (**19**).

This alternative method enables a calibration of the oscillograph at resonant frequency to be dispensed with. On the other hand, it is apt to be less precise than the preferred method; because Δ is found as the difference of two identified frequencies. With sharp vibrators, these two quadrantal frequencies lie close together, and their difference is thus subject to relatively considerable error. In complete and thorough tests of an oscillograph, however, it may be worth while identifying all three values ω_o, ω_1, and ω_2, besides finding θ_s and θ; so as to secure mutual checks.

Another method for finding Δ and ω_o, without the use of the oscillographmeter, when the vibrator is sharp, is to make an oscillogram of a series of naturally decaying oscillations of the mirror, impressing on the same record a known fairly high frequency, from the movement of a second vibrator separately excited. The rate of decay in amplitude is given by the relation

$$\Theta = \Theta_i \, \epsilon^{-\Delta t} \qquad \text{max. cy. radians} \quad (57)$$

where Θ_i is the initial displacement, at $t = 0$, and Θ the displacement at time t seconds

whence $$\Delta = (1/t) \operatorname{logh} \Theta_i/\Theta \qquad \text{hyps. per sec.} \quad (58)$$

The frequency of vibration, as determined from the record, is

$\frac{\omega_f}{2\pi}$ cycles per second.

This method can be applied with fair success, to sharp vibrators. It cannot ordinarily be applied when B is greater than 0.2. This means that in ordinary liquid-damped oscillographs, this alternative is unsatisfactory.

It might be supposed that since a calibration is desired near to zero frequency, a continuous-current calibration would suffice. It has been found, however, that specific deflections with continuous currents are considerably in excess of those measured with low-frequency alternating currents, apparently because the elastic system has a greater opportunity to stretch under a steady impressed torque than under alternating torques.

A fairly typical and commercial form of oscillograph has its behavior represented in Fig. 16. This oscillograph is damped by a specially prepared liquid, supplied with the instrument. By means of the oscillographmeter, and Lissajous figures, the following frequencies are identified, $f_1 = 2663 \sim$, $f_o = 3300 \sim$, $f_2 = 4077 \sim$. The specific deflection θ_o, at resonance, was observed to be 5.35 mirror radians per r.m.s. absampere. The specific deflection θ_s at 60 $\sim$, was also observed to be 2.25 mirror radians per r.m.s. absampere. The ratio θ_o/θ_s is 2.38; so that $\Lambda = 4.67$ or $B = 0.21$. From ω_o and B, the curve of Fig. 16 has been drawn, by the use of (40) and (41) or of (54). In addition, a number of values of the specific deflection were observed experimentally at different frequencies. These are marked as crosses on the diagram. If these observations were in complete accordance with the theory, the crosses should coincide with the curve. It will be noted that θ_d has the value 5.4 mirror radians per absampere at 3140 $\sim$, which is 160 $\sim$ below f_o. In this instrument, at 1000 $\sim$, the size of the deviation factor is 1.09, or the instrument overindicates 9 per cent at 1000 $\sim$, with respect to the low-frequency calibration. The slope of the deviating factor is also $\beta = -8$ deg. by (40).

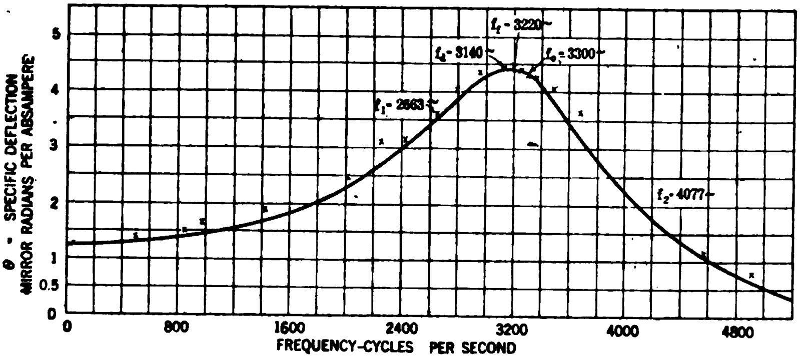

FIG. 16—SPECIFIC DEFLECTION VS. FREQUENCY
h Vibrator in Light Damping Liquid. $\Delta = 4450$ hyps. per second

It may be observed that a curve of specific deflection θ as ordinates, against frequency f as abscissas, always rises to a maximum at f_d, unless B exceeds unity. The curve is always dissymmetrical about the maximum ordinate, being steeper on the high-frequency side. But if the curve of θ as ordinates be drawn against frequency squared, or f^2, as abscissas, the curve will be symmetrical about the maximum ordinate, as far as f_2, *i. e.*, as far as symmetry is possible.

Fig. 17 gives the curves of specific deflection θ, against impressed frequency f, for a vibrator in four successive damping fluids; *viz.*, air, glycoline, mineral oil and castor oil. The results are as follow:

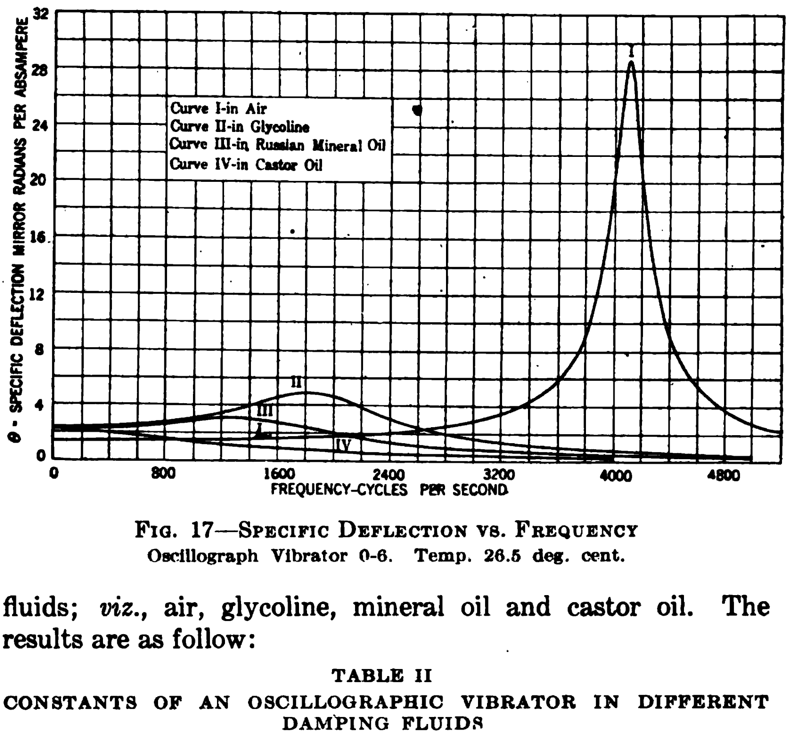

FIG. 17—SPECIFIC DEFLECTION VS. FREQUENCY
Oscillograph Vibrator 0-6. Temp. 26.5 deg. cent.

TABLE II

CONSTANTS OF AN OSCILLOGRAPHIC VIBRATOR IN DIFFERENT DAMPING FLUIDS

Curve	Damping fluid	f_0 cycles/sec	ω_0 rad./sec.	B_0 numeric	f_1 cycles/sec.	f_2 cycles/sec.	Δ hyps/sec.	f_d cycles/sec.	Resonant range cycles/sec.
I	Air	4,110	25,820	0.024	4,013	4,207	609	4,110	194
II	Glycoline	1,891	11,880	0.26	1,523	2,513	3,100	1,827	490
III	Mineral Oil	1,614	10,140	0.43	1,060	2,460	4,400	1,272	1,400
IV	Castor Oil	1,302	8,180	1.1	505	3,365	8,980	0	2,860

Whereas in air, the vibrator had a bluntness of 0.024, or a sharpness of 41.7, in castor oil, the bluntness was increased to 1.1 or a sharpness of 0.9. The effect was also to change the resonant frequency f_0 from 4110 ~ to 1302 ~.

In case III, the resistance r of the vibrator to motion is 0.43 of that which would render the motion aperiodic. In case IV, however, the resistance is ten per cent greater than that necessary for strict aperiodicity. The case is therefore one of over-damping, or ultraperiodicity. It is developed in greater detail in Fig. 17. The curve is drawn from the formulas already given, and the crosses indicate the observations. It will be seen that $f_d = 0$. At resonance, the specific deflection is only about half that at zero frequency.

It may be noted that except in the air-damped case, the value of θ_s is substantially the same for all the damping fluids. This condition seems to be typical.

A considerable number of oscillographs have been investigated by means of the oscillographmeter, and the general results are typically represented by Figs. 15 to 17. It appears that a bluntness of about 0.6 is perhaps the most desirable, in a long-range oscillograph, in the sense that the deviation factor is then moderate, over a considerable range of impressed frequency. Even then, however, the constants ω_o and B should be measured, and the correcting factor worked out, whenever accurate oscillographic measurements are attempted. In general, an oscillograph should have a bluntness between $B = 0.5$ and $B = 0.8$, in order to avoid large correcting factors.

Tests of the Auxiliary Vibrator. An exploration test of θ versus f for one of the oscillographmeters used in this research, is graphed in Fig. 18. It was also tested by the oscillographmeter method. The vibrator is seen to be very sharp. The three principal frequencies are $f_1 = 2511 \sim$, f_o 2514.5 $\sim$, f_2 2518 $\sim$. The resonant range is 7 cycles per second, and the bluntness $B_o = 0.0014$, $\Lambda_o = 718$. In order to present the peak value of θ on the same sheet as the earlier values, the observations are plotted to two different scales. It is evident that provided the resonant frequency is slightly avoided, the phase departure of the auxiliary mirror from zero or 180 deg. in the v.m.t. will be small.

Precautions to be taken. In supporting the auxiliary vibrator in front of the oscillograph, it is important that the auxiliary steel frame does not come so close to the poles of the oscillograph magnet, as materially to disturb the magnetic field in which the oscillograph vibrator swings. At the same time, the auxiliary and oscillograph mirrors have to be brought within a distance of about 1.3 cm., in order to secure good Lissajous figures, without making the auxiliary mirror unfavorably long. If the magnetic field of the oscillograph is somewhat weakened, this will not appreciably affect the identifiable frequencies f_1, f_0, and f_2. It may affect θ_s and θ_0, however; so that where there is reason to believe that the oscillograph's magnetic field has been affected by the proximity of the oscillographmeter, the specific deflections θ_s and θ_0 should be measured with the oscillographmeter removed.

It should be noted that any dissymmetry in the construction of a vibrator, such as unequal dimensions, or unequal tensions in the two strips composing it, or a dissymmetrical mounting of the mirror, may give rise to plural resonances in the action of the instrument. A number of such cases have been observed. A good airdamped oscillographic vibrator requires good mechanical workmanship. When the vibrator is liquid damped these plural resonances often disappear.

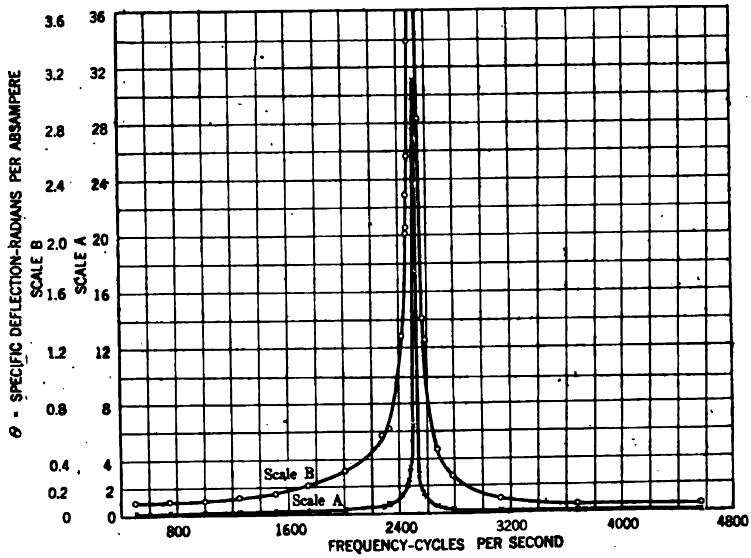

FIG. 18—SPECIFIC DEFLECTION VS. FREQUENCY
Auxiliary Vibrator A-1. Temp. 22.5 deg. cent.

Measurements Necessary for Evaluating A, m, r and s. In order to evaluate the fundamental constants of an oscillograph vibrator, it seems necessary to obtain an additional independent datum, such as is supplied by measuring the motional impedance of the vibrator at resonant frequency. This motional impedance is a pure resistance, and, unlike that of the telephone receiver, has no reactive component at resonant frequency. It is, however, relatively very small. Whereas the resonant motional impedance of a telephone receiver is sometimes over 1000 ohms, and is often over 100 ohms, that of a vibration galvanometer near $f_o = 1000 \sim$ may be near 10 ohms*; that of an ordinary liquid-damped oscillograph is in the neighborhood of 0.025 ohm. Airdamped oscillographs have larger motional impedances.

If we call Z the measured motional impedance of the oscillograph vibrator, at resonant frequency, expressed in absohms, and θ_m the observed max. cy. mirror displacement at resonant frequency per max. cy. absampere; then*

$$A = \frac{Z}{\theta_m\ \omega_o} \qquad \frac{\text{dynes perp. cm.}}{\text{absampere}}\ \angle \qquad (59)$$

$$r = \frac{A}{\theta_m\ \omega_o} \qquad \frac{\text{dyne perp. cm.}}{\text{mirror radian per sec.}} \qquad (60)$$

$$m = \frac{r}{2\ \Delta} \qquad \text{gm.} - \text{cm.}^2 \qquad (61)$$

$$s = m\ \omega_o^2 \qquad \frac{\text{dyne perp. cm.}}{\text{mirror radian}} \qquad (62)$$

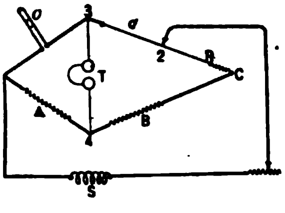

FIG. 19—SLIDE-WIRE BRIDGE
For measuring the motional impedance of an oscillograph vibrator O

An analysis of several oscillographic vibrators is given in Table III.

Technique for Measuring Motional Impedance. A convenient form of slidewire Wheatstone bridge is indicated in Fig. 19, for the purpose of measuring the small motional impedance of a liquid-damped oscillograph. Two equal anti-inductive resistances A and B have about one ohm each. The oscillator, supplying the adjustable impressed frequency, is brought to slider 2 and junction 1. The slider 2 can be adjusted to a fraction of a millimeter, over a manganin slide wire, having a linear resistance not exceeding 0.01 ohm per cm. A pair of low-impedance head telephones T, enables a re-

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*Bibliography 13.
*Bibliography 13.

TABLE III. DATA FOR SOME OSCILLOGRAPH VIBRATORS TESTED

Vibrator Damping fluid	V_1 Glycoline	V2 Glycoline	V3 Glycerine	V4 Glycoline	V5 Glycoline	V6 Glycoline
Span, cm	....	1.26	1.26	1.18	1.25	1.25
Strip width, cm	....	0.0350	0.0350	0.0350	0.0372	0.0372
Strip thickness, cm	....	0.0025	0.0025	0.0025	0.0028	0.0028
Strip spacing, center to center, cm	....	0.055	0.055	0.056	0.052	0.052
Mirror length, cm	....	0.150	0.150	0.130	0.180	0.180
Mirror width, cm	....	0.065	0.065	0.080	0.060	0.060
Mirror thickness, cm	....	0.014	0.014	0.014	0.014	0.014
Strip tension, kilodynes per strip	....	51.5	51.5	74.5	39.1	96.0
Air-gap of vibrator well in cm	....	0.28	0.28	0.23	0.15	0.15
D-c. resistance, ohms	1.591	0.725	0.725	0.670	1.610	1.610
A-c. resistance at the resonant freq., ohms	1.650	0.753	0.745	0.733	1.739	1.739
f_0, ~	2280	2366	1707	2950	2170	2760
f_1, ~	1862	2010	1246	2520	1670	2240
f_2, ~	2790	2780	2340	3480	2815	3390
ω_0, rad/sec	14,320	14,840	10,720	18,520	13,630	17,320
ω_1, rad/sec	11,680	12,620	7,825	15,730	10,480	14,050
ω_2, rad/sec	17,520	17,460	14,705	21,840	17,650	21,300
Z_m, absohms	0.032×10^9	0.0219×10^9	0.0054×10^9	0.02085×10^9	0.0201×10^9	0.030×10^9
θ, rad/absamp	9.15	8.55	2.62	4.41	12.4	8.38
Δ, hyp. rad/sec	2920	2420	3440	3055	3580	3655
A, dyne-p-cm./absamp	244.5	173.0	192.5	255.0	119.0	207.0
r, dyne-p-cm./rad. per sec	1.865×10^{-3}	1.364×10^{-3}	6.85×10^{-3}	3.124×10^{-3}	0.705×10^{-3}	1.428×10^{-3}
m, gm.-cm ^{2}	31.95×10^{-8}	28.3×10^{-8}	99.6×10^{-8}	51.15×10^{-8}	9.85×10^{-8}	19.56×10^{-8}
s, dyne-p-cm./rad	65.5	62.1	114.5	175.3	18.2	58.6
B_0	0.203	0.1295	0.320	0.1645	0.262	0.211

TABLE IV. OSCILLOGRAPHIC RESULTS

			From observations of f_0, θ_0 and θ_s						From Lissajous figures			
Vibrator	Damping fluid	Temp deg. cent.	Δ	f_0 ~	fd ~	ff ~	f_1 ~	f_2 ~	f_1 ~	f_2 ~	w_0	B_0
1	2	3	4	5	6	7	8	9	10	11	12	13
0—1	Castor oil	27.5	5,750	1,486	721	1,170	827	2,657	813	2,570	9,388	0.615
0—3	Air	22.5	1,010	3,235	3,220	3,228	3,079	3,401	3,100	3,420	20,320	0.050
0—4	Air	27	1,430	2,985	2,970	2,978	2,767	3,223	2,760	3,215	18,755	0.096
0—5	Air	28.5	823	2,880	2,875	2,878	2,751	3,013	2,760	3,000	18,095	0.0455
0—5	Castor oil	28	5,700	1,151	Imag	707	561	2,375	513.5	2,475	7,230	0.79
0—6	Air	25	609	4,110	4,110	4,110	4,013	4,207	4,010	4,240	25,830	0.0236
0—6	Glycoline	26	3,100	1,957	1,827	1,891	1,523	2,513	1,464	2,645	12,300	0.252
0—6	Russian Min'l Oil	28	4,400	1,614	1,272	1,454	1,060	2,460	993	....	10,140	0.434
0—6	Castor Oil	27	8,980	1,302	Imag	Imag	505	3,365	517	....	8,172	1.10
0—7	Air	26	92	2,635	2,335	2,635	2,620	2,650	2,615	2,650	16,560	0.0055
0—7	Glycoline	27	1,320	1,674	1,650	1,662	1,475	1,895	1,444	1,957	10,520	0.125
0—7	Russian Min'l Oil	25	3,000	1,614	1,460	1,540	1,202	2,158	1,246	2,133	10,140	0.288
0—7	Castor Oil	27	6,240	1,196	Imag	665	560	2,544	555	2,520	7,515	0.83
0—8	Air	27	1,620	4,210	4,195	4,203	3,962	4,468	3,830	4,570	26,460	0.0613
0—8	Russian Min'l Oil	26	4,150	2,010	1,780	1,900	1,455	2,775	1,404	2,755	12,630	0.33
0—8	Glycoline	27	3,142	2,323	2,215	2,270	1,880	2,880	1,932	2,820	14,600	0.215
0—8	Castor Oil	27	12,400	1,674	Imag	Imag	615	4,565	634	....	10,520	1.18
G	Light damping liquid	....	4,450	3,300	3,140	3,220	2,663	4,077	2,520	4,070	20,740	0.214

sistance balance to be obtained on the oscillograph vibrator O. An extraneous resistance R, connects the end of the slidewire and junction C. A zero balance is first obtained at the resonant frequency f_o, and then at any frequency well removed from resonance. The change in the setting of the slider thus produced enables the motional impedance Z absohms to be evaluated.

Acknowledgments. The authors desire to express their acknowledgments to the American Telephone and Telegraph Co., under an appropriation from which, the experimental researches here reported were carried out; also to Mr. H. G. Crane of Boston, who has had much experience with air-damped oscillographs, for his care, skill and courtesy in preparing for us the improved form of oscillographmeter shown and described in the paper. It is believed that he is willing to furnish similar instruments on demand.

SUMMARY

1. Every mechanical oscillograph vibrator is subject to a vector correction factor, at other frequencies than that at which it is calibrated, and especially at high frequencies.

2. The mechanical laws governing an oscillographic vibrator are precisely similar to those governing a simple alternating-current branch circuit of fixed C, L and R, under variable impressed frequency. The velocity-impedance graph is a straight line. The displacement-impedance graph is a parabola.

3. Every oscillogram, intended for accurate analysis, is completely defined in regard to its correction for frequency by two constants; namely, f_o the resonant frequency, and B_o the oscillatory bluntness. These two constants, or their equivalent data, should be measured, and recorded on the oscillogram.

4. If the two constants f_o and B_o,—the resonant frequency and bluntness of an oscillograph,—are measured, its behavior and correction factors for all frequencies can be computed relatively to its calibration at any particular frequency.

5. In ordinary liquid-damped oscillographs, f_o commonly lies between 1000 $\sim$ and 5000 $\sim$. The greater the damping, the lower f_o tends to become.

6. The bluntness of resonance B_o indicates the ratio of the frictional resistance present to the resistance necessary for strict aperiodicity. When $B_o = 1$, the vibrator is just aperio-

dic. The bluntness of an air-damped vibrator may be as low as 0.0014. Oscillographs damped in castor oil may readily have bluntness greater than 1, or be ultraperiodic. The best bluntness for a long-range oscillograph, in order to have relatively small corrections to be applied at moderately high frequencies is, perhaps, near $B_o = 0.6$.

7. The value of f_o for an oscillograph may conveniently be determined by means of a simple permanent-magnet auxiliary vibrator, operating from the same source, and so arranged as to supply a critical and recognizable form of Lissajous optical figure at resonance.

8. Having identified f_o for an oscillograph, the bluntness B_o may be determined, by measuring the specific deflection at a low frequency and also at the resonant frequency.

9. A number of actual oscillographs have been tested, and typical results, obtained from them, are reported.

10. In order to obtain the fundamental constants A, m, r and s, of a vibrator, it is desirable to measure its motional impedance at resonance, in addition to securing the measurements for f_o and B.

11. In analyzing the oscillograph of a periodically recurring complex harmonic wave into its Fourier components, the analysis is first made, as though the oscillograph had no error with change of frequency, and then each evaluated harmonic component should be corrected, both in magnitude and in phase position, by applying the correction factor for that frequency, in view of the values of f_o and B for the instrument.

12. A number of curves of deviation factors for different values of B have been worked out, and are presented for use, in order to supply approximate corrections by inspection.

LIST OF SYMBOLS EMPLOYED

A Torque constant of a vibrator, or the torque produced by unit current

$$\left(\frac{\text{dyne perp. cm.}}{\text{absampere}}\angle\right).$$

$\alpha = \tan^{-1}\left(\frac{L\,\omega - 1/C\,\omega}{R}\right)$ Slope of an impedance to current (radians or degrees).

$B = \Delta/\omega_o = B_o$ Bluntness of resonance of a vibrator or branch circuit based on τ_o (numeric).

$B_s = 2\,B_0$	Bluntness of resonance of a vibrator or branch circuit based on τ_s (numeric).
β	Slope of a displacement admittance, varying between − 0 deg. and − 180 deg. (degrees)
C	Capacitance of a condenser (farads).
$c = 1/s$	Slackness of a vibrator $\left(\frac{\text{radians}}{\text{dyne perp. cm.}}\right)$.
Δ	Damping constant of a vibrator, or hyp. angular velocity of decay $\left(\frac{\text{hyps.}}{\text{sec.}}\right)$.
$D = y'/y_s'$	Deviation factor, or ratio of vector displacement at ω to that at ω_s (numeric $\angle$)
E	Maximum cyclic e.m.f. impressed on a branch circuit (volts $\angle$).
F	Maximum cyclic torque impressed on a vibrator (dynes-perp-cm.).
f, f_0, f_1, f_2 .	Frequency, with reference to resonance and quadrantal values (cycles per sec.).
G	Conductance of a branch circuit (mhos).
$g = 1/r$	Mechanical conductance of a vibrator (mechanical abmhos).
I	Alternating current supplied to a branch circuit (max. cyclic amperes $\angle$). also alternating current supplied to a vibrator (max. cyclic absamperes $\angle$).
Θ	Maximum cyclic displacement produced in a vibrator mirror (radians $\angle$).
Θ_i	Initial cyclic displacement produced in a vibrator mirror (radians $\angle$).
$\dot{\Theta}$	Maximum cyclic angular velocity produced in a vibrator mirror $\left(\frac{\text{radians}}{\text{sec.}}\ \angle\right)$.
$\theta = \Theta/I$	Specific mirror deflection, or max. cy. displacement per r.m.s. absampere, produced in a vibrator $\left(\frac{\text{radians}}{\text{absampere}}\ \angle\right)$.
$\theta_s,\ \theta_0$	Specific deflections produced at a low frequency, and at resonant frequency $\left(\frac{\text{radians}}{\text{absampere}}\ \angle\right)$.

θ_m	Max. cy. displacement per max. cy. absampere produced in a vibrator $\left(\frac{\text{max. cy. radians}}{\text{max. cy. absamp.}} \angle\right)$.
$j = \sqrt{-1}$	
$K = 1/D$	Correction factor for the max. cy. displacement at any impressed ang. vel. ω, to that at ω_s (numeric $\angle$).
L	Constant inductance of a branch circuit (henrys).
m	Moment of inertia of a vibrator system (gm − cm.^{2}).
Q	Quantity of electricity cyclically displaced in a branch circuit (coulombs).
R	Constant resistance of a branch circuit (ohms)
r	Resistance to motion of a vibrator $\left(\text{mechanical absohms or } \frac{\text{dyne-perp.-cm.}}{\text{radian per sec.}}\right)$
s	Stiffness coefficient of a vibrator $\left(\frac{\text{dynes perp. cm.}}{\text{radian}}\right)$.
t	Elapsed time (seconds).
$\tau = \tau_0 = 1/\Delta$ $= \frac{2L}{R}$	Oscillatory time constant of a circuit or system $\left(\frac{\text{seconds}}{\text{hyp. radian}} \text{ or seconds}\right)$.
$\tau_s = \tau_0/2 = L/R$	Non-oscillatory time constant of a circuit (seconds).
$u = \omega/\omega_0$	Ratio of an impressed angular velocity to the resonant angular velocity of a system (numeric).
$\Lambda = \omega_0/\Delta$ $= \Lambda_0 = 2\,\Lambda_s$	Sharpness of resonance of a branch circuit or vibrator based on τ_0 (numeric).
$\Lambda_s = \Lambda_0/2$	Sharpness of resonance of a branch circuit or vibrator based on τ_s (numeric).

v.m.t.	Vibromotive torque (dynes perp. cm.).
$Y = 1/Z$	Admittance to alternating current in a branch circuit (mhos $\angle$).
$y = 1/z$	Admittance to angular velocity in a vibrator $\left(\text{mechanical abmhos or } \frac{\text{radians per sec.}}{\text{dynes perp. cm.}}\ \angle\right)$
$Y' = 1/Z'$	Admittance to displacement in a branch circircuit $\left(\frac{\text{coulombs}}{\text{volts}}\ \angle\right)$.
$y' = 1/z'$	Admittance to displacement in a vibrator $\left(\frac{\text{mirror radians}}{\text{dyne perp. cm.}}\ \angle\right)$.
Z	Impedance to alternating current in a branch circuit (ohms $\angle$).
z	Impedance to angular velocity in a vibrator $\left(\text{mechanical absohms or } \frac{\text{dynes-perp. cm.}}{\text{radians per sec.}}\ \angle\right)$.
Z'	Impedance to displacement in a branch circuit $\left(\frac{\text{volts}}{\text{coulombs}}\ \angle\right)$.
z'	Impedance to displacement in a branch vibrator $\left(\frac{\text{dynes perp. cm.}}{\text{radian}}\ \angle\right)$.
ω	Impressed angular velocity of circuit or vibratory system $\left(\frac{\text{radians}}{\text{sec.}}\right)$.
$\omega_0, \omega_1, \omega_2$	Impressed angular velocity of resonance and quadrantal values $\left(\frac{\text{radians}}{\text{sec.}}\right)$.
$\angle$	Sign of planevector or complex quantities.

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DISCUSSION ON "OSCILLOGRAPHS AND THEIR TESTS" (KENNELLY, HUNTER AND PRIOR), NEW YORK, N. Y., FEBRUARY 20, 1920.

F. S. Dellenbaugh, Jr.: The paper in addition to a very valuable method of calibrating oscillographs, presents a most interesting change in the philosophy of attacking problems of this sort. It was not so very long ago, 25 years at the most, when electrical circuits were so little understood that mechanical analogies were used as devices to explain electrical action. Now the electrical circuit can be analyzed so simply that the method of attack is completely reversed, and electrical analogies are used to explain mechanical action.

It also is the first time, to the best of my knowledge, that the term Displacement Impedance has been used, and an equation derived for it.

It has been seen from the illustrations that the locus of this impedance is a parabola in the first and second quadrants, with $y = o$ at $x = 2$ and extending to $x = -6$. The upper half of the parabola only is given however, and if we complete the curve and examine the meaning of the additional branch we find a more generalized application of the impedance equation.

The equation of the parabola is:

$$Z' = (S - L\,\omega^2) + j\,R\,\omega$$

So plotted that $(S - L\,\omega^2)$ forms the abscissas and $j\,R\,\omega$ forms the ordinates. Thus any line drawn from the origin to the curve represents the impedance vector and its projections on the axes give its coordinates in the form of a complex number.

It is evident that in order to have the lower part of the parabola, R must become negative, since $j\,R\,\omega$ must be negative, and ω cannot be negative, as that would also affect the abscissas. Therefore the generalized curve involves $-R$. In the electrical circuit this of course means a generated e. m. f. In the mechanical circuit this means negative friction, or an applied external force.

If we substitute a term "e" representing a generated voltage in the electrical circuit, and specify that it must be of the same frequency and in phase with the applied e. m. f. we obtain the equation:

$$V = i\,Z = i\,R + e + j\,i\,(L\omega - S/\omega)$$

From which

$$i\,Z'' = i\,(S - L\,\omega^2) + j\,(R\,i\,\omega + e\,\omega) = i\,(S - L\,\omega^2) + j\,\omega\,(R\,i + e)$$

Where V is the applied voltage and i the resulting current, the whole equation being put in the voltage form in order to introduce e unmodified. It is of course possible to replace any voltage e by a resistance drop $i\,R'$, and if we let:

$$R' = -K\,R$$

Then

$$Z'' = (S - L\,\omega^2) - j\,R\,(K - 1)\,\omega$$

If K is greater than 1.

Thus for various values of K we get a family of curves all having the same general form as the locus under consideration and converging from both sides of the X-axis to a straight line coincident with the X-axis when K is unity.

When $K = 2$ we get the lower half of the same curve that is already represented in the positive ordinate portion presented.

The impedance now is:

$$Z'' = (S - L\,\omega^2) - j\,R\,\omega$$

That is, the displacement of the current which will pass through this circuit is equal and opposite to that which would have passed through the circuit represented by the original equation for Z'. In other words the system is delivering power at the same rate at which it previously absorbed it, and all phase relations, must, *a priori,* remain the same, except that the current has shifted 180 deg.

Following out the same line of reasoning as applied in the original analogy we find for the mechanical case that the lower half of the parabola is the locus of the mirror displacement on an oscillograph which must be obtained by applying an external force as a torque producing forced oscillations of the mirror of the same magnitude and the opposite phase as would have held for the free case, the value of the force being twice the friction and always opposite it in direction.

In other words the oscillograph will now deliver as a generator current of the same characteristics as it previously absorbed as an instrument, but opposite in sign.

This does not have any immediate useful application but extends the scope of the analogy. It is interesting to note that a generator similar to this type has actually been proposed, and I believe constructed, in connection with producing small amounts of high-frequency power for testing circuits in radio telegraphy. It is also interesting to note that the efficiency of such a generator may be made quite high, as there is no inherent limitation other than the usual friction and resistance losses.

As a conclusion from this expansion it might be stated that the lower half of the parabola should be added and the complete curve drawn, for although in the immediate application its values are fictitious, other cases of similar mechanical analogy might give it real importance.

B. W. St. Clair: I think that this is one of the best contributions to the oscillograph art we have had in a number of years. Anyone who has had to do with general testing, and especially with development work will appreciate that there are many limitations to our present oscillographs. There are occasions when it has been necessary to give unsatisfactory answers to inquisitive engineers about the faithfulness with which reports represented the happenings in some complicated circuit or machine.

The oscillograph has proven to be one of the finest engineering instruments we possess and it has enabled us to solve many problems that could not have been approached without it. Its range of usefulness, however, has not been as extensive as might be desired on account of the rather low natural period of the vibrator. For that reason we have had to effect a compromise between sensitivity and natural period. In cases where the sensitivity had to be much increased, the natural period was often lowered so far as to make the accurate recording of the higher harmonics of a 60-cycle wave impossible. The normal vibrator takes considerable current for its operation, and, there are many instances where we have wished we might increase its sensitivity a hundred fold. It is possible in extreme cases to increase the sensitivity fifty fold. But this can only be done by a serious lowering of the natural period. This, of course, represented a compromise between accuracy of recording and ability to get measurable deflections. Until the advent of Dr. Kennelly's paper it was impossible to correct the oscillogram for limitations of the vibrator at high frequencies.

There is another application of Dr. Kennelly's work that is going to be very helpful in modern oscillograph practise. It is the ease with which it is now possible to determine the damping factors of our vibrators. Previously, every time it was necessary to make a serious change in the vibrator, either in tension on the strings, or the size of the mirror, or the dimensions of the strings, it was necessary to make elaborate tests to determine how near aperiodic the damping was. It involved the making of films at high speeds showing the way the vibrator returned to zero when suddenly disconnected from a storage battery. Several times in cases where it was necessary to use increased tension, the damping was so much less than critical that the vibrator made appreciable deflections on the negative side of the zero line. In general, it has been our experience that with oil damped vibrators the damping is excessive; that is, there is too much frictional resistance for the vibrator to be aperiodic. It will now be possible to determine how far our damping is from critical by making a determination of Dr. Kennelly's "bluntness constant." This is a much easier thing to do than the older test for damping.

This question of damping is a very important one, especially on impulse work. A number of cases have come to our attention where erroneous results were given because our damping was not sufficiently great. There have been cases where it has been necessary to increase the vibrator tension almost to the breaking point, and to decrease the size of the mirrors almost to the vanishing point in order to get a sufficiently high period for some particular impulse test. Experience showed that every vibrator under these conditions needed checking for damping. The determination of the bluntness constant will be easier than the making of films of the damping. We cannot use Dr. Kennelly's methods in their present form directly for the correction of impulse results, but they will be useful for checking damping, which is so very important in this kind of work.

Again it will be possible to make this determination at the point where the oscillograph is to be used. In many cases it is necessary to use the instrument in hot engine rooms where the damping is far from what it was in the laboratory.

N. E. Bonn: Mr. St. Clair brings out the fact that it requires a tremendous amount of skill to accurately operate the oscillograph. You not only have to take pictures and develop them, but you have got to measure quantities such as B and F in the damping, and the resonant frequency of the oscillograph element. The oscillograph is limited in its usefulness to a certain amount of energy, that is, since we must use the vibrator very near the critical damping point, the use of a varying coil is necessary which means that you cannot detect very small amounts of electric current. The sensitivity is materially decreased. In certain cases I can see how the oscillograph cannot be used, because it may be that the natural frequency of the vibrator, by which you make the measurements, is just as high as, or perhaps higher than, the natural frequency of the oscillograph meter.

It might be well to mention here that there are certain classes of work in which the old point to point method is still preferable to the oscillograph, because it is much more simple, and there is no chance of errors which have to be corrected, especially if you consider some of the more modern types of the point to point methods, such as the Rosa curve tracer. That is an instrument that gives absolutely correct results and at the same time reduces the labor of getting the wave shape to a fraction of its former value. It is capable of plotting curves in a comparatively short time, with very great accuracy, because it follows the potentiometer method of measurement.

We have taken a further step in that direction and now we think we will be able to develop an instrument which will be entirely automatic, which will automatically plot the coil—of course, if the coil is periodic, it must be a periodic phenomenon or must be capable of being made periodic by some external means. The instrument which is being developed is going to be altogether automatic, so that the only thing to be required will be to press a button and throw a switch, and the instrument will automatically give you the true shape of the curve you are looking for within certain frequencies. I think that 2000 frequency is the upper limit of the point to point method of wave-shape determination, because of the impossibility of getting a contact of very short duration.

M. A. Rusher (by letter): I agree with the authors of this paper that for certain extremely accurate tests in which high frequencies are involved, more attention should be given to the matter of calibrations than has usually been given. One who is not well acquainted with the use of the oscillograph is likely, however, to infer from this paper that the operation of the oscillograph is inherently bad, which is not true. New uses are being made of it everyday with perfectly satisfactory results.

Assuming that the apparatus and tests here presented are the best possible, it is still true that it has no application to all of those tests involving transient phenomena. Moreover, it is not necessary to apply any such correction to the wide range of investigations involving direct current and alternating current of low frequencies only. Even in the case of wave shape records, which seems to be the case which has received the most attention, I believe that results will show that the corrections are small except in relatively few cases.

The tests suggested do not seem to be so readily made as indicated. Unless they are made under the exact conditions of test in regard to temperature of damping liquid, etc., I do not see that they are of much benefit. It appears to me that it would be necessary to remove the complete galvanometer from the oscillograph and either place it in a special box with special optical system and the oscillographmeter or else have these properly mounted in a dark room where the necessary Vreeland Oscillator is available for the test. Tests of resonant frequency should be made with the damping liquid at different temperatures and a curve plotted between temperature and resonant frequency.

By knowing the temperature of the damping liquid in any later test, the proper corrections could then be applied.

In order to test the oscillographmeter, advantage can be taken of the fact that the resonant frequency without damping is the same as the natural free period of the vibrator. This can be very easily determined by taking an oscillogram at the instant of disconnecting a direct current from the vibrator circuit, at the same time recording a wave of known frequency from another vibrator. The period of the vibration without damping can be found by comparing the two waves over any given period. It would simplify the matter of corrections if all could be referred to this natural free period of vibration as it can be determined without any special apparatus. I believe that when standard vibrators with the ordinary damping liquid are used, it would only be necessary to make the tests suggested by the authors on a few vibrators at different temperatures of damping liquid and from these, constants could be determined which would apply to any vibrator of the same type. The accuracy would be almost as great as if the tests were taken on each vibrator at sometime previous to using it in the oscillograph.

A. E. Kennelly: Mr. Dellenbaugh has shown us that the parabolic curve which is only a half parabola in the paper, is really part and parcel of a whole parabola, the lower part being the generator part, and the upper part the motor part. Mr. Dellenbaugh is the father of the lower half of this parabola. Cutting the parabola in two in the middle is always an unscientific process, from the geometrical standpoint, and a complete parabola is always preferable if only on theoretical grounds.

Mr. Hunter has contributed largely to this work by a great deal of patient observation, and I am sorry he did not extend his remarks.

Mr. Bonn has referred to the wave tracer, and there is room in electrical laboratories for all kinds of instruments. We want oscillographs and wave tracers, and there is always room for various types, and I am glad to know that a wave tracer is likely to be developed which does not take too long to get around the cycle. As a rule, by the time you get around the cycle, when you start to trace the waves the second time, one is apt to find the wave has changed, and when you start to trace it the third time, one finds that the wave is still further changed. If you could get the tail end before the head changed, you might get a complete curve, and it is gratifying to know that we may get the tail and head to agree with the new instrument that is suggested.

The oscillographmeter is not limited by its particular frequency of resonance. All that is necessary is that its resonance shall not be near to the resonance of the tested instrument. It may be above or below and if it is used above the natural frequency, the results are as good as if used below. We have used it both ways, and it is hard to tell the difference.

The remarks of Mr. St. Clair are very interesting. I am afraid that test of his may still be necessary to determine the damping for impulse work, because the work described in this paper applies mainly to sustained oscillations and is not directed to impulse oscillograms which differ as much from sustained oscillograms and wave forms as free oscillations in electric circuits differ from sustained oscillations. There is another chapter to be contributed by somebody in dealing with impulse oscillograms.

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THE ACCURACY OF COMMERCIAL ELECTRICAL MEASUREMENTS

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The paper discusses the accuracy required in commercial electrical measurements, and the means of obtaining it, namely, proper choice, installation, use, and maintenance of instruments. Conditions of use, external disturbing influences, and features of design and construction affecting the accuracy are considered in detail. The best types of instrument for measuring voltage and current are mentioned.

The sources of error in electrodynamic wattmeters are discussed, including the effect of instrument transformer errors. The principal factors affecting the accuracy of watthour meters are given. In conclusion, some improvements which should be soon forthcoming are mentioned.

1. INTRODUCTION; DEFINITIONS

THE object of this paper is to discuss the need for accuracy in practical electrical measurements,* the means by which it may be obtained, and the effect of external conditions and manner of use upon the results. The discussion is limited mainly to commercial measurements, and is intended more especially for the average user who is neither ignorant of the subject nor a laboratory expert. It is hoped, however, that the latter, as well as designers and makers of instruments, may find some points of interest.

It is not often realized to what extent measurement runs through the whole structure which the electrical engineer has created. We have at our disposal such varied means for electrical generation, distribution, conversion and utilization that we are apt to think of measurement only in terms of electrical instruments and meters. Yet almost any device we can mention, as for example an incandescent lamp, requires in its manufacture a great variety of measurements, each of which must be made with some definite accuracy if the lamp is to give a satisfactory performance. The control of a large electric system by a load dispatcher is another example. Here the state of affairs in the various parts of the system is shown by the instrument readings, and we may liken the meas-

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*This paper is written from the laboratory viewpoint, but the effects of service conditions and operation are rather fully discussed. It is not possible to secure as high accuracy in the field as in the laboratory, and in comparing service performance with laboratory performance this fact should be kept in mind.

uring apparatus in this case to the nervous system by which the brain is kept constantly advised of the condition of the entire body.

Accuracy should be clearly distinguished from sensitivity. An instrument is sensitive when a relatively small change in the quantity under measurement produces an appreciable motion of the index; it is accurate when its indications differ only slightly from those of suitable standard instruments. A well-made ammeter is sensitive to 0.1 per cent at or near full-scale reading, but may fall short of being accurate by several per cent because of bad adjustment.[1]

While initial accuracy is important, sustained accuracy in service is what counts in the long run, and it is quite possible in some cases to get better sustained accuracy from certain types of instruments than from others of much higher initial accuracy but of more delicate and intricate construction.

The accuracy of an instrument may be expressed in several ways. The simplest and in many respects the most unsatisfactory is to state that the instrument is accurate within a certain percentage. Such a statement can apply to only a part of the scale. For example, it was at one time customary for the maker to certify that precision direct-current portable voltmeters were accurate to 0.2 per cent. At full scale (say 150 divisions) this amounts to 0.3 of a division, and such an accuracy is quite possible. However, at 50 divisions the same percentage error is only 0.1 division, which is approaching the limit of reading. Such a guarantee should be qualified by stating the part of the scale over which it applies. Even then, this permits errors of increasing amount, expressed in divisions, as the upper end of the scale is approached, which from the user's standpoint is an unnecessary limitation.

A method much superior to the preceding is to certify that the instrument is correct to within a given percentage of the full scale value. Such a guarantee is stated (or understood) to apply over the entire scale, and in uniform-scale instruments amounts to saying that no division mark is out of its proper position by more than a certain linear distance. For instruments which are to be used for general testing or experimental work and will therefore be used over a considerable

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[1] For a more detailed discussion of the terms sensitivity and accuracy, see "Variance in Measuring Instruments," by F. J. Schlink, Scientific Paper No. 328 of the Bureau of Standards.

part of the scale, such a guarantee seems to be a satisfactory one for all concerned.

2. ACCURACY REQUIREMENTS

It is not easy to specify just what accuracy is required for a given class of electrical instruments. It is a waste of money to expend labor in adjusting apparatus to a higher accuracy than the work requires, and it is no less so to vitiate the results of an important test by the use of one or more instruments of insufficient accuracy. Aside from their extra cost, instruments of needlessly high accuracy are sometimes less rugged than cheaper ones of sufficient accuracy.

However, it is not usually practicable for the maker to determine which individual instruments will be used for approximate measurements and which for work requiring greater accuracy. In general, therefore, he must adjust all instruments of a given type and grade to an accuracy appreciably greater than the lowest tolerable for some particular purpose. For his own guidance and that of the user, he should have standard requirements for instruments of definite grades, and all instruments should be clearly marked to show their grade. This need was recognized in England ten years ago by the Engineering Standards Committee, which issued the "British Standard Specification for Ammeters and Voltmeters." A revised edition has just been issued, covering also wattmeters, frequency and power factor meters. Graphic ammeters, voltmeters, and wattmeters are the subject of a separate specification. Ammeters, voltmeters, and wattmeters are divided into three grades, namely, Sub-Standards, First Grade, and Second Grade. Each instrument must be marked to show the grade to which it belongs.[2]

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[2] The accuracies required by the British Standard Specification are as follows:

Kind and Grade		Accuracy
Voltmeters, sub-standard	±	0.2% of full scale value.
Ammeters, sub-standard	±	0.5% of full scale value.
Voltmeters, first grade	±	1% of the reading, above half scale;
	±	0.5% of full-scale value, below half scale.
Ammeters, first grade	±	2% of the reading, above half scale;
	±	1% of full-scale value, below half scale.
Voltmeters, second grade	±	2% of the reading, above half scale;
	±	1% of full-scale value, below half scale.
Ammeters, second grade	±	4% of the reading, above half scale;
	±	2% of full-scale value, below half scale.

In the case of ammeters with external shunts, the error of either the shunt or the millivoltmeter with its leads must not exceed one-half the specified tolerance for the given grade.

In an interesting paper on "Guarantees of Accuracy of Instruments for Industrial Electrical Measurements," G. Campos[3] points out that the radically different method of use of voltmeters and ammeters requires recognition in the specification of accuracy for the two. He says:

> While it is desirable for a voltmeter to be accurate within 1 per cent in the very limited region in which it is used, and even a greater accuracy is useful in many cases, it is on the other hand entirely superfluous that such values be maintained when by some accident the voltage drops to half the normal, at which time it is more than sufficient for the operator or the user to follow roughly the variations, either the coming up, or the renewed fall of the voltage; in such cases 2 or 3, or even 4 per cent accuracy is practically sufficient.

Campos's suggestion implies that the cost of voltmeters could be reduced, or the accuracy over the working part of the scale could be increased for the present cost, if a much lower accuracy were permitted outside the working range.

He also points out that the unnecessary requirement of too high an accuracy not only means a useless increase in cost but may lead to the impairment of some essential qualities. He makes the suggestion that in addition to the more precise but more delicate instruments used for the normal operation of a plant there ought to be provided emergency instruments, especially ammeters, which will give rather rough but reliable indications in case of short circuit or similar accidents.[4]

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[3] *L'Elettrotecnica*, vol. 4, pp. 123-127; 1917.

[4] This applies only to abnormal currents which last for an appreciable time. Direct-current ammeters having this feature have been constructed by Jules Richard of Paris. In this ammeter the moving coil is controlled by the usual two spiral springs over, say, the lower two-thirds of the scale. When the coil moves farther, a third spring also opposes the motion, and thus the current required for a given increment of angular motion is increased. For example, the lower two-thirds of the scale may be graduated from 0 to 100 amperes, while the upper third extends from 100 to 300 amperes. This type of ammeter is used, for example, in connection with motors, the running current being observed on the lower two-thirds of the scale and the large momentary starting currents on the upper third. If an ammeter of the usual construction is used for such a purpose, the angular deflection given by the running current is too small for accurate reading.

3. How Accuracy may be Obtained

A required degree of maintained accuracy in service requires attention to all of the following points: (1) choice of the most suitable operating principle; (2) selection of a competent and experienced maker; (3) correct installation; (4) intelligent use; (5) suitable maintenance and testing.

(a) *Choice of Operating Principle.* For nearly all direct-current purposes the permanent-magnet moving-coil instrument is the most desirable. This type is so well known that no description need be given here. It has a practically uniform scale, which facilitates the mechanical sub-division of the scale between the points laid out by comparison with a standard, and also makes it easier to estimate fractions of a division. This type is made in a great variety of sizes and styles, from which it is possible to select instruments for all but very special requirements.

For alternating current the question of choice is not so simple. There are available five types, namely, moving-iron, electrodynamic,[5] induction, hot-wire, and thermocouple instruments. Electrostatic instruments are available for the measurement of alternating voltage and power, but on account of their delicacy they are more suitable for the laboratory than for commercial work, and they are used only when it is essential or very desirable that no current be taken by the voltage element.

In choosing a-c. ammeters and voltmeters, one finds two schools of thought favoring the moving-iron and the induction type respectively. The advocates of the moving-iron type point out that well made ammeters on this principle are useful with only slight changes in accuracy over the whole commercial range of frequency, and that even at say 500 cycles the change in accuracy as compared with that at ordinary frequency is only a few per cent. Those who champion the induction type point out the relatively low torque and torque-weight ratio and the delicacy of the parts of the moving-iron and electrodynamic types, and contrast these with the ruggedness, high torque and torque-weight ratio, and ease of repair by ordinary operators which characterize the induction in-

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5. The use of the shorter adjective form "electrodynamic" is suggested as preferable to the use of the somewhat clumsy noun "electrodynamometer" as an adjective. The French have used "électrodynamique" for a number of years, setting us a good example.

strument. They also point to the widespread use of the induction watthour meter and its practical advantages over the electrodynamic (direct-current) watthour meter. The advocates of the moving-iron and electrodynamic instruments, on the other hand, point out that induction instruments can be adjusted for only one frequency (two in the case of induction ammeters), and hence would be relatively more affected by departures from normal frequency. The matter is not one that can be decided off-hand, but needs to be considered in connection with the circumstances of use, including the kind of labor available for maintenance of instruments. However, it is not amiss to point out that for experimental or other work where the frequency must be varied, the induction type is unsuitable.[6] Other features affecting the choice are the effects of stray magnetic fields, ability of the ammeters to withstand short circuits, and scale length per unit area of space occupied on panels, which favor the induction type, and low impedance of the ammeters which favors the moving-iron type.

For laboratory use at usual frequencies, when the desired accuracy is better than the moving-iron type will give on various frequencies and wave forms, the electrodynamic type is preferable because it can be checked on reversed direct current, and its error due to inductance can be readily computed for any given frequency.

For the measurement of high-frequency currents there is a choice between the hot-wire (expansion) instrument and the more recent thermocouple instrument. The hot-wire instrument has a number of failings, including zero shift, liability of damage from overloads and rough handling, and sluggishness of response to changes of the current. The hot-wire voltmeter is practically independent of the frequency of the circuit. The comparison of a 150-volt hot-wire voltmeter with an electrostatic voltmeter of similar range, first on direct current, then on 180,000 cycles, gave no indication of any effect of frequency, within the limit of reading. The thermocouple instrument has a permanent-magnet moving-coil system which is connected to a thermocouple or a bridge

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[6] This may seem like a needless warning, but the writer has in mind the case of a laboratory intended for commercial research and investigation which obtained a large outfit of induction instruments because their limitations were not recognized.

arrangement[7] of thermocouples. It appears probable that the operating advantages of the thermocouple instrument will cause it eventually to supersede the hot-wire instrument. For some purposes, however, the hot-wire instrument has an advantage over the bridge form of thermocouple ammeter in that the former indicates correctly the value of a direct current regardless of its direction, while the thermocouple ammeter may even deflect below zero for one direction of the current.

(b) *Scale Law.* This term may be used to denote the manner in which the length of a division varies over the scale. If this length is constant, the scale is said to be linear. If the scale divisions increase in length going up scale, and in such a way that n times the electrical quantity gives n^2 times the angular deflection, the instrument has a quadratic or square-law scale. The latter are found in the precision ammeter and voltmeter made on the Kelvin balance principle, and the torsion head electrodynamic voltmeter. Many instruments have scale laws which are neither linear nor quadratic; for example, some electrodynamic voltmeters have a scale which for the first half of the angular deflection resembles the quadratic, but which shows a decrease in length of a division from about the middle on to the upper end of the scale. The reason for this departure will now be given, with a discussion of the scale law as affecting the initial and sustained accuracy.

Strictly speaking, no instrument scale is exactly linear or quadratic. Neglecting pivot friction, a permanent-magnet moving-coil instrument would give a scale law more and more nearly linear as the angular length of the scale is reduced. Over the whole scale there will be departures from uniformity of length of the divisions, because of variations in the strength of the magnetic field from point to point along the gap. The electrodynamic voltmeter or ammeter will tend more and more toward the quadratic scale as the angular length of the scale is reduced. There are two distinct causes at work; first, the general operating principle, second, the geometrical relations between the fixed and the moving coils which determine for each position of the moving coil the strength and direction of the magnetic field whose interaction with that of the moving element produces the torque. The latter cause

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[7] The bridge arrangement of thermocouples is described by P. Gossen, *E. T. Z.* vol. 31, pp. 143-4; 1910.

may be called the "geometry" of the operating system, and the way in which it operates may be seen from the following illustration.

In an electrodynamic voltmeter in which a moving coil turns within a fixed coil, there are two positions of the moving coil in which no current, however large, can produce torque. One occurs when the coil is turned below its normal zero position until the plane of the moving coil is parallel to the plane of the fixed coil. The other position is above the full scale point, with planes parallel as in the preceding. The latter is a stable position with current flowing, the former is unstable. In the stable position the magnetic fluxes set up by the two coils are additive, while in the unstable they are opposing. Between these two extreme positions the torque corresponding to a given current is not constant, but depends on the form and relative dimensions of the coils. If an arrangement of coils could be used such that the torque for a given current is constant over a certain part of the range of motion of the moving coil, and the scale be limited to that range, then the instrument will have a quadratic scale if made as a voltmeter or an ammeter, or a linear scale if made as a wattmeter. Much effort has been spent in devising forms of coils that would give this result, and some unusual and difficult forms have been used; for example those of Bruger and Raps[8]. However, further research has shown[9] that with proper relative dimensions the desired results can be had with sufficient approximation using simple circular or rectangular coils of very reasonable form. In any case the moving coil must not approach too near the limiting positions of zero torque.

By varying the coil geometry it is possible to give the scale special features. For example, a well-known American portable wattmeter is designed so that the divisions near the zero are 2.7 times as long as those near the upper end of the scale. This scale is approximately a logarithmic[10] one, and tends to give equal percentage accuracy on all deflections.

As an extreme case of what can be done in altering the natural scale law by special geometry may be mentioned a

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[8] Bruger, *E. T. Z.* vol. 15, pp. 331-4, 1894; Raps, *E. T. Z.* vol. 20, pp. 665-670, 1899.

[9] Sack, *E. T. Z.* vol. 28, pp. 268-271, 1907.

[10] Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., p. 9.

form of electrodynamic ammeter and voltmeter[11] made by Hartmann and Braun. In this instrument, which inherently would tend to the quadratic scale, a special arrangement of coils gives a scale which is nearly linear over more than nine-tenths of the range of current or voltage measured. The instrument possesses great interest as showing what can be done in the way of meeting special requirements with relatively simple coil geometry. The scale of such an instrument is shown in Fig. 1. In general, however, instrument scales should follow the natural law of the instrument, if an efficient design is desired.

While agreeing with the theoretical basis of the logarithmic scale, the writer wishes to call attention to a practical defect of electrodynamic voltmeters and wattmeters having such scales. The defect consists of an appreciable effect upon the accuracy when the index is slightly bent and the resulting zero error is removed by shifting the spring abutment, as is done by turning the zero adjusting device with which most modern instruments are equipped. If the geometry is such that the instrument has a linear scale as a wattmeter, or a quadratic scale as ammeter or voltmeter, and if further there is a small angular distance α below the zero and a similar distance above the full scale mark which could be used as extensions of the scale, if desired, with no change in length of division as a wattmeter, then a bending of the index by an angle not exceeding α, either above or below the zero, followed by a return of the index to zero by using the zero adjustment, will not affect the accuracy of the instrument. We may define the desired geometry for the electrodynamic type

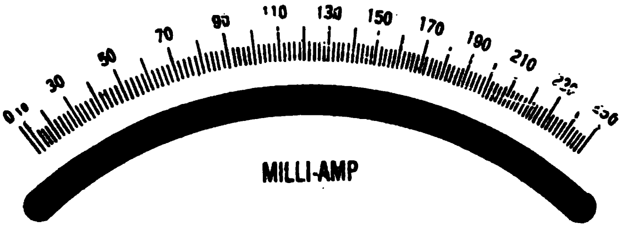

FIG. 1—LINEAR SCALE OBTAINED IN ELECTRODYNAMIC INSTRUMENTS BY SPECIAL COIL GEOMETRY

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[11] Bruger, *E. T. Z.* vol. 25, pp. 822-5, 1904, *Phys. Zs.* vol. 4, pp. 876-881, 1903.

mathematically by saying that if M is the mutual inductance between the fixed coil and the moving coil and θ the angle by which the moving coil is deflected from its position of rest, then over the whole length of the scale and including the angular margin α at each end of the scale, $dM/d\theta$ should be constant. When a wattmeter has other than a uniform scale[12] $dM/d\theta$ varies from point to point. If in such an instrument the index be bent and the zero error be then corrected by moving the spring abutment, the values of $dM/d\theta$ corresponding to 10, 20, . . . divisions of the scale are now different from the normal, and the instrument shows errors which may be serious. The existence of this source of error has been known[13] for a number of years, and its numerical magnitude for two well-known types of instrument is shown by the results of tests recently made in the writer's laboratory. Four instruments were included, namely, a wattmeter and a voltmeter having nearly linear and quadratic scales respectively, and a wattmeter and a voltmeter for which $dM/d\theta$ varied considerably over the scale, giving large divisions at the lower end of the scale in the wattmeter and preventing the usual widening of the divisions toward the upper end of the scale in the voltmeter. The scales are shown in Fig. 2. Test was made (a) with all instruments in normal condition; (b) with pointers bent 3 mm. ($=1.7$ to 2 deg.) below the normal position; (c) 6 mm. ($=3.4$ to 4 deg.) below the normal position; (d) 3 mm. above the normal position; (e) 6 mm. above the normal position[14]. In each case the points tested were 0.2, 0.4, 0.6, 0.8, and 1.0 times the full-scale value.

The results for the wattmeters are shown in the curves of Fig. 3, from which it will be seen that the errors introduced

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[12] This assumes that the springs give a counter torque which varies directly as the angle through which the moving coil deflects from its position of rest. While this is not strictly true, it is sufficiently close to the facts for the purpose of the present discussion. For data on the variation of springs from Hooke's law, see Bradshaw, *Elec. Jour.* vol. 3, pp. 390-6, 1906.

[13] H. B. Taylor, *Elec. Jour.* vol. 2, pp. 480-1, 1905.

[14] Actually, the pointers were not bent, but the same result was had as would have been given by the process described. The test was made by setting the pointer over marks 3 mm. and 6 mm. above and below each scale point tested, after the pointer had been brought to a rest position 3 mm. or 6 mm. above or below its normal zero position, by shifting the spring abutment.

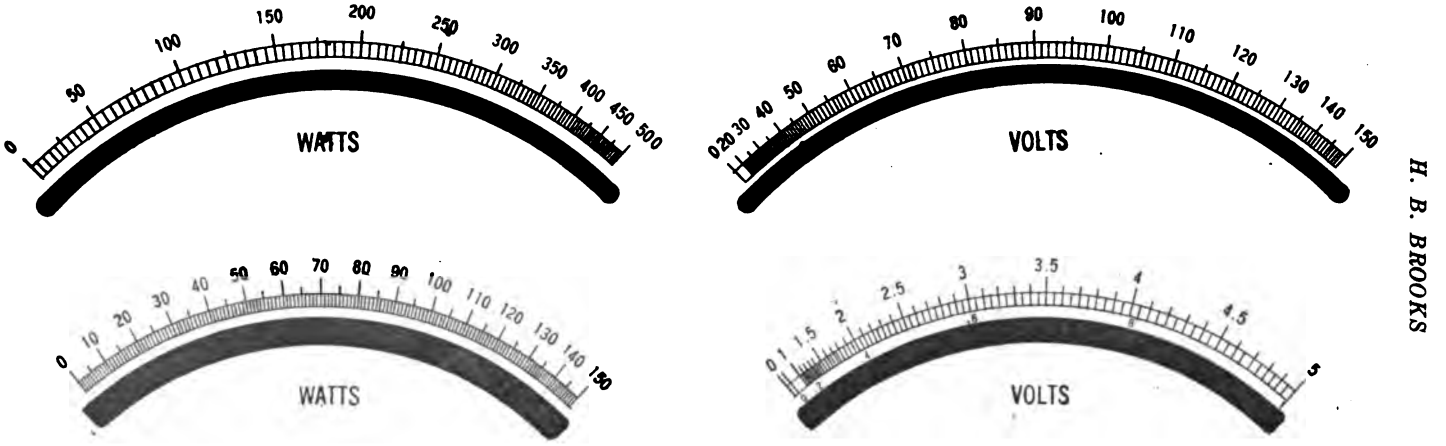

Fig. 2—Linear and Non-Linear Wattmeter Scales, and Quadratic and Non-Quadratic Voltmeter Scales, Used in Pointer-Bending Tests

by bending the pointer 3 mm. are from 0.0 per cent to 1.1 per cent of the full-scale value for the uniform scale wattmeter, 0.0 per cent to 2.7 per cent for the non-uniform. For 6 mm. bending, the errors are from 0.0 per cent to 1.9 per cent for the uniform scale, and from 0.0 per cent to 6.2 per cent for the non-uniform. The average error (without regard to sign) for the five scale points is 0.25 per cent for 3 mm. bending, 0.45 per cent for 6 mm. for the uniform scale wattmeter, 1.1 per cent and 2.45 per cent respectively for the non-uniform scale wattmeter. It will be seen from the curves that the effect of a bent pointer on the accuracy is greatest at the highest and lowest points. Taking the remaining three points, from 0.4 to 0.8 of full scale, as the part most generally used, the average errors for the two amounts of bending are 0.1 per cent and 0.15 per cent for the uniform scale, 0.8 per cent and 1.9 per cent for the non-uniform. These figures show the superiority of the uniform scale for wattmeters, when the question is one of sustained accuracy in use under practical conditions, as contrasted with mere initial accuracy. There are advantages

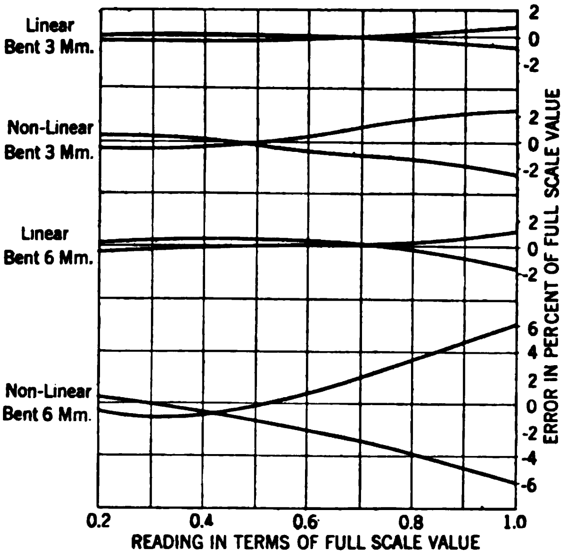

FIG. 3—ERRORS PRODUCED IN WATTMETERS HAVING LINEAR (UNIFORM) AND NON-LINEAR SCALES BY BENDING THE INDEX AND RETURNING IT TO ZERO BY THE ZERO ADJUSTER OR EQUIVALENT

for the maker as well. The uniform scale is easier to lay out with a given degree of accuracy, and instruments with $dM/d\theta$ constant will give rise to fewer complaints of inaccuracies developed in service.

Comparing the results of the bending of the pointer in the voltmeters, shown graphically in Fig. 4, we see that over the whole range from 0.2 to full scale the errors caused by 3 mm. and 6 mm. bending are respectively 0.0 per cent to 0.5 per cent and 0.1 per cent to 1.0 per cent of full-scale value for the quadratic scale, and 0.1 per cent to 2.1 per cent and 0.1 per cent to 2.8 per cent for the non-quadratic. Taking the region from 0.6 to 0.8 of full-scale as that most used, the average errors are 0.1 per cent and 0.15 per cent for the quadratic, 0.2 per cent and 0.5 per cent for the non-quadratic. Bending the index in the down-scale direction with respect to the coil produced positive errors at the lowest scale point tested, changing to negative at roughly one-half of full-scale value. The converse is true for bending up scale. These statements apply to all four of the instruments tested.

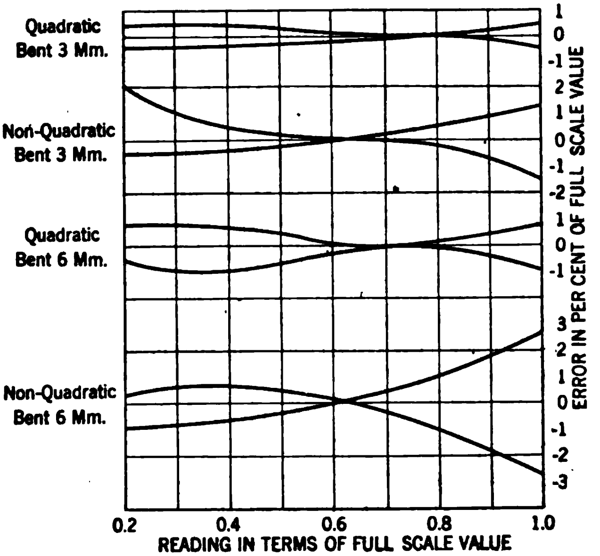

FIG. 4.—ERRORS PRODUCED IN VOLTMETERS HAVING QUADRATIC (SQUARE-LAW) AND NON-QUADRATIC SCALES BY BENDING THE INDEX AND RETURNING IT TO ZERO BY THE ZERO ADJUSTER OR EQUIVALENT

The voltmeter and the wattmeter are two of the most important electrical instruments, the former because of its relation to lamp life and efficiency, the latter because its use in checking standard watthour meters directly affects the revenues. It is the writer's opinion that in future designs makers should strive to realize the condition ($dM/d\theta$ = a constant) which gives the desirable forms of scale.

(c) *Design and Workmanship.* After choosing the proper type of instrument and a suitable scale law, one is still very much dependent upon the maker's knowledge, skill and facilities. For example, poor design in a d-c. voltmeter, such as an abnormally large air gap, may result in a loss of accuracy during service. Even with good design, the important questions of quality of materials and workmanship still remain; for example, one must have good magnet steel, properly heat treated, magnetized, and aged, in order to ensure a degree of permanency which will give satisfactory service. Such matters can best be judged by the general performance of a given maker's line over a period of years, and even if instrument makers were to adopt the radical suggestion recently made[15] concerning electric motors, and all build to the same designs, so that all makes of instruments of a given type and size would be interchangeable, much room for competition could still exist on the questions of quality, price, and sustained accuracy.

(d) *Proper Installation.* Electrical instruments and meters have very small operating forces as compared with gas and water meters, for example, and their parts are necessarily relatively delicate and carefully made. It is therefore essential that they should be carefully handled before and during installation in order to avoid damage to pivots and jewels. Instruments and meters should not be mounted on switchboards until all other work on the board is completed, in order to avoid shocks caused by hammering, drilling, etc. They should be installed where they can be easily read, and where they will not be subjected to extremes of temperature, vibration or shocks, strong stray magnetic fields, or moisture. If it is impossible to avoid all of these, one must expect more frequent repairs or replacements, with reduced accuracy in service.

An important precaution, which is often overlooked, is to have all contact surfaces clean and firmly brought together. Neglect of this precaution in the case of contacts carrying large currents, as in ammeter shunts, results in excessive

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[15] Burke, *Elec. World.* vol. 73, pp. 172-5, 1919.

heating of the shunt and consequent errors. Also, particularly in the case of millivoltmeters and separate-shunt ammeters, bad contacts at the binding posts or the shunt terminals may give rise to large and variable errors. The leads connecting the instrument and shunt are part of the instrument, and may have a resistance equal to 5 to 10 per cent of the total, and any change in their resistance directly affects the accuracy. The joints between such leads and the terminals in which they end should be carefully examined to see that they are securely soldered. When leads for portable millivoltmeters are in frequent use, some of the strands are very liable to break and thus affect the resistance. Excessive errors have been found by the writer in separate-shunt ammeters where the leads had been repaired by the user, without securing a good joint. A bad joint is aggravated by the use of rubber gum or friction tape, as the sulphur from the tape corrodes the joint surfaces, giving bad contact.

(e) *Maintenance and Testing.* It is difficult to lay down general rules for the frequency with which instruments should be checked, as a great deal depends on the nature of the service. The following table, summarized from the Electrical Meterman's Handbook, is presumably the result of the experience of some of the largest central station companies:

Name of Instrument	How Often Checked
Standard Cell	Intercompare weekly, and one to standardizing laboratory semiannually.
Resistance Standard	Check working standards against reference standards at least semiannually. Latter to standardizing laboratory at least once every two years.
Potentiometer	Once a year.
Volt Box	Once a month.
Secondary Standard Deflection Instruments	Compare with primary standards at least once every two weeks.
Working Standard Deflection Instruments	Every two weeks.
Portable Watthour Meters ("rotating standards")	Direct-current, at least once a week; alternating-current, at least once in two weeks.
Stop Watches.	Daily.
Calibrated Resistances (used in meter testing)	Weekly.

The preceding table does not include switchboard instruments, for which no data on frequency of checking is available, to the writer's knowledge.

Users of musical instruments realize that they must be tuned occasionally if they are to give satisfactory results, and also that some types, such as the violin, require very frequent tuning as compared with others. Users of electrical instruments should realize that these also need occasionally to be brought into accord with standards, and that this should be done more frequently when the service they render is more exacting or important.

4. ACCURACY GUARANTEES

It is the practise of most American makers to publish either in their catalogues or on certificates accompanying the individual instruments, statements or guarantees of the accuracy of their product. The accuracy is usually expressed as a percentage of full scale value, and this method would seem to be quite suitable with the possible exception of switchboard voltmeters, frequency meters, ammeters for constant-current circuits, or other instruments for measuring a quantity which in practise is held at a constant value. The advantage of allowing a greater error outside of the working range in the latter class of instruments is that the maker can give high accuracy over the limited working range at a lower cost.

Instruments with non-uniform scales, such as a-c. voltmeters, have a part of the scale near the zero which is practically useless. For such cases, the guarantee should state to what part of the scale the stated accuracy applies.

The British specification requires a certain percentage accuracy from full scale down to half scale, and a fixed percentage of full scale value below this point.

The wording of the guarantees of American makers varies considerably in definiteness even between different instruments of the same maker. Thus: one maker referring to 9-inch switchboard instruments states that "The d-c. instruments are accurate to within 1 per cent of full scale value under all normal working conditions," while for similar a-c. instruments he writes "The indications of all (a-c.) instruments are guaranteed to be correct within 1 per cent of full scale value when used on circuits of any frequency from 15 to 140 cycles, irrespective of any wave form, power factor or temperature likely to be encountered in commercial practise." Other makers avoid definite statements and use such general expressions as "...... instruments are accurate to a high degree. They may be used on circuits of any frequency, wave form or power factor and are free from heating errors," or in the words of another maker "These instruments will be found of particular value as reference standards in laboratories, or in any place where it is desirable to check portable or switchboard instruments with standards of absolute accuracy."

While the question of specifying the accuracy of an instrument presents some complications, it is easily seen that the accuracy of instrument transformers, especially of current transformers, is still more difficult to cover in any simple manner. The following is a typical form of guarantee of one maker for voltage transformers:

> Are designed to have their actual ratio exactly equal to their marked or nominal ratio for a secondary load of 25 volt-amperes (which is the load resulting when practically the maker's entire line of switchboard instruments is connected to the secondary), but are normally built for a secondary capacity of 50 volt-amperes * * * For any load from no load to full load, the actual ratio will not differ from the nominal ratio by more than 1 per cent for either inductive or noninductive loads.

It is suggested that if makers would each publish curves of ratio and phase angle for their current transformers on at least two standard secondary burdens, comparison of the different makes as to suitability for a given service could more readily be made.

Examination of the catalogues of several American makers shows the following accuracy statements or guarantees expressed in per cent of full scale value: Switchboard instruments, 2 per cent in three-inch and five-inch diameter, 1 per cent for seven-inch and nine inch; medium grade d-c. portable instruments 0.5 per cent; standard d-c. portable instruments, 0.2 per cent; standard portable electrodynamic instruments, 0.25 to 0.4 per cent for single-phase, 0.5 per cent for polyphase; portable moving-iron instruments, 0.5 per cent; portable induction ammeters, 0.5 to 1 per cent; laboratory standard instruments, 0.1 to 0.2 per cent. Instrument transformer accuracies are given as from 0.5 to 2 per cent on stated loads.

5. EFFECT OF CONDITIONS OF USE ON ACCURACY

(a) *Introductory.* All instruments are to some extent susceptible to the influence of surrounding conditions, such as varying temperature, stray magnetic fields, electrostatic effects, etc. Some conditions may affect the accuracy seriously although no permanent change takes place in the instrument, which may show the same performance when checked in the laboratory before and after use, while other conditions may permanently injure the instrument. It is therefore important for the user of instruments to understand their operating principles, the location of their magnets and coils, whether they are magnetically shielded, etc., as only with such knowledge can he most effectively locate and protect the instruments so as to secure the best obtainable accuracy.

In discussing the effects of conditions of use on accuracy, watthour meters are separately treated in section 11.

(b) *Care in Handling.* Because of their necessarily small operating forces and delicate moving parts, electrical instruments and meters should be handled very carefully if sustained accuracy is to be expected. If they are to be shipped, they should be carefully packed. Indicating instruments, which have relatively sharp pivots, require more care in handling than a-c. watthour meters, which have rounded pivots. While the preceding statements may seem trite to many, it is remarkable in how many ways the facts are overlooked by those who deal with instruments. Property clerks, unless restrained, will stamp numbers on instruments, and even experienced laboratory men will hammer on a bench upon which are pivoted instruments, or use instruments directly on tables which are set in strong vibration by motors or other machines. If instruments must be used where such vibration exists, they should be set on pads of felt or folded cloth. It has been suggested[16] that the shelves on which instruments are stored should be provided with a thick felt covering, and that this will improve the service accuracy and reduce repair bills.

(c) *Overloads.* Instruments subjected to overloads may have their accuracy impaired or destroyed. Some of the forms of damage which may result from overloads are: burning out of windings, bending of pointers, alteration of value of resistors or shunts, annealing and deforming of springs, change in shape of moving coils and consequent lack of balance of the moving system. Some of these, such as deformation or annealing of springs, can usually be discovered by inspection, while others

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[16] F. P. Cox, *Elec. Rev.* (N. Y.), vol. 52, p. 589: 1908.

may require tests. Wattmeters are especially liable to be damaged because they have not only a watt limit but also limits of current and of voltage. With low power factors, the attempt to get a good reading may result in overloading the current or voltage coil.

To avoid overloads when using portable instruments, it is advisable to lay out the work carefully in advance, calculating the approximate values of current, voltage or power. When instruments have more than one range, it is advisable to try the highest range first, as there will then be the least risk of damage if an abnormal condition exists. All lines and connections should be run and secured in such a way as to avoid accidental contacts which may produce short-circuit currents through the instruments. Leads attached to instruments should not be allowed to hang over the edge of the bench, as this is liable to result in dragging the instrument off onto the floor.

Instruments which have been overloaded should be put under suspicion of inaccuracy, and should be checked before further use. Switchboard instruments should be made on more substantial lines than portable or laboratory instruments, as exposure to momentary overload is unavoidable.

Permanent-magnet moving-coil instruments may suffer magnetic damage by exposure to strong fields, such as those set up by short circuits. This damage leaves no mark, and should be guarded against by proper location of the instruments.

(d) *Fraction of the Scale Used.* It is not possible for the maker to locate each mark on the scale where it should be; also, the user must estimate the position of the pointer between scale divisions. Since the combined error amounts roughly to a constant angular displacement, it will introduce a relative error into the result which increases as the deflection diminishes. For this reason, it is desirable to use as large a part of the full deflection as is practicable. In the use of portable instruments this question may be decided from the requirements at the moment and the ranges available. In choosing switchboard instruments, however, one must be governed also by the unusual requirements which may arise. On railway circuits where heavy overloads are encountered, and where the measurement of light loads is of minor importance, it is necessary to have instruments of larger capacity than would be used, for example, on lighting feeders. Wherever accuracy is essential, however, as in the testing of meters, or plant acceptance tests, ranges should be such as give not less than half and preferably two-thirds of full scale deflection.

An important limitation to the foregoing statement is that in some cases the power required to operate instruments of lower range may occasion errors in other parts of the measuring apparatus which may be difficult to allow for. For example, if a one-ampere ammeter is used with a current transformer in order to get larger deflections for currents below one ampere, it should be remembered that such an ammeter has about 25 times the impedance of a five-ampere ammeter of the same type and make, and that the increase in impedance may affect the performance of the current transformer so as to make the observed results less accurate than if a five-ampere ammeter had been used. Another instance is that of low-range a-c. voltmeters, which are of low resistance. There may be cases where the current they take from the source will introduce errors of more significance than the reading error which would occur if a voltmeter of greater range were used.

(e) *Variation of Room Temperature.* The net effect of a change of room temperature upon the reading of an instrument is the resultant of a number of changes, some of which tend to annul each other, and others, such as those caused by change of dimensions of the parts, are of negligible effect. It is desirable that the user should know the general facts about this matter in order that errors may be avoided or minimized when tests are of necessity made under abnormal temperatures.

In a direct-current voltmeter, an increase of temperature of 1 deg. cent. produces the following effects: The strength of the spring is reduced 0.04 per cent; the magnetic flux density in the air gap is reduced by about 0.02 per cent[17]. The weakening of the spring tends to increase the reading, and the lowering of the flux density tends to reduce it. Hence for a given current through the coil, the effect of 1 deg. cent. rise of temperature is to increase the reading by 0.02 per cent. However, the moving coil is of copper, and its resistance increases 0.4 per cent per degree C, thus tending to reduce the reading for a given applied voltage. If we add to the copper coil a resistance which is 19 times as great as that of the coil and is unaffected by temperature, the resistance of the whole circuit will increase

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[17] This is an average of the values found for six instruments of three makers. The individual values ranged from — 0.01 to — 0.03 per cent.

0.02 per cent per degree cent. A voltmeter so made will therefore be compensated for temperature. If less than the above amount of added resistance be used, as may be the case for low ranges, the reading of the instrument on a given voltage will decrease as the temperature rises, while if more added resistance is used the reading will increase.[18].

In a direct-current ammeter of the shunted type the effects on magnet and springs are as above stated, but there are some limitations not found in the voltmeter. For example, it is not feasible, in general, to add to the moving coil a series resistance 19 times that of the coil because such a construction would require too large a drop of potential between the shunt terminals, thus increasing the size, weight, and cost of the shunt and also the power loss in it. In practise, it is usual to make shunts of relatively low drop (50 to 75 millivolts) for switchboard use, where temperature compensation is not of particular importance, while for portable instruments of greater accuracy drops from 100 to 200 millivolts are commonly used. It might appear that the ammeter of low drop might be compensated for temperature by making the shunt of a material which increases in resistance with temperature at such a rate as to supply the higher drop which the millivoltmeter requires for a given deflection. Such a procedure is not feasible because of the relatively large difference in the temperatures of the millivoltmeter and the shunt when the latter is carrying a load. It is the usual custom to make the shunts of an alloy [19] having a negligible temperature coefficient, and to minimize the temperature effect on the millivoltmeter by reducing the relative drop on the copper coil as much as is consistent with good design in other respects. The important thing for the user to remember is that the temperature errors of d-c. switchboard ammeters, while not of serious consequence for switchboard purposes, are such as to make them unsuitable for accurate work, such as the testing of watthour meters or acceptance tests of apparatus, unless

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[18] This assumes the average value of —0.02 per cent for the temperature coefficient of the magnetic field. If exact compensation must be assured, the individual instrument must be investigated. The average value, however, gives results satisfying all but extreme requirements.

[19] It is becoming general practise to use manganin, or similar alloy, for both portable and switchboard ammeter shunts. In some cases where a lower grade of performance is admissible, and first cost is important, switchboard ammeter shunts are made of cast iron.

the temperature of the instrument is determined and proper allowance is made for it. It is preferable for such work to use instruments whose temperature corrections are small.

In the moving-iron ammeter an increase of temperature affects the magnetic conditions in the iron in such a way as to decrease the operating torque by about the same percentage that it reduces the spring strength. These ammeters are thus very nearly independent of temperature changes.[20] The temperature coefficient of a moving-iron voltmeter thus depends mainly upon the temperature coefficient of resistance of its circuit, which depends on the ratio of the resistance of the copper coil to that of the manganin or similar alloy in series with the copper. For example, a well-known American portable moving-iron voltmeter having ranges of 150 and 300 volts has a total resistance of 5100 ohms, of which 130 ohms are in the copper coil. Neglecting the effect of temperature change upon the series resistor, the effect of a rise of temperature of 10 deg. cent. would be to make the voltmeter read lower by $10 \times \frac{130}{5100} \times 0.4$ per cent, or about 0.1 per cent. This applies to the 300-volt range. For the 150-volt range the corresponding effect would be 0.2 per cent.

In an electrodynamic ammeter with spring control and with the fixed coil and the moving coil in series, the only appreciable effect of temperature on the readings is that caused by the change in the strength of the springs, which is 0.04 per cent per degree cent. For any given position of the coils, the operating torque varies as the square of the current; the scale, however, is marked to read the first power of the current. Such an ammeter will therefore read 0.02 per cent higher on a given current for each degree cent. rise of temperature. An electrodynamic voltmeter is simply a low-range ammeter as just described, plus a series resistor of suitable value. If this has 19 times the resistance of the copper, the temperature coefficient of resistance of the instrument will be 0.02 per cent per degree cent. and will just offset the effect of temperature changes on the spring strength. The instrument will be compensated for temperature as a voltmeter.

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[20] This statement is based on the experience of the Bureau of Standards with several instruments of each of two prominent American makers.

In an electrodynamic wattmeter a similar condition exists, but since the torque is proportional to the first power of the quantity indicated on the scale, the effect of the spring change with temperature (0.04 per cent per degree cent.) enters unchanged. If the resistance of the voltage circuit remained unchanged as the temperature varied, the wattmeter would read 0.04 per cent higher per degree cent. of temperature rise. This requires, for compensation, that a series resistor of negligible temperature coefficient be used, having nine times the resistance of the copper. Commercial wattmeters often have more than enough added resistance for temperature compensation, and would actually be better in this respect if a part of the added resistance were of copper. Attention is called to this matter because the effect of temperature on the spring strength has been frequently overlooked, and in consequence the statement has been made that as much series resistance as possible should be used in order to minimize the temperature error. If copper be added to the potential circuit to minimize the temperature coefficient, it should be in the form of fine wire or strip wound on a thin card, so as to respond quickly to temperature changes, and it should be mounted as near the springs as practicable. It is of theoretical advantage, at least, to have both springs above the moving coil so that the ascending current of warm air from the coils will maintain the springs at nearly the temperature of the moving coils.

For induction instruments no general statement concerning temperature effects can be given. In simple instruments on this principle the effect of temperature may be rather large. In the form made by an American maker on the "series-transformer" principle it is stated that approximate independence of temperature changes is inherent in the design of ammeters and voltmeters. Induction wattmeters (now made only for switchboard use) have an inherent temperature error which has been reduced by the use of a rotor of special alloy instead of aluminum. It is not practicable to secure complete compensation in this way without reducing the operating torque to an objectionably low value.

In the hot-wire instrument a change of room temperature changes the length of the working wire, and also changes the distance between the supports of the ends of the wire. If the base plate supporting the wire be made of material having the same coefficient of expansion as the wire, the instrument will theoretically be compensated for room temperature changes. However, the wire takes up the temperature of its surroundings very quickly, but the base plate does not, so that a considerable lag effect occurs. In the best modern hot-wire instruments the performance in this and in other respects has been improved by substituting platinum-iridium for the platinum-silver so long used for the working wire. The platinum-iridium has a much smaller coefficient of expansion, but may be operated at a much higher temperature.

While the use of the hot-wire instrument is limited to laboratory and radio purposes, it may be well to point out that it is subject to an external condition which does not affect other electrical measuring instruments, namely, variation in the atmospheric pressure. While the subject has not been studied with any degree of thoroughness, some experiments made seven years ago by the writer, using a Hartmann and Braun hot-wire voltmeter, showed that a reduction of air pressure from 75.2 cm. to 55.9 cm. (29.6 to 22.0 inches) increased the reading on a given voltage by 2.7 per cent. For intermediate values of pressure, the change in the reading was directly proportional to the change of pressure.

(*f*) *Self-Heating of Instruments.* In the preceding discussion of effects of room temperature it has been assumed that temperatures within the instrument were uniform throughout. Most instruments contain sources of heat in the form of coils traversed by currents, and it is therefore possible for differences of temperature to be set up while the instrument is in circuit. This affects the accuracy of the results obtained.

In d-c. millivoltmeters the heating of the coil is entirely negligible. In d-c. voltmeters of moderate range it is small. For example, the power expended in a 150-volt d-c. voltmeter of 15,000 ohms resistance at full scale deflection is 1.5 watts. Since this is nearly all expended in the series resistor, the effect on the accuracy should be very small.

In a-c. voltmeters and the voltage circuits of wattmeters the resistance for a given voltage is much lower than in d-c. instruments, and the rate of heat liberation is much greater. It is good practise to partition off and ventilate the series resistor, in which nearly all the heat is liberated. In switchboard instruments the series resistor is often supplied in a separate case for back of board mounting.

Direct-current ammeters with separate shunts, being essentially millivoltmeters, have negligible internal heating, but may have errors caused by the heating of the shunts. This may occasion appreciable errors if the resistance material used in the shunts has an appreciable temperature coefficient or thermoelectric force against the material of which the shunt blocks are made. The copper-nickel resistance material variously known as constantan, *Ia Ia*, advance, eureka, and ideal has a very small temperature coefficient of resistance, and would be very well adapted for ammeter shunts if it were not for its high thermoelectric power which makes it unsuitable for the purpose.[21] Manganin meets both requirements.

A considerable part of the heat produced in switchboard ammeter shunts is carried away by conduction to the cables or bus bars, from which it escapes by radiation and convection. Even if the opportunity for escape of heat is the same at each end of the shunt, there will still be a difference of temperature between them. This is caused by the flow of current through a circuit made of dissimilar metals, and is known as the Peltier effect. It causes a thermal electromotive force in such a direction as to increase the reading of the ammeter. In cases of unequal heating of the ends of the shunt, caused by bad contacts or by difference in the size of bus bars, the resultant thermal electromotive force may either increase or decrease the reading of the ammeter. Thermal electromotive force may be detected by breaking the main circuit, when the index will fail to return to zero if appreciable thermal electromotive force is present. This effect may be distinguished from zero shift (due to spring fatigue) by disconnecting one lead from the instrument, when the position of the index will change by an amount proportional to the thermal electromotive force.

The use of copper bus bar as an ammeter shunt is no longer recommended, and the same is true of the use of internal copper shunts in self-contained d-c. ammeters, except for small ranges where the heating is small. In the latter case the effect of room temperature on the shunt and on the copper moving coil is the same, and no error results. Such construction is not advisable for currents exceeding say 15 amperes.

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[21] For the construction of coils of high resistance, such as series resistors and multipliers for voltmeters, constantan has some advantages over manganin.

In the moving-iron ammeter the power loss (in good designs) is small, and the effect of temperature changes is small. In the moving-iron voltmeter the conditions are somewhat less favorable, as the heating of the copper coil increases the resistance of the circuit.

Electrothermic instruments, including the expansion (hot-wire) and the thermocouple types, are unique in that their operation depends on internal heating. The effect of this heating in hot-wire instruments has been referred to. No published data seems to be available on the limitations of thermocouple instruments, such as effect of varying room temperature and of standing in circuit.

(*g*) *Stray Magnetic Fields.* Unidirectional stray fields are set up by conductors carrying direct currents, by permanent magnets, and by direct-current electromagnets. The strength of the field, upon which depends the magnitude of any error produced by it in the indications of an instrument, varies directly as the strength of current in the conductor and decreases with the distance from the conductor to the given point. Thus the necessity for precaution increases as currents are greater and as the instrument is located nearer to the conductor.

It is important to remember that the magnets used in direct-current instruments of the usual unshielded types set up a strong stray field which is capable of appreciably affecting the accuracy of other instruments near by. Conductors carrying moderate or large currents to the instruments set up stray fields which may occasion errors. The iron framework of switchboards, magnetized by large currents in bus bars, may set up strong stray fields at a considerable distance from the bus bars. Even unmagnetized masses of iron placed too near some types of instrument will affect the working field of the instrument and thus introduce errors. The iron pipe framework of testing benches has given rise to disturbances of this kind.

Unidirectional stray field affects the accuracy of the permanent-magnet moving-coil, the moving-iron, and the electrodynamic types of instruments.[22] The first-named type,

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[22] McCollum has suggested that the hot-wire instrument would theoretically be affected by stray magnetic field of the same frequency as that of the current in the wire. This effect can be observed, but would be very small in most practical cases.

because of its large value of working field strength, is in general less affected by a given value of stray field than the other two. The smallness of the air gap is also an important feature, and of two otherwise similar direct-current instruments, the one having the smaller air gap will be less affected.

The following table is given by Farmer:[23]

EFFECT OF STRAY MAGNETIC FIELDS ON D'ARSONVAL TYPE VOLTMETERS

Stray field, lines per cm.^{2}	Error at ½ of full-scale deflection, per cent	
	Shielded	Unshielded
5	0.5 to 1.0	2
10	0.75 to 1.75	3.5 to 5.5
15	1.0 to 8.0	6.0 to 7.5
20	1.25 to 3.25	7.5 to 10

Moving-iron instruments are affected by unidirectional stray field, even when operating on a-c. circuits. Moving-iron instruments on the repulsion principle are stated to be considerably more affected by stray field than those on the attraction principle.[24] Electrodynamic instruments are affected by unidirectional stray field only when they are used on direct-current circuits. This is in contrast to the moving-iron instrument, which is affected by either unidirectional or alternating stray fields when used on either d-c. or a-c. circuits.

Alternating magnetic fields are set up by alternating currents in conductors, by a-c. electromagnets, transformers, etc. In general, such fields are not liable to attain the values which occur with direct current, on account of the larger values of voltage and hence smaller currents for the same power in a-c. plants.

Alternating magnetic fields affect the readings of permanent-magnet moving-coil instruments only when the field strength is great enough to reduce permanently the strength of the

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[23] F. M. Farmer, Electrical Measurements in Practise, p. 54.

[24] In a test of nine moving-iron ammeters, reported in *Electrician*, vol. 76, pp. 120-123, 1915, the stray field was set up by two conductors located six inches away from the instrument, each carrying 2000 amperes. The change of reading due to stray field was 0.4, 1.5, and 1.9 per cent in the three attraction-type ammeters, and 3.5, 7.4, 12.3, 9.4, 6.8, and 5.7 per cent in the six of the repulsion-type.

magnet.[25] Other types, including moving-iron, electrodynamic and induction instruments, have their accuracy affected only during the time of exposure to the field. Moving-iron instruments used on a circuit of a given frequency are affected by alternating fields of this or any other frequency, or by unidirectional fields. Electrodynamic instruments, however, are affected only by fields of the same frequency as that of the circuit on which they are used, with the further restriction that the effect depends on the direction and time-phase of the field with respect to the current in the moving coil. That is, with the moving coil in a given position, there is a direction of the stray field in which no torque is produced on the coil; also, for any position of the coil there will be no torque if the stray field is in time-quadrature with the current in the coil. It is stated[26] that the stray-field error in unshielded non-astatic electrodynamic wattmeters may be as much as 25 per cent with an a-c. field of five lines per cm.^{2}, and 75 per cent with ten lines, and that a suitable shield will reduce the effect of a field of 20 lines per cm.^{2} to practically zero without introducing eddy current or other errors.

If the stray field is of constant direction, it will be possible to turn an unshielded electrodynamic wattmeter into such a position that no deflection occurs when voltage only is applied. If the position of the index is noted, and the instrument is then used to measure power, no error from stray field will occur if the wattmeter is turned so that the index points along the given direction.

Induction instruments, because of their strong working fields, are only slightly affected by stray fields of the values found in practise.

To avoid errors due to stray field, we may (1) keep instruments at a sufficient distance from heavy conductors or other sources of the disturbance; (2) eliminate the effect of stray field by reversing the connections to the instrument or by turning the instrument through 180 deg. between readings,

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[25] An extreme case of this kind is given by E. P. Peck, (*Elec. World*, 51, p. 1220; 1908) in which a d-c. voltmeter was connected to a 600-volt circuit and placed within 18 inches of heavy bus bars. After a severe short-circuit, which caused very large alternating currents to flow through these bars, the voltmeter read 350 volts when the applied voltage was 600. It should be noted that damage to an instrument of this nature leaves no external mark.

[26] F. M. Farmer, Electrical Measurements in Practise, p. 159.

taking the mean as the correct value; (3) construct the instrument so as to be astatic as to other than its own working field; (4) surround the working parts with a magnetic shield. In the most careful work it is well to make use of more than one of these methods, if practicable. These precautions will now be briefly considered in detail.

Opinions differ somewhat as to the separation necessary between permanent-magnet moving-coil instruments in order to avoid error from interaction.[28] Since it is often necessary to assemble a number of instruments in a relatively small space, it is important to know just how far to separate them in order to secure a desired degree of accuracy. For this reason some experiments were made recently in the writer's laboratory on permanent-magnet moving-coil instruments. The results are given in the curves of Fig. 5. In obtaining data for these curves the instrument *L* at the left was tested at two points by a potentiometer while other instruments were removed from its vicinity. The readings so given were taken as the normal or "zero error" values. Another instrument (an unshielded standard portable voltmeter) was then placed alongside the first, and successively at various distances to the right, the tests of the first instrument being repeated for each position of the disturbing instrument. The curves show the error in per cent as a function of the distance in inches between the centers of the moving coils.

With unshielded instruments side by side, (condition 1) the error is 0.8 per cent, and in the worst position, with pole-pieces adjacent, it is 2.0 per cent. As the disturbing instrument is moved away the error drops until it becomes practically zero when the centers of the coils are about 20 inches

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28. F. P. Cox states (*Elec. Review*, N. Y., 53, pp. 589-591, 1908) that, "Normal earth's field introduces an error of about 0.1 per cent in unshielded instruments, and such instruments should not be used nearer together than 18 inches. If placed side by side these instruments react on each other and an error of 1 per cent may very readily be introduced."

H. B. Taylor (*Elec. Jour.*, 2, pp. 474-481, 1905) says,

"A space of from two to three feet may be taken as a safe distance to allow between direct-current meters of the ordinary portable type. When space is very limited, two instruments can often be brought closer together without causing trouble, by placing one of them with its scale inverted with respect to the other, so that the neutral parts of the magnets are nearest to each other and the pole pieces are as far apart as possible".

apart. The same distance applies to conditions 2 and 3. For the iron-cased instruments, curves 4 and 5 show that the maximum error with instruments as close together as possible is much less than in the unshielded instrument, and drops to a negligible value with a separation of the coil centers of

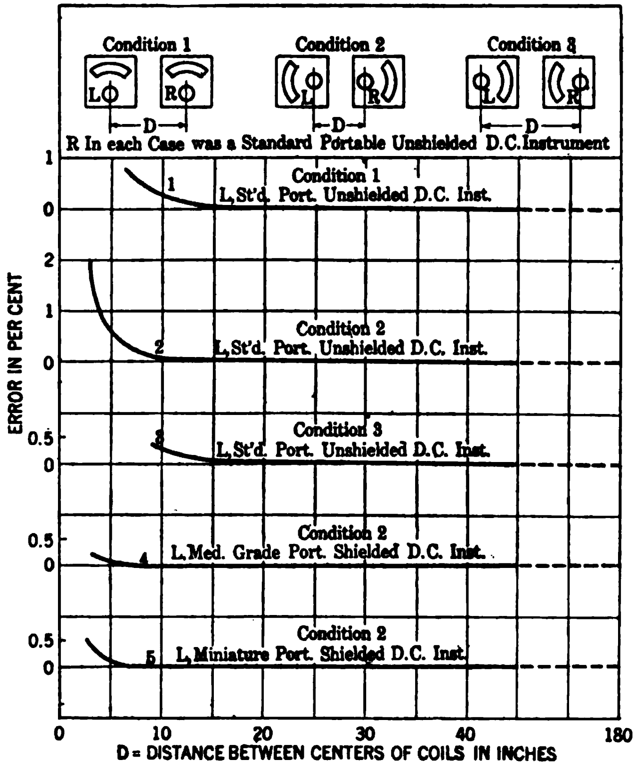

Fig. 5.—Errors Produced in D-C. Instruments by the Stray Field from an Unshielded Portable D-C. Instrument

about eight inches. It is usually recommended that when portable instruments must be used in places subject to stray magnetic fields the mean of two readings be taken, the instrument being turned 180 deg. about the axis of rotation of the moving element for the second reading. In the case of unshielded moving-iron and electrodynamic instruments it is probably better to leave the instrument in one position and take the mean of two readings, the current through the instrument being reversed for the second reading. Instruments of these types should not be used for d-c. measurements if permanent-magnet moving-coil instruments are available.

To avoid the effects of stray fields, astatic instruments are made by combining two operating systems on a single axis. For example, two permanent-magnet moving-coil systems are used, with the moving elements coupled to add their torques and with the two magnets arranged to have opposite polarities. Another astatic arrangement consists of two moving-iron systems, the polarities of the two solenoids being opposite.[29] Astatic electrodynamic instruments have been made, but have not come into any extended use, probably because of the adoption of magnetic shielding by the leading American makers. It should be noted that "astatic" instruments are strictly astatic only for fields which are uniform over the space occupied by the coils, or for non-uniform fields which may happen to produce the same torque on each coil. The field about a conductor carrying a current is not uniform, but decreases inversely as the distance from the conductor. If an astatic instrument is placed near the conductor, one coil of the astatic system may be so much closer to the conductor than the other as to give an appreciably greater torque on the nearer coil, and hence an error will be caused by the stray field. In using astatic instruments the same precautions should be taken as for simple instruments, if the maximum possible accuracy is desired.

A special form of permanent-magnet moving-coil instrument[30] has been developed to reduce to a minimum the disturbing effect produced by the stray field from a vertical bus bar, since such vertical bars are the most frequent source of trouble. This is accomplished by placing the magnet so that the lines of force of the stray field run through the steel from side to side of the bar, rather than along its length. It is stated that even very strong stray fields, which would demagnetize the magnets of instruments as ordinarily made, will produce little or no effect when they pass through the

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[29] Edgcumbe, Industrial Electrical Measuring Instruments, 2d edition, p. 142.

[30] P. MacGahan, Nat. Elec. Lt. Assoc., 36th Convention, Hyd.-Elec. & Trans., Technical, pp. 602-604, 1913.

steel at right angles to the direction of the permanent magnet flux, even if the instrument is entirely unshielded. It is also stated, in the same connection, that even heavy shielding will not prevent demagnetization when short-circuits occur in nearby conductors on large power systems.

Connections to ammeters and the current terminals of wattmeters and watthour meters should be run as close together as practicable in order to avoid forming a loop. This precaution is not important for small currents, but for a given design (constant ampere-turns for all ranges) the effect of stray field at the operating system, caused by the current in a loop of given form and size, increases directly as the ampere range. This may be seen from the fact that the current coils of large range instruments have only a few turns, so that even one turn in a nearby loop will have an appreciable effect on the readings.

It seems to be the consensus of opinion of American makers that electrodynamic voltmeters, ammeters and wattmeters should be protected from stray field by a laminated iron shield. In one moderate priced type this shield is open at top and bottom, but in the highest grade instruments the shield completely surrounds the operating system except for a limited opening at the top. In one make the laminated shield is surrounded by a drawn steel shield, and in the polyphase wattmeter the shielding of the two systems from external disturbances is supplemented by a laminated iron shield between the two systems to prevent interaction. At least two companies supply shielded portable moving-iron ammeters. In checking shielded electrodynamic instruments on direct current it is necessary to take readings with the current first in one direction, then reversed, the mean being taken as the normal value. The necessity for this procedure in unshielded instruments is caused by the interaction of the earth's (or local) magnetic field with the current in the moving coil, and in the shielded instrument it is due to a slight residual polarity in the shield.

Stray magnetic fields may be detected and their intensity determined by using an unshielded electrodynamic wattmeter with only the voltage circuit energized, or an electrodynamic voltmeter with its fixed coils cut out of circuit. The excitation should be from a d-c. source for unidirectional field, and from an a-c. source of the proper frequency for alternating field. In either case the instrument must be oriented until the deflection is a maximum, and in the a-c. case, the phase of the exciting current must be adjusted for maximum deflection. To calibrate the apparatus, the experiment may be repeated with the moving coil at the center of a coil of few turns of large diameter (say two or three feet radius.) The field intensity over the space occupied by the moving coil may be taken as constant, and is given with sufficient accuracy by the formula

$$H = \frac{0.2\,\pi\,NI}{r} = \frac{0.63\,NI}{r} \tag{1}$$

where N is the number of turns in the coil, I the current in amperes, r the radius of the coil in centimeters. If the radius is expressed in inches, the constant becomes 0.25 instead of 0.63. The formula assumes that the cross section of the coil is small compared with the radius. In the a-c. case, H and I are effective values. The voltage used in calibrating the instrument should be the same as that used in measuring the stray field, or if not, an allowance must be made.

(h) *Electrostatic Effects.* Electrostatic attraction or repulsion of the moving system of an instrument may seriously affect its accuracy. This effect is most pronounced in instruments of relatively low torque, such as electrodynamic instruments, portable and some forms of switchboard moving-iron instruments, and in permanent-magnet moving-coil instruments, especially those having long pointers. It is possible with many of these instruments to cause the index to move one-fourth of the way up scale by rubbing the cover glass with a dry cloth. This should be avoided when readings are about to be taken, or, if necessary to clean the glass, the charge on the glass should be dissipated by breathing on it.

In the use of an electrodynamic wattmeter, it is very important to make the connections so that the fixed coil and the moving coil are at nearly the same potentials. With improper connections, errors may arise from electrostatic action between the coils, and there is also danger of breakdown of the insulation. This danger is especially great when a multiplier is improperly used with a wattmeter to extend the voltage range. It is suggested that the manufacturers might well agree on a standard method of marking the terminals of wattmeters and that more stress should be laid by the maker on the matter of proper connections, which are in nearly all cases discussed in instructions separate from the instruments. The fundamental principle of correct connection as regards the two coils might well be briefly set forth in the calibration card attached to the instrument.

Another condition which may occasion error from electrostatic action is the testing of a wattmeter on separate sources of current and voltage, or in its use with current and voltage transformers. In the latter case, the grounded side of the secondary circuit of the voltage transformer should run directly to the moving coil, in order that the latter may be at the same potential as the current coil. In testing the wattmeter on separate d-c. sources, some writers recommend joining the two coils at the instrument terminals. As this may cause trouble if the circuits are accidentally connected elsewhere, the writer prefers to join the two points to a d-c. voltmeter whose maximum reading is not less than the voltage used in testing the wattmeter. The index of this voltmeter should remain at zero. If a deflection occurs, leakages or accidental contacts are present in the test circuits or the sources. Another method suggested is to make the connection between the coils through a piece of fine fuse wire.

Another class of electrostatic trouble has been noted in voltmeters of relatively high range with metal covers, namely, electrostatic action between the pointer and the insulated cover.[31] This action has been noted in voltmeters of 750-volts range, and is greater in instruments with long pointers. The pointer is at the potential of the moving coil, which in railway systems with grounded negative would be 500 volts or more above earth, since the moving coil (in one well-known make at least) is connected to the + terminal of the instrument. The potential of the metal cover is not definite, but depends on circumstances as to leakage from the instrument circuits to the cover and the insulation of the latter from ground. It is evident that in general the potential will be lower than that of the pointer, and the cover may even be at earth poten-

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[31] F. M. Farmer, Electrical Measurements in Practise, p. 54.

tial, thus placing full line potential between cover and pointer, and making very appreciable errors. This difficulty has been noted in testing such voltmeters at the Bureau of Standards, and the method of avoiding it is to connect the cover to the + binding post during the test. Since the makers have not yet adopted means for avoiding the difficulty, so far as the writer is aware, and since the user is very liable to encounter it, attention is here called to it. A remedy would be to connect the cover to the moving coil, but this would be objectionable because of the possibility of accidental contact between the cover and other conductors or the operator. One possible remedy would consist in lining the cover with insulating material within which would be a lining of thin foil or sheet metal which would be connected to the moving coil and to the magnet system and scale plate. Where the voltmeter is to be connected to grounded circuits, the same result could be attained by connecting the cover to the moving coil and to the grounded side of the circuit, using a built-in reversing switch with the moving coil to bring the deflection up scale regardless of which pole of the circuit is grounded. The latter method requires circumspection on the part of the user, and from this standpoint is not as good as the former.

Use has been made of a conducting lining in wattmeters and other instruments having hard rubber tops, but so far as the writer is aware no such precaution has been adopted for direct-current instruments.

Electrostatic fields from high-voltage conductors in the vicinity of instruments are theoretically capable of affecting the indications of the instruments unless the latter are either in metal cases or in glass cases lined with a grid of conducting material suitably connected to the movement. This effect seems unlikely to occur in practise because of the necessity of keeping high-voltage conductors at a distance from observers. It will be greatest in electrostatic instruments[32] because of their low torque and the fact that the operation of the instrument depends on electrostatic forces. The trouble can be avoided by suitable screening.

Electrostatic difficulties may occur in the use of reflecting galvanometers, due to differences of potential between the

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[32] K. Edgcumbe, Industrial Electrical Measuring Instruments, 2d. ed., p. 186.

moving coil and the magnet system and the case. Both the latter should be connected to one terminal of the coil and well insulated from the support on which the galvanometer is placed.

(*i*) *Variation of Frequency, Wave Form and Power Factor.* The effect of ordinary variations of frequency from the normal value is negligible in moving-iron, electrodynamic, and electrothermic instruments. The effect on induction instruments, unless carefully designed and compensated, is much greater. Even the most carefully compensated induction instruments must be used within a rather narrow range of frequency, especially wattmeters. Induction wattmeters may be compensated for moderate variations of frequency at unity power factor, or for large variations of power factor at normal frequency, but will show appreciable errors if both frequency and power factor differ from normal and from unity respectively.

Variations of wave form from the sine shape, when not excessive, produce very small effects on electrodynamic instruments and no effect on electrothermic instruments. The effect on moving-iron ammeters depends on the magnetic quality of the iron and the flux density at which it is worked. If the permeability is practically constant throughout the working range, the torque will always be proportional to the square of the current, which is the condition to be fulfilled if the instrument is to indicate the effective value of the current. As the iron approaches saturation the torque tends to vary as the first power of the current, which makes the instrument tend to indicate the average value of the current rather than the effective value. It will be seen that the quality of the iron and the correctness of the design have much to do with the proper functioning of moving-iron instruments, and it is doubtless due to the lack of these essential conditions that early forms of moving-iron instruments functioned badly on change of wave form and in other respects, and thus created a prejudice against the type which perhaps is not yet entirely dispelled.

Induction instruments, from the principle of their operation, are evidently subject to error due to change of wave form. There seems to be very little published data on this matter.

(*j*) *Mechanical Balance of Moving System.* Unless the axis of rotation of the moving system of an instrument is exactly vertical, any lack of mechanical balance will introduce a gravitational torque which will affect the accuracy of the instrument. It is partly for this reason that laboratory instruments are made with vertical axis, and this feature is one of the strong points of the horizontal edgewise instrument. One of the first things to examine in an instrument about to be used on important work is the condition of the moving system as to balance. Rough usage, or severe overloads, or excessive heating from any cause, is very apt to unbalance the moving system.

Switchboard instruments may be checked for balance by turning them into different positions about an axis parallel to the axis of rotation of the moving system. The same test is sometimes applied to portable instruments, and is satisfactory if approximate results are desired. For the most accurate balancing of portable instruments it is better to place the instrument in various positions on a plane support which is inclined say 10 deg. to 20 deg. to the horizontal. The difference between the two procedures lies in the fact that in the latter method the lower jewel and pivot come into contact at more nearly the working point of the pivot. When the instrument is checked for balance with axis horizontal, the point of contact of pivot and jewel is different from the working point, thus changing the axis of rotation slightly and giving an inaccurate indication of balance.

In American instruments of the better class the balancing is effected by varying the position of small weights which are adjustable in two directions at right angles. In balancing such an instrument, place it on a sloping support, first in the position which makes the pointer horizontal, then in another which is at right angles to the first. In the first position, any unbalance which appears can be removed by moving the weight which is adjustable along the direction of the pointer, and in the second position any unbalance in the line at right angles to the pointer may be corrected. If these two adjustments are carefully made, the instrument should be in balance when turned into any position on the sloping support, and should therefore be in the best possible balance for use on a nominally horizontal support.

Maintenance of balance in service requires that the moving system be as rigid as is consistent with lightness, and that the temperature shall not attain to values at which windings soften and alter their shape. It is evident that a given small weight placed on a moving system originally balanced will be able to produce less error as the operating torque of the instrument is greater. This constitutes an additional reason for the use of the highest practicable value of the ratio of torque to weight.

6. Features of Design Affecting Accuracy

In the following section will be discussed certain features of instrument design which have a more or less direct influence on the accuracy of the results, especially those features which can be checked by inspection.[33]

The designer of electrical instruments must fulfill commercial conditions as well as mechanical and electrical requirements. Cheapness and ease of manufacture, assembly and repair, and the ability to withstand a reasonable amount of handling in shipment, are matters which can not be disregarded if the product is to meet competition. The writer has endeavored to keep this fact in mind in the present discussion.

(*a*) *Shunts and Resistors.* These are always sources of heat when in use, and provision should be made for getting rid of the heat without raising the temperature of the operating system undesirably. It is possible to have shunts and resistors built in with the moving system with no ventilation, provided the power lost in the instrument is small, say only a few watts.

(*b*) *Permanent Magnets.* A general principle applying to the permanent magnets used in instruments and meters is that the magnetic circuit should be as nearly closed as possible. Since a gap must exist in order that the magnet may be of use, the practical question is to set a limit which shall give a reasonable degree of permanency, and endeavor to design the instrument within this limit. The following empirical

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[33] There is relatively little in print on the detailed design of electrical instruments. Edgcumbe's "Industrial Electrical Measuring Instruments," 2d edition, contains much information on the design of permanent magnets, springs, moving coils, fixed coils, current transformers, etc. See also a paper on "Calculation of Moving-Coil Measuring Instruments", by F. Janus, *E. T. Z.* 28, pp. 560-3, 1905.

relation was first given by Hookham.[34] If A is the area of cross section of the air gap, L the distance between the pole pieces, a the cross section of the magnet, and l the length of the magnet (mean length of the lines of force in the steel) then A/L should equal about 70 times a/l. This ratio of A/L to a/l may be called the permanency factor of the magnetic circuit. Heinrich and Bercovitz[35] restate Hookham's formula, but say that the permanency factor should not be less than 100. In the usual construction of permanent-magnet moving-coil instruments, L is equal to the sum of the lengths of the two air gaps, or in general, it is the total length of air gap.

The Hookham permanency factor of a magnet is a purely geometrical relation, and does not take into account other matters which affect permanency, such as composition of steel, heat treatment, aging treatment, and flux density in the steel. It is, however, a useful guide in design and in comparing types of instrument. Values of this factor for switchboard voltmeters, as given by Heinrich and Bercovitz, range from 35 in poorly designed instruments to 500 in the best ones. In addition to the great advantage of a higher permanency, the use of a high value of permanency factor also results in less stray flux from the magnet, more of it being utilized, and the instrument is also less affected by the presence of adjacent masses of iron or by stray fields.

(c) *Springs*. The quality of the springs used in an electrical instrument has an important bearing on the accuracy of the measurements. Tempered steel springs as used in watches would be the most desirable if they were not magnetic. It is necessary to use non-magnetic material, and the one which has been most generally used is phosphor-bronze. On account of the high resistivity of the latter, it is necessary to employ other bronzes in millivoltmeters where the resistance of the instrument must be kept low. Since in general the elastic properties are less desirable when a large percentage of copper is used, the requirements as to low resistivity and high elasticity are conflicting, and a compromise must be made.

The elastic performance of the spring will depend upon its design, for any given material. The thicker the spring, for a given length and angular deflection, the greater the

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[34] On Permanent Magnets, by George Hookham, *Phil. Mag.*, 5th Series, vol. 27, p. 186; 1889.

[35] *Handbuch der Elektrotechnik*, vol. 2, part 5, p. 29.

stresses in the inner and outer surfaces of the material. It is stated[36] that the stress should not exceed 600 kg. per cm.^{2} for phosphor-bronze, and that for a deflection of 90 deg. this is equivalent to saying that the length of the spring must be not less than 1500 times its thickness. If this condition is not complied with, or if the spring material is of inferior quality for the purpose, the instrument will show an undue amount of zero shift, which will reduce the accuracy of the measurements.

Another kind of zero shift is caused by the gradual uncoiling of the springs over a period of years. This effect is noticeable in instruments employing but one spring, such as the moving-iron type. In moving-coil instruments it is customary to use two spiral springs wound in opposite directions, so that the tendency in one spring to unwind would offset a similar tendency in the other spring.

Zero adjusters are becoming more generally used, and are very convenient. As ordinarily made, however, the range of adjustment is far greater than that necessary to correct for the slight amount of zero shift likely to occur in the reasonable use of the instrument. This excess of range of adjustment introduces the liability of error, since the zero adjuster may be used to bring a bent pointer back to zero, thus introducing errors over the whole scale which in some types may be objectionably large. While it might be some additional trouble for the maker, the service accuracy and reliability would be improved by limiting the range of zero adjustment to a small amount, say not over 1 deg. or 2 deg. of arc.

A permanent shift of zero in an instrument with a uniform scale can be corrected for, but transient zero shift occurring during the use or test of an instrument can not be accurately allowed for because it is not possible to know the effective zero position corresponding to given readings.

(*d*) *Ratio of Torque to Weight of Moving Element.* In order that the frictional error of an instrument shall be kept below the limit of reading, it is necessary to have good workmanship in the pivots and jewels, and also a value of the ratio of torque to weight which is not less than a specified amount for the given kind of instrument. Janus[37] gives the rule that

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[36] Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., p. 31.

[37] Janus, *E. T. Z.* vol .28, p. 560; 1905.

the quotient of the torque in centimeter-grams divided by the weight in grams should not be less than 0.17; Heinrich and Bercovitz[38] give the value 0.05. Both of these cases refer to a full-scale deflection of 90 deg., and do not specify any particular class of service. A more definite statement[39] is that of Edgcumbe, namely, that the ratio of torque in centimeter-grams to weight in grams, for 90 deg. deflection, should not be less than 0.05 for portable instruments or 0.10 for switchboard instruments, the higher value for the latter being necessary because the axis is horizontal, the pivots usually are coarser, and the service is more severe. For switchboard service MacGahan[40] considers that 0.15 is the minimum satisfactory value.

The presence of frictional error or "sticking" may be observed by testing the instrument in comparison with a suitable standard, bringing the instrument under test carefully to a given reading without overshooting, first from lower, then from higher values. The difference between the means of sets of readings approached from the two directions will be twice the frictional error.

In this country it is more usual to express the torque of electrical instruments and meters in millimeter-grams.[41] The corresponding values of ratio of torque to weight then become ten times the value above given.

It should be noted that an instrument may have a high ratio of torque to weight and still be objectionable because the weight is excessive. In fact, it appears that as the weight on the pivots is reduced the value of torque-weight ratio may be lower for a given limit of frictional error.

The importance of high torque-weight ratio and light weight of moving element in watthour meters is now generally recognized, and a statement of these values is customary in announcing new designs. The corresponding practise in indicating instruments seems to be only beginning.[42]

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[38] Heinrich and Bercovitz, *Handbuch der Elektrotechnik*, vol. 2, part 5, p. 14.

[39] Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., p. 32.

[40] *Elec. Jour.* vol. 8, p. 1100; 1911.

[41] The use of the millimeter-gram is required by the A. I. E. E. Standardization Rules (No. 236).

[42] A recent catalog issued by an American maker gives values of torque and weight for direct-current and moving-iron instruments.

(*e*) *Damping Devices.* The usual methods of damping involve either eddy currents or fluid friction. Eddy-current damping is the ideal method in the permanent-magnet moving-coil instrument, and is realized by winding the moving coil on a conducting frame (usually of aluminum) in which eddy currents are set up whenever the coil moves. Eddy-current damping is also used in two of the leading American makes of electrodynamic instruments.

The most commonly used fluid-friction damper is the air damper, which is adapted to both switchboard and portable instruments. Recording instruments with moving systems of large inertia require more damping than can be supplied by an air damper, and oil damping is used. Another typical case requiring oil damping is that of the electrostatic voltmeter with its large moment of inertia and small operating forces. The inconveniences attending the use of oil make it undesirable where one of the other methods can be used.

Damping devices, if free from static friction, as they should be, can affect the accuracy of measurements only indirectly. If the damping is too small or too large to give critical motion of the moving system, larger errors may be made in attempting to read the momentary value of a fluctuating load. For loads which fluctuate very greatly, overdamping is sometimes used in order to smooth out the fluctuations and give an average value.

The one disadvantage of air damping is that it requires very small clearances between the vane or piston and the chamber in which it moves. Compared with the eddy-current damper consisting of an aluminum vane moving between the jaws of a permanent magnet, the air damper adds less weight and less moment of inertia to the moving system. The magnet of the eddy-current damper introduces a stray field, and is in turn exposed to the demagnetizing action of the alternating current in electrodynamic instruments, and for this reason it could hardly be used but for the fact that such instruments of the better class are now generally provided with magnetic shields around the operating system.

(*f*) *Instrument Losses.* The torque and torque-weight ratio of some forms of instrument may be increased by designing the windings for greater current densities and consequently greater internal losses. This procedure is open to several objections. The increased losses cause greater temperature differences within the instrument, and thus tend to produce larger errors due to self-heating. They also affect the conditions in the circuit. Increased losses imply greater resistance in series windings and lower resistance in voltage windings. When instrument transformers are used, their accuracy may not be so good when the instruments have higher losses. Finally, the margin of permissible overload is cut down as the losses are increased. In general, instruments should be designed with the smallest losses compatible with adequate torque.

When using instruments under conditions such that the power lost in the instruments needs to be taken into account, the connections should in general be made so that the error would be a minimum if not allowed for.[43] For example, if the power taken by a 100-volt 1-ampere lamp is to be measured, using an ammeter of 0.1 ohm resistance and a voltmeter of 10,000 ohms resistance, we may connect the voltmeter either around the lamp or around both the lamp and the ammeter. In the former case the error is 1 per cent, since the ammeter reads the sum of the lamp current and the voltmeter current. In the second case the voltmeter reads the lamp voltage plus the drop through the ammeter, and the error is only 0.1 per cent. If the voltage across the load is small compared with the rated voltage of the instruments, it may be better to put the shunt instruments across the load only. For example, if the above ammeter and voltmeter are used to measure the power taken by a 10-ohm resistor on a 10-volt circuit, the error would be 0.1 per cent with the voltmeter connected across the load, or 1 per cent if across both load and ammeter. Similar considerations hold for the case of measurement of small values of power with an uncompensated wattmeter.

7. EFFECT OF DETAILS OF CONSTRUCTION ON ACCURACY

(*a*) *Good Connections.* It is of importance in all instruments, and especially in millivoltmeters and in instruments operated from current transformers, that the electrical connections shall be well made and capable of remaining so in use. Binding posts should be pinned or otherwise secured so that they will not turn, and the electrical connection should be made independent of the attachment of the post to its support, so that the loosening of the post will not introduce

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[43] H. B. Taylor, *Elec. Jour.*, vol. 2, pp. 474-481; 1905.

a bad contact. Neglect of this principle has given rise to inaccuracy and uncertain readings, not only in instruments, but also in current transformers, where a defective contact affects the ratio and phase angle.

One of the best methods of checking the connections of an instrument is to measure its resistance with a Wheatstone bridge, using one cell (or at most only a few cells) to supply the current. Such a measurement of the voltage circuit of a wattmeter, for example, will often reveal the existence of a bad contact at the key which would not be detected in the ordinary use of the instrument, but which might become bad enough to introduce unsuspected errors later.

(*b*) *Contact Keys.* These should be made so as to close with a slight wiping motion to keep the surfaces clean. The contact points should be of platinum or some other metal or alloy suitable for the purpose. Built-up keys having the parts separated by thin sheets or washers of insulating material are more liable to give trouble by leakage or insulation breakdown between the parts than keys in which the metal parts are independently carried by the hard rubber or other insulating support.

(*c*) *Spring Abutments.* Instruments occasionally develop during shipment apparent zero errors and friction, which are found to be caused by the catching of one or more turns of the spring on some projecting part of the spring abutment. The latter should be made in such a way that this cannot occur.

(*d*) *Scales.* Cardboard scales often give trouble by loosening, especially in moist climates, and they should preferably be held by screws or rivets in addition to the usual cement. Buckling of the scale may bring it near enough to the needle to introduce friction.

8. THE MEASUREMENT OF VOLTAGE

(*a*) *Direct-Current Circuits.* While some other types can be used for this service, the permanent-magnet moving-coil voltmeter is the acknowledged standard except for certain low-priced instruments used for testing dry cells, for which the moving-iron type is used.

While there is no theoretical limit to the maximum range for which direct-current voltmeters may be made, in practise the limit is set by the high resistance of the multiplier and the consequent large shunting effect of leakage paths. For example, an ordinary portable d-c. voltmeter to measure up to 15,000 volts would require a multiplier of about 1.5 megohms resistance, and if a leakage path occurs, with a resistance of 150 megohms, an error of 1 per cent will result. The same leakage path across the series resistor of a 150-volt instrument would produce an error of only 0.01 per cent. By subdividing the multiplier and applying the guard-wire principle to protect each subdivision from leakage, the error in measuring the high voltage can be reduced as much as is desired, but at the cost of complication, extra cost, and added power loss. For switchboard use there are on the market d-c. voltmeters with oil-insulated series resistors up to 15,000 volts.

In using high-range voltmeters, or in checking them by the potentiometer method, considerable care is necessary to avoid errors due to electrostatic action on the index of the voltmeter and to leakage currents through the galvanometer used with the potentiometer. It is probable that many unsuspected errors occur in the use of potentiometers having delicate reflecting galvanometers. This is especially liable to occur in humid weather. As an illustration, the results of an experiment carried out some years ago at the Bureau of Standards may be cited. Two five-dial potentiometers of the high-resistance type, with reflecting galvanometers, were used with volt boxes to measure the voltage of a storage battery, which was about 500 volts. One of the potentiometer outfits was carefully shielded by the guard-wire method, the other was used in the ordinary manner. The shielded one gave very steady readings, as would be expected, while the other showed variations up to 1.5 per cent, depending upon the condition of the battery as to insulation from ground. The temperature during this experiment was 26 deg. cent. and the relative humidity 90 per cent. By placing the unshielded potentiometer and all its accessories, including the galvanometer, on hard rubber blocks, the maximum error was reduced from 1.5 per cent to 0.3 per cent. While this experiment was carried out with potentiometers, it is evident that similar difficulties may arise with high range voltmeters.

The lower limit of measurement with portable d-c. instruments of regular construction is about 20 millivolts (for full scale deflection) if temperature corrections are applied, or 200 millivolts for instruments requiring no temperature correction. This applies to instruments compensated by the addition of resistance in series with the moving coil. By the use of suitable resistances in parallel and in series, compensated instruments are made for as low as 45 millivolts for full scale deflection.

When a d-c. voltmeter is connected to a battery or other source having an appreciable internal resistance, the reading of the voltmeter is lower than the e.m.f. of the source by an amount which depends on the relative magnitudes of the resistance of the source and of the voltmeter. If the resistance of the source is unknown, the magnitude of the error is unknown. The following method will show whether such an error exists and will make it possible to compute from two readings the true e.m.f. of the source.

Connect the voltmeter to the two points between which the measurement is to be made; let its reading be denoted by V_1. Then shunt the voltmeter by a resistance equal to its own resistance; let the corresponding reading be V_2. If the internal resistance of the source be negligible, the two readings will not differ. If they differ, the e.m.f. E of the source may be computed by the formula

$$E = \frac{V_1 V_2}{2\,V_2 - V_1} \tag{2}$$

This method is not limited to the simple case of a source with an unknown resistance in series, but is quite general, and may be stated as follows. If A and B are any two points on a network of conductors containing unknown but constant direct electromotive forces and resistances, and if E is the value of the potential difference between A and B, then the use of a d-c. voltmeter in the manner above described will give the value of E. In other words, a voltmeter used in this way will give the same result (within the limits of accuracy of the method and the instrument) as a null measurement with a potentiometer. This is true, whether the voltmeter be connected directly to the points A and B, or through "pressure wires" of appreciable resistance.

The accuracy of this method is greatest for values of V_2 nearly equal to V_1, and decreases as the ratio V_2/V_1 becomes less. For the case of internal resistance of the source equal to that of the voltmeter, $V_2 = 2V_1/3$, and an error of 0.1 per cent in V_1 and V_2 will cause errors of $+0.4$ per cent and -0.3 per cent respectively in the calculated value of E.

In using this method, if V_2 differs from V_1 by only a small amount ΔV, say a few per cent, the value of E is given well enough by the approximate formula

$$E = V_1 + \Delta V \tag{3}$$

A modification[44] of the above method consists in getting the reading V_1 as above, and then inserting a resistance in *series* with the voltmeter, equal to the resistance of the voltmeter. The unknown e.m.f. E may then be found by the formula

$$E = \frac{V_1 V_2}{V_1 - V_2} \tag{4}$$

A possible application of the above method would be to avoid error in the use of thermocouple pyrometers due to resistance in the leads. The temperature is measured by the reading of a millivoltmeter connected to the couple, and the reading for a given temperature will vary with the lead resistance. By providing such millivoltmeters with a resistance which can be connected in parallel by a key, the two readings may be used to determine the true electromotive force of the couple. Another obvious use is for the measurement of voltage at feeder ends by means of pressure wires of appreciable resistance.

With the increasing need for accuracy in temperature measurement and control, methods have been devised to eliminate the effects of lead resistance. In the instrument known as the pyrovolter the moving coil of the millivoltmeter is first used as a galvanometer to indicate when the IR drop through a resistance coil is equal to the value of the electromotive force E to be measured. By pressing a key, the circuits are then changed so that the moving coil now carries the current I. The scale is marked to read the IR drop, and hence the quantity to be measured. The instrument thus draws no current from the source, and within limits, is equivalent to a millivoltmeter of infinite resistance; since it therefore has something in common with the potentiometer, it might be called a "potentiometric millivoltmeter." There is a limit to the amount of resistance permissible in the leads, beyond which the sensitivity of setting the IR drop to equality with E is impaired, and with it the accuracy of the results. It must also be kept in mind that the result depends directly upon the constancy of magnet and springs, effect of local disturbing fields, etc.

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[44] Suggested by Dr. P. G. Agnew.

A more recent device for the same purpose is called the "compensated indicator." It consists of a millivoltmeter having a variable rheostat in series, also a key by which a shunt circuit may be connected across the moving coil and a part of the series resistance cut out. In operation, the variable rheostat is adjusted until no change in reading occurs when the key is depressed. The reading then gives the value of the electromotive force of the couple. The compensated indicator and the pyrovolter have other applications besides in pyrometry. They may be used to measure the potential difference between two points of a network, or the electromotive force of a battery, without the necessity of allowing for the current taken by the instrument, as is necessary with ordinary indicating instruments.

(*b*) *Alternating-Current Circuits.* For this work the type of voltmeter to be used depends on the nature of the service, as previously discussed. Voltmeters are made for direct connection to a-c. circuits up to 750 volts, and in some forms up to 2300 volts, but the latter practise is not recommended.

The measurement of alternating voltages less than one volt is difficult. A high-grade electrodynamic voltmeter of 1-volt range has a resistance of 2 ohms, so that it draws 0.5 ampere from the source. If such an instrument were rewound for a maximum reading of 0.25 volt, it would require a current of two amperes, which would be too large to pass through the springs. For such a range, an unshunted hot-wire ammeter of say three to five amperes range may be used, but the current taken from the source is excessive. When it is desired to measure the current taken by a load at low voltages of the order of 0.25 volt, the current required by a hot-wire ammeter used as a voltmeter may introduce a serious error, which may be difficult to allow for when the phase angles of the load and the voltmeter are different. To overcome this, the following method may be used.

Fig. 6 shows a load *C* for which it is desired to find the current for a given applied voltage. *A* is a two-range ammeter having the terminals of the two sections of the winding accessible.[45] The total current taken by the load and the voltmeter is passed through one section, and the voltmeter

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[45] Two-range ammeters with links for changing the range, and hence accessible terminals of the two parts of the winding, are available.

current is passed through the other section in reverse direction. The voltmeter current thus cancels itself so far as the reading of the ammeter is concerned. The voltmeter corrections should be determined with the ammeter coil and the leads up to the points *B* and *D* included as a part of the voltmeter.

(*c*) *Voltage Transformers.* These are practically identical with distributing transformers of small capacity. For moderate voltages, no special difficulties are present in their design, such as are encountered in the current transformer. The rated output of the voltage transformer is made smaller than the permissible output as limited by heating considerations, in order to limit the change of voltage with change of load to a sufficiently low value. It is good practise to choose the winding ratio so that the "voltage ratio" (ratio of primary impressed voltage to secondary terminal voltage) will be slightly below its nominal value at no load and above it at full load, so that the ratio will be correct at some intermediate load approximating average conditions of use.

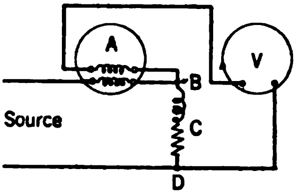

FIG. 6

The secondary terminal voltage is in general not in exact opposition to the primary voltage, but is displaced from it by a small angle which for the lack of a better term[46] is called

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[46] Strictly speaking, the phase angle of a voltage transformer would be the angle 180 deg. ± γ, where γ is the angle as defined above. In the same way, the phase angle of a condenser should be defined as the angle between the impressed sinusoidal voltage and the current, and is nearly 90 deg. In practise, however, the "phase angle" of a condenser is taken as the small angle by which the above differs from 90 deg. Various names have been suggested for the small angle by which a vector quantity departs from its ideal position; for example, "angle of defect." This is objectionable because it implies either that the apparatus has a defect, or that the angle may be too *small*, whereas it may be too large. Since the word "bias" may mean a swerving to one side or another of a desirable or normal direction, its use in such expressions as "phase bias," "bias angle," or "angle of bias" would be etymologically correct.

the "phase angle." This quantity however, does not have to be considered in connection with the measurement of voltage, and will be discussed under "Measurement of Power."

The most accurate method of determining the ratio and phase angle of voltage transformers is one which is practically an a-c. potentiometer arrangement. This method has been devised independently by several workers in this field.[47] It consists in applying the primary voltage to a high non-reactive resistance and in balancing the secondary voltage against the drop over a portion of this resistance. The ratio of the total resistance to the portion is equal to the "voltage ratio." It is obvious that the method is theoretically an accurate one, since resistance measurements (particularly resistance comparisons) may be made with great accuracy. However, in the present case one of the resistances is quite large for high voltages, so that the insulation must be carefully maintained, and the high resistance should preferably be divided into a number of sections, each protected by an electrostatic screen to prevent capacity currents from flowing between the sections themselves and between the sections and surrounding objects. At the Bureau of Standards, the testing of voltage transformers by this method has so far been restricted to an upper limit of 30 kilovolts. The ratio of voltage transformers for voltages considerably above this value cannot be measured as accurately as is desirable, and it seems probable that the ratio as fixed by the ratio of turns may depart considerably from the values which would be given by computation based on the performance of the same iron core wound for lower voltages. The reason is that at high voltages there are capacity currents between the primary sections themselves and between sections and other parts, such as core and case. These currents are superposed on the exciting current in the primary winding, and alter both ratio and phase angle in a way which does not at present admit of calculation. The attempt has been made to measure the high secondary voltage of step-up transformers by a voltmeter connected to a special low-voltage winding (the so-

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[47] Agnew and Fitch, *Bull. Bur. of Standards*, vol. 6, p. 281, 1909; *Elec. World*, vol. 54, p. 1042, 1909.

L. T. Robinson, Trans. A. I. E. E., vol. 28, p. 1005, 1909.

F. A. Laws, *Elec. World*, vol. 55, p. 223, 1910.

Sharp and Crawford, Trans. A. I. E. E., vol. 29, p. 1517, 1910.

Agnew and Silsbee, Trans. A. I. E. E., vol. 31, p. 1635, 1912.

called "test coil," "voltmeter coil," or "tertiary coil") located at the center of the high-voltage winding, and evidence is adduced to show that in certain cases good agreement was obtained between two transformers equipped with voltmeter coils. The use of such a coil will give accurate results, in general, only if the coil is in every way a representative sample of the high-voltage winding. It may be possible to locate the voltmeter coil so that the flux through it is the average flux through the high-voltage winding, but the effect of capacity between coils can not be readily cared for. Differences in ratio of 0.6 per cent have been observed in 100 kv. transformers due to these capacity effects and it seems probable that they may often be larger. It seems evident that there is a great need for means of accurately measuring high voltages, which shall operate directly on the high voltage. Such apparatus, even if not applicable to routine measurements, would be of great value in settling such questions as the accuracy of results obtained with the "voltmeter coil."

(*d*) *Crest Voltmeters.* With the constantly rising values of transmission voltages, the insulation of transformers and other high-voltage apparatus becomes more and more important. Since the evidence indicates that the breakdown of insulation is determined by the crest value of a voltage wave, rather than the effective value, it is necessary to have means for measuring crest voltages. This is the more necessary because distortion of the voltage wave is liable to occur in the testing of insulation, on account of the electrostatic capacity of the load.

The needle gap has been used for years for measuring crest voltage, and the sphere gap is a more recent apparatus which is more reliable and better adapted to high voltages. The needle gap is very much affected by distortion of the electrostatic field caused by surrounding objects, and the sphere gap is similarly affected, though to a smaller extent.

Direct-reading methods of measuring crest voltage have been devised, and instruments based on some of them are on the market. One of these[48] uses hot-cathode valves to rectify the charging current of a condenser, with a d-c. instrument to measure the current. This method is correct only when the voltage wave has but one maximum and one minimum per

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[48] L. W. Chubb, TRANS. A. I. E. E., vol. 35, p. 109; 1916.

cycle. Another form[49] is essentially an oscillograph element which throws upon a screen a band of light the length of which is proportional to the peak voltage. Of these two instruments, the former is used directly on the high-voltage, while the latter is necessarily connected to the low-voltage secondary (or tertiary) winding of a transformer, and hence any uncertainty as to the ratio of transformation affects the accuracy of the result.

There is reason to believe that the corona voltmeter[50] is capable of giving accurate measurements of crest voltage, and that its calibration may be computed from measured dimensions and the electric strength of air under given conditions of temperature and pressure. Its establishment as a standard instrument by which other crest voltmeters may be checked seems to depend only upon further work in which high voltages shall be measured and their wave forms determined by methods which are free from the limitations and uncertainties of those now available.

9. THE MEASUREMENT OF CURRENT

(*a*) *Direct-Current Circuits.* The permanent-magnet moving-coil ammeter is the standard for all but small low-priced instruments used for dry-cell testing and for automobiles. For the latter the polarized vane[51] type is largely used.

The practise of American makers seems to be to supply medium size switchboard ammeters with self-contained shunts up to about 75 amperes. Above this, (and also below, on order) instruments with separate shunts are supplied. One maker lists seven-inch instruments self-contained up to 200 amperes. Small instruments, such as three-inch round and the smallest fan-shaped instruments, are self-contained up to 15 to 30 amperes. Large illuminated-dial, also some fan-

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[49] Middleton & Dawes, TRANS. A. I. E. E., vol. 33, p. 1204; 1914.

[50] Whitehead and Pullen, TRANS. A. I. E. E., vol. 35, pp. 809-833, 1916.

[51] This instrument, brought out by Deprez in 1880, contains a pivoted soft-iron vane in the field of a permanent magnet. A coil carrying the current to be measured sets up a field at right angles to the field of the permanent magnet, and the direction of the resultant field thus varies with the current. The vane assumes the position of the resultant field. This construction was used in the Thomson-Rice ammeter and voltmeter in the early days of electric lighting. In recent years the need for a simple and very cheap instrument for automobile use, capable of showing reversal of current flow, has caused this type to be made in large numbers.

shaped instruments, are made only for external shunts. The widest variation of practise is found in standard portable ammeters, which are supplied self-contained up to 500 amperes by one maker, while two others limit internal shunts to 30 amperes in order to avoid errors due to heating. This is a matter which affects the accuracy of electrical measurements, and it is therefore advisable to compare briefly the self-contained ammeter with the separate-shunt ammeter.

In the self-contained ammeter the shunt is not open to the air[52], and hence its rise of temperature under load is greater than it would be if separate from the instrument. While the effect of this heating on the shunt itself may be made small by the use of suitable alloy, the heat liberated raises the temperature of the moving coil, springs, and other parts, thus introducing opportunity for error, since even compensated millivoltmeters may have errors if their parts are at different temperatures. Another objection to internal shunts for large currents is the magnetic effect of the current in the loop formed by the conductors which connect the instrument into the circuit. Large conductors are difficult to handle, and cannot be readily twisted together to make them non-inductive, as can be done with smaller wires. A further advantage of the separate-shunt ammeter is that its range can easily be altered by the use of other shunts.

On the other hand, the separate-shunt ammeter has the disadvantage that its moving-coil circuit is completed through four binding-post contacts, which may become dirty or corroded, and two flexible leads, which may be inadvertently exchanged for others[53] of similar appearance, or which may have their resistance affected by the breaking of strands.[54]

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[52] This does not apply to the self-contained portable ammeter of 500 amperes capacity above referred to. This line of ammeters has provision for ventilation of the shunt, in some of the higher ranges.

[53] Instrument makers are not always sufficiently careful in this matter. In a shipment of instruments sent to the Bureau of Standards for test were a portable millivoltmeter and a labratory standard millivoltmeter, with leads which were of different length and resistance and which bore no numbers or other marks to show with which instrument they were to be used.

[54] Durand, Congress of the Applications of Electricity, Marseilles, 1908, p. 620, states that he has often found d-c. ammeters reading 10 to 20 per cent low from this cause. This evidently occurred with ammeters having a low resistance in the instrument, with low shunt drop. He gives this difficulty as one reason for avoiding low values of shunt drop, and he prefers 100 millivolts.

One American manufacturer has for several years supplied small portable ammeters with three self-contained ranges on the plan of the Ayrton-Mather galvanometer shunt. This is an arrangement which deserves to be more widely known and used. It has the advantages of avoiding flexible leads, as well as binding-post and switch contacts in the millivoltmeter circuit. There is one common + terminal, with a − terminal for each range, and the change from one range to another is made by shifting the − lead from one − terminal to another. Thus the only contact which is shifted is in the line circuit and has no effect on the accuracy of the instrument. It would be an advantage to the users of standard portable self-contained ammeters if all makers would furnish them with multiple ranges on this plan.

It was once common practise to make direct-current ammeters with non-interchangeable shunts and leads. It is now the prevailing custom of American makers to adjust all shunts of a given class to a uniform drop, which varies from 50 to 100 millivolts. The Meter Committee of the National Electric Light Association investigated the matter of standardization of shunts by sending a circular letter to ten instrument makers. This letter contained sixteen recommendations, some of which related to mechanical details such as width of slots and number of bolt holes, while others specified the use of a standard millivolt drop, the accuracy of adjustment (1 per cent), and the electrical properties of the shunt metal, namely, a practically zero temperature coefficient and no thermoelectric effect against copper. The Committee suggested the tentative adoption of the following values of millivolt drop; switchboard type instrument shunts, 50 and 60 millivolts; portable instrument shunts, 100 and 200 millivolts. The Committee concluded from its analysis of the replies that a large percentage of shunts was being manufactured in accordance with its recommendations, and that further conference with the makers might lead to further progress.

The matter of standardizing all shunts of a given maker has a bearing on instrument accuracy. If shunts of a given kind and make are only roughly adjusted, say within 10 per cent of a nominal value, the adjustment of the instrument resistance must be individually made to suit a particular shunt. In service, such instruments may be connected to the wrong shunts through oversight. If two shunts differing in drop by 10 per cent, which were correctly adjusted to their respective millivoltmeters, are interchanged, the instruments will differ by over 20 per cent when connected to measure the same current. A case of this nature is reported in which the error resulted in the overloading of one generator and the underloading of another, with generally unsatisfactory results, for more than a year before the trouble was discovered and rectified.[55]

The range of measurement by means of d-c. ammeters is very wide. Excluding the very sensitive unipivot instruments and considering only regular two-pivot instruments of good torque-weight ratio, capable of operation in any position, the lower limit is perhaps illustrated by a standard portable instrument giving a deflection of 150 divisions for 300 microamperes. No upper limit of current exists for the shunted ammeter, except that fixed by the requirements of the user. Shunts of 20,000 amperes rating are regularly listed by several American makers, and larger ones may be had on special order.

The construction and accurate adjustment of very large shunts presents some difficulties. The usual practise in making shunts of small and medium capacities is to solder all of the sheets of alloy into two terminal blocks. This requires that all of the sheets shall be soldered into a given block at one heat. This becomes very difficult in large shunts, and the accurate adjustment is not easily accomplished because of the lack of resistance standards of such very low values and the impossibility of getting enough current to give a sufficient drop for an accurate test. It is possible to avoid this by making up the shunt of sections in parallel, each section being individually adjusted and provided with potential wires which have resistances proportional to the resistance of the shunt and large in comparison with the latter. The sections may then be bolted or soldered together at each end to form a shunt of large capacity. The free ends of the potential wires leading from each terminal are joined together to form the two potential points of the large shunt. With this arrangement, the separate sections are readily adjusted, and small irregularities in the division of current among the

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[55] McTighe, *Elec. Jour.* vol. 3, pp. 358-9, 1906.

different sections will not materially affect the resistance of the shunt as observed from these potential points.[56]

The method of connecting the copper bars to the ends of large shunts has an appreciable effect on the accuracy. Changes amounting to as much as 10 per cent have been reported.[57]

When a d-c. ammeter is connected into a circuit to measure a current, the reading of the ammeter, corrected for any error it may have, does not give the value of the current which was flowing in the circuit before the ammeter was inserted. This is because the addition of the ammeter increases the resistance and therefore diminishes the current. While this effect is small in many or most practical cases, there are occasions where the current is flowing in a low-voltage low-resistance circuit, and the use of a high-resistance ammeter, such as one of the hot-wire type, may introduce an appreciable error. The following method makes it possible to determine the value of the current I which was flowing before the ammeter was inserted:

Let the reading of the ammeter be I_1. Then insert in series with it a resistance equal to its own resistance; let the reading now be I_2. Then the current originally flowing is

$$I = \frac{I_1 I_2}{2\,I_2 - I_1} \tag{5}$$

This result is of the same form as that for the similar case of the voltmeter. It is possible to derive a slightly simpler formula by shunting the ammeter with an equal resistance, but this procedure is not to be recommended, especially for low-resistance ammeters.

(*b*) *Alternating-Current Circuits*. For this work, moving-iron ammeters are probably the most generally used, because of their simplicity, relative independence of frequency, and the fact that if necessary they can be checked (at least approximately) on direct current.

In the laboratory the moving-iron ammeter is very conven-

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[56] The principle involved in this method is discussed by Wenner from a different point of view; *Bulletin of Bur. of Standards*, vol. 8, p. 575, 1912, Scientific Paper No. 181.

[57] Rosa, TRANS. A. I. E. E., vol. 24, p. 223, 1905; Durand, Congress of the Applications of Electricity, Marseilles, 1908, pp. 625-6.

ient for measurements not of the highest accuracy. For more accurate measurements on ordinary commercial frequencies, the electrodynamic ammeter has a number of advantages, including the possibility of accurate test on reversed direct current. For higher frequencies it is necessary to use hot-wire or thermocouple instruments. The writer has been informed by some users of thermocouple instruments that they are more robust and give much better satisfaction than the hot-wire instruments. They are displacing hot-wire instruments in the most important commercial application of the latter, namely, in radio work.[58]

Moving-iron ammeters are made in ranges from about 0.1 ampere up to 600 amperes. Above the latter value it is necessary to use current transformers. This is frequently done for much lower ranges, and is always done when the circuit is of high voltage, in order to insulate the instrument from the circuit. The accuracy of the measurement is then a composite of the individual accuracies of the ammeter and the transformer. The ratio of transformation (primary current divided by secondary current) is in general not quite equal to the nominal value as marked on the name plate, and it varies with the frequency, primary current, and secondary burden. This matter is discussed in more detail in the following paragraphs.

(*c*) *Current Transformers.* An ideal current transformer would give a secondary current equal to $1/n$ of the primary current (where $n/1$ is the marked ratio) for all values of primary current, frequency, wave form, and secondary burden. Furthermore, the phase displacement between the primary current and the secondary current would be 180 deg. under all conditions.

Because a component of the primary current is required to magnetize the core, the secondary ampere-turns are in general smaller than the primary ampere-turns, and the angle between them differs from 180 deg. by a small angle. By using a slightly smaller number of secondary turns than the number required by an ideal transformer for a given ratio, the secondary current may be brought near to the desired value for a given set of conditions. The phase angle cannot be so readily controlled, and since it depends largely on the vector relation between the secondary current and the com-

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[58] G. Y. Allen, *Elec. Jour.* vol. 16, pp. 494-500, 1919.

ponent which magnetizes the core, the usual method of keeping the phase angle down is by the use of proper material and good design.[59]

For a given current transformer the ratio will vary with the primary current and frequency, and with the secondary burden.[60] There is no method in use at present by which the performance of a current transformer under one set of conditions can be readily computed from its performance under another set. It is therefore necessary, in accurate work, to test the transformer under the conditions of use. In general, an increase in the secondary burden will increase the values of the ratio and the phaseangle for all values of secondary current and will also increase the rate of change of ratio and of phase angle with changing secondary current. The secondary burden should therefore be reduced to a minimum. A decrease in frequency acts in much the same way as an increase in the secondary burden.

The requirements which current transformers must meet differ widely. If only an ammeter of low impedance is to be operated at one stated frequency, the current ratio only is of importance. If the transformer and ammeter may be calibrated and used together, a relatively large variation in ratio with change of secondary current is not objectionable, as it can be taken care of in the marking of the ammeter scale. However, if a current transformer is to be capable of satisfactorily operating one or a number of ammeters, the latter being calibrated independently of the transformer, it is necessary to use what might be called a higher grade of transformer. To produce it, in general one must use more iron in the core, or more copper in the windings, or both. If, further, it is necessary to operate a wattmeter or a watthour meter, and the power factor of the system is appreciably below unity, a still better grade of transformer must be used, since in addition to good ratio performance a small phase angle is required.

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[59] For a clear and practical discussion of instrument transformer design see Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., pp. 312-338.

[60] The term "secondary burden" is recommended by the Institute as a substitute for the somewhat ambiguous terms load, secondary load, or secondary connected load. It is further recommended that secondary burden be expressed quantitatively as the resistance in ohms and inductance in henrys of the external circuit connected to the secondary winding of the transformer.

In the above statements, *grade* is intended to apply to the materials and the electrical and magnetic design, not to the mechanical features of construction, nor to the ruggedness of the windings, breakdown strength of insulation, etc. These latter features, while not directly related to the inherent accuracy as determined by design and materials, are of great importance because of their relation to service accuracy and reliability. Current transformers are often subjected in service to severe electric stresses due to high-frequency line surges, and to mechanical stresses caused by electrodynamic action between the windings when short circuits occur. Their construction therefore requires careful attention to details.

Using the word *grade* in the above qualified sense, it would seem desirable to have a set of performance specifications for various standard grades of current transformers.[61] These specifications would include the permissible deviation of the current ratio from the marked value over prescribed ranges of secondary current, secondary burden, and frequency, and would set limits for the phase angle under the same conditions. This would remove the uncertainty that now exists in comparing or selecting current transformers, since the present name-plate rating in watts or volt-amperes gives but little definite information which may be used as a basis of comparison, especially between the transformers of different makers.

Some general statements may be made in regard to the suitability of current transformers for accurate measurements. It is stated that,[62]

> A well-designed current transformer will have from 600 to 1200 ampere-turns in the primary at its rated full-load current. There is no objection to increasing the upper limit to almost any extent when the current to

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[61] The "Code for Electricity Meters" of the Joint Meter Committee of the A. E. I. C. and the N. E. L. A. contains specifications for the acceptance of types of current and voltage transformers. While this is limited to the use of such transformers as auxiliaries to watthour meters, and does not consider other grades, it would serve as a beginning for more inclusive specifications. The advance in the art of making transformers is such that the limits of tolerance for ratio and phase angle given in the Code are too broad for present practise.

Edgcumbe, in Industrial Electrical Measuring Instruments, 2d ed., pp. 337-8, gives concise specifications for the accuracy of current and voltage transformers, with tolerances considerably smaller than those of the Code for Electricity Meters.

[62] Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., p. 321.

be measured exceeds 1200 amperes, but there is no advantage in doing so for smaller currents, and the cost of construction is increased. These figures refer to transformers intended for a secondary load of from 15 to 40 volt-amperes at normal full-load current and at a frequency of 50. For other frequencies the permissible volt-ampere load varies in direct proportion with it, and for heavier loads an increase in the ampere-turns is desirable.

Another general statement is that current transformers having hinged or split cores should not be used with wattmeters or watthour meters, on account of the large phase angle which results from the large reluctance of the air gap in the core, and to the small value of full-load ampere-turns (125 to 250). These transformers are intended only for the approximate measurement of current in station or outside lines where it is necessary to link the transformer with the cable without cutting the latter. Because of the low value of ampere-turns, such transformers are used with ammeters specially calibrated with them, and they should be calibrated for the frequency on which they are to be used. Similar statements apply to the bushing type of current transformer of low full-load ampere-turns and excessive length of magnetic circuit.

Low-range instruments should not be substituted for five-ampere instruments with the expectation of better accuracy at low loads, unless the transformer has been tested with such instruments. The impedance of instruments of a given type increases approximately as the inverse square of the ampere rating.

The secondary circuit should not be opened while current is flowing, because the voltage at the secondary terminals may become dangerously high. The high value of flux may cause excessive heating of the core with liability of damage to the insulation. Furthermore, the magnetic condition of the core will be affected, with a resulting change in the ratio and phase angle, especially at small values of secondary current. The same abnormal condition of the core will occur if a direct current of sufficient strength flows through either of the windings. If magnetization occurs, the transformer may be brought back to normal condition (demagnetized) by operating at a flux density above the knee of the magnetization curve and gradually reducing the flux to a very low value. A method usually given for accomplishing this result consists in passing at least half the rated primary (alternating) current through the primary winding with a resistance of 10 to 20 ohms or more connected to the secondary in series with the secondary instruments. This resistance should then be gradually reduced to zero by steps of one ohm or less. Loose or corroded contacts, or excessive resistance in the secondary should be carefully avoided, and the resistance of the current windings of secondary instruments should be measured by a bridge to detect possible variable resistance due to loose or dirty connections.

When it is necessary to secure the best possible accuracy from wattmeters or watthour meters operated from current transformers, it is not advisable to connect relays or circuit-breaker trip coils in the same secondary circuit. In case it should be necessary to use the same transformer for instruments, relays, and trip coils, a high-grade transformer of liberal secondary rating should be used, and a reduced degree of accuracy should be expected. For important metering installations the current transformers operating the meters should have no other secondary burden.

When the secondaries of current transformers in a polyphase line are interconnected, it is difficult to calculate the effective secondary burden on the individual transformers in order to apply corrections for ratio and phase angle. For this reason, interconnection of secondaries is not advisable where the highest accuracy is desired.

The question of permanency of the constants of instrument transformers is one that has been up for a number of years. No systematic study of the question appears to have been published. The following statement has been made:[63]

> The permanency of commercial (instrument) transformers is such that their ratios may be relied upon to remain unchanged for a period of five years (but not longer without calibration) provided that the transformers have not been subjected to overloads, primary short circuits, or open-circuiting of the secondary of the current transformer under load.

Assuming that none of these abnormal conditions had been present, and that no contacts had occurred inside the windings, the only thing that could cause changes in ratio or phase angle is a change of the magnetic properties of the core. The kinds of transformer steel in use prior to the advent of silicon steel showed changes of this kind which in some cases were

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[63] Electrical Meterman's Handbook, 2d ed., p. 327; taken from "Code for Electricity Meters," 1912 ed., 123.

very marked. It is generally understood that modern high grade silicon steels are practically non-aging. In view of the fact that methods for testing current transformers are being devised[64] which are adapted to use by central stations, it is to be hoped that data will be kept for a sufficient time to enable the question of instrument transformer permanency to be settled.

Multiple-range current transformers in which the same primary windings are grouped in various connections by means of links or plugs usually have practically identical phase angles and proportional current ratios on the different connections. Current transformers in which the primary winding is put in place by the user[65] sometimes show slightly different values of ratio for different locations of the primary windings. If the primary leads are kept well centered in the hole and the return portions several inches away from the transformer these variations will usually be less than 0.1 per cent.

10. The Measurement of Power

Direct-current power may be measured by voltmeter and ammeter, or with an electrodynamic wattmeter. The latter requires that the mean of reversed readings be taken in order to eliminate the effect of local fields on unshielded wattmeters, or of residual magnetic polarity in shielded ones. For the measurement of a-c. power the wattmeter is the most suitable instrument.

(*a*) *The Electrodynamic Wattmeter.* For the accurate measurement of power at usual commercial frequencies this instrument is practically alone in the field.[66] This refers more particularly to portable or laboratory instruments which must be capable of use on various frequencies, power factors, and wave forms. These instruments are of great commercial importance because they are used to check watthour meters,

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[64] P. G. Agnew, *Bull. of the Bureau of Standards*, vol. 11, p. 347, 1914 (Scientific Paper No. 233); *Electrical World*, vol. 62, p. 898, 1913; O. A. Knopp, *Electrical World*, vol. 67, p. 92, 1916; F. B. Silsbee, *Bull. of the Bur. of Standards*, vol. 14, p. 317, 1917 (Scientific Paper No. 309.)

[65] This kind of current transformer needs a standard name. It is called by the following names at present: through-type, inserted primary type, hole type, form R, and doughnut type.

[66] The induction wattmeter was formerly made as a portable instrument, but it was subject to the limitation of a fixed frequency, and had a rather large temperature coefficient. At present, all portable wattmeters made in this country are of the electrodynamic type.

either directly or through portable watthour meters ("rotating standards"). Their accuracy limitations therefore directly affect the customer's bill and the company's income, and they are well worth careful study.

In principle, an electrodynamic wattmeter consists of a fixed coil, through which the load current flows, and a moving coil through which flows a current which is proportional to the voltage across the load. The moving coil is usually pivoted within the fixed coil, and its motion is opposed by a spiral spring. The fixed coil is wound with comparatively few turns of thick wire, or in some instruments with copper strip or bar, while the moving coil is wound with many turns of fine wire, and in series with it is a relatively large noninductive resistance. When currents flow through both coils, the moving coil tends to move so as to make the mutual inductance of the two coils a maximum. In the torsion-head form of wattmeter the moving coil is deflected by the action of the currents, but is brought back to the zero position by turning a head which winds up a spring attached to the moving coil. The zero position in such wattmeters is chosen as the position of zero mutual inductance. In the ordinary deflection wattmeter the mutual inductance is zero at about half scale deflection, and changes sign as it goes through the zero position.[67]

The torque of a wattmeter is at every instant proportional to the product of the currents in the two coils. If the moving coil had no inertia and no damping, its deflection at any moment would thus be proportional to the instantaneous power. Because of its inertia its deflection indicates the average power, which is the quantity usually sought.

(*b*) *Losses in Wattmeter Windings; Compensating Coils.* In using a simple wattmeter as above described, the voltage circuit (moving coil plus added resistance) may be connected either directly across the load, or so that the drop through the series coil of the wattmeter is also included. In the first case, the wattmeter reading includes the power loss in the voltage circuit, and in the second case it includes the loss in the

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[67] The Thomson inclined-coil wattmeter is an exception. In it the mutual inductance is positive over the whole scale, increasing toward the full scale point. The point of zero mutual inductance is at a considerable angular distance below the zero of the scale, and is a point of unstable equilibrium when currents are flowing through the two coils. The mutual inductance increases in both directions from its zero position.

current coil. The choice of one or the other of these connections depends upon the circumstances in the same way as in the use of a voltmeter and ammeter to measure power. The use of a compensating winding on the fixed coil makes it possible to connect the voltage circuit of the wattmeter directly across the load terminals and still get a reading of power which does not include the loss in the voltage circuit. The additional complication is justified by the convenience of not having to apply corrections, but it introduces some risk of error due to wrong connections. To avoid this, it is well to check the correctness of connections experimentally. When using a compensated wattmeter to measure the power taken by a load, break the load circuit by removing the wire from one terminal of the load, while leaving the voltage circuit excited. If the connections are correct, the wattmeter will show zero deflection, unless it is not entirely compensated. If the connection of the voltage circuit is incorrectly made, so that the compensating coil is not in circuit, the wattmeter will read up scale by an amount equal to the watts loss in its voltage circuit.

In testing a compensated wattmeter on separate sources of current and voltage, the compensating winding should be out of circuit. To test this point, apply an alternating voltage to the voltage circuit, with no current flowing in the current circuit. If the connections are correctly made, there will be no deflection; otherwise, a deflection down scale will result. This test can not be made so conveniently on direct current, on account of the effect of the local magnetic field.

Serious errors may result in the measurement of small values of power with compensated wattmeters of low range, unless the compensating winding has been very carefully located so as to have the same effect upon the moving coil as that of the fixed coil, for all positions of the moving coil.[68] A case came under observation some years ago at the Bureau of Standards which illustrates this point. A wattmeter of 150 watts rating was checked for compensation in the manner above described, and was found to be very closely compensated. It was then tested by the method of separate sources, and found to be reasonably accurate. In service, however, using

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[68] A. L. Ellis, Compensating Wattmeters, TRANS. A. I. E. E., vol. 31, pp. 1579-1590; 1912.

the compensating winding, the results of measurements on standard lamps showed very large inconsistencies, and the wattmeter was again investigated for compensation, not only at the zero point, but at points up scale by shifting the spring abutment. It was found that the compensation held good only at the zero point, and was very far from correct at other points. Investigation showed that the compensating coil had been wound up separately as a coil of short axial length compared to that of the current coil, and had been located at one end of the current coil, inside of the latter. The instrument was later rewound, using a cable of insulated strands, the center one being selected as the compensating winding. This method gave practically exact compensation for all positions of the moving coil. It may be pointed out that one of the ways of improving compensation for such losses is to have the losses as small as possible. For this and other reasons, the losses in both coils of a wattmeter ought to be kept as low as possible, while maintaining sufficient torque.

(c) *Effect of Self Inductance in the Voltage Circuit of a Wattmeter.* Self inductance in the voltage circuit of a wattmeter is unavoidable, as the torque depends upon the mutual inductance of the moving coil and the fixed coil. The self inductance of the moving coil affects the accuracy of a measurement of a-c. power by reducing the value of the current in the voltage circuit and by making it lag behind the applied voltage by a small angle. Since the magnitude of these effects does not depend simply on the inductance but on the ratio of inductance to resistance, it is evident that the resistance which is placed in series with the moving coil should be non-inductive, and as large as possible consistent with other requirements. By winding resistance wire in a single layer on mica or other insulating cards, resistors are made which have extremely small inductance and capacity, and this construction is much used.

(d) *Correction Factor for Self Inductance of the Voltage Circuit.* If a wattmeter is used to measure the power of a sinusoidal alternating current I which lags by an angle θ behind the e.m.f. E, and if the inductance and resistance of the voltage circuit of the wattmeter are L and R, so that $\tan \alpha = \omega L/R$ is the tangent of the angle of lag of the current in the voltage circuit, then the correction factor C by which the reading of the wattmeter must be multiplied to correct it for the effect of inductance in the voltage circuit is

$$C = \frac{1 + \tan^2 \alpha}{1 + \tan \alpha \tan \theta} \tag{6}$$

In a wattmeter which is fit to use, tan α will be quite small at ordinary frequencies, so that its square can be neglected, and we have the simpler form,

$$C = \frac{1}{1 + \tan \alpha \tan \theta} \tag{7}$$

(*e*) *Additive Correction for Self Inductance of Voltage Circuit.* While this expression is usable for moderate values of tan θ, it becomes less applicable as θ approaches 90 deg., near which value the correction factor becomes infinite. It is therefore preferable to make use of an additive correction, as first proposed by Drysdale.[69] To make use of this correction it is necessary to know the volt-amperes EI and the sine of the load phase angle, with L and R as before. Then the correction C', which must be added to the wattmeter reading, is

$$C' = - \frac{\omega L}{R} \cdot EI \sin \theta = - EI \sin \theta \tan \alpha \tag{8}$$

In both of these correction formulas, tan α is positive for inductance in the voltage circuit, and θ is counted positive when I lags behind E. Cases have arisen in which the series resistor in the voltage circuit, being wound as a bifilar coil, had enough electrostatic capacity to more than neutralize the effect of inductance, so that the current in the voltage circuit led the impressed voltage. In such cases tan α is taken as negative.

The additive correction may be written

$$C' = - (EI \tan \alpha) \sin \theta$$

which shows that for a given wattmeter and a given value of volt-amperes the correction *in watts* is zero at unity power factor ($\theta = 0$) and increases as a sine function up to a maximum of $-EI$ tan α at $\theta = 90$ deg. At 50 per cent power factor, with a given value of volt-amperes, the error due to inductance in the potential circuit, expressed in watts, has thus already reached 87 per cent of its maximum value. Since the error in a good wattmeter, as expressed in displacement of the needle

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[69] C. V. Drysdale, *Electrician* (London) vol. 46, pp. 774-8, 1901; vol. 76, pp. 523-5, 1916.

from its true position at a given power factor for the value of volt-amperes giving full scale deflection at unity power factor, is at most only a few tenths of a division, it may be seen that this error is not serious in most cases, and that the error in divisions for all points below 50 per cent power factor may be considered approximately equal to the error at 50 per cent power factor.

In the formulas for the additive correction and the correction factor the angle θ appears, and while it might be measured by a power factor meter, it is usually not known except from the values of volts and amperes, and the watts as read from the wattmeter. The latter being subject to error due to the various phase angles, the value of θ so found is only an approximate one. It is possible to use this value to get a first approximation to the true power, from which a more accurate value of θ can be found and used to get a second approximation. At this point we may note a further advantage of the additive correction as compared with the correction factor. The latter contains the term $\tan \theta$, which increases very rapidly as θ approaches 90 deg. Hence any error in determining θ will affect the accuracy of the correction factor very much. In contrast to this, the additive correction contains the term $\sin \theta$, which has reached 87 per cent of its maximum value when θ is 60 deg., and which increases to its full value more and more slowly as θ approaches 90 deg. It may be said that for a wattmeter which is fit to use, the apparent power may be used to determine θ for substitution in the additive correction formula, without introducing appreciable error.

As an illustration, assume $L = 4$ millihenrys $= 0.004$ henry, $R = 4000$ ohms, frequency 60 cycles ($\omega = 377$), and E volts and I amperes at unity power factor give full scale deflection of 150 divisions. Then the maximum error, which occurs at zero power factor, will be

$$\begin{aligned} E\,I \tan \alpha &= E\,I\,\omega L/R \\ &= E\,I\,\frac{377 \times 0.004}{4000} \\ &= 0.000377\,E\,I \\ &= 0.06 \text{ division} \end{aligned}$$

For power factors of 50 per cent and 86.7 per cent the error will be 0.05 and 0.03 division respectively.

The preceding example refers to a modern high-grade portable wattmeter with a maximum voltage rating of 125 volts. At low power factors, use may be made of the overload capacity of the windings. When these are pushed to their limits the maximum self-inductance error will be 0.2 division. Wattmeters of similar type made specially for use on very low power factor will have a maximum self-inductance error of about 0.4 division. The wattmeters above considered all have 150-division scales.

As another example, consider a wattmeter having an inductance of 9 millihenrys and $R = 1000$ ohms, full scale reading of 250 divisions for $E\,I$ volt-amperes at unity power factor. The maximum error will be

$$\begin{aligned} E\,I \tan\alpha &= E\,I\,\omega\,L/R \\ &= E\,I\,\frac{377 \times 0.009}{1000} \\ &= 0.00339\,E\,I \\ &= 0.82 \text{ division} \end{aligned}$$

In all of these statements on the error due to inductance in the potential circuits, it has been assumed that this cause only is acting. Other causes will be separately considered, and finally the manner in which these errors combine to affect the result.

(*f*) *Compensation of Self Inductance of Voltage Circuit.* The effect of inductance in the voltage circuit may be compensated over quite a range of frequency by shunting a portion R/n of the series resistor by a condenser of capacity C. The inductance of the voltage circuit being L and its resistance R, the following relation[70] should be satisfied

$$C = \frac{L}{R^2/n^2} \tag{9}$$

While this expression is not rigorously exact, it is sufficiently exact for all practical purposes in wattmeter compensation.

The formula shows that an infinite number of combinations of C and R/n may be used. It is therefore important to see whether there is any advantage in choosing some particular value. Since for a given wattmeter and given voltage range L

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[70] Sumpner, *Jour. Soc. Teleg. Eng.*, vol. 16, p. 334, 1887; Drysdale, (London) *Elec.* vol. 46, p. 774, 1901; Rosa, *Bull. of Bur. of Standards*, vol. 3, pp. 48-50, 1906.

and R are determined, we have

$$C/n^2 = L/R^2 = \text{constant.} \tag{10}$$

Thus if we make n large, that is, place the condenser around only a small fraction $1/n$ of R, the capacity of the condenser must be large, and conversely, if the condenser is shunted around a large part of R (n small) the capacity required decreases as n^2. In addition to bringing the current in the voltage circuit into phase with the applied e.m.f., as a secondary effect the resistance R is decreased by an amount ΔR and the current I for a given voltage increased by an amount ΔI, thus requiring a correction to the wattmeter reading[71]. It may be shown that for any value of $\omega^2 C^2 R^2/n^2$ which is small compared with unity we may write

$$\frac{\Delta R}{R} = - \omega^2 C^2 R^2/n^3 \tag{11}$$

$$\frac{\Delta I}{I} = - \frac{\Delta R}{R} = \omega^2 C^2 R^2/n^3 \tag{12}$$

The change in current being inversely proportional to n^3, it is advisable, from this viewpoint, to shunt a large portion of the resistance with a relatively small value of capacity. A practical disadvantage is that if the insulation of the condenser should puncture in service, the unshunted part of the voltage circuit, including the moving coil, would be liable to burn out. It is therefore preferable to compromise by shunting say a third of R with capacity. In any given case, the limitations should be examined.

(*g*) *Correction Factor for Self and Mutual Inductance of Wattmeter.* The effect of the mutual inductance between the fixed coil and the moving coil of a wattmeter is usually stated to be negligible. On the other hand, makers of wattmeters in which the moving coil is always in the position of zero mutual inductance when readings are taken have sometimes put this feature forward as an advantage because it eliminates errors due to mutual inductance. The writer does not know of any definite published data as to the magnitude of this effect. It seemed desirable to have a formula for determining the error, since in special cases of high frequency or low voltage ranges it might

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[71] This matter was brought to the writer's attention some years ago by Dr. P. G. Agnew.

be appreciable. The following discussion is based on an equation derived by Laws,[72] which, slightly rearranged, is as follows:

$$P = P_w\left(1 + \frac{\omega^2 (L_p \pm M)^2}{R_p^2}\right) - \frac{I_l^2\, \omega^2 (L_l \pm M)(L_p \pm M)}{R_p}. \tag{13}$$

in which P = actual power taken by the load, the loss in current coil of wattmeter being included.

P_w = reading of wattmeter (corrected for scale error).

ω = 2π times the frequency.

L_p = inductance of voltage circuit of wattmeter.

R_p = resistance of voltage circuit of wattmeter.

M = mutual inductance between current circuit and voltage circuit of wattmeter.

I_l = current taken by load.

L_l = inductance of load, including current coil of wattmeter.

Also, let

R_l = resistance of load, including current coil.

θ = lag angle of load.

The connections are understood to be made so that the voltage applied to the voltage coil includes the drop in the series coil, which thus virtually becomes part of the load.

The above equation may be put into the form

$$P = P_w\left(1 + \frac{\omega^2 L_p^2}{R_p^2} + \frac{\omega^2 M^2}{R_p^2} - 2\,\frac{\omega L_p}{R_p} \cdot \frac{\omega M}{R_p}\right)$$

$$- I_l^2\, \omega\, \frac{L_l\, \omega L_p - \omega L_p M - L_l\, \omega M + \omega M^2}{R_p} \tag{14}$$

It will be noted that in the equation (14) we have put the − sign before quantities containing M, instead of the ± sign given by Laws. The reason for this will now be given.

In most wattmeters of the deflection type the mutual inductance changes sign at approximately half-scale deflection. In the full-scale position the fluxes of the two coils are in the same general direction. If we consider the analogous case of an electrodynamic ammeter having the two coils in series, we have the known condition that the torque is in such a direction

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[72] F. A. Laws, *Electrical Measurements*, pp. 315-16. A similar equation can be derived for the case where the voltage circuit of the wattmeter is connected in parallel with the load only.

as to increase the total self inductance L, which is

$$L = L_1 + L_2 + 2\,M_{12} \tag{15}$$

where L_1 and L_2 are the self inductances of the fixed coil and the moving coil, and M_{12} is the mutual inductance between them. On this basis, we ascribe the + sign to M when the coil is above the position of zero mutual inductance, in which position the reactance voltage in each circuit due to its self inductance is in the same direction as the voltage induced by the current in the other circuit. If this convention as to signs be adopted, equation (**14**) will have a + sign before the terms containing M when the moving coil is below the point of zero mutual inductance, and a − sign when the coil is above this position. We may therefore write equation (**14**) in general with a − sign as an operator before the terms containing M, with the understanding that M is a quantity which is inherently + or − according as the moving coil is above or below the point of zero mutual inductance.

For $\dfrac{\omega L_p}{R_p}$ in equation (**14**) we may write $\tan \alpha_L$, where α_L is the angle of lag in the voltage circuit, the mutual inductance being assumed zero. Also, for $\dfrac{\omega M}{R_p}$ we may write $\tan \alpha_M$, since ωM, when mutiplied by the load current I_l gives a quantity $\omega M I_l$ which may be called the mutual reactance voltage in the voltage circuit, and which is analogous to $\omega L I_p$, the reactance voltage due to the self inductance of the voltage circuit. Also, for ωL_l we may substitute $R_l \tan \theta$. Making these changes

$$P = P_w\,(1 + \tan^2 \alpha_L + \tan^2 \alpha_M - 2 \tan \alpha_L \tan \alpha_M)$$
$$- I_l^2 R_l \tan \theta \,.\, (\tan \alpha_L - \tan \alpha_M)\,(1 - M/L_l) \tag{16}$$

Since $I_l^2 R_l = P$,

$$P\,[1 + \tan \theta\,(\tan \alpha_L - \tan \alpha_M)\,(1 - M/L_l)]$$
$$= P_w\,(1 + \tan^2 \alpha_L + \tan^2 \alpha_M - 2 \tan \alpha_L \tan \alpha_M)$$

from which

$$P = P_w\,\frac{1 + \tan^2 \alpha_L + \tan^2 \alpha_M - 2 \tan \alpha_L \tan \alpha_M}{1 + \tan \theta\,(\tan \alpha_L - \tan \alpha_M)\,(1 - M/L_l)} \tag{17}$$

If M be assumed $= 0$, $\tan \alpha_M = 0$, and this equation becomes

$$P = P_w \frac{1 + \tan^2 \alpha_L}{1 + \tan \theta \tan \alpha_L} \tag{18}$$

which is the common form of the expression for correcting for the error due to self inductance. If self inductance be assumed $= 0$, or compensated by a condenser shunting a resistance, equation (17) becomes

$$P = P_w \frac{1 + \tan^2 \alpha_M}{1 - (1 - M/L_l) \tan \theta \tan \alpha_M} \tag{19}$$

This equation is seen to be similar in form to equation (18), but contains the term $1 - M/L_l$ in the denominator.

In all wattmeters fit to be used, $\tan^2 \alpha_L$, $2 \tan \alpha_L \tan \alpha_M$, and $\tan^2 \alpha_M$ should be absolutely negligible, so that the equations may be used in the following simplified forms:

Self and mutual inductance present:

$$P = P_w \frac{1}{1 + (1 - M/L_l) \tan \theta (\tan \alpha_L - \tan \alpha_M)} \tag{20}$$

Self inductance present alone:

$$P = P_w \frac{1}{1 + \tan \theta \tan \alpha_L} \tag{21}$$

Mutual inductance present alone:

$$P = P_w \frac{1}{1 - (1 - M/L_l) \tan \theta \tan \alpha_M} \tag{22}$$

As L_l approaches zero, the term $(1 - M/L_l) \tan \theta$ in the preceding equations approaches the indeterminate form $\infty . 0$. This may be avoided by putting equation (17) into either of the following forms:

$$P = P_w \frac{1 + \tan^2 \alpha_L + \tan^2 \alpha_M - 2 \tan \alpha_L \tan \alpha_M}{1 + (\tan \theta - \frac{R_p}{R_l} \tan \alpha_M)(\tan \alpha_L - \tan \alpha_M)} \tag{23}$$

$$P = P_w \frac{1 + \tan^2 \alpha_L + \tan^2 \alpha_M - 2 \tan \alpha_L \tan \alpha_M}{1 + \left(\tan \theta - \frac{M \omega}{R_l}\right)(\tan \alpha_L - \tan \alpha_M)} \tag{24}$$

These equations do not require the value of L_l. The terms in the numerator after unity may be dropped in practical use of the equations.

In order to show the order of magnitude of the mutual-inductance and self-inductance effects, we may take a modern high grade switchboard wattmeter of 5 amperes and 110 volts rating, capable of 100 per cent overload in current, used on a 60-cycle load of 0.50 power factor, 550 watts, 1100 volt-amperes, current lagging. For this wattmeter, $R_p = 3400$ ohms, $L_p = 3.4$ millihenrys, $M = +94$ microhenrys at full scale, and $R_l = 5.5$ ohms. We then have

$$\tan \alpha_L = \frac{2\pi \times 60 \times 0.0034}{3400} = 0.000377$$

$$\tan \alpha_M = \frac{2\pi \times 60 \times 0.000094}{3400} = 0.000010$$

$$\tan \theta = \tan 60 \text{ deg.} = 1.732$$

Using equation (21),

$$P = P_w \frac{1}{1 + (1.732 \times 0.000377)}$$

$$= P_w \frac{1}{1.00065} = P_w (1 - 0.065 \text{ per cent.})$$

The error due to self inductance alone is thus 0.065 per cent, which is below the limit of reading for a switchboard instrument, but is just barely appreciable on a portable instrument.

Using equation (24), neglecting the second order terms in the numerator, we have for mutual inductance alone

$$P = P_w \frac{1}{1 + \left(1.732 - \frac{0.000094 \times 377}{5.5}\right)(-0.00001)}$$

$$= P_w \frac{1}{1 - 0.000017} = P_w (1 - 0.002 \text{ per cent}).$$

This is far below the limit of reading, even for the best portable instruments. With high frequencies, low current ranges, or low volt ranges it would be advisable to check the effect of both self and mutual inductance.

(*h*) *Effect of Eddy Currents in Fixed Parts of Wattmeter.* If conducting masses are near the current coil of a wattmeter, eddy currents will be induced in them when current flows through the current coil. For sinusoidal currents, the e.m.f. induced will be 90 deg. behind the flux due to the fixed coil.

The eddy current will lag behind the induced e.m.f. by an angle which at ordinary frequencies is probably small for high resistance alloys, but which may be relatively large for copper. The moving coil is deflected by the resultant flux, namely, the vector sum of the main flux and the eddy current flux. Since the latter lags behind the main flux by more than 90 deg., the resultant flux is of somewhat smaller magnitude than the main flux and lags behind the latter. The effect upon the reading of the wattmeter is thus two-fold; (a) the decrease in magnitude tends to reduce the wattmeter reading; (b) the lagging of the resultant flux may tend to increase or decrease the reading, depending upon the vector relations of the voltage and current in the load. The effect of lagging the resultant flux by an angle α_E is evidently the same as would be produced by a cause (such as excess capacity effect in the voltage circuit) which advances the current in the voltage circuit by the angle α_E with respect to the applied voltage. Thus one part of the effect of eddy currents is of opposite direction to the effect of inductance in the voltage circuit. If the latter effect is to cause the current in the voltage circuit to lag by an angle α_L, then both effects may be taken care of by replacing α and α_L in the preceding formulas by $\alpha_L - \alpha_E$. This "equivalent phase angle"[73] of the wattmeter may also be extended to include the phase angles of current and voltage transformers used with the wattmeter.

The above discussion does not take into account the eddy currents set up in fixed conducting masses by the current in the moving coil. Aside from the fact that the motion of the latter would complicate the matter, the ampere-turns of the moving coil at rated voltage are only a few per cent of those of the fixed coil at rated current, and hence the eddy currents set up by the former may be neglected.

A few instances of appreciable error at ordinary frequencies resulting from eddy currents may be instructive. A new wattmeter in which the two current coils were supported by brass castings was found to have errors indicating eddy currents. On investigation it was found that in assembling the instrument certain insulating washers had been omitted, thus forming a closed conducting path around these castings. The error was 1 per cent at power factor 0.75, 60 cycles. Another

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[73] L. T. Robinson, in Pender's Handbook for Elec. Eng., 1st edition, p. 1819.

instance, in which the eddy current error was 1.5 per cent at 87 per cent power factor, 60 cycles, was traced to short-circuited turns in the compensating winding. A similar result would be given by a short circuit in the current coil which would cut out some of the turns from the path of the load current and give a path for eddy currents. Wattmeters as received from the maker, or which are about to be used for important tests, should be checked on low power factor, in addition to the test with reversed direct current.

(*i*) *Effect of Eddy Currents in Moving Parts of Wattmeter.* Eddy currents in the moving coil or its metal fittings will introduce errors of the same sort as those due to mutual inductance, but they may be of appreciable magnitude. Eddy currents in the moving coil can occur only when one or more turns have been accidentally short-circuited. An idea of the magnitude of the effect may be gained by connecting the terminals of the moving coil together, excluding the series resistor, and passing rated current at a commercial frequency through the current coil. The interaction between this current and the current induced in the moving coil will deflect the coil, sometimes by several scale divisions, toward the point of zero mutual inductance. This effect decreases as the point of zero mutual inductance is approached, and for a given coil position it varies as the square of the current in the current coil. The observed effect is caused by eddy currents in *all* the turns of the moving coil, usually several hundred, and the deflection for a part of the coil short-circuited will be in proportion to the number of turns short-circuited. No case of this kind has come under the writer's notice, but as the matter can be readily checked, it should be kept in mind as a possible source of error in power measurements. In the manufacture of wattmeters and a-c. voltmeters all coils should be tested before assembly to detect possible short-circuited turns.

(*j*) *Methods of Testing Wattmeters at the Bureau of Standards.* Electrodynamic wattmeters are tested by the Bureau of Standards on direct current, using the mean of reversed readings in order to eliminate error due to local magnetic field or residual polarity in iron shields. A further test which is usually made is the determination of the "a-c—d-c. difference," which is made as follows: The current coil of the wattmeter is connected in series with the current coil of a standard reflecting electrodynamometer, the voltage coil of the wattmeter in parallel with that of the electrodynamometer. Alternating current is passed through the current coils and alternating voltage of the same frequency, adjustable in phase, is applied to the voltage coils. The current and voltage are regulated to give a stated deflection of the wattmeter, and the electrodynamometer is read. Using direct current and voltage, the wattmeter is then brought to the same deflection as before, first with current and voltage of a given direction, then with both reversed. As a check, the a-c. reading is then repeated. The mean of the two a-c. readings, minus the mean of the two d-c. readings, is then divided by the mean of the d-c. readings in order to obtain the relative difference (usually expressed as a percentage) in the reading of the wattmeter on d-c. and on a-c. This difference gives an index of the quality of the wattmeter as to freedom from errors due to self inductance and eddy currents. High grade portable wattmeters, tested on commercial frequencies at the volt-amperes giving full scale deflection for unity power factor, give a-c.-d-c. differences of not over a few tenths of a division. Wattmeters which have been carefully compensated for inductance give practically no difference.

(*k*) *Multiple Ranges in Wattmeters.* Multiple ranges are secured in wattmeters by making the current winding in parts (usually two) which can be joined in series or in parallel, and by bringing out taps from the series resistor in the voltage circuit. It is important that the separate parts of the current winding should be alike in resistance and in their magnetic effect upon the moving coil. This can be checked, in instruments having accessible links for changing the range, by putting the two parts of the current winding in series opposing, and exciting the voltage circuit from a source approximately in phase with the current. A deflection indicates a difference in the coils. Strictly, this test should be carried out throughout the scale, by shifting the spring abutments so as to bring the coil into various initial positions over the scale.

(*l*) *Polyphase Wattmeters.* A polyphase wattmeter consists of two single-phase wattmeter systems with their moving elements on a common spindle. Because of the limited separation between the elements, some provision for preventing interaction between the fixed coil of each element and the moving coil of the other element is necessary if the instrument is to give accurate readings. In certain instruments, for example, some graphic wattmeters, no such provision is made, but in high-grade portable polyphase wattmeters a magnetic screen is interposed between the elements, or each element is enclosed in a magnetic shield. The latter arrangement not only prevents interaction but shields each element from outside magnetic disturbing forces.

Polyphase wattmeters can hardly be given as high a degree of accuracy as the single-phase wattmeter of the same type and grade, because it is practically impossible to adjust the two elements to follow the same scale law. By careful workmanship, the two elements may be brought very nearly into agreement as to scale law and sensitivity, as is shown by the fact that one well-known maker guarantees portable single-phase wattmeters to 0.25 per cent and the corresponding polyphase wattmeters to 0.5 per cent. The necessity for having the two elements as nearly identical as possible arises from the fact that in measuring polyphase power the parts of the torque contributed by the two elements become more and more unequal as the power factor of the load is reduced. At power factor 0.50 all the torque is furnished by one element, and in the ordinary use of the wattmeter it might be either element indifferently. It is thus essential that the deflection corresponding to a given power be the same for each element, and that this equality of elements should hold for all parts of the scale. A check of this feature may be made on a single-phase circuit by putting the current coils of the two elements in series so as to oppose their torques and the voltage coils in parallel. A deflection shows an inequality of the torques of the two elements, and such deflection should be only a small fraction of 1 per cent. This test should be repeated for a number of points over the scale, because the equality of elements may be exact at the zero position and not at other points. The spring abutments should be shifted in order to bring the coil successively into the various positions. It is best to precede this test by a test for interaction between elements, which is carried out by exciting the voltage coil of one element from a single-phase supply and passing rated current, approximately in phase with the voltage, through the current coil of the other element. The resulting deflection, if any, ought to be very small, say not over 0.1 division.

(*m*) *Measurement of Polyphase Power*. The measurement of polyphase power may be made in various ways, which are described in text-books [74] and will not be considered in detail here. The two-wattmeter method seems to have the preference. This presents no difficulties on a two-phase circuit, since each element measures the power in one phase. On a three-phase circuit, however, the two elements measure the power of three phases, and care is necessary to have the proper connections and to interpret the readings correctly, because at power factors below 0.50 the reading of one wattmeter must be subtracted from that of the other. The power factor under which each element is working is not the same as that of the load, and hence with unbalanced load corrections for inductance of the voltage circuit of the wattmeter and for the phase angles of instrument transformers are more difficult to apply. The use of a separate wattmeter in each phase avoids these difficulties, but requires one more instrument and access to a neutral point, which may however be an artificial one.

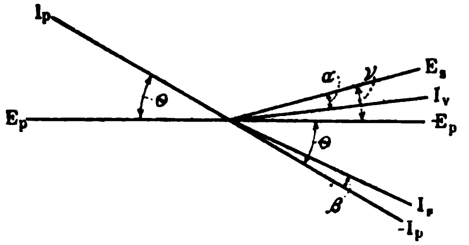

FIG. 7

(*n*) *Use of Instrument Transformers with Wattmeter.* The use of instrument transformers with a wattmeter introduces the necessity for taking account, not only of the departure of the current ratio and voltage ratio from their nominal values, but also of the phase angles. If the latter were zero or negligible, the power under measurement would be the reading of the wattmeter, corrected for scale error, multiplied by the product of the actual values of current ratio and voltage ratio. If the phase angles are not negligible, the manner in which they affect the result may be seen from Fig. 7. In this, I_p and E_p represent the primary current and primary voltage respec-

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[74] Standard Handbook for Electrical Engineers, 4th ed., pp. 156-8; Pender's Handbook for Electrical Engineers, 1st ed., pp. 1823-1826; Laws, Electrical Measurements, pp. 330-340; Farmer, Electrical Measurements in Practise, pp. 160-4; Edgcumbe, Industrial Electrical Measuring Instruments, 2nd ed., pp. 201-8, 222-4.

tively, and the power to be measured is their product multiplied by cos θ. In the actual measurement, however, we must use the secondary voltage and current, E_s and I_s, in place of the primary quantities. The current in the voltage circuit of the wattmeter, I_v, lags behind E_s by the angle α. The reading of the wattmeter is thus proportional to

$$E_s I_s \cos \left[\theta - (\alpha + \beta - \gamma)\right]$$

instead of to $E_s I_s \cos (\theta - \alpha)$ as in the case of a wattmeter used without transformers and having an angle of lag α in the voltage circuit. It may be seen that the phase angles of the voltage circuit and of the two transformers may be lumped into one equivalent phase angle. The angular relations shown in Fig. 7, while exaggerated as to magnitude, are in the directions usually found, as follows: The secondary voltage leads the reversed primary voltage; the current in the voltage circuit of the wattmeter lags behind the secondary voltage at its terminals; the secondary current leads the reversed primary current. It is usual to give the + sign to the angle α when the current in the voltage circuit lags behind the voltage applied to it. There is a lack of uniformity of practise, however, as to the sign which should be prefixed to the transformer phase angles. The convention used by the Bureau of Standards is to measure from the reversed primary quantity to the secondary quantity, and to apply the sign which would be given in trigonometry, namely, that angles described by a vector moving in a counterclockwise direction are +. The usual case, namely, secondary current or voltage leading the reversed primary current or voltage, would be given a + sign. The contrary convention has also been advocated.[75]. Using the convention of the Bureau of Standards, and referring to Fig. 7, it will be seen that the angle β of the current transformer is to be added to the angle of lag α in the wattmeter, and from this sum is to be subtracted the angle γ of the voltage transformer. The remaining angle may be used as the equivalent angle of the whole combination, in applying the correction formulas.

The preceding discussion of power measurement with wattmeter and instrument transformers applies to the simple case of a single-phase circuit, or in the measurement of three-phase

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[75] L. T. Robinson, TRANS. Am. Inst. Elec. Eng., vol. 28, pp. 1005-1052; Pender's Handbook for Electrical Engineers, 1st ed., p. 1640.

power by a wattmeter in each phase. When a three-phase balanced load at unity power factor is measured with two wattmeters, one wattmeter measures half the load under conditions as in a single-phase circuit of 0.866 power factor, current leading, while the other operates under the same power factor, current lagging. If the equivalent angles α are alike for the two wattmeters with their instrument transformers, the errors introduced will be equal and opposite, and (neglecting a term $\cos \alpha$ which differs from unity by only a negligible amount) the sum of the readings of the two wattmeters will give the actual power. In general, for a balanced polyphase load in which the phase angle is θ and the equivalent angle of each wattmeter and its transformers is α, the total correction to be added to the wattmeter reading is

$$C_1' + C_2' = \sqrt{3} E_s I_s \tan \alpha \sin \theta \qquad (25)$$

If three wattmeters were used, each would measure the power $\frac{\sqrt{3}}{3} E_s I_s \cos \theta$, and the correction to each would be $\frac{\sqrt{3}}{3} E_s I_s \tan \alpha \sin \theta$. The sum of the three corrections is thus equal to the correction above given for the measurement of the same total power by the two-wattmeter method. Nothing is gained in the way of increased accuracy by the use of three wattmeters.

For example, if the power factor of the load is reduced to 0.50, the power should all be indicated by one wattmeter, while the other should indicate zero. For this case we must use the wattmeter correction in the additive form, since the correction factor becomes useless at zero power factor. The first wattmeter operates under the condition of 0.866 power factor, current lagging by 30 deg. If the current and voltage in each case are I_s and E_s, the correction to the first wattmeter will be, by equation (8)

$$C_1' = - E_s I_s \tan \alpha \sin 30 \text{ deg.}$$
$$= - 1/2\, E_s I_s \tan \alpha.$$

Similarly for the second wattmeter,

$$C_2' = - E_s I_s \tan \alpha \sin 90 \text{ deg.}$$
$$= - E_s I_s \tan \alpha$$

The sum of these gives the total correction, namely,

$$C_1' + C_2' = -\frac{3}{2} E_s I_s \tan \alpha.$$

$$= \sqrt{3} E_s I_s \left(- \frac{\sqrt{3}}{2} \tan \alpha \right)$$

$$= - \sqrt{3} E_s I_s \tan \alpha \sin 60 \text{ deg.}$$

This is the result that would be obtained by applying the additive correction formula to the total volt-amperes $\sqrt{3} E_s I_s$, and shows that the correction to the polyphase measurement is the same as would apply to a single-phase measurement of the entire amount of power at 0.50 power factor, using a wattmeter and transformers having the equivalent angle α.

The magnitude of the phase angle in commercial voltage transformers may be estimated from the following data of tests made by the Bureau of Standards on 55 transformers during the period 1908–1918: Phase angle at no load, average + 9 min., maximum + 69 min., minimum + 2 min. The value of 69 min. is exceptional, the maximum, if this were omitted, being 29 min. The phase angles at full non-inductive load were: algebraic average, – 21 min; extreme values, – 38 min. and + 62 min.; omitting the last-named the next largest positive value was + 26 min.

While it is difficult to give any useful general statement regarding the magnitude of the phase angle of current transformers, since it depends on so many factors, it may be said that good current transformers, used within their rated limits of frequency and secondary burden, will have a phase angle of not over 1 deg. to 1 deg. 30 min. at 60 cycles and not over 1 deg. 30 min. to 3 deg. at 25 cycles, both at 0.5 ampere secondary. At 5 amperes secondary, the phase angle will be about one-fourth to one-third of these values. In the preceding statements, the reversed secondary current is understood to lead the primary current. In some cases, especially when the secondary burden is highly inductive, the reversed secondary current may lag behind the primary current for a part of the curve including the five-ampere point. Good transformers with small secondary burden, say a wattmeter or a watthour meter and connecting leads having a resistance of a few tenths of an ohm, will average about one-half of the above values.

(*o*) *Use of Iron in Electrodynamic Wattmeters.* On account of the relatively low torque of the usual electrodynamic wattmeters, the attempt has been made to use iron in the coils. This construction has been investigated by Drysdale, Sumpner and others,[76] but does not appear to have gained any footing in this country. It seems evident that such constructions would be unsuitable for precise testing, especially where more than one frequency is necessary, and that they would be limited to use on switchboards at a given frequency. One advantage of some of these constructions is that they are but little affected by stray magnetic field.

(*p*) *Induction Wattmeters.* These wattmeters are similar in principle to the induction watthour meter. They are inherently less accurate than electrodynamic wattmeters, but for switchboard use they have the advantages of simple construction, ruggedness, high torque, no moving wire, and long scale. They have the disadvantages of being useful on only one frequency, and a much larger temperature coefficient than electrodynamic wattmeters. Their accuracy is also affected by voltage and wave form variations, which do not appreciably affect electrodynamic wattmeters. While they are entirely suitable for the *operation* of a plant, they should not be used in such work as acceptance tests, or the accurate testing of watthour meters.

(*q*) .*Electrostatic Wattmeters.* These wattmeters have found very limited application in this country, being confined to such special laboratory work as the determination of small amounts of power at very low power factors, such as the dielectric losses of insulating materials. They are not nearly as convenient to use as electrodynamic wattmeters, and their use would seem to be justified only for very special work for which ordinary methods are inapplicable.

(*r*) *Hot-Wire Wattmeters.* This form of wattmeter contains two hot wires, one carrying a current proportional to the sum of the line voltage and the drop at the terminals of a shunt carrying the load current, and the other a current proportional to the difference of the line voltage and the shunt drop. This instrument seems to have little to recommend it, as it is stated[77] to have rather more pronounced faults

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[76] 76. Edgcumbe, Industrial Electrical Measuring Instruments, 2nd edition, pp. 219-222.

[77] 77. Edgcumbe, Industrial Electrical Measuring Instruments, 2nd edition, pp. 237-8.

than the usual hot-wire ammeters and voltmeters. It seems to be a possibility, however, for the approximate measurement of power at high frequencies.

11. THE MEASUREMENT OF ELECTRICAL ENERGY

(*a*) *General Discussion.* The measurement of electrical energy is of great commercial importance. It is not possible in this paper to do more than briefly refer to some of the principal factors which affect the accuracy of energy measurements. The many questions involved in the maintenance of accuracy of electric energy meters are being covered by the large organizations of central station companies, and the results appear in the *Meterman's Handbook* and the reports of the N. E. L. A. Committee on Meters.

In contrast to the large variety of types of indicating instruments, the direct-current energy meters in this country are limited to the commutator motor meter and the mercury motor meter, while the induction meter is the only one in use for a-c. work. In spite of this limited number of types, the problems involved in maintaining meter accuracy are very numerous, and it seems doubtful whether any other measuring device has had so much labor spent upon it in a given time. Too much can not be said in favor of the policy of cooperation which prevails among the large central stations, through their national association, in the way of pooling information and experience with meters for the common good.

What has been said of indicating instruments as to good design, workmanship, proper installation, maintenance, and testing, applies with even greater force to meters. In the following discussion it is assumed that the reader is familiar with the details of meter construction and operation.

(*b*) *Accuracy Curves of Watthour Meters.* It is desirable that the curve showing the "per cent registration"[78] as a function of the load should be a straight line from a very few per cent of full load to very heavy overloads. This can be realized only imperfectly in the meters now available, with

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[78] No single satisfactory word seems to have been proposed for this quantity, which may be defined as the percentage of the energy actually passed through the meter which is registered by it. Considered as an instantaneous quantity, it may be defined as the ratio (expressed as a percentage) of the actual speed of the meter disk at any moment to the speed of a correct meter of the same range and constants, measuring the same load.

the possible exception of the Aron pendulum meter, which has apparently not found any application in this country. The departures from the ideal curve are generally as follows: Zero registration for very small loads, a per cent registration in excess of 100 per cent at from say 25 to 75 per cent load, and a continually decreasing per cent registration as the load increases beyond 100 per cent of rating. The shape of the accuracy curve in the light-load region (say 5 to 10 per cent of rated load) is different in different types and makes of meters. In some cases, as in portable watthour meters, it may be necessary or desirable to adjust the meter so that the curve from 25 per cent to 100 per cent of rated load will be as flat as possible, even if this does cause the meter to be several per cent fast in the region of 5 to 10 per cent load. This is because a portable watthour meter ought not to be used below say 25 per cent of rated load, but should have such ampere ranges that commercial ranges of meters can be tested at light and at full load without using the standard at less than 25 per cent load. This is important in both a-c. and d-c. meters to avoid frictional errors in the standard, especially in the d-c. meter, and for the further reason that the d-c. portable watthour meter is especially subject to errors from local magnetic fields, which errors become relatively greater as the load decreases.

From the central station standpoint, the under-registration at light loads and at overloads is important, because it is necessary in most cases to use a meter of much smaller rating than the connected load, in order to get approximate accuracy on very light but long-hour loads. When occasion arises for operating a large part of the connected load, the meter is slow.

(c) *Effect of Room Temperature on Meter Accuracy.* In a properly made d-c. commutator meter, and also in mercury meters, a rise of temperature increases the resistance of the voltage circuit at practically the same rate as the resistance of the drag disk, so that the only remaining variable is the strength of the magnetic field cut by the disk. The change in the accuracy of a good meter of this type should not exceed

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79. Values of temperature coefficient for five American makes of d-c. watthour meter are given by Fitch & Huber in *Bulletin of the Bureau of Standards*, vol. 10, pp. 166-8, 1913; Scientific Paper No. 207.

0.1 per cent per degree cent.[79] Cases where a much higher value has been found are usually traceable to the use of resistance alloy instead of a pure metal for the series resistance in the voltage circuit.

In the induction meter the conditions are inherently favorable for low temperature coefficient, because the same disk forms a part of the motor device and the drag device, so that any variations in its temperature have no effect of the first order. The other sources of temperature coefficient are the drag magnets and the electromagnets. Change of temperature of the voltage winding and the lag compensating circuit also have a slight effect. The actual temperature coefficient of induction meters is small. No recent determinations have been made by the Bureau of Standards, but data from tests made a number of years ago shows that the temperature coefficient is under 0.1 per cent per degree cent. for all but very abnormal conditions of power factor.

(*d*) *Effect of Self Heating on Meter Accuracy.* In the d-c. commutator meter the effect of self heating is quite marked. The armature and its series resistor have a high resistance-temperature coefficient, and in consequence will increase in resistance after the line voltage is applied, until a steady state is reached. Because of this fact, it is necessary to have the meter at operating temperature before determining its accuracy, and the same precaution is necessary with the portable watthour meters used to check service meters.[80] A second effect of self heating in the d-c. meter is the increase of resistance of the armature due to the heating of the series coil by the load current. This heating increases as the square of the load current, and its tendency is to reduce the per cent registration of the meter more and more as the load increases.[81] The use of the full-load adjustment to make the meter correct at full load in spite of this effect gives the accuracy curve the same shape as in the induction meter, namely, over-registra-

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[80] An exception to this statement is the General Electric type CB-5 portable watthour meter, in which the usual pure metal drag disk is replaced by an alloy disk of lower temperature coefficient. This calls for a correspondingly low temperature coefficient in the voltage circuit, with resulting smaller change in accuracy as the meter warms up after being put in circuit.

[81] Fitch & Huber, *Bulletin of the Bureau of Stanards*, vol. 10, pp 162-6, 1910; Scientific Paper No. 207.

tion from say 25 to 75 per cent of rated load, and an increasing under-registration beyond full load.

In the d-c. mercury motor meter the power taken by the current element is so low (0.2 watt at full load, in the meters tested by Fitch and Huber) that the effect of self heating from this source is small, and of opposite sign from that in the commutator meter.

In the induction meter the losses are relatively much smaller than in the d-c. meter, and the combined motor and generator action of the disk would make the meter practically unaffected by heating of the disk from the windings even if it were much greater. The effect of self heating may be said to be negligible in induction meters.

(*e*) *Effect of Stray Magnetic Fields on Meter Accuracy.* The d-c. commutator meter of the usual form is seriously affected by even moderate values of stray field, because of the relatively low flux density of the working field set up by the series coils (less than 100 gausses at full load). The effect of a given value of stray field upon the accuracy increases as the load on the meter decreases. It is therefore standard practise to adjust such meters in position after installation, and it is necessary to avoid locations where the magnetic field is liable to change. The meter should not be placed near structural iron which may become permanently magnetized by a short circuit through the meter. The mercury motor meter is only slightly affected by stray field. The induction meter is not affected by the earth's or other unidirectional magnetic field, but only by alternating fields of the same frequency as that of the current in the meter. It is relatively much less susceptible to error from stray field than the d-c. commutator meter.

In commutator meters of special construction the effects of stray field are reduced. The following devices are used: four-pole arrangement of the series coils and armature windings; the use of two bipolar armatures of opposite polarity; and the use of two sets of bipolar field coils in connection with a special form of armature having radiating strips of silicon steel to intensify and direct the flux set up by the armature coils. In some large capacity switchboard meters the drag magnets are enclosed in an iron box to shield them from stray field.

(*f*) *Effect of Change of Voltage on Meter Accuracy.* In d-c. meters, an increase of voltage raises the temperature of the voltage circuit without a corresponding change in the temperature of the drag disk. In consequence, such meters run more slowly at higher voltages, and vice versa.[82] The effect is more pronounced in the mercury motor meter than in the commutator meter. No simple method of obviating this effect seems to be available. The use of a voltage circuit and drag disk of lower temperature coefficient would not be feasible for service meters, as the torque developed by such a disk at a given speed in a given magnetic field is lower in proportion to its lower conductivity.

The current taken by the voltage circuit of an induction watthour meter on a given voltage is determined very largely by its reactance. The increase of resistance of the winding has only a small effect. On account of the presence of iron in the magnetic circuit, the reactance will change somewhat as the voltage changes. The data available indicate that induction meters are considerably less affected by change of voltage than direct-current meters, at least for commercial ranges of voltage variation.

(*g*) *Effect of Change of Frequency, Power Factor, and Wave Form on Meter Accuracy.* By proper design, induction watthour meters may be made insensitive to small changes of frequency on either side of the normal, provided the power factor is near unity.[83] Departures from normal frequency at low power factors introduce larger errors for which no simple method of compensation seems to be at hand. This is an argument for the practise of holding the frequency constant, as by the method using a standard clock. With frequency variation eliminated, errors at low power factor will not occur if the meters are properly lagged.

In some tests of American a-c. watthour meters at the

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[82] Fitch & Huber, *Bulletin of the Bureau of Standards*, vol. 10, pp. 166-8, 1910; Scientific Paper No. 207.

[83] In some tests made by the Bureau of Standards on American induction watthour meters a number of years ago, a variation of ± 10 per cent in frequency at unity power factor affected the accuracy by from 0.1 per cent to 0.7 per cent in different makes. The same variation in frequency, at power factors of 0.75, 0.50, and 0.25, changed the accuracy by about 1.5 per cent, 3 per cent, and 5 per cent respectively.

Bureau of Standards a number of years ago, the change from a sine wave form to one containing 30 per cent of third harmonic, at normal voltage and frequency and unity power factor, affected the accuracy by amounts varying from a few tenths of a per cent up to nearly 3 per cent. Durand[84] cites a case coming under his notice in a French installation where the watthour meters were correct at a certain hour of the day, while at other hours the errors attained to 15 per cent. In the latter case the network was supplied from another alternator having considerable difference of wave form from the one operating in the first case. It is a fortunate circumstance, from the standpoint of meter accuracy, that the tendency of recent years toward better control of frequency has been accompanied by the effort to design commercial alternators to give a good approximation to the sine wave form.

Another cause for the variation of wave form is the varying load on a line supplied from a given generator. When the load is heavy the wave form of the line voltage may approximate closely to that of the generator, but when the load is light the charging current of the line may considerably distort the wave form of the voltage along the line. It seems reasonable therefore to conclude that in designing induction meters the factors which tend to accuracy on varying wave form should be given due weight.[85]

(*h*) *Use of Instrument Transformers with Watthour Meters.* The effects of the phase angle and ratio errors of instrument transformers used with watthour meters are identical with the corresponding effects on wattmeters. A few points of difference in details may be noted. The watthour meter, unlike the wattmeter, has adjustments which permit the regulation by the user of the accuracy and of the phase angle. Since the ratio of the transformer, the accuracy of the meter, and the phase angle all vary with change of circuit conditions and of secondary current, it is obvious that the adjustment of the whole can be made only for some average load and conditions. The most convenient method of adjusting a meter with its transformers, when the primary voltage and current are not too high, is obviously by the use of a precision watt-

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[84] A. Durand, Le Compteur Électrique, International Congress of the Applications of Electricity, Turin, Vol. 2, pp. 743-776, 1912.

[85] W. H. Pratt, *General Electric Review*, Vol. 18, pp. 277-281, 1915.

meter connected directly in the primary circuit. This method ceases to be useful, even for primary voltages now considered relatively low, on account of such disturbing elements as leakage across insulating surfaces, electrostatic action on pointers, and electrostatic capacity between sections of the series resistance and their surroundings,[86] all of which are difficult to eliminate or allow for. There is also the added hazard to those making the test. It is therefore necessary in practise to use wattmeters with high grade instrument transformers, having small errors which are determined by laboratory methods.

(*i*) *Portable Watthour Meters* (*"Rotating Standards"*). These meters are substantially equivalent to service meters except as to the register, connections, and case. In using them, it should be kept in mind that they are subject to all the limitations of the service meter, and should be carefully handled and frequently checked. Portable d-c. watthour meters in particular must be used with great care if results of reasonable accuracy are to be expected. If line conditions are fairly steady, more reliable results can probably be obtained with the ammeter, voltmeter, stop watch method. If the current and voltage are very unsteady, the portable watthour meter must be used. Care should be taken to avoid error due to the local magnetic field. A magnetic compass placed at the point where the portable watthour meter is to be used will show the direction of this local field, and the portable watthour meter should be placed with the planes of its series coils parallel to the direction previously indicated by the needle. Care should be taken to have the portable watthour meter at least two feet away from the compass, in order that the stray field from the drag magnets may not produce an error in the determination of the direction of the local field. Direct-current instruments should be at least five feet away from the compass for the same reason.

Portable a-c. watthour meters give much better results than the d-c. meters because of the absence of commutator, lighter weight of moving element, larger ratio of torque to weight, and greater independence of stray fields. They are subject to the same general limitations as the corresponding

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[86] Edgcumbe, Industrial Electrical Measuring Instruments, 2nd edition, pp. 216-7.

a-c. service meters. There are two ways of looking at this fact. According to one, it is an advantage to have the portable meter affected by abnormal frequency, low power factor, etc., in the same way as the service meter, because when this is the case the test of service meters under conditions which happen at the time to be abnormal will give the same result as a test with the conditions normal. The weak point in this argument is that if conditions are more often abnormal than normal, the test fails to show the deviation of the service meter from normal accuracy. It also frequently happens that the service meters are of more than one type, and are variously affected by a given departure from normal conditions. On the whole, it seems better to make the accuracy of the portable watthour meter as independent of voltage and other conditions as possible, so that every test of a service meter shall show its accurracy at the time.

The current range of many portable watthour meters is varied by changing the grouping of a sectional current winding. When bad contacts occur at the links, plugs, or contact surfaces of a drum controller, this introduces little error in the results when the meter is used in the ordinary way. However, when the meter is used on the secondary of a current transformer, such bad contacts may introduce serious errors by putting more than the normal resistance in the secondary circuit, with resulting change of ratio and phase angle. Some metals which have been used in drum controllers for this purpose have shown a tendency to oxidize and introduce high resistance into the circuit. Meters having such means for changing the range should be checked by measuring the resistance of the current coils with a bridge, before using them with current transformers.

(*j*) *Permanency of Meter Drag Magnets.* Since a change of 1 per cent in the strength of field of the drag magnets affects the accuracy by 2 per cent, it is evident that permanency of the magnets is a very important matter. These magnets, especially in direct-current meters, are liable to be seriously altered by the magnetic field from the series coils when short circuits occur. This effect may be minimized by arranging the magnets with their long dimension parallel to the plane of the field coils and keeping them as far from the field coils as the design will permit.

The shortage of tungsten during the war compelled makers to use steels which depend upon other materials, such as chromium, for their coercive force. While there is reason to believe that with careful control of all processes of manufacture it is possible to make chrome steel magnets of high quality, it will probably be advisable to keep a closer watch than usual on the accuracy of meters manufactured during the war, since the permanency of magnets is better established by actual use than by laboratory tests.

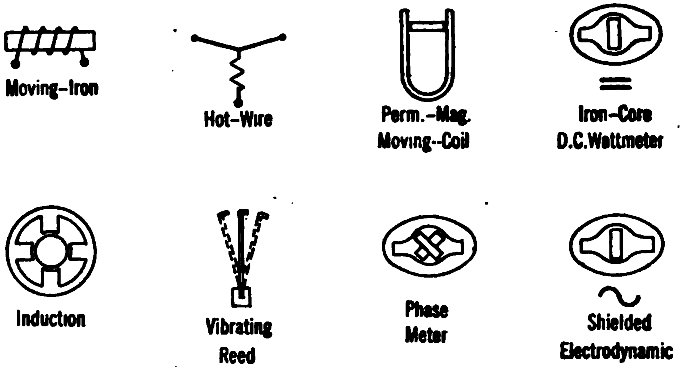

FIG. 8.—SYMBOLS PLACED ON INSTRUMENTS BY A FOREIGN MAKER TO DENOTE THE OPERATING PRINCIPLE

12. FURTHER DEVELOPMENTS DESIRED

(*a*) *Better Understanding of Accuracy Requirements by the User*. This would benefit both user and manufacturer, and tend to simplify the purchase and sale of apparatus. It is suggested that the makers can aid in this matter by devoting more space in their bulletins to the question of the accuracy needed for different classes of work, the importance of accuracy in certain classes of measurement, and the uselessness and expense of needless accuracy. More definite statements of accuracy obtainable with the instruments should be given. Some makers publish definite accuracy guarantees, while others do not. This condition, coupled with competition, is a hardship to the user and to the maker who gives guarantees.

It is suggested also that every instrument should bear an inscription or symbol which will clearly show the operating principle.[87] This has been done for switchboard instruments by a foreign maker, as shown in Fig. 8.

(*b*) *Standardization.* The art of metering energy has been greatly advanced by the preparation and use of the "Code for Electricity Meters" prepared by the Electrical Testing Laboratories under the auspices of the Joint Meter Committee of the National Electric Light Association and the Association of Edison Illuminating Companies. The British Engineering Standards Association has prepared similar specifications for indicating instruments, a revised edition of which has recently been issued. Perhaps a classification may sometime be realized which would bear the same relation to the instrument field that the code of the Joint Meter Committee bears to the watthour meter field. Some efforts along this line have already been made by the Joint Meter Committee. An interesting step in the same direction was undertaken during the war by cooperation between the makers and the radio services of the Government in the standardization of electrical instruments for use with radio apparatus.[88]

(*c*) *Magnet Steel.* A magnet steel is needed which will have a greater coercive force than the present steels, this coercive force not being accompanied by an undue reduction of permeability or residual induction. Such a steel would be valuable for the drag magnets of watthour meters, especially for direct-current meters. It would also be useful for indicating instruments, especially for the small sizes which are coming into extensive use. The improvement in the steel could be used to get increased strength and permanency with the same amount of steel, or the same strength without sacrificing permanency with a smaller amount of steel, as might be desired in the case of instruments and magnetos for airplane use, where weight must be minimized. Other applications would include polarized relays, and similar apparatus employing permanent magnets.

(*d*) *Spring Materials.* It would be desirable to have

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[87] As an illustration may be mentioned the case of a high-grade portable shielded moving-iron ammeter which is put out in a style identical with a number of electrodynamic instruments of the same maker, but with nothing to show the operating principle. Such an instrument was used for a series of tests by a trained laboratory man, on the assumption that it was an electrodynamic instrument.

[88] G. Y. Allen, *Elec. Jour.* vol. 16, pp. 494-500, Nov. 1919.

non-magnetic alloys suitable for springs for electrical instruments, which would show less elastic fatigue than those now available. For some applications, a low specific resistance is also very desirable. On account of the thinness of instrument springs (only a few thousandths of an inch in some cases) and the fact that the strength of the spring varies as the cube of the thickness, it is important that the alloys used should be very resistant to oxidation or attack by sulphur or other impurities in the atmosphere. It would seem that gold plating would protect springs from this deterioration, but the question requires some investigation to determine whether any detrimental secondary effects might result from the plating process.

(*e*) *Voltage Transformers for High Voltages.* Such transformers as now made are large and expensive, and because of electrostatic capacity of the primary windings with respect to each other and the core and case, their ratio cannot be inferred with accuracy from the performance of the same core worked at the same flux density but with windings for lower voltages for which the capacity effects are much smaller and methods of measurement of ratio and phase angle are at hand. Improvements are desired in the way of reducing weight and cost, and in correcting for electrostatic effects or determining and allowing for such effects on the ratio and phase angle.

(*f*) *Current Transformers.* The use of current transformers with a watthour meter implies heavy current or high voltage or both, and hence a commercially important amount of power. In all such cases, and especially in the larger installations, we ought to have current transformers whose phase angle is much smaller and more nearly constant over a wide range of operating conditions than at present. The ratio should also be very constant, and within a small fraction of 1 per cent of the marked value. With such transformers, and with good voltage transformers, meters could be lagged and adjusted on the secondary side, thus greatly reducing the labor and skill required.

Improvements are needed in current transformers which of necessity have a low value of primary ampere-turns, such as cable-testing and bushing transformers; also, in those which must be made with an abnormally long magnetic circuit, namely, air-insulated high-voltage current transformers for outdoor use.

(*g*) *Instruments for Plant Acceptance Tests.* For such tests instruments are needed which will give much greater accuracy and reliability under service conditions than can be had with present instruments. Such instruments could be larger and more expensive than the present portable instruments, because on some contracts a fraction of 1 per cent bonus or penalty would pay for them. The deflection potentiometer, with volt box and current shunts, is an example of the kind of instrument required, but is available only for direct current and voltage. Instruments of similar capabilities ought to be available for a-c. power at least.

(*h*) *D-C. Watthour Meters.* Further improvements are needed in the commutator meter which will decrease the attendance required, give reasonable immunity from stray fields, and permit operation from shunts without introducing errors from temperature differences. Some method should apparently be devised to shield the motor element from stray field and at the same time protect the drag magnets from the effect of short circuits through the meter.

(*i*) *A-C. Watthour Meters.* No figures are available on the various operating characteristics of the meters now being manufactured. They seem to have approached pretty close to practical lower limits of size, weight and cost, though from the station standpoint no doubt still further improvements in the latter would be welcomed.

For important installations it is desirable to have induction meters in which first cost[89] has been subordinated to greater accuracy, especially on overloads. Such meters could be larger and heavier, and could require more power in their windings, provided they gave improved results. It does not seem logical to use for such work a type of meter designed to be used by the million on small installations.

Some evident methods of improving the present watthour meter for this purpose may be mentioned. The braking flux due to permanent magnets should be largely increased, thus reducing the relative braking effect of the current-coil flux, which latter causes the present accuracy curve to droop

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[89] See discussion of paper on "Guarantees of Accuracy of Instruments," by G. Campos, *L'Elettrotechnica*, vol. 4, p. 123, 1917.

heavily on overloads, and also reducing the relative braking effect of the voltage-coil flux which is one of the causes of change of accuracy with change of voltage. To improve the accuracy on varying frequency at low power factors, the use of iron of much better magnetic quality suggests itself. Possibly the use of silicon steel melted in vacuum and properly heat treated would give the desired result. In any event, enough iron should be used in the voltage magnet to enable it to be worked at the most favorable induction.

The use of a much greater braking flux would have the further advantage of giving a lower full-load speed, with corresponding reduction in wear of pivots and jewels.

DISCUSSION ON "THE ACCURACY OF COMMERCIAL ELECTRICAL MEASUREMENTS" (BROOKS), NEW YORK, N. Y., FEBRUARY 20, 1920.

W. H. Pratt: The subject of the paper is "The Accuracy of Commercial Electrical Measurements". That is a very broad topic, and it suggests that we ought to consider the meaning of "commercial". As I interpret the paper, it is intended to include the measurements which are just below the accuracy of refined laboratory work, but certainly acceptance tests scale very high and other lines of measurement which are evidently included do not rank at a lower stage.

There is one feature of this paper which I think should be particularly noted, and that is, due to its very nature and the repeated warnings that occur from page to page, of how to avoid errors, it is possible that it may give the reader the impression that instruments are not very good things, whereas I know that the author meant no such thing, and I know that his purpose in writing the paper was to show how good the available instruments are, and to point out how by discriminating use of our available devices even more accuracy than is customarily obtained, can be achieved.

On page 501 there is a discussion of scale law. I think that this must be viewed in a very broad way. If an instrument is to be used in a certain manner, and can be given a certain type of supervision, undoubtedly one form of scale distribution might be preferred to another, which could be better suited, were the facilities for following the performance of the instrument through its life different.

On page 529 there is mention made of the static protection in certain a-c. instruments, to prevent the effect of electrical charges disturbing the indications of the needle, and somewhere in the paper, near that point, it is stated that certain d-c. instruments are not so protected. I fail to remember cases where that is not effectively taken care of.

On page 534 the suggestion is made that if zero adjusting devices are arranged so as to give a very limited range of adjustment, this adjustment could be better effected. It seems to me that it would be very difficult, almost impossible, to do what is here suggested. One or two degrees, truly, is a very considerable displacement of zero, and would not occur in any ordinary instrument, except as a result of accident, nevertheless, other considerations, such as general repairs to the instrument, might easily dictate that considerably more than one or two degrees of adjustment should be provided.

On page 571 there is a reference to the use of polyphase wattmeters, and it is pointed out that the general accuracy is not as great as with single-phase instruments. Abstractly that is true, but we must remember that there are a great many types of measurements where the load is somewhat unsteady, where the result will be much more accurate if absolutely simultaneous addition is effected than if addition were effected by observing the reading of two separate instruments. On the whole, though, it is simply a general statement that the measurement of the more complex polyphase circuit is more difficult than that of a simpler single-phase circuit.

At another point in the paper it is stated that the moving iron instruments are not adapted to d-c. measurements where the moving coil instrument is available. There are some special types of measurements where this general statement fails to apply, for instance, in circuits carrying rectified direct currents, on which perhaps arc lamps and incandescent lamps are simultaneously connected. It is very important, if very high efficiency incandescent lamps are used to measure the current on the basis of the r. m. s. effect rather than the average basis. In such cases, even if current would ordinarily be described as direct current, the moving-iron instrument might be the necessary instrument to employ.

Under "Measurement of Energy", I think it is well to point out that there are available in this country a number of watthour meters which are of very high accuracy, considering the extreme range of conditions that must be met. Furthermore, this device was developed, with no limit to the expense on investigation, the results are such that added expense put into the structure in all probability would bring about entirely incommensurate returns in the way of added accuracy.

The accuracy is, of course, not 100 per cent, in fact, one limitation of this whole subject, and one that gives it its particular aspect, is that with measuring devices, we attempt to obtain the absolute truth, which we never can obtain, that is, we are always a little wrong.

On page 584 a criticism is made of the use of chromium steels for damping magnets and magnets in general, and it points out that the devices made during the war should be very carefully watched. It is only fair to state that manufacturers of devices using permanent magnets were fully acquainted with the chromium steels for many years before they made the change, but there were certain manufacturing advantages connected with the use of tungsten steel. During the war it was necessary to go to the chromium steels, and once the change has been made, there is no apparent reason for changing back. Almost the only shortcoming of chromium steel is that you cannot get quite as high a strength with a given amount of material. On the other hand, magnetic problems in general are such that by using a trifle more material you can get back to where you were.

On the last pages of the paper there are a number of suggestions made as to topics which should be taken up for improvement. I think it proper to say that these and many other investigations have been undertaken, and that we will all welcome the finding of new materials which would permit the general improvement of measuring devices. There are few devices that are in use that have been given a more careful scrutiny with regard to the details than measuring devices. It is mentioned on the last page that there should be improvement in d-c. watthour meters. I think that it is again only fair to say that this topic has been most extensively investigated, and if you should refer to the records of the large users of this device, you will find that there is not a great deal to be gained, even with perfect accuracy of the meter.

The suggestion was made that springs might be advantageously gold-plated. We must remember that part of the spring which is most important in the indications of an instrument is its outer skin, and that the suggestion of gold-plating does not seem to suggest an improvement in this respect.

Again the possibility of making use of more expensive meters for the measurement of large blocks of a-c. power is mentioned. Here again it must be pointed out that the devices which are in use are the result of extremely expensive investigation, and there have been very few sacrifices in obtaining accuracy to what might be called commercial considerations.

A fact that electrical measuring devices are small, and that the work which is put into them is a very considerable part of the whole cost, carries with it the meaning that there is very little reason for scrimping materials in order to save costs, and thereby reduce the character of the performance.

J. R. Craighead: In discussing terminology on page 543, Mr. Brooks objects to the use of the term "phase angle" in reference to instrument transformers because this "phase angle" really differs by 180 deg. from the real phase angle between primary voltage or current and secondary voltage or current. This has always seemed to me an unnecessary refinement. The potential or current at the secondary terminals of a transformer has either 180 deg. relation or zero phase relation, approximately, to that at the primary terminals provided you do not pay too much attention to what is inside the box. If you take simply a case of primary terminals, and secondary terminals, the phase relation may be taken as either reversed or not reversed. The method of handling connections in meter installations, pays no attention to reversal in the transformer but goes by the marked terminals. For practical purposes, therefore, the term "phase angle" may be properly used as an abbreviation of "phase angle between primary and reversed secondary" voltage or current.

Mr. Brooks discusses the limitations in the building and also in the testing of potential transformers and compares them to small power transformers. Now in the first place the potential transformer is only superficially like a small power transformer; in practically every design feature it has an individuality of its own; its exciting current is likely to be extremely different from the exciting current of the power transformer of the same size, its rating, as Mr. Brooks states, is very much below that which would be determined by its heating, its regulation is based on different requirements. The extension of the design to the higher voltages, consequently, does not necessarily offer exactly the same difficulties although they may be no less than those that are found in power transformers.

In respect to the testing, the author is right in stating that the higher you go in voltage the more difficult it becomes to test with accuracy for either ratio or phase angle, but it is possible to reach test values, somewhat beyond the 30,000 volts mentioned and still obtain results which are apparently consistent. The results also show that, at least, for a moderate range of voltages, the performance of a potential transformer can be fairly definitely related to its core and winding, without excessive variation due to the capacity effect.

We have conducted tests up to 45,000 volts for about ten years, with seemingly accurate results on both ratio and phase angles. We are at present building an extension of the outfit to enable us to make tests up to 70,000 volts, and while I have no tests as yet in the higher range, I believe it is quite certain we shall get reasonably accurate results up to that point.

With respect to the crest voltmeter, I believe Mr. Brooks does not mention the use of the kenotron with electrostatic voltmeter for the measurement of high potentials. This is a very acceptable and valuable means of measurement, for potentials as high as we can provide electrostatic voltmeters. On account of the leakages and corona effect in most electrostatic voltmeters, that limit is not yet very high. We have used them up to voltages of 25,000 crest value with comparatively small errors, but above this point the errors increase rather rapidly.

The kenotron voltmeter is also usable on the tertiary coil of test transformers, to which reference is made in another part of the paper. Oscillograph tests have shown that with a suitably designed tertiary coil, suitably placed, and with normal conditions of operation of the transformer, the accuracy is entirely satisfactory for the ordinary limits of high potential testing. I do not believe the tertiary coil is, or ever will be, the full equivalent of the potential transformer because the variation of load in the power transformer must of necessity cause a wider variation under testing conditions that would be obtained with the potential transformer itself, but it is quite a practicable instrument for the ordinary grades of accuracy.

On page 553 Mr. Brooks makes the statement that the ruggedness of windings, etc., are not directly related to the inherent accuracy. I am afraid I shall have to disagree with that. One of the most difficult questions that we have to meet is the design of current transformers for carrying very large overload without injury; that is, a transformer using normal full-load current of perhaps 100 amperes may have to be so constructed that it will carry 200 times normal load for two seconds or something of that kind. This has an important and direct effect on the accuracy of the transformer. It is possible to build such a transformer to standard accuracy, only by very greatly exaggerating the size of all the parts. It is possible to build a transformer within standard size only by considerably diminishing the accuracy and ability to carry secondary burdens.

The same is true regarding excessive requirements for insulation. As soon as the necessary dimensions of the opening of the core are increased by the special requirements, a change becomes necessary in the design of the apparatus, if the same phase angle and ratio are to be maintained.

Mr. Brooks states certain limits of design for satisfactory current transformers. While suggestions of this kind are of value to describe apparatus, they should not be used as arbitrary restrictions on the designer. A statement is quoted in the paper which limits current transformers of standard accuracy to a range of ampere turns from 600 to 1200 except for single-turn transformers. It is possible to build a good transformer for certain purposes below the minimum figure. It is possible to build a very excellent transformer for other purposes above the maximum figure, and most current transformers built at the present time to meet standard accuracy and standard requirements are above the maximum figure. This is especially noticeable in the case of the bus bar type of current transformers, where they run to 20,000 amperes, which would mean that a single turn corresponds to that many ampere turns.

In reference to the precautions to be used in operating current transformers, the following statement is made on page 554: "The secondary circuit should not be opened while current is flowing, because the voltage at the secondary terminals may become dangerously high. The high value of flux may cause excessive heating of the core with liability of damage to the insulation." Some little time ago a section was inserted in the Standardization Rules of the A. I. E. E. requiring that all current transformers should be so built that they can operate continuously with the secondaries open, without permanent damage. Practically all American transformers at the present time are now being built that way—the insulation is sufficient so that it is not damaged by the application of the voltage developed at the secondary of the transformer by normal primary current.

Magnetization of the core of a current transformer by the opening of the secondary with current in the primary is still a trouble which must be taken care of. The transformer must be demagnetized after the occurrence. Further, the voltage developed on the secondaries is always high enough to be unpleasant, and in some cases dangerous to the operator, if he happens to get across it.

Mr. Brooks discusses phase-angle errors of wattmeters, as affected by transformers and recommends Formula 6 on page 560, the so-called tangent formula, for correction of phase-angle errors. Another formula based on the cosines is widely used, and has some advantages over the tangent formula.

If $\cos \theta$ is the true power factor.

$\cos \theta_2$ is the apparent power factor shown by voltmeter, ammeter and wattmeter.

α is the equivalent phase angle of the wattmeter

β and γ are respectively the phase angles of the current and potential transformers.

Thus $\theta = \theta_2 + \alpha + \beta + \gamma$

True watts = apparent watts $\times \dfrac{\cos\theta}{\cos\theta_2}$

The convention as to positive and negative angles differs from that described by Mr. Brooks. θ_2 is a positive angle if the current is lagging. α, β and γ are positive if under lagging current conditions they increase the reading of the wattmeter, negative if they decrease it. Thus for current transformers a secondary current leading the reversed primary current gives a positive phase angle (β). In potential transformers a secondary voltage leading the reversed primary voltage gives a negative angle (γ). If the current in the wattmeter potential circuit leads the voltage applied, the angle (α) is negative.

The reason for preferring the cosine formula is that it is scientifically correct to start with, and most direct. You simply take a thing that has wrong power factor in it, and you divide it by that power factor, which is in the denominator, and multiply it by the true power factor, which is in the numerator, and that gives the correct value. In practise the correction factors $\dfrac{\cos\theta}{\cos\theta_2}$ are tabulated for various apparent power factors and values of $\alpha + \beta + \gamma$, rendering their application extremely simple.

There is also the advantage that this formula works with entirely satisfactory accuracy at extremely low power factors, where the tendency of the tangent formula, on account of the exaggeration of the tangent, as it approaches 90 deg., is to become incorrect. The same advantage which Mr. Brooks notes for the additive formula on this page, is obtained directly by the multiplying formula using the cosines.

On page 562 Mr. Brooks speaks of compensating for phase angle in the wattmeter. I note, in passing, that compensation can also be obtained with multiple circuits taken from the secondary of the current transformer or, better by a tertiary winding placed in the current transformer, to be loaded with compensating loads. These means while practicable, are rarely adopted, except in experimental work of a high class, because they are unnecessary.

Mr. Brooks also issues a caution on page 584 regarding the shifting of portable watthour meters from one current range to another, by means of series multiple

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(See paper by L. T. Robinson, TRANS. A. I. E. E., Vol. 28 page 1011-1012.)

connections, and calls attention to the fact that a bad connection affects the secondary burden of the current transformer and is a sufficient cause of error, to be worthy of consideration. There are a number of current transformers, particularly those used for testing, in which there is a series or multiple connection on the primary, allowing the current to have several ratios. If these connections are bad, so that the current flows through completely to one side of the transformer, while another coil receives little or no current, there will be a certain amount of variation of the ratio and phase angle of the transformer. The exact amount depends on the design. This is nearly always under one per cent, but with a very poor design, might be more than that.

This is also true in potential transformers. Where it is necessary to change the double-ratio potential transformer from one ratio to another, by connections on the high-tension side, it is necessary to see that a good connection is made. We have found one or two cases in which an error of as high as one or one and one-half per cent may be caused if failure of the multiple connection makes practically only one half of the transformer operative.

Now, just a word in regard to page 575, where Mr. Brooks mentions the results actually obtained in a large number of tests. He states that the phase angle at no-load of a certain number of commercial voltage transformers ranged from a maximum of 69 minutes to a minimum of two minutes with an average of 9 minutes. Modern design tends towards a small angle under that condition, and it is practicable to obtain transformers for either 25 or 60 cycles, which will not exceed ten minutes of phase angle at that point. Probably the transformer which showed 69 minutes was made some years ago, because up to a few years ago there was a large number of transformers being made using such high flux density that their no-load phase angles were extremely large.

The phase angles mentioned at full non-inductive load represent ordinary practise at the present time; that is, about a degree maximum negative phase angle, and a much smaller phase angle in the opposite direction.

E. P. Peck: Instruments, I have found, are generally considered troublesome things, and they do not receive very much attention, because they are not understood. They are quite often overlooked and neglected, not properly cared for.

As to some of the experiences I have had with testing instruments, a case very much in point was an acceptance test on a plant for five 12,500-kv-a. generators. The plant was designed and built. I got word that an acceptance test was to be made on the plant in two days, and I was to furnish the instruments for the test. I knew it was to be an important test from the size of the plant, and took all the precautions that could be taken, in a commercial laboratory to take care of such a test.

We calibrated two complete sets of instruments, and took them ninety seven miles up in the country for the test. When we got there, we found the contract was worded in such a way that an increase of efficiency in the units of one per cent meant $1000 bonus, and a decrease of efficiency of one per cent meant $1000 deducted from the contract price, so that one-tenth of one per cent error in the test meant $100. and that looked big to me;– particularly big, in those days when it took me three months to get $100 to put in laboratory equipment. There was $1000 one way or the other, on one small division of the indicating wattmeter.

The plant had been built, it cost several million dollars, three specialists had been obtained to measure the water end of the plant, and the electrical end was considered of such little importance, that only two days' notice was given, and no special instruments were purchased or thought of. They used merely the instruments on hand in a central station laboratory.

In this test we used every precaution we could think of, particularly after the contract had been read with the test in mind.

The indicating wattmeters that we used in the test were checked before they were taken up to the point where the test was made, and checked after they were taken back to the laboratory. We found there was a difference of 0.4 of a division in the reading of the meters, on the instrument test made before the acceptance test, and on the instrument test made after the acceptance test. It amounted to $400 one way or the other, depending on whether the meter changed on the way to the plant where the test was made, or changed on the way back from the plant. We averaged the thing, and gave either the other company $200 or took $200 ourselves.

The correction necessary on account of errors due to current and potential transformers, was over 2.5 per cent. It might have been easy to have overlooked current and potential transformer errors on that test, in which case $2500 would have been taken by one party or the other.

F. V. Magalhaes: Mr. Brooks has analyzed very thoroughly the magnitude of the errors that creep in due to bent pointers. Bent pointers can occur easily even with rather careful use of instruments, but I do not believe that Mr. Brooks has pointed out the guard for the error, namely, a periodic checking of instruments for scale check in addition to their electrical accuracy. These scale checks should be made periodically irrespective of the previous known history of the instrument. Conditions that may produce bent pointers are not always reported, and it is safer to keep a continuing record of the scale of the instrument by periodic checks. The point may be obvious, but has not been pointed out in the paper.

On page 555 Mr. Brooks comments on the question of permanency of the constants of instrument transformers. He says the question has been up for a number of years, and quotes from the Electrical Meterman's Handbook. I should like to state that while at the time the Handbook was compiled the recommendation for periodic interval of ratio test was five years, a sub-committee of the Meter Committee of the National Electric Light Association has been working practically continuously since that time, collecting data from the various utilities represented in that association relative to instrument transformers. The last two reports of the Meter Committee of the National Electric Light Association presented some of these results although no final conclusions were drawn. In general, a period of ten years looks to be a fairer period than five years for periodic ratio check. Some State Commissions have already changed their rule from five years to ten years for the periodic tests.

Mr. Brooks on page 588 in enumerating various things that he feels should be brought to the attention of the manufacturers for development, makes a comment as follows: "The use of a much greater braking flux would have the further advantage of giving a lower full-load speed, with corresponding reduction in wear of pivots and jewels."

With meters having heavy moving elements, it is entirely possible to use diamond jewels with a good grade of steel pivot, and obtain satisfactory service. It is a question whether the lowering of the speed would not increase errors at light load.

One other point—I feel it is a matter of regret that the title of the paper "Commercial Electrical Measurements" could not have been stretched to include stop watches. A stop watch is used on a great many of the field tests involving electrical instruments, and personally I would have been very grateful if Mr. Brooks had analyzed the weaknesses and faults of stop watches as well as the errors that might creep into their use. In the field at the present time it is about all that can be used. There are some improved methods of timing used in the laboratory, but personally I would have been glad to have had Mr. Brooks include an analysis of the stop watch and its proper use with the other instruments that are used with it.

F. H. Bowman: Mr. Brooks uses the term "permanent-magnet moving-coil" type of instrument. That is distinctly descriptive. That is nothing more or less than what is known as the d'Arsonval instrument. It strikes me that "d'Arsonval instrument" is entirely descriptive. We have analogy for that in the other terms, such as the watt, the farad, and others, and "d'Arsonval" instrument is a shorter term than the one used.

Mr. Brooks speaks also of an electro-dynamic instrument, or electro-dynamometer instrument. That can quite well be shortened to dynamometer instrument, which is a reminiscence of the old Siemens dynamometer, the origin of the instruments of the type described.

Furthermore, we have the phrase "moving iron instrument", which to me is one which has a core of iron. There might be recognized the repulsion type of iron instrument, where there is a repulsion type of moving iron. We could have a moving type of iron instrument and the repulsion type of iron instrument, which, although they both use iron in their structure are, to some extent, different.

The next point I wish to raise is with reference to the question of the uniform versus the non-uniform type of scale. Mr. Brooks' conclusion is that in future designs engineers should lead toward the one that, with a wattmeter gives a perfectly uniform scale. I think for a voltmeter it will give a scale tending more to the quadratic or square scale. That, with certain types of instruments, lends itself to a uniform scale. There will be no difficulty in making the d'Arsonval galvanometer, or anything else, substantially uniform under those circumstances. Call it either dynamometer wattmeter, or, as Mr. Brooks says, electrodynamic type of instrument. That gives all of our scale, and you would have to go to expedients to get it other than uniform.

On the other hand certain types of instruments will not lend themselves to the quadratic scale, but are a sort of mongrel.

When I am buying an instrument, rather than have the manufacturer attempt to sell me one which he has pulled and hauled and twisted, so that he gets one not actually uniform, but pulled into a semblance of uniformity, at the sacrifice of torque or efficiency, I would prefer the natural scale, and, in fact, I think it is an axiom that when you try to digress from a natural law you are apt to get into difficulties, which leads me to believe if you have a certain type of instrument, it is well to stick to the natural type scale for that instrument, especially for that particular kind of instrument you are working with. The scale which is pointed in just the portion that you wish to use it, for example, in the voltmeter around the 100 volt point. You do not care about the zero part of the scale and care little or nothing about the divisions from 130 degrees up. You are interested in the voltages from 100 to 115 and naturally prefer to have that rectified as far as possible.

As regards shifters, I am not sure whether Mr. Brooks favors them, and would like him to enlighten us. In any event, we can see the trend of instrument design is toward the use of zero shifters, for both a-c. and d-c. instruments. I would prefer an instrument that would give a slightly greater adjustment than the one or two angular degrees specified in the paper. I would prefer if the zero shifter itself was made mechanically strong, particularly that it was constructed so that it would have a uniform rotation, that is, it would not be a zero shifter to go a little one way and go against a stop and a little in the other way and go against a stop. The zero shifter is naturally weak, and when you put it against an obstruction it sometimes fails.

As a conclusion to the paper, there is a paragraph headed "Further developments desired." Now, if these further developments, which Mr. Brooks points out, are the goal to which the instrument designers should strive, then expressing that idea, as Mr. Brooks has, advances the art. If, however, a reading of that gives one the impression that there are radical defects in instruments and meters as now put out, then I believe that harm has been done to the art.

For example, on the subject of magnets. Tungsten has in the past been frequently not obtainable. Chromium is obtainable, and chromium makes a good magnet; there is no doubt that a chromium magnet is as permanent as a tungsten magnet. The question could be brought up—why was not chromium used in the beginning? That could be answered, no doubt, but the fact remains that chromium magnets are as permanent as tungsten magnets, other things being equal.

As regards spring material, I believe there is little trouble experienced with the fatigue of springs. I would consider it poor practise to gold-plate a spring, because the spring is naturally thin, and as Mr. Brooks points out, the torque depends on the cube of the thickness, and if on the outside of the spring we add a little soft electro deposit of gold, we are doing what we should not do, making the outer surface soft where it ought to be very hard.

Alexander Maxwell: In reading Mr. Brooks' paper, and others which have been presented before dealing with commercial electrical measurements, one notices almost invariably that the electrical measurements discussed are the comparatively refined measurements made in commercial laboratories; the accuracy of field measurements is not often discussed. Certainly it is difficult to discuss field measurements, as it becomes mainly a discussion of personal errors, but nevertheless it is important to keep in mind the lower accuracy attainable in the field. In the field you have great extremes and sudden changes of temperature, poor light, observers in uncomfortable postures, and many other factors all tending to reduce the attainable accuracy. I think that it should go into the record in any discussion of commercial measurements that field measurements are always appreciably less accurate than measurement made with the same instruments in the laboratory.

Regarding the use of symbols to indicate the operating principle for commercial instruments, it occurs to me that the direct and simple thing to do would be to mark the type of movement on the scale or on the case in plain English. The idea is good, and perhaps symbols might be used, so long as they were better than the ones shown in the paper.

H. H. Sticht: Referring to Mr. Brooks' classification of instruments, I am sure, speaking as a manufacturers' agent, that most manufacturers would be glad to see such classification made, because in a great many cases instruments are sold under false pretense.

Reference is made to the periodical checking of instruments. The use of electrical measuring instruments in industrial plants is increasing very rapidly, but in this class of users it is not generally recognized that instruments should be tested. Although the Meterman's Handbook mentions this, I think Mr.

Brooks has done well to bring it to the attention of engineers and electricians who are not connected with central stations.

Mr. Brooks' suggestion for the zero adjuster is very good, particularly for the industrial man, as distinguished from the central station man, who as a rule does not have a standard instrument for checking purposes. If such a man keeps on bringing the pointer back to zero, by means of the adjuster, whenever he bends the pointer, he will never find out that his instrument is incorrect and therefore continue to use it as an accurate instrument. On the other hand, in the instrument business as in everything else you get what you want; in many cases the consumer wants as large a zero adjustment as possible, and the manufacturer has to make it. During the war we had a number of instruments rejected, because they had but two per cent zero adjustment. I think that this is the reason why most manufacturers put in more than the amount specified in the paper.

As far as symbols are concerned, I think something should be placed on the scale, as suggested by Mr. Brooks. I do not care whether it is a symbol, or whether the name is there, but to the large number of users in industrial work a voltmeter is a voltmeter. It is for this reason that I believe a symbol or name on the scale would have a good effect, since a man would then know if he were working with a d'Arsonval, soft iron, or dynamometer instrument.

B. W. St. Clair: I want to say a good word for the present product of the American instrument makers. There are plenty of test instruments in the market today that are thoroughly reliable, and accurate. If rightly handled and carefully treated they can be depended upon for very consistent and accurate results. I have watched the performance of some five or six thousand of them in use every day on all kinds of testing, and, except in those cases where they are misused or abused, their performance is perfectly satisfactory. This applies to the better grade of instruments, for it must be acknowledged that there are some types much advertised that are subject to many troubles.

The first point I wish to mention has to do with instrument scales and zero adjustments. Zero adjustments were never put on instruments, and especially a-c. instruments, to correct for bending of the needles. The proper cure, and in fact the only cure for a bent needle is to straighten it. If an attempt is made to bring the needle back to zero with the zero shifter the needle will not line up with the divisions.

Under these conditions it is, of course, possible to make large errors in readings. Zero adjustments are designed to take care of changes in the spring regulators or to correct for internal changes in the spring that affect its zero position.

As regards scales of a-c. instruments it seems to me that there are some advantages to logarithmic scales. It makes possible the taking of readings at most any part of the scale with equal accuracy. Alternating-current voltmeters are used generally for voltages between 100-120, 200-240, etc., and it would seem a desirable practise to make the readability over this range as great as it can be by closing up the scale beyond these points. Voltmeters are hence an exception to this logarithmic scale. In wattmeters where the total scale range will be needed, the logarithmic scale enables a given amount of testing to be gotten out with a minimum number of instruments. In Fig. 2 of the paper are shown two wattmeter scales, one evenly divided and one logarithmic. For ordinary testing it would be possible to use the logarithmic scale from 100 to 500 watts or a range of values of 5 : 1. On the other hand it would only be possible to use the evenly divided scale from 50 to 150 watts or a range of 3 : 1. In the first case the precision of reading in per cent would be approximately constant while in the latter it would increase from a relatively low value to a maximum at full scale.

I cannot agree with the author that it is easier to mark uniform scales than logarithmic. In the manufacturing of high grade instruments, each instrument has its own individually calibrated scale, whether uniform or logarithmic. The actual dividing of the scale into the proper divisions between the cardinal points is done by jigs, and it is just as easy to divide a non-uniform scale as a uniform one.

I also want to say a good word for the compensating wattmeter. I am afraid you may have received a wrong impression as to its ability to hold its calibration. It is true that the compensating feature is one that requires very careful adjustment, but once that adjustment is made it is very permanent. It is something that depends only upon the geometry of the instrument, and the constancy of a resistance. Both these factors are highly permanent ones. This compensating feature is independent of springs, magnets or other things whose permanency might be doubted.

The question of pivots and jewels is one that is discussed whenever meter and instrument men get together. However, it is one of the least sources of trouble in either a meter or an instrument if they are treated with a certain degree of attention and respect. Instrument bearings never wear out. Meter bearings very seldom do. I have seen diamond meter bearings carrying 150 grams load at 180 rev. per min. that have made 600,000,000 revolutions with no sign of wear. While instrument bearings do not wear out they are easily damaged. To reduce the friction torque to a minimum it is, of course, necessary to reduce the radius of the area of contact between the jewel and the pivot as near to zero as possible. In well made bearings this may be so small that even with a moving element weight of from 5 to 10 grams the pressure per square centimeter is of from 100,000 to 250,000 kilograms. Because of the small area of contact and the enormous pressure that the contact is under, rough treatment imposes enormous crushing forces upon the pivot even with very light moving elements. A very little jar is sufficient to upset the end of the pivot, *i. e.*, "mushroom" it. This results in an enormous increase of the area of contact and a big increase in the friction torque. The ability of an instrument to withstand jars, depends upon the quality of engineering and the manufacturing skill that goes into the bearing. For this reason the oft repeated phrase,—"the ratio of torque to weight" is a very fictitious criterion of instrument quality. I am afraid we have been too prone to judge an instrument by this quantity without paying due regard to other factors of more importance. It is possible to secure high torque at the sacrifice of other desirable qualities. Besides torque such things as freedom from temperature errors, frequency errors, low power factor errors, and the like must be considered. In a-c. instruments (and it is only there that there is any excuse for relatively heavy moving elements) the uncertainty of reading on account of the bearing friction is much reduced by the slight vibration given the entire moving system.

Then a word about magnets. There are other things in magnet steel that should be considered in the selection of materials for instrument magnets, besides coercive force and permanence. High permanence is desirable, for high permanence means high torque. Large coercive force is also desirable for that means large permanence under demagnetization. But more important especially than coercive force, is the manner in which the B —H. curve crosses the B axis. Other things being normal, that steel which approaches nearest to a right angle intersection of this axis has the highest permanence when made up into an instrument. The demagnetizing forces that an instrument meets, are in general not 40 or 60 H, but are relatively weak forces, and it is therefore, more necessary to have the steel show small changes under these forces than it is to have it show a high absolute coercive force. Another point that needs consideration is the ease of working and the necessity of working to close limits of temperature, and other factors. Steels which show high breakage losses during hardening and grinding are not desirable manufacturing materials.

Our present steels are very permanent. d'Arsonval types of instruments, when well made, and when not abused, show inappreciable yearly changes of calibration. Watthour meter magnets, whose variation in strength affects the accuracy of the meter as the square of flux changes cause no apparent speeding up from year to year. The point is this: Our present steels are very trustworthy when properly made and treated. The measurement art is not suffering because of low grade permanent magnets. Better materials are always acceptable, but there is no cause for a general worry on the part of meter and instrument users that their measuring equipment is in unstable calibration. There has been much agitation over the general substitution of chrome steel for tungsten steel during the war. Chrome steel gives magnets that are just as permanent as tungsten steel. There is little choice between them from a magnetic standpoint. In general, the average permanence of chrome is slightly lower than tungsten steel, and the coercive force slightly higher. It is very doubtful if there will ever be a general return to the use of tungsten steel.

A. L. Ellis: About the matter of bent pointers and the zero adjuster for correcting the trouble, it should be borne in mind that a good instrument does not have a pointer that is easily bent. The thin tip on the pointer is necessary in order to be able to read the scale closely, but the remainder of the pointer should be strong enough, and is strong enough, in practically all instruments, to maintain its form. Applying zero adjusters to instruments is a matter of choice. If the zero adjuster is put on an a-c. instrument it should only be used, as Mr. St.Clair has pointed out, for correcting a shift of the control spring, and not a shift due to a bent pointer. The zero adjuster would be useful, however, in certain types of a-c. instruments, notably switchboard instruments, where it could be used as an adjustment for correcting a scale error at a particular point when the instrument is being checked, but in this case it would probably be useful only for voltmeters.

With reference to the frequency with which instruments should be checked, much depends upon their use. Frequently a high grade instrument will be kept in a laboratory and referred to only for rough measurements, in which case it would not be necessary to check the calibration in many years simply because a high degree of accuracy is not required. In cases where instruments are used to turn out quantity production in a manufacturing plant, frequent checking is required to insure uniformity of product and guard against an error due to accidental damage of the instrument.

With regard to the scale divisions—the character of the scale on an instrument can be varied somewhat at will, depending upon the position of the moving and stationary parts with respect to one another at the starting point. This matter has been fairly well discussed.

With reference to magnet steel—chromium magnet steel, as a matter of fact, gives just what Mr. Brooks asks for as compared with tungsten, that is, greater coercivity. Having that, you naturally expect a more permanent magnet.

As to the electrostatic field from the case, the metallic cases in most instruments are grounded to the moving system so that there should not be any trouble from this source.

With reference to the interference of unshielded d-c. instruments with one another, it should be borne in mind that one can easily get into trouble where only one instrument is used. The unshielded d'Arsonval instrument is in error 0.3 of a division at full scale deflection, (1/5 of 1 per cent) depending upon whether the instrument faces east or west, due simply to the normal earth's field. It may be much greater in New York City.

One word about springs. Phosphor bronze control springs give no trouble whatever as regards oxidation if the instrument designer and instrument user keep sulphur away from them.

Harvey P. Sleeper: I think there are perhaps one or two points to be considered in the use of a polyphase meter to measure the power in a three-phase circuit; for example, if we use the two wattmeter methods, you then have two meters in place of one to read. Of course, that is a difficult operation for one person, particularly when the pointers may land at different points on the scale.

The matter of the sectional shunt is interesting to me. It is stated in the paper, and very correctly, that the trouble is confined in the manufacture of heavy capacity shunts. This is, of course, true, but it occurs to me that the trouble which is encountered in balancing the potential between the sections of the shunt by use of the potential leads could be offset by putting the additional effort into the manufacture of the shunt as a single unit, that is, not in sections.

With regard to the matter of a compensating wattmeter, it might be of interest to the operating man to suggest that it is only on the smaller capacity meters that the compensated winding really comes into any practical effect, and that the power consumption on the heavy capacity windings is really negligible.

In regard to the symbol suggested for the marking of instruments to indicate their type, I think that is an excellent idea. A meter should not be a mystery to a man, and he should be able by inspection to tell what type of movement it is, and I think this system of marking offers a very simple solution of it.

H. B. Brooks: Taking up Mr. Pratt's remarks, I wish to say that I did not intend to give a bad impression by saying so much about the *errors* of instruments, in fact, I strove to be diplomatic, and I said "factors affecting the *accuracy* of the instrument." I do not wish to be an alarmist, but I feel that if we understand our enemy, we have him half conquered, and if the user understands just where he may get into trouble with an instrument, he will know what to do to avoid that trouble.

Mr. Pratt and several others have said that instruments are protected electrostatically if the case is connected to the moving system. Direct-current voltmeters in which the brass case is set on a mahogany base, do not have this connection, at least those I have knowledge of have no connection between the circuits and case. The reason is obvious. The brass case is where it may be touched by the operator and the leads may drop on it. The suggestion in my paper would obviate the difficulty—it is to put an insulating lining in the case and inside of that a metal lining which is connected to the circuit, and in this way you can be free from electrostatic trouble, and free from trouble from accidental contact.

A number of speakers have touched on the matter of zero-adjusting devices, and it has been stated that my suggestion is difficult to carry out. By having a double adjustment, so that within the instrument you could make a relatively large shift—which would be used by the makers,—and then another secondary adjustment of one or two degrees, which the user can effect, the maker can use his discretion in setting his part of the double adjustment, and that will give him all the latitude he needs for manufacturing conditions. Under these circumstances the user would have only a small margin. If the pointer should be bent it should be necessary to open the case and straigthen it out, and it should not be possible to pull a bent pointer back on to zero with the external zero adjuster.

Several speakers, including Mr. Pratt, have mentioned that polyphase wattmeters, in practise, often give more accurate results than two wattmeters. I agree with that. I do not wish to say a word against the polyphase wattmeter, but only to point out that it is not commercially possible for the maker to get as good an accuracy with it as with the single-phase wattmeter of similar type.

Mr. Pratt speaks of the use of a moving-iron instrument for certain circuits, because we are more concerned with the root-mean-square value, and that the iron instrument is better. I agree with him there. Ordinarily, however, when we make measurements of current in a direct-current circuit, we are interested in the arithmetic mean, as in charging a storage battery from a pulsating circuit.

Mr. Pratt states that watthour meters have been developed with no limits to the expense. He referred particularly to direct-current watthour meters, and it was to those that I referred when I suggested that they looked too much like Dr. Thompson's device. My answer would be, evidently we have worked that principle to the limit, and it seems we need some entirely new development in the direct-current metering art. I might take this illustration from Dr. Whitney. Originally we had speaking tubes, and we might have conducted a research into the various factors on speaking tubes, to find out what was the best type of tube, and best material, but someone came along and said—"Let us shrink the tube to a wire," and Graham Bell reduced the tube to a wire. Somebody else got interested in the matter—"Let us do away with the wire entirely and talk through space." Apparently we need to do something of that kind with the direct-current metering art and get some radically new proposition.

Mr. Pratt and others spoke of the chromium steel magnets. I regret that my statements were not sufficiently clear on this point. I did not intend to reflect on chromium steel magnets, because I have been informed that about the only limitations are that the manufacturing operations must be kept within sharper limits. You have not the same wide latitude as to heat treatment and temperatures as with tungsten steel, and the feeling is, apparently, that if you will take pains in the manufacture, you can make good chrome steel magnets. I simply wanted to caution users of meters manufactured in war time to keep the point in mind, and to see what chromium magnets would do in service.

In regard to the goldplating of springs, I would say that the thickness of gold which would be necessary to protect against corrosion is so microscopically small that its effect upon the elastic properties would probably be negligible. The bronze will give the necessary elasticity, and a very thin film of gold will protect it. Mr. Ellis referred to the fact that phosphor bronze springs would not corrode if we would keep sulphur away. Some makers, at least, have put sulphur in instruments by using hard rubber parts, and at one time soft rubber was used, which is also capable of giving out sulphur. I have seen cases in which the sulphur from rubber tubing had blackened the springs. Instruments are frequently used in laboratories where the products of combustion of illuminating gas, which contain sulphur, are allowed to diffuse into the atmosphere. Therefore, I think we should have some protection for the springs. Mr. Pratt mentioned the fact that watt-hour meters have been developed after much investigation. I do not wish to champion the "super-meter" unduly, but I think this suggestion of the Italian engineer is a novel one, and we should consider the advisability of spending $30 or $40 or $50 for a meter to measure large quantities of energy which involve the accounting for thousands of dollars per month. The matter might well be investigated to see how much better meters could be made if we increased the braking flux and used a superior grade of steel in the electromagnet.

Mr. Craighead spoke of the use of the term "phase-angle" in connection with instrument transformers. I agree that if you get away from the physics of the transformer, and consider it as an iron box, with primary and secondary leads, you can think of your secondary voltage as in phase with the primary voltage. That is easy, but this matter comes up in other ways. For example, we have condensers which, on a sinusoidal voltage, should take a current leading by 90 deg., but actually the current does not lead exactly 90 deg., and the small angle by which it departs from 90 deg., we have been calling the phase-angle. We need an expression which shall signify the little angle by which a quantity departs from some ideal angle. At the Bureau of Standards we have talked that over a number of times, and have suggested such terms as "angle of defect," which sounds bad, and this suggestion of angle of bias" is only thrown out to see what you think of it.

Mr. Craighead spoke of the limitations on building and testing high-voltage potential transformers and I am interested to know that he has gone to 45 kilovolts and expects to go to 70 kilovolts. The figure of 30 kilovolts which I mentioned was what we have been able to do, and where we felt we ought to stop for the present. I do not doubt that we might go further than that, and with transmission voltages going up faster than we can keep up with them, it seems evident that we must consider radical improvements in methods of measuring high voltage.

Mr. Craighead spoke of the use of the kenotron and electrostatic voltmeter to get the crest value of voltage. Limitations on the space available for my paper prevented me from saying much about the crest voltmeter, though it is an important subject in which I am much interested. Dr. Silsbee of the Bureau of Standards is using that method for measuring the peak of the voltage induced in magneto armatures and seems to find it a good method.

Mr. Craighead takes me to task for saying that we should have a small phase angle in current transformers. I agree with him that a zero phase angle is desirable, but I did not want to ask for too much, and asked for a small constant angle. The value zero is certainly small, and I will be glad to accept that in any transformer which Mr. Craighead turns out for me.

Mr. Craighead said that the ruggedness of the windings is not related to the inherent accuracy. Looking at it from the understanding he got from my statement, I agree with him, but I did not mean it in the way in which he took it. Suppose I put it in this way. Let two iron cores identical in form and material be submitted to two persons, one a manufacturer who has had experience and will put in the right kind of insulation in the right way to make it a good rugged transformer, and let the other be wound with the same number of turns of the same size of wire, by an amateur. Then what I have called the inherent accuracy, which depends on iron performance and turns, etc., will be the same for both transformers. The service accuracy after a short circuit hits the transformers will be very different, and doubtless the accuracy of the well-made transformer will be greater, but I am speaking only of the inherent performance which will be shown on a laboratory test.

Mr. Craighead spoke of my quotation from Mr. Edgcumbe in regard to the number of ampere turns necessary for a good current transformer. I am aware that is a sort of empirical way of stating the case. But the user has few criteria for selecting transformers, and if he can put in his specification that the ampere-turns must not be less than 1000 or 1200, he is, at least, protected from a manufacturer who might put out a 500-ampere-turn transformer. That is merely a start, in the way of telling the user what he ought to have, in order to get good results. The value 1200 ampere-turns, which I quoted, is expressly stated not to be an upper limit.

I am glad Mr. Craighead corrected me in regard to the statement with reference to opening the secondary circuit. That caution has been common from the time I can remember working with current transformers, and I am glad the rules now require the transfomer to be so built that it can have the secondary open. Nevertheless, we should strive to educate the user to keep the secondary circuit closed.

In regard to the matter of correction formula for wattmeters, I think Mr. Craighead's formula has perhaps some points in which it does not quite come up to the Drysdale additive correction. The great advantage of the Drysdale formula is that it gives you so much at a glance. If you will put the term cos α equal to unity, the remaining formula tells at a glance the magnitude of the error in watts, and you can readily express it in divisions at full volt-amperes.

The compensation of phase angle in the current transformer, of which Mr. Craighead spoke, using a tertiary winding on the transformer, is a device that is well-known; at least it is on record in the patent literature. It requires to be adjusted by some means which would enable you to know when you had zero phase angle. This limitation would tend to prevent its general application.

In regard to the matter of multiple ranges in current transformers, we have found at the Bureau of Standards that in such transformers from good makers, the relative ratios are very accurately what they should be when the links are changed to give different connections. If the sections of the primary winding are properly distributed over the core, you might as an extreme case have three sections open out of four, and all the current going through the remaining section, with practically no error, and the only disadvantage would be the excessive heating of that one section.

The excessive value of 69 minutes was for a 25-cycle potential transformer and I think Mr. Craighead has suggested the reason, namely, that the transformer was designed with too high a value of the flux density.

In regard to the matter of conventions as to algebraic sign to be used in connection with wattmeter measurements, the important thing is to agree on some standard convention and then to have everybody use it.

Mr. Peck gave an interesting illustration of an acceptance test in which a wattmeter differed by 0.4 per cent, before and after the test. This tends to confirm the statement that we need a radically better type of alternating-current instrument for such work.

Mr. Magalhaes states that bent pointers can occur easily and that scale checks should be frequently made. I agree with that, and would suggest that we use the type which will give the least error if the pointer does get bent, and also that we should in addition make the scale checks.

Mr. Bowman apparently does not like the term "permanent-magnet moving-coil." I do not like the length of it myself, but have not been able to think of any other term that properly describes the type. The term "moving coil" would also apply to electrodynamic wattmeters and voltmeter, and is therefore ambiguous unless further qualified. I do not think the term d'Arsonval should be used for several reasons. In the first place, the d'Arsonval galvanometer was anticipated many years by Lord Kelvin's siphon recorder, which is simply the d'Arsonval galvanometer, plus a little glass tube carrying ink. Personally, if I were to give the name of any man to that instrument, it would be the name of that great pioneer who thirty years ago showed us how to make servicable instruments—Dr. Weston. Instead of applying the names of individuals to types of instruments, however, I prefer descriptive terms, such as hot-wire, moving-iron, and the like.

I think the term "electrodynamic" is preferable to dynamometer because dynamometer is a noun, and when we have a good adjective, we are hardly justified in using a noun as an adjective. Furthermore, the derivation of the word "dynamometer" suggests the measurement of force and does not bring in the idea of electric action. The term "electrodynamic" is not long and includes everything.

Mr. Bowman spoke of the fact that we need another term rather than moving-iron for the repulsion type of instrument. As I see it, both the so-called plunger type, in which a solenoid attracts a core, and the repulsion type in which there is one soft-iron piece within a coil, which is repelled by another, are simply two kinds of moving-iron instruments,—the essential thing is that each has a piece of iron which moves.

I agree with Mr. Bowman that the natural scale law of the instrument should not be unduly modified at the expense of efficiency. This has been shown by the experience of a maker of induction ammeters, who at one time forced these instruments to give an approximately uniform scale, but who has returned to the quadratic scale.

I heartily favor the use of zero adjusters. They are a great convenience. I only want the maker and user to bear in mind the difficulties that may arise, and guard against them.

Mr. Maxwell spoke of the fact that field measurements have not been touched on, and that there is an unknown extra amount of error in the use of an instrument in the field as compared with its use in the testing room. That is true, and I hope that those who have had extensive experience with field tests will collect and publish definite data on this subject in the near future.

I think that symbols to denote the operating principle of instruments are better than words, for this reason—we are going to manufacture things not for America only, but for the world, and our export trade in measuring instruments should greatly increase. I think, therefore, that some symbol that suggests to the user what the instrument is like is much better, and if properly chosen it can be understood by a man who cannot read the language of the instrument maker. I do not advocate the particular symbols shown in my paper, which are given merely to show what has been done.

I am glad to note that Mr. Sticht advocates the idea of a classification of instruments. Ten years ago, a prominent instrument engineer advised us that he wished we would lay out such a classification because, as he put it, it would relieve the manufacturer from the embarrassment of claiming everything for all types of instruments.

I am sorry to know that the government has rejected instruments because they had only two per cent zero adjustment. It is evident that the question needs to be investigated so that general agreement may be had.

Mr. St. Clair stated that the subdivision of the logarithmic scale between cardinal points is as easy as that of the uniform scale. The trouble with this procedure is that if the divisions between two given cardinal points are made of uniform length, some are too long, some too short. This may be discovered by noticing the abrupt change in the length of a division which occurs as each cardinal point is passed. These intermediate divisions are seldon if ever checked, so that if the cardinal points check correctly the instrument passes for an accurate one in spite of the errors which occur in actual use between cardinal points. This difficulty is absent in the uniform-scale instrument.

In regard to the usual idea concerning torque-weight ratio, I am convinced that it is not logical, and some other than a linear relation should exist. Experience seems to show that as we go to lower and lower values of weight, we can go to lower values of torque-weight ratio. I cannot make an exact statement of the matter, but believe that if the matter were properly studied we should find that some other than the simple torque-weight relation exists. It is the opinion of some of us who have considered the matter a little that a better rule than the present one is that for a given degree of excellence of pivots and jewels the ratio, torque divided by weight *squared*, should equal a constant.

Mr. Ellis stated that instrument pointers are strong and not liable to bend. That is true of certain modern types, but on the other hand, there are older forms in use, and many of them have pointers which are easily bent, and until they are eliminated we should keep the matter in mind.

Mr. Sleeper referred to the method of constructing large shunts, which I have given, and states that difficulty would be involved in balancing the parts of the shunt. This method has been used in the construction of a large shunt which had to be very precise, and no difficulty was found in carrying it out, and the mechanical difficulties which this method avoids are very great.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

SHORT-CIRCUIT PROTECTION FOR DIRECT-CURRENT SUBSTATIONS

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The author includes an outline of the progress made in protection of direct-current machinery from short circuits since the publication of a paper at Atlantic City on this subject.

The improvements mentioned include the refinement and perfection in details of the flash barriers; a new design of high-speed circuit breaker for both direct-current substations and electric locomotives; a new high reluctanc ecommutating pole for 60-cycle synchronous converters and a new design of protected brush holder. An instructive analysis of conditions during direct-current short circuits is shown by several photographs, oscillographs and diagrams. Special reference is made to operating results on the electric zone of the Chicago, Milwaukee and St. Paul Railroad.

THE investigation of means to prevent flashing of direct-current machinery and development of suitable equipment has been continued since the presentation of the paper on "Protection from Flashing for Direct-Current Apparatus," by Mr. J. L. Burnham and the writer at the Atlantic City Convention in 1918.* This paper outlined in a general way the results obtained with quite a number of different methods of reducing or preventing flashing under extreme overload or short-circuit conditions.

This study indicated that a special form of flash barrier with arc coolers and a new form of high-speed breaker with current-limiting resistance had proved the most promising development. Tests showed that the two types of protection provided complete protection from a "dead" short circuit caused by short-circuiting the terminals of a machine without external resistance.

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*TRANS. XXXVII Part II, 1919, pages 1341-1365.

These two types of protection have been further perfected and are now in regular commercial use. They are used either separately or together and in many instances considered standard railway practise.

The improved type of high-speed breaker described has been perfected in all details as shown in Figs. 1 and 2.

Thirteen of these breakers were installed by the Chicago, Milwaukee & St. Paul Railroad as part of the electrification of their Coast and Cascade Divisions. Fig. 2 shows the breaker with the arc chute installed but with covers removed. This view shows the calibrating rheostat used to set tripping point of the breaker. Description of this breaker will not be repeated as general theory of operation and design is described in the paper referred to, and more detailed description of the perfected breaker will be described in article by Mr. J. F. Tritle in the April number of the *General Electric Review*.

This breaker was used instead of the first type of circuit breaker which has given successful operation during the past three years in the fourteen substations of the 440-mile original electrification of the Chicago, Milwaukee & St. Paul Railroad. This new breaker has the advantage of lower cost and greater simplicity.

One of these breakers is used with each of the 2000-kw., 3000-volt, synchronous motor-generator sets in the Tacoma, Renton, Cedar Falls, Hyak and Cle Elum substations and the remaining five breakers on each of the new gearless type passenger locomotives. On account of the lower cost of these breakers and advantages of using the "unit" system throughout, each of the sets is protected by its own high-speed breaker instead of one breaker per substation, the arrangement in the original installation.

The general connections, location of circuit breaker, etc., are shown in Fig. 3.

The circuit breakers for the substations and locomotives are exactly alike with exception of interlocking and calibration for tripping points.

The circuit breakers were given a very exhaustive test in connection with one of the 2000-kw. sets in test before shipment, with very successful operation in all details. It was found that the generators could be short-circuited with only sufficient cable in the circuit to connect the different meter shunts, short-circuiting contactor and high-speed breaker without damaging machines in any way and with practically no flashing at the brushes. Figs. 4 and 5 show photographs of two of these short circuits, one of which gave so slight a flash that it can hardly be seen while the other shows very slight flashing in the flash barriers. Fig. 6 shows oscillogram of one of these short circuits giving a very good idea of the high speed of the breaker and the protection afforded. It will be noted that the current was limited to about 7000 amperes and reduced to three times load in 0.016 seconds. The current starts to decrease in 0.0081 of a second and it will be noted that the area repre-

FIG. 1—1500-AMPERE, 3000-VOLT, D-C. HIGH-SPEED CIRCUIT BREAKER

senting the load which would be likely to cause flashover is very small and of such short duration that very little gas or arc could be formed. Fig. 7 has been prepared to show graphically the much greater

Fig. 2—1500-Ampere, 3000-Volt D-C. High-Speed Circuit Breaker With Asbestos Lumber Casing Removed Showing Rheostat Used to Obtain Different Tripping Points

protection afforded by circuit breaker of such high speed over that obtained with the usual type of breaker. This figure shows the current curve of a 2000-kw., 3000-volt machine on short circuit when protected by the high-speed breaker and by a standard 3000-volt breaker designed for higher speed than usually obtained with regular carbon-break 600-volt breakers. The area of each curve above the load which would cause flashing has been cross-hatched to show the ratio between the two areas which gives a very good idea of the value of the great speed and the reason there is so little flashing. During these tests about one photograph out of four was similar to Fig. 4, while the others were of the character shown in Fig. 5.

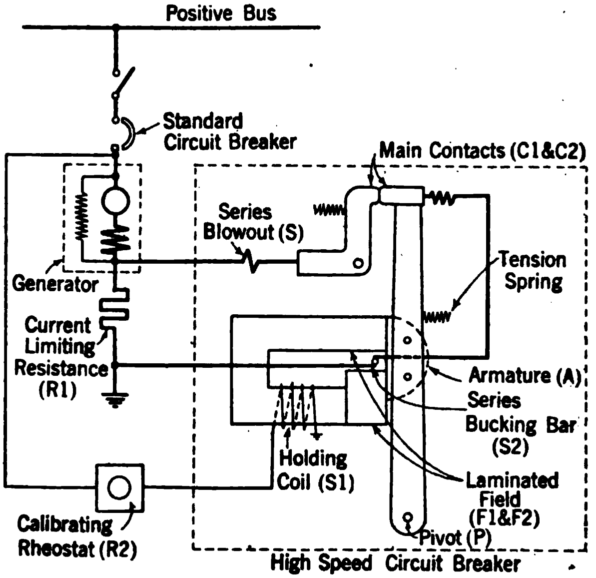


Fig. 3—General Connections of High-Speed Circuit Breaker

A special reliability or endurance test was made as part of the acceptance tests of the breaker during which about 65 short circuits of different magnitude, fifteen of which were "dead" short circuits, were applied at intervals of about 2½ minutes without cleaning the commutators or giving them any attention whatever. At the conclusion of these tests, five "dead" short circuits were thrown on the set within ten minutes. At the end of these tests, commutators were in excellent condition without any need of cleaning or attention of any kind.

These circuit breakers are now in regular operation and reports received of their operation have been very gratifying.

The application of the high-speed circuit breaker to direct-current electric locomotives is another distinct advance as in addition to protecting the apparatus on the locomotive, it prevents the short circuits from affecting the substations.

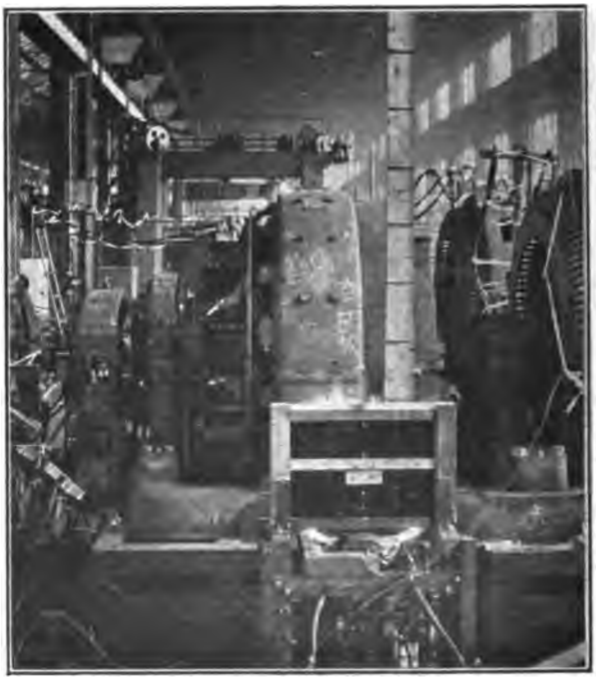


FIG. 4—SHORT CIRCUIT ON 2000-KW., 3000-VOLT MOTOR-GENERATOR SET PROTECTED BY HIGH-SPEED CIRCUIT BREAKER AND FLASH BARRIERS. TRIPPING POINT 2550 AMPERES. LINE RESISTANCE 0. CURRENT LIMITING RESISTANCE 1.2 OHMS

If both the substations and locomotives are equipped with this type of high-speed circuit breaker, current under maximum conditions would never reach a value much greater than 7000 amperes. The value of the maximum short-circuit current decreases very rapidly with distance from substation as shown in Fig. 8. These records were taken with a 1500-kw., 3000-volt

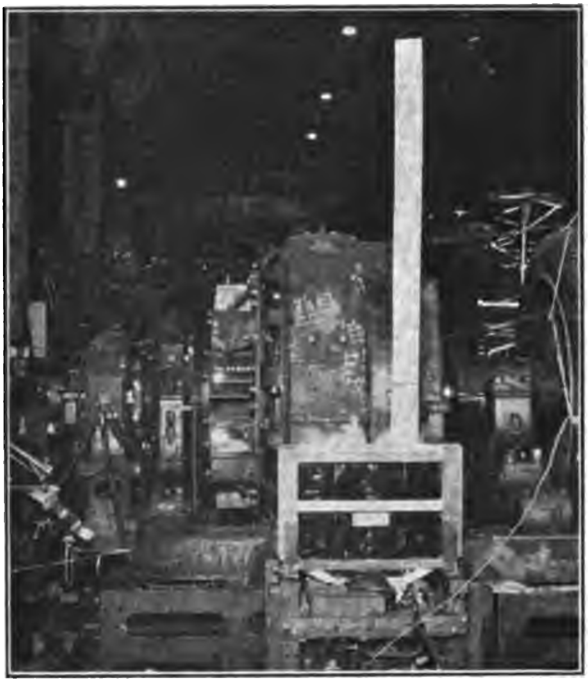


FIG. 5—SHORT CIRCUIT ON 2000-KW., 3000-VOLT MOTOR-GENERATOR SET PROTECTED BY HIGH-SPEED CIRCUIT BREAKER AND FLASH BARRIERS. TRIPPING POINT 2940 AMPERES. LINE RESISTANCE 0. CURRENT LIMITING RESISTANCE 1.2 OHMS

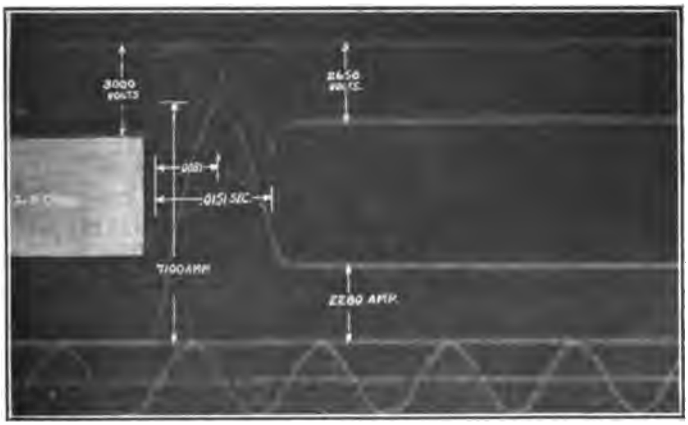


FIG. 6—SHORT CIRCUIT ON 2000-KW., 3000-VOLT MOTOR-GENERATOR SET PROTECTED BY HIGH-SPEED CIRCUIT BREAKER AND FLASH BARRIERS. TRIPPING POINT 2250 AMPERES

motor generator set connecting the overhead trolley consisting of two No. 4/0 wires directly to the 100-lb. track rails and closing the circuit at the substation. It will be noted that the maximum peak current is only 3600 amperes with short circuit 4800 ft. from the substation and 2860 amperes, 9600 ft. from the substation as compared with 7000 amperes "dead" short circuit across the terminals of the machine.

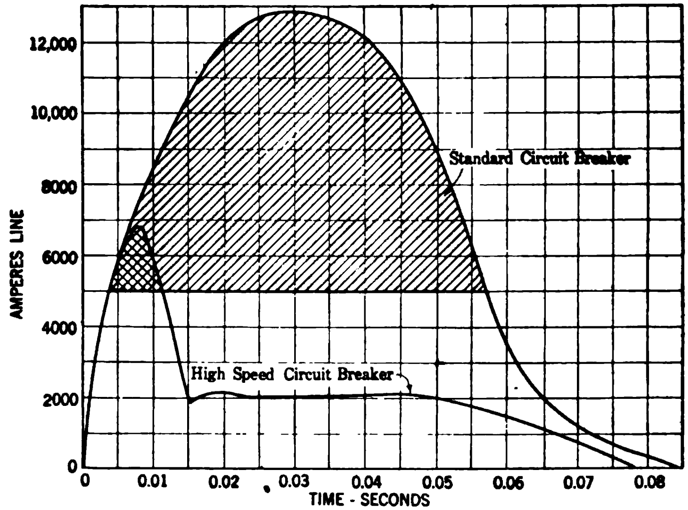


Fig. 7—Short Circuit on 2000-Kw., 3000-Volt Motor Generator Set Protected by High-Speed Circuit Breaker with Curve Showing Current Which Would be Obtained with Standard Circuit Breaker. Ratio of the Area of the Two Curves Above 5000 Amperes Gives Indication of the Protection Afforded

The value of this type of protection was proved very conclusively during these tests due to the accidental grounding of one of the switchboard busbar insulators. The high-speed breaker inserted the current limiting resistance so quickly that some time was required to locate the trouble as burning at the grounded point was so slight as to be hardly noticeable, indicating that even under such extreme conditions current is reduced so quickly that sufficient time is not allowed to cause current to generate sufficient heat to cause destruction of the current-carrying parts.

Another of the incidental advantages of this type of protection is the elimination of disturbances on the a-c. side of synchronous converters or motor-generator sets ordinarily caused by d-c. short circuits due to the fact that the load is decreased so quickly that momentum of the armatures supplies the energy and the load is not increased materially on the a-c. side. The overload relays are therefore not affected, increasing very greatly the general operating efficiency of a substation, eliminating time required to start up set from the a-c. side, etc. After the occurrence of a short circuit it is only necessary for the operator to close the high-speed breaker and then the main switchboard breaker which is interlocked with the high-speed breaker after which the main switch is thrown in following regular switching practise. If the short circuit still persists, the high-speed breaker will again open but with no flashing or damage to brushes or

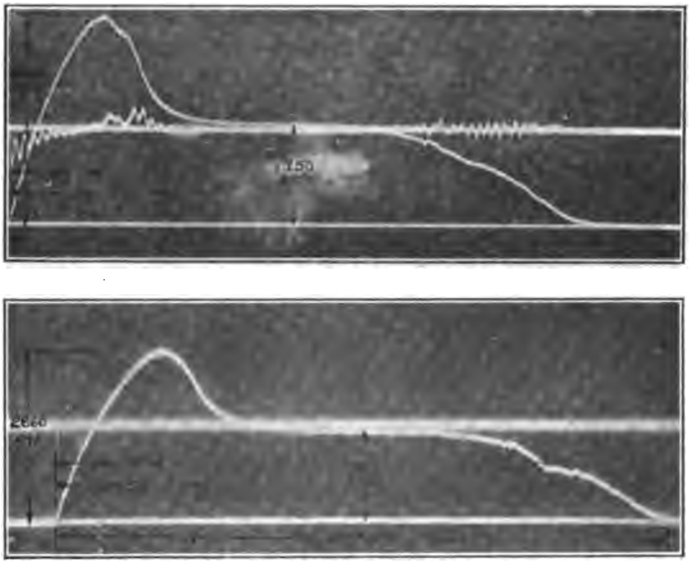


FIG. 8—SHORT CIRCUIT ON 1500-KW. 3000-VOLT MOTOR-GENERATOR SET AT DIFFERENT DISTANCES FROM SUBSTATION. GENERATORS PROTECTED BY HIGH-SPEED CIRCUIT BREAKER AND FLASH BARRIERS

commutator and greatly decreased duty on the regular breaker.

The flash barriers described in the original paper have not been changed in any essential details, improvements being along the line of simpler construction, ease of removal for inspection and improvement in appearance.

These barriers with iron wire arc coolers are standard equipment on all 3000-volt motor-generator sets as well as on all machines used in connection with automatic substation control. Fig. 9 shows one of the 2000-kw., 3000-volt sets for the Milwaukee electrification equipped with these barriers, while Fig. 10 shows barriers for 600-1200-volt, 60-cycle, 500-kw. synchronous converter.

FIG. 9—2000-KW., 3000-VOLT D-C. SYNCHRONOUS MOTOR-GENERATOR SET EQUIPPED WITH FLASH BARRIERS

Flash barriers and high-speed circuit breaker are standard equipment on 1500 to 3000-volt substations and it is believed this practise will be extended to include 600 and 1200-volt apparatus.

Another advance in short-circuit protection is the protected type of brush holder recently perfected as shown in Fig. 11. This brush rigging is protected on all sides where flashing might occur by asbestos lumber so that an arc cannot readily hold between brush holders of opposite polarity and prevents the formation of iron or copper vapor which might cause a flash to the frame and cause damage to the brush rigging or commutator. A removable cover is provided for inspection and removal of brushes. It is made of an iron sheet for convenience as there has been no tendency for the arc to strike this part of the brush rigging during tests or in actual operation. It will be noted that this type of brush rigging lends itself very readily to the addition of flash barriers as shown in Fig. 10.

This type of brush rigging has been standardized for all 600-volt, 60-cycle synchronous converters.

The use of the high reluctance commutating pole is a very promising improvement which has just been made in 60-cycle, 600-volt synchronous converters and has been standardized for all 60-cycle machines. Tests indicate that the use of these poles raises the flashing point at least 50 per cent. In actual commercial use the improvement is greater than indicated by this figure as a very great proportion of short circuits which originally caused flash-over would not cause flashing on machines equipped with this new type of commutating pole winding.

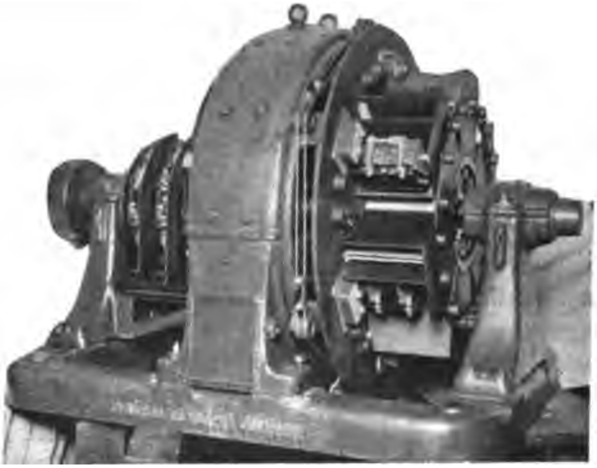


FIG. 10—500-KW., 600/1200-VOLT 60-CYCLE SYNCHRONOUS CONVERTER EQUIPPED WITH FLASH BARRIERS

These improvements are of particular value for 60-cycle converters which are inherently more sensitive than 25-cycle converters.

Brief attention should be called to the great protection afforded by tapping the feeder at some dis-

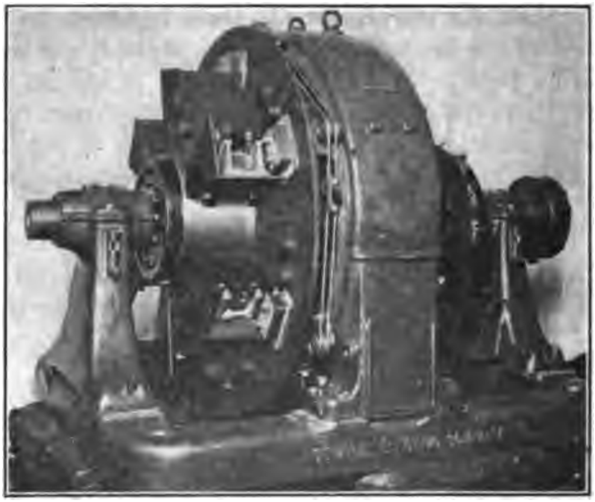


Fig. 11—500-Kw., 600/1200-Volt 60-Cycle Synchronous Converter with Protected Type of Brush Holder

tance from the substation. This is undoubtedly the cheapest type of protection which can be used but cannot be relied upon to prevent flashing over under extreme short circuits. A very slight amount of permanent resistance such as would be given by such an

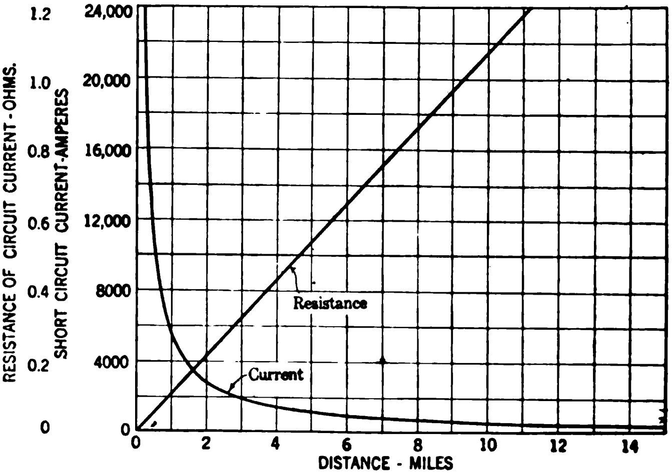


Fig. 12—Curve Showing Ratio of Short-Circuit Current to Distance From Substation

arrangement greatly reduces the number of flash-overs with very little loss of energy or voltage. Under ordinary conditions it is believed that the distance to the first tap need not be greater than 2000 ft. A greater distance than this causes an appreciable loss of energy and drop in voltage. Fig. 12 shows very clearly the great benefit of a small amount of resistance in reducing the maximum possible current on a short circuit.

If complete immunity is desired from short circuits, the high-speed circuit breaker and barriers undoubtedly offer the best known solution. With this protection, feeder taps can be connected to the overhead trolley directly at the substations, reducing losses to a minimum. Maintenance on the substation apparatus will also be decreased as burning from short circuits undoubtedly causes most of the wear and deterioration on brushes and commutator. Another particular advantage of this type of protection is that it can be applied to old generators or synchronous converters of any voltage without change in the machine itself.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

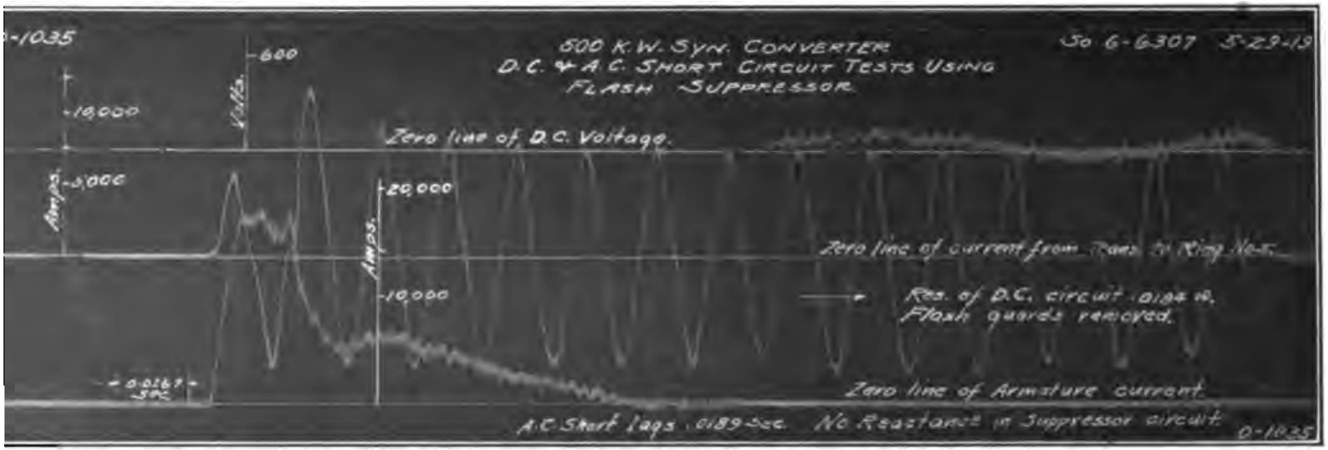
FLASHING OF 60-CYCLE SYNCHRONOUS CONVERTERS AND SOME SUGGESTED REMEDIES

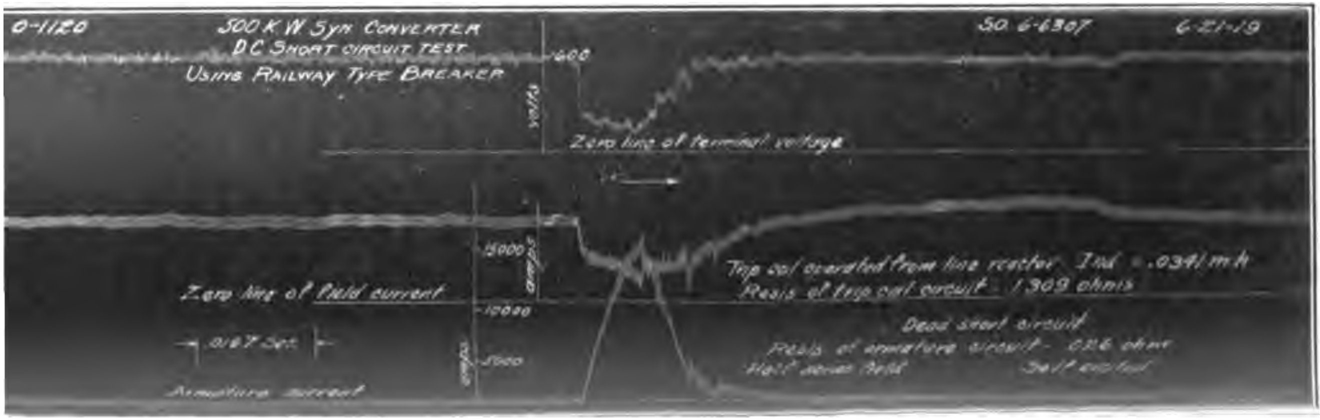
BY MARVIN W. SMITH
Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

WITH the increasing application of 60-cycle synchronous converters to railway service, the question of flashing is receiving considerable attention. A series of tests has recently been made, and others are still in progress, with a view to determining the possibilities of protection from flashing by various methods, including the "quick-acting breaker," "flash suppressor," "flash guards" and various modifications and combinations of these. The converter that has been used for these tests is a standard 500-kw., 600-volt, six-phase, 60-cycle, 1200-rev. per min. machine. It was supplied with power from a 5000-kv-a. generator through three standard high-reactance transformers. These transformers had a reactance of approximately 17· per cent at normal load and 10 per cent on short circuit.

Reasons why 60-cycle converters are susceptible to flashing. The protection of the 60-cycle converter is more difficult than the protection of any other class of commutating machines due to the inherent limitations of distance between neutral points on the commutator. The limiting distance between neutral points is fixed by the frequency and peripheral speed of the commutator. For instance, considering the maximum allowable peripheral speed as 5500 feet per minute, the distance between neutral points in the case of a 60-cycle machine will be approximately 9.2 inches regardless of the speed or number of poles. This may be readily seen from the following simple relation:

$$\text{Dist. between neut. points} = \frac{\text{circum. com.}}{\text{poles}}$$


0-1035
500 K.W. Syn. Converter
D.C. & A.C. Short Circuit Tests Using
Flash Suppressor
So 6-6307 5-29-19
600
Volts
10,000
5,000
Amps.
20,000
Amps.
10,000
Zero line of D.C. Voltage.
Zero line of current from Trans. to Ring No. 5.
Res. of D.C. circuit .0194 Ω.
Flash guards removed.
Zero line of Armature current.
0.0167 Sec.
A.C. Short lags .0189 Sec. No Reactance in Suppressor circuit.
0-1035


0-1120
500 K.W. Syn. Converter
D.C. Short circuit test
Using Railway Type Breaker
So. 6-6307 6-21-19
600
Volts
Zero line of terminal voltage
5000
Amps
10000
5000
Amps
Zero line of Field current
.0167 Sec.
Armature current
Trip coil operated from line reactor Ind. = .0341 m.h
Resis of trip coil circuit = .1309 ohms
Dead short circuit
Resis. of armature circuit - .026 ohm
Half series field Self excited

$$= \frac{\text{periph. speed}}{\text{rev. per min.} \times \text{poles}} = \frac{\text{periph. speed} \times \text{poles}}{\text{poles} \times 120 \times \text{freq.}}$$

$$= \frac{\text{periph. speed}}{120 \times \text{freq.}}$$

An interesting and rather surprising feature shown by the tests is the extremely rapid rate of increase of the direct current on short circuit as compared with direct-current generators. For example, tests made on the generators of the 2000-kw. sets for the Chicago, Milwaukee and St. Paul Railroad, show the initial rate of increase in current on short circuit to be approximately 1,100,000 amperes, or 1650 times full load, per second. Also short-circuit tests on a 1000-kw. 600-volt direct-current generator show an initial rate of increase in current of approximately 2,700,000 amperes, or 1620 times full load, per second. On the 500-kw. converter, however, the average initial rate of increase is approximately 3,300,000 amperes, or 4000 times full-load current per second. This large difference between the rates of increase of short-circuit current for the d-c. generators and the converter cannot be accounted for by the difference in the inductances of the machines. In fact, the calculated inductances of the generators, which are of considerable lower frequency than the converter, are lower than the calculated inductance of the converter. The difference may be due to the fact that the alternating current which is a motor current building up in the opposite direction from the direct current, produces, or tends to produce flux linkages in the armature winding which oppose those of the direct current, with the result that the induced voltage opposing the rise of direct current is reduced. In other words, a higher rate of increase in current is necessary to produce the required counter voltage, which accounts for the higher rate of increase in current in a synchronous converter than in a direct-current generator. Reference to oscillogram Fig. 1 shows that the current reaches its maximum value in approximately 0.006 seconds. This rapid increase in short-circuit current together with the limited distances between brush-holder arms make the 60-cycle converter particularly susceptible to flashing trouble.

CONDITIONS CAUSING FLASHING

Flashing does not appear to be entirely dependent upon the point or value at which the current is arrested. Oscillograms in Figs. 2 and 3 show dead short circuits on the machine when protected by a modified high-speed railway-type breaker. The current was limited to approximately 14,500 amperes, 17½ times full load, the rate of current increase being limited to some extent by the use of a small reactor as a shunt for the trip circuit of the breaker. (The maximum short-circuit current value of this machine is approximately 19,000 amperes or about 23 times full load). However, at this speed the machine was completely protected, except for slight pitting of the brush holders, even at this high current value. The voltage records show clearly that the machine cleared itself immediately after the complete opening of the breaker, which took place in approximately 0.015 seconds. Fig. 4 shows an overload only one-fifth the above value in connection with the ordinary low-speed breaker, from which it is evident that the machine bucked over immediately after the breaker opened even at this lower value of current. When a machine is on short circuit a large percentage of the voltage is consumed internally. The heavier the short circuit the larger is the percentage of the voltage thus consumed. Then so long as the d-c. breaker is closed the voltage between brush arms and hence the tendency to flash is a minimum. The voltage between neutral points on extreme overload and short circuit with the d-c. breaker closed, is also dependent somewhat upon the brush pressure. The higher the brush pressure the lower the contact drop and hence the external voltage between neutral points. This contact drop may be quite appreciable if the brush pressure is low enough to allow the force of the arc to lift the brushes off the commutator, which means a lower percentage of the voltage will be consumed internally, with a result that the voltage on the commutator will be higher. Assuming a reasonable brush


300 K.W. Syn Converter
30 G 6307
6-21-19
D.C. Short Circuit Test
Using Railway Type Breaker
Trip coil generated from line reactor Ind. = .0341 m.h.
Res. of trip coil circuit = 7.309 ohms
Dead short circuit
Res. of armature circuit .0266 ohm
No series field
Self excited
Zero line of terminal voltage
Zero line of field current
.0167 Sec
Armature current
C-1122
600
-18000
-3000

Fig. 3

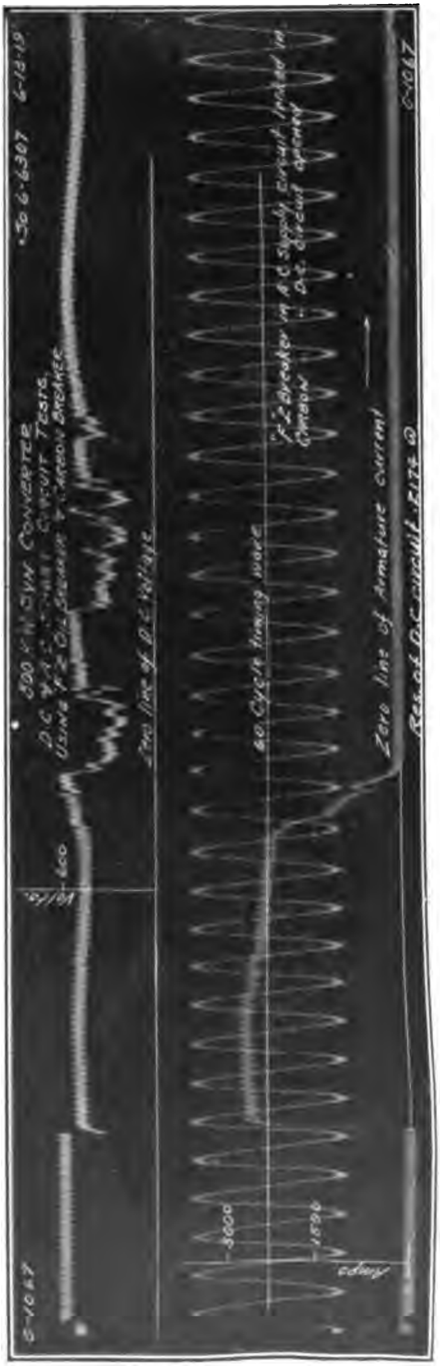

300 K.W. Syn Converter
D.C. Wattless Short Circuit Tests
Using F-2 Oil Breaker & Carbon Breaker
30 G 6307
6-18-19
Zero line of D.C. Voltage
60 Cycle timing wave
F-2 Breaker in A.C. Supply Circuit, Jacked in
D.C. Circuit opened
Zero line of Armature current
C-1067
600
3000
1590

Fig. 4


500 K.W. Syn. Converter
D.C. & A.C. Short Circuit Tests,
Using "F" 2 Oil Breaker & Carbon Breaker
50-6-6307
Zero line of D.C. Voltage
60 Cycle timing wave
"F" 2 Breaker in A.C. supply circuit opened
Carbon " " D.C. circuit locked in.
Zero line of Armature current
Res. of D.C. circuit .116 Ω
O-1071
600
volts
5000
2500
amps
O-1071


500 K.W. Syn. Converter
D.C. & A.C. Short Circuit Tests
Using "F" 2 Oil Breaker & Carbon Breaker
50-6-6307 6-10-12
Zero line of D.C. Voltage
.0167 Sec.
Zero line of Current in Ring No. 3
Res. of D.C. Circuit .0905 Ω
Carbon Breaker set to trip @ 3500 Amps.
Zero line of Armature Current
600
volts
6000
3000
amps
6000
3000
amps
O-1072

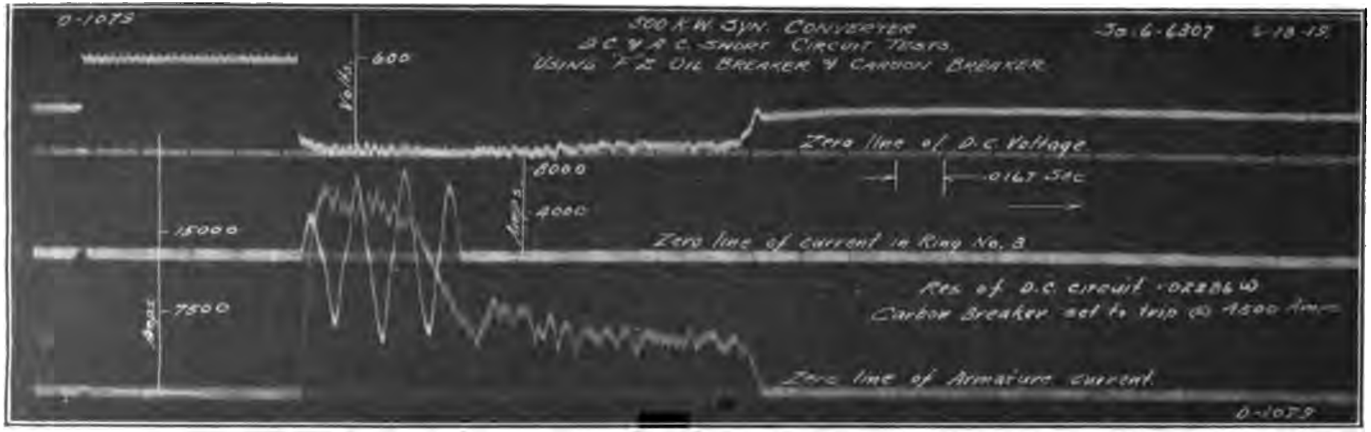

D-1079
500 K.W. Syn. Converter
D.C. & A.C. Short Circuit Tests.
Using "F2" Oil Breaker & Carbon Breaker.
3016-6307
5-18-15.
600
Volts
8000
4000
Amps.
15000
7500
Amps.
Zero line of D.C. Voltage
.0167 Sec
Zero line of current in Ring No. 3
Res. of D.C. circuit .02286 ω
Carbon Breaker set to trip @ 4500 Amp
Zero line of Armature current.
D-1079

FIG. 11

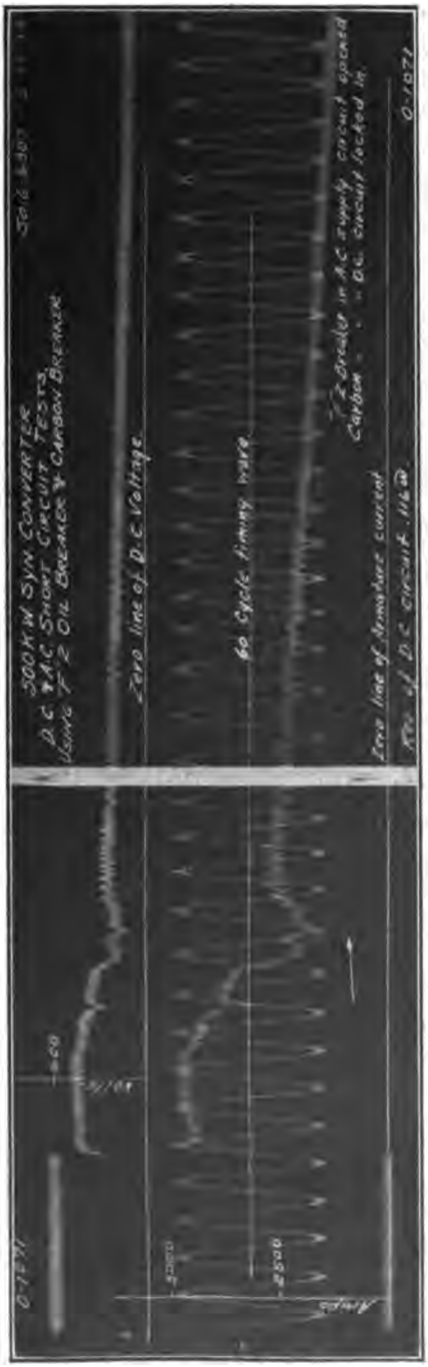

500 K.W. Syn. Converter
D.C. & A.C. Short Circuit Tests
Using "F"2 Oil Breaker & Carbon Breaker
So. 6-8307
Zero line of D.C. Voltage
60 Cycle timing wave
Zero line of Armature current
Res. of D.C. circuit .116 ω
Breaker in A.C. supply circuit opened
Carbon " " D.C. circuit locked in
O-1071
600
Volts
3000
2500
Amps

FIG. 9


500 K.W. Syn. Converter
D.C. & A.C. Short Circuit Tests
Using "F"2 Oil Breaker & Carbon Breaker
So. 6-8307
Zero line of D.C. Voltage
.0167 Sec.
Zero line of Current in Ring No. 2
Res. of D.C. circuit
Carbon Breaker set to trip @ 5000 Amps.
Zero line of Armature current
O-1072
600
Volts
6000
3000
Amps

FIG. 10

gas is again dependent upon several other variables, such as, the inherent commutating characteristics of the machine (which is a measure of the local short-circuit currents under the brush), the value of the load current and the time it has been flowing, the grade of brush, etc.

It is the writer's opinion in view of the above tests that a moderately high-speed breaker is no better than an ordinary slow-speed breaker. In the case of a moderately high-speed breaker probably as much gas as possible has been formed over the commutator by the time the breaker opens, *i. e.* after this point the gas is dissipated as fast as it is generated. Then a very slow-speed breaker may be even better than a moderately high-speed breaker due to the fact that the voltage will have time to die down appreciably before the breaker opens, especially, if the a-c. supply is opened in the meantime.

Opening the a-c. breaker is of course undesirable because the machine has to be synchronized again. However, this procedure minimizes the flashing considerably. Figs. 10 and 11 cover an overload of approximately seven times full load and a dead short circuit respectively taken under this condition. As shown by the a-c. wave, the a-c. supply was tripped off approximately 0.05 seconds after the instant of the d-c. short circuit. The opening of the d-c. breaker was purposely delayed as long as possible (by changing spring adjustments) so as to allow time for the current and voltage to die down due to the decreasing speed as well as the dying down of the field flux. In the case of the dead short circuit it may be seen that the current has dropped to almost one-fourth of the maximum short-circuit value and the voltage has dropped to less than one-half normal value when the d-c. breaker opens. The machine immediately clears itself upon the opening of the d-c. breaker as shown by the voltage record. During the period prior to the opening of the d-c. breaker there was of course considerable flame on the commutator but due to the limited voltage on the commutator (with the d-c. breaker closed) the arc did not have the tenacity to hang on, and by the pressure and contact drop, it is practically impossible for a machine to buck over and hang on between arms on dead short circuit so long as the d-c. breaker is closed, for the voltage on the commutator is practically killed just in the same manner as in the case of the flash suppressor referred to later. It is usually the opening of the d-c. breaker that does the damage. Tests in connection with both the low-speed and high-speed breakers and, in fact, ordinary operating experience show that a machine rarely bucks over until the d-c. breaker opens. See Figs. 4, 5 and 6. There may be considerable flame on the commutator due to the heavy current, but it does not have the tenacity nor power to cause any serious damage until the voltage is restored. Figs. 1 and 7 covering oscillograms taken in connection with the flash suppressor in which approximately one cycle elapsed between the time of the d-c. short circuit and the application of the a-c. short circuit, show clearly that the machine did not buck over before the a-c. short circuit was applied. During this time the commutator rotated through a distance corresponding to two neutral points. Figs. 8 and 9 show overloads of 3 and 6 times full load respectively when the d-c. breaker was tied in and did not open at all. However, the d-c. supply was tripped off about 0.06 seconds after the instant of short circuit as shown by the slight dip in the current and voltage wave. It is evident from the steady nature of the voltage wave that the machine did not buck over and hang on between arms, although there was considerable flash on the commutator.

Therefore, the ideal circuit breaker is not necessarily one which opens the circuit before the machine bucks over (for it usually does not buck over until the breaker opens) or before the current reaches a certain value, but one which opens the circuit before sufficient gas and volatile matter has been formed over the commutator to cause the machine to buck over when the voltage is restored by the opening of the breaker. It may be said that flashing is roughly a function of the voltage and distance between neutral points and the amount of gas or volatile matter over the commutator. This gas is again dependent upon several other variables, such as, the inherent commutating characteristics of the machine (which is a measure of the local short-circuit currents under the brush), the value of the load current and the time it has been flowing, the grade of brush, etc.

It is the writer's opinion in view of the above tests that a moderately high-speed breaker is no better than an ordinary slow-speed breaker. In the case of a moderately high-speed breaker probably as much gas as possible has been formed over the commutator by the time the breaker opens, *i. e.* after this point the gas is dissipated as fast as it is generated. Then a very slow-speed breaker may be even better than a moderately high-speed breaker due to the fact that the voltage will have time to die down appreciably before the breaker opens, especially, if the a-c. supply is opened in the meantime.

Opening the a-c. breaker is of course undesirable because the machine has to be synchronized again. However, this procedure minimizes the flashing considerably. Figs. 10 and 11 cover an overload of approximately seven times full load and a dead short circuit respectively taken under this condition. As shown by the a-c. wave, the a-c. supply was tripped off approximately 0.05 seconds after the instant of the d-c. short circuit. The opening of the d-c. breaker was purposely delayed as long as possible (by changing spring adjustments) so as to allow time for the current and voltage to die down due to the decreasing speed as well as the dying down of the field flux. In the case of the dead short circuit it may be seen that the current has dropped to almost one-fourth of the maximum short-circuit value and the voltage has dropped to less than one-half normal value when the d-c. breaker opens. The machine immediately clears itself upon the opening of the d-c. breaker as shown by the voltage record. During the period prior to the opening of the d-c. breaker there was of course considerable flame on the commutator but due to the limited voltage on the commutator (with the d-c. breaker closed) the arc did not have the tenacity to hang on, and by the time the d-c. breaker opened the current and voltage had reduced to such an extent that the arc could not establish itself.

INFLUENCE OF COMMUTATING CHARACTERISTICS ON FLASHING

Oscillograms covered by Figs. 12 and 13 which were taken at lighter loads and where the a-c. supply was cut off prior to the opening of the d-c. breakers, show clearly by the dip in the current and voltage wave that the machine bucked over slightly before the d-c. breaker opened, which appears contrary to previous conclusions. However, at this light load the voltage on the commutator is not appreciably reduced. Furthermore, with the a-c. supply cut off, the neutralizing effect of the a-c. armature reaction upon the d-c. armature reaction is absent. Therefore, the converter, operating as a straight d-c. generator with the increased effect of the armature reaction upon the commutating-pole circuit, is enormously under-compensated, with the result that the sparking under the brush is greatly increased which together with the high voltage between neutral points causes the machine to flash over even with the d-c. breaker closed. This again emphasizes the importance of the inherent commutating characteristics of the machine.

The relative strength of armature and commutating-pole fields has a considerable influence upon the commutation and hence upon the flashing of a synchronous converter on sudden changes of load and extreme overload or short circuit. Under normal load conditions the a-c. m. m. f. opposes the d-c. m. m. f. In the interpolar space the resultant armature reaction is only about 15 per cent of the d-c. armature reaction, and is in the same direction as the d-c. reaction. The commutating-pole field ampere-turns under this condition are just sufficient to buck down this resultant m. m. f. and in addition to force sufficient flux across the commutating-pole gap to generate the required counter voltage for commutation. However, at the instant of short circuit the converter acts largely as a d-c. generator delivering the first rush of current from its own inertia, with the result that the machine is enormously under-compensated. In fact, with the relatively low a-c. armature reaction at the instant of short-circuit, the resultant armature ampere-turns may be even greater than the commutating-pole ampere-turns, which means that the commutating-pole flux is actually reversed. This of course depends upon the relative amount of the commutating-pole m. m. f. expended in bucking down the armature m. m. f. and in forcing the flux across the gap. The larger the proportion of the ampere-turns expended in the commutating-pole air gap, or its equivalent, the less will be the effect of the armature reaction upon the commutating-pole flux and hence upon the commutation and flashing of the machine. Furthermore, on extreme overload or short circuit the machine falls back in phase similar to a synchronous motor carrying load with the result that the a-c. armature m. m. f. lags (in space position) with respect to the d-c. armature m. m. f. which is fixed in position by the position of the d-c. brushes. Therefore, even after the first instant of short circuit and after the a-c. m. m. f. has established itself the resultant armature reaction is still proportionately higher than the increase in load due to the relative shift in the position of the a-c. and d-c. armature reaction. This produces a result similar to the conditions in the first instant of short circuit, *i. e.* an under-compensated machine. Trouble from this source can also be minimized by increasing the strength of the commutating-pole field relative to the armature.

Sudden changes in the applied frequency to the converter is equivalent to the machine being out of phase and produces similar results from the standpoint of flashing. In fact, our attention was first called to the importance of the relative strength of the armature and commutating-pole fields by a case of trouble from this source in connection with flashing on a 1000-kw. 600-volt 25-cycle converter at the plant of the Philips Sheet & Tin Plate Co., at Wierton, West Virginia in the early part of 1917. This converter is connected to the secondary of an induction motor which drives a rolling mill and which is subjected to very sudden changes in load. These sudden changes in load produce sudden


0-1073
500 K.W. Syn Converter
D.C. & A.C. Short Circuit Tests
Using 1-2 Oil Breaker & Carbon Breaker
SO 6-6307
6-13-19
Zero line of D.C. Voltage
Zero line of Current in Ring No. 3
Res of D.C. circuit .1762 Ω
Carbon Breaker set to trip @ 8000 Amp.
Zero line of Armature current.
volts
800
amps
4000
2000
amps
3000
6000

Fig. 12


0-1166
500 KW Syn Converter
Short Circuit Test
Using Carbon Breaker
SO 6-6307
7-16-19
Milwaukee Brush Holder
25 cycle timing wave
Resis of Armature circuit .216 ohms
Grounded
Terminal voltage
volts
600
3000
2000
1000
amps
Armature current

Fig. 13

changes in speed of the induction motor with a resulting sudden change in the frequency applied to the converter which causes flashing in a manner referred to above. A change was made in the commutating-pole circuit to strengthen the commutating-pole field in an effort to eliminate the flashing trouble. This change was to place, approximately, three-quarters of an inch of brass liners at the back of the commutating pole. New commutating-pole windings to supply the additional ampere-turns required as well as new commutating poles to allow space for the brass liners, were also required. These brass liners being of conducting material, also have a tendency to damp out the pulsations of the commutating-pole flux caused by the pulsating armature reaction. This change practically eliminated the flashing trouble on this machine and showed a field for development along this line.

Fig. 14

Quick-Acting Circuit Breaker

Tests made in connection with a modified high-speed railway-type breaker have been previously referred to see Figs. 2 and 3. This breaker has sufficient speed to protect the machine in most cases but is not suitable for this size machine on account of its limited continuous current carrying capacity. Its high speed is due to some extent to its over-rated application in this case.

A 2000-ampere high-speed breaker now being developed and which is still in the experimental stage was also tried out with some success. However, even with this breaker complete protection could not be obtained. Illustrations in Figs. 14 and 15 show front and rear views respectively of this breaker. On dead short circuit this breaker has a speed of, approximately, 0.014 second for complete opening. The arc tips begin to open in approximately 0.004 second. Dead short circuits on the machine when protected by this breaker are shown in Figs. 16 and 17. In these cases the rate of initial increase in current is limited to approximately 2,100,000 amperes per second, about 65 per cent of the normal rate of increase. This reduction in rate of current increase is such that the speed of the breaker is sufficient actually to limit the maximum value of the short-circuited current. This lower rate of increase in the direct current is due to a 0.15-millihenry reactor which was put in the circuit. The primary purpose of the reactor is to shunt the trip coil of the breaker so as to give a greater rush of current through the trip coil and thereby increase the speed of operation of the breaker. Although this reactor was effective in increasing the speed of the breaker as well as limiting the rate of increase in the direct current, a very undesirable and vicious voltage was induced in the coil upon the opening of the d-c. breaker, which in some cases appeared to re-establish the arc across the breaker.

This breaker was about as much underrated as the modified railway breaker was overrated on this application and it is believed that it would have a slightly higher speed on higher currents than dealt with in this case. A smaller breaker with lighter moving parts is now being considered on which it is expected to obtain higher operating speed.

Some very interesting high-speed photographs of the machine when flashing and when protected by this breaker are reproduced in Figs. 18 and 19. The camera used in taking these photographs has twenty-two stationary lenses with a rapidly revolving shutter in the rear which opens them in rapid succession. The time interval between each exposure is of course dependent upon the speed of this shutter which is variable. The progression of the phenomena is in the order of the numerals on the plate. The pictures marked with the sub letter *S* were taken at the same instant (by two different lenses) as the picture marked with the corresponding number without the sub letter *S*. When properly mounted these pairs of pictures

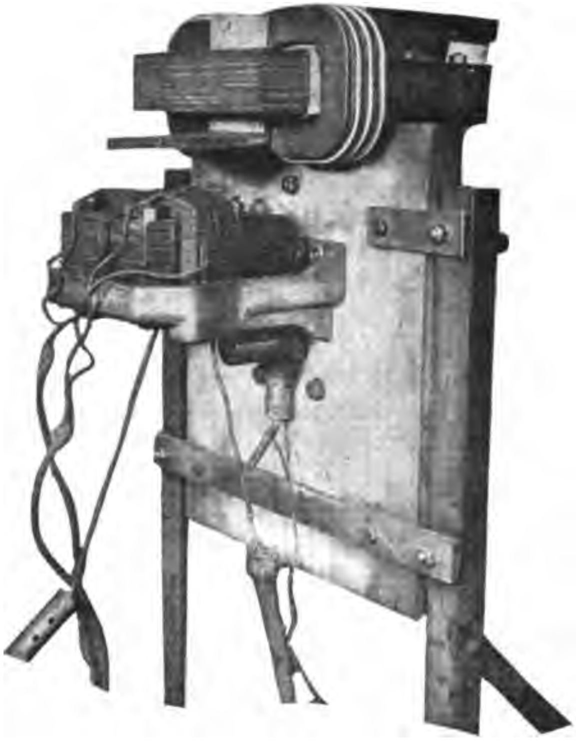


Fig. 15

can be viewed to a great advantage through a steroscope which brings out the third dimension or depth of the picture.[1]

Fig. 18 which corresponds to oscillogram in Fig. 16 shows a picture of both the machine and breaker when flashing. There is approximately 0.0018 second interval between each picture. Pictures 1 and 17 which show the beginning and the end of the phenomena shown on this plate overlap and are shown together. The small light spot in the lower left hand corner of picture 1 shows the beginning of the arcing. The breaker is not yet visible. In picture 2 the arcing on the machine is increasing the several spots of light representing sparking under the several brush arms, and the beginning of the breaker opening is shown in the upper right-hand side of the picture. In picture 3 the flashing is still progressing, and so on to picture 7 where the breaker has completely opened and the machine flashed over. The machine continues to flash throughout the length of this series of pictures. Fig. 19 which corresponds to oscillogram Fig. 17 shows a closer view of the flashing on the machine alone.

FLASH SUPPRESSOR

The flash suppressor as applied to converters is simply a high-speed switch actuated by the short-circuit current, much in the same manner as a circuit breaker, which short-circuits either all or a part of the collector rings (usually three or six in the case of a six-phase machine.) This reduces or kills the voltage on the commutator to such an extent that it prevents the machine from flashing over. A detailed explanation of the flash suppressor is given in an article on this subject by Mr. F. T. Hague and Mr. N. W. Storer in the May 1918 issue of the *Electric Journal*; and its particular application to the generators of the Chicago, Milwaukee and St. Paul motor-generator sets is given in an article by Mr. Daivd Hall in the January 1920 issue of the *Electric Journal.*

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[1] A detailed explanation of the construction and operation of this camera is given in an article by Mr. J. W. Legg in the December 1919 issue of the *Electric Journal.*


8.29.19
SO 6-6303
HIGH SPEED CIRCUIT BREAKER
MAGNETIC BLOWOUT TYPE
WITH 500 KW 600V ROTARY
Zero line of terminal voltage
time
Zero line of trip coil current
Load setting Dead short
.15 mh reactor for tripping
New arc chute
Multi-Exposure High Speed Camera Plate No 12
Zero line of current in breaker
1500
1000
500
amps
600
volts
-10000
amps
.0167 Sec.
0-1250


8.29.19
SO 6-6303
HIGH SPEED CIRCUIT BREAKER
MAGNETIC BLOWOUT TYPE
WITH 500 KW 600V ROTARY
Zero line of terminal voltage
time
Zero line of trip coil current
Load setting Dead short
.15 mh reactor for tripping
New arc chute
Multi-Exposure High Speed Camera Plate No 14
Zero line of current in breaker
1500
1000
500
amps
600
volts
-10000
amps
.0167 Sec.
0-1260

The operation of the flash suppressor means practically a short circuit on the a-c. system through the converter transformers. The alternating current drawn on a dead d-c. short circuit where the high reactance transformers are used for compounding purposes, was only 15 to 20 per cent higher with the suppressor in operation than without. Thus on dead d-c. short circuit the suppressor did not add so much to the duty of the machine and a-c. system. However, each overload on the d-c. side above a predetermined value for the suppressor to operate meant a short circuit on the a-c. side. Another undesirable feature was the fact that the machine fell out of step and had to be synchronized again. In an effort to limit the heavy alternating current drawn by use of the flash suppressor and so eliminate this objection, tests were made with inductance in the local suppressor circuit for the purpose of finding out the maximum value of inductance which could be used to limit the alternating current and still reduce the d-c. voltage sufficiently to give protection on the d-c. side. For each setting of inductance the d-c. short-circuit load was increased to the point of prohibitive flashing. It was found that in order to give protection the value of this inductance had to be such that upon the opening of the d-c. breaker the d-c. voltage would not rise above the value it had dropped to due to the d-c. short circuit, *i. e.* there must be no appreciable rise in voltage when the d-c. breaker opens. Also the tests showed that reasonable protection could be had at about eight times full d-c. load without drawing over nine times full-load alternating current, or at about 12 times full direct current without drawing over 10 to 12 times full-load alternating current.

With the idea of eliminating the undesirable feature of having to re-synchronize the machine after the operation of the flash suppressor, a three-pole oil circuit breaker was put in the local leads of the suppressor circuit in connection with the above tests, so as to open the suppressor circuit and throw the machine back on the line. The trip circuit for the oil breaker was energized immediately after the opening of the d-c. breaker by means of an auxiliary contact on this breaker. Figs. 20 and 21 show tests in which the flash suppressor circuit was opened and the machine thrown back on the line. It was found that the suppressor circuit could be opened and the machine thrown back on the line in this manner without any particular disturbance up to the point where the inductance in the suppressor circuit must be decreased (for protection to the d-c. side) sufficiently to cause the machine to fall out of step. This point is at approximately ten times full load. Some evidence of the tendency of the machine to fall out of step is shown by the slight oscillation in the d-c. voltage wave in Fig. 21. The oscillogram in Fig. 22 shows a short circuit of approximately 13 times load and the pulsations in the d-c. voltage wave show clearly that this machine is out of step. The frequency of these pulsations corresponds to the slip frequency on the commutator and shows the extent to which the machine is out of step. When the suppressor circuit is opened and the machine thrown back on the line, when it is out of step, it flashes over due to phase displacement on the a-c. side regardless of the value of the d-c. load.

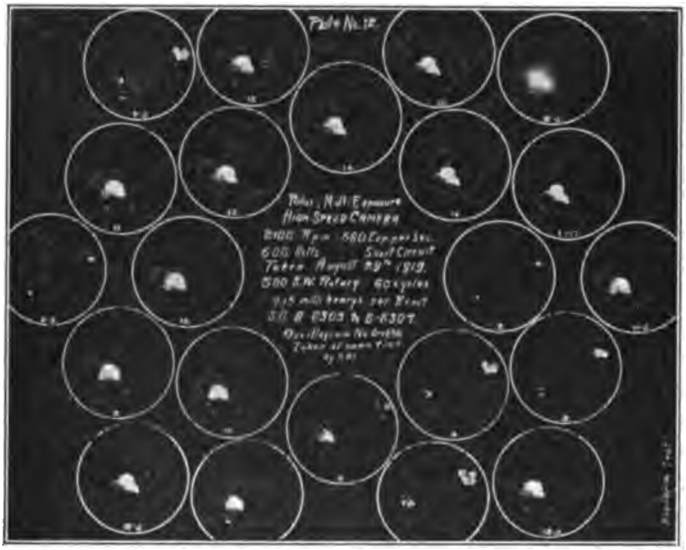


Fig. 18

All of the above tests show clearly that the flash suppressor will give ample protection from flashing to the converter as far as the a-c. side is concerned. However, there are several problems connected with the application of the flash suppressor to synchronous converters, such as, the increased duty on the collector ring, disturbance to the a-c. supply systems and the machine's falling out of step, which have not yet been worked out.

FIG. 19

FLASH GUARDS OR BARRIERS

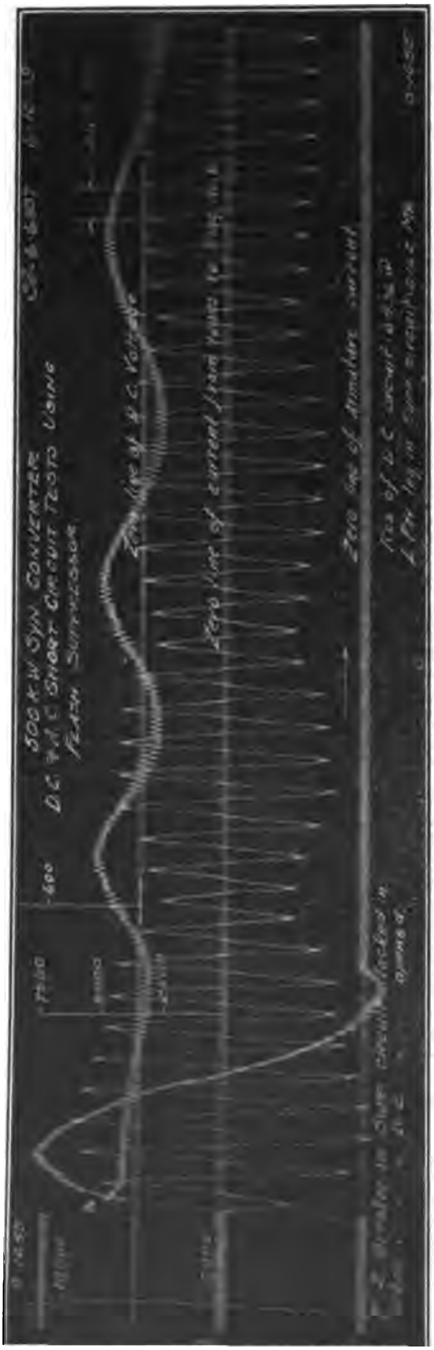
Various forms of flash guards have so far proved to be unsuccessful. The general construction of these guards consisted of two continuous end rings, one at the back of commutator next to commutator necks and one at the front of commutator, with barriers between arms extending down to within approximately 1/32 in. of the commutator. These guards were made of asbestos lumber, a composition of asbestos and asphalt. Considerable trouble was experienced at first from the arc going under the front guard ring to the commutator *V* ring. This was remedied by making this front ring counterbored so that it extended over the face of the


500 KW Syn Converter
D.C. & A.C. Short Circuit Tests Using
Flash Suppressor
50-6-8307 6-10-19
Zero line of D.C Voltage
.0167 Sec.
Zero line of current from trans. to Ring No 6
Res of D.C circuit .185 Ω
1 Per leg in Supp circuit .0062 Ω
Zero line of Armature current
F.E. Oil Breaker in Supp. circuit opened
Carbon " " D.C. "
3000
4000
2000
Volts
Amps
0-1052
0-1053

Fig. 20


500 KW Syn Converter
D.C & A.C Short Circuit Tests Using
Flash Suppressor
50-6-8307
.0167 Sec.
Zero line of D.C Voltage
Zero line of current from trans. to Ring No 6
Res of D.C circuit .07028 Ω
1 Per leg in Supp circuit .0036 Ω
Flash guards removed
A.C. Short lags .012 Sec.
Zero line of Armature current
F.E. Oil Breaker in Supp circuit opened
Carbon Breaker in D.C circuit opened
800
5000
2500
4000
Volts
Amps
0-1050

Fig. 21


500 K.W. Syn Converter
D.C. Short Circuit Tests Using
Flash Suppressor
Zero line of D.C. Voltage
Zero line of current
Zero line of Armature current
600
7500
C.S. 6307


500 K.W. Syn Converter
Short Circuit Test
Using Cannon Breaker
SO 6307
7-16-19
0-1173
600
Terminal voltage
Milwaukee Brush-holder
Electric timing wave


500 KW Syn Converter
Short circuit test
Using Carbon Breaker
SO 6-6307
7-16-19
Milwaukee Brush-holder
25 cycle timing wave
Resis of Armature circuit .1494 ohms
Grounded
Terminal voltage
volts
600
Armature current
4000
2000
amps
0-1172
0-1172

Fig. 24


500 KW Syn Converter
Short circuit test
Using Carbon Breaker
SO 6-6307
7-17-19
Milwaukee Brush-holder
25 cycle timing wave
Dead short Resis of Armature circuit .08127 ohms
No Ground
Armature current
Terminal voltage
volts
600
20000
16000
amps
0-1191
0-1191

Fig. 25


7-23-10
SO 6-6307
Dead short
Grounded
500 KW. Syn Converter
Short circuit test
Using Carbon Breaker
25 cycle timing wave
Terminal voltage
Armature current
20000
10000
600
0-1198

FIG. 26

commutator a small distance as well as below the front end of the commutator. However, even this did not prevent the machine from flashing; with the arcing space confined by these guards the arc seems to become explosive and force itself to parts that it otherwise has no tendency to go. Furthermore, after several flashes the surfaces of these guards become so carbonized and covered with metal particles that they actually aggravate the flashing rather than prevent it. In fact, in several instances the guards were so carbonized and metal-smeared after the flash-over, that streamers would run along the surface at no load and normal voltage. All tests indicated that these barriers should be of an open construction and as far away from the source of the arc as possible so as to give it sufficient room to expand and dissipate itself without becoming explosive. In other words the barriers should be next to the parts to be protected (as the pedestal and leading side of brush holders) and as far away from the arc as possible. This fact is borne out by oscillogram in Fig. 23. In oscillogram shown by Fig. 24 the machine flashed over to the front pedestal (no flash guards on machine). In an effort to keep the arc from jumping to the pedestal, the front ring, which had been slightly carbonized on a previous flash-over was then bolted back on front of the brush-holder brackets and the short circuit repeated as shown by Fig. 23. This time the arc jumped along the surface of the front ring between the two front brush holders on the two lower arms and practically demolished these two brush holders. Later this ring was fastened to the pedestal, away from the commutator and the source of the arc, and gave complete protection clear down to dead short circuit except for slight pitting of the brush holders.

This shows very clearly that the flash guards increase the tendency of the machine to flash over after they have become carbonized. No suitable material for this purpose which will not carbonize has been found so far. Best results have been obtained by protecting the pedestal with an arc shield and leaving as much space as possible on the commutator for the arc to expand. This also gives free access to the commutator, the lack of which is one great disadvantage and objection to any form of flash guard. This same protection can be offered more readily and easily by insulating both pedestals much in the same manner as for the elimination of bearing currents. This, however, has the disadvantage that a dangerous voltage may exist between the pedestals and ground in case the armature becomes grounded. With the pedestals insulated this machine can be dead short-circuited with the ordinary carbon breaker with no particular damage to the machine except a slight pitting and marking of the brush holders. Figs. 25 and 26 show dead short circuits with the machine grounded and ungrounded under the above condition. Evidence is again shown of the machine falling out of step as discussed under the heading of flash suppressors. The fact that the d-c. voltage is pulsating undoubtedly prevents the arc from hanging on. It goes out when the d-c. wave passes through zero. For this reason a dead short circuit on a converter in connection with a slow-speed breaker does not appear to be as vicious or tenacious as a short circuit at a point just before the machine falls out of step and the d-c. voltage is maintained in one direction. This point is about 10 to 12 times load in the case of this machine. This is also about the point of maximum *E I* output.

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AUTOMATIC RAILWAY SUBSTATIONS

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This paper reviews the broad range of conditions to which railway automatic substations have been applied and also discusses the economies and operating advantages effected by their use. A description is given of the modern equipment with details of its operation. Special reference is made to improvements in design of control apparatus, to the positive sequence of starting the machines and the protection afforded the apparatus against overloads or other irregularities either outside or internal to the stations.

PRESENT day railway conditions are such that any improvement leading to the reduction of invested capital or decreased operating charges is of vital importance. Automatic substations as shown by several years' operation have resulted in decided economies and improved operating conditions, and as the subject has not been presented before the A. I. E. E. since an account[1] of the original installation was prepared a review of the modern automatic equipment seems desirable.

The first automatically controlled railway substation was placed in service during December 1914, on the Elgin & Belvedere Electric Railway. The station equipment prior to being made automatic consisted of a single 300-kw., 600-volt, 25-cycle, three-phase synchronous converter with three single-phase 110-kv-a., 26,000-370-volt self-cooled transformers and standard manually-operated switchboard apparatus. The individual devices comprising the first equipments were with but few exceptions, those which had previously been developed by electrical manufacturers for use in other applications. This condition was a

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[1] Allen and Taylor Vol. XXXIV, Part 2, page 1801.

decided advantage to the operating companies since developmental charges were eliminated, rendering the equipments comparatively low in cost, which in addition to the successful operation of the early installations made possible the rapid growth of automatic application to electric railways.

Shortly after the initial installation on the Elgin & Belvedere Electric Railway, the two remaining substations on that road were made automatic, followed by the Potomac Electric Power Co. which made automatic a 500-kw. substation. About that time (1916) the Des Moines City Railway and Interurban Railway Co., two adjacent roads, outlined and have practically completed a program involving automatic substations, which has resulted in the most notable example of their use up to the present time. These roads now have in operation a total of three 300-kw. and nine 500-kw., 600-volt automatic synchronous converter equipments. The confidence inspired in the minds of operating engineers and the rapid adoption of this comparatively new phase of electric railway operation was in no small measure accelerated by the successful performance of this rather broad application.

The range of requirement to which the new scheme could be successfully applied was demonstrated when in 1917 two 300-kw., 600-volt, 25-cycle synchronous converters operated in series on 1200 volts by the Milwaukee Railway & Light Co. were automatically equipped. Two automatic substations each containing one 600-kw. 1500-volt d-c. induction motor-generator set were also installed in 1919 by the Salt Lake, Garfield & Western Railway. The Rhode Island Co. have for some time been operating a station containing two 300-kw., 600-volt converters in parallel, while in other localities portable automatic substations have been functioning successfully. Railway converters now automatically equipped range in size from 200 to 1500 kw. with motor-generator sets from 300 to 2000 kw. The total capacity operating in this manner and including those in process of installation is estimated to be 45,500 kw. while the number of automatic equipments involved is approximately 79, 59 of which having been applied to 300- and 500-kw. sizes, and the remainder cover the range of various types and sizes of installations briefly referred to.

While it is the intention to confine the scope of this paper to railway activities, it is interesting to note that a hydroelectric station on the Iowa Railway & Light Company's system containing three 500-kv-a. synchronous generators has been operated automatically since 1917 while the Interstate Light & Power Co. in the same year placed in operation a 3000-kv-a. automatic synchronous condenser. The Union Electric Light & Power Co. is applying the scheme to a 2000-kw., 250-volt lighting synchronous converter. Railway operation, up to the present time, has presented apparently the most attractive field for the use of automatic features, but a well grounded start in other lines has been made and it is only reasonable to expect considerable activity in the lighting and hydroelectric branch of the industry.

Without question a direct reduction in operating expense has been the prime motive for the purchase by railway companies of automatic substations. The saving is effected in several ways, although it is a variable and depends on conditions under which a particular station is operated as well as the number and capacity of machines in the station.

Labor Saving. Considering the several items of saving more or less in their order of importance, the matter of eliminating operator's wages stands out most prominently. In those localities where three eight-hour shifts are in force in single-unit substations the labor saving alone from automatic operation will often wipe out the original cost of the equipment in two to three years. Where only one or two station attendants are employed, as generally is the case on the average interurban system, the net return is not so great, but usually is sufficient to represent a desirable return on the investment in extra equipment incident to automatic operation. When two or three machines are installed in a single station an equal number of complete automatic units are necessary, requiring two or three times the initial investment of a single-unit station. This fact makes it more difficult to justify automatic operation of such a station if only the saving of station attendant's wages is considered. A further study, however, of other savings and advantages inherent to automatic operation as applied to a specific case may show economy of sufficient magnitude to warrant two-unit automatic substations.

The procuring of competent labor constitutes one of the trying difficulties which operating companies have had to contend with during recent years. A natural result of such a condition as applied to manually operated substations is a poor handling of apparatus with a corresponding increase in maintenance costs and less efficient operation. The substitution of automatic stations for the manually operated types eliminates this difficulty in the proportion to the extent of substitution and also reduces the ill effects resulting from labor difficulties. While these features are assets, they are somewhat intangible and not easily capitalized.

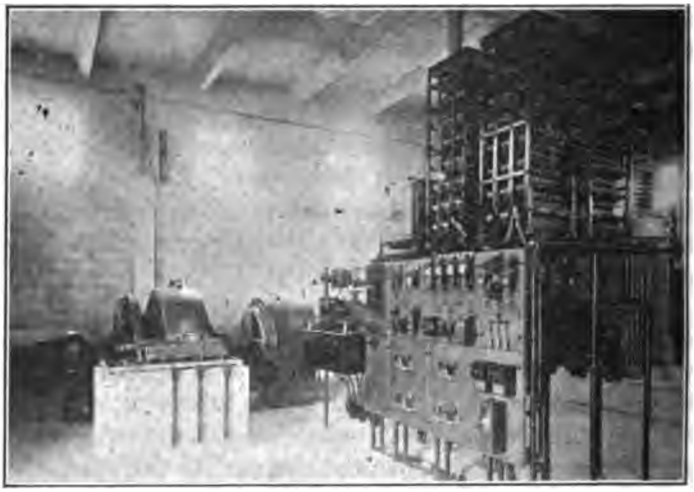

FIG. 2—INTERIOR VIEW OF AUTOMATIC SUBSTATION CONTAINING 600-KW. 1500-VOLT D-C. INDUCTION MOTOR-GENERATOR SET—SALT LAKE, GARFIELD AND WESTERN RY.

A complete elimination of all labor charges against these stations is not feasible since a regular system of inspection must be maintained. Some companies include this work as a portion of a patrolman's or other workman's duties, while other companies which operate several of these stations employ a man who devotes his entire time to this phase of the work. Those stations having high-tension aluminum-cell lightning arresters require daily visits by an inspector to charge the arresters, at which time a casual inspection of the automatic equipment may be performed so as to preclude any minor irregularities becoming serious. Where a reliable high-tension arrester is used which does not require charging such as the oxide film type, two or three hasty inspections per week have been found ample. At intervals of approximately two weeks a thorough inspection should be made and a detailed report prepared to indicate the condition of every device. The amount charged against each station for this service, including maintenance as reported by several companies, varies considerably but will average approximately $300.00 per year.

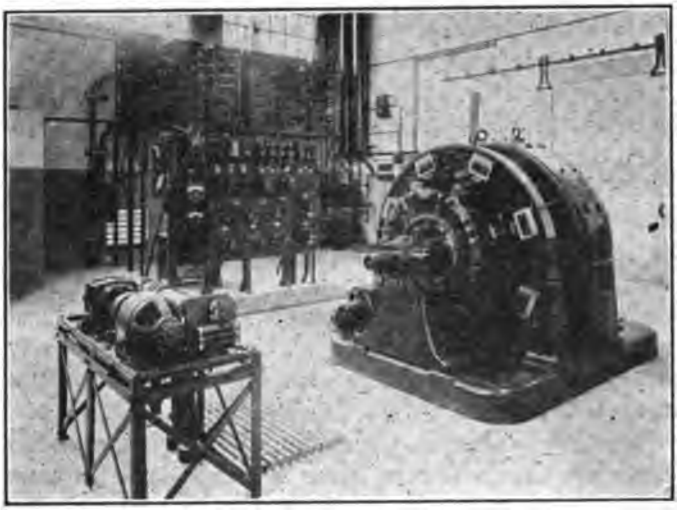

FIG. 3—INTERIOR VIEW OF 1000-KW. 600-VOLT SYNCHRONOUS CONVERTER AUTOMATIC SUBSTATION—PACIFIC ELECTRIC RAILWAY

Light Load Savings. Saving in light load losses, due to automatically shutting down a station when little or no power demand exists, is most noticeable on interurban roads maintaining an infrequent headway. Often a station in manual operation will run idle from a quarter to half the total operating time particularly so if the schedule results in train passing points in the immediate vicinity of the substations. In the event passing points are between stations the machine shutdown period will be somewhat reduced: The total energy saving from this source as applied to single-unit interurban stations depends upon the track layout, characteristics of the schedule and energy required to idle the machine which for a 500-kw. converter equipment is approximately 16 kw.

Energy saving in interurban stations having two machines automatically operated may work out to be very attractive especially if the second machine is used to help out on rush service or move occasional heavy express or freight trains. With automatic operation the second machine would cut in only when a demand for additional power existed and would drop out when not required. Prompt action in this respect is a decided factor in economy which while not entirely overlooked in the average manually operated substation, is a possible saving considerably neglected.

Two Unit Stations. Combining the savings accruing from the elimination of labor and light-load losses in two-unit parallel-operated automatic stations, together with other advantages, the arrangement is justified as evidenced by the operation of the two-unit station on the Rhode Island Company's system as well as a station now being installed by the Dayton & Troy Electric Railway Co. containing two 300-kw., 600-volt converters and a station on the Cleveland, Railway Co.'s system containing two 1500-kw., 600-volt converters.

Coal Saving. The elimination of coal or perhaps considerable energy used by electric radiators for heating the station or operator's booth is another direct saving attributed to automatic equipments. In the more northern localities this item is sufficient to warrant consideration especially in view of the scarcity and price of coal prevailing in recent years.

Building Design. Building design best suited to accommodate automatic substations as shown in Fig. 4 is often materially different than that common to manually operated stations. Simplicity, with resulting lowered initial cost, seems to predominate. The structure is arranged to house all apparatus indoors, which with locating the windows out of reach affords a protection from vandalism. Ample ventilation is provided by louvers located in the side walls near the floor level and ventilators in the roof.

Fig. 4—Exterior View of Polk Boulevard Automatic Substation Containing 500-Kw. 600-volt Synchronous Converter—Des Moines City Railway

Load Factor Improvement. It is a well-known fact that the load factor on the average interurban substation is exceptionally low. Otherwise expressed, this means an unusually large investment in electrical apparatus compared to the average all day power delivered from the station. This condition arises because sufficient capacity must be installed to accommodate not only the regular service, but also to take care of excess demands usually of short duration such as results from accidental bunching of trains, snow plows or occasional heavy freight trains. These irregularities because of their rather brief demands on any one station do not, by heating limitations necessitate greatly increased substation capacity, but they do severely tax the station apparatus from the point of commutation and short period capacity. Since in the past there has been no alternative but to provide large capacity machines to protect against these contingencies, the condition of low load factor resulted in high investment charges for the average energy delivered.

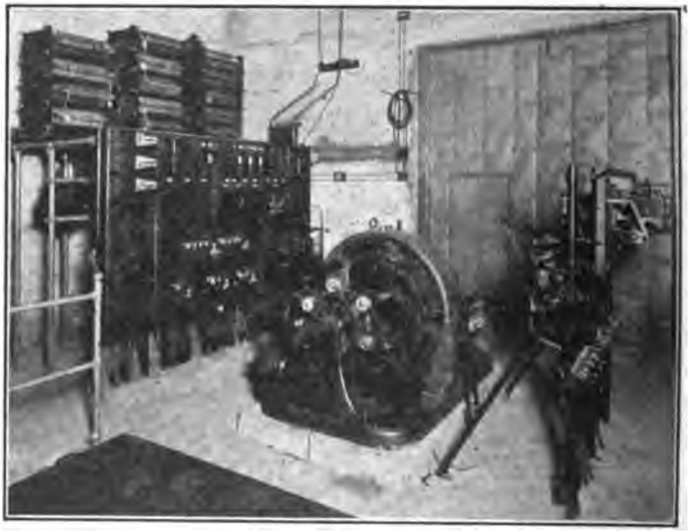

Fig. 5—Interior View of 200-Kw. 600-Volt Synchronous Converter Substation on Cincinnati Lawrensburg and Aurora Electric Street Railway

The automatic substation functions in a manner partially to remedy the situation so that the average load approaches the maximum load demand on the station, which condition permits increased traffic being served by a given substation or conversely permits a smaller substation capacity to serve a given traffic. This condition is brought about by inserting a high-capacity resistance in the direct-current side of the machine at times of severe overload. The direct purpose of the resistance is to limit the current delivered to a reasonable value, which prevents the abuse of apparatus arising from any load demand. The effect of this device is to cut off the high current peaks so frequent to substations serving rush service, locomotives, heavy freight trains or other requirements where the momentary demands tax the commutation capacity of the machines. With the peak loads limited to a safe value, it then becomes possible to permit the average load to approach somewhat nearly the continuous capacity of the machine which condition results in load factor betterment. Caution must be exercised, however, not to exceed the limit in this direction since the resistors may be in circuit sufficiently to interfere too greatly with the average trolley voltage, thereby decreasing train speeds as well as incurring an undesirable energy loss from heating in the resistors. Based on this principle as well as the fact that the average heating of transformers is low in the smaller interurban automatic substations due to frequent light-load shut-downs, it has become generally standard practise in these stations to provide transformers having a capacity equal to 80 per cent of the converter rating. In justification of the idea it may be noted that there are in operation at

FIG. 6—PORTABLE AUTOMATIC SUBSTATION CONTAINING 500-KW. 600-VOLT SYNCHRONOUS CONVERTER—KANSAS CITY RAILWAYS CO.

least two railway systems where full advantage of the principle is realized with exceedingly satisfactorily results.

Feeder Saving. Reduction in feeder copper is effected by the application of automatic substations based on the fact that elimination of labor cost in such stations permits a closer spacing and smaller capacity than would be the case with the manually operated type. For the same line characteristics then less feeder copper is required. The installation of automatic stations has in several instances resulted in the removal of sufficient copper to offset materially the cost of the station. Frequently, conditions arise on existing roads when it is necessary to bolster up some particular section of the system due to rapid growth in service requirements. An automatic station is often the cheapest means of producing the desired result since it is more economical to operate than a manual equipment.

Electrolysis. The effects of electrolysis may be to an extent eliminated by the use of automatic substations since their economic spacing is less than those manually operated. Such an arrangement shortens the negative return with a consequent reduction in potential difference between the rail and adjoining water mains. The National Bureau of Standards recognize this benefit in connection with the principle of automatic substation installations.

As previously stated the first automatic installation consisted of devices which with but few exceptions had already been developed by manufacturers for application to other service such as power station, car equipment, steel mill and similar industrial requirements. Experience soon taught, however, that successful automatic substations required an ultra-reliable class of apparatus since the failure of a single device means a shut-down of considerable duration because it necessitates sending a man from some distant point to investigate the trouble. This fact has been realized by the manufacturers and based on experience during recent years a class of apparatus has been developed for these stations which embodies a degree of reliability consistent with the exacting demands of present-day railway service.

Details of Operation. The functions of starting and connecting the machines to the line upon power demand and finally shutting them down after the demand for power has ceased are all carried on in their proper sequence without any assistance whatsoever from an operator. In present day practise the great majority of these stations are controlled entirely by the automatic equipment in accordance with the above statement, but a few are, for specific reasons, remotely controlled by dispatchers with the aid of a pilot wire. A remote controlled station as generally applied may be considered in the same class with a purely automatic station since the control current merely replaces the automatic devices which determine when the station is to start or stop. The remaining apparatus which performs the actual switching operations are identical in both cases.

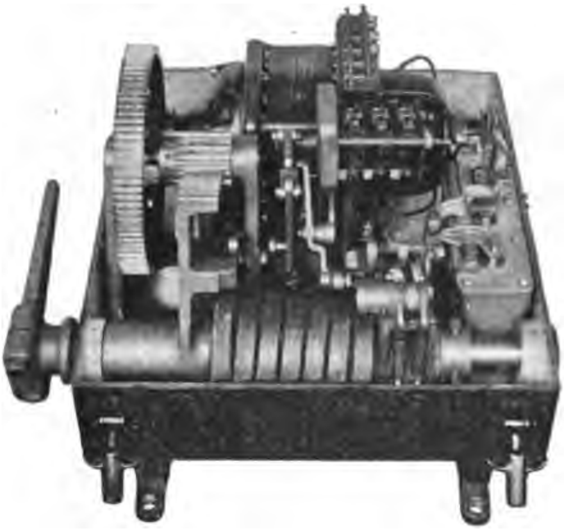

FIG. 7—MOTOR-OPERATED OIL CIRCUIT BREAKER MECHANISM FOR AUTOMATIC SUBSTATIONS

The type of automatic equipments in the most extensive use consists of a group of relays, grid resistors and standard contactors, which together with a motor driven drum controller shown in Fig. 8 perform the usual function of starting, stopping and protecting the machines against irregularities without the aid of an attendant. In general, relays are used where the functions of starting, stopping and protecting the machines depend upon voltage, current or independent time values. During starting and stopping, however, numerous operations must be performed in a definite sequence, which if not strictly adhered to, is conducive to service interruptions. The motor-driven drum controller is used to obtain this fixed time relation of events and to substitute, wherever possible, a type of contact more substantial than can be used with relays. This device also includes a small d-c. generator which at the proper time during the starting operation separately excites the converter field, thereby definitely and immediately insuring the correct polarity.

Fig. 8—Motor-Operated Controller and Exciter for Automatic Substations

Protective devices having the following duties are provided to perform the functions ordinarily left to the discretion of the operator.

1. To limit the overloads.
2. To limit the temperatures.
3. To shut down the machine.
    (a) When a-c. or continuous d-c. short circuits occur.
    (b) Upon failure of alternating current.
    (c) Upon failure of any device.
    (d) In case of excessive speed.
    (e) Upon reversal of direct current.

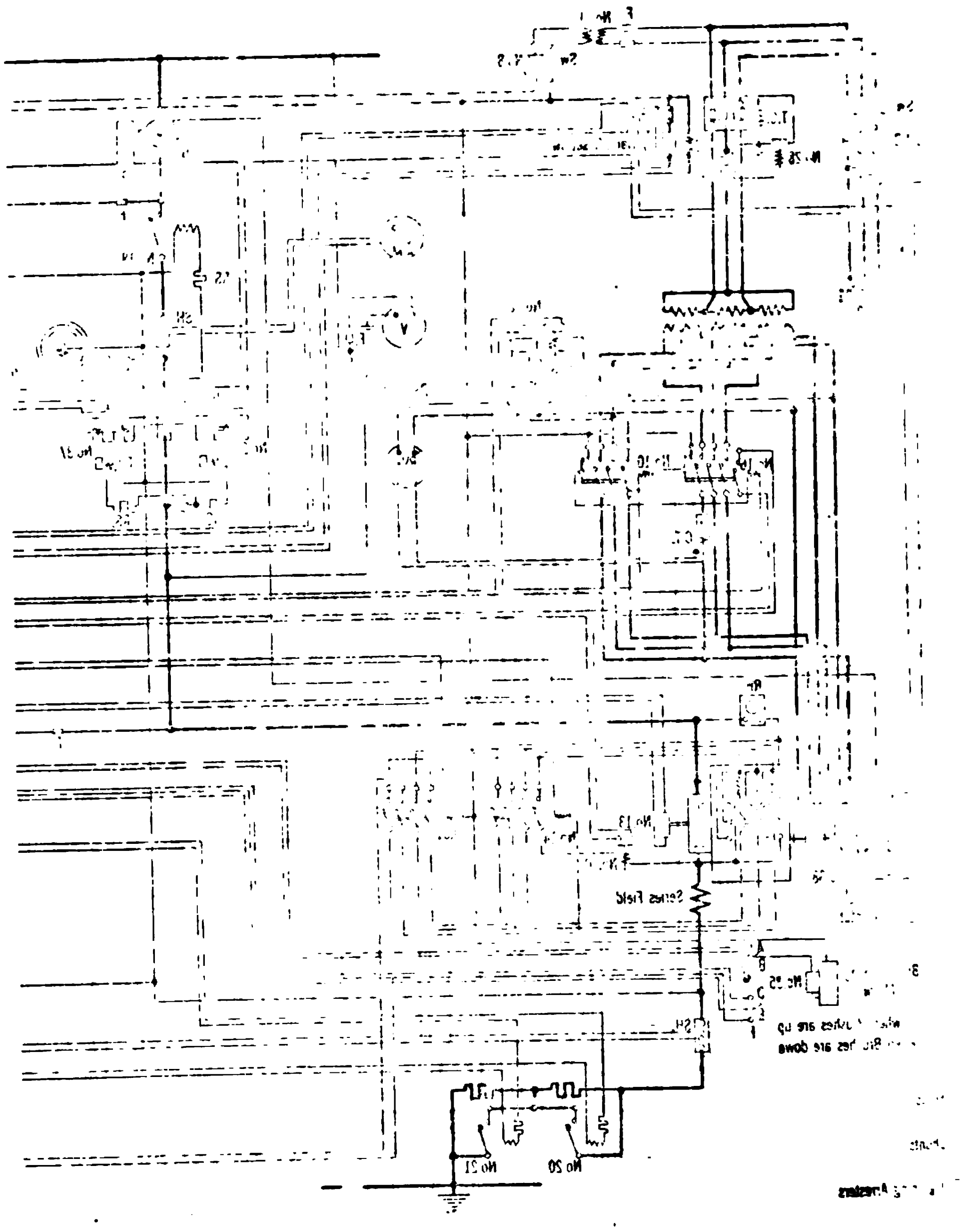

FIG. 1—TYPICAL WIRING DIAGRAM FOR

(1) Contact-making voltmeter. (2) Time delay starting relay. (3) Time delay shutting [illegible] relay. (4) Contactor for making control circuit connections. (5) Contactor for making circuit connections. (6) [illegible] for making control circuit connections. (7) Three-[illegible] circuit breaker. (8) 250-volt a-c. control switch. (10) [illegible] contactor. [illegible] control transformer, 220 volts. (12) [illegible] switch [illegible] (13) [illegible] control [illegible] saved). (14) Converter field contactor shunt excitation. (15) A-c. running

4. To prevent machine starting.
   (a) During low a-c. voltage.
   (b) During single-phase a-c. supply.

By referring to Fig. 1 which is a typical wiring diagram of an automatic 500-kw., 600-volt equipment, the sequence of operation may be followed. For convenience of reference the principal devices have been numbered or otherwise labeled. It will be noted that the 220-volt a-c. control bus is continuously excited from the control transformer No. 11 and the operating coil of contact-making voltmeter No. 1 is always connected between trolley and ground.

Assuming a particular station is shut down and a train is approaching. As it increases its distance from the next station on the line it will eventually cause the trolley voltage to drop and at a predetermined value, usually 450 volts, contact-making voltmeter No. 1 opens, de-energizing the operating coil of relay No. 2, which had been previously held open by excitation from the 220-volt a-c. control bus through relay No. 1. The closing of No. 2 closes relay No. 3 causing it to pick up and close contactor No. 4 provided the hand reset switch and contacts of a-c. low-voltage relays No. 27 are closed. Relays No. 2 has a dashpot to prevent momentary fluctuation of low voltage from producing false operations of the machine. With the drum controller No. 34 in the "off position" as would be the case before the machine starts, contactor No. 4 completes a circuit through segments No. 13 and No. 16 on the drum controller and the limit switch of the brush-raising device which closes contactor No. 6, thereby starting rotation of the motor-driven drum controller. Controller segment No. 15 soon closes contactor No. 5 which in turn energizes the motor-operated oil switch mechanism causing the main converter transformers to become energized by the closing of oil circuit breaker No. 7. The operating coil connection of contactor No. 5 is then transferred from segment No. 15 to No. 14. This circuit passes through an auxiliary switch on the oil circuit breaker to insure the return of all devices to their normal position should the breaker open for any reason. When segment No. 2 makes contact, starting contactor No. 10 is closed connecting the converter to the low-voltage taps provided the a-c. supply is delivering three-phase current as determined by relay No. 32. Shortly the drum controller stops rotating because of the gap in segment No. 16 and waits if necessary for the converter to come up to speed. At approximately synchronism, speed-control switch No. 13 closes, bridging by aid of segment No. 20 the gap in segment No. 16, causing the controller again to start rotating so as to complete the function of connecting the machine to the line.

Next segment No. 3 closes contactor No. 31 connecting to the converter fields a 250-volt supply obtained from the small generator on the drum controller, thereby immediately ensuring proper polarity. Contactor No. 31 is then opened by segment No. 3 and the self-exciting field contactor No. 14 closed by segment No. 4 and running contactor No. 16 closed by segment No. 5 connecting the converter to normal secondary a-c. voltage. Starting and running contactors No. 10 and No. 16 are both mechanically and electrically interlocked with respect to one another to insure against accidentally short-circuiting a portion of the transformer secondary winding. Segment No. 26 next starts the motor-operated brush rigging causing the converter brushes to be lowered which completes the operation of preparing the machine for connection to the d-c. bus. Segment No. 7 is next energized with 600 volts direct current and shortly thereafter segment No. 8 closes the d-c. line contactor No. 18 whose control circuit is in series with converter field relay No. 30, polarized relay No. 36 and auxiliary switches on running contactor No. 16 and control contactor No. 4, thereby ensuring before closing No. 18 that the converter has proper polarity, correct field and full voltage a-c. running connections.

As soon as the line contactor closes the machine delivers load to the bus through the load limiting resistors which, however, are soon short-circuited by contactors No. 20 and No. 21 operated by segments No. 9 and No. 10. The drum controller is then stopped by segment No. 17. When connection to the bus is made through No. 18 the flow of current closes relay No. 37, which will cause relay No. 3 to remain closed regardless of relay No. 1 whose function started the station. In other words the control of the station is now dependent on the contacts of No. 37 which will remain closed so long as a predetermined current is being delivered to the bus. Should the current fall below a set value, relay No. 37 will open and cause relay No. 3 to drop out after a certain period of time and shut down the station. Relay No. 3 has a dashpot and is timed so that momentary low values of current causing No. 37 to open will not shut down the equipment.

When the station does shut down relay No. 3 opens contactor No. 4 causing running contactor No. 16 and d-c. line contactor No. 18 to drop out and disconnect the machine. Contactor No. 5 opens after contactor No. 4 which operation establishes through an auxiliary contact a circuit to contactor No. 6, thereby starting the controller and running it to its "off position." While doing this, however, segment No. 24 trips out the oil circuit breaker and segment No. 25 causes the converter brushes to be raised in preparation for starting upon the next load demand.

The preceding covers briefly the necessary operation in starting and stopping the machines, but thereremain the equally important functions of protecting the equipment from irregularities caused by disturbances on either the a-c. or d-c. side of the station or within the apparatus itself. Briefly these contingencies are taken care of as follows:

In the event a heavy d-c. overload occurs, relay No. 24 will pick up and open contactor No. 20, thereby inserting resistance in the circuit. Should the overload increase to a greater value, relay No. 25 will operate and insert more resistance, and in stations not provided with individual feeder protection a third step of resistance is provided to limit still greater overload demands. The value of resistance used is such as to permit short circuit in the immediate vicinity of the station without injuring the machine. The resistor capacity, which determines the length of time heavy overloads can be carried by the resistors without serious heating, is to a degree a function of the service requirements but the duration of those irregularities cannot be foretold with accuracy and the practise of providing liberal capacity in the resistors has not only proved desirable but very necessary. In some stations individual feeder protection which consists of an overload relay No. 23, a contactor No. 19 and a resistor in each feeder circuit is installed, thereby localizing to a degree the function of overload protection to each feeder. With such an arrangement only two sections of resistance are used in the machine circuit.

Protection from overheating the machine, its bearings and load limiting resistors is obtained by use of temperature relays No. 38 and No. 33 arranged to shut down the station immediately should such a condition arise.

A reversal of operation as a d.c. motor is prevented by relay No. 29 and over-speed by speed-limit switch No. 12-A. Both of these devices necessarily operate a control circuit which immediately opens contactor No. 4 and shuts down the station. A shunt-trip hand-operated d-c. circuit breaker No. 15 is in series with No. 18 and only used to protect against the possibility of the line contactor freezing closed. Should this condition occur the converter would motor from the d-c. end upon the a-c. end being disconnected and the excessive speed resulting would trip the circuit breaker No. 15 through the operation of speed switch No. 12.

In case a short circuit occurs on the a-c. side of the equipment, the definite time-limit overload relay No. 28 will trip out the main oil circuit breaker, shutting down the station and at the same time opening the hand reset switch which necessitates reclosing by hand before the station can be started again. This feature insures an inspector visiting the station to investigate the cause of the serious a-c. overload.

Low a-c. voltage relay No. 27 which is calibrated for a definite value is connected so as not to permit the station to start or to shut it down if running should the high-tension voltage become so low as to interfere with proper operation.

If for any reason a single-phase condition exists on the secondary side of the transformer during starting operations, relay No. 32 will lock out starting contactor No. 10 and prevent the converter from being connected to the transformer.

Polarized relay No. 36 protects against the possibility of the machine ever being connected to the line in the reverse direction. Unless proper polarity has been established before connecting the machine to the bus, line contactor No. 18 will not close.

In stations containing a motor-generator set instead of a synchronous converter, certain modifications to the equipment are necessary to accommodate the starting operations, but the scheme of operation with few exceptions is similar to the converter equipments. Oil-immersed starting and running contactors are used because of the higher transformer secondary voltage and a certain amount of overload protection is obtained by inserting one or two steps of resistance in the generator field circuit in addition to two steps of series resistance in the main d-c. circuit. This arrangement reduces initial cost since the field resistance and its contactors are of small capacity. An energy saving in resistor heat loss is also accomplished. The 250-volt generator on the drum controller becomes unnecessary in the case of a motor-generator automatic equipment.

The recent development of flash barriers for railway machines affords a device of much benefit to automatic substations and these devices are now included by one manufacturer as a regular part of the equipment. Where conditions are particularly severe and much trouble is experienced from flashing, barriers and a quick-acting circuit breaker will practically protect the machine.

The recent development in automatic equipments has largely consisted of perfecting and making more reliable the present type of station as well as arranging for and applying the principle in other applications such as lighting, mining and hydraulic generating stations. Among specific instances of improved design may be mentioned the elimination from the equipment of all disk-type interlocks and the substitution of substantial finger-type auxiliary contacts. Relay contact elements have been improved where necessary so as to provide a quick make and break action and on d-c. circuits blow-out coils have been added. An improved type of motor-operating mechanism as shown in Fig. 7 for oil circuit breakers is now in use in which compactness and reliability are the outstanding features. D-c. contactors having a rupturing capacity sufficient to handle any load conditions have been developed and are performing their function with complete success. Reverse current and under-load relays of substantial construction and capable of accurate calibration at low current values have been designed to accommodate the conditions of automatic substation operation.

The use of automatic control in railway substation has in a comparatively short time expanded to where it is firmly established in city and interurban railway operation. The successful experience of the past has resulted in larger capacity stations serving heavy traffic being made automatic. The adaptation of this type of control to electric trunk line service at 2400 or 3000 volts direct current as well as more extended use in the strictly industrial field is not far beyond the horizon.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

AUTOMATIC SUBSTATIONS FOR HEAVY CITY SERVICE

BY R. J. WENSLEY
Westinghouse Electric & Manufacturing Co.

WHEN automatically controlled substation equipment was in the development stage, the general consensus of opinion was that its greatest field lay in relieving the smaller interurban systems of a greater portion of their substation operating labor. While many such systems have installed one or more automatic substations, they have not displayed the interest in the matter that was expected. In many cases the officials have professed their total inability to finance the new apparatus even though the investment would show a very attractive saving.

On the other hand, it is very gratifying to those directly interested in the subject, that the larger city street railway systems are eagerly taking up the question of rebuilding their distribution systems by taking full advantage of the decided savings made possible by automatic operation of the converting equipment. Orders received from this source have more than made up for the failure of the smaller roads to equal expectations.

The earlier street railway systems made little or no attempt to lay out a distribution system with economy or efficiency. Any location available was seized on as a power house site. Cables were run the shortest possible distance to the trolley and from that point the trolley was usually the only means of carrying the current. As service demands increased due to larger cars and closer schedules, the voltage drop became too great for satisfactory service even by the rather lax standards of those days. To remedy this the trolley voltage was gradually increased from the early standard of 500 volts to 550 and then to 600 volts and the feeders were run parallel to the trolley so as to help the voltage at the ends of the lines.

As the traffic demands increased another factor entered into the distribution problem, that of electrolytic destruction of underground metallic structures such as water pipes, gas mains and so forth. This necessitated heavy copper feeders in parallel with the rail to prevent the returning current from seeking to travel over the underground piping system. The negative booster was also freely used to compensate for the drop in these return feeders. This of course was practically pure waste so far as efficient utilization of the power was concerned since the entire output of the boosters was expended in heating the cables. It was however a good preventive of electrolytic troubles and therefore widely used in spite of its wastefulness.

All this time the source of power supply was almost invariably a slow-speed d-c. machine operating at 550 to 600 volts, and direct-connected to a reciprocating engine. These units grew to unwieldy size in the attempt to keep up with the ever increasing demands of the public for more transportation facilities. The engine driven alternator and then the turbo alternator came to the front as the most economical method of power generation and the railway substation followed soon after. There were of course, railway substations in operation long before the turbine became popular but these were exceptions to the rule.

Some of the larger cities soon installed a distribution system composed of one or more central steam plants with a number of large substations located in various parts of the city. Owing to the cost of operating labor these were kept down to the least possible number even at the expense of considerable investment in feeder copper. There are many of the old systems still in operation with an unwieldy amount of feeder copper in the air and with I^2R losses amounting into the millions of kilowatt hours per year. Poor trolley voltage with consequent slowness of schedule speeds is a natural result of this. The possible savings to be had from reduced copper losses, reduction in running time and decreased amount of motor repairs by rebuilding the distribution system would in many cases enable a property to make a better showing of earnings than it is making at the present time.

The development of automatically controlled substation equipment has now placed in the hands of the distribution engineer a possible method of reconstruction that is little short of ideal. Substations can now be scattered about without the specter of heavy and ever rising operating labor expense to be watched. The increase in the number of substations enables the holding of good trolley voltage without the heavy expense in large feeder capacity as has previously been the case. Trouble due to electrolysis is greatly minimized and in many cases entirely eliminated by the increase in number of substations.

These substations are of course not one hundred per cent efficient so care must be used in applying this remedy that it not be overdone. Sizes of machines and stations must be carefully calculated so that the load factor will be as high as possible. In deciding on the best sizes of equipment to install, several factors must be considered. Interchangeability is always desirable and therefore there should be as few different sizes of machines as consistent with economical operation. Wherever possible the machines in any one station should be of the same size so that their places in the operating schedules may be interchanged. In most city applications two machines will prove to be the most desirable number for an automatic station with space provided for a third machine to take care of future growth. In most cases this problem of added capacity can be better solved by the installation of additional substations when the load grows too great for the original installation of converting equipment. This latter method keeps down the feeder copper requirements.

Where two machines are selected for a given location each one must be capable of carrying the normal off peak load without assistance. The combined capacity must be sufficient to take care of the peak load. In some cases this will result in more capacity than necessary during the off peak period, and in other cases just

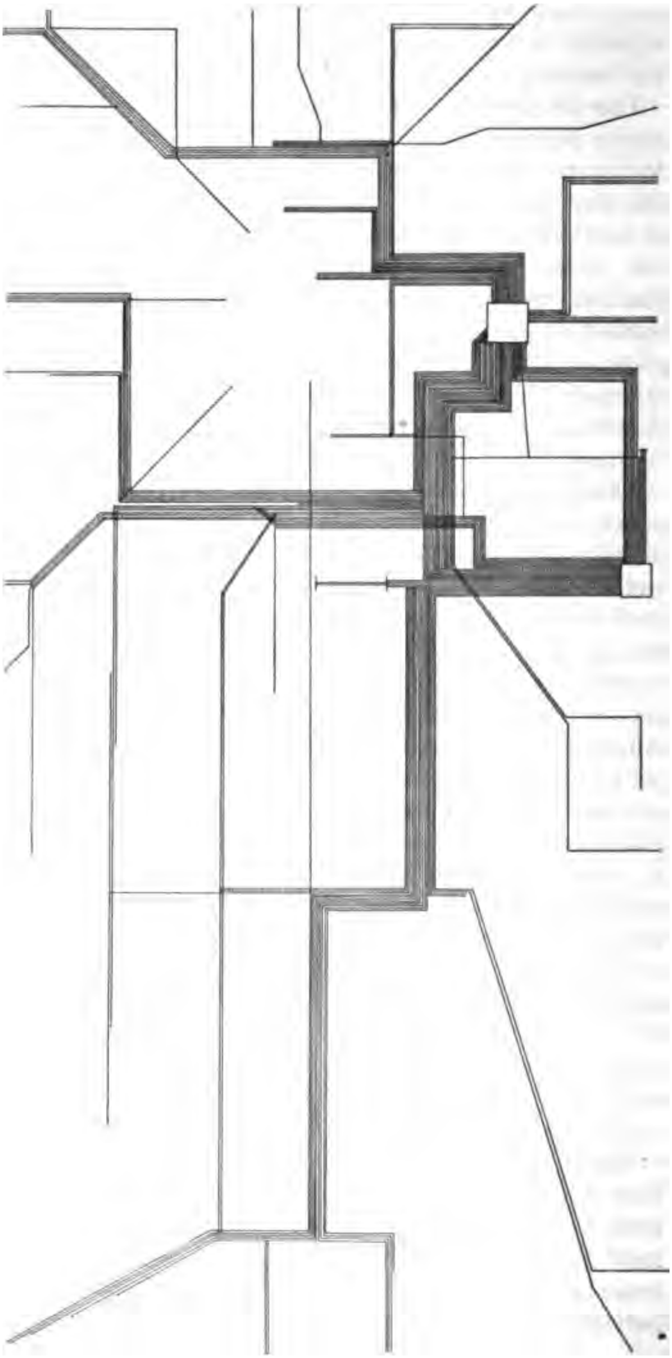

FIG. 1

the reverse, depending on the off peak load being less or greater than half the peak.

By selecting machines of combined continuous capacity equal to the r. m. s. value of the peak load, a reasonable reserve capacity is allowed since the two-hour rating will probably allow one machine to pull over the peak if the other is out of service. To help in such cases, a limited number of feeders should be run through from one station to another so that the feeder may be opened in the station in trouble thus transferring a portion of its load to an adjacent station. The automatic load limiting resistances supplied as part of a railway automatic substation will also help out in such cases at the expense of the trolley voltage.

To illustrate the way in which the problem of applying automatic substations to city service should be approached, a city of about 300,000 population has been selected to serve as an example. This city is now served by an excellent example of the old method of centralized distribution. Fig. 1 shows in a general way the stupendous amount of feeder copper that has been installed in the attempt to give good service. The two plants are about three-quarters of a mile apart on the same side of the city and about one mile from the loop district where lies the greatest load concentration.

The heaviest loaded lines are on the opposite side of the city from the power plants. The lower plant is the oldest one and is mainly equipped with reciprocating units. The total d-c. capacity of this plant is about 8000 kw. The upper plant is a modern turbine station which supplies power for several interurban systems. In this plant are located three 2000-kw. synchronous converters. This gives a total d-c. capacity of about 14,000 kw.

Most of the feeders shown in the cut are 1,000,000 cir. mils. There is a total of 25,800,000 cir. mils in feeders both positive and negative leaving the old plant, 23,000,000 cir. mils leaving the new plant and 8,000,000 cir. mils in tie lines between the plants. One of the heaviest loaded lines runs from the loop

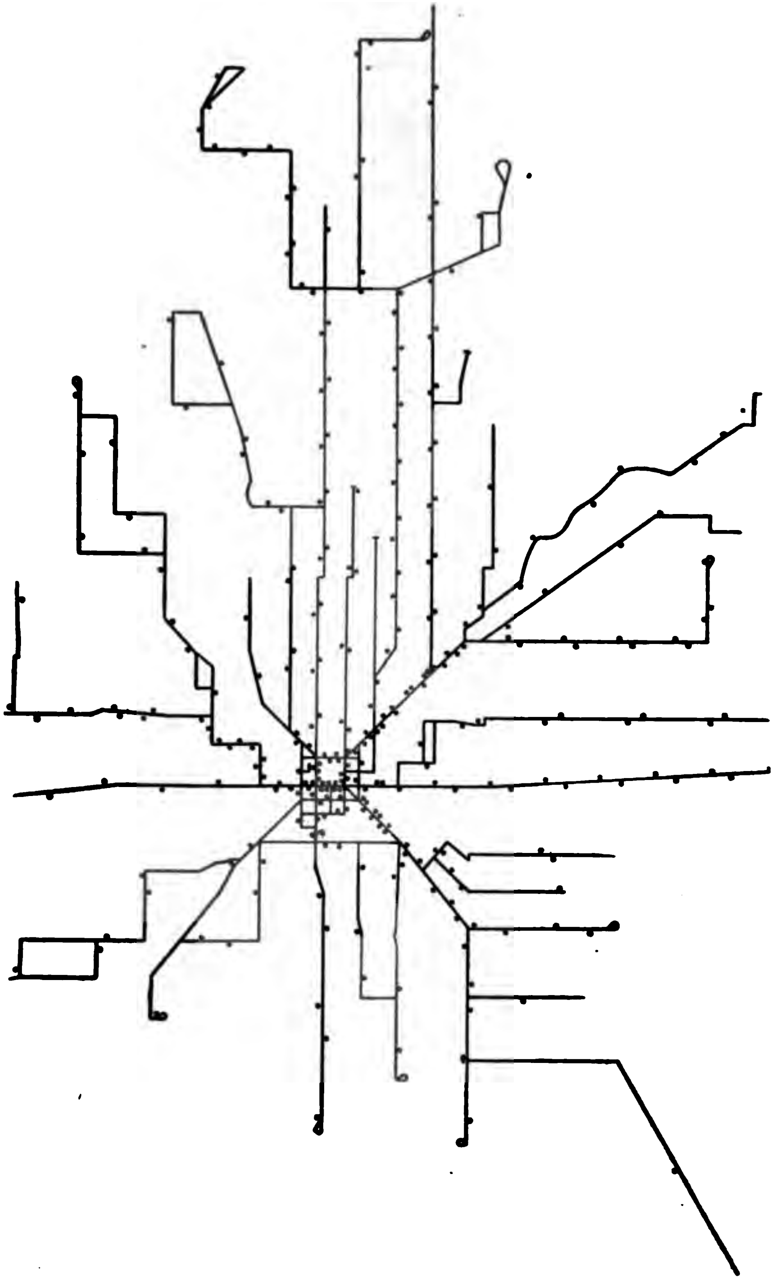

FIG. 2—SPOT MAP—AUTOMATIC SUBSTATIONS

directly away from the plants with the end of the line about seven miles distant measured along the feeder run.

Of this total feeder capacity about 40,000,000 cir. mils are in the positive feeders, the remaining being in the return feeders. The average length of feeder to the center of its load is about two miles. The peak load on the two plants is about 22,000 amperes. Assuming for estimating purposes that the rail and return feeder drop will be somewhere near equal, we find that at peak load the average voltage at the load centers will be 475 volts with 600 volts on the station bus bars. The I^2R losses reach the enormous value of 2750 kw. or 20 per cent of the total d-c. input, over the peak period. The average trolley voltage at the car will be somewhat less than that given for the load centers. In many cases the voltage on the car is so low that it is almost impossible to read news print during the evening rush hour.

This city operates about 300 cars during the periods of heavy traffic most of which are double truck, 20 tons in weight with two 60-h.p. motors geared for 27 miles per hour. The schedule speed is about six miles per hour during the evening rush hour and even this can hardly be held. Due to the low average voltage the cars cannot accelerate rapidly as they should and bunching soon occurs which only aggravates the trouble.

The problem is to lay out a new distribution system which will reduce the transmission losses to a minimum, raise the average trolley voltage to a reasonable value, keep the stray earth currents to a minimum, and at the same time avoid raising the expense for operating labor to an undue amount. All this must be accomplished at the lowest possible net cost.

As a preliminary to deciding on the size and location of substation equipments a spot map of the system should be prepared based on the rush hour schedules. A sample of this is shown in Fig. 2. A factor which upsets the spot map as shown is the fact that there are fourteen interurban lines entering this city all of which operate fairly heavy cars; 45 tons weight with

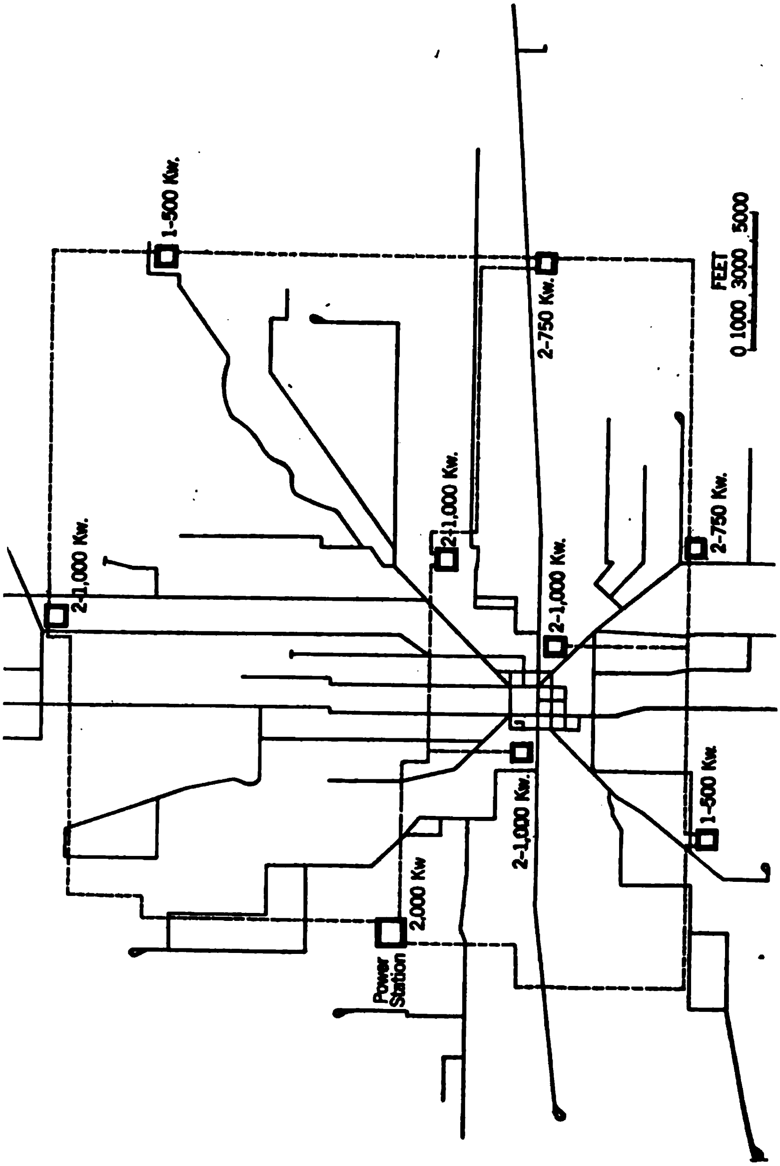

Fig. 3—Proposed Substation Layout—Automatic Substations

four 90-h.p. motors geared for sixty miles per hour would represent the average. In laying out the power supply it has been assumed that two of these cars are at the center of each line which they use for entering the city.

Certain other conditions such as heavy seasonal loads must be considered. There are in this city three parks that are heavily patronized and a fair ground that is a very heavy peak for one week in the year. Such considerations as these have been used to modify the strictly mathematical method of locating the new substations. This method is the one described in Richey's handbook as having been used to lay out the feeder system in Chicago, and consists of assuming that the distributed load on each line is concentrated at the center, then, treating the loads as actual weights, find the center of gravity. This locates the substation in the ideal position to feed its section. This ideal location is of course seldom practical and due consideration must be had for local conditions.

In figuring the spot map shown in Fig. 2 the average current per car has been taken as 53 amperes. The interurban cars have been figured as equal to two city cars. The city has been arbitrarily divided into nine districts for the purpose of apportioning the substation capacity. Care was taken in making this division to keep each section down to less than 2000 kw. in possible load as this seemed to be the largest size station that would be advisable on this size of system, having due regard for investment in feeder copper. The center of gravity was then found for each section and the substation located as near this point as a study of local conditions would warrant.

The two substations feeding the congested portion of the city were located on each side of the loop district just far enough away to be on the edge of the underground district. This is desirable from the standpoint of economy in installation, although the most desirable location from a standpoint of copper loss and feeder copper would be right in the center of the loop. This would however result in excessive investment charges for real estate and underground construction. Locations east and west of the center were chosen rather than north and south because of real estate values.

The final location decided for these substations is shown in Fig. 3. The high-tension line locations were picked out by the superintendent of distribution of the street railway company with a view to using as much of the present pole line as possible and of selecting streets for the necessary new lines where there would be the least opposition from the residents and the city authorities. The feeders form a ring bus around the city with a tie line across the center.

The intention is to sectionalize the feeder in each substation and equip the section breakers with reverse-power relays so that any section will be cut out in case of trouble, thus leaving the entire system in operation with the exception of the damaged portion of the line. Owing to the local situation with regard to the telephone companies and the city authorities the maximum voltage which it would be advisable to use above ground is 6600. While this is not the most economical voltage for the amount of power and the distance, it has been adopted because of the great expense of underground high-tension feeders at 11,000 or 13,200 volts.

At this voltage the feeder belt may be of 4/0 wire, and the cross tie line of 350,000-cir. mil cable which will give approximately 5 per cent power loss in the a-c. system at full load.

The substations as laid out have the following capacity machines installed: Three with two 1000-kw. converters each; Three with two 750-kw. converters each; Two with one 500-kw. converter each; One of the existing 2000-kw. machines to be retained at one of the steam plants. Total converting capacity installed 13,500 kw. Total peak load at present time 13,200 kw.

While this is apparently only sufficient nominal continuous capacity for the peak, it must be remembered that 20 per cent of this peak is lost with the present feeder arrangement so that if the feeder losses can be reduced to 5 per cent this will leave an actual surplus capacity of 15 per cent. This taken with the two-hour overload rating of the converters will give ample margin of reserve capacity.

The feeder calculations are based on maintaining an average voltage of 550 at the car during the evening rush hours. This property now has the equivalent of approximately 110 miles of 1,000,000-cir. mil copper in the distribution system. With the new substation locations we find that the rail resistance is within allowable limits in nearly all parts of the system. In only a few places will negative feeders be required and these will be quite short. With the widely distributed sources of d-c. energy, the carrying capacity of the trolley wires themselves become an important factor, in the distribution of the 600-volt current.

Of the present feeder system it will be possible to remove approximately seventy miles of 1,000,000-cir. mil cable or its equivalent. At 18 cents per pound this will amount to $205,380 which is the first important item of credit for the new installation.

The old d-c. plant should be scrapped entirely and the proceeds applied on the new equipment. It is estimated that the second hand and scrap value of this plant will be at least $100,000.

The I^2R losses on this system were calculated last year and the value of the current lost in the feeder system only, not including local track and trolley losses, was figured as being $100,000. These figures are not vouched for by the writer but as can be readily imagined from the description of the feeder system, this amount does not seem unreasonable. Since approximately one third of the feeder copper is to be left in service we will assume that the copper losses in the remaining feeders will be in proportion to the amount now in service. This will give us a saving in copper losses amounting to $66,000 annually.

If the rush hour schedules of each car are so speeded up as to save five minutes on the rush hour trip morning and evening, with 300 cars and with platform labor at 40 cents the annual possible saving will be $14,600.

The automatic substation buildings should be of the most simple type of construction possible with no heating apparatus and with no accommodations for operators. If extensive repair work becomes necessary in the winter time a portable electric heater can be installed. By constructing this type of building absolutely without ornamentation, the cost should not exceed $40 per kw. complete. Twenty-five miles of a-c. line will be required which can be partly installed on existing poles. The cost of this should not exceed $5000 per mile.

Gross cost of the new system;	
11,500 kw. of substations at $40	$460,000
25 miles of line at $5,000	125,000
Reconstruction of feeder system	25,000
Power house equipment, transformers, etc	100,000
Total cost of automatic substations	$710,000
Credit power plant scrap	100,000
Credit copper removed	205,380
Net cost of automatic substations	$404,620
Fixed charges at 15 per cent	$75,693
Annual operating labor	7,000
Gross annual charges against automatics	$82,693
Savings due to the new equipment;	
Annual saving in copper loss	$ 66,000
Annual saving in platform labor	14,600
Annual saving in labor due to shutting down of old power plant	50,000
	$130,600
Annual cost of substations	$ 82,693
Net operating credit annually	$ 47,970

In addition to the credit shown above, the system would have an increased capacity of 15 per cent or 20 per cent over the present one and would be attracting more patronage by better schedule speeds and better lit cars. It would also be in ideal position to take care of expansion in its business which is almost certain to come as the city in question is growing rapidly and is attracting many new and large manufacturing establishments all of which are locating well toward the outskirts of the city thus requiring increased transportation facilities.

If manually operated substations were to be installed it would not be possible to show an operating credit since only three or four stations at the most would be installed on account of the cost of operating labor. More of the old feeder copper would be required and the saving in copper loss would not be as large. More negative copper would be required to keep the return drop within allowable limits.

The entire equipment should be automatic in operation except the one converter retained in the power plant. One of the converters in one of the loop stations would be set for continuous operation so that night operation would be taken care of. With the heavy amount of feeder copper still remaining, it is doubtful if the voltage drop caused by the night cars would start the outlying substations.

If in practise this assumption should prove not warranted, then it would be necessary to install time switches or pilot wires to prevent these outlying stations from operating at greatly reduced load with consequently poor efficiency during the night.

Many modifications of this layout are possible without materially altering the saving shown. The writer feels that many such interesting problems in automatic control will be met with in the next few years and that its use will spread even faster than the expectations of those who have been following its development. Automatically controlled substations have at this date been applied to railway work, both city and interurban, to Edison three-wire systems, to large industrial plants, to synchronous condenser equipments, and to coal mines.

Heavy steam railroad electrifications have not as yet been tackled although several such jobs have been estimated and it will only be a relatively short time until such an equipment will be installed on one or another of the heavy electrifications.

From an engineering standpoint there seems to be great possibilities in this field.

DISCUSSION ON "SHORT-CIRCUIT PROTECTION FOR D-C. SUBSTATIONS" (LINEBAUGH), "FLASHING OF 60-CYCLE SYNCHRONOUS CONVERTERS AND SOME SUGGESTED REMEDIES" (SMITH), "AUTOMATIC RAILWAY SUBSTATIONS" (PETERS) AND "AUTOMATIC SUBSTATIONS FOR HEAVY CITY SERVICE" (WENSLEY), Pittsburgh, Pa., March 12, 1920.

F. D. Newbury: I wish to discuss the two papers on flashing of converters. While the important results from this study of flashing will undoubtedly appear in improvements in the operation of the converters, one very interesting incidental benefit is the better appreciation of the phenomena of flashing.

Most of us, I am sure, in thinking of flashing, and the relation of speed circuit breakers to the problem, have had the idea that if a breaker could be produced that would open the circuit a little bit quicker than the ordinary types, the problem would be solved. Mr. Smith's oscillograms show conclusively that a moderately high speed breaker may increase the difficulty.

The circuit must be opened very quickly indeed or better not open it at all. Flashing, in a number of instances, in Mr. Smith's tests, did not occur until after the breaker opened even when the circuit remained closed for sufficient time for the current to reach its maximum value and remain there for an appreciable time. The real problem then is the development of very high speed breakers that will limit the current to a non-flashing value.

One reason why high speed is particularly important with the 60-cycle converter is the extreme rapidity with which the current increases on short circuit. If Figs. 7 and 8 of Mr. Linebaugh's paper showing the current increase when a d-c. generator is short-circuited be compared with the oscillograms in Mr. Smith's paper showing the current increase when a 60-cycle converter is short-circuited, the marked difference in rate of current increase will be observed.

If in the case of a 60-cycle converter the circuit increases to a flashing value before the high speed breaker opens sufficiently to limit the current and prevent the formation of conducting gases, then the circuit will be completed by the arc at the commutator and the opening of the breaker will not stop the flashing. This is clearly shown by the high speed camera pictures in Fig. 18 of Mr. Smith's paper.

There is one important point on which the two authors do not agree—this is the effectiveness and advantages of flash barriers. Even granting they are effective, we will all be better satisfied, I believe, if the answer to this flashing problem can be found in some other direction. Barriers that more or less completely box up the brushes greatly increase the difficulty of proper care and maintenance of brushes and commutator and for this reason they are undesirable, in my opinion, from an operating standpoint.

The improvement in performance that comes with the higher reluctance commutating pole has no accompanying disadvantage other than the slightly increased converter cost. It is easy to understand why the higher reluctance commutating pole does improve the converter performance on short circuit. One of the reasons for greater flashing in synchronous converters is the d-c. generator action at the instant of short circuit. Inasmuch as the converter normally requires only 15 to 20 per cent of the commutating-pole ampere turns, as the equivalent d-c. generator the converter is considerably under compensated when it functions as a generator. The use of larger gaps—or equivalent reluctance—and the accompanying additional winding tends to correct this condition.

I am glad to see the matter of feeder taps referred to by Mr. Linebaugh because, after all, that is the most satisfactory way of obtaining the necessary degree of protection in many cases. The loss involved in extended feeder taps is surprisingly small unless the length of tap is carried to an extreme. This of course is due to the small load factor of the ordinary small substation with which this question usually arises. A moderate amount of resistance between the converter and the trolley is good insurance against flashing trouble, that can be obtained at moderate cost.

S. Q. Hayes. I had a very interesting coincidence happen in connection with this meeting today regarding automatic substations. Yesterday I happened to receive from the Brown Boveri Company, in Switzerland, one of their publications for December,1919, and in it they stated that the Brown Boveri Company had been experimenting with a full automatic rotary converter substation at Riehen, near Basle, Switzerland. This supplies the Basle-Lorrach tramway. This is probably the first automatic substation in Europe, and has been working satisfactory, so it is stated.

This station starts automatically in the morning, shutting down at night, and opens and recloses its breakers temporarily in case of open circuit in the primary, or of overload or short circuit in the secondary. In case of a long period of open circuit, an alarm is given relative to the interruption of power.

It is stated that the station operates through a combination of relays and contactors which control various switches. The automatic starting in the morning and shutdown at night is accomplished by a clock of simple construction. They state that they are going to give a more complete description in the course of a couple of months.

C. H. Jones: I will relate a few of the experiences of the Chicago, North Shore & Milwaukee Railway, which is operating automatic substations, and has been operating them for about two years and a half now. We now have five of these stations. Three of them are 500-kw., one 300-kw. and one 1000-kw. stations. They are all in heavy interurban traction service. The car equipment is 47-ton, steel cars, 440-h.p. motors per car, operated in from one to four-car trains.

During the two and one-half years of experience with these stations we have had very fine success and we think very favorably of them, in fact, the operation has been considerably more satisfactory than manually-operated stations ever were. You can absolutely depend on them but there is this point—you cannot expect an automatic station to go on forever without getting some attention. It is useless to think you can put in an automatic station and lock the door and leave it forever and expect satisfactory service. Like every other piece of apparatus it needs careful operation and careful maintenance. The amount of maintenance which should be given varies considerably in the minds of many operators. Personally, I believe that it is absolutely necessary to inspect your station at least once a day. It is also very desirable to know what your station has been doing between the inspection period.

This matter last referred to is one concerning which the manufacturer has not yet seen fit to furnish the operator with means of ascertaining, that is, there is no recording device which indicates what has taken place between the inspection periods provided. I believe that to be absolutely necessary and in order to obtain some such record, we installed in our station quite a number of small mechanical counters connected to the various pieces of apparatus, such as contactors, or controllers, oil switch operating mechanism and things like that.

These counters register every time an operation of a particular piece of apparatus takes place, and by checking up one counter against another in the course of time you will get a very good indication of the general action of your station.

You will find at times that a certain piece of apparatus has failed. The real cause of it you do not know, and as long as it does not repeat itself, you are not particularly interested, there may be a failure at some time or other, but if you continue to get a failure on certain pieces of apparatus, it is time to make a closer inspection and observation of the station, in order to find out what may be causing these minor failures. That has been our case and the counters have paid for themselves many times over, but I believe that the manufacturers will do a great deal toward helping the operators, if they will provide some such device as I have referred to when the apparatus is first installed. The operator will eventually find out he ought to know these things, but it may take him some little time before he discovers it, and if these devices are provided in the first place, it will improve the general operation of the automatic substation equipment.

Up to this time, all of our five stations have been 25-cycle, but we now have on order a couple of 60-cycle equipments. One of these equipments is to change over a present 25-cycle station, and the other one is a new 1000-kw. 60-cycle equipment complete.

With reference to the papers on prevention of flashing in the converter we have had some little experience along that line, not so much in the automatic stations, as in the hand-operated stations. The older type of rotary converter, the non-inter pole type, was not subject to flashover; in fact, we have some old type 500-kw., which to my knowledge, and the knowledge of many of the men who have been with the company longer than I have been, have not been known to flash over. In the case of the new commutating pole rotaries, they are susceptible to the flashing. In the case of the later, 1,000-kw. equipments, we have had them provided with flashing barriers, and I will say that the converter has never flashed over. Many times we have heard them squealing, just as if they were going to flash over, but they never do go over. This occurs principally when the rotary is located right close to the starting point of heavy four-car trains that draw down something like 1000 amperes per car on starting. Rotary converters with barriers have never flashed over even under the above named conditions.

Sometime ago we installed two converters, before the flash barriers were furnished for them, and they were continually flashing over, when operating conditions such as I have stated, occurred. We put flash barriers on them and they have not flashed since, so personally I think a great deal of the flash barriers. Some of my friends do not think so much of them.

David C. Hershberger: In the application of automatic stations we find that the machines have a greater margin in capacity than with present manual control. Cases have come to the speaker's attention where machines are now loaded up to their limit, but by applying automatic control, have a greater leeway in capacity, because they are better able to carry over the peak loads. This is made possible by the use of a load-limiting resistance, whereby the trolley voltage is reduced at the substations which are carrying an excessive load. If adjacent stations are in operation, it results in shifting part of the load to these stations. However, if the adjacent stations are not operating, it merely means that the peak load demanded from the station which is operating is reduced. This is especially true on the interurban systems, where the continous capacity of the machine is not such an important factor as it would be on a city system, so that in that respect it does influence the application of machines, especially as regards sizes.

Another phase which needs consideration is that of inadequate d-c. distribution systems. One instance came to our attention where with manual substation control the system of distribution was inadequate. By re-locating some of the stations and re-distributing the units, it was found that the trolley voltage could be materially improved, thus avoiding the necessity of installing additional feeder. On some installations it is possible to take down copper and affect an economy, but where the operation of trains and rolling stock is so hampered by low voltage, that it would be necessary to put up additional feeder lines, the investment in automatic control is well worthy of investigation.

Still another requirement which has come to our attention is that of controlling auxiliary circuits. Many of the present manually operated stations have a number of auxiliary circuits, such as lighting circuits, circuits for supplying industrial loads and pump motor circuits, etc., that are hand controlled. The automatic control of such circuits has been solved, and they are handled very satisfactorily. For instance, take a circuit where the industrial load is variable, and where there are overloads which would trip the breakers, they can be arranged to close, say three times and then lock out until they have been reset. This practise is followed by some companies in manual operation, and it is just as feasible with automatic operation as it is with manual operation. A pump motor circuit, for instance, can be controlled by a float-type switch. Street and other lighting circuits are now successfully handled by time clocks.

The labor situation touched on by Mr. Peters demands careful consideration. When the system is shut down, due to substation labor trouble, it results in a financial loss. The liability to shut down with automatic stations is less than with manual stations, and therefore the saving which would result from the use of the automatic stations in this way should be credited to them.

C. A. Butcher: Very little has been said of high-tension switching. On the interurban roads there is much competition with the service furnished by steam roads; many of the interurban lines are supplied at the present time through long distance transmission lines, and in a great many cases steam plants are maintained. Often during stormy periods the long distance transmission lines have gone out of service, and it has been necessary to transfer to the steam generating plant and maintain service until the transmission line can be restored. That means that automatic substations operating from these long distance transmission lines are thrown out of service, and unless we have duplicate lines, it is necessary to wait until the steam plant has been put into operation to excite that line.

A number of the larger interurban roads throughout the country are operating on a duplicate system of transmission lines. In some cases they are supplied from separate sources, and in some cases from the same source. In order to take care of the automatic substations on transmission lines, which are paralleled over the right of way, it is necessary to provide some scheme of automatic transfer for the high-tension switch. This has been very effectively taken care of by the addition of a number of relays which adjust themselves to the conditions. If the transmission lines to which the station is normally connected go out of service, a set of relays falls out and tests the other circuit, and if that is in working order the station automatically is immediately transferred to that line, and when the high-tension long distance line is restored to service, it is ordinarily required that these stations be transferred to their normal connection. The same relays when the high-tension line is restored, automatically switch the substations immediately back to that connection.

The automatic service restoring relay system has been in use for a number of years, but it does not give the immediate operation the service companies would like to have. If, for example, a power feeder is knocked off the boards, usually the operator is instructed to wait one minute before he closes the circuit, and after he makes three attempts to fix the circuit and fails, he tries no longer, but reports the outage to the dispatcher and the difficulty is then turned over to the trouble gang, to clear up.

In order to give that same service automatically, there has been developed a motor-operated relay, which automatically closes the circuit breaker periodically. That is, it will close it in one minute, five minutes, or any predetermined length of time, depending upon how the relay is set, and will close it any number of times before the line is definitely locked out of service, and then by some means, either by pilot wires or otherwise, indication is given to the load dispatcher, who immediately gets into touch with the maintenance crew, and the circuit is taken care of. That is in addition to some of the problems already brought up, and must be taken care of in many existing stations, and will undoubtedly have to be taken care of in a great number of new installations.

Donald Bowman: Mr. Wensley in presenting his subject is very optimistic as to the future possibilities of the automatic substation, as applied to low-tension direct-current service in our big cities.

In the operation of the low-tension direct-current substations in our big cities, where everything is underground, and where the total capacity of the system may exceed 100,000 kw. the element of continuity of service is of equal importance with operating costs, because if continuity of service cannot be maintained, the central station will be given a very serious black eye.

In considering the installation of a number of direct-current substations, which will function at different times, according to load, I can readily see how such operation would be satisfactory if everything were carried out as per schedule, but at different times such inadvertent things occur as manhole fires and burning out of units in generating stations and substations, which seriously derange the normal sequence of operations. At those times it becomes necessary to have emergency switching equipment which will operate on both the high-tension and low-tension distribution systems. In the past, these emergency operations have been developed according to the needs of the moment, and have not been arranged as per schedule in advance.

I would like to ask Mr. Wensley to state how some of these unforseen contingencies such as the burning out of units in generating stations and in substations, burning out of transmission lines in manhole fires, and other failures of a similar nature can be successfully handled with the automatic low-tension direct-current substation.

J. F. Tritle: As there were no questions in regard to Mr. Linebaugh's paper, I would like to add a few points in regard to the high-speed circuit breaker, shown in Fig. 3. Mr. Smith in his paper called particular attention to the fact that a moderately high-speed circuit breaker may be even worse than a low-speed circuit breaker. We have found this to be exactly the case in our experiments, tests indicating that a circuit breaker to prevent flash-over, should operate, stop the current rise, and reduce it below the normal flashing value, in something less than the time required for a commutator bar to pass from one brush holder to the next. On 60-cycle machines, this means a speed of something like 0.008 second for the complete cycle.

Most of the oscillograms in Mr. Linebaugh's paper show the results of tests on 2000-kw., 3000-volt motor-generator sets for the Chicago, Milwaukee & St. Paul electrification, which were eight-pole, 514-revolution machines giving approximately 0.015 second for a commutator bar to pass from one brush holder to the next.

The results obtained were so encouraging that the investigation was extended to 600-volt machines. The latest test and experiences indicate that the form of breaker shown in Fig. 3 will quite positively protect up to date 600-volt, 60-cycle synchronous converters. On dead short circuits, the main contact tips, with the breaker adjusted to trip on three times normal load, started to break contact in from 0.002 to 0.003 seconds after the short occurs. The maximum current peak occurs somewhere around 0.006 to 0.007 seconds and we can get the current down to normal or zero in about 0.010 seconds. With the 300-kw. machines the maximum current peak is about 5000 amperes; in the 500-kw. machines around 8000 amperes, and in the 1000-kw. machines approximately 10,000 amperes.

We recently put five dead short circuits on a 1000-kw. 60-cycle converter in a period of approximately six seconds; in other words, only about 1.4 seconds between short circuits. A contactor was used in series with the high-speed circuit breaker for closing the circuit and the high-speed breaker automatically tripped and completely ruptured the circuit similarly to an ordinary circuit breaker, except for the much greater speed of operation.

Our experience also agrees with Mr. Smith's in regard to the tendency towards flashing after a moderately high-speed circuit breaker opens, rather than while the short is on the machine. I remember a series of tests on a 60-cycle machine in which simultaneous oscillograph records were taken of the d-c. and the a-c. current which showed this tendency very clearly. If the short circuit was cleared by the high-speed circuit breaker in less than 0.01 seconds, there was practically no flashing on the commutator and no pulsation on the a-c. end. However, if the short or heavy overload was left on the machine longer than 0.01 seconds, the machine got out of phase a maximum angle in about six or seven cycles after the short circuit occurred and back in phase again in about 17 or 18 cycles. The flashing was materially worse if the circuit was opened by a moderately high-speed breaker while the a-c. current was near its maximum value, in greatest out-of-phase position, than if opened by a relatively low-speed breaker while the a-c. current was near its minimum value for the in-phase position. When in phase no appreciable synchronizing current is required but if out of-phase when the load is tripped the synchronizing current is still required. This current excites the commutating pole sufficiently to give high voltage under the brushes which causes severe sparking and flashing.

A complete line of 60-cycle machines has now been developed having high reluctance poles and radial type protective brush rigging. Complete protection against d-c. flash-overs is apparently afforded these machines by the high-speed circuit breaker and as far as we can forsee at the present time, the flash barriers will not be necessary on them.

One of the principal points which I desire to leave with you is that operating companies need have little concern in the future about flashovers from direct-current short circuits on 60-cycle machines, it being principally a question of the kind of protection provided. Mr. Jones has described the results obtained with flash barriers when properly maintained. There seems to be general agreement about the effectiveness of high reluctance commutating poles. Our tests and experience indicate that practical immunity from flashover can be obtained with the high-speed circuit breaker. The breaker is simple, reliable and strong and has been practically standardized for use in connection with our 1500- and 3000-volt d-c. machines. A line of 600-volt breakers is also being developed as rapidly as possible.

L. D. Bale: The steady growth of Cleveland has made it necessary to provide additional cars to take care of the demand for increased transportation facilities.

With the placing of these new cars in service, together with the fact that at present there exists several locations where insufficient trolley voltage is maintained during the critical peak hours to handle the present schedule makes it necessary either to provide additional feeders or substations. To attempt to rectify this low voltage condition and provide for additional cars by means of feeders would involve a heavy expenditure to say nothing of accompanying losses, therefore, additional substations were decided upon, these stations having an aggregate capacity of 12,000 kw. made up of eight 1500-kw. 60-cycle rotary converters.

Taking advantage of the advance in the art, making it possible to operate substation equipment automatically, enables the location of a greater number of smaller stations having smaller distributing areas than would be economically possible if it were necessary to operate such stations manually, therefore, four automatic substations are to be built each containing two 1500-kw. converters respectively. Each of these stations having practically a capacity load within one mile radius.

Aside from the fact that this particular type of 1500-kw. 60-cycle rotary converter has been standardized by The Cleveland Railway Company, two units are to be used in each of these stations rather than one large unit because of the higher efficiency procurable by maintaining the rotary capacity in service at any time, as near as possible to that of the existing load, also with two units in the station instead of one, greater reliability of service may be expected.

The fact that with each of these stations, practically a capacity load originates within its individual distributing zone, taken with the savings made possible through automatic operation, in addition to a much more economical distributing system and the elimination of possible labor disturbances, the elimination of considerable existing feeder copper and its accompanying losses, the better return-circuit conditions obtainable and the determination of the company to eliminate wherever possible the use of manual labor, has been the deciding elements in the adoption of this type of substation of comparatively large size.

As to definite figures in connection with these stations, I should like to wait until such time as at least one or two of the automatic stations are completed and in operation, that we may have an opportunity to secure reliable costs. All data of this nature at the present time are in the form of estimates which under existing conditions should not be quoted.

E. W. Cook: Referring to section of paper under Details of Operation. No provision has been made to prevent the immediate restarting of the machine following shut down by reverse d-c. current even if the trolley voltage is normal and graphic meter records taken on the 1000-kw. substation shown in Fig. 3 of

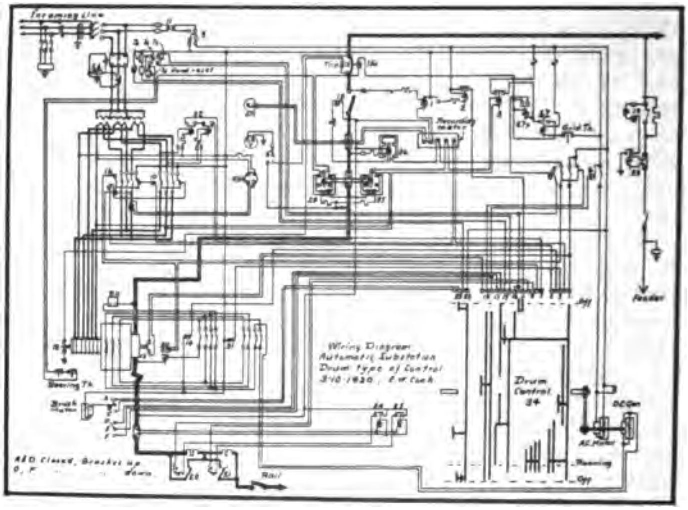

FIG. 1—WIRING DIAGRAM FOR AUTOMATIC RAILWAY SUB-STATION

1. Contact making voltmeter, contact closed on normal trolley voltage.
2. Time-delay starting relay, contact closed when coil is de-energized.
3. Time-delay shutting-down relay, contact opened when coil is energized.
5. Contactor for making control circuit connections.
6. Contactor controlling drum control motor.
7. Three-phase oil circuit breaker.
7a. Interlock closed when oil breaker is closed.
7b. " " " " " " open.
7c. " " " " " " closed.
7d. Hand reset switch in parallel with 7a.
8. 220-volt control bus knife switch.
10. A-C. starting contactor.
11. Control transformer 220-volt bus.
12. Over-speed limit switch.
13. Under-speed control switch.
14. Converter field contactor self-excitation.
15. Main d-c. breaker.
16. A-C. running contactor.
18. Main line d-c contactor.
19. Feeder load limiting contactors.
20. Converter " " "
21. " " " "
23. Feeder " " relays.
24-25. Converter load limiting relays.
27-27x. A-C. low-voltage relay.
28. A-C. abnormal overload relay (hand reset).
29. D-C. reverse-current relay.
30. Converter shunt field relay.
31. Converter shunt field contactor separate excitation.
32. Single-phase protective relays.
33. Grid resistor thermostats.
34. Motor operated drum controller.
36. Polarized relay.
37. D-C. underload relay.
38. Bearing thermostats.

the paper indicate that under certain load conditions the machine could be "motored out" and restarted on an average of once every two minutes for more than an hour at a time.

This restarting is caused by the time delay, about five minutes, of relay No. 3 which allows the machine to be motored out four or five times before No. 3 can open and it is only necessary to have a momentary load of a few hundred d-c. amperes to reset No. 3 and start the cycle all over again.

In attempting to overcome this feature it was noticed that No 4 control circuit contactor is unnecessary and could be eliminated by transferring its work to No. 5.

Referring to Fig. 1, the sequence of operation would be as follows:

Low d-c. voltage opens No. 1 and after a definite time delay No. 2 closes, energizing drum fingers 11 and 12 and the coil of No. 5 through the contacts of No. 27x, 15a, 29, 38 and 7d; No. 5 closes and through drum segments 1-16 and interlock on brush operating device closes No. 6 which starts drum control motor.

Shortly after the drum starts to revolve segments 18-19 make contact ready to turn drum to "off position" if No. 5 should open, a moment later segment 13 makes contact and 11-12 break contact leaving No. 5 locked closed through its main a-c. contact and contacts of No. 27x, 15a, 29, 38 and 7d; any one of these relays or interlocks could open No. 5 and it would not reclose until the drum is turned to the off position and then only by the closing of No. 2 which normally occurs when No. 1 is opened by low d-c. voltage.

This arrangement prevents restarting after being "motored out" unless normally started by low d-c. voltage.

As the drum continues to revolve segment 15 closes the oil circuit breaker and a little later the holding circuit of No. 5 is transferred from segment 13 to segment 14 through interlock 7c on oil breaker, if 7c fails to close No. 5 is opened, drum is set in "off" position and if d-c. voltage remains low this cycle would continue to be repeated. Assuming normal starting, segment 2 through interlocks of No. 32 and 16 closes No. 10 and starts the converter, a few seconds later the holding circuit of No. 6 is transferred from segment 16 to 20 through underspeed switch No. 13 and the drum motor stopped until converter reaches normal speed when drum motor starts and soon returns the holding circuit of No. 6 to segment 16.

Segment 3 closes No. 31 and fixes the polarity of the converter, segment 4 closes No. 14 making the converter self-excited; No. 31 and 14 are mechanically interlocked.

Segment 5 through interlock on No. 10 closes No. 16 main running contactors, segment 26 lowers the brushes and this transfers the holding circuit of No. 6 from segment 16 to 17 and would stop the drum if brushes are not lowered. Segments 7, 8, 9 and 10 are energized from the positive lead of converter, segment 8 closes No. 18 through interlocks on No. 30, 16, 5 and 36; segments 9 and 10 close resistor shunt contactors; No. 21-20 and segment 17 breaks contact stopping the drum in the running position.

Shutting down by low d-c. load: No. 37 closes and energizes No. 3 from one phase of the converter, after a definite time delay No. 3 opens the holding circuit of No. 27x which opens No. 5 which in turn opens No. 16, 18 and as drum turns to the off position No. 7 is opened, brushes raised and the converter is shut down; No. 3 closes shortly after No. 16 is opened.

By reverse d-c. current: No. 29 opens the holding circuit of No. 5 which shuts down the converter as described under low d-c. load.

By low a-c. voltage: No. 27 shunts the holding coil of No. 27x which opens and in turn shuts the converter down by opening No. 5.

Abnormal a-c. overload: No. 28 trips No. 7 and opens hand reset switch 7d, leaving converter shut down until manually restarted.

Hot resistor grids: Thermostats open holding circuit of No. 27 which opens No. 27x and No. 5 shutting down the converter until thermostats cool and reset.

Hot bearing: Thermostats open No. 5 and converter is shut down until manually restarted.

Additional modifications and equipment: Polarized relay No. 36 should stop the drum until the d-c. voltage is equal to or more than the line voltage; this is very desirable where the control is applied to motor generator sets.

No. 1 and 2 might be combined to reduce the number of relays required and similarly No. 27-27x.

A suitable recording graphic meter is a very necessary part of the equipment; this meter should record on a single strip chart the d-c. amperes and voltage and have pens to indicate when No. 10, 18 and 37 operate.

J. L. Burnham: This paper presents data very similar to that given in a paper on Protection from Flashing for Direct-Current Apparatus presented at the June 1918 annual convention. In that paper it was stated that complete protection from flashing could be secured by the use of the high-speed circuit breaker and flash barriers of a peculiar construction with cooling screens to condense the vapor formed by the arc. It was pointed out at that time that a flash barrier without means for disposing of the conducting vapors formed by an arc would be worse than no barriers. This seems to be completely confirmed in this paper.

Mr. Smith has stated the advantages that were secured by the use of high reluctance commutating poles in a special installation. He also states that the high reluctance was also accompanied by extra damping which may have had something to do with improving the performance under the special conditions described. No mention is made of having tried high reluctance poles on 60-cycle converters for railway service. The writer first pointed to their advantages in a discussion on commutating pole converters in 1910 (TRANS. A. I. E. E. 1910, Vol. XXIX, Part 2, pp. 1663-4). A number of tests were also made in 1918 which showed the great advantage of high reluctance in the commutating pole, particularly when carried to a point where the excitation required is in excess of the direct-current armature reaction. In Fig. 2 an oscillographic record is given showing very plainly what takes place when about six times full load is thrown on and tripped off with a breaker of ordinary speed. From the values of current on this oscillogram the excitation on the commutating pole, resulting from its field windings and armature reactions are plotted, Fig. 3, for com-

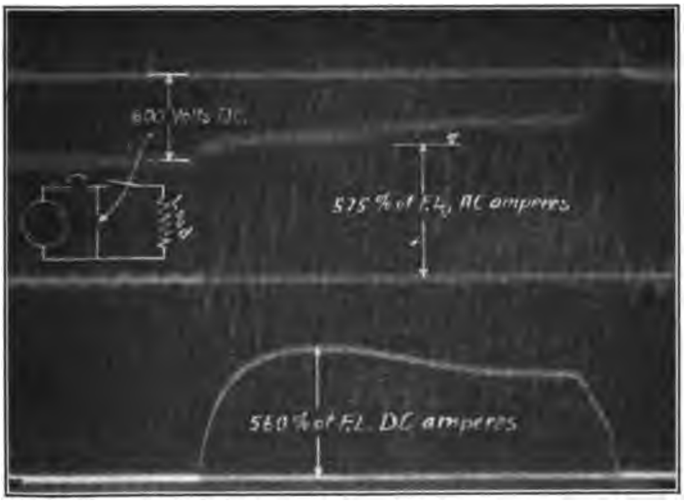

FIG. 2

parison to what is actually required for best commutation. The percentage departure from the correct value, with the usual all steel commutating pole is about 70 per cent as compared to about 20 per cent for the high reluctance commutating pole with a large portion of non-magnetic material next to the magnet frame. As usually short circuits may be several times greater than this load it will be seen that the commutating field might actually be reversed with the all magnetic pole, whereas it would be changed only in strength with the high reluctance pole.

A comparison by test of the loads to cause flashing under similar conditions with the all magnetic and with high reluctance poles show that the latter will increase the amount of short-circuited load about 50 per cent before flashing occurs.

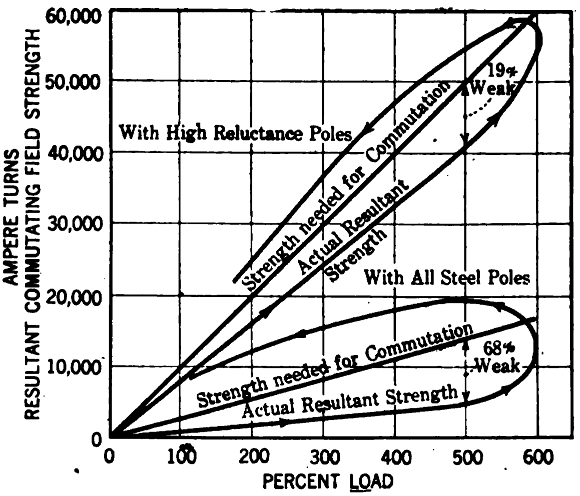

Fig. 3—Transient Effect of Armature Reaction in a Synchronous Convertor During Short Circuit

Tests that have been made with high-speed breaker and high reluctance commutating pole on 600-volt, 60-cycle converters show that complete protection against any amount of d-c. short circuit is obtained.

From Mr. Smith's paper I understand that he has not been able to obtain complete protection with the high-speed breaker. From the figures he gives on speed I believe his results are due to the breaker not being fast enough and also to the fact that all steel commutating poles are used. Mr. Smith has given the factors which cause flashing as "The inherent commutating characteristics of the machine, the value of the load current and the time it has been flowing, the grade of brush, etc." With the use of high reluctance commutating poles I have found that the inherent commutating characteristics of the machine under short circuit are decidedly improved for reasons given above and also that the effect of saturation is reduced. With the high-speed breaker the time the excessive load is on the machine should be sufficiently short to avoid an appreciable formation of gas or volatile matter between neutral points on the commutator and to cause a minimum pulsation. These factors have been kept in mind in the development of a line of 60-cycle railway converters designed by the writer in the last two years. The complete line is now being built with high reluctance poles and several sizes are in operation. For complete protection from flashing of this line of machines high-speed breakers which will completely open the circuit in 0.01 second from the beginning of short circuit are available. Still higher speed (0.006 to 0.007 of a second) is obtained by connecting across the high-speed breaker a resistance which will limit the current to a value easily handled by the converter.

The action of the so-called suppressor described in the paper is interesting in that protection under certain conditions may be obtained without seriously increasing the short-circuit current on the a-c. side. However, the same a-c. short circuit is applied to the machine by the suppressor regardless of the value of the d-c. short circuit which may be only a moderate overload. Mr. Smith states that this scheme is good for only a few limited combinations of load and is, therefore, not applicable for protection in commercial work.

Marvin W. Smith: I agree with Mr. Trittle that the best solution for flashing seems to be in the direction of the high-speed breaker, but we have found, in order to give complete protection, that the breaker must be of very high speed. We found, however, several cases where flashing did not occur before the opening of the breaker, even though the commutator had moved a sufficient distance for the bar to pass from one brush arm to the other. The fact is mentioned in connection with Fig. 1, where reference is made to the flash suppressor, in which the short circuit on the a-c. side was delayed even greater than one cycle. During this time the commutator bar passed a distance of two brush arms, and the machine did not flash over during that time, but does flash over at the opening of the d-c. breaker. There is no question but what, as Mr. Trittle has mentioned, there is considerable advantage in the higher reluctance commutating pole, and we had rather see the problem solved in that way, than by flash barriers or some other means which obstruct the commutator and prevent accessibility.

F. W. Peters: The economies made possible by the use of automatic substations in concentrated service have been presented very effectively by Mr. Wensley. In this connection he spoke of the recent Cleveland installation, or rather the decision to install this large concentrated converter capacity, automatically operated, in the heart of the city. In the past, the majority of automatic applications have been in conjunction with interurban substations where the capacities are relatively small and a correspondingly low cost of automatic devices could return, by labor saving, an appreciable amount on the capital invested. When we deal, however, with city service involving several large units in a station, the initial automatic equipment cost is increased at a rate much greater than the corresponding saving in labor and, therefore, it becomes difficult to determine the economy of such an installation. In view of this fact, I would request Mr. Wensley to enlarge upon this point and bring out just what is attracting the managers of concentrated city service to adopt automatic operation in large stations, and I have in mind particularly the Cleveland Railway Substation involving 3—1500-kw. automatically operated converters.

The question of using automatic equipment in industrial applications, particularly of direct current, is a point well taken and you gentlemen no doubt are familiar with cases where large factories have their generating units in one particular portion of the plant from which power is distributed in a very uneconomical and cumbersome way, by the use of heavy bus copper. It appears that by distributing the conversion units around a factory, direct improvement may be brought about from an efficiency standpoint and also by the elimination of extensive bus layout, and disadvantage encountered due to concentration of power at one point where a short circuit tends to pull out everything.

It is very gratifying to hear what Mr. Jones, Engineer of the Chicago, North Shore and Milwaukee Railway has to say about the operation of automatic substations. He has had considerable experience with this type of station and I am sure we could report still more progress were all engineers equally interested and painstaking. The exhaustive records of automatic substation operation as obtained by Mr. Jones are very valuable and have at all times been open to review by manufacturers for the purpose of making modifications and improvements wherever possible.

Mr. Jones' comments on flash barriers are very well taken and it brings this to my mind—flash barriers, from their very construction, show that they depend largely for their success upon careful adjustment, that is, protection from flashing is obtained by diverting the arc in such a way that it will be extinguished without damaging the equipment. Naturally care must be taken to properly guide the arc in a safe path. In certain cases, flash barriers have been installed and even automatic stations have been started where the idea seemed to be "now I have it, lets lock it up and let it run." That fortunately is not always the case and where a reasonable amount of maintenance has been given to the apparatus, it has, of course, resulted in a much more satisfactory operation than in those cases where the equipments have been neglected. Sometimes operators have not taken the trouble to thoroughly understand the principle on which the automatic equipments function.

Mr. Hays spoke of an automatic equipment brought out by the Brown, Boveri Company, and I am sure it will be interesting to read the details of their arrangement at such time as we have that privilege. Europe is a little slow in so far as automatics are concerned as evidenced by the fact that Australia has had several equipments in successful operation for sometime and a complete automatic equipment is now in transit to New Zealand.

Mr. Cook in his discussion presents a condition of false "motoring out" which the more modern equipments have been modified to protect against. In so far as combining in one device the functions as now performed by two or more devices, we encounter the limitations as determined by manufacturing conditions where it is often much cheaper to use a combination of devices already developed, than to design a new piece of apparatus whose limited production would necessitate higher costs.

The flexibility of automatic substations has been emphasized by Mr. Butcher, and in no case has it been impossible to meet practical operating conditions. Frequently local arrangement of the distribution or operation necessitates treatment as a special case, and progress, if compared to the complications of other automatic applications such as steel mill and automatic operation of car equipments, should not be considered as having reached its limits, in spite of the many accomplishments of the last few years.

R. J. Wensley: Taking up Mr. Bowman's questions first, regarding continuity of service in large Edison 3-wire systems, and heavy city service, this is a real problem and Mr. Bowman has touched on one of the biggest problems that the automatic substation has to meet. We have, at the present time, as part of standard production, a well-developed line of selective power relays. We can do almost anything in the way of sectionalizing a system by eliminating bad feeders, thus stopping back flow of current, and clearing out from any tie bus any set of feeders that may be giving operating trouble. We can feed into heavy city substations through several different tie buses, from several different directions. We can by the use of selective reverse-power relays, definitely clear off of these buses, feeders which may be giving trouble, so as to leave uninterrupted the power supply on these buses.

This, of course, does not cover all possible cases. There are operating contingencies that come up that cannot be foreseen.

The system with which Mr. Bowman is connected, the Commonwealth Edison Company of Chicago, probably has the most complicated Edison network in the world. I do not think there is any other system that has the same kilowatts in converting and battery capacity on one bus. Such a system could not be made fully automatic in the true sense of the word. They have a load dispatcher. There is no particular reason why the principle of the automatic telephone cannot be applied to such an emergency, so that the load dispatcher, instead of having a telephone report made to him of the condition of a circuit breaker, can have full control over the circuit breaker from his own office. He can press a button and close the circuit better than an operator half a mile down the street. There are special means in automatic substations to enable large Edison systems to go back into service after total outage which are much superior to manual operation. Of course, these same features can be applied to manual stations, but they are inherently a part of the automatic station. I refer to the load limiting resistances and the resistance schemes of getting back and establishing service.

Suppose they have an extensive and disastrous feeder burnout, and the entire Edison network is put out of commission. It is almost impossible for any one station, or any one group of stations to take that load up. It practically becomes necessary to get out the entire operating force and send them out through the town to pull the main switches of all big buildings, before service can be restored.

With the automatic station, we have load limiting resistances, and cannot take more than load and one-halt on the converter. After an outage the load dispatcher can restore service by starting all stations as nearly as possible at the same time. The load limiting resistance prevents overloading of the first converters thrown on the bus.

The automatic control is much quicker and more positive than any operator. The automatic control can supply power while the operator is deciding which switch to throw. This is not merely talk. It has been done time and again. I have seen the railway type of automatic control on 300-kw. non-interpole converter put on the line in 12 seconds. The operator cannot think that quick, or cannot throw switches that quick. I do not mean to say that we can take a 2000-kw. booster type of converter and do that. We can take a system of the type Mr. Bowman represents and install automatic control, and if the load dispatcher attacks the problem intelligently, I will guarantee that three minutes after the power supply is resumed the Edison system will be in full operation. This is in sharp contrast to a period of three hours as certain instances have shown are required to put the system back where they do not have automatic operation.

Just a word regarding flash-over protection and automatic substations. So far in the railway substations we have not gone into the high-speed breaker as part of our automatic control, to any great extent. We have talked about it, and in a few cases tried them out, but the average automatic substation today is not operating with high-speed breakers. We are operating with what is decidedly a low-speed breaker. It is only a contactor having no accelerating spring and no triggers or tripping devices. It falls open by gravity, assisted by the pressure of the contact. The overload relay is of the low-speed sort, and no attempt is made to get high-speed operation.

We limit the short-circuit current by running feeder taps out to the furthest convenient distance from the station. The short-circuit current dies down to some extent, before the first contactor opens. This contactor does not open until a good many hundredths of a second after the short circuit first strikes the machine. Then, instead of suddenly opening the circuit thus allowing abnormal rise in voltage between commutator bars we insert a single section of the load limiting resistance and limit the short-circuit to four or five times the machine rating. The next step then cuts in, and then a third step cuts in, thus limiting the current to about load and one-half. The statement has been made by both of the authors of the papers on flashing that the flash-over in most cases occurs after the breaker opens. Since we do not open any breaker, but instead reduce the current gradually we do not expect much trouble from flashing.

In this way we hope in automatic stations to get around the necessity for employing the complicated high-speed breakers, and have confined ourselves to the most simple form of contactor.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

THE BALDWIN-WESTINGHOUSE CHICAGO, MILWAUKEE & ST. PAUL ELECTRIC LOCOMOTIVES

BY N. W. STORER
General Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

REQUIREMENTS

IN the summer of 1917, the Chicago, Milwaukee & St. Paul Railway Company issued specifications covering the apparatus necessary for the electrification of their line from Othello to Seattle and Tacoma. The following were the principal requirements for passenger locomotives as finally determined by the Railway Company:

Power Supply. To be 3000 volts direct-current.

Route. Either the line between Harlowton and Avery, or the newly electrified line from Othello to Tacoma and Seattle.

Grades. The limiting grades are 2 per cent compensated for a distance of 20 miles from Piedmont to Donald on the east slope of the Rocky Mountains, and 2.2 per cent compensated for 17.8 miles from Beverly Junction to Boylston on the western division.

Alignment. The sharpest curve on the main line is 10 deg., but the locomotive must negotiate a 16 deg. curve in the yards satisfactorily.

Load. Twelve steel coaches weighing 950 tons.

Speed. The locomotive to be designed for a speed of approximately 25 mi. per hr. up a 2 per cent grade, about 35 mi. per hr. on 1 per cent grade, and to have a maximum speed of 65 mi. per hr.

Mechanical Design. The locomotive to have a four-wheel guiding truck at each end, of the "Woodard" type.

The driving wheels to be not less than 60 in. in diameter.

Regeneration. Trains to be held on down grades by regenerative braking.

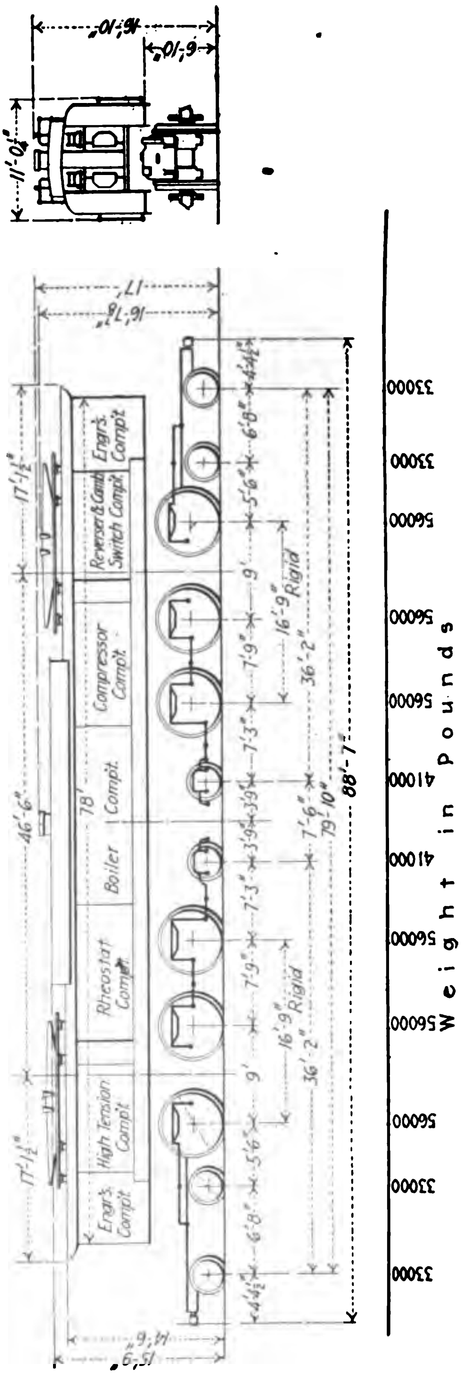

FIG. 2—OUTLINE SHOWING EQUALIZATION SCHEME, WEIGHT DISTRIBUTION AND WHEEL BASE

Train Lighting. Current to be supplied from the locomotive for lighting the train and charging the train storage batteries at a voltage of 60 to 85.

Train Heating. The locomotive to be equipped with an oil-fired boiler, to be supplied by the Railway Company, to furnish steam for heating the train.

Storage capacity to be provided for 30,000 lb. of water and 750 gallons of fuel oil.

Thermostats. Thermostats to be provided that will automatically start the blowers for cooling the main motors when the motor temperature reaches a predetermined value, in order to limit their operation to that time when they are actually needed.

Fig. 1—Baldwin-Westinghouse 3000-Volt D-C. 275-Ton Passenger Locomotive for Chicago, Milwaukee & St. Paul Railway Co.

Description

A portion of the order for locomotives was secured by the Westinghouse Electric & Manufacturing Company, and this paper covers the brief description of the Baldwin-Westinghouse locomotive as furnished.

General. A view of the complete locomotive is shown in Fig. 1, and a diagram showing the wheel arrangement, axle loading and equalization, in Fig. 2.

The locomotive is built in a single unit, having one long cab, carried on running gear of the 4-6-2 - 2-6-4 type.

The locomotive weighs 275 tons.

Main Running Gear. The running gear consists essentially of two Pacific type running gears, coupled back to back. One-half of the running gear is shown in Fig. 3. This shows the running gear complete with the motors mounted and the air conduit carried on top of the motors.

The side frames are steel castings, joined over the four-wheel trucks by a heavy "A" frame casting; also by heavy cross-ties between the drivers which also support the motors, carry the center pin and carry the coupling between the two running gears.

Each half running gear has six spring-supported plungers on which the cab rests. There are two supports at each end and two in line with the center pin.

Fig. 3—One-Half Locomotive Running Gear, Showing Driving Motors, Coupling Bar and Air Conduit

By the use of shims, the distribution of weight between the two ends of each running gear may be adjusted as desired.

The equalization, as shown in Fig. 2, is of the standard three-point type; the leading bogie being cross equalized with the leading pair of drivers and the pony axle being equalized with the two adjacent driving axles on the sides.

Extra points are provided in the equalizing levers so that practically any distribution of weight that is desired can be attained.

The driving wheels are 68 in. in diameter; the journals 8½ in. by 14½ in., located outside of the wheels.

The drawbars, with Minor fraction draft gears, are carried in the "A" frame casting, previously mentioned.

The coupling between halves of the running gear consists of a long bar of a box section. This is shown in Fig. 3. The coupling pins are 10 in. in diameter and are located well inside the pony axles. The pins are hollow, filled with oil-soaked waste and have oil holes provided that keep the pins well lubricated.

Bogie Trucks. The four-wheel trucks are of the "Woodard" type with outside journals; 36-in. rolled steel wheels and cast steel side frames. The journals are 6½ in. by 14 in.

Pony Trucks. The two-wheel truck is of the well known "Rushton" side-bearing type, also with outside journals and 36-in. wheels. The journals are 6½ in. by 14 in.

Brakes. Brake shoes are provided on all drivers. A modified form of the 14-EL Westinghouse Air-Brake Company equipment is used.

Cab. The cab is 78 ft. 0 in. long, 10 ft. 2 in. wide; is strong and rigid so that it can be lifted at the ends. The main strength lies in the two bridge girders extending from end to end. The heavy cross-braces and the side members and top of the raised deck down the middle of the cab, form a construction that is light, but stiff. The cab is divided by cross partitions into compartments, one at each end for the engineer, and the others for the various parts of the cab equipment.

Locomotive Capacity. The total motor rating of the locomotive is 4200 h. p. on the one-hour basis. The continuous rating is 3400 h. p. The tractive effort and the speed are given on the nameplate of the locomotive as follows:

Rating	Tractive effort		Speed	
	Full field	Short field	Full field	Short field
1-hour rating	66,000	57,000	23.8	27.2
Continuous rating	49,000	40,800	26.0	30.4
Weight on drivers	168 tons			

ELECTRICAL EQUIPMENT

Driving Motors. The six driving motors are of the twin armature type. See Fig. 4. Both armatures are contained in a single frame, arranged to secure the maximum economy of weight. The fields are of the standard four-pole type with four salient poles and four inter-poles for each armature. There are brush-arms on each commutator which are easily accessible.

Each armature is wound for 750 volts, but the two armatures and the two sets of field windings are connected permanently in series so that the rating of the complete motor is based on 1500 volts.

FIG. 4—700-H. P. 1500/3000-VOLT D-C. TWIN ARMATURE MOTOR

The motor is designed for field control by means of inductive shunts.

The one hour rating is 700 h. p. The continuous rating is 567 h. p. with forced ventilation and 400 h. p. without blowers.

The characteristic curves, when running as a series motor, are shown in Fig. 5. Curve sheet Fig. 6 shows the performance when regenerating.

Quill Drive. The motors are mounted rigidly on the cross ties of the running gear, one directly above each driving axle. Each motor is geared to a quill centered in bearings in the motor frame and surrounding the driving axle with a clearance all around when axle and quill are concentric of 1¾ in. The quill is connected to the drive wheels by long helical springs which are clamped rigidly at the ends in castings which are bolted one to the quill flange and the other to the drive wheel. There are seven springs at each end, worked in compression in one wheel, while those in the other are in tension. All springs with clamps are interchangeable. They are easily accessible for inspection, and any spring may be removed without disturbing any other part of the running gear.

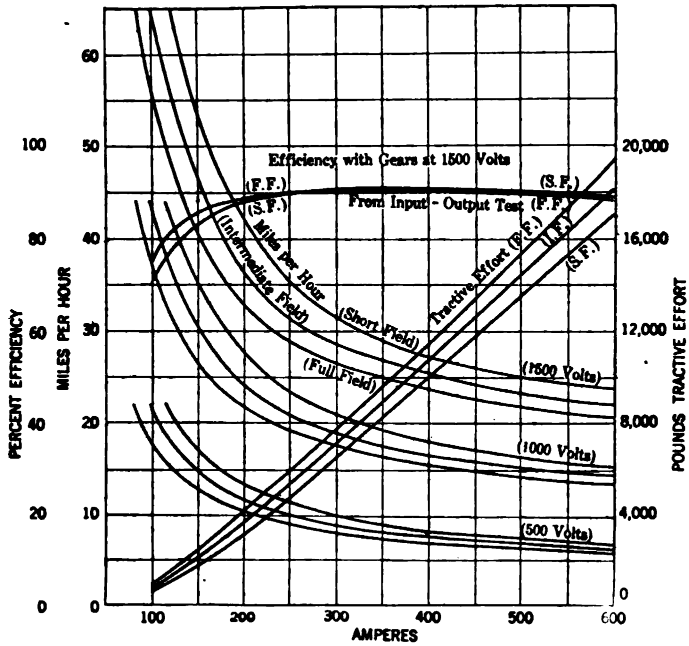

Fig. 5—Motoring Performance Curve of Driving Motor at 1500, 1000 and 500 Volts

This drive is similar to the well-known quill drive of the geared locomotives of the New York, New Haven & Hartford Railway.

Main Motor Control. A schematic diagram of the main motor circuits with starting resistances and all stabilizing resistance and exciters for regenerative braking is shown in Fig. 7. The six motors are arranged to be connected in three combinations:

1. All in series, giving one-third speed.

2. Three in series, two in parallel, giving two-thirds speed.
3. Two in series, three in parallel, giving full speed.

Inductive shunts are applied to the fields on all three of these positions.

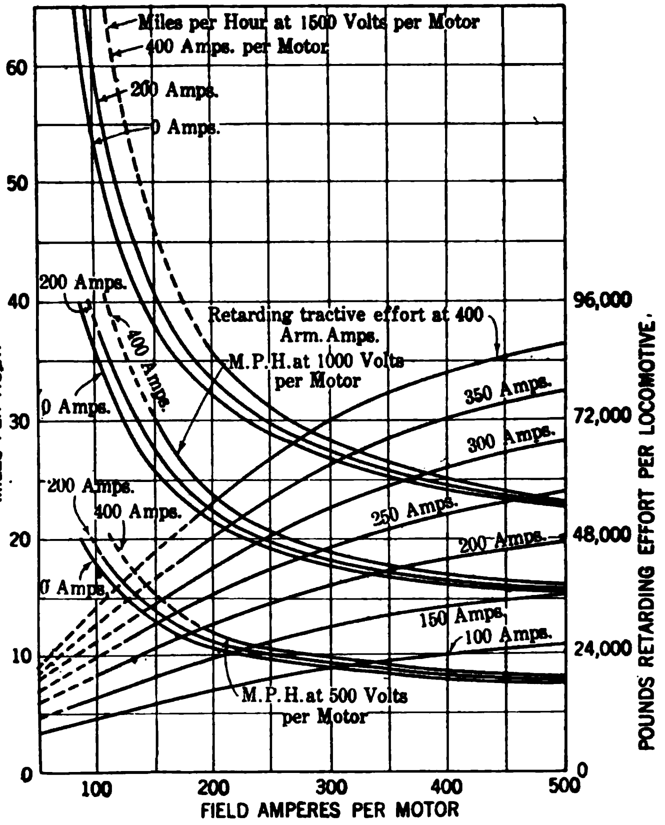

Fig. 6—Regenerating Performance Curves of Locomotive with 3000 Volts on Line. Speeds with First, Second and Third Motor Combinations. Current Shown is Current per Motor

Shunt transition is used in passing from one combination to the next.

Regenerative Braking. When regenerating, the motors are separately excited from two axle-driven generators which are carried on the inside axles of the two four-wheel trucks and geared to them like ordinary interurban railway motors. These generators are separately excited and the field strength of the main motors is controlled by varying the fields of the exciters.

The scheme that is used for regeneration includes the use of stabilizing resistance, which is connected in series with the exciter armature, main motor field circuit, and also with the main motor armature circuit, so that the field excitation is dependent, to a certain extent, on the armature current. A schematic diagram of this is shown in Fig. 8.

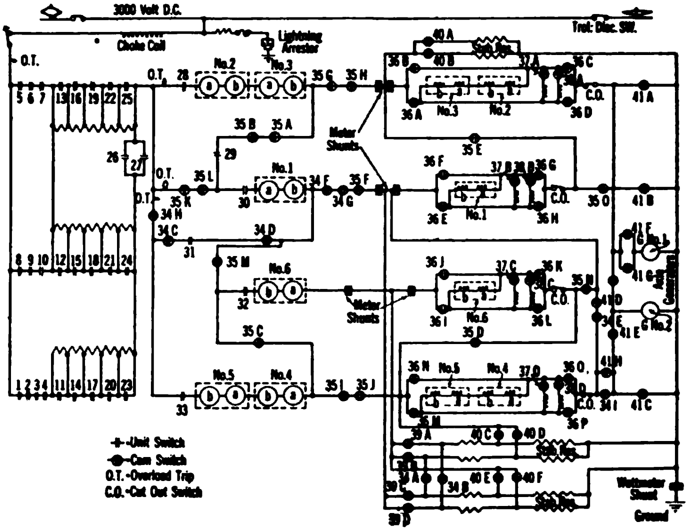

FIG. 7—SCHEMATIC DIAGRAM OF MAIN MOTOR CIRCUITS

Master Controller. The master controller is shown in Figs. 9 and 10. It has four control drums and four operating handles.

1. The speed drum, which controls the resistance switches and line switches; field shunts during motoring and the exciter voltage during regeneration.
2. The reverser drum, which performs the usual function.
3. The motor combination drum, which has three positions, each corresponding to one of the three motor combinations.
4. The regenerative drum, which changes the connections from motoring to regenerating and vice-versa.

The master controller is arranged so that the controller may be thrown from the "off" position to the second or third speed combination, if desired, without passing through the lower combinations.

The method of operation is as follows:

To Start the Train. Motor combination lever in first speed position; regenerative lever in motoring position; the reverse lever in forward position.

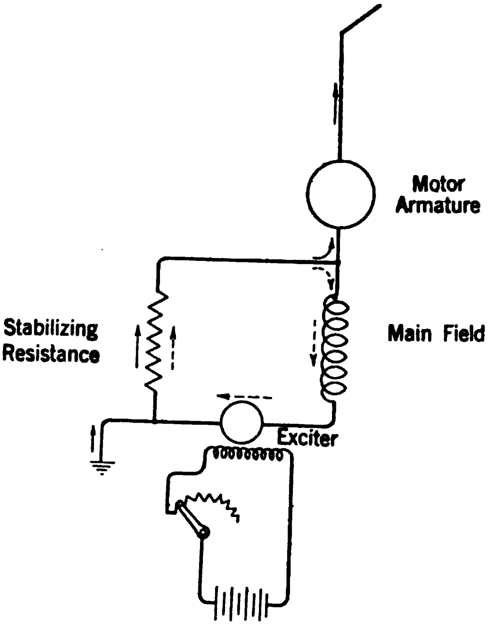

FIG 8—SIMPLIFIED SCHEMATIC OF REGENERATION CONNECTION

Move the speed lever to the first notch and gradually notch up until the train starts and then continue until the resistance is all cut out and the train is running with the motor field shunted in the first motor combination.

Then throw the motor combination lever to the second position; back off the speed handle to the first position (at the same time pressing a button in the end of the lever so that it cannot pass the first position). At that point, with the resistance all in circuit and the three legs of resistance in parallel, the change-over takes place; one-half of the motors being shunted momentarily and then thrown in parallel with the other half. The resistance is then notched out as before and the motor-fields shunted.

Then the motor combination handle is thrown to the third position and the speed handle backed to the fourth notch when the change is made from the second to the third combination, and the resistance speed handle again notched forward. With the resistance all cut out, the motors may be run with full field or first or second shunt.

Fig. 9—Master Controller with Cover Removed, Showing Drums for Speed Control, Reversing, Motor Combinations and Regeneration

The engine-man is prevented from accidentally starting the locomotive from rest, in the second or third combination, by an interlock in the controller which prevents the line switches from closing when the motor combination lever is in the second or third position, unless he pushes a button in the top of the master controller.

It is, therefore, necessary when applying current with the locomotive at speed, to press the button when it is desired to go immediately into the second or third combination.

There are thirteen resistance steps, all of which are available in both first and second speed combinations. On the third combination, there are ten steps that are useful; making a total of thirty-six resistance steps and two field shunting notches in each combination, or a total of forty-two steps in the master controller. The ease of manipulation of this arrangement of master controller has already been established in service.

FIG. 10—TOP VIEW OF MASTER CONTROLLER

Main Resistance. The main resistance is arranged in three groups, each of which has five switches, each short-circuiting a section of resistance. These three groups are connected all in series for the first combination, and three in parallel for the second and third combinations. The sequence of closing the resistance switches is the same on all combinations. The resistances are always connected in series before the master controller reaches the "off" position.

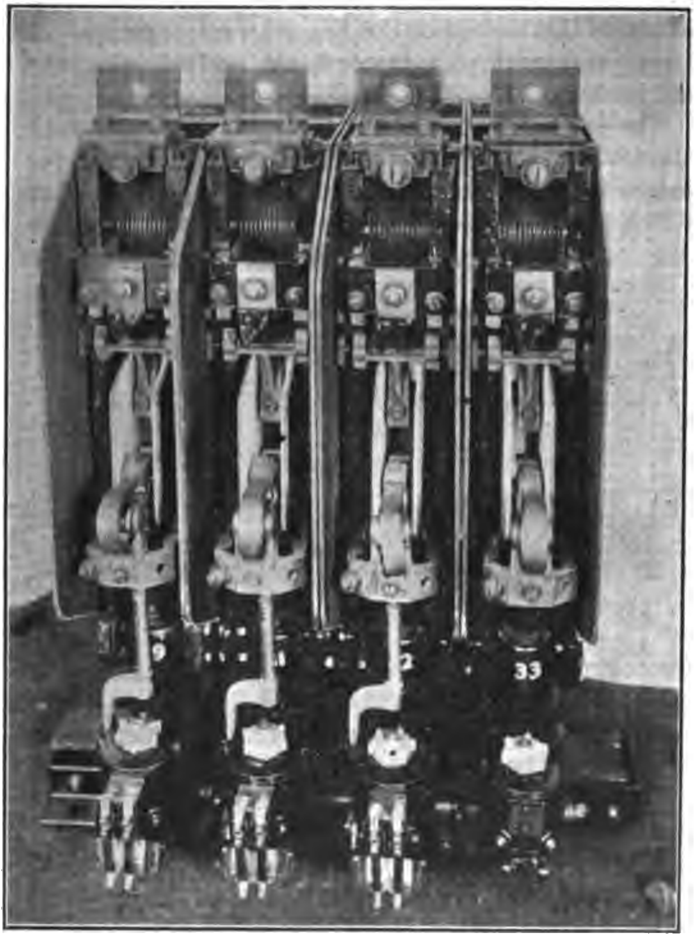

Fig. 11—Rear View of Group of Four High-Voltage Unit Switches

Overload Trips. The overload trips are arranged to open the resistance switches and insert the entire resistance in series before the line switches are opened.

Regeneration. The operation to begin regeneration is as follows:

1. Throw the regenerative handle to the regenerative position on the master controller.

2. Place the motor combination handle in position corresponding to the speed of the locomotive.

3. Move the speed lever to the first step. This connects the motors to the line with all resistance in series and with the motor-fields excited with the minimum exciter voltage.

The exciter voltage is automatically increased until the motor voltage equals the line when the speed lever is moved to the sixteenth notch of the controller, after which the exciter voltage and speed can be changed by the engine-man by moving the speed handle alternately back and forth between the sixteenth and the seventeenth notches, if the exciter voltage is to be decreased; or between the sixteenth and the fifteenth, if it is to be increased. The motor-operated rheostat will move one step for each time the controller is moved.

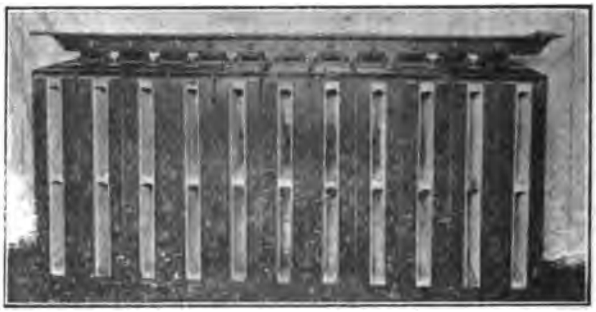

Fig. 12—Front View of Unit Switch Group

To change the motor combination during regeneration, throw the speed handle to the "off" position; move motor lever to the new position and start regeneration as before.

Unit Switches. These are shown in Figs. 11 and 12, and are indicated on the diagram by two vertical dashes. The switches are numbered from "1" to "33". The switch is provided with a very powerful magnetic blowout with arc chutes of arc-resisting material, and an arc splitter. It is electro-pneumatically operated and is designed so that it is very thoroughly protected from insulation troubles and for ease of inspection and over-hauling.

Cam Switches. The cam switch groups are shown in Figs. 13 and 14, and indicated on the diagram by two vertical dashes enclosed in a circle. The switches are designated by numbers and letters. The number corresponds to the group in which the switch is placed and the letter corresponds to the particular switch in the group.

Those of Nos. 37 and 38, which are the groups operating the field shunts, also Nos. 39 and 40 which control the stabilizing resistance during regeneration, are provided with magnetic blowout; but the switches for the reverser, which never close or open the circuit with current on, have no arc chutes or blow-outs. Groups Nos. 34 and 35, which make the different motor combinations, have barriers placed between adjacent switches which have a large difference in potential, but no magnetic blow-out, since these switches are never used for opening the circuit under load. The groups are operated electro-pneumatically.

FIG. 13—CAM OPERATED SWITCH GROUP WITH MAGNETIC BLOWOUTS

Auxiliaries. Power for the auxiliary motors, control circuits, train lighting, motor excitation, etc., is derived from three sources:

1. *Motor-Generator.* (See Fig. 16). The motor receives current from the line; the generator delivers

current at a constant voltage of 85. This is required primarily for train lighting.

2. *Storage Battery.* Of the MV-25 ironclad Exide type containing 38 cells. This battery has a capacity of 300 amperes for approximately one hour.

3. *Two Axle-driven Generators.* Which are designed primarily to furnish current for exciting the main motors during regeneration.

The storage battery is always available to supply current for locomotive lights, control circuits, and the air-compressor motor for short periods. When the locomotive is in service, the generator of the motor-generator set is always in parallel with the battery, and from this dual source is always taken the power for lighting the locomotive and train, control circuits, cab floor-warmers, small blower motors for inductive field shunts, boiler blower and exciting current for the axle generators. In addition, the current for the air-compressor motor is supplied from this dual service whenever the locomotive is standing still or regenerating.

The axle generators are, as stated, used primarily for exciting the main motors during regeneration. At other times, when the locomotive is in motion, the axle generators are automatically connected to the auxiliary circuits from which the compressor and main blower motors take power. At this time the voltage is automatically maintained at 90.

Air-Compressor. A two-stage compressor with inter-cooler, having a displacement of 150 cubic feet per minute, is provided. It is a double-acting, upright type, driven by an industrial type motor.

Blowers. Two blowers are provided to supply forced ventilation to the driving motors. They are driven by motors that are duplicates of the compressor motor.

Auxiliary Control. The axle generator circuits are shifted from one circuit to another by a group of the same type of cam operated switches as are used in the main motor circuits.

The field rheostats for the generator of the motor-generator set and the axle generators are operated by small motors. As the fields of the two axle generators are connected in series, only one rheostat is required for them.

The switches for controlling the blower and compressor motors and the high-voltage motor are magnetically operated.

The motor of the motor-generator set is protected from overload or short circuit by a small permanent resistance in series and a set of three expulsion type fuses which blow in series, the first two inserting additional resistance in the circuit, and the last opening only after a relatively high resistance has been inserted and the current consequently limited to a low value.

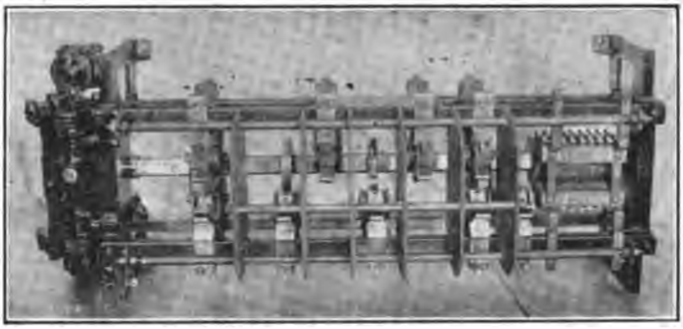

FIG. 14—CAM OPERATED SWITCH GROUP WITH BARRIERS

Meters. A Sangamo wattmeter is provided which has separate dials for integrating the motoring watts and regenerating watts. A full set of motor ammeters is provided, and also a voltmeter reading line volts. An ampere-hour meter is provided for the storage battery.

Train-Heating Plant. A boiler with a capacity for evaporating 4000 lb. of water per hour is provided for heating the train and engine-man's cab. The boiler and water storage tanks are located in the middle of the cab and occupy a relatively large proportion of the space. The tank for fuel oil is located directly beneath the boiler, and the water tanks, two in number, just fore and aft of the boiler.

Pantagraphs. Two pantagraphs of the double slid-

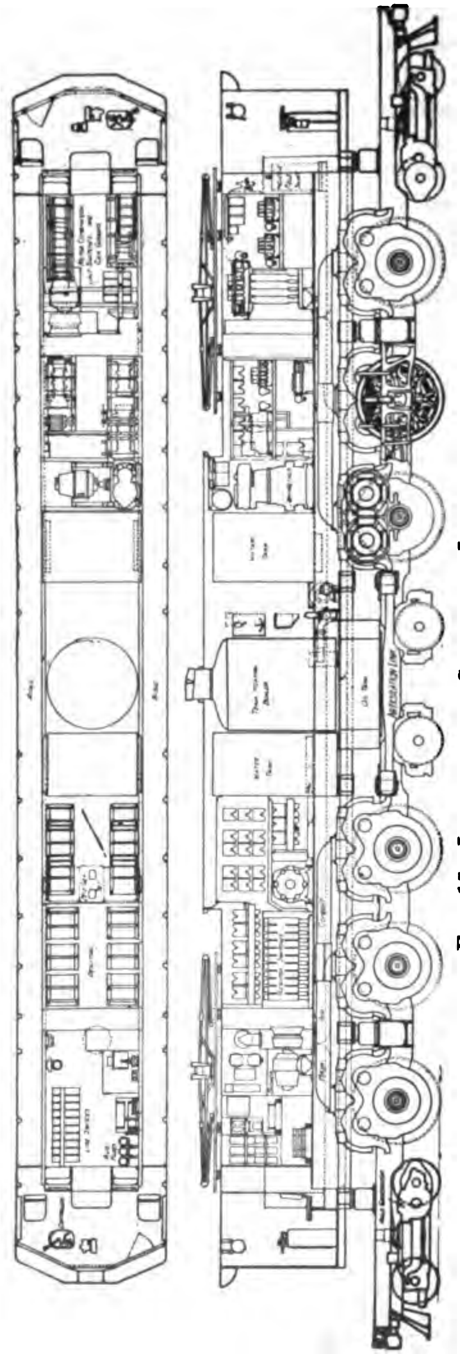

Fig. 15—Longitudinal Section of Locomotive

ing shoe type are provided. These are raised by air and lowered by gravity.

Wiring. All cable is in steel conduit, and all exposed connections are solid copper bar or strap.

Mechanical Design. This locomotive has been designed to possess certain mechanical features which have developed through years of experience with steam locomotives. These characteristics are believed to be advantageous when applied to electric locomotive design as well. They may be summarized about as follows:

1. Distribution of weight, fore and aft, with respect to wheel base.
2. Height of center of gravity.
3. Length of rigid wheel base.
4. Guiding trucks.
5. Equalization.
6. Coupling between truck frames.
7. Diameter of driving wheels.
8. Non-spring-borne weight on driving axles, etc.

The principal means that contribute to good operation are:

1. The location of the mass of the locomotive as close to the center as possible.
2. High center of gravity of that portion of the locomotive carried on the running gear.
3. A long rigid wheel base to insure stability and also to decrease the transfer of weight due to drawbar pull.
4. Guiding trucks, located well outside the mass of the running gear, to contribute to stability, as well as leading the locomotive into curves.
5. Proper equalization to secure stability and weight distribution.
6. The minimum permissible restraint to free movement of each axle and portion of the running gear.
7. Large diameter of drivers and low "dead" weight on axles.

In designing a locomotive cab, it is very advantageous to concentrate the heavy equipment between the center pins. A good rule is to have the radius of gyration with the cab swinging from one center pin, shorter than the distance between center pins.

In the running gear, it is desirable to have the radius of gyration measured from the center pin, less than half the wheel base. The closer the weight is concentrated to the center pin, the less tendency there is to nose and the easier the duty on the wheel flanges and track. The steam locomotive seldom has a good fore and aft distribution of weight, but its high center of gravity compensates for it.

As concerns the size of drivers and dead weights, it may in general be said that, other things being equal, the effect on track is inversely proportional to the square of the wheel diameter, and directly proportional to the dead weight.

FIG. 16—3000-VOLT-85-VOLT MOTOR-GENERATOR SET—TOP HALF OPENED

This locomotive embodies all the foregoing features to a remarkable degree. Particular attention was given to weight distribution. The cab has the boiler, water and oil tanks, storage batteries, air-compressor, resistors, motor-generator set, and the heavier parts of the control equipment, concentrated between the center pins. The driving motors are mounted above the axles on the running gear, thus getting the weight well inside the wheel base, but placing it relatively high. The height of the center of gravity of the complete locomotive is 68 in., a value that corresponds well with that of a steam locomotive.

The Pacific type of running gear with its long rigid wheel base and the guiding trucks is a particularly stable design and is especially good with the weight distribution that obtains on this locomotive. The height of the center of gravity of trucks with motors mounted is 43¾ in.

The quill drive, which is a further development of the one used on the New Haven locomotives, gives each driving axle perfect freedom to move vertically the full distance permitted by pedestal jaws without affecting the motors or frames, except through springs. The only "dead" weight carried is the weight of wheels, axles, journal boxes and spring clamps; a total of 7032 pounds for the wheels, axles and spring clamps which are rigidly fastened together, and 770 pounds for the journal boxes. This weight with the larger diameter of drivers (68 in.), and the total weight of 56,000 lb. per axle, gives a combination that is much better than has been considered very good practise on steam locomotives.

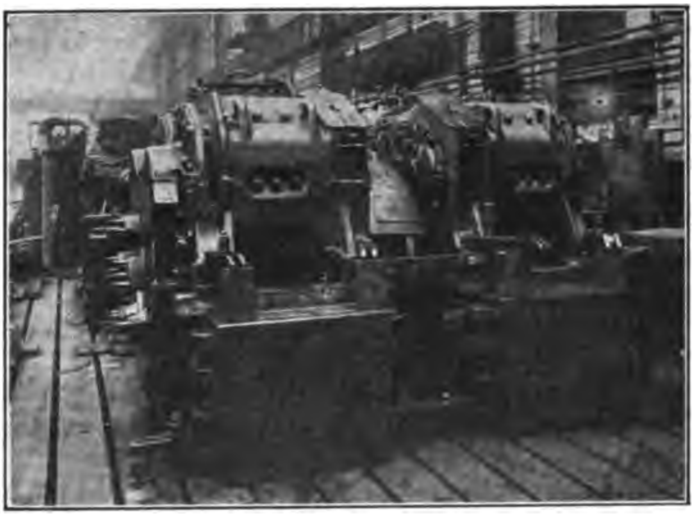

FIG. 17—TWO 700-H. P. MOTORS COUPLED TOGETHER FOR TEST

Great effort has been made to cushion the locomotive against shock, either while running or from bumping. The apparatus in the cab is especially well protected since the cab rests on spring-supported plungers which are in series with the main semi-elliptic springs. The cab is protected against bumping strains by floating center pins, which, while held rigidly against lateral motion, are cushioned against longitudinal motion by heavy springs. It has been the practise heretofore with this general type of locomotive to have one center pin rigid in the running gear frame and allow the other one to move freely in a longitudinal direction. It is felt that it is much better to allow a slight relative motion between the two center pins, but to prevent the bumping shocks by spring cushioning.

ELECTRICAL DESIGN

Main Motor. This was constructed of the twin armature type for several very good reasons:

1. The two armatures can both drive the same gear.

A single armature motor would require twin gears, each of the same gear face as the present single gear in order to keep the tooth pressure down to the same value.

2. The space between wheels available for motors is limited and must be divided between gears and the motor itself.

The use of a second gear would take just that much space out of the armature core, and thus make a very large diameter necessary to get the output required.

3. A large diameter motor and double gears would increase the weight and cost of the locomotive.

4. A large diameter would make the motor extend farther up into the cab, thus necessitating raising the floor above the motors and cutting down the space available for other apparatus.

5. The use of two armatures makes it easy to design the motors so as to secure the advantages of low-voltage armatures for commutation.

6. Small armatures are easier to handle and cheaper to maintain than the large ones would be.

The capacity of the motor was fixed by the number of driving axles. Six driving axles were used as this seemed to be the smallest number that could be used with the limiting weight on the drivers. It was especially fortunate, as it led to the adoption of the double Pacific type of running gear with its excellent riding characteristics.

The characteristic curves of the motor, shown in Fig. 5 are taken from test; the efficiency being measured by the "input-output" method with all losses included. In making this test, two motors were connected together through gears, quills and quill springs; one motor being run as a motor, driving the other as a generator. See Fig. 17. The current from the generator was fed back through the motor by boosting its voltage, and the additional current required was supplied from an outside source. The measurements of the power supplied by the booster and from this outside source constitute the total losses in the two machines. The test was made on motors which had been operated only a few hours, consequently, the friction losses are probably higher than they would be after being in service for awhile; but the electrical and mechanical losses are thus included in the total, and the efficiency thus derived is the same as if the motors were on the locomotive. It is worthy of note that the efficiency thus measured is materially higher than that calculated by the A. I. E. E. Rules from "no-load" losses and the fixed percentage of losses for gears and other load losses. These tests show plainly that those losses assumed for the Institute Rules are too high for large motors.

Fig. 4 shows the pinion end of the motor, and on the motor frame are shown the shields which protect the motor from melting snow falling into the outlet openings in the motor frame.

The motor armatures have very effective fans mounted on them which give the motors quite a high continuous rating without the use of any forced ventilation. It is expected that the blowers will not be used at all with normal train weight, except on heaviest grades.

that there is scarcely any comparison possible. The half speed is too low for a running speed and too high for switching. The one-third speed is an excellent switching speed, and the two-thirds speed is a good running speed and is especially good for regenerating on heavy grades.

Another considerable advantage lies in the decreased rheostatic losses on this locomotive compared with one having the ordinary series-parallel control. The three speeds alone make a very considerable reduction in rheostatic losses; but when combined with the field shunting positions on each combination, reduce the rheostatic losses to less than one-half of what would be obtained with the ordinary series-parallel control. This, of course, means a very decided reduction in the weight of resistance necessary, as well as an increase in efficiency.

The combination of unit switches and cam switches gives a perfectly successful, and at the same time, the lightest combination of switches possible. The unit switches are used in the high-voltage circuit wherever they are required to open the circuit with any current; thus the line switches, resistance switches and safety switches in the motor combination circuits are of the unit type and are interchangeable, except for interlocks which are easily transferred. The reverser switches, field shunt switches and those for regenerative change-over, exciter and auxiliary change-over are of the cam type, and operated by compressed air. These switches are assembled in groups of from four to sixteen switches; the sixteen-switch group being the reverser.

Shunt transition is used in passing from one motor combination to another as it is simple and gives sufficiently smooth transition to be very satisfactory. In changing from the first to the second combination, one-half of the motors are shunted momentarily; but in going from the second to the third speed, only one-third of the motors are shunted.

The sequence of switches is so simple that relatively few of the unit switches have interlocks on them, but the cam groups are all interlocked so that it is impossible to get false operation. The switches in any given cam group have the mechanical interlock provided by the cam shaft.

One of the fundamental principles on which this locomotive was designed was that the motor circuit should not be opened, either in normal operation or under emergency, until the current had been cut down by the introduction of resistance.

The specifications describe how this is done on this locomotive. It is considered to be a very valuable, if not an absolutely necessary, feature. It not only divides the arcs among a large number of switches and, therefore, promotes safety, but it prevents the generation of high voltages from suddenly rupturing a heavy current.

The line switches are located in a separate compartment from the resistance switches so that the four switches in series are always kept free for the final break.

Another fundamental principle in the regeneration was that the motors should never be connected to the line without the maximum starting resistance in series between motors and the line. This is a valuable provision in motoring, but is necessary in regeneration to prevent undesirable surges in the train.

This locomotive has approximately 17 ohms of resistance on the first step, consequently, the maximum current that could flow from the line would be less than 200 amperes, if there were no voltage generated in the motors at all. If this current is divided between two or three circuits, as would be the case on the second or the third combinations, the torque resulting, especially with the very small current in the motor field, would be negligible. The exciter rheostat is so arranged as to increase the voltage of the exciter immediately on closing the line switch until the flow of current through the resistance is practically zero, which means that the voltage of the motors equals that of the line. This method gives perfect safety in connecting the motor to the line and the stabilizing resistance and the axle-driven exciters add still more to the safety.

The stabilizing resistance protects the motors against sudden changes in line voltage, while the axle-generators protect against variations in grade, which would otherwise tend to vary the speed.

In case of a sudden reduction in line voltage, which would be followed by a heavy increase in the regenerated current, the increased voltage drop in the stabilizing resistance resulting from this, immediately cuts down the field current and generated voltage of the main motor sufficiently to limit the increase in regenerated current to a safe value.

In the case of the passage of a train from a sharp curve on a compensated grade to a tangent, there would be a tendency for a sudden heavy increase in speed, because of the steeper grade and the decrease in train resistance. As the speed increases, the voltage of the axle-generators automatically increases and, therefore, the field current of the main motors, which holds the train with a very slight increase in speed.

This scheme of regeneration offers a high efficiency as the only extra losses are those in the exciter and in the stabilizing resistance, both of which are small.

Auxiliaries. It has been the experience with high-voltage d-c. equipments that the auxiliary motors are very much more difficult to design than the main motors. More trouble is ordinarily experienced with auxiliaries than with the main current apparatus and in order to make such apparatus reliable, a great deal more expense is involved than would normally be warranted. It was the policy on this locomotive to limit the high voltage to the minimum possible number of circuits and the minimum amount of apparatus. It was impossible to avoid it altogether on the moving apparatus, due to the fact that the train lighting must be supplied for hours at a time in case of any emergency when the train is held up by a wreck or because of storms. The motor-generator set, which is designed primarily for this purpose, is the only piece of moving apparatus among the auxiliaries which has the high voltage applied to it. The voltage selected for the auxiliary circuits was primarily decided by the voltage required for train lighting.

While 85 volts is lower than would otherwise be selected for the auxiliary motors, it is perfectly safe and is a voltage that is so low that the motors can be forgotten, as far as the questions of insulation and commutation are concerned. The auxiliary motors are, of course, very much smaller than would be the case, if the high voltage were applied to them.

A storage battery, from the standpoint of operation, is always a desirable thing on the locomotive as it offers an opportunity for making a complete inspection of control apparatus by furnishing current for the lights and control, and for keeping up the air pressure without having the high-voltage current on the locomotive at all. Having a source of power to furnish light on the locomotive is a very desirable feature.

The combination of the storage battery and the axle-generators will furnish ample auxiliary power to take a locomotive to the end of a run, if the motor-generator set fails. The same combination will take a train safely down the longest and steepest grade with the air-brakes, if power is for any reason cut off the line. If the train is going up a grade and power is cut off, the battery will furnish air to hold the train with the brakes for a considerable time, so that it will not be necessary to block the wheels as is the ordinary practise, unless power is off for a considerable time.

Low-voltage motors are so small, comparatively speaking, that although there is a great difference in the power required for the compressor and the main blower motors, it was felt that the advantage of having the motors interchangeable was worthy of the increase in cost of the blower motors. The design of the compressor adopted is favorable to this, so that the motor is practically a standard industrial type of motor.

Thermostat. The thermostat, which is used for starting the blower motors, in accordance with the requirements, consists simply of a cast-iron resistance made up in tubular form, which is in series with the main motor armature. This resistance is enclosed by several micarta tubes so that it has substantially the same thermal characteristic as the main motor. There is a Sylphon tube contained within the resistance tube, which expands when a predetermined temperature is reached and closes the magnet circuit of the blower motor switch, thus starting the blower when that temperature is reached. The apparatus is very simple and rugged; and while, of course, it cannot follow the temperature of the motor absolutely, it is sufficiently accurate for the purpose.

CONCLUSION

The initial operation of these locomotives has given every reason to anticipate that good riding qualities, the advantage due to low-voltage auxiliaries, the flexible speed control, stability of regeneration, safeguards against short circuits, and the simplicity and ruggedness of the individual parts of the locomotive, form a combination that will be a great success in the service for which it was designed.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

PASSENGER LOCOMOTIVES FOR CHICAGO, MILWAUKEE & ST. PAUL RAILWAY

BY A. F. BATCHELDER
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AND
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DECEMBER 9th, 1915, may be considered the date of the initial electrical operation over the electrified lines of the Chicago, Milwaukee & St. Paul Raliroad. During the following winter the electrification was extended over 440 miles of route from Harlowton, Montana, to Avery, Idaho, a section which crossed the Belt Mts., the Rocky Mts., and the Bitter Roots. The locomotives for this initial electrification were of the geared type, designed and built especially with a view to the most economical operation of the freight service. The locomotives for passenger service differed from the freight locomotives only in the details where it was absolutely necessary to meet the operating requirements, such as changing the gear ratio to increase the speed and providing car heating and lighting equipment.

Three years later, in 1918, the successful operation of the original equipment had convinced the railroad company of the economical advantages of electric operation, and they decided to equip an additional section extending over the Cascade Mts. between Othello, Washington, and Tacoma, Washington, a distance of 212 miles. In choosing the equipment for the new extension, it was decided to give special emphasis to the requirements of passenger service and to purchase locomotives which were primarily designed with this in view, taking advantage of any details which would assist in the proper and economical operation of passenger trains. For the freight service it was decided to retain the geared locomotives that were in use on the

Harlowton-Avery Division, changing the gear ratio where necessary to meet freight conditions, and using only locomotives of the new design for passenger service.

To meet the specification for passenger locomotives, the General Electric Company has designed, completed, and tested, a locomotive which appears to embody the necessary qualifications and to successfully fulfill the requirements, both from electrical and mechanical standpoints. In designing the locomotive, particular attention has been given to the features affecting safety, reliability, efficiency, convenience of operation, effect on track, and cost of maintenance. The locomotive has especially good riding qualities; it has no apparent effect on the alignment of the track, and to a marked degree, it is free from transverse movements or oscillation which would tend to create lateral pressures against the rails.

It is the intention of this paper to give a description of this locomotive which differs in many ways from the locomotives which are now in operation on the Harlowton-Avery Division, to indicate the reason for choosing this design, and to call attention to some of the principal features which differ from usual practise. Briefly stated, the service requirements for the locomotive are to operate a 950-ton passenger train over the mountain divisions of the Chicago, Milwaukee & St. Paul Railway at speeds of 25 mi. per hr. up 2 per cent grades with maximum operating speeds of 60 mi. per hr. on the level, and to provide regenerative braking on the down grades at speeds consistent with safe operation. Fig. 1 is a photograph of the locomotive in three-quarter view. Fig. 2 is an outline drawing of the side elevation, giving the general dimensions. Fig. 3 is a section through the apparatus cab, showing the location and arrangement of the principal pieces of apparatus.

It will be seen that the running gear is composed of four individual trucks; two end trucks having three axles each, and two center trucks having four axles each. These trucks are connected together by special articulation joints. The motor armatures are mounted on the axles and the motor fields are carried on the truck frames.

The superstructure is made in two sections of similar design with a third section between them. The third or central section contains the train heating equipment, which consists of an oil fired steam generator, together with water and oil tanks. This unit is complete in itself, and is carried over supports attached to the two middle trucks. It can be readily removed for repairs without interfering with any other part of the locomotive. It is placed between the two operating cabs in order to be easy of access to the engineer's helper or fireman, from either location.

FIG. 1—THREE-QUARTER VIEW OF LOCOMOTIVE

The two end sections are similar to each other in appearance. The operator's cab in either section is on the inner end next to the heater cab above described, in order that the operator can be convenient to the heater and in order to allow a maximum space for apparatus in the apparatus cab or outer end section. Another advantage of this arrangement of cabs is that the operator can have access to any section of the locomotive requiring his presence without passing through a section containing high-tension apparatus. The engineer's or operating cab contains a main or master con-

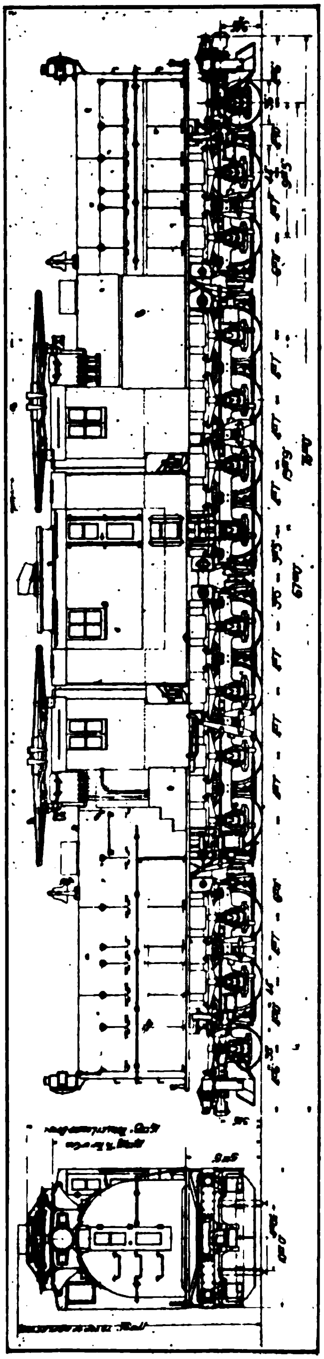


FIG. 2—OUTLINE DRAWING AND DIMENSIONS OF LOCOMOTIVE

troller, the air brake valves and handles, and an instrument panel, containing air gages, ammeters, and speed indicator. The engineer uses either of the two operating cabs according to the direction in which he is running.

A door gives access from the operating cab to the apparatus section which extends with a cylindrical top to the extreme end of the locomotive. The cylindrical construction naturally adapts itself to the protection of the apparatus included and in addition to this, it has the advantage of allowing a clear vision for the operator from his normal operating position. Contained in this apparatus section are the resistors and contactors to control the power circuits of the locomotive. The starting resistors are placed in two rows on each side of the central passage just above the floor of the superstructure and are covered at the sides by removable covers which when opened will allow the separate resistor boxes to be slid out upon the longitudinal running board outside of the apparatus cab. The air compressor for the air brakes, the motor-generator set for train lighting, and the storage battery for marker lights and emergency control, stand upon the same level as the resistors, and can be removed or replaced in a similar manner. Above the resistors are located the contactors with their arc chutes facing a central aisle two feet wide. This allows ample arcing space and room for inspection of contactors. Above the contactors is the cylindrical roof of the locomotive with trap doors for inspection of the back connections and insulation and for removing the contactors in case replacement is necessary. The whole design and arrangement of this apparatus cab lends itself to a maximum economy of cost and material, as well as to convenience of inspection and repair of apparatus.

MOTORS

The motors are of the well known bi-polar gearless design which were adopted by the New York Central Railroad 14 years ago, and which have proved by fourteen years' service, operating heavy passenger trains between Grand Central Station and Harmon, to be well suited for the service. This motor has demonstrated its remarkable reliability and low cost of maintenance.

To insure light weight per axle, flexibility in control. good truck arrangement for curving as well as for high-speed running, twelve motors are chosen, each of relatively small capacity. They are especially designed to withstand high temperature, being insulated with mica and asbestos.

Fig. 4 shows the motor armature complete, built directly on the axle with the wheels pressed and keyed in place. The continuous rating of each motor at 1000 volts and with 120 degrees rise by resistance is 266 h.p., corresponding to 3500 lb. tractive effort at the rim of the drivers at a speed of 28.4 mi. per hr. Forced ventilation is employed for cooling. The armature core is provided with holes for the passage of ventilating air. Ventilating blowers are located above each motor armature and deliver air at the commutator end of the motor where it divides, a part passing through the armature and a part back through and around the field coils where it escapes upwards and is afterwards used for ventilating the starting resistors.

This type of motor gives very high power efficiency in average operation, it having no journal bearings or gearing. It lends itself nicely to simple and compact locomotive design as the frame is made use of to furnish the entire path for the magnetic flux. The pole pieces and field coils are fastened to the cross transoms of the trucks and the magnetic flux passes horizontally in series through all twelve motors, finding a return path through the locomotive frame. The articulation joints between the trucks are made in such a manner that large surfaces are in contact to provide an easy path for the flux. The pole pieces are made flat in order to prevent the pole pieces from coming in contact with the armature during the vertical movement of the truck frame on its springs or when removing or assembling the armatures. A minimum clearance of 1/8 in. on each side is allowed between the armature and the pole piece tips. The brushholders are bolted to the transom allowing the brushes to move up and down with the fields as the frame rides on the truck springs.

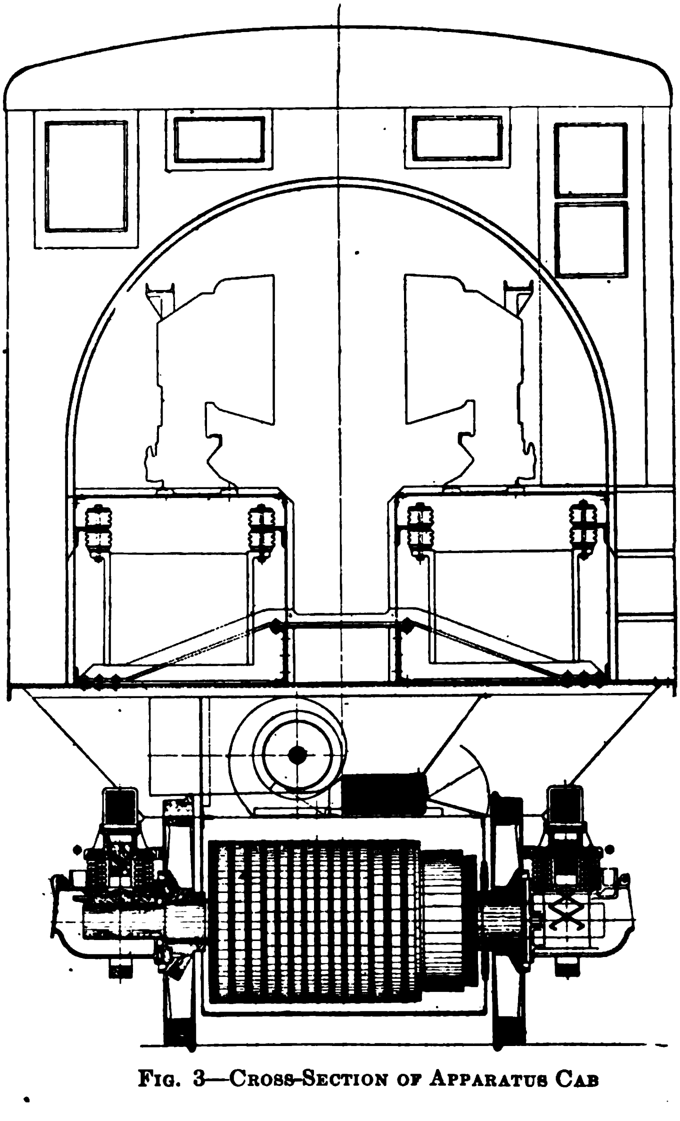


FIG. 3—CROSS-SECTION OF APPARATUS CAB

CONTROL

In choosing the control apparatus special care has been taken to use individual pieces of apparatus best suited to the particular requirements. Where single independently operating switches are necessary as on the resistance notches, electro-magnetic control is used. Where several switches are required to operate at one time as in changing from series to parallel motor connections, banks of switches with electro-pneumatic cam control are used, thus insuring positive operation, eliminating interlocks, and simplifying the wiring.

The control for motoring is arranged for four motor combinations.

The first combination has nine rheostatic steps, one full field step, and one tapped field step, with twelve motors in series across 3000 volts.

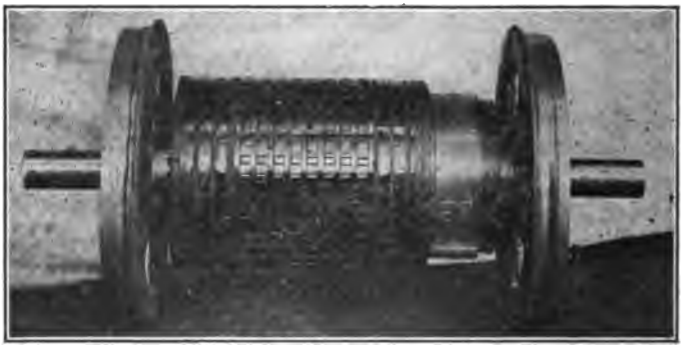


Fig. 4—Bi-Polar Gearless Armature and Wheels

The second combination has six rheostatic steps, one full field step, and one tapped field step, with six motors in series and two sets in multiple.

The third combination has eight rheostatic steps, one full field step, and one tapped field step, with four motors in series, and three sets in multiple.

The fourth combination has eight rheostatic steps, one full field step, and one tapped field step, with three motors in series, and four sets in multiple.

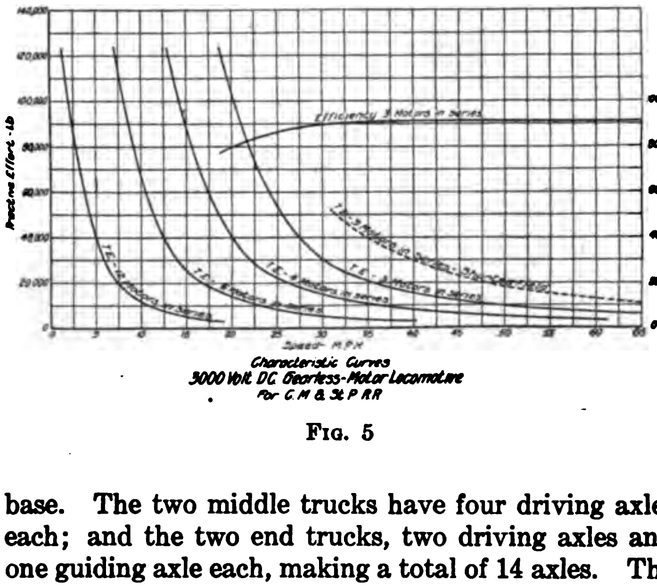
This results in a total of 39 control steps with a choice of eight operating speeds, exclusive of the resistance steps. The locomotive characteristics on the various steps are clearly shown in Fig. 5.

The regeneration of power for braking is accomplished in a simple manner by using some of the motors for exciting the fields of the others, which in turn are used as generators to return power to the line.

As a provision against short circuits, or extreme overloads, a quick acting circuit breaker is provided in the apparatus cab which will protect the circuit in less than 1/100 of a second.

MECHANICAL CONSTRUCTION

For flexibility in curving, the running gear is made up of four trucks, each of a relatively short wheel base. The two middle trucks have four driving axles each; and the two end trucks, two driving axles and one guiding axle each, making a total of 14 axles. The trucks are connected together with articulated joints which allow of no relative lateral movement between them, so that each truck positively leads the following truck. This is for the purpose of reducing flange wear on curves and lateral oscillation on tangent track.



Characteristic Curves
3000 Volt DC Gearless-Motor Locomotive
For C M & St P RR

FIG. 5

The most important problem that has to be faced in the design of a locomotive for high-speed passenger service is the problem of limiting as far as possible the lateral oscillations of the locomotive structure which tend to distort the track, and to minimize the effect on the track of such oscillations as occur. If a locomotive were built with a rigid wheel base as long as the total wheel base of the present locomotive (67 ft.), the lateral oscillations could not reach any large angular value. However, on account of the long wheel base, such a locomotive would be incapable of taking curves. By articulating the wheel base, as we have done, the locomotive is capable of accommodating itself to track curvature, but at the same time, on account of the articulation between trucks, and the consequent guiding effect of one truck on another, the lateral oscillations on tangent track are minimized in the same manner as would be done by the use of a long rigid wheel base.

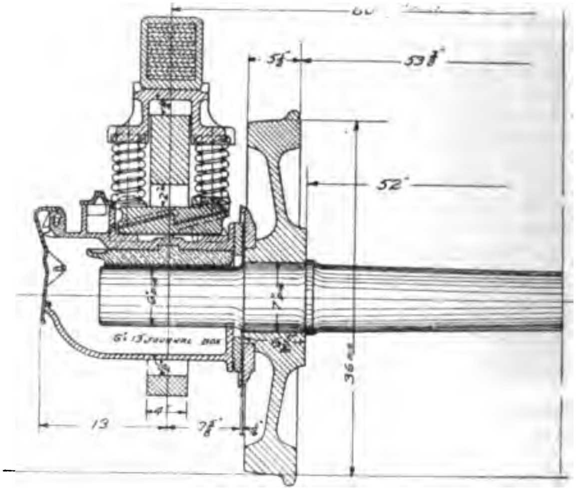


Fig. 6

To soften any lateral blow that may be given against the rail, the leading and trailing axles are allowed a movement of one-half inch relative to the truck frame, either way from their central position. This movement takes place against a resistance introduced by wedges above the journal boxes which tend to hold the box in its central position and to give a dead beat action opposing the motion. This wedge construction is illustrated in Fig. 6. To further protect the track from lateral displacement on the ties, the outer end of the superstructure is carried on rollers, bearing on inclined planes upon the truck frames, while the inner end of the superstructure is rigidly fastened to one of the middle trucks. This construction tends to hold the leading and trailing trucks in their central position. When a blow is delivered by the leading or trailing truck against the rail head, the superstructure, is displaced laterally across the outer truck. In such a sideways displacement, the weight of the superstructure rolls up on the inclined plane on that side, and thus transfers weight to the rail that is affected and increases the adhesion of the rail to the tie. This action really has two results. It not only increases the holding power between rail and tie at that point, but it introduces a time lag and increases the time and distance during which the pressure is delivered to the rail head.

Fig. 7—Side View of Complete Locomotive

As a matter of record, it should be said that the first of these new locomotives was delivered to the railway company at Deer Lodge, Montana, on December 14th, 1919, and was put in operation handling passenger trains between Deer Lodge and Avery.

For convenience of reference a table is attached giving a summary of the principal dimensions and characteristics of this locomotive.

Total weight	521,200 lb.	
Total weight on drivers	457,800	
Weight per driving axle	38,150	
Dead weight per driving axle	9,500	
Weight per idle axle	31,700	
Dead weight per idle axle	3,560	
Length overall	76 ft.	0 in.
Width overall	10 ft.	0 in.
Height over cabs	14 ft.	11⅝ in.

Height over pantograph, locked down	16 ft. 8 in.
Total wheel base	67 ft. 0 in.
Max. rigid wheel base	13 ft. 9 in.
Diameter of driving wheels	44 in.
Diameter of idle wheels	36 in.
Size of journals	6 in. by 13 in.
Dimensions of operator's cab	5 ft. by 10 ft.
Dimension of heater cab	14 ft. 11 in. by 10 ft.
Heater capacity	4,000 lb. steam per hr.
Water capacity	30,000 lb.
Oil capacity	6,000 lb.
Compressor capacity	150 cu. ft. per min.
Number of motors	12
Type of motor	(bi-polar) GE—100
Diameter of armature	29 in.
Clearance between bottom plate and top of rail	5¼ in.
Working range of pantograph	9 ft. 0 in.

Locomotive Rating	*Tapped field*	*Full field*
Total horsepower, 1 hr. motor rating	3,600	3,500
Total tractive effort 1 hr. motor rating	37,500	48,500
Speed mi. per hr	35.6	27.1
Total horsepower continuous	3,200	3,200
Total tractive effort continuous	32,000	42,000
Speed, mi. per hr	37.8	28.4

DISCUSSION ON "PASSENGER LOCOMOTIVES FOR CHICAGO, MILWAUKEE AND ST. PAUL RAILWAY" (BATCHELDER AND DODD) AND "THE BALDWIN-WESTINGHOUSE CHICAGO, MILWAUKEE AND ST. PAUL ELECTRIC LOCOMOTIVES" (STORER), PITTSBURGH, PA., MARCH 12, 1920.

S. Q. Hayes: There is one point I would like to ask Mr. Dodd. One of the illustrations of the G. E. locomotive—the one looking through the apparatus cab—apparently showed all the control leads lying loosely ontop of the contactors off on the left hand side. I was wondering whether that were temporary, or whether that was their permanent position in place.

R. L. Wilson: In the early days of the application of electric traction to railroads, I had some experience with it as a practical man. In those days the electric locomotive was so complicated—or it seemed that way to me, at any rate—that there not did seem to be any hope of anybody but an engineering apprentice understanding it. Since that time, I am informed that a man with ordinary intelligence can handle it. From what I have seen tonight it still looks complicated. However, the reliability of all these devices has been proved.

Now, to me, the main point that has been brought out tonight is the absolute divergence of opinion, if you like to so term it, as to how to meet a given problem, particularly mechanically. In looking at the different illustrations, we see that the locomotive described by Mr. Storer has a running gear very similar to the ordinary Pacific type of steam locomotives, which everybody is familiar with, and which apparently has proved satisfactory in a mechanical way, so far as running and the effect on the track is concerned. I imagine that fact is what lead to the adoption of that particular type of running gear, by the engineers who designed the locomotive in question. What led to the other design I do not know, unless it would be to gain electrical efficiency at high speeds. It seems to me that inherently that design would be expected to react on the track more than the other, and that certain devices would have to be introduced in the design and construction to overcome that inherent tendency.

I was much interested in Mr. Dodd's description of what the devices were, the wedges which are on the leading truck and the rollers rolling on an inclined plane elsewhere. That idea of a roller was used,—or something corresponding to it—in the first engines which were installed on the New York, New Haven & Hartford system, which had a tendency to oscillate. In that case it was not a roller rolling an inclined plane, but rather a cam effect at each corner, so that when the cab tended to oscillate, it raised the weight on this cam, which tended to bring the cab back into position and stop the oscillation. It seems to me that the fact that these rollers come into play, and act only on one side, and leave the other loose would not be a good thing, although it may work out otherwise.

Now, as Mr. Storer said, the low speed motor is inherently subject to higher losses, copper losses particularly, and I noticed in the paper here that the tractive effort of the locomotive that Mr. Dodd described is 46,000 lb., and that somewhere else it says something about the heating limit being 120 deg., corresponding to 3500 lb. tractive effort. As I understand, the specifications call for 25 miles an hour up a two per cent grade or thereabout, which corresponds to a tractive effort of about 50,000 lb., which would bring an overload on a motor designed for 46,000 lb. tractive effort.

I was wondering, in going up a long grade, which would take an hour to ascend, what temperature the motors would likely attain.

E. H. Martindale: In the paper presented by Mr. Dodd the statement is made: "The continuous rating of each motor at 1000 volts and with 120 deg. rise by resistance is 266 h. p., corresponding to 3500 lb. tractive effort at the rim of the drivers at a speed of 28.4 mi. per hr." I ask Mr. Dodd what the maximum operating temperature is which is allowed on the motor.

R. J. Wensley: I would like to ask Mr. Dodd a a question. A couple of years back, or possibly more, there were one or two disastrous wrecks on the New York Central with the low gravity motors, and there were some comments made at the time as to the effect of the low gravity motors on the track, and some claimed that the low gravity motor was to blame for the wreck, claiming that it caused the locomotive to climb off the track. I never heard that satisfactorily explained. Possibly Mr. Dodd can do that.

F. D. Hall: Considerable stress has been laid on the superior efficiency of the gearless motor at high speed. What is the average speed of the locomotive in its day's run?

R. E. Ferris: I would like to ask Mr. Dodd if he anticipates any trouble from snow when the motor is mounted on the axle, and in close to the truck, as the snow conditions are very severe on the Milwaukee Road.

A. M. Candy: Mr. R. L. Wilson brought out a question which I think should prove quite interesting namely, the reference to the complexity of the modern electric locomotive. About ten years ago I had the pleasure of being numbered amongst a group of some 25 or 30 young men who were assigned to work on the Pennsylvania terminal electrification in New York City. When the locomotives for that service were placed in operation a few weeks previous to the official opening of the terminal, there was some question as to whether or not the average railroad man(the locomotive engineer and fireman) could handle these locomotives satisfactorily. As a result, one of us was assigned to each locomotive to be on board at all times when it was in service, so that if anything went wrong of a minor nature the train service would be delayed as little as possible. Each of us prepared himself with a small notebook, in which we had all the schematic diagrams and main wiring diagrams of locomotive, together with the names and numbers of all the wires, control fingers, interlock contacts, etc. We had the sequence of the various switches and the names and numbers of practically all of the control wiring committed to memory, so that at a moment's notice we could tell exactly where to look for trouble in the event of failure of any particular switch or any of the auxiliary equipment. We did an unlimited amount of tutoring with the engineers to teach them this information, and it was quite customary for us to quizz one another as to the method of procedure assuming certain failures should take place. In fact we vied with each other to devise imaginary troubles and the simplest means of correcting or eliminating the failures. The most of us were located on this job and worked faithfully for approximately one year. During all that time, however if my memory serves me correctly, there was not one of us who ever got an opportunity to display the knowledge which he had, because the apparatus, even in the minutest detail, never failed to function properly. Of course, the locomotives were inspected at regular intervals, but the strength and simplicity of the equipment was such that even though the locomotives were operated a great many more miles per inspection than is customary with steam locomotives, they did not cause any train detentions.

It seems to me that in so far as the subject of general electrification of steam railroads is concerned at least, it would be better for us to consider the electric locomotive from the standpoint of being a perfected complex mechanism rather than an experimental complicated mechanism, and that it would be better to point out the advantages of the electric locomotive versus the steam locomotive, rather than to argue extensively among ourselves as to which particular type or design of locomotive is most satisfactory.

There is one other point which I desire to mention, and that is the one brought out by Mr. Dodd in reference to his locomotive, namely, the very free and unobstructed view presented to the engineer through the cab window, looking out along the arch shaped dome over the switching and resistance units. Judging from the illustration thrown upon the screen, it strikes me that the view is certainly not any more obstructed than that obtaining for the average steam locomotive. I do not believe that Mr. Storer mentioned this point in connection with his locomotive, and therefore, I think it would be in order if he would make a few statements relative to this feature.

L. J. Hibbard: Mr. Dodd, in describing his locomotive, mentioned the fact that each main motor had a separate blower motor. I would like to have him tell me what type of motor they use for running each separate blower, and if they expect to have any trouble on these separate blower motors.

Calvert Townley: If I may, I would like to emphasize the remarks brought out by the next to the last speaker. It is well known, that every electrification of steam railroads has been successful. We have now been electrifying roads for a number of years with different types of locomotives. Some of them are more efficient than others, some have lower maintenance cost than others, they vary in degrees of success, but everyone of them has been successful, and as a matter of fact, notwithstanding the large number of types which have been put into use, there are not anything like as many electric locomotive types as there are standard steam locomotive types, and the railroads, accustomed as they are to dealing with steam locomotives, do not think anything of having a large number of different types of steam locomotive on their lines for use for different purposes.

In much of our discussion of this subject, we argue about the value of respective locomotives, and we are in danger of giving our steam railroad friends the idea that the electric locomotive development is doubtful, that we are not sure of ourselves, and that perhaps they had better wait a while and see what happens—

that is the attitude they are likely to take under such circumstances.

Of the two locomotives described tonight, one will perhaps show a better efficiency than the other, one may show a lower cost of maintenance than the other, but I feel sure that each of them will do the work of the Chicago, Milwaukee and St. Paul Railway, to the satisfaction of the railroad company. My reason for speaking in this way is not to discourage discussion, I hope the discussion will bring out every feature of possible advantage—but I think we should emphasize the uniform success which has attended all electrification. Do not let the steam railroad people think we are not sure of ourselves.

If the electrification of steam railroads, ever gets started, it will call for so great a number of locomotives, and so rapidly that if all the existing facilities of the different companies, be devoted to that purpose, they will be nowhere near sufficient. Therefore, it is perhaps well that they do not go too fast. No steam railroad man with an electrified section, would now consider going back to steam, even though from our view point he may have been most reactionary, and looked askance at the electrified part of his lines. After he has become familiar with the electrified system, he recognizes its many advantages, tremendously increased traction, greater speed improved traffic possibilities and other incidental advantages which the electric locomotive affords to an extent not practical with the steam locomotive. The poorest electric locomotive compared with the best steam locomotive easily exhibits a decided margin in its favor.

C. M. Davis: I ask Mr. Storer, to explain the detail of the exciting motor on the leading truck. As I understand it, this motor is geared to the truck axle in much the same way that the ordinary railway motor is geared—this being the case, the gearing, of course, works just the reverse to what it does in the case of the railway motor, when the motor is used to drive the car, that is, a large gear in this case is driven by small pinions.

I should like to have brought out the method of accomplishing this gearing, whether springs are used to soften blows on the teeth when running at high speeds, 60 miles an hour, or what provision is made for taking care of that particular feature.

S. T. Dodd: Mr. Hayes asked about some leads shown in one of the illustrations. The leads were temporary only. The picture was taken before completion of the cab and was used by me to show the location of apparatus before installation of the outside part of the cab.

Another question was asked as to service capacity. I can only say that the motors have ample capacity to perform the service specified by the railroad company and outlined in the paper.

Mr. Ferris asked about the possibility of snow troubles. There will be no trouble on this score as we know from our experience with the New York Central gearless locomotives. If I had been prepared for such a question, I could have shown pictures of electric locomotives running through snow piled up to the windows. Possibly he misunderstands the construction of the gearless motors and thinks they are entirely unprotected. As a matter of fact, there are protecting sheets under the armatures and across the ends so that the armatures are practically as well protected from weather as in the ordinary railway motor. You know that with modern insulation we have very little trouble in that respect from railway motors.

Mr. Candy has referred to the complexity of the electric locomotives. The question of complexity of any mechanism depends largely on the amount of analysis to which you subject it. A steam locomotive is a complex piece of apparatus if you go into a careful analysis of all its features from the chemical composition of the coal to the output at the drivers. I am afraid that electrical engineers too often look at problems from the electrical standpoint and do not pay sufficient attention to the mechanical standpoint. We build locomotives to do service and that is what we should keep in mind rather than the details that go to make up the final combination. Maintenance of equipment tells the whole story, and in that respect the electric locomotive has a record of which to be proud.

The question has been raised as to the center of gravity of these locomotives. The center of gravity of the locomotive as a whole is 57 in. above the rail head. In the first place, I wish to point out that the location of the motor is not the only thing that determines the height of center of gravity. The weight of drawbar and couplers, the end and side frame castings, the cab construction and other mechanical features fix the height within rather narrow limits and the location of the motors is of secondary importance. In the second place the location of the center of gravity has very little to do with the riding qualities of a double ended locomotive. With such a locomotive equipped with guiding trucks at each end, the locomotive is steered by these guiding trucks and the thrust against the rails is determined by the construction of those trucks rather than by the location of the center of gravity of the main body of the locomotive. This is the reason that in my paper I emphasized the construction of the leading and trailing ends of the locomotive. I wished to show the precautions we had taken to cushion the track thrust when running at high speeds.

Two very different types of locomotives have been presented tonight. It is a source of congratulation both for the users and manufacturers of electric locomotives that such should be the case. In the United States at the present time there are only about 350 electric locomotives in service on steam roads. If we include the trolley roads there may be 700 electric locomotives in service. At the same time there are 63,000 steam locomotives in service. We are just beginning to touch the job of the construction and application of electric equipment to the transportation problem. The sooner we learn how to build locomotives which will meet all the conditions of service, the better it will be for all of us, and I agree with Mr. Storer that it is a splendid thing for all of us to obtain this experience and find out the lines along which we must progress because the big developments are still ahead of us.

N. W. Storer: While appreciating very much the discussions we have had tonight, I regret very much that some of the steam railway engineers who are présent did not take part in it. It is all very well for electrical engineers to discuss electric locomotives and to extol their virtues, but we need the side lights that the railway engineers can throw on the subject.

Mr. Townley made an excellent point in bringing to our attention the fact that the electric locomotives that have been built have all done their work better than any steam locomotives that have been put on the road. We should keep that fact in mind.

In the few remarks I made to draw out Mr. Dodd, I had no intention to disparage the gearless type of locomotive. I am more than happy to have that locomotive built, and also one of our own manufacture, which is so absolutely different, and to have them both tried out on the same road. It makes little difference which one does the work the best. It is going to be to the advantage of the electric railway industry to have all types of locomotives thoroughly tested out, in order that the best ones may finally be determined. We are very glad that the two types are to be operating side by side, and if the low friction losses in the gearless type of locomotive, and the absence of gears and other connections between the armatures and the axles, are sufficient to over-balance some of the other features which are not so desirable and which need not be mentioned by me, it may solve some of the difficulties in connection with electric locomotives, one of the greatest of which is the transmission of the torque of the motor to the driving wheels.

I agree very largely with what Mr. Dodd has said about the kick of the locomotive and about the rolling of the cab with the weight up high. There is no question about it, where the cab weight is up high, as it is in our locomotive, it steers a straight course regardless of variations in the road-bed. It is certainly easier on the road-bed to have the weight high than it is to have it down close to the track.

Mr. Davis has asked a question about the performance of the little axle-driven generator. As pointed out in the paper, this is geared directly to the axle like an ordinary interurban motor, and is about the same size. The output of the generator is never more than 25 to 30 kw., so that the duty on the gears is very limited and no special provision has been made to avoid vibration. We do not anticipate any trouble from it.

Presented at the 350th meeting of the American Institute of Electrical Engineers, Boston, Mass., April 9, 1920.

FIXATION OF ATMOSPHERIC NITROGEN THE SILENT ELECTRIC DISCHARGE PROCESS-I

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AND

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THE very great increase in the demand for nitrates during the war for the manufacture of explosives and fertilizers, together with the shortage of imported Chile saltpeter, which was the principal source of nitrates before the war, has been emphasized sufficiently in previous papers. This condition of increased demand and curtailed supply led not only to the establishment of many manufacturing plants for the production of nitrates by well-known methods, but also to laboratory research and commercial development in connection with promising new processes.

Such a process, for obtaining nitric acid from the air by means of the silent electric or corona discharge at high voltage, has been investigated throughout a period of several years at Purdue University under the auspices of the Engineering Experiment Station of that institution. Improvements in the yield, resulting from a given input of electrical energy, are being made quite frequently as the investigation progresses and many of the peculiarities of both electrical and chemical reactions of the oxides of nitrogen, combined with ozone, are being disclosed as phenomena incidental to the process under consideration. It has, nevertheless, been considered advisable to present, as a progress report, the results obtained thus far in order that, in the spirit of co-operative research, discussions and suggestions of value will result which in turn will hasten the development of the process. With this object in view, the results of the investigation being carried on at Purdue University are presented in this paper.

It is well to point out at first the distinction between the process herein described and the arc method of nitrogen fixation. In the former, air is passed through an electro-static field of such intensity that marked corona and some static sparks are produced, but no power arc is established and the temperature is increased but slightly above that of the incoming air. While the arc process of nitrogen fixation is dependent principally upon the very high temperature of the power arc, the silent discharge reaction is the result of electrical ionization of the air at a relatively low temperature.

The fact that electro-static brush discharges and sparks will produce, in a confined volume of air, various oxides of nitrogen and ozone, has been known for many years. Oxidation of these lower oxides of nitrogen by an excess of ozone results in nitrogen pentoxide, $N_2 O_5$. This gas, when absorbed in water, produces nitric acid from which the desired nitrates may readily be formed.

EARLIER INVESTIGATIONS

The investigation, to which this paper forms an introduction, was the result of the translation from the German, by Messrs. G. N. Unger, G. W. Payne and F. S. Weimer, of an article entitled, "The Formation of Nitric Oxides by the Silent Electric Discharge in a Siemen's Tube." The latter was submitted by Hugo Spiel for a doctor's degree at The Technical High School, Vienna, in 1909. Spiel gives credit for the first publication of the effect of this silent electric discharge upon air as follows:

> The first observations of the silent electric discharge in literature appeared about 1870, following the work of Andrew, Hauzau, Jean, Thenard and Boillit, with ozone and nitric acid. It concerned the decomposition of carbonic acid and turned the general interest to the chemical effect of the electric discharge in gases. In 1873 the French physicist, Moncel, published a work entitled, "Concerning the Condensed Discharge of the Induction Coil," in which he claimed to have discovered the above phenomena in 1853.

Following a review of the previous work done by others upon the problem, Spiel described a series of laboratory research investigations performed by him upon fixed volumes of air and various artificial nitrogen-oxygen mixtures. Spiel's apparatus consisted of a small glass or quartz Siemen's tube, using acidified water electrodes upon either side of an annular air space of only 3 mm. thickness. The electro-static discharge, within this space, was produced by an induction coil and condenser having a sparking distance of 40 cm., and a frequency of approximately 40 cycles per second.

As a result of the exposure of a confined volume of air to the electro-static discharge, at different pressures and voltages, Spiel noted during a single test; first, a decrease in pressure and later, at the minimum pressure, a reversal of the reaction. This reversal was indicated by the presence of brown fumes of $N O_2$. If the electric discharge was continued beyond this point, the pressure increased to the initial, or to a greater pressure, when equilibrium was established.

The conclusions reached by Spiel have been summarized in the appendix of this paper, because of the bearing they have upon this investigation and the inaccessibility of the original article.

EARLY INVESTIGATIONS AT PURDUE UNIVERSITY

In planning a further investigation of this process,[1] it was decided that although the work must necessarily be performed with laboratory apparatus upon a comparatively small scale, when contrasted with commercial arc plants, the equipment should be designed and constructed in such a way as to provide for a continuous flow of air through the electric field. There is no reason apparent, therefore, to prevent the construction and continuous operation of a commercial plant based upon the same principle.

To produce the so-called "silent electric discharge" in a column of air, it is, of course, necessary to apply

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[1] Thesis entitled "The Fixation of Atmospheric Nitrogen by the Silent Electric Discharge at High Voltage," 1917 by G.W. Payne, G. N. Unger and F. S. Weimer.

an electric potential between conducting plates or concentric pipes, Fig. 1, sufficient in magnitude to exceed the dielectric strength of air under the particular pressure at which such air is being forced through the apparatus, *i. e.*, the potential gradient must be such that the molecules of the air are torn apart into ions and neutral atoms which may reunite in new combinations. Such ionized air is a comparatively good conductor of electricity and unless special precautions are taken in the design of the tube, a power arc will result at one point and the potential will be lowered at other points in the field, thus eliminating the reaction.

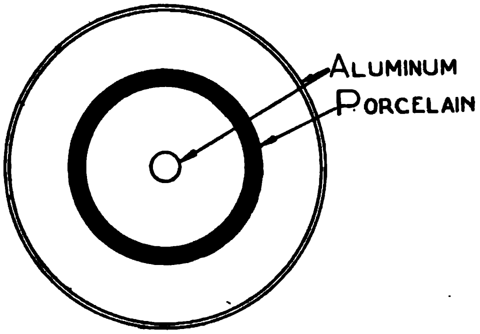

FIG. 1—ELEVATION OF CORONA CHAMBER

Original Apparatus. The apparatus used in this first investigation is illustrated in Fig. 2. Following the passage of the air from the compressor on the left through air chamber, meter and drying apparatus in sequence, it passes into the discharge tube. This consists of a vertical aluminum tube 37.8 in. (96 cm.) in length and 2.91 in. (7.4 cm.) internal diameter, inside of which was mounted concentrically a glass tube of slightly greater length, 2.5 in. (6.35 cm.) diameter and 0.125 in. (0.318 cm.) thickness. This glass tube provides the barrier of higher dielectric strength than air to prevent the arc from forming. The potential is applied between a coaxial aluminum rod of 0.625 in. (1.59 cm.) diameter and the outer aluminum tube. The air passes upward between the glass and aluminum tube, and thence the gases pass downward between the central rod and the glass tube, emerging from the bottom into the mixing chamber. Receptacles are provided at the right with glass beads and distilled water or a solution of $N/10$ potassium hydroxide (KOH)for the fine subdivision and absorption of the gases. Traps are placed upon either side of the drying bottles to prevent the sulphuric acid, (H_2SO_4) which is used as the drying agent, from being carried into the meter or discharge tube. A similar trap is located between the mixing chamber and absorption apparatus to prevent the absorption water from being forced back into the mixing chamber. Manometers and thermometers are installed, as indicated in the figure, for the purpose of determining pressures and temperatures of entering air and effluent gases, respectively.

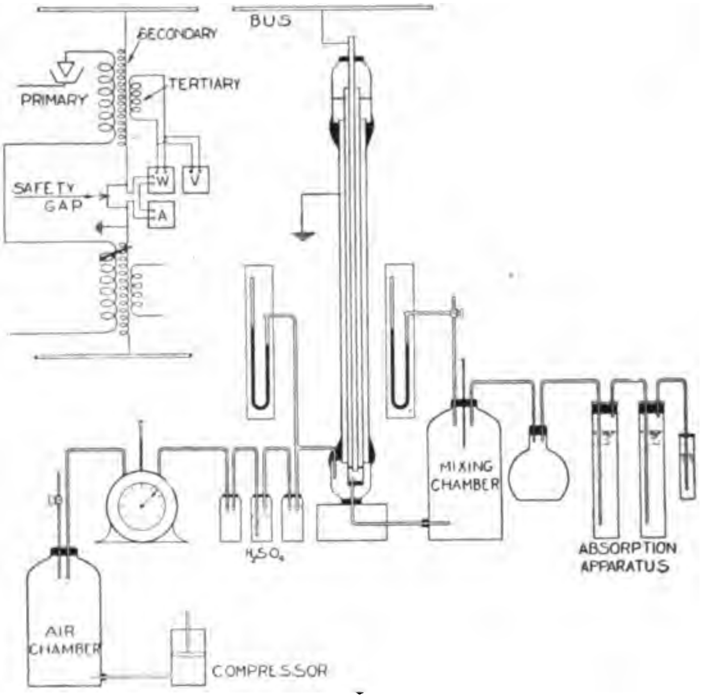

FIG. 2—ORIGINAL APPARATUS

Power Measurement. A potential in the neighborhood of 30 kilovolts, alternating at a frequency of 60 cycles, was furnished by a 200-kilovolt, 50-kv-a., air-cooled transformer. The tertiary coil of this transformer, whose potential was calibrated against the A. I. E. E. standard sphere gap in the secondary circuit, provided a convenient means of reading voltage. The potential terminals of the wattmeter were also connected to the tertiary coil. An ammeter and the current coil of a wattmeter were connected, as indicated in the figure, between the neutral point of the transformer winding and ground. Only one side of the transformer was used to supply the potential to the discharge tube. Protective film cut-outs, connected around the meters, prevented accident in case the ground wires of the meters or tube should become disconnected. A Rowland dynamometer, calibrated over a wide range of power factor as a wattmeter, was used in these early tests for measuring the power input to the discharge tube. The difference between the readings of this dynamometer, with and without the tube connection, provided a measure of the net power supplied to the tube, quite apart from any transformer or corona losses on connecting bus bars. This method of power measurement is the same as that used in determining corona loss on transmission lines previously reported to the Institute.[2]

With the methods of gas absorption and power measurement outlined, it was a comparatively simple matter to titrate the nitric acid formed or determine the amount of alkali neutralized thereby per kw-hr. of energy expended.

Conclusions of Original Investigation. The results of three (3) tests by Messrs. Unger, Payne, and Weimer are listed in Table I.

A study of Table I indicates that thus far the yield by the silent discharge method is very much smaller than those of the arc processes. Some of the conclusions derived from the investigation as indicated seemed however, to warrant further research along similar lines.

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[2] "Corona Losses between Wires at High Voltages," C. Francis Harding, Transactions A. I. E. E. Vol. XXXI (1912), page 1035.

1. The reaction is not necessarily a thermal one, but nitrogen pentoxide (N_2O_5) is formed at low temperatures by the action of the corona at high voltages.

2. Although this reaction is accompanied by much free ozone (O_3), the lower oxides of nitrogen, so objectionable in the arc process, are apparently not present to any great extent, except in case of reversal of the reaction.

3. Since the effluent gases consist principally of a mixture of nitrogen pentoxide (N_2O_5), ozone (O_3) and air, the absorption is more readily accomplished than with the arc process.

TABLE I.

NITRIC ACID YIELDS.

Test	Grams HNO_3 per kw-hr.
No. 1	4.17
2	2.55
3	5.55

4. The gases emerge at comparatively low temperatures. Little energy is therefore carried away in the form of heat and no apparatus is needed to make efficient use of this heat energy.

5. The process is a continuous one and, as such, is more adaptable to commercialization than previous investigations of the silent discharge process which have been limited to a single confined volume of air.

IMPROVED APPARATUS

Encouraged by the possibilities of this process, in spite of the relatively low yields available thus far, the work was taken over upon a larger scale by the Engineering Experiment Station with the assistance of Messrs. H. W. Asire, S. S. Green and H. C. Thuerk.[3]

Apparatus operating upon the same principle, but much larger and more commercial in design, was constructed as indicated in Fig. 3.

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[3] Thesis entitled "The Fixation of Atmospheric Nitrogen by the Silent Electric Discharge Method," 1918 by H. C. Thuerk and S. S. Green.

The corona discharge takes place inside an aluminum tube five feet (1.52 m.) long and six inches (15.2 cm.) in diameter, mounted in a vertical position, the lower end being about 18 inches (45.7 cm.) from the floor. A 5/8-inch (1.59-cm.) aluminum rod is placed on the axis of the tube and extends 18 inches (45.7 cm.) above the end of the tube. The lower end of the rod is screwed into the end of a ½-inch (1.27-cm.) bakelite tube 15 inches (38.1 cm.) in length. A perforated bakelite disk, about four inches (10.2 cm.) in diameter, is arranged so that it may be screwed onto the outside of the bakelite tube. This disk furnishes the support for a glazed porcelain tube of three inches (7.62 cm.)

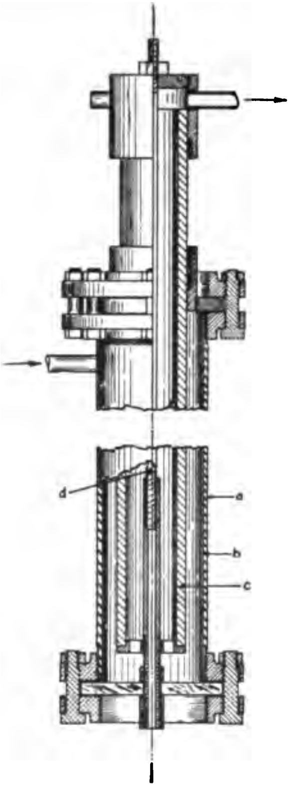

FIG. 3—DISCHARGE TUBE

a.—Wrought iron casing.
b.—Aluminum lining.
c.—Porcelain dielectric.
d.—Aluminum electrode.

inside diameter and ½ inch (1.27 cm.) thickness, the upper end of which projects beyond the end of the aluminum tube, but not as high as the aluminum rod. The top of the tube is closed with a bakelite cap placed over the end of the porcelain tube. A collar, also of bakelite, is inserted between the aluminum and the porcelain tubes. The joints between the porcelain and the bakelite are sealed with a rosin, beeswax and sealing wax compound, which is not appreciably affected by the gases. The bottom of the tube is sealed with a piece of thick plate glass, through the middle of which the bakelite tube is passed. By the use of a mirror, placed on the floor at the proper angle, a view of the inside of the tube is afforded.

Surrounding the aluminum tube is a length of wrought iron pipe, having flanges screwed on either end in such a way that the bakelite collar at the top and the plate glass at the bottom may be clamped securely.

In order that the temperature of the tube may be raised at will, a heating element, consisting of several hundred feet of iron wire, is provided. The heating element, wrought iron pipe and flanges are covered with a magnesia covering which decreases the radiation.

Air enters the tube near the top, and passes downward between the aluminum and porcelain tubes, through the holes in the bakelite disk and up through the inside of the porcelain tube, exhausting through a bakelite tube screwed into the cap over the end of the porcelain. The path of the air is shown in Fig. 4.

A mercury manometer, connected to the inlet pipe, indicates the relative pressure in the discharge tube. The temperatures of the incoming and outgoing gases are measured by alcohol thermometers whose bulbs are in actual contact with the gases. Mercury thermometers were not used on account of the breaking up of the mercury column due to the static charges formed. The mercury is also a conductor which is, of course, a serious disadvantage, and makes their use unsatisfactory.

The aluminum rod is connected to the high-tension transformer, while the outer iron casing is grounded.

With from 50 to 60 kv. impressed on the discharge tube, the space both inside and outside of the porcelain tube may be filled with corona discharge.

Absorption Apparatus. The absorption apparatus consists of an absorbing chamber, separator and precipitator. This apparatus will be more easily understood by reference to Fig. 5.

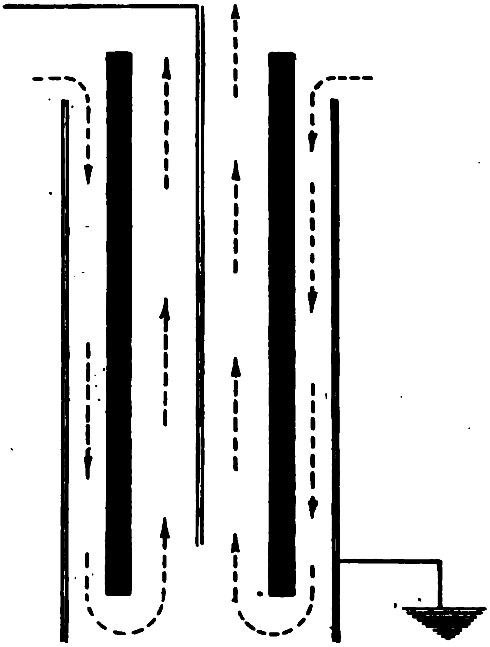

FIG. 4—PATH OF AIR IN DISCHARGE TUBE

The gases, leaving the discharge tube, bubble through a solution of sodium hydroxide (NaOH) of known strength, contained in the absorption chamber. The bubbles of gas are made to come into contact with quite a large wetted surface as the chamber is partially filled with very short lengths of small glass tubing. The use of the tubing makes possible relatively high air velocities with but little back pressure. The absorbing liquid is titrated both before and after the run, in order to ascertain the amount of HNO_3 absorbed.

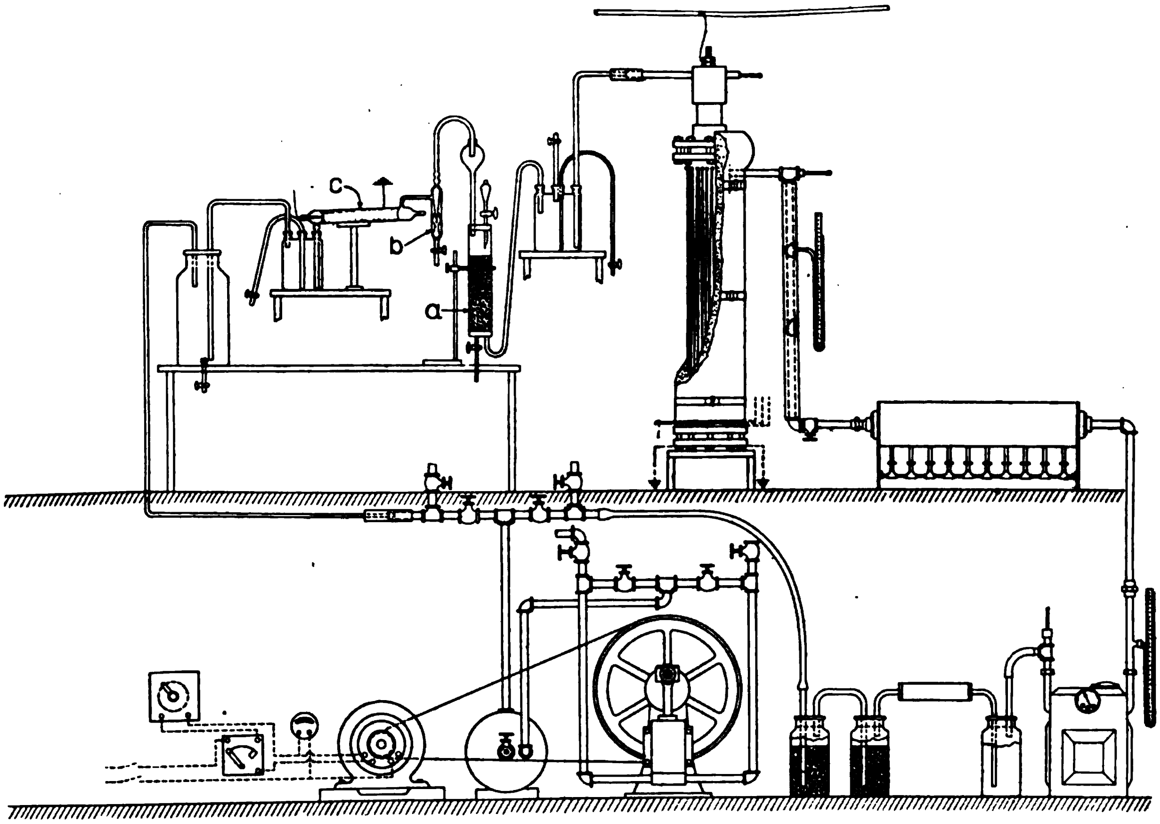

FIG. 5—IMPROVED APPARATUS

a.—Absorption chamber. b.—Separator.
c—Precipitator.

At certain air velocities, a fog is formed over the absorbing liquid and, in order to recover the acid carried with it, a separator, Fig. 5, working on the principle of the steam separator, is connected in series with the absorption chamber. In this separator the gas is forced to turn through an angle of 180 deg., while moving at a high velocity with the result that the heavier particles are thrown down, and collected in the bottom of the chamber.

From the separator, the gases pass to the precipitator, Fig. 5, in order that the finer particles of fog may be collected. This device is patterned after that of Cotrell. A glass tube, about 1½ inches (3.81 cm.) in diameter and 23 inches (58.4 cm.) in length, is coated with tinfoil up to within about six inches (15.2 cm.) of either end. The air passes in and out through the two side outlets blown in the glass near either end. A small (1/16-inch (1.59-mm.) diameter) aluminum wire, threaded throughout its entire length, is sealed into two small glass tubes about four inches (10.2 cm.) in length, which in turn are sealed into either end of the precipitator tube. Thus, quite a long creepage distance is secured, which is necessary when the surfaces become wet with nitric acid. This tube is operated with an induction coil giving a one-half inch (1.27 cm.) spark between needle points.

With the induction coil connected to the precipitator in such a way that the larger lobe of the wave is impressed on the threaded wire, the tube seems to work well. The smaller lobe is more or less excluded by point plate action.

Air Control. A small piston air pump is connected to a system of valves, in such a way that the discharge tube may be operated under pressures above or below the atmosphere, by a simple manipulation of the valves. The valves are provided with a pointer and dial, so that settings may be duplicated at any time. The fluctuations due to the pump are smoothed out by the use of an air tank, whether the system is working under pressure or vacuum. The air pump is driven by a d-c. motor, arranged with armature control of the speed. A voltmeter is connected across the armature circuit to act as an indicator of the motor speed.

Air leaving the pump bubbles through sulphuric acid, contained in two chambers in series, and then passes on through a tube containing soda lime. By this process the air is rendered practically free from moisture and carbon dioxide.

A special gas meter, having a capacity of 100 cu. ft., (28.32 liters) is connected to a settling chamber through which the air passes after leaving the soda lime chamber. By use of the large dial with which the meter is provided, readings are possible to one one-hundredth (0.01) of a cu. ft. (0.283 liter). The temperature and pressure of the air at the air meter are measured because the air meter is not capable of withstanding all of the pressures used in the discharge tube.

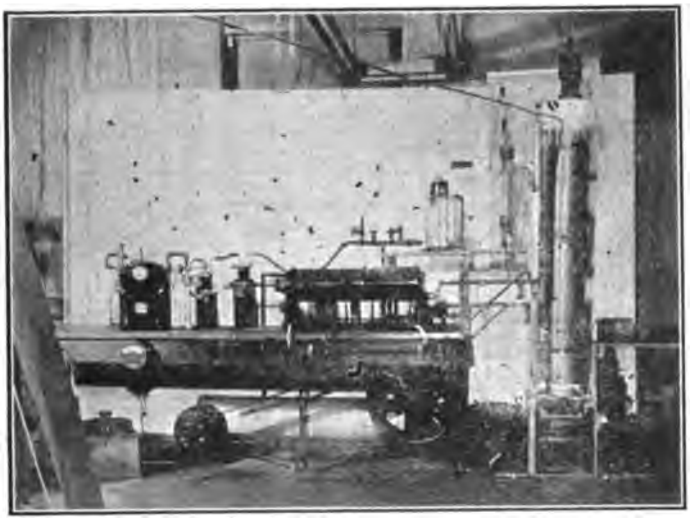

FIG. 6—EXPERIMENTAL APPARATUS

Upon leaving the air meter, the air passes through an air heater, where its temperature may be raised as much as required. This heater consists of a long tube, heated by ten gas burners, properly insulated to prevent excessive radiation. In some of the earlier work, the heater was used in conjunction with the heating element on the discharge tube to hold a constant temperature. It was found however, after several trials, that small temperature changes did not seem to affect the yields materially, and as a result all of the tests recorded in this paper were made at room temperature.

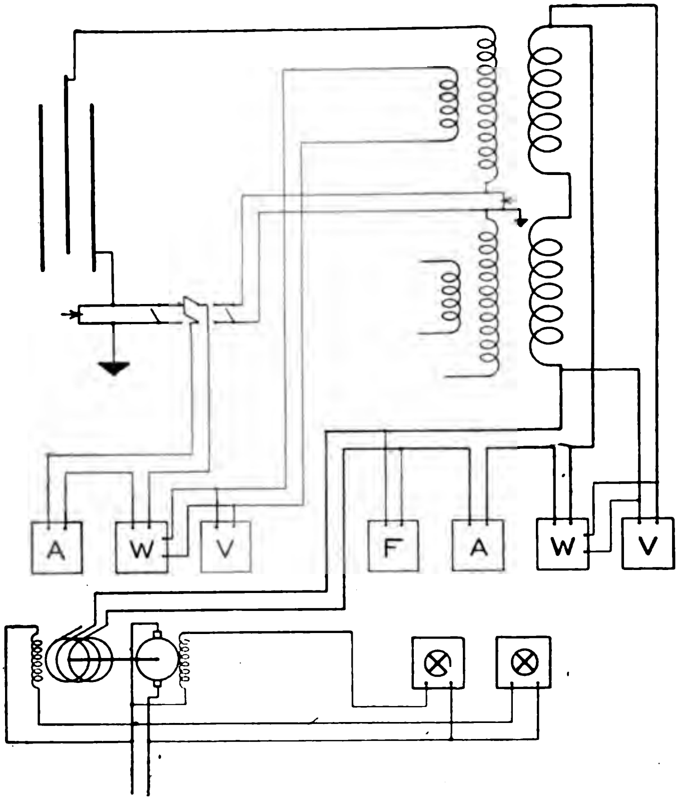

FIG. 7—DIAGRAM OF CONNECTIONS

This was of course held as nearly constant as possible. It is planned to make a set of runs in which the temperature will be made the independent variable. Upon leaving the gas heater the air passes through a valve, used for regulating the tube pressure, and then enters the discharge tube.

Power Measurement. The high-tension winding of the transformer was grounded at its neutral point. The other terminals were connected to high-tension buses. Each half of the secondary may be operated up to 100 kv. The excitation for the transformer is furnished by an alternator which gives practically a sine wave.

A test table has been arranged, equipped with various meters and rheostats. On this table, switching arrangements, Fig. 7, are provided so that the current in either circuit, *i. e.* from transformer to ground or, from discharge tube to ground, may be read on the same milliammeter. These currents differ because of the capacity of the transformer coils to ground, and also because of the actual electrostatic leakage occurring in the transformer.

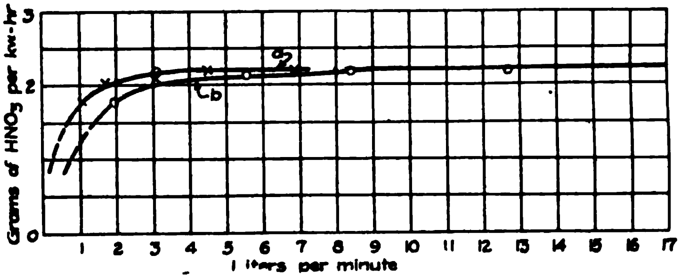

Fig. 8—460 mm.

a.—n. t. p. b.—actual.

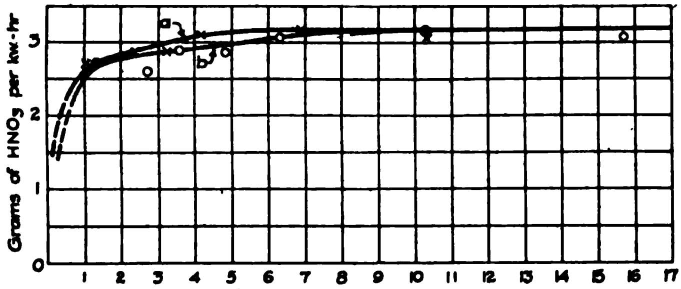

Fig. 9—560 mm.

a.—n. t. p. b.—actual.

A special low-reading wattmeter, whose potential coil is connected to the tertiary coil on the transformer, has its current coil in series with the milliammeter in order that readings of watts in tube circuit and transformer circuit may be taken independently. There is some considerable variation in the readings of the wattmeter in the two circuits with varying pressure in the discharge tube. This variation in wattmeter readings appears to be a function of the static sparks in the tube, and is greater than can be accounted for by capacity effects alone. This matter will be the subject of future investigation.

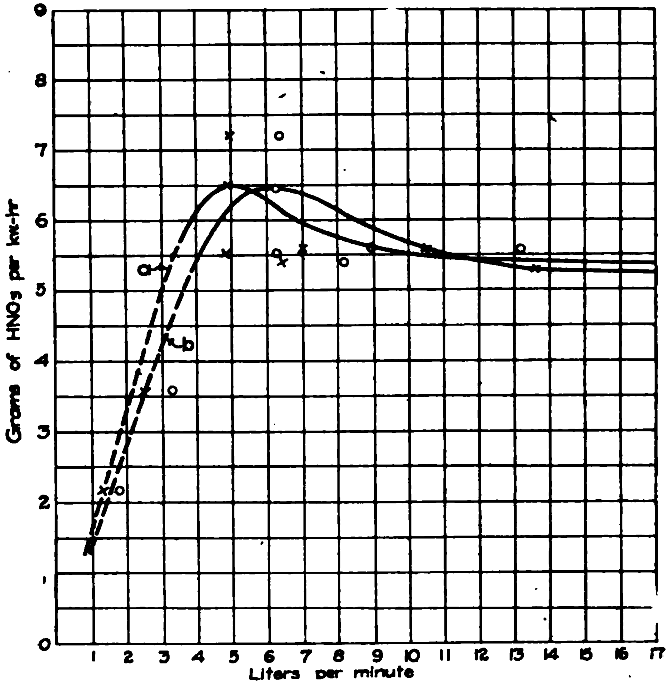

FIG. 10—660 MM.

a.—n. t. p. b.—actual.

Meters were connected in the primary circuit of the transformer for measuring the input power. The net power measured from the primary side has checked that observed on the secondary side within 10 per cent in practically every run made.

Results of Tests. A series of runs at pressures of 460, 560, 660, 710, and 780 mm. has been made. The power in the transformer circuit to ground was held constant during all of these runs. The frequency was 60 cycles. Each run was 45 minutes in length. For each pressure a series of runs was made with air velocity as the variable. Figs. 8, 9, 10, 11, and 12 show the yield of HNO_3 per kilowatt-hour of energy expended in the tube. The curves marked "actual velocity" refer to the actual velocity of a particle in passing through the discharge tube.

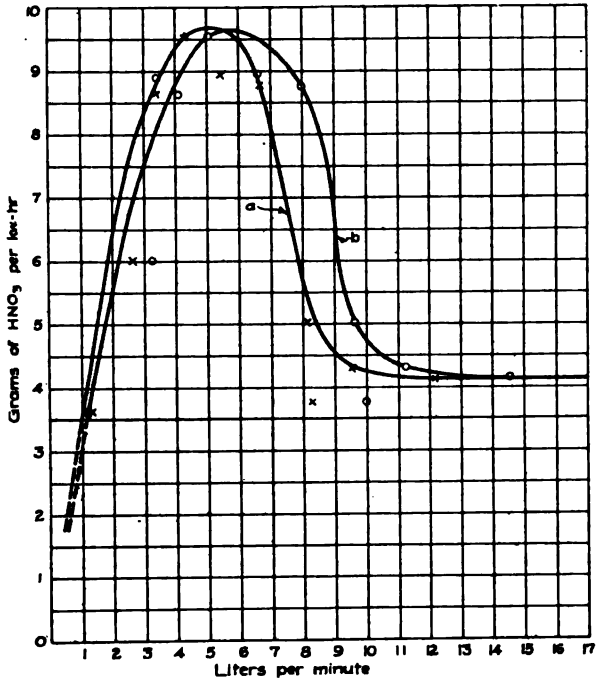

Fig. 11—710 mm.

a.—n. t. p. b.—actual.

Figs. 13 and 14 show the relation between pressure and yield with constant air velocity. From these curves it may be seen that the yield at the lower velocities is limited, but that at the higher velocities the limit has not yet been approached.

From the equations for the formation of nitric acid, it may be seen that the concentration of N_2O_5 will be half that of *N O* for any given value of nitric acid.

$$2NO + O_3 = N_2O_5$$
$$N_2O_5 + H_2O = 2HNO_3$$

The N O first formed is oxidized to N_2O_5 by the excess of ozone present so the N_2O_5 may be contrasted with 2 N O for the same yield of nitric acid.

The relation between per cent concentration of fixed nitrogen, calculated as N O, velocity and pressure

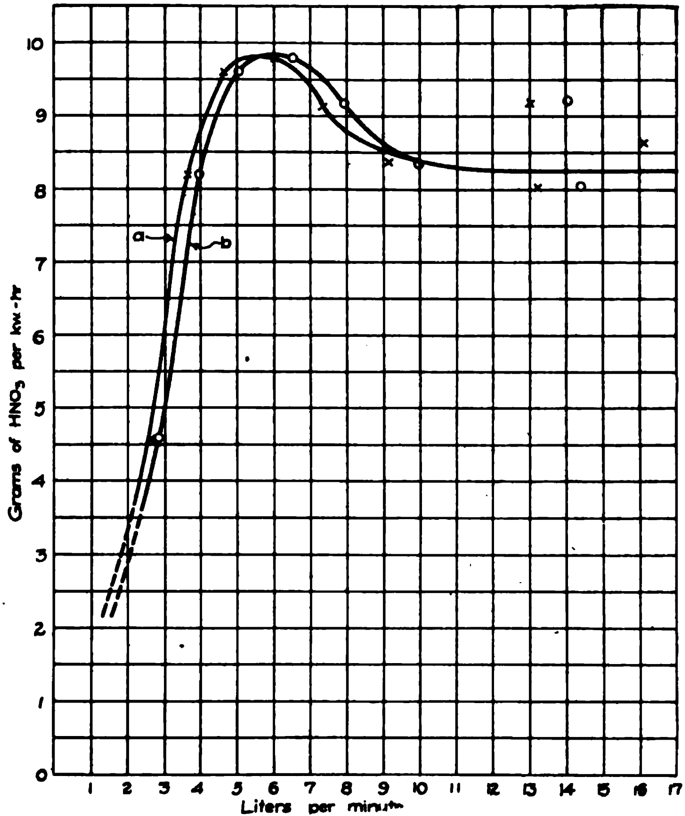

FIG. 12—780 MM.

a.—n. t. p. b.—actual.

is given in Figs. 15 and 16. These curves show that high concentration is to be secured at low velocity, but as seen in Figs. 13 and 14 at the expense of energy input. The concentration curves are therefore somewhat misleading from a commercial standpoint.

It seems probable that all of the velocity-yield runs

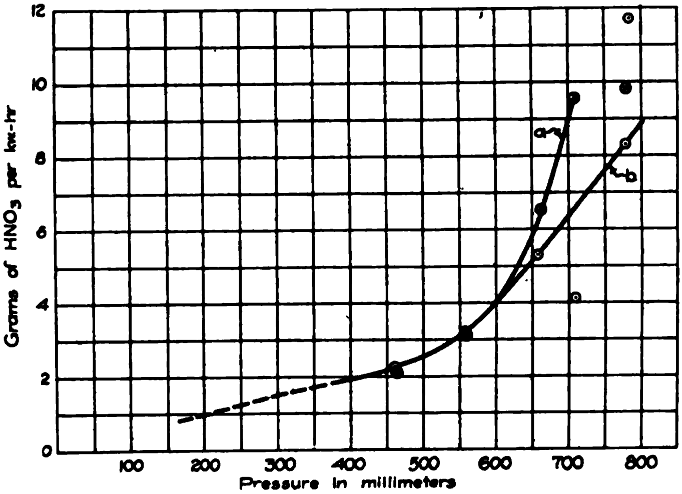

FIG. 13—PRESSURE YIELD CURVES
Constant Air Velocity (n. t. p.).

for the different pressures should follow the same general form. At low pressures the amount of gas to be dissolved was low and the efficiency of absorption higher. At the higher pressures, with more gas to take up, it is likely that the absorption was incomplete,

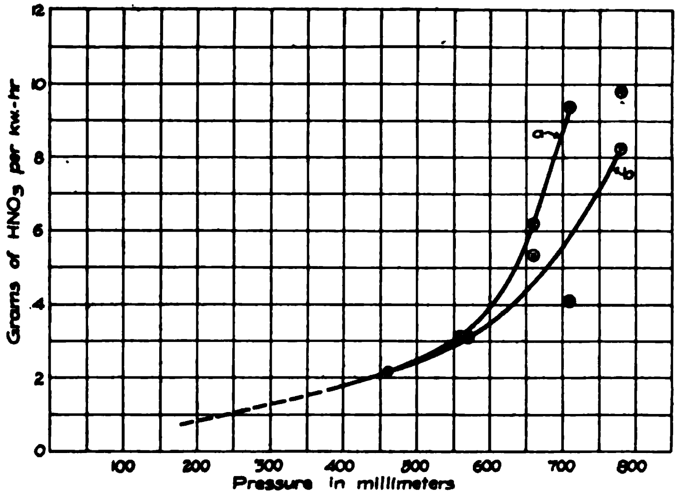

FIG. 14—PRESSURE YIELD CURVES
Constant Air Velocity (actual).

face of the absorption liquid. In the low pressure tests slight fog is formed, while at the higher velocities no fog appears. In many cases the fog is an initial condition disappearing after the first ten or fifteen minutes of operation. As the pressure is increased the fog becomes stronger and in general remains longer, until at 780 mm. pressure the fog, at a velocity of six liters per minute, is so dense that it is not possible to see through three inches of it. The fog is found to contain some nitric acid, its concentration being subject to considerable variation. The fog formation is being investigated further.

Meter and Pressure Variation.—At certain air velocities the readings of all the meters connected in the transformer and tube circuits and, to a lesser degree those in the primary circuit show a cyclic variation. At low pressures the effect is very slight, but with increasing pressure it becomes more pronounced. The greatest variation at each pressure occurs at an actual velocity of six liters per minute. At velocities above and below this, the indications rapidly become steady. With but one exception, these variations do not extend over a period of more than 40 minutes. In most cases they do not occur after the first 20 minutes of operation.

When the variations are most pronounced, the manometer connected to the discharge tube shows a disturbance corresponding to that of the electric meters. The frequency seems to be about two per second, although it is not by any means constant.

From the velocity yield curves given in Figs. 8, 9, 10, 11, and 12, it is to be noticed that the curves begin to flatten out at about the critical velocity of six liters per minute.

In the discharge tube, nitrogen pentoxide (N_2O_5) is being formed, while at the same time, if subjected to discharge for too long a period, it is again broken down. These two actions are therefore opposing one another and both are a function of time. This action may have some bearing on the observed variations.

Another effect which should be mentioned has been termed "Initial effect." During practically every run made, in order to hold the wattmeter deflection constant, the primary voltage had to be reduced as the run progressed. This average potential decrease during runs amounted to about 3 per cent. The tertiary coil voltage did not show as high a percentage variation and in some cases no change was noted. Most of the observed changes took place in the first fifteen minutes of operation.

Analysis of Losses. The energy consumption in the discharge tube may be divided into four parts.

1. Warming up the Material of the Tube. Using the weight of the iron and aluminum parts of the tube, and the specific heats of the metals, an expenditure of 14.14 watthours will be required per degree cent. temperature rise.

2. Loss in the Porcelain Dielectric. In order that some idea of the magnitude of the dielectric losses in the porcelain might be obtained, a series of heat runs on a regular discharge porcelain tube was made. The outside of the tube up to within about a foot (30.5 cm.) of either end was covered with tinfoil, the corona discharge from the tinfoil edges being prevented by pressboard collars sealed over them. A snug fitting metal tube three inches (7.6 cm.) in diameter was slipped inside the porcelain, its length being such that the dielectric field was not distorted by its ends. The tinfoil was grounded, the inner tube being connected to the high-tension supply. Alcohol thermometers placed on the tube indicated the temperature of the tube. From curves taken during this test the power loss in the porcelain dielectric will be of the order of 10 watts, its exact value depending upon the value of the current flowing, in any particular case. From the observed temperatures of the discharge tube and effluent gases the temperature rise of the porcelain is greater than that which would be caused by its own dielectric loss. On account of this the energy calculated as being used in warming up the porcelain will exceed the value obtained by test.

3. Heat Carried away by Discharge Gases. The amount of heat carried away by the air is very small, being of the order of 0.20 watthour per 1000 liters of air per degree cent.

4. Energy used in the Chemical Reactions. Using the figures given by Berthelot[4] the amount of energy required for the formation of enough $N_2 O_5$ to produce one gram of nitric acid is 95.4 calories, or 0.11 watthour. For the ozone 667 calories or 0.78 watthours are required.

Assuming that the wrought iron tube and the aluminum tube are at the same temperature, and that the porcelain tube is at the temperature of the discharge gases the following energy relations are secured, based upon their respective weights, specific heats and assumed temperatures.

Metal parts average rise 5.6 deg. cent. loss = 79.2 watthours
Porcelain tube average rise 9.35 deg. cent. loss .... = 27.1 watthours
Effluent gases 9.35 deg. rise—478 liters loss = 0.9 watthour
Chemical reactions 2.13 grams $H N O_3$ (per hour) .... = 0.24 watthour
0.96 grams ozone = 0.78 watthour

108.2

The power read on the wattmeter when connected in the tube circuit was 134 watts, leaving a difference of 25.8 watthours to be accounted for. Some of this may be due to temperatures existing inside the discharge tube higher than those used in the calculations. This error is not likely to be very large, however.

Conclusions. This work is but well begun, and no prediction can be made with any degree of certainty, yet the results obtained thus far are quite promising. Improved absorption apparatus has already indicated that increased yields may be expected. The process is very simple, and the first cost and maintenance of a plant would not be high. The difficulties of operation associated with the use of high temperatures are obviated.

Referring to Anderegg's calculations,[5] based upon his

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[4] Smithsonian Physical Tables, 6th Edition.

[5] "The Calculation of the Efficiency of the Silent Discharge Process for Nitrogen Fixation," F. O. Anderegg, Science, 50, 49 (1919) and Chemical Abstracts *13*, 3090 (1919).

theory of the reaction, the ideal yield from the silent discharge process, if all the electrical energy were available for the reaction, would be 250 grams of nitric acid per kilowatt-hour. The theoretical possibilities from the thermal reaction, according to thermodynamical calculations, represent a yield of only 134 grams of acid per kilowatt-hour at a temperature of 4200 deg. cent. If the former process can be developed to the present efficiency of the arc process, the yield will be greater and the first cost and operating expenses will probably be much less because of the simpler apparatus involved.

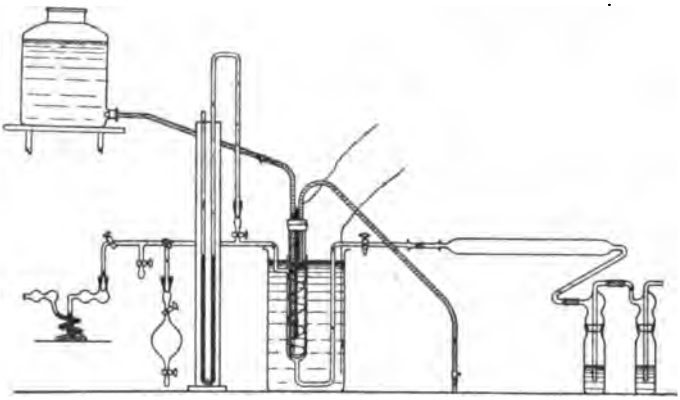

FIG. 17—SPIEL'S APPARATUS

In general, those reactions approximating most closely natural phenomena are found ultimately to be most satisfactory and economical. Nature provides dilute nitric acid for the fertilization of the soil by the ionization of the air as a result of electric discharges in the atmosphere. The gases thus produced are absorbed by the moisture of the clouds or rain and are distributed over the land. Such a process is identical in principle with that under discussion in this paper.

This work has been accomplished only by the generous cooperation of many interested investigators. Dr. F. O. Anderegg in particular, Assistant Professor of Chemistry at Purdue University, has followed the work in detail and has made many suggestions of value. The hearty appreciation of the authors is extended to him, to other collaborators mentioned in the paper, to Professor H. C. Peffer, Head of the School of Chemical Engineering, and Mr. Emerson Pugh, instructor in the School of Electrical Engineering. The departments of Chemistry and Physics have also rendered services which are hereby recognized with appreciation.

The Engineering Experiment Station at Purdue University will continue the investigation of the "Fixation of Atmospheric Nitrogen by the Silent Discharge Process."

APPENDIX

In Table I will be found a summary of the results of Spiel's experiments upon fixed volumes of air. The change of primary current of the induction coil indicated in the second column of the table, no doubt varied the voltage of the secondary and therefore the energy input to the Siemen's tube over a considerable range, but no record of such energy or the yields of nitric acid were reported.

TABLE I.
SUMMARY OF SPIEL'S RESULTS

No. of experiment.	Prim. current.	Pressure mm.	Temperature deg. cent.	Time reversal minutes	Time equilibrium minutes	N O concentration vol. per cent
1	1.9	715 -744	12 -15.3	120	175	0.7
2	1.9	716 -748.5	14 -14.2	110	..	4.4
3	1.9	481.5-503.5	14.3	39	51	0.6
4	1.9	481.5-502	15	43	..	5.6
5	1.9	345.5-353.5	16 -16.1	14	85	0.5
6	1.9	344 -352	15.5-15.7	11	..	6.6
7	3.0	729.5-746	17 -17.1	30	50	0.4
8	3.0	721.5-746.5	16.1-16.2	31	..	4.2
9	5.0	731 -760	14 -14.2	11	80	0.1
10	5.0	731 -750	14	11.5	..	3.6

The analyses were made after the equilibrium was reached for the experiments of odd number. In the even numbered experiments, the analyses were made as near the reversal point as possible.

CONCLUSIONS

The conclusions derived by Spiel as the result of his work on air are quoted as follows:

1. "With the discharge in enclosed volumes of air the final equilibrium concentration gives nitric oxide below one volume per cent for these conditions."

2. "Decrease of the original pressure essentially alters the equilibrium concentration."

3. "Increase in the primary current decreases the equilibrium concentration."

4. "The pressure decrease, which the gas undergoes, as is to be seen from curve, Fig. 17, varies between 8 and 34 mm. This pressure decrease depends not only upon the formation of ozone, but also upon the formation of oxides of nitrogen for, as can be seen from the red color of the gas at the time of reversal, and as has been proven by the determinations of concentration, much more oxidized nitrogen is present before and at the time of reversal, than at equilibrium."

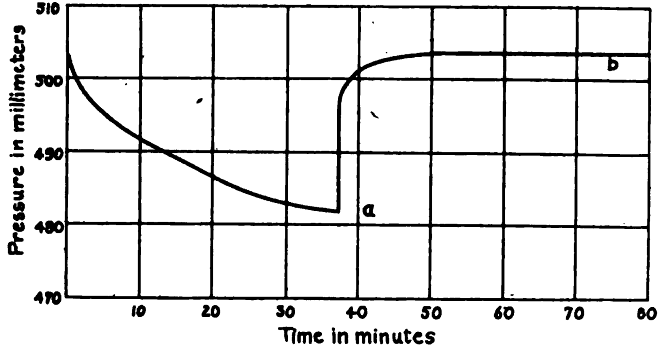

FIG. 18

a.—5.6 vol. per cent N O.
b.—0.6 vol. per cent N O.

This abrupt and strong reversal is a very peculiar phenomenon. It has been checked by Anderegg in the chemical laboratory of Purdue University. The reversal has been shown by Spiel to be affected by changes of temperature. An investigation is being undertaken by R. E. Nelson to see what effect the maintenance of a very constant temperature will have on the reversal.

5. "The concentration at the time of reversal in experiments at atmospheric pressure is about six

times the final concentration. For lower pressures this ratio is raised, the final concentration being lowered; the reversal concentration increases rapidly. With 500 mm. pressure at the time of reversal about 10 times, with 350 mm. about 12 times as much NO is present as in the final equilibrium."

6. "With increased current the reversal as well as the final concentration decreases considerably."

7. "The time after which reversal occurs varies between 12 and 110 minutes; the latter time, which was obtained with air at atmospheric pressure and 1.9 amperes, agrees with the results of Hantefeuille-Chappius."[6]

The conclusions reached by Spiel resulting from experiments with initial mixtures of air and various proportions of oxygen and nitrogen are to the effect that the concentration is increased and the time required to produce a reversal of the reaction is lowered by higher oxygen concentrations. He further decides the "The Silent Electric Discharge is not concerned with a thermal effect."

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[6] *Comptes rendus* 92 (80), 1881; 92 (134), 1882.

DISCUSSION ON "NITROGEN FIXATION BY THE SILENT DISCHARGE METHOD" (HARDING AND MCEACHRON), BOSTON, MASS., APRIL 9, 1920.

Mr. Finch: I should like to ask Mr. Harding if he made any runs at a higher pressure, say 2 or 3 atmospheres, at the same time increasing the velocity?

Mr. Benjamin: I might state that in the demand for argon for filling electric light bulbs in 1916, I was called upon by my company to develop a method for the obtaining of argon. In connection with that I naturally turned to the Berklinite process to remove the nitrogen from the air and eventually developed an apparatus which is extremely simple, being a reaction chamber provided with 4 electrodes. These electrodes were nickel. Two diametrically opposite electrodes were arranged to receive a discharge current from an a-c. transformer; it was only a 5-kw. transformer giving a secondary voltage of 30,000 volts. This sets up a spark. Another two diametrically opposite electrodes were placed fairly close so that their axes were at right angles to their high potential electrodes, and these received a 30-volt d-c. current. The circuit could not be established until the a-c. arc was formed; in the reaction chamber at the top, we arranged a spray which sprayed the interior of the walls of the reacting chamber with sodium hydroxide which was fed in. The gases consisted of air and oxygen. We found a very peculiar condition. Theoretically we take it that 8 parts of air and 9 parts of oxygen would form a complete reaction, but we had very poor results, practically no results, until we supplied a surplus of oxygen, amounting to 11 parts of oxygen to 8 parts of air, and then the reaction became violent, the entire reaction chamber was filled with a yellow glow which you could not detect as caused by the d-c. arc or the a-c. arc, being a natural sodium flame effect. Our products were completely converted by circulating sodium hydroxide until the gases were completely consumed and formed sodium nitrate, the residual gas being the rarer elements of the air, such as argon, helium. The other remaining gases we did not attempt to analyze. This process was extremely simple, as it consisted of just one reaction chamber and air circulating system for the sodium hydroxide.

K. B. McEachron: The results, as you can see from the curves, indicate that the higher pressures and higher velocities give the best yields. We have not done any work at any higher pressures than those reported in the paper. In case the pressure was increased materially the voltage would also need to be increased to produce a satisfactory corona. For several reasons this is not feasible with the present design of tube, one of the most important limitations being that the creepage distance at the top of the tube is such that the voltage cannot be increased very materially without danger of flashing over.

We have considered the advisability of introducing certain absorbing liquids, as sodium hydroxide or water vapor, into the discharge chamber. We have found however that dry air is very much better than moist air, and all of the air we use has been dried. The trouble with moist air is that sparks form very readily which seem to hinder the reaction. If the air contains much moisture, a portion of the yield of nitric acid will adhere to the walls of the discharge chamber, thus diminishing the observed yield. The presence of nitric acid inside the discharge tube is also likely to cause a short circuit at the bottom of the tube, where the surface of the insulating materials becomes more or less conducting.

Mr. Tucker: The authors refer to a higher yield than that indicated in the paper; could they give us any figures on that?

K. B. McEachron: We are not quite ready to give any definite values for the increased yields referred to. I might say, however, that we have obtained yields more than twice those mentioned in the paper. The reason why we do not want to give this information is on account of the possible error in our power measurements. To measure 100 or 200 watts with an electric potential of 50,000 or 60,000 volts is a problem which offers considerable difficulty. Until we have more definite information regarding our power measurements, we do not wish to put these data on record. We hope to obtain a check on our measurements by the oscillographic method, which method we are using now.

Mr. Benjamin: I might state that I have experienced trouble with porcelain insulation in that respect and have resorted to lava, I believe made somewhere about Chattanooga, that gave perfect results and no crazing on the surface whatever such as we have experienced with porcelain, and gave perfect electrical insulation.

K. B. McEachron: The process described in the paper should prove to be a valuable one, if the yields can be made to equal or exceed those of the arc process. A large amount of work will necessarily have to be done in order that the conditions which produce the best yield may be determined. When new data of sufficient value has been obtained, further reports will be made to the Institute.

Presented at the 359th meeting of the American Institute of Electrical Engineers, Boston, Mass., April 9, 1920.

MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS

BY T. D. YENSEN

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Part 1. This investigation was undertaken to determine whether any iron-nickel alloys could be found having a higher saturation value than pure iron. Alloys were prepared containing 0-100 per cent Ni.

Pure Fe-Ni alloys do not forge readily, and to make them forgeable it is necessary to add alloying elements like Mn or Ti.

The results show that the saturation value decreases slowly with increase in Ni content up to 20 per cent Ni; then rapidly to 30 per cent; again rises rapidly to 50 per cent and falls off gradually toward 100 per cent Ni. At no point does it exceed that of pure iron.

For values of H between 100 and 400 the permeability is about 5 per cent higher for 6 to 8 per cent Ni than for pure iron, but this advantage is offset by the large increases in hysteresis loss.

Alloys containing 35 to 70 per cent Ni have high permeability at low and medium densities and low hysteresis loss, the highest permeability occurring for 50 per cent.

30 to 50 per cent alloys are characterized by a nearly straight line B-H curve from the origin to B = 2000 to 4000 gausses and also by low retentivity and coercive force, properties which are of value in connection with certain electromagnetic meters.

Part II. Previous investigations on commercial iron-nickel alloys have shown that 25 to 35 per cent alloys have irreversible magnetic and electrical transformation points occurring below ordinary temperatures. The present investigation confirms these results for pure alloys. A 30 per cent alloy, annealed and cooled to room temperature, had its saturation value, $4 \pi I_s$ increased from 2500 to 17,800 gausses and its electrical resistance decreased from 81 to 32 microhms per cu. cm. after being cooled to liquid air temperature and reheated to room temperature. Alloys containing 15, 35 and 50 per cent nickel showed practically no change after the above treatment. After allowing all transformations from the austenitic state to the α state to take place the curves for $4 \pi I_s$ and for electrical resistances both have definite cusps for 34.5 per cent nickel, corresponding to the compound Fe_3Ni, thus giving evidence of the existence of this compound. It is pointed out that the irreversible transformation causes an enormous increase in the hysteresis loss.

diam., 1.83 cm. (23/32 in.) inside diam. and 0.95 cm. (3/8 in.) long.

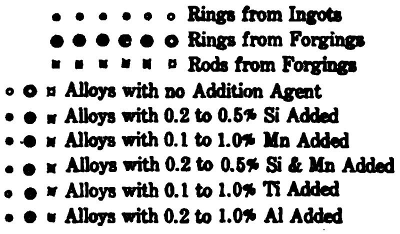

FIG. 1—KEY TO POINT-NOTATION OF IRON-NICKEL ALLOYS

2. Rings from forgings, 2.46 cm. (31/32 in.) outside diameter, 1.83 cm. (23/32 in.) inside diameter and 0.95 cm. (3/8 in.) long.

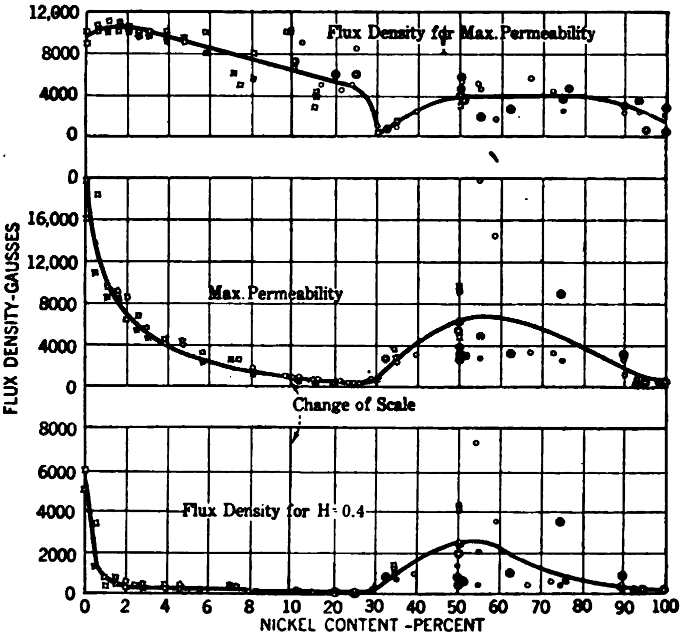

FIG. 2—MAXIMUM PERMEABILITY

3. Rods from forgings 1.27 cm. (1/2 in.) diam. and 30.5 cm. (12 in.) to 35.5 cm. (14 in.) long.

The annealing was done at 900 to 950 deg. cent, in rare cases at higher temperatures, in a vacuum furnace, under a pressure of less than 0.1 mm. of mercury, the test pieces being buried in granulated MgO, and the temperature measured by means of a Pt-Pt Rh thermo couple. The cooling was extended over a period of 24 hours and was done at a rate of approximately 30 deg. per hour.

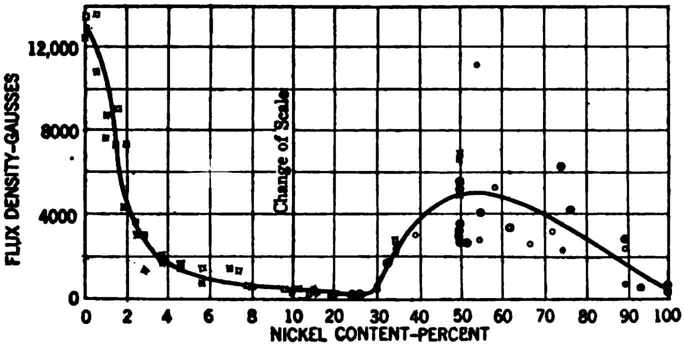

FIG. 3—FLUX DENSITY FOR $H = 1$

Testing. The magnetic testing of the rods was done by the Burrows compensated double bar and yoke method,[5] being the most approved method thus far

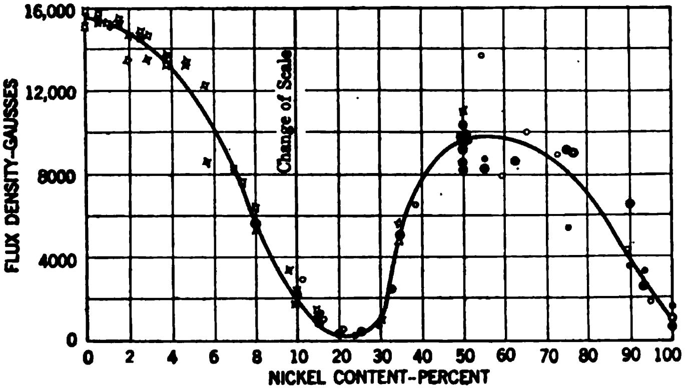

FIG. 4—FLUX DENSITY FOR $H = 4$

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[5] Bull. *Bur. of Stand.* Vol. 6, No. 1, Rep. No. 117. Bull. Eng. Exp. Sta. Univ. of Ill. No. 72 (1914), No. 83 (1915), TRANS. A. I. E. E., 33, I. 451 (1914), 34, II., 2601 (1915).

developed for rods. The rings were tested by means of the ordinary two-winding method, no other method being either simpler or more accurate.

Results. The results are embodied in the following tables and curves. No attempt has been made by means of separate curves to show the effect upon the properties of pure iron-nickel alloys by the addition of other elements necessary to make them forgeable. In most cases the effect of nickel upon iron is so marked as to obscure any effect due to other elements. Only in the region from 50–70 per cent are the alloys markedly susceptible to mechanical and heat treatment and to other alloying elements, and in this region there is much uncertainty as to the magnetic properties at low flux densities. This is plainly seen in Figs. 2, 3, 4 and 10 for maximum permeability, for permeabilities for $H = 0.4$, 1 and 4, and for hysteresis loss, respectively. In spite of this uncertainty all points have been plotted together and the curves drawn through the centers of density of the points. The curves thus obtained for $H = 0.4$, 1, 4, 20, 100, 400 and for the saturation intensity, $4\pi I_s$, have all been replotted in Fig. 9 showing at a glance the variation in permeability for various magnetizing forces with variation in nickel content. These curves might be

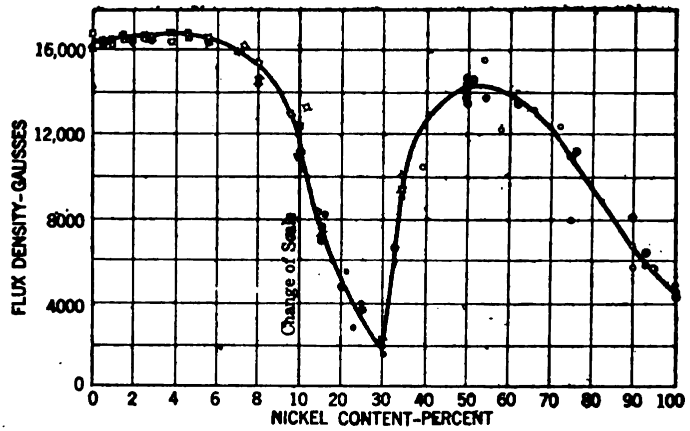

Fig. 5—Flux Density for $H = 20$

TABLE I—MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS. ELECTROLYTIC IRON AND NICKEL, MELTED AND ANNEALED IN VACUO

Specimen No.	Ni-content per cent	Kind testpiece	Annealing Temp. deg. cent.	Electrical resist. microhms per cu. cm.	Max. permeability	B for μ_{max}, gausses	Density, B, in Kilogausses for H =							Satur. value, $4\pi I_s$	Hysteresis loss for B = 10,000, ergs/cu. cm./cycle	Retentivity for B = 10,000, gausses	Coercive force for B = 10,000 gilberts/cm.
							0.2	0.4	1	4	20	100	400				
2 Ni 201	0.50	Forged Rods	920	11.5	18,200	10,000	0.6	3.4	13.6	15.7	16.5	18.3	21.2	22,700	1,175	9,200	0.39
203	1.01[1]	" "	920	12.7	9,350	11,000	0.2	0.9	8.7	15.2	16.2	18.4	21.4	22,500	..	...	..
205	1.48	" "	900	13.8	9,100	10,000	0.2	0.8	9.0	15.4	16.7	18.6	21.5	22,700	..	..	..
207	1.96	" "	940	14.9	8,200	10,000	0.2	0.6	7.2	13.4	16.7	18.6	21.6	22,600	1,900	8,700	0.65
209	2.44	" "	940	16.1	6,680	10,000	0.15	0.4	3.6	14.9	16.7	18.75	21.65	22,600	..	..	..
211	2.84[1]	" "	920	17.1	5,650	10,000	0.15	0.4	3.0	14.6	16.8	18.6	21.65	22,700	..	..	..
213	3.85	" "	902	19.4	4,350	10,000	0.1	0.2	1.6	13.2	16.4	18.6	21.8	22,360	3,300	8,300	1.10
215	4.62[1]	" "	902	20.3	4,100	9,000	..	0.3	1.6	13.3	16.7	18.7	21.2	22,300	..	..	..
294	5.68[1]	Ring from ingot[2]	923	..	4,500	3,220	0.2	0.6	4.5	11.0	14.5	16.7	19.1	20,000	2,440	6,600	0.80
217	5.67	Forged rods	920	22.0	3,300	10,000	..	0.1	1.3	12.2	16.7	19.0	21.7	22,400	..	..	..
219	7.33[1]	" "	1,064	24.8	2,130	5,000	0.1	0.3	1.2	7.6	16.2	19.1	21.6	22,600	4,225	5,700	1.45
221	8.00	" "	920	25.2	1,710	8,000	..	0.1	0.5	6.4	15.4	19.1	21.6	22,100	..	..	..
237	9.61[1]	" "	920	27.1	810	10,000	..	..	0.3	3.4	13.0	18.7	21.0	21,200	9,270	5,600	3.3
295	11.10[1]	Ring from ingot	923	..	855	8,900	..	0.1	0.3	2.9	13.3	19.1	21.5	22,000	9,450	5,800	3.70
225	14.92[1]	Forged rods	960	29.8	460	3,000	..	..	0.2	1.5	8.2	17.3	21.0	21,600	12,500	4,200	5.00
296	15.87[1]	Ring from ingot	923	..	475	5,000	..	..	0.2	1.1	8.1	15.2	19.7	22,300	13,080	4,250	5.10
297	21.03[1]	" " "	923	..	276	4,500	..	..	0.1	0.5	5.4	15.2	20.0	22,200	30,300	5,200	13.00
233	34.81[1]	Forged rods	950	80.4	3,600	1,000	0.8	1.4	2.8	5.7	10.0	12.0	12.2	..	910	2,400	0.40
298	38.92[1]	Ring from ingot	923	..	3,060	2,300	0.3	1.0	3.0	6.4	10.4	12.4	12.2	11,900	1,030	2,400	0.45
2,102	54.05[1]	" " "	923	..	19,600	5,200	3.6	7.3	11.1	13.7	15.6	15.6	16.0	15,600	580	7,600	0.18
2,101	57.99[1]	" " "	923	..	14,500	1,888	2.6	3.6	5.2	7.9	12.2	13.8	14.2	13,800	194	3,400	0.14
2,100	66.96[1]	" " "	923	..	3,200	5,500	0.2	0.4	2.6	10.0	13.1	13.6	13.9	13,500	1,330	3,400	0.56
2,104	72.63[1]	" " "	923	..	3,230	4,200	0.2	0.6	3.1	8.8	12.4	12.6	12.9	12,500	922	2,500	0.40
285	75.80[1]	Ring from forging	931	..	4,230	4,400	0.1	0.7	4.2	9.0	11.1	11.4	12.0	12,000	627	2,350	0.34
															(B =4000)	(B =4000)	(B =4000)
2,103	89.83[1]	Ring from ingot	923	..	2,310	2,200	0.2	0.5	2.2	4.3	5.7	6.2	6.2	6,000	592	2,150	0.53
287	95.05[1]	Ring from forging	931	..	500	560	..	0.1	0.5	1.9	5.6	7.0	7.6	7,410	3,015	2,450	2.85
288	99.88[1]	" " "	931	..	400	400	..	0.1	0.4	1.0	4.4	6.0	6.8	6,550	5,650	2,450	5.20

1. As per Chem. Anal. other alloys as per amount Ni added. 2. Low sp. gr.—not plotted.

termed the magnetic iso-gilberts for the iron-nickel alloys. To this set of curves the following general statements can be applied:

1. The saturation intensity, I_s decreases slightly—not more than 1 or 2 per cent—from pure iron to 20 per cent nickel. Between 20 and 30 per cent nickel I_s decreases to one-tenth of its previous value, increases again rapidly beyond 30 per cent, reaches a maximum for 50 per cent nickel and finally decreases gradually toward 100 per cent nickel, the value for pure nickel being between ⅛ and ¼ that for pure iron.

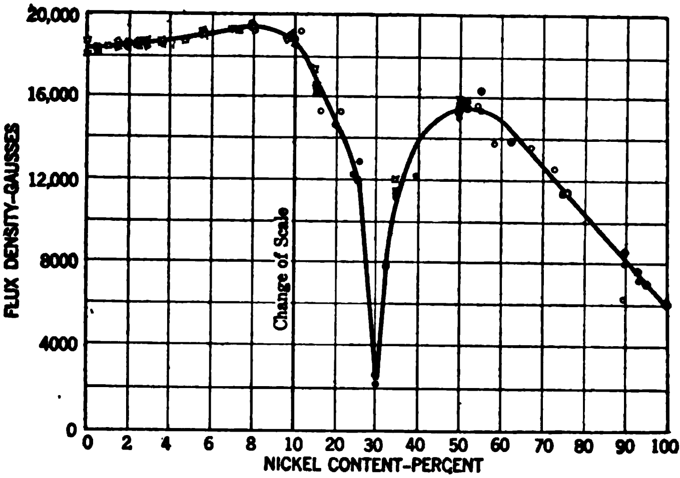

Fig. 6—Flux Density for $H = 100$

2. The iso-gilberts for high values of H are of the same general shape as that for I_s, except that there is a slight increase—about 5 per cent—in permeability between 0 and 8 per cent nickel.

3. For low values of H the iso-gilberts drop rapidly from pure iron and remain very low until the 30 per cent point has been passed; reaching a maximum between 50 and 60 per cent nickel, for which in some cases a permeability has been reached approaching that for pure iron; and finally decreases to very low values for 100 per cent nickel.

The low saturation value occurring in the region of

TABLE II

MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS. ELECTROLYTIC IRON AND NICKEL, MELTED AND, ANNEALED IN VACUO. EFFECT OF SI AND MN AS ADDITION AGENTS

Specimen No.	Ni-content per cent	Si-added per cent	Mn-added per cent	Kind Testpiece	Annealing temp. deg. cent.	Electrical resist. microhms per cu. cm.	Max. permeab. μ_m	B for μ_m, gausses	Density, B, in kilogausses for H =							Satur. value, $4\pi I$	Hysteresis loss for $B = 10,000$, ergs/cu. cm./cycle	Retentivity for $B = 10,000$, gausses	Coercive force for $B = 10,000$, gilberts/cm.
									0.2	0.4	1	4	20	100	400				
2 Ni 202	0.50	0.2	..	Forged rods	920	12.7	11,000	10,000	0.2	1.4	10.8	15.3	16.4	18.2	21.1	22,500	1,570	9,100	0.52
204	1.00	0.2	..	" "	900	14.3	8,330	10,000	0.15	0.5	7.6	15.2	16.6	18.4	21.4	22,500	..	..	..
206	1.48	0.2	..	" "	900	15.1	8,530	11,000	0.2	0.7	7.2	15.3	16.8	18.3	21.6	22,500	..	..	..
208	1.96	0.2	..	" "	940	16.5	6,060	10,000	0.15	0.4	4.2	14.7	16.6	18.5	21.5	22,500	2,420	8,700	0.80
210	2.44	0.2	..	" "	920	18.0	5,650	10,000	0.15	0.4	3.0	14.6	16.8	18.4	21.5	22,200	..	..	..
212	2.90	0.2	..	" "	920	19.1	4,550	10,000	0.1	0.2	1.2	13.4	16.5	18.5	21.4	22,500	..	..	..
214	3.85	0.2	..	" "	902	20.7	4,500	9,000	0.1	0.3	2.0	13.6	16.9	18.6	21.5	22,500	3,030	8,100	1.02
216	4.76	0.2	..	" "	920	21.4	4,000	9,000	..	0.2	1.4	13.2	16.9	18.8	21.75	22,800	..	..	..
218	5.67	0.2	..	" "	920	25.8	2,130	8,000	..	0.1	0.6	8.5	16.4	19.1	21.7	22,240	..	..	..
220	7.00	0.2	..	" "	1,064	25.7	2,400	6,000	0.2	0.5	1.3	8.1	16.0	19.2	22.0	22,600	3,760	5,400	1.30
273	8.00	..	0.1	Forged rod	900	26.2	1,300	5,200	..	0.1	0.5	5.2	14.4	19.2	21.9	21,800	5,990	5,900	2.36
				" ring	900	..	1,350	5,400	..	0.1	0.5	5.5	14.6	19.4	22.2	22,700	6,340	5,800	2.20
272	10.00	0.2	0.1	Forged rod	900	28.9	640	6,400	..	0.07	0.2	2.2	12.4	18.5	21.5	21,850	11,200	5,300	4.10
				" ring	900	..	700	7,000	..	0.05	0.2	2.1	11.1	18.8	21.9	22,200	11,100	5,400	4.30
238	9.52[1]	0.2[2]	..	Forged rods	920	29.6	556	10,000	..	0.1	0.3	2.7	10.6	17.0	19.9	20,400	10,150	5,400	3.90
226	15.0	0.2	..	" "	960	31.0	450	4,000	..	..	0.2	1.4	7.5	16.5	20.9	21,300	12,300	4,200	5.00
232	30.0	0.2	..	Forged rods	950	82.0	500	250	0.1	0.2	0.4	1.0	2.1	2.6	3.1	2,700	169	200	0.40
				Ring from ingot	1,100	..	570	250	0.1	0.2	0.5	1.0	1.9	2.6	..	..	220[3]	200[3]	0.60[3]

TABLE II—*Continued*

Specimen No.	Ni-content per cent	Si-added per cent	Mn-added per cent	Kind Testpiece	Annealing temp. deg. cent.	Electrical resist. microhms per cu. cm.	Max. permeab. μ_m	B for μ_m, gausses	Density, B in kilogausses for H = 0.2	0.4	1	4	20	100	400	Satur. value, $4\pi I$	Hysteresis oss for B = 10,000 ergs/cu. cm./cycle	Retentivity for B = 10,000, gausses	Coercive force for B = 10,000, gilberts/cm.
283	32.6[1]	..	1.0	Ring from forging	931	..	2,320	720	0.5	0.6	1.7	3.4	6.5	7.9	8.7	8,570	334[6]	950[6]	0.38[6]
234	34.5	0.2	..	Forged rods	950	81.3	2,780	1,500	0.3	1.1	2.5	4.9	9.2	11.4	11.8	11,410	1,040	2,000	0.40
				Ring from ingot	1,100	..	2,210	1	0.2	0.8	2.1	5.0	9.0	11.2	..	..	1,800	1,500	0.80
261	50.0	..	0.75	Forged rod	900	..	9,600	2,870	0.2	4.0	6.8	10.9	14.5	15.7	15.9	15,540	810	4,800	0.29
				" ring	917	6	3,200	4,500	0.9	0.5	2.9	9.1	14.0	15.2	15.7	15,350	1,810	3,600	0.75
262	50.0	..	1.0	Forged rod	900	45.	9,600	2,000	0.2	4.0	6.8	10.9	14.1	15.1	15.5	14,900	810	4,800	0.29
				" ring	917	..	6,060	4,870	0.9	2.4	5.5	10.2	14.5	15.4	16.0	15,450	724	2,300	0.30
263	50.0	0.5	0.5	Forged ring	931	..	3,570	4,000	1.0	0.8	3.4	9.3	13.8	15.5	16.3	15,900	2,315	5,300	0.89
264	50.0	0.25	0.5	" "	900	..	2,940	4,000	0.4	0.8	2.8	8.5	14.2	15.5	15.7	15,800	1,390	2,500	0.60
281	51.4[1]	..	1.0	" "	931	..	3,110	5,600	0.2	0.6	2.7	9.8	14.7	15.8	16.3	15,960	2,155	4,400	0.80
289	54.9[1]	..	0.5	Ring from ingot	931	..	2,860	4,300	0.1	0.3	2.7	8.7	13.7	14.3	14.8	14,500	1,177	2,550	0.68
				Forged ring	923	..	5,000	2,000	0.8	2.0	4.0	8.2	13.7	15.2	15.4	15,100	579	2,300	0.32
279	62.2[1]	..	1.0	Forged ring	931	..	3,300	2,800	0.3	1.0	3.3	8.5	13.4	13.9	14.5	14,160	1,581	3,700	0.55
290	74.8[1]	..	0.5	Ring from ingot	931	..	2,200	2,200	0.1	0.4	2.2	5.3	7.9	8.8	9.2	8,840	706[4]	2,550[4]	0.68[4]
				Forged ring	923	..	9,000	3,600	1.4	3.5	6.2	9.0	11.0	11.4	11.7	11,300	541	3,650	0.24
291	89.9	..	0.5	Ring from ingot	931	..	1,040	2,400	..	0.1	0.6	3.6	6.7	8.0	8.5	8,210	1,453[4]	2,400[4]	1.42[4]
				Forged ring	923	..	2,660	2,800	0.3	0.9	2.7	6.5	8.0	8.5	8.7	8,300	1,222[6]	2,700[6]	0.60[6]
292	93.3[1]	..	0.5	Ring from ingot	931	..	800	2,400	..	0.1	0.3	3.1	5.8	7.2	7.6	7,230	2,055[4]	2,700[4]	1.8[4]
				Forged ring	922	..	630	3,400	..	0.1	0.3	2.4	6.3	7.6	7.8	7,450	4,480[5]	3,400[5]	2.7[5]
293	99.9[1]	..	0.5	Ring from ingot	931	..	500	2,000	..	..	0.4	1.5	4.7	6.2	6.8	6,490	3,640[4]	2,600[4]	3.2[4]
				Forged ring	923	..	316	2,750	..	0.1	0.4	0.8	4.3	6.0	6.3	6,000	5,870[4]	2,400[4]	5.6[4]

1. As per Chem. Anal. other alloys as per amount added.
2. C—0.061 per cent not plotted.
3. For $B = 2{,}000$
4. For $B = 4{,}000$
5. For $B = 6{,}000$
6. For $B = 8{,}000$

30 per cent nickel has been shown to be due to the fact that the magnetic transformation point of these alloys lies below ordinary temperatures. K. Honda in a recently published paper[6] has shown that cooling the alloys in liquid air and then heating them to room temperature raises I_s from the minimum value of about 200 to nearly 1000. At the same time the minimum point has been shifted from 28 to 30 per cent to about 34.5 per cent, the latter corresponding to the compound Fe_2Ni.[7]

Fig. 10 shows the variation of hysteresis loss for $B = 10{,}000$ gausses with nickel content. As might be expected from the iso-gilberts of the previous curve, the loss increases rapidly even with small additions of nickel, and the loss becomes very high for alloys containing 15 to 25 per cent, reaching a maximum of nearly 50,000 ergs per cu. cm. per cycle for the 25 per cent alloy. Between 30 and 70 per cent the loss is as low or even lower than for pure iron, but above 70 per cent the loss again increases rapidly reaching 20,000 for pure nickel.[8]

Fig. 10 shows the retentivity to decrease with increasing nickel content reaching a minimum of only 200 for the 30 per cent alloy. With further increase in nickel content the retentivity again in-

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[6] The Thermal and Electrical Properties of Nickel Steels, *Sci. Repts.*, Tohoku Imp. Iniv. 7, pp. 59-66. (1918).

[7] By the above treatment the iso-gilberts would, of course, also be changed, and the entire set of curves would appear very differently. This matter is discussed in Part II of this paper.

[8] All losses are based on a flux density of 10,000 gausses. In case of the 30 and 80 to 100 per cent alloys having saturation values lower than 10,000 the losses were obtained for as high densities as possible, and the hypothetical loss for 10,000 calculated by means of the formula

$$W_{10} = W_B \left(\frac{10{,}000}{B} \right)^{1.6}$$

where W_B is the loss actually obtained by test for the flux density B. The 1.6 law may not be strictly applicable in the case of the alloys mentioned, but is believed to be sufficiently accurate for the purpose of comparison.

TABLE III

MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS. ELECTROLYTIC IRON AND NICKEL, MELTED AND ANNEALED IN VACUO. EFFECT OF Ti AND Al AS ADDITION AGENTS

Specimen No.	Ni-content, per cent	Ti-added per cent.	Al-added per cent	Kind testpiece	Annealing temp. deg. cent.	Electrical resist. microhms per cu. cm.	Max. permeab. μ_m	B fo μ_m, gausses	Density, B, in kilogausses for H =							Satur. valve, $4\pi I_s$	Hysteresis loss for B = 10,000, ergs/cu. cm./cycle	Retentivity for B = 10,000, gausses	Coercive force for B = 10,000, gilberts/cm.
									0.2	0.4	1	4	20	100	300				
2 Ni 236	9.9	0.20	..	Forged rods	920	27.5	605	10,000	..	..	0.2	1.7	11.0	18.9	21.5	22,600	10,700	5,300	4.10
247	15.0^{1}	0.20	..	Forged rods	900	30.8	405	4,050	..	..	0.1	1.3	7.3	16.5	20.5	21,600	15,150	4,400	5.70
				" ring	917	..	382	4,000	..	..	0.05	0.9	6.9	16.2	20.2	21,900	13,900	3,900	5.90
248	20.0^{1}	0.20	..	Forged ring	917	..	240	6,000	..	..	0.05	0.4	4.8	14.7	19.4	22,200	28,200	4,900	12.00
249	25.0^{1}	0.20	..	" "	917	..	187	6,000	..	..	0.05	0.3	3.6	12.0	15.9	18,400	36,900	5,000	15.00
251	50.0^{1}	0.50	..	" "	917	..	5,700	4,000	0.6	2.0	5.2	9.8	14.5	15.7	16.0	15,600	852	3,000	0.40
252	50.0^{1}	1.00	..	Forged rod	900	47.5	4,960	4,960	0.4	1.3	5.0	9.5	13.9	15.2	15.6	15,200	1,385	4,200	0.62
				" ring	917	..	3,200	3,200	0.4	1.0	3.2	8.2	13.6	15.0	15.6	15,300	1,270	2,600	0.60
2 Ni 240	23.8	..	0.20	Ring from ingot	950	..	200	5,000	..	..	0.03	0.2	2.8	12.1	16.3	19,200	10,860^{2}	2,300^{2}	12.80^{2}
241	25.1	..	0.20	" " "	950	..	172	8,600	..	..	0.05	0.3	3.8	12.9	17.0	19,700	8,950^{2}	2,400^{2}	10.20^{2}
242	29.8	..	0.20	" " "	950	..	400	400	..	..	0.4	0.8	1.5	2.1	2.8	2,860	278^{3}	150^{3}	0.50^{3}

1. Per cent Ni added, alloy not analysed chemically.
2. For B = 4,000
3. For B = 1,500

creases, reaching a maximum between 50 and 60 per cent nickel.[9]

Fig. 12 shows the variation in electrical resistance checking very closely the results obtained by Burgess and by Honda.[10] The curve has been drawn through the points for pure iron-nickel alloys for all cases in which alloys were available in the form of rods. In every case it will be noticed that the alloys containing a small amount of silicon have a decidedly higher resistance than the alloys without silicon, as might be expected. The curve reaches a maximum for 30 per cent nickel corresponding to the composition of the alloy having the lowest magnetic saturation value. Honda has obtained a decrease in electrical resistance corresponding to the increase in the

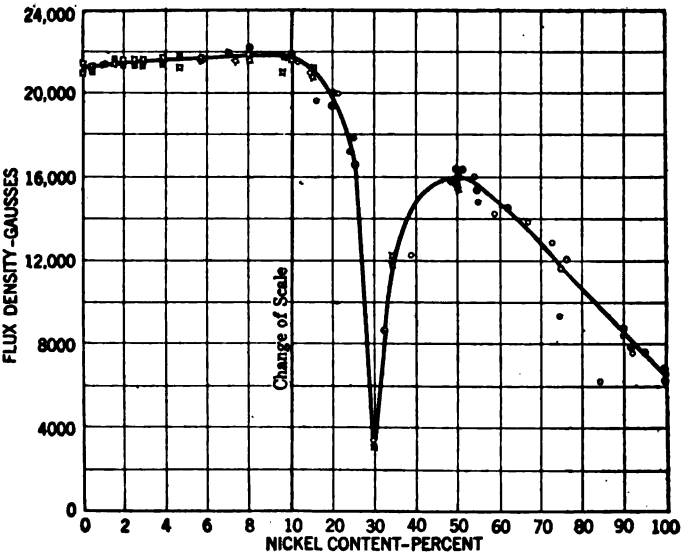

Fig. 7—Flux Density for $H = 400$

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[9] No correction has been made in case the alloy was tested at a lower density than 10,000, but the error is supposed to be small in every case. Examples: 2 Ni 285, containing 75 per cent nickel was tested at 10,000 gausses, showing a retentivity of 2350, while 2 Ni 290, having the same nickel content, tested at B = 4000 gave a retentivity of 2550.

[10] Loc. Cit.

saturation value above referred to, by cooling the alloys in liquid air and then heating to room temperature. For the 30 per cent alloy the resistance decreased from a value of approximately 90 to 30 microhms, whereas no change is shown for the 34.5 per cent alloy corresponding to Fe_2Ni, making this the alloy having the maximum resistance.[11]

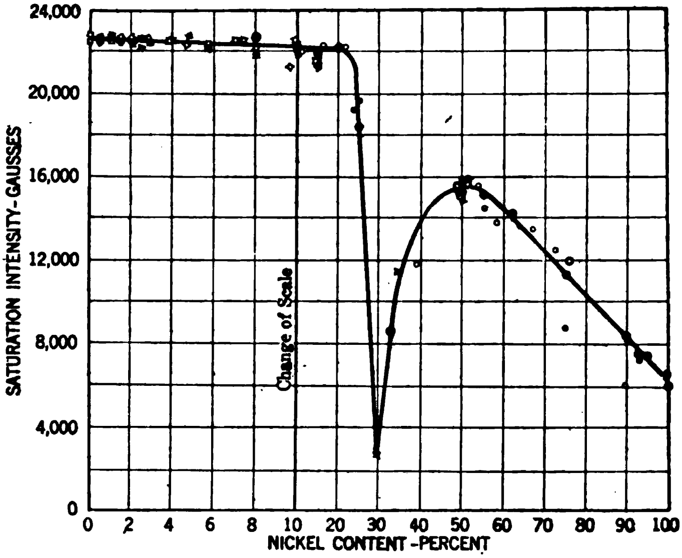

Fig. 8—Saturation Intensity of Magnetization $4\pi I_s$

The specific gravity shown in Fig. 13 was obtained incidentally as a means of obtaining the true cross section of the ring test pieces. The values increase uniformly from 7.9 for pure iron to 8.9 for pure nickel.

PART II

Reversible and Irreversible Transformations

Introduction. In Part I of this paper it was pointed out that the properties of alloys in the neighborhood of 30 per cent nickel would be very different after

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[11] Further data on this subject is contained in Part II.

being cooled to liquid air temperature from after being cooled merely to room temperature. It was further pointed out that this difference is due to the fact that these alloys—in addition to the reversible transformation point occurring above room temperature—have an irreversible transformation point occurring below room temperature. These transformations have been studied a great deal in recent years, the pioneers in the field being Hopkinson (1), LeChatelier (2), Osmond (3), Guillaume (4) and Dumas (5).[12] The alloys investigated by these scientists all contained foreign elements to some extent, particularly carbon and manganese. As a matter of fact manganese was considered an essential constituent, because without it, the alloys could not be

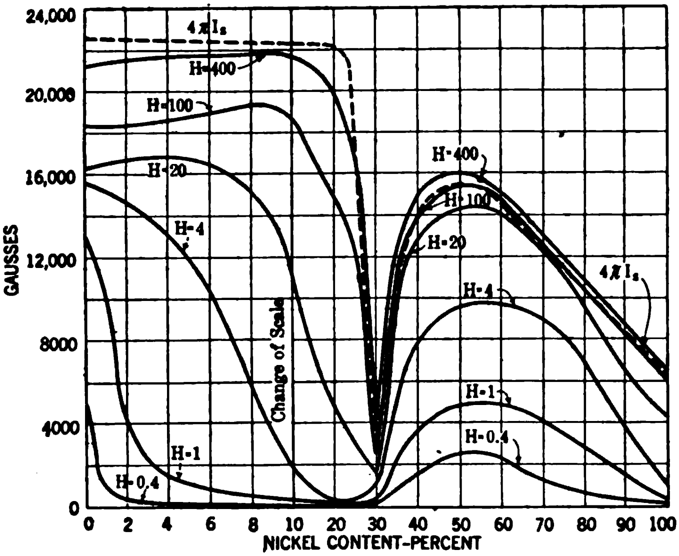

Fig. 9—Saturation Intensity $4\pi I_s$ and Flux Density for Various Magnetizing Forces

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[12] (1) *Proc. Royal Soc.*, 46 and 47, p. 23, 1889, 48 p. 1., 1890.

(2) *Comptes Rendus*, 90 p. 285, 1890.

(3) *Comptes Rendus*, 118, p. 532, 1894; 121, p. 684, 1895; 128, pp. 306, 1395, 1513, 1899.

(4) *Comptes Rendus*, Jan. 25, April 5, June 18, July 26, 1897.

(5) *Comptes Rendus*, 139, p. 42, 1899, *Jour. Iron & Steel Inst.*, II, p. 225.

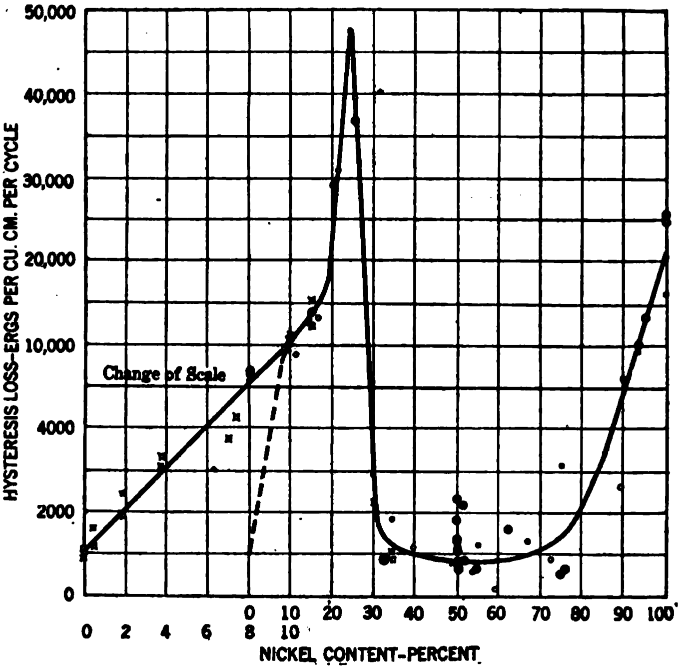

FIG. 10—HYSTERESIS LOSS FOR $B = 10,000$

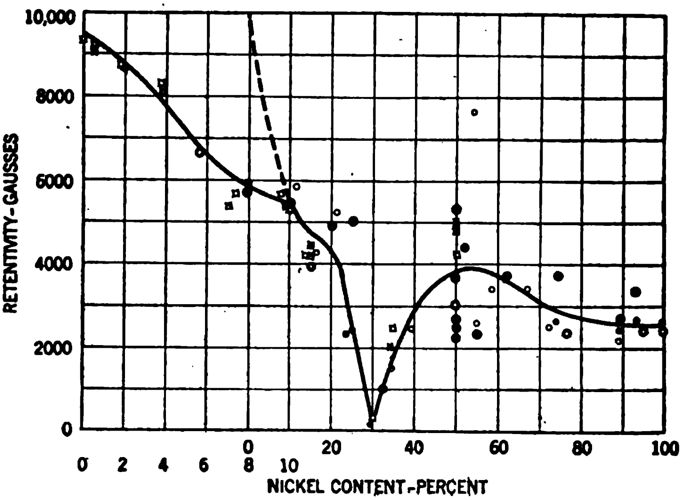

FIG. 11—RETENTIVITY FOR $B = 10,000$ GAUSSES

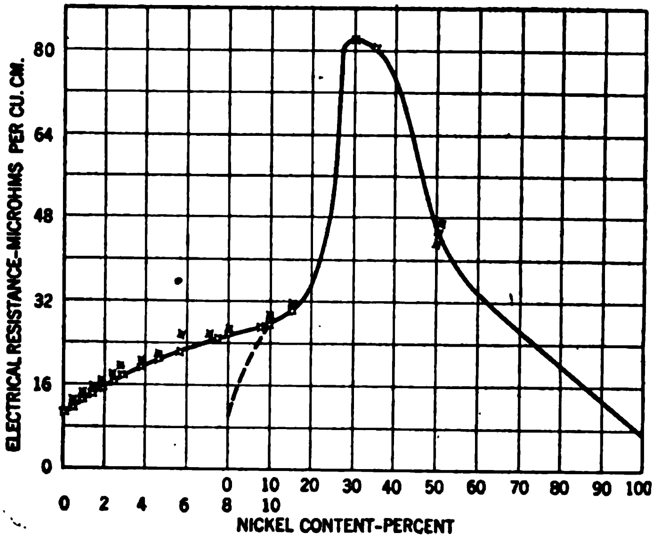

Fig. 12—Electrical Resistance

forged. It is very unfortunate, however, for the interpretation of the results of the above investigations that these two elements referred to are the ones that affect the transformation points the most. Thus carbon is 20 times as effective as nickel in lowering

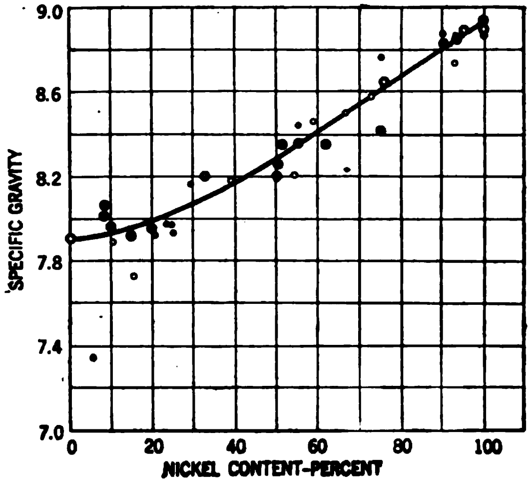

Fig. 13—Specific Gravity

the transformation points, and manganese, while less effective in this respect than carbon, still is much more effective than nickel. Dumas[13] realized this when he stated that "No nickel steel is non-magnetic at ordinary temperatures except steels containing carbon and manganese," and he consequently attempted to keep these elements as low as possible. The results of his experiments are shown in Fig. 14. It will be noted that from 27 to 31 per cent nickel Dumas found both reversible and irreversible transformations.

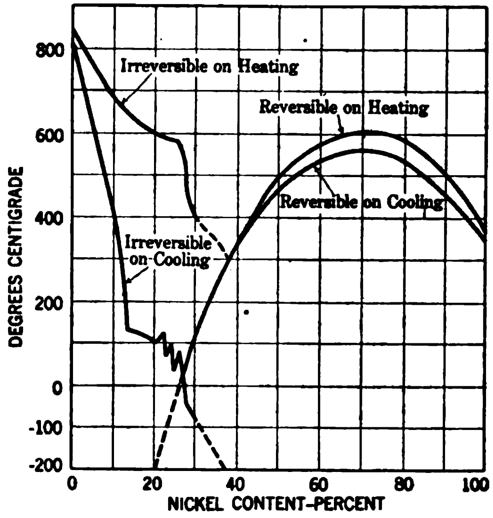

Fig. 14

Of the two, however, the irreversible was by far the more intense. The broken curves indicate probable transformations in pure alloys that are too feeble in comparison with the transformations that have already occurred to be readily detected. However, by introducing carbon into the alloys, the irreversible transformation points may be lowered sufficiently without lowering the reversible points, to reveal the latter. Dumas cites the example of a 23 per cent alloy, which when pure has an irreversible point on cooling at + 75 deg. but apparently no reversible

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[13] Loc. Cit.

point. By adding 0.85 per cent and 1.41 per cent Mn to this alloy the irreversible point is decreased to −188 deg., and on cooling from 0 deg.—the alloy is now non-magnetic—the reversible point is revealed at −150 deg. On the assumption that the reversible points are unaffected by C or Mn, Dumas concludes that the pure Fe-Ni alloys between 20 and 37 per cent nickel have both reversible and irreversible transformation points.

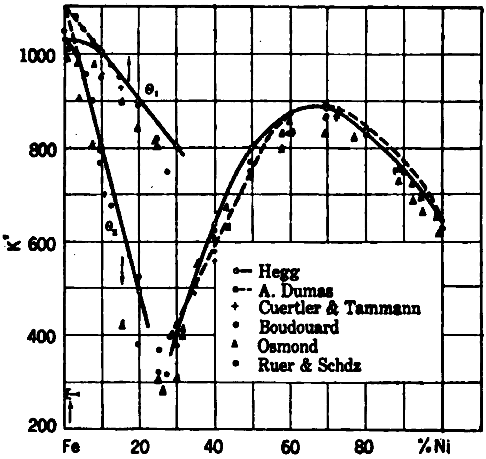

FIG. 15—ABSOLUTE CENTIGRADE TEMPERATURES OF MAGNETIC TRANSFORMATION (CURIE POINTS) FOR PURE FERRONICKELS (HEGG)

In a recently published circular[14] the Bureau of Standards has compiled all the important data published prior to 1916 relating to iron-nickel alloys and includes an extensive bibliography. Fig. 15 gives the magnetic transformation according to Hegg reproduced from this circular, giving also points determined by Dumas and by Osmond magnetically, by Guertler and Tammann and by Rurer and Schuz metallic graphically and by Boudouard thermoelectrically. The ordinates of this curve are in absolute degrees centigrade (= 273 + deg. cent.)

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[14] Circular No. 58. Invar and Related Nickel Steels April 4, 1916.

TABLE IV

MAGNETIC AND ELECTRICAL PROPERTIES OF Fe-Ni ALLOYS. REVERSIBLE AND IRREVERSIBLE TRANSFORMATIONS

Condition	Max. permeability, μ_m	Flux density for μ_m, gausses	Flux density (kilogausses) for H (gilberts/cm.) of				Saturation value, $4 \pi I_s$	Hysteresis loss for B = 10,000—ergs/cu. cm./cycle	Electrical resist.—microhms/cu. cm.	Max. permeability, μ_m	Flux density for μ_m, gausses	Flux density (kilogausses) for H (gilberts/cm.) of				Saturation valve, $4 \pi I_s$	Hysteresis loss for B = 10,000 ergs per cu. cm. per cycle	Electrial resistance. microhms per cu. cm
			1	10	100	400						1	10	100	400			
			30 per cent Ni (2 Ni 232)						Rings			34.5 per cent Ni (2 Ni 234)						
Annealed at 1,100 deg. c. in Vacuo..	560	220	0.4	1.5	2.8					2,210	1,500	2.0	7.2	11.2				
Cooled to —70 deg.	..	..	..	..	..							0.5	5.3	13.1				
Cooled to —70 deg. and heated to + 20 deg.	232	4,650	0.1	2.0	10.3							2.1	7.0	11.1				
Cooled to —185 deg.	279	5,580	0.1	2.3	13.0							0.3	5.3	13.4				
Cooled to —185 deg. and heated to + 20 deg.	244	4,880	0.1	1.9	12.0							0.6	4.7	11.2				
			30 per cent Ni (2 Ni 232)						Rods			34.5 per cent Ni (2 Ni 233 and 234)						
Annealed at 900 deg. c. in Vacuo....	500	250	0.4	1.5	2.6	3.1	2,700	2,230[1]	82	3,200	1,300	2.7	8.0	11.7	12.0	11,500	1,000	81
Cooled to —70 deg. and heated to + 20 deg.	213	4,550	0.1	1.8	10.2	13.4	14,600	31,100	38	..	..	2.6	7.7	11.6	12.0	..	..	80
Cooled to —185 deg. and heated to + 20 deg.	250	5,750	0.1	1.8	12.5	16.4	17,800	..	32	3,120	1,560	2.6	7.6	11.7	12.1	11,710	..	78
			15 per cent Ni (2 Ni 225)						Rods			50 per cent Ni (2 Ni 261 and 262)						
Annealed at 900 deg cent. in vacuo.	460	3,000	0.2	4.9	17.3	21.0	21,600	12,500	30	9,600	2,870	6.8	13.0	15.4	15.6	15,200	810	44
Cooled to —70 deg. and heated to + 20 deg.	494	3,600	0.2	4.8	17.2	21.0	..	..	30	..	..	6.8	12.9	15.2	15.5	..	..	44

[1] Calculated from the loss for $B = 2,000$ by means of the 1.6 law $= W_{10} = W_2 (10/2)^{1.6}$.

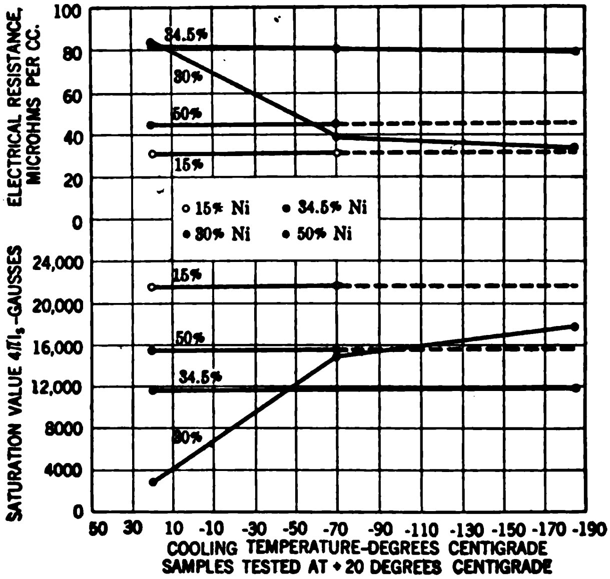
ELECTRICAL RESISTANCE, MICROHMS PER CC.
100
80
60
40
20
0
SATURATION VALUE $4\pi I_s$ -GAUSSES
24,000
20,000
16,000
12,000
8000
4000
0
34.5%
30%
50%
15%
○ 15% Ni
● 34.5% Ni
● 30% Ni
● 50% Ni
15%
50%
34.5%
30%
50
30
10
-10
-30
-50
-70
-90
-110
-130
-150
-170
-190
COOLING TEMPERATURE-DEGREES CENTIGRADE
SAMPLES TESTED AT + 20 DEGREES CENTIGRADE

Fig. 16

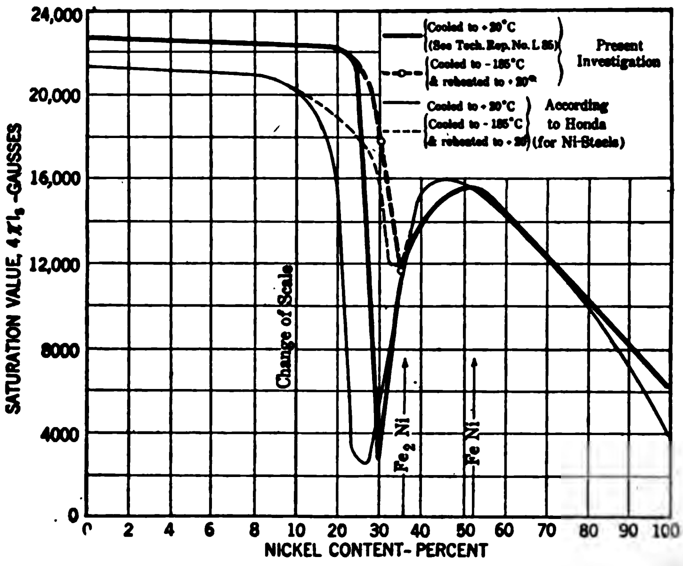
24,000
20,000
16,000
12,000
8000
4000
0
SATURATION VALUE, $4\pi I_s$ -GAUSSES
Cooled to + 20°C
(See Tech. Rep. No. L 85)
Cooled to - 185°C
& reheated to + 20°
Present Investigation
Cooled to + 20°C
Cooled to - 185°C
& reheated to + 20°
According to Honda (for Ni-Steels)
Change of Scale
Fe_2 Ni
Fe Ni
0
2
4
6
8
10
20
30
40
50
60
70
80
90
100
NICKEL CONTENT- PERCENT

Fig. 17

More recently Honda[15] has studied the magnetic and electrical properties of nickel steels at ordinary temperatures and after quenching in liquid air. The latter treatment caused remarkable changes in the region of 25 to 30 per cent nickel, but no appreciable changes for other nickel contents, thus confirming the results already referred to. The results are shown in Figs. 17 and 18. On account of the foreign elements present in Honda's steels it seemed desirable to repeat his tests with pure alloys, and thus round out the investigation of the iron-nickel alloys covered in Part I.

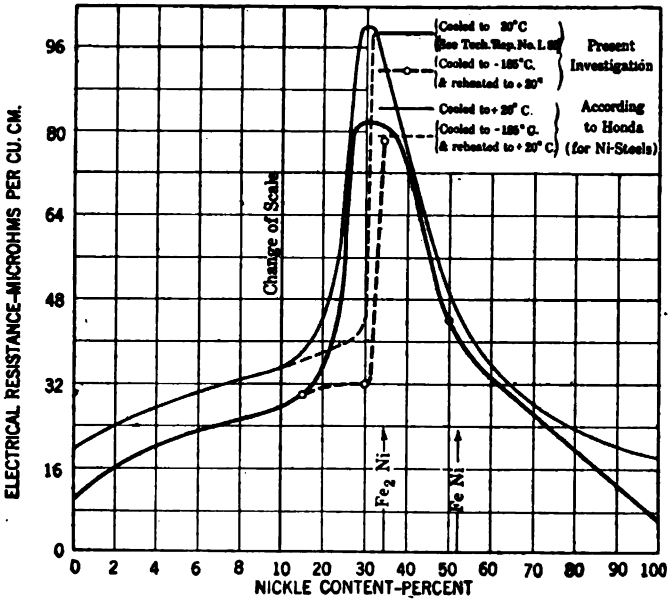

FIG. 18

Procedure. The samples used were selected from those used in the previous investigation, the preparation of which is fully described in Part I. The test pieces used are as follows:

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[15] The Thermal and electrical properties of Nickel Steels *Sc. Reports* Tohoku Imp. Univ. 7, p. 59, 1918.

2 Ni 225—15 per cent Ni, 2 Rods 1.27 cm. (½ in.) diameter.
2 Ni 232—30 per cent Ni, 2 Rods as above and 1 ring 3.83 cm. (1½ in.) inside diam. and 4.47 cm. (1¾ in.) outside diam.
2 Ni 233—34.5 per cent Ni, 1 Rod and 1 Ring as above.
2 Ni 234—34.5 per cent Ni, 1 Rod and 1 Ring as above.
2 Ni 261—50 per cent Ni, 1 Rod and 1 Ring as above.
2 Ni 262—50 per cent Ni, 1 Rod and 1 Ring as above.

As a preliminary the rings were tested successively at room temperature, in CO_2 snow (– 70 deg. cent.), again at room temperature, then in liquid air (– 185 deg. cent.) and finally at room temperature. Then the rods were tested at room temperature, after having been cooled to – 70 deg., and after having been cooled to – 185 deg. The cooling to – 70 deg. was done by immersing the test pieces in gasoline, gradually adding CO_2 snow until no more would melt and maintaining the temperature for about half an hour by adding more snow from time to time.

FIG. 19—2 NI 232. 30 PER CENT NI COOLED TO ROOM TEMPERATURE. (100 DIAMETERS)

Results. The results are tabulated in Table IV and shown graphically in Figs. 16, 17 and 18. It will be noted that while there is practically no change in the properties of the 15, 34.5 and 50 per cent alloys the 30 per cent alloy has undergone a marked change. Its saturation value, $4 \pi I_s$, was increased from 2700 to 14,600 by cooling to – 70 deg. and to 17,800 by cooling to —.185 deg. Similarly, the electrical resistance was decreased from 82 to 38 and to 32 by the above treatments. It is probable that the change would have gone to completion at – 70 deg. if the testpiece had been held there sufficiently long and that the irreversible transformation point therefore lies between 0 and — 70 deg. thus confirming Dumas' results.

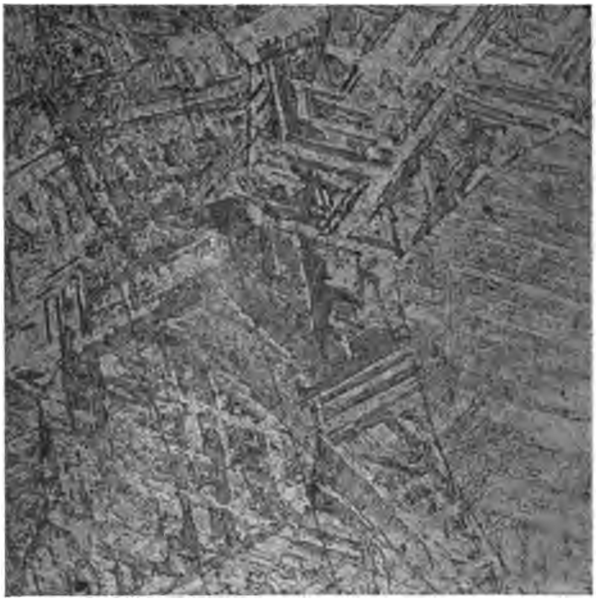

FIG. 20—2 Ni 232 30 PER CENT Ni COOLED TO —70 DEG. CENT. AND REHEATED TO ROOM TEMPERATURE. (100 DIAMETERS)

While only a few points were determined, they are sufficient to demonstrate that the cusp of both curves, after the completion of the transformations, occurs for approximately 34.5 per cent nickel instead of for 30 per cent as the previous investigation would lead one to believe. This points strongly towards the existance of the compound Fe_2 Ni, corresponding to 34.6 per cent nickel, suggested by earlier investigators. The existence of the compound FeNi corresponding to 51.5 per cent nickel, is more doubtful, because there is no definite indication of it in the electrical resistance curve.

It is interesting to note that while the saturation value of the 30 per cent alloy was increased to a remarkable extent by being cooled to below the irreversible transformation point, the permeability at low flux densities was decreased and the hysteresis loss enormously increased, the latter 15 times. In other words, the reversible transformation produces a material with low hysteresis loss and low magnetic saturation, while the irreversible transformation produces a material with 6 times the saturation value, but with 15 times the hysteresis loss.

It is also interesting to note that there apparently is no change in the microstructure of the 30 per cent alloy accompanying the marked magnetic and electrical transformation. Fig. 19 shows the microstructure of the alloy cooled to room temperature, and Fig. 20 after cooled to – 70 deg. cent. and reheated to room temperature. The latter corresponds to magnetic and electrical properties widely different from those of the former, and yet the microstructures at 100 diameters are identical as far as can be seen. It is therefore apparent that the magnetic and electrical transformations are atomic or ionic in their nature.

Discussion on "Electric and Magnetic Properties of Nickel-Iron Alloys" (Yensen), Boston, Mass., April 9, 1920.

S. L. Gokhale (by letter): In this discussion I intend to limit my remarks to two points only, *viz.*, saturation values and methods of measuring them.

For measurement of magnetization, with magnetizing force up to 400 gilberts, Mr. Yensen has used the Bureau of Standards method, popularly known as Burrows method. He has not specified how the saturation values have been measured, and extrapolation according to Kennelly's law, does not in all cases fit the figures given by him. According to Mr. Yensen, the saturation value of iron is not improved by the presence of nickel. In this respect his findings are opposed to those of Burgess and Aston, who seem to have produced three samples having saturation values higher than pure electrolytic iron, by introducing 7 to 10 per cent of nickel, as has been observed by Mr. Yensen on page 792.

Burgess and Aston have used for their tests, methods which are now superseded by the Burrows method. Their tests stop at $H = 100$, and it would be scarcely fair to place their findings along side those of Mr. Yensen which are made with a better method and extend over a much greater range.

Computing the saturation values for different alloys by the extrapolation method, using data in Mr. Yensen's paper, I find that the quality of iron improves by the introduction of nickel up to about 7 per cent of nickel, in the presence of 0.2 per cent of silicon and with only about 4 per cent of nickel when there is no silicon. For complete computation giving the saturation values, by extrapolation from Mr. Yensen's data, see Tables I and II attached.

This raises the question of the validity of the extrapolation formula, for the determination of saturation values. The importance of the saturation values of materials both for theory and for practical engineering application is too obvious to need further comments, and it is scarcely necessary to emphasize the need of some direct and reliable method for obtaining the necessary data. To this task I have devoted a considerable part of my time during the last six years.

The experimental work has just been completed (that is, in the only sense in which an experimental work can ever be completed). A method for measurement of saturation values, has been developed giving results correct to 0.3 per cent. A paper for publication explaining the method is in progress and will

TABLE I

SATURATION VALUES FOR IRON—NICKEL ALLOYS

With pure Iron and Nickel
By extrapolation method based on Kennelly's Law.
From data by T. D. Yensen, (page 797).

Specimen	Nickel per cent	B for $H = 400$	$H = 100$	$B - H$ for $H = 100$	$H = 400$	ρ for $H = 100$	$H = 400$	$\Delta\rho$	Sat. $\frac{\Delta H}{\Delta\rho}$	Sat. (Yensen)
2 Ni.										
201	.50	18.3	21.2	18.2	20.8	.00550	.01923	.01373	21830	22700
203	1.01′	18.4	21.4	18.3	21.0	546	1903	1357	22100	22500
205	1.48	18.6	21.5	18.5	21.1	540	1895	1355	22150	22700
207	1.96	18.6	21.6	18.5	21.2	540	1885	1345	22300	22600
209	2.44	18.75	21.65	18.65	21.25	536	1880	1344	22330	22600
211	2.84′	18.6	21.65	18.5	21.25	540	1880	1330	22550	22700
213	3.85	18.6	21.8	18.5	21.4	540	1870	1330	22550	22360
215	4.62′	18.7	21.2	18.6	20.8	537	1923	1386	21650	22300
294	5.68	16.7	19.1	16.6	18.7	601	2140	1539	19500	20000
217	5.67	19.0	21.7	18.9	21.3	529	1880	1351	22200	22400
219	7.33′	19.1	21.6	19.0	21.2	526	1885	1359	22090	22600
221	8.00	19.1	21.6	19.0	21.2	526	1885	1359	22090	22100
237	9.61′	18.7	21.0	18.6	20.6	537	1940	1403	21350	21200
295	11.10′	19.1	21.5	19.0	21.1	526	1895	1369	22000	22000
225	14.92′	17.3	21.0	17.2	20.6	581	1940	1359	22100	21600
296	15.87′	15,2	19.7	15.1	19.3	661	2070	1409	21300	22300
297	21.03′	15.2	20.0	15.1	19.6	661	2040	1379	21800	22200

TABLE II

SATURATION VALUES FOR IRON—NICKEL ALLOYS

Iron and Nickel with 0.2 per cent silicon
By extrapolation method based on Kennellys' Law
From data by T. D. Yensen pages 779 and 800.

Specimen	Nickel per cent	B for $H = 100$	$H = 400$	$B - H$ for $H = 100$	$H = 400$	ρ for $H = 100$	$H = 400$	$\Delta \rho$	Sat. $\frac{\Delta H}{\Delta \rho}$	Sat. (Yensen)
Ni										
202	0.50	18.2	21.1	18.1	20.7	.00553	.01933	.01380	21750	22500
204	1.00	18.4	21.4	18.3	21.0	546	1905	1359	22100	22800
206	1.48	18.3	21.6	18.2	21.2	550	1885	1335	22400	22500
208	1.96	18.5	21.5	18.4	21.1	543	1895	1350	22200	22500
210	2.44	18.4	21.5	18.3	21.1	546	1895	1340	22300	22200
212	2.90	18.5	21.4	18.4	21.0	543	1905	1362	22000	22500
214	3.85	18.6	21.5	18.5	21.1	540	1895	1355	22150	22500
216	4.76	18.8	21.75	18.7	21.35	535	1872	1337	22490	22800
218	5.67	19.1	21.7	19.0	21.8	526	1878	1354	22150	22240
220	7.00	19.2	22.0	19.1	21.6	524	1850	1326	22600	22600
273	8.00	19.2	21.9	19.1	21.5	524	1860	1336	22500	21800
272	10.00	18.5	21.5	18.4	21.1	543	1895	1352	22200	22700

NOTE: Sample No. 273 contain 0.1 per cent manganese but no silicon content.
Sample No. 272 contain 0.1 per cent manganese and 0.2 per cent silicon
The remaining samples contain 0.2 per cent silicon.

be published at the earliest possible date. This method briefly stated is as follows: The sample under test, generally a rod ½ in. diameter and about 12 in. long, or an equal sized bundle of strips is magnetized by a coil capable of producing 3000 gilberts. The magnetic circuit is completed by a heavy yoke of soft iron. The magnetizing coil is connected in series with the primary coil of a variable mutual inductor. The potential coil surrounding the sample near its middle is connected in series with the secondary coil of this variable inductor, and also in series with a galvanometer of high sensitivity. See Fig. 1.

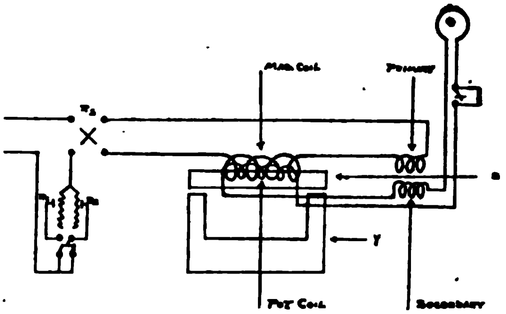

Fig. 1

S = Sample Under Test. Y = Yoke. R_s = Reversing Switch. R_1 and R_2 = Rheostats for Controlling Independently, Two Points on the Saturation Curve. Other Details Omited in Diagram.

The magnetizing current is first adjusted to give a force of about 2200 gilberts, exact adjustment or measurement not being necessary; this adjustment is made by the rheostat R_1. The magnetizing force is now raised to about 3000 gilberts, the necessary adjustment being made by the rheostat R_2. If at $H = 2200$ the sample is saturated, it will remain saturated at $H = 3000$; the increase of flux in this case will be due to increase of H only. If also the design of the permeameter is such as to make H, proportional to the magnetizing current within the limits of H, 2200—3000, this increment in H can be neutralized by a proper setting of the mutual inductor, and conversely if the mutual inductor can be so arranged as to give a zero deflection for all upward changes in the magnetizing current, the sample should be regarded as saturated.

A reversal of the magnetizing current will under these circumstances give an effective reversal of the intrinsic flux $B-H$ only; or in other words the ballistic deflection of the galvanometer under these conditions will give directly the saturation value, without any intermediate determination of H or without any computations or corrections dependent on H. In this method it is therefore, not necessary to measure the value of H, either accurately or approximately, but if desired, it may be computed by the formula

$$H N A = \frac{M I}{10}$$

Where H is the magnetizing force producing saturation.
N is the number of turns of potential coil.
A is the area enclosed by the potential coil.
M is the inductance of the variable mutual inductor for the particular setting under test.
I is the magnetizing current.

During the experimental period, saturation values have been obtained for a large number of iron alloys, both by the method outlined above, and by the extrapolation method, the necessary data for the latter being obtained by the Burrows method. As a general rule there is a discrepancy between the results obtained by the two methods; in extreme cases this discrepancy has reached 10 per cent, but as a general rule it does not exceed 3 per cent.

Such a discrepancy may be due partly to errors inherent in Burrows method and partly in the extrapolation formula. The possibility of error in the data obtained directly by the method outlined above, is not to be ignored, but evidence of such error has not so far appeared. Incidentally I may mention at this point, that the high saturation values $4 \pi J$ shown by Mr. Yensen are very unusual.

Prof. B. O. Pierce has given results of test on four samples of American Ingot iron; the highest saturation value obtained was 21780. (*Proceedings of the American Academy of Arts and Sciences; June, 1913*, p. 139). An analysis of his data indicates an error in the measurement of H, which when corrected for, reduces the saturation value to 21475, which is in close agreement with the result of my tests on similar material. That conclusion is also confirmed by the researches of Hadfield and Hopkinson (*Journal I. E. E.* Dec. 1910—page 251), who obtained for the purest available sample, a saturation value of 21100.

In view of these facts I think we ought to make sure of our data, and therefore, give more attention to the methods of test before drawing any important conclusions.

T. D. Yensen: Replying to the discussion contributed by Mr. Gokhale in regard to the saturation values of Iron-Nickel alloys and methods of measuring them I very much regret that I did not mention in my paper the method that was actually used by us, as this might have saved Mr. Gokhale's time in preparing his tables. We actually did use the extrapolation method based on Kennelly's law. The reason that Mr. Gokhale has obtained values that differ from those given in my paper is that he makes use of the inductions for $H = 100$ and $H = 400$ in order to compute the slope of the $\rho - H$ curve, whereas $H = 100$ lies below the bend in the $\rho - H$ curve in every case that has come to my notice. This is illustrated in the accompanying table and Fig. 2 for Alloy 2Ni 204. Here it will be noticed that

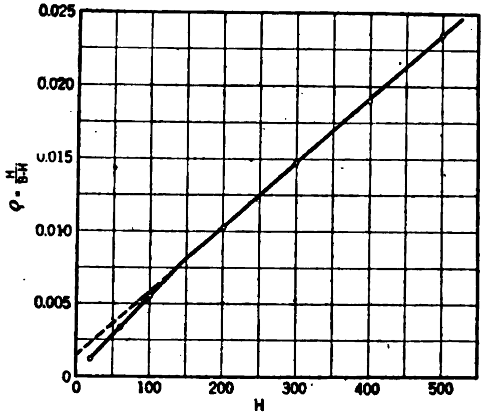

FIG. 2—COMPUTING SATURATION VALUE FOR SPECIMEN 2 Ni 204 BY KENNELLY'S LAW

Using Values for $H = 100$ and $H = 400$ } Incorrect

$$\text{Sat.} = \frac{\Delta H}{\Delta \rho} = \frac{300}{0.0135} = 22200$$

Using Curve

$$\text{Sat.} = \text{Slope of Curve} = \frac{500}{0.02195} = 22800 \text{ Correct.}$$

H	B	B-H	ρ
20	16600	16600	0.0012
100	18430	18330	0.0055
200	18870	18670	0.0102
300	20730	20430	0.0147
400	21430	21030	0.0190
500	21810	21310	0.0235

the $\rho - H$ curve is a straight line down to $H = 200$ but that for lower values of H, the points lie on another straight line having a greater slope than the former. It is, therefore, evident that, by making use of the values for $H = 100$—as Mr. Gokhale has done—saturation values are obtained that in every case are too low. All of my values were computed from the slopes of the $\rho - H$ curves above $H = 200$ as shown in the figure, and may therefore be considered as substantially correct, and my conclusion that Ni does not improve the saturation value of pure iron still holds.

If saturation values computed from the data obtained by Burgess and Aston show an increase by the addition to pure iron of small percentage of Ni (7-10 per cent) this may be due to one of two factors, or both, namely; (1) The relatively high percentage carbon in their pure iron, and (2) inaccuracy of testing, inherent in the method used. However, my results check theirs in one particular;—Ni up to 7 per cent increases the permeability of pure iron at high inductions (for $H = 100$ to 400); but—this does not mean that the saturation values have been increased. This distinction should be kept clearly in mind.

In regard to the saturation values of American Ingot iron, to which reference is made by Mr. Gokhale, I may say that we have tested many samples of this iron and of Armco iron (which is practically the same) and have obtained saturation values of 22,000 for the material as received. When this material is vacuum treated however, the saturation values can be raised to 22,700 or practically the same as that obtained for pure vacuum fused electrolytic iron. The conclusion seems therefore natural, that the reason for the lower values obtained by Professor Pierce for Ingot and by Hadfield and Hopkinson for the purest available iron (which I believe was Swedish charcoal iron) is impurities in the samples, either gaseous or solid.

The method for measuring the saturation value directly as proposed by Mr. Gokhale—although the principle may not be new— is undoubtedly valuable in so far as it may be used as a check on the extrapolation method, but otherwise I believe the latter is sufficiently accurate and simple for most purposes.

I hope I have succeeded in removing any further skepticism that may exist in regard to the reliability of the results.

In conclusion I wish to acknowledge my indebtedness to Mr. Thomas Spooner and his assistants, for the magnetic testing.

Presented at the 30th Annual Convention of the American Institute of Electrical Engineers White Sulphur Springs, W. Va., June 29, 1920.

AN ENGINEERING ANALYSIS OF THE LABOR PROBLEM

Presidents Address

BY CALVERT TOWNLEY

THE Constitution prescribes that the President must deliver an address at the Annual Convention,—whether you want to hear it or not. It has been sometimes customary for the Executive who is about to retire from office to summarize the achievements of his term. It has seemed to me that these have been better chronicled as they occurred in the pages of your JOURNAL. The pastime of prophecy has some attractions, but is of doubtful value, and somewhat dangerous as well. The advances in the art are well covered by many valuable technical papers of the year including those which are to be submitted at this Convention. There remain broad public questions regarding which engineers are peculiarly well qualified to have opinions and to influence the opinions of others and it has therefore seemed to me that I might perhaps come nearer justifying the constitutional provision by dealing with one of these.

One of the most important problems which the American people now have to solve is that of the relations between the employer and the employee classes, generally referred to as capital and labor. A large measure of our national success will depend upon this solution, and as engineers are even more directly affected by and therefore more intimately connected with industrial prosperity than are the other professions, for example, of law, medicine and theology, it is more obviously their duty to assist in the solution of the problem as much as they can. Although this subject has been voluminously discussed by numberless people between employer and employee and, because they could escape doing so, provided no means by which the workmen could be penalized for violation, except by the union itself. This flagrant flouting of a fundamental did not for a time have any serious effect, but the blunder once made was never thereafter corrected and it is probable that even today it would be difficult to find a labor leader who could be made to see the seriousness of this mistake, much less to endeavor to rectify it. I pass quickly over a long period of years after the first organization of unions during which unionism had its ups and downs and came to be regarded, if not with equanimity, at least with tolerance by the employer class, and come to the recent period of the World War. Here an entirely new condition was created—an imperative demand for men came up almost over night. It simply had to be satisfied at no matter what price. At the same time not only was the supply diminished by those required for the armed forces but immigration stopped as well. The effect on the price of labor was axiomatic—it rose. The unions being the vehicles through which the demands of many classes of workmen could most readily be expressed at once became active. The law of supply and demand was fighting on their side and they made the most of it. Wages and prices rose to hitherto unheard-of figures. Labor had too much power; its demands had to be met whether or no, and of course it abused its power. Then the unions made another mistake. They credited too much of their successes to their organization and failed to appreciate the part played by the law of supply and demand, also they underestimated the power of resistance which could be engendered by oppression. In their turn they became the oppressors—insistent, arrogant. Where their principal weapon, the strike, was not effective as against the employer class in one industry, they enlisted the workmen in other industries, and who had no grievance, in sympathetic strikes. They asserted their power to dominate the public convenience, safety and health in order to coerce their opponents into submission to their will. Were it necessary I could recite many specific instances to prove this fact but I assume it to be so generally known and accepted that a recital would be superfluous. Now this is the condition to day and it is one which has caused much grave concern and has produced many strange ideas. We hear that a new order of things has come to pass; that the "rights" of labor must now be respected; that the workman will hereafter have a greater share of the products of his toil; that he must share in the management of industry and have a recognized place in government, and the like. To my mind the facts warrant none of these assumptions. There is no new order,—economic laws are the same as they always have been. They are as ruthless and as inexorable as are the laws of physics. Neither has human nature changed. The unions are in the saddle, but, to use a hunting expression, they are riding for a fall. Please let me remind you again that I am not blaming the unions, or any one else. I am trying to confine myself to statements of facts and to logical opinions based thereon according to the precept and training of the engineer. Suppose we analyze the fundamentals.

In our system of govermnment where every man can vote and have his vote counted, and with our multitude of newspapers informing everybody of what is going on, there is little chance that a condition can arise such as that of China or of Russia or even of Germany. Further while we talk a great deal of politicians and parties, we know that on any really vital question the people are going to make up their minds themselves and will elect men who will carry out the wishes of the majority. In other words the great majority rules.

The number of workmen in labor unions is variously estimated to be from ten to forty per cent of the men employed in trades where there are unions. These figures represent the extreme claims of the contending parties. We shall probably not be far wrong if we take a compromise figure of say 20 per cent. In the largest class of all, the farmer there are no unions and similarly none in many other avocations, so if we consider the total voting strength of the country, the percentage of men in unions is certainly not over 10 per cent and probably not over 5 per cent. Of this small proportion a few are the leaders, a larger number enthusiastic followers and the great majority largely governed by conditions. By that I mean they are loyal unionists when the union is succeeding but desert on small provocation when it is defeated. This is almost obviously true because the workmen are banded together to accomplish selfish purpose and no other. The individual workman cares no more for his fellow as a class than does the employer. It follows therefore that a union must continue to succeed or it will disintegrate and disappear. The history of the last thirty years has recorded many once powerful unions even the names of which are now almost forgotten. The "Knights of Labor" and "The Amalgamated Association of Iron and Steel Workers" are two examples which illustrate this point. The strength of organization contending against the mob is very great and it is probably true that so small a proportion as ten per cent or even as five, would prevail under these conditions. That in fact is the situation confronting the nation today. This small organized percentage supported by the operation of the law of supply and demand has prevailed, but looking into the future, it seems clear that there is only needed a sufficient incentive to induce the unorganized 95 per cent, or even a small part of them, to get together in opposition to the five per cent in order that the latter may be overwhelmed. This incentive has been or will be furnished by union oppression. I say "has been or will be" because the abuse of too great power always grows. Therefore if the unions shall still continue to gain their ends, even in the face of their present insolence and ruthless disregard of others, their arrogance and self confidence born of such successes, will inevitably lead them on to acts of greater and yet greater oppression until they force their opponents to combine against them for defense or even for self preservation. The moneyed class never has had, and probably never will have much sympathy or cooperation from the general public for the very obvious reason that the general public is jealous of it. Sympathy goes naturally to those who are worse off. The moneyed class being better off than the great majority therefore comes in for sympathy with a minus sign, that is, jealousy. An incentive which will induce the public to oppose the unions can therefore be sufficient only when the reason is strong enough to overcome this jealousy. That reason becomes sufficient when the need for self preservation is made clearly apparent. Already there are indications that the people will not much longer submit to domination by the unions. The Governor of Massachusetts defied them, was shortly thereafter reelected by an abnormally large majority and has now been nominated on the Republican National ticket for Vice-President which latter distinction it is safe to say he owes almost entirely to this incident. The State of Kansas has enacted a law to curb union interference with the peace and comfort of her citizens and her Governor has become a popular National figure in consequence. The temper of much of the daily press has changed, and where formerly there was much said about the so-called "living wage" and criticism of "capitalistic greed" "there now appear articles about the vicious cycle of mounting wages and costs", "loafing on the job" and the "need to teach the people thrift and economy". These and other signs are merely symptoms of the incentive to resistance that may be expected to be superinduced by oppression. The aroused public would not be just to the unions, and an attack once thoroughly launched might be expected to go further than it should. However I am not here discussing justice or how either party should behave but rather what they have done in the past and under pressure of human nature and economic laws most certainly will do in the future.

Whether or not an aroused public does curb union domination, the present high wage era will not be radically affected. That is a condition controlled, not by the unions at all, but by the law of supply and demand. Even the complete abolition of the unions while it might check further increases and perhaps bring about some recessions, and while it would reduce living costs some what by cutting out interruptions, and tend to increase production, would not itself either increase the supply of workmen available nor decrease the demand for their services. Indeed while I have discussed the fundamentals of barter between the employer and the employee and have endeavored to show the natural reactions resulting from barter with either party under duress, it is not at all certain that conditions will become so extreme as to bring about the reaction described. That is to say, instead of the abuse of power by the unions being curbed by the organized opposition of other classes of our people it may be reduced or even thwarted by the operation of the law of supply and demand. Lest I may be misunderstood I might here state that because I have been confining myself to a discussion of the relations between the employer and the employee and have had much to say about the employee's abnormally high wages, I do not want to be understood as giving that condition as the only cause of the prevailing high prices. Advantage has very generally been taken of the opportunity to increase profits by those who had wares to market, which is only another way of saying again that selfishness is a universal trait of which no class or classes has a monopoly. A bettering of conditions may be affected by a decrease in the demand for or an increase in the supply of labor. Already there is a marked reduction in the sales of certain products resulting from an unwillingness or the inability of many people to pay the exorbitant prices asked. Government reports of falling exports indicate that the expected lessening of the foreign market for our high priced products is approaching and it is not at all unlikely that this may be followed by heavy increases in our imports which will displace American made goods. This result may come about both because foreign nations are getting into their stride of production again and because the high prices prevailing here have naturally created an attractive market. A combination of these and other conditions will reduce the demand for our products and consequently for the workmen to produce them. The immigration authorities of the port of New York report their facilities overtaxed to handle the large numbers coming to our shores, and say that even those numbers would be far larger if there were only more ships, to carry the people who want to come. It is natural to suppose that as the heavy burdens of after-war taxation are brought home to the foreign people an increasing percentage will seek to escape them by coming here to live, and also that the gradual restoration of trans-Atlantic shipping to its normal schedules will afford the greater facilities thus demanded. This augmentation of population—largely workmen of course, will swell our supply and likewise tend to correct any shortage that may exist.

If my analysis of the facts is correct, it is clear that the present domination by organized labor is temporary and also that the era of high prices will pass. Therefore, no material permanent change in either our social order or in our industrial structure is to be anticipated. In the contest between brains and brawn, waged since the world began, brains have always won and brains always will. Free play for the natural forces of trade may be counted upon to exercise a beneficent influence and it should be hampered and interfered with by government restrictions as little as possible. We cannot of course determine from history or from any facts at hand, how long a time it will take for conditions to become normal again but what we need now is clear thinking, courage and patience. I know that we can rely on the engineer for clear thinking and I haven't the slightest doubt about his courage, but I am not so sure about his patience. You should be leaders of thought in your several circles. You can help to allay much of the present anxiety about the so-called unrest and the apprehension as to the future; therefore I feel warranted in asking you to give your close consideration to the subject matter of what I have said this morning.

Presented at the 30th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29, 1920.

ANNUAL REPORT OF THE ELECTROCHEMISTRY AND ELECTROMETALLURGY COMMITTEE

To the Board of Directors.

THE chairman of the Electrochemistry and Electrometallurgy Committee for the year 1917–1918 suggested to the Board of Directors of the American Electrochemical Society that it should approach the A. I. E. E. with the idea of forming a joint committee to consider matters that were of common interest to the two societies. Such a committee could consider papers which might be of importance to both societies, could bring to each society's attention matters of common interest and make arrangements for occasional joint meetings. The result was that the Board of the American Electrochemical Society appointed Messrs. Fink, Parmelee and Schluederberg "to cooperate with" this Committee.

Nothing was done in this matter in the year 1918-1919; but this year it was taken up again with the result that a joint meeting of the A. I. E. E. and the A. E. S. was held in Boston April 9th, 1920. The meeting was successful, the combined registration amounting to about 425, with the result that the seating space of the rooms used for meeting was somewhat overtaxed.

It is believed that the occasional holding of joint meetings with the A. E. S. similar to that held this year is desirable and it is suggested that the plan be followed up next year when an even more successful joint meeting might be held. The weak point of the April meeting was the failure to print in advance some of the papers presented at the meeting by members of the A. I. E. E. The blame for this rests with

the chairman of your committee who had not provided for his unforseen detention in Europe during the first part of the year, with the result that sufficient time was not given to authors to prepare their papers before the meeting. The printing of the papers in advance of the meeting is important so as to stimulate discussion and the exchange of views between the members of the two societies which have so much ground of common interest.

FRANCIS A. J. FITZGERALD, *Chairman.*

ANNUAL REPORT OF THE COMMITTEE ON ELECTRICAL MACHINERY

To the Board of Directors.

THE Committee on Electrical Machinery placed before itself the aim to cover as nearly as possible the range of work in which there had been changes of importance which had not yet become a matter of record in the TRANSACTIONS of the Institute.

The Committee joined with other committees in the presentation of papers at the Midwinter Convention on the general subject of super power stations for the eastern coast states. This subject will receive renewed attention from time to time.

The polyphase alternating-current commutator motor for variable speed operation received comprehensive treatment in two papers, one by B. G. Lamme, presented before the Schenectady Section, and the other by John I. Hull, to be presented at the Annual Convention.

The subject of temperature measurements in large alternating-current generators and allied questions received a complete and exhaustive treatment in four papers to be presented at the Annual Convention. Rationalization of temperature measurements and uniformity of policy have been established as a result of these papers and of the investigations which led up to them.

A mathematical paper by R. E. Gilman discusses the losses due to eddy currents in stranded conductors embedded in slots, thus carrying on further the fundamental work begun by A. B. Field.

A symposium of papers on the design and the operating characteristics of welding machinery has been arranged for the Annual Convention so as to make a record in the TRANSACTIONS of the progress made in this important direction.

In presenting this brief report, the Chairman desires to point out that he has considered it the essential function of his Committee to trace, through the papers solicited and the meetings arranged, the progress of the art which it was desired to record. Thus it is unnecessary to deal again here with the subject matters comprehensively presented in the papers.

In conclusion, the Chairman wishes to thank the members of his Committee for their support and co-operation.

B. A. BEHREND, *Chairman*

ANNUAL REPORT OF THE PROTECTIVE DEVICES COMMITTEE

To the Board of Directors:

EARLY in the year the Protective Devices Committee appointed a number of sub-committees to investigate, study and report on various subjects and these are discussed in some detail in the following summary.

SUBCOMMITTEE ON TERM, "INSTANTANEOUS RELAY"

E. E. F. CREIGHTON, *chairman*

This Subcommittee resulted from the criticism of the term "instantaneous" as applied to a relay, it being contended that the term did not properly describe the device. A number of other terms were suggested of which the term "quick acting" appeared to have the preference. On account of the wide diversity of opinions among the members of the committee on the subject, no definite recommendation has been made, and it was agreed that the Chairman of the Subcommittee would summarize the various opinions on the subject and submit them for publication in the JOURNAL.

SUBCOMMITTEE ON SCHEMES OF RELAY PROTECTION

P. H. CHASE, *chairman*

Following the recommendation in the Report of the Committee last year, a number of letters were written to the executives on a selected list of operating companies, and about thirty-five replies have been received from companies who were willing to cooperate in the work assigned to this subcommittee, and one of their engineers delegated as the correspondent on the subject. Questionnaires regarding schemes of relay protection have been forwarded to these correspondents. The United States and Canada has been divided into six geographical sections and each section has been assigned to a member of the subcommittee who is with an operating company in that district. These members are charged with examination and classification of replies received and with distribution of the information to the other members of the subcommittee. It is felt that the work of the subcommittee to date has been very largely of a preliminary nature and that its value will be more apparent later in the year.

SUBCOMMITTEE ON OIL CIRCUIT BREAKERS AND SWITCHES

H. R. WOODROW, *chairman*

This Subcommittee is cooperating with similar committees of the National Electric Light Association and the Power Club, and is securing information on the subject from a limited number of the larger operating companies. There has not been sufficient time since the request for information was issued, to secure answers from the various companies and to summarize the results. The subject of high-tension fuse protection has also been referred to this Subcommittee.

SUBCOMMITTEE ON SYSTEM DISTURBANCES

D. W. ROPER, *chairman*

It was the opinion of a number of members of the Committee that the serious system disturbances experienced by the operating companies were due in a large measure to the defective action, or perhaps the absence, of suitable protective devices, and that a study of such disturbances might indicate the lines along which the future work and study of the Protective Devices Committee should properly be directed. The members of the Subcommittee are interchanging information on this subject and in recent months have been somewhat embarrassed by the amount of data available. An unusually large percentage of the larger operating companies have experienced severe system disturbances and in some cases the behavior of the protective devices was an important factor in the disturbances. The Subcommittee is now endeavoring to formulate some standard method of reporting on such matters, so that the reports can be properly condensed and summarized and be made available for the purposes of the Protective Devices Committee.

SUBCOMMITTEE ON REACTORS

F. E. RICKETTS, *chairman*

In the last few years there has been a marked improvement in the design of reactors such as, methods of winding to reduce eddy currents, improvements in structure to increase the insulation strength, and to work the copper to the best advantage. In some cases this improvement in design has been taken advantage of to reduce the size of the reactor so that the coils of today are small when compared with those of equal rating built a few years ago. This, however, is not a feature that should be carried to extremes as the current limiting reactor is, fundamentally a protective device and this function should be fully recognized in its development. Every effort should be toward the perfection of the design to avoid failure of the device itself and at the same time avoid the reduction of any of the protective features.

The application of reactors has been reviewed at different times in papers presented before the Institute and various schemes of connection have been proposed; but there is still some difference of opinion as to the proper place to insert reactors to obtain the best results, and there are many factors entering into this question. Each application requires a separate study but in general the best protection for the service is obtained by installing the reactors in individual feeders, and this is the practise followed by most companies provided the necessary space is available. However, some engineers prefer to sectionalize the system by inserting reactors in the bus, but when this is done, consideration should be given to the possbility of increasing the chance of the different sections getting out of synchronism which becomes very much of an operating menace where the amount of reactance in the circuits of the generators is much greater than that necessary to give the greatest synchronizing power.

SUBCOMMITTEE ON RESISTORS FOR USE WITH POTENTIAL TRANSFORMER FUSES

R. N. CONWELL, *chairman*

One of the larger companies in New York has made some elaborate and complete short-circuit tests on potential transformer fuses installed with and without protective resistors in series. Following these tests a number of companies in and around New York have adopted the standard practise of installing protective resistors in series with the potential transformer fuses which are connected to their high-tension buses for the purpose of limiting the current, and in this manner preventing damage to apparatus or interruption to service which had previously occurred. A summary of the results of the investigations, with the practise of the several companies is being prepared for submission to the Publication Committee for use in the JOURNAL. A canvass by the Chairman brought forth the information that certain other companies are using a different type of fuse which has given excellent results in service, and which does not require the use of a protective resistor in series.

PRIMARY CUTOUTS FOR LINE TRANSFORMERS

This subject was discussed at some length in the meetings of the Committee. It was the opinion of a number of the members of the committee that are connected with operating companies that the development of primary cutouts for line transformers had not kept pace with the development of the transformers themselves. One company reported they had considerably more trouble originating in the primary cutouts than in the transformers which these cutouts were intended to protect. Several of the larger companies install the larger sizes of overhead line transformers and all sizes of subway types of transformers in manholes without any primary cutout as their experience indicates that they have fewer interruptions in this manner than they would if they attempted to use cutouts with these transformers. It is the opinion of the members of the Committee that if the larger manufacturing companies would devote as much engineering skill to improvements in primary cutouts as they do to improving the design of line transformers, this difficulty which is one of long standing, would very quickly be eliminated.

During the discussion of the subject of primary cutouts it was brought out that one of the largest companies had, some years ago, abandoned the use of all types of cutouts or lightning arresters which were enclosed in an iron case. This was found necessary on account of the troubles with these devices during rain storms and wet snow storms, and that the difficulty had increased with the size of the generating or transforming units which supplied the current to the lines on which these devices were installed. During the past year another large operating company has taken exactly the same action and for the same reason. The Committee therefore, recommends that operating companies take note of this situation as with the increase in the size of their generating and transforming units they will ultimately be called upon to take similar action and they should, therefore, investigate types of such devices without the metal enclosing case with a view to its adoption for their future work.

During the year the Committee has undertaken to make investigations which will not be completed during the fiscal year. The Committee members who are taking an active part in this work all feel that the work is not only of particular interest to the members of the subcommittees, but should ultimately result in something of value to the Institute. It is recommended that the investigations of these subjects be continued next year, and that the members of the subcommittees who have been actively engaged in these several investigations be continued on the Protective Devices Committee during the ensuing year.

D. W. Roper, *Chairman.*

ANNUAL REPORT OF COMMITTEE ON INSTRUMENTS AND MEASUREMENTS

To the Board of Directors:

THE Instruments and Measurements Committee submits the following report covering its own activities during the past year and the developments and progress in that part of the electrical field covered by its title.

A considerable number of papers was arranged for during the year and at the Mid-Winter Convention in February, one morning session, one afternoon session and part of a second afternoon session were assigned to this Committee. A most interesting and valuable group of papers was presented. The sessions were very well attended and the discussion was limited only by the available time.

The papers presented at the sessions referred to above are as listed below:

1. "Measurements of Projectile Velocities" by Dr. P. E. Klopsteg and Major A. L. Loomis.
2. "A New Form of Vibration Galvanometer" by Dr. P. G. Agnew.
3. "Precision Galvanometer for Measuring Thermo E. M. Fs." by Messrs. T. R. Harrison and P. D. Foote.
4. "Notes on Synchronous Commutators" by Professor J. B. Whitehead and Mr. T. Isshiki.
5. "Oscillographs and Their Tests" by Professor A. E. Kennelly and Mr. A. A. Prior.
6. "The Accuracy of Commercial Electrical Measurements" by Mr. H. B. Brooks.

Without entering into a complete review of all the papers or the discussion, brief mention should be made of the paper on the Check and Calibration of Oscillographs and the paper on the Accuracy of Commercial Electrical Measurements. The oscillograph is rather generally used as a means of practically ultimate analysis of electrical functions and phenomena, and the paper provides the means of checking the functions and characteristics of the oscillograph itself. The contribution to the papers of the Institute entitled "The Accuracy of Commercial Electrical Measurements" is a most thorough and comprehensive analysis, review and statement of the characteristics and limitations of the many commonly used electrical instruments. This paper will, without doubt, serve as a ready reference for many years and should be of value to many engineers in other than actual testing work. It covers in a thorough and intelligent manner the errors that may be expected in the use of various instruments under various conditions.

The definition of power factor in polyphase circuits is a matter that has been active and of interest from many angles throughout the past year. While it has been the direct work of a Special Joint Committee of the N. E. L. A. and the A. I. E. E., its work falls within the general purview of this committee and a brief mention in this report seems entirely proper The Special Joint Committee canvassed the manufacturing, operating, scholastic and purely engineering branches of the electrical industry and prepared a report of its findings. It was intended to have this report presented at the Mid-Winter Convention last February. It was impossible, however, to analyze and intelligently condense in the form of a report, the volume of material received. It was decided, therefore, to have the report ready for presentation at the Summer Convention of the A. I. E. E. and to have a session assigned to the general subject. Arrangements will be made for discussion by various interested individuals to present the point of view of various interests affected by the settlement of the question. It was hoped that this method of procedure would provide an adequate form for the use of the many interested individuals and permit the ultimate acceptance of a satisfactory solution of the question.

Your committee has surveyed the field of manufacture of instruments and measuring devices and finds in general a few reports of new developments, some reports on standardization and adaptation to commercial usage of the designs dictated by the war and practically unanimous reports from all of the manufacturers of extreme difficulty in contending with the shortage of material and labor turnover in the production of the standard and accepted lines of apparatus.

The various companies manufacturing watthour meters and demand devices, practically all report the completion of experimental models, finished designs or actual production of devices for measuring kv-a-hr. and demand devices for measuring the demands of kv-a. and kw. and power factor. These various devices may make the measurements separately or simultaneously on a single record. Further there is a steady continuation in the development and standardization of maximum-demand devices to complete the series of devices suitable for easy adaptation to d-c. watthour meters, a-c. watthour meters and various frequencies.

Aside from the adaptation to various types of meters there is the continuation of the development providing devices actuated on the time basis, either on spring driven clocks, spring driven clocks electrically wound or motor driven clocks.

The Warren motor referred to in last year's report of this committee continues to be developed in a most satisfactory manner. It is being used for the control of frequency on generating systems through the master clock. It is also being installed in many forms of graphic and recording instruments used for various purposes throughout the stations of generating and distributing systems.

In the field of indicating instruments, both for direct- and alternating-current use, it is found that the designs which were developed particularly for war purposes, have been modified sufficiently to meet the regular demand and there are now available lines of both switchboard and portable types of instruments that were not available two or three years ago. These instruments are in general of a very satisfactory type and serve a very useful purpose in providing other sources of supply of instruments which did not exist some years ago.

The greatly increased activity in the measurement of high frequencies used in wireless work caused the development of vacuum thermal converters. These devices serve very satisfactorily for other purposes than for wireless work inasmuch as they permit the precise measurement of extremely small values of alternating current, a field of measurement which has not until recent times been covered by other than laboratory instruments.

In canvassing the field of telephone and telegraph engineering for instruments and measurements, the committee finds in answer to inquiry practically nothing to report, but mainly the continuation of developments which were started during the war period and previously reported.

Reports from those companies specializing in the manufacture of laboratory and precision instruments indicate a very valuable development of this type of instrument for certain special as well as general purposes. The line of development has been to produce on a commercial scale automatic precision electrical instruments, either of the indicating or the graphic type. Wheatstone bridges and potentiometers are normally considered as indicating instruments or devices manually operated by means of dial switches or plug contacts. One of the companies manufacturing this type of apparatus reports the development of an automatic potentiometer and an automatic Wheatstone bridge, both used for recording or controlling processes or a combination of the two. These automatic equipments are supplied for the control of chemical and electrochemical processes and some installations of these are in actual use. For chemical and electrochemical work the automatic Wheatstone bridge can be operated on a commercial 60-cycle a-c. circuit and a special a-c. galvanometer has been especially developed for this equipment.

Two of the important applications of the a-c. Wheatstone bridge equipment, are the recording of surface condenser leakage in power plant operation and the controlling of the finish point during the evaporation process in plants making condensed milk. The potentiometer equipment is also used for accurately recording and even controlling the acidity and alkalinity of solutions through the measure of the potential between two electrodes immersed in the liquid.

The recording or indicating potentiometer when used in connection with thermocouples produces a very satisfactory and accurate record of temperature. The measurement of temperatures is, of course, directly related to design and operating requirements of electrical apparatus and these devices serve very satisfactorily indeed to analyze the daily or weekly cycles of thermal performance of cables, transformers, turbine generators or in fact any of the various commonly used types of apparatus in generating and distribution systems.

S. G. RHODES, *Chairman.*

ANNUAL REPORT OF THE TELEGRAPHY AND TELEPHONY COMMITTEE

To the Board of Directors:

FOLLOWING out the plan adopted last year as to the form of annual report to be submitted by the committee the following report includes references to technical progress only. The report is made up of contributions forwarded by each member of the committee.

STANDARDIZATION

During the past year considerable progress has been made in standardizing telegraph and telephone practises of the railroads of this country and Canada. Committees composed of representatives of the railroads, the commercial telegraph and telephone companies, and various manufacturers of apparatus, have prepared specifications for materials, apparatus and methods which, while conforming to the generally accepted standards in each case, include special features adapting them to railroad requirements. It is expected that these specifications will be formally adopted by the American Railroad Association in the near future, and that the desired approach to uniformity of practise will soon follow.

TOLL LINE CONSTRUCTION

Toll circuits may be composed of either open wire on pole lines, aerial cable, or underground cable. Any one of these will give high grade transmission when improved equipment and methods of operation are used. Therefore, the tendency in toll line construction is to build plant which will be most economical and at the same time reduce the cases of interference to service from storm breaks and various other troubles to a minimum.

Underground cables properly installed and maintained are open to the least interference to service.

However, it is not economical to place underground construction unless more than two full-size cables are required. Generally, underground cables cannot be justified except between the larger centers and their surburban sections.

Aerial cables are liable to less interference than open wires, but economy restricts their use largely to cases where the existing number of circuits and the rate of increase require extensive rebuilding, or additional pole lines to continue on an open wire line basis. Where the number of circuits will not justify aerial cable, open wire circuits must be constructed and maintained.

UNDERGROUND CABLE CONSTRUCTION

No radical changes have recently been made in engineering or construction methods involved in connection with laying of underground cables. The maximum size of cable used contains 1200 pairs of No. 24 B. & S. gage wires. Experimental work is being carried on for the purpose of increasing the number of wires without increasing the diameter of the cable sheath.

Troubles in underground cables have been due in part to disorderly arrangement of cables in manholes and the manner in which the splices were made. Great stress is now being laid upon the importance of having the manholes in good condition, supporting the cables in an orderly manner, and making splices with extreme care. The cost of labor required for installing underground cables is relatively small compared with the total cost of the cable in place, and therefore it is felt that all reasonable efforts should be made to install and splice the cables in such manner as to reduce the cable troubles to a minimum.

INDUCTIVE INTERFERENCE

The Railroad Commission of the State of California has recently published a comprehensive work entitled, "Inductive Interference," in which is given a large amount of technical information gotten together by the Joint Committee on Inductive Interference, which investigated this subject in California over a period of about five years. This work makes available to interested parties a great deal of information of fundamental importance.

The activity in both the power and the communication fields since the war, has demanded renewed close attention to the study of inductive interference.

The growing use of electric arc furnaces which usually produce great distortion of wave form has given rise to serious new problems in noise interference prevention.

The methods of overcoming interference with communication circuits from abnormal conditions on adjacent power circuits have not been so well developed as the methods of overcoming disturbances from the normal operation of power circuits. Constant increase in the amounts of energy carried by power circuits has also made these problems more difficult. Especial attention must be given, therefore, to this phase of interference, particularly along the lines of limiting the number of abnormal conditions, reducing the length of time in which the circuits are abnormal and in limiting the magnitude of unbalanced power currents during such abnormal periods.

Information on the subject of interference from electric railways has been increased by two or three important investigations, but is still far from complete.

PRINTING TELEGRAPH TENDENCIES

While the invention of systems of printing telegraphs has been active substantially since the introduction of Morse's telegraph in the year 1844, the use of printing telegraphs on a commercial scale in America—aside from stock and news ticker systems—has been extensive only since about the year 1900. During the past twenty years improved printing telegraph systems have been extensively applied in moving traffic over trunk circuits where the volume of business is heavy and continuous. On the lines of the Western Union Telegraph Company practically 80 per cent of the trunk line traffic is so despatched. The Postal Telegraph-Cable Company, on the other hand, makes no use of printing telegraph systems at the present time.

On railroad telegraph lines printing telegraphs are used also to a large extent.

The next stage of development—now very likely close at hand—is the application of printing telegraphs to wider uses and to circuits having a much lighter than trunk line traffic load. Extending to light-load circuits the use of printer systems makes simplicity of operation, low first cost, and low maintenance cost of first importance.

With the object of meeting the requirements for simplification, work is now under way with tape printing devices. Also, it is probable that two or three other systems of simplified telegraph printing will be given trials in service before long.

RADIO SIGNALING

The year 1919 has seen the application of discoveries and inventions which were necessarily veiled in secrecy during the years of war. Under stimulus of war necessity remarkable progress was made in the application and extension of the known principles of the art, standardization of radio apparatus and of its quantity production and use. These advances were made possible by the cooperation of industrial companies, their engineers, and engineers and scientists with academic connections. The efforts of these individuals or groups were ably coordinated and stimulated by the military and naval branches of the Government.

Progress during the past year has been more in the nature of a consolidation of the scientific advances of the war time than in the nature of an entry into new fields. The entire art of radio communication now stands on a firmer basis, with a greatly enlarged personnel of trained engineers and research investigators.

Radio telephone communication was maintained between the seat of Government and the steamship George Washington, on which President Wilson made his European voyages. In these demonstrations the radio station at New Brunswick, N. J., was used.

The guidance by radio of airplanes engaged in peace-time activities is well illustrated in the directing towards landing places of the planes of the Post Office Department. Communication with planes on long voyages was accomplished in the memorable voyage of the N C-4.

High-speed radio telegraphy has been greatly accelerated by the use of the Hoxie recorder and the Alexanderson magnetic amplifier. Direction-finding by radio compasses and its corollary of directive-receiving has been placed upon a firm scientific basis.

The possibility of selecting signals with reference to their direction has been successfully applied by Weagant, Wood, Taylor, Alexanderson, and others, to the selection of desired signals and the discrimination against atmospheric disturbances and signals of foreign stations. These principles have also permitted engineering advances in duplex, or two-way operation of pairs of stations, and in increased possibilities of multiplex transmission and reception. Knowledge and experience along the lines of duplex and multiplex operation has given added interest to the questions of regulation of wave length, range, and of the effect upon radio development of legislative action.

In the manufacturing field there has been progress not only in standardization of apparatus and in better performance, but also in the production of equipment for high-power continuous-wave stations; compact and simple apparatus for medium-power stations and short-range sets suitable for amateur use.

The withdrawal of military restrictions has resulted in a stimulus to amateur operation. By the encouragement of amateurs and of the research departments of the universities and engineering schools, the art should attract additional workers and provide for its future the necessary increase in an enthusiastic and well-trained personnel.

AUTOMATIC TELEPHONY

During the past year there was begun a movement which may become general looking to the converting of manual telephone exchanges to automatic operation. The same movement is said to be under way in foreign countries. From the mechanical standpoint there were no outstanding improvements in automatic telephony brought into public use. Progress took the form of a general refinement of details necessary to meet operating conditions.

DONALD MCNICOL, *Chairman.*

ANNUAL REPORT OF THE COMMITTEE ON TRANSMISSION AND DISTRIBUTION

To the Board of Directors.

THE Committee on Transmission and Distribution submits the following report for the year 1919-1920:

GENERAL

Early in the year the Board's Committee on Technical Activities suggested that it might be advantageous to change somewhat the character of the Annual Reports of the Technical Committee so that each of these reports would become an authoritative source from which could be obtained a complete resumé of developments in its particular field.

This matter was thoroughly discussed at a meeting of the Technical Committee Chairmen, who were of the opinion that it would be inadvisable to take such a step

The chief objections to the plan may be summed up as follows:

1. The preparation of a really complete and authoritative annual resume by each Technical Committee would require a great deal more time than could possibly be devoted to it by the Committee members.
2. Several publications prepare annual digests of developments and an attempt by the Institute to prepare similar statements would be more or less in the nature of duplication.
3. The preparation of a summary by the Committee instead of the present method of reviewing papers presented, would eliminate the personal element. This might tend to take away the incentive which now exists for the preparation of papers, as under present conditions, the preparation of papers is solicited, and when abstracted for the Annual report, full credit is given to the author.

It therefore seems that the advance in the art is best recorded by the present methods, and that those who desire a summary of developments, obtain this summary in a more accurate form than would be possible if the Committee's Annual Report were to deal in generalities.

During the year your committee in conjunction with the National Electric Light Association Underground Systems Committee, formed a joint Subcommittee on Cable Specifications.

This Subcommittee, after a number of conferences in which practically all the cable manufacturing companies were represented, completed its standard specification for paper-insulated lead-covered cable. This specification, which is accompanied by an appendix giving very complete notes explaining the various clauses, is printed in full as part of the report of the Committee on Underground Systems of the National Electric Light Association. The specification is also printed in pamphlet form for general distribution among cable engineers.

As a result of this standardization, cable manufactured under these specifications will be of the first quality and in accordance with the latest and best practise.

In addition to the work on preparation of cable specifications, this Joint Subcommittee is also preparing data and carrying on investigations of the dielectric losses in cables, relation of heating of cables to their load-carrying capacity, and such other information as is essential to the proper operation of underground cable systems.

In this connection, the Subcommittee has plans under way for the construction of an experimental conduit line about 200 feet in length. With the facilities offered by this experimental line the following phases of the problem will be investigated:

1. Relative temperature drops taken up by the cable insulation, air surrounding the cable, duct walls and surrounding soil.
2. Determination of watts lost as related to the temperature curve under various conditions.

3. Best arrangement of ducts to reduce heating to a minimum.

4. Determination of a reliable method of rating cables.

PAPERS ON TRANSMISSION AND DISTRIBUTION

There has been considerable discussion during the past year of the desirability of erecting high-voltage tie lines to connect existing or proposed, large power developments.

A paper presented at the Pacific Coast Convention in Los Angeles, September, 18th, 1919, by Messrs. R. W. Sorensen, H. H. Cox and G. E. Armstrong, deals with the high-tension tie line in so far as the state of California is concerned.

The authors summarize the power resources of the state, estimate the probable 1926 demand, and discuss the advantages of an all state tie line. The paper points out that the fundamental problems in 220,000 volt transmission are well understood, and that as the need of such a line is imperative, arrangements should be made without delay for the working out of details, and the construction of the line.

The super-power generation and transmission problem for industries and railroads on the North Atlantic Seaboard has been very thoroughly covered in a paper begininng on page 10, by Mr. W. S. Murray.

The author's plan, which has the endorsement of the important engineering societies, provides a means whereby the present estimated machine capacity of 17,000,000 horse power, divided 10,000,000 for industrial and 7,000,000 for railroad purposes, in the region between Boston and Washington, can be lifted from a present load factor of 15 per cent to a load factor of 50 per cent or more.

By utilizing the more economical generating apparatus at such a high load factor, it is expected that present fuel consumption can be cut in half and that the railroads can be relieved of carrying a considerable amount of the coal which is annually transported to the seaboard.

In conjunction with Mr. Murray's paper there is also a symposium giving the views of a number of eminent engineers on the subject of super-power generation and transmission.

In the April, 1920 JOURNAL, Mr. H. H. Plumb has contributed an extremely interesting paper on a rational method of determining the economic voltage for a long transmission line. The paper presents a rapid solution for the size of copper conductor based on Kelvin's law, and with this as a foundation, develops the solution of the economic voltage for copper, copper-clad steel, aluminum and other conductor materials.

Mr. W. A. Del Mar in the January, 1920 issue of the JOURNAL, has worked out a method by which different insulating materials may be compared even when the tests are made at different voltages, and on cables of different sizes. This method consists in expressing the specific quality of the insulation, in terms of a quantity which may be termed the coefficient of dielectric loss, and which is the product of the power factor and the specific inductive capacity.

TENDENCIES IN HIGH-VOLTAGE TRANSMISSION

The probable magnitude of the power requirements of the near future, together with a quickened appreciation of need for maximum economy in its production has led to widespread and serious consideration of projects for bulk transmission of power to load centers from distant energy sources. It is of interest to note that similar conditions exist and similar projects are being considered in many foreign countries. The large amounts of power and the long distances involved in such projects are beyond the economic range of transmission voltages thus far in use. In consequence there has been, during the past year, considerable study and discussion of the feasibility, design, cost and characteristics of transmission at a materially high voltage.

The study and discussion have centered largely around 220 kilovolts which has appeared to represent a sufficient increase in transmission capacity to meet the requirements of projects thus far under discussion, and at the same time involve no such large step beyond present practise as to introduce problems not susceptible of present commercial solution.

Development of trunk transmission at such a voltage as 220 kv. would involve new problems arising from the high voltage, the large amounts of power involved, and the high service standards which the importance of the transmission would demand. The various studies which have been made indicate that these problems differ in degree, but not in character, from those satisfactorily handled in present practise. The production of equipment for such a voltage has been the subject of thorough study and investigation by the manufacturers, and definite assurance is given that equipment entirely adequate for the service can be supplied. For transformers in suitably large units for voltages of the order of 220 kv. it is stated that no radical departures are involved in either design or construction. As regards oil circuit breakers, where the problem is one of high capacity rather than of high voltage, it is claimed that satisfactory designs have been worked out. In general, for extra high voltage installations the conclusions from various studies have been that design policy should tend to extreme simplicity of layout and staunchness of equipment. One feature of this policy would probably be omission of arresters or similar protective equipment. While present type of arresters could presumably be adapted for such a voltage, it appears to the opinion of those who have thus far studied the problem that they would be neither needed nor efficient. The idea appears to be becoming quite general among operating engineers that arresters of the usual type, with a series gap, afford little protection against dangerous low-voltage high-frequency surges, while the need for protection against excess potentials becomes progressively less for the higher operating voltages. Both theoretically and in the light of such experience as is available, circuits operating near the corona limit will tend to be self protecting against excess potentials as a result of corona dissipation, while atmospheric lightning disturbances in general appear to have relatively less destructive effects upon lines and apparatus for the very high voltages.

It appears probable that the one problem of 220-kv. transmission to which present practise cannot satisfactorily be extended without considerable modification is that of line insulators. While it is believed that present commercial types of suspension disk insulators could be made to give acceptable service, they are not regarded as likely to prove wholly satisfactory. Even if it could be assumed that the deterioration which has constituted such a serious defect with the earlier disks has in recent types been adequately overcome, it would still appear that both the electrical and mechanical characteristics of these insulators are not well adapted to the conditions of extra high voltage service. With long strings there is an excessive concentration of potential on the end disks and very little would be added to the total flashover of the string by increasing the number of disks. For 220 kv. this condition appears to be so pronounced that probably some remedy would have to be devised. It has been suggested that grading by using several types of disks with different electrostatic capacities would effect an improvement in the potential distribution over the string, although maintenance would be somewhat complicated, while another suggestion is for the installation of suitably shaped electrostatic shields at the ends of the strings. A difficulty considered even more serious is that mechanically these disks are not strong enough, in a single string, to support with adequate margins of safety the extreme loads which have been assumed as called for by 220 kv. service, due to the large size of conductor, the long spans probably demanded by considerations of economy and the heavy design loadings which the high service standards would dictate. The complication attending the use of two-string or three-string assemblies for suspension units is considered highly undesirable. In view of these limitations of the present commercial type of high-voltage insulator, the need for an improved or radically different design for higher voltages has been strongly emphasized. Some of the insulator manufacturers appear to be making extensive investigations and experiments in this field.

The long lines and high load capacities per circuit, conditions which call for extra high voltage service, have led to the recommendation that voltage be controlled mainly at the receiving end, either by synchronous condenser installations operated as an integral part of the circuit or, where practicable, by the use of excess generator capacity in the receiving system. For a given amount of power to be transmitted, ample condenser capacity, by improving voltage regulation, would reduce the number of circuits otherwise needed and by reducing the values of line current would permit of economical use of smaller conductors.

For 220 kv. transmission the expensive character of substation apparatus and the greater insurance of immunity from operating troubles which is afforded by simplicity of system would both favor keeping to a minimum, the number of connections. It may be presumed that the tendency in developing such a system will be to use the high voltage for trunk transmission only, existing lower voltage transmission systems or extensions thereof being employed in general to reach customers and minor or secondary load centers.

The practise of interconnecting power supply systems in adjacent territories, which, during the war, gained new impetus as a result of the need for obtaining the maximum effective output from existing generating capacity, has been further extended, and numerous plans are in progress or under discussion for interconnections of considerable magnitude. The great interconnected system in the Southeastern states has been extended and the interconnections reinforced.

This movement toward interconnection is hastening the tendency, already evident, toward uniformity and simplicity in transmission standards and practises. Standardization of voltages in multiples of 11 kv. is well established, accepted steps being 22 kv., 33 kv., 44 kv., 66 kv., 88 kv., 110 kv., 132 kv., and 154 kv., with 220 kv. projected as the next step. Certain earlier standard steps, such as 60 kv. and 100 kv., are no longer used except in connection with existing systems. The advantage of keeping these standard voltage steps to a minimum are so great, even more from the point of view of future interconnection than of economy in manufacture, that certain of the steps in the above list may fall into disuse. Eighty-eight kv. for instance, does not seem likely to be continued as a standard, while possibly either 33 kv. or 44 kv. and either 132 kv. or 154 kv., may tend to drop out. Sixty cycles, likewise, is becoming firmly established as the standard frequency for the United States, and its virtually exclusive use for general power supply can be predicted for the near future. The practise of using a grounded neutral connection for the higher voltages is considered to have clearly demonstrated its superiority and is rapidly supplanting the isolated delta connection, while for larger systems at least the preference for the grounded neutral connection is evident also in case of the lower voltages.

In general it is coming to be clearly recognized that interconnection of power supply systems is not to be merely an occasional expedient but the normal condition, and that it will tend to become more and more extensive, adjacent systems being connected into groups, and these groups being joined by trunk transmission lines into systems covering a number of states, probably leading eventually to an interconnected system of nation-wide proportions. With this tendency toward widespread interconnection so unmistakeable, an important obligation rests upon the engineering profession to so plan present work as to enable it to fit into such development with a minimum of waste, particularly in such matters as standardization of voltages and frequencies in adjacent territories. The establishment of a comprehensive power supply system, district or regional, will mean much to the future of the electrical power industry and to the industrial future of the country, in which the question of power supply is clearly destined to become one of the determining factors.

TENDENCIES IN UNDERGROUND CABLE PRACTISE

The lack of accurate information on certain elements influencing the operation of underground cables, has in the past caused this branch of the transmission and distribution to be neglected to some extent, and some of the operating companies have been put to a great deal of trouble and expense on this account. At present, however, there is a growing interest in this subject which has already resulted in a marked improvement in the manufacture of cables, so that the carrying capacity of cables manufactured today is materially greater than that of cables manufactured a few years ago.

The principal improvement is the reduction of dielectric loss at relatively high temperature. This has been accomplished principally as the result of a thorough study of the properties of impregnating compounds. It is now possible to manufacture cable the rating of which will be decreased only a few per cent under temperature as high as 100 deg. cent.

These refinements in manufacture make the three-phase underground cable fairly satisfactory for voltages up to 25,000, and for loads per circuit up to 10,000 kw., but if these limits are to be exceeded it will probably be necessary to go to single-conductor construction.

During the past year there have been a great many serious burnouts on underground systems where trouble was communicated from one cable to another until sometimes a whole duct line was involved, and at present there is a great deal of attention being given to methods for the prevention of the spreading of such troubles.

Results have been improved by changes in the design of duct lines, such as the wider separation of ducts especially where they enter the manhole. This wider separation tends to prevent the communication of trouble from one cable to another which is very dangerous where there are a great number of important cables in one line. For the same reason it is now the prevailing practise to use some form of conduit construction that gives an individual wall of concrete around each duct, as concrete is a very good arc-resisting material.

Protection of cables from fire in manholes by cement covering is still the prevailing practise. Methods of applying the cement varies to some extent, but the results are uniformly satisfactory.

There is also a recognized advantage in installing separate duct lines for cables operating at widely different voltages in order to avoid the possibility of limiting the load on some of the cables on account of the temperature limits of cables working at another voltage. This temperature influence is very important and should always be considered when there is a possibility of extreme limitations being placed on some cables by the influence of others.

As regards sizes and types of underground cable, manufacturers report a steadily growing demand for cables of 350,000 to 500,000 cir. mils. There is also a tendency on the part of customers to specify sector type cable on account of the smaller overall diameter for a given copper cross section.

The installation of these large size cables makes it more difficult to bend and train the cable in manholes and in some cases requires a modification in the design of manholes to avoid sharp bends.

The rectangular manhole with duct openings in the center of, and flush with one wall, compels the bending of the cable at too short a radius. Oval or rectangular manholes can be used, but as these shapes are not entirely satisfactory, it has been proposed that the rectangular manhole be set at 45 degrees with the center line of the conduit, and the duct entrance be recessed so as to permit of training through the manhole with long radius bends.

The Hochstadter type of cable in which the individual conductors are covered with a thin metal foil, and the assembled conductors sheathed without belt insulation, is now being supplied commercially.

Users of split-conductor cable report satisfactory operating results and state that the advantages of the selective protection obtained by the use of this type of cable more than offset the disadvantage of the smaller current-carrying capacity for a given duct size.

E. B. MEYER, *Chairman.*

ANNUAL REPORT OF THE IRON AND STEEL INDUSTRY COMMITTEE

To the Board of Directors,

DURING the Convention held at Lake Placid in June, 1919, Dr. C. P. Steinmetz in discussing the work of the technical committees of the Institute expressed the belief that the annual reports of such committees should be planned to provide for the use of the entire membership of the Institute, a comprehensive record of the present status and recent progress of electrical engineering in the particular industry or branch of the art to which each committee had been organized to apply itself.

Accordingly, Mr. Wilfred Sykes, the Chairman of the Board's Committee on Technical Activities, suggested to the various 1920 Technical Committees that their reports "should cover notable installations or developments during the year and in view of the fact that, in the past, little has been done in this respect, that the reports for 1920 might be a review of the state of the art up to date."

Guided by these suggestions it has been the aim of the Iron & Steel Industry Committee, to at least prepare as a foundation, the present status of electrical application to the iron and steel industry, upon which subsequent committees might base annual reports showing recent progress. The field is an extremely broad one dealing as it must, with practically every process in the transformation of the raw material into the many forms of finished product. Electricity is utilized in the handling of the ore and provides the means of handling the finished products as well as the final heat treatment of the more highly developed products. From the mechanical point of view, it has been stated that from the ore to the finished product the hand of man has no occasion to touch the materials and it can be stated further that throughout this same transformation, practically no step is taken in which electricity is not playing an increasingly important part.

While the iron and steel industry in America, dates back into the country's earliest history, electricity only comparatively recently began to play its important part, to be more exact, about 1891. At present the total generating capacity required by the industry is approximately one million kilowatts and each new development in the art of steel making opens up new fields for electrical application so that in many important respects, the making of steel is becoming an electrical industry.

Supplement A has been prepared to indicate the major applications of heat and power entering into the making of iron and steel products from the raw materials, and to show the extent to which electrical heat and power have been or may be applied. As to the auxiliary applications of power in the handling, manipulating, and finishing of the materials and products, electricity is almost universally used as supplementary data will show.

It was decided to consider the mining of the raw materials as outside the scope of the Iron & Steel Industry Committee and it has dealt only with the material after arrival at the blast furnace. Reference again to Supplement A will indicate the processes to which the Committee has applied itself. Furthermore, it has not been intended to prepare a handbook for electrical engineers engaged in the iron and steel industry, but rather provide a source of general information for the large proportion of the Institute membership engaged in the many other lines of electrical engineering.

Using Supplement A as a basis, memoranda have been or are being prepared along the following general lines, believing that brief consideration of each subject separately, could be used to better advantage and more readily added to than to attempt a consecutive description.

Blast Furnace Group

Memorandum A 1—The status of iron ore smelting in the electric furnace.
Memorandum A 2—Synthetic electric furnace pig iron.
" A 3—The electrical precipitation of dust of blast furnace gases.

Electric Melting Furnace Group

*Memorandum B 1—Statistical data—The growth of the application of electric furnaces.
* " B 2—Types and electrical characteristics.

Electric Heat Treatment Furnace Group

Memorandum C 1—Electrical heated soaking pits.
" C 2—Carbon resistor type furnaces.
" C 3—Typical heat treating plants (Carbon resistor).
* " C 4—Metallic resistor type furnaces.

Rolling Mill Group

*Memorandum D 1—Statistical data.
* " D 2—Types of Drives—Non Reversing — Adjustable, Speed — Reversing.

Miscellaneous Group

*Memorandum E 1—The electric hydraulic system as proposed for large forge presses.

The Committee has prepared and sent to all the principal Iron and Steel Companies, the following questionnaire and the results will be tabulated as a general indication of the extent and characteristics of both electric power supply and application. This data will appear in the form of supplement B at an early date.

ELECTRICAL STATISTICS

Name and location of plant..

POWER SUPPLY (period January to December, 1919, inclusive)

Purchased from central stations........................ kw-hours
Generated by own steam units from coal-fired boilers....... kw- "
" " " " " from gas or waste heat boilers kw- "
(including exhaust pressure turbines)
" " " gas engines using waste gases........... kw- "

MOTOR EQUIPMENT a-c....phase...cycle...volts d-c....volts

Number of motors............. ....
Total horse power............. ....

GENERATING CAPACITY a-c.....volts. d-c.......volts.

Average kw. capacity in use—kw..... kw.......
Average load factor......... %.... %......
{ Do not include sub-station d-c. apparatus }

MEETINGS AND PAPERS

No attempt has been made to secure papers or arrange Institute meetings for the consideration of iron and steel Industry subjects, owing to the extensive interlocking of the memberships of the A. I. E. E. and the Association of Iron & Steel Electrical Engineers which holds monthly meetings of high grade in Pittsburgh, Cleveland, Chicago, Birmingham and Philadelphia. In some cases joint meetings between the local sections of these organizations have been successfully held and an extension of this practise is recommended. The nature of the papers presented at the A. I. & S. E. E. meetings is made clear by the list supplementing this report (Supplement C) and it is recommended that closer relations be established between the two organizations through the publication, in the JOURNAL of the A. I. E. E., of notices before and after the A. I. & S. E. E. meetings, together with other items of mutual interest.

It is further recommended that the Board of Directors consider the scope of the work of the Association of Iron and Steel Electrical Engineers and with the appointment of the iron and steel industry committee for the coming year, formulate a definite policy as to the nature of the Committee's activities in order to avoid duplication of effort on the part of the two organizations, the memberships of which are closely interlocked.

The Board of Directors of the A. I. E. E. has authorized this Committee to discuss with the Board of the

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*In preparation for issue at an early date.

A. I. & S. E. E. the question of co-operation in the matter of Standardization Rules and it is regretted that no opportunity for such conference has presented itself. Such action is strongly recommended at an early date.

Many valuable papers relative to electric furnaces in the iron and steel making processes are presented in the meetings of the American Electrochemical Society and are recommended for notice and possibly publication in the A. I. E. E. JOURNAL.

The Committee wishes to acknowledge with thanks, the hearty co-operation it has received from the individuals and companies appealed to in the gathering of the data presented herewith.

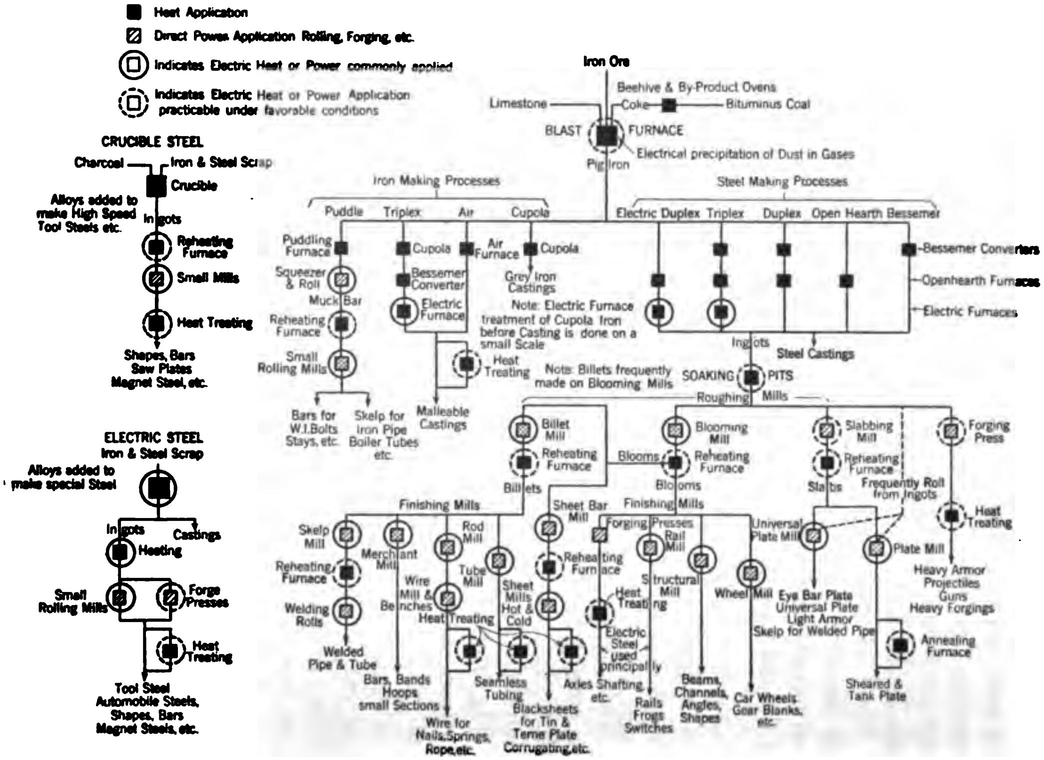
MEMORANDUM A 1

THE STATUS OF IRON ORE SMELTING IN THE ELECTRIC FURNACE

The electric method of smelting of iron ore is in no sense competitive from a commercial point of view with the blast furnace, as commonly used in the manufacture of pig iron, in America.

Pig iron of high grade can be made in the electric furnace, and is so made in considerable quantities, but only at localities that combine the conditions of having ore and charcoal accessible to dependable low cost electrical energy, not exceeding twelve to fifteen dollars per horse power year. The industry has therefore, been confined largely to Norway, Sweden, Italy, with smaller installation in Japan, Switzerland, California and Canada. British Columbia is being considered as having the necessary requirements.

Three-phase furnaces are commonly used and power factors of from 65 to 95 per cent are possible with frequencies as high as 60 cycles, but 25 cycles is preferable in the large units, such as used in Sweden. The load factor can be made high and the units vary from 2000 to 7000 kw. in capacity. The furnace voltage is low, being below 100 volts, and voltage regulation is accomplished by means of taps in the high-tension side which limits the use of very high trans-



ELECTRICITY IN THE MAKING OF IRON AND STEEL PRODUCTS

NOTE—Practically all auxiliary applications of power for the transportation, manipulation, hoisting, pouring, shearing, finishing, magnetic lifting, etc. of the steel and iron materials, raw materials and machinery or tools used, are electrical.

mission voltages without intermediate stepdown transformers.

The furnaces used in Sweden where the greatest amount of electric pig iron is produced (approximately 70,000 tons in 1918) consist of a furnace shaft quite similar to a small blast furnace independently supported above a relatively shallow crucible of larger diameter which is covered by a circular arch through which the carbon electrodes project into the charge which is fed to the top of the furnace as in a blast furnace.

MEMORANDUM A 2

SYNTHETIC ELECTRIC FURNACE PIG IRON

Conditions of shortage of pig iron and the unusual availability of steel turnings incident to the war period, developed the manufacture of synthetic pig iron in electric furnaces by re-carburizing fusion of steel turning or scrap. This process was used principally in Canada and France, the latter country having made at Levet, Nanterre and Limoges over 150,000 tons during the war period. The future of this industry depends upon cheap water power, availability of materials and the relative cost of coal.

Typical Furnaces in Canada. A small 250-kw., single-phase furnace at Orillia, Ontario, produced six to seven tons of low phosphorus pig iron per twenty-four hours. A six-ton, three-phase furnace at St. Catherines, Ontario, with 1200-kw. transformers produced approximately 20 tons per day, with a 700-kw. input and with full 1200-kw. input—35 tons per day. For each hundred weight of turnings charged, there was added 5 per cent in the form of carbon and silicon and the yield was approximately 95 lb. of pig iron. The furnaces were of the enclosed type as usual to the making of steel and the current was of standard frequency. The load factor and power factor can be made high.

The French Furnaces. Open type furnaces of greater capacity, are used and are principally of single-phase construction with one moveable electrode and the other imbedded from below in the hearth. Standard frequencies with high load factor and power factor prevail. A 2500-kw. furnace produces 80 to 100 tons per day with a power consumption of 675 kw-hr. per ton. Complete foundries for the casting of shells were in some cases located adjacent to the pig iron furnace and by a further treatment in single-phase electric mixing furnaces of closed type, with an expenditure of approximately 50 to 100 kw-hr. per ton, the metal was made to meet the specifications for such castings (carbon 2.90, silicon 1.75, manganese 0.50, sulphur-trace, phosphorus 0.05).

These single-phase mixer furnaces are operated in groups of three and have one adjustable electrode above with the other electrode imbedded in the hearth from below.

MEMORANDUM A 3

THE ELECTRICAL PRECIPITATION OF DUST OF BLAST FURNACE GASES

The Cottrell system of electrical precipitation has been applied in two instances to the cleaning of blast furnaces gas. The advantages are such that such equipment is likely to become common among the blast furnaces.

In applying this system the gas piping from the blast furnace is arranged so that the gas is conducted through a group of vertical pipes from bottom to top and the high-tension electrode in the form of chains or wires held taut by weights, are supported in the center of the pipes from insulators at the top. The dust and fume particles are repelled by the charged electrode and are deposited on the sides of the pipes from which it is jarred by vibration of the pipes at suitable intervals (once each hour or more). This jarring can be accomplished automatically by relays adjusted to voltage drops and amperage increases which occur when the accumulation of dust becomes excessive. During the shaking down of the dust the precipitating apparatus is shut down for the necessary two or three minutes required.

A typical installation, put into service during 1919, takes three-phase, 60-cycle, 220-volt current from the power system, two 25-kv-a. transformers in open delta being ample for the entire power requirement which is 15 to 20 kv-a. to clean 45,000 to 50,000 cubic feet of gas per minute. The 220-volt current is applied to a small synchronous motor operating at 1800 rev. per min. and driving a mechanical rectifier, and is also fed through transformers which step the voltage up to 35,000 to 50,000 volts, adjustable by taps. The rectifier converts this high-tension current to the unidirectional current required by the precipitating electrode. The equipment can be made to suit any standard frequency.

A second installation has been made more recently in connection with a blast furnace and is of about one-half the capacity of the first equipment.

MEMORANDUM C 2

CARBON RESISTOR TYPE FURNACE FOR STEEL TREATING

Furnaces of this type are usually rectangular in shape with doors at each end for the receipt and delivery of the materials to be treated. Parallel with the length of the furnace, one on each side, are located two resistor troughs. The resistance elements composed of broken carbon are thrown loosely into these carborundum fires and troughs which are supported on brick pillars along the side wall and protected by being recessed back from the space into which the steel materials are introduced. The heat from the resistance element is radiated to the walls and top of the furnace which is lined with refractory materials which become heated and in turn heat the charge. Carbon or graphite electrodes at each end of the troughs provide the means of introducing the current and the control of the current, hence the heat is effected by varying the voltage impressed at the electrodes, this variation being obtained by means of numerous taps on the secondaries of the special transformers.

The materials to be treated are in some cases put into the furnace on cars for a period treatment and in other cases the operation is continuous, the materials being slowly pushed or conveyed through the furnace.

Advantage is invariably taken of the possibility of making these furnaces automatic, both as regards the cycle of temperature treatment and the charging and discharging.

The following list of furnaces, together with the dates of installation, will indicate the development and application in steel treating:

Plant	Date	Application	No.	Electrical capacity each	Tons of steel heated per 24 hours
No. 1	1915	Annealing castings	1	150 kw.	12
2	1916	Annealing castings	1	300 kw.	24
3	1916	Treating draw-bar knuckles	2	1–600 kw. 1–300 kw. Tandem operation	144 total
4	1916	Annealing steel castings	1	600 kw.	12
5	1917	Treating small steel parts for aeroplanes and tractors	1	150 kw.	24
6	1917	Treating auto gears	1	40 kw.	4
7	1918	Duplicate of plant	3		
8	1918	Duplicate of plant	5		
9	1918	Annealing R. R. axles	1	150 kw.	12
10	1918	Treating cast steel anchor chain	2	1–600 kw. 1–300 kw. Tandem operation	150 total
11	1918	Treating crank shafts for liberty motors		1-600 1-300 Tandem operation	144 total
12	1918	Annealing steel	1	600 kw.	75
13	1919	Roller bearing and ball race treatment	1	150 kw.	12
14	1919	Annealing steel sheets	1	600 kw.	150
15	1919	(2) Duplicate of plant	10		
16	1920	Treating bolts and nuts	1	40 kw.	4
17	1920	Roller hardening	1	75 kw.	5

MEMORANDUM C 3

TYPICAL HEAT TREATING PLANTS

(Carbon Resistor Type)

Plant A. Automatic furnace handling roller bearing and ball races.

Materials to be treated are placed in metal baskets holding approximately 125 lb. each and fed into one end of rectangular carbon resistor type of furnace maintained at approximately 1525 deg. fahr. by an input of 150 kw. A clock contactor regulates, over a variable range, the rate of flow of material through the furnace and ordinarily 1000 to 1200 pounds of material is handled per hour.

The discharge of the heated material is into a drawing or quenching tank, the oil of which is maintained at a temperature of approximately 350 deg. fahr. by means of immersed nichrome resistors requiring about 40 kw.

The temperature control of the furnace and drawing tank is by pyrometers and the adjustable clock contactor devices make the operation entirely automatic as to the temperature, the time in the furnace and the time in the tank. Motor-driven conveyers, door operating mechanisms and oil pumps are all under the control of the time and temperature measuring devices.

Plant B. Annealing furnace for treating low carbon cold rolled strip.

The annealing furnace is approximately 225 ft. long and 22 ft. wide with the heated chamber in the middle. Two parallel tracks pass through the furnace and heated chamber, and cars of steel strip each holding about 20 tons of material are fed on to tracks from opposite ends, the purpose being to absorb in the cold carload on one track the heat from the heated carloads on the other track on the way to the discharge point.

The temperature attained in the electrically heated chamber is approximately 1200 deg. fahr. A movement of the cars takes place every six hours, discharging one twenty-ton car of material at each end or approximately 150 tons of annealed material per 24 hours with an electrical input of approximately 120 kw-hr. per ton.

Plant C. Annealing furnace for treating alloy steel.

Arrangement is similar to Plant B except furnace is shorter and the nature of the steel requiring a 40-hour treatment at about 1400 deg. fahr. The ten cars in the furnace at one time each hold 30 tons of steel or a total of 500 tons, 120 tons of which is in the

middle electrically heated chamber. The delivery is approximately 72 tons per day with an electrical input of approximately 250 kw-hr. per ton of metal annealed.

SUPPLEMENT C

REPRESENTATIVE PAPERS RECENTLY READ BEFORE THE ASSOCIATION OF IRON & STEEL ELECTRICAL ENGINEERS MAIN AND SECTION MEETINGS.

Organization of Electrical Department in the Iron & Steel Industry.
  Inspection and Operation.
  Educational Training for Employees.
  Electrical Repair Shop.
  Storeroom and Spare Parts.
Records and Tests.
A-C. *vs.* D-C. Motors for Rolling Mill Table Drives.
Recent Improvements in Industrial Control.
Present Status of Electric Furnaces in the Steel Industry.
Electric Heat Treatment.
Overload Protection for Motors.
Overload Protection on Cranes.
General Specifications for A-C. Motors for Main Roll Drive.
Present Status of Arc Welding in the Iron & Steel Industry.
Safety Rules for Government of Employees Working on Electrical Equipment.
Babbitt and Babbitting.
Grounded Neutral.
Current-Limit Reactance.
Welding, Electric *vs.* Gas.
Electric Rolling Mill Drives.
Steel Plant Power Generation from Waste Heat and Coal.
Electrical Equipment of the Largest Plate Mill in the World
Electrical Installations at Trumbull Steel Co.
Steam *vs.* Electric Driven Mills.
Electrical Features of a Modern Steel Plant.
Steel Mill Electrical Repair Shop Practise.
Influence of Gear Ratio on Speed of Operation, Motor Heating and Contactor Wear in Auxiliary Steel Mill Drives.
Automatic Electrolytic Oxy-Hydrogen Plant.
Heating of Underground Cables.

ANNUAL REPORT OF THE INDUSTRIAL AND DOMESTIC POWER COMMITTEE

To the Board of Directors,

THE Industrial and Domestic Power Committee has continued its earlier plan of work without change or modification. We are convinced that it is sound. We know the results now appearing will justify the study. We strongly urge its continuance.

Eleven subcommittees are analyzing electrical power applications in eleven different branches of industrial activity. The analysis consists of a study of the various processes involved, the movements in the processes and the electrical application to the movements. No standardization studies are considered. This is a province of other committees. The work is entirely ethical. In no way is our work overlapping studies by other Institute committees, or by committees external to the Institute.

A similar study of the electrical power applications in the use of prominent industrial tools is being conducted.

The work is being reported by monographs. Each monograph comprises a section of subcommittee study, complete in itself and yet forming a part of the whole. Each subcommittee is planning its own work to its own ideas, but all in conformity to the general plan. In the final execution of the plan, a progression of monographs carefully edited and approved by the Industrial and Domestic Power Committee will reach headquarters and will be available for presentation to Section or Institute meetings; for publication in the JOURNAL or for sale by headquarters at a nominal fee. With the interest being generated in the plan, we look confidently for extremely valuable returns; for treatises on industry of practical value. They in no sense can replace the work of the consulting engineer.

They should be a guide in the study and planning of industrial applications.

In our work we have been helped by men not of the main committee, but regularly appointed by the President to our several subcommittees. These appointments have been cheerfully accepted, and such results as are being obtained are because of their service, and we may suggest that this plan has brought into active Institute service men whose abilities for such service were not otherwise being reached.

The personnel of the several subcommittees follows:

1. SUBCOMMITTEE ON MOTORS WITH PARTICULAR REFERENCE TO SPEED-TORQUE CHARACTERISTICS.

A. M. Dudley, *Chairman.*

R. H. Tillman,	A. C. Lanier,
Wilfred Sykes,	John C. Parker.

2. SUBCOMMITTEE ON DOMESTIC POWER APPLICATION.

H. Weichsel, *Chairman.*

James Dixon,	C. L. Kennedy,
A. F. Welch,	L. L. Keilholtz.
Bernard Lester,	Edgar D. Doyle.

3. SUBCOMMITTEE ON APPLICATIONS IN PRINTING INDUSTRY.

W. C. Kalb, *Chairman.*

W. E. Date,	John D. Nies,
J. C. Lincoln,	Carl F. Scott.

4. SUBCOMMITTEE ON APPLICATIONS TO CRANES AND HOISTS.

R. H. McLain, *Chairman.*

H. W. Eastwood,	E. Friedlaender,

James A. Shepard.

5. SUBCOMMITTEE ON APPLICATIONS TO MACHINE TOOLS.

H. D. James, *Chairman.*

T. E. Barnum,	W. C. Yates,
W. T. Snyder,	R. H. Goodwillie.

6. SUBCOMMITTEE ON APPLICATIONS TO PASSENGER AND FREIGHT ELEVATORS.

R. H. Goodwillie, *Chairman.*

H. D. James,	H. P. Reed,
David L. Lindquist,	Charles H. Roth.

7. SUBCOMMITTEE ON APPLICATION IN TEXTILE INDUSTRY.

H. W. Cope, *Chairman.*

C. T. Guildford,	S. B. Paine,
J. C. Ramsay,	D. H. Sadler.

8. SUBCOMMITTEE ON APPLICATIONS IN CEMENT INDUSTRY.

R. B. Williamson, *Chairman.*

H. Weichsel,	C. A. Kelsey,
J. F. Siegfried,	Arthur Simon,
S. A. Staege,	M. R. Woodward.

9. SUBCOMMITTEE ON APPLICATION IN WOODWORKING INDUSTRY.

L. E. Underwood, *Chairman*
Truman Hibbard, Robert L. Smith
S. A. Staege.

10. SUBCOMMITTEE ON APPLICATION OF ELECTRIC ENERGY IN INDUSTRIAL HEATING.

E. V. Buchanan, *Chairman.*
T. E. Penard, H. O. Swoboda,
W. S. Scott, H. A. Winne.

11. SUBCOMMITTEE ON APPLICATIONS IN RUBBER INDUSTRY.

W. E. Date, *Chairman.*
M. Berthold, A. C. Bunker,
B. T. Mottinger, C. A. Rice,
H. F. Schippel.

Most of the subcommittees have made reports of their activities for the term. It is impossible to give these complete in this report without making it unwieldy. It is impractical to condense them. They should have individual consideration, and their reading amplifies this picture of our work. For this reason they are forwarded herewith for the information and consideration of the Board of Directors. The subcommittees in the main have worked hard and conscientiously to develop the plan, the main committee acting as a clearing house, and in an advisory capacity. All subcommittee chairmen are members of the parent committee. But the burden of the work has fallen to the subcommittees, and their work merits every appreciation.

The outstanding feature in electrical industrial development during the present term has been magnitude of production. Demand has continued greater than supply taxing combined production to the greatest limit. Probably the development and increase in use of fractional horse power motors for domestic and small industrial use has been the greatest contributing factor in this total. In this fractional field, quantity of production especially is to be emphasized. In general, quantity demand has been the real problem, the design and application of new productions being forced to second place.

In larger horse power fields, the increasing use of the synchronous motor is to be noted. As a corrective of the power factor of industrial loads it continues not only desirable but a growing necessity to meet a continual problem. In this connection, the use of the static condenser has continued as a power factor corrective.

The use of the direct-current motor in its several types continues accurately as does the use of the induction motor. In general there is continued progress in the standards of application of the several types of motors and in the recognition of these standards and away from misapplication, and great credit must be accorded to the discerning intelligence of the engineers connected both with the production and use of motors that has cooperatively tended to bring this about.

The Industrial and Domestic Power Committee particularly appreciates the advice of your committee on technical activities, together with the help constantly accorded from our Secretary's office at headquarters. As a committee, we have tried to recognize fully the trust imposed and bespeak for our successors your full cooperation in the work we are undertaking.

A. G. PIERCE, *Chairman.*

ANNUAL REPORT OF THE MARINE COMMITTEE

To the Board of Directors,

IT will no doubt be recalled the inception of the Marine Committee was in 1913. Little actual work was accomplished by this Committee until last year, when the compilation of a set of rules for electrical installation on shipboard was started, together with dissemination of knowledge relative to that subject to the electrical engineers of the newly created ship yards.

The immensity of the work, that of preparing the Marine Rules started, was the more fully appreciated as the work progressed, and while the work of last year's committee was very commendable, the volume of the work completed was performed by this year's committee. The number of members constituting the committee this year was greatly reduced to facilitate the work and curtail expenses as much as possible.

Eight meetings of the committee were held. The first meeting was given over to the appointment of the various Committees and the outlining of the work of each. The Ship Installation, Marine Propulsion and Historical Committees constituted the main divisions. The first two of these committees were supplemented by subcommittees to look after the vast amount of detail work involved in the activities of the main committee.

In last year's Annual Report of the Marine Committee it was suggested that the unfinished work of that Committee be continued this year. This unfinished work consisted of the completion of the rules for governing the installation of electrical apparatus on shipboard. The work on these rules that was not completed consisted of the following:

Revision of Rules prepared last year.

Control Equipments.

Alternating-Current Apparatus Control.
Radio Installations.
Running Lights, etc.
Fire Alarm Systems.
(not completed by subcommittees.)
Tabulation of Generating Sets.
Storage Battery Installation.
Gyro-Compass Installation.
Gyro Stabilizing Installation.
Tabulation of Wires and Conductors.
Appliances in vicinity of Compass.
Inspection Report to Classification Societies.

The Rules for Electrical Installations, as stated above, have now been completed and are in the hands of the Secretary to be passed on by the Board of Directors. These Rules will be published in pamphlet form and it is the intention that they be sold at a nominal figure.

Too much credit cannot be given to the various members of the Ship Installation Committee and the Chairman, Mr. G. A. Pierce, in particular for the work accomplished, as we believe these rules to be the most complete and comprehensive ever formulated by any society, covering as they do practically every phase of the electrical field as applied to ship work and extending the field of the Institute to practical installations. This being an innovation on the part of the Institute, the attitude of the ship owners, builders and classification societies relative to their adoption is eagerly awaited. Several new features are also introduced, which will probably extend the field of this work to a marked degree.

The rules as now completed have been referred to the various classification societies and the emergency Fleet Corporation, all of which were given opportunity to comment. This method of action has gone far to put in composite form the work as viewed from the different angles. When published, copies of the rules are to be forwarded to the Institute of Electrical Engineers of England, France and Italy.

As the work of the Ship Installation Subcommittee neared completion, it was found that, due to the magnitude of this work, it would be necessary to have someone care for the preparation of it, in order that it could be put in final form for publication. It was therefore decided to appoint an Editing Committee, whose duty it would be to correlate the vast amount of information contained in these rules. The work of this committee was no easy task and we therefore take this opportunity of complimenting it on its labors in connection with this work.

The Marine Propulsion Committee has prepared its historical and data work and it is suggested that this be put in form to be published in the JOURNAL of the Institute, as a means of drawing out discussion and additional data as regards this subject. It was decided to await experience on electrically propelled ships, soon to be put in service, rather than introduce data based on theory only. It is suggested that this feature be given careful thought and consideration during the coming year, as many new ideas will probably result from the experience gained by that time.

HISTORICAL COMMITTEE

The Historical Subcommittee has prepared a Historical Review of the Use of Electricity on Shipboard. This review accompanies and forms a part of this report. A more complete account will be prepared for publication in the JOURNAL of the Institute. This is considered a very important work, as the evidences of the use of electricity in the marine field are only fragmentary and should be placed in a form of record, as a matter of history. When this is accomplished, it will then be a very simple matter to add, from time to time, the outstanding events in their chronological order. This historical feature is one that should not be lost sight of in the work of future committees.

In line with the requirements of the rules as now completed, governing the installation of electrical apparatus on merchant ships, it is suggested that next year's committee take up the matter of approval of various fixtures, fittings, etc., with the end in view of having these meet the requirements of the new rules. It is believed that the preparation of General

Construction Specifications, covering various appliances should be performed by the Marine Committee.

One of the ideas brought forth during the year, was the consideration of terminal facilities at marine piers. This is more or less a new activity, as very little has been done thus far to facilitate the handling of cargoes at these terminals. The field is large and is therefore due a considerable amount of thought.

In conclusion, I feel that particular comment should be given to large attendance of the members of this committee at the regular meetings and the splendid spirit of co-operation existing between these various members. It is only in such harmony that the greater results, for which we all hope, are to be attained.

ARTHUR PARKER, *Chairman.*

HISTORICAL REVIEW OF THE USE OF ELECTRICITY ON SHIPBOARD

The marine field has probably been one of the most backward in the adoption and development of electrical apparatus which in the main, we believe has been due to the traditional conservatism of the seafaring man. While electricity was used to a very limited extent as far back as 1882 for interior communication, watertight bells, buzzers, annunciators, etc., having been installed by the Chas. Cory & Son Co. on the *Santa Rosa* about this time, and in the Navy electrical instruments were installed on the *Trenton* in 1883, apparently one of the first, if not the very first, electric light installations was also made on the *Trenton* about this time, at New York Navy Yard. All early lighting installations were Edison bipolar generators belt driven to an Armington-Sims or other reciprocating type engine. Voltmeters were not used until after the early '80s, and field regulation was not indulged in until about the same time, all of the early installations simply having a pilot light on the generator for gauging the voltage and candle power of the lamps.

Early wiring was installed in wood mouldings, and we believe that Habirshaw wire was considered the standard in the early installations, which was rubber covered with tape outside. Branches were in most cases spliced and the first junction branch boxes were used about 1885-1886. Fuses were used about the same time, usually using an ordinary single fuse placed in the branch circuit on a small insulating base which was inserted in the moulding.

The first marine telephones, we believe, were installed on the U. S. Cruiser *New York* (the first *New York*) about the year 1890. They were single direct circuits, *i. e.*, two telephones used in place of voice tubes. The instruments were of the "Bell" type and installed by Chas. Cory and Sons. The first "loud speaking" marine telephones were installed on the *Korea* and *Siberia* and the U. S. S. *Charlestown,* about 1895.

The first complete central energy (operator's) switchboard telephone systems were installed on the S. S. *Le Grande Duchesse* built at Newport News about the years 1897-1898. Pneumatic electric bells and annunciators were used quite extensively on ships during the period 1878-1890.

While the records are not clear we have every reason to believe that electric bells were installed in the Navy on ships at least ten years previous to the first electric light installation on the *Trenton.*

The most conspicuous example of the use and rapid development of electricity on shipboard has been in the American Navy, which has not only been for years a larger user of electrical machinery than the merchant marine, but also has led the navies of the world in this respect.

Lighting equipments were installed in the Navy on several of the early vessels, beginning with the *Trenton,* and certain power applications installed, such as the ammunition hoists on the Cruiser *New York* and the Battleships *Massachusetts, Indiana* and *Oregon,* turret turning equipment on the Cruiser *Brooklyn,* and certain minor auxiliaries such as ammunition hoists on the gun boats *Wilmington* and *Helena.* The extensive use of electrical apparatus for lighting and power purposes was not made however, until the Battleships *Kearsage* and *Kentucky* were constructed in 1898-1900, these vessels having seven 50-kw. engine-driven, 80-volts generating sets, which were operated in series on a 3-

wire system to give 80 volts for lighting, and two voltages 80 and 160, for power motors. The major portion of the auxiliaries on these battleships were electrically operated, including the winches, boat cranes, ventilation fans, broadside ammunition hoists and turret auxiliaries. The anchor windlass and steering gear however, were steam driven.

After the *Kearsage* and *Kentucky* some succeeding battleships continued in many of their auxiliaries the use of steam drive, and the *Kearsage* and *Kentucky* are the only instances in the Navy of the use of the 3-wire control.

About 1902 the Navy Department devised an electric steering gear operated on the follow-up system and purchased apparatus of different manufacturers to effect installation on one of the small monitors which had then been recently constructed. The results obtained from this equipment were not satisfactory however. This question remained under discussion for a number of years until about 1909, when a contract was placed for supplying electric steering gear for the Cruiser *Des Moines* with the Cutler-Hammer Mfg. Co. For this purpose the use of an electric motor taking its power direct from the dynamo mains and operated by an automatic controller was advocated. The results of this trial equipment were so satisfactory that it eventually resulted in the use of electricity for the steering of all capital vessels in the Navy, the later installations, beginning with the Battleship *New Mexico*, employing the use of the electro hydraulic gear instead of the direct motor drive, and to the exclusion of the steam drive.

Beginning with the Battleships *Nevada* and *Oklahoma*, electrical equipments were provided for the anchor hoist gear, which proved so generally satisfactory that all subsequent capital ships have specified electrical drive for this auxiliary, thus practically completing the application of electricity to all auxiliaries on capital naval vessels.

The use of electricity for lighting and auxiliary power purposes in the Navy having proved so generally successful, the General Electric Co. was granted contract for the installation of electric drive on the Collier *Jupiter* in 1911 which proving satisfactory, a contract was given to the same company for the installation of electric drive on the Battleship *New Mexico*, which continuing the satisfactory results obtained, all battleships and battle cruisers authorized since the *New Mexico* have included the electric drive, and undoubtedly it will remain the means of propelling capital ships of the Navy.

In the submarine branch of the Navy various means of supplying the necessary power for propulsion on early boats when submerged were used, such as hand power, compressed air, and steam. With the development however, of the storage battery,, the application of electrical apparatus to the submarine made rapid advancement and made the modern type of submarine possible. In the modern submarine, besides using motors and storage batteries for propulsion purposes, all auxiliary machinery such as hydroplane, steering gear, pumps, windlasses, machine tools, periscope hoists, etc., are electrically operated, electricity also being used for lighting, sound detecting devices and interior communication devices of various kinds. The growth of electricity in the case of submarines may be noted from the fact that the early successful types were equipped only with electric lighting, bell signals and motors for propulsion under 50-h.p., whereas in the most recent electric propulsion, motor power has reached about 3000 h.p. As a matter of historical interest, the first application of the use of propulsion motors for submarines arranged with two armatures on the same shaft was made by the Lake Torpedo Boat Co. on submarines built in Russia for that Government and designed in 1906 and furnished with contactor type of controllers arranged for series parallel operation of the armatures as supplied by the Cutler-Hammer Mfg. Co., this type of motor and system of control being now extensively used in the later submarines of the American Navy.

The American Navy therefore, at the present time, is a user of electricity for practically all purposes for lighting and power and extensively for interior communication work.

While the American Navy has led all other navies of the world in the use of electricity, Europe seems to have led this country in the trial of electrical auxiliaries for use on merchant vessels, and while electricity has been used generally for lighting and some interior communication purposes on merchant vessels for years, it has been employed but to a very limited extent for power purposes until the advent of the oil-engine-driven ship, the most notable instances of this kind being the fleet of vessels constructed by Burmeister and Wain of Copenhagen, the *Christian X* being one of the first vessels so constructed by this company, and its first voyage to this country created marked attention. Since that time it is understood that some 70 or more vessels of this type have been constructed and projected by that company.

In 1916 and '17 a number of merchant vessels were projected in this country, including tankers and cargo vessels employing electrical auxiliaries, among others being the Tanker *La Brea* with electric operated pumps and the Tanker *Solitaire* just completed by the Texas Co., having oil-engine drive and electrical auxiliaries throughout, including electric heating. Other instances are six tank vessels constructed by the Pennsylvania Shipbuilding Co. propelled by geared steam turbines and having electrical auxiliaries.

At the present time there are a number of oil-engine-driven cargo vessels with electric auxiliaries and others with electric drive and electric auxiliaries being projected in this country, and it is believed from this time the field will rapidly broaden until a great many of the merchant vessels will be so propelled and operated.

Extensive progress has been made recently in the merchant field in the use of the electric drive, the first installation on record in this country being two fire boats constructed in 1908 on the Great Lakes. Since these first installations in 1908, a total of 28 electrically propelled vessels have been constructed or projected, all but three of these being built or building in this country. Among these may be mentioned

ELECTRIC PROPULSION OF SHIPS (Not Including Naval)

Date	Name of ship	Type of ship	Type of propulsion	Size in tons	S H P	Propeller R P M	A. C. or D. C.	Voltage	Gen. units	Motor units	Remarks
1910	Jos. Medill	Fireboat	Steam electric	....	500	190	D C	250	2	2	Twin screw
1910	Graeme Stewart	Fireboat	Steam electric	....	500	190	D C	250	2	2	Twin screw
1913	Tynemouth	Cargo	Diesel electric	3400	500	78	A C	500	2	1	Method abandoned as Diesel engine misfit
1916	Mjolner	Cargo	Steam electric	800	850	90	A C	440	2	2	Geared single screw
1918	Wulsty Castle	Cargo	Steam electric	6400	1500	76	A C	650	2	2	Geared single screw
1918	Panoil	Tanker	Steam electric	1400	620	150–180	A C	440	1	2	Twin screw
1918	Mexoil	Tanker	Steam electric	1400	620	150–180	A C	440	1	2	Twin screw
1918	Fuel Oil	Tanker	Steam electric	1400	620	150–180	A C	440	1	2	Twin screw
1919	Mariner	Trawler	Diesel electric	500	400	200	D C	250	2	1	2–125 V. gen. Single screw 1–250 V. motor
1920	Crudoil	Tanker	Steam electric	1400	620	150–180	A C	440	1	2	Twin screw
1920	Elfay	Schooner yacht	Diesel electric	313	90	360	D C	125	1	1	Single screw
Building	Guinivere	Yacht	Diesel electric	642	550	220	D C	250	2	1	Single screw
Building	Powhaton	Passenger ship	Turbo Elec.	3440	3000	100	A C	1150	1	1	Synchronous motor
Building	4 ships	Coast guard cutters	Turbo electric	1600	2600	120	A C	1150	1	1	Synchronous motor
Building	United States Shipping Bd. 12 ships	Cargo	Turbo electric	9600	3000	100	A C	2300	1	1	Induction motor
Building	United Fruit Company	Cargo & Pass.	Turbo electric	6550	3000	100	A C	1150	1	1	Synchronous Motor

the four tankers constructed by the Pan American Petroleum Transport Co., the trawler *Mariner* equipped at the New London Ship and Engine Bldg. Co.'s works, the passenger ship *Cuba*, the 12 ships projected by the Shipping Board, and the cargo vessel *Wulsty Castle* constructed in England. This growing use of electric drive within the last two or three years would indicate a still further and more rapid adoption.

While the above gives a preliminary and general outline of the history and development of electricity on shipboard, a more complete and detailed historical review of this subject will be compiled during the coming year for submission to, and publication by, the Institute.

H. L. HIBBARD

ANNUAL REPORT OF THE LIGHTING AND ILLUMINATION COMMITTEE

To the Board of Directors,

I BEG to submit on behalf of the Lighting and Illumination Committee the following report for the year 1919-20.

ACTIVITIES OF THE COMMITTEE

Your Lighting and Illumination Committee, at its meeting on October 10th., 1919, decided to conduct a symposium on "Distribution Systems for Street Lighting" at one of the Institute meetings of the year, and to schedule, if possible, a paper on "An Analysis of Daylight Saving" at the Midwinter Convention. These plans resulted in holding the national meeting in Chicago on January 9th and 10th under the auspices of this Committee, at which three papers were read dealing with street lighting distribution systems. One of these papers on series systems was presented by W. P. Hurley, another, on multiple systems, by Ward Harrison, and a third, on constant potential series systems, by Charles P. Steinmetz.

The Chicago meeting was very successful and the discussions at both sessions were as complete as time permitted. At the afternoon session, prepared discussion was presented by C. H. Shepard of the Lincoln Park Board, Chicago; by N. B. Hinson, of the Southern California Edison Co.; by F. F. Fowle and W. F. Parker. In the evening, following Dr. Steinmetz's paper, illustrated discussion was presented by F. A. Vaughn together with a description of the new group lighting system now used in Chicago for underground distribution for residence street lighting, by Deputy Commissioner Nixon. The newest things brought out in this discussion were the Chicago group lighting system and the conduit return system used in Southern California.

At the Midwinter convention in New York in February 1920, a paper on "Daylight Saving" was presented by Preston S. Millar under the auspices of the Lighting and Illumination Committee. This paper contained a comprehensive analysis of the subject and the general purpose kept in view by the author in the preparation of the paper is indicated by the following quotation from the opening paragraph of his remarks:

> In accepting the invitation of the Lighting and Illumination Committee to present a paper on "Daylight Saving," the author stipulated that in order to consider the subject comprehensively, a considerable part of the paper would have to be devoted to matters remote from electrical engineering. The economic and sociological aspects of daylight saving surpass in importance the effect upon use of artificial light. Any treatment which should ignore these important features would lack perspective and would be likely still further to increase confusion on a subject which is greatly in need of clarification. Accordingly this paper includes a brief survey of daylight saving in its several aspects.

Your Lighting and Illumination Committee decided this year, as last year, to send a circular letter to the Chairmen of all of the Institute Sections, suggesting that one meeting of the year in each Section might profitably be devoted to an illumination topic. The following quotations from this year's letter will indicate the point of view taken by the Committee:

> At the last meeting of the Lighting and Illumination Committee of the American Institute of Electrical Engineers held on October 10, 1919, in Philadelphia, it was decided to suggest to the Chairmen of the various Institute Sections that it might be desirable to include on the program for the current year one meeting devoted to some illumination topic.
>
> Where the Local Section of the Institute is in a territory having a Local Section of the Illuminating Engineering Society it would be desirable in connection with the meeting devoted to illumination to have said meeting as a joint session between the two organizations.

The responses to this letter were widespread and a number of excellent papers dealing with various aspects of lighting and illumination were presented before some of the Local Sections during the course of the year.

DEVELOPMENTS IN THE LIGHTING FIELD

Under date of January 16, 1920, the Chairman of the Board's Committee on Technical Activities sent a letter to the Chairmen of the Technical Committees, which contained, among other items, the following:

> It was brought out very strongly at the last annual convention in the discussion of the Development Committee's report that the reports of Technical Committees should cover the activity in the particular line covered, so that anyone reading these reports from year to year would obtain a review of the progress of the art.
>
> These reports should cover notable installations or developments during the year and it is suggested in view of the fact that in the past little has been done in this respect, that the reports might be a review of the state of the art up to date. One plan that was suggested which seemed to have considerable merit, was that the Chairman of the Committee might apportion to different members of the Committee certain parts of the field covered by the Committee's activities, and these should be edited and combined to be the report of the Committee.

Acting on this suggestion, the Chairman of your Lighting and Illumination Committee has addressed the members of the Committee in an effort to gather the important developments in the lighting field. These have been edited and combined so as to be the main portion of this report. It should be stated in advance, however, that the developments in the field of street lighting distribution, which were covered in the papers read before the Chicago meeting in January, will not be discussed in this report, since they will be found completely treated in the TRANSACTIONS of the Institute. The same statement applies to the important question of "Daylight Saving" which was covered in the paper before the Midwinter convention, and which will be found in the Institute JOURNAL for February, 1920.

GENERAL NOTES

As one of the results of the war, in its effects on the lighting field, a number of interesting papers have been presented during the past year. The titles of several of these papers will indicate the rather unusual character of the material which has been gathered under war conditions. Among these may be mentioned the following: "The Science of Marine Camouflage Design," "Painting Battleships for Low Visibility," "Camouflage," "The Principles of Camouflage" and "Industrial Lighting and its Relation to the War."

Following the war, the past year has witnessed an unusual increase in the interest taken in lighting in general. It has been felt that the past twelve months have, in fact, marked an awakening among the users to the possibilities of artificial light. This has been evidenced by the efforts made by various societies, organizations and associations, to have placed before them the facts as to what light can do in the way of increasing production, decreasing accidents and of improving the morale of employees in industrial plants. It would appear that a transition period is in progress in the lighting field, if one can judge by the trend of the developments during the year. Among these developments the following may be mentioned:

1. A widespread recognition of good lighting as an important aid to manufacturing.
2. The increasing use of the foot-candle meter as a means of checking illumination intensity in various parts of a lighted room.
3. Progress of industrial lighting codes.
4. Progress in automobile lighting regulations in several states.
5. Development of the bowl enameled Mazda C lamp to give better diffusion. The latest lamps of this type have a feathered edge of the enameling to prevent a sharp "cut off" line.
6. The proposal of a plug outlet for permitting the convenient change of ceiling and wall fixtures.
7. The increasing tendency on the part of fixture manufacturers to make and advertise ready-to-hang types of fixtures. These fixtures are principally for commercial and industrial lighting but there is also some tendency to invade the residence lighting field, which seems to be an improvement both from the scientific and the commercial standpoints.
8. The formation of the fixture manufacturers association, with an attempt (as mentioned in item 6 above) to perfect a method of fixture hanging which will permit electric light fixtures to be hung as easily as a picture and moved, by the renter, as easily as a portable lamp can be moved, thus encouraging the renter to install improved fixtures as a substitute

for bad designs often found to be in use when a house or an apartment is rented.

9. Progress in the enforcement of the industrial and automobile headlight codes. New York, Connecticut, California, Wisconsin and Pennsylvania have adopted the I. E. S. S. A. E. automobile headlight specifications, and it is reported that in all of these states, except New York, there are practical attempts at enforcement, which are gradually producing results that are favorable.

10. An important development in street lighting units is the "Duoflux" standard. This unit contains two lamps (in one type, one 1000 candle-power and one 250 candle-power) and by means of a relay located in the casing of the fixture, the 1000 candle-power lamp will be extinguished at midnight; and the 250 candle-power lamp lighted.

DEVELOPMENTS IN LIGHTING UNITS

The RLM standard dome reflectors have during the year become more widely adopted and nearly all the leading steel reflector manufacturers recommend these standards for industrial lighting.

Among the new fixtures and auxiliaries developed during the year, there may be mentioned the enclosing type of interior units which consist of a combination reflector, a diffuser and an enclosing globe in one piece. Another interesting development has been that of simplified units of the semi-indirect type, and the increased application of the incandescent lamp for moving picture projection. In street lighting, the single unit of high candle power has quite largely replaced the cluster form of post, the former being considered superior both from the standpoint of efficiency as well as of appearance.

Luminous arc lamp efficiencies have been increased by the compounding under high pressure of the ingredients of the electrodes. It is reported that by this means, 30 to 40 per cent more light is produced than with the previous standard electrodes, with at least equal life. For the same light as formerly, increased life of the electrodes is expected.

IMPORTANT LIGHTING INSTALLATIONS

The unusual lighting effects secured at the Chicago and the Buffalo electrical shows in 1919 may be mentioned. At Chicago, the so called "Palace of Aladdin" was housed in a structure 50 feet high, and interesting lighting effects secured with glass jewels, painted mirrored glass and flood lights. At Buffalo, 5000 *white* Mazda lamps were employed along with nearly 5000 illuminated disks distributed among the roof girders. The application of spectacular lighting of this general nature, was seen at its best, perhaps, in the "Jewel Portal for the Victorious Army" in New York and the "Altar of Victory" in Chicago. The use of 30,000 so called jewels in each of these remarkable lighting displays, contributed largely to the spectacular effects secured.

The effect of the war on industrial lighting has been to stimulate the interest taken in better factory lighting the past year. The tendency to employ illuminating engineers for the handling of industrial lighting schemes, rather than to entrust this work to the electrical department as heretofore, has been noticed. In one large industry a research is being conducted to ascertain the effects of lighting on production. The large increase in wages in the industries, with very little increase in the cost of factory lighting, has reduced the cost of good lighting in terms of wage equivalents. This same fact has also made lighting the more important in terms of its effects on increased production and reduced spoilage.

A notable example of modern factory lighting installations is contained in a booklet recently issued by one of the lamp manufacturers. It would have been very difficult a short time ago to have published a book of this kind without relying upon the artist to touch out undesirable features and to paint in desirable ones. This entire booklet contains unretouched photographs of industrial lighting installations and is a commentary upon the improvements which have been made in this part of the illumination field.

In street lighting a number of important installations have been made. Among these may be mentioned the main street lighting in Salt Lake City consisting of 70 standards, each equipped with three 6.6-ampere ornamental luminous arc lamps. These units are spaced about 100 feet and the overall height is 29 feet. Although this particular system was put into operation prior to the current year, it represents an example of intensive street lighting which is coming more and more into prominence. More recent systems of intensive street lighting are the Triangle Lighting in San Francisco, first put into operation in January 1919, and the Broadway system in Los Angeles, put into operation in January 1920. The latter installation consists of 134 two-light ornamental luminous arc standards 106 feet apart and 27 feet high. The installation cost was about $6.50 per front foot of property, and the annual operating cost is about $1.00 per front foot.

The Broadway system at Saratoga Springs is of interest in that about a mile of street is to be lighted by 69 of the new "Duoflux" units mentioned above. Each of these units contains two lamps, one of 1000 candle power and one of 250 candle power. This installation, which is to be put into operation about June 1920, will cost about $32,000 for its installation and for operation, about $10,350 per annum.

CONTRIBUTIONS TO THE ART

Important papers contributed during the year include a discussion entitled "Coefficients of Utilization" by Ward Harrison and Earl A. Anderson, published in the March 20, 1920 issue of the *Transactions* of the Illuminating Engineering Society. This paper presents a method for the direct determination of coefficients of utilization applying to installations of all ordinary types of lighting units in rooms of varied proportions and different ceiling and wall colors. Actual colors for ceiling and wall surfaces covering 32 different shades with the corresponding reflection factors are included in the printed paper.

A paper on "Opportunities of Extending Lighting through New Applications" was read by R. M. Searle before the annual convention of the Illuminating Engineering Society in October 1919, which contains references to the following divisions of the lighting field: Street Lighting, Display Lighting, Flood Lighting, Lighting of Freight Terminals, Industrial Lighting, Stairway Lighting and Lighting for Community Affairs, this list giving an idea of some of the fields in which developments may be expected.

The following quotation from an address of President S. E. Doane before the Thirteenth annual convention of the Illuminating Engineering Society is of interest:

> The war has taught us that we have been regarding electric lighting, to use a phrase that one of my associates has given me, as a janitor service, whereas we should have looked at lighting as an item in the cost of production or in the cost of sales, and regarded it as any other factor in the cost. Then we would have examined, as have men within the last year or two, under what light we could get the maximum of visual acuity or speed of visual reaction. During the war time we have had some demonstrations. Mr. Durgin has given the most and the best of the practical demonstrations of how to apply this knowledge that visual acuity increases with the intensity of light and the speed of reaction increases production.
>
> Its application to production has been spectacular, and the results have been remarkable. As a matter of interest and as a measure of our opportunities I would like to use some figures from the lamp industry. Sixty-five per cent of the output of the lamp manufacturers, according to our estimates, is used in that portion of our business which would be affected by this knowledge. In other words, more than half of the electric lighting of this country can be affected and will be affected by the better knowledge we have of the production or increase in visual acuity and speed of visual reaction under intensive lighting.

For those who may be especially interested in the progress of the lighting and illumination art, attention is directed to the annual report of the Committee on Progress published each year by the Illuminating Engineering Society. The last report of this kind, covering 80 pages, will be found in the *Transactions* of the Illuminating Engineering Society for November 20, 1919.

C. E. Clewell, *Chairman.*

ANNUAL REPORT OF THE POWER STATION COMMITTEE

To the Board of Directors,

DURING the year 1919-1920 the Committee held two meetings, mainly devoted to discussion of papers presented, in person by the writers for comments and suggestions. The contributions to the activities of the Institute under the auspices of this Committee were as follows:

A symposium on steam turbine design, with the following papers:

Present Limits of Speed and Power of Single-Shaft Curtis Steam Turbines, by Eskil Berg.

Present Limits of Speed and Power of Single-Shaft Steam Turbines, by J. F. Johnson.

Present Limits of Speed and Output of Single-Shaft Turbo Generators, by F. D. Newbury.

A plea for standardization in statistical records of state and governmental bodies reporting efficiencies of power plants, with the presentation of the following paper:

Essential Statistics for General Comparison of Steam Power Plant Performance, by W. S. Gorsuch.

A symposium on excitation, with the papers to be presented at the Annual Convention as follows:

Considerations which Determine the Selection and General Design of an Exciter System, by J. T. Barron and A. E. Bauhan.

Factors in Excitation Systems of Large Central Station Steam Plants, by A. A. Meyer and J. W. Parker.

Exciters and Systems of Excitation, by H. R. Summerhayes.

Application of D-C. Generators to Exciter Service, by C. A. Boddie and F. L. Moon.

Exciter Practise in the Northwest, by J. D. Ross.

Generator Excitation Practise in the Hydroelectric Plants of the Southern California Edison Company, by H. H. Cox and H. Michener.

The committee also considered the question of making suggestions for the activities of the incoming committee for the ensuing year, and recommended for consideration the following:

Safe Maximum Limit of Operating Capacity for Each Section of Bus.

Reactive Component Dispatching.

Auxiliaries in Steam and Hydroelectric Plants.

Fighting Generator Fires.

Trend of Modern Power Station Design.

PHILIP TORCHIO, *Chairman.*

ANNUAL REPORT OF THE ELECTROPHYSICS COMMITTEE

To the Board of Directors,

THE Electrophysics Committee desires to receive suggestions for future work. It seems well, therefore, for this purpose to re-state from the report of last year the policy that this Committee has endeavored to carry out. It is as follows:

1. To encourage original papers of high technical standard, marking advances in electrophysics.
2. To have each year a broad, interesting, general lecture, free from mathematics, dealing with modern physics.
3. To promote a more complete cooperation and mutual understanding between the engineer and the physicist.

It is our object to keep open the "line of communication" between the pure physicist and the strictly applied physicist or engineer.

A joint meeting was held with the American Physical Society in Philadelphia, October 10-11th. The Physical Society session gave a notable discussion on the present status of theories of atomic structure. A review of this subject designed for engineers has been printed in the JOURNAL. The A. I. E. E. papers showed how these theories had been practically applied in the study of crystal structure and in the solution of vacuum tube problems. Other papers covered a variety of subjects marking advances in electrophysics.

Talks were given in an evening session by past presidents of the two Societies. It was the object of these talks to promote cooperation. The titles are significant—"The Indispensability to Each Other of Pure and Applied Science" by H. A. Bumstead; "Pure Science and Industrial Research" by J. J. Carty.

Additional technical papers have been given at other sessions.

F. W. PEEK, JR., *Chairman.*

ANNUAL REPORT OF THE TRACTION AND TRANSPORTATION COMMITTEE

To the Board of Directors,

TWO meetings were held under the auspices of the Traction and Transportation Committee; one at the mid-winter convention in New York in February, and the other at a regular meeting of the Institute in Pittsburg, March, 1920.

At the New York meeting there was presented and discussed a symposium paper entitled: "The Economic Supply of Power for the North East Atlantic Seaboard." This paper made reference to an item before Congress calling for a survey of the situation between Boston and Washington looking toward the establishment of a national policy in the matter of the generation and distribution of power through the means of which large economies could be effected as applying to labor, fuel and other materials.

The meeting passed a unanimous vote recommending that the Board of Directors of the Institute appoint a special committee to be known as the "Committee on Super Power System." This committee has since been appointed.

Since the above meeting, Congress has approved the proposed survey and appropriated $125,000 for the purpose.

The Pittsburg meeting included the presentation and discussion of papers relating to the design of the Minneapolis and St. Paul Electric locomotives by the General Electric Co. and by the Westinghouse Electric and Mfg. Co.; also papers concerning automatic substations and protection to substations.

Both of these meetings were largely attended and much valuable information and data were presented, all of which has appeared or will appear in the TRANSACTIONS.

There seems to be no disagreement on the part of any members of the committee in the conclusion (1) that super power systems stand as future guarantors for economical generation and distribution of power and (2) that the electric locomotive has demonstrated its ability as applying to passenger, freight or switching movement, to outclass the steam locomotive in capacity and control.

WM. S. MURRAY, *Chairman.*

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29–July 2, 1920.

CLASSIFICATION OF LARGE TURBO GENERATOR FAILURES

BY PHILIP TORCHIO

Chief Electrical Engineer, New York Edison Co.

THE experience with this class of electrical apparatus is in a measure less satisfactory than could be desired. The design difficulties are undoubtedly greater than encountered in the design of low-speed generators as used in hydroelectric plants, but the exacting service requirements of large steam power undertakings demand the maximum of reliability from its generators. To meet these requirements, designing engineers must ultimately produce apparatus more dependable. At the same time users must install and operate their generators under conditions that will be most favorable to their maintenance.

For the only object of reviewing past failures to aid us to avoid repetitions in the future, I have accepted the invitation of the Machinery Committee to give a summary of typical generator failures of which I have a record or report from different installations in this country.

The size of generators investigated ranged from 5000 kw. to 30,000 kw., the larger units being of more recent manufacture, while the smaller date back from twelve to sixteen years.

The total failures, several occurring on the same unit, amounted to fifty-five, of which thirty-three occurred in armatures, sixteen in fields, four in armatures and fields, and two in terminals.

The classification of all these generator troubles is as follows:

Armature failures due to:		
Mechanical damages	3	
Heating of windings	17	
Heating of iron	2	
Loose laminations	1	
Moisture in cooling air	3	
Corona shield breaking	2	
Heating at end turn clamping	1	
Overload damaging end turns	2	
Causes undiscovered	2	33
Field failures due to:		
Open-circuited connections	3	
Grounding	7	
Grounding caused by bus short-circuit	4	
Loosening of damper windings	2	16
Armature and field failures due to:		
Moisture	1	
Undiscovered	3	4
Terminal failures due to:		
Moisture	2	2

From the above analysis, it appears that at least 50 per cent of armature failures are due to heating, damaging the insulation. To avoid similar troubles, designing engineers should use insulating materials of proved dependability, maintain low copper temperatures by proper subdivision of copper and transposition of strands in windings and provide liberal ventilation throughout the machine. The operating copper temperatures should be limited at or closely to 100 deg. cent. Too great range of operating temperatures is bound to cause generator failures.

It may be appropriate here to emphasize a broad general principle to which perhaps we have not given full consideration in the past, and that is that insulated electrical apparatus subjected to too wide operating temperature ranges may be seriously impaired by mechanical bulging and damages to the insulation and the incasing structure, without inherent deterioration of the insulation itself. We found that the insulation, while excellent to withstand the maximum temperatures, was mechanically damaged by the expansion and contraction of the copper which it surrounded. For instance, mica insulation installed in hydroelectric generator armatures, which have only a small fraction of length of a turbo generator armature and are operated at a practically steady continuous load, last indefinitely at high temperatures, while mica insulation, under same or lower temperature conditions, in steam turbo generators operated at partial loads for several hours, then loaded to a maximum and later shut down every twenty-four hours, will fail.

To weed out cracks or latent defects due to defective workmanship at factory or in installation, the test voltage for windings should be raised to three times full voltage plus 1000.

The large number of field failures on generators of older design using fibrous insulation were practically eliminated in more recent designs by the use of mica and asbestos tape insulation.

Experience seems to indicate that a solid forged field has greater resistance to field stresses under bus short circuits than laminated field structures or even structures made up of end plates and several central plates a few inches in thickness.

The loosening of damper windings seems to have been satisfactorily overcome, from the experience of the last few years.

From the standpoint of the users, it appears important that great care be exercised in the supply of cooling air to prevent moisture or condensation depositing on the windings.

DISCUSSION ON "CLASSIFICATION OF LARGE STEAM TURBO GENERATOR FAILURES" (TORCHIO), WHITE SULPHUR, W. VA., JUNE 30, 1920.

W. J. Foster: Mr. Torchio speaks of limiting the temperature in the copper to 100 deg. cent. That is an extremely difficult thing to obtain. At the present time it is regarded by most designers as impossible, with the specifications as made out, calling for certain outputs at certain power factors.

I simply wish to point out that there is hardly a user of turbo generators who cannot radically improve his temperatures by operating his machines at better power factor. I am not going to argue as to whether that is practicable or not, but I wish to point out the desirability of it.

Most 60-cycle, large turbo generators are 1800 revolutions and 80 power factor and will operate at temperature rises of only about one-half, somewhere between 50 and 60 per cent of the present temperature rises in the rotors, if operated at unity power factor with the same energy output. That is extremely desirable when we consider the length of life. I think that statement is a safe one to make—regardless of construction—even with the very best possible construction—longer life may be expected at such low temperatures as Mr. Torchio recommends.

There is one point in the paper in which I would like to take issue with Mr. Torchio—or rather I would insist upon a limitation—and that is, where he asks for an insulation test of three times normal plus 1000 volts. That is very well and practicable for generators of 6600 volts and thereabouts, since it is evident that all you need to do is to take your 11,000 or 13,200-volt design and wind it for 6600 volts. That is entirely practicable in those cases, but when it comes to the higher voltage machines, it is a very serious matter, and I regard it as practically impossible at the present time in the larger turbo generators.

In that connection, I will take advantage of the situation here and refer Mr. Torchio and other interested members to the little table on page 961 of my paper for this session, and ask them to compare columns 2 and 4, in which I regard the design represented in column 2 as having about as large a slot as should be incorporated in a machine, due to magnetic limitations and eddy current limitations. If you use that slot, and put in a coil with an insulation for three times normal, plus 1000 volts, you get the temperature rise shown in column 4 as compared with column 2.

F. D. Newbury: Mr. Torchio presents a list of turbo-generator failures from which it would appear that armature winding heating is the major cause of failure.

My data and experience do not show that armature heating is the chief cause of failure, although it should not, by any means, be overlooked. This history of Westinghouse turbo generators impresses one with the importance of operating hazards such as fires of external origin, abnormal voltage surges, and water carried into the generator.

The investigation I have made covers every Westinghouse turbo generator from 15,000 kv-a. rating upwards, that has been placed in operation up to January, 1920. It happens that these generators, without exception are mica insulated, with mica insulation between strands and between conductors as well as between coil and core. This should be remembered in considering the record of these generators in regard to failures caused by armature heating.

The first Westinghouse generators larger than 10,000 kv-a. were placed in operation in 1913. These were generators roughly 20,000 kv-a. in size, two pole in the case of 25 cycles and four pole in the case of 60 cycles. The design of such large two and four pole units in 1911 and 1912 was highly experimental and while these units met the standard temperature rise of 50 deg. by thermometer, we have since found that the true copper temperature of the strands nearest the air gap gave at least 200 deg. rise in some cases. Experience has shown how to avoid these high local temperatures mainly by reduction in eddy current losses.

I also wish to draw your attention to the inclusive and representative nature of the record in connection with heating troubles. It covers all generators of large size of a given type and it is safe to draw general conclusions, bearing in mind the type and detailed design of the armature insulation involved.

During these seven years' experience there have been 22 cases of major armature trouble, involving 12 different installations or designs and 16 individual units. By major armature trouble is meant trouble serious enough to result in the failure of one or more armature coils.

The classification of armature failures as to cause is in some cases a difficult matter. The events leading up to the trouble may be unknown and the evidence is very often burnt up. It has been thought best, therefore, to separate these cases into three classes as follows:

1. Where the manufacturer has accepted responsibility.
2. Where the operating company has accepted responsibility.
3. Where the cause is unknown or in dispute.

1. Eight cases were chargeable to design, workmanship or other cause for which the manufacturer accepted responsibility.

(a) Five cases (of these eight) were caused by defects in manufacture that developed very shortly after installation.

(b) Three cases (of these eight) concerned details of design that led to failure after four or five years operation. Two of these cases (in one design) involved high local temperatures caused by eddy currents in the top strands of the coil.

2. Eleven cases were chargeable to operating hazards for which the operating company accepted the responsibility.

(a) Four cases (of these 11) were caused by fires originating outside of the armature winding. In three of these cases the fire started in the cables just outside of the generator; in the fourth case the fire started in a series transformer accidentally open-circuited.

(b) Four cases (of these 11) occurred in one installation and were caused by abnormal voltage surges that caused the outside surfaces of the insulation to catch fire. In addition to these four cases that resulted in coil failures as many more fires started that were put out before such damage was done. This trouble disappeared after the generator neutral was grounded.

(c) One case (of these 11) was caused by ice or water (it occurred on Feb. 28) carried into the generator from the air washer.

(d) Two cases (of these 11) were caused by enforced operation under conditions that were known to be unsafe. One case involved unsafe overloads and the other case involved continued operation after it was known the armature needed minor repairs.

3. Three cases involved unknown causes or the cause was in dispute.

In two of these cases the operating company believed armature coil heating to be responsible.

This classification of armature breakdowns shows that out of 19 cases, where causes were agreed to, 11 cases were caused by operating hazards for which the generator can in no way be held accountable. The majority of these cases were caused by fires of external origin.

If all the cases in which armature heating was involved (even by suspicion) are grouped, there are only five cases out of 22 breakdowns. Two cases (involving one of the first designs) were caused by design proportions; in two others temperature was not the primary cause of breakdown, but the operating companies believed it to be a contributory cause; and in the fifth case enforced overloading was the primary cause.

R. B. Williamson: I would like to endorse Mr. Foster's remarks about the effect of power factor on turbo generators. The rotor of a turbo generator is relatively small, and the surface available for getting rid of heat is also small. With inductive load, the lagging current that the machine is compelled to furnish calls for increased exciting current in the rotor and very materially increases the heating. Thus the output which can be obtained from the machines at unity power factor, with a given rotor heating, is much greater than it would be, at, say, 80 power factor.

The question of voltage test of three-times normal plus 1000, as Mr. Foster pointed out, would entail no hardship with machines for 6600 volts, or possibly 8000 or 9000 volts, but with generators for 11,000 or 13,200 volts, it is a difficult matter to design machines for such high-voltage test. As Mr. Foster pointed out in his paper, a machine insulated for such test would involve thicker insulation and much higher internal temperatures. The latter might lead to trouble due to creepage of the coils or some effect of that kind, even if the higher temperature were not sufficient to damage the insulation.

W. F. Dawson: I do not want you to get the idea that it is only the mica insulation that will stand the gaff. I have been responsible for something like 2000 turbo alternators, and probably 90 per cent were not insulated with mica in the armature winding, and I can say very truthfully that there have been practically no failures in those generators from any causes of workmanship, material or manufacture. In the beginning we had some trouble due to the fact that the sharp corners of the armature slots bore against the insulation, but with the eased off slots, used by most of the American manufacturers, and the insulation properly applied to the coils, we have had almost unbroken success.

The great difficulty that was noted in Europe some ten or fifteen years ago, and particularly pointed out by Mr. Highfield was due in a large measure, as

Mr. Newbury has just mentioned, to the so-called pulled in windings. I believe even today some manufacturers in Europe adopt that method, and it is very handy from the manufacturing standpoint, but it is just about as futile in respect to keeping up insulation as anything can be. These windings were filled with air voids, and the potential gradient was steepest on the untreated cotton of the individual wires, and failure was bound to result, but with our American practise, we are in a very much better position. Of course, these machines I mentioned, are not high-voltage machines. We have adopted mica insulation wherever the voltage exceeds 4000, but I have put out enormous quantities of machines for industrial work at voltages of 4000 and under, with all treated fabric insulation, and I should say that the success has been, at least, 99 per cent.

Robert Treat: At the bottom of page 904 Mr. Torchio makes this statement:

"For instance, mica insulation installed in hydroelectric generator armatures, which have only a small fraction of length of a turbo-generator armature and are operated at a practically steady continuous load last indefinitely at high temperatures . . . . ."

I would like to inquire whether Mr. Torchio has in mind any definite limiting temperature which in his mind constitutes a "high temperature" as here used.

The writers opinion, based on a careful study of operating experience and based on a detailed knowledge of internal temperatures and coil and insulation design, is that breakdowns caused by armature heating are in reality due to abnormal local temperatures that have values of the order of several hundred degrees; and that temperatures of 100 deg. or 150 deg. ordinarily discussed in connection with guarantees have very little to do with the problem. This statement of course, applies only to windings completely insulated with mica within the slots.

This record of breakdowns brings to the surface another fact that is reassuring to the companies operating high-voltage units. If these generators be classified according to voltage it is found there are twice as many generators wound for 11,000 volts, or higher voltage, as there are generators wound for lower voltages. But these 22 cases of armature breakdown are equally divided between these two voltage classes. This means that the percentage of armature breakdowns in the high-voltage generators under discussion is only one-half that of the lower voltage generators. This record would be very different with treated cloth insulation (in large high-voltage generators) that is subject to cumulative heating on account of increasing dielectric losses with temperature; and with partially closed armature slots, still used by some European designers.

This record does not prove that high-voltage generators are necessarily safe or less subject to breakdown except in so far as breakdowns may be caused by voltage surges that are independent of generator voltage. Obviously a 10,000-volt surge would be dangerous in a 2400-volt generator but would be harmless in a 13,000-volt winding. The record does prove that 11,000 and 13,000-volt windings are just as reliable as low-voltage windings and, further, that breakdowns are generally caused by trouble unrelated to line voltage.

The facts brought out by this record may be summarized as follows:

1. The majority of armature breakdowns is caused by operating hazards originating outside the generator.
2. Armature heating in armatures completely insulated with mica is a minor cause of breakdown.
3. Large high-voltage generators, with mica insulation and open armature slots, are as reliable and probably are more reliable than large low-voltage generators.

James Lyman: We appreciate the almost insurmountable difficulties in obtaining all that we would like in the matter of insulation tests, and in the matter of temperatures, but what were apparently insurmountable difficulties five or ten years ago, the designing engineers of our large manufacturing companies have overcome, and it is to be hoped that they may, in the line of evolution, gradually work toward the limits that Mr. Torchio has given.

Mr. Newbury has stated that one-half as many burnouts occurred in the case of large generators wound for 11,000 volts and above, as have occurred in the same number of similar generators wound for lower voltages. Cannot we, therefore, assume that the insulation was quite equal to the service, but that the weakness in these machines was the high temperatures they attained?

Undoubtedly, mica insulation, when not subjected to any mechanical strain, is quite equal to the extreme temperatures that have been mentioned, but in turbo generators, especially in the very large units, the windings are subject to great mechanical strains, and I suspect that where very high temperatures are allowed, the burn-outs are due more to expansion and contraction, the movement of the mica insulation, which mechanically is not very strong, than to the insulating qualities of the mica.

Philip Torchio: Practically all the limitations which have been pointed out hinge on temperature limits and voltage tests, both of which are closely interrelated. In fact, Mr. Foster has no hesitation in applying the three-times normal voltage test if the machine, instead of being rated 100 per cent, is rated at only 75 per cent, because then he can put on the thicker insulation to stand the higher voltage test and still dissipate the heat and maintain a low temperature.

That prompts me to comment that the users are not requesting the large ratings and the last drop of capacity from the machines—they want safety and continuity of service. I do not propose to stand before you and say I would want to retard progress. We must make progress, and we share with the designers and the manufacturers the responsibilities and the burden of any experiment, but we must emphasize one point; that these large units are of such vital importance that the question of getting out the last 10 per cent or 15 per cent of capacity is of no importance whatsoever to the users in general, and by users in general I do not mean the operating companies alone, but I mean the customers who use the power. They want continuity of service and reliability, and in the end the apparatus with the longest life, the most reliable apparatus, is the apparatus which will give the cheapest service.

Along the same line of reasoning is my reply to Mr. Foster's reference to power factor. The operating companies are making a strenuous effort to improve power factor, and I want to take this opportunity of saying that it is not for sentimental or selfish reasons that we do try to get the power factor improved, but it is really because the low power factor is an obstacle to the progress of power generation, as Mr. Foster has forcibly demonstrated.

In answer to Mr. Treat's question as to what I mean by high temperature, I would state that as far as it refers to pure mica insulation, not mica built up with a binder, I mean temperatures of, say, 185 deg. cent., as prevailing on the 2200-volt generators at Niagara, which have been in satisfactory service for over 20 years. I had in mind such temperature as far as the limitation of pure mica itself to stand the voltage goes, provided it is not disturbed by local stresses, by periodic and frequent expansion and contraction, and by vibration and pounding due to the periodic beating of the strands in the windings of the coils. In the latter case and with built up mica, I would limit the temperature to about 100 deg. cent. to preserve the integrity of the binding material.

I would like to make some qualifying comments on Mr. Newbury's very interesting presentation, as my analysis of troubles also covers many of Mr. Newbury's troubles, but to enter into these details it would take up too much time. The important point is that the art is advanced by the lessons of past errors.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29–July 2, 1920

VENTILATION AND TEMPERATURE IN LARGE TURBO GENERATORS

BY B. G. LAMME

Chief Engineer, Westinghouse Elec. & Mfg. Co.

IN the turbo alternator, the problems of air circulation, or ventilation, and of insulation have been carried further than in any other type of large apparatus. There are fundamental reasons for this. In the first place it may be said that there are four effective materials concerned in the turbo-generator operation, namely the iron for magnetic purposes, the copper in the windings, the insulation encasing the winding, and the ventilating air. The iron and copper have long been carried up to their practical limits, so that the major part of recent developments has been in the insulation and in the ventilation. In the second place the physical limitations of the materials having been reached, in many ways, it is no longer possible to increase the physical dimensions, as has been the case in lower speed machines.

As present insulations have been known and utilized fairly well up to their limits, for some years, it follows that the more recent growth has been largely along lines of improved methods of heat dissipation, which, in the final consideration, is largely the problem of air circulation.

Therefore, beginning with the problem of ventilation in general, it may be said that, broadly, there is but one method in use today, namely forcing a large quantity of air through the machine, at a high velocity due to unavoidable restriction of the air paths. The air-forcing means in all cases consists of powerful fans or blowers driven either by separate motors, or by the rotor shaft of the turbo alternator itself. There are advantages in favor of either arrangement, but these do not concern the problem broadly. The real problem is to force, through effective cooling channels or paths, sufficient air to take up the generator heat, without unsafe temperature limits being reached. It is the design of these cooling channels or paths that much of the modern turbo-alternator engineering has been expended. In the following treatment of the general problem, these various methods will be considered.

The types of ventilating channels may be classified broadly as *radial* and *axial.* Very few machines use either one exclusively. In axial ventilation the air travels in the general direction of the shaft, whereas in the radial arrangement the movement is in a right angle direction. The air-gap ventilation in the turbo generator is axial in most cases, while the internal ventilation of the rotor body or core is usually the axial and radial combined. In the same way the ventilation of the turbo stator core is usually a combination of the axial and the radial; for, even in the so-called radial types, the cooling air through the gap passes axially over the armature bore, before passing out radially. In the same way, in the so-called axial types the air through the axial core channels passes radially out at the core center.

In addition to the purely axial and radial arrangements, may be mentioned a third one, namely, circumferential ventilation, or air circulation around the machine circumferentially. This has been used to a certain extent in certain European types of machines, and in combination with axial and radial arrangements, it appears in certain American types. In fact, in the axial air gap ventilation, the circumferential action also occurs naturally, due to the speed of the rotor, the air traveling in a spiral path from the ends toward the center of the machine.

RADIAL VENTILATION

In the usual radial ventilation in the modern high-speed turbo alternator, the air enters the air-gap at each side, or end, of the core, and as it travels toward the center part, it is deflected outward at intervals, through the numerous radial ducts. In most cases the entire cooling air enters the gap at the two ends, and, therefore, its highest velocity is at the entrance.

As part of the air breaks away to pass out the first of the radial ducts, the volume of air in the gap decreases, thus successively lessening the pressure required to overcome the gap resistance. There is thus fairly high pressure at *a* in Fig. 1, due to the great restriction of the air path section at this point, and this tends to shunt part of the air out through the radial ducts over *a*.

At the center of the machine, due to the velocity of the ingoing air, there is "banking up" of the air, with corresponding pressure tending to drive it out radially. Here the velocity is thus converted into static pressure. Obviously, therefore, the air flow in the different ducts along the core, being due to different conditions of pressure, may be quite unequal. The real problem, therefore, is to obtain equal, or proper, air flow through the various ducts.

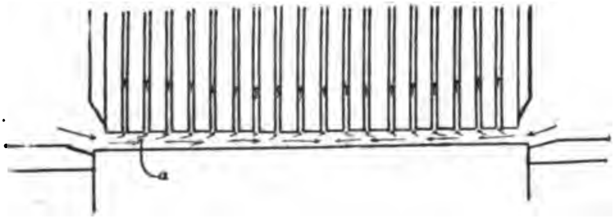


Fig. 1

An extension of this problem is found where the width of the core is greater than the permissible air flow can cool. Assume, for instance, with the largest permissible gap the design will allow, and the largest allowable diameter of rotor, that a core width of 80 in. can be ventilated by air from the two ends. Now what can be done with a core width of 100 in. or 120 in., which would be required for a 25 or 50 per cent larger output? One answer would be that it is impossible to use cores of such width. But that would put a stop to a certain trend of development and, therefore, some solution involving an important modification must be adopted. One of these is to put air in, radially, at the middle part of the stator core, and to allow it to flow axially *toward the ends,* in the air gap, and then out radially, as shown in Fig. 2. Here is a new problem, in that the entering air must be forced in against the rotation of the rotor core, and possibly against the velocity effects of the air from the end gaps. In consequence the driving pressure may have to be considerably different from that for forcing the air in at the end gaps. This thus further complicates the problem of the proper distribution of the air in the various ducts. Nevertheless this general arrangement has been used with considerable success.

The above description of the difficulties of the problem has been given, not as a criticism of this particular arrangement or method, but simply to illustrate some of the very real difficulties encountered in large turbogenerator design. More or less serious difficulties are found with *all* known methods, judging from the variable results obtained in different designs, even where the method itself is unchanged. The exact calculation of air pressures, and air flow in complex channels or passages, especially where a high-speed rotating element is present, is beyond the capabilities of the most experienced calculator, and, therefore, the experimental factor enters to a certain extent in all new designs. Good approximations can be made, in many cases, but no good designer will "fool himself" into thinking he can calculate the ventilation on a new construction with any great exactness. Even if he gets the right quantity of air through his machine, he cannot be sure it will distribute properly through the many parallel air paths, such as represented by the radial ducts, in

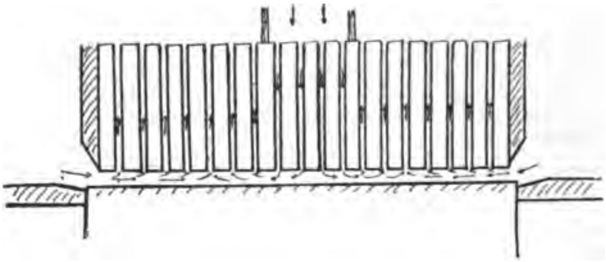


FIG. 2

the radial scheme, or the many axial ducts in the axial method.

AXIAL VENTILATION

As stated before, axial ventilation is usually partly radial. In this method, as usually applied, air not only passes axially along the air gap and out at radial ducts at the center of the core, but axial ducts, or channels, parallel to the shaft or air gap, are in the stator core, back of the armature slots, for allowing air-flow in parallel with the gap. The intention is to force sufficient air through the gap to cool the rotor, and the stator tooth tips, while air through the stator ducts will further cool the stator teeth and armature core. In the early designs one large central duct was used, this being changed later to two or three narrow ones, while in the latest types, there may be a dozen or more radial ducts, taking up a considerable portion of the central zone of the stator. With this latter arrangement, therefore, the central part might be considered as radially ventilated, while the end parts are axially ventilated. The real merits and demerits of this later construction, compared with the so-called radial type can only be judged properly when the quantitative conditions of the problem are taken into account. This will be considered, later in the paper, under the treatment of losses, heat flow, radiating surfaces, etc. Without knowing these quantitative conditions it is impossible for any one to say, in any particular case, whether any one method or arrangement is materially better than another.

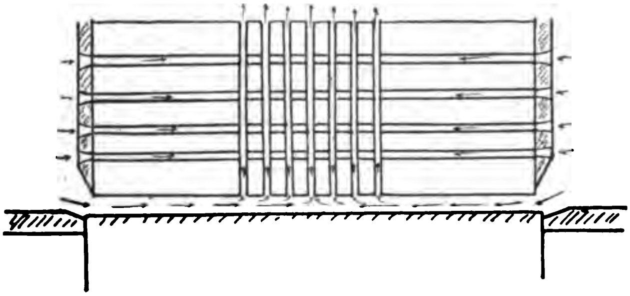


FIG. 3

A variation of the above axial scheme consists in supplying air, under pressure, to the central radiating ducts which then passes through the stator channels and the air gap, to the two ends. Or, again, the ventilating fan may operate reversed so that it *pulls* the air through from the central ducts. One advantage of the latter arrangement is that the friction loss due to the fan heats the outgoing instead of the ingoing air. This method will be referred to again under heating conditions.

It will be seen that with the axial arrangement, like the radial, the problem is to obtain the best air distribution with a large number of parallel paths and with different resistances of the various paths. One advantage of the axial arrangement is that part of the parallel paths,—those in the stator core,—are under possible control at the inlet.

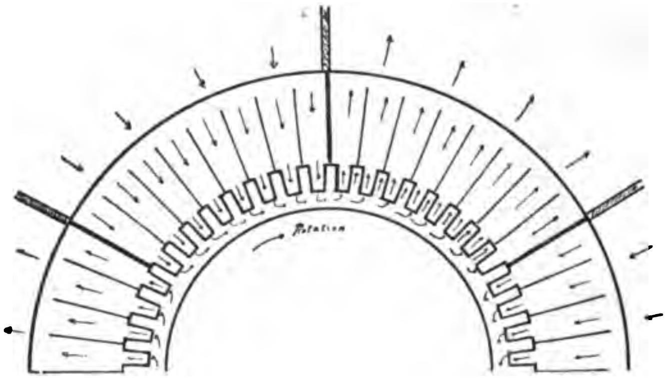


FIG. 4

CIRCUMFERENTIAL VENTILATION

This method should be considered, as it has been used commercially, to a considerable extent. In practise it is in reality a combination of radial and circumferential methods. As usually applied, there are radial passages through the stator core, but divided, circumferentially, into groups, so that the air passes inward to the air gap, in certain axial zones, and then circumferentially around the gap to the next zone where it passes out radially, as shown in Fig. 4.

At first glance this looks like an ideal arrangement, but a little study will show that it also has its difficulties, just like the other methods, for there is no way to insure proper division of the air among all the parallel paths. For instance, if one zone covers eight teeth, then due to crowding of the air in the gap, due to the rotor velocity, the pressures might be balanced in some of the tooth passages so that there is practically no air flow in those particular elements. This would mean very unequal temperatures in the tooth belt as a whole, and so the problem thus narrows down to one of proper air distribution, just as in all other arrangements.

A modification of the above, which has been used, combines radial and circumferential methods, as shown in Fig. 5. In this the air from the outer core surface is simply deflected through the stator core, and cools the tooth belt mainly by conduction through a relatively long path. The stator gap surface and rotor must be cooled by the usual methods.

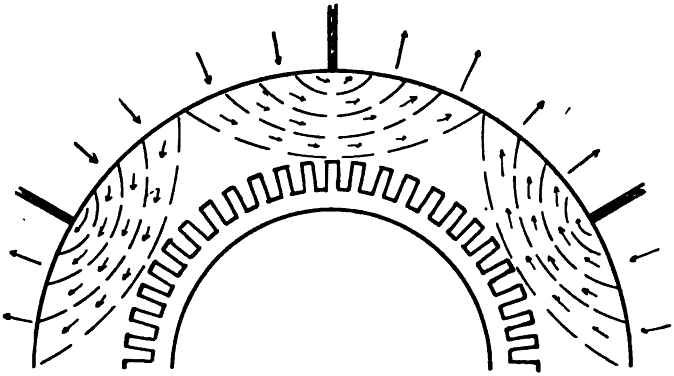


Fig. 5

In addition to the above there are various other combinations of radial, axial and circumferential methods, each of which possess certain merits, but a description of which is unnecessary here.

Heat flow from the Armature Copper

There are three principal sources of heat in such apparatus, namely, the losses in the copper windings, the iron losses, and the friction of the air at the high velocities used. Considering first the armature windings, all the heat generated must flow through the insulation, the relative rate of flow at any part being dependent principally upon the temperature differences. In addition to the transverse heat flow, through the insulation, there will be flow along the copper itself from hotter parts toward cooler places, if such exist, which is almost always the case. In the end windings, for instance, the heat can flow directly from the copper, through the insulation, to the cooling air. Moreover, local or eddy currents in the copper are liable to be least in this part, due to the absence of the iron. Consequently, in the end windings the actual copper temperatures are higher than the surrounding air temperatures only by the amount of temperature drop through the insulation including the contact drop.

However in the core, or buried part of the coil, the conditions are quite different. Here the heat, to a great extent, must pass from the copper, through the insulation to the embedding iron, and then through the iron to the cooling surfaces and to the cooling air which may or may not be at as low temperature as that around the end windings, usually not, as will be explained later. Obviously the temperature of the copper in the buried part must be at a materially higher temperature than in the end parts. But as copper is a pretty good heat conductor, this difference in the copper temperature tends to equalize itself by heat flow along the coil itself, thus tending to lower the higher temperature and increase the lower. This action can be quite effective in equalizing the temperatures in the various parts in those machines where the heat paths are not too long compared with the section of the conducting path and the amount of heat to be conducted. In narrow core machines, with short end windings, the equalization may be within a few degrees, but in the usual wide turbo-generator cores, with the very long span end windings, this equalization only shows prominently in the dividing zone between ends and core. In other words there is a very considerable longitudinal heat flow at this part, but it is relatively small far in the core or far out on the end windings. This may be illustrated diagrammatically by Fig. 6. Here the temperature of the core with respect to the copper is dependent upon the load conditions. With heavy load this flow may be from the buried copper to the iron at all parts of the core width, while at light load the flow may be from the iron to the copper over the whole core width. At intermediate loads there may be heat flow from the iron to the copper near the ends of the core, and from copper to iron in the midportions.

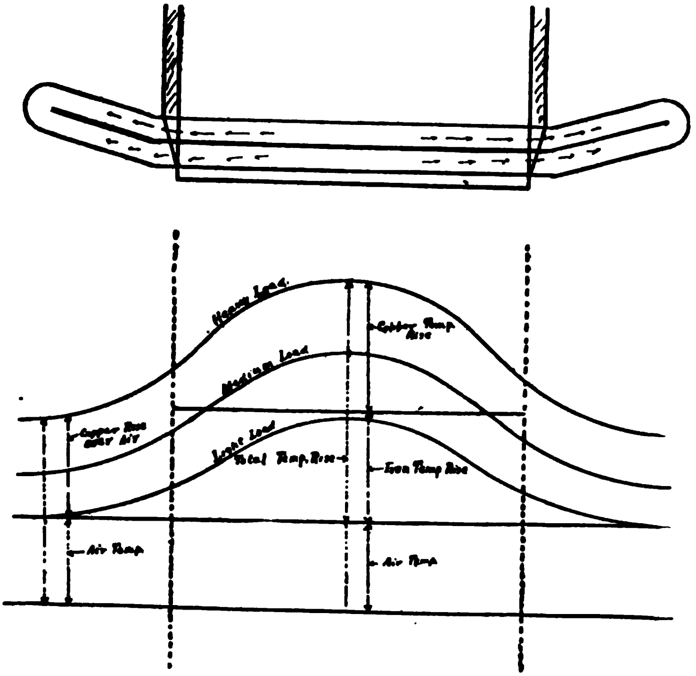


FIG. 6

Considering the heavy load conditions it is obvious, in a wide core machine, that the center parts should show very materially higher temperatures than the end copper, such higher temperatures being due to two or more drops, (1) through the insulation, (2) through the iron and the surface contact, and (3) to the difference between the cooling air in the core and that around the end windings. This latter effect may be very considerable, as will be brought out later. It should be evident that if the temperature rise or insulation drop, as determined at the end windings, should be 40 deg. cent. for instance, then the temperature rise in certain of the embedded parts should be higher by the amount of the adjacent iron temperature, plus the rise in the cooling air at those parts. If the iron temperature should be 40 deg. cent. above the adjacent cooling air, and the latter should be only 10 deg. cent. above the outer or end air, then this represents 50 deg. cent. additional temperature to be added to the insulation drop from the copper to the iron, or a total of 40 deg. + 50 deg. = 90 deg. These should not be taken as representative figures, but are simply assumed, to bring out the general fact. The

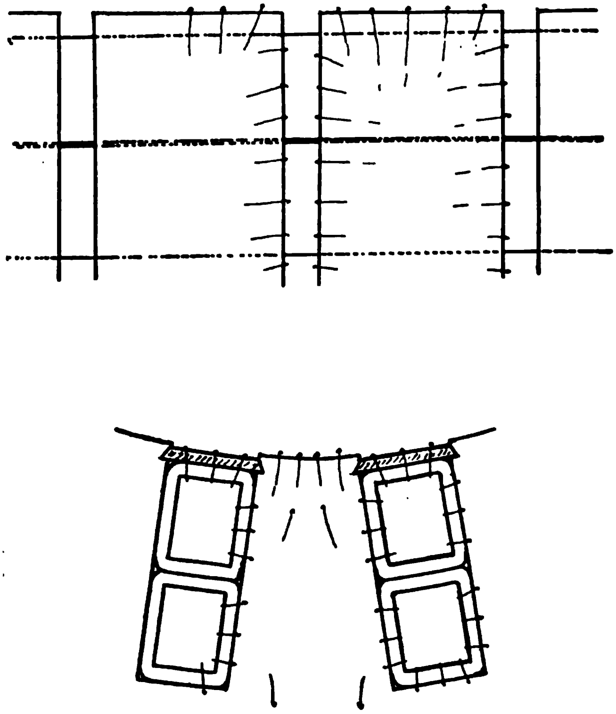


Fig. 7

object is to show that the central portion of the core will necessarily be considerably hotter than the ends, and that an end winding temperature measurement alone is of no value in such apparatus.

HEAT FLOW IN THE ARMATURE IRON

In practically all methods of turbo-generator ventilation there are two general paths of heat flow in the armature iron, namely, along the laminae, and transversely. With the radial arrangement of ducts it is usually considered that the heat flow is across the laminations, to the ducts, which is true in certain parts of the core, but is not entirely so in what may be called the *tooth belt*, as distinguished from the body or core back of the teeth. However, it is the tooth belt with which we are mostly concerned, for here are the embedded armature coils. Here the heat flows partly *across* the laminae to the radial duct surfaces, but in many cases quite a large part flows *along* the laminae to some cooler parts, such as the tooth tips, or the core behind the teeth. One reason for the longitudinal flow lies in the much better heat conducting path along the iron, compared with that across the thin laminae with their good coating of insulating (and, therefore, poor heat conducting) varnish.

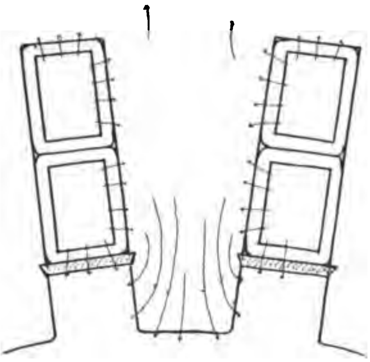


FIG. 8

The flow to the tooth tips is especially large in those cases where the armature coils are sunk some considerable distance below the gap surface, as shown in Fig. 8.

Here the tooth tip surface is very materially increased, and the heat dissipation is largely at the edges of the laminations. Experience has shown very considerable improvement in iron temperatures with this sunken coil arrangement.

In the body of the core, back of the slots, the heat flow, in the radial slot arrangement, will be mostly across the laminae. Due to this the ducts should be spaced at quite frequent intervals, there being, usually, one duct for at least each 2 in. of core width. Even with a 2-in. spacing, temperature differences of as much as 10 deg. cent. may be noted in cases, between the center of the pack and the end laminae.

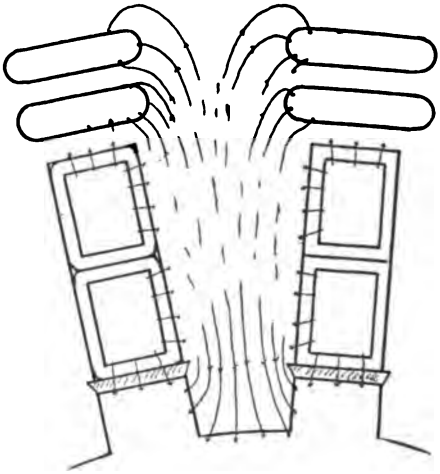


Fig. 9

In the axial arrangement of ducts, the heat flow is largely along the laminations, except at the central part where radial ducts are used.

At the tooth tips and air gap, obviously the heat dissipating conditions are very much as in the radial duct arrangement, especially where the coils are well below the surface. There is also a strong flow outward to the core and the axial ducts back of the coil slots, as indicated in Fig. 9.

From off-hand inspection, one would say that this method of heat dissipation is less effective than that with radial ducts, as the paths of heat flow are longer. But it is purely a quantitative problem, as the heat flow *across* the laminae is not nearly as effective as along them, possibly in the ratio of 1 to 10, so that the longer path may actually represent considerably less temperature drop than the shorter transverse one. On this basis, it then narrows down to the effective air surfaces for dissipating the heat, and upon the amount of air which can be utilized.

The primary reason for the axial method of ventilation is to get away from some of the physical restrictions of the purely air gap method of air supply. In the latter method the entire air supply, in most cases, is through the air gap, thus limiting the permissible volume of air which can be supplied. The axial arrangement, as usually supplied, simply adds core ventilating passages in parallel with the air gap. Sufficient air is forced through the gap to cool the rotor and also to exert a scouring action on the tooth tips, thus assisting materially in cooling the tooth belt. Additional air is forced through the core ducts, back of the tooth belt, to assist in cooling the teeth and to cool the core proper. As stated before, the effectiveness of the method depends largely upon the duct surface exposed and the quantity of air supplied, just as with other methods of ventilation. The real temperature drops are not so much in the iron itself, as from the exposed surfaces to the air, or what may be called the contact drop.

EFFECTS OF HEATING OF THE VENTILATING AIR

As the ventilating air takes up the heat, it rises in temperature and becomes correspondingly less effective as a cooling element. In other words, any addition to the cooling air temperature means a corresponding addition, in degrees, to the temperature of the iron and copper. Therefore, the effectiveness of the air cooling depends to some extent upon the course of the air through the parts to be cooled. Theoretically, if any given part needs the most effective ventilation, then the coolest air should be fed directly to that part, with as little rise as possible. None of the present methods accomplish this completely, but some appear to be more effective than others.

Consider first, the radial arrangement, where the air enters at each end, through the air gap. The air is heated by friction as it passes along, as about half the total windage loss, in such machines, is in the fan and the other half is in the air passages. Also as the air passes over the rotor and stator gap surfaces, it takes up heat from these parts. In consequence, there is a gradual accession in air temperature, from the ends of the core, towards the center. This adds a corresponding increase to the temperatures of the iron and copper parts. The increase in the air temperature at the central part will be, presumably, about half the total air rise, or possibly more, as the loss due to the air friction in the gap, plus the rotor gap loss, plus the tooth loss dissipated at the air gap, may be

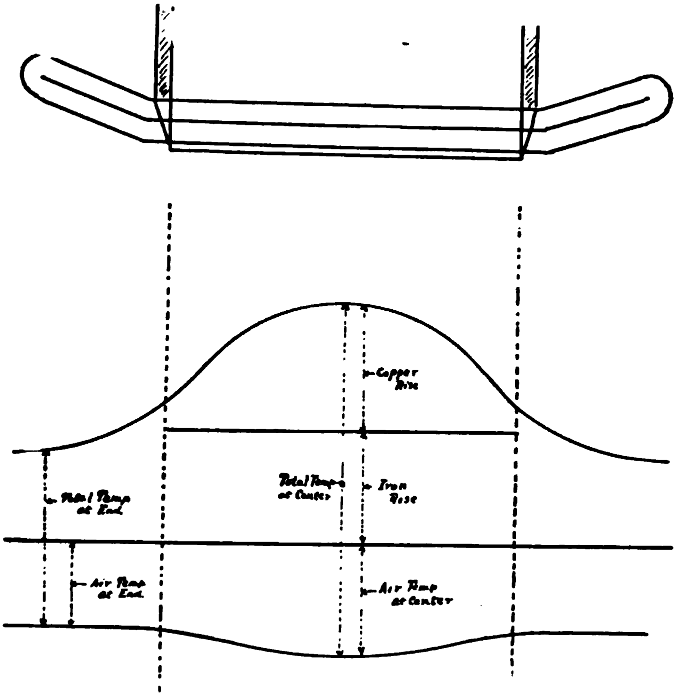


FIG. 10

taken as equal to, or greater than, the core and tooth losses dissipated directly into the radial ducts.

With auxiliary central duct ventilation, as described before in connection with Fig. 2, the rise in the air temperature at the tooth belt, is probably about midway between the temperatures of the entering and exit air.

In the axial method, of the more usual type, similar conditions hold, and the temperature rise of the air is somewhat greater at the central parts, than the usual radial arrangement, due to the fact that much the larger part of the heat dissipation to the air occurs by the time the air has reached the central portion. In consequence, the air temperature rise, at the center, is due to the core, as well as the tooth losses, considering only the core channel. In the air gap the temperature increase of the air is due to air friction and to the stator and rotor gap surface heat dissipation, just as in the radial arrangement. However, it must be borne in mind that the total volume of air through the gap is smaller than in the radial method, which should mean smaller friction loss. Taking into account both the decrease in loss, and the reduced volume of air, the increase in the air temperature in the gap, at the center, may or may not be larger than in the radial arrangement,—it is purely a question of proportion.

With the more recent types of axial construction, where the central portion of the stator embodies a large number of radial ducts, the end parts of the core being supplied with axial channels, it is a question whether there is much difference between the axial and radial arrangements, as far as air temperatures are concerned.

The above is on the basis of air flowing *towards* what are nominally the hotter parts of the machine. Let us now look at the matter on the basis of the air flow being in the opposite direction, with the axial method of ventilation. At first thought, this would not appear to change the conditions. However, there is one important difference. Here the temperature rise of the air is *least* at the central part of the core, and is higher near the ends, whereas the copper temperatures, as shown in Fig. 6, are naturally higher toward the center, and lower at the ends, due to longitudinal heat flow from the buried windings toward the ends. The resultant of the two effects is to tend to equalize the copper temperatures in the core, by bringing down the higher points and increasing the lower. There is, however, one disadvantage in this arrangement, in that there will be higher temperatures in the end windings, so much so that mica end insulation may be required, which usually is not so desirable on the curved end parts. Of course, there are possible remedies for this condition, such as an auxiliary cooler air supply for the ends, as from a separate fan, or fans.

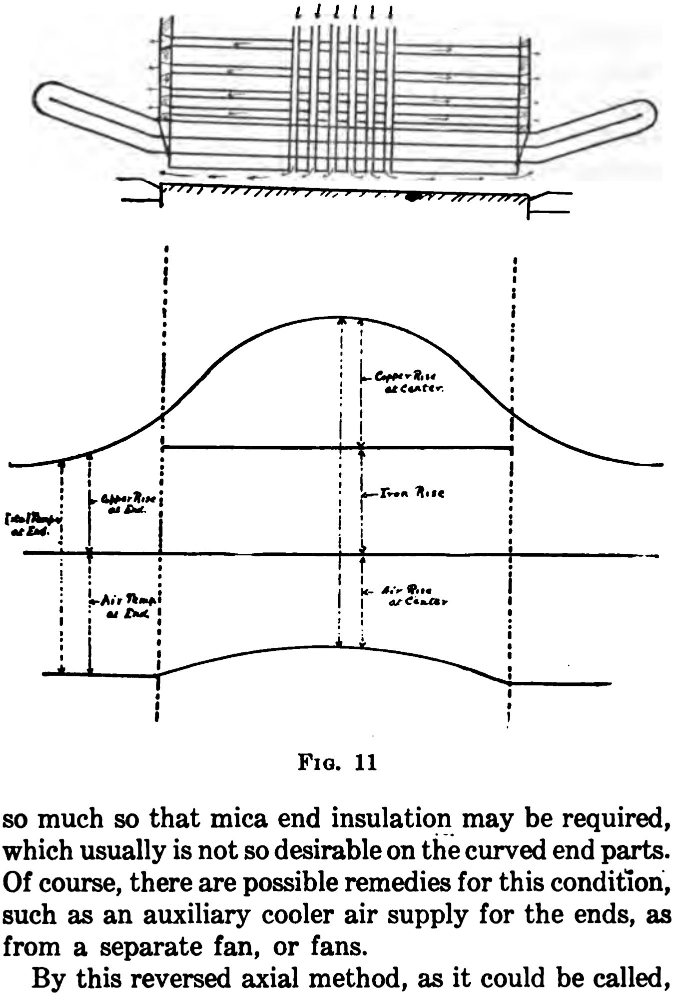


FIG. 11

By this reversed axial method, as it could be called, a reduction of the central copper temperatures of 10 deg. to 20 deg. might be possible, depending upon the individual designs. However, at some intermediate point *a* in the core there should be no gain, while at the ends there may be an increase of possibly 20 deg. If the hottest central part, with the usual arrangement, is 10 deg. hotter than the intermediate point *a* of no-temperature-change, then the actual peak or hottest spot temperature of the stator, as a whole, may be materially lowered by this arrangement. The rotor ventilation, however, presents a more difficult problem, as will be referred to later.

VENTILATION AND TEMPERATURE PROBLEMS OF THE ROTOR

In the foregoing, the problems of the stator have been considered, but the rotor must not be forgotten. The actual copper loss in the rotor of the modern huge capacity turbo-alternators may be as much as 80 to 100 kw., and most of the heat resulting from this must be radiated from the rotor air gap surface. In addition there are eddy current losses in the rotor iron, sometimes relatively large, due to the open stator slots, etc. And yet the excitation, or copper losses in turbo alternators are very small, in per cent, compared with other types of rotating electrical machines. In a 30,000-kw. turbo-alternator, which is by no means the largest size built, one quarter of one per cent, expended in the exciting windings, means 75 kw. loss in the rotor, which must be dissipated. The rate of heat dissipation from the gap surface of the rotor is often extremely high. Four to five *watts per square inch* of gap surface is not unusual. Comparing this with the 0.5 to 1.0 watt per sq. in., common in connection with dissipation of the excitation heat in more moderate speed machines, it may be seen that the above figures indicate a very abnormal rate of heat dissipation to air.

The writer believes he was one of the first to use purely air gap ventilation for the cooling of high-speed turbo rotors, and his early tests gave decided evidence of the great effectiveness of this method. However, there is naturally a limit to the effectiveness of such cooling, for there is some point where the improvement is quite small, with materially increased air velocities. Possibly air "cavitation" comes in, as one of the conditions which influence this result. The effectiveness of the air, in cooling the rotor, depends upon the scouring action of the air on the rotor surface, the hot layers in contact with the metal being continually torn off and replaced by cooler air. Experience shows that this action does not appear to be proportionally effective with air velocities above a certain point. Apparently the ventilating air along the air gap is crowded out against the stator surfaces, and thus possibly may become more effective in cooling the stator, with the higher velocities. Under such conditions, sinking the stator coils well below the gap surface should be especially beneficial.

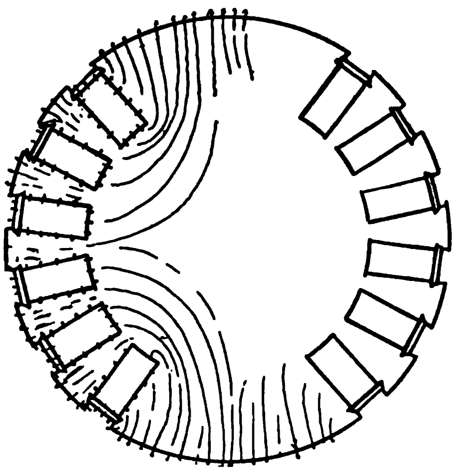


Fig. 12

It was stated that a large part of the rotor losses is dissipated as heat at the rotor gap surface. But much of this loss occurs at a considerable distance from the surface, and the resulting heat must be conducted through various paths to the cooling air. This constitutes quite a problem in itself.

Fig. 12 illustrates the arrangement of the rotor coils and core, in cross-section, in the usual radial slot type of rotor, with a two-pole machine. From this illustration it may be seen that there are two main paths of heat flow from the rotor coils, namely, outward directly toward the surface, and inward to the central body or core and then along the metal of the core to the pole surface and to the ends of the core. Obviously these latter paths are very much longer than

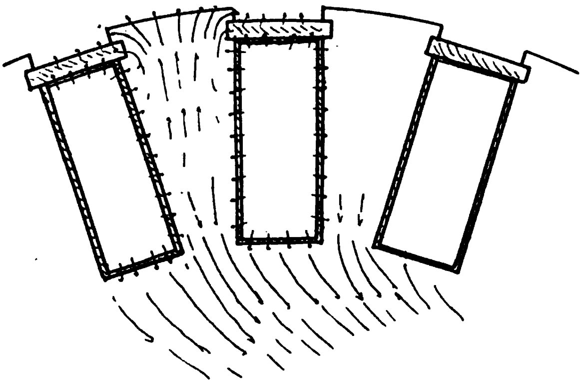


Fig. 13

the former, but to compensate for this, their sections are relatively larger, so that the density of heat flow

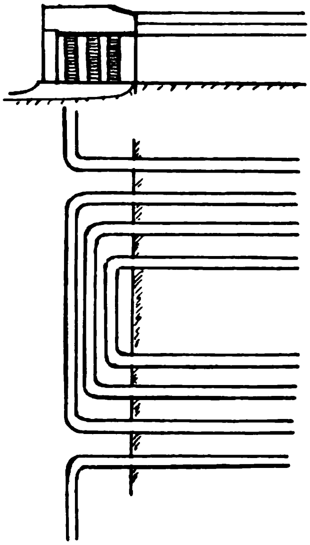
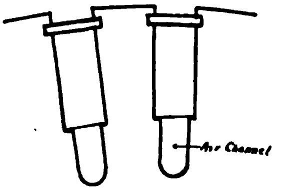


Fig. 14

is correspondingly lower. It is largely a question of relative proportions.

Let us consider now, an individual rotor slot and its winding, as shown in Fig. 13. Here the first temperature drop of importance is from the copper to the surrounding iron, and the supporting wedges. The amount of drop is dependent upon the density of heat flow, thickness and grade of insulation, etc.

The division of heat flow between the direct outward path and the various inward paths is dependent upon the relative heat conducting characteristics of these paths, as determined by their lengths, cross sections, materials, etc. The problem of the heat flow and distribution is somewhat similar to that of the axial type of stator, except that it is more complicated. As the purpose of this paper is to illustrate the problem, rather than to solve it quantitatively, the above description is considered sufficient.



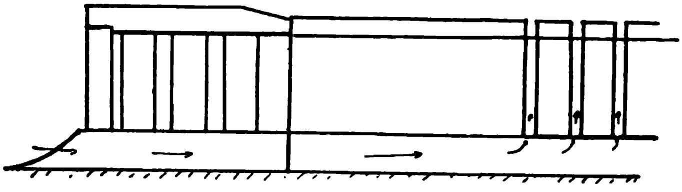


FIG. 15

In the end windings of the radial slot type of rotor still other problems of ventilation and heating are encountered. The usual arrangement of the end windings is as illustrated in Fig. 14, which shows six slots per pole. Here the end windings are exposed to the air, and should show ideal conditions for ventilation, provided good air circulating conditions are obtainable, which is not often the case. A heavy supporting end ring is required over the windings, and the coils must be well insulated from this. This enclosing ring is a direct hindrance to radial ventilation between the coils. Numerous ventilating holes through the supporting ring will aid, but in turn brings up problems of surface leakages and creepage distances, especially where the end windings have no outer wrappings, which is frequently the case. Outer wrappings over these end windings tend to bulge and crush out of shape, collect dirt, etc., and, in consequence, some manufacturers omit them altogether, and more or less completely enclose the end windings, except from below, believing that better insulating conditions can be obtained in this manner. To compensate for the reduced ventilation of the windings, it is not unusual to arrange ventilating channels into and through the rotor core, directly under the rotor coils, as shown in Fig. 15, suitable radial outlets being supplied near the core center.

With reversed ventilation as described in connection with Fig. 11, it is difficult to say just what course the rotor ventilating air would follow.

IRON LOSSES IN TURBO GENERATORS

The stator or armature core losses in large turbo-generators follow the same laws as in other types of rotating apparatus (except possibly in degree), due largely to the very abnormal magnetic proportions. In the largest synchronous converters, outputs of 300 kw. per pole are considered quite high; in large high-speed water-wheel generators, synchronous condensers, etc., 3000 kw. or kv-a. per pole is unusual; but, in the modern capacity turbo alternators, 8000 to 10,000 kv-a. per pole is accepted practise for 60 cycles, and up to 20,000 kv-a. per pole for 25 cycles. Such enormous outputs per pole can only mean abnormal magnetic proportions, compared with other classes of electrical apparatus. In consequence, special problems are encountered, in the way of losses, both in the iron and in the copper parts.

One of the special conditions is the relatively high voltage between laminations, compared with ordinary types of machines. For comparative purposes this may be expressed in terms of volts per inch of core. In ordinary types one volt per inch width of core is quite high, whereas in some of the modern turbo-alternators this figure is doubled, or even more in some instances. This, together with the enormous plate surface per pole, tends naturally to exaggerate eddy current losses in the iron. The laminae must be insulated better from each other, than in ordinary machines. The effects of the burred edges of the plates, due to punching, are more pronounced. For instance, a two-mil (0.002-in.) burr is considered quite small, in punching, yet with 17-mil thickness of plates, this burr represents about 12 per cent of the plate thickness, and several times the depth of the insulating coating on the plates. Such burrs, under great pressure, tend to cut through and make metallic contact between plates. Such minute contacts may seem to be of minor importance, and correctly so in machines with low volts per inch of iron. But in the huge turbo-alternators, with their higher voltages between plates, the effects of undue burr are quite considerable.

In the same way, extreme care must be taken to line up the stator plates or punchings, so that little or no filing is necessary in the coil slots. If the building is uneven, so that individual laminae project into the slots, these high points should be removed, otherwise the coils are liable to loosen in the slots, in time, through concentrated pressure, with consequent wear of the insulation at the projecting laminae. Fairly smooth slots are an essential condition, as shown by bitter experience. On the other hand, if one attempts to level down the high laminae by filing, as has been not uncommon practise in other types of machines, there is more or less danger, due to possible dragging over of adjacent laminae, with consequent metallic contact. It is almost unbelievable what high temperatures may be developed locally by such burring over at the edges of the laminae. In two instances directly within the writer's personal knowledge, such burring resulted in actual fusion of the metal at the filed or burred place. This fusion meant actual temperature of many hundreds of degrees centigrade, or far above any workable point. Yet, even under this condition, one of these turbo generators went through its complete shop test without evidence of trouble, and it was only by the merest accident that the damaged point was discovered. In the other case, three adjacent laminae had been burred together in some way, possibly due to filing, and had *welded* together at their edges, in the coil slot, and had so damaged the mica coil wrapper that a burn-out occurred in a few hours' time. Here the local temperature must have been far above 600 deg. cent., for incandescence, which begins between 550 deg. and 600 deg. is far too low to cause such welding. In both of the above cases the writer's inference is that the local temperatures attained approximated 1000 deg. cent. It was only due to the mica, that the breakdown did not occur immediately, in both cases.

Pressure apparently has a considerable effect on the losses in turbo-generator cores, possibly due to the establishment of better contact between adjacent plates, the burr cutting through the insulating coating on the plates. Quite often, in assembling the stator core, the resulting pressure is greatest next to the ends, and, in practise, greater losses and higher temperatures have resulted. To lessen the loss, by breaking up the contact between plates, etc., it is sometimes the practises to add separating sheets of suitable paper at frequent intervals throughout the core, the intervals being shortened near the core ends. This has proved quite effective, in the past, in equalizing the losses and temperatures due to core pressure. In one case where the core losses were unduly high, and there was excessive temperature in the end sections, the writer had the stator iron removed for about 6 in. at each end, and then replaced, adding sheets of fairly heavy "express" paper every 1/8 in. With the original amount of iron replaced in the core, thus representing greater compactness and higher pressure than before, the stator losses were down to normal, and the temperatures were quite well equalized. It might be mentioned that this particular stator used radial type ventilation with air ducts at quite frequent intervals. The above is simply indicative of a number of quite similar experiences. Some criticism has been raised regarding the durability of such paper, but long experience shows that where the stator iron has an ultimate temperature lower than that at which the paper deteriorates, apparently no trouble whatever results from its use. It should be borne in mind that this method of using the paper is not the equivalent of the tissue like paper coating on the individual lamina, sometimes used abroad in place of a varnish coating on the plates. This heavy paper at intervals is in addition to the coating on the plates, and is for the purpose of adding a large number of definite positive breaks in the metallic structure, these breaks being wider than the worst burrs which are liable to occur in the punchings.

While the effect of the punching of the plates is to produce burrs, which may be eliminated to a great extent by further treatment, it also has a harmful effect in increasing the magnetic losses in the iron, apparently due to the shearing effect. The action of punching or shearing seems to affect the material in proximity to the sheared edges. Consequently the harmful effect is largest, where the sheared edges bear a high ratio to the total iron, that is, at the teeth. Annealing after punching quite often shows a gain sufficient to warrant the additional expense. In some types of apparatus, however, it does not appear to be worth while.

ARMATURE COPPER LOSSES IN TURBO GENERATORS

Like the iron losses, the armature copper losses follow the laws of ordinary machines, except in degree. There are three principal losses to be considered,— (1) the usual I^2R due to the load current,— (2) eddy currents in the conductors due to the flux across the armature slots set up by the armature load current, and (3) the eddy currents in the coils set up by the main field, or gap, flux entering the top of the armature slots.

Loss (1) is taken care of by a suitable section of conductor. Loss (2) may be quite large in large turbo-alternators, due to the fact that the ampere-wires per slot are quite large in many cases, the number of slots being small due to the physical limitations of the dimensions of the machines. This means relatively deep ots and coils, and a tendency for excessive local currents, unless unusual subdivision or lamination of the conductors is resorted to, this being done quite completely in modern practise.

Loss (3) due to gap or field flux penetrating the slot and armature conductor is dependent upon the amount of such flux, which in turn depends upon the proportions of the slot with respect to the gap, etc. At first glance, one would be inclined to say that, with a small air gap, the gap flux would penetrate the armature slot more than with a wide gap, but, in fact, the opposite is the case. With the large gap, the relative lengths of the flux paths into the slots, are shorter, compared with the direct iron-to-iron paths, than with a short gap, other conditions being equal, and consequently the fringing into the slots will be greater, see Fig. 16. An obvious means for lessening this flux is to sink the armature conductors a considerable distance below the gap surface, and this is quite common practise in modern machines. Fairly large air gaps are necessary, from the design standpoint, on all large turbo generators; so that, without special precautions,

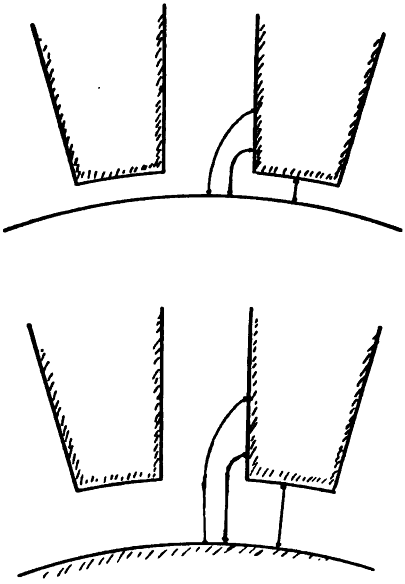


FIG. 16

such as sunken coils, well laminated conductors, etc., the losses from the gap flux would be much larger than in usual types of machines.

The lamination of the conductors of turbo generators has been quite a problem. Various methods of subdivision and of insulation of the individual strands, have been tried. Enamels and varnishes of various sorts, cotton coverings, and taping with mica paper, have been used. Due to the high temperatures inside the coils themselves, which almost invariably occur in large turbo generators, the use of mica on the individual strands appears to be the only really safe and satisfactory method just as mica in the outside wrapper represents the best practise.

Due to the fact that the upper coil in the slot naturally has greater eddy current losses, there is liable to be some difference between the temperatures of the upper and lower coils, which has an influence on the temperature measurements. This, however, belongs in the next subject treated.

In order to sub-divide the large conductors more completely, as well as for reasons of design, it is a not infrequent practise to sub-divide the armature circuits into two or more parallels, by which means certain transpositions of the conductors and their elementary strands can be obtained, with consequent reduction in the eddy current losses.

VOLUME OF COOLING AIR, AND WINDAGE LOSSES

Reference has been made, repeatedly, to the difficulties in obtaining the requisite cooling air. It is the enormous volume of air required that is back of much of the difficulty. As a rough approximation; let us assume the total generator losses, including windage, as 4 per cent of the rated capacity; or the loss in kilowatts is equal to 0.04 × kw. capacity. Also accepting the well-known relation that one kw. loss will raise the temperature of 100 cubic feet of air 18 deg. cent. in one minute, then four cubic feet of air per minute is required per kw. capacity of the machine for 18 deg. cent. rise of the air, or three cubic feet per kw. for 24 deg. cent. rise. This is only a crude quantitative statement of the problem, but it indicates that for a

30,000-kw. or kv-a. generator, for instance, a total of about 90,000 cubic feet of air per minute is required. This volume, in practise, will actually lie between 75,000 and 100,000 cubic feet, depending upon the total losses (including windage itself), and upon the air temperature rise allowed. In any case it is very large and the real problem is to obtain sufficient cross section of air inlet without unduly high air velocities. Assuming a maximum velocity of 9000 feet per minute, the minimum cross section of the air path must be about nine square feet, or equivalent to an opening three ft. by three ft.,—an apparently impossible figure from casual observation of the air gap and other air channels through the machine. But even 9000 ft. air velocity is very high and means quite considerable pressures for forcing the air through the machine, with consequent high windage losses.

Whether the core loss proper be taken care of by axial or radial duct ventilation, in either case a large volume of air is required to take up the total gap losses and this alone means comparatively large windage losses. This is an inherent condition, and the difference, with the various methods of ventilation, is only one of degree.

With any reasonably well designed method of ventilation, it is possible to get the requisite air through the machine with the desired cooling effect, provided suitable pressure is obtainable. However, the higher the pressure with a given volume of cooling air, the higher will be the losses, other conditions being approximately the same. In consequence, there is a limit to the permissible pressure which can be used.

The efficiency of the fan which provides the pressure, is an element of the problem, but when one considers that more than one-half of the total windage loss is inside the machine and independent of the fan itself, it is evident that the fan efficiency is not a dominating feature, although it is of importance.

The problem of artificial cooling is more complex in the turbo generator than in any other type of electric machine, and it may be said that more engineering effort has been, and is being, spent on this than any other problem in this class of apparatus. Each step upward, in capacity, or in speed, opens the problems anew, for in each of the preceding advances the designers have usually strained to the utmost to meet the necessities of the case.

To the uninitiated, the volume and the weight of air required to cool the modern turbo generator is always a matter of surprise. Without the figures before one, it is almost unbelievable that a modern large capacity turbo generator *puts through itself practically its own weigh tin cooling air in forty to sixty minutes.* Obviously, unless this air is kept very clean, the total amount of dirt and other foreign substance which pass through the machine, in even a few days' time, will be enormous, and a very small percentage of this, deposited inside the machine, may soon seriously clog the ventilating passages and thus interfere with the operation of the machine. This indicates why air washers and other devices for preventing, or lessening, the admission of dirt, have become almost a necessity in such machinery.

TEMPERATURE DETERMINATIONS

There are two special problems in the temperature determination of large turbo generators, namely, location of the hottest part, and measurement of the temperature in such a manner, or by such method, as will give a fairly close approximation to the actual results. The former problem is easier than the latter, as it simply involves a multiplicity of measurements on a given type or construction of machine, until a reasonably accurate location is determined, which may be applied on other machines of the same type. Uniformity of materials and construction is involved in this.

The second problem is difficult. If one could apply temperature indicators directly to the copper itself, of course the actual highest copper temperatures could be obtained directly. But this is not practicable except under what may be called laboratory conditions. The requisite continuity of the insulation over the copper makes difficult any measurement directly on per itself. We have, therefore, adopted certain approximate methods, which have proved fairly satisfactory, but which must be used with judgment, if a reasonably accurate approximation is to be obtained. The results are dependent upon the method of application, the type of ventilation used, and various other considerations.

The method now used most extensively depends upon the placing of the temperature indicator between upper and lower coils in the same slot, and, therefore, is predicated upon the two-coils-per-slot arrangement, which, however, is now the common practise in this country. The theory of the method is based upon the principles of heat flow from the two coils.

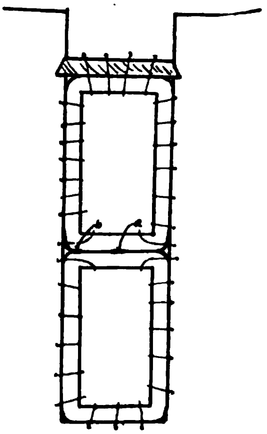


Fig. 17

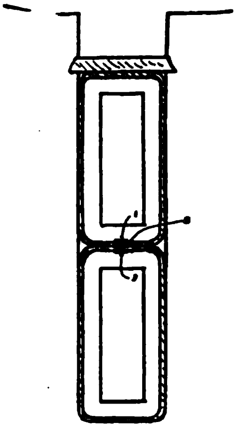


Fig. 18

Let Fig. 17 illustrate the usual two-coils-per-slot arrangement. The arrows shown indicate the general paths of heat flow from the two coils. Assuming the copper temperatures of the two coils as equal, and the width of the coils to be considerable, then at the mid-point *a* between the upper and lower coils the temperature should be practically midway between the upper and lower copper temperatures, and an indicator at this point should show about the same temperatures as the copper. This is not exactly true, for there is some heat flow along the insulation from the midpoint *a*, so that this point will probably be slightly lower than the true mid-temperature between the coils.

A point *b* between the coils, but at the corner next the iron, should have a temperature very nearly equal to that of the iron itself. Its temperature should thus be very much lower than that of *a*. Between *b* and *a* there should be intermediate temperatures, depending upon the distance from the center point *a*.

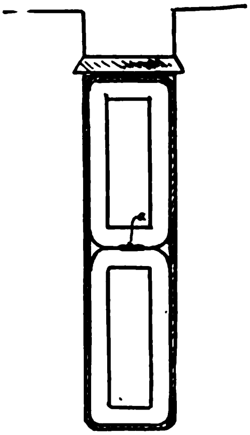


Fig. 19

It should be obvious, therefore, that a temperature indicator covering practically the full width of the slot, will show some average value between *b* and *a*. Therefore, for greatest accuracy the indicator should be as narrow as possible, and be placed at *a*.

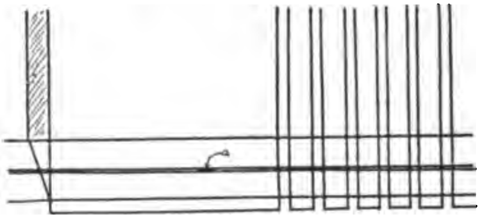


Fig. 20

Another consideration involves the method of applying the usual slot cells. In common practise the turbo-armature coils are "pushed home" in hard fibre cells placed next to the iron. This serves as a protection for the insulation proper, in putting the coil in place, and also makes a tighter fit in the slot. There are two general methods of applying these cells. In one, the cell is placed over the lower coil, and then folded over its upper side, either during manufacture of the coil, or before the upper coil, with a corresponding cell, is put in place. This is shown in Fig. 18.

In the second arrangement, the two coils are set in a single cell, which is then folded over the top one, as shown in Fig. 19.

It will be noted that with the two-cell arrangement, there are three places to put the temperature indicator, —(1) below the upper coil and inside the cell,—(2) above the lower coil, but inside the cell, and (3) between the upper and lower cells. On position (1) the indicator will tend to show more nearly the temperature of the upper coil, while in (2) it tends to indicate more nearly the lower coil temperature. When placed in position (3) it should be intermediate between (1) and (2), provided this position is thoroughly shielded from external influences, which is not always the case, as will be explained further on.

In the single cell arrangement, there is but one location for the indicator between the coils, unless, as is found in some constructions, a separating or spacing strip of appreciable thickness is placed between the upper and lower coils. In such case, if placed above the strip, the indicator should show more nearly the upper copper temperature. Obviously, with this single cell, the indicator is well shielded from external influences, such as the ventilating air, that might come in contact with the measuring device at the radial air ducts.

Considering next, the application of such indicators to the axial and the radial types of stators, it will be noted that if the indicator be located in the axially ventilated portion of the core, as indicated in Fig. 20, at point *a*, for instance, there is no chance for the ventilating air to come in contact with the indicator, regardless of whether the single or double slot coil arrangement is used. In other words, the indicator is naturally well protected. However, if placed in the central radially ventilated section or zone, the conditions may be quite different, but not necessarily so, depending upon the construction and application of the indicator, etc. Here the coils are partly embedded in iron and lie partly in the radial ventilating ducts. The indicated result will depend upon whether the measurement is taken at *a* or *b*, for instance, in Fig. 21. This difference, however, may be quite small if the detector is thoroughly shielded from the ventilating air in the duct. It may be suggested that, as the coils are ventilated in the duct, the indicator should be ventilated also. But we are not after the temperature of the outside or exposed surface of the insulation, but are attempting to approximate the internal copper temperatures. *The indicator should thus be as well shielded, if possible, as the copper inside the coils.* The single slot cell arrangement thus appears to be more accurate than the double cell, with the indicator outside the cells, when the indicator is placed close to, or bridges one or more ventilating ducts. Thus, in the axially ventilated type of stator, the ventilating device should not be used in the central radially ventilated zone, unless unusual precautions are taken to avoid the discrepancies which are liable to result. The same may be said of the radially ventilated stator as a whole.

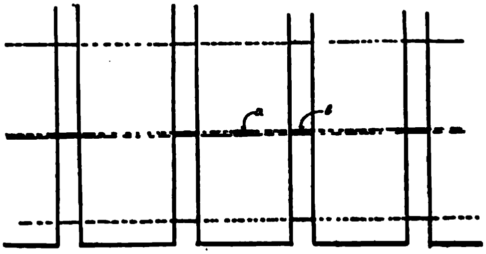


Fig. 21

It should be evident, from the above, that a certain amount of judgment and skill is necessary to obtain the closest possible approach to the true copper temperatures. Furthermore, the problem is complicated by the fact that the upper coil in the slot is liable to be hotter than the lower one, due to higher eddies in the upper copper. At best, therefore, the method as a whole is only an approximation, but it gives so much closer to the correct result that any of the measuring methods in use only a few years ago, that it marks a decided step in advance in generator temperature measurements.

As far as temperatures in the stator are concerned, they are not of serious importance except as they have an influence on the copper and insulation temperatures, and even the copper temperature, in itself, is of importance only as it affects the insulation. In fact, it is solely the insulation durability which we are after, in the last analysis. The improvement, and refinements in design, betterments in the grade of insulation, innovations in ventilation, etc., all are directed to the one end, not the temperature itself, but the factor of safety of the insulating materials.

DISCUSSION ON "VENTILATION AND TEMPERATURE PROBLEMS IN LARGE TURBO-GENERATORS" (LAMME), WHITE SULPHUR, W. VA., JUNE 30, 1920.

R. B. Williamson: On page 936, Mr. Lamme calls attention to a point in connection with large turbo generators that I think has not heretofore been brought out very forcibly; namely, the necessity of very thorough insulation of the laminations. This is a point that is important in all alternators but specially so in turbo generators.

Consider one section of the stator core. Each lamination is dovetailed into the stator frame and on each side of the section we have a ventilating spacer in the form of brass castings, or in the shape of steel strips welded to the laminations forming the ventilating duct walls. In a turbo-generator with a deep core behind the teeth and a relatively large flux in this part, there is considerable voltage between the two sides of a given core section. This may be of the order of 6 to 10 volts, and the vent spaces together with the yoke casting form a single-turn secondary in which the only thing that prevents current flowing is the varnish or other insulation between laminations.

With low-speed generators, such as water-wheel or engine-type machines, the core depths back of the teeth is small. Consequently the voltage between the two sides of a core section is proportionally smaller and there is not so much danger of circulating current as in turbo generators.

I recall one case of a turbo generator, where the insulating varnish broke down, and the current flowing across the laminations was such that it was possible by running up the magnetization to make some of the teeth red hot. If, therefore, great care is not taken in varnishing laminations and stacking the core, it is possible to lose all the advantage, so far as core loss is concerned, that may be gained by a careful selection of iron and in addition there will be the danger of local hot spots.

As Mr. Lamme points out, in some cases it is customary to put a thick layer of paper in the center of each core section, this breaks up the circuit between the two sides of the core section, and at the same time does not prevent the flow of heat towards the ducts on each side. In order to get the laminations in the best condition, it is desirable to anneal them, not so much to reduce the loss, but to remove the burrs, oil, etc., and leave the surface of the steel in good condition to receive the coating of insulating varnish.

W. J. Foster: Mr. Lamme states that the maximum flow of air is something like 80,000 to 100,000 cu. ft. per minute in a 30,000-kw. unit, which stated in terms we can understand, amounts to the weight of the generator itself passing through it every 40 to 60 minutes.

Recently we sent out a questionnaire to the engineers of different air conditioning companies with regard to the quantity of dirt that is to be found in the neighborhood of a modern steam turbine power house. The replies lead us to make this statement—that at the average power house, the quantity of dirt and impurities of all kinds is one hundred thousand millionths of the air itself. That is a pretty small fraction when you write it out in decimals, but when you multiply the quantity of air, it gives this answer—that every day where you pass 80,000 cubic feet of air through, you pass one and one quarter cu. ft. of dirt.

This dirt, fortunately, most of it passes right through, but if there is oily vapor or extreme moisture, or if the impurity itself is of the nature of carbon dust, which is apt to appear more than other forms of impurities, it is not long until the passageways are somewhat clogged up, and the surfaces are put in such condition that external fires that we are hearing so much about nowadays may start.

Mr. Lamme does not bring in one form of radial ventilation. On pages 959 and 960 of my paper you will find a modification of the radial ventilation which results in equalizing the temperature throughout the length of the machine.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29–July 2, 1920.

TEMPERATURES IN LARGE ALTERNATING-CURRENT GENERATORS

BY W. J. FOSTER

Engineering Dept., General Electric Co., Schenectady, N. Y.

WHAT shall be considered a large generator—5000, 10,000, 15,000 or 20,000 kv-a? Shall it be the rating alone that is considered or shall we take such factors as speed into account?

Undoubtedly, a large proportion of the general public thinks of a large machine as one that occupies a large space compared with other machines used for the same purpose. They judge the largeness by physical dimensions alone. At the same time, it is safe to say the more intelligent of the general public think of size in terms of the work that can be done. Probably a machine of 10,000 kw. or higher, is regarded by them as a large machine. To the engineer, largeness involves the difficulties inherent in the design and construction. A 1000-kv-a., 10,000-cycle alternator is a large one; a 5000-kv-a. generator at 3600 revolutions per minute is large. Considered strictly with reference to the temperature problem, the engineer would hardly consider a 20,000-kv-a. generator a large one if the periodicity and potential were those in regular use in commercial work, and the speed 100 revolutions per minute, or thereabouts. However, for the purpose of this article we will consider 20,000 kv-a. as a large generator.

There are two principal factors in the temperature problem in every case; first, the total losses, or the amount of heat energy to be disposed of and its concentration, and, second, the means that can be provided for dissipating the heat in such manner as not to cause damage to any part of the machine. The problem may be attacked along the lines of reduction of losses or of devising such constructions that the heat may be more effectively dissipated.

In a rotating dynamo electric machine three sources of heat are always involved; first, hysteretic losses in the magnetic material; second, the resistance to flow of current losses in windings, and, third, the frictional losses in the bearings and of the windage. The first two are electrical in their nature; the last is mechanical.

Combined with hysteresis losses in the magnetic material are more or less eddy current losses. The total losses in the magnetic material are dependent upon such factors as the degree of lamination employed, the character of the insulation between laminations, the amount of pressure employed in clamping up the cores, as well as the character of the steel employed. In like manner, the resistance losses in the copper are frequently accompanied by eddy current losses, the amount of which is dependent upon such factors as the stranding of the conductor, the pitch of the winding and the arrangement of the turns. The windage losses, or the losses that result from either the fan action of rotating parts or from the disk action or rubbing of a revolving body on the surrounding air, are dependent upon the peripheral speed and the details of design of the parts that are producing fan action.

In considering the temperature problem, the first and most fundamental consideration is the relation of the space occupied by the object, to the total heat losses to be dissipated. A 20,000-kv-a. 100-rev. per min. machine compared with one of same output at 1800 rev. per min., has its losses generated in a space eight times as great in terms of cubical space occupied, or approximately two times in the projected area occupied. We may well think of the temperature problem in terms of heat losses to space occupied. Below a certain value of this constant it is absurd to use ventilation housings, no matter how great the rating of the machine, as such housings have the t of preventing the natural means of heat dissipa- *viz.*. convection and radiation, and ventilation housings in such cases result in higher temperatures, unless forced draft is provided, which results in a decrease in the efficiency of the unit and can be justified only on the score of reduction of noise, or some similar reason.

Second in importance to the space factor comes the heat density factor. By this we mean the quantity of heat energy passing through a unit area of material.

The third factor is the thermal conductivity of the various materials,—a factor which depends not only upon the thermal properties per se of the materials but also upon the manner in which the materials are put together.

CLASSIFICATION OF MACHINES WITH RESPECT TO VENTILATION

Attempts have been made at standardization of the various classes, but I think it safe to say nothing yet has been suggested that appeals to engineers in general as entirely satisfactory. Possibly it is desirable to have a large number of classes of machines to fill in the gap between the extremes of the lowest speed small capacity machine that requires no special provision and may be said to depend upon *natural* ventilation alone, and the highest speed machine that requires the most careful artificial ventilation. It is difficult to classify the types that have already been developed to fill in this gap as they blend into one another.

The points to be kept in mind in the design and construction of machines in general, not particularly those standing at the extreme ends, as outlined above, are; first, obtaining supply of cooling air from a region well removed from the region into which the outlet air is dumped; second, the placing of barriers or bafflers to assist the flow of air and to prevent re-circulation; third, the providing of ample cross section in all parts of the paths of flow of cooling air and the avoidance of sharp contrasts in the cross-sectional area of the paths, especially the avoidance at any point of greatly reduced cross section that would introduce great resistance; fourth, the avoidance of "churning of air," or internal circulations, which are often hard to prevent by reason of the irregular shapes of the different parts of the machine.

CLOSED AND SEMI-CLOSED VENTILATED MACHINES

In almost all large generators—whether hydraulic or steam turbine, it is necessary to pipe air either to the machine or away from it, or both to and away from. Probably the most common practise is to pipe air to the machine, allowing it to escape through the stator frame into the dynamo room,—the escape often arranged so as to be upwards, which is preferable on account of the greater comfort to the operators, the reduction in noise and the slight reduction in temperature obtained by the lower temperature of the air immediately around the machine.

The author wishes to call attention to certain advantages that would result in reversing the common practise of the present time, in the ventilation of large generators in hydraulic units, and to take the air in directly from the room and pipe it away either to some point in the building removed from the machine or to out of doors. The advantages of this arrangement have already appealed strongly to the operators of some of the largest hydraulic generators, and such a system is now in use in a few plants. It is a much simpler matter to draw air into the rotor direct from the room at the two ends of the generator than to provide the necessary space for the air conduits and the housings required either at the one end or the two ends, which almost invariably involve greater distance between bearings and, consequently, an increase in both the diameter and length of the shaft and corresponding parts. A great advantage of the scheme of piping air away is the more comfortable temperature of the dynamo room in hot weather. It is never necessary to be in an atmosphere of higher temperature than that existing out of doors, whereas, in case of the more common practise, the air surrounding the machine has its temperature raised several degrees above that of out of doors, due to the heat that has been added to it by passing through the machine.

WATER COOLING

Water is an ideal agent for cooling purposes. At first thought it seems strange that it has not been made greater use of in removing heat from large machines. A small quantity of water, on account of its specific capacity, would suffice to remove heat from a large generator, but the difficulty is in arranging jackets that would prove safe and could be located in close enough proximity to the parts in which the losses are generated to remove such losses without a considerable drop in temperature through the intervening walls of material. It is apparent at once that water-cooling is much better adapted to the stationary than to the revolving parts. While it is possible to arrange for a flow of water through the revolving parts, it is probably not possible to so arrange the flow of water as to cool the surfaces where the most of the heat is generated. Hence, a system of water-cooling would be dependent upon the joint action of air-cooling and

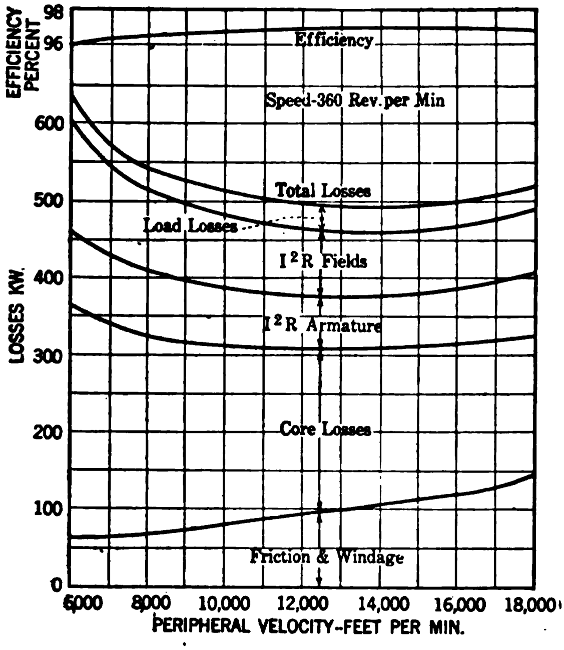


Fig. 1

would require a design that would re-circulate the air surrounding the rotor, in such manner as to most effectively carry the heat from the surfaces of the rotor to the surfaces of the water piping in the stator. Another objection is the danger of injuring a machine in case the water circulating system becomes leaky. Still another is the danger of too great an accumulation of dampness, due to condensation at certain times, of the moisture in the air on the water-cooled parts. It is doubtful whether water-cooling can ever be a competitor of air-cooling in dynamo electric machines.

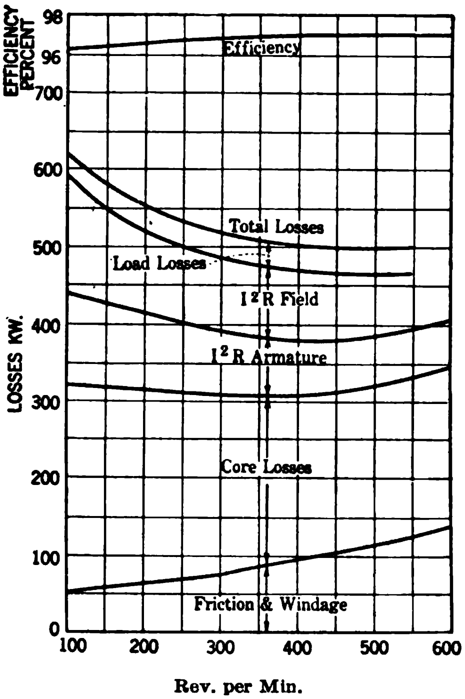


FIG. 2

PERIPHERAL SPEED OF ROTOR

It is possible that the advantage electrically of higher peripheral speed in almost all designs of large generators, is not fully appreciated by many designers themselves. The losses in both the iron and the copper are almost universally less at the higher peripheral speeds in any practical problem. Assuming the same characteristics electrically in all respects, such as saturation curve, armature reaction per unit pitch, etc., the losses in the armature teeth are less at the higher peripheral speed. They are exactly inversely as the peripheral speed, while the core losses proper remain practically constant. Copper losses in both armature and field are less at the higher peripheral speed, unless the speed is carried to an absurd limit. The windage losses will be increased. Hence, considered electrically, the most efficient design will be that where the windage losses begin to increase so rapidly as to offset the combined reduced losses of core and windings. For the purpose of illustration, I have worked out a 20,000-kv-a. 60-cycle, 360-rev. per min., three-phase, 11,000-volt generator at peripheral speeds varying from 6000 to 18,000 feet per minute—see Fig. 1.

For the purpose of illustrating the variation in the segregated losses, as affected by different rotative speeds, I have added curves—see Fig. 2, showing such losses for 20,000-kv-a. generators throughout the range 100 to 600 revolutions per minute. It should be understood that these generators are designed with identical electrical characteristics and have the same temperature rises, *viz.*, those corresponding to the A. I. E. E. Standard for Class "A" insulation. They are what the author considers normal in design for the output at the several speeds.

The 100-rev. per min. generator has a peripheral velocity of about 7500 feet per minute, the 600-rev. per min., of 15,000 feet per minute. The first has its losses generated in a space of approximately 2200 cubic feet, the last in approximately 550 cubic feet. The total losses of the first are 620 kw., or 270 watts per cubic foot of space occupied; of the last, 495 kw., or 900 watts per cubic foot of space occupied. Hence, the ventilation problem is quite different in the two cases. The first might be of the open type, drawing its ventilating air in from the room and returning it directly into the room; the last must be enclosed,—preferably totally enclosed.

VENTILATION DUCTS IN ARMATURE CORES

The common practise for ventilating armatures is to provide at short intervals in the laminated core, a narrow passage,—usually ⅜ in. or ½ in., for the air to be driven through radially by the fan action of the rotor, or in special cases by an external fan. The flow of air is in any special case dependent upon the details of construction, such as the character of the space blocks, how located with respect to the coils in the slot: the niceties introduced at the entrance from the airgap in the way of treatment of retaining wedges of the windings, the exact location of the end of the spacers, etc.

Good results are usually obtained by having a ventilating duct about every two inches,—the most efficient spacing being dependent upon such factors as the radial depth of the core, length of core, and the pressure of the cooling air. In long cores the spacing may be graded and the sections of core at middle made smaller than at the ends. This for two reasons,—first, some of the heat at the ends travels to the head of the core where the cooling conditions are usually good and, second, the ventilating air gathers up heat as it passes in from the head towards the middle and, hence, is not as good a cooling medium when it enters the ventilation duct as the air in the ducts nearer the head. When fans are mounted at the two heads of rotor, and end housings placed on the stator so as to establish a good air pressure, the pressure in the ventilating ducts increases from the head to the middle—hence, the quantity of air passing through is greatest in the duct at middle, decreasing toward the heads. The curve in Fig. 3, entitled "Velocity of Air at Entrance to Air Duct," was plotted from air pressure readings made on a large turbo-generator with equally spaced ventilating ducts. The other curves were determined from consideration of the heat dissipation problem. In like manner, Fig. 4 shows curves for a later turbo-generator with stator core sectionalized in such manner as to equalize temperatures throughout the length of core.

GIVING DIRECTION TO COOLING AIR

It is quite wonderful what improvements are sometimes accomplished in cooling machines by very simple expedients. It is sometimes advisable to arrange a machine so that it is obliged to take all of the cooling air in at one end and put it out at the other. This, as a rule, is especially helpful to the rotor.

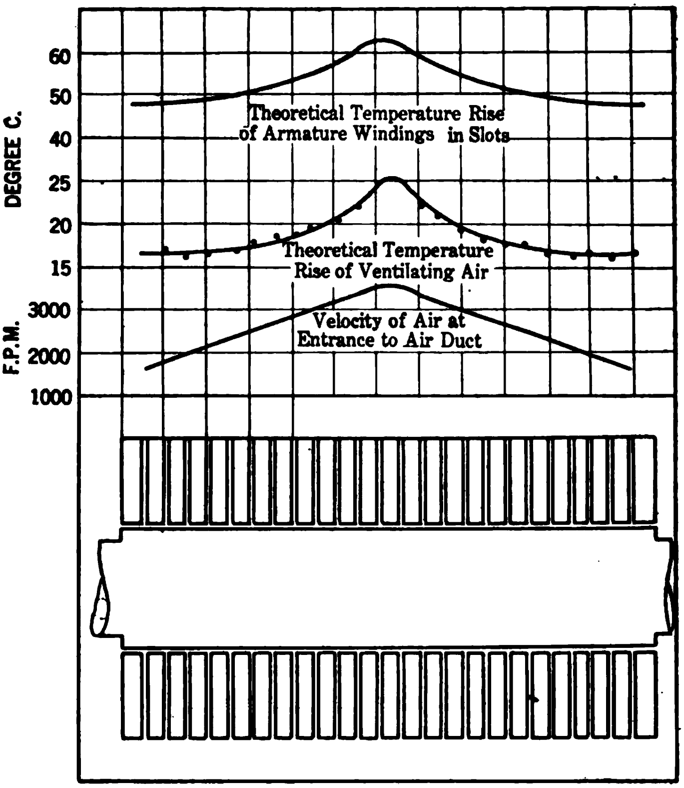


FIG. 3

But, in general, machines arranged with a radial system of air ducts through the core, should draw the air in equally from both ends. Often a machine that, seems on a casual glance to be symmetrical as to the two ends, proves a surprise in taking practically all its air from one end. In such case little, if any, air passes outward through the radial air vents,—in fact, sometimes the air will pass inwards in some of the vents. The remedy is usually very simple,—any little barrier interposed in the path of the axial flow will restore the desired circulation and often reduce the temperature several degrees.

The poles themselves act as the fan blades on the rotors of many salient pole generators, and no fans or fins for additional fan effect are required. It may not be generally known that even in cases where carefully designed fans similar to those used in large cylindrical rotor turbo generators are employed, the poles themselves contribute more to the blower action than the fans. The problem of ventilation in such machines is more complicated than in cylindrical rotor machines, where the blower action is more largely due to fans designed for the purpose.

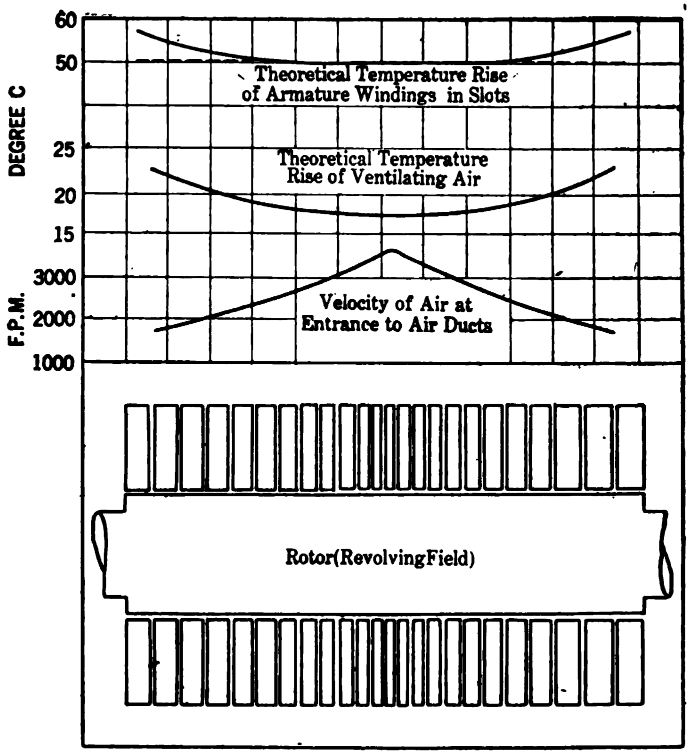


FIG. 4

HEAT FLOW

The most efficient ventilation of a large electric generator requires circulation of the cooling air in such manner as to bring it in contact with large surfaces of the solid materials in which heat is being generated and close to the sources of the heat generation.

The heat resistance of the various materials entering into the construction, such as copper, the magnetic steels, and the various insulating materials, is quite well-known.

An analysis of heat flow in a 30,000-kv-a. generator from the inside of armature coil at middle point of core to the ambient cooling air is given in the following table for four designs:

(1) 4000-volt mica insulated coils to withstand A. I. E. E. high potential tests;

(2) 11,000-volt mica insulated coils to withstand A. I. E. E. high potential tests;

(3) 11,000-volt mica insulated coils with copper density same as in 4000-volt design;

(4) 11,000-volt mica insulated coils to withstand high potential test of three times normal, instead of two times, plus 1000 volts (A. I. E. E. Standard) with coils of same external dimensions as those of (2), so as to be assembled in same slots.

	1	2	3	4
Drop through insulation...	21 deg.	30 deg.	48 deg.	67 deg.
" " core.......	6	4	6	5
" at surface...........	16	11	16	14
" in cooling air.......	15	15	15	15
Total drop.....	58	60	85	101

HEAT STORAGE

Heat storage must be reckoned with when ratings for intermittent loads are to be given to a generator, but when continuous load service is under consideration, as in nearly all large commercial generators, the heat capacity properties are chiefly of scientific interest, except when the duration of heat runs in acceptance tests is under consideration. Fig. 5 shows curves of time required to reach constant temperature, in the case of an 18,750-kv-a. turbo generator at overload corresponding to about 20,000-kv-a. which may be taken as typical of the modern large cylindrical rotor generator. This set of curves represents three runs under widely different conditions. The curves "Rise in Field Winding" and "Rise in Armature

Winding" were determined in the same run. The curve "Unexcited" shows the rate of temperature rise when the heat is generated by the windage alone, as measured by the detector embedded in armature slots. The curve "Excited to Normal Volts" shows the rate of rise measured in same manner, when the heat is that of core losses on open circuit in addition to the windage. Fig. 6 gives curves of temperature rise in a single run on a salient pole generator. Comparing the two sets of curves, it is interesting to note the quicker rise in the field winding of the salient pole machine, where nearly all the heat passes directly into the cooling air from the surface of the bare copper, over that of the cylindrical rotor machine where the field winding of each pole consists of several coils embedded in slots in the magnetic material.

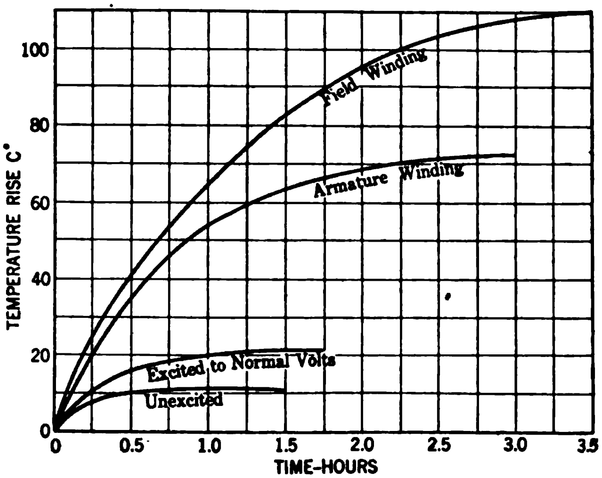


Fig. 5

HIGH VS. LOW TEMPERATURE GENERATORS

Undoubtedly any machine, electric generator or strictly mechanical machine, such as steam turbine, would be better off if it could always maintain the same temperature in all its parts. Change of temperature beyond certain limits repeated often enough, results in deterioration in most electric generators. This is due primarily to an effect of heat that is mechanical in its nature, *viz.*, a change in size. Much can be done to minimize the deleterious effects of change in size of the various parts, by introducing constructions in detail parts that automatically adjust for changing size. But it is extremely difficult in certain parts to protect materials of quite frail mechanical nature, like many insulations, from the effects of change in compression or what is more serious, slight movements of different degree in different places. Looked at in this way, it is desirable to have a generator of low temperature rise. But it is not always convenient or possible to build low temperature generators if machines are to be produced equal in capacity to prime movers. Furthermore, high temperature machines are justified in cases where increased efficiency or lower cost will more than offset the shorter life.

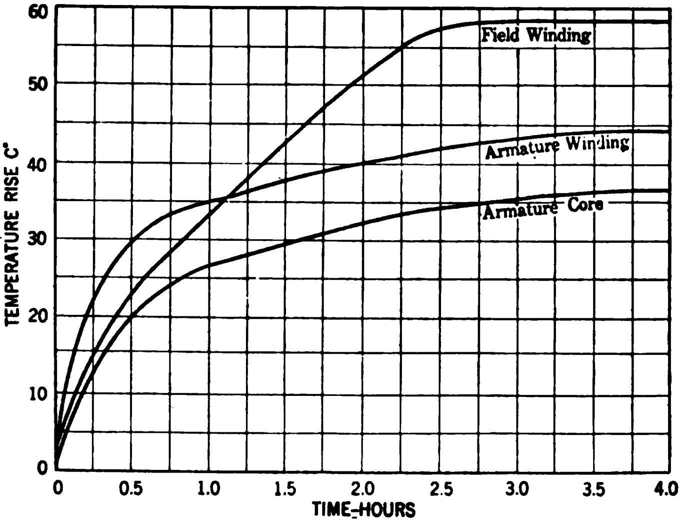


Fig. 6

With reference to relative efficiency and cost, it is quite apparent that the high temperature machine has the advantage in the case of a generator whose insulations are suitable for the higher temperature, and whose efficiency is still rising with increase of load, as in most alternating-current generators, and whose cost is only slightly increased by enlarging the shaft and other mechanical parts involved.

It may be laid down, as a rule, that the high temperature machine costs less, but it must not be taken for granted that its efficiency is better. In fact, most of the lower speed machines of low temperature, have better efficiency than corresponding machines of high temperature, unless the designer has been grossly careless in taking care of the ventilation.

It is possible to design most machines for low temperature rise without great additional cost, and have decidedly better efficiency. This condition obtains especially in connection with large salient-pole generators of low speed and low or medium potentials. As a rule, machines of this class, designed for 50 deg. cent. rise, permit of decided increases in the amount of copper in both armature and field, without any change except slightly larger slots in the armature. In addition, a higher grade of magnetic steel than that called for from temperature considerations, may be used. Often a full per cent in efficiency at full load may be gained at an increase in cost of 10 per cent to 15 per cent. In other cases as much as ½ per cent efficiency can be gained. The resulting generators may have only 35 deg. cent. or 40 deg. cent. rise at rated load.

Possibly the author is on dangerous ground in discussing the advantages of generators that do not conform to the Standardization Rules of the A. I. E. E. But it is not for a moment his intention to reflect in the slightest on the standards that have been set, but he wishes to point out gains to the user to be had by following along more conservative lines in certain cases. Again, he realizes the possibility of trouble to himself and his ilk from urgent requests that may be in store from buyers of generators, when generators are under consideration that cannot economically be built for temperatures below the Standards of the A. I. E. E. in the high and low temperature classes. It is well to add that designing certain sizes for low temperatures, where only slight gain or no gain whatever in efficiency results, often involves hardship and results in certain risks being taken that are wholly unjustified.

DISCUSSION ON "TEMPERATURE IN LARGE A-C. GENERATORS" (FOSTER), WHITE SULPHUR, W. VA., JUNE 30, 1920.

F. D. Newbury: I would like to ask Mr. Foster if these figures have been checked by direct measurement on the actual generator, and by direct measurement, I mean the measurement of the copper temperature by embedded detectors located within the insulations, and therefore in contact with the copper? In our experience and as brought out in detail in the paper of Mr. Fechheimer and myself, we have found it impossible to arrive at any knowlegde of the true copper temperature except by such direct measurement.

In this connection I would like to point out in connection with Mr. Foster's statement that "heat conductivity of insulation is well-known", that that may be true in a sense, but it is not true in the sense that we know a definite value that can be applied to a particular armature coil for a particular type of insulation.

In the case of tests made in the laboratory with built-up samples of insulation, supposedly made by the same process, with the same materials, and even by the same workmen, such samples will give results as to heat conductivity that vary by as much as fifty per cent of the smaller value. We know average values, but we do not know, with reasonable accuracy, the values that can be applied to a particular case. So that is why I say that we have found it necessary to measure copper temperatures directly by thermocouples in contact with the copper in order to draw conclusions that are of any value.

We have also found in the case of the detector method with the detectors located between coils, that that method is subject to wider variations, and in some instances is really less reliable than the resistance method. Of course, these new methods give entirely different results—one gives the local temperature, the other gives the average temperature of the complete winding, both inside and outside the core.

I would like to ask Mr. Foster if in a machine such as the one for which he has given us the temperature drop X, what the relative rise by resistance would be, because until we have some way of checking the probable error of the figure we cannot very well discuss them.

W. I. Slichter: The appearance of a heating unit which had been running for a year or so at temperatures of 500 deg. regularly, and occasionally temperatures of 600 deg. cent. and which consisted of ordinary resistance wire insulated by mica, struck me as so good that I wonder whether we might not very soon expect to increase the limit which we are now discussing of 185 to 200 deg. cent., for our mica insulation.

Philip Torchio: I would take issue with Prof. Slichter about suggesting temperatures like 185 degrees for mica insulation in turbo generators. I would go back to my limit of 100 degrees. For other apparatus, like heating units, I would go up to 500 or 600 degrees, or anything the Professor wishes.

I would like to make just a brief comment on the remark of Mr. Newbury on the heat conductivity of materials varying considerably, as much as fifty per cent., and also the statement by one of the writers that the heat conductivity varied considerably with and against the grain, varying as much as from one to 3.5.

I have had somewhat similar difficulties in trying to determine the heat conductivity of concrete due to the surface drop in making measurement. Furthermore, with windings insulated with mica, if the different layers of mica are not perfectly uniform, they create air spaces which will give all kinds of differences; in reality it is not the conductivity of the material that varies so much, but rather the physical condition of the layers, how they are put together, and also the differences in the measurements in determining the surface drop while the measurements are made.

A. S. Loizeaux: From what has been said this morning about temperatures in large turbo generators, it would seem that high temperature itself is probably responsible only for a rather small part of the failures, and that there is a considerable amount of experience of very recent origin that goes to add to that evidence. I think that the support of end windings, or the lack of rigid support, is probably responsible for failures that might ordinarily be laid to the door of high temperature. A very small movement, gradually resulting in breaking the insulation, is responsible, I think, for some failures.

The difficulty in supporting the end windings is a real one. I have found cases where they have been decidedly bent due to a heavy short circuit.

Another thing mentioned in the discussion of fires originating in current transformers. I think we should warn every operating company to take the current transformers out of the air ducts, if they are in there at present, because an explosion of a current transformer or the burning of it will easily set fire to the generator windings. Also the terminals and the main leads of the generator should be adequately spaced and insulated and supported. In order to prevent damage from heavy short circuits, the supports should be close together and extremely rigid.

Selby Haar: The objection to the high temperatures, as suggested by Prof. Slichter, was that it resulted in extremely low efficiencies, because of the large increase in copper loss at high temperatures. This hardly effects large turbo generators because they have high efficiencies under almost any circumstances, but the situation is far different for some other classes of machinery in which copper space is limited.

S. R. Bergman: The influence of the temperature on the efficiency of high-speed generators is small. In such machines the core loss, friction and windage losses are predominant, whereas, the copper loss is relatively small. An increase of 50 deg. to 100 deg. cent. in the temperature would, therefore, have but a small effect on the efficiency.

The reverse is true in low-speed machines in which the copper loss is predominant. In such machines an increase of 50 deg. to 100 deg. cent. in the temperature would have a marked effect on the efficiency.

W. J. Foster: It is all right, perhaps, in the sense in which Mr. Bergman means it, that the addition of some copper losses is of small consequence—that is true as far as efficiency is concerned, because the I^2R is ridiculously small, as pointed out by Mr. Lamme in his paper, but just the same the local heating is there, just as it is in the laminated core. If you happen to have laminations pressed together so they solidify along the slot for an inch or two, you may have the temperature go to red heat. That does not hurt the machine, as far as efficiency is concerned; it is only 0.001 of a per cent, perhaps, but it will destroy the machine, since it will cause a fire, so we must look out in the windings to keep temperatures down to a certain point that will be safe for their operation.

Right along that line Mr. Loizeaux brings up the point of the support. I do not know what Mr. Loizeaux has in mind when he speaks of rigid support, but I would like to point out the great difficulty in this whole matter of support of end windings is not to have it too rigid, but a little flexible, just to take the blows that Mr. Loizeaux points out occur, due to short circuits, improper phasing, and everything of that kind. I agree with Mr. Loizeaux that they should be very effectively supported, so that they can stand in the best manner the strains that they will meet in commercial service.

Another point brought out by Mr. Loizeaux was the different conditions that may prevail. That is a thing in regard to which the manufacturers are now having some contention with customers' engineers, some of whom are reaching the point of specifying, or asking the manufacturer to guarantee, that a generator if thrown in 180 deg. or 90 deg. out of phase, anywhere between proper phase-in and the worst phase-in, will be good for that kind of operation, under the conditions that arise in the power houses. Furthermore, they expect that guaranty to apply for a considerable period, possibly ten or twenty years.

We must remember that guaranties of that kind, to be reasonable, should be defined and limited, such, for instance, as that a generator will stand short circuits, at normal voltage, or maybe ten or twenty per cent above voltage. The manufacturer can agree to a reasonable guaranty of that kind, and have the test applied.

Now, with regard to the little table of temperature drop, about which Mr. Newbury asked, page 961, he has pointed out, there are probably two factors in there that involve uncertainties. He points out a statement which has been made by me, as to the fact that temperature drops in different types of material have been determined and are well-known, I would say that, in this particular case the temperature drop has been determined from the actual armature coils, but not in the machines.

F. D. Newbury: Do you excite the coils with direct current or alternating current?

W. J. Foster: Direct current. The only factor I am getting at is the temperature drop in the insulation. When it comes to the factor of how much heat there is there to get out, that is really where the uncertainty comes in, and in this case, the 20,000 kv-a. turbo generator of 11,000 volts, the allowance for eddy current losses was based on the famous classical paper of A. B. Field, with such modifications as were necessary. It is practically his formula but slightly modified, such modifications being based on experience.

The thickness of the insulation was another thing that Mr. Newbury asked for, and the character of the detector. The thickness of the insulation in that particular case is a trifle over 0.1 inch.

Regarding Mr. Newbury's further point as to whether the temperatures have been checked up by the drop through the windings as a whole. I wish to state that I have always regarded the determination of temperatures by that method as a good one. The Institute has never seen fit to standardize it, but I think it is a very resonable request for the buyer to make, that the temperature shall be determined by the drop in resistance at the end of heat run as well as by the temperature detectors. In the case of machines of the size for which these data are given, we have always found that that temperature rise was less than that revealed by the temperature coil, but I do not have the data on the largest generators, for the reason that we have never operated these in the testing department at home, and it is very difficult to obtain a machine outside for testing purposes.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29-July 2, 1920.

SOME PRACTICAL EXPERIENCE WITH EMBEDDED TEMPERATURE DETECTORS IN LARGE GENERATORS

F. D. NEWBURY AND C. J. FECHHEIMER
Both of the Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

WE have now had some six years' experience with the built-in detector method of temperature measurement as applied to large a-c. generators and have accumulated experience that seems to warrant reconsideration of the Standards Rules relating to this subject.

When this method was first proposed and discussed in 1913 and 1914, it was believed that it provided a way by which the temperature of the copper—inside the insulation—could be very closely approximated by a device installed outside the insulation. When the detector—be it thermocouple or resistance coil—is properly installed, it does measure a temperature much nearer the inside copper temperature than does any other established method, but if the detector is not properly installed, even though located between coil sides, it will give considerably lower results.

Designing and operating engineers are interested in armature coil temperatures from two different standpoints; they are interested in the effects of heat on the insulation and from this standpoint are concerned that the temperature at no point in the entire winding shall exceed the safe operating temperature of the insulation located at that point of highest temperature; they are also interested in the linear expansion of the straight part of the armature coil due to temperature changes and, from this standpoint, they are interested in the *average* temperature of the entire length of the coil located in the armature core-slots.

The ideal measuring device from the standpoint of the chemical effect of heat on insulation is a detector, such as the thermocouple, that will measure local temperatures; although, obviously such a device, to be of service, must be located where the highest temperatures are to be found. The best device to gage the effect of heat in causing the winding to "creep" and chafe the insulation is a device such as a long resistance coil, that will average the temperature along the length of the armature coil.

In this paper some errors of measurement with the embedded temperature detector are discussed. Some pertain to details concerned with proper installation of the detectors, and while they, at first glance, may appear to be unimportant details, they are, according to our experience, of such importance as to warrant very careful study.

A discussion of errors of measurement requires that some standard of reference be established. In connection with the present subject the only satisfactory reference temperature is the temperature of the copper inside the insulation as obtained by direct measurement. In general, the statements made in this paper are based on tests in which such actual copper temperatures have been measured by thermocouples placed in contact with the copper, the leads of the thermocouples being carried through the armature coil insulation. These inside temperatures can be measured accurately and can be checked consistently without difficulty. Practically all the factors that lead to errors when temperature detectors are located on the outer surface of the insulation disappear when a flat ribbon thermocouple is placed in metal-to-metal contact with the copper conductor.

When a detector is placed between the two coil sides in a slot, the detector measures the actual temperature inside the insulation providing:

(1) that there is no flow of heat from the adjacent sides of the copper in the upper and lower coils to the slot sides; and

(2) that there is no difference in temperature between the upper and lower coil sides. In practically all machines both are present, to a greater or less extent. These two effects upon the reading of the detector are important, and therefore will be discussed.

FLOW OF HEAT TO THE SLOT SIDES

Recent measurements of thermal conductivities* show that the longitudinal conductivity of insulating material is considerably greater than the transverse, approximately 1.5 to 4 times. When built up in the usual manner around a coil the transverse conductivity is considerably decreased so that the longitudinal conductivity may be 3 to 10 times the transverse.

In Fig. 1 is shown a portion of a slot with the usual two coil sides and the approximate lines of heat flow as well as the isothermal (equipotential) lines. As drawn, the assumption was made that the insulating material was isotropic, not of greater conductivity in one direction than the other. It is evident that the path of heat flow from the portion of the copper under consideration is in part transverse to, and in part longitudinal with, the insulation. In accordance with this figure, there is heat flow from the adjacent sides of the copper in the two coils to the slot sides, which heat flow is, in the actual case, augmented by the relatively high longitudinal thermal conductivity. The particular isothermal line *abc* is at that temperature which is recorded by the detector at *b*, and the drop in temperature from the copper to the detector is approximately $\frac{(ga)}{(gf)} \times$ temperature difference between copper and iron. The actual case is too complicated to admit of accurate mathematical analysis, but an idea of the difference between the copper and detector temperatures can be gained from

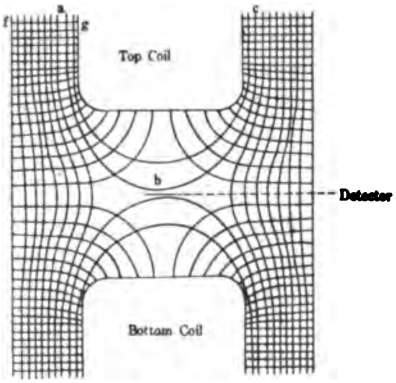


FIG. 1

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*"The Thermal Conductivity of Insulating and Other Materials" by T. S. Taylor, Electric Journal, Dec., 1919.

data on the 12,000-kv-a. alternator and small model contained in this paper.

The importance of this factor depends largely on the distance between copper conductors in the upper and lower coil sides. It may have little effect in low voltage windings but may be of considerable importance in 11,000- to 13,000-volt generators in which the copper of the two coil sides is separated by at least 0.5 inch of insulation.

DIFFERENCE IN TEMPERATURE BETWEEN THE UPPER AND LOWER COILS

In practically all a-c. machines there is a difference in temperature between the copper in the two coils, and then the detector cannot read higher than the average of the two. Although a number of factors may be contributory to such difference, most are of minor influence, and only eddy currents due to the load current will be considered.

It is well recognized that, owing to the rate of change of leakage flux, due to the load currents in the conductors, the current density in the conductors is a minimum at the bottom of the lower coil, and increases with increasing depth of conductor. Thus, the loss in the upper coil is greater than in the lower coil, and consequently the temperature of the upper coil is usually higher than the lower. A striking example of this was shown by the tests on the original Niagara generators described in a paper five years ago.* Those tests showed that with a load of 980 amperes, the maximum temperature of the top coil was 224 degrees, of the bottom coil 168 degrees (the average of which was 196 degrees), whereas the couple between coils showed 185 degrees. Thus, the difference between the top coil temperature and that given by the couple between coils was 39 degrees. While this is admittedly an extreme case due to the use of solid bars in a two-layer winding, eddy current conditions, even when all possible precau-

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*"Experimental Data Concerning the Safe Operating Temperature for Mica Armature—Coil Insulation." F. D. Newbury, A. I. E. E. Vol. XXXIV, p. 2747.

tions are taken, are an important factor of design in the larger 60-cycle generators of the present day.

The eddy current loss in the top coil is influenced by the throw of the coils. Thus, if there are three phases with the usual six-phase grouping, and the throw of the coils is two-thirds of full pitch, the load currents in the upper and lower coils are displaced from each other by 60 electrical degrees. The density of the leakage flux interlinking the upper coil is reduced by the displacement and the eddy loss is lowered thereby. If the throw of the coils is one-third of full pitch with three-phase winding (six-phase grouping), the currents in the upper and lower coils are out of phase by 120 electrical degrees. The reduction in eddy loss in the upper coil is greater when the displacement between currents is changed from 60 to 120 degrees than when it is changed from 0 to 60 degrees.

If in a three-phase machine the throw of the coils is between full pitch and two-thirds pitch in some of the slots the currents in the two coils are in phase with each other, and in other slots the currents are displaced by 60 degrees. Hence, there is a difference in eddy loss in the top coil in some slots from that in other slots. Similarly, with a throw between one-third and two-thirds, the eddy loss differs in some slots from that which obtains in other slots.

In a two-phase machine with a throw of coils between one-half and full pitch, some slots have coils in which the currents are in phase and other slots have coils in which the currents are displaced by 90 degrees. If the throw is less than 90 degrees, in some slots the currents are in phase opposition, which gives the minimum loss.

In a three-phase, two-pole, 60-cycle, turbo generator that has been in service for about six years, with fairly deep conductors in the coils, there are 36 slots, and the throw of the coils is from slot No. 1 to slot No. 12. Thus, the throw is between one-third and two-thirds of full pitch. Tests made showed that with 60 electrical degrees phase displacement the temperature rise by thermocouple between coils was 63 degrees cent., whereas with 120 degrees phase difference, the temperature rise was 56 degrees; a difference of seven degrees.[*] It is therefore evident that the temperature detectors should be located in slots in which there is minimum phase difference between the currents in the upper and lower coils.

PROTECTION OF DETECTOR FROM THE COOLING AIR

With the usual two coil per slot arrangement there are two methods of wrapping the mechanically protecting slot cell around the coils, viz.:

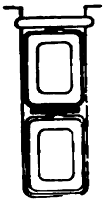

FIG. 2

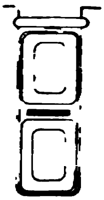

FIG. 3

(1) To place these cells on coils individually as shown in Fig. 2, or,

(2) To use a single slot cell to wrap both coils as indicated in Fig. 3.

With the usual radial scheme of ventilation (that is, with air ducts at intervals axially), it would seem that there is likelihood of the cool air passing between coils and cooling the detector below the temperature that would otherwise be read. This cooling air would not, however, have an appreciable effect upon the temperature of the coils, because the percentage increase in surface exposed to the cooling air is negligible. Tests made upon a 12,000-kv-a. alternator and more especially upon the model, results of which are given in this paper, bear out this supposition.

TESTS ON A 12,000-KV-A., 6600-VOLT, THREE-PHASE, 60-CYCLE, 150-REV. PER MIN., VERTICAL ALTERNATOR

The stator of the alternator had the following dimensions:

Internal diameter = 192 inches
External = 209 inches
Core width = 33 inches
Number and size radial vents = 13 – ½ inches
Number and size of slots = 324 – 0.79 in. x 3.45 in.
Number of conductors per slot = 4

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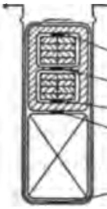
[*] The temperatures were measured by means of thermocouples between coil sides, not on the bare copper.

Size of conductor =4 (0.162 in.x0.25 in. bare)
4 (0.129 in.x0.21 in. asbestos covered)
Section of conductor =0.264 sq. in.
For arrangement of conductor see Fig. 4.
Connection, two-circuit star.
Throw of coils 1 and 7.

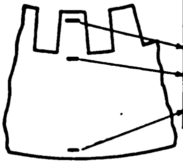
In order to make tests with thermocouples on the bare copper, these couples were installed in coils near the neutral point, and the neutral was grounded during the tests. In addition to these couples, the two-cell, and the one-cell arrangements were used—two cells from the center line of the machine to the right, and one cell to the left. The thermocouples were placed in the packages of iron adjacent to the central vent. Thermocouples were also embedded in the iron, as shown in Fig. 4, they being axially at the middle of the package to the right of the central vent.



12,000 KV.-A ALTERNATOR TEMPERATURE RISES. DEGREES C.

6600 V. 960 A.				7260 V. 960 A.			
Max.	Min.	Av.	No. of Detec.	Max.	Min.	Av.	
78.3	71.3	74.1	6	91.3	84.3	86.8	
77.0	70.3	73.0	6	89.0	74.7	84.0	
72.6	66.5	69.5	6	84.0	78.6	81.4	
64.0	52.9	56.9	12	69.0	57.0	63.3	Single Slot Cell
58.7	43.5	51.5	5	63.5	56.5	6 07	Double Slot Cell
54.0	42.8	47.9	6	66.3	54.2	6 01	

THERMO-COUPLES IN IRON NEAR MIDDLE OF CORE



6600 V. 960 A.				7260 V. 960 A.		
Max.	Min.	Av.	No. of Detec.	Max.	Min.	Av.
48.0	44.3	46.3	3	60.5	53.4	56.6
49.0	40.4	45.0	3	58.3	47.1	52.3
45.1	40.5	43.0	3	53.8	50.5	52.2

ARMATURE COILS

	7260 V. 960 A.	
	Max.	Min.
Ends 10 Thermometers	40	35
Temp. Rise by Resistance	54.2	

FIG. 4

Two heat runs were made: the first at 6600 volts, 960 amperes per phase until constant; then, without shutting down, the voltage was raised to 7260, the current being kept at 960 amperes. Both runs were made at nearly zero power factor, lagging current.

The summary of these heat runs is given in Fig. 4. Before drawing conclusions it will be interesting to estimate the temperature drops through the insulation and to compare these calculated results with the test.

The ingoing air temperature was 31 deg., so that for the first heat run the total temperature of the top coil was about 107 deg. total. The calculated eddy current loss for the top coil is 48 per cent of I^2R loss.* The I^2R loss at 0 deg. cent. with 960 amperes per inch of length per coil is 1.09 watts. The average surface (taken as bare copper surface plus three times the distance from copper to iron) is 3.31 sq. in., the thickness of the single wall = distance copper to iron is 0.154 inch; the thermal conductivity of the insulation we shall take as 0.003 watts per inch cube per degree cent. Then the watts per square inch at 107 deg. =

$$\frac{(1 + 0.00427 \times 107)\ 1.48 \times 1.09}{3.31} = 0.707$$

The thermal drop from the copper to the iron is approximately: $\dfrac{0.707 \times 0.154}{0.003} = 36.2$ deg. cent.

The temperature rise of the iron 48.5 deg. cent. in the first run, is taken as the mean of the maximum thermocouple readings in the teeth and just back of the teeth. Taking the air in the vents at the teeth to be 10 deg. above the ingoing air (for second run 13 deg.) the weighted mean temperature of iron and air is:

$$\frac{48.5 \text{ deg.} \times 26\tfrac{1}{2} \text{ in.} + 10 \text{ deg.} \times 6\tfrac{1}{2} \text{ in.}}{33 \text{ in.}} = 40.8 \text{ deg. rise.}$$

The temperature of the copper above the ingoing air is then: 36.2 deg. + 40.8 deg. = 77 deg.; whereas the average temperature rise by test of the top coil in the slot which showed maximum temperature was 76 deg. The calculations do not allow for axial flow of heat in the copper to the ends, but the decrease in temperature at the middle due to longitudinal flow in this machine is not great.†

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*See paper by R. E. Gilman on Eddy Currents, page 977.

†This subject of relative longitudinal and transverse heat flow has been investigated mathematically by one of the authors and a paper on this subject will be submitted to the Institute in the near future.

Summarizing the calculations we obtain Table I.

TABLE I

	6600 Volts, 960 Amperes	7260 Volts, 960 Amperes
Weighted mean of tooth and air temp.	40.8	50.4
Thermal drop through insulation	36.2	37.1
Calculated copper temp. rise = sum of above	77.	87.5
Average temp. rise of the coil in slot having max. temp. (test)	76.	88.1

This close agreement between the calculated and observed drop in temperature through the insulation under two different load conditions indicates the overall consistency and reasonableness of the test results and permits us to draw conclusions from them with added confidence. Attention is especially called to fact that the drop through the insulation cannot be altered (except for slight change in loss due to resistance being modified by change in total temperature) by change in ventilation. The only way the heat can be dissipated from the copper is by conduction through the insulation; some of the heat may flow longitudinally but still must be conducted through the insulation on the coil ends and then be liberated by convection from the surface.

The readings may be summarized still further as in

TABLE II

		6600 Volts, 960 Amperes Degrees	7260 Volts, 960 Amperes Degrees
Average temp. rise of top coil in slot having max. temp.		76.	88.1
Couples between coils	Max. temp. rise single slot cell	64.	69.
	Max. temp. rise double slot cell	58.7	63.5
Average temp. rise bot. of slot		47.9	60.1
Average temp. rise teeth		46.3	56.6

From Table II it will be seen that:

(1) There is a very material difference between the true copper temperature and the temperature as measured by means of detectors between coil sides; 12 deg. and 19 deg. for the 6600- and 7260-volt heat runs respectively. These differences are due to a combination of effects such as heat flow to the sides and difference in temperature between upper and lower coils. Expressed as a percentage of the total drop in temperature from the copper to the weighted mean of iron and air (76 deg.

– 40.8 deg. for first run), these differences are 34 per cent for the first and 51 per cent for the second heat run.

(2) The temperature reading is lower with the two-cell arrangement than with one cell. This amounts to 5.3 deg. for the first and 5.5 deg. for the second heat run. The differences between the maximum temperature on the copper and the maximum temperature by detector with two cells expressed as percentages of the total drop are 49 and 65 for the first and second runs respectively. The various figures are put in more convenient form in Table III.

TABLE III

	6600-Volt Run.		7260-Volt Run.	
	Degrees	Per Cent Total Drop in Insulation	Degrees	Per Cent Total Drop in Insulation
Difference between max. copper temp. and max. detector reading with one cell........	12.	34	19.1	51
Difference between max. copper temp. and max. detector reading with two cells.......	17.3	49	24.6	65

(3) There is a considerably greater difference between maximum and minimum readings with detectors exposed to cooling air as in the two-cell arrangement than with detectors in contact with copper or with detectors between coils but protected from air currents. This is shown by the following figures from Fig. 4.

	Max.	Min.	Ratio
Couples on copper......................	78.3	71.3	0.91
Couples bet. coils one-slot cell............	64.	52.9	0.83
Couples " " two-slot cell............	58.7	43.5	0.74

This is a matter of considerable importance as it is a measure of the reliability of the readings by the several methods. The greater variation in the case of the two-slot cell arrangement is possibly due to the greater effect of poor contact between the coil surfaces and the detector when the detector is exposed to air currents.

(4) A comparison of the average temperature rise by couple at the bottom of the slot and the average temperature rise of the teeth, shows that the former is not much higher than the latter. The thermocouple at the bottom of the slot measures the iron temperature at that spot and cannot read the copper temperature.

The difference between the two readings is quite small, but that can be readily understood when one considers that there is considerable flow of heat from the coil, which must be accompanied by a thermal drop in the iron; the longitudinal thermal conductivity of the iron is only about one-ninth that of copper.

Further conclusions may be drawn by reference to Fig. 4.

(5) The temperatures by thermometers on the end windings give practically no information in regard to the maximum copper temperatures.

(6) The temperature rise by resistance is at best, the average resistance of the complete winding and therefore conveys no information in regard to maximum temperature, and little information as regards the average temperature in the slots.

TESTS WITH ARMATURE MODEL

In Fig. 5 are shown the details of a model designed to reproduce the temperature conditions in a radially ventilated armature core from which data of interest can be obtained more conveniently than in the actual generator. This model consists of sheet-steel laminations having two armature slots and built up into a core of four packages with ½-in. air ducts between. Two flat coils (connected in series) wound with small wire are located in the two slots. A second sheet steel core, one inch thick, is placed over the slots to complete the magnetic circuit. The coils were insulated with mica of a thickness sufficient for a 6600-volt generator.

In Fig. 6 are shown elevations of the model when placed in a wooden box with a small Sirocco fan used for driving air through the vent-ducts.

In Fig. 5, C_1, C_2, etc., I_1, I_2, etc., all refer to thermocouples, the "C" couples being placed between coils, thus measuring coil temperatures, and the "I" couples being imbedded in the iron. R_1, R_2, R_1', R_2', are resistance exploring coils. Couple C_3 is placed within R_1; C_3' is within R_1'; C_4 is within R_2; and C_4' is within R_2'. C_1, C_5, and C_5' are above the resistance exploring coils. As shown R_1 and R_1' are substantially the entire width of the slot; R_2 and R_2' are each about 9⁄16 in. wide. In

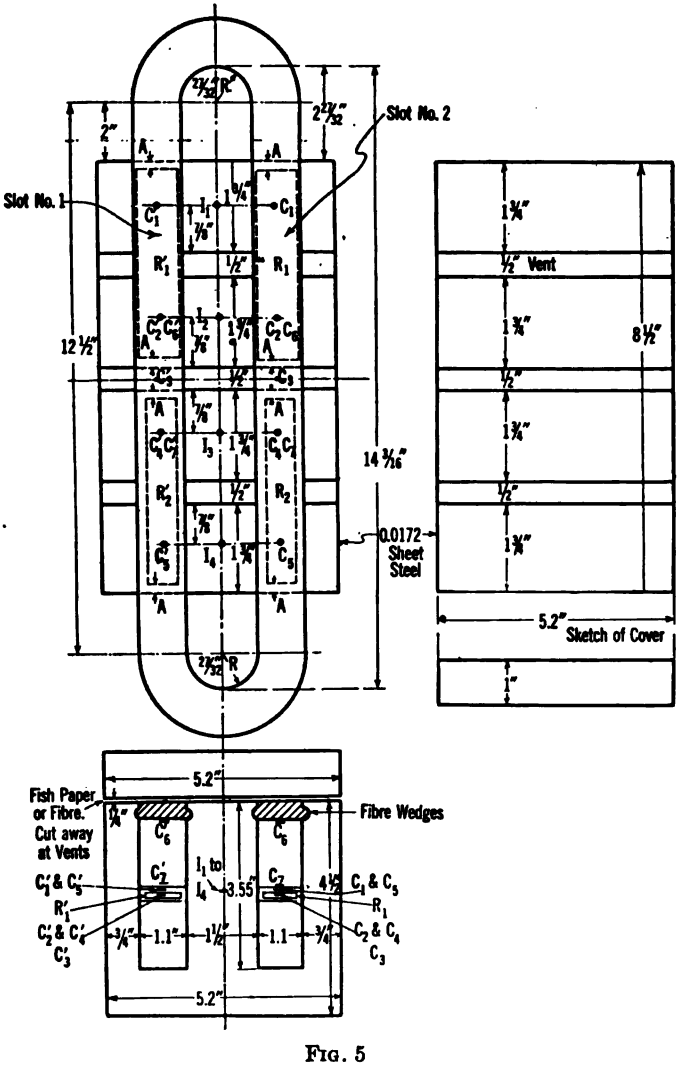

FIG. 5

slot No. 1, the slot cell was made in two parts as in Fig. 2. In slot No. 2 only one slot cell was used for parts of two coils in a slot as in Fig. 3. Thus, in slot No. 1, the air in the vent-ducts could strike the detectors and affect their indication, as is proved by the results. In order to show that there was no appreciable difference between the two slots, data are also included taken with all space closed between coils (litharge and glycerine was used). In Table IV the data from heat runs with 20 amperes 60 cycles are given.

TABLE IV
TEMPERATURE RISES, DEGREES CENT.
Test with Alternating Current, 20 Amperes, 60 Cycles, Approximately 374 Volts

Amperes per sq. inch = 1870.
Approx. tooth density = 92,000 lines per sq. inch.
" core " = 73,000 " " " "
" gap " = 51,200 " " " "
Eddy current loss top coil = 0.39 x I^2R.

Without Packing Between Coils		With Packing Between Coils	
Two-Slot Cells	One-Slot Cell	Two-Slot Cells	One-Slol Cell
Slot No. 1	Slot No. 2	Slot No. 1	Slot No. 2
C1′ = 30.6	C1 = 44.6	C1′ = 47.5	C1 = 43.5
C2′ = 34.5	C2 = 45.2	C2′ = 43.	C2 = 39.5
C3′ = 38.6	C3 = 48.2	C3′ = 43.	C3 = 41.0
C4′ = 34.6	C4 = 50.4	C4′ = 46.5	C4 = 44.0
C5′ = 32.	C5 = 48.5	C5′ = 45.5	C5 = 45.0
C6′ = 55.2	C6 = 54.5	C6′ = 54.5	C6 = 54.
C7′ = 58.	C7 = 59.6	C7′ = 58.5	C7 = 58.5
R1′ = 32.5	R1 = 45.	R1′ = 43.7	R1 = 41.
R2′ = 33.	R2 = 48.8	R2′ = 44.9	R2 = 42.
Entire winding by resis. = 54 deg.		53 deg.	
11 = 38.9	13 = 30.5	11 = 41.	13 = 30.
12 = 31.5	14 = 38.8	12 = 30.5	14 = 35.5

It will be noted that the temperatures in the slot with two cells is materially lower than with one cell, whether thermocouples or resistance coil method be used. Table V was made up in the same manner as Tables I and II; thermal conductivity was taken as 0.003 watts per inch^3 per deg. cent. The air tempera-

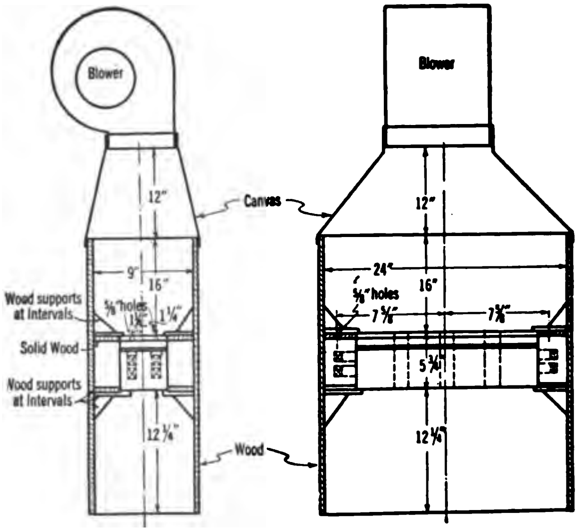

Fig. 6

ture rise was taken as 3 deg. above ingoing air for figuring weighted mean of tooth and air temperatures.

TABLE V

Weighted mean of tooth and air temp		= 26
Thermal drop through insulation		= 29.5
Calculated copper temp. rise = sum of above =		= 55.5
Average temperature rise of copper top coil, test		= 56.6
Couples between coils	Max. temp. rise single slot cell	= 50.4
	Max. temp. rise double slot cell	= 38.6

Figuring in same way as for the 12,000-kv-a. alternator, and using 58.8 deg. for the internal temperatures (the average of the C_7 couples), Table VI is obtained:

TABLE VI

	Degrees	Per Cent of Total Drop in Insulation
Difference between copper temp. and max. detector reading with one cell	8.4	25.6
Difference between copper temp. and max. detector reading with two cells	20.2	61.5

From the tests on the model it will be noted that:

(1) The cooling effect of air which passes between coils with the two-cell arrangement very materially lowers the reading of the detector, whether thermocouples or resistance coils are used.

(2) The air which passes between coils with the two-cell arrangement has no influence upon the internal temperatures (compare tests without packing with those with packing; also compare C_6 and C_7 readings in two different slots in the test without the glycerine and litharge packing).

(3) With any detector arrangement between coils, temperatures lower than the maximum are indicated.

(4) Wide resistance coils R_1 and R_1' showed slightly lower temperatures than did the narrow coils, due undoubtedly to heat flow from sides. (Although the same argument applies to thermocouples, they can more readily be made narrow, and usual practise is to make them of smaller width than is mechanically feasible with resistance coils).

(5) Thermocouples showed slightly higher temperatures than the resistance coils, but the difference was too small to warrant much discussion other than as noted under (4) above.

The fact that the temperature rise by resistance agreed fairly well with the average of the thermocouples inside the top coil may be accounted for by:

(1) The ventilation of the ends was very poor (air was allowed to pass through 5/8-inch hole) and therefore its temperature was probably higher than the slot portion, thus raising the average temperature; and

(2) Very little time was lost between shut-down and taking of reading of resistance. Neither of these conditions can be obtained in a large alternator—the ends are very effectively cooled, and there is a considerable lapse of time after shutting down before readings can be taken. The latter implies that in a machine, the average temperature is higher than can be measured by resistance.

ERRORS IN DETERMINATION OF TEMPERATURES BY PLACING DETECTORS BETWEEN COIL AND IRON OR UNDER WEDGE

Previous reference to errors by this method has been made. A test on another machine will be given to assist in showing the futility of attempting to measure the copper temperature with detectors, say, between bottom of coil and slot, and between coil and wedge.

A 3750-kv-a., 480-volt, three-phase, 60-cycle, 3600-rev. per min. turbo generator was wound with one coil side per slot, one turn per coil, the connections having been two-circuit star. The arrangement of the conductor in the slot is shown in Fig. 7, there having been 72 strands, each (0.102 in. by 0.205 in. d. c. c.) ribbons per conductor. (The temperature was low enough to permit of use of Class A insulation.) The coils were turned over at the ends; that is, a strand at the top of a coil in one slot became the bottom strand in the other slot in which the other part of the coil was placed. In addition the strands were transposed depthwise, as indicated schematically in Fig. 8. There were 36 slots total, so that the number

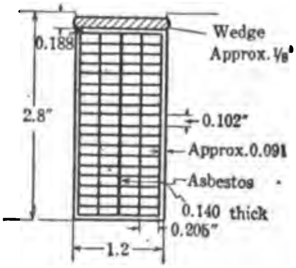

FIG. 7

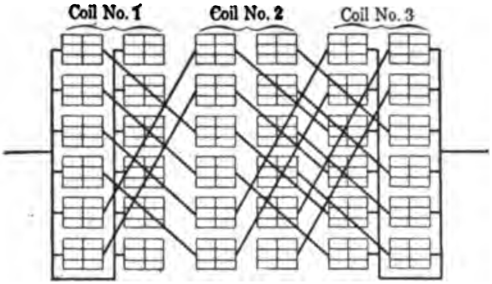

FIG. 8

of coil-sides per pole per phase per circuit was three. The three coils in such a phase group formed a group for transposition. Thus, starting at the beginning of a phase group, all the strands were joined together at one end of the coil, but at the other end the 72 strands were arranged in 12 groups, of six strands each. The top left group at the beginning of coil No. 1 becomes the bottom left group at the end of coil No. 1. To simplify the sketch in Fig. 8, the left sides of the 12 groups of coil No. 1 are shown joined in parallel; these are the beginnings of the coil group. At the other end of coil No. 1, the position of the groups are actually inverted, but they are not so shown. Then, the top left group which is actually the bottom left in coil No. 1 is joined to the third left group in coil No. 2; and the other end of that group is joined to the fifth left group in coil No. 3, etc.

Thermocouples were placed at the middle of the straight parts of the coil, before the coils were insulated, and couples were also placed at the bottom and top of slot, as shown in Fig. 9. In the same figure the final temperature rises are indicated for heat run at 4100 amperes per phase, normal voltage, zero power factor.

Figuring the drop through the insulation in the same manner as previously, allowing 0.091 in. for the thickness of the insulation; 0.3 for the watts per sq. in. at 0 deg.; 0.2 for the eddy loss as fraction of I^2R; 1.32 for the increase in resistance due to temperature; 0.0025 for the thermal conductivity;* the drop is:

$$\frac{1.2 \times 1.32 \times 0.091 \times 0.3}{0.0025} = 17.3 \text{ degrees.}$$

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*The lower value of conductivity was chosen at the lower voltage because of the greater relative influence of air pockets with the thinner insulation.

The weighted mean of iron (34 deg.) and air (say 10 deg.) (for relative lengths of 3.33 to 1) is 28.4 deg. The mean of the three temperatures inside the coil is 47 deg., and the drop is (47 − 28.4 =) 18.6 deg. as compared with 17.3 deg. calculated.

It will be noted that:

(1) The detector at the bottom of the slot or under the wedge does not indicate the temperature of the copper.

(2) That the maximum temperature of the copper is probably not at either the top or at the bottom of the coil.

(3) The cool currents of air above the wedge may cause the detector below it to read lower temperature than one at the bottom of the slot, even though the copper at the top is at higher temperature.

As a matter of fact, as previously pointed out in commenting upon the tests on the 12,000-kv-a. alternator, the detector at the bottom of the slot reads the temperature of the iron with which it is in contact; the detector placed below the wedge reads the temperature of that portion of the wedge, and neither can take into account the drop through the insulation. The A. I. E. E. Standardization Rule number 356 allows for a correction factor of 10 deg. cent. plus 1 deg. cent. per 1000 volts above 5000. Such correction factor is necessarily more or less meaningless.

Thus, the tests and calculations on the 12,000-kv-a. alternator showed that the drop through the insulation was approximately 36 degrees with 91 per cent load or about 43 degrees with full load. With high voltages, especially in long-core 60-cycle machines of large output, the drop is necessarily greater, due to the heavier wall of insulation; in such machines the drop is of the order of 50 degrees cent.

In our opinion it is not possible to decide upon correction factors which are applicable, even for comparison, with the one coil per slot arrangement. In point of fact, machines built at the present time in large or moderate sizes (in which detectors are incorporated), have with very few exceptions, two coil-sides per slot. Therefore, it is our opinion that the method of measuring copper temperatures by means of detectors at the bottom of the slot or under the wedge, should not be recognized in the Standardization Rules.

LOCATION OF THE HIGHEST COPPER TEMPERATURE

It will be seen from an inspection of Fig. 4 that the temperatures at the top of the upper coil are slightly higher than at the middle of the upper coil, but that there is a larger difference in temperature between the middle and the bottom of the coil than between the top and the middle of the coil. In the model (see Table III) the temperatures at the bottom of the upper coil are higher than at the top. In the 3750-k.v.-a. generator (see Fig. 9) the temperature at the side was substantially the same as at the top (test showed 1 deg. higher at the side), but the temperature of the copper at the bottom of the coil was appreciably lower than at the side. In all three cases, the effect of eddy currents was to increase the loss toward the top of the coil; but the fact that the temperature at the top was lower than might have been anticipated was in every case because the thermal conductivity of the compact wedge was relatively high, and with the effective cooling of the outside of the wedge, the temperature at the inside was lowered. (See Fig. 9 and note the reduced temperature below wedge.)

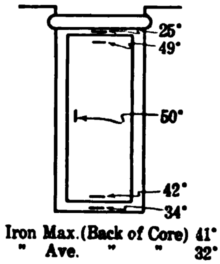

FIG. 9

The fact of the matter is that, if the temperature of the bare copper is to be measured, detectors should be located at various depths within the upper coil, and the maximum temperature taken to be the maximum copper temperature. If this is done in a number of coils in the machine and if the detectors are so located axially that their positions correspond to the axial position of maximum core temperature (to be determined by test on similar machines before manufacturing the coils) the maximum copper temperature will be measured with possibly a very small percentage error.

SUMMARY, CONCLUSIONS AND REMARKS

The points brought out in this paper may be summarized as follows:

(1) With detectors placed between coil sides, the difference in eddy currents in the upper and lower coils causes the detectors to read lower than the maximum.

(2) With detectors placed between coil sides, the flow of heat to the slot sides causes the detectors to read lower than the maximum.

(3) With detectors placed between coil sides, the phase difference in currents in the particular slot influences the reading, and the detectors should be located in slots in which there is minimum phase difference.

(4) With detectors placed between coil sides the possible circulation of air about the detector may have a material influence upon the reading of the detector. The slot cell should be so arranged that air currents cannot influence the reading of the detector. (See Fig. 3.) The single slot-cell provides excellent protection as the detector is then located in dead-air space.

(5) The air currents which lower the reading of the detector with the two-cell arrangement have an inappreciable influence upon the internal copper temperature.

(6) All heat from the copper must flow transversely through the insulation.

(7) The thermal drop through the insulation in large, long-core, high-voltage 60-cycle machines is of the order of 50 deg. cent.

(8) The thermal drop through the insulation cannot be appreciably lowered—at least in long-core machines—by improvement in ventilation.

(9) With any arrangement of detectors between coil sides, temperatures lower than the maximum are read, with the possible and rare exception of copper temperatures equal to or lower than the iron temperatures.

(10) With wide detectors between coil sides lower readings are obtained than with narrow detectors. The detectors should be made as narrow as is mechanically feasible.

(11) With detectors placed on the bare copper, the maximum temperature is not necessarily read at the top of the upper coil.

(12) A detector placed between coil side and iron or between coil side and wedge reads only the iron or wedge temperature at the particular spot; the reading gives no information in regard to the thermal drop through the insulation and the actual copper temperature.

(13) Thermometers placed on the end windings give no information in regard to the highest copper temperature in large high-voltage generators.

(14) Resistance measurements taken after completion of the heat run, convey no information in regard to maximum temperature, and but little information in regard to the average temperature in the slots.

(15) The machine usually cools too much after shutdown and before taking readings to obtain accurate data on the average temperature of the winding.

Whereas the only known way to measure the maximum copper temperature is by means of detectors on the bare copper, that method is not usually commercial, and therefore we must continue to employ detectors between coil sides. If a one-cell arrangement is used (Fig. 3), the detector if fairly narrow reads temperatures which, while they are slightly lower than the average of the top and bottom coil temperatures, afford a basis of comparison for the maximum temperature which is helpful to the purchaser and operator in judging on the one hand the value of the points of superiority, and on the other hand whether or not the machine is being operated at too high load. It is true that the method cannot cover all discrepancies, it cannot, with any simple correction factors, allow for the variation in eddy current loss in the various proportions of machines; it cannot cover the thermal drop due to the heat flow to the sides of the slot; it cannot allow for the uncertainty of locating the hottest spot over which there was lengthy discussion in the Standardization Committee in 1914, but it is a means of judging, and the only means that we now have that is entirely commercial. Inasmuch as any correction factor would be of comparatively little significance, it would, in our opinion, be well to omit such factor entirely, it being understood that in any case the reading by detector is probably somewhat lower than the maximum copper temperature.

The machines built at the present time in large or moderate sizes in which detectors are incorporated, have with very few exceptions, parts of two coils per slot. An occasional machine is built in which part of only one coil per slot is used, but such machines are usually wound for a lower voltage than is possible with the two-coil winding.* Therefore, it is our opinion that the method of measuring maximum copper temperatures by means of detectors at the bottom of the slot or under the wedge, should be omitted from the Standardization Rules.

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*This statement may not apply to European practise in which partly closed slots are frequently employed and "shoved-through" type of coil used.

DISCUSSION ON "SOME PRACTICAL EXPERIENCE WITH EMBEDDED TEMPERATURE DETECTORS IN LARGE GENERATORS" (FECHHEIMER AND NEWBURY), WHITE SULPHUR, W. VA., JUNE 30, 1920.

C. J. Fechheimer: In order that additional proofs might be secured of the influence upon the reading by detectors placed between coil sides of (a) heat flow to the slot sides, (b) width of detector, (c) difference in losses in the upper and lower coils due to eddy currents, an armature model similar to the one described in our paper was built, as shown in Fig. 1. The construction of this model differed from the previous one only in that additional insulation was placed between coil sides consisting of 14 layers, each layer 0.015 inch thick, of treated cement paper and mica, and in the arrangement of the thermocouples. Those indicated as C_3 to C_{24}

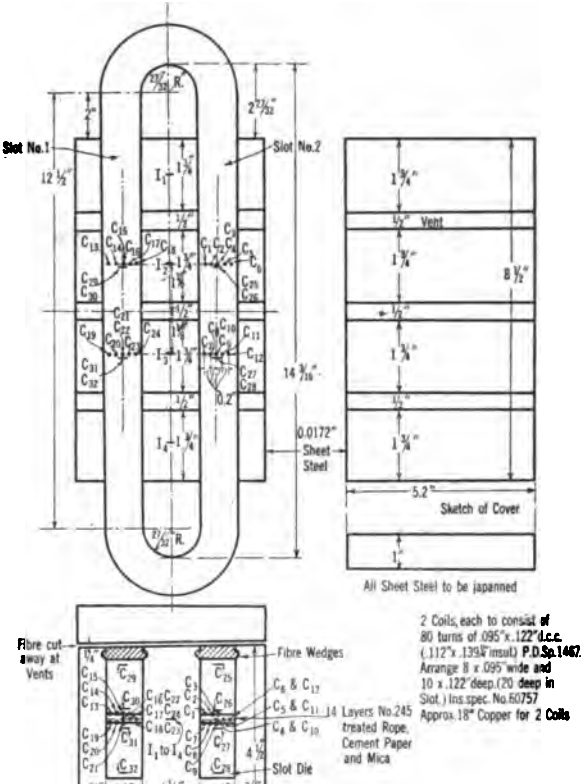
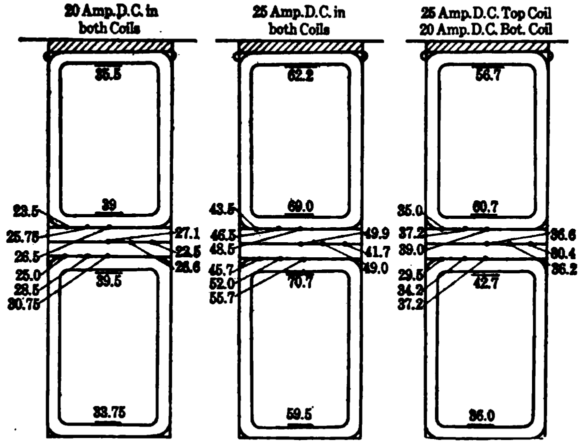


FIG. 1

inclusive were made of No. 27 B. & S. gage copper and advance wire; those inside the coils C_{25} to C_{32} inclusive were made of 0.005 in. by 0.25 in. copper and advance ribbons.

A series of tests is being conducted, and at present, we are prepared to give data on three. These tests were made with direct current, so that there was no eddy current loss; in order to initiate the greater loss in the upper coil which usually obtains when alternating current flows, different values of direct current were used in the two coils. It will be noted in Fig 1 that there are thermocouples in corresponding positions slots 1 and 2 and in Fig. 2 the averages of the temperature rises, for symmetrical points, above the ingoing air in degrees centigrade are recorded. In Fig 2 the temperatures are averaged from two thermocouple readings except those at the middle of the extra insulation between coils, the recorded temperature rises then being the average of four readings.



Rise by Resistance Top Coil 42.7° Bot. Coil 44.5° Rise of Iron End Packages 13° Middle Packages 10.5° Air Rise 2.5°.

Rise by Resistance Both Coils 79° Rise of Iron End Packages 24.5° Middle Packages 20° Air Rise 3.5°.

Rise by Resistance Top Coil 66.5° Bot. Coil 48.7° Rise of Iron End Packages 17° Middle Packages 13.7° Air Rise 2.5°.

FIG. 2

A study of the 20 ampere and 25 ampere heat runs will show that they are consistent with each other, if due allowance is made for the increase of the resistance of copper with temperature, and for the weighted mean of iron and air temperatures. The third heat run with different values of current in the two coils is also as consistent with the first two runs as may be expected with temperature tests, due allowance having been made for iron temperatures.

It will be noted that:

(1) There is a large difference in temperatuare be tween the inside and outside of the coil insulation.

(2) There is a comparatively small difference in temperature between the outside of the coil insulation at the center of the slot and the middle of the extra insulation.

(3) There is a marked temperature gradient from the center of the slot to the slot sides.

(4) With different values of current in the two coils there is a greater departure of temperature at the mid-point between coils from the copper temperature in the upper coil than with equal currents in the two coils.

(5) Although the internal temperatures of the two coils are substantially the same at symmetrical spots with equal currents, there is a considerable difference between the temperature on the outside of adjacent coil sides.

(6) The internal temperatures for symmetrical spots in the two runs with equal currents in the two coils differed very little from each other.

(7) The internal temperatures at the top of the upper coil and at bottom of the lower coil were lower in all three runs than the internal temperatures on the sides of the coils adjacent to each other.

The data prove the statements given in the paper, that there is appreciable heat flow to the slot sides; that the detector between coils does not read the true copper temperature; and that the width of the detector influences the reading thereof.

Conclusion (5) brings out the fact that there may be discrepancies of considerable magnitude as a result of unequal thermal conductivities in the insulating wall. Although there may have been minor factors which affected the temperatures at the outside of the coils, the difference in thermal conductivities, due perhaps to better surface contacts between layers of insulation on one coil side than the other, permitted greater rate of heat flow from the upper coil than from the lower. That the internal temperatures were only slightly influenced thereby, is probably because the rate of heat flow from the other sides were nearly equal in the two coils. The temperature rises of symmetrical points in the two slots also were different. Thus, in the 25-ampere run, the temperature rises at positions C_3 and C_{15} (Fig. 1) were 44 and 53 degrees respectively, an average of 48.5; and for C_9 and C_{21}, 57 and 54.5 degrees, an average of 55.7 degrees.

Although sufficient data are not at present available, if conclusion (6) were applicable generally, the temperatures on the copper could be taken in a smaller number of places in a machine than with the usual method with detectors between coil sides. Data in Fig. 4 of the paper also show that the percentage difference is smaller for internal differences than between coil sides.

Conclusion (7) should be considered in conjunction with that portion of the paper which relates to the "location of the highest copper temperature". It is again seen, that, owing to the relatively high thermal conductivity of the fibre wedge (the wedge was thinner than ordinarily) the rate of heat dissipation from the top of the upper coil was sufficiently great to reduce the temperature below that of the copper at the bottom of the same coil.

While we have various ideas in regard to what should be the limiting temperatures in machines, we cannot reach any decision as to the limit until we agree upon a method of measuring temperatures. Thus, it has been suggested that we go back to the old method of measurement of temperature by resistance after shut-down. As stated in the paper, that method does not give any information in regard to the maximum copper temperature, nor does it even convey information as to the average temperature of the buried copper in the slot; because that method only measures the average temperature of the entire winding after shut-down. This will be evident when we consider that the end windings are usually much cooler than the buried portions.

The time that elapses after shut-down, and before taking resistance readings, is usually sufficient to permit of a not inconsiderable reduction in average temperature.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29-July 2, 1920.

EDDY CURRENT LOSSES IN ARMATURE CONDUCTORS

By R. E. GILMAN
Westinghouse Electric Mfg. Co.

THE recent increase in the size of generators particularly of the turbo type has compelled a more complete investigation of the effect of stranded conductors in reducing eddy current losses. A reduction in losses has been necessary to secure safe temperatures inside the armature coil, since in larger units with few poles the ampere conductors per slot are necessarily large and the use of deep slots is forced upon the designer. Further the present practise of guaranteeing hot spot temperature or the safe heating temperature for the insulation, lends general interest to the subject and it seems advantageous for the engineering public to have more detailed information as to the magnitude of eddy current losses and as to some of the ways in which these losses can be controlled by the designer.

In looking up the literature on the subject of eddy currents in slot wound conductors, it appears that there is very little information of practical value in print.

One notable exception to this general statement is a paper, entitled "Eddy Currents in Large Slot Wound Conductors," by A. B. Field, presented at the 22nd Annual Convention of the A.I.E.E. in 1905. This paper covers only special applications of conductors. The limitations imposed in Mr. Field's paper are that the conductors are either solid or infinitely laminated and that the currents of all conductors in same slot are in phase.

In modern designs the stranding of conductors is limited to a finite number of strands depth-wise and there is often a large difference in loss between solid bars and infinitely standard bars of the same total cross section. So it becomes of interest to investigate the effect of, say, three or five strands. In the second place in large two-pole machines, for example, special strand groupings are sometimes desirable for mechanical reasons. And lately it is almost universal practise to use short-cord windings and therefore have conductors in the same slot carrying currents of different time phase. For these reasons the writer ventures to put forward the discussion which follows, as it may be of interest as additional data on a subject which has never been thoroughly discussed.

It is proposed to cover the following ground:

1. Develop formula for the calculation of the density in any point of a current-carrying wire (strand or conductor) provided the magnitude of the current in that wire and the total current in all the wires below it in same slot are known, together with the phase relation of these currents.

2. The calculation of this density will involve the evaluation of four constants of integration which are discussed in Part 2.

3. The development for the general case of the loss in a solid bar as compared to the same bar with uniformly distributed current.

4. The development of the loss ratio in an infinitely stranded conductor with various arrangement of groupings and the discussion of the effect of end windings in this case.

5. Approximation formula for the loss ratio of solid bars and the discussion of the limitation of its application.

6. Mathematical solution of current values in various strands for any number of strands and any arrangement of these strands. Development of formula for the equivalent loss ratio for a stranded conductor.

7. Examples of the application of the formulas to various grouping arrangements in the case of two strands.

8. Examples, same as in section 7, but using three strands.

9. Comparison of results in the case of solid bars,

with two strands, three strands and infinite strands, and the development of the law for any number of strands.

10. Test data covering the comparison of calculations and tests with various equivalent thicknesses of strands, various ratios of length of buried coil to free ends, and various number of strands per conductor from one to six.

In slot-wound conductors surrounded on three sides by iron there are two fluxes which produce eddy currents. The first flux is the one due to magnetomotive-force of the field and enters at the top of the slot adjacent to the gap. The strength of this field can be regulated by proper design proportions, such as overhanging tooth tips, sunk coils, or magnetic wedges and the use of low flux densities in the teeth. This source of eddy currents is not the subject of this paper and will not be further discussed. The second flux is that due to the m.m.f. of the currents in the coil itself. A little consideration will show that this second flux crosses the slot transversely from tooth to tooth through the body of the conductors and that it causes currents to flow along the top edge of the coil (air gap side) throughout the length of the iron and return along the bottom edge. The eddy currents themselves give an additional m.m.f. and react upon the system in such a way that the net result is a current density which is always of normal frequency, but varies in magnitude and phase as a function of the distance from the bottom of the conductor. In any plane that is parallel to the bottom of the conductor the magnitude and phase of the current are constant.

This is evident at once if we assume that the flux due to the armature current passes at right angles to the slot depth transversely across the slot. This assumption is made, also the further assumption that the reluctance in the iron circuit to this transverse flux is zero and that after a conductor reaches the end of the iron core that all induced fluxes link all the elements of any conductor equally and produce no e.m.fs. to cause eddy currents in the conductor. This latter condition is not strictly true.

1—CALCULATION OF DENSITY AT ANY POINT IN A CONDUCTOR

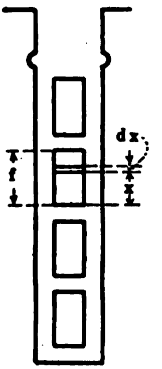
Let us consider a conductor carrying a current which may have any position in a slot with respect to other conductors which also carry currents.

Let Δ = The instantaneous current density at any distance x from the bottom of conductor

x = The distance from the bottom of conductor of Δ

f = The depth of conductor

a = The width of slot



r_1 = The ratio of copper width to the slot width

$a\, r_1$ = The copper width

ρ = The specific resistance of copper, which is taken as 0.625×10^{6} ohms per cu. inch at 0 deg. cent.

$\omega = 2\pi \sim$

$\sim$ = The frequency of current

V = The instantaneous value of the e.m.f. of self induction per unit of length produced at x by the transverse flux.

At the point x the e.m.f. of self induction is V; at a point $x + d\,x$ this e.m.f. is $V + dV$.

The current density at x is Δ, the current is $\Delta \times$ section, the resistance is $\frac{\rho}{\text{section}}$ per unit length. The resistance drop is $\Delta\, \rho$ at x and $(\Delta + d\,\Delta)\,\rho$ at $x + d\,x$. In the element of the conductor of width $a\, r_1$, depth $d\,x$ and length unity the four e.m.fs. given above form a closed circuit if the iron core is of unit length. If the core is of a length l then the factor l is multiplied into each value.

Since the sum of all e.m.fs. in a closed circuit must be zero we get at once

$$(V + d\,V) - V = (\Delta + d\,\Delta)\,\rho - \Delta\,\rho$$

or

$$\frac{d\,V}{d\,x} = \frac{d\,\Delta}{d\,x}\,\rho \qquad (1)$$

Let the total flux up to $x = \phi$

Then at $x + d\,x$ it is $\phi + d\phi$

If the induction per unit area at x is β then $d\phi = \beta\, d\,x$ and if t is time

$$\text{Then } d\,V = \frac{d}{dt}\beta\, d\,x \text{ and } \frac{d\,V}{d\,x} = \frac{d\beta}{dt} \text{ and } \frac{d^2\,V}{d\,x^2} = \frac{d^2\beta}{d\,x\,dt}$$

Further the current between x and $x + d\,x$ is $a\,r_1\,\Delta\,d\,x$ and produces an increment of flux density $d\,\beta$

$$\text{Whence } d\,\beta = \frac{4\,\pi\,a\,r_1\,\Delta\,d\,x}{10\,a} \text{and } \frac{d\,\beta}{d\,x} = \frac{4\,\pi\,r_1\,\Delta}{10}$$

$$\frac{d^2\,\beta}{d\,x\,dt} = \frac{d^2\,V}{d\,x^2} = \frac{4\,\pi\,r_1}{10}\ \frac{d\,\Delta}{dt} \text{ in absolute units}$$

If we use volts, ohms, amperes, etc., and Δ is the density per unit area

$$\text{Then } \frac{d^2\,V}{d\,x^2} = \frac{d^2\,\beta}{d\,x\,dt}\,10^{-8} = 4\,\pi\,r_1\,\frac{d\,\Delta}{dt}\,10^{-9} = \frac{d^2\,\Delta}{d\,x^2}\,\rho \qquad (2)$$

Let $\alpha^2 = \dfrac{2\,\pi\,r_1}{\rho}\,\omega\,10^{-9}$ and we get from (2)

$$\frac{\omega}{2}\,\frac{d^2\,\Delta}{d\,x^2} = \frac{\alpha^2\,d\,\Delta}{dt} \qquad (3)$$

This is the differential equation for Δ in terms of x and t as variables and the solution of this equation gives us Δ.

From the conditions of our problem we know that the frequency of Δ is the same as the frequency of the current in the conductor, also that Δ is of uniform value in any plane of the conductor which is parallel to the bottom of the slot.

Let Δ_m be the maximum value of the density at the point x under consideration, also let the instantaneous value of current in the conductor be $J \sin \omega\, t$ and let θ be the phase angle between J and Δ_m. Then $\Delta_m \sin (\omega\, t + \theta)$ is the instantaneous value of the density at our point x.

$\Delta_m \sin (\omega\, t + \theta) = \Delta_m \sin \omega\, t \cos \theta + \Delta_m \cos \omega\, t \sin \theta$ by expansion this can be rewritten $\Delta_m \sin (\omega\, t + \theta) =$

$$\text{A} \sin \omega\, t + \text{B} \cos \omega\, t = \Delta \qquad (4)$$

A and B are functions of x and J and Δ is a function of x, J and t

From equations (3) and (4)

$$\left[\frac{d^2 A}{dx^2}\sin \omega t + \frac{d^2 B}{d x^2}\cos \omega t\right]\frac{\omega}{2} =$$

$$\left[A \cos \omega t - B \sin \omega t\right]\alpha^2 \omega$$

$$\left[\frac{d^2 A}{dx^2} + 2\alpha^2 B\right]\sin \omega t + \left[\frac{d^2 B}{dx^2} - 2\alpha^2 A\right]\cos \omega t = 0$$

and therefore $\frac{d^2 A}{dx^2} = -2\alpha^2 B$, $\frac{d^4 A}{dx^4} = -2\alpha^2 \frac{d^2 B}{dx^2}$, $\frac{d^4 A}{dx^4} = -4\alpha^4 A$ $\frac{d^2 B}{dx^2} = 2\alpha^2 A$, $\frac{d^4 B}{dx^4} = 2\alpha^2 \frac{d^2 A}{dx^2}$ and

$$\frac{d^4 B}{dx^4} = -4\alpha^4 B \tag{5}$$

$\frac{d^4 A}{dx^4} = -4\alpha^4 A$ and $\frac{d^4 B}{dx^4} = -4\alpha^4 B$ are simultaneous equations of the fourth degree in x only and their solution involves four constants of integration for each equation. Since, however, the equations are simultaneous these constants for the two equations are related to each other so that it should be possible to get a general solution of the original equation (3) involving only four constants of integration. The two equations of (5) are linear in x with constant coefficients of the form

$$A_0 \frac{d^n y}{d x^n} + A_1 \frac{d^{-n1}y}{dx^{n-1}} \quad A_{n-1}\frac{d y}{d x} + A_n y = 0 \tag{6}$$

Where A_0, A_1, etc., are constants. For y substitute ϵ^{mx}) where m is a constant to be evaluated. Since $\frac{d}{dx}\epsilon^{mx} = m\epsilon^{mx}$ and $\frac{d^2}{dx^2}\epsilon^{mx} = m^2 \epsilon^{mx}$, etc., we get after rejecting the factor ϵ^{mx}.

$$A_0 m^n + A_1 m^{n-1} + \text{- - -} + A_{n-1} m + A_n = 0 \tag{7}$$

and any value of m which satisfies (7) corresponds to some particular value of ϵ^{mx} which is an integral of (6). Therefore if m_1, m_2, m_3, etc., satisfy (7) then $y = c_1 \epsilon^{m_1 x} + C_2 \epsilon^{m_2 x} + etc.$, is the complete integral of (6).

From (5) we can write $D^4 + 4\alpha^4 A = 0$ if we put D for $\frac{d}{dx}$ and $A = C_1 \epsilon^{m_1 x} + C_2 \epsilon^{m_1 x} + etc.$, where m_1, m_2, and m_4 are the $\sqrt{2}\,\alpha$ multiplied by the four roots of

$\sqrt[4]{-1}$ or $\pm\left(\frac{1+j}{\sqrt{2}}\right)$ and $\pm\left(\frac{1-j}{\sqrt{2}}\right)$

$$A = \epsilon^{ax}\left[C_1\,\epsilon^{jx} + C_2\,\epsilon^{-jx}\right] + \epsilon^{-ax}\left[C_3\,\epsilon^{jx} + C_4\,\epsilon^{-jx}\right]$$

Let $C = a_1 + j\,a_2$ then $C_1 + C_2 = 2a,\quad C_2 = a_1 - j\,a_2$ and $C_1 - C_2 = 2\,j\,a_2$. Hence we can write the solution of equations (5) as follows:

$$\left.\begin{aligned} A &= 2\,\epsilon^{ax}\left[a_1 \cos\alpha\,x - a_2 \sin\alpha\,x\right] + 2\,\epsilon^{-ax}\left[a_3 \cos\right. \\ &\qquad \left.\alpha\,x - a_4 \sin\alpha\,x\right] \\ B &= 2\,\epsilon^{ax}\left[b_1 \cos\alpha\,x - b_2 \sin\alpha\,x\right] + 2\,\epsilon^{-ax}\left[b_3 \cos\right. \\ &\qquad \left.\alpha\,x - b_4 \sin\alpha\,x\right] \end{aligned}\right\}\ (9)$$

But $\frac{d^2 A}{dx^2} = -\,2\alpha^2 B$ and combining with equations (9)

$$\frac{d\,A}{dx} = \alpha\,\epsilon^{ax}\left[(2a_1 - 2a_2)\cos\alpha\,x - (2a_1 + 2a_2)\sin\alpha\,x\right]$$

$$-\,\alpha\,\epsilon^{-ax}\left[(2a_3 + 2a_4)\cos\alpha\,x + (2a_3 - 2a_4)\sin\alpha\,x\right]$$

$$\frac{d^2 A}{dx^2} = 2\alpha^2\,\epsilon^{ax}\left[-\,2a_2\cos\alpha\,x - 2a_1\sin\alpha\,x\right] +$$

$$2\,\alpha^2\,\epsilon^{-ax}\left[2a_4\cos\alpha\,x + 2a_3\sin\alpha\,x\right] = -\,2\alpha^2 B \qquad \text{———}$$

$$-2\alpha^2\,B = -\,2\,\alpha^2\,\epsilon^{ax}\left[2\,b_1\cos\alpha\,x - 2\,b_2\sin\,\alpha\,x\right] -$$

$2\alpha^2\,\epsilon^{-ax}\left[\,2\,b_3\,\cos\,\alpha\,x\,-\,2\,b_4\,\sin\,\alpha\,x\,\right]$ This reduces to $\epsilon^{ax}\left[-\,2\,a_2 + 2\,b_1\right]\cos\alpha\,x + \epsilon^{ax}\left[-\,2\,a_1 - 2\,b_2\right]\sin\alpha\,x$ $+\ \epsilon^{-ax}\left[2\,a_4 + 2\,b_3\right]\cos\alpha\,x + \epsilon^{-ax}\left[2\,a_3 - 2\,b_4\right]\sin\alpha\,x$ $= o$ Separating terms in $\sin\alpha\,x$ and $\cos\alpha\,x$

$\epsilon^{ax}\left[-\,2\,a_2 + 2\,b_1\right] + e^{-ax}\left[2\,a_4 + 2\,b_3\right] = 0$ and

$\epsilon^{ax}\left[-\,2\,a_1 - 2\,b_2\right] + \epsilon^{-ax}\left[2\,a_3 - 2\,b_4\right] = 0$

Note: Through typographical error a has been used instead of α throughout paper in the exponent for ϵ.

And since these equations hold for all values of x

$$a_1 = -b_2 \quad a_2 - b_1 \qquad a_3 = b_4 \text{ and } a_4 = -b_3$$

Substituting in equations for A B and

$\Delta = A \sin \omega t + B \cos \omega t$ the values of A and B from

$$(9) \quad \Delta = \epsilon^{ax} \Big[2 a_1 \cos \alpha x - 2 a_2 \sin \alpha x \Big] \sin \omega t + \epsilon^{-ax} \Big[2 a_3 \cos \alpha x - 2 a_4 \sin \alpha x \Big] \sin \omega t + \epsilon^{ax} \Big[2 a_3 \cos \alpha x + 2 a_1 \sin \alpha x \Big] \cos \omega t + \epsilon^{-ax} \Big[-2 a_4 \cos \alpha x - 2 a_3 \sin \alpha x \Big] \cos \omega t,$$

which reduces to

$$\Delta = 2 \epsilon^{ax} \Big[a_1 \sin (\alpha x + \omega t) + a_2 \cos (\alpha x + \omega t) \Big] - 2 \epsilon^{-ax} \Big[a_3 \sin (\alpha x - \omega t) + a_4 \cos (\alpha x - \omega t) \Big] \qquad (10)$$

This is the complete solution of Δ involving the four constants of integration a_1, a_2, a_3 and a_4. The evaluation of these constants is given in Part 2.

2—EVALUATION OF CONSTANTS IN THE SOLUTION OF Δ

The value of the constants a_1, a_2, a_3 and a_4 can be secured from the limiting conditions imposed upon the current in any strand or conductor as the use may be.

The most general condition is the one where the current J in the wire under consideration can have any value and any phase angle with respect to some reference current; and also the current I below the wire can have a value and phase in no way related to J.

Taking a wire which may be a strand of the conductor or a conductor itself: Let its depth be f, its current J, its phase displacement be θ. Let $\Delta_c \sin(\omega t + \theta)$ represent the mean density if the current is uniformly distributed over the wire. Let I be the current below the wire and let its phase angle be ϕ.

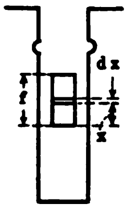
Then our limiting conditions are

$$\Delta_c \sin(\omega t + \theta) = \frac{1}{f} \int_0^f \Delta \, dx \qquad (11)$$

$$\left[\frac{d\,v}{d\,x}\right]_{x=0} = \rho\left[\frac{d\ \Delta}{d\,x}\right]_{x=0} = 4\,\frac{\pi\,\omega}{a}\,I\cos\,(\omega\,t + \phi)\,10^{-9} = \left[\frac{d\,B}{dt}\right]_{x=0} 10^{-8} \tag{12}$$

The development of equation (12) is as follows:

For any element of a conductor $d\,x$ the current above this element produces a flux which links this particular element and all elements below it, therefore producing no variation in these elements or $\frac{d\,v}{d\,x} = 0$ due to this flux.



We have then to consider only the current in the element and the current below it.

Let the depth of the wire be f.

Consider only the current up to the wire. We have called this current I and its phase ϕ. We have then a flux density at X and all points above it of β_1

$\beta_1 = \frac{4\,\pi}{10a}\,I\sin(\omega\,t + \phi)$ and $\phi_1 = \frac{4\,\pi}{10a}\,I\sin\,(\omega\,t + \phi)\,(f - x)$ is the flux due to I which links the strand above the element $d\,x$.

The e.m.f. due to ϕ_1 in volts is V_1.

$V_1 = \frac{d\phi_1}{d\,t}\,10^{-8} = \frac{4\,\pi}{a}\,I\,\omega\cos(\omega\,t + \phi)\,(fx)\,10^{-9}$ and $\frac{d\,V_1}{d\,x} = -\left[-\frac{4\pi}{a}\,I\,\omega\cos\,(\omega\,t + \phi)\,10^{-9}\right]\frac{V_1}{d\,x}$ is negative as ϕ_1 decreases when X increases. It is readily seen that when X is zero that V_1 is the only e.m.f. producing eddy currents and we derive our equation (12) under the condition that x is zero. We have from equation (10)

$$\rho\,\frac{d\ \Delta}{d\,x} = 2\rho\,\alpha\left\{\left[a_1 - a_2 - a_3 - a_4\right]\sin\,\omega\,t + \left[\,a_1 + a_2 - a_3 + a_4\right]\cos\,\omega\,t\right\}_{x=0} \tag{13}$$

Combining (**12**) and (**13**) and remembering

$$\alpha^2 = \frac{2\,\pi\,\gamma_1}{\rho}\,\omega\,10^{-9}$$

we get

Note: On this page and in remainder of paper the letter r should be substituted for γ.

$$(a_1 - a_2 - a_3 - a_4) \sin \omega t + (a_1 + a_2 - a_3 + a_4) \cos \omega t$$
$$= \frac{\alpha}{\alpha \gamma_1} I \cos(\omega t + \phi) \qquad (14)$$

and therefore grouping all terms together which contain $\sin \omega t$ or $\cos \omega t$ and equating each group to zero we get

$$a_1 - a_2 = a_3 + a_4 - \frac{\alpha I}{\alpha \gamma_1} \sin\phi$$
$$a_1 + a_2 = a_3 - a_4 + \frac{\alpha I}{\alpha \gamma_1} \cos\phi \qquad (15)$$

From (11) $\Delta_o \sin(\omega t + \theta) = \frac{1}{f}\int_o^f \Delta \, d x$ and (10) for value of Δ we get

$$\sin \omega t \Big[a_1 \epsilon^{\alpha f} (\cos \alpha f + \sin \alpha f) + a_2 \epsilon^{\alpha f} (\cos \alpha f - \sin \alpha f) \overset{c}{+} a_3 \epsilon^{-\alpha f} (-\cos \alpha f + \sin \alpha f) + a_4 \epsilon^{-\alpha f} (\sin \alpha f + \cos \alpha f) - \sin \omega t (a_1 + a_2 - a_3 + a_4)$$
$$+ \cos \omega t \Big[a_1 \epsilon^{\alpha f} (\sin \alpha f - \cos \alpha f) + a_2 \epsilon^{\alpha f} (\sin \alpha f + \cos \alpha f) + a_3 \epsilon^{-\alpha f} (\cos \alpha f + \sin \alpha f) + a_4 \epsilon^{-\alpha f} (\cos \alpha f - \sin \alpha f) + \cos \omega t (a_1 - a_2 - a_3 - a_4) = \Delta_0 \alpha f$$
$$\Big[\sin \omega t \cos \theta + \cos \omega t \sin \theta \Big] \qquad (16)$$

All terms in (16) contain $\sin \omega t$ or $\cos \omega t$. Collecting each set separately and equating to zero and substituting the values of $- \alpha \frac{I}{a \gamma_1} \sin\phi$ from (15) and also $J = a \gamma_1 \Delta_{\alpha f}$ and we find that (16) reduces to

$$\left.\begin{aligned}
&\frac{\alpha}{a\gamma_1}\Big[J \cos \theta + I \cos\phi \Big] = (a_1 + a_2) \epsilon^{\alpha f} \cos \alpha f \\
&+ (a_1 - a_2) \epsilon^{\alpha f} \sin \alpha f + (a_3 + a_4) \epsilon^{-\alpha f} \sin \alpha f \\
&- (a_3 - a_4) \epsilon^{-\alpha f} \cos \alpha f \frac{\alpha}{a\gamma_1} \Big[J \sin \theta + I \sin \phi \Big] \\
&= (a_1 + a_2) \epsilon^{\alpha f} \sin \alpha f - (a_1 - a_2) \epsilon^{\alpha f} \cos \alpha f + (a_3 \\
&+ a_4) \ldots \epsilon^{-\alpha f} \cos \alpha f + (a_3 - a_4) \epsilon^{-\alpha f} \sin \alpha f.
\end{aligned}\right\} \quad (17)$$

$$\frac{\epsilon^{\alpha f} + \epsilon^{-\alpha f}}{2} = \cosh \alpha f \quad 1 \frac{\epsilon^{\alpha f} - \epsilon^{-\alpha f}}{2} = \sinh \alpha f.$$

If we combine (15) and (17) and if we let $\cosh \alpha f \sin \alpha f$

$= p$ and $\sinh \alpha f \cos \alpha f = q$, we can after rearranging obtain the following values for a_1, a_2, a_3, a_4

$$\begin{aligned} a_1 = K \Big[& (p - q)\,(J \sin\theta + I \sin\phi) + (p + q)\,(J \cos\theta \\ & + I \cos\phi) + (p - q)\, I\, \epsilon^{-\alpha f} \sin(\alpha f - \phi) \\ & - (p + q)\, I\, \epsilon^{-\alpha f} \cos(\alpha f - \phi) \Big] \end{aligned}$$

$$\begin{aligned} a_2 = K \Big[& (p + q)\,(J \sin\theta + I \sin\phi) - (p - q)\,(J \cos\theta \\ & + I \cos\phi) + (p + q)\, I\, \epsilon^{-\alpha f} \sin(\alpha f - \phi) \\ & + (p - q)\, I\, \epsilon^{-\alpha f} \cos(\alpha f - \phi) \Big] \end{aligned}$$

$$\begin{aligned} a_3 = K \Big[& (p - q)\,(J \sin\theta + I \sin\phi) + (p + q)\,(J \cos\theta \\ & + I \cos\phi) - (p - q)\, I\, \epsilon^{\alpha f} \sin(\alpha f + \phi) \\ & - (p - q)\, I\, \epsilon^{\alpha f} \cos(\alpha f + \phi \Big] \end{aligned}$$

$$\begin{aligned} a_4 = K \Big[& - (p + q)\,(J \sin\theta + I \sin\phi) + (p - q) \\ & (J \cos\theta + I \cos\phi) + (p + q)\, I\, \epsilon^{\alpha f} \sin \\ & (\alpha f + \phi) - (p - q)\, I\, \epsilon^{\alpha f} \cos(\alpha f + \phi) \Big] \end{aligned}$$

$$\text{Where } K = \frac{1}{4}\,\frac{\alpha}{a\,\gamma_1}\,\frac{1}{p^2 + q^2}$$

3—Excess Loss in Conductor Due to Eddy Currents

Considering the case covered in Part 2 of a wire carrying a current J with a displacement θ and located in a slot over the current I with a displacement ϕ we can determine the copper loss in the wire. To obtain this loss we require the average value of Δ^2 with respect to x and the mean value with respect to time. Let this value be known as Δ^2_1.

$\frac{1}{f}\int_0^f \Delta^2\,dx$ is the square root of the mean squared value of Δ with respect to x. The time of one cycle is from zero to $\frac{1}{\sim}$ where $\sim$ is the frequency. $\frac{1}{\sim} = \frac{2\pi}{\omega}$

The complete operation is indicated as

$$\frac{\omega}{2\pi}\,\frac{1}{f}\int_0^f dx \int_0^{\frac{\pi\, 2}{\omega}} \Delta^2\, dt = \Delta^2{}_1$$

From 10

$$\Delta = 2\,\epsilon^{\alpha x}\Big[a_1 \sin(\alpha\, x + \omega\, t) + a_2 \cos(\alpha\, x + \omega\, t)\Big]$$
$$-\,2\,\epsilon^{-\alpha x}\Big[a_3 \sin(\alpha\, x - \omega\, t) + a_4 \cos(\alpha\, x - \omega\, t)\Big]$$

For convenience let $\alpha\, x + \omega\, t = A$ $\alpha\, x + \omega\, t = B$ when

$$\Delta = 2\,\epsilon^{\alpha x}\Big[a_1 \sin A + a_2 \cos A\Big] - 2\,\epsilon^{-\alpha x}\Big[a_3 \sin \beta$$
$$+\, a_4 \cos \beta\Big]$$

$$\Delta^2 = 4\,\epsilon^{2\alpha x}\Big[a^2{}_1 \sin^2{}_1 A + 2\, a_1 a_2 \sin A \cos A + a^2{}_2 \cos^2 A\Big]$$
$$-\,8\,\Big[a_1\, a_3 \sin A \sin B + a_1 a_4 \sin A \cos B\Big]$$
$$+\Big[a_2\, a_3 \cos A \sin B + a_2\, a_4 \cos A \cos B\Big]$$
$$+\,4\,\epsilon^{-2\alpha x}\Big[a^2{}_3 \sin^2 B + 2a_3\, a_4 \sin B \cos B + a^2{}_4$$
$$\cos^2 B\Big]$$

$$\text{Sin}^2 A = \tfrac{1}{2} - \cos\frac{2A}{2}$$
$$= \tfrac{1}{2}\Big[1 - \cos(2\alpha\, x + 2\,\omega\, t)\Big]$$
$$= (a)\,\frac{\omega}{2\pi}\int_0^{\frac{2\,\pi}{\omega}} (a)\, dt = \tfrac{1}{2}$$

$$2 \sin A \text{ ocs } A = \sin(2\alpha\, x + 2\,\omega\, t)$$
$$= (b)\,\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} (b)\ dt = 0$$

$$\text{Cos}^2 A = \tfrac{1}{2}\Big[1 + \cos(2\alpha\, x + 2\,\omega\, t)\Big]$$
$$= (e) \quad \text{etc.} \quad = \tfrac{1}{2}$$

$$\text{Sin } A \sin B = \tfrac{1}{2} \Big[\cos 2\,\omega\,t - \cos 2\,\alpha\,x\Big]$$

$$= (d) \qquad \text{etc.} \qquad = -\tfrac{1}{2}\cos 2\alpha\,x$$

$$\text{Sin } A \cos B = \tfrac{1}{2} \Big[\sin 2\,\alpha\,x + \sin 2\,\omega\,t\Big]$$

$$= (e) \qquad \text{etc.} \qquad = \tfrac{1}{2}\sin 2\alpha\,x$$

$$\text{Cos } A \sin B = \tfrac{1}{2} \Big[\sin 2\,\alpha\,x - \sin 2\,\omega\,t\Big]$$

$$= (f) \qquad \text{etc.} \qquad = \tfrac{1}{2}\sin 2\alpha\,x$$

$$\text{Cos } A \cos = B\ \tfrac{1}{2} \Big[\cos 2\,\alpha\,x + \cos 2\,\omega\,t\Big]$$

$$= \ 8 \qquad \text{etc.} \qquad = \tfrac{1}{2}\cos 2\alpha\,x$$

$$\text{Sin}^2 B = \tfrac{1}{2} \Big[1 - \cos(2\alpha\,x - 2\,\omega\,t)\Big]$$

$$= (h \qquad \text{etc.} \qquad = \tfrac{1}{2}$$

$$2\sin B\cos B = \sin(2\alpha\,x - 2\,\omega\,t)$$

$$= (i) \qquad \text{etc.} \qquad = 0$$

$$\text{Cos}^2 B = \tfrac{1}{2} \Big[1 + \cos(2\alpha\,x - 2\,\omega\,t)\Big]$$

$$= (j) \qquad \text{etc.} \qquad = \tfrac{1}{2}$$

The average integrated value of Δ^2 between $t = O$ and $t = \frac{2\pi}{\omega}$ or $\frac{\omega}{2\pi}\int^{\frac{2\pi}{\omega}} \Delta^2\,dt = 2\,\epsilon^{2ax}\,(a_1^2 + a_2^2) + 4\,a_1\,a_3\cos 2\,\alpha\,x - 4\,a_1\,a_4\sin 2\,\alpha\,x - 4\,a_2\,a_3\sin 2\,\alpha\,x - 4\,a_2\,a_4\cos 2\,\alpha\,x + 2\,\epsilon^{2ax}\,(a_3^2 + a_4^2)$
and this expression can be called Δ^2_t

$$\frac{1}{f}\int_0^f \Delta^2_t\,d\,x = \Delta_1^2$$

This can be reduced to the following

$$\alpha\,f\,\Delta_1^2 = \epsilon^{2af}(a_1^2 + a_2^2 - \epsilon_1^{2af}\,(a_3^2 + a_4^2) + 2\sin 2\,\alpha\,f\,(a_1\,a_3 - a_2\,a_4) + 2\cos 2\,\alpha\,f\,(a_1\,a_4 + a_2\,a_3) - (a_1^2 + a_2^2 - a_3^2 - a_4^2) - 2\,(a_1\,a_4 + a_2\,a_3) \qquad (19)$$

If in formula (19) we substitute the values of a_1, a_2, a_3 and a_4 from (18) we can reduce to the following

$$\alpha\,f\,\Delta_1^2 = 2\,K^2\,(p^2 + q^2)\Big[\,8\,\Big\{J\cos\theta\,I\cos\phi + J\sin\theta\,I\sin\phi + I^2(\sin^2\phi + \cos_2\phi)\Big\}\,\Big\{\sinh\alpha\,f\cosh\alpha\,f$$

$$- \sinh\alpha f \cos\alpha f - \cosh\alpha f \sin\alpha f + \sinh\alpha f \cos\alpha f \Big\}$$

$$4 + \Big\{ J^2 (\sin^2 \theta + \cos^2 \theta) \Big\} \Big\{ \sinh \alpha f \cosh \alpha f$$

$$+ \sin\alpha f \cos\alpha f \Big\} \Big] \qquad (20)$$

$$K = \tfrac{1}{4} \frac{\alpha}{a\gamma_1} \frac{1}{(p^2 + q^2)} (p^2 + q^2) = \frac{\cosh 2\alpha f - \cos 2\alpha f}{2}$$

also since Δ_1 is the square root of the mean squared value of Δ and Δ_0 is the maximum value of the average density in the wire therefore the ratio of loss is

$$R = 2 \frac{\Delta_1{}^2}{\Delta^2} \text{ whence (20) is}$$

$$R = \frac{\alpha f}{a^2\gamma_1{}^2 f^2 \Delta^2_0} \left[4 \frac{\{ J \; I \cos (\theta - \phi) + I^2 \}}{\operatorname{Cosh} 2 \alpha f - \operatorname{Cos} 2 \alpha f} \right.$$

$$\frac{\{ \sinh \alpha f - \sin \alpha f \}}{\operatorname{Cosh} 2 \alpha f - \operatorname{Cos} 2 \alpha f} \frac{\{ \cosh \alpha f - \cos \alpha f \}}{\operatorname{Cosh} 2 \alpha f - \operatorname{Cos} 2 \alpha f}$$

$$\left. \int + \frac{\{ \sinh 2 \alpha f + \sin 2 \alpha f \} J^2}{\operatorname{Cosh} 2 \alpha f - \operatorname{Cos} 2 \alpha f} \right] \qquad (21)$$

$a r_1 f \Delta_o = J$ and if we consider the case where the current J and I are in phase then $\cos (\theta - \phi)$ is unity. Let $\left[\frac{I}{J} + 1\right] = m$. Then for this special case we have conditions given in Mr. A. B. Field's paper and obtain the same formula.

$$\frac{R}{\alpha f} = \frac{4 (m^2 - m) (\sinh \alpha f - \sin \alpha f) (\cosh \alpha f - \cos \alpha f) + (\sinh 2 \alpha f + \sin 2 \alpha f)}{\operatorname{Cosh} 2 \alpha f - \operatorname{Cos} 2 \alpha f} \qquad (22)$$

m in this case is the number of conductors in a slot counting from the bottom of the slot up to and including the conductor under consideration.

4—INFINITELY STRANDED CONDUCTORS

The most general type of conductor is a stranded one. The limits of stranding extend from the solid bar or one strand to infinite strands. We have developed the general case of the solid bar. Let us consider the other limiting case or an infinitely stranded bar. The one self evident fact is that with zero depth of strand there is no circulating current within the strand, consequently any circulating current within the conductor must travel the full length of the conductor rather than just the length of the core as in the case of a solid bar. If L_t is the length of the conductor and L_1 the length of the iron we can let $\gamma = \frac{L_t}{L_1}$. Then equation (1) becomes in the case of an infinitely stranded bar

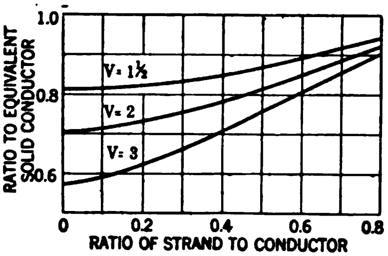
$$L_1 \frac{dv}{dx} = L_t \frac{d\Delta}{dx} \rho \quad \text{or} \quad \frac{dv}{dx} = \gamma \frac{d\Delta}{dx} \rho \tag{23}$$

Our general equation (2) can now be written

$$\frac{d^2V}{dx^2} = \frac{d^2\beta}{dxdt} 10^{-8} = 4\pi \frac{d\Delta}{dt} 10^{-9} = \frac{d^2\Delta}{dx^2} \gamma\rho \tag{24}$$

A little consideration will show at once that

$$\frac{d^2V}{dx^2} = \frac{d^2\Delta}{dx^2} \gamma\rho \text{ from (23) and } \frac{d^2V}{dx^2} = 4\pi \frac{d\Delta}{dt} 10^{-9}$$



independently of the value of I below the conductor under consideration. See equation (12) and note that

$$\frac{d^2{}_1V}{dx^2} = \text{zero}$$

With our general equation for Δ as given in (24)

that is $\frac{d^2\Delta}{dx^2} \gamma \rho = 4\pi \frac{d\Delta}{dt} 10^{-9}$ or $\frac{\omega}{2} \frac{d^2\Delta}{dx^2} = \frac{x^2}{r} \frac{d\Delta}{dt}$ (24)

our limiting conditions are the same as for equation (2) namely equation (11) and (12) and the only difference in the solution of the heating equivalent is the substitution in (24) of $\frac{\alpha^2}{\gamma}$ for α^2 in equation (3).

For an infinitely laminated conductor of the depth f we can take our losses from the solid bar of a depth of $\frac{f}{\sqrt{\gamma}}$. In the above we have taken the case where all strands are short-circuited at each end of one conductor or half a turn. In stranded conductors there are in general use two kinds of coils which differ from this construction. In these coils the strands are short-circuited at the start and finish of the completed coil. The coil itself may consist of one or more complete turns. The two types of coil differ in the relative location of the strands in successive conductors. In this paper the types are designated as "straight up" shown in Fig. 1 and "turned over" Fig. 2.

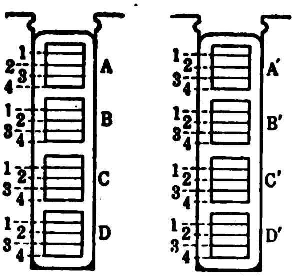


Fig. 1

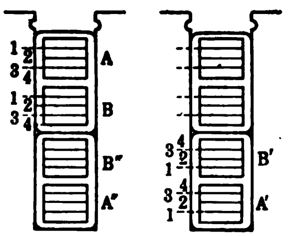


Fig. 2

In Fig. 1 the winding order is strand 1 coil A to strand 1 coil A' to strand 1 coil B to strand 1 coil B' and so forth.

The relative location of strand 1 in coil A A' B B' etc. is always the same.

In Fig. 2 the winding order is strand 1 coil A to strand 1 coil B' to strand 1 coil B to strand 1 coil B'. And the relative position of strand 1 in coil A and B is the same but is reversed with respect to the bottom of the slot in A' B'.

In the case shown in Fig. 1 we see that V varies in value depending upon whether we are considering conductor A B C, etc., but we find that $\frac{d}{dx}\left[\Sigma V - \gamma n \rho \Delta\right] = 0$ where n is the number of conductors per slot depthwise. $\frac{d^2V}{dx^2} = 4\pi\gamma \frac{d\Delta}{dt} 10^{-9}$ for any value of V with the same value of X no matter which conductor is considered.

Our fundamental equation for Δ is the same as in the previous case, that is $\frac{d^2V}{dx^2} = 4\pi \frac{d\Delta}{dt} 10^{-9} = \frac{d^2\Delta}{dx^2} \gamma \rho$ (24)

and our limiting conditions are $\Delta_o \sin \omega t = \frac{1}{f}\int_o^f \Delta\, dx$ and when $x = o$

$$r n \rho \left(\frac{d\Delta}{dx}\right)_{x=o} = 10^{-9}\, 4 \frac{\pi}{\alpha} \omega J \cos \omega t \left[1+2+3+ \quad +n-1\right]$$

$$\text{or} \quad \gamma \rho \frac{d\Delta}{dx} = 10^{-9}\, 4 \frac{\pi \omega}{\alpha} \left(\frac{n-1}{2}\right) J \cos \omega t \qquad (25)$$

These limits result in a solution for R the same as (22) except that $\frac{n-1}{2} J$ in (25) replaces I in equation (12). J is the current in the conductor. Therefore in place of (22) we get an equation in all respects similar except that $\frac{n-1}{2}$ replaces $m - 1$ or $\frac{n+1}{2}$ replaces m

$$R = \frac{4\left(\frac{n^2-1}{4}\right)\left(\begin{matrix}\sin h\alpha f\\ -\sin \alpha f\end{matrix}\right)\left(\begin{matrix}\cos h\alpha f\\ -\cos \alpha f\end{matrix}\right) + \left(\begin{matrix}\sinh 2\alpha f\\ +\sin 2\alpha f\end{matrix}\right)}{\alpha f \quad \cosh 2\alpha f - \cos 2\alpha f} \qquad (26)$$

and R is the equation of the ratio for the average loss of all conductors in the same slot.

Considering the case in Fig. 2 limited to infinite stranding and no phase displacement in currents in the conductors shown in the top and bottom coils in the same slot and we find

$$\frac{d}{dx}\left[\Sigma V - \gamma n \rho \Delta\right] = o \text{ or } \Sigma \frac{dv}{dx} \qquad n \rho \frac{d\Delta}{dx}$$

$\frac{d^2 V}{dx^2} = 4\pi\gamma \frac{d\Delta}{dt} 10^{-9}$ as before and our limiting equations are $\Delta_o \sin \omega t = \frac{1}{f}\int_o^f \Delta\, dx$ and $\Sigma \frac{dv}{dx}$ for lower half of slot when $X = O$ or $-$

$$10^{-9} \frac{4\pi}{\alpha} \omega J \cos \omega t \left[0 + 1 + 2 + \quad + \left(\frac{n}{2} - 1\right)\right]$$

and $\Sigma \frac{dv}{dx}$ for upper half of slot when $x = f$ or

$$-10^{-9} \frac{4\pi}{\alpha} \omega J \cos \omega t \left[n + (n-1) + (n-2) + \quad + \frac{n}{2} + 1\right].$$

$$\text{Therefore } \gamma \rho \frac{d\Delta}{dx} = 10^{-9} - \left(\frac{n+2}{4}\right) J \cos \omega t \frac{4\pi\omega}{\alpha} \qquad (27)$$

These limits result in a solution for R the same as (22) except that $-\frac{n+2}{4}$ replaces $(m-1)$and R is the equation for the ratio of the loss of all conductors in the same slot—

$$R=\frac{\alpha f\left(\frac{n^2-4}{16}\right)4\left(\begin{matrix}\sinh\alpha f\\ -\sin\alpha f\end{matrix}\right)\left(\begin{matrix}\cosh\alpha f\\ -\cos\alpha f\end{matrix}\right)+\left(\begin{matrix}\sinh 2\alpha f\\ +\sin 2\alpha f\end{matrix}\right)\alpha f}{\cosh 2\alpha f-\cos 2\alpha f} \qquad (28)$$

In both of these cases $\frac{f}{\sqrt{\gamma}}$ is used in place of f to get the proper loss ratio.

The development of this case of infinite stranding has been gone through is in order to show that R is of the same general form for all cases considered and that an equivalent depth of conductor can be used.

5—Approximate Formulas for Losses in Solid Bars

From formula (21)

$$R=\left[\frac{I}{J}\cos(\theta-\phi)+\frac{I^2}{J^2}\right]$$

$$\left[4\alpha f\frac{(\sinh\alpha f-\sin\alpha f)(\cosh\alpha f-\cos\alpha f)}{\cosh 2\alpha f-\cos 2\alpha f}\right]$$

$$+\left[\frac{\alpha f\sinh 2\alpha f+\sin 2\alpha f}{\cosh 2\alpha f-\cos 2\alpha f}\right]$$

We can for convenience let

$$M_s=4\,\alpha f\frac{(\sinh\alpha f-\sin\alpha f)\;(\cosh\alpha f-\cos\alpha f)}{\cosh 2\alpha f-\cos 2\alpha f}=$$

$$2\alpha f\frac{(\sinh\alpha f-\sin\alpha f)}{\cosh\alpha f+\cos\alpha f}\quad O_s=\alpha f\frac{\sinh 2\alpha f+\sin 2\alpha f}{\cosh 2\alpha f-\cos 2\alpha f}$$

Let us investigate these equations for M_s and O_s.

If we develop sinh x_1 sin x etc. in series we get

$$\text{Sinh}\,x=x+\frac{x^3}{\lfloor\underline{3}}+\frac{x_5}{\lfloor\underline{5}}+\frac{x^7}{\lfloor\underline{7}}\text{ etc.}$$

$$\text{Sin}\,x=x-\frac{x^3}{\lfloor\underline{3}}+\frac{x^5}{\lfloor\underline{5}}-\frac{x^7}{\lfloor\underline{7}}\text{ etc.}$$

$$\text{Cosh}\,x=1+\frac{x^2}{\lfloor\underline{2}}+\frac{x^4}{\lfloor\underline{4}}+\frac{x^6}{\lfloor\underline{6}}\text{ etc.}$$

$$\text{Cos}\,x=1-\frac{x^2}{\lfloor\underline{2}}+\frac{x^4}{\lfloor\underline{4}}-\frac{x^6}{\lfloor\underline{6}}\text{ etc}$$

If for convenience we use x for αf, M becomes

$$M_s = 2\ \frac{\left[\frac{x^3}{\lfloor 3} + \frac{x^7}{\lfloor 7}\text{etc.}\right] x}{1 + \frac{x^4}{\lfloor 4} + \frac{x^8}{\lfloor 8}\text{ etc.}} = \frac{x^4}{3} - 0.01x^8 \text{ approx.} \tag{29}$$

For all practical purposes when x is less than 1

$$M_s = \frac{x^4}{3} = \frac{1}{3}(\alpha f)^4$$

Similarly for O_s

$$O_s = \frac{\left[2x + 8\frac{x^3}{\lfloor 3} + \frac{32x^5}{\lfloor 5}\text{etc.}\right] x + \left[2x - \frac{8x^3}{\lfloor 3} + \frac{32x^5}{\lfloor 5}\text{ etc.}\right] x}{}$$

or since $x = \alpha f$

$$\left[1 + \frac{(2x)^2}{\lfloor 2} + \frac{(2x)^4}{\lfloor 4}\text{etc.}\right] - \left[1 - \frac{(2x)^2}{\lfloor 2} + \frac{(2x)^4}{\lfloor 4}\text{etc.}\right]$$

$$O_s = 1 + \frac{8}{90}(\alpha f)^4 \tag{30}$$

for all practical purposes when $\alpha f < 1$

In modern day machines we will find that αf is less than unity $\alpha^2 = \frac{2\pi\gamma_1}{\rho}$ $2\pi \sim 10^{-9}$ in *c.g.s.* system $\rho = 625\ 10^{-9}$ per cu. in. at zero deg. cent.

$$\alpha^2 = \frac{4\pi^2 \times 2.54}{625} \sim \gamma_1 \text{ at } 0° \quad \alpha = \frac{.4}{\sqrt{1 + .00427\,\theta}} \sqrt{\sim \gamma_1}$$

for θ deg. cent.

For high voltage machines γ_1 is approximately 0.6 and for $\sim\ = 60\alpha$ at 100 deg. cent. would be $0.334 \times 6 = 2$ abt.
αf is < 0.75 for a 3/8 in. deep strand even in case of 60 cycles.

Therefore for all cases which we care to consider values of O_s and M_s can be taken from formulas (29) and (30).

6—SOLUTION FOR CURRENTS IN STRANDS

In the paper up to the present stage we have considered only the two limiting cases of a stranded conductor, that is the solid bar and the infinitely laminated bar. We have found that an equivalent solid bar can be substituted for an infinitely laminated one, and that the equivalent depth of substituted bar is $\frac{1}{\sqrt{\gamma}}$ times the depth of the stranded conductor; γ is the ratio of total length of the conductor to the embedded length.

If we express in rectangular coordinates this relation using the values of the equivalent depth of bars as ordinates and the ratio of strand depth to conductor depth as abscissas we obtain two limiting points for each value of γ. It is reasonable to assume that these terminal points can be connected with a curve at least under certain limiting conditions and we shall now investigate this possibility.

So far we have a formula from which we can calculate the density of current at any point of a conductor carrying a current J of known value and of known phase θ provided we know the value of the current I below J and the phase of I or ϕ.

This formula is (10).

$$\Delta = 2\,\epsilon^{\alpha x}\Big[a_1 \sin(\alpha\, x + \omega\, t) + a_2 \cos(\alpha\, x + \omega\, t)\Big]$$

$$+\ 2\,\epsilon^{-\alpha x}\Big[a_3 \sin(\alpha\, x - \omega\, t) + a_4 \cos(\alpha\, x - \omega\, t)\Big]$$

a_1, a_2, a_3, and a_4 are constants dependent on the value of the current J and its phase angle θ in the conductor or strand and of the current I and its phase angle ϕ for the current below the strand. When we know J, θ,.I and ϕ we can write the loss ratio in the strand from formula (21)

$$R = \frac{1}{J^2}\Big[\Big\{ J\, I \cos(\theta - \phi) + I^2 \Big\} M s + J^2\, O_s\Big]$$

If we take the case of a stranded conductor where the number of strands is finite we can calculate the currents and phase displacements for the strands and the relative loss in each strand. Let us take the phase angle for a conductor current as θ_0 and the current as J_0. Let the conductor have any number of strands (say n) and designate the sin and cos components of these strands as J_n sin θ_n, J_{n-1} sin θ_{n-1}, J_n cos θ_n, J_{n-1} cos θ_{n-1}, etc. For convenience we will write these respectively ${}_sJ_n$, ${}_sJ_{n-1}$, ${}_cJ_n$, ${}_cJ_{n-1}$, etc.

In any stranded conductor between the points of common short circuit of all strands which go to make up a conductor, if any two strands are adjacent at any point they will be adjacent throughout their entire length. It is possible therefore to sum all the voltages in adjacent strand edges and equate them since the insulation thickness between strands can be taken as negligible. In this way 2 (n-1) equations can be obtained in solving 2 n unknown quantities (${}_sJ_n$ ${}_sJ_{n-1}$ - - - - ${}_cJ_n$ ${}_cJ_{n-1}$ - - - etc.) and the other two equations needed to evaluate all unknown quantities are $\sum_1^n {}_cJ_x = J_0 \cos \theta_0$ and $\sum_1^n {}_sJ_x = J_0 \sin \theta_0$. We have therefore a method for obtaining R for any stranded conductor provided we are able to manipulate it.

In Part 4 we considered the case of infinitely stranded coils and in Figs. 1 and 2 illustrated "straight up" and "turned over" coils. It may be well to point out here that there are various modifications of these types; for example, all strands which go to make up a conductor can be short circuited at the end of a half turn, a full turn or only at the end of a coil, where the coil may consist of one or more turns; also in the case of coils shown in Fig. 2, Part 4, the currents in the conductors $A'' B''$, etc., may be of different phase from those in A B, or in the event of only one coil per slot turned over there will not be any conductors $A'' B''$.

Treatment of all these factors will be considered in the remainder of this paper. The final formulas will give the equivalent loss ratio for such a group of conductors as will be included in a common insulating cell from ground. That is in the case of one coil side per slot the ratio for the entire slot will be given. While in the case of two coil sides per slot the upper and lower coil sides of this will be given separately.

Let us investigate the case of a coil wound straight up. The strand arrangement is then as shown in Fig. 1, Part 4. The winding order is strand 1 coil A to strand 1 coil A' etc. We can consider a full turn as our unit or since each conductor has relatively the same position in the second slot we can consider a single wire as the unit. Let l_t be length of the conductor, l_s the length of the iron, and $\frac{l_t}{l_i} = \gamma$. Then the length of the end winding $= \frac{l_t - l_i}{l_i} = \gamma - 1$.

In the bottom edge of the n^{th} strand and the top edge of the (n-1) strand the summation of e.m.fs. is zero, provided we neglect the flux that leaks through the insulation between strands. We will also assume that no unbalanced e.m.fs. are induced in the strands beyond the iron length of the conductor. Let V be the induced e.m.f. per unit length in the core; use ${}_{n-1}V_t$ to represent this e.m.f. at the top of the (n-1) strand, and ${}_nV_b$ to represent this e.m.f. at the bottom of the n^{th} strand, etc.

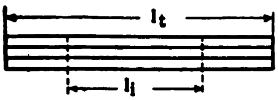
$$\text{Then } l_i\, {}_{n-1}V_t + \rho\, {}_{n-1}\Delta_t\, l_i + J_{n-1} \sin(\omega t + \theta_{n-1})\, \rho \frac{l_t - l_i}{a \gamma_1 h} =$$

$$l_i\, {}_nV_b + \rho\, {}_n\Delta_b\, l_i + J_n \sin(\omega t + \theta_n)\, \rho \frac{l_t - l_i}{a \gamma_1 h}$$

$$\text{or } {}_{n-1}\Delta_t + J_{n-1} \sin\ (\omega t + \theta_{n-1}) \frac{\gamma - 1}{a \gamma_1 h} = {}_n\Delta_b + J_n \sin$$

$$(\omega t + \theta_n) \frac{\gamma - 1}{a \gamma_1 h} \tag{31}$$

Where a is the width of the slot h is the depth of the strand, $a\, \gamma_1\, h$ is the section of the strand.



Equation (31) can be combined with equations (10) and (18) to give us relations involving J_{n1} and J_n θ_{n-1} and θ_n. From the general equation for Δ (10) we can conveniently express ${}_{n-1}\Delta_t$ and ${}_n\Delta_b$ at the two intervals of time represented by $\omega t = 0$ and $\omega t = \frac{\pi}{2}$. At the top of the strand in (10) αx is replaced by αh therefore $\Delta_t =$ $2\ \sin \alpha h\, (\epsilon^{ah} a_1 + \epsilon^{-ah} a_3) + 2 \cos \alpha h\, (\epsilon^{ah} a_2 - \epsilon^{-ah} a_4)$
when $\omega t = 0$ (32)
$\Delta_t = 2 \cos \alpha h\, (\epsilon^{ah} a_1 + \epsilon^{-ah} a_3) - 2 \sin \alpha h\, (\epsilon^{ah} a_2 + \epsilon^{-ah} a_4)$
when $\omega t = \frac{\pi}{2}$

At bottom of the strand in (10) αx is zero.
$\Delta_b = 2\,(a_2 - a_4)$ when $\omega t = O$ $\Delta_b = 2\,(a_1 + a_3)$
when $\omega t = \frac{\pi}{2}$ (33)

We can now put the values of a_1, a_2, a_3, and a_4 in equations (32) and (33).

In equation (18) we had formulas for a_1, a_2, etc. The value of J in (18) is the strand current and I is the total current below the strand. It will be noticed that these equations can be rewritten as follows:

$$\begin{aligned}
a_1 = K\ \{ &(p-q)\,{}_sI_t + (p+q)\,{}_cI_t - [(p+q)\,\epsilon^{-ah}\sin\alpha h \\
&+ (p-q)\,\epsilon^{-ah}\cos\alpha h]\,{}_sI_b + [(p-q)\epsilon^{-ah}\sin\alpha h - \\
&(p+q)\,\epsilon^{-ah}\cos\alpha h]\,{}_cI_b \\
a_2 = K\ \{ &(p+q)\,{}_sI_t - (p-q)\,{}_cI_t + [(p-q)\,\epsilon^{-ah}\sin\alpha h \\
&- (p+q)\,\epsilon^{-ah}\cos\alpha h]\,{}_sI_b + [(p+q)\,\epsilon^{-ah}\sin\alpha h \\
&+ (p-q)\,\epsilon^{-ah}\cos\alpha h]\,{}_cI_b \qquad (34) \\
a_3 = K\ \{ &(p-q)\,{}_sI_t + (p+q)\,{}_cI_t + [(p+q)\,\epsilon^{ah}\sin\alpha h \\
&- (p-q)\,\epsilon^{ah}\cos\alpha h]\,{}_sI_b - [(p-q)\,\epsilon^{ah}\sin\alpha h \\
&+ (p+q)\,\epsilon^{ah}\cos\alpha h]\,{}_cI_b \\
a_4 = K\ \{ &-(p+q)\,{}_sI_t + (p-q)\,{}_cI_t + [(p-q)\,\epsilon^{ah}\sin \\
&\alpha h + (p+q)\,\epsilon^{ah}\cos\alpha h]\,{}_sI_b + [(p+q)\,\epsilon^{ah}\sin \\
&\alpha h - (p-q)\,\epsilon^{ah}\cos\alpha h]\,{}_cI_b
\end{aligned}$$

Where ${}_sI_t$, ${}_cI_t$ are the sin and cos components respectively of all currents from the bottom of slot up to and including the current in the strand under consideration. Similarly I_b is the summation of currents below the strand under consideration.

By substitution (34) in equation (32) we get

$$\begin{aligned}
&2\sin\alpha h\ [\epsilon^{ah}a_1 - \epsilon^{-ah}a_3] + 2\cos\alpha h\,[\epsilon^{ah}a_2 - \epsilon^{-ah}a_4] \\
&\quad = 4K\ [\tfrac{1}{2}(d+e)\,{}_sI_t - \tfrac{1}{2}(d-e)\,{}_cI_t - (p+q) \\
&\quad {}_sI_b + (p-q)\,{}_cI_b] = \Delta_t,\ \omega t = 0 \\
&2\cos\alpha h\ [\epsilon^{ah}a_1 + \epsilon^{-ah}a_3] - 2\sin\alpha h\ [\epsilon^{ah}a_2 + \epsilon^{-ah}a_4] \\
&\quad = 4K\ [\tfrac{1}{2}(d-e)\,{}_sI_t + \tfrac{1}{2}(d+e)\,{}_cI_t - (p-q) \\
&\quad {}_sI_b - (p+q)\,{}_cI_b] = \Delta_t,\ \omega t = \frac{\pi}{2} \\
&2(a_2 - a_4) = 4K\,[(p+q)\,{}_sI_t - (p-q)\,{}_cJ_t - \tfrac{1}{2}(d+e) \\
&\qquad {}_sI_b + \tfrac{1}{2}(d-e)\,{}_cI_b] = \Delta_b,\ \omega t = o \\
&2(a_1 + a_3) = 4K\,[(p-q)\,{}_sI_t + (p+q)\,{}_cI_t - \tfrac{1}{2}(d-e) \\
&\qquad {}_sI_b - \tfrac{1}{2}(d+e)\,{}_cI_b] = \Delta_b,\ \omega t = \frac{\pi}{2} \qquad (35)
\end{aligned}$$

In these equations d is $\sin 2\alpha h = 2\sin\alpha h\cos\alpha h$

e is $\sinh 2\alpha h = 2\sinh\alpha h\cosh\alpha h$

also $(d+e) = 2\,[\sinh\alpha h\sin\alpha h(p-q) + \cosh\alpha h \cos\alpha h(p+q)]$

$-(d-e) = 2\,[\sinh\alpha h\sin\alpha h(p+q) - \cosh\alpha h \cos\alpha h(p-q)]$

The above expressions (35) for Δ_t and Δ_b when ωt is zero and $\frac{\pi}{2}$ are general equations and are true for any strand provided the proper currents are used in obtaining the values of ${}_sI_t$, ${}_cI_t$ and ${}_sI_b$, ${}_cI_b$. Consider the case of an even number of conductors per slot say m when we have two coil sides per slot. Let us evaluate the expression for the x and $(x-1)$ strands of the y^{th} coil above the center line of the slot. Where the currents in the bottom half of the slot are displaced by the angle β

$$\left.\begin{aligned}
{}_sI_t &= \sum_o^x {}_sJ + \frac{m}{2} J_o \sin\beta + (y-1)\,{}_sJ_o \\
{}_cI_t &= \sum_o^x {}_cJ + \frac{m}{2} J_o \cos\beta + (y-1)\,{}_cJ_o \\
{}_sI_b &= \sum_o^{x-1} {}_sJ + \frac{m}{2} J_o \sin\beta + (y-1)\,{}_sJ_o \\
{}_cI_b &= \sum_o^{x-1} {}_cJ + \frac{m}{2} J_o \cos\beta + (y-1)\,{}_cJ_o
\end{aligned}\right\}\quad \text{If } \theta_o \text{ is taken as zero, then } {}_sJ_o = 0,\ {}_cJ_o = J_o. \qquad \textbf{(36)}$$

At such time as ωt is zero, if we substitute values from (35) and (36) in (31) we obtain

$$\begin{aligned}
&-(p+q+2c)\,{}_sJ_x - (p+q-2c-d-e)\,{}_sJ_{x-1} \\
&\quad -(2p+2q-d-e)\sum_o^{x-2} {}_sJ + (p+q)\,{}_cJ_x + (p-g \\
&\quad -d+e)\,{}_cJ_{x-1} + (2p-2q-d+e)\sum_o^{x-2} {}_cJ \\
&\quad = \Big[2(p+q)-(d+e)\Big]\Big[\frac{m}{2} J_o \sin\beta\Big] - \Big[2(p \\
&\quad -q)-(d-e)\Big]\Big[(y-1)\,J_o + \frac{m}{2} J_o \cos\beta\Big]
\end{aligned}$$

At such time as $\omega t = \frac{\pi}{2}$ we get

$$\begin{aligned}
&-(p-q)\,{}_sJ_x - (p-q-d+e)\,{}_sJ_{x-1} - (2p-2q \\
&\quad -d+e)\sum_o^{x-2} {}_sJ - (p+q+2c)\,{}_cJ_x - (p+q-d \\
&\quad -e-2c)\,{}_cJ_{x-1} - (2p+2q-d-e)\sum_o^{x-2} {}_cJ = \Big[2(p \\
&\quad -q)-(d-e)\Big]\frac{m}{2} J_o \sin\beta + \Big[2(p+q)-(d+e)\Big] \\
&\quad \Big[(y-1)\,{}_cJ_o + \frac{m}{2} J_o \cos\beta\Big]
\end{aligned}$$

$$\text{Where } 2c = \frac{\gamma-1}{\alpha\gamma_1 h}\,\frac{1}{4K} = \frac{\gamma-1}{\alpha h}(p^2+q^2) \qquad \textbf{(36A)}$$

These two equations can be simplified and rewritten

$$\left.\begin{aligned}&(p+c)\,{}_sJ_x-(d-p+c)\,{}_sJ_{x-1}-(d-2p)\sum_{0}^{x-2}{}_sJ\\&+(q+c)\,{}_cJ_x-(e-q+c)\,{}_cJ_{x-1}-(e-2q)\sum_{0}^{x-2}{}_sJ\\&=K_1\quad(q+c)\,{}_sJ_x-(e-q+c)\,{}_sJ_{x-1}-(e-2q)\\&\sum_{0}^{x-2}{}_sJ-(p+c)\,{}_cJ_x+(d-p+c)\,{}_cJ_{x-1}+(d-2p)\\&\sum_{0}^{x-2}{}_cJ=K_2\end{aligned}\right\}\quad(37)$$

$$\left.\begin{aligned}\text{Where }K_1&=(d-2p)\left[\frac{m}{2}\,J_o\sin\beta\right]+(e-2q)\\&\quad\left[{}_cJ_o\,(y-1)+\frac{m}{2}\,J_o\cos\beta\right]\\K_2&=(e-2q)\left[\frac{m}{2}\,J_o\sin\beta\right]-(d-2p)\\&\quad\left[{}_cJ_o(y-1)+\frac{m}{2}\,J_o\cos\beta\right]\end{aligned}\right\}\quad(38)$$

Since we have taken any two strands the x and the (x-1) in the y^{th} conductor above the center line of a slot where there are two coil sides per slot carrying current with phase displacement or β°, therefore, we have a perfectly general solution for a strand in any position above the center line of a slot, for a coil wound straight up. For a strand below the center line y is counted from the bottom of the slot and terms $J_o\sin\beta$ and $J_o\cos\beta$ are zero. So we can write for this case

$$\left.\begin{aligned}&K_1-(e-2q)\left[(y-1)\,{}_cJ_o\right]\quad(e-2q)\left[\left(\frac{m}{2}-\right.\right.\\&\qquad\left.\left.y\right){}_cJ_o\right]\\&\qquad\qquad\qquad\text{or}\\&K_2=-(d-2p)\left[(y-1)\,{}_cJ_o\right]-(d-\\&\qquad2p)\left[\left(\frac{m}{2}-y\right){}_cJ_o\right]\end{aligned}\right\}\quad(38A)$$

if y is counted from center line of slot.

Obviously these same limitations apply to a case of one coil per slot where y is counted from bottom. It is also quite apparent that if we strand continuously that the only term which will change in K_1 and K_2 is the coefficients of ${}_cJ_o$ and that this will have the average value of $\sum_1^m (y - 1)$ or $\frac{m-1}{2}$ in the case of one coil per slot.

Equations (37) and (38) with modifications enable us to solve for the sin and cos components for any strand in any coil wound straight up.

The turned over coil can be solved in exactly the same manner. In Fig. 3 we show the relative position of the x and $(x-1)$ strands in the usual type of coil where two coil sides per slot are used.

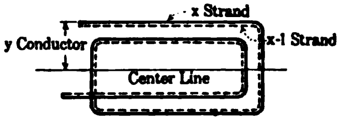


Fig. 3

Consider all strands which go to make up a conductor short-circuited each end of a full turn and assume that the currents in the bottom half of any slot are $\beta°$ out of phase with the currents in the top of the slot.

We get an expression for the e.m.fs. in the bottom edge of the x and top edge of the (x-1) strand summed around closed circuit similar to (31).

$$
{}_{n-1}\Delta_t + {}_{n-1}\overline{\Delta_b} + 2\, J_{n-1} \sin(\omega t + \theta)_{n-1} \frac{\gamma - 1}{a \gamma_1 h} = {}_n\Delta_b + {}_t\overline{\Delta}_n + 2\, J'_n \sin(\omega t + \theta_n) \frac{\gamma - 1}{a \gamma_1 h} \tag{39}
$$

in which ${}_{n-1}\overline{\Delta_b}$ is the density in the bottom of the (n-1) strand of the y^{th} conductor below the center line of slot, etc.

This equation can be combined with (35), using proper values for ${}_sI_t$, ${}_sI_b$, etc., for the strands above and below the center line of the slot.

In the case of a turned over coil these values for the x^{th} strand of the y^{th} conductor above center line are

$$
{}_sI_t = \sum_o^x {}_sJ + (y - 1)\, {}_sJ_o + \frac{m}{2} J_o \sin\beta
$$

$$
{}_cI_t = \sum_o^x {}_cJ + (y - 1)\, {}_sJ_o + \frac{m}{2} J_o \cos\beta
$$

$$
{}_sJ_b = \sum_o^{x-1} {}_sJ + (y - 1)\, {}_sJ_o + \frac{m}{2} J_o \sin\beta
$$

$$
{}_cI_b = \sum_o^{x-1} {}_cJ + (y - 1)\, {}_sJ_o + \frac{m}{2} J_o \cos\beta
$$

and for the same strand below the center line

$$
\begin{aligned}
{}_sI_t &= \left(\frac{m}{2} - y + 1\right) {}_sJ_o - \sum_{o}^{x-1} {}_sJ \\
{}_cI_t &= \left(\frac{m}{2} - y + 1\right) {}_cJ_o - \sum_{o}^{x-1} {}_cJ \\
{}_sI_b &= \left(\frac{m}{2} - y + 1\right) {}_cJ_o - \sum_{o}^{x} {}_sJ \\
{}_cI_b &= \left(\frac{m}{2} - y + 1\right) {}_sJ_o - \sum_{o}^{x} {}_cJ, \text{ etc.}
\end{aligned}
$$

Substituting these values in 39 and combining with 35 we obtain when ωt = zero.

$$
\begin{aligned}
&-(p + q + 2c)\, {}_sJ_x - (p + q - d - e - 2c)\, {}_sJ_{x-1} - (2p + 2q - d - e) \sum_{o}^{x-2} {}_sJ + (p - q)\, {}_cJ_x + (p - q - \\
&d + o)\, {}_sJ_{x-1} + (2p - 2q - d + e) \sum_{o}^{x-2} {}_cJ = \left[2(p + q) - (d + e)\right] \left[\frac{m}{4} J_o \sin\beta\right] - \left[2(p - q) - (d + e)\right] \\
&\left[(y - \frac{m}{4} - 1)\, {}_cJ_o + \frac{m}{4} + J_o \cos\beta\right]
\end{aligned}
$$

and when $\omega t = \frac{\pi}{2}$

$$
\begin{aligned}
&-(p - q)\, {}_sJ_x - (p - q - d + e)\, {}_sJ_{x-1} - (2p - 2q - d + e) \sum_{o}^{x-2} {}_sJ - (p + q + 2c)\, {}_cJ_x - (p + q - d - e - 2c)\, {}_cJ_{x-1} \\
&- (2p + 2q - d - e) \sum_{o}^{x-2} {}_cJ = \left[2(p - q) - (d + e)\right] \left[\frac{m}{4} J_o \sin\beta\right] + \left[2(p + q) - (d + e)\right] \left[(y - \frac{m}{4} - 1)\, {}_cJ_o \right. \\
&\left. + \frac{m}{4} J_o \cos\beta\right]
\end{aligned}
$$

These two equations rearranged and rewritten are the same as (37) except the values of K_1 and K_2 will be for a turned over coil which is short-circuited at the end of each turn.

$$
\left.
\begin{aligned}
K_1 &= (d - 2p)\left[\frac{m}{4} J_o \sin\beta\right] + (e - 2q)\left[(y - \frac{m}{4} - 1)\, {}_cJ_o + \frac{m}{4} J_o \cos\beta\right] \\
K_2 &= (e - 2q)\left[\frac{m}{4} J_o \sin\beta\right] - (d - 2p)\left[(y - \frac{m}{4} - 1)\, {}_cJ_o + \frac{m}{4} J_o \cos\beta\right]
\end{aligned}
\right\} \quad (40)
$$

For the case of one coil per slot turned over short-circuited at the end of each turn.

$$\left.\begin{aligned} K_1 &= (e-2q)\left[y - \frac{m}{4} - 1\right] {}_cJ_o \\ K_2 &= -(d-2p)\left[y - \frac{m}{4} - 1\right] {}_cJ_o \end{aligned}\right\} \text{Since } J_o \sin\beta \text{ and } J_o\cos\beta \text{ are zero, } \frac{m}{2} \text{ in this case is number of conductors per slot.} \qquad (40A)$$

For continuous stranding two coils per slot, the coefficient of ${}_cJ_o$ is the only term to change and this becomes the average value of $\sum_o^{\frac{m}{2}} \left(y - \frac{m}{4} - 1\right)$

$$\left.\begin{aligned} K_1 &= (d-2p)\frac{m}{4} J_o\sin\beta + (e-2q)\left[\frac{m}{4} J_o\cos\beta - \tfrac{1}{2}\,{}_cJ_o\right] \\ K_2 &= (e-2q)\frac{m}{4} J_o\sin\beta - (d-2p)\left[\frac{m}{4} J_o\cos\beta - \tfrac{1}{2}\,{}_cJ_o\right] \end{aligned}\right\} \qquad (40B)$$

And for one coil per slot continuous stranding,

$$K_1 = (e-2q)\left[-\tfrac{1}{2}\,{}_cJ_o\right] \quad K_2 = -(d-2p)\left[-\tfrac{1}{2}\,{}_cJ_o\right] \qquad (40C)$$

We have now determined the equations necessary for the solution of the sin and cos components for any strand in any of the coil arrangements of the types shown in Figs. 1 and 2. The solution is made at once from equation (37) and the conditions that $\sum_o^n {}_sJ = \text{zero}$ and $\sum_o^n {}_cJ = J_o$ by the aid of determinants.

Equation (37) will be simplified if we note that when $c = 0$ we have a solid bar and that when c is not zero that $(p + c)$ and $(q + c)$ are substitutes for p and q and $(d + 2c)$ and $(e + 2c)$ for d and e. We can omit all reference to c, therefore provided we introduce it later on as indicated above.

Our determinant for two strands would be.

$$\left\| \begin{array}{lllll} p, & -(d-p), & q, & -(e-q) & K_1 \\ 1, & 1 & o, & o & o \\ q, & -(e-q), & p, & (d-p) & K_2 \\ o, & o, & 1, & 1 & J_o \end{array} \right\|$$

And the solution is

$$\left.\begin{aligned}
{}_sJ_2 &= \frac{K_1 d + K_2 e + J_o(p\,e - d\,q)}{d^2 + c^1} \\
{}_sJ_1 &= -\,{}_sJ_2 \\
{}_cJ_1 &= \frac{-\,K_{1\,e} + K_2 d + J_o(d\,p + e\,q)}{d^2 + e^2} \\
{}_cJ_2 &= J_o - {}_cJ_1
\end{aligned}\right\} \quad (41)$$

Where K_1 and K_2 come from terminal conditions for three strands our determinant is

$$\left\| \begin{matrix}
p & (d-p) & -(d-2p) & q & -(e-q) & -(e-2q) & K_1 \\
o & p & -(d-p) & o & q & -(e-q) & K_1 \\
1 & 1 & 1 & o & o & o & o \\
q & -(e-q) & -(e-2q) & p & (d-p) & (d-2p) & K_2 \\
o & q & -(e-q) & o & -p & d-p & K_2 \\
o & o & o & 1 & 1 & 1 & J_o
\end{matrix} \right\|$$

We will use the following expressions in the solution

$E = d\,p + e\,q$ $\qquad W - F(Q + R)$

$F = p\,e - d\,q$ $\qquad X = Q^2 + R^2 + 2\,F^2 - Z\,E^2$

$Q = d^2 + e^2$ $\qquad Y = F^2 + R^2 - E^2$

$R = p^2 + q^2$ $\qquad Z = E(Q - R)$

$V - 2\,E\,F$

In the equations for E, F, Q, R, etc. we must remember that for laminated windings we have to add c to p and q and 2 c to d and e in evaluating these symbols.

With this understanding we give the equations for the sin and cos components of the three strands. The evaluation of these terms depends on the terminal conditions affecting K_1 and K_2

$$\left.\begin{aligned}
{}_sJ_3 &= \frac{W\,J_o + [K_1(d-p) + K_2(e-q)]\,[Q+R+2\,E]}{X} \\
{}_sJ_2 &= -\,\frac{W - V}{X}\,J_o \\
{}_sJ_1 &= -\,\frac{V}{X}\,J_o - \frac{[K_1(d-p)+K_2(e-q)]\,[Q+R+2E]}{X} \\
{}_cJ_3 &= \frac{X-Z}{X}\,J_o + \frac{[K_1(e-q)-K_2(d-p)]\,[Q+R+2E]}{X} \\
{}_sJ_2 &= \frac{Y+Z}{X}\,J_o \\
{}_cJ_1 &= -\,\frac{Y}{X}\,J_o - \frac{[K_1(e-q)-K_2(d-p)]\,[Q+R+2E]}{X}
\end{aligned}\right\} \quad (42)$$

If we let Jx represent the current in any strand the x*th* and J_o the conductor current then we can write from (21)

$\frac{J_x^2}{n^2} R = \left[\left\{ J_x I_b \cos(\theta_x - \phi_b) + I_b^2 \right\} M_s + J_x^2 O_s \right]$ as loss ratio for *xth* strand and for the conductor

$$\left.\begin{array}{l} \frac{J_o^2}{n^2} R = \frac{1}{n} \sum_o^n \left\{ \left[{}_sJ_x\, {}_sI_b + {}_cJ_x\, {}_cI_b + {}_sI_b^2 + {}_cI_b^2 \right] M_s + \right. \\ \quad \left. \left[{}_sJ_x^2 + {}_sJ_x^2 \right] O_s \right. \end{array} \right\} (43)$$

By substituting the values of sin and cos components of the strand currents from equations (41) and (42) in equation (43) we can secure the average value of the losses for various stranding conditions shown by different values of K_1 and K_2

7—EDDY CURRENTS IN CONDUCTORS OF TWO STRANDS

The average loss in top and bottom coils in a slot having two coil sides is given below, covering the case where the conductors are solid, and where the currents in the bottom half of the slots are displaced from those in the top half by the angle β.

The expression for loss in any one bar is·

$$R = \frac{1}{J_o^2} \left\{ \left[J_o I_b \cos(\theta_o - \phi_b) + I_b^2 \right] M_s + J_o^2 O_s \right\} \quad (44)$$

If the bar is located above the center line of the slot in the position y, then

$$J_o \cos\theta_o = J_o \qquad J_o \sin\theta_o = \text{o}$$

$$I_b \cos\phi_b = \frac{m}{2} J_o \cos\beta + (y-1) J_o \quad I_b \sin\phi_b = \frac{m}{2} J_o \sin\beta$$

$$R = \left\{ \frac{m}{2} \cos\beta + (y-1) + \frac{m^2}{4} + m(y-1)\cos\beta + (y-1)^2 \right\} M_s + O_s \quad (45)$$

This is the equation for the loss ratio in the *y*th bar above the center line of the slot.

The loss ratio for the top half of the slot is $\frac{2}{m} \sum_o^{\frac{m}{2}} R = R_t$

$$R_t = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos\beta \right] M_s + O_s \quad (46)$$

The general equation for the loss ratio in the bottom half of the slot is the same as (**45**) except that the terms containing sin β and cos β are zero.

$$R_b = \left[\frac{m^2 - 4}{12}\right] M_s + O_s \qquad (\mathbf{47})$$

The $\sum\limits_1^m (x) = \dfrac{m(m+1)}{2}$

and the $\sum\limits_1^m x^2 = \dfrac{1}{6}\, m(m+1)(2m+1)$

We have developed the formulas (46) and (47) as we wish to use them for standard reference as regards the form for the equations developed for conductors of two strands.

Calculation of the loss ratio for any conductor, say the y conductor, above the center line of the slot, with m total conductors, each consisting of two strands in parallel, is given below. The general case is chosen where the strands are short-circuited at the end of each conductor, and when there are two coil sides per slot with the current in the bottom half of the slot, β degrees out of phase with the currents in the top half. The loss ratio for any strand is given in (44). We have, therefore, for the first and second strands of the y conductor above the center line of the slot

$$\mathrm{M}_s \Big[{}_sJ_2 \left({}_sJ_1 + \frac{m}{2}\, J_o \sin\beta\right) + {}_cJ_2 \left({}_cJ_1 + \frac{m}{2}\, J_o \cos\beta\right.$$

$$+ (y-1)\, J_o + \left({}_sJ_1 + \frac{m}{2}\, J_o \sin\beta\, \mathrm{M}^2\right)$$

$$+ \left({}_cJ_1 + \frac{m}{2}\, J_o \cos\beta + (y-1)\, J_o\right)^2 \Big]\, {}_sJ^2_2 + {}_cJ_2^2]\, O_s$$

$$+ M_s \Big[{}_sJ_1 \left(\frac{m}{2}\, J_o \sin\beta\right) + {}_cJ_1 \left(\frac{m}{2}\, J_o \cos\beta\right.$$

$$\left. + (y-1)\, J_o\right) + \left(\frac{m}{2}\, J_o \sin\beta\right)^2 + \left(\frac{m}{2}\, J_o \cos\beta\right.$$

$$\left. + (y - 1J_o)\right)^2 \Big] + \Big[{}_sJ_1 + {}_cJ_1 \Big]\, O_s = R\frac{J_o^2}{n}$$

Since ${}_sJ_1 + {}_sJ_2 + = O$ and ${}_cJ_1 + {}_cJ_2 = J_o$ we can reassemble the above and simplify to an expression in which we keep separate the terms which involve cos β and sin β

$$\frac{R\, J_o^2}{n} = M_s \Big[J_o\, {}_cJ_1 (2y - 1) + J_o^2 (2y^2 - 3y + 1)$$

$$+ J_o m\,({}_cJ_1 \cos\beta + {}_sJ_1 \sin\beta) + J_o^2 \Big(\frac{m}{2}\cos\beta$$

$$+ \frac{m^2}{2} + 2(y-1)\,m\cos\beta\Big)\Big]$$

$$+ [J_o^2 - 2\,J_o\,{}_cJ_1 + 2\,{}_cJ_1^2 + 2\,{}_sJ_1^2]\,O_s \qquad \textbf{(48)}$$

In evaluating the above ${}_sJ_1$ and ${}_cJ_1$ are taken from (**41**) and k_1 and k_2 are taken from (**38**) for the top half of the slot.

Let $\dfrac{p\,e - d\,q}{d^2 + e^2} = H$ and $\dfrac{d\,p + e\,q =}{d^2 + e^2} = L$

Then (**48**) after substitution becomes

$$R\,\frac{J_o^2}{n} = {}_1J\,M_s\Big\{(L\,m^2 + 2\,L\,m\cos\beta + L)$$

$$+ (y-1)(4\,L\,m\cos\beta + 4\,L) + (y-1)^2\,4\,L\Big\}$$

$$+ J_o^2\Big\{1 + m\cos\beta - 4\,L\,m\cos\beta - 2\,L$$

$$+ 2\,H^2 m^2 + \frac{m^2}{2} + 2\,L^2 m^2 - 2\,L\,m^2 + 2\,L^2$$

$$+ 4\,L\,m^2\cos\beta + 2\,H^2 + 4\,H^2 m\cos\beta$$

$$+ (y-1)(2\,m\cos\beta - 8\,L\,m\cos\beta - 8\,L$$

$$+ 8\,L^2 m\cos\beta + 8\,L^2 + 8\,H^2 m\cos\beta$$

$$+ 8\,H^2 + 2) + (y-1)^2(2 + 8\,L^2 - 8\,L + 8\,H^2)\Big\}\,O_s$$

and $R_t = \dfrac{m}{2}\,\overset{\Sigma}{\underset{1}{}}\,R$ which reduces to

$$\frac{R_t}{n} = \Big[\frac{4}{3}\,L\,m^2 + L\,m^2\cos\beta - \frac{1}{3}\,L\Big]\,M_s$$

$$+ \Big[(H^2 + L^2)\Big(\frac{8}{3}\,m^2 + 2\,m^2\cos\beta - \frac{2}{3}\Big) - \Big(\frac{8}{3}\,L\,m^2$$

$$+ 2\,L\,m^2\cos\beta - \frac{2}{3}\,L\Big) + \frac{2}{3}\,m^2 + \frac{m^2}{2}\cos\beta$$

$$+ \frac{1}{3}\Big)\Big]\,O_s$$

If we reduce this formula to the same form as (**46**)

$$\frac{R_t}{n} = 4\Big[\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta\Big]\,\Big[L\,M_s\Big] + L\,M_s$$

$$+ 4\Big[\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta\Big]\,\Big[2\,(H^2 + L^2) - 2\,L$$

$$+ \tfrac{1}{2}\Big]\,O_s + \Big[2(H^2 + L^2) - 2\,L + 1\Big]\,O_s$$

$$R_t = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos\beta\right] 4\,n \Big[L\,M_s$$

$$+ 2(H^2 + L^2)\,O_s - 2\,L\,O_s + \tfrac{1}{2}\,O_s\Big]$$

$$+ n\Big[L\,M_s + 2(H^2 + L^2)\,O_s - 2\,L\,O_s + O_s\Big] \qquad (49)$$

$$\text{Let } 4\,n\Big[L\,M_s + 2(H^2 + L^2)\,O_s - 2\,L\,O_s + \tfrac{1}{2}\,O_s\Big] = M^1{}_c$$

$$\text{Let } n\Big[L\,M_s + 2(H^2 + L^2)\,O_s - 2\,L\,O_s + O_s\Big] = O^1$$

$$R_t = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos\beta\right] M^1{}_c + O^1{}_c$$

Similarly, for the bottom half of the slot, our general term is the same as (48), except that those terms which have a factor sin β or cos β are zero.

$$R\,\frac{J_o{}^2}{n} = M_s\Big[J_o\,{}_cJ_1(2\,y - 1) + J_o{}^2(2\,y_2 - 3\,y + 1)\Big]$$

$$+ \Big[J_o{}^2 - 2\,J_o\,{}_cJ_1 + 2\,{}_cJ_1{}^2 + 2\,{}_sJ_1{}^2\Big]\,O_s$$

s the general term for the bottom half of the slot. In evaluating this expression, ${}_sJ_1$ and ${}_sJ_2$ are taken from (41), and K_1 and K_2 from (38A).

$$R_b = \left[\frac{\mathrm{M}m^2 - 4}{12}\right] M^1{}_c + O^1{}_c$$

which is of the same form as equation (47). Equations (49) and (50) can be simplified by following out the same method of development which we have used in Part 5 and by limiting $\alpha\,h$ to values less than unity.

$$H = \frac{(p + c)(e + 2\,c) - (q + c)(d + 2\,c)}{(d + 2c)^2 + (e + 2\,c)^2}$$

$$L = \frac{(p + c)(d + 2\,c) + (q + c)(e + 2\,c)}{(d + 2\,c)^2 + (e + 2\,c)^2}$$

$$2\,c = \frac{\gamma - 1}{\alpha\,h}(p^2 + q^2) = 2\alpha\,h(\gamma - 1)$$

$$p^2 + q^2 = \frac{\cosh 2\alpha\,h - \cos 2\alpha\,h}{2}$$

$$= \tfrac{1}{2}\Bigg[\Bigg(1 + \frac{2^2(\alpha\,h)^2}{\underline{|2}} + \frac{(2\alpha\,h)^4}{\underline{|4}} + \Bigg)$$

$$-\left(1-\frac{(2\alpha h)^2}{\lfloor\underline{2}}+\frac{(2\alpha h)^4}{\lfloor\underline{4}}-1\right)\Bigg]$$

$$= 2(\alpha h)^2 + \frac{8}{90}(\alpha h)^6,\ \text{etc.},\ = 2(\alpha h)^2\ \text{approx.}$$

$$H \quad = \frac{4(\alpha h)^4\gamma}{8(\alpha h)^2\gamma^2} = \tfrac{1}{2}\frac{(\alpha h)^2}{\gamma}$$

$$L \quad = \frac{4(\alpha h)^2\gamma^2}{8(\alpha h)^2\gamma^2} = \tfrac{1}{2}$$

$p \quad = \cosh\alpha h \sin\alpha h.$

$q \quad = \sinh\alpha h \cos\alpha h.$

$d \quad = \sin 2\alpha h.$

$e \quad = \sinh 2\alpha h.$

When the stranding of the straight up coil is continuous, the general term is the same as equation (48), omitting the β terms and counting y from the bottom of the slot.

$$R\frac{J_o^2}{n} = M_s\left[J_o\,{}_cJ_1(2y-1) + J_o^2(2y^2-3y+1)\right]$$

$$+\left[J_o^2 - 2J_o\,{}_cJ_1 + 2\,{}_cJ_1^2 + 2\,{}_sJ_1^2\right]O_s$$

$$K_1 = (e-2q)\left(\frac{m-1}{2}\right)J_o$$

$$K_2 = -(d-2p)\left(\frac{m-1}{2}\right)J_o$$

$$R_{av}\frac{J_o^2}{n} = \frac{1}{m}\sum_1^m R$$

$$R_{av} = n^2\left[\frac{m^2-1}{12}\right]M_s + \left[\frac{m^2-1}{4}\right]M^1_c + O^1_c$$

For a coil of two strands "turned over" and short-circuited at the end of each full turn with currents in the bottom half of the slot displaced β° K_1 and K_2 are taken from (40). The general equation for the loss ratio in the y conductor above the center line is (48), and after substituting and $\sum_1^{\frac{m}{2}} R$, the average value in the top of the slot will be

$$R_t = n\,M_s\left[\frac{H\,m^2}{4}\sin\beta\right] + n^2 M_s\left[\frac{m^2-1}{3} - \frac{m^2-4}{12}\right.$$

$$\left.+\frac{m^2}{4}\cos\beta\right] + \left[\frac{m^2-4}{12}\right]M^1_c + O^1_c$$

The general equation for R in bottom of the slot is the same as (48), except that J_1 and J_2 are interchanged, and that all terms containing sin β or cos β are zero.

$$R_b = n\,M_s\left[-\frac{H\,m^2}{4}\sin\beta\right] + M_c\left[\frac{m^2-4}{12}\right] + O_c$$

In the case of turned-over coils, where the stranding is continuous from the start to the finish of the coil, and there are two coils per slot, K_1 and K_2 are taken from (40B).

$$R_t = n\,M_s\left[\frac{H\,m^2}{4}\sin\beta\right] + n^2\,M_s\left[\frac{m^2-1}{3} - \frac{m^2-4}{16}\right.$$
$$\left. + \frac{m^2}{4}\cos\beta\right] + \left[\frac{m^2-4}{16}\right] M^1{}_c + O^1{}_c$$

$$R_b = n\,M_s\left[-\frac{H\,m^2}{4}\sin\beta\right] + n^2\,M_s\left[\frac{n^2-4}{12}\right.$$
$$\left. - \frac{n^2-4}{16}\right] + \left[\frac{m^2-4}{16}\right] M^1{}_c + O^1{}_c$$

In the case of one coil per slot turned over, short-circuited at the end of each turn, K_1 and K_2 are from (40A); $\frac{m}{2}$ is the number of conductors per slot.

If $2\,m_1 = m$ where m_1 is the number of conductors per slot, then

$$R_{av} = n^2\,M_s\left[\frac{m_1-1}{3} - \frac{m_1-4}{12}\right]$$
$$+ \left[\frac{m^2-4}{12}\right] M^1{}_c + O^1{}_c$$

For one coil per slot turned over, and stranded continuous, K_1 and K_2 are taken from (40C).

$$R_{av} = n\,M_s\left[\frac{m^2-1}{6}\right] + n\,\tfrac{1}{2}\,O_s.$$

Where m is twice the number of conductors per slot,

$$R_{av} = M_s\left(\frac{s^2-1}{3}\right) + O_s$$

Where s is the total number of strands per coil depthwise.

8—EDDY CURRENTS IN CONDUCTORS OF THREE STRANDS

Consider the case of two coils per slot where there are m total conductors and the currents in the bottom conductor are displaced by an angle from those in the top conductors, let each conductor consist of three strands depthwise and let all strands be short-circuited at each end of the conductor. Take the case of the yth conductor above the center-line of the slot

$$M_s \Bigg\{ {}_sJ_3 \Bigg[{}_sJ_2 + {}_sJ_1 + \frac{m}{2} J_o \sin\beta \Bigg] + {}_cJ_3 \Bigg[{}_cJ_2 + {}_cJ_1$$

$$+ (y - 1) J_o + \frac{m}{2} J_o \cos\beta \Bigg] + \Bigg[{}_sJ_2 + {}_sJ_1$$

$$+ \frac{m}{2} J_o \sin\beta \Bigg]^2 + \Bigg[{}_cJ_2 + {}_cJ_1 + (y - 1) J_o$$

$$+ \frac{m}{2} J_o \cos\beta \Bigg]^2 \Bigg\} + \Bigg\{ {}_sJ_3^2 + {}_cJ_3^2 \Bigg\} O_s + M_s \Bigg\{ {}_sJ_2 \Bigg[{}_sJ_1$$

$$+ \frac{m}{2} J_o \sin\beta \Bigg] + {}_cJ_2 \Bigg[{}_cJ_1 + (y - 1) J_o + \frac{m}{2} J_o \cos\beta \Bigg]$$

$$+ \Bigg[{}_sJ_1 + \frac{m}{2} J_o \sin\beta \Bigg]^2 + \Bigg[{}_cJ_1 + (y - 1) J_o$$

$$+ \frac{m}{2} J_o \cos \mathrm{B} \Bigg]^2 \Bigg\} + \Bigg\{ {}_cJ_2^2 + {}_sJ_2^2 \Bigg\} O_s$$

$$+ M_s \Bigg\{ {}_sJ_1 \Bigg[\frac{m}{2} J_o \sin\beta \Bigg] + {}_cJ_1 \Bigg[(y - 1) J_o$$

$$+ \frac{m}{2} J_o \cos\beta \Bigg] + \Bigg[\frac{m}{2} J_o \sin\beta \Bigg]^2 + \Bigg[(y - 1) J_o$$

$$+ \frac{m}{2} J_o \cos\beta \Bigg]^2 \Bigg\} + \Bigg[{}_sJ_1^2 + {}_cJ_1^2 \Bigg] O_s = \frac{R J_o^2}{n}$$

Since ${}_sJ_1 + {}_sJ_2 + {}_sJ_3 = 0$ and ${}_cJ_1 + {}_cJ_2 + {}_cJ_3 = J_o$ we can rewrite this

$$M_s \Bigg\{ \Bigg[- {}_sJ_1 \, {}_sJ_3 + J_o (2 \, {}_cJ_1 + {}_cJ_2) - {}_cJ_1 \, {}_cJ_3$$

$$+ 2 (y - 1) J_o (2 \, {}_cJ_1 + {}_cJ_2) + (y - 1) J_o^2$$

$$+ 3 (y - 1)^2 J_o^2 \Bigg] + \Bigg[(2 \, {}_sJ_1 + {}_sJ_2) \, m J_o \sin\beta$$

$$+ \tfrac{3}{4}\, m^2 J_o^2 (\sin^2\beta + \cos^2\beta) + \frac{m}{2} J_o \cos\beta$$

$$+ (2\,{}_cJ_1 + {}_cJ_2)\, m\, J_o \cos\beta + 3\, m\, (y - 1)\, J_o \cos\beta \Big]\Big\}$$

$$+ \Big\{ {}_sJ_1^{\,2} + {}_sJ_2^{\,2} + {}_sJ_3^{\,2} + {}_sJ_1^{\,2} + {}_cJ_2^{\,2} + {}_cJ_3^{\,2} \Big\} O_s = \frac{R\, J_o^{\,2}}{n} \qquad (60)$$

This equation is for the loss in the yth conductor above the center-line of the slot; y is counted from the center-line upwards. If we want to consider a term below the center-line it is the same for straight up coil except that y is counted from the bottom, the β terms are zero. For a turned over coil below the center-line the expression changes in the M_s term due to the fact that the position of J_3 and J_1 are interchanged, that $\left(\frac{m}{2} - y\right)$ replaces $(y - 1)$ and also that β terms are zero.

Bottom term is

$$M_s \Big\{ - {}_sJ_1\, {}_sJ_3 + J_o (2\,{}_cJ_3 + {}_cJ_2) - {}_cJ_1\, {}_cJ_3$$

$$+ 2 \left(\frac{m}{2} - y\right) J_o (2\,{}_cJ_3 + {}_cJ_2) + \left(\frac{m}{2} - y\right) J_o^2$$

$$+ 3 \left(\frac{m}{2} - y\right) {}^2J_o^2 \Big\} + \Big\{ {}_sJ_1^{\,2} + {}_sJ_2^{\,2} + {}_sJ_3^{\,2}$$

$$+ {}_sJ_1^{\,2} + {}_cJ_2^{\,2} + {}_cJ_3^{\,2} \Big\} O_s = \frac{R\, J_o}{n} \qquad (61)$$

For the case of a conductor short-circuited at each end we have the values of ${}_sJ_1$, ${}_sJ_2$, etc., from formula (42) where K_1 and K_2 are defined by (38) for the top half of the slot, and by 38-a for the bottom half.

The following transformations are convenient

$$\Big[(d - p)(d - 2p) + (e - q)(e - 2q)\Big]$$

$$\Big[Q + R + 2E\Big] = X + Y - Z$$

$$\Big[(d - p)(e - 2q) - (e - q)(d - 2p)\Big]$$

$$\Big[Q + R + 2E\Big]_{\cdot} = W + V$$

$$\frac{X+Y-Z}{X} = 1 - \frac{Z-Y}{X}$$

$$\frac{Z-Y}{X} = \frac{E(Q-R) - (F^2+R^2-E^2)}{Q^2+R^2+F^2-2E^2}$$

Where d and e are written for $(d+2c)$ and $(e+2c)$ and p and q for $p+c$ and $q+c$. By developing in series it will be found that the above reduce to

$$\frac{Z-Y}{X} = \frac{4(\alpha h)^2\gamma^2\Big[8(\alpha h)^2\gamma^2 - 2(\alpha h)^2\gamma^2\Big] + 16(\alpha h)^4\gamma^4 - 4(\alpha h)^4\gamma^4}{64(\alpha h)^4\gamma^4 + 4(\alpha h)^4\gamma^4 - 32(\alpha h)^4\gamma^4} = 1$$

$$\text{and } \frac{X+Y-Z}{X} = 0$$

The average value $\sum_{1}^{\frac{m}{2}} R$ in formula (60) gives

$$n M_s \left\{ \frac{WV+XY-YZ}{X^2} + \left(\frac{W+V}{X}\right)^2 \left[\frac{m^{2-1}}{3} \right.\right.$$

$$\left.\left. + \frac{m^2}{4}\cos\beta\right] + m^2 + \frac{3}{4} m^2 \cos\beta \right\} + \left\{ \left(\frac{X-Z}{X}\right)^2 \right.$$

$$+ \left(\frac{Y+Z}{X}\right)^2 + \left(\frac{W-V}{X}\right)^2 + \frac{W^2+V^2+Y^2}{X^2}$$

$$\left. + 2\left(\frac{W+V}{X}\right)^2 \left(\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta\right) \right\} O_s\, n = R_t$$

$$\left[\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta\right] \left\{ M_s\, n \left[\left(\frac{W+V}{X}\right)^2 + 3\right] \right.$$

$$\left. + 2\left(\frac{W+V}{X}\right)^2 O_s\, n \right\} + \left\{ \frac{WV+XZ-YZ}{X^2} M_s \right.$$

$$+ \left[\left(\frac{Y+Z}{X}\right)^2 + \left(\frac{X-Z}{X}\right)^2 + \left(\frac{W-V}{X}\right)^2\right.$$

$$\left.\left. + \frac{W^2+V^2+Y^2}{X^2}\right] O_s \right\} n = R_t \qquad (61)$$

$$\text{Let } n\left\{ M_s\left[\left(\frac{W+V}{X}\right)^2 + 3\right] + 2\left(\frac{W+V}{X}\right)^2 O_s \right\} = M_c''$$

$$\text{and}\left\{\frac{WV+XZ-YZ}{X^2}+\left[\left(\frac{Y+Z}{X}\right)^2+\left(\frac{Y-Z}{X}\right)^2\right.\right.$$

$$\left.\left.+\left(\frac{W-V}{X}\right)^2\frac{+W^2+V^2+Y^2}{X^2}\right]O_s\right\}n=O''_c$$

$$\text{Then } R_t=\left[\frac{m^2-1}{3}+\frac{M^2}{4}\cos\beta\right]M''_c+O''_c \qquad (62)$$

Which is of the same form as formula (51) for two strands. Similarly

$$R_b=\left(\frac{m^2-4}{12}\right)M''_c+O''_c \qquad (63)$$

The same form as (52)

For three strands wound continuous in a straight up coil with one coil per slot use equation (60) except that the β terms are zero and that Y is measured from the bottom of the slot.

To get the average term we have $\frac{1}{m}\sum_1^m R$ and the values of k_1 and k_2 are taken from

$$k_1=(e-2q)\left(\frac{m-1}{2}\right)J_o,\qquad k_2=-(d-2p)\left(\frac{m-1}{2}\right)J_o$$

The equation reduces to

$$n^2M_s\left(\frac{m^2-1}{12}\right)+\left(\frac{m^2-1}{4}\right)M''_c+O''_c=R \qquad (64)$$

For three strands per conductor with a turned over coil short-circuited at end of each turn. k_1 and k_2 are taken from (40).

R for top of the slot is taken from (60) and for bottom of slot is taken from (61).

$$\left.\begin{aligned}R_t&=n\,M_s\left\{\frac{W+V}{X}\,\frac{m^2}{4}\sin\mathrm{B}\right\}+n^2M_s\left\{\frac{m^2-1}{3}\right.\\&\quad\left.-\frac{m^2-4}{12}+\frac{m^2}{4}\cos\beta\right\}+\frac{m^2-4}{12}M''_c+O''_c\\R_b&=n\,M_s\left\{-\frac{W+V}{X}\,\frac{m^2}{4}\sin\beta\right\}+\frac{m^2-4}{12}M''_c+O''_c\end{aligned}\right\} \qquad (65)$$

For three strands, turned over coil with continuous stranding when there are two coils per slot take k_1 and k from 40A. *R* for the top half of the slot is taken from (60) and for the bottom from (61).

$$\left.\begin{aligned} R_t &= n\,M_s\left\{\frac{W+V}{X}\,\frac{m^2}{4}\sin\beta\right\} + n^2 M_s\left\{\frac{m^2-1}{3}\right.\\ &\quad \left. -\frac{m^2-4}{16} + \frac{m^2}{4}\cos\beta\right\} + \frac{m^2-4}{16}\,M_c'' + O_c''\\ R_b &= n\,M_s\left\{-\frac{W+V}{X}\,\frac{m^2}{4}\sin\beta\right\} + n^2 M_s\left\{\frac{m^2-4}{12}\right.\\ &\quad \left. -\frac{m^2-4}{16}\right\} + \frac{m^2-4}{16}\,M_c'' + O_c'' \end{aligned}\right\} \tag{66}$$

For three strands, turned over coil, short-circuited each end with one coil per slot take

$$k_1 = (e - 2\,q)\,(Y - \frac{m}{4} - 1)\,J_o$$

$$k_2 = -\,(d - 2\,p)\,(Y - \frac{m}{4} - 1)\,J_o$$

$$\frac{m}{2} = \text{number conductors}$$

$$R = n^2 M_s\left\{\frac{m^2-4}{12} - \frac{m^2-16}{48}\right\}$$

$$+\frac{m^2-16}{48}\,M_c'' + O_c'' \quad \text{if } 2\,m_1 = m$$

$$R = n^2 M_s\left\{\frac{m_1^2-1}{3} - \frac{m_1^2-4}{12}\right\}$$

$$+\frac{m_1^2-4}{12}\,M_c'' + O_c'' \tag{67}$$

For continuous stranding, three strands per conductor, one coil per slot and turned over, take

$$k_1 = (e - 2\,q)\,(-\tfrac{1}{2})\,J_o$$
$$k_2 = (d - 2\,p)\,\tfrac{1}{2}\,J_o$$

The average value of R is

$$n M_s \left[\frac{WV+XZ-YZ}{X^2} + \frac{m}{4} - 1 - \tfrac{1}{4} \left(\frac{W+V}{X} \right)^2 \right]$$

$$+ \left[\left(\frac{X-Z}{X} \right)^2 + \left(\frac{Y+Z}{X} \right)^2 + \left(\frac{W-V}{X} \right)^2 \right.$$

$$\left. + \frac{W^2+V^2+W^2}{X^2} - \tfrac{1}{2} \left(\frac{W+V}{X} \right)^2 \right] n O_s \qquad (68)$$

To reduce the above expression to a convenient form it is desirable to use the development in series for $p - q - d - e$, etc., and to limit αh to values < 1. The following formula will be found convenient:

$E = 4(\alpha h)^2 \gamma^2$

$F = 4(\alpha h)^4 \gamma$

$Q = 8(\alpha h)^2 \gamma^2$

$R = 2(\alpha h)^2 \gamma^2$

$V = 32(\alpha h)^6 \gamma^3$

$W = 40(\alpha h^6 \gamma^3$

$X = 36(\alpha h)^4 \gamma^4$

$Y = -12(\alpha h)^4 \gamma^4$

$Z = 24(\alpha h)^4 \gamma^4$

$\frac{X+Y+Z}{x} = 0$

$\frac{W+V}{x} = 2\frac{(\alpha h)^2}{r}$

$\frac{Y+Z-Y}{x} = 2$

$\frac{X-Z}{X} = \frac{1}{3}$

$\frac{W-V}{X} = \frac{2}{9} \frac{(\alpha h)^2}{\gamma}$

$\frac{W}{X} = \frac{10}{9} \frac{(\alpha h)^2}{\gamma}$

$\frac{V}{x} = \frac{8}{9} \frac{(\alpha h)^2}{r}$

$\frac{Y}{x} = -\frac{1}{3}$

$\frac{W\,V}{X^2} = \frac{80}{81} \frac{(\alpha h)^4}{\gamma^2}$

$\frac{X\,Z}{X^2} = \frac{2}{3}$

$\frac{-Y\,Z}{X^2} = \frac{2}{9}$

$M_s = \frac{1}{3}(\alpha h)^4$

$L = \frac{1}{2}$

$H = \frac{(\alpha h)^2}{2\gamma}$

$O_s = 1 + \frac{8}{90}(\alpha h)^4$

Substituting in (68) we get.

$$3 M_s \left[\frac{m^2}{4} - \frac{1}{9} \right] + \left[1 + \frac{2}{9} \frac{(\alpha h)^4}{\gamma^2} \right] O_s = R$$

$\frac{m}{2}$ is the number of coils per slot. If S is the number of strands per slot depthwise then $m = \frac{2}{3} S \quad m^2 = \frac{4}{9} S^2$ and

$$M_s \left[\frac{s^2 - 1}{3} \right] + O_s = R \text{ average},$$

where S is number of strands depthwise. (69)

9—Summary of Formulas

We have shown that the formula for the loss in windings due to eddy currents has the same general form for solid bars and bars of two and three strands and for infinite number of strands.

Consider whether we can develop the equivalent depth of a solid conductor to replace the stranded one

$M'_c = 4\,n\,[L\,M_s + 2\,H^2\,O_s]$

$$M'_c = n\left\{ M_s\left[\left(\frac{W+V}{X}\right)^2 + 3\right] + 2\left[\frac{W+V}{X}\right]{}^2O_s\right\}$$

$$M' = 4\left[M_s + \frac{3\,M_s}{\gamma_2} + \frac{24}{90}\,\frac{M^2{}_s}{\gamma_2}\right] \text{ approx.}$$

$$M''_c = 3\,M_s\left[\frac{12\,M_s}{\gamma^2} + 3\right] + 72\,\frac{M_s{}^2}{\gamma^2}$$

Since $M_s = \dfrac{(\alpha\,h)^4}{3}$ let $M = n^4\,M_s$ where M corresponds to $n\alpha\,h$. Then $M = M_c\left[\dfrac{1}{n^2} + \dfrac{n^2-1}{n^2}\,\dfrac{1}{\gamma_2}\right]$

$$M''_c = M\left[\frac{1}{n^2} + \frac{n^2-1}{n^2}\,\frac{1}{\dot{\gamma}^2}\right]$$

We have therefore a value from our M curve which is to be multiplied by $\left[\dfrac{1}{n^2} + \dfrac{n^2-1}{n_2}\,\dfrac{1}{\gamma^2}\right]$ or we can use the value $\alpha\,f\left[\dfrac{1}{n^2} + \dfrac{n^2-1}{n_2}\,\dfrac{1}{\gamma^2}\right]^{\frac{1}{4}}$ to get the equivalent solid conductor to replace the stranded one. O'_c and O'' follows the same law, a proof is not readily demonstrated but in any event the error is small.

If we consider the limiting conditions where the number of strands are one or infinite we find that our equivalent depth fits both cases. For a solid bar the term containing γ is zero and for an infinite number of laminations $\dfrac{\alpha\,f}{\sqrt{\gamma}}$ is the equivalent solid bar a result found previously in Part 4. As a check on the approximations made, the writer calculated the values M'_c, M''_c, O^c_1 and O''_c for two values each of $\alpha\ h$ 0.3 and 0.4 and for $\gamma = 1$, 2 and 3. The tabulated results of these figures which were made on a calculating machine and are correct to five decimal places are tabulated for comparison with the results taken from curves and approximate methods outlined above.

		Calculated $M'c$			Curve Value $M'c$	
		$r = 1$	$r = 2$	$r = 3$	$r = 2$	$r = 3$
0.3	0.6	0.042838	0.018883	0.014390	0.0189	0.0144
0.4	0.8	0.134074	0.059240	0.045107	0.0590	0.0455
		Calculated $M''c$			Curve Valu $M''c$	
0.3	0.9	0.21345	0.07436	0.04570	0.0732	0.0456
0.4	1.2	0.63709	0.23439	0.14299	0.2320	0.1440
		Calculated $O'c$			Curve Value $O'c$	
0.3	0.6	1.011413	1.005425	1.00430	1.0053	1.0040
0.4	0.8	1.035742	1.017034	1.013501	1.0160	1.0120
		Calculated $O''c$			Curve Value $O''c$	
0.3	0.6	1.05671	1.02036	1.01347	1.0195	1.0120
0.4	1.2	1.17066	1.06326	1.04231	1.0620	1.0390

These results show a sufficiently close agreement to permit of their use in any calculations.

It may be of interest to develop the formula for the loss in the end windings under the assumptions which we have made.

It will be noted that the loss in the ends is proportional to

$$[{}_sJ_1^2 + {}_sJ_2^2 + {}_sJ_3^2 + \qquad + {}_cJ_1^2 + {}_cJ_2^2 + {}_cJ \ + +]$$

and that this is most readily evaluated as follows:

For two strands we have

$$[K'] \, [L \, M_s + 2 \, H^2 \, O_s] \, 4n + n \, [L \, M_s + 2 \, H^2 \, O_s + \tfrac{1}{2} \, O_s]$$

as loss ratio

Where $[K']$ is coefficient of M'_c and the above can be written $[K'] \; M'_c + O'_c$. The loss on the ends is proportional to the coefficient of O_s and is for two strands,

$$\frac{O_c}{O_s} - \frac{n \, L \, M_s}{O_s} + 4 \, n \, 2 \, H^2 \, [K'] = \text{loss ratio on ends}$$

$$= \frac{O_c}{O_s} + \frac{n^2 - 1}{n^2} \frac{M}{\gamma^2} [K'] \quad M \text{ is curve value for } \alpha f.$$

For three strands,

$$\frac{O_c}{O_s} - \frac{3 \, M_s}{O_s} \left[\frac{80}{81} \frac{(\varkappa \, h)^4}{\gamma^2} + \frac{8}{9} \right] + \frac{n^2 - 1}{n^2} \frac{M}{\gamma^2} \, [K'] \text{ which is}$$

the same as the above, since $3 \, M_s \left[\frac{80}{81} \frac{(\alpha \, h)^4}{\gamma^2} + \frac{8}{9} \right]$

is negligible. Checking up on this value we find that for one strand per conductor our loss is unity and for infinite strands is of the same value as in the straight part of the coil.

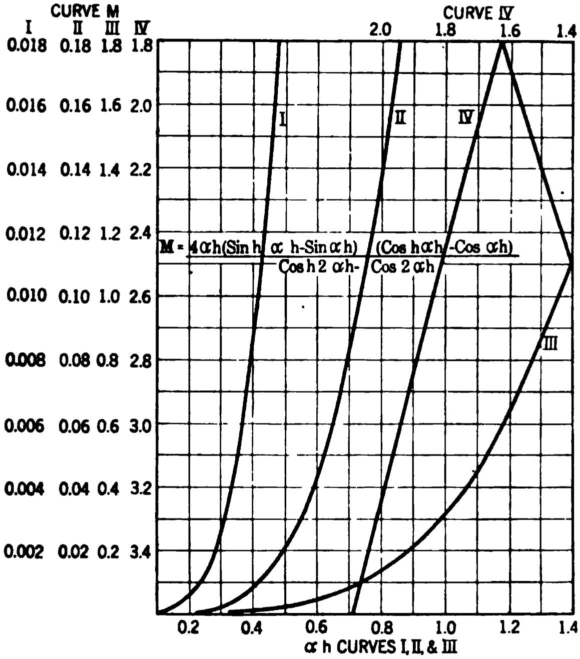
Approximate loss on ends is

$$\frac{O_c}{O_s} + \frac{n^2 - 1}{n^2}\frac{M}{\gamma^2}\left[K^1\right] = \text{ratio}$$

K^1 is coefficient of M_c in loss for embedded length

M is curve value for αf

O_c is curve value for αf equivalent



O_s is curve value for αh

A general summary for loss in the embedded part of coil is

m = the no. of conductors depthwise in a slot

n = the no. of strands depthwise per conductor (equal strands)

γ = the ratio of total conductor length to the embedded part

s = the total number of strands, depthwise

$\alpha = \dfrac{0.4}{\sqrt{(1 + .00427\,\theta)}}\sqrt{\sim r_1}$ = ratio of copper width to slot width

θ = total temperature of the copper above zero

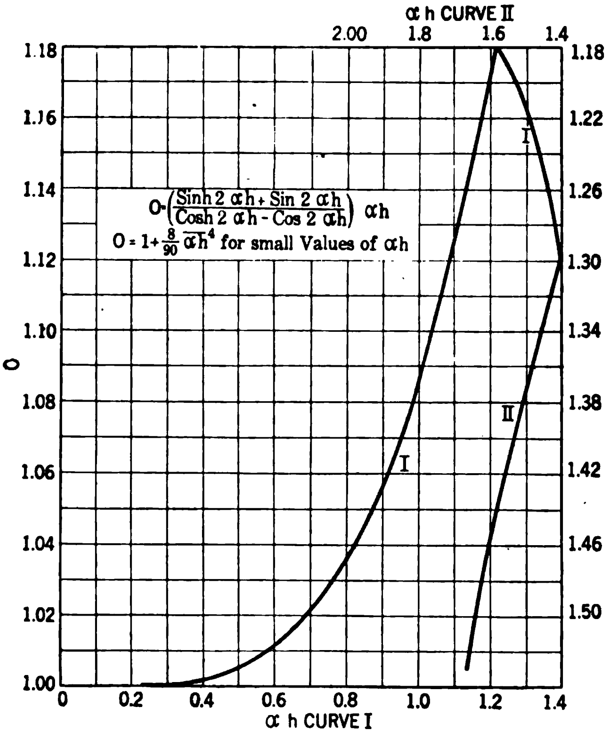
h = the depth of the strand in inches

f = the depth of the conductor in inches

M_s = the curve value for $\alpha\, h$

M_c = the curve value for the equivalent value of $\alpha\, f$

O_c = the curve value for the equivalent value of $\alpha\, f$



β = the angular displacement between the conductor currents of the top and the bottom coils in a slot.

M = the curve value for $\alpha\, f$

K^1 = the coefficient M_c

$$M_s = \frac{(\alpha\, h)^4}{3} \text{ for small values of } \alpha\, h$$

$$O_t = \text{curve value for } \alpha\, h = 1 + \frac{8}{90}(\alpha\, h)^4$$

The equivalent value of a conductor is

$$\alpha f_e = \alpha f \left[\frac{1}{n^2} + \frac{n^2 - 1}{n^2} \frac{1}{\gamma^2}\right]^{\frac{1}{2}}$$

and is given in the curve.

R_t is the average loss ratio in the top half of a slot when there are two coils per slot.

R_b is the average loss in the bottom.

R is the average loss for the entire slot.

In the calculations for two strands per conductor we found in (54) and (55) the expression $n\, M_s \frac{m^2}{4} H \sin \beta$. This expression occurs in the formulas for the top and the bottom half of the slot with reversed sign and can be neglected in the total heating. In any event its value is

$$2 \frac{(\alpha h)^4}{3} \frac{m^2}{4} \cdot \frac{(\alpha h)^2}{2r} \sin \beta$$

and is negligible for small values of αh; similarly in (65)

$$n\, M_s \left\{\frac{W + V}{x} \cdot \frac{m^2}{4} \sin \beta\right\} = \frac{3(\alpha h)^4}{3}\left[2 \frac{(\alpha h)^2}{r} \frac{m^2}{4} \sin \beta\right]$$

and is negligible

Either of these expressions can be written

$$\frac{1}{2}\, n\, (n - 1)^2 M_s \frac{m^2}{4} \sin \beta$$

which is zero when $n = 1$ and zero when $n = \infty$.

The formulas for loss are

(A) When the conductors are solid bars (2 coils per slot)

$$R_t = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos \beta\right] M_c + O_c$$

$$R_b = \left[\frac{m^2 - 4}{12}\right] M_c + O_c$$

(B) When the conductor is stranded and shorted at each half turn

$$R_t = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos \beta\right] M_c + O_c$$

$$R_b = \left[\frac{m^2 - 4}{12}\right] M_c + O_c$$

(C) For a continuous stranded coil wound straight up

$$R = n^2 \left[\frac{m^2-1}{12}\right] M_s + \left[\frac{m^2-1}{4}\right] M_e + O_e$$

(D) For a turned over coil—shorted each full turn with two coils per slot

$$R_t = n^2 M_s \left[\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta - \frac{m^2-4}{12}\right] + \left[\frac{m^2-4}{12}\right] M_e + O_e$$

$$R_b = \left[\frac{m^2-4}{12}\right] M_e + O_e$$

(E) For a turned over coil continuous stranding

$$R_t = n^2 M_s \left[\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta - \frac{m^2-4}{16}\right] + \left[\frac{m^2-4}{16}\right] M_e + O_e$$

$$R_b = n^2 M_s \left[\frac{m^2-4}{12} - \frac{m^2-4}{16}\right] + \left[\frac{m^2-4}{16}\right] M_e + O_e$$

(F) For one coil per slot turned over and shorted each turn

$$R = n^2 M_s \left[\frac{m^2-1}{3} - \frac{m^2-4}{12}\right] + \left[\frac{m^2-4}{12}\right] M_e + O_e$$

(G) For one coil per slot turned over with continuous stranding $R = M_s \left[\frac{s^2-1}{3}\right] + O_e$ (approx.)

These formulas will stand the check imposed by the limiting values of $n = 1$ and $n = \infty$.

A, B, C, D and E are all equal for $n = 1$ that is for solid bars. Also the loss ratio on the end windings is 1 for $n = 1$.

For $n = \infty$, $n^2 M_s = O$ and we find that formula C checks value in (26) and E checks (28).

Also we see D and E are equal for $m = 2$ and the loss ratio on the ends for $n = \infty$ is the same as the loss ratio in the slots.

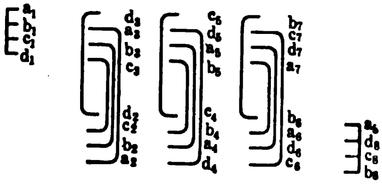
A study of these formulas shows that for a given total copper depth in a slot with two coils per slot that the minimum loss is secured by increasing m and n to the limit.

n is limited by mechanical considerations in manufacture and m for machines with a great many poles can be controlled to some extent by the use of multiple turn coils and by transposing the winding in parallel circuits. Such a procedure is in common use but introduces the danger of a line voltage between adjacent conductors with the increased chance of break down. For turbo generators, where only two or four poles are common, a transposition scheme has been developed to give the effect of increasing m without introducing multiple circuits. This arrangement consists in subdividing the strands of a conductor depthwise into a number of groups which is a multiple of the slots per pole per phase then interconnecting these groups in such a way that equal voltages will be produced in all the various circuits in parallel.

Consider the case of four slots per pole per phase shown below:

```
                 1 3 5 7
top coil         x x x x o o o o . . . . x x x x
bottom coil                          x x x x
                                     2 4 6 8
```

The successive positions of a coil are shown as 1, 2, 3, 4, 5, 6, 7, and 8 this being the sequence of conductors in series. Each single conductor is then arranged in four groups of strands, so that a section of the coil group would show as below.

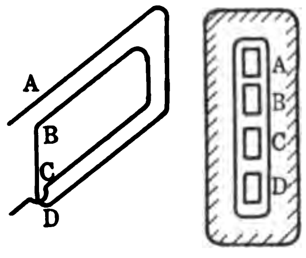


The voltage of the groups a, b, c, d, taken through eight conductors are all equal, and a little consideration will show that equal currents must flow. Hence we have the effect in two coils per slot of securing the same current distribution as if we had had eight conductors, each carrying one-quarter of the total current. The loss is given by formula (D), Part 9, and is only slightly greater than the loss for (E).

This method of winding has been found very useful in reducing the eddy losses in coils of large current capacity.

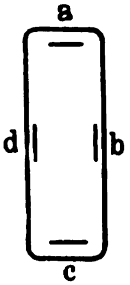
10—COMPARISON OF CALCULATIONS AND TESTS

In order to compare the formulas with test results a model was made up with an adjustable coil and tested with various combinations of windings. Four conductors *A B C D* each consisting of six strands in parallel were wound as shown in the sketch, that is, one end of each coil open and one closed. An iron core surrounded the conductors as shown with this arrangement the m. m. f. of *A B* opposed that of *C* and *D* and coils *A* and *B* have the same relation to each other as the top and bottom coils in a slot.



There is no iron loss except that due to the transverse flux across the slots. With the six strands in each conductor were provided connectors so that the following possible arrangements could be secured: *A* coil of six turns of one strand, three turns of two strands, two turns of three strands or one turn of six strands. In addition, the connectors on the open end were provided with taps so that it was possible to get the effect of all strands of a conductor in series throughout the multiple turn coil or else a short circuit could be made at the end of each full turn or one-half turn. By adjusting the length of the iron we could introduce the factor γ into our formula as a variable. By changing the frequency we could vary αh and αf. So on a single model we were able to check all the formulas except the case of one coil per slot and those covering two coils per slot where the currents in the top and bottom coils were not in phase.

Tests were made as follows: the coils were provided with thermocouples inside on the copper around the periphery of the coil and direct current was passed through the coil until constant temperatures were reached. A calibration curve between temperature and current was then made up. The average temperature of the coil was taken by multiplying the temperature at (*a*) by the surface at (*a*) temperature at (*b*) by surface (*b*), etc., and using the average temperature so obtained. This procedure eliminates any variation in the heat conduction through the insulation. Alternating current was then passed through the coil with various groupings and the temperatures of the top and bottom coils secured corresponding to these temperatures. The direct currents were taken from the calibration curve. These values are compared with the equivalent calculated current obtained from the loss ratio R_t or R_b as the case may be. Since the iron temperature varies with the alternating currents a correction is necessary; the correction formulas are given below. The relation between current and temperature for a given iron temperature is



$$I_2 = I_1 \left[\frac{\theta_2\{1 + (\alpha + \theta_1)\,0.00427\}}{\theta_1\{1 + (\alpha + O_2)\,0.00427\}}\right]^{\frac{1}{2}}$$

I_1 is known amperes corresponding to total temperature $(\alpha + \theta_1)$.

α is iron temperature. θ_1 is rise above iron with amperes I_1.

θ_2 is rise above iron for same unknown current and I_2 is unknown current.

Neglecting conduction of heat along the copper the current corresponding to some other iron temperature is

$$I_2 = I_1 \left[\frac{1 + 0.00427(\alpha_1 + \theta)}{1 + 0.00427(\alpha_2 + \theta)}\right]^{\frac{1}{2}}$$

Where I_2 is desired current for θ_o rise above its iron temperature α_2 and I_1 is the known current for the iron temperature α_1 and rise θ above the iron. For a 10 deg. change in iron temperature, this correction is about 1 per cent. The agreement between the current value taken from the calibration curve and that calculated from the formulas is unusually close, and indicates that the formulas can be relied upon in practise.

The physical dimensions of the test model were as follows:

Each strand 3/16 in. by 5/8 in., copper six strands per coil.

Iron length was 40 in. when $r = 2$.

Iron length 26¾ in. when $r = 3$.

Opening in iron widthwise was 1.178 in.

The curves for M_s, M_c, O_s, O_c and for the evaluation of αf_e are attached as a part of the paper.

It is evident that these formulas can be used in a number of ways for the special investigation of particular problems in design. For example, curves can be made up showing that beyond a certain point it is useless to increase the copper depth in a slot in order to secure a lower temperature. The heating effect of third harmonics in delta-connected windings can be investigated. Also, such problems as are involved in the calculation of the losses in the damper windings of single-phase machines, and phase-converting apparatus. The applications are for the most part obvious.

Current Connection Short. Circ. Value of r.	D-C Series	60 Series	60 Two Strands			60 Three Strands		
	None 2	None 2	None 2	1T 2	½T 2	None 2	1T 2	½T 2
Amperes	250	210	420	420	385	600	600	500
Temp. Rise	78.5°	71.9°	73.3°	74.6°	75.2°	71.7°	74°	74°
Iron Temp	53°	49°	44°	44.5°	49°	48°	54°	48°
D.C. Amps	....	242	495	496	494	725	731	736
Temp. Correct	....	1.8°	1.8°	1.8°	2.3°	1.8°	1.8°	3.7°
Calc. R	....	1.402	1.428	1.43	1.695	1.497	1.495	2.25
D.C. Equiv	....	245	497	500	500	735	738	752
Amps. from R	....	249	502	502	501	734	733	750
Temp. Rise	71°	52.3°	53.6°	54.8°	46.2°	52.7°	55°	34.2°
D.C. Amps	....	222	450	454	416	666	675	552
Temp. Correct	....	1.8°	1.8°	1.8°	2.3°	1.8°	1.8°	3.7°
Calc. R	....	1.065	1.10	1.108	1.173	1.168	1.179	1.214
D.C. Equiv	....	217	444	445	410	655	660	.....
Amps. from R	....	216.6	440.5	442.5	406	650	651	551
Diff. Temp	7.5°	19.6°	19.7°	19.8°	29°	19°	19°	40°

Current Connection Short Circ. Value of r.	D.C. 60 6 Strands		60 3 Strands		60 6 Strands		40 6 Strands	
	None 2	½T 2	1T 3	½T 3	1T 3	½T 3	1T 3	½T 3
Amperes	1140	730	624	565	1210	900	1330	1100
Temp. Rise	78°	70°	76°	78.5°	81.9°	82.5°	83.8°	80.3°
Iron Temp	47.5°	49°	47.5°	54°	53°	54.5°	62°	62°
D.C. Amps	1500	1430	745	749	1524	1522	1526	1492
Temp. Correct	1.8°	4.3°	1.9°	3.1°	2.2°	4.4°	1.9°	3.8°
Calc. R	1.718	4.225	1.444	1.741	1.521	2.79	1.232	1.771
D.C. Equiv	1524	1480	752	761	1524	1558	1535	1520
Amps. from R	1492	1500	750	746	1494	1502	1477	1465
Temp. Rise	58.5°	23.5°	54.7°	43.5°	57.5°	33.4°	63.1°	37.8°
D.C. Amps	1392	933	678	611	1375	1082	1418	1133
Calc. R	1.47	1.491	1.122	1.124	1.23	1.241	1.101	1.115
D.C. Equiv	1380	846	669	593	1350	1013	1393	1082
Amps. from R	1382	891	662	600	1342	1002	1396	1162.
Diff. Temp	19.5°	46.5°	21.5°	35°	24.4°	49°	20.7°	42.5°

Temperature correction in degrees is given by ½ $\frac{D \times 0\ 0.625}{3.5}$. D is the difference in temperature rise between the top and bottom coil; 0.625 is the copper surface in the top coil adjacent to the bottom coil; 3.5 is the periphery of the coil

The comparison of the calculated current from the formula to the corrected current value taken from the calibration curve is secured by taking the values for the d-c. equivalent and for amperes from *R* and *R*

DISCUSSION ON "EDDY CURRENT LOSSES IN ARMATURE CONDUCTORS" (GILMAN), WHITE SULPHUR, W. VA., JUNE 30, 1920.

S. L. Henderson: The eddy current theory as worked out in Mr. Gilman's paper has made possible a much more accurate study of the losses in armature coils. He has developed formulas covering all the most commonly used types of coils, and it is possible therefore, to calculate the loss in different coil arrangements and to find for a given application that coil giving the minimum temperature drop through the insulation. As an example, in the application of these formulas we will consider several coil designs for a 31,250-kv-a., 13,200-volt, three-phase machine on both sixty and twenty-five cycles, and from this study several interesting results can be shown.

A number of curves have been worked up showing the relation between loss and temperature drop with varying depth of copper. These curves have been calculated on the basis of a rise on the iron surrounding the coil of 55 deg. cent., and a temperature of inlet air of 40 deg. cent., giving a total temperature of iron of 95 deg. cent. The length of the machine being great enough, it was assumed that there is no heat transfer from the point under discussion to the end windings and all the heat conducted through the insulation. The assumption has also been made that the watts per square inch transmitted is uniform around the slot. For purposes of calculating the temperature drop, the surface through which the heat flows has been taken as the mean between the bare copper and the periphery of the slot. The temperature of the iron and the thermal conductivity of the insulation has been taken constant at all points. Since the temperature drop is the unknown and this must be known in order to calculate the eddy current factor and the temperature coefficient of the copper, it was first necessary at each point to calculate a preliminary curve of three points by assuming three temperature drops and working backward to find the loss and current for each temperature. A curve was then plotted between current and temperature drop and from this curve the temperature drop was taken for the rated current.

In connection with the assumption of the temperature of the iron, this has little effect on the loss in the coil, and the consequent drop through the insulation, because with decrease in temperature, for instance, the temperature coefficient of the copper decreases, but the eddy factor increases and results in the product of the two being very nearly constant. The figures of temperature drop are practically independent of ventilation.

In Fig. 1 is shown a number of curves of a two-turn per coil winding, two coils per slot. Each conductor consists of three in width of 0.182 in. wide by 0.091 in. thick. Starting in with eight strands in depth, and increasing in steps by one strand on each conductor up to thirteen strands in depth per conductor, the losses and temperature drop at each point have been plotted against total copper depth in the slot. From these curves it is evident that beyond a certain point the temperatures are not decreased by increasing the copper depth, and that a point may be reached where an increase in copper really increases the copper temperature. In contrast, if this same coil were used on twenty-five cycles, we reach mechanical limitations in the coil with increasing coil depths before reaching a point of constant temperature. The curves for this

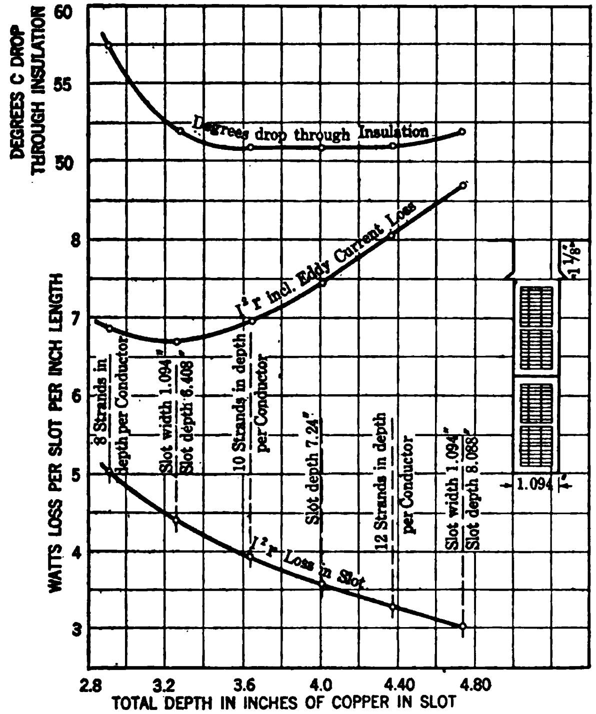


FIG. 1—BASED ON 31,250 KV-A., 13,200 VOLTS, 3-PHASE, 60 CYCLES, 1370 AMPERES. TWO PARALLELS

coil on twenty-five cycles are shown in Fig. 2. The depth of the slot given on the curves includes the dimension of $1\frac{1}{8}$ in. covering the wedge and open section of slot above the wedge.

If now we had used a four-turn coil, which would be possible by connecting the coils per phase in twice as many parallels, at considerable gain in temperature drop, as shown in Fig. 3, is obtained, being approximately 19 deg. improvement at a point of 4.36 in. total copper depth of slot.

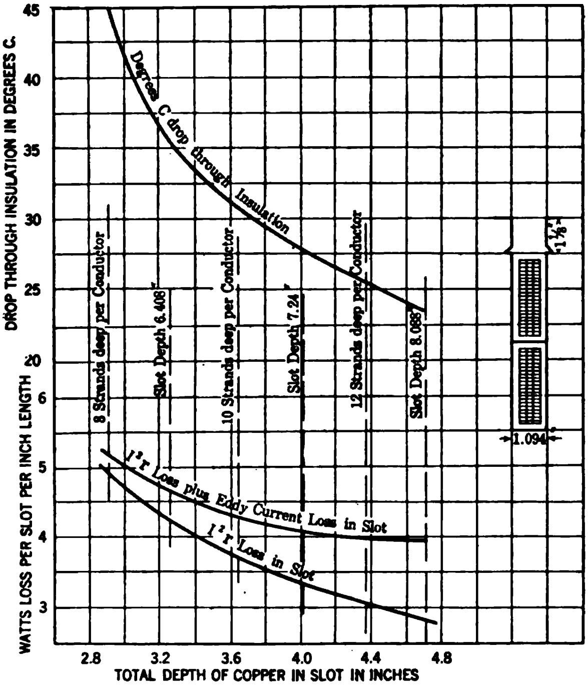


FIG. 2—31,250 KV-A., 13,200 VOLTS, 3-PHASE, 25 CYCLES. FOUR CONDUCTORS PER SLOT. STRANDING CONTINUOUS

One other possibility in the design of the coil lies in the use of a transposed coil, instead of a multi-turn coil. In this type of coil, the strands must be a multiple of the slots per pole per phase, in this case six. Fig. 4 is, therefore, calculated at 18, 24, and 36 strands in depth per coil. This coil is slightly better than the four-turn coil, and also reaches a point beyond which there is no gain in temperature. The point of 24 strands in depth per conductor in the transposed coil can be compared with the four-turn coil and six strands per conductor, or the two-turn coil with twelve strands in depth per conductor, as all three coils have the same copper depth. The four-turn coil and the transposed coil have practically the same temperature drop, differing only because of a slight difference in thickness of insulation, due to the absence of conductor insulation in the transposed coil. Either of these arrangements is considerably better than the two-turn coil. All these curves show an increase of total loss in the coil before a corresponding increase occurs in the drop through the insulation which means that for a period the surface of the coil increases at a faster rate than the loss.

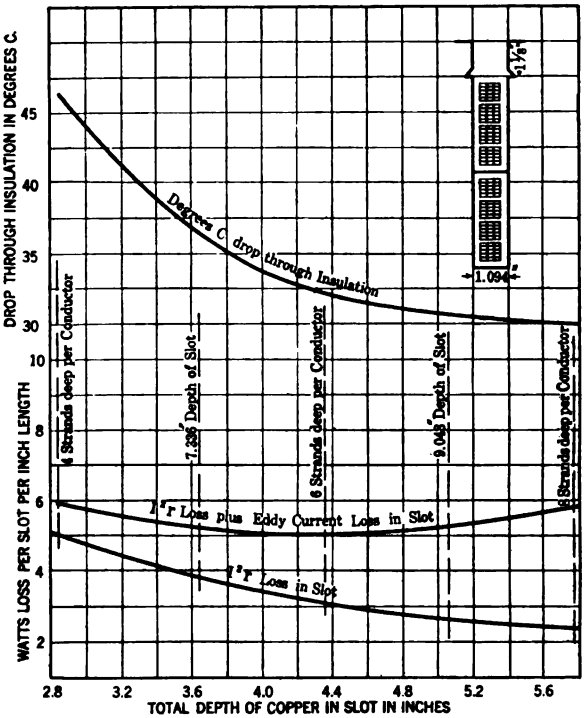


FIG. 3—31,250 KV-A., 13,200 VOLTS, 3-PHASE, 60 CYCLES. EIGHT CONDUCTORS PER SLOT. STRANDING CONTINUOUS. FOUR PARALLELS

All these curves have been carried farther than it is mechanically possible to build coils for large machines. Being continuous coils, not made in halves and connected together in front and back, they become too heavy to be handled without danger of injuring the insulation. Also, the great depth of the coil results in a considerable difference in length of chord in the bottom and top of the slot and makes it almost impossible to enter the coil at the top of the slot without seriously twisting it, and damaging the insulation.

It would be possible to use coils of this great depth if they were formed in halves and then connected together in front and back. It is practically impossible, however, to connect each strand individually, owing to the large number of strands, and if all the strands are connected solidly together on both the front and rear ends, the eddy current loss is increased tremendously. For instance, if the four-turn coil of seven strands in depth per conductor were connected front and back, the eddy current factor would be increased from 2.12 for a continuous coil to 5.7, and the drop through the insulation increased from 31 deg. cent. to 93 deg. cent. This is for a depth of slot of 9.048 in. which is deeper than has been used, and, were

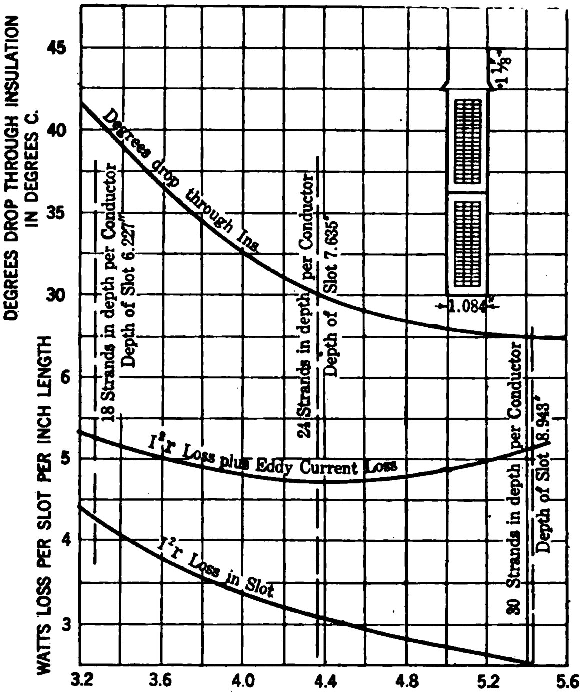


FIG. 4—31,250 KV-A., 13,200 VOLTS, 3 PHASE, 60 CYCLES. TWO CONDUCTORS PER SLOT. EACH CONDUCTOR TRANSPOSED.

it used, would probably necessitate a two-piece coil.

From a consideration of the curves, it is theoretically possible on sixty cycles, and with 13,200-volt insulation, to obtain a drop through the insulation of approximately 30 deg. cent. as a minimum. Practically, however, this result cannot be obtained because of mechanical limitations. So far, a slot deeper than 6 to 6.75 in. has not been used, and for such a slot, and by using a transposed coil or a multi-conductor coil, the temperature drop from bare copper to iron will be approximately 40 deg. cent.

These facts have considerable practical importance in view of the demand in some quarters for low temperatures and the feeling that the designer advocates higher temperatures merely to reduce sizes and costs. These figures show that the designer faces definite physical limitations in large high-voltage 60-cycle turbo generators, and true copper temperature rises approaching 100 deg. cannot be avoided.

V. Karapetoff (by letter): The author's mathematical deductions may be considerably simplified by handling the functions in the exponential form, as J. J. Thomson does, and some other writers on skin effect and eddy currents. Let the *real* current be $I \cos \omega t$; assume that in addition to this an *imaginary* current $j I \sin \omega t$ flows through the same conductor. The total *complex* current is then

$$I (\cos \omega t + j \sin \omega t) = I \epsilon^{j\omega t} \qquad \textbf{(a)}$$

Thus, one can carry all differential equations and their integration in the form $I \epsilon^{j\omega t}$, and then *in the end* separate the real from the imaginary part, and use the former only. Suppose we have another current, J, which lags behind I by an angle ϕ. Its expression is

$$J \epsilon^{j(\omega t - \phi)} = J \epsilon^{-j\phi} . \epsilon^{j\omega t} = K \epsilon^{j\omega t} \qquad \textbf{(b)}$$

where $K = J \epsilon^{-j\phi}$ is a *complex constant.* Thus, by using *complex amplitudes,* quantities differing in phase from one another can be represented by means of the same exponential $\epsilon^{j\omega t}$.

Consider Mr. Gilman's eq. (3) and let the density be expressed as

$$\Delta = u \epsilon^{j\omega t} \qquad \textbf{(c)}$$

where u is a *complex function* of x, but does not contain t. This means that the eddy currents are everywhere of the frequency ω, that their phase and amplitude depend upon x, and that in addition to the real currents we assume certain imaginary currents to flow, which are disregarded in the result. Substituting the value of Δ from eq. (c) in eq. (3) we get, after reduction,

$$\frac{d^2 u}{d x^2} = 2 j \alpha^2 u \qquad \text{(d)}$$

One can readily see how much simpler this differential equation is than the simultaneous equations which the author gets on p. 1002. The well known solution of (*d*) is

$$u = P \epsilon^{\beta x} + Q \epsilon^{-\beta x} \qquad \text{(e)}$$

where $\beta = \alpha \sqrt{2 j}$ and P and Q are complex constants of integration. This simple deduction takes the place of all the mathematics on pp. 1002 to 1004.

The expression $\sqrt{j}$ corresponds to the rotation of a vector by 45°, or

$$\sqrt{j} = \cos 45^\circ + j \sin 45^\circ = \frac{1}{\sqrt{2}} (1 + j);$$

hence

$$\beta = \alpha \sqrt{2 j} = (1 + j)\, \alpha \qquad \text{(f)}$$

Thus expression (e) becomes, after being combined with (*c*);

$$\Delta = \epsilon^{j \omega t}\, u = P\, \epsilon^{\alpha x}\, \epsilon^{j(\alpha x + \omega t)} + Q\, \epsilon^{-\alpha x}\, \epsilon^{j(-\alpha x + \omega t)} \qquad \text{(g)}$$

This is identical with eq. (10) in the paper, although it contains only two constants of integration. Each constant being of the complex form $a + j b$, there are really four *scalar* constants. Eq. (11) becomes

$$u_0 f = \int_0^f u \, d x \qquad \text{(h)}$$

where

$$u_0 = u_0^1\, \epsilon^{j \theta} \qquad \text{(i)}$$

and u_0^1 is the scalar density (average).

Integrating eq. (h) we get

$$\beta f u_0 = P (\epsilon^{\beta f} - 1) - Q (\epsilon^{-\beta f} - 1) \qquad \text{(j)}$$

which gives one relationship between the constants P and Q.

To deduce eq. (12) we have, in accordance with the author's expression near the middle of p. 1005:

$$\beta_1 = \frac{4 \pi}{10 a} I\, \epsilon^{j(\omega t + \phi)}$$

and

$$\rho \left(\frac{d\,\Delta}{d\,x} \right)_{x\,=\,o} = \frac{d\,\beta_1}{d\,t}\ 10^{-8}$$

Hence, omitting the factor $\epsilon^{j\omega t}$ on both sides, we get

$$\rho\ \beta\,(P - Q) = j\ \frac{4\,\pi\,\omega}{a}\ I\ \epsilon^{j\phi}\,10^{-9} \qquad \textbf{(k)}$$

From eqs. (**k**) and (**j**) the constants P and Q may be readily computed, thus making pp. 1006 and 1007 unnecessary. The trick is not to use trigonometric functions explicitly, although they are understood throughout the foregoing deduction.

Coming now to the power computations, on p. 1008, no integration is needed with respect to t, since the current is sinusoidal at all points. Expression (**e**) may be readily changed to the form $u_r\ \epsilon^{j\varphi}$, where u_r is a *real* quantity and a function of x, and where angle φ is also a function of x. The value of u_r is then the amplitude of the current density, so that

$$(\text{effective current})^2 = 1/2 \int_0^f u_r^2\,d\,x \qquad \textbf{(l)}$$

This will do away with the long expressions on pp. 1008 and 1009.

INDEX.

NOTE: For complete topical and synoptical index see end of Part II.

PAPERS

INDEX TO AUTHORS