TRANSACTIONS

OF THE

AMERICAN INSTITUTE

OF

ELECTRICAL ENGINEERS

JULY TO DECEMBER, 1919

VOL. XXXVIII, PART II

PUBLISHED BY THE
AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
33 WEST THIRTY-NINTH STREET
NEW YORK, N. Y., U. S. A.
1919

CONTENTS

MEETING AT LAKE PLACID, N. Y., JUNE 27, 1919

MEETING AT LOS ANGELES, CAL., SEPTEMBER 18-20, 1919

MEETING AT NEW YORK, N. Y., OCTOBER 1, 1919

MEETING AT PHILADELPHIA, PA., OCTOBER 9-10, 1919

MEETING AT NEW YORK, NOVEMBER 14, 1919

MEETING AT NEW YORK, DECEMBER 12, 1919

MEETING AT SAN FRANCISCO, FEBRUARY 7, 1919

MEETING AT PITTSBURGH, PA., FEBRUARY 11, 1919

PRESENTED BY PUBLICATION ONLY SEPTEMBER 1920

PRESENTED AT CALCUTTA, INDIA, JANUARY 2, 1919

INDEX

NOTE: For complete topical and synoptical index see end of volume

PAPERS

INDEX OF AUTHORS

C. A. Adams

[illegible]

THE DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE

BY R. W. ATKINSON

[illegible]

In the case of single-conductor cables, by means of simple mathematical calculations it is readily possible to calculate from the dimensions and data, the [illegible] quantities of the dielectric system [illegible], the corresponding properties of the entire cable. Also the voltage gradient at the conductor surface or elsewhere in the insulation may easily be determined in terms of the applied voltage and [illegible] or the unit quantities may be determined from measurements [illegible].

Very few corresponding [illegible] are available. The reason for this is the difficulty of the mathematical work required [illegible]. Some of these [illegible] have been [illegible] results of electrical measurements [illegible] of conducting paper [illegible] a solution [illegible] three-conductor cable. Also there are [illegible] results

C. A. Adams

Presented at the 35th Annual Convention of the American Institute of Electrical Engineers, Lake Placid Club, N. Y., June 27, 1919.

THE DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE

BY R. W. ATKINSON

ABSTRACT OF PAPER

The data given pertain particularly to the field of three-conductor three-phase cables when supplied with three-phase voltage, and are primarily the solution by physical measurements of some of the geometric problems of the three-conductor three-phase cable.

Data are given so that it is possible, from electrical measurements on three-conductor cable, to determine certain specific quantities as permittivity, resistivity, etc. of the dielectric of three-conductor cables in the same way as can readily be done for single-conductor cables, from geometric considerations.

Also, there is shown the potential and stress distribution in a three-conductor cable. The most extensive data are based on measurements made with electrodes in an electrically conducting liquid, thus simulating a homogeneous dielectric. Also exploring electrodes were built into some actual three-conductor cable, and measurements were made when three-phase voltage was supplied to the conductors.

IN the case of single-conductor cable, by means of simple mathematical calculation it is readily possible to calculate from the fundamental data, the unit quantities of the dielectric, such as permittivity, thermal or electrical resistivity, the corresponding properties of the cable itself. Also the voltage gradient at the conductor surface or elsewhere in the insulation may easily be determined in terms of the applied voltage and the cable dimensions, or the unit quantities may be determined from measurements upon cables.

Very few corresponding data for three-conductor cable are available. The reason for this is the difficulty of the mathematical work required for this solution and also the complicated form of those results which have been obtained. By means of electrical measurements in a tank of conducting liquid containing electrodes representing the conductors and sheath of a cable a solution is obtained of many of the problems of the three-conductor cable. Also there are reported the results of a few measurements of voltage gradients and of potentials in an actual three-conductor cable.

A great many data are available regarding the effect of stranding of single-conductor cable upon the gradient and the dielectric strength. We have added somewhat to the data now available on voltage gradient. For various reasons, the practical effect on dielectric strength of the increase of gradient due to stranding is not in proportion to that increase. Two reasons probably predominate: the change in gradient affects only a very small thickness of dielectric; it probably usually occurs that a better mechanical construction results when paper wrappings are applied to a stranded than to a solid conductor. Further data regarding this matter are of very considerable importance.

It is intended to leave most of the application of these data for presentation at a later time, though some of the directions in which it is useful will be pointed out briefly. It is important to distinguish from the beginning between two distinct uses for the data given. First, there is the correlation of the capacitance of the cable with the permittivity of the dielectric, the resistance of the cable with the resistivity of the dielectric, etc. Second, there is the determination of voltage gradient. Concerning the first there is a very definite and immediate application. The field of application of the second is much less definite though actually far more extended. The distribution of the voltage gradient in various parts of a cable is one of the several fundamental considerations governing the voltage rating of the cable.

Measurements of stresses and capacities, both three-phase and single-phase, were begun in the laboratory of the Pittsburgh factory of the company with which the writer is associated, in 1913, under the direction of C. W. Davis. At this time were begun the experiments with a tank of liquid, or solid electrolyte similar to that hereinafter described, and the results obtained were substantially in accordance with the results herein published. Later experiments were made at Perth Amboy in 1917 and again in the spring of 1919 by the writer. In the successive series of tests some of the earlier measurements were repeated and the field of study was extended, and apparatus was devised which would permit of a still more accurate measurement of the stresses than those obtained in 1913. This paper deals largely with the actual measurements made in the last two series of tests, though a brief summary of part of the earliest series is added.

DETERMINATION OF CAPACITY RELATIONS

The determination of capacity relations will first be described. The experimental work consisted in the measurement of resistance between electrodes placed in a tank of electrolyte with the electrodes so arranged that they occupied the same relative position to each other as do the separate metallic parts of a three-conductor cable.

The terms conductors and sheath will be applied to the electrodes representing them, and the terms conductor and belt insulation to that part of the electrolyte occupying the position that the conductor and belt insulation would occupy in a cable. Also the terms "filler" or "filler space" will be applied to the part of the section corresponding to that part of the cable section usually occupied by jute or paper "filler." The term cable will be applied to the complete set of electrodes and electrolyte in any one arrangement.

The capacity relations were determined in the following manner. The conductors and sheath were arranged in their relative positions in a wooden tub containing fairly high resistance water. Measurements were made of the resistance between electrodes when they were connected in various ways. The following were the connections used.

1. One conductor *vs.* another conductor. (Sheath and third conductor free).
2. One conductor *vs.* two conductors. (Sheath free).
3. One conductor *vs.* sheath. (Other conductors free).
4. One conductor *vs.* one conductor and sheath. (Third conductor free).
5. One conductor *vs.* the other two conductors and sheath.
6. Two conductors *vs.* sheath. (Third conductor free).
7. Two conductors *vs.* sheath and third conductor.
8. All conductors *vs.* sheath.

One of the same conductors was then placed in the center of the same sheath, thus forming the equivalent of a single-conductor cable, and a reading taken of resistance between conductor and sheath. The ratio of the resistances for the three-conductor connections to the resistance of the single-conductor to sheath gave the ratio of capacities for the three-conductor connections as compared to the capacity of a single-conductor cable having the same size conductor and sheath. The single-conductor capacity can easily be calculated. The matter may be explained in another manner. The measurement with concentric tubes furnishes data for ready calculation of the resistivity of the electrolyte. The three-conductor measurements thus are made with "dielectric" of known resistivity and conductors of known dimensions.

The above measurements were made with various proportions of belt to conductor insulation and with various ratios of conductor to sheath diameter.

The resistance measurements were made using single-phase 60-cycle current, by measuring the applied voltage and the energy consumed in the resistance of the electrolyte. For the latter measurement there was used the wattmeter connection of a Rowland dynamometer, the connection being such that the resistance of the fixed coils was included in the measurements. Approximately two volts from an auto-transformer was applied to the tubes, and watt readings were taken for the various connections, from which the effective resistance between the electrodes could be calculated. The dynamometer was calibrated by substituting a variable known resistance for the electrolyte.

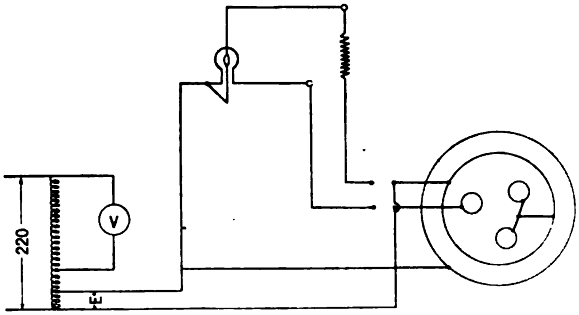


FIG. 1—WIRING DIAGRAM FOR DETERMINING CAPACITY RELATIONS

The bottom of the tub was covered with a layer of paraffine which prevented end effect due to possible conduction in the water-soaked wooden bottom, and also served as a flat level bottom on which to rest the tubes.

To check the general accuracy of the method and the applicability of the results, a large number of measurements were made with concentric tubes. Resistance measurements were made between many combinations of pairs of different sizes. Also after a measurement was made between such a pair of rings, a third one was placed between the two and concentric with them and measurements repeated. By this means it was shown that the resistances obeyed the theoretical law.

TABLE I.

Sheath Diam.	Cond. Diam.	Ratio thickness belt insulation to thickness conductor insulation
15.4 in.	3.62 in.	0.
"	"	0.25
"	"	1.0
15.4	2.50	0.
"	"	0.25
"	"	1.0
15.4	1.90	0.
"	"	0.25
"	"	1.0
11.8	2.5	0.
"	"	0.25
"	"	1.0
11.8	1.9	0.
"	"	0.25
"	"	1.0
9.9	2.5	0.
"	"	0.25
"	"	1.0
9.9	1.9	0.
"	"	0.25
"	"	1.0

These measurements showed that end effect actually had been eliminated by the initial arrangement.

Another disturbing factor was detected and eliminated through the agency of these tests. The tubes or rings seemed to have a high surface resistance possibly due to polarization.

Water of fairly low resistivity was being used (a very weak saline solution). The resistance was increased largely by the use of distilled water mixed with a very small amount of tap water. This however only partly removed the difficulty, as the surface resistance, though relatively smaller, was larger absolutely, and large enough to cause the results to be slightly in error. It was found that if the tubes were thoroughly polished with fine sandpaper or emery cloth at sufficiently short intervals this resistance could be entirely eliminated. In measurements of stress, described later, even a much lower surface resistance would have been of importance, but careful tests showed it was possible to entirely free the results from this type of error. Attempts were made to use some rings made of galvanized iron, but this material did not give satisfactory results and was abandoned.

Measurements were made with the sizes and arrangements of electrodes shown in Table I.

All tubes were of copper and 8½ in. high. The readings for different connections for any one cable checked the relations given by Russell within 2 per cent.

These relations are as follows;

TABLE II.

(1) Capacity between 1 and 2 $= 1/2\,(a - b)$
(2) " " 1 and 2, 3 $= 2/3\,(a - b)$
(3) " " 1 and S (2 and 3 insulated) $= \dfrac{(a - b)\,(a + 2b)}{a + b}$
(4) " " 1 and S, 2 (S insulated) $= \dfrac{(a - b)\,(a + b)}{a}$
(5) " " 1 and S, 2, 3 $= a$
(6) " " S and 1, 2 (3 insulated) $= \dfrac{2\,(a - b)\,(a + 2b)}{a}$
(7) " " 1, S, and 2, 3 $= 2\,(a + b)$
(8) " " 1, 2, 3, and S $= 3\,(a + 2b)$

With each cable, actual measurements were made of all the 8 combinations, though the tests on No. 1 and No. 5 were made with the greatest care. From these, the values of *a* and *b* were calculated, and following this, the values of the other six combinations. Check with measurements was made as indicated above.

APPLICATION OF DATA OF CAPACITANCE AND CHARGING CURRENT

The data of these measurements are shown only in the final form, Fig. 2. Specifically, Fig. 2 gives the charging current of different sizes of cable within the range of present practise. The values are based on a frequency of 60 cycles and a specific

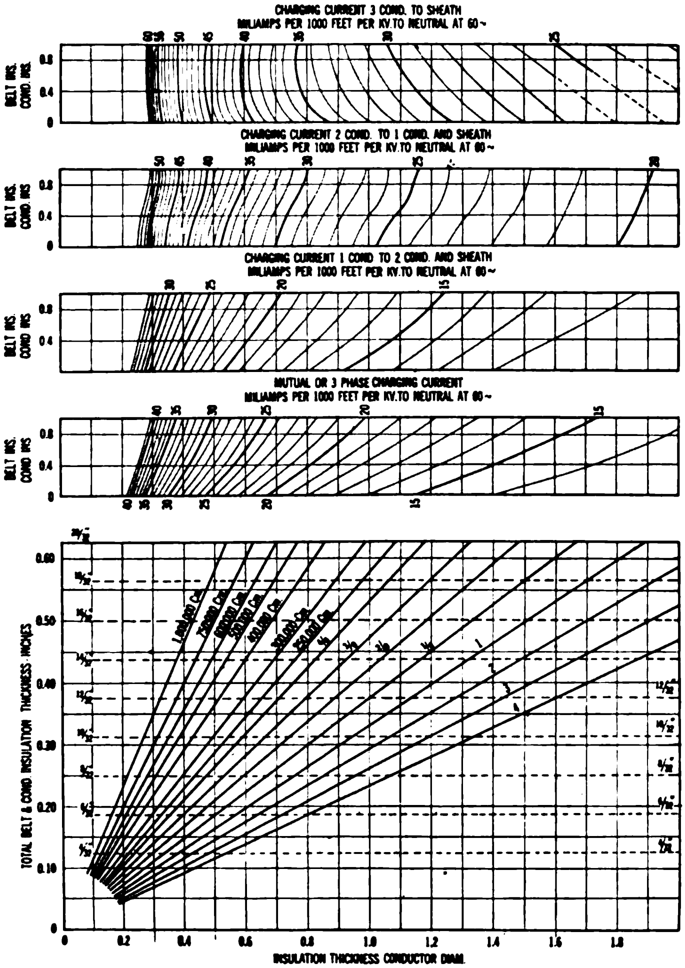


FIG. 2—CHARGING CURRENTS FOR ROUND THREE-CONDUCTOR CABLES BASED ON SPECIFIC INDUCTIVE CAPACITY OF 3.27 AND A FREQUENCY OF 60 CYCLES

Note:—The abscissa shown applies to all 5 sets of curves.

inductive capacity of 3.27, the latter being a somewhat arbitrary figure but actually about the normal value for impregnated paper.* By the use of constants given below, the chart may be used for the determination from unit quantities of the various other properties of a three-conductor cable.

To find microfarads per mile multiply the values taken from the curves by 0.014.

To find insulation resistance of any cable in megohm-miles divide the number of megohms resistivity (cm. unit) of the dielectric by the value obtained from the curve Fig. 2 and multiply by 2.06×10^{-5}.

To find the temperature rise across the insulation due to one watt per foot of copper I^2R in a 3-conductor cable, divide 0.1097 by the charging current given for three conductors to ground, and multiply by the thermal resistivity in watt-cm. units. (This is of the order of 500 to 1000 for most insulating materials.)

The number of milamperes per kv. to neutral per 1000 ft. is also equal to the number of volt amperes per 1000 ft. with one kilovolt of three-phase voltage between conductors. This forms a very convenient basis for comparison of dielectric loss data on cables of different size conductors and different thickness of insulation and with three-phase voltage supply. When the power factor and specific inductive capacity of any insulating material are found by tests on one or more cables, the three-phase losses in kw. per 1000 ft. on any other cable with that insulating material may be determined by multiplying the volt-amperes per 1000 ft. as obtained from the chart, by the power factor, by the ratio of the specific inductive capacity to 3.27, and by the square of the number of kilovolts between conductors.

It will be observed that though all the capacity measurements indicated are with single-phase current supply, data are given for three-phase charging current. There is a very simple theoretical relation between the three-phase connection and single-phase connection No. 1, but we have not relied solely upon that. The measurements may be considered as supplying information as to charging current of the single-phase connections only. By measurements on many cables it is known that the charging current per conductor with three-phase voltage is the same as that for single-phase voltage, connection No. 1,

---

*See Appendix A for instructions as to how to use Fig. 2.

when the voltage from conductor to sheath is the same in each case. (In case of connection No. 1 the third conductor is at sheath potential, whether actually connected to sheath, or free). This is the theoretical relation said above to exist.

Sometimes it is important to know the charging current which may flow in the case of a system operating with ungrounded neutral, when one of the lines becomes grounded. The value of the current is of importance for various reasons external to the cable itself. Also, on account of large increase in the kilovolt-amperes of the charging current, there will be at least a proportionate increase of dielectric losses. These particular data are calculated from the other data given by the use of the same method Russell gives for calculating the relationship of the various single-phase capacities. The data follow:

Let C_5 and C_8 represent respectively the single-phase charging currents per kv. for one conductor against the other two and sheath, and for three conductors against sheath. For normal three-phase operation then, the charging current per kv. to ground on each conductor is equal to $1.5\,C_5 - 0.167\,C_8$. With a grounded conductor the charging current on each of the free conductors per kv. is equal to $\sqrt{0.752\,C_5^2 + 0.027 C_8^2}$, which, for practical purposes is identical with $0.91\,C_5$. The charging currents per kv. on the grounded conductor and on the sheath, are, respectively, equal to $0.0866\,C_5 - 0.288\,C_8$ and $0.577\,C_8$. With a given operating voltage E between conductors, the charging current values are therefore as follows:

For each conductor on normal three-phase operation	$E(0.866C_5 - 0.096C_8)$
For each free conductor when one of the conductors is grounded	$0.91EC_5$
For the grounded conductor	$E(0.866C_5 - 0.288C_8)$
For the sheath when one of the conductors is grounded	$0.577EC_8$

The total volt-amperes due to the charging current will be, on normal operation $\sqrt{3}\,EI$, which is equal to

$$E^2\,(1.5C_5 - 0.167C_8)$$

and when one conductor is grounded

$$E^2(1.5C_5 + 0.167C_8)$$

Let us take as an example a three-conductor cable with 4/0 conductors, having 6/32 in. insulation on each conductor and 6/32 in. on the belt, and at, say, an operating voltage of 11 kv.

$C_5 =$ 0.0202 amperes per kv. per 1000 ft.

$C_8 =$ 0.0365 amperes per kv. per 1000 ft.

The charging current with ordinary operation is, therefore, 0.154 amperes.

With one of the phases grounded, the charging current for each free conductor is 0.203 amperes, for the grounded conductor 0.077 amperes, and for the sheath 0.231 amperes.

The total volt-amperes for the case where the neutral is grounded will be 2930, and for the case where one of the conductors is grounded 4400. On the basis of the same power factor, the dielectric loss, as well as the volt-amperes of the charging current, in the second case will be 50 per cent higher than in the first.

EXPLANATION OF NATURE OF FIELD WITH THREE-PHASE VOLTAGE SUPPLY

Fig. 4 is presented largely to illustrate the general nature of the field in a three-conductor cable upon which three-phase voltage is impressed. There are two sets of curves shown—one showing lines of equal phase, and the other showing lines of equal maximum potential. Each set, to be complete, should comprise the entire 360 deg. of section, but, because of symmetry, the 120 deg. included by each set gives information which is exactly as complete. The method used for the determination of these data is not dissimilar to the one previously described in connection with the measurement of single-phase capacity relations.

The isophase line shows the phase relations of the potentials existing in the cable section. Taking the phase of the potential of the left hand conductor as the basis of reference, and calling it 0 deg., the phase of the potential of the right hand conductor will be 120 electrical degrees away. The phases of the potentials at all points included by the 120 geometric degrees between the radii through the centers of these two conductors are as indicated by the isophase lines. Another way of considering this set of curves is as follows: At some instant the 0 deg. line will be at zero potential. After the lapse of an interval of time represented by one electrical degree, the 1-deg. line will be at zero potential; after the lapse of another, and exactly equal interval, the 2-degree line will be at zero potential; until, finally, after a total interval of time, represented by 120 electrical degrees, has elapsed, the 120-degree line will be at zero potential. It will be noted that the change of phase is much more rapid (that is, a given distance corresponds to much more phase change) as the radius bisecting the center line of conductors (the 60-degree line) is approached. This difference is very pronounced at the sheath, less so at the conductors, and, as the center of the cable is approached, becomes nearly unnoticeable. At the very center the variation does not exist at all,—in other words, the electrical degrees coincide exactly with the geometric degrees.

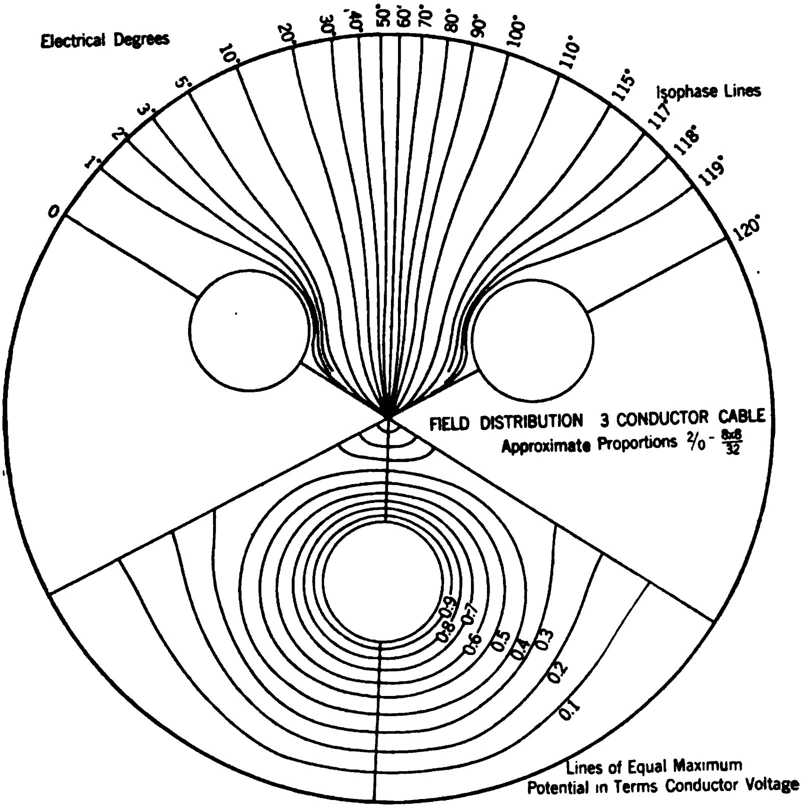


FIG. 4—LINES OF ISOPHASE AND LINES OF EQUAL MAXIMUM POTENTIAL

The lines of equal maximum potential are the loci of points having the same maximum potential above ground at some time during the cycle. These curves should not be confused with equipotential lines, which represent instantaneous values of potential, and would be, for a three-phase field, different at each instant of time. Wide spacing between the lines of maximum potential does not indicate low stress. Particularly is this true near the position midway between two conductors. Here the lines of maximum potential are rather widely spaced, but points a very short distance apart reach their maxima at such different instants that the voltage gradient is actually very high. The curves for potentials very close to the maximum potential of the conductor are circular in form, and concentric with the conductor. For lower potentials the lines are no longer circular but still completely surround the conductor. Finally, a value of potential is reached, below which there are two lines for each value of potential,—one coming within the space between conductor and sheath, and the other coming entirely within the space between conductors, and symmetrical about the center of the cable. As zero potential is approached these lines becomes circles concentric with the cable.

Either set of curves taken by itself is useful as general information, but nothing really definite can be deduced from it. The two sets taken together, however, completely specify the three-phase field of the cable. By the use of the two sets, the potential above ground and its phase relation may be found for any point in the cross section of the cable at any instant of time. Stress data may be obtained by determining the voltages and their directions at two points which are close together. Stress data thus obtained, however, would not be of a sufficient degree of accuracy to be of great value.

For accurate stress determination it is necessary to measure the actual difference of potential between two points, a known small distance apart, rather than to correlate the voltages between the two given points and a given basis of reference. The method used for making these measurements is described in the next section. The data given in Fig. 4 might readily be calculated from the stress data recorded later, which were gotten by this method.

STRESSES AS MEASURED WITH ELECTROLYTE TANK

The stresses in the three-phase field were measured in the following manner: An exploring device was made up as shown in Fig. 5. Two steel rods about 1/16 in. in diameter and 8½ in. long were fitted into small hard rubber blocks so as to form a frame. Half way between the steel rods, and with their plane perpendicular to the plane of the rods, two No. 26 bronze wires were held taut. These bronze wires, which were approximately 0.25 in. apart, served as the potential electrodes. The potential between the wires divided by the distance apart is quite accurately equal to the stress at the point half way between them.

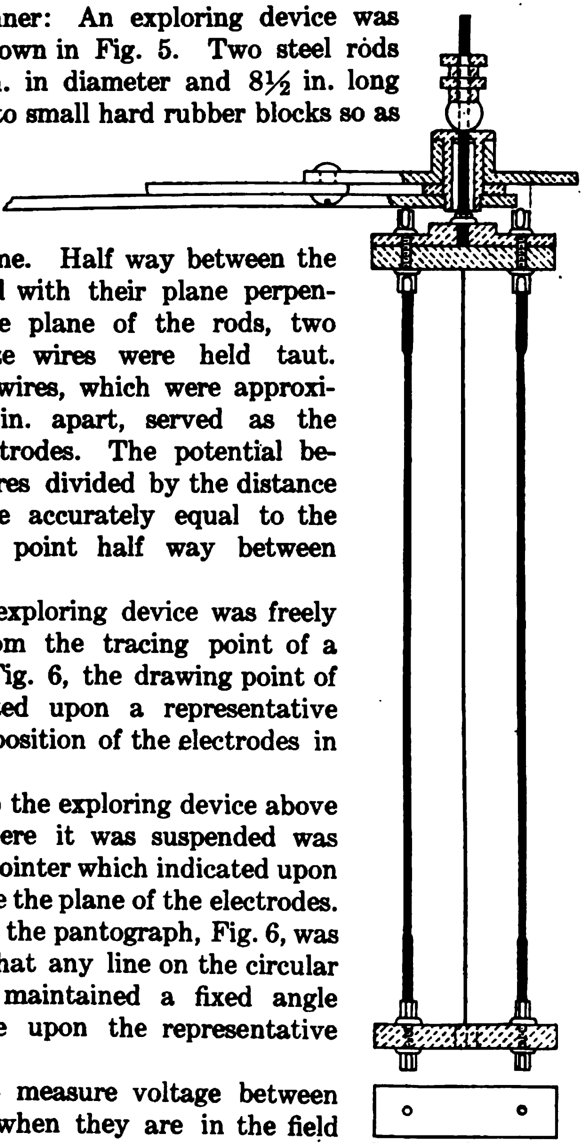


FIG. 5—EXPLORING DEVICE

The whole exploring device was freely suspended from the tracing point of a pantograph, Fig. 6, the drawing point of which indicated upon a representative drawing the position of the electrodes in the tank.

Fastened to the exploring device above the point where it was suspended was fixed a small pointer which indicated upon a circular scale the plane of the electrodes. The rigging of the pantograph, Fig. 6, was so arranged that any line on the circular scale always maintained a fixed angle with any line upon the representative drawing.

In order to measure voltage between these points when they are in the field between the three conductors, upon which is impressed three-phase voltage, it is necessary to measure the magnitude and phase displacement with respect to a fixed voltage, and to do so without distortion of the electrical field by the measuring current. The second requirement demands that a null method be used, or that the instrument for measuring the voltage be of sufficient sensitivity so that the current which it takes will distort the field by a negligible or a very small and known amount. It will be seen below that the latter arrangement has been used with very satisfactory results. To secure the correct measurement of the voltage regardless of its phase and to measure the displacement, it was first proposed to make use of a phase-shifting transformer, but the method to be described proved actually simpler in operation and fully as accurate in results.

The potential across the electrodes was measured by means of a Rowland electro-dynamometer, the field of which was separately excited with fifty amperes, as shown in Fig. 7. The connections were so arranged that it was possible to obtain currents in the field circuit that were equal and ninety degrees out of phase with each other by merely throwing a switch (No. 1). These currents were obtained by taking taps from a star-connected three-phase transformer, one side of the switch being connected across one leg of the transformer winding, and the other across approximately 58 per cent of the windings of the other two legs. The neutral of the transformer was grounded. The potential circuit contained 40,000 ohms non-inductive series resistance.

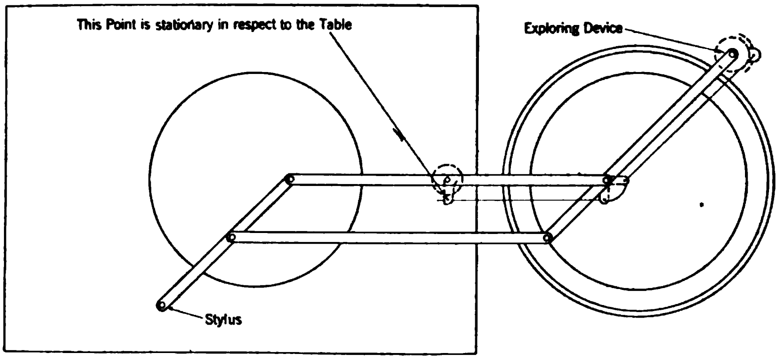


FIG. 6—PANTOGRAPH

The reactances of the fixed coil and of the auto-transformer were compensated by a capacity of 16 microfarads shunted across a variable resistance. The compensation was accomplished in the following manner: A voltage in phase with the voltage across one side of the main field switch was impressed on the moving coil circuit. When switch No. 1 was thrown on that side, the field current was approximately in phase with the current in the potential circuit, and a large deflection resulted. With switch No. 1 thrown in the opposite direction, the field current was nearly 90 degrees out of phase, and a very small deflection resulted. The resistance which shunted the condensers was then varied until this deflection was zero.

To insure that the voltages on the two sides of the switch were at the proper 90-degree relation with each other, the circuits were then interchanged, so that voltage from the other side of No. 1 switch was required to produce zero deflection. It was found that the phase relation was correct within ½ per cent.

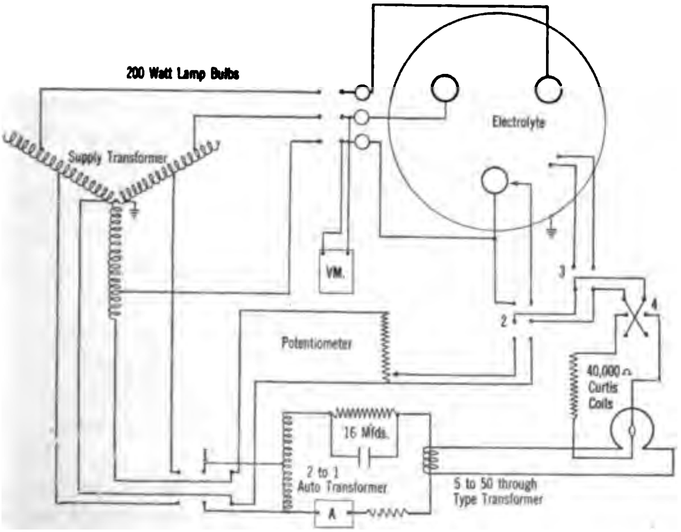


FIG. 7—WIRING DIAGRAM OF CONNECTIONS FOR STRESS MEASUREMENTS

The procedure of the stress measurements was then as follows:

An accurate template was made of thin Bakelite fibre, so that the conductors and sheath always maintained the same relative position.

A large circular piece of plate glass was placed in the bottom of the electrolyte tub and leveled by means of a spiri level. The electrodes were then placed in the tub and held in their positions by the template. The tub was then filled with water to within a half inch of the top of the electrodes.

Three-phase voltage of approximately 50 volts to neutral was then applied to the three conductors and the exploring device suspended in the water. The measurements were made in the field surrounding conductor *A*, whose voltage was in phase with the field current when the field switch (No. 1) was closed to the left.

A full size drawing was then made which exactly corresponded to the template, and various circles and lines were drawn upon it as shown on Fig. 8, the intersections of this line indicating the points at which it was desired to measure the stress.

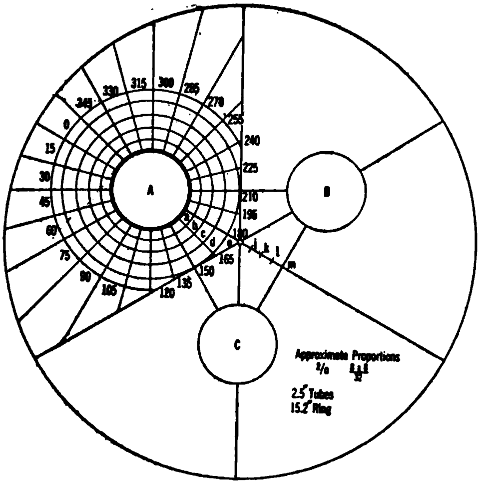


Fig. 8—Cross-Section of Cable Showing Relative Positions of Conductor and Sheath and Points at Which Stresses Were Measured

The drawing was then lined up as follows:

The exploring device was hung somewhere near the center of the cable. The two electrode-wires were short-circuited and the voltage between these and the sheath was measured by means of the dynamometer. At exactly the center of the cable the voltage to sheath is zero. The electrodes were moved about until the dynamometer indicated zero with the field switch on either side. The center of the representative drawing was then placed exactly under the drawing stylus and fixed to the board with a thumb tack.

The exploring device was then placed between the two conductors *B* and *C*. At a point exactly half way between these conductors the potential to sheath is in phase with the voltage of conductor *A* to ground. The switch was then thrown on the side that is out of phase, and the exploring device moved until the deflection was zero. The drawing was then swung about with its center as an axis until the line half way between conductors *B* and *C* was under the stylus. The drawing should then be exactly lined up with the tank. Checks were made by placing the exploring device near one of the conductors and visually comparing its position relative to the conductor with the position of the stylus to the conductor on the drawing.

The circular scale for measuring the direction of the plane of the electrodes was then adjusted by measuring stress along the 180-degree line. The stresses along this line are in phase with the voltage of conductor *A*. Normal to this line there is a large phase change. With the field current out of phase with the potential of conductor *A* the dynamometer will read zero only when the plane of the exploring electrodes coincides with the 180-degree line. With the stylus on the 180-degree line near conductor *A* the exploring device was turned about its axis until the deflection became zero. The zero on the circular scale was then adjusted to coincide with the pointer. Check was made to insure that the actual position was as above stated.

Readings were then taken by placing the stylus at the intersections of the radial lines and the circles shown in Fig. 8. Readings were also taken at the other points indicated. All readings were taken with the plane of the exploring electrodes radial to the conductor, except along the 180-degree line, where the stresses were also measured normal to the line.

Stresses close to the conductor were found by measuring between the conductor and a No. 40 wire fastened by hard rubber cleats 0.125 in. from the conductor. This conductor could be revolved in the template and the stresses on all sides of the conductor could be accurately compared.

To take a reading, the stylus was placed on a point on the drawing at which it was desired to measure the stress. The exploring device was rotated until its electrodes were radial to the conductor. Readings were taken of voltage and dynamometer deflection with the main field switch first on one side and then on the other. This gives both components of the stress as compared to the voltage to neutral of conductor *A*. From these readings the stress and phase angle could easily be calculated.

Voltage was read across the points indicated on Fig. 7. This voltage was frequently compared with the voltage directly on the conductors. The three-phase field would have been slightly distorted had it been attempted to keep the voltmeter connected to the conductor while making stress measurements, due to the slight additional drop across the series lamp bulbs.

Calibration of the instrument was made by means of a potentiometer connected as shown in Fig. 7. A deflection of the dynamometer of 8.08 divisions per volt was obtained with 40,000 ohms series resistance and 50 amperes field current. All results were placed on a common basis of field current and conductor voltage.

The deflections along the 180-degree line were plotted, and the average of this curve between the conductor and center of cable was found. The average voltage between the conductor and center of cable was the voltage on the conductor divided by the distance. The average deflection divided by the calibration constant and by the distance between the exploring electrodes must exactly check the average voltage. From this relation it is possible to find the exact distance between electrodes.

We have mentioned the possibility of field distortion due to the current taken by the apparatus used for measuring the potential difference of exploring electrodes. The effect of this upon the measurements described above was investigated in the following manner. With the exploring electrodes maintained in a given electric field in the tank, the resistance in the metering circuit was varied through a wide range. The reciprocals of the deflections were plotted as ordinates against the external series resistance as abscissas. The resultant "curve" was a straight line cutting the axis at a negative value. This value is equal to the effective resistance between the electrodes If this resistance be added to the external resistance in determining the calibration constant, we fully correct for whatever small distortion there may occur, for it follows from the above that the same voltage is obtained regardless of the series resistance, and therefore the value is the same as for infinite resistance, or zero current and distortion.

The calibration of the electrode fastened to the tube was found by measuring the stress in a single-conductor concentric field. The stress can here be quite accurately calculated and the distance between the No. 40 wire and the tube was found from the relation that the stress at a point half way between the wire and the conductor must equal the deflection divided by the calibration constant of the instrument divided by the distance between the wire and conductor. The internal resistance between this electrode and the conductor was found in the same way as for the other. The calibration constant to be used for any given condition is that given above, which applies to 40,000 ohms total resistance, divided by the ratio of the actual total resistance of the potential circuit (including internal resistance between electrodes) to 40,000. This ratio does not exceed 1.02 or 1.03.

All deflections were put on a common basis by dividing by the average deflection between the conductor and center of cable along the 180-degree line, thus giving the stresses in terms of the average along the 180-degree line from conductor to center of cable.

The two components of the stresses were added at right angles, and the angles by which the stresses lead the voltage of conductor *A* to ground were calculated.

The values were plotted in Figs. 9 to 18 inclusive, which will be described in detail below.

The field between the conductor and the sheath was next investigated. The electrodes were placed on the shortest line between conductor and sheath, and the deflection read. The other two tubes were then removed without disturbing the position of the remaining electrode or its voltage to ground. The deflection was not changed by the removal of the other conductors. Other spacings and sizes of tubes were used, but never did the removal of the two tubes change the amount of the deflection. From this, we can draw the conclusion that the field on the shortest line between the conductor and sheath is a single-phase field and is independent of the presence of the other two conductors.

Determination was then made of the effect of the amount of belt upon the stresses on the inside of the cable. Readings were taken with the tubes so arranged as to simulate a cable with equal belt and conductor insulation. Then without changing the position of the conductors, the sheath was replaced by a smaller one. This new ring was even smaller than one that would correspond to the sheath of an unbelted cable. Most of the readings were then repeated. The change in sheaths produced practically no change in stress within the portion of the field bounded by the three lines joining centers of conductors.

Description of Figs. 9 to 16

Fig. 9 shows the stresses along the line joining the center of conductor *A* and the center of cable in terms of the average stress between the conductor and the center. This average represents the total voltage of conductor *A* divided by the distance between the conductor and center of cable.

In all the following curves, as well as most of the subsequent discussion, stresses are given in terms of this same unit. For a round conductor, this distance between conductor and center is equal to 1.154 times the conductor insulation (half the distance between surfaces of conductors) plus 0.077 times the conductor diameter.

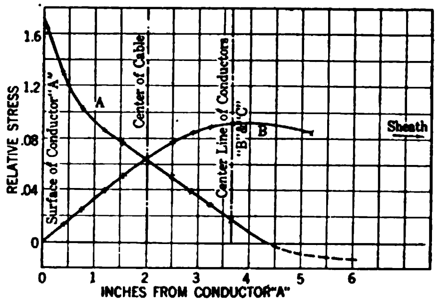


Fig. 9—Stresses along 180-Degree Line of Fig. 8 in Terms of Average between Conductor and Center of Cable (small tubes)

Curve A—Stresses in direction of 180-degree line

Curve B—Stresses normal to 180-degree line

Curve *B* shows the stress normal to the line of centers of conductor *A* and cable. Here the maximum stress occurs near the line joining the centers of conductors *B* and *C*.

Fig. 10 shows the stresses around circle *a*, which is 1/16 in. away from the conductor. All stresses were measured radial to the conductor. It will be noticed that the maximum stress occurs on the line which passes through the center of the cable, though there is very little difference along the arc inclosed by the center lines of conductors.

Another maximum occurs at the outside of the conductor on the shortest line between the conductor and sheath. This maximum is dependent upon the total thickness of insulation, including conductor and belt.

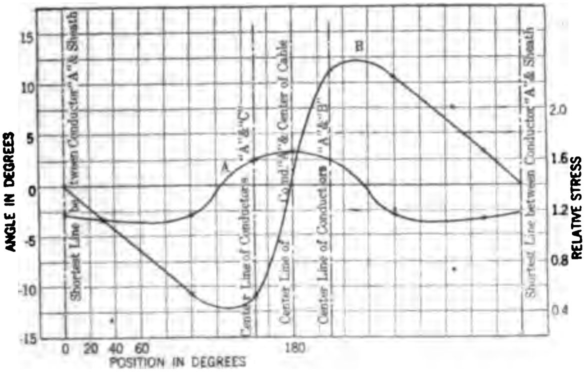


FIG. 10—STRESSES ON CIRCLE (A) OF FIG. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)

Curve B—Angle by which stress leads voltage of conductor A to neutral

Figs. 11, 12, 13 and 14 show the stresses on the various other circles shown. It will be noted that as the distance between the conductor and circle increases that the point of maximum stress moves toward the line joining centers of conductors. Also the point of maximum phase difference moves farther past the line joining centers of conductors.

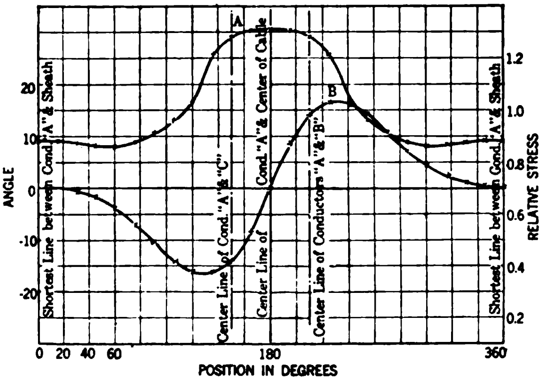


FIG. 11—STRESSES ON CIRCLE (B) OF FIG. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)

Curve B—Angle by which stress leads voltage of conductor A to neutral

Fig. 15 shows the stresses on the outside of the conductor along the shortest line from the conductor to sheath. This is a single-phase field and the maximum stress can be calculated by a formula given by Russell, as shown below. The value obtained from the curve checks within 2 per cent the calculated value.

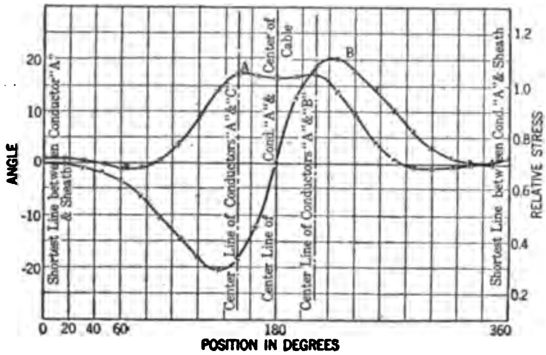


FIG. 12—STRESS ALONG CIRCLE (C) OF FIG. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)
Curve B—Angle by which stress leads voltage of conductor A to neutral

Fig. 16 is given in this paper because of its value in supplying data outside the range of the experimental values. The lowest curve gives the maximum stress toward the sheath of a three-conductor cable in terms of the average stress between conductor and sheath. The middle curve give

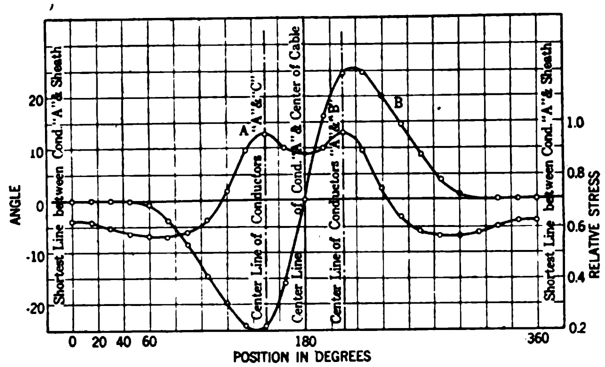


FIG. 13—STRESS ALONG CIRCLE (D) OF FIG. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)
Curve B—Angle by which stress leads voltage of conductor A to neutral

for a single-conductor cable the maximum stress in terms of the average. The upper curve simply gives the ratio between the maximum stress of a single-conductor cable and that towards the sheath of a three-conductor cable having the same size conductor and the same distance between conductor and sheath. The ordinates for this last curve are obtained by dividing the ordinates of the middle curve by those of the lower curve. For the solution of all problems to which Fig. 16 is applicable, any two of the three curves shown give the complete data.

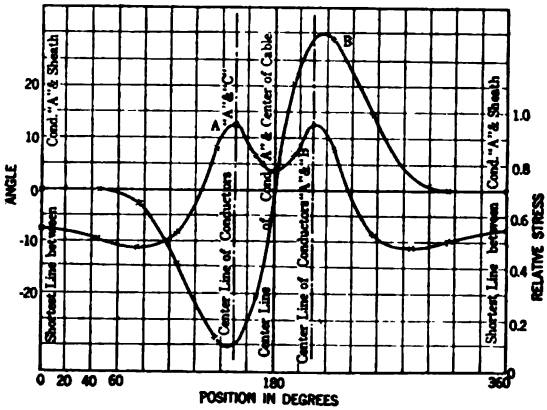


FIG. 14—STRESS ALONG CIRCLE (E) OF FIG. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)

Curve B—Angle by which stress leads voltage of conductor A to neutral

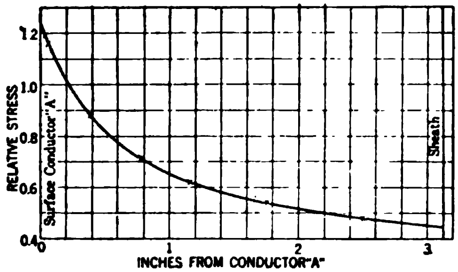


FIG. 15—STRESSES ALONG 0 DEGREE (FIG. 8) IN TERMS AVERAGE BETWEEN CONDUCTOR AND CENTER OF CABLE ALONG 180-DEGREE LINE (SMALL TUBE)

The middle curve is a graphical solution of the common formula showing the relation between maximum and average stress of a single conductor cable. This is

$$\frac{\text{maximum stress}}{\text{average stress}} = \frac{R - r}{r = \log R/r}$$

r being the radius of the conductor, R the radius of the cable (= log) being natural logarithm, = 2.303 times the common logarithm).

The lowest curve is a solution of the rather lengthy formula given by Russell for the maximum stress between eccentric cylinders. Since we have found that this maximum is unaffected by the presence of the other two conductors of a three-conductor cable, we may expect our values to check a solution of this formula, and, as stated, have found they do so. There are several variables in this formula, and an exact solution for all conditions would require a series of curves. However, the use of the coordinates taken has enabled us to plot a single curve which is correct within about one per cent for any point.

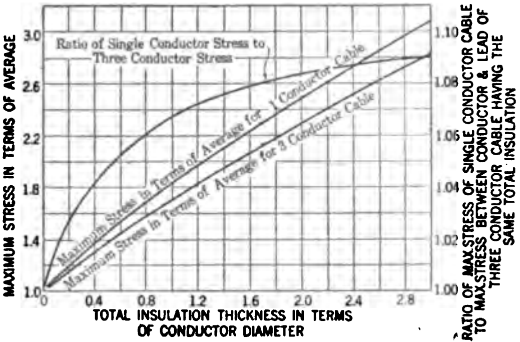


Fig. 16—Curves Showing Relation between Stress of Three-Conductor Cable and Stress of Single-Conductor Cable and between Maximum and Average Stress of Each

Fig. 17, Curve *A*, gives the stresses between the conductors on the line joining their centers. Curve *B* gives the angle by which the stress leads the voltage of conductor *A* to neutral.

The vector diagram, Fig. 18, was drawn primarily as a check on the method used for obtaining stresses. Incidentally, from it may be determined voltages between a conductor or the center of the cable, and any point on the line joining the centers of the two conductors.

The horizontal line *A X* is drawn to represent the direction of the voltage of conductor *A* to neutral. The arc *A Z B B* was obtained by the addition of potential differences obtained by the integration of stresses (shown in Fig. 17) along the line joining the centers of the conductors *A* and *B*.

The potential at a point along the center line *A B*, a short distance from conductor *A*, was determined, being proportional to the area included below curve *A* of Fig. 17. The vector representing this potential was plotted as the first step of the arc *A Z B B*, Fig. 18, the angle with the horizontal being determined by curve *B* of Fig. 17. Next, the voltage between this first point and a second was obtained in the same manner and a new portion of arc drawn in. (It will be noted that the individual sections of arc are actually drawn as chords). This process was continued for the entire line between conductors *A* and *B*.

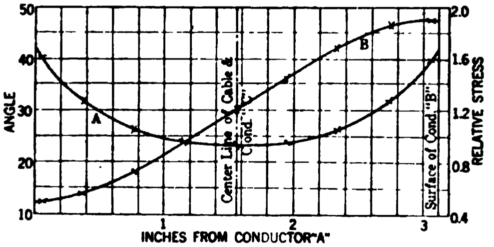


FIG. 17—STRESSES ALONG LINE JOINING CENTERS OF CONDUCTORS IN TERMS OF AVERAGE BETWEEN CONDUCTOR AND CENTER OF CABLE ALONG 180-DEGREE LINE (15.2 IN. RING—2.5 IN. TUBE)

Curve A—Stresses

Curve B—Angle by which stress leads the voltage of conductor A to neutral

The vector represented by any of the lines drawn from *A* and terminating on the arc *A Z B B* is the difference of voltage between conductor *A* and some definite point on the center line of *A* and *B*, but, since both the abscissa and ordinate of the point on the arc are used to represent potential, it is not possible to show on that curve alone the location of the point on the center line of *A* and *B*. Therefore, the line *A B* is drawn so that if a vertical line is drawn from any point on the arc to the line *A B*, the intercept between *A X* and *A B* represents the distance from the conductor *A* to the point the potential of which is represented by the chord drawn from *A* to the said point on the arc. Thus, the distance *B X* represents the distance of the conductor *B* from the conductor *A*, the potential of *B* being represented by *A B B*.

Also the line *W Y* is equal to half the line *B X*, and thus the chord *A Z* represents the voltage from conductor *A* to a point on the center line of *A* and *B*, and halfway between them.

The line *A O* represents the voltage between conductor *A* and center of cable, or ground. Also, the voltage to ground of any other point is represented by a line to the point *O*, thus *B B O* represents the voltage to ground of the conductor *B*, as determined by the measurements of stress given in Fig. 17.

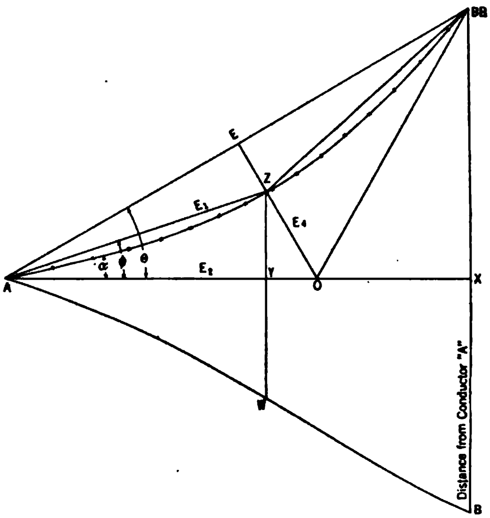


FIG. 18—VECTOR DIAGRAM OBTAINED BY SUMMATION OF STRESSES ALONG LINE JOINING CENTERS OF CONDUCTORS

The value obtained from the mathematical integration of the stress along the line joining the centers of conductors *A* and *B* was found to be 1.72 times the voltage of conductor *A* to ground and at an angle of almost exactly 30 degrees from this voltage, a very satisfactory agreement with the actual values of 1.73 and exactly 30 degrees.

The voltage *O Z* is found either from this diagram or by the integration of the stresses along the line joining the center of the cable and the mid-point of the line joining the centers of conductors *B* and *C*. The values obtained by these two methods are 0.325 and 0.355, respectively, in terms of voltage of conductor A to neutral, and at an angle of 60 degrees from that voltage.

The voltage from conductor A to the point midway between conductors A and B on the line joining their centers is the chord $A\ Z$, and is equal to 0.87 times the voltage of conductor A to neutral, and is at an angle of 18.6 degrees from this voltage.

Data given thus far have applied to measurements on a single "cable." Similar measurements were taken of stresses with a different size of conductor. A sheath 15.2 in. in diameter was used and 5.82 inch conductors were so placed that the conductor and belt insulation were equal.

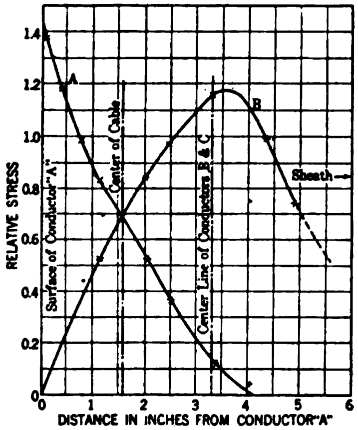


Fig. 19—Stresses along 180-Degree Line in Terms Average between Conductor and Center of Cable (large tubes)

Curve A—Stress along 180-degree line
Curve B—Stress normal to 180-degree line

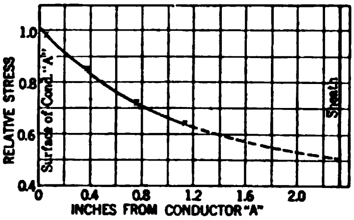


Fig. 20—Stresses along 0-Degree Line in Terms Average between Conductor and Center of Cable along 180-Degree Line for 15.2 in Ring and 3.82 in. Tubes

Not as many readings were taken for this "cable," Enough were taken however to show that the maximum stress occurred at the conductor on the line towards the center of cable and to determine stresses at the points of maximum importance. Stresses were taken all along the lines joining the center of conductor A and center of cable.

In Fig. 19, curves A and B, respectively, are shown the stresses along (parallel to), and normal to, the line joining center of conductor and center of cable.

Fig. 20 shows the stresses on the shortest line between the conductor and sheath. The maximum of this curve checks within 2.5 per cent the value calculated from Russell's formula as shown in Fig. 16.

USE OF SOLID ELECTROLYTE

In the Pittsburgh experiments of 1913 and 1914 use was made of solid as well as liquid electrolytes. Cable cross sections were prepared of solid electrolytes with cast-in exploring electrodes in which the specific resistivity of the electrolyte occupying the filler space differed from that of the electrolyte representing the conductor and belt insulation. The extreme limit of range of resistivity for the fillers was obtained first, by using solid insulators, and second, by making the fillers of copper.

Also, thermometers were cast into the solid electrolyte so as to note the change in temperature in the cross-section due to the energy loss. One of the solid electrolytes used melted at a relatively high temperature and it is of interest, in passing, to note that after three-phase voltage had been applied for a short period to the tank, filled with such solid electrolyte, all of the electrolyte in the triangle formed by the straight lines connecting the centers of the conductors became molten, thus giving a direct analogy to the over-heating of ordinary 3-core cable, as indicated by the characteristic charring of the central filler space, when going to destruction by overheating under three-phase voltages. This phenomenon has been noted by Mr. C. W. Davis as early as 1911.

The characteristic charring of three-core cable was observed by Hōchstadter on what we understand was mineral base compound about the same time, although not reported to us until about 1913. It has also been noted on vegetable compound cable by Clark and Shanklin and by Bang and Louis, and reported by them in the A. I. E. E. TRANS. 1917, p. 431 and 447 and for some time engineers of operating and manufacturing companies have generally recognized this condition.

The electrolyte experiments of 1913-1914 were part of an investigation into the causes of this type of failure of three-core cable. Supplementing those experiments, covering the same period approximately, a comprehensive set of dielectric loss measurements were made at high and low voltage, both single-phase and three-phase, on mineral compound and vegetable compound three-core cable. As a result of these experiments, the opinion was reached that the arrangement of conductors and insulation in any ordinary three-core cable is such as to make it inherently weak, the tendency of the center to overheat being increasingly difficult to overcome the higher the operating voltage.

From the user's standpoint especially, however, it is very desirable to retain, insofar as possible, the advantages and economy resulting from having the three conductors under one lead sheathing rather than the more costly alternative method of using three single-conductor cables, each separately lead covered, with attendant disadvantages of extra cost due to the increased amount of lead sheathing, and the larger ducts or greater number required.

M. Höchstadter (*London Electrician,* May 19, 1916) has described a way to avoid the disadvantages of the ordinary three-conductor type of cable by converting the cable into what is substantially three single-conductor cables without the attendant disadvantages of disproportionate increase in cost. The conductors may be made sector shaped so that all the space within the single lead sheathing is economically used, thus securing a still further gain in economy, since it is obvious that three single-conductor cables separately sheathed could not conveniently be made in sector form and installed in such a way as would permit of their fitting together in order to occupy minimum duct space. The thin metal foil or tape suggested by M. Höchstadter is preferably made of copper, thus securing the most effective dissipation of heat.

As part of the characteristic charring of the central portion of a three-conductor cable of normal type is due to internal corona, it is obviously desirable to prevent this as far as possible, and what appears to be one of the most effective methods of preventing the occurrence of minute air or gas spaces in the highly stressed portions of cable dielectric is the suggestion of H. W. Fisher in 1911 to apply the previously dried and impregnated tape to the conductor while the conductor itself is submerged under oil. The adoption of this method should make internal corona, or ionization, of relatively little importance even at the highest voltages likely to be reached for many years to come.

Numerous tests were also made with the electrodes representing the conductors, of different shapes. Certain advantages were found as regards distribution of stress and dielectric loss with a conductor of approximately semi-circular form with slightly rounded corners, and with the approximately flat side toward the sheath. It is conceivable that such a special form would be available in special cases where large conductors are involved, but on account of the constructional difficulties and larger diameter of this form, a modified sector conductor is the most suitable practical alternative for regular use.

Referring again to the use of solid electrolytes, it may be of interest to report the use of such electrolyte in the form of pieces simulating simple forms of solid insulators, such as those made of porcelain, in use in certain types of cable terminals. By immersing such molded solid electrolyte in liquid electrolyte of higher resistivity there was obtained a direct representation of a porcelain insulator surrounded by air, and we were thus enabled to plot the equi-potential lines from the point where they leave the surface of the solid electrolyte. This method may be of interest to the designers of porcelain insulators, although complicated forms may prove themselves difficult to cast and make self supporting without undue deformation.

Using the same electrolyte method, and following the analogy of heat flow, isothermal lines for various shaped conductors in three-conductor cables were plotted in the Pittsburgh experiments of 1913-1914. These show that under the I^2R loss alone the center of a three-conductor cable will be lower in temperature than the temperature of the conductors by a small relative amount. In a cable using three sector shaped conductors, less relative difference was found between the temperature of the center of the cable and the temperature of the conductors than where round conductors were used.

Thornton has explored the field of a three-conductor cable by means of the Hele-Shaw method. He suggested the existence of a triple-frequency field at the center of the cable. In the same series of Pittsburgh experiments data were also secured regarding this point, and also later in measurements with actual cable, at Perth Amboy. All of the measurements made failed to show any such effect as described by Thornton. It is to be expected, however, that measurements carried on with cable at voltages above the ionization point would show pronounced harmonics in the potential wave of stresses at different points.

MEASUREMENT OF STRESSES IN ACTUAL CABLE

For this test a specially constructed cable was used. This cable was in nearly every way like a commercial three-conductor cable with round conductors. Its main differences were that at definite points very thin wires (No. 40 A. W. G.) were laid parallel to the conductor, and that the cable was assembled without "lay".

The cable consisted of 4/0 stranded conductors with 9/32 in. of paper insulation on each conductor and 5/32 in. on the belt. The separate conductors were made up as follows: Insulation was first applied to a thickness of 1/32 in. Eight No. 40 wires were then placed at equal intervals around the conductor, and parallel to its axis. The same was done after a total insulation thickness of 3/32 in., 6/32 in., and 9/32 in, had been applied. This made a total of 32 wires per conductor, or 96 for the cable. Immediately over each set of eight wires two layers of paper were applied by hand. The rest of the insulation was applied in the ordinary manner by machinery.

A three-conductor sector cable with the same size conductors and the same thickness of insulation as the other was also constructed in exactly the same manner. The number of No. 40 wires per layer, however, instead of being eight were only six,—one at each corner of the sector and one half-way between each pair of corners. The total number of wires for each conductor were then 24, and for the cable 72.

The small wires were placed where they were so as to furnish a means of measuring the potentials at these points. Hereafter, then, they will be termed "potential wires." The points at which the potential wires were placed will be termed "potential points." The three conductors of the cable will be designated as I, II, and III. The four layers of potential wires of each conductor, starting at the layer nearest the conductor, will be called, respectively, *A*, *B*, *C*, and *D*.

Potential data were obtained, in both the circular and the sector cable, on all four rows. Measurements of voltages on *A* and *B* were taken between each potential wire and its conductor, while values on *C* and *D* were taken between potential wire and sheath. Chiefly because data on *B* and *C* would give relatively little additional information, only results gotten on *A* and *D* are given in this paper.

The problem of measurement of the voltage of the potential wires in the three-conductor cable with three-phase voltage applied is quite similar to that encountered in the measurement of potentials and stresses in the electrolyte tank. As before, there is the problem of measuring voltages of various unknown phase relations. This was accomplished in a manner somewhat similar to the one previously described. The current which could be taken without field distortion was much smaller, so small that only a null method seemed available. Also, it was desirable to keep the detecting apparatus at ground potential.

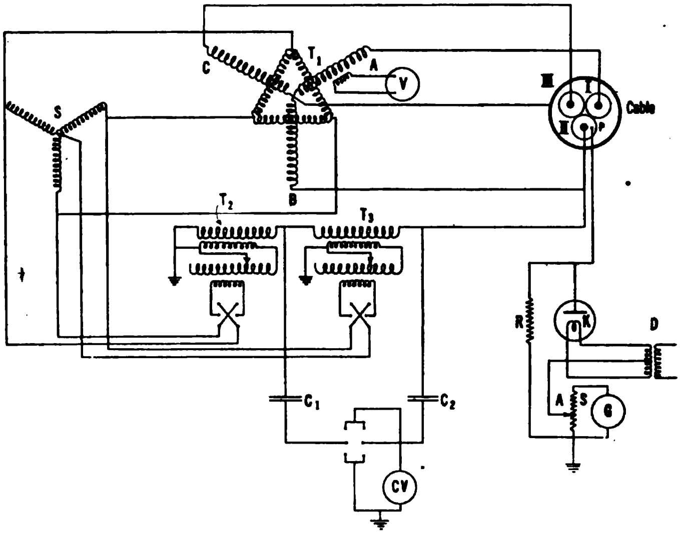


Fig. 21—Diagram of Connections for Determination of Potential Inside of Cable

S—three-phase supply
T_1—three-phase transformer supplying cable
T_2 and T_3—single-phase balancing transformers
V—Voltmeter for T_1
C V—crest voltmeter for T_2 and T_3
C_1 and C_2—condensers used with C V
D—detecting device
K—rectifier (Kenotron)
R—20 megohm resistance shunting K
A S—Ayrton shunt for G
G—d-c. galvanometer
P—potential wire

This explanation will make clearer the reasons for the methods adopted.

Fig. 21 shows the diagram of the connections used. The current supply was taken from a 40 kv-a. three-phase transformer, the primary being delta connected. The secondary is star connected and gives a rated voltage of 95. There are taps on each leg, of 21 per cent and 42 per cent of the full voltage. The delta primary of the three-phase high-tension transformer,

T_1, was connected to the 42 per cent taps, and as the normal rating of this transformer per phase is 440 to 25,000 volts, this gave a voltage per phase of approximately 2300, or a voltage between conductors of about 4000. This voltage was impressed on the cable.

The balancing voltages were obtained from two single-phase transformers, T_2 and T_3, each equipped with a regulating transformer, thus allowing a selectivity of voltage. The primary of T_2 was connected to the 21 per cent taps of the two phases of the supply which supplied phase A of transformer T_1, thus putting T_2 in phase with phase A. The primary of T_3 was connected between the 42 per cent tap of the third supply phase and neutral, thus making the voltage of T_3 90 degrees away from that of T_2. The normal rating of T_2 is 440 to 62,500 volts, with a capacity of 300 kv-a. There are 480 steps in the regulator, so that, with a primary voltage of about 20 volts there are six volts per step. The normal rating of T_3 is 440 to 50,000 volts, with a capacity of 200 kv-a. There are 400 steps in the regulator, so that, with a primary voltage of about 23 there are about seven volts per step. The secondaries of T_2 and T_3 were connected in series, the free end of T_3 being connected to one of the phases or to the neutral of T_1, and the free end of T_2 grounded. T_1 is not grounded at any point, so that the potential above ground of the phase of T_1, or of the neutral, to which the large transformers T_2 and T_3 are connected, is governed entirely by the voltage of these. This arrangement enables the detecting device to be always near ground potential.

To determine the potential at any point P_2, proceed as follows: Connect the potential wire at P to the detecting device. Connect the conductors of the cable to T_1 and the balancing transformers to either the conductor of P or to the sheath. Raise the voltage of T_2 and T_3 in the proper direction until the detecting device shows minimum deflection, which indicates that P is at ground potential. The voltage produced by T_2 and T_3 will then be accurately the difference of potential, in direction as well as in magnitude, between P and its conductor or sheath. This being a null method, the stress distribution, when balance is obtained, is in no wise altered by the measuring apparatus.

The detecting device consisted of a rectifier for which a 100,000-volt, 0.1-ampere kenotron was used in connection with a galvanometer equipped with an Ayrton shunt. The rectifier was shunted by a water resistance of the order of twenty megohms. The rectified current is necessarily unidirectional, and so the proper balance was considered obtained when an increase or decrease in voltage on either T_2 or T_3 increased the deflection. The minimum reading of the galvanometer was not zero, due doubtless mainly to the thermoelectric (Seebeck) effect between the plate and filament of the kenotron, but this did not alter the point of balance.

Voltages were measured as follows: The voltage of T_1 was measured by a voltmeter connected to a tertiary coil of the transformer. The voltages of T_2, and of the combination of T_2 and T_3, were measured by means of a crest voltmeter. By separate measurement the voltage per step of the regulator of T_3 was found to be quite uniform, and its value determined. The voltage of T_3 was therefore calculated from the number of steps on its regulator. Thus, the actual potential difference, as well as its two components, was accurately measured.

ACCURACY CHECKS ON MEASUREMENTS ON ACTUAL CABLE

To determine what error, if any, was due to end effect, the connector of the rectifier was brought within a quarter of an inch from the end of the cable and the full voltage applied to the cable by means of T_1, T_2 and T_3 being at their zero position. The galvanometer showed a deflection which would be c used by less than one step on either controller, indicating that the maximum error is less than 2 per cent in the measurement of the smallest voltage read.

The sensitivity of the detecting device was such that after balance was obtained, an unbalance of one step (6 or 7 volts) on either transformer would definitely increase the galvanometer deflection.

The voltages on T_2 and T_3 should be at right angles to each other. To check this point, both transformers were raised to the same potential and readings taken of the separate voltages and of the hypothenuse voltage. This check was made at several different values with the reversing switches of both transformers in the up position, with both in the down position, and with one up and the other down.

The measured and calculated values checked exactly in some cases, and in others varied by as much as 3 per cent. This variation probably accounts for some small phase angle errors in the results.

A single-conductor cable was prepared with potential wires in the same manner as the three-conductor cables, and was supplied with voltage from phase *A* of T_1, T_2 and T_3 were used as balancing transformers, and the potentials determined at various distances from the conductor. In the ideal case T_2 should have been exactly in phase with phase *A*, and so the balance point should have been with zero voltage on T_3. However, T_3 was required to the extent of changing the phase of T_2 about 1½ degrees on all the measurements. Correction was made for this error in plotting the curves.

The balancing transformers were connected successively with the three phases of T_1, the balancing transformers being grounded directly and T_1 being grounded through a condenser and the detecting circuit. The voltages were then balanced and readings taken on *V* and *C V*. These readings checked each other as closely as the instruments could be read.

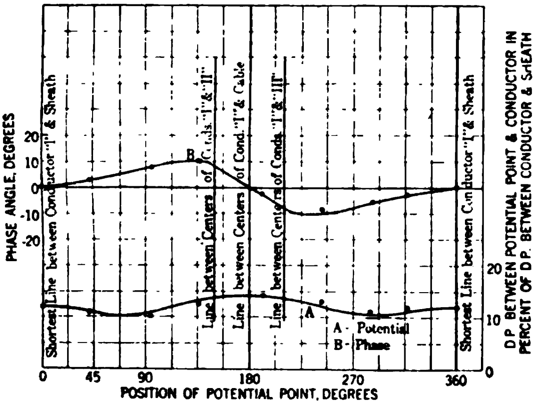


Fig. 22—Phase and Potential at Layer A, Conductor I, Circular Conductor Cable

After the three-phase tests were performed, the cable with circular conductors was dissected, and one of the insulated conductors wrapped with foil, and measurements were made with single-phase on layer *A* of potential wires. Calculation from values thus gotten indicated these potential wires to be 0.031 in. away from the conductor. Micrometer measurements indicated this distance to be 0.032 in. The dissection also showed that the potential wires were quite uniformly spaced around the conductor, but that they were not absolutely parallel with its axis, the rotation in the entire length being not more than half the distance between wires.

Dissection of the sector cable showed layer *D* of potential wires to be placed quite closely to their proper position, but that layer *A* was altogether misplaced. The actual positions were therefore accurately measured and used in Fig. 25. Micrometer measurements indicated the insulation thickness between the conductor and the first layer of potential wires to be 0.325 in. at the small diameter and 0.335 in. on the long diameter.

The curves, Figs. 22 to 25 inclusive, are, as far as possible, drawn from values obtained on one conductor only. The circles indicating the points for this conductor have been filled in solid. On all curves except Fig. 22 points also have been shown which were gotten on the other conductors.

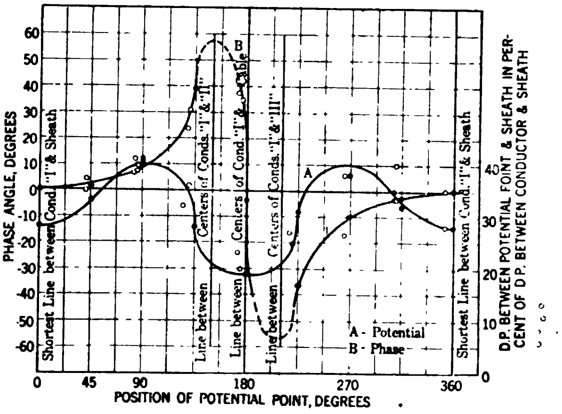


FIG. 23--PHASE AND POTENTIAL AT LAYER D, CONDUCTOR I, CIRCULAR CONDUCTOR CABLE

In the case of the circular conductor cable, the relative positions of the potential points were found by dissection, but the exact location of the group with reference to a fixed line of the cable, such as the line between conductor center and cable center, was found from preliminary plotting of the phase measurements. It was known that the zero of the phase curve must occur at the point nearest the center of the cable and again at the point nearest the lead. For the final curve the reference point of scale of abscissas was so taken that the phase curve would pass through zero on the said line.

In the case of the sector cable, the relative positions of the potential points were, as in the other case, found by dissection.

However, since the shape of the cable furnishes a reference point, there was no difficulty in determining the reference point of the scale of abscissas from the measurements of the dissected cable. The zero of the phase curve is here seen to pass zero

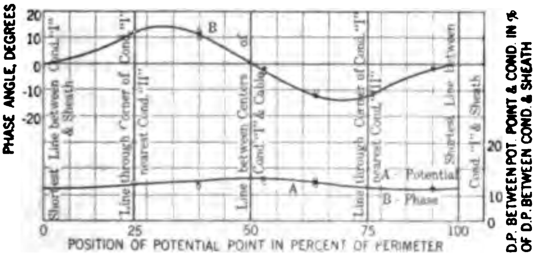


FIG. 24—PHASE AND POTENTIAL AT LAYER A, CONDUCTOR I, SECTOR CABLE

on the line of centers of conductor and cable, as was assumed in the case of the round conductor.

Curves were drawn of phase and of potential on layers *A* and *D* of both the circular conductor (Figs. 22 and 23) and sector (Figs. 24 and 25) cables. The ordinates of the phase

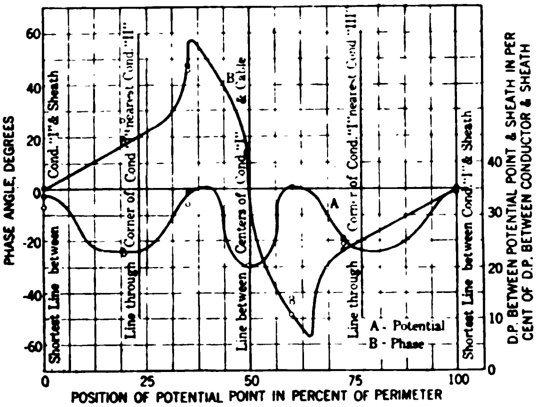


FIG. 25—PHASE AND POTENTIAL AT LAYER D, CONDUCTOR I, SECTOR CABLE

curves are in electrical degrees, positive and negative; positive representing out of phase in one direction and negative in the other, as in similar curves from the tank experiments. The ordinates of the potential curves are in per cent of the voltage between conductor and sheath. The abscissas of the curves on the circular-conductor cable are in circular degrees, starting with 0 degrees at a point nearest the sheath, thus making the point nearest the center of the cable 180 degrees. In the case of the sector cable the zero is also taken at the point nearest the sheath, but the abscissas are in per cent of the perimeter at the layer of potential wires concerned. These curves should be symmetrical about their middle point, and any slight discrepancy may be attributed to non-symmetry of the three phases.

EFFECT OF STRANDING UPON STRESS

The data here recorded on the effect of stranding on voltage gradient are not extensive but supplement previous data. Tests were made on a seven-strand and a nineteen-strand conductor, that is one containing six wires and one containing twelve wires in the outer row. The ratio of the stress in the case of the stranded conductor to that with a round conductor of the same area of section and with the diameter of the sheath the same in each case, is given to compare with the value determined mathematically by Jona. The value 1.23 found for seven-strand conductor checks exactly with the value given by Jona. We find the same value for the 19-strand, though Jona states the value will be larger, but he does not solve his equation for this case.

TABLE III
SHOWING EFFECT OF STRANDING UPON VOLTAGE GRADIENT

No. of Strands of conductor	$\frac{\text{Insulation thickness}}{\text{conductor diameter}}$	Ratio of stress with stranded conductor to that with round conductor	
		A	B
7	1.47	1.23	1.26
7	0.50	1.23	1.18
19	1.07	1.21	1.22
19	0.71	1.22	1.20

A. Same outside diameter over insulation, area round solid equal to total area of strands.
B. Same outside diameter over insulation, same diameter of solid and stranded conductor.

The most generally useful basis of comparison is the ratio of the stress of the stranded conductor to that of a round conductor of the same outside diameter and with the same insulation thickness. These values are given in the last column of the table. Little difference between the two styles of a stranding is apparent, though the data are not on exactly the same basis. On ordinary thicknesses of insulation the increase of stress due to stranding is roughly 20 per cent. For very thick insulation relative to conductor diameter, the increase is greater, and for thin insulation the increase is less.

Whitehead has shown that the effect of twisting the strands, as is actually done in cable practise, is to decrease the effect of stranding upon the gradient. Though his tests were with insulation thicknesses very great as compared to cable insulations, doubtless there is an important effect of this kind for thinner insulation. The data here given are upon untwisted strands, and should be supplemented by determination of the effect of twisting. Data here given are for single-conductor cables in a single-phase field.

PRACTICAL APPLICATION OF DATA

A brief outline of the manner in which the accompanying data may be applied to important problems is given.

The application of the resistance in electrolyte tank measurements has been indicated in the data given in connection with Fig. 2. This curve has been in use for some time in our laboratory and serves admirably to link together measurements of capacity, energy loss, etc. on different sizes, three-conductor cable as well as single-conductor cable. An interesting and important fact has been learned by the comparison thus allowed between the permittivity of the dielectric of three-conductor and single-conductor cable. It has been found that there is a definite difference, greater than the possible error of the curves, in the direction of lower permittivity for the dielectric of the three-conductor form. This is attributed to a low permittivity of the filler material in actual three-core cable as compared with the uniform permittivity of the dielectric occupying the filler spaces in the tank of electrolyte, as we know that the filler of an actual cable may have lower permittivity than the paper on the conductor, under certain conditions. A more comprehensive investigation of this has been begun. The bearing of this upon the stresses in the cable is apparent and will be discussed further.

The application of these data to temperature rise measurements of three-conductor cable is similar and of corresponding convenience and value.

A table is given comparing the stresses of a three-conductor cable under three-phase voltage with the stresses of single-conductor cables of certain specified insulation thicknesses. Tests are reported on two sizes, one with relatively thin and one with relatively thick insulation. It will be observed that the maximum stress next the conductor, toward the cable center, is so close to that in a single-conductor cable having the same insulation thickness as the distance between conductor and center of cable, that quite accurate calculation of the maximum stress of any three-conductor cable may be made readily from these data.

TABLE IV.—STRESSES AT IMPORTANT POINTS IN SECTION

	r	*A*	*B*	*C*
Max. stress at conductor surface on line toward center of cable.............	0.625 0.288	1.71 1.43	1.01 1.04	2.65 1.88
Stress at center of cable.............	0.625 0.288	0.65 0.685	0.385 0.496	1.01 0.902
Stress at mid point between conductors on center line of conductors in direction of center line..........................	0.625 0.288	0.92 1.18	0.545 0.855	1.42 1.55
Stress at edge of conductor insulation on center line of conductor and center of cable.............................	0.625 0.288	0.77 0.83	0.455 0.602	1.19 1.09
Ditto but normal to the center line... .	0.625 0.288	0.51 0.52	0.302 0.277	0.790 0.685

A. Stresses in terms of average stress between conductor and center of cable along line joining these points.

B. Stresses in terms of maximum stress of single-conductor cable having the same insulation as the distance between the conductor and center of three-conductor cable.

This distance is equal to 1.154 times the conductor insulation plus 0.077 times the conductor diameter.

C. Stress in terms of minimum stress of single conductor having same insulation as *B*.

r. Ratio of conductor insulation to conductor diameter.

Data have been given showing the agreement between values determined from Russell and those measured, for maximum stress between conductor and sheath. A curve (Fig. 16) constituting a solution of Russell's formula is given.

Stresses at and near the center of the cable and at various points on the surface of the conductor insulation are seen to bear a fairly definite relation to the stress at the sheath of the same single-conductor cable. At the center, the stress is quite closely the same as that at the sheath of the single-conductor cable, though the stress at the other points is considerably greater.

When the stresses of the three-conductor cable are compared with those of a single-conductor cable, it should be remembered that in many parts of the three-conductor cable there is not a simple alternating field, but it may be considered to consist of two alternating fields superimposed at right angles and producing a rotating electric field somewhat analogous to the rotating magnetic field of an induction motor.*

We have also summarized in Table V the data on the actual three-conductor cable so that they may be compared with those given in Table IV, as the result of the measurements with the conducting liquid.

TABLE V

MEASURED STRESSES AT IMPORTANT POINTS IN ACTUAL CABLE

The data here given are for the circular conductor cable for which curves have been shown. For this cable $r = 0.532$.

		AA	*A*	*B*
Stress at conductor on line joining center of conductor and center of cable.	From data	.87	1.70	1.05
Stress at conductor on shortest line between conductor and sheath.....	From Data	.75	1.46	.90
	From Russell's Formula	.70	....	.84

A, *B* and *r*—same as table above.

AA. Stress in terms of maximum stress of single-conductor cable with insulation thickness equal to conductor insulation of the 3-conductor cable.

NOTE. Data shown in column *AA* were obtained by comparing the three-phase stresses with stresses obtained by actual single-phase measurements on one of the conductors of the three-phase cable after it had been dismantled. Data shown in columns *A* and *B* were derived from the values in column *AA*.

It will be seen that the stresses next the conductor are somewhat greater than those determined by the previous method. We are not able to state with certainty the reason for the difference, though we believe it too great to be accounted for by inaccuracy of measurements. If the permittivity of the filler material were higher rather than lower than that of the main insulation, it would result in giving such a result. As stated previously, it would be expected that this effect would be in the opposite direction, since for the low voltage at which measurements were made the filler was that thought to have

---

*For instructions as to the use of Table IV in connection with Fig. 16 for obtaining stresses at different points in the section of a 3-conductor cable, see Appendix B.

a low permittivity. It is possible that the presence of the potential wires, due to the fact that they were not exactly parallel to the axis of the cable, distorted the field sufficiently to cause the increase of stress above the value found by the other method.

It is important to consider the data on stresses in connection with the paper given before this Institute in February, 1919, by Shanklin and Matson. They conclude that the maximum allowable operating pressure for a given cable occurs at a certain gradient that is very definite for a given insulating material. If we accept this conclusion, we have at once from the data presented herewith a basis for the relative rating of cables with different sizes of conductor and different thicknesses of insulation. However, the application of the data by one of the same authors indicates that important modification of the said conclusion must be made. In the 1917 paper, a cable, No. 1/0 with 9/32 in. insulation on the conductors and 7/32-in. insulation on the belt, is mentioned as having operated successfully at 24 kv. for some years. The maximum voltage gradient in this cable is more than 27 kv. per cm.—much in excess of 22 kv., which they conclude in the 1919 paper to be the limit for that class of material, as based on experiments on cable presumably of much more recent manufacture, and therefore not of poorer construction. Of course, also, there are many other cables operating successfully at gradients much in excess of 22 kv. per cm.

In bringing out this point in connection wth the paper of Shanklin and Matson, I do not want to be misunderstood, as I consider their paper an important contribution to the problem of cable insulation; and though I feel that the theory offered must be subjected to material modification, yet it may be part of the foundation upon which may grow a material evolution in the cable industry.

While considering this paper by Messrs. Shanklin and Matson, it may be well to call attention to the possibility that, in some cases, the ionization which they found, occurred actually in the filler spaces rather than next to the conductors. It is not possible to give specific figures, as we do not now have sufficient data of stresses in the filler space with single-phase voltage, which is the condition under which their measurements were made.

SECTOR CABLE

We are not attempting to give as specific data on sector cable as have been given for round. Time and space make a barrier, and furthermore, specific data will have less general application, for the terms "sector" and "modified sector" as applied to cable conductors are given to shapes differing quite widely in form and thus in characteristics, and presentation of complete data on even one specific form of conductor would require even more space than for round conductors. Some approximate general statements will however be of value.

To determine capacitance of cables with conductors of sector shape, take the data for a cable with the same insulation thickness and with round conductors of about 50 per cent (two sizes A. W. G.) greater area than the sector conductor. Actually, for different insulation thicknesses relative to conductor sizes, and for capacities of the different combinations listed, there is much range in the ratio of areas of round conductors and sector conductors having the same capacitance, the range of this ratio being from about 1.2 to 2.0, hence the rule just given is not more than a first approximation though it will seldom give capacities greatly in error.

The measurements made on the sample of actual sector cable indicate that the maximum stress is somewhat lower than that for the corresponding round conductor cable, but tests in the electrolyte tank indicate that relatively slight change of shape can influence the value of maximum stress sufficiently to vary it from lower to higher than for round conductor. The stress on the outer surface next the sheath was in all cases much less than for the round conductor.

Attention has been called above to the fact that these data throw much light on the inherent weakness of the ordinary form of three-conductor cable. The stresses in the filler spaces and tangential to the layer insulation are somewhat less than the maximum, which occurs at the conductor surface. However, because of the weakness of the filler insulation and the weakness of the conductor insulation for tangential stress, these stresses become of great importance. Other stresses in the filler space are very large as compared with the average stress throughout the insulation, and large even as compared with the maximum values at the conductor surface. Improvement in this portion of the dielectric has not in the past kept pace with improvement in the insulation near the conductors and probably can still not do so in the future. Doubtless, therefore, stresses in this portion of the cross section will become more and more the limiting condition in cables of the usual three-conductor form. Furthermore, it must be borne in mind that the stresses given are for the condition of dielectric of uniform permittivity. The stresses in the filler space will exceed materially the values here found, where the permittivity of the filler material is lower than that of the remainder of the insulation.

These experiments were carried on in the factory of the Company with which the author is associated, and the writer wishes to express appreciation for the facilities extended by his Company, and for the cooperation of various individuals. Thanks are due Mr. C. W. Davis and Mr. H. W. Fisher for many valuable suggestions and comments, to Mr. Donald Simons in connection with the early work done in the Pittsburgh Laboratory, and to Mr. A. M. Hagen and Mr. L. Meyerhoff for their faithful and valuable assistance in the performance of experimental work and in the preparation of this paper.

APPENDIX A.

To obtain the charging current for any desired connection of a three-conductor cable from Fig. 2, proceed as follows:

On the lowest set of curves follow the horizontal line representing the value of total (belt plus conductor) insulation, until the curve for the given size conductor is intersected. This will give the ratio of total insulation thickness to conductor diameter. Now calculate the ratio of belt to conductor insulation for the given cable. From the point of intersection gotten above go vertically upwards until the horizontal line representing the calculated ratio of belt to conductor insulation, on the set of curves for the particular type of charging current desired, is reached. This point of intersection gives the value of charging current sought in terms of milliamperes per 1000 ft. at 60 cycles and a permittivity of 3.27.

For example, let it be required to find the charging current 3 conductors to sheath on a 3-conductor cable, having 4/0 conductors with 10/32 in. insulation on each conductor and 4/32 in. on the belt. The total insulation here is 14/32 in. and following the 14/32 in. line on the lowest set of curves we find that it intersects the 4/0 curve at a ratio of total insulation thickness to conductor diameter of 0.83. The ratio of belt to conductor insulation is 0.4. On the 0.4 line for 3 conductors to sheath we find that an abscissa of 0.83 corresponds to a charging current of 34.3 miliamperes per 1000 ft., which is the value sought.

APPENDIX B

The values given in Table IV puts us in position to determine stresses at various points in the cross-section of a 3-conductor cable when operating 3-phase. We will here outline the method for obtaining the stress at one particular point by the use of Table IV in connection with Fig. 16, but practically the same method can be used for obtaining stresses at other points in the cable section.

The particular point chosen is that point on the conductor surface which is nearest to the cable center. As stated in the paper, this stress is approximately equivalent to the stress of a single-conductor cable having the same conductor diameter and an insulation thickness equal to the distance from the conductor surface to the center of the cable. After finding this distance, the maximum stress at the surface of the conductor for this equivalent single-conductor cable may be found by the usual formula, or more conveniently by the middle curve of Fig. 16. As shown, however, in Column B of Table IV, this value is not absolutely correct, and a slight correction must be made depending upon the ratio of conductor insulation to conductor diameter, as shown in the above table. The correction for values of the ratio between those given in Table IV may be found by interpolation. Great care however must be used in exterpolating from these values, but, as very few commercial cables have ratios except between these limits, the question will ordinarily not arise.

To illustrate the above by example, let us consider a 3-conductor cable, No. 1 A.W-G. strand, 9/32 in. × 7/32 in. Putting sizes into decimals, we have a 0.328 in. diameter conductor, with insulation 0.281 in. plus 0.219 in. Obtain the equivalent single-phase insulation 1.155×0.281 plus $0.078 \times 0.328 = 0.350$. The ratio of equivalent insulation thickness to conductor diameter is $\frac{0.350}{0.328} = 1.07$, and from the middle curve of Fig. 16, for 1.07 we find the ratio of maximum to average stress is 1.88. The average stress with say 14 kv. between conductors is $\frac{14}{\sqrt{3} \times 2.54 \times 3.50} = 9.08$ kv. per cm. The maximum stress is therefore $1.88 \times 9.08 = 17.01$ kv. per cm. To make the final corrections referred to above, divide the actual conductor insulation by the conductor diameter, or $\frac{0.281}{0.328} = 0.856$. This is a higher value than either of the values given in the top of column B, Table IV. The correction is less than 1 per cent therefore and can be neglected.

To find the maximum stress in the direction of the sheath, for a three-conductor cable, either under three-phase voltage, or as is done in cable testing, under single-phase voltage, the method is very similar. Find the average voltage stress per cm. of the insulation between the conductor and the sheath. Find the ratio of the total insulation thickness (*i.e.* conductor insulation plus belt insulation) to conductor diameter and find the value corresponding to this ratio in the *bottom* curve of Fig. 16. Multiply the average stress by this figure and maximum stress at the surface of the conductor is obtained.

As illustration, let us take the same cable. The total insulation is 0.281 in. plus 0.219 in. = 0.500. The average gradient with 14,000 volts three-phase between conductors is

$$\frac{14}{0.500 \times 2.54 \times \sqrt{3}} = 6.35$$

$\frac{0.500}{0.328} = 1.52$, and the ordinate corresponding to this in Fig. 16, bottom curve is 2.02. $2.02 \times 6.35 = 12.7$ kv. per cm., which is the maximum stress at the surface of the conductor toward the sheath.

DISCUSSION ON "HIGH-TENSION SINGLE-CONDUCTOR CABLE FOR POLYPHASE SYSTEMS" (CLARK AND SHANKLIN) AND "THE DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE", (ATKINSON), LAKE PLACID, N. Y., JUNE 27, 1919.

H. W. Fisher: The paper of Messrs. Clark and Shanklin takes me back to the year 1907 or 1908 when I had been making some experiments to determine the change of impedance on single-conductor lead-covered cables with and without lead cover short-circuited. I found in some cases the impedance was greater with the lead cover short-circuited; in other cases it was less. I discussed this matter with some of our very prominent engineers at the time and they said my experiments must be wrong as they would expect the impedance to decrease as it does in a transformer with the secondary short-circuited.

About that time, I spent a considerable time in trying to find a competent young man to take charge of the Experimental Dept. of my Company and I selected Mr. R. W. Atkinson whom I asked to give a solution of the above mentioned problem. In a paper which I read at Frontenac, in the year 1909, I presented Mr. Atkinson's graphical and mathematical solution of the problem with which the solution, in the paper we are discussing, is in accord.

Just about that time we were asked to quote on a 60-mile transmission in South America where it was very advisable to install cables instead of aerial lines on account of tropical conditions, and as we had previously been making experiments on dielectric loss, commencing about the year 1907, we were prepared at that time to discuss the problem fully.

The problem was as follows:—Frequency, 25 cycles; proposed transmission voltage, 42,000; power factor of load, 80 per cent; system operated with grounded neutral; number of circuits, 2 at least, and line loss to be 5 per cent.

Our calculations very soon showed we could not use three-conductor cables for the service and therefore we had to figure on single-conductor cable. We considered 60,000-volt and 42,000 volt transmission, and owing to the very much higher cost of 60,000-volt cables, we found that 42,000 volt-transmission was the more economical. It was impossible to operate at 5 per cent loss and the figured loss was between 12 and 15 per cent for the different kinds of cable.

The recommendation that stresses in the insulation of commercial high-voltage paper insulated cables made under present day conditions be limited to so low a value as 19.5 kv. per cm. is not in accordance with our experience. I know of a number of cases in operation where the stress at the surface of the conductor is considerably more than that, and I do not think that there should be a limit in the stress specified, because this would tend to curtail development in the art by manufacturers who might be able to improve their products so as to furnish cable which could be operated at a considerably higher stress at the surface of the conductor.

C. W. Davis (read by H. W. Fisher): Höchstadter in his paper of 1910, published in the *E. T. Z.*, discusses at length the merits of single-conductor and three-conductor cables and gives a large amount of data on dielectric loss measurements up to 40 kv. or 50 kv., both single-phase and three-phase current. The results of this work induced German engineers to use single-conductor cables on the Prussian State Railway as described by Pfannkuch in *E. T. Z.* of 1913.

Klein's careful tests on single-conductor cables at stresses at surface of conductor up to 60 kv. per cm. reported in *E. T. Z.* for 1913. show for the first time in published data, so far as we know, the effect of ionization, although the significance of this was not apparently known to Klein at the time.

In 1913, from the tests he made on four out of six cables, I found that ionization or internal corona began at from 25 kv. to 30 kv. per cm. in solid insulation or about 75 kv. to 90 kv. in air spaces. At that time (1913) I suggested a law of loss for evenly distributed air spaces including a term (based on the proof of Monash in 1907) varying as the square of the voltage and representing the inherent loss in the solid insulation as well as another term based on Peek's law of corona for the loss in the air spaces at voltages above the start of ionization or internal corona.

Wm. A. Del Mar: Messrs. Clark and Shanklin assign much importance to the tangential stresses pointed out by Höchstadter. These stresses occur in a three-conductor cable because at any instant the drop of potential between any conductor and the outside of the insulation surrounding it, is unequal along different radii from the center of the conductor, due to the different voltages between that conductor and the other conductors and sheath. This variation of potential along the surface of the insulation of the conductor gives rise to circumferential leakage currents and dielectric losses. All this is based on the assumption that the spaces between the insulated conductors are filled with a relatively imperfect insulating material, such as impregnated jute. If paper fillers are used there will be no well defined surface on which this phenomena can occur, and the limitation pointed out by Höchstadter will not directly apply.

The remarks by Messrs. Clark and Shanklin regarding the impracticability of grading cables are much to the point. It has long been known that it is commercially impracticable to accurately predetermine the specific capacitance of varnished cambric or rubber insulation.

The general proposition of using single-conductor cables in place of triplex cables offers an interesting field for speculation. I am of the opinion, however, that single-conductor systems will not come into extensive use because of the failure of triplex cables at 30,000 volts. I believe that future development will result in raising the voltage limit for triplex cables.

Like Mr. Fisher, I am rather surprised at the limiting stresses which the authors give for paper insulated cables and I can't help wondering whether a fallacy has not been introduced by basing these stresses upon the results of tests on concentric air films, as it is possible that air bubbles near the surface of the conductor will not break down as easily as flat films.

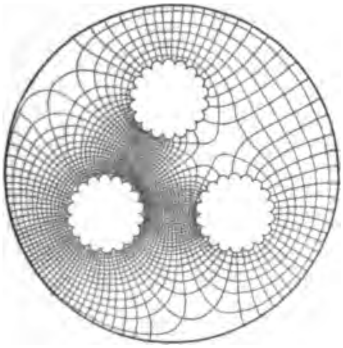


Fig. 1.—Equipotential Lines and Lines of Electric Stress in Round Conductor Cable

Referring to Mr. Atkinson's paper, I wish to state that for several months the Research Laboratory of the company with which I am connected has been carrying out a series of tests, similar to those recorded by the author, and using substantially the same methods. The primary purpose of the tests recorded in this discussion was to determine the best form of sector cable from the point of view of reducing di-

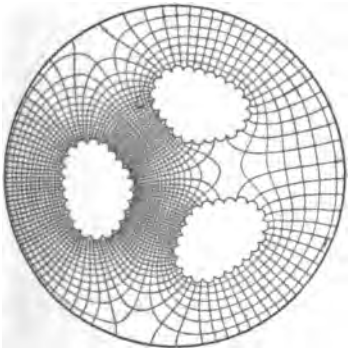


Fig. 2—Equipotential Lines and Lines of Electric Stress in Sector Cable

electric stresses and dielectric losses to a minimum, and increasing the heat dissipation to a maximum.

Mr. Atkinson states that tests in the electrolyte tank indicate that relatively slight change of shape of sector cable can influence the value of the maximum stress sufficiently to vary it from lower to higher than for round conductors. This agrees with our observation, and we therefore, made tests on various modified sectors for the purpose of ascertaining the best form of sector cable from the electrical point of view. A close approximation to the ideal form thus found, being easy to manufacture, subsequent tests were made upon conductors of this form.

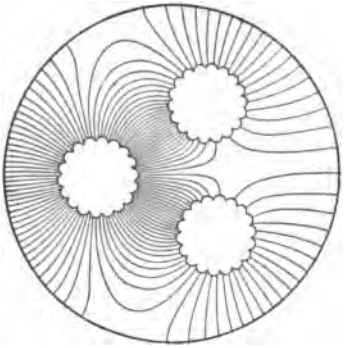


Fig. 3—Lines of Electric Stress in a Round Conductor Cable

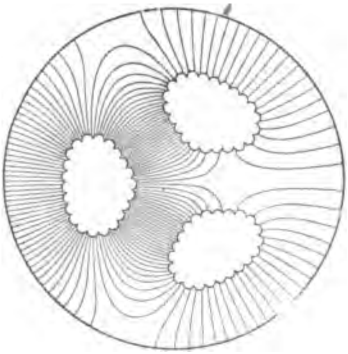


Fig. 4—Lines of Electric Stress in a Sector Conductor Cable.

Figs. 1 and 2 show the equipotential lines and stress lines in three conductor round and sector cables, respectively, and Figs. 3 and 4 show the stress lines without the equipotential lines. It will be noted that the maximum density of stress lines is about 15 per cent greater in the round than in the sector cable. Furthermore, the stress along the line joining the centers of round conductors, checks Mr. Atkinson's curve *A*, Fig. 17, very closely, while in the case of the sector cable, the corresponding curve would have only about one-half the sag.

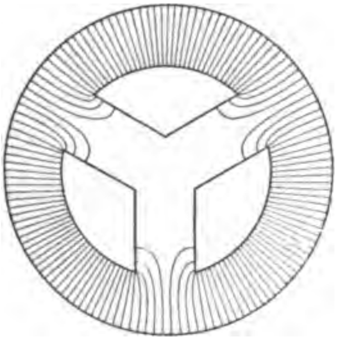


Fig. 5—Lines of Heat Flow in a Thermally Ideal Sector Cable

Mr. Atkinson mentions the application of the electrolyte method for the determination of lines of heat flow in cables. Fig. 5 shows the lines of heat flow for a thermally ideal sector cable. Such a cable is electrically impracticable, due to the high stresses that would occur at the sharp points. Fig. 6 shows corresponding lines for a round conductor cable and it is interesting to note the much greater average length of path that the heat must travel in flowing from conductor to sheath. Fig. 7 shows the heat flow from a sector cable, indicating that an electrically good sector cable is intermediate between a thermally ideal sector cable and a round-conductor cable. The relative heat conductance from conductors to sheath are as follows:

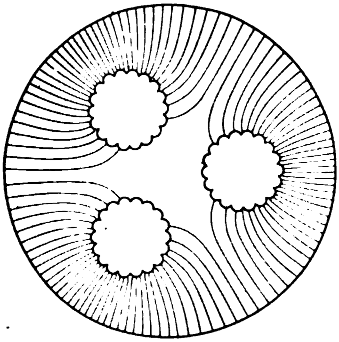


Fig. 6—Lines of Heat Flow in a Round Conductor Cable

Thermally Ideal Sector Cable	100
Electrically Good Sector Cable	110
Round-Conductor Cable (Stranded)	120

Figs. 8 and 9, are similar to Figs. 6 and 7 except that isothermal lines are added. It should be noted that these curves apply only to cables wherein the dielectric loss is negligible and therefore, do not represent conditions in a 60-cycle high-voltage cable at normal operating temperature.

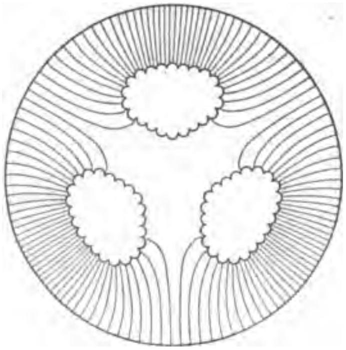


Fig. 7—Lines of Heat Flow in an Electrically Good Sector Cable

The method of test differed from that used by Mr. Atkinson in the employment of 300-cycle current in conjunction with a telephone receiver and resistance bridge. The ratio of the bridge resistance, when balance is obtained, is equal to the ratio of the potential drops between the movable contact needle and each of the two electrodes. A diagram of connections is shown in Fig. 10.

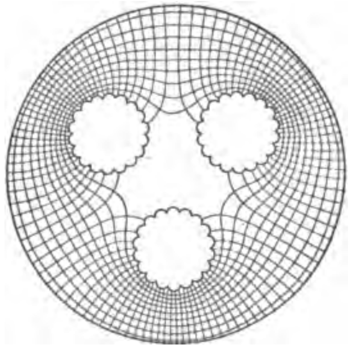


Fig. 8—Isothermal Lines and Lines of Heat Flow in a Round Conductor Cable

The tests recorded above are from a general research on dielectrics, which is now being carried out under my direction by my associates, Mr. C. F. Hanson and Mr. L. Cockaday.

Mr. Atkinson states that the dielectric losses in a cable may be taken to be proportional to the charging current. This is practically true for cables having conductors of similar form. It is not true when comparing round conductor with sector cables, as the effect of increase of charging current in the latter may be neutralized by the elimination of spots wherein dielectric stresses are excessive. As the dielectric loss, at any point, is proportional to the square of the dielectric stress at that point, it is obvious that making the stress more uniform will have a marked effect in decreasing the dielectric loss.

DETAILS OF TEST

1. Electrolyte = Hydrant water.
2. Depth of electrolyte = 1.5 in. (6 cm.).
3. Size of tank = 36 in. by 42 in. and 6 in. deep, (91½ cm. x 107 x 15.2 cm). Painted white with lines ruled 1 cm. apart on the bottom. Plate glass placed on bottom for insulation. The numbers on the lines could be read clearly through the electrolyte and glass.

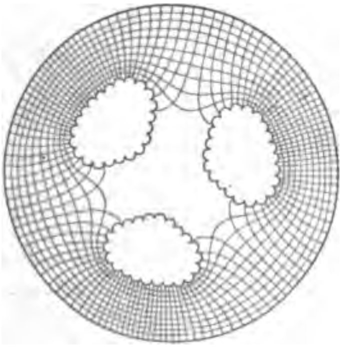


FIG. 9—ISOTHERMAL LINES AND LINES OF HEAT FLOW IN A SECTOR CABLE

4. A chart ruled to be a duplicate of the bottom of tank was used in plotting the readings taken with the exploring needle.
5. Size of model round conductor = 6.70 in. (17 cm.) diameter. Size of model cable = 28.0 in. (71 cm.) diameter.
6. Exploring needle = 0.4 mm. platinum wire sealed in end of glass tube.
7. Model conductor represents a 350,000 cir. mil cable, insulation $= \frac{7 \times 7}{32}$ in.
8. R_1 and R_2 are resistance boxes, R_1 and R_2 = 10,000 ohms.
9. R and 2 R, a 3220-ohms rheostat.
10. K R_1 and K R_2, a 52-ohms rheostat.
11. The observer's stand and telephone receiver were kept at the same potential by varying K R_1 and K R_2. This balance was obtained by disconnecting telephone from exploring needle and connecting it on to the slider of the rheostat K R_1 − R_2 which was adjusted to obtain a minimum in the telephone receiver.

This prevented current from passing from telephone receiver through the observer, thereby eliminating any error due to the high-frequency current producing an error in balancing with the exploring needle.

12. Balance with needle point was obtained with 300-cycle current. Practical silence was obtained in the telephone on balance.
13. As a bridge method was used, readings are independent of variations of line voltage.

D. W. Roper: Referring to the paper by Messrs. Clark and Shanklin in which they suggest the use of a sectional sheath on two-conductor cables, this may be all right for normal operating conditions but I think it would be a little awkward for some of the abnormal conditions such as the failure of the cable. In the earlier years in Chicago, when the Fisk St. Station had just started and there was no problem regarding electrolysis, the lead sheaths connecting the transmission lines were not bonded together and it was discovered that when we had a transmission line burnout that the voltage of the lead sheath was raised to such an extent that there was arcing and burning of the lead sheath wherever there was actual contact with the manhole brackets or where the sheaths were in contact.

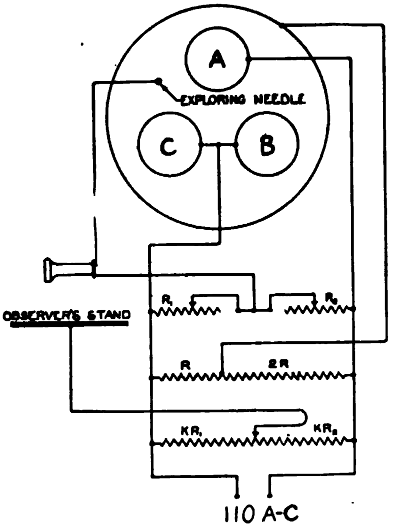


FIG 10—BRIDGE CONNECTIONS FOR FINDING EQUIPOTENTIAL LINES

It would seem, therefore, very probable that in the case of a burnout of one section of a single-conductor cable with a sectional lead sheath, that this portion of the sheath, in spite of the fact that there was a ground connection, would be raised to such a point as to cause arcing or pitting or perhaps the jumping of the insulation between sections.

The paper speaks of hot spots, apparently referring to hot spots in the conduit due to local heating conditions, but slurs over the fact that there are also hot spots in the cable entirely independent of the hot spots in the conduit. In other words, although a cable appears to be of quite uniform construction throughout, as a matter of fact the construction is not uniform —far from it! So that the dielectric loss, particularly in the resinous compound cables, is far higher at some points than at others, so when you approach the limiting temperature at which the cable can be safely operated or if that temperature is exceeded, then breakdowns in the cable will occur due to the dielectric loss being higher in some points of the cable than in others although the surrounding conditions may be identical.

That might be thought to be a disadvantage but in some respects it might be considered a safety valve because experience has shown that when breakdowns occur at that point due to overloading of the cable, that the damage to the cable is entirely local and that there is no sign of charring or injuring in any way of the paper insulation at other points between the point of breakdown and the source of supply. It is entirely possible that as we get into the use of cables with lower dielectric loss that this question of the local hot spots may not be so important and if that actually results, then it is equivalent, you might say, to screwing down the safety valve that you have with the present compound cables.

The question of heating of the cables and the limiting of the load is an interesting one but as a matter of fact the company with whose operation I am familiar has never yet pulled a switch on a transmission line that was overloaded. The fact that the line is overloaded is an indication that some emergency has arisen so that it is more important than ever that the cable be maintained in service as long as possible and it is their universal practise on such occasions not to pull the switch but to allow the load to be carried until other arrangements can be made to carry the load or until the cable fails and the switch thereon opens automatically.

In the case of the dielectric loss, I think the shape of the curve is just as important as the value of the dielectric loss at some particular temperature. The thing that is desired is that the dielectric loss should not increase at or near the maximum loads which are likely to be carried in emergencies and that the resulting hot spots which result, will not cause breakdowns at too low loads.

With regard to the statement on page 959, I think that is made a little bit too rigid where it says, at the top of the page, "Eddy currents in stranded-cable conductors have been shown both theoretically and by actual measurement to be negligible." This is the case, I believe, for ordinary values of current but under exceptional conditions it is quite possible to get kinetic current loss, which is very important and serious.

We recently had a case of trouble in Chicago, an excess current for some eighteen minutes on a stranded-cable conductor which made up a reactance coil between sections of the bus. That was an eight hundred thousand circular mils, stranded-cable conductor, barrel wound in three layers. After the trouble was over it was found on examination that the top and bottom turns of the reactance coil was still bright but that the turns in the middle had been oxidized by the heating and there were places four inches away from the conductor in the middle of the coil where the varnish on the wood adjacent had been blistered by the heat.

Referring to the diagrams which have been passed around by Mr. Del Mar showing the isothermal lines and the relative heat conductivity of round and sector cables, it would be very interesting if he could add to his discussion some figures which would indicate the relative carrying capacity of round and sector-shaped conductors of the same size.

Philip Torchio: I am very glad to have a paper like the one of Mr. Clark and Mr. Shanklin on record, especially with the further comments which Mr. Shanklin has made that 88,000 volts operating voltage can be used. However, I would like to say a few words on the plan of the paper to prevent confusion and, perhaps, misconstruction of the intent of the writers. I believe that the writers have gone a little too far in showing the advantages of the single-conductor cables.

The comparison is made between a three-conductor cable versus the single-conductor cable, using three three-conductor cables compared with one circuit of three single-conductor cables. Also the comparison is made, I believe, for three-conductor cables, round conductor. If they had used the sector cable they would have had a smaller diameter; also, as the thermal characteristics of the sector cable would be better than those of the round conductor cable, these changes would have, to a large extent, modified the relative values, as arrived at in the paper, of the saving due to the use of single conductors.

In practical application, I would not consider the installation having three independent circuits, each consisting of a three-conductor cable as merely equal to a three-phase circuit consisting only of three single-conductor cables.

I think in the first layout with one cable out you still have two-thirds, with two cables out you still have one-third, but in the single-conductor cables if you lose one you lose the whole circuit.

The object in making these remarks is not in any way to detract from the value and the essence of this paper. The paper gives a limit of construction for three-phase cables of 30 kv., which, possibly, if the sector cable had been considered, Mr. Clark and Mr. Shanklin might have raised to 40 kv., but anyway we are limited in the three-phase cable while we have a much wider range in the single-conductor cable.

That is the essential and important value brought out by the paper. But we must not conclude that perhaps we can accomplish similar results at lower voltages. Although single-conductor cables are used for the lower voltages, as we have been using them quite generally in this country in central station work for wiring up generators to the switchboard, the single-conductor leaded cable has many disadvantages. If you keep the lead sheath open-circuited, high voltages are caused on the sheath and if you ground the sheath then you have large currents flowing in the lead sheath.

Another point is that when you use low-voltage cables where the currents are high, in case of short circuits, the mechanical stresses are enormous. As a matter of fact, in our own generating stations we provide frequent supports, say four feet apart, throughout the run of the cable unless it is in pipe.

Difficulties with these stresses would be serious in manholes if such cables, for general distribution, were not properly planned and properly protected against these stresses.

H. L. Wallau: I have a few figures here that may be of interest. They refer to a test made on two lengths of one million cir. mil conductor, 61-strand, low-tension cable, separated 26 in. (67 cm.) center to center. The cables were in a dry subway and each cable was 1,140 feet (347.4 m.) long. The far ends of the conductors were sweated together, the near ends of the sheaths permanently bonded, and the far ends of the sheaths cross connected through cables with bolted lugs so that the sheath circuit might be opened or closed.

One hundred and twenty volt, 60-cycle current was applied to the conductors. The current in the conductor was 650 amperes. With the sheath circuit closed the sheath current was 225 amperes; with the sheath circuit open the induced voltage was 95 volts. This shows that the induced current for distances of 1,000 ft. (304.8 m.) between bonds is apt to be material and add quite some to the heat generated in the cable.

In fact in our own station, where we have long (500 ft.) runs between generator terminals and switches, the use of leaded cable was found to be objectionable and a cambric, non-leaded, single-conductor cable used instead.

However, we did look into the question of grounding the sheaths of leaded cables at the middle of the run and inserting a limiting resistance in the ground connection, to prevent an abnormal flow of current in case of breakdown.

Such an installation was found would be satisfactory, but in the particular case in mind the resistance required would have been about 0.3 of an ohm and of large physical dimensions. There was no space available in the manholes, for a resistance of this size, so the plan was not carried out.

R. W. Atkinson: The paper by Messrs. Clark and Shanklin contains much of value, as is to be expected from investigators of their standing. It is, therefore, with some hesitation that I venture to point out where they have fallen into error in making some assumptions and in drawing conclusions from their data not borne out by careful investigation. Attention is drawn also to the importance of data already on record concerning matter for which formulas are developed.

From reference to Fig. 16 and Fig. 19, it seems that dissipation characteristics of duct systems have been given and assumed without regard either to the number or kind of cables involved. Fig. 16 is given as showing "representative heating characteristics in conduit lines." For example, in the right hand curve, 10 watts per foot of cable is considered to give 57 deg. cent. rise of temperature above the surrounding soil temperature. No distinction is made, as to the size or construction of the cable or its insulation, and no allowance is made for the fact that much higher temperature rises are found where cables are grouped. It is most unfortunate that data should appear in this form with the weight of the authority of these authors.

Furthermore in the application of these data, as given in Fig. 19 in the comparison of single-conductor and three-conductor cables, conclusions are drawn which are very seriously in error. In Fig. 19, it is stated that a final temperature of 82 deg. or a temperature rise of 57 deg., is attained with a loss per foot of cable of 10 watts, with either of the two described systems. One of these systems consists of three single-conductor cables, the other consists of three three-conductor cables. In order to determine the probable error of the assumption that the temperature rise of the two would be the same, and would be that assumed by the authors, calculation was made from our own data, as to the probable temperature rise, basing these calculations on fairly average duct conditions, and on fairly representative thermal resistivity of the insulating materials. On the basis of the relatively small amount of duct rise which there would be with only one cable instead of three, I calculate that the rise for the three-conductor cable would be 53 deg. and for the single-conductor cable 79 deg. With a greater relative rise of duct structure, there will be less relative difference between the three-conductor and the single-conductor cable, but it is immediately obvious that the particular single-conductor cable will rise to a very much higher temperature for the same amount of loss per unit length, than will the three-conductor cable. Furthermore, where three cables of either type are involved, it seems that the estimate of temperature rise for even the three-conductor cable is too low. Thus, the rating even for the latter is too high and that for the single-conductor cable is not only altogether too high, but is out of proportion to and not comparable to the rating given for the three-conductor cable.

In the last paragraph, beginning on page 951, it is apparently assumed by the authors that where there occurs a loss in the sheath equal to 28 per cent of the conductor loss, this can be fully compensated by an increase of conductor area of 28 per cent. This is erroneous as it will actually require about 39 per cent more copper, as presently will be explained. This is of comparatively little consequence, but on page 953, reference is made to an increase in copper of from 50 per cent to 100 per cent for two certain cables. Apparently these are the two cables on page 943 where about 50 per cent in one case is the increase in loss and about 100 per cent is the increase in loss in the other case. Now if there is 50 per cent loss in the sheath, that loss in the sheath is not going to be affected by changing the size of the conductors. Thus in order to have the same total loss as without the sheath loss, the loss in the conductor must be reduced by 50 per cent, that is its area increased by 100 per cent. Where the sheath loss is 100 per cent, the conductor loss would need be reduced to zero. It seems therefore, that large sheath losses become prohibitive at much lower values than indicated by the authors. The situation is not materially different for very small sheath losses, than is indicated by the authors.

In their sentence at the top of page 937, I believe the authors have underestimated the value of previous work that has been done, in determining the voltage or current, or loss produced in the sheath of single-conductor lead-covered cables. The formulas of Berg, which they mention, are approximate as said, but do give results which apply fairly well within the range considered in the paper now under consideration.

In 1909, in a paper presented at Frontenac by Mr. Fisher, a solution appears which gives results in agreement with the formulas developed by the present authors, within a fraction of one per cent. Of the two formulas which are necessary for this solution, one is identical with the corresponding formula given in the present paper. The other formula is very much shorter and simpler than the corresponding formula which is now given; in fact, it is as simple as the formula of Berg. The simplicity of the older formula is due to the fact that the calculation is made on mean instead of outside diameter, thus eliminating all that part of formula 6, page 962, contained in the bracket following the minus sign. With this omission and the substitution of mean for outside diameter formula 6 becomes identical with the older formula.

Besides the approximation mentioned by the present authors at the bottom of page 959, another approximation was used in deriving the older formula, the assumption that the sheath could be considered as the equivalent of a thin shell of the same resistance and of a diameter equal to the mean diameter of the actual sheath. This latter approximation is even closer than the other. On the whole there seems no warrant for using the long formula on page 962 instead of the older and simpler one.

Solution of these problems of voltage, or current induced in lead sheaths is greatly facilitated by a set of curves presented by the writer in the discussion of a paper by Mosman presented before the Institute in 1913.

A very interesting and important commercial application of single-conductor cables for carrying alternating current is the use by some operating companies, of two, three, or four such cables in a single duct on 2200-volt distribution. Such cables are used instead of multi-conductor cables on account of the greater facility of splicing while the conductors are "alive". It is interesting that the losses with any size of conductor usual for this construction are of small consequence and do not reduce the carrying capacity. That the loss is small is due to the very short spacing. Where the spacing is so short, neither the formulas given by Messrs. Clark and Shanklin, nor the older formulas mentioned above, are strictly applicable. They do serve, however, to show that the losses are comparatively small, and actual measurement of these losses confirms this.

W. A. Del Mar: Referring to the relative carrying capacity of round and sector cables, the curves of heat dissipation which I showed this morning indicate that the heat resistance from the conductor to the sheath of a sector cable is about 10 per cent less than that of a round-conductor cable. The heat resistance from the conductor to the air is made up of two important parts, that from the conductor to the sheath and that from the sheath to the air.

Mr. Atkinson, in a previous paper, has shown that the total heat resistance of a triplex cable is divided approximately in the proportion of two-thirds from conductor to sheath and one-third from sheath to air. Suppose we take the heat resistance from conductor to sheath as $66\,^2/_3$, and from that from sheath to air as $33\frac{1}{3}$, a total of 100. If we decrease the heat resistance from conductor to sheath 10 per cent, making it 60, and if we increase the heat resistance from the sheath to air, by about 10 per cent, making it $36\,^2/_3$, which would correspond approximately to the lesser diameter of the sector cable, the total heat resistance would be reduced to about $96\,^2/_3$ per cent. Now the current causing a given temperature rise is inversely as the square root of the heat resistance, so that a simple calculation shows that a sector cable of the type described will have a carrying capacity of something like one to two per cent greater than that of a round-conductor cable. Substantially, they are equal.

E. B. Meyer: The paper by Messrs. Clark and Shanklin on high-voltage single-conductor cables for polyphase installations is a step in the right direction toward solving the problem of underground cable construction, where the transmission of large blocks of power at high voltage is under consideration.

The use of three-conductor cable as pointed out by the authors seems to be limited to about 30,000 volts on account of the properties of the insulating material and the unwieldy size of the cable itself.

Three-conductor cables as at present manufactured are limited in capacity to about 12,000 kv-a. and if the operating voltage is to be increased materially, the use of single-conductor cables appears to be the readiest solution unless some radical improvement in three-conductor cable construction can be effected.

On the other hand, opinion as to the desirability of concentrating heavy loads in a few large cables, is by no means unanimous and as I have some doubt as to the safety of such a course, I would be glad to hear some discussion on this point.

The question with which most central station companies are interested at this time is the problem of reducing failures in underground cables, as with the increase in capacity and extension of electrical systems, disturbances are more destructive, and it will be interesting to know whether by the use of single-conductor cable the maintenance cost per mile of circuit will be materially reduced.

I question whether the maintenance cost of a single-conductor installation would be as low as that of a three-conductor installation.

Probably the most serious objection to the single-conductor installation would be the necessity of splitting up the cable sheath into sections and installing grounded bonds. This would introduce troublesome complications and would probably add materially to the cost of cable maintenance.

I am heartily in sympathy with the authors comments regarding the methods of determining the maximum safe loading of a cable system. It appears that the strictly logical way to arrive at the proper loading of any cable is to determine by actual temperature survey what may be called the constants of each main duct line. With this information available the rating of cables based on maximum allowable operating temperatures is comparatively simple.

A large amount of data is gathered each year by operating companies relating to temperature rises of conduit systems under different conditions of installation and it would be of extreme value to the industry as a whole for such data to be made generally available. It is only in this way that the rule-of-thumb methods of rating cables can be discarded for a more logical method based on an accurate knowledge of conditions.

In Mr. Atkinson's paper on the dielectric field in three-conductor cables, I am pleased to note the author's statement that the maximum stress in sector cable is lower than for a corresponding round-conductor cable, the stress in the outer surface next to the sheath being in all cases much less than for round conductor. This seems to offset the erroneous theory which has been advanced by a number of engineers that sector cable is not desirable for transmission at voltages above 15,000.

It is to be regretted that time did not permit the author to include the subject of dielectric losses in three-phase cables, as this subject is a very live one at the present time. It is known that the magnitude of these losses and their variation with temperature has a very marked effect on the operating characteristics of the life of a cable. This subject is a most vital one and is receiving considerable attention by a number of central station engineers and while it appears that no fixed rules can be formulated at present, it is desirable that this subject be given careful consideration.

G. B. Shanklin: Mr. Del Mar's discussion of our paper is much to the point. I am in full accord with the sentiment expressed in his belief that future development will result in raising the voltage limit for multi-conductor, cable possibly as high as 40,000 volts. This goal may ultimately be reached, but it is a development that must be approached with caution. It is one of those developments whose factor of safety cannot be even approximately determined until it is actually tried out under operating conditions, at least with the knowledge we have at present.

For some time past our laboratory has been investigating dielectric losses in three-conductor cable, hoping to throw some light on the causes of the excess loss and deterioration in the central triangle of the cross section, a factor that plays a big part in limiting the voltage rating of this type of cable. Our work has consisted mainly of comparing actually measured 3 ϕ losses with those calculated from measurements with 1 ϕ voltage. These measurements were made on cables whose cross section were at uniform temperature. Now, the only difference between 3 ϕ and 1 ϕ stresses occurs in the central triangle and if the loss in this section was an appreciable part of the total then it would be expected that the 3 ϕ readings would give higher values than the 1 ϕ calculations. We have tested cable from the very best grades to those so poor in quality that they could hardly be classed as insulated cable but we have never found any difference between the measured and calculated 3 ϕ losses, provided the temperature conditions were exactly the same.

I have come to the conclusion that the generally accepted view of an inherent excess 3 ϕ loss in this central triangle is, in the sense as usually interpreted, not strictly true. The deterioration of this section under actual operating conditions is found only infrequently and in extremely poor grades of cable. This leads me to believe that it is primarily caused by an accumulative heating action that renders the central triangle much hotter than the rest of the cross section, and that without this accumulative heating action there is no more deterioration in the central triangle than in the rest of the cross section.

With the cross section at uniform temperature we have never found any indication of ionization starting in the central triangle at a voltage lower than that required to start it at the surface of the conductors, where the stress is greatest. It is probable that when the central triangle becomes excessively hot there is a tendency for the stresses to re-distribute themselves, and in doing so to concentrate at certain points, causing, ionization at a lower voltage than otherwise. This, of course would cause deterioration but it should be classed as an effect of the accumulative heating mentioned above and not as a cause.

It is consideration of these factors that encourages me to believe that higher voltage ratings of three-conductor cable will ultimately be reached. The main requirement will be the lowest possible dielectric losses, generally throughout the cable cross section, but particularly in the central triangle. The fact remains, though, that we are rapidly approaching the upper limit of voltage rating for three-conductor cable. Whether the final goal will be 35, 40 or even 45 kv. we must if we wish to go beyond this goal resort to single-conductor cable.

Messrs. Fisher, Del Mar, Atkins on and several other gentlemen have criticized the limiting stress of 19.5 kv. per cm. recommended in our paper. I will confess full responsibility for this recommendation. Mr. Clark has always been skeptical of this limit and agreed to its inclusion in the paper more for the sake of having a target to be shot at than anything else. He thinks it ultra-conservative in view of the fact that three-conductor cables have in some cases operated satisfactorily for years at maximum stresses as high as 28 kv. per cm.

Different types and makes of cable, like all other apparatus, have their own individual merits and peculiarities. It was a mistake, perhaps, to select an arbitrary value of maximum stress and I feel that an explanation is due. In the first place, it was not proposed for general application and I think it was made clear in the Institute paper, "Ionization of Occluded Gases in High-Tension Insulation," page 489, that it represented a *minimum* value.

We have tested many types of cable and have found that their limiting safe stress as far as ionization is concerned, has varied from 10 to 40 kv. per cm. The reasons for selecting 19.5 kv. per cm. (which is only an average of the range from 19 to 20 kv. per cm., as first recommended) were:

a. A start had to be made some where. It is difficult to form a judgment of performance without some criterion or basis to estimate from.

b. As far as we could judge our data seemed to indicate 19.5 kv. per cm. as a fairly good dividing line between acceptable and unacceptable grades of paper cable, that a cable showing ionization at an appreciably lower stress than this could hardly be considered as satisfactory.

c. When a cable is highly stressed we have not only ionization to consider but also high dielectric losses with consequent reduction in safe current carrying capacity and greater possibility of accumulative heating at "hot spots." This is the more pronounced the higher the voltage rating and certainly when we approach 40-kv. ratings we must pay more attention to this factor than we have in the past. Quite often it will be found more economical to operate a high-tension cable at a stress of, say, 19.5 kv. per cm. than at 28 kv. per cm. because of the greater carrying capacity and factor of safety. When the thickness of insulation is increased the question of heat dissipation introduces another factor. Obviously, the economical balance between dielectric and copper losses, heat dissipating ability of the cable, and possibility of ionization will be governed by the grade of insulation, the thermal conditions of the duct line and the relative costs of the materials used in the cable. No set rules can, therefore, be applied.

We agree with Mr. Roper in his criticism of sectionalized sheathing for single-conductor cable. He is right in stating that it would, "be a little awkward for some of the abnormal transient conditions, such as failure of the cable." Throughout the paper we have tried to make it clear that short-circuited continuous sheathing is much superior to sectionalized sheathing.

It is quite difficult to answer the many points in Mr. Atkinson's discussion. He has apparently misintrepreted and taken issue with practically every thing in our paper. Some of his criticisms are really helpful and add to the paper in throwing light on a very difficult problem. I will attempt to answer a few of the main issues.

Fig. 16 is given as showing representative heating characteristics in conduit lines. Mr. Atkinson does not think Fig. 16 is representative and says: "No distinction is made as to the size or construction of the cable or its insulation, and no allowance is made for the fact that much higher temperature rises are found where cables are grouped." We purposely avoided, for the sake of simplicity, any detailed discussion of the theory of heating in duct systems, but simply assumed the thermal conditions as such that Fig. 16 held for our particular problem. Incidentally, Fig. 16 is, as far as our experience shows, fairly representative of the heating conditions met with in the average duct system. I believe our assumptions would have been made clearer to Mr. Atkinson if we had represented the temperature in Fig. 16 as that of the duct air and then added on the temperature drop through the cable insulation but that would have involved complications that would have made our paper uselessly long.

The greater part of Mr. Atkinson's discussion is a detailed criticism of our assumption that the temperature drop through the insulation of the single- and three-conductor cables compared in Fig. 19 is approximately the same. Now, anyone who studies this problem carefully can see that it is impossible to deal with it theoretically unless certain approximations are accepted. To begin with it must be assumed that the cable cross sections are at uniform temperature, that is, the copper and lead sheath are at the same temperature. Obviously, this assumption gives approximately correct results only over that range of heating conditions where the cable takes up only a small part of the total temperature drop. In water-cooled or forced air-cooled duct systems or with buried cable it would not, of course, apply very well but in the ordinary still-air duct systems, and especially at "hot spots" where poor heat conductivity of the soil is encountered, it applies much better.

I am sure that our assumptions have not led to any such grave errors as Mr. Atkinson's discussion would seem to indicate. For instance, he states in reference to Fig. 19 and cables (a) and (c), "assuming a loss of 10 watts per duct foot for each of these two cables and fairly average duct conditions, I calculate that the rise for the three-conductor cable (a) would be 53 deg. cent. and for the single conductor cable (c) 79 deg. cent.

Now, let us see if his calculations are correct. Instead of assuming "fairly average duct conditions" let us assume that the total temperature drop is between copper and lead sheath, as would be the case if the cables were immersed in water. For the same heat dissipation the temperature drops of the two cables will be inversely proportional to their thermal conductances between copper and sheath. The thermal conductances are directly proportional to the electrostatic capacities so we need only to find the ratio between their electrostatic capacities. This is easily obtained from Formula 6 (bearing in mind that the total capacity between copper and sheath is 3 C_1), and the ordinary formula for concentric cylinders. The ratio found in this way is;

$$\frac{\text{thermal conductance of cable (a)}}{\text{thermal conductance of cable (c)}} = 1.40$$

In other words, the three-conductor cable has 40 per cent better heat conductance. With the cables immersed in water and dissipating equal quantities of heat let us assume the drop through the three-conductor cable as 53 deg. cent. The drop through the single-conductor cable will be, $53 \times 1.4 = 74.2$ deg. cent. This agrees so closely with Mr. Atkinson's value of 79 deg. cent. that it shows the fallacy of his assumption of "fairly average duct conditions."

Under average still-air duct conditions the drop from copper to sheath can be roughly assumed as 20 per cent of the total drop. Then, if the total drop of cable (a) is 53 deg. cent the drop between copper and sheath is 10.6 deg. cent. With cable (c) in a similar duct and dissipating an equal quantity of heat the total drop will be 57 deg. cent. and the drop between copper and sheath 14.6 deg. cent. The difference is only 4 deg. cent. and our assumptions, therefore, hold reasonably well.

Messrs. Torchio and Meyer both express the same thought in a different way in their comparison of the merits of single- and three-conductor cable. It is true that in the case of single-conductor cables the failure of one cable throws three times the carrying capacity out of commission than in the case of equivalent three-conductor cables. It is partly for this reason that single-conductor cable shows promise of the greatest usefulness only where there are such large blocks of power to be transmitted that the temporary loss of three cables instead of one will not prove serious.

R. W. Atkinson: This use of paper filler instead of jute has been mentioned. This, however, can only partly overcome the weakness of that portion of the ordinary three-conductor cable which is occupied by such filler. This is because paper is better than jute to an important extent, only for stresses normal or nearly normal to the surface of the paper. Most of the paper of the filler will be subject to stresses tangent or nearly tangent to its surface, and thus the important weakness of the jute filler is also shared by the paper filler.

Mr. Del Mar takes exception to the statement concerning the proportionality between charging current (volt-amperes) and dielectric loss. It is worthy of special comment that this proportionality exists only where the dielectrics are actually identical in properties. There may exist wide variations of the dielectric properties though the same grade of paper is saturated with compound of the same ingredients. It should further be noted that such a dielectric as paper will have different properties according as to whether the stress is perpendicular or parallel to the direction of the layers.

Also if the stresses in any specimen are beyond the range where the losses vary as the square of the voltage, then as indicated by Mr. Del Mar, that specimen cannot be compared directly with another specimen having lower stresses.

Study of the effect of these factors is facilitated by the data reported in the paper of the author. That is, effects of change of shape and dimensions may be differentiated from effects of differences in the properties of the material.

Mr. Roper has described an exceedingly interesting and important viewpoint concerning emergency overloading of cables. I refer to the fact that his company has never yet pulled a switch on a line because of overload. His arguments against doing that particular thing are very forceful and it may be recognized that the operating engineers have a real problem to solve to take care of such emergencies.

I wish to point out however that unless some provision is made, high-tension cables, as compared with most other electrical apparatus, are made to occupy an extremely hazardous position. Thus, a generator (or the turbine which runs it) sets for itself a very definite limitation of load which cannot be exceeded. To some extent most other apparatus does this in one way or another. Thus the low-tension cable does so by voltage drop; or its short-time overload capacity may be greater than that of its switches, unless these are very liberally rated. But a high-tension cable with very low dielectric losses will have no limitation short of destruction of the cable. Thus, if the policy be carried to the limit, we have, to use Mr. Roper's simile, a case of firing a boiler without a safety valve and with no regard to the steam gauge.

Presented at the 35th Annual Convention of
the American Institute of Electrical Engineers
Lake Placid Club, N. Y., June 27, 1919.

PROBLEMS OF 220-KV. POWER TRANSMISSION

BY A. E. SILVER

ABSTRACT OF PAPER

The dependence of industrial progress upon an adequate supply of electrical power, together with the vital need for a rational policy of conservation of the country's fuel resources, points to a probable early demand for transmission of large blocks of power from distant energy sources—coal fields and water powers. Two hundred and twenty kv. is suggested as a logical voltage for such high-capacity, long-distance transmission, and the important problems introduced by large concentrations of power, high voltage and high service standards are discussed. The economic and technical considerations underlying design of a 220-kv. system are outlined, and general designs are developed for a typical 220-kv. transmission line.

The studies made establish confidence in the conclusion that 220-kv. transmission is feasible as an immediate commercial proposition. Established principles of design and present types of equipment, with proper adaptation to the new conditions, are applicable to 220-kv. service. While all essential problems seem assured of acceptable solution, attention is directed to certain points as to which further investigation and experimental research are needed to determine most effective designs.

It is hoped that the paper may in some measure aid in the working out of this advance in the art by promoting constructive discussion and stimulating the needed investigations.

INTRODUCTORY

ECONOMIC development of the country's power resources is fundamental to the industrial progress of our national life. The growing recognition by our economists, engineers and industrial leaders, and even by the public, of the significance of the power supply problem must soon result in an insistent demand upon the engineering profession for large and bold strides in the solution of the attendant engineering problems.

The vital dependence of industrial advancement upon an adequate power supply has been strikingly illustrated in our struggles to meet the industrial demands of the war emergency. The experiences of this trying period have resulted, also in a quickened understanding of the need for conservation of our natural power resources. The labor and commodity situation accentuates the demand for greater unit productiveness, possible only through greater use of power in industrial operations, and for prompt progress toward broadly economical methods of power supply. A major step toward conservation of fuel resources and general commercial economy is recognized to lie in gradual substitution of electric transmission of energy for railroad transportation of fuel to be used in local generation of power. Interconnection of electric power systems, both prior to and during the war, has demonstrated the marked benefits obtainable by taking advantage of diversity of loads and other attendant economies and the well established movement toward general and extensive interconnection is still gathering headway. The relation of the railroad electrification problem to fuel conservation is well recognized and for years has been consistently expounded by able engineers.

These influences lead directly toward large scale development, at the source, of our natural energy reservoirs, the coal fields and the potential water powers. An essential feature of such development will be the transmission electrically of great blocks of power, over increasingly great distances, to strategic centers of distribution in or near present and future industrial districts. This trend of the power industry has long been recognized, and important steps toward its realization are a certain and not distant eventuality. Further elaboration of the subject is unnecessary to the purposes of this paper.

THE FIELD FOR 220-KV. POWER TRANSMISSION

Visualization of the demands of this evolution points to the need of trunk electric transmission service of a capacity and range of greater magnitude than thus far developed or required. Increasing distances and increasing quantities of power require an increasing voltage for economic transmission. The quantities and distances involved in the broad field outlined in the preceding paragraph pass beyond the economic range of existing transmission voltages. The practical working out of this next step in the transmission art is a problem now definitely facing the engineer and manufacturer.

Scarcely more than ten years ago an operating voltage of 100 kv. was remarkable; today 130 kv. may be considered standardized and 150 kv. is in sufficiently extensive use to have established its reliability.

Two hundred and twenty kv. appears a logical choice for the next step in the transmission voltage schedule. Why 220 kv., it may be asked? Because, from an appraisal of the general situation, a voltage of this order is considered adequate for the immediately pending needs of the industry and commensurate with expected growth in transmission service demands for a considerable period. Its suitability to a variety of conditions and the probable extent of its use would assure it a place in the schedule of transmission voltages which commercial needs are rapidly standardizing. Furthermore, such a voltage, while representing a step beyond present usage sufficient to afford a distinct economic advantage, does not reach so far into uninvestigated fields but that the problems of development and design can be approached with full confidence of early commercial solution.

The particular numerical value of 220,000 is in accord with the well established practise of standardization in multiples of 11,000. In some instances, it may be an incidental convenience that this voltage is the double of the extensively used 110 kv.

An illustration of the advantage for long transmission distances of 220 kv. over the highest present system voltage, 150 kv., is given in Figs. 1 and 2. This comparison is based upon a transmitted load of 500,000 kw. The same relative advantages will obtain for larger loads, and, above a certain minimum, for smaller loads.

The field of 220 kv. is not broad. Its economic application is primarily to large blocks of power and long transmission distances. It is in no sense a panacea for transmission problems generally. It presumably will infringe to some extent upon the present fields of the lower transmission voltages, but will by no means tend to supersede their use, in fact it will considerably enlarge the field of usefulness of such secondary transmission voltages as 66 kv. and 110 kv. It is not a universally suitable medium for extensive interconnection of power systems.

Power from steam-electric stations in the coal fields or from large hydroelectric stations would advantageously be transmitted over 220-kv. lines to terminal substations at important load centers or at the hubs of secondary transmission networks serving industrial areas. The introduction of transmitted power, in amounts limited only by the load demands, will constitute a strong stimulus to expansion of these networks.

Power equalization between load centers might frequently best be accomplished through extension of these secondary transmission lines. Interconnection at 220 kv. would be expected only where the equalizing duty reaches a large magnitude, where there is no existing secondary transmission system suitable to serve as a basis for inter-connecting lines, or where interconnecting 220-kv. lines might function also as a supplementary or important reserve link in a main 220-kv. trunk transmission system.

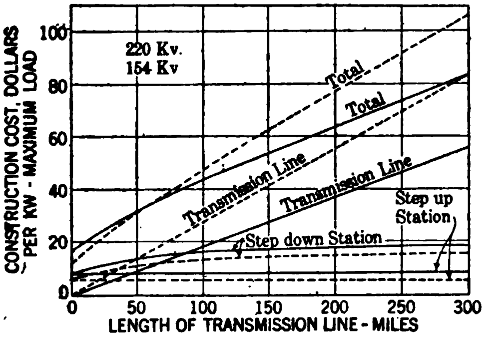


Fig. 1—Economic Comparison of 154-kv. and 220-kv. Transmission—Construction Cost—Seven Lines 154 kv. and Four Lines 220 kv.—500,000 kw. Delivered

Costs include those of lines, step-up and step-down substations and synchronous condensers.

Costs of line per mile 154 kv. $20,000, 220 kv. $23,500—all costs based on early 1919 prices.

Size of conductor 92,900 cm. steel—716,000 cm. aluminum.

Voltages high side of transformers, receiver end 150 kv. and 200 kv., sending end 170 kv. and 225 kv.

To a marked extent, and especially in the earlier stages of its introduction, power transmitted at 220 kv. will be high load-factor power. The initial investment in a 220-kv. system, including as essential elements the step-up and step-down stations, will be of such magnitude that there will be a strong inducement to utilize the investment as nearly continuously as practicable, thus reducing the unit transmission cost of energy supplied. The natural economic tendency in introducing power transmitted from distant energy sources will be to supply base load, leaving the peak loads to existing local generating stations. It is to be expected that high-voltage transmission from energy sources, fundamentally economic as it is believed to be, will in general for years contribute only a part of the power supply of any district. The relegation of existing stations to partial operation or reserve service will be gradual, and, even when transmitted power becomes the main reliance, presumably it will usually prove more economical to maintain local stations for reserve and short peak load service than to provide necessarily expensive transmission capacity for such short periods of actual use.

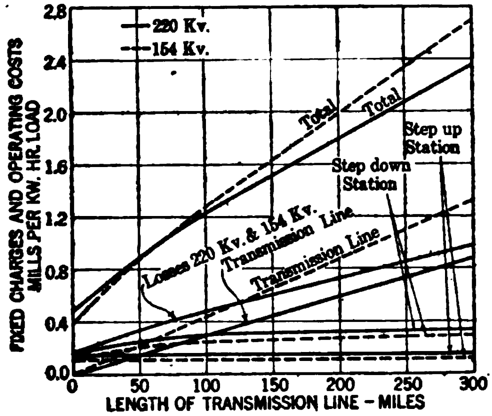


Fig. 2—Economic Comparison of 154-kv. and 220-kv. Transmission—Fixed Charges and Operating Costs—Seven Lines 154 kv. and Four Lines 220 kv.—500,000 kw. Delivered

Costs and losses include those of lines, step-up and step-down substations and synchronous condensers.

Costs of line per mile, 154 kv. $20,000, 220 kv. $23,500—all costs based on early 1919 prices.

Size of conductor 92,900 cm. steel—716,000 cm. aluminum.

Losses based on 0.95 load factor and 0.85 power factor delivered load.

Cost of energy 5 mils per kilowatt-hour.

Fixed charges and operating expenses of transmission lines 13 per cent, substations 15 per cent.

Voltages high side of transformers, receiver end 150 kv. and 200 kv., sending end 170 kv. and 225 kv.

A characteristic of the field of 220-kv. transmission which exercises a determining influence upon designs and costs is that it is a field of high service standards. A 220-kv. system, with the generating stations for which it would be the outlet, would represent a tremendous amount of power. The economic importance of reliability and continuity of this power, in view of the great volume of industrial enterprises and public utilities which would be dependent upon it, is of so high an order that new standards of care in design and conservatism in construction are imposed. The aim, and a not unrealizable aim, is to make high-voltage, large-capacity transmission for all practical purposes equal in dependability to local generation of power.

SCOPE OF PAPER

Based upon such a conception of the field of 220-kv. transmission, it is the aim of this paper to carry through an analysis of a typical transmission problem, sufficiently specific as to assumptions to afford a basis of designs, but not limited to any exact geographical location. Conditions underlying designs are analyzed, certain salient features of the design of lines and apparatus are discussed, proposed types of construction are considered and an effort is made to point out the problems of the designer and the manufacturer, particularly in those applications where there is apparent need of more thorough investigation and research before particularized conclusions can be drawn with confidence. It is not intended that the paper be in any sense exhaustive as to completeness of conclusions or solution of details of the problem, which for any specific case will involve extensive and thorough studies and investigations based upon specific load characteristics, geographical conditions and other basic considerations.

The studies have been guided by the desire that any recommendations should be capable of prompt commercial execution. In other words, it has been the aim to outline a type of construction built up essentially of established factory products in such way as to insure initially successful results. At the same time an attempt has been made to point out the short-comings which must be tolerated and the apparent opportunities for more efficient solutions.

It is the desire and hope that this tentative development of the problem in outline will afford a basis and incentive for full and constructive discussion by those interested in the advancement of the art of power transmission. It is hoped also that it may encourage designing engineers and manufacturers to undertake needed investigations into insufficiently explored fields as the foundation for developing designs for suitably improved apparatus and line materials. There would seem to be promising opportunities for distinct and beneficial departures from prevailing practises.

DESIGN FEATURES OF 220-KV. TRANSMISSION LINES

General Assumptions. Before considering the specific designs which are suggested for 220-kv. transmission lines, a brief statement will be made of the basic underlying assumptions as to loads to be transmitted, as to frequency and type of system to be adopted and as to climatic loadings and corona conditions to be assumed.

As to amount of load to be transmitted, it is assumed that even an initial 220-kv. system would be laid out on a basis of two or more main generating stations connected to load centers by a number of circuits. The load per 220-kv. circuit has been assumed to be from 100,000 kw. to 125,000 kw. A lower load per circuit than 100,000 kw. would entail a considerable sacrifice of the economy obtainable through use of 220 kv. No discussion is offered as to the maximum economic load per 220 kv. circuit, *i. e.*, as to the point above which additional circuits should be provided, since in any initial system the number of circuits would be determined from considerations rather of reliability insurance or load distribution than of maximum inherent economy. Where the studies involve a specific transmission distance, 250 miles has been assumed for purposes of illustration. The economic range of 220 kv. as to distance is very large.

The frequency of a 220-kv. trunk transmission system should be 60 cycles. For a general power distribution system furnishing lighting and power service in cities and industrial centers, the superiority of 60 cycles over 25 cycles has been well established, and for a transmission system delivering power to such a distribution system, the decision as to frequency unquestionably must follow the requirements of the load. In some districts, for which 220-kv. transmission may come up for consideration, it presumably will be found that both 60 cycles and 25 cycles are in use, possibly that 25 cycles in amount of load is still predominant. Even in these cases, however, the tendency in new development will be found away from 25 cycles, and it would be serious economic error to compromise so important an undertaking as a 220-kv. transmission system with a frequency approaching obsolescence.

High-capacity long-distance transmission will inevitably play a significant part in electrification of main line railroads. In instances where alternating-current electrification is an important factor in a project, there may be greater seeming inducements in favor of 25 cycles, but even in such cases it is believed that consideration of the general and industrial portions of the load, of the probable tendency of future growth and of the trend toward wide-spread interconnection will lead to a decision for 60 cycles.

From the standpoint of line performance alone, use of the lower frequency would afford better conditions of voltage regulation, but with synchronous condensers, which must be regarded as an integral part of a modern high capacity transmission system, the problem of satisfactory regulation at the higher frequency is not serious. Incidental considerations, none of them relatively important, are greater cost of 25-cycle equipment, lower reactance of a 25-cycle system and hence, heavier circuit breaker duty, and lower corona loss for 25 cycles.

The step to 220 kv. will by no means be the ultimate development in power transmission. Eventually a demand may be expected for transmission capacities and distances beyond the economic range of this voltage, and the advance then to be evolved may conceivably abandon 60 cycles for some very low frequency, for direct current or for some other revolutionary change in the transmission art. Such remote eventualities, however, do not affect the considerations governing the step to 220 kv.

Transformer connections, at all installations, should be grounded Y for the 220-kv. windings. The question of Y *vs.* delta connections has in the past evoked considerable discussion for each new undertaking, whatever the specific requirements of the system, and debate may arise in connection with projected 220-kv. transmission. Modern thought and experience shows a consistent tendency toward the use of grounded Y connections over the whole range of higher voltages. For a 220-kv. system, in addition to a distinct gain in dependability of operation, the requirements for line and equipment insulation, with their attendant effect upon the size and cost of equipment, particularly transformers, gives the grounded Y arrangement a marked advantage.

The simultaneous conditions of maximum climatic conductor loading used as a basis for line design have been assumed as follows:

a. Wind pressure of 8 lb. per sq. ft. of projected area, corresponding to an indicated wind velocity of 72 mi. per hr.;

b. Ice of 1½ inches radial thickness on all wires;

c. Temperature of 0 deg. fahr.

For checking clearances of conductors a maximum temperature of 120 deg. fahr. has been assumed.

These loadings are, of course, far in excess of those used as a basis for ordinary transmission line design. It is not believed that there is any reasonable likelihood of the line being subjected to such a simultaneous combination of conditions. From the standpoint of vertical tower loads; there is a real possibility of occasional ice loadings as heavy as or even heavier than assumed, and accordingly a reasonable margin of excess vertical strength is called for. Justification for these apparently extreme loading assumptions is found in the high service standards which, as noted, must in general apply in 220-kv. service. In view of the economic importance of continuity of service, it is considered that this basis is not unreasonable for a region where severe climatic conditions obtain, and in particular where heavy sleet is to be expected. Where climatic conditions are more mild, a lighter loading basis should naturally be assumed. In the planning of any particular project, this question of assumptions as to line loading, and the underlying economic conditions, should receive most careful consideration.

Corona formation and corona loss enter as a significant factor in the design of transmission lines at this high voltage. The conditions assumed for this study are:

a. Average elevation, 1000 ft. (normal barometer 28.9 in);

b. "Storm factor", or percentage of time during which conductors would be subject to increased corona losses due to rain, snow, sleet or fog, 12.5 per cent;

c. Conditions during "storm" periods, 28.4 in. barometer, average temperature 55 deg. fahr.

Corona conditions likewise will require special study for each particular installation, particularly the matter of "storm factor." The ordinary source of data will be Weather Bureau records. The published reports, however, while complete as to amount of precipitation and number of days of which precipitation occurs, do not give in summarized form the actual duration of storm conditions. A reasonably correct value of the factor for any district may be obtained from study, extending over a considerable period, of the original records of individual storms at several stations in the district. The dividing line between "storm" and "fair" conditions is, of course, not clearly defined, and hence preciseness of results is difficult to attain.

Conductor Materials and Types. Apparently the choice of conductor materials and types for 220 kv. service is limited, at least from the standpoint of immediate availability, to three alternatives,

1. Aluminum with steel core.
2. Copper with steel core.
3. All copper.

Aluminum without steel core does not possess sufficient mechanical strength for use on the span lengths in current use for high-voltage transmission. From the standpoint of mechanical strength, copper may be used either with or without steel core. A copper cable with a core of some other type than steel strand is a possibility; hemp cores have been used, but experience, while inconclusive, appears to indicate that there may be injurious chemical action; a semi-hollow copper cable with internal spacers of wood or metal has been suggested, but its feasibility has not been demonstrated and there seem to be no real benefits. Such a dead weight "filler" decreases the effective strength of the cable.

As to the satisfactory performance of steel cored cables, there appears to be but one point open to question; *i. e.*, possible electrolytic action between the galvanizing coat of the steel and the main conductors. Composite aluminum cables have been coming into increasing use during recent years, and from the experience gained, there is growing assurance of freedom from electrolytic action which would materially impair the durability of the cable. Aluminum and zinc are not far separated in the electro-chemical series, and aluminum is electro-positive to the zinc, so that there is less reason for anticipating trouble. There has been less experience with copper cable with galvanized steel core, but theoretically the conditions are somewhat less favorable, since copper is farther separated from and electro-negative with respect to zinc. It may be noted, however, that no injurious action has been observed in the extensive use of galvanized fittings with copper cable. Further experience with such composite copper cables is awaited with interest. In case injurious electrolytic action is established with galvanized cores, there appear to be a number of possible remedies, such as the use of copper plated steel cores. A sufficiently heavy copper plating would, however, be more expensive than galvanizing.

In general, for equal conductivity, the relative physical advantages of the three types of conductors may be summarized as follows. Certain features noted will later be discussed in more detail.

Aluminum-steel vs. copper-steel

1. Less corona loss, due to larger diameter
2. Skin effect presumably approximately equal
3. Greater area exposed to wind loading, hence, greater transverse tower strength and greater clearances required
4. Less tensile strength, hence more limitation upon height and spacing of towers
5. Less weight (unimportant)

All-copper vs. composite cable

1. More corona, greater than either of above
2. All material effective as a conductor
3. Skin effect more serious in larger sizes, owing to high priced material in the core
4. Less area exposed to wind loading
5. Less total tensile strength than either of above
6. Less weight than copper-steel
7. Homogenity of material, hence certain advantages in construction, more positive assurance of durability and higher scrap value

It will be seen that aluminum-steel, due to corona limitations, has a greater relative advantage for smaller line loads, which economically require smaller sizes of cable than for larger line loads.

For any particular case and any given magnitude and character of load to be transmitted, the choice of conductor type will depend upon a complex economic balance between the cost of materials involved, the losses due to resistance, corona, and skin effect, and the mechanical characteristics of the cables. Different loads may call for different types.

For the purpose of developing tower designs, a cable of 716,000 cir. mils of aluminum and 93,000 cir. mils of steel has been used in the studies which follow, the considerations upon which this selection was made will be discussed later.

Electrical Characteristics of Conductors. At such a high voltage as 220 kv., one of the primary considerations in selecting size and type of conductor is corona formation and corona loss in its relation to conductor diameter. The subject has been extensively treated by Peek and others before the Institute and the methods of calculation outlined by these authorities have been employed in the studies which follow. In these methods of analysis, it is necessary to assign values to the principal factors affecting corona. The physical and climatic conditions assumed have been stated earlier. The conductor arrangement has been taken as a flat horizontal configuration

TABLE I

CORONA LOSS DATA ON CONDUCTORS FOR 220-KV. CIRCUITS 20-FT. HORIZONTAL SPACING

(See Fig. 3)

	Aluminum-Steel				Copper	Copper-Steel			
Conductor Designation (See Table 2)	A	B	C	D	E	F	G	H	K
Dia. of cable—Ins.	0.952	1.036	1.092	1.196	0.814	0.857	0.904	0.946	1.004
Fair Weather Disruptive Critical kv. (e_0)									
Center wire	129	139	146	157	114	118	124	129	135
Outer wires	142	154	161	174	126	130	137	142	150
Storm Disruptive Critical kv. (e_0)									
Center wire	103	111	117	126	91	94	100	103	108
Outer wires	114	123	129	139	100	104	109	114	120
Total kw. Loss—3 Wires—250 Mi. (100,000-*kw. Circuit*) *Storm Factor* 12.5 *per cent*									
60 per cent load factor	464	162	66	16	1986	1253	758	493	254
75 per cent load factor	425	141	57	12	1835	1152	694	452	227
90 per cent load factor	378	119	47	8	1664	1032	620	404	195

with conductors separated 20 ft. Proper allowance has been made for the unbalanced disruptive critical voltages due to this horizontal configuration. The conductor irregularity factor, taking into account the effect of weathering of the conductor with age and the irregular surface resulting from stranding, has been assumed as 0.87.

In Table I are shown, for a number of conductor sizes, the fair weather and storm disruptive critical voltages and the total corona loss for 250 miles of circuit. Corona losses for various load factors are shown graphically in Fig. 3. In determining the values of line voltage to be used in corona calculations, effective r. m. s. values of load were obtained from hypothetical daily load curves prepared for various load factors, and the mean voltages corresponding were determined from typical line regulation curves (Figs. 4, 5 and 6). It is believed that this method of analysis affords a reasonably and sufficiently accurate estimate of corona losses.

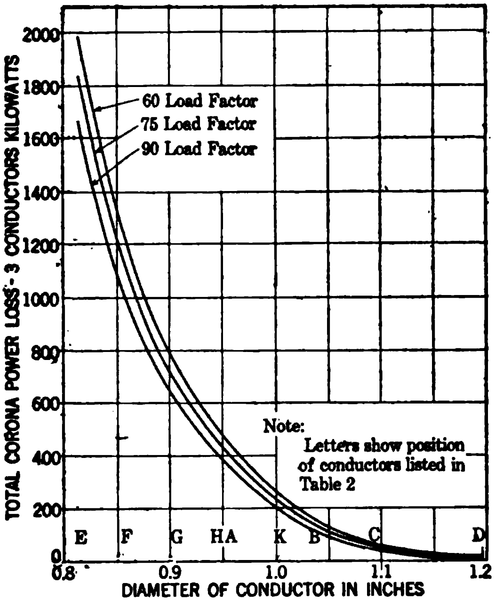
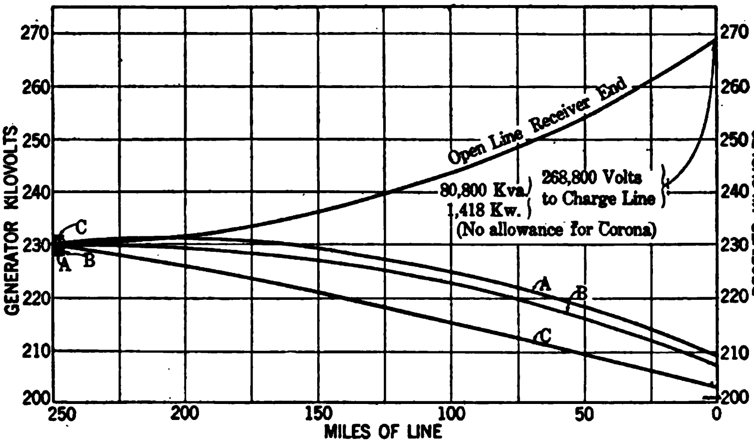


Fig. 3—220-kv. Corona Power Loss for 250-Mile Circuit (See Table I for Explanatory Data)
Generating Voltage Held Constant at 230 kv.

The electrical characteristics of composite cables, other than as regards corona, are somewhat more complicated of analysis than for a cable of homogeneous material, particularly as to the effects of the steel core on the internal inductance and effective resistance of the conductor. In the preparation of these studies, access has been had to data from certain experiments recently conducted in regard to these effects. It is understood that the investigators who have been carrying out these experiments anticipate presenting their findings in some detail to the engineering profession at an early date. It may be stated that this investigation indicates that in the practical



RESULTS

Curve	Rec'r kw.	Cond'r kv-a.	High voltage				Equiv. low voltage gen.		Losses (excl. corona loss)	
			Receiver		Generator					
			kv.	% p. f.	kv.	% p. f.	kv.	% p. f.	kw.	%
				250 mile line						
A	0	*49,800	209.0	* 0.18	230.0	† 4.61	227.9	† 4.69	2,646	..
B	50,000	† 7,500	207.2	*78.00	230.0	† 96.15	228.1	†97.28	2,780	5.6
C	100,000	†81,000	203.0	*98.72	230.0	*100.0	231.3	*99.67	11,927	11.9

NOTE: *Indicates lagging quantities.
†Indicates leading quantities.

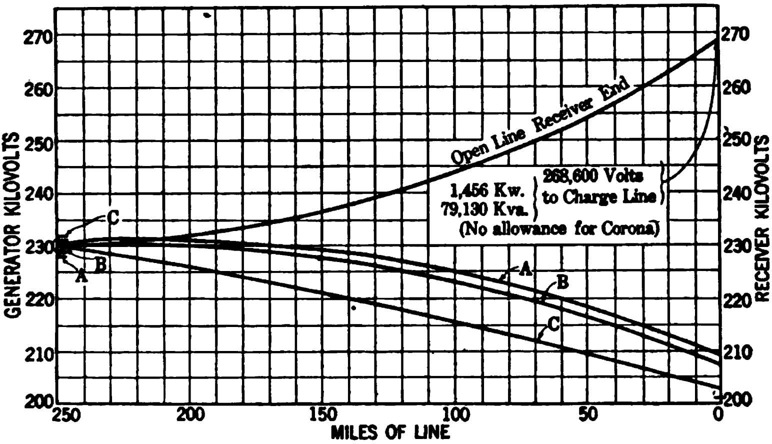
FIG. 4—220-KV. TRANSMISSION LINE CHARACTERISTICS—RELATIONS BETWEEN KW., KV., P. F., AND CONDENSER KV-A.—CONDUCTOR—716,000 CM. ALUMINUM—92,900 CM. STEEL

CALCULATION ASSUMPTIONS

Steel core aluminum conductor—716,000 c. m. aluminum and 92,900 c. m. steel—20 ft. horizontal spacing
Constant receiver volts—200,000 (low side receiver transformer, includes transformer drop).
Constant generated volts—230,000 (high side generator transformer)
Loads—delivered at 75 per cent power factor
Transformer bank of 50,000 kw. capacity with—resistance 0.5 per cent, reactance 12.0 per cent
Two banks of transformers in parallel at each end of line.
Losses include those of line, transformers and synchronous condensers

application of inductance and resistance formulas and calculations to transmission line studies, the effect of the steel core may be neglected.

Skin effect assumes appreciable proportions for the large sizes of conductor called for by 220-kv. transmission, particularly in case of the larger sizes of all-copper cable. The calculation of skin effect in composite cables is somewhat more complicated and burdensome than in the case of a homogeneous conductor.



RESULTS

Curve	Rec'r kw.	Cond'r Kv-a.	High voltage				Equiv. low voltage gen.		Losses (excl. corona loss)	
			Receiver		Generator					
			kv.	% p.f.	kv.	% p.f.	kv.	% p.f.	kw.	%
			250 mile line							
A	0	*49,000	209.0	* 0.18	230	† 5.48	227.9	† 5.56	2,704	..
B	50,000	† 7,600	207.2	*78.24	230	†96.89	228.6	†97.70	2,691	5.4
C	100,000	†81,600	202.9	*98.80	230	*99.97	231.7	*99.51	11,843	11.8

NOTE: *Indicates lagging quantities.
†Indicates leading quantities.

FIG. 5—220-KV. TRANSMISSION LINE CHARACTERISTICS—RELATION BETWEEN KW., KV., P. F. AND CONDENSER KV-A.—CONDUCTOR—450,000 CM. COPPER—308,200 CM. STEEL

CALCULATION ASSUMPTIONS

Steel core copper conductor—450,000 c. m. copper and 308,200 c. m. steel—20 ft. horizontal spacing.

Constant receiver volts—200,000 (Low side receiver transformer—includes transformer drop)

Constant generated volts—230,000 (high side generator transformer)

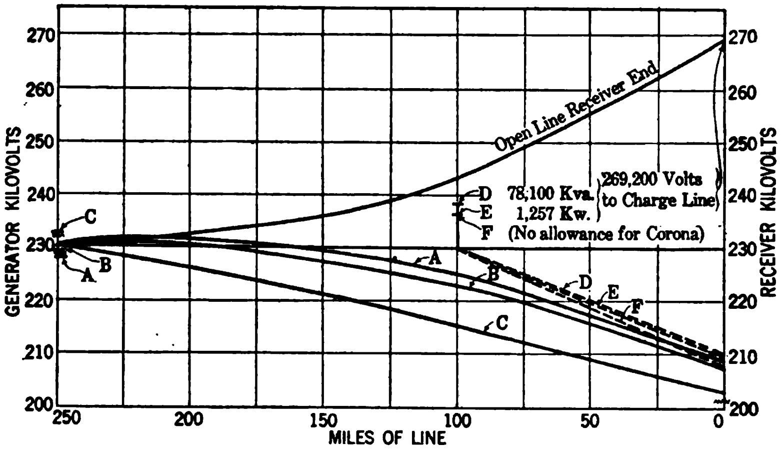
Loads—delivered at 75 per cent power factor.

Transformer bank of 50,000 kw. capacity with—resistance 0.5 per cent—reactance 12.0 per cent

Two banks of transformers in parallel at each end of line.

Losses include those of line, transformers and synchronous condensers

The operating characteristics of a 220-kv., 250-mile line are illustrated by Figs. 4, 5, and 6, which show for an aluminum-steel, a copper-steel and an all-copper conductor the relations between power transmitted, generator, receiver and line voltages, power factor, condenser load and resistance losses.



RESULTS

Curve	Rec'r kw.	Cond'r kv-a.	High voltage Receiver		High voltage Generator		Equiv. low Voltage gen.		Losses (excl. corona loss)	
			kv.	% p. f.	kv.	% p. f.	kv.	% p. f.	kw.	%
			250 mile line (solid line)							
A	0	*49,000	209.0	* 0.17	230	† 5.2	228.0	† 5.3	2,607	..
B	50,000	† 7,000	207.5	*77.8	230	†97.3	228.6	†98.1	2,168	4.3
C	100,000	†80,000	203.0	*98.5	230	*99.9	232.5	*99.3	10,562	10.6
			100 mile line (dotted line)							
D	0	*57,500	210.3	* 0.20	230	* 2.0	236.3	* 2.09	3,158	..
E	50,000	* 4,900	209.4	*68.5	230	*81.1	236.2	*79.1	1,478	3.0
F	100,000	†54,000	208.0	*91.5	230	*92.8	238.2	*90.0	6,242	6.2

NOTE *Indicates lagging quantities
†Indicates leading quantities.

FIG. 6—220-KV. TRANSMISSION LINE CHARACTERISTICS—RELATION BETWEEN KW., KV., P. F. AND CONDENSER KV-A.—CONDUCTOR—500,000 CM. COPPER

CALCULATION ASSUMPTIONS

500,000 cm. copper (no steel core—20 ft. horizontal spacing.
Constant receiver volts 200,000 (low side receiver transformer—includes transformer drop)
Constant generated volts 230,000 (high side generator transformer)
Loads—delivered at 75 per cent power factor.
Transformer bank 50,000 kw. with resistance 0.5 per cent, reactance 12.0 per cent
Two banks of transformers in parallel on each end of line.
Losses include those of line, transformers and synchronous condensers

Necessary explanatory details are shown in tabulations accompanying the curves. These studies show that effective voltage regulation of such long distance, high capacity lines may be obtained by means of large, but not impracticable, synchronous condenser installations.

A study of the relative economy of various sizes and types of conductors is shown in Fig. 7, with accompanying explanatory data in Table II. This study shows, for a load of 100,000 kw. per circuit delivered at 0.75 power factor and at load factors of 60 per cent, 75 per cent and 90 per cent, that the combined annual costs of such items of the transmission system as would be materially affected by the size and type of the conductor, *i. e.*, interest, taxes and amortization charges on cost of conductor, annual value of lost power on the line due to resistance and corona and of power absorbed by transformers and condensers. The curve falling lowest on the scale, at any number of years which may be assumed as the life of the line, represents the most economical of the conductors considered.

This curve is presented merely as an example of the general method followed in studying conductor economy. It does not represent the degree of refinement which would be warranted in making final determination of the economical conductor for an actual 220-kv. installation. Other items for which allowance should be made in such a study are the effect of conductor size and type upon cost of line structures and insulators, and the possible scrap value of the conductor. Amortization should preferably be calculated by the annuity or "sinking fund" method rather than by the simpler straight line method. It should be noted, however, that these additional refinements tend in some respects to offset one another, a feature which gives added justification for their omission from a preliminary study. Obviously, in any case great refinement in the technical assumptions is not called for until reasonably close values can be assigned to cost of conductor materials, for which the market will presumably be unstable for some time to come, to the equivalent costs of the power losses, and to the percentages to be employed for return on investment, taxes, etc.

For the purpose of developing the line designs which will be presented later, a cable was chosen consisting of 716,000 cir. mils of aluminum (54 strands of 0.1151-inch diameter) and 93,000 cir. mils of steel (7 strands of 0.1151-inch diameter). Particular attention, however, is called to the fact that these designs were started and this conductor chosen as a basis of study about a year ago, when prices were much higher than at present, and further that the choice was made upon the basis of a lower load factor than, in the light of further study, seems reasonable to assume as likely to obtain on the usual 220-kv.

transmission system. It is fully recognized that at present price levels and for a high load factor, the economical size of conductor would be larger than this, and that possibly the economic advantage would fall to a different type.

Mechanical Characteristics of Aluminum-Steel Conductor. The large and heavy cables required for 220-kv. transmission, and the heavy conductor loadings which the importance of 220-kv. service requires as a design basis, together with the unusual general precautions which must be taken to assure reliability, call for a new order of refinement in the mechanical features of line design. Careful study with a high degree of imaginative foresight is required to provide against unsafe loads being imposed upon insulators and towers or clearances being dangerously reduced as a result of unusual or unexpected con-

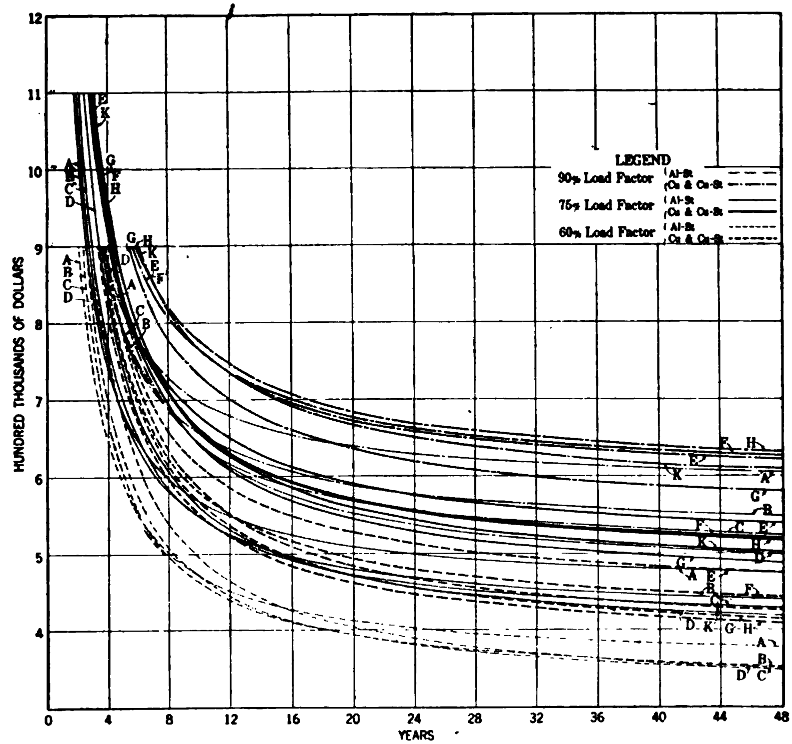


FIG. 7—ECONOMIC COMPARISON OF 220 KV. TRANSMISSION CONDUCTORS—ANNUAL COST CURVES—250 MILE, 100,000 KW. SINGLE CIRCUIT LINE (SEE TABLE II FOR EXPLANATORY DATA)

tingencies, and against possible unpreventable failures becoming cumulative and occasioning extensive damage.

The mechanical characteristics of steel-core aluminum cables are complicated, and calculation of stresses and sags under varying loading and topographical conditions presents an involved and difficult problem. This is due to the fact that the co-efficients of expansion and moduli of elasticity are different for the two metals and consequently are not accurately determinable for the composite cables. The cable manufacturers, however, have investigated these characteristics and have developed practical, though laborious, methods of calculation of conductor stresses and sags. These methods have been followed and are considered in the main to give results of an accuracy satisfactory for design purposes.

TABLE II

DATA RELATING TO ECONOMIC COMPARISON OF 220 KV. TRANSMISSION CONDUCTORS

(See Fig. 7)

ANNUAL COST CURVES FOR 9 DIFFERENT CABLES AT 3 LOAD FACTORS

1. Annual cost curves are plotted, dollars as ordinates, years as abscissas, and show the annual cost for any period up to 48 years. This is explained as follows:

Annual Cost Includes
  1. Depreciation expressed as first cost divided by number of years chosen.
  2. Yearly interest and taxes taken as 8 per cent of first cost.
  3. Annual value of Lost Power taken as 5 mills per kw-hr.

First cost includes only cost of finished Aluminum-Steel Cable at 44.9c. per lb. for aluminum and 12.2c. per lb., for steel f. o. b. factory. First cost of finished copper or copper-steel cable @ 27.7c. per lb. for copper and 12.2c. per lb. for steel f. o. b. factory. All other items of construction costs have been eliminated as not materially affecting the relative positions of the curves.

Lost power includes line $I^2 R$, transformer I^2R, condenser loss and corona loss.

2. Cable Data.

Curve designation (See Fig. 7)	Kind of cable	First Cost	Circular Mils			Strands		Diam. of cable in.
			Alum. or copper	Steel	Total of cable	Alum. or copper	Steel	
A	Al.-St.	$1,175,700	605,000	78,000	683,500	54	7	0.952
B	Al.-St.	1,362,800	716,000	92,900	808,900	54	7	1.036
C	Al.-St.	1,513,500	795,000	103,100	898,100	54	7	1.092
D	Al.-St.	1,814,000	954,000	123,700	1,077,700	54	7	1.196
E	Copper	1,764,800	500,000	....	500,000	37	..	0.814
F	Cu.-St.	1,733,300	450,000	105,000	555,000	30	7	0.857
G	Cu.-St.	1,924,400	500,000	116,600	616,600	30	7	0.904
H	Cu.-St.	1,790,200	400,000	274,000	674,000	54	37	0.946
K	Cu.-St.	2,011,700	450,000	308,200	758,200	54	37	1.004

The basic data for calculations have been taken as:

	Aluminum	Steel
Elastic Limit (lb. per sq. in.)...........	14,000	130,000
Modulus of elasticity (lb. per sq. in.) ..	9×10^6	30×10^6
Co-efficient of expansion (per degree fahr)............................	12.8×10^{-6}	6.4×10^{-6}

Elastic limit of composite cable 17,300 lb.
(716,000 cir. mils aluminum, 93,000 cir. mils steel)

Within any limits thus far investigated, it appears economical under the design loading, to utilize the full strength of the conductor up to its elastic limit, *i. e.*, to keep sags to a minimum, thus enabling use of shorter towers or longer spans at the expense of stronger supporting structures at angles and dead-end points. With a new aluminum-steel cable, when the loading has been reached which will stress the steel core to its elastic limit, the aluminum will have passed its elastic limit and in consequence have been permanently stretched. The full working strength of the cable, 17,300 lb., will then have been developed, and it may then be said to have reached its final stretched condition, *i. e.*, to have received its "permanent set". As the heavy design loading is removed, the action of the aluminum strands will be to loosen infinitesimally on the steel core and to take no stress. In any subsequent applications of the design loading, the core will again be stressed to the elastic limit of the steel, and the aluminum will coincidentally reach its own elastic limit stress. At lighter loadings the aluminum will be a dead load upon the steel core, which will carry all of the stress.

This introduces an interesting feature in that the characteristics of the composite cable when *new* are distinctly different from its characteristics after it has received its "permanent set," and, for loadings less than the design loading, will follow different tension-sag curves. This relation is shown graphically in Fig. 8.

The problem of predetermining stringing sags for a suspension insulator line, in which there will be considerable differences in lengths of adjoining spans and in elevations of the ends of individual spans, involves considerations of unbalanced tower loads, abnormal insulator loads and reduced tower and ground clearances under varying conditions of conductor loading. It is necessary, initially, to assume a somewhat different mental attitude toward the problem of conductor stringing than that which largely has been customary. For the conditions of 220-kv. design, with heavy design loadings and large conductors under heavy tension, the problem assumes greater importance than in transmission line practise heretofore. In usual practise on suspension insulator lines, the conductor is strung to a certain predetermined tension, derived from tension-sag curves on the basis of the *average span* length between dead-end points, and the suspension insulators are then tied in in a vertical position. It is assumed that when the design loadings occur the conductor tensions and attendant loads upon insulators and towers will continue to maintain the condition of proper balance of stresses within reasonably close limits. Probably, with most present lines, the resulting unbalanced effects under design loading will not be serious.

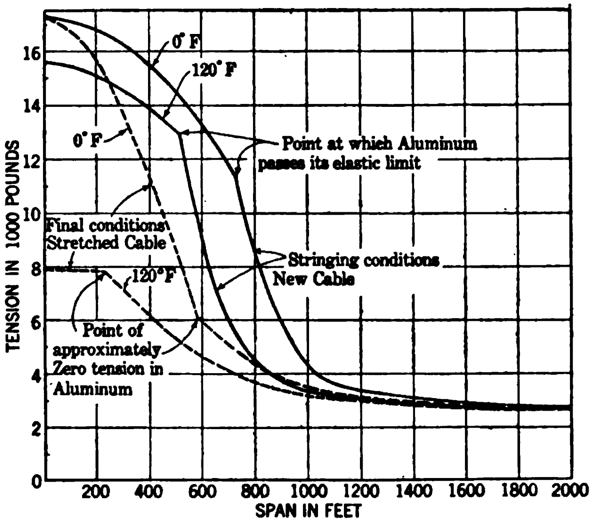


FIG. 8—TENSION—TEMPERATURE CURVES—DEAD ENDED SPANS—STRINGING AND FINAL CONDITIONS—CONDUCTOR—ALUMINUM 716,000 CM.—STEEL 92,900 CM.

Steel reinforced aluminum cable—716,000 c. m. aluminum—92,900 c. m. steel—elastic limit of cable and maximum allowable stress—17,300 lb. with 1½ inch ice and 8 lb. wind at 0 deg. fahr.

Solid line curves represent stringing tensions of *new* cable, without load, before first application of maximum loading.

Dotted line curves represent final tensions of *stretched* cable, without load, after application of maximum loading has given a permanent set to the aluminum.

nder the assumed 220-kv. design conditions, preliminary study has shown that such a method of stringing will result, under the extremely heavy design loading, in great longitudinal deflections of suspension insulators intended to operate purely as suspension units, and will consequently impose unsafe tensions upon these insulators, with attendant unbalancing of tower loads and possibly dangerous reduction in tower clearances, in addition to over-stressing and stretching the conductor itself.

The theoretically proper point of view is that, under design loading conditions, the conductor should be uniformly stressed to its elastic limit and insulators should be hanging normally. Assuming the line to be in this condition under maximum load, the inverse of the condition just described will occur as the load is removed and the conductor contracts in length, that is, the suspension insulators will assume certain definite deflections at towers between spans of different lengths and at adjacent towers at different elevations, resulting, of course, in a non-uniform tension in the conductor itself. It is this unbalanced condition without load which careful design should aim to produce in initial stringing. This method of stringing might, however, in some cases result in excessive insulator deflections and tensions under the non-loaded condition of the conductor, and some compromise between the two methods may be necessary to safeguard the clearances of the unloaded conductors.

A method of predetermining for stringing conditions the proper conductor tensions or sags in individual spans and the proper insulator deflections is accordingly a desirable refinement. Some approximate treatment of the subject, at least, will probably be necessary. It should be noted that the 220-kv. designs suggested in this paper contemplate the use of very long suspension insulator strings, and that this length of string tends to lessen the amount of abnormal insulator and tower stresses and reduction of tower clearances which may be set up. If, as is hoped, improvement in insulator design should enable the adoption of a shorter insulator, an accurate method of conductor stringing will become correspondingly more important.

A purely mathematical treatment of the problem appears complex in the extreme, although approximate methods of analysis seem to be feasible. The working out of such approximate methods offers an interesting and valuable subject for research. An exceedingly instructive experimental investigation could be made along these lines, either by means of a series of spans of a full-sized line capable of being artificially loaded, or by means of a miniature properly proportioned model. One troublesome feature of such an experimental determination of these phenomena would seem to be that of readily obtaining and controlling the necessary temperature range, or of compensating or correcting for temperature variation by some indirect means. Such an investigation would fulfill a most valuable function in supplying empirical constants to be used as a basis for mathematical treatment and in verifying the results of approximate methods of analysis.

The problem of actual stringing is further complicated by the use of a composite cable, owing to the shifting of tensions between the aluminum and the steel, and especially to the radically different characteristics of the cable before and after it has been stretched to receive its "permanent set", as previously described and as illustrated by Fig. 8. This difference in characteristics leads to the possibility of two different methods of stringing a composite cable, *i. e.*, it may be strung as received from the factory, or it may be stressed, prior to sagging and tying in, to its full strength, 17,300 lb., thus giving it nearly all of its "permanent set" (it would receive all of its "permanent set" if the stretching were done at 0 deg. fahr.) In the latter case a series of tension-sag curves based upon the "permanent set" condition of the cable would be used for stringing. In the case of stringing the new unstretched cable, it would be necessary to develop a special series of tension-sag curves, while the "permanent" tension-sag curves would be used in locating towers and checking clearances.

For a new cable, in the absence of special data, the division of stress between the aluminum and the steel is indeterminate. Such special data may, however, be obtained, presumably by experiment. It probably will be found that the conditions of the cable as it comes from the factory is sufficiently uniform to enable such special stringing curves to be used consistently. The application of methods predetermining insulator deflections, as discussed above, would involve even further complication if these special unstretched tension-sag curves were to be used for stringing.

From the operating standpoint, however, it would seem preferable to string the cable without preliminary stretching.

The design loading, with the large margin of safety which it is assumed to contain, will probably rarely be reached or approximated on the greater portion of the line. The smaller initial sags would then in practise, for the greater part, never be increased to the "permanent" sags by natural causes. Hence there would be obtained the advantage of smaller normal operating sags, with the unstretched aluminum acting as a reserve to increase them suitably when, or if, the design loading should occur.

Possibly a combination of the two methods might be worked out, whereby the cable would be stressed before tying in to some definite tension sufficient to insure giving the aluminum part of its "permanent" set. It would then be sagged in by unloaded tension-sag curves based upon the definite relation between stresses in the aluminum and the steel as established by this preliminary application of tension.

The practical importance of the theoretical considerations involved in stringing irregular spans supported by suspension insulators, and the extent to which refinements may and should be carried will in the last analysis be governed by consideration of the practical limits of field application, taking into account the many variable physical and personal factors. The intent is to point merely to the interesting and apparently effective possibilities of theoretical and experimental research, pending further analysis and study. No field stringing curves embodying the refinements suggested have yet been developed, even in approximate form. For purposes of clearance determinations, of tower design and of study of tower economics, the use of "average span" tensions and sags probably embodies sufficient accuracy, and has tentatively been used.

Adequate splices for aluminum-steel cables no longer are considered to present a problem. Satisfactory types have been developed and are in successful use on existing lines. In making these splices, the aluminum is cut back from the ends of the steel core, the core is then spliced by means of a soft steel sleeve, twisted on by means of special wrenches, and the sleeve and a considerable length of the aluminum strand at each end are then covered by a heavy aluminum sleeve which is solidly compressed on the conductors between the dies of a portable oil operated jack. Such splices are reported to develop the full strength of the cable, the splice itself having considerable excess strength.

Overhead Ground Wires. The line designs presented provide for two overhead ground cables of 5/8-inch diameter high-strength steel, 16,000 lb. elastic limit for the cable. The justification for the use of ground wires in 220-kv. transmission is debatable, but no well-formed conclusions seem possible except as a result of practical comparative experience. Their use on present high-voltage steel tower lines is nearly universal, remarkably so in view of the meagre and inconclusive character of the data as to the benefits derived. At 220 kv., with the high insulation provided and in view of the diminishing importance which, with increasing line voltage, it is believed can be attached to induced lighting disturbances, it is wholly conceivable that the protection afforded may be found to be of disproportionately small value. The cost of using ground wires is undoubtedly a large item, particularly in view of the extremely heavy design loadings assumed and the consequent tower stresses which they occasion.

In view, however, of the great importance of 220-kv. service, it has been deemed conservative, until further experience is obtainable, to make provision for the use of ground wires. The plan of installation would be to omit ground wires from a considerable portion of one line, in districts where lightning conditions were severe, while ground wires would be used on a parallel line. Comparative performance data would be a guide to subsequent procedure.

Types of Insulators Available. There is no type of insulator as yet developed which has thus far demonstrated its ability to give adequate, or even reasonably satisfactory results on high-voltage lines. This condition, however, applies nearly as much to the high voltages in current use, 110 kv. to 150 kv., as it does to 220 kv. It does not in any way affect the feasibility of 220-kv. transmission. It is confidently believed that 220-kv. line insulation can, with existing types of insulators, be made as safe and dependable as can the line insulation of present installations. In fact, where foresight in design and careful maintenance are employed, the present unsatisfactory insulator situation makes its effects evident far less in impairment of service reliability than in the high first cost, direct and indirect, of precautions against insulator failure and in high maintenance expense and operating inconvenience. The economic value of 220-kv. service is so high that greater expense and attention than at lower voltages are warranted in measures to guard service.

The probable advent in the near future of such voltages as 220 kv. should serve, nevertheless, to emphasize the economic necessity of developing line insulators to a point consistent with the other elements of the electric power generation and transmission situation. The baneful effects of the insulator situation are evident not merely in the expense and trouble occasioned, but in a certain degree of popular suspicion, in essentials unwarranted, of the general idea of high-voltage transmission.

It is believed, as has just been noted, that practicable 220-kv. insulators can be obtained from present established types. There are commercially available three such types, all based upon the principle of a series string of disks, in practise of about 10-inch diameter, *i. e.*, the standard cemented cap and pin type, the Hewlett type and the newer Jeffrey-Dewitt type. Numerous other designs for high-voltage insulators have been suggested, some of the series unit type and some in one piece. Certain of these designs appear to offer real promise, but none of them have been developed to the stage of commercial production, or even to a point where their service performance can be predicted.

The following studies and 220-kv. line designs have been based upon the standard 10-inch cemented cap and pin type units. This implies no disparagement of the other types mentioned; in fact, these appear in some respects to offer significant advantages. The standard type has been taken as a basis of study because it has been used by far the most extensively and for long periods, and, therefore, more data are available in regard to its characteristics. Any general designs of towers and fittings developed for these units could equally well be used for the other types. The studies which follow, in conjunction with the general record of experience and investigation, indicate that there are certain characteristics, largely inherent, of these standard disk insulators which render them far from ideal for service at extra high voltage with heavy conductors. In certain respects this would be true of any insulator built up of a large number of disks. It is hoped that manufacturers will soon be able to develop an insulator, which will be more suitable for extra high-voltage use and will offer assurance of greater strength and permanence, both electrically and mechanically.

Electrical Characteristics of Disk Insulators. The designs suggested make provision for a string of 15 standard units for regular suspension service on a 220-kv. grounded neutral line. This relatively large number of units affords a considerable margin for deterioration and, with proper care in maintenance, should assure a degree of reliability in service commensurate with the economic importance of 220-kv. service.

It should be noted that a string of 15 standard units, with the necessary connecting pieces and fittings, will be nearly nine feet long. Such a length of insulator obviously involves great expense in obtaining the necessary tower clearances and heights, and it is also the determining feature in fixing conductor separation. This serves again to call attention to the need of more efficient and suitable insulators. An improvement in insulator design which would justify shortening the string, in addition to improving the electrical characteristics of the insulator itself, would enable material saving in tower costs. A wholly new insulator, having no greater length than necessary to insure requisite air clearances from conductor to support, say four or five feet, would enable a correspondingly greater and a very significant saving in tower costs. Such an insulator at moderate price might readily open the door to a variety of new types and arrangements of supporting structures.

The primary electrical characteristics are those of arc-over and puncture. It is essential to reliable service that units be employed with the largest obtainable ratio of puncture voltage to arc-over voltage. The particular importance of this high ratio for 220 kv. will be evident in the light of certain data which will be presented later in regard to concentrations of electrical stress upon individual units of the string.

The 60-cycle arc-over characteristics of long strings of standard disk insulators are shown in Fig. 9, which gives curves of arc-over voltage for wet and dry conditions. These curves are based upon published test data for shorter strings, extended mainly on a theoretical basis with the assistance of such fragmentary test data as have been available.* It is pertinent, and of great interest, to note that the dry arc-over curve flattens out as the number of units in the string increases, approaching a condition where increasing the number of disks adds practically nothing to the dry arc-over voltage, while on the other hand the wet arc-over curve follows nearly

---

*Peek, A. I. E. E. Trans. 1912, Vol. XXXI, p. 907,—"Electrical Characteristics of the Suspension Insulator."

a straight line characteristic and for long strings reaches higher values than the dry arc-over curve. These characteristics, as is well understood, are due in the case of dry arc-over, to the action of the charging current on the system of distributed series and shunt capacities which the insulator string constitutes, the resultant effect being a concentration of potential on certain units of the string. In the case of wet arc-over, the effect of the charging current is lost in the greater effect of the large leakage current.

The dry arc-over characteristic thus appears to constitute the controlling feature of insulator design, in so far at least as 60-cycle characteristics are determining. The fact that an insulator has a higher arc-over value under rain conditions is of no advantage, since this condition obtains for but a small fraction of the time, and in particular since heavy lightning discharges, with possibility of resultant normal frequency surges on the line, are more likely to occur just before a rain storm than during it.

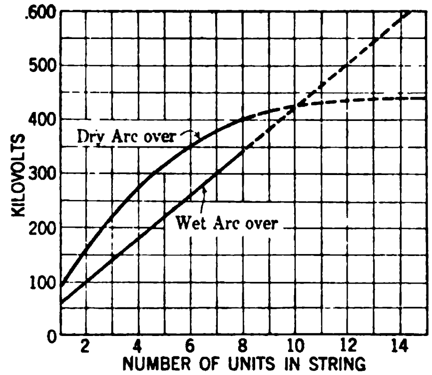


FIG. 9—TYPICAL 60-CYCLE ARC-OVER CHARACTERISTICS—SUSPENSION INSULATORS

The distribution of voltage stress over the units of a string of fifteen standard 10-inch disks and the concentration of stress on certain units, notably those nearest the conductor, is shown by curve *A* in Fig. 10, which is based in the main upon published test data.* At a line voltage of 220 kv., (127 kv. to ground), this concentration reaches a degree which not only interferes seriously with the efficient and economic use of the insulator units, but which brings the stress on the unit next to the conductor to a point higher with respect to its strength than the standards of practise set for 220-kv. service make desirable.

Two methods are recognized as offering relief from this excessive concentration. The first is the grading of the insulator units used in the string. This would be accomplished by making up the string of units of two or more distinct types, differing in size or diameter or in some other feature which would cause them to have different condenser capacities, those with the larger capacities being placed nearest to the conductor. The results of one form of such grading are shown in curve *B*, Fig. 10 which is based upon published test data.* It is evident that by such grading considerable and, for ordinary purposes, ample improvement in voltage distribution can be effected. From a practical standpoint, this expedient involves a certain amount of complication and expense in construction and of expense and inconvenience in operation due to the necessity of maintaining stocks of each of the different types of units

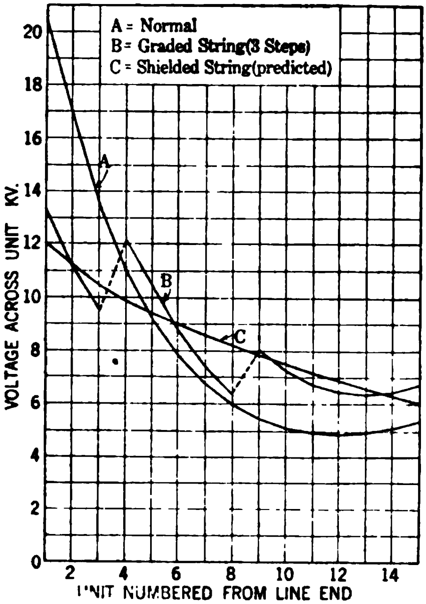


FIG. 10—TYPICAL 60-CYCLE VOLTAGE DISTRIBUTION—SUSPENSION INSULATORS—220 KV. GROUNDED NEUTRAL SYSTEM

---

*A. I. E. E. 1916 TRANS., Vol. XXXI, Part I, p.745, R. H. Marvin, "A New Method of Grading Suspension Insulators."

and of insuring their proper use in maintenance replacements. This disadvantage, while of some consequence, cannot be regarded as prohibitive in view of the benefits which might be expected to accrue.

A second method of relieving this excessive concentration of stress consists in installing below or around the disk nearest the conductor suitably designed metallic shields or rings. The effect of such shields in improving the stress gradient may be even more marked than that of grading the insulator units. The stress distribution obtainable by this method has been predicted in curve *C* of Fig. 10, the values for which have been assumed from the fragmentary test data at hand. The use of such shields or rings at the lower end of the insulator string would obviously tend in itself to increase the requirements for tower clearances, but the shortening of the insulator string, which a successful application of this expedient would justify, might presumably compensate.

In general either grading or shielding or a combination of the two appears to be feasible. Neither would appear to require any very elaborate investigations and tests to determine effective designs free from possibility of secondary complications of any moment. The conditions with the 15-unit string of standard units are so unsatisfactory that probably some alleviating measures should be adopted. Of the two described, probably grading could be developed to a point ready for actual use most quickly and with least experimental investigation. It is wholly possible that a considerable grading effect might be worked out through selection from present commercial types of disks.

A feature of the insulator situation which complicates the question of voltage stress distribution and which will have some effect upon methods of carrying out remedial measures is the fact that in order to obtain adequate mechanical strength, as will be discussed later, two or three strings of standard disks must be used in parallel at suspension points, and proportionately more at tension points. So far as is known, no investigation has been made of the effect of parallel strings upon potential gradient.

The belief is widely entertained that arcing horns or rings or other discharge devices are of sufficient benefit to warrant their use. They would fulfill several functions, the first and primary function being to protect the insulator from the destructive heat of an arc. This function assumes particular importance on a large capacity, dead-grounded neutral system. For this purpose the devices should be so shaped and placed as to hold the arc securely away from the insulator string. Other functions are protection of the conductor from the possible burning by a high-power arc, and reduction of the likelihood of insulator puncture. It may be possible to combine in one device in some effective and economical manner the functions of a discharge horn or ring and of an electrostatic insulator shield.

The foregoing discussion of electrical stresses on insulators refers, as will be noted, to such stresses as may be produced at the normal line frequency of 60 cycles. Under conditions of high-frequency oscillations and of steep wave front phenomena generally, the insulator voltage stress characteristics will be quite different. In particular, the high-frequency voltage stress distribution over an insulator string is understood to follow approximately a straight line characteristic. The importance to be ascribed to high-frequency phenomena as a disturbing factor in 220-kv. operation is a problem of a highly speculative character. Such evidence, however, as is available, and the trend of theoretical opinion, tend toward the conclusion that high-frequency becomes of diminishing relative significance as the line voltage is increased, particularly when transformer neutrals are dead grounded. The suggestion has been offered that, with a line operating near the corona limit, high-voltage surges at high frequency, representing usually small amounts of energy, tend to dissipate themselves in corona, corona dissipation of energy being more rapid at high frequencies. In any event it is believed improbable that a line sufficiently well-insulated to withstand low-frequency high-power disturbances is likely to encounter trouble from high-frequency disturbances.

Mechanical Characteristics of Disk Insulators. The widespread dissatisfaction with the present insulator situation is believed to have arisen largely on mechanical grounds.

A very disturbing feature is the much discussed insulator deterioration. The progressive failure, after a relatively short period of years in service, of the cemented type disk insulator is well recognized and has been almost universally experienced. Existing high-voltage lines are facing the prospect of continuous and difficult tests and expensive maintenance, both attended by interference with operation. Whether the causes be also electrical, chemical or ceramic, the results are unquestionably serious. In this respect, however, the difficulties at 220-kv. would differ from those at low voltage only in the greater number of disks involved.

From the standpoint of deterioration, the non-cemented types of disk insulator, which avoid the effects of dissimilar expansion of the component materials, seem to offer and to be demonstrating marked advantages.

The working stress for standard cemented cap and pin type disks permitted by conservative practise is only about 2500 lb. With the large conductors and extremely heavy design loadings required for 220-kv. service, this low mechanical strength immediately presents itself as an embarrassingly serious limitation. The designs which have been worked out in this study call at normal suspension points for two strings in parallel with spans up to 700 ft. and for three strings in parallel with spans in excess of 700 ft. The complication and expense of hardware and the difficulties involved in clearances are obvious, not to speak of the direct cost of the insulators. If these insulators were to be used at dead-end points, tension assemblies of at least six strings would be required, at the unconservative design load of 2900 lb. per string. A failure in one string, by unbalancing the load distribution, would seriously jeopardize the whole assembly.

For service at dead-end points, however, the unsuitability of the standard disk is so pronounced that it is considered probable that resort would be made at once to an entirely different type of insulator.

Considerable promise is offered by a relatively new type of insulator, a wooden rod insulated with compound and enclosed in a suitably petticoated procelain shell. Any desired mechanical strength can readily be secured in an insulator of this type, so that the full dead-end tension would be carried on one unit. Probably two units would be used in series to obviate an excessively long porcelain tube. Such tension insulators are in limited use on 120-kv. and 150-kv. lines. They are a new development, however, and in the absence of long service demonstration there naturally arises a question as to their electrical permanence. Experiments with somewhat similar types of insulator have shown unfavorable results in the way of disintegration of the wooden rod under long applied high electrical stress. On the other hand, oil insulated wooden rods in circuit breakers have, with few exceptions, been found to be durable. Possibly some other material, less susceptible than wood to disintegration, might be employed for the rod. Any cracking of the porcelain shell or loosening of joints, which would permit escape of the insulating compound, would of course result in failure of the insulator. In the light of present knowledge, however, the lack of service trial appears no more serious than the known complication of huge assemblies of the standard type of disk insulator.

Summarizing the situation as to availability of present commercial disk insulators for 220-kv. service, it is evident that, in addition to known drawbacks, there exists considerable question as to fundamental characteristics which must to some extent be removed before confidence can be established in assumptions made and designs based upon these assumptions. Investigations should be conducted, with possible resultant developments, as to at least three features of the applicability of disk insulators to 220-kv. service, *i. e.:*

1. Tests to confirm or establish the dry and wet flashover characteristics, and accompanying potential gradients, of strings of ten to fifteen disk units, both singly and with several strings in parallel. These characteristics should be investigated both at 60 cycles and at high frequency;
2. Experiments to determine how grading or shielding or both can most effectively be applied to strings of present types of disks;
3. Continued investigation of insulator deterioration problem.

It is to be hoped that those concerned with the design and manufacture of insulators will be inclined to undertake investigations of this character. Of even greater ultimate importance, of course, are efforts looking to the development of a new type of insulator, more efficient electrically, with greater strength mechanically and of unquestionable permanence.

Clamps and Fittings. Reliable and satisfactory clamps are now on the market for composite cables of somewhat smaller sizes. It is not expected that any significant difficulty will be encountered in the design of clamps suitable for a cable of the size proposed. The clamps will presumably be of generally similar design to that of those now in use on some of the highest voltage lines in the country, of which illustration is given in Fig. 11.

In the design of all cable clamps for use at 220-kv., especial care would be required in avoiding of sharp projections, ridges or points which might afford opportunity for formation of corona or static discharges. Sleeve protectors should be used with all types of clamps. At the ends of the clamps, bells of sufficiently large radius should be provided to avoid any chance of conductor crystallization from continued vibration.

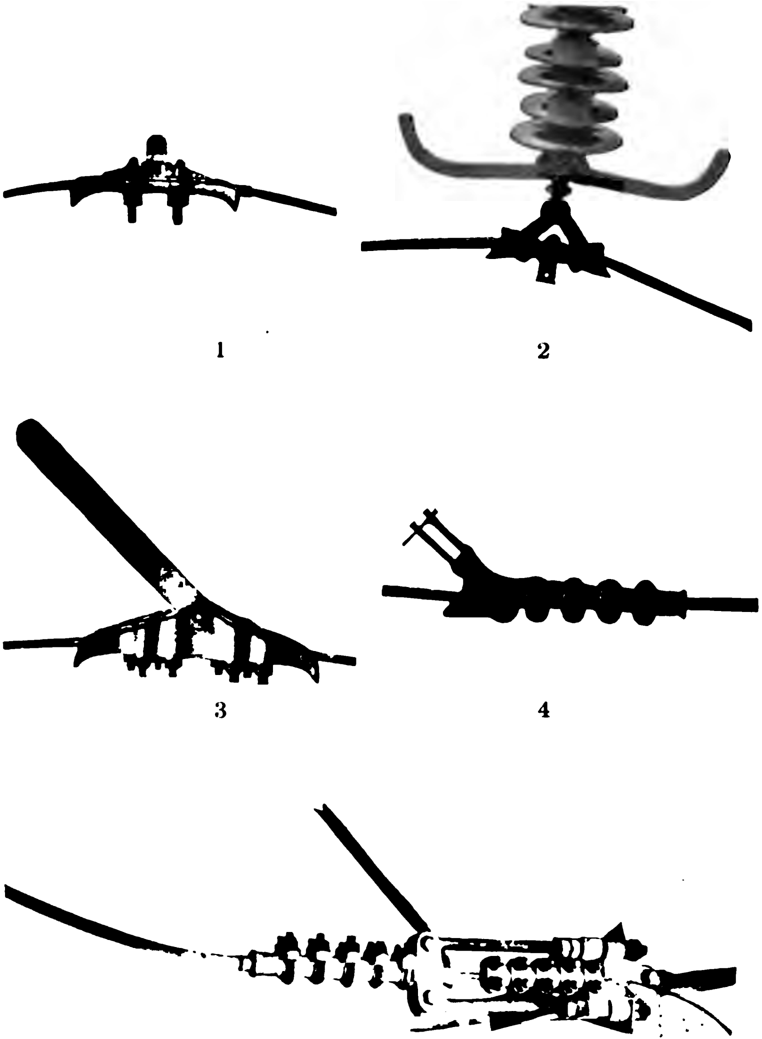
In the case of suspension clamps, attention should be given to obtaining long smooth clamping surfaces, and any corrugations should be of large radius. The clamp should hold the cable with sufficient strength to prevent slipping under normal conditions, and in case of conductor breakage slipping should not occur except at the tower adjacent to the break. With the great length of the suspension insulator string, this requirement should not be difficult to meet.

Semi-tension clamps will have an important field of use, since it is the aim to go to extremes in avoiding points where full dead-end tension will come upon insulators. This type of clamp will accordingly be used at smaller angle and stabilizing points. It should hold the conductor, under all conditions, with no appreciable slipping, a requirement which may involve separate clamping of the steel core. A somewhat similar angle or side tension clamp will be needed for use at points where the line makes a considerable horizontal angle.

For tension clamps, the best present practise is to provide for separate clamping of the core, as illustrated in Fig. 11. For large cables worked to their elastic limits, as required for 220-kv. service, this method of clamping will be essential.

Equalizer yokes and connectors present more of a problem than for present lines, since, as noted earlier, even at suspension points two or three insulator strings in parallel would be used, while if disk insulators were to be used at tension points, not less than six parallel strings would be necessary. Whether the present patterns of cast yokes are satisfactory for such heavy duty service is questionable. Pressed steel or some design built up of structural shapes would appear preferable.

Jumpers at dead-end towers will obviously be long, more than 20 ft., and probably some form of jumper guide or anchor will be necessary. This has been satisfactorily accomplished in existing lines by means of auxiliary weights, auxiliary insulator strings or rigid guides of structural steel attached to the yokes.



1 2 3 4 5

FIG. 11—CONDUCTOR CLAMPS [SILVER]

EXAMPLES OF PREVAILING PRACTISE

Nos. 1, 2 and 3........Suitable Type for Suspension
Nos. 3 and 4..........Suitable Types for Semi-Tension
No. 5................Suitable Type for Tension

Transmission Towers. Before proceeding to discussion of the problems of tower types and designs, it may be noted that for present purposes consideration has been confined to single circuit lines, primarily for the reason that at the start a 220-kv. system would be developed gradually, one or two circuits at a time, probably to a considerable extent over different rights-of-way. When, however, 220-kv. becomes established and conditions call for three or more parallel lines, or when networks have been established such that absolutely continuous service of any one line becomes relatively somewhat less important, then the question of using double-circuit structures will deserve careful investigation. The advantages will consist in a considerable saving in tower cost and some saving in right-of-way, against which would lie the relatively minor chance of failure of a structure from extraordinary causes involving two circuits instead of one.

The design of two circuit 220-kv. structures would open interesting possibilities. The large conductor separation, about 20 ft., employed in the accompanying designs, adopted largely as a result of the long insulator strings used, would severely handicap the conventional type of double circuit tower,—three conductors in a generally vertical plane on each side of the tower,—owing to the great height and great weight of steel required. A shorter insulator would tend to reduce this handicap. There are other feasible types of double-circuit tower for 220-kv. service, in some respects more promising, in particular a tower with three legs transverse to the line carrying all six conductors in a horizontal plane. This type makes the high transverse strength easy to obtain, and suggests the possibility of building two legs to carry one circuit initially and adding the third leg and the second circuit later. The design of a double-circuit tower would, of course, be determined primarily by economy of steel. A discussion of structures for multiple circuit lines is, however, beyond the scope of the present study.

The choice of structure material, whether wood, or steel, is primarily one of total economy, considering first cost, depreciation and maintenance. The long economic life which presumably a 220-kv. line would represent, causes durability of material to assume even greater importance than in present practise, so that for most localities, wood construction would not be economical, even assuming that the requisite strength for the heavy loads could be secured without resorting to unduly short spans.

As to type of structure to be employed, the conventional *rigid* tower has been adopted as the most available for construction in the immediate future. The rigid tower type of line has been thoroughly studied and tested by experience, and when properly designed, its performance record has been satisfactory, at least with the smaller conductors and lighter loadings thus far used. It is recognized, of course, that the very great stresses existing in such a line as is here proposed tends to disturb the balance of considerations which has deter-

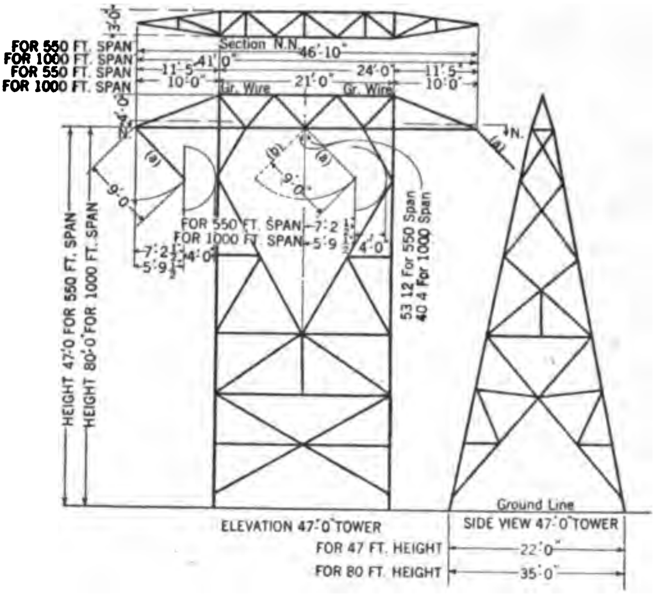


FIG. 12—220-KV. STEEL TRANSMISSION TOWER—CLEARANCE DIAGRAM TYPE A (SUSPENSION)—FOR VERTICAL ANGLE 5 DEG. AND HORIZONTAL ANGLE 0 DEG. TO 2 DEG. (FOR DESIGN LOADING AND STRESSES SEE TABLE III.)

CONDITIONS CAUSING MAXIMUM SWING OF INSULATORS TAKEN FOR DETERMINING TOWER DIMENSIONS

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	550	——> 2 deg.	——> 8 lb.	0 in.	0 deg.
b	550	<—— 2 deg.	<—— 8 lb.	0 in.	0 deg.

TABLE III
DESIGN LOADING AND STRESSES

TYPE A TOWER (SUSPENSION)
VERTICAL ANGLE 5 DEG. AND HORIZONTAL ANGLE 0 DEG. TO 2 DEG.
CONDUCTOR, ALUMINUM 716,000 CIR. MIL.—STEEL 92,900 CIR. MIL.
GROUND WIRE 5/8 IN. STEEL STRAND SUSPENSION INSULATORS
WIND 8 LB. ICE 1½ IN. AT 0 DEG. FAHR.
(FOR TYPE A TOWER CLEARANCE DIAGRAM SEE FIG. 12)

Span (Normal and max. allowable with maximum angles)	550 ft.	700 ft.	800 ft.	1000 ft.
Height of cross arm above ground	47 ft.	57 ft.	63 ft.	80 ft.
Transverse Loading				
Wind on tower	750	850	1,000	1,250
Wind on 3 conds. with 1½ in. ice at 2.69 lb. per ft.	4,400	5,650	6,440	8,050
Wind on 2 gr. wires with 1½ in ice. at 2.42 lb. per ft.	2,650	3,400	3,860	4,850
Pull, 3 conds. due to 2 deg. hor. angle	1,800	1,800	1,800	1,800
Pull, 2 gr. wires due to 2 deg. hor. angle	1,200	1,200	1,200	1,200
Total load	10,800	12,900	14,300	17,150
Test load—125 per cent	13,500	16,000	18,000	21,500

Torsional Loading

It is assumed that one conductor is broken and the pull of the conductor in the adjoining span is decreased due to the tendency of the insulator string to swing in the direction of the pull.

Assumed maximum load	8,000	9,000	10,000	11,000
Test load—125 per cent	10,000	11,300	12,500	13,800
Vertical Loading (At each conductor and ground wire support)				
Weight of conductor at 0.9 lb. per ft.	500	600	750	900
Weight of 1½ in. ice at 4.72 lb. per ft.	2,600	3,300	3,800	4,750
Due to 5 deg. vertical angle	1,500	1,500	1,500	1,500
Insulators and hardware	400	400	400	400
Men	200	200	200	200
Repair tackle	800	800	800	800
Total load at one support	6,000	6,800	7,450	8,550
Test load—125 per cent	7,500	8,500	9,300	10,700

Design Stresses

Above test loads include all factors of safety so that structural steel should be stressed as follows:

	Test load	Working load max. condition
In tension	30,000 lb. per sq. in.	24,000
In compression	$\left(30{,}000 - \dfrac{100\,L}{R}\right)$ lb. per sq. in.	$\left(24{,}000 - \dfrac{80\,L}{R}\right)$
Bolts in shear	25,000 lb. per sq. in.	20,000
Bolts in bearing	50,000 lb. per sq. in.	40,000

Values L/R to be used shall be:
a. For main members not greater than 120
b. For secondary members not greater than 200
c. For redundant members not greater than 250

Minimum thickness of metal ¼ in. All material to be galvanized.

Test loads as specified for transverse, torsional and vertical loading to be applied separately.

mined the present rigid tower type of line design, and further study, in conjunction with actual experience in 220-kv. construction, may later indicate that greater economy is obtainable with a different type of construction. For immediate purposes, however, it has been deemed that a conservative attitude should be adopted toward innovations not specifically called for by the conditions of 220-kv. service.

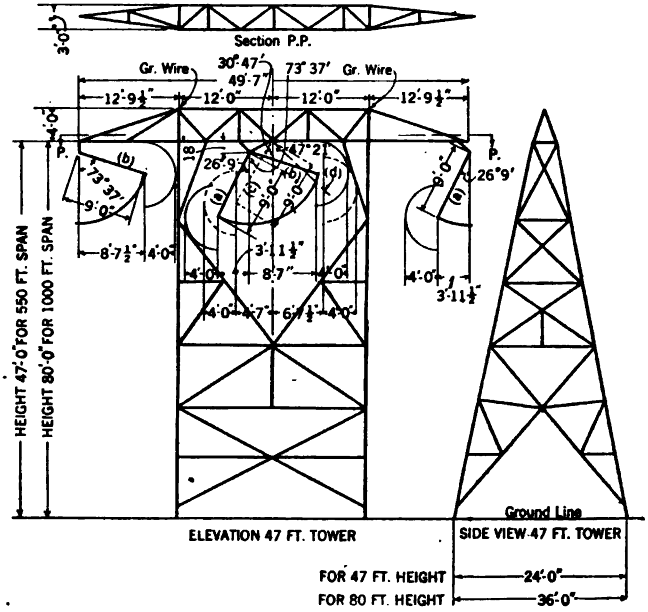


FIG. 13—220-KV. STEEL TRANSMISSION TOWER—CLEARANCE DIAGRAM—TYPE B (SUSPENSION)—FOR VERTICAL ANGLE 10 DEG. AND HORIZONTAL ANGLE 2 DEG. TO 10 DEG. (FOR DESIGN LOADING AND STRESSES SEE TABLE IV)

CONDITIONS CAUSING MAXIMUM SWING OF INSULATORS TAKEN FOR DETERMINING TOWER DIMENSIONS

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	1000	——> 2 deg. plus	<—— 8 lb.	0 in.	120 deg.
b	550	10 deg.	——> 8 lb.	0. in.	0 deg.
c	1000	0 deg.	<—— 8 lb.	0 in.	0 deg. or 120 deg.
d	550	2 deg. minus	——> 8 lb.	0 in.	0 deg.

It is growing practise in heavy line construction to provide a

TABLE IV
DESIGN LOADING AND STRESSES

TYPE B TOWER (SUSPENSION)
VERTICAL ANGLE 10 DEG. AND HORIZONTAL ANGLE 2 DEG. TO 10 DEG.
CONDUCTOR, ALUMINUM 716,000 CIR. MILS. STEEL 92,900 CIR. MILS.
GROUND WIRE $^5/_8$ IN. STEEL STRAND SUSPENSION OR SEMI-TENSION INSULATORS
WIND 8 LB. ICE 1½ IN. AT 0 DEG. FAHR.
(FOR TYPE B TOWER CLEARANCE DIAGRAM SEE FIG. 13)

Span (Normal)	550 ft.	700 ft.	800 ft.	1000 ft.
Span (maximum allowable with maximum angles)	700 ft.	900 ft.	1000 ft.	1200 ft.
Height of cross arm above ground	47 ft.	57 ft.	63 ft.	80 ft.
Transverse Loading				
Wind on tower	850	1,000	1,250	1,500
Wind on 3 conds. with 1½ in. ice at 2.69 lb. per ft.	5,600	7,250	8,050	9,700
Wind on 2 gr. wires with 1½ in. ice at 2.42 lb. per ft.	3,350	4,350	4,850	5,800
Pull 3 conds. due to 10 deg. horizontal angle	9,000	9,000	9,000	9,000
Pull 2 gr. wires due to 10 deg. horizontal angle.	6,000	6,000	6,000	6,000
Total load	24,800	27,600	29,150	32,000
Test load—135 per cent	33,500	37,300	39,400	43,200

Torsional Loading

It is assumed that two conductors are broken and that the pull of the conductors in the adjoining span causes unbalanced loading in the tower.

Load at any one conductor or ground wire support	17,300	17,300	17,300	17,300
Test load—135 per cent	23,400	23,400	23,400	23,400
Vertical Loading (At each conductor and ground wire support)				
Weight of conductor at 0.9 lb. per ft.	600	850	900	1,100
Weight of 1½ in. ice at 4.72 lb. per ft.	3,300	4,250	4,700	5,700
Due to 10 deg. vertical angle	3,000	3,000	3,000	3,000
Insulators and hardware	400	400	400	400
Men	200	200	200	200
Repair tackle	800	800	800	800
Total load at one support	8,300	9,500	10,000	11,200
Test load—135 per cent	11,200	12,800	13,500	15,100

Design Stresses (Same for Towers Types C. D. and E. Tables V, VI and VII)

Above test loads include all factors of safety so that structural steel should be stressed as follows:

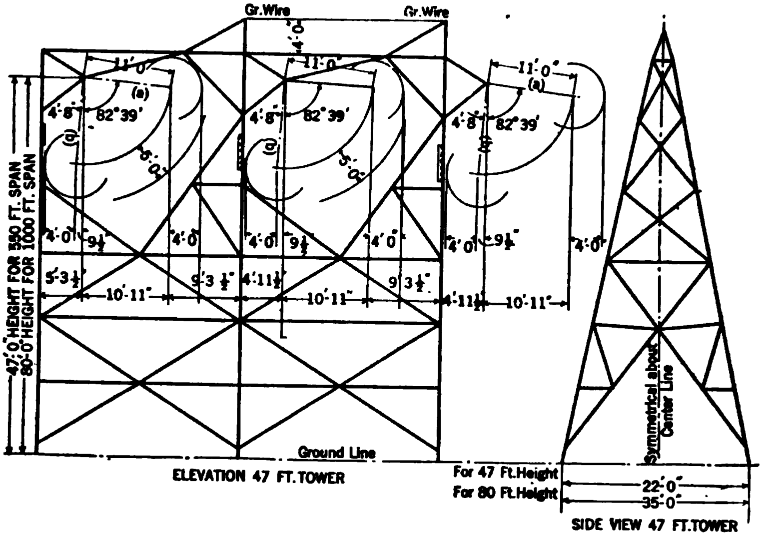
	Test loads	Working load max. condition
In tension	30,000 lbs. per sq. in.	22,000
In compression	$\left(30,000 - \frac{100\,L}{R}\right)$ lbs. per sq. in.	$\left(22,000 - \frac{75\,L}{R}\right)$
Bolts in shear	25,000 lbs. per sq. in.	18,500
Bolts in bearing	50,000	37,000

Values of L/R to be used shall be:
a. For main members not greater than 120
b. For secondary members not greater than 200
c. For redundant members not greater than 250

Minimum thickness of metal $^1/_8$ in. All material to be galvanized.

Test loads as specified for transverse, torsional and vertical loading to be applied separately.

series of standard towers of different strengths, adapted to safe and economical use under varying conditions of span lengths and angles. The extremely heavy loading basis assumed for a 220-kv. line makes it feasible, and desirable from the standpoint of economy of steel, to carry this differentiation to a greater extent than has been customary heretofore. The tower design studies which follow are based upon a series of five



ELEVATION 47 FT. TOWER

SIDE VIEW 47 FT. TOWER

FIG. 14—220-KV. STEEL TRANSMISSION TOWER—CLEARANCE DIAGRAM—TYPE C (SIDE TENSION)—FOR VERTICAL ANGLE 15 DEG. AND HORIZONTAL ANGLE 10 DEG. TO 25 DEG. (FOR DESIGN LOADING AND STRESSES SEE TABLE V)

CONDITIONS CAUSING MAXIMUM SWING OF INSULATORS TAKEN FOR DETERMINING TOWER DIMENSIONS

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	550	——> 25 deg.	——> 8 lb.	0 in.	0 deg.
b	1000	10 deg.	<—— 8 lb.	0 in.	120 deg.

standard types of tower for use under the varying conditions presented by a 220-kv. line over a rolling terrain.

The determination of economic balance between height and weight of towers and normal length of span is an interesting and a fundamentally important problem. Studies of tower designs and of this economic balance for four different normal span lengths are shown in some detail in Figs. 12, 13, 14, 15 and

16, with accompanying data in Tables III, IV, V, VI and VII. The procedure followed in carrying out this study of economic span length will be described briefly. As a basis of study a typical section of profile, as shown in Fig. 17, (a section 23.3 miles long from an actual surveyed route), was selected as a fair example of average 220 kv. line location in rolling country,

TABLE V
DESIGN LOADING AND STRESSES

TYPE C TOWER (SIDE TENSION)
VERTICAL ANGLE 15 DEG. AND HORIZONTAL ANGLE 10 DEG TO 25 DEG.
CONDUCTOR. ALUMINUM 716,000 CIR. MIL.—STEEL 92,900 CIR. MIL.
GROUND WIRE $^5/_8$ in. STEEL STRAND
SUSPENSION OR SEMI-TENSION INSULATORS
WIND 8 LB.—ICE 1½ in. AT 0 DEG. FAHR.
(FOR TYPE C TOWER CLEARANCE DIAGRAM SEE FIG. 14)

Span (Normal)	550 ft.	700 ft.	800 ft.	1000 ft.
Span (max. allowable with maximum angles)	900 ft.	1100 ft.	1200 ft.	1400 ft.
Height of crossarm above ground	47 ft.	57 ft.	63 ft.	80 ft.
Transverse Loading				
Wind on tower	1,200	1,400	1,500	1,750
Wind on 3-conds. with 1½ in. ice at 2.69 lb. per ft.	7,250	8,875	9,700	11,300
Wind on 2-ground wires with 1½ in. ice at 2.42 lb. per ft.	4,350	5,325	5,800	6,750
Pull, 3 conductors due to 25 deg. horizontal angle	22,500	22,500	22,500	22,500
Pull, 2 ground wires due to 25 deg. horizontal angle	14,500	14,500	14,500	14,500
Total load	49,800	52,600	54,000	56,800
Test load, 135 per cent	67,200	71,000	72,900	76,700

Torsional Loading

It is assumed that two conductors and one ground wire are broken and that the pull of the unbroken conductor and ground wire in the adjoining span causes unbalanced loading on the tower.

Load at any one conductor support	17,300	17,300	17,300	17,300
Test load—135 per cent	23,400	23,400	23,400	23,400
Load at any one ground wire support	16,000	16,000	16,000	16,000
Test load—135 per cent	21,600	21,600	21,600	21,600
Vertical Loading (at each Conductor and Ground Wire Support)				
Weight of cond. at 0.9 lb. per ft.	810	990	1,080	1,260
Weight of 1½ in. ice at 4.72 lb. per ft.	4,250	5,200	5,670	6,600
Due to 15 deg. vertical angle	4,500	4,500	4,500	4,500
Insulator and hardware	1,200	1,200	1,200	1,200
Men	400	400	400	400
Repair tackle	1,200	1,200	1,200	1,200
Total load at one support	12,360	13,490	14,050	15,160
Test load—135 per cent	16,700	18,200	18,900	20,500

Design Stresses (Same as for Type B Tower, See Table 4)

and the economic study was based upon this profile. For each of four normal span lengths; *i. e.*, 550 ft., 700 ft., 800 ft. and 1000 ft., four actual approximate line designs, based each upon

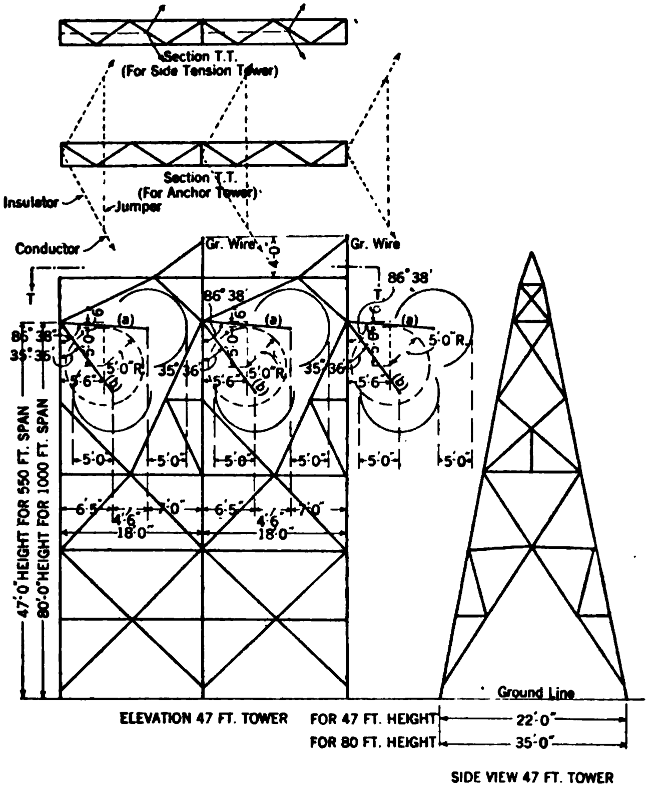


Fig. 15—220-kv. Steel Transmission Tower—Clearance Diagram—Type D (Side Tension and Anchor)—Vertical Angle 25 deg.—Horizontal Angle (Side Tension) 25 deg. to 60 deg. (Anchor) 60 deg. to 90 deg. (For Design Loading and Stresses See Table VI)

Conditions Causing Maximum Swing of Insulators Taken for Determining Tower Dimensions

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	550	——> 60 deg.	——> 8 lb.	0 in.	0 deg.
b	1000	25 deg.	<—— 8 lb.	1½ in.	32 deg.

its own set of specific tower locations, was carried out, and the cost of each line was estimated. These costs, as plotted in Fig. 18, show maximum economy with normal spans of 800 ft. with

TABLE VI

DESIGN LOADING AND STRESSES

TYPE D TOWER (SIDE TENSION AND ANCHOR)
VERTICAL ANGLE 25 DEG.
HORIZONTAL ANGLE (SIDE TENSION) 25 DEG. TO 60DEG., (ANCHOR) 60 DEG. TO 90 DEG.
CONDUCTOR, ALUMINUM 716,000 CIR. MIL.—STEEL 92,900 CIR. MIL.
GROUND WIRE $^5/_8$ in. STEEL STRAND
SUSPENSION, SEMI-TENSION OR TENSION INSULATORS
WIND 8 LB.—ICE 1½ IN. AT 0 DEG. FAHR.
(FOR TYPE D TOWER CLEARANCE DIAGRAM SEE FIG. 15)

Span (Normal)	550 ft.	700 ft.	800 ft.	1000 ft
Span (maximum allowable with maximum angles)	1100 ft.	1300 ft.	1400ft.	1600 ft.
Height of gross arm above ground	47 ft.	57 ft.	63 ft.	80 ft.
Transverse Loading				
Wind on tower	1,200	1,400	1,500	1,750
Wind on 3 conductors with 1½ in. ice at 2.69 lb. per ft.	8,875	10,500	11,300	12,900
Wind on 2 ground wires with 1½in ice at 2.42 lb. per ft.	5,325	6,300	6,800	7,750
Pull 3 conductors due to 90 deg. horizontal angle	73,400	73,400	73,400	73,400
Pull 2 gr. wires due to 90 deg. horizontal angle.	45,300	45,300	45,300	45,300
Total load	134,100	136,900	138,300	141,100
Test load—135 per cent	181,000	184,800	186,700	190,500
Longitudinal Loading (Line terminal tower)				
Pull due to 3 conductors	51,900	51,900	51,900	51,900
Pull due to 2 ground wires	32,000	32,000	32,000	32,000
Total load	83,900	83,900	83,900	83,900
Test load—135 per cent	113,300	113,300	113,300	113,300

Torsional Loading

It is assumed that two conductors and two ground wires are broken and the pull of the conductors and ground wires in the adjoining span causes unbalanced loading in the tower.

Load at anyone conductor support	17,300	17,300	17,300	17,300
Test load—135 per cent of actual	23,400	23,400	23,400	23,400
Load at any one ground wire support	16,000	16,000	16,000	16,000
Test load—135 per cent of actual	21,600	21,600	21,600	21,600
Vertical Loading (At each conductor and ground wire support)				
Weight of conductors at 0.9 lb. per ft.	1,000	1,200	1,300	1,450
Weight of 1½ in. ice at 4.72 lb. per ft.	5,200	6,150	6,600	7,550
Due to 25 deg. vertical angle	7,300	7,300	7,300	7,300
Insulators and hardware	1,200	1,200	1,200	1,200
Men	400	400	400	400
Repair tackle	1,200	1,200	1,200	1,200
Total load at one support	16,300	17,450	18,000	19,100
Test load—135 per cent of actual load	22,000	23,600	24,300	25,800

Design Stresses (Same as for Type B Tower, See Table IV) the rigid type of tower assumed, while longer spans or shorter spans are more expensive. The 800 ft. normal span was selected as a basis for further studies.

The method of carrying out these line designs involved, first, determining the height of tower corresponding to the span length selected. To determine this height, a ground clearance of 25 ft. under design loading was assumed (corresponding with an 800-ft. span to about 35 ft. at 60 deg. fahr) and a sag-clearance template developed for the aluminum-steel conductor used. On level ground, for the 25 ft. clearance, this template gave the standard tower heights for the four normal span lengths under consideration as 47 ft., 57 ft., 63 ft. and 80 ft. For each height of tower, the five standard designs were developed in outline and, with the sag-clearance template, tower locations were spotted on the typical profile selected. With the locations determined, the required type of tower was selected from the

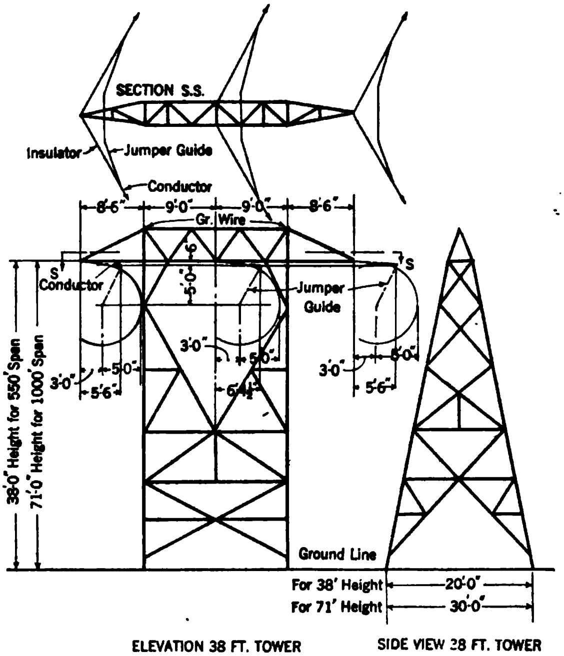


FIG. 16—220-KV. STEEL TRANSMISSION TOWER—CLEARANCE DIAGRAM—TYPE E (ANCHOR)—VERTICAL ANGLE 0 DEG. TO 15 DEG. HORIZONTAL ANGLE (ANCHOR) 0 DEG. TO 60 DEG. (FOR DESIGN LOADING AND STRESSES SEE TABLE VII)

TABLE VII
DESIGN LOADING AND STRESSES

Type E Tower (Anchor)
Vertical Angle 0 Deg. to 15 Deg. and Horizontal Angle 0 Deg. to 60 Deg.
Conductor Aluminum 716,000 Cir. Mil.—Steel 92,900 Cir. Mil.
Ground Wire 5/8 in. Steel Strand
Tension Insulators
Wind 8 Lb.—Ice 1½ in. at 0 Deg. Fahr.
(For Type E Tower Clearance Diagram See Fig. 16)

Span (normal)	550 ft.	700 ft.	800 ft.	1000 ft.
Span (maximum allowable with maximum angles)	1000 ft.	1200 ft.	1300 ft.	1500 ft.
Height of cross arm above ground	38 ft.	48 ft.	54 ft.	71 ft.
Transverse Loading				
Wind on tower	850	1,000	1,250	1,500
Wind on 3 conds. with 1½ in. ice at 2.69 lb. per ft.	8,070	9,700	10,500	12,100
Wind on 2 gr. wires with 1½ in ice at 2.42 lb. per ft.	4,830	5,800	6,300	7,300
Pull—3 conductors due to 60 deg. horizontal angle	51,900	51,900	51,900	51,900
Pull—2 gr. wires due to 60 deg. horizontal angle	32,000	32,000	32,000	32,000
Total load	97,650	100,400	101,950	104,800
Test load—135 per cent	131,800	135,500	137,600	141,500
Longitudinal Loading (Line terminal tower)				
Pull due to 3 conductors	51,900	51,900	51,900	51,900
Pull due to 2 ground wires	32,000	32,600	32,000	32,000
Total load	83,900	83,900	83,900	83,900
Test load—135 per cent	113,300	113,300	113,300	113,000

Torsional Laoding

It is assumed that two conductors and two ground wires are broken and the pull of the conductors and ground wires in the adjoining span causes unbalanced loading in the tower.

Load at any one conductor support	17,300	17,300	17,300	17,300
Test load—135 per cent	23,400	23,400	23,400	23,400
Load at any one ground wire support	16,000	16,000	16,000	16,000
Test load—135 per cent	21,600	21,600	21,600	21,600
Vertical Loading (at each conductor and ground wire support)				
Weight of conductors at 0.9 lb. per ft.	900	1,080	1,170	1,350
Weight of 1½ in. ice, at 4.72 lb. per ft.	4,700	5,670	6,136	7,080
Due to 15 deg.—vertical angle	4,500	4,500	4,500	4,500
Insulators and hardware	1,200	1,200	1,200	1,200
Men	400	400	400	400
Repair tackle	1,200	1,200	1,200	1,200
Total load at one support	12,900	14,050	14,600	15,700
Test load—135 per cent	17,400	19,000	19,700	21,200

Design Stresses (Same as for Type B Tower, See Table IV.)

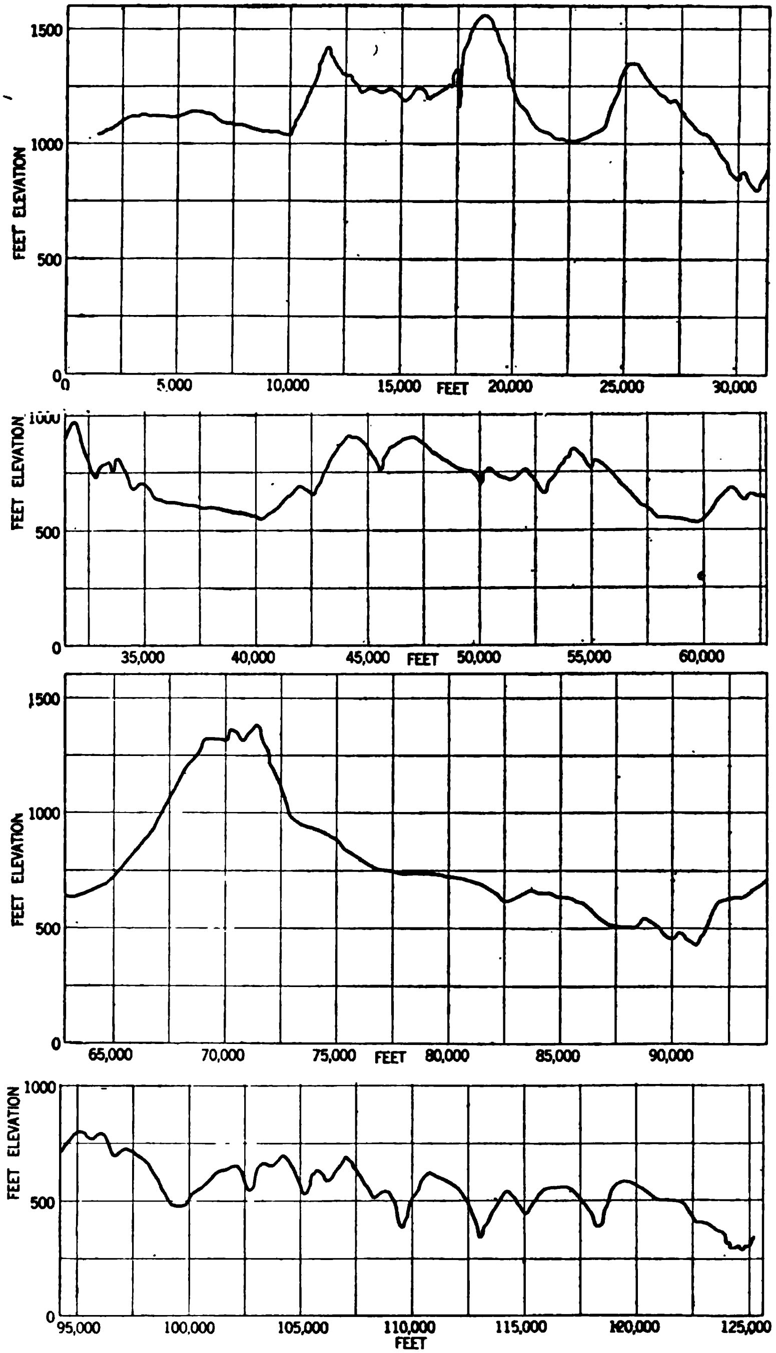


Fig. 17—Section of Profile of Typical Transmission Right of Way Used as Basis for 220-kv. Line Studies to Determine Economic pan

F-SUSTAINING, HORIZO

Quan.	Unit Used
1	mi. 1
6.810	mi.
5.350	each
1.160	"
0.172	"
0.086	"
0.042	"
712	cwt.
35.6	ton
35.6	"
....	mi.
5.35	each
1.16	"
0.172	"
0.086	"
0.042	"
....	mi.
6.60	twr.
0.210	"
....	each
....	"
0.085	"
0.085	"
0.042	"
....	mi.
6.81	twr.
6.81	"
6.81	each
1	mi. 1
1,152	each
18	"
2	"
2	"
45	"
42	"
45	"
202	"
51	"
140	cwt.
7.0	ton
,152	disc
,152	"
1	mi. 1
1	" "
1	mi. 1
1	" "
1	mi. 1
6.08	twr.
1	mi. 1

Article	ot span	
Type of tower	C	
Avg. span on typical profile (a)	43	
Towers typical profile (b)	4	
Towers per mile	0.172	
Height of tower	63	
Weight of tower	50	36,
Cost of steel per lb	12	
" " tower	28	4,
Max. allow. hor. angle	25	
Max. allow. vert. angle	15	
Weight of towers per mile	20	3,
" " " " " total	1,180	

Article	
Type of tower	
Towers per mile	0
No. insul. strings	
Total number	
	Mile
Insulators (c)	302
Susp. clamps	1.7
Semi-ten. clamps	1.7
Tension clamps	....
Equalizer yokes	34
Ex. H. equal. yoke	7
Links	34
Clevis Eyes	54
Arcing horns	14

Article	
Type of tower	
Towers per mile	0
No. insul. strings	
Total number	
	Mile
Insulators (c)	292
Susp. Clamps	1.6
Semi-ten. clamps	1.6
Tension clamps	....
Equalizer yokes	32.4
Ex. H. equal. yoke	6.5
Links	32.4
Clevis eyes	51.9
Arcing horns	1.3

NOTES:—(*) Part affected by span only. (a) The kind of tow
an actual profile of a line 23.3 miles in le

		1000-foot span				
D	E	A	B	C	D	E
743	743	1,015	1,060	657	657	657
2	1	88	25	4	2	1
0.086	0.042	3.800	1.080	0.173	0.085	0.042
63	54	80	80	80	80	71
700	23,500	12,810	20,250	29,810	49,780	34,120
12	12	12	12	12	12	12
405	2,821	1,536	2,435	3,572	5,980	4,095
90	60	2	10	25	90	60
25	15	5	10	15	25	15
160	990	48,690	21,880	5,160	4,236	1,434
....	....	....	....	81,300	....	....

700-foot span

C 0.216 4 per		D 0.086 2 × 6 per		E 0.086 2 × 6 per		Total
Tower	Mile	Tower	Mile	Tower	Mile	
204	44	612	55	612	53	1,040
....	....	....	....	....	....	22
....	....	....	....	....	....	2
e 3	0.7	6	0.5	6	0.5	2
12	2.6	60	5.1	60	5.2	86
6	1.3	12	1.1	12	1.1	11
12	2.6	60	5.1	60	5.2	47
30	6.5	96	8.2	96	8.3	194
8	1.7	24	2.0	24	2.1	59

1000-foot span

C 0.173 4 per		D 0.085 2 × 6 per		E 0.042 2 × 6 per		Total
Tower	Mile	Tower	Mile	Tower	Mile	
204	36	612	53	612	26	920
....	....	....	....	....	....	13
....	....	....	....	....	....	2
e 3	0.5	6	0.5	6	0.3	2
12	2.1	60	5.1	60	2.5	42
6	1.1	12	1.0	12	0.5	32
12	2.1	60	5.1	60	2.5	42
30	5.2	96	8.2	96	4.0	161
8	1.4	24	2.0	24	1.0	29

ers, shown in this horizontal row, a template for given normal span was applied to

[SILVER]

known angles and span lengths involved at each tower site. The results of this study are given in Table VIII, showing for each normal span length the numbers required of each type of tower and a comparison of line costs, exclusive of conductors and ground wires for which the costs would be constant.

It is believed that this method of analysis, *i. e.*, using an actual or typical profile, affords a more accurate and reliable basis of determining economic normal span lengths and tower heights than the simpler method of considering only straight lines on a level profile. Further refinements in the way of studying the effect of using a different height or more than one height for the heavier towers might be attempted, but it is questionable whether they would be warranted, in view of the many uncertainties involved, particularly in field work. It will be noted that this study of economic balance is carried out on the basis of the high prices obtained in early 1918. A lower price level would have some effect upon the results, as would a different size or type of conductor and a different type of insulator, particularly if it were stronger and shorter.

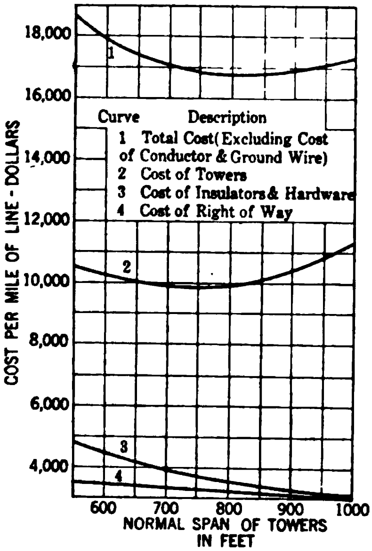


Fig. 18—220 kv. Transmission Line—Determination of Economic Span—Relation of Span Length to First Cost

In the tower designs study has been made of the general form, of the dimensions of the main members and of approximate weights, but the designs have not been carried into the details. No designs have been prepared of tower footings, side hill extensions, etc., which are distinctly contingent upon local conditions and which have no direct or material effect upon the general line designs. Figs. 12 to 16 show the general types of towers proposed, with details as to dimensions, and as to working and test loads. Estimated weights are shown in Fig. 19. The working loads were computed from the design loadings under the most extreme conditions of angles and span lengths for which the tower is to be used. In addition to the main margin of safety provided in the design loading, a further margin is introduced in the difference between working and test loads, *i. e.*, 25 per cent for suspension towers and 35 per cent for angle and dead-end towers. It may be noted that a still further margin of safety will result when a tower is employed in a location imposing less than its maximum designed working load, a condition which will obtain for the major percentage of the line.

After the study of tower locations was carried out, it became evident that there was economic justifications for a sixth type

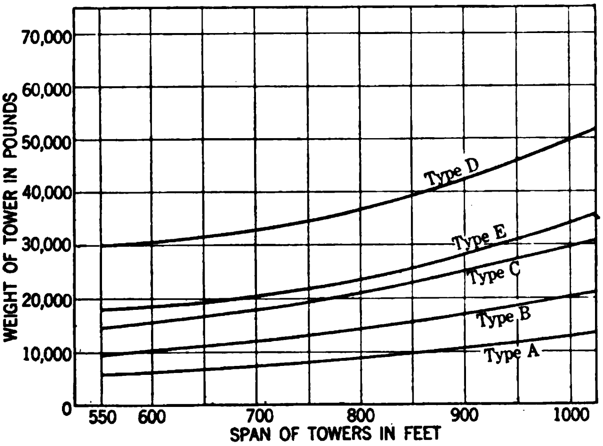


FIG. 19—220-KV. SINGLE-CIRCUIT STEEL TRANSMISSION TOWERS— ESTIMATED WEIGHTS

Type A—See Fig. 12; Type B—See Fig. 13; Type C—See Fig. 14; D—See Fig. 15; Type E—See Fig. 16

of tower, a special light weight type to be used primarily on short spans in tangents. This tower is illustrated in Fig. 20. An economic improvement in the design shown for this light weight type might be effected by shortening the tower to correspond to the smaller sags obtaining in short spans.

It is believed desirable that sample towers of each design should be subjected to thorough experimental tests before quantity production is started. Tower design involves approximations, and the heavy loads and consequent great weights of 220-kv. towers justify more than usual effort to obtain an economically consistent design. In the method of making such tests, more elaborate and refined methods than hitherto employed might be developed to advantage. It is desirable to ascertain not only the point and manner of failure under test, but also the simultaneous stress conditions existing in all important tower members.

Tower foundations will be largely a special problem for each locality. The conditions governing present practise will apply, the only new considerations being the unusually heavy loads involved and the strong emphasis which must be placed upon dependability. In general, for most soils, earth foundations would probably be adequate for straight suspension towers, while concrete foundations would be necessary for full tension and heavy angle towers and probably for semi-tension and

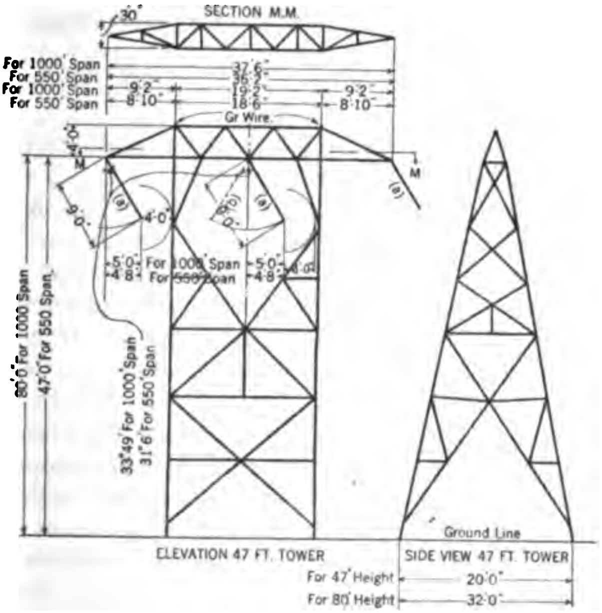


FIG. 20—220 KV. STEEL TRANSMISSION TOWERS—CLEARANCE DIAGRAM—TYPE AA (SUSPENSION)—FOR VERTICAL ANGLE 5 DEG. AND HORIZONTAL ANGLE 0 DEG.

CONDITIONS CAUSING MAXIMUM SWING OF INSULATORS TAKEN FOR DETERMINING TOWER DIMENSIONS

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	1000	0 deg.	8 lb.	0 in.	0 or 120 deg.
b	1000	0 deg.	8 lb.	0 in.	0 or 120 deg.

light angle towers. Where concrete foundations are used, the tower should be carefully grounded.

Protection of towers from deterioration is of more than ordinary importance owing to the great investment represented by a 220-kv. line and the long economic life which it presumably will possess. Galvanizing would seem essential and it should be heavy and carefully applied. Protection of joints and of steel at the ground line will call for every feasible precaution.

DESIGN FEATURES OF 220-KV. STATION AND SUBSTATION EQUIPMENT

While, in general, the character and arrangement of equipment at generating stations and substations is not a part of the primary scope of this paper, there are certain features in connection with this equipment which directly concern the feasibility and the economic and general advantages of transmitting power at this high voltage. Brief consideration of these features is essential to a study of 220-kv. transmission.

The problems of interest, new or assuming particular importance in connection with equipment for a 220-kv. system, are of two types, those arising directly from the high voltage *per se* and those resulting from the relatively enormous capacities and amounts of power involved in a system large enough to call for the use of such a voltage. The handling of electrical potentials of 220,000 volts does not appear to involve any disturbing complications or uncertainties. In fact, the manufacturers do not recognize that any serious problem exists. Current design principles and materials now in ordinary use will be employed, the principal difference from present high voltage equipment being the greater amounts of insulation and the larger clearances required. The step to 220 kv. is relatively no greater than that previously taken from 66 kv. to 110 kv., or from 110 kv. to 154 kv. Certain of the manufacturers have already developed designs, and assert readiness to undertake commercial production of 220-kv. equipment on short notice.

The problems attendant upon the huge capacities which use of 220-kv. transmission makes possible and feasible,—generating stations of several hundred thousand kilowatts, interconnected systems of a half million or a million kilowatts,—are distinctly of a major order. From the purely physical standpoint, the principal problems are those of switching and protection from short-circuit stresses. Both of these, it is believed, can be handled satisfactorily. The more general questions of simplicity and reliability of operation and of efficiency and general economy attain a new order of importance and will probably lead to some significant departures from present practise in smaller scale systems.

Design for such large capacities and high voltage centers around one cardinal principle, *simplicity and intrinsic strength of equipment rather than flexibility and external protective measures.* The principal equipment units represent such high values, both in service and in investment, that the maximum of continuous service should be obtained, and more careful attention and large expense are fully justified in assuring virtual elimination of failures in service. The accompanying arrangement diagram of an assumed typical system, Fig. 21. indicates the extent to which the aim of simplicity is advocated in reduction of bus arrangements to a simple and even rudimentary form and in elimination of superfluous oil switches. In particular, attention may be called to the fact that no spare units, either generators or transformers, are provided, and none are contemplated. The investment in the large units will be so large that outage should be represented only by the irreducible minimum of apparatus troubles and maintenance, not by

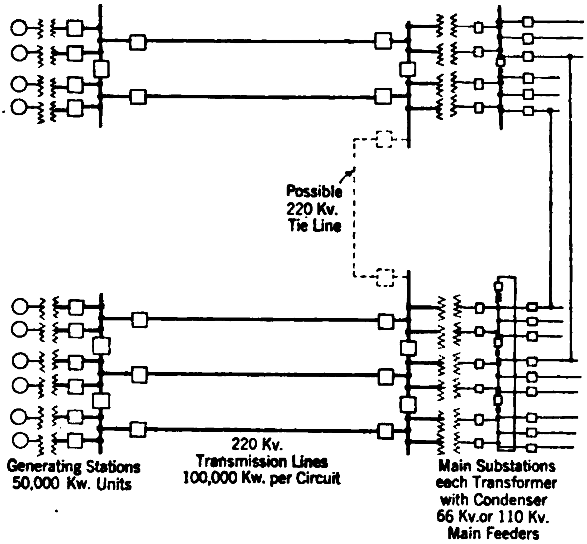


FIG. 21—TYPICAL ARRANGEMENT DIAGRAM—220-KV. SYSTEM

the nearly continuous idleness of spare equipment. This complete omission of reserve units of major apparatus is of course predicated upon adequate reserve capacity being available in local generating stations near the load centers, as has been discussed previously.

The obtaining of strength and high reliability of equipment is primarily a problem of construction economics rather than of system design, although the latter is an important element. Certain features of the problem are high reactance of generators and transformers, ample insulation of transformers and an efficient oil circulation system, high mechanical strength of oil switches, avoidance of low-voltage buses and low-voltage paralleling, and an effective and dependable system of relay sectionalizing.

Study of the design and operation of 220-kv. equipment has brought out certain general principles or features which may be noted before taking up consideration of particular pieces of apparatus:

1. *High-Voltage Switching.* All line switching, automatic or manual, should be done on the *high-voltage* side of transformers. This statement may come as a jolt to certain established ideas, but the further the switching problem has been studied, the more clearly does it appear that high-voltage switching is not only more simple but is more safe. The amounts of power involved, particularly under abnormal conditions, are so tremendous that the current values obtaining at lower voltages impose switch duties and heavy stresses generally which could be handled only with great difficulty and at an expense materially higher than would be required at 220-kv., where the currents involved are relatively small.

From the standpoint of automatic sectionalizing and continuity of service, high-tension switching is obviously desirable, since defective circuits in tripping out will leave the transformers in service to continue supplying load from parallel circuits or alternative routes.

2. *Transformer Connections.* All 220-kv. transformers, on the high-voltage side, should be connected in Y with the neutral grounded, as shown graphically in Fig. 22. The high potentials involved and the great lengths of line to be interconnected constitute conditions which might easily result in uncontrolable surges if the high-voltage circuits were isolated from ground. While grounding at one end of a line would materially alleviate

these conditions, more adequate stabilization of voltage, with consequent greater economy in equipment and security in operation, can be obtained by grounding at all points where transformers are located. It may be of interest to note that the voltage stresses on a 220-kv. grounded system (normal voltage to ground 127 kv.), will be less than for some certain existing isolated delta systems. There obtains also the general advantage of grounded Y operation that automatic sectionalizing is made more positive and reliable, a vital consideration on so large and important a transmission system.

The neutral should be grounded without resistance. The purpose of a resistance in the neutral connection is to limit current flow into a ground on a line or in apparatus. For lower voltages where excessive magnetic stresses would result, the expedient is frequently desirable, but for 220 kv., where the short-circuit current values are relatively small and provision for handling the resultant stresses relatively simple, any benefit which might be gained would not be justified by the expense and complication which the use of resistance would involve, and it would in any case be more than offset by the impairment of the function of the ground in stabilizing potentials.

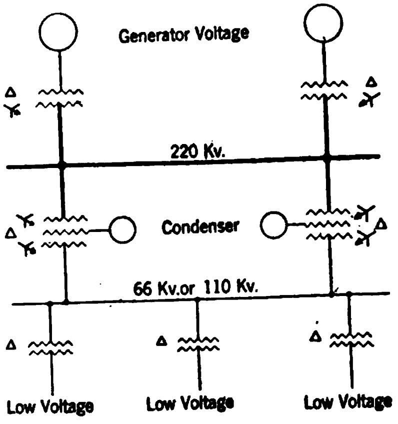


FIG. 22—DIAGRAM OF TRANSFORMER CONNECTIONS—220-KV. SYSTEM

3. *Protective Equipment.* No equipment of any character is contemplated for protection against over-voltages. As a fundamental principle it is believed that investment ca

applied more effectively and with greater ultimate economy in providing greater margins of safety in the apparatus against the stresses which are likely to be encountered, than in attempting to shield weaker apparatus by methods now known. For such a voltage as 220 kv., protective equipment would be proportionately more costly than in case of lower voltages and its efficacy more questionable.

High-voltage protective devices are of two general types, those intended to protect against abnormal potentials at normal or low frequencies, and those intended to protect against high-frequency surges or steep wave fronts which might give rise to localized high-potential stresses in the windings of transformers and in other apparatus. For protection against high potentials, electrolytic arresters and other so-called lightning arresters have been used extensively in systems of lower voltage. On a 220-kv. system with dead-grounded neutrals, it is not believed that any abnormal potentials are likely to be encountered which apparatus insulation cannot and should not be able to withstand. For protection against high-frequency effects, series reactances have been used and elaborate arrangements of reactances, condensers and resistances have been proposed. While probably high-frequency constitutes more of a real danger, it is considered that it can be met most effectively and economically by providing adequate insulation strength.

220-*kv. Transformers.* The questions which naturally will be the first to arise in consideration of transformers for a 220-kv. system are those of size of units and three-phase vs. single-phase units, the two questions being to some extent interdependent.

Considerations of simplicity in station arrangement and of economy in operation favor, in general, transformers of the largest size permitted by the conditions of each installation or by limitation of manufacture. Theoretically there seems no limit to the capacity of a transformer unit, but beyond a certain capacity the provision of strength to withstand the mechanical stresses of short-circuit current becomes a matter of such complication and expense that an economical limit will come into effect. At the present stage of the art, the manufacturers advocate that transformers be not attempted beyond 50,000 to 60,000 kv-a. for three-phase units, or 35,000 to 40,000 kv-a. for single-phase units. Transformer cores and windings for these capacities can be shipped assembled from the factories. Advance quotations and estimates from manufacturers show progressively lower unit costs up to these sizes. The economic advantage of large units depends not so much, however, upon lower unit costs as upon considerations of space, bus construction, number of switches, etc., all of which involve relatively less expense for larger units. The adverse difference in cost of handling facilities is not great.

As to choice between three-phase and single-phase units, from the standpoint of simplicity and cheapness of installation, three-phase units would be preferable. As to factory cost and efficiency, there appears on the basis of information at present available to be little difference, three-phase transformers being somewhat cheaper for same capacity of bank and somewhat more expensive for same capacity of unit. The great weight which should be given the principle of providing high intrinsic reliability of main units, rather than providing spare units, materially reduces one of the principal advantages claimed for single-phase transformers, that of the less capacity required in reserve units. For large installations, three-phase units offer advantages. For relatively small substations, it is believed that equally satisfactory service at less cost can be rendered by a bank of two three-phase units as compared with one bank of single-phase units with a spare. Certainly a bank of three three-phase units would be superior, and the cost should be no greater.

At generating stations, operating and economic considerations strongly favor the generator and its transformer equipment as a unit. If the generator capacity should exceed the practicable limit of three-phase transformer capacity, there apparently would be a close margin of choice between using a single-phase bank or two three-phase transformers operating as a bank and controlled by one circuit breaker.

In past high-voltage practise, probably one of the most troublesome features in operation has been the bushings. Experience indicates, however, that satisfactory bushing designs have now been developed and can readily be applied to 220-kv. service. Bushings will of course be identical and interchangeable for transformers and oil switches. Transformers will have but one high-voltage bushing per phase, the neutral ends of the winding not being insulated from the tank.

The saving in equipment costs which has been noted as one of the reasons for selecting grounded Y rather than delta for the 220-kv. transformer connections, is very marked in the case of transformers. Insulation is less throughout the whole transformer and extra insulation is required at only one end of the windings rather than at both, cores and tanks are correspondingly smaller, efficiency is higher, amount of oil required is less and bushings are lighter and shorter.

The large sizes of installations involved in a 220-kv. system, together with the necessity for careful attention to maintaining insulation strength and transformer reliability generally, will not improbably result in a radical change in the present method of handling and cooling transformer oil, *i. e.*, the abandonment of cooling of oil in each transformer and gravity circulation of oil through the windings in favor of forced oil circulation and external cooling. The idea of external cooling is by no means new, but whereas for the relatively small installations of present practise it appears to involve unjustified complication and expense, for installations of the size here contemplated this condition will probably be reversed. The apparent advantages, briefly summarized, are as follows:

a. Tanks smaller and cheaper due to elimination of cooling coils and smaller volume of oil in transformer.

b. Positive circulation of oil and possibility of more accurately directing it in its flow through the windings to the points needed and in amounts needed.

c. More effective cooling due to low temperature which may be obtained in incoming oil and more rapid circulation.

d. Oil kept absolutely dry and always at maximum insulation strength due to possibility of sealing oil system against the atmosphere, a most important advantage for extra high-voltage operation.

e. Oil may readily be filtered continuously to any extent desired.

f. System as a whole more reliable and probably cheaper.

Such an external transformer oil system might be developed to handle all oil in a common system, or to keep the oil for each transformer bank separate. The common system would be somewhat cheaper and simpler, and hence preferable if adequate precautionary measures could be worked out to prevent contamination of the entire oil system as a result of a failure in one transformer. The system in either case would include cooling devices located wherever desired, probably in a pond or stream, oil pumps, filters, a storage tank at a suitable elevation to provide circulating head, and at some point an expansion device which would maintain a reasonable pressure regulation under conditions of varying volume of the enclosed oil due to temperature changes. The entire oil system would be effectively sealed against the atmosphere. Vital parts of the oil system would presumably be installed in duplicate, the expense being small in proportion to total cost of the station.

A system of external cooling of oil may, at generating stations, open the way for a significant increase in overall efficiency. Transformer losses will amount to about 1.25 per cent of station output. Possibly half of this heat may be recovered from the oil by passing it, on its way to the final cooling coils, through some device installed in the steam system between the condensers and the economizers. While no studies have been made of the economic feasibility of such a scheme, the physical possibility opens an interesting problem.

220-*kv. Oil Circuit Breakers.* A system such as here contemplated, with huge generating stations, numbers of them interconnected, and large synchronous condenser capacity at substations, will necessarily involve unprecedented concentrations of energy at switching points, particularly in case of short circuits. The high duty required of circuit breakers to meet this condition is a vitally important problem in the design of such a system. It is, as has been noted, early evident in a study of the switching problem, that switching should be done wherever possible on the high-voltage side of transformers, since the high voltage can more easily be handled than the high currents at lower voltage. Two hundred and twenty-kv. short-circuit current values are not extreme, and in a 220-kv. circuit breaker the large clearances and switch openings necessarily involved by the high voltage contribute directly to giving the rupturing capacity required by the current to be handled.

Magnitudes of 220-kv. circuit-breaker duty will, of course, depend upon the size and arrangement of the 220-kv. system. With a system of the capacity and arrangement shown in Fig. 21, the duty of a circuit breaker at the bus of either the larger generating station or the larger substation would, according to approximate calculations, be from 1,000,000 kv-a. to 1,500,000 kv-a., depending upon the length of the 220-kv. lines. At the smaller generating station or substation the duty would be somewhat less. No difficulty should be involved in obtaining circuit breakers for this duty at 220-kv. With the gradual expansion which might be expected in such a power supply system, with other generating stations and substations added, extensive interconnections made in secondary networks and probably a considerable 220-kv. network established, there might be expected circuit breaker duties of 3,000,000 to 4,000,000 kv-a. Manufacturers foresee no insurmountable difficulty in designing 220-kv. circuit breakers for duties of this magnitude. The duty on such an extensive system might be reduced by adoption of some degree of sectionalized operation of the 220-kv. lines, but it is hoped that the need for such an expedient can be avoided.

Two types of circuit breakers have been offered by the manufacturers for 220-kv. high duty service. One type consists of a massive circuit breaker, each phase in one tank of heavy boiler plate, with two breaks in series for moderate duty and four breaks in series for heavy duty. The tank is built according to the principles of boiler design to withstand the internal pressure generated by the opening under oil of the rated kv-a. loads, as predetermined by test and calculation. The tank is not designed to withstand explosions in the space above the oil, this hazard being eliminated by provision for thorough ventilation or other expedients to prevent formation of explosive mixtures in this space.

Another type of 220-kv. circuit breaker consists of two breaks in series per tank, one such tank being used for conditions of moderate duty and two tanks in series for heavy duty. The four series breaks in the double-unit circuit breaker are operated simultaneously. These tanks likewise are designed to withstand the stresses generated under the oil under conditions of maximum duty, but in this case the full stresses are not permitted to come upon the tank. Each break is located in a carefully designed explosion chamber, of relatively small diameter and of any required strength, which confines the initial force of the explosion and serves both to reduce the pressures which can be set up in the main tank, and to utilize this force as an aid in extinguishing the arc. With this type of circuit breaker, one tank unit would probably be installed initially when the 220-kv. system was small, and a second unit added later when the growth of the system brought about power concentrations in excess of the rupturing capacity of the ingle unit.

It is believed that either of these types of circuit breaker will give satisfactory service on a 220-kv. system.

220-kv. Air-Break Disconnectors. Air-break disconnectors for 220-kv. service present a problem which, while distinctly of a minor character in relation to the general problems of 220-kv. construction, is troublesome, and for which no wholly satisfactory designs have thus far been offered. The difficulties attending development of such disconnectors do not concern feasibility, since a disconnector which will operate acceptably can be built after existing designs, but concern rather avoidance of unnecessary expense, space requirements, and general complication. The problem is one of making a firm high-capacity contact, without incurring hazard or undue inconvenience to operators or to service, under any sort of weather conditions, between two elements supported by long, heavy, cumbersome and somewhat fragile insulators.

These 220-kv. disconnectors would be used to disconnect an oil switch from or connect it to a live line or bus. It is not contemplated that such a switch need be able to break the charging current of a 220-kv. transformer, although a switch capable of this duty would be valuable if it could be developed. It might be used to take the place of a 220-kv. circuit breaker in some cases or to give added flexibility in cases where transformer circuit breakers were not provided.

In situations where disconnectors need not be operated with either pole alive, as for instance between transformers and transformer circuit breakers at the generating station in Fig. 21, the cumbersome 220-kv. disconnector would not be needed, and some form of simple readily removable link would be used.

Generating Station Arrangement. Simplicity should be the dominating principle in the electrical layout of the generating station. The generator and its transformer should be a unit and there should be no low-voltage bus and no low-voltage paralleling. For stations of the size contemplated, a bus at generator voltage would involve such tremendous duty on oil switches, such elaborate sectionalizing reactances and, in general, such expense and hazard to reliability of operation, all without any material benefit, that it unquestionably should be omitted.

Preferably, although not necessarily, a specific bank of generators should be assigned to each 220-kv. circuit, thus enabling segregation of circuits at the generating stations, if desired. The economic capacity of a 220-kv. circuit is roughly 100,000 kw. to 150,000 kw., so that usually this would mean two, or possibly three, generators per circuit. In case conditions governing generating station practise should favor larger generators, say of the order of 100,000-kw., and if satisfactory units of this capacity should be developed, simple station layout would be possible by using one generator to each 220-kv. circuit.

In case it were necessary for a 220-kv. generating station to deliver power also at a lower voltage, for instance 66 kv., it would still be desirable to avoid a bus at generator voltage. The preferable arrangement for such a station would be a bus at each line voltage, each fed by its own generators. These two busses might or might not, depending upon the particular conditions obtaining, be interconnected by transformers.

The omission of the low-voltage bus would not complicate the problem of auxiliary station service, since good practise already provides that such service shall be supplied from a special generator rather than from the station bus.

Synchronizing at 220-kv. can be effectively accomplished, it is believed, by an adaptation of the static synchroscope.

Main Substation Arrangement. Two hundred and twenty-kv. substation layout will be influenced largely by the particular local conditions of each installation. The usual type will probably serve to step down from 220 kv. to a secondary transmission voltage, such as 66 kv. or 110 kv. In such a case it will, in general, be necessary to have both a primary and a secondary bus system,—at 220 kv. a simple bus with sectionalizing circuit breakers, at secondary voltage, where greater flexibility would seem desirable, probably a ring bus. Owing to the high current values which would obtain at the secondary voltage, (it should be kept in mind that the large condenser capacity will aggravate short-circuit conditions), it appears advisable to provide sectionalizing reactances in this bus.

Synchronous condensers, of such capacity as may be demanded by length of 220-kv. line, amount, load factor and power factor of load, will be provided for each transformer bank. In the usual case, these condensers will be connected to a third winding in the main transformers. This method of connecting the condensers appears to be simpler, cheaper, and equally reliable as compared with connecting the condensers to separate transformers on the low-voltage bus, although the corrective effect would not apply to the secondary windings. Three-winding 220-kv. transformers, with the tertiary winding of 50 per cent of the capacity of the 220-kv. winding, would cost about 15 per cent more than two-winding transformers. The voltage of this tertiary winding and of the condenser would be as high as may be found practicable for satisfactory operation of this type of apparatus.

In cases where the secondary substation voltage were within range of generator operation, 11 kv., 13 kv. or 22 kv., the condensers might be connected directly to the secondary bus.

Relay System. The station and substation arrangements which have been proposed are predicated upon a relay system which may be depended upon for effective and consistent automatic operation, *i. e.*, for insuring that a faulty piece of apparatus, transmission line, or low-voltage feeder will be cut out correctly, promptly and in such a manner as to avoid interruption to the other elements of the system. Complete multiple operation of all 220-kv. apparatus, stations and lines is feasible only with a thoroughly dependable relay system. In view of the encouraging developments in the relay field during recent years, and of the fact that on such an important system much effort can be concentrated in working out a solution, it seems reasonable to assume that an adequate relay system will be available.

Complete 220-kv. multiple operation is essential to obtaining full economy and service reliability from the system, and any resort to sectionalized operation, due to lack of adequate relays, unreliability of circuit breakers or any other cause, should be looked upon as a distinct failure to develop the full possibilities of the system and as a temporary expedient to be dispensed with as soon as possible. With complete multiple operation, transmission lines are used to maximum efficiency, there is full flexibility in shifting of load between generators or stations, and, with dependable automatic disconnecting, trouble in any part of the system may be isolated with a minimum of disturbance to load.

CONSTRUCTION COST

The following estimates are intended to give an indication of the installation cost of construction carried out along the lines of the assumptions and recommendations in this paper. Local conditions, of course, may require a considerable variation in arrangement and cost, particularly of substation apparatus. These estimates are based on early 1919 prices, and apply to station capacities of 200,000 kw. and larger at 220 kv.

Step-Up Substation. Outdoor transforming and switching structures and equipment (220-kv. apparatus and connections only), installation and indirect expense.		$8 to $9 per kw.
Step-Down Substation. Outdoor transforming and switching structures and equipment (does not include low-tension feeder bus or feeder switching equipment) synchronous condensers of capacity for length of connected lines, building, control equipment, installation and indirect expense		$15 to $20 per kw.
Transmission Lines. Single circuit steel towers suspension insulators.		
Towers	$8000 per mi.	
Insulators and Hardware	2800 per mi.	
Conductors and Ground Wire	5000 per mi.	
Special Structures	1000 per mi.	
Right-of-way	3000 per mi.	
Indirect expense	3700 per mi.	
Total		$23,500 per mi.
Total Cost 220-kv. Transmission.		
100 mi		$40 to $45 per kw.
200 mi		60 to 65 per kw.
300 mi		80 to 85 per kw.

CONCLUSIONS

1. It is generally recognized that the country's industrial advancement will require increasingly great amounts of electric power, particularly in view of the relation between use of power and unit productiveness of labor, and that this power must be supplied in accordance with a rational policy of conservation of our greatest national asset, our fuel resources. This points to the necessity of large scale transmission of power from distant points of generation at the energy sources, coal fields and water powers.

2. Two hundred and twenty kv. appears a logical choice for such large-capacity, long-distance transmission, which is clearly beyond the economic range of present transmission voltages. Two hundred and twenty kv. is high enough to meet pending

requirements in power supply evolution but not so far beyond existing practise as to involve question of its immediate feasibility.

3. Design of a 220-kv. system brings up important problems, new in character or significance. These problems are due rather more to the large amounts of power involved than to the high voltage.

4. Two hundred and twenty-kv. power supply will require unprecedentedly high service standards, load factors will be high, particularly at the start, and the frequency should be 60 cycles. For full economy the load for each 220-kv. circuit should be 100,000 kw. or higher.

5. The selection of type and size of conductor is an important economic problem involving a balance of many factors. Corona does not enter as an especially serious limitation owing to the large size of conductors otherwise required. Research is needed as to best method of stringing conductors.

6. Insulators of present commercial disk types can be made to give acceptable service, but there is need of an insulator more efficient electrically, stronger mechanically and of unquestioned durability. There is opportunity for valuable research and development work in this field.

7. The familiar single-circuit rigid type of steel tower appears to be most conservative for immediate use, owing to its reliability having been demonstrated by experience. The field offers promising possibility for development of other types of single-circuit and multiple-circuit towers. The economical span length (here taken as 800 ft. normal), may vary with the factors of each situation. A series of standard towers of graduated weights will be advisable.

8. The basic idea of 220-kv. station and substation design is simplicity and intrinsic strength of equipment, rather than flexibility and external protective measures.

9. Transformer connections should be Y on 220-kv. side with neutral dead grounded. Development of 220-kv., large capacity transformers presents no undue difficulties. Frequently the design might advantageously employ forced oil circulation with external cooling. Present types of bushings, suitably adapted, seem satisfactory for 220-kv. service.

10. The 220-kv. system should be laid out for multiple operation and switching should be done at 220-kv. A thoroughly dependable and carefully coordinated relay system is a basic requirement.

11. Development of circuit breakers presents no serious difficulty from standpoint of voltage, and it is believed that proposed designs will be adequate for the duty which will be demanded.

12. At generating stations each generator and its step-up transformer bank should be treated as a unit. There would be no low-voltage bus and no low-voltage circuit breaker.

13. At substations each transformer would be a unit with a synchronous condenser supplied from a tertiary winding.

14. The studies which have been made and the tentative designs which have been built up serve to establish confidence in the conclusion that 220-kv. transmission is immediately feasible as a commercial proposition.

15. It is hoped that public presentation of these 220-kv. studies may serve in some measure to facilitate the working out of this advance in the transmission art, and that in particular it may promote interchange of ideas, bring out constructive criticism and stimulate needed investigations.

The author wishes to offer acknowledgment and express appreciation of the assistance of his associates and of various engineers and manufacturers whose ideas have been sought and freely used in the preparation of this paper.

DISCUSSION IN "PROBLEMS OF 220-KV. POWER TRANSMISSION (SILVER), LAKE PLACID, N. Y., JUNE 27, 1919.

W. S. Murray: I think Mr. Silver has made a contribution apropos of the times and for it I have had in mind for some time a very concrete application.

The time is before you for very large things and it would seem to me that 220,000-volt transmission will offer an opportunity to exercise ourselves in that field.

The amount of waste due to improper generation of power, its transmission and application, is something enormous—far beyond, I think, the conception of many of you here present. I can epitomize such a statement by saying that I believe that a careful investigation will show that in the Boston-Washington territory, 450 miles long and say 100 miles inland, the amount of waste is about $300,000,000 per annum.

The plan of the super-power transmission to save that waste, is to establish on the coast, stations in which there will be units not smaller than 35,000 to 40,000 kw., each of these stations aggregating 50,000 to 1,000,000 kilowatts as a whole, and at the same time developing such hydroelectric points on rivers and steam stations at the mouths of mines as will economically justify themselves; all of this power so generated being transmitted to the super-power line to give an economy of between ten and eleven pounds of steam per kw-hr., the consumer securing it say somewhere between twelve or thirteen pounds.

The average rate for the generation of power throughout territory taking into account the steam power factory drive is somewhere between twenty-five and thirty pounds of steam, per kw-hr. Thus this rate would be cut in two.

In my experience with railroad electrification there is no question that the present economy of the electric engine over the steam, taking passenger, switching and freight, into consideration is in a ratio of two to one. In the case of steam driven factories it would be much greater than that.

The load factor of a steam railroad, say on the order of the New York Central or the New York, New Haven & Hartford, can be made to be 70 per cent. On the New Haven there are opportunties on its main line division of making it 75 per cent. As we all know, the load factor of the central station is good if it is 35 per cent to 40 per cent and so, in combination with the load factor of the railroads and that of industrial power and lighting, we might hope to get an average load factor of something like 50 per cent.

The idea of economy alone is good but the plan is even far more reaching when we consider that we must create very shortly some form of common carrier for power. I think that you will be surprised when I tell you that the cargo space used by coal shipment today on the Northeast Atlantic seaboard is 40 per cent and this means that our industrialism, which is expanding tremendously (all the earmarks of it are before us now), the necessary transportation to take care of it falling far behind, and therefore if we can take this enormous coal shipment off the rails and put it up overhead, we will take a tremendous burden off of the railroads.

There are two great things that enter the consideration of this matter. One is the question of the coal men who will only have to carry one pound of coal against two, if the economies I have suggested are correct. The answer is not to say to the coal man that his railroad is going to be knocked out of its dividends if it is a coal carrying road, but that his road is going to become the highway for new industries. New industries will spring up along that road and the space created by eliminating a part of the coal, will give space for the new industrial commodities which can be hauled at a higher rate.

In other words, it is a constructive and not a destructive policy.

The other most important question is the matter of the obsolescence of the plants already established. What are we going to do with them? The answer to that, I think, is not a difficult one. We are going to use those plants and we will use them this way: On peak load they will be a most valuable asset in the regional system. On light load they will be most valuable as condensers to take care of the idle or magnetizing current in order that the larger generating units may generate at maximum efficiency the real power. Thus instead of facing obsolescence their life will be extended in this new field.

Another question: What will be the corporate status of the present companies? The primaries of today will become the secondaries of tomorrow. The power will be generated in bulk and be transmitted to the present companies who will maintain their present entity as distributors and so far as franchise rights, etc., are concerned, will remain exactly as they are today.

If an investigation is made, and I trust will be, and we find, as I have stated that there are $300,000,000 being wasted in this zone, it seems to me that as we are now mining some 75,000,000 tons of coal anthracite, and we have only got 75 years of it left, and as we have jumped from 100,000,000 of bituminous coal mined per annum to 580,000,000 that every engineer must fight for this conservation plan.

It is very well to say that we have 3,000,000,000 tons of coal, which we have, in this country but where is it? It is not in a place where it can be economically transported. Much of it is away out in the Alaska fields. Of course we have the Pennsylvania, Illinois and Ohio fields but if you can see the acres that have been taken out of them you will see the force of the necessity of our economizing, I think, as much for ourselves as for the sake of posterity.

I want to leave a final thought with you in this statement:

When the matter of this investigation is presented to Congress an appropriation will have to be made to cover the work, and I think that we ought to all of us consider this not a local matter but a national one. It seems to me that this viewpoint is fair enough. The Senator from Ohio or the Senator from Nevada or even the Senator from California might get up and say "Well, why not make a super-power transmission in our territory? Why should we not vote for the Northwest Coast instead of the Atlantic seaboard?"

I think that that would be a very short-sighted attitude because the finishing shop of the United States is on the Northeast Atlantic seaboard for American industry. There is very nearly 80 per cent of the country's industry concentrated right here. Now, if we are going to maintain the American standard of wage and living while competing with the world —and this is the great angle of vision we should take now— that is where the American product must continue to be finished and placed on shipboard.

I was very happy indeed at the Boston Section to have that branch of the American Institute of Electrical Engineers, vote a resolution of approval of Secretary Lanes' proprosed appropriation to cover this investigation and as I said before, we can just visualize this great line constructed as described, into which is fed a kilowatt hour at an expenditure of ten pounds of steam keeping in mind for economies sake the wonderful contributing factors of load factor, to say nothing of breakdown service which will be established between plants. Surely this is a sound vision, it is certainly not impractical.

I don't believe I could leave a better slogan with you in closing than saying, that lately we have been spending billions for destruction for preservation, now let's spend billions for construction for conservation.

W. M. Dann (Pittsburgh): The transformers that Mr. Silver speaks of are certainly very interesting on account of their size and the high operating voltage. They are the largest units that have so far been proposed seriously, but I believe the manufacturers feel no hesitancy at all in building units that will be dependable and reliable in service.

On page 1091 referring to the choice between three-phase and single-phase units, it seems to me that all the arguments are in favor of single-phase units, particularly since the three-phase unit as large as 100,000 kv-a. seems to be impractical at the present time. For a bank of that size three 35,000-kv-a. single-phase units will be cheaper than two 50,000-kv-a. three-phase units. The switching would be simplified, the floor space required would be less and there would be some gain in efficiency.

The high intrinsic reliability which Mr. Silver refers to can almost be said to be characteristic of power transformers nowadays because they are actually responsible for very little real trouble. However, 100 per cent reliability is an ideal which we will have to admit has not yet been reached and it seems to me from the point of view of insurance that it would be unwise to figure on operating such a big and important system without spare transformer capacity. If this is conceded, the arguments in favor of single-phase units are even stronger.

The statement is made that for relatively small substations equally satisfactory service can be rendered at less cost by a bank of two three-phase units as compared with one bank of single-phase units with a spare. It seems to me that the arguments are in favor of single-phase units particularly since spare units are provided with the single-phase and not with the three-phase transformers.

Operating the generator and the transformer equipment as a unit will very likely meet with pretty general approval. Here again I believe that the choice of units is in favor of the single-phase transformer.

The suggestion that the future may see a radical change in the present method of handling and cooling transformer oil is certainly a very interesting one. Heretofore the forced cooling of transformer oil has not been very common and not very popular and the reason it has not been popular is the expense and complication and the lack of real necessity for the forced cooling system. It may be that in connection with transformers of this unusual size, experience may bring up some new points which will make it seem a little more desirable.

On page 1092 is given a list of apparent advantages for the forced cooling system and just to take the opposite side of the argument for the moment, I would like to submit a few points to go with this list:

(a) Tanks are smaller and cheaper due to eliminating the cooling coils. The saving in size is really not very great. It appears only in floor space because the height is practically fixed by the insulation and the length of the bushings for 220,000 volts. In other words, the height of the transformer is practically the same whether the cooling coils are internal or external. The saving in cost, if copper cooling coils are taken into consideration, amounts to from 5 per cent to 8 per cent. If iron cooling coils are taken into account, the saving is considerably less. This is not a net saving, for the cost of the external system with its pump and motor is greater than the internal cooling coils.

(b) Assuming that the ducts in the windings are free from restrictions and that there are no places where the oil does not flow naturally in response to the thermal head, there is nothing more positive in circulation or more positive in cooling than the natural flow of oil set up by the heat of the windings and it would seem that forced cooling should be unnecessary unless some new and compelling reasons are developed to justify it.

I question whether the rapid circulation of oil in a water-cooled transformer is fully appreciated, for it is really more rapid under the natural conditions than with forced cooling unless the external piping and pump are unreasonably large. The thermal head in a transformer forms one of the most efficient pumps. Anyone who has ever looked into a transformer when operating under load will remember the boiling of the oil at the surface due to the strong circulation.

(c) The temperature of the oil at the bottom of the case is dependent only on the size of the cooling plant, irrespective of whether it is internal or external, and low temperature of the ingoing oil is not specially characteristic of the forced cooling system.

(d) The oil must of course be kept absolutely dry, and if it is conceded that we can't get reliable cooling coils placed inside the transformer, then the present almost universal practise would have to be abandoned. It seems to me that we can count on reliable coils and perhaps the risk of getting water into the oil is not very much greater than the risk of getting air into the external system through the piping and pump, and that of course would be a very serious thing with these high voltages.

(e) Continuous filtering of the oil can be carried out whether the transformer is cooled by natural cooling or by forced cooling. It would be impracticable to filter the main flow of oil supplied to a transformer of this size because the volume is too great.

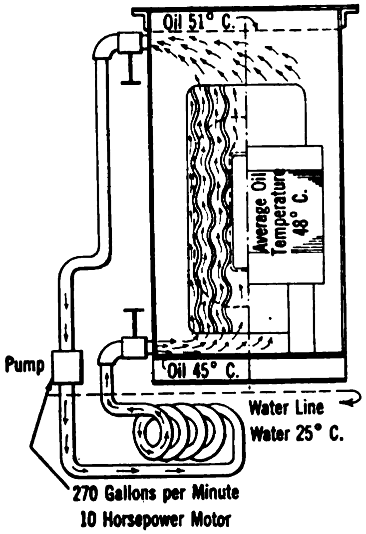
(f) It may be that when we get used to large systems, having these high-voltage transformers, the forced circulation of the oil may appear to be more desirable and perhaps cheaper than it has heretofore, but experience up to the present time really does not indicate it.

I happen to have had some experience with a 14,000-kv-a. three-phase transformer which is cooled with a forced circulation of the oil through an external system. This is quite a little smaller than a 35,000-kv-a. or 40,000-kv-a. single-phase unit but its test results are interesting and it seems as though they ought to be useful in considering the forced cooling of large transformers. This transformer is supplied with 270 gallons of oil per minute and is circulated by means of a pump driven by a 10-h.p. motor. The maximum temperature at the top of the oil is 51 deg. cent. and at the bottom, before it enters the case, 45 deg. making a difference of 6 deg. from bottom to top.

Now, if that same transformer were cooled in the more usual way with water circulated in internal cooling coils, it would require approximately 60 gallons of water per minute and a pump requiring not over 2 h.p. to drive it. The temperature of the oil at the top would be 51 deg. cent., the same as before, but at the bottom it would be 47 deg., a difference of only 4 deg. from bottom to top. The temperature of the water is 25 deg. cent. in both cases.

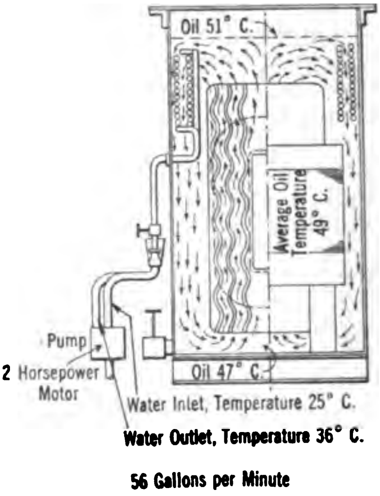
The interesting points here are that:

(1) The difference of temperature of the oil from bottom to top as a water-cooled unit is only two-thirds of what it is as a forced-cooled unit. This means that the circulation of the oil is about 50 per cent more rapid and 50 per cent greater in volume when the transformer is cooled with the natural circulation. This is a good illustration of the rapid flow of oil in a water-cooled transformer, which I said a moment ago perhaps is not fully realized. The thermal conditions tending to produce the rapid circulation of the oil in a water-cooled unit, are all present when it is operated as a forced-cooled unit, but the restriction to the flow of the oil in the piping and the external cooling coils is great enough to pull the flow down to about two-thirds of what it would be as a water-cooled unit. In other words, the efficient pumping effect of the thermal head plus the external pump are not sufficient to produce the flow that would result from the thermal head alone simply on account of the friction head of the external system and the only way to bring the circulation up to a par with the natural water-cooled circulation would be to spend an unreasonable amount of money on the external piping, pump and motor. Now this may seem surprising to some but when one stops to think that it requires very little pressure to make the oil flow up through the windings and to return it down the sides of the case and that it requires a very considerable pressure to circulate it



FORCE COOLED TRANSFORMER

FIG. 1



WATER COOLED TRANSFORMER

FIG. 2

through the external piping and cooling coils, the results are not so surprising.

(2) The temperatures are more uniform under the natural cooling condition than with the forced cooling condition in this transformer. This follows as a natural result of the greater flow of oil.

(3) If enough money were spent in pump and motor to double the flow of oil that exists under the forced cooling conditions the gain in maximum oil temperature would be only 1.5 deg. over the natural water-cooled condition and the expense of this undertaking would be practically prohibitive.

Looking at this problem from the point of view of simplicity, it would seem to be a better proposition to go into the tank directly with the cooling medium and carry the heat away rather than to provide means for removing all of the oil with its heat units from the tank, carry it to some external point and there extract the heat and then return the oil to the tank.

Really, the big problem in cooling a large transformer is the problem of getting the heat out of the coils into the cooling stream of oil, and usually the cooling of the oil itself can very easily be taken care of without resorting to the forced cooling system.

After we have studied this problem of cooling unusually large transformers during the next year or two, perhaps the forced cooling system may show up to a little better advantage. I think it will be very interesting to watch the developments of the next few years.

Philip Torchio: I am representing in my employment, central stations. I want to say from the start that the central station as an industry don't care how the power is generated. Whether they generate it themselves or if they buy it from outside sources, it is the same to them. The central stations are purveyors of power to the public and they are not concerned about the question of selfish motives to have it generated locally under their direct control or from an outside entity, provided that equal cost can be secured and equal reliability of service can be given to the customers.

So, in commenting on the interpretation of Mr. Silver's paper in the broad generalization and application to the solution of the power problem of the nation, I want to put an emphatic denial on record that the central stations have any selfish motive in raising any question or any objection to such plan. In fact, the question of securing economical advantages of tie connections has been one of the characteristics of the central station industry.

In the particular field of activity of my people, we have probably been leaders in the interconnection of systems. We have had interconnection between the stations of the lighting companies and the railway companies since 1899 and they have been added thereto from year to year so that all the stations in that territory are now interconnected in one system.

In commenting on the general problem of power supply to the nation, as the time is short I want to refer you to a discussion I presented in January 1918, on Mr. Jackson's paper. In that discussion I brought out emphatically the essential necessity for the states east of the Rocky Mountains to develop to the highest degree the steam power plants. The question of utilization of water power is secondary. We have very little water power available east of the Rocky Mountains, except at one location, the Niagara and St. Lawrence River, and for the application of those powers I also made certain suggestions, which I think should be given serious consideration by the Congress Committee if such a Committee is appointed.

Now in our daily studies of supplying power to important centers or important industries, we are meeting daily with the question "Can you as safely transmit it ten miles away? If you have to cross a river, wouldn't it be better for us to have a station?" Now here comes a suggestion to go 250 or 300 miles. I don't think that the public is much concerned on the theoretical savings that can possibly be made by having the power delivered from a distant point, two, three or four hundred miles, versus the power generated locally, which may be a difference of one-tenth of a cent per kilowatt hour. They are more concerned that the power be reliable, continuous.

In 1911 I presented a paper to the Turin International Congress on the latest development of high-tension transmission which was then 120,000 volts. Being a delegate of the Association of Edison Companies, I transmitted to the Association the paper presented at the Congress, with a supplemental report, headed "What the Central Station Can Gain from the Latest Development of High-Voltage Transmission." While the report did not state specifically the location, still it covered the territory between Boston and New York, a part of the territory that Mr. Murray has described. I recommend that that paper be considered, because it gives the values of what you can gain and visualizes the relative advantages.

Returning to the point of reliability of service, Mr. Murray says the plan contemplates using the existing stations for peak loads. I assume that in a district like Greater New York the peak load would occur in the winter months, usually between November and February, and I imagine that during that period the stations would be under steam. I don't understand what Mr. Murray had in mind about the summer months. I imagine that he would use the transmitted power when the loads are light, as during the summer months, when the loads are normally about 60 or 70 per cent of the maximum winter load. It may interest you to know that on June 20th, 1919, a few days ago, at four o'clock, out of the clear sky, suddenly darkened, we got the highest load in the history of our company.

Now how could the customers be supplied from a peak station, shut down? It would be impossible. We will have to keep those stations under steam. Now when you keep stations under steam you have a great loss in banking fires and radiation, and those losses must be considered.

A valuable paper has been presented to the Smithsonian Institution by Mr. Pogue and another writer, emphazing the necessities of this power problem and the necessities of having power transmission lines the same as we have passenger and freight communication lines.

But I think one thing is overlooked, that we must use coal to generate our power. It makes very little difference in the saving of coal if we generate the power at the mine or if we generate it at tide-water or in Chicago. The only possible saving is of lightening the load on the railroads.

The point I have made in my quoted remarks is that by keeping up the development of the central stations as we are doing it, securing for our increment power the high efficiencies of which Mr. Murray speaks, and we concentrate all our combined efforts along the lines which the central stations are following, we will get out from coal all there is in it, the same as if we generated the power at the mine. Our goal must be to concentrate all efforts to prevent the small uneconomical plant from generating power. Those are the people that use the thirty, forty and fifty pounds of steam. We must also stop the trunk railroads from using from one hundred and twenty-five to one hundred and fifty millions of tons of coal a year at 5 per cent efficiency.

Now if we generate that power in our stations and we realize with these intercompany ties that I was proposing eight years ago, we will secure all the advantages that Mr. Murray points out, and we will give to the customer a more reliable service than if we tried to supply it with power over long distance transmission lines.

The time is short and I want only to emphasize this feature, that the central stations are not advocating from a selfish standpoint the higher development of their plants, but they are logically fulfilling a line of economic development. If we carry out that development we will reduce from one-half to two-thirds the freight for power coal on our railroads. Then the gradual electrification of trunk railroads will double their carrying capacity and the question of congestion of traffic will be solved in a logical way and in a saving way.

S. W. Mauger and **R. M. Spurck:** The importance of the subject and the advantages shown by Mr. Silver in the use of 220-kv. transmission, warrants some comments from the standpoint of the designing engineer and we are therefore presenting the following remarks on line insulators, switching devices and relays, confirming Mr. Silver's conclusions.

1. Although it is believed that the values shown in Fig. 9 for the dry flash-over voltages of insulator strings, is somewhat

low for strings of 8 units and more, the values which it is believed are more nearly correct, are not enough higher to neglect the additional advantages that can be obtained from grading. Such grading appears to be practically feasible. The use of shields, however, offers many mechanical difficulties that would make the results obtainable not worth the added complication.

Very little reduction in the flash-over voltage of strings of units is expected when such strings are combined in multiple. Attention is called to the probable mechanical difficulties involved in shielding strings of multiple units.

We believe that a great part of the insulator trouble noted by Mr. Silver, has been confined to insulators having cemented-on metal fittings. Practically no trouble has been experienced with the link type of insulator, which has no cement and whose design is such that the insulating section is maintained in compression.

The completion of the development of new designs of hardware promises further extension in the use of existing designs of link insulators which already have a service record, indicating that they are suitable for 220-kv. transmission with the moderate mechanical loads ordinarily required.

For higher mechanical loads, a similar insulator of much more rugged design and capable of handling the heaviest mechanical load estimated for the present 220-kv. transmission problems is being developed.

2. Eliminating low-tension switching is a big step in reducing the complication involved with such large units as are required for economical operation at 220-kv. and the saving will go a long way toward paying for the increased cost of the high-voltage switching apparatus.

This, of course, can only apply to generating stations, but in substations the voltage to which the transformers are stepped will be high enough to obviate the very heavy currents.

As Mr. Silver brings out, the handling of very heavy currents involves more difficulty aside from the question of space, than the handling of very high voltages.

3. Mr. Silver makes an apparently bold recommendation regarding protective equipment, but with the great saving in line conductor cost, we can possibly afford to spend a little of this saving in making our insulation proof against over voltage. This is at least worthy of careful consideration.

4. As far as oil circuit breakers and line disconnecting switches are concerned, we have found no design problems that cannot be worked out and the performance of the apparatus predicted with a fair degree of accuracy from designs and performance data on similar apparatus for lower voltages, namely, those voltages between 100 kv. and 155 kv.

In oil circuit breakers for 220-kv. in order to obtain the desired insulation, it is necessary to have exceedingly large tanks on account of the large breaks between contacts and large clearances between contacts and tank, and we believe therefore, that it is advisable to have a smaller and very strong arcing chamber which will relieve the tanks from stresses occasioned by opening large amounts of power. These explosion chambers also are very efficacious in breaking the arc because in their design the pressure of the gas is utilized to a good advantage in blowing fresh oil through the arc stream, thereby assisting in effecting a prompt extinction of the arc. Such breakers can be supplied with two or four breaks per phase, with four break design having all four breaks in one oil vessel or preferably two breaks in each of two oil vessels.

5. 220-kv. air break disconnectors must necessarily occupy considerable space, which of course involves difficulty in design and operation as well as expensive structures for supporting the switches. We cannot hope for any change in these conditions, but it is felt that the conditions can be met with entirely operative and rugged designs, even if necessary to open the exciting current of a transformer. It is not recommended, however, that this should be done except on the small capacity transformers. The following types have been suggested:

(a) Combination knife-blade arc-horn type with two blades in series operated simultaneously and mounted on rigid insulators. The blade opens upward.

(b) Underhung-blade type supported from link suspension insulators, the blade opening downward.

(c) Knife blade type with hinge end supported partly on insulating bushing of transformer or oil circuit breaker and partly on a separate rigid insulator which is revolved or otherwise moved to operate the switch and the other end supported by a rigid insulator on the tower structure. The blade opens downward.

The type mentioned above first is the only type we would recommend for opening with exciting current flowing. The second type with the proper mechanical design can probably be made to give the most reliable service from standpoint of insulation, because link insulators without cement can be used, thereby eliminating troubles attributed to cement.

6. It is gratifying to know that Mr. Silver recognizes the great progress that has been made in relay development. From the standpoint of relay application, there is no more reason to anticipate difficulty with a 220-kv. system than with one of lower voltage.

J. C. Parker: I take it that the comments by Mr. Torchio are not intended in disparagement of the very large social vision that Mr. Murray has given us of the implications of this paper on the 220-kv. transmission, rather that they are intended as a warning and a possible encouragement and incentive to the profession as a whole to tackle the large proposition of the super-power development as an integral matter, rather than as a specialized line of development.

I think it is very satisfactory to the electrical engineers that Mr. Murray has pointed out to us an opportunity for a social relation for our technical work. I think electrical engineers in the past have had occasion to feel that particularly the civil engineers have had a greater social relation in such things as matters of transportation and sanitation than have we electrical engineers.

The problem of coal conservation as affected by these super-voltages and super-power lines, it seems to me, divides itself naturally into two parts, one which is technical, the other which is more or less political. Those two parts are incidentally concerned with the things that are internal to the industry and peculiarly within the hands of the electrical engineers, and those things which are external. Whatever we may do in the way of technical improvement in getting higher voltage lines, lines capable of transmitting at high voltages and at high economies, will be utterly nullified, as pointed out by Mr. Torchio this morning, unless an inducement may be created for the ultimate user to take the power so economically generated and transmitted.

Mr. Torchio is quite right in the statement that if you wipe out the entire marginal cost of steam power production you have done no good toward getting economical coal usage, unless the ultimate consumer can find a selfish interest in such utilization. Now that does not mean that these higher voltage lines, these large projects necessarily fail to attain the end. It simply means that we must be spurred toward very large efforts in the direction of securing the one thing that is essential, continuity of service. With these very large systems, with large centers of population dependent on the service, continuity becomes preeminently the requirement, and much more so than even in our metropolitan systems of today. I don't think that that need discourage us. There is opportunity, there is hope for the right sort of development.

Mr. Silver has pointed out one line that must be pursued, that of simplicity. Much of the discouragement in our high-tension transmissions in the past has been due to the fact that we did not sharply differentiate between a transmission system and a distribution system. These super-power lines cannot be tapped for every small community. We will probably have intermediate voltage lines for distributing to the smaller communities.

I believe also that Mr. Murray pointed out the importance of a multiplicity of generating points on a relatively simple system. Where that sort of thing has been done with such voltages as 140 kv., continuity of service has been attained. On the other hand, the profession as a whole, if it is going to accomplish this very large social program, must devote to it a great deal of effort. We must make our utilization apparatus simpler and cheaper. We must make the means of distribution much more economical, with a marginal generation at as low as 0.2 of a cent in the pre-war days, the distribution companies were the thing that stood between cheap coal utilization and sales to the customers. There is a big problem for even the application engineer whereby he ties himself to this program of national conservation.

W. S. Murray: Mr. Torchio raised some very interesting and very valuable points, indeed, and I thought it possible that an impression might have been conveyed that in the study of this matter we rather considered we had plenty of hydraulic power in this region. Now I want to draw your attention to the fact that at first blush the study shows that there is a demand of some seventeen million horse power, of which ten is industrial and seven is required for the railroads.

Now the actual amount of hydroelectric development available for what you might call, economic application, is somewhere around a million, and so there is not a very great deal of hydroelectric power available as it stands. In other words, just as Mr. Torchio pointed out, the steam will carry the heavy end—93 per cent steam, 7 per cent hydroelectric. May be later as we extend the line up into Maine and South down to Richmond, we will strike into the Northern and Southern hydroelectric powers, and the percentage of water power will be increased, but it is primarily now a steam problem, a problem of getting the highest economies out of steam.

That was just the point I wanted to bring up for fear you thought perhaps we were thinking of the development of the hydroelectric feature.

Next is with regard to the point Mr. Torchio raised concerning a day that might come along in the middle of a low load season which would reach right up to the maximum of the high. We have got to look at this thing away out there and not right in front of us. We have got to keep our minds far ahead and with this super-power system installed one or two units quickly thrown into service would meet the conditions Mr. Torchio has described. It is just because of the lack of it today that we are at times embarrassed. His argument "against" is truly "for" when we view this matter broadly.

I agree that these great steam stations are preferably to be located in the large centers, such as New York, or Boston, or Philadelphia, etc. Mr. Torchio is absolutely right along those lines, but we must not lose sight when we have done that, of the fact that this tremendous super-power transmission at 220-kv. is the interlink between all those large centers, and provides a means to supply the very customers that Mr. Torchio pointed out we must reach and therefore, while entirely agreeing with him that his load centers are correct it should be remembered that the super-power transmission ties in those load centers and make them more effective, permitting us opportunity to create a high load factor for a regional instead of a city plant.

Now just remember the house. Then, after the house, the city. Then the district, then the territory, and now the region and that is all this proposition is; to establish a twelve pound line in this region. We are on the verge of large things, and it is within the vision of those large things and within the vision of practicability that we do the larger things in a more efficient way than the smaller.

J. C. Clark: The San Francisco Section has organized a local Committee on Railway Electrification, including in its membership a number of the best electrical engineers of the community. The Committee was organized about the first of April, 1919, and has been holding meetings every two weeks since that time. One object of the Committee is to gather all the data they can get in the way of actual figures of power consumed in railway operation, such things as maximum load swings, demands upon lines to carry trains at all times, and load factors. In this latter connection it is quite surprising and gratifying to hear Mr. Murray state that the load factor on his regional trunk, due to railroad operation, will run as high as 75 per cent. No such high load factor has been predicted by our people for the California mountain divisions.

Among other subjects which are being studied is the amount of potential water power in the Pacific Coast States, especially in California; the amount of fuel oil used by railroads; and more technical topics such as the practicability of dispatching trains with special attention to load factor.

The plan of the Committee is not to engage in political activity unless called upon. The Committee wish to be prepared so that they will have all the facts and figures at their disposal which they may need in order to assist any legislative body in studying the problems of conservation of fuel in their district. Fuel oil in California is going very fast.

Some of us do not look for the initiative in railway electrification to come from the railways. It is rather remarkable that the matter of railway electrification has been studied in the past almost exclusively from the railway standpoint. It is felt that we need not hope, certainly not in the near future, for any considerable move for railroad electrification from our railroads. One reason that has been apparent to some of us is that, in these days of regulation of such utilities as railroads, it is a growing practise to insure a reasonable return to the railroads, or any utility, on its investment.

Now, if the railroads are assured, say, 8 per cent, it perhaps removes one of the incentive toward more economical operation. If that return is assured in any case, it is not quite apparent why they should strive to improve their methods—at least not from the point of view of the fuel conservationist. It seems to many of us to be up to the public to insist upon railway electrification, and with the attitude of being prepared for a public movement for railway electrification, we are getting together all the data we can to enable us to be in a position to offer such assistance to legislative bodies as may be asked.

Mr. Silver expresses some apprehension about insulators. He has a good reason for this in view of the universally bad experience with the cemented type of insulator. Fortunately, however, there are some very good operating data, covering a period of perhaps ten years, on another type of insulator. I refer to the original suspension insulator known as the Hewlett. The failures in the cemented type of insulator have not been due primarily to electrical causes but to mechanical cracking and absorption of moisture. Electrical failures follow. The cracking may be caused by expansion of the cement or of tightly fitting metal parts. The presence of the cement greatly aids in the absorption of moisture by porous porcelain in contact with it. The absorption is apparently due to breathing. In the Hewlett type the metal parts are strung loosely through glazed cable ways. I have recently had occasion to examine many insulators of this type having a ten year service record with inappreciable loss. Electrical, mechanical, and porosity tests showed no depreciation. Some of the old original units that gave perfect service were poorly made compared to present standards. This speaks well for the design.

It may be necessary to grade the strings of insulators used on these high-voltage lines to lower the stress on the line unit.

In regard to the transformers, I believe decidedly, that all high-voltage neutrals should be grounded without resistance. With proper precautions there should be no difficulty with telephone operation. The connection will in general be Y-delta to prevent tripple frequency.

There are some advantages and many disadvantages in high-tension switching. The number of these switches should be reduced to a minimum.

As transmission voltages are increased, the number of times that lightning voltages exceeding operating voltage are induced on the line in a given season will decrease.

The question naturally arises whether it will pay to install lightning arresters. While it is probably true that a lightning arrester gap on a 220-kv. line would discharge only at infrequent intervals, great damage might be done if it were absent during these intervals. If it is assumed that the transient voltage is limited to the gap setting, it can be shown that about five times the insulation is necessary to make the apparatus equally safe without a lightning arrester. When the arrester is absent the voltage is limited by the line insulation. The turn insulation probably receives relatively greater transient stresses on very high-voltage lines than on moderate voltage lines. A lightning arrester is decidedly desirable if its cost is commensurate with the cost of other apparatus. Consideration must also be given to the enormous energy involved.

F. C. Hanker: The field of the 220-kv. system is apparently limited, as pointed out by Mr. Silver, to trunk line service where we have long distance transmission and heavy blocks of power. That would be the natural conclusion, based on present analysis of costs, the development of the necessary transforming and switching equipments involved in the line construction. It would naturally hold for pioneer installations but if we look back on the history of transmission voltage, we find parallel situations in the steps of 110-kv. to 154-kv-a. It has only been a few years since 110-kv. was discussed as a high voltage for transmission, while today it is so common that it may be considered almost in the class of distribution voltages.

The frequency of 60 cycles selected for the studies is in line with present tendencies toward the establishment of this frequency as the standard frequency of power supply in this country. A general analysis of the factors entering into a decision as to the best frequency to adopt in any specific case will show that the increase in 60-cycle systems is a natural one and is one that will undoubtedly continue at an accelerated rate.

The existing transmission systems that are operating at high voltages above 110-kv., have layouts that in general require regulation at one load center so that it is usually possible to adjust operating voltages at the generator station to correspond to the requirements at the load center and in this way keep the investment in synchronous condensers to a minimum. In the case of trunk line service where a mumber of generator stations and different load centers are involved, the maintenance of satisfactory voltage regulation at all points becomes more of a problem and it is usually more economical to supplement the synchronous condenser installation with synchronous boosters or the equivalent, such as induction regulators. This is particularly true where systems tie in with existing city systems that have standards established, making it necessary to maintain closer voltage regulation than is permissible on some of the pioneer lines such as exist in the West.

With the relatively high investment in lines and substation apparatus it is important that full capacity be available. This makes regulating apparatus at tap points desirable in order to reduce the circulating currents to a minimum. We have reviewed cases where parallel lines were involved in which it was desirable to provide independent regulation for each circuit. This was due to unequal loading at tap points that would have resulted in excessive circulating currents had the two lines been paralleled at both ends and regulation obtained by synchronous condenser equipment. It would also have cost very much more to control by the use of synchronous condensers than by boosters or equivalent regulating schemes. This condition would obtain on projects such as have been proposed for trunk lines along the Atlantic Coast tieing in the different existing power stations. To be of value it should be possible to transfer power in both directions, which would mean, in the case where synchronous condensers were used, installations at each end of capacity to give the full range of regulation. This condition can be better fulfilled by the use of sufficient synchronous condenser capacity to provide for operation at the most economical power factor and take care of the additional regulation by some other method.

With the installations under consideration, the capacity of the regulators that would be required would be larger than have been built of the usual type of induction regulator. There has been developed, however, a combination of the step type and induction type that would be feasible for the capacities involved. In this regulator the objectionable features of the straight step type regulator have been eliminated in that no switching is done with circuits having difference of potentials, and as a result, sparking is avoided. The short-circuit stresses that have been a limiting feature in the design of induction regulators are avoided, in that only a small percentage of the total short-circuit current is handled by the regulator.

The kv-a. required to charge the lines of the length that have been considered in the studies becomes of importance due to the effect on the generator excitation. A normal design of generator such as would be installed would have a short-circuit ratio of approximately unity, that is, with the excitation required to give normal voltage at no load, the sustained short-circuit current would be approximately full-load current. In the studies made, 80,000 kv-a. is given as required to charge the line so that it would only be necessary to supply 20 per cent external excitation to excite the full 100,000 kv-a. in generators to normal voltage. There will undoubtedly be times when only one generator will be available to supply a line, and under this condition, the charging kv-a. would exceed the capacity of generating apparatus. This would take the control of the voltage out of the hands of the operators and result in abnormal voltages on the transmission line.

This condition is not of importance when operating under the usual loads, but must be considered where high-voltage switching is involved.

In commenting on the design features of 220-kv-a. station and substation equipment the paper points out that the studies have developed that current design principles and materials now in ordinary use will be employed. This is encouraging after reviewing the analysis that has been made of the problems encountered in laying out the other parts of the system. While the margin on safe operating voltage is probably greater than is apparently the case with the line construction, it is recognized that new problems exist and the manufacturers appreciate the importance of these problems. They are not content to be satisfied with the present conditions but feel that just as careful analysis and research is fully warranted and should be undertaken to ensure the success of this higher voltage system.

The problems have been classified as of two types, those due to the high voltage of itself and those due to the enormous capacities involved. The insulation problems in the stations are concentrated where greater expense is justified in providing the margin in safety that is essential.

The conclusion that all line switching should be done on the high-voltage side will probably find opposition but if future investigation confirms this statement it will simplify the switching problem. In accepting the recommendation, it should be recognized that this method of operation will subject the transformers to higher stresses than in the case of low-tension switching. It is true that the potentials resulting from switching are of the same order as those the apparatus would have to stand from static disturbances or other transients that may occur on transmission lines, but they probably occur more frequently. On the 110-kv. and 154-kv. systems, we have heard of no serious results from the high-voltage switching, yet this point must be taken into consideration in laying out a system on the higher voltage. With the operating conditions that have been assumed it would be well to consider arrangements wherein the step-up transformers and the step-down transformers are considered as a unit and operated as such. This does not necessarily mean that the apparatus must be operated in parallel on the low-tension side at the generating station, as the layout can be so arranged as to provide for what is essentially unit operation.

In investigating breakdowns that have occurred on lower potential circuits, we have obtained results that indicated that the current was ruptured in the oil breaker at a rate several times the normal frequency. If this condition holds for low voltages with relatively short travel of the contacts and short arcs, it must be more severe at high potentials with correspondingly long arcs.

H. R. Summerhayes: Figs. 1 and 2 of Mr. Silver's paper show a comparison of the cost of the line construction, total operating cost for 154,000 volts and 220,000 volts, and at first it seems as if more of a gain should be made on 220,000 volts in spite of the increased cost of the towers, etc., which is incurred by the higher voltage, but in reality the comparison is between 170,000 and 220,000 because 170,000 as stated in the footnote is the voltage at the generating end of the line, just as 220,000 is the generating end voltage, and I think that the comparison is really between those two voltages and not between 150,000 and 200,000.

R. P. Jackson: Mr. Silver's statement that there is no type of insulator as yet developed that has thus far demonstrated its ability to give adequate, or even reasonably satisfactory results on high-voltage lines is an exceedingly severe indictment of the suspension insulator considering the amount of power now being actually carried by conductors hung on such insulators. "Reasonably satisfactory" is a relative term and if defined as being as satisfactory for example as the steel towers on which the insulators hang, the statement is doubtless correct.

In reading Mr. Silver's paper, it is obvious that he has some serious misgivings as to the insulators for the high-voltage high-power lines proposed, and with good reason. A link of nine to eleven ft. between the line and the tower appears altogether too long. Six feet would be possible as a pillar to support disconnecting switches. Further, a strain tower taking up the tension of the three lines in both directions apparently requires six strings, each of fifteen insulators for each conductor anchor. This means 6 × 15 × 6 = 540 insulator disks per strain tower. This strikes one as a monstrosity.

In the first place it would seem to the writer that the use of 15 disks in series is unduly conservative on a solidly grounded neutral line that limits the voltage to 130,000. Ten, or at the most twelve, should be sufficient with say a 7 in. spacing. This would give a string about 6.5 to 7 ft. long and reduces the inequality in stress distribution.

When it comes to 2900 pounds as a maximum load on the ordinary disk insulator, Mr. Silver is not unduly conservative. Such an extraordinary proposition calling for such loads as Mr. Silver describes demands something larger and stronger in an insulator than anything now on the market.

The present types of insulators have been developed to meet certain prevailing conditions. In fact, some operating men state that for their use the usual cap and pin insulator is unnecessarily strong. It is undesirable to use a size and type of insulator developed for the low stress conditions of ordinary transmission lines for these new and much more severe conditions. It certainly ought to be possible to make up an insulator good for twice the stress of the ordinary unit, thus reducing the parallel strings for a conductor anchor from six to three.

Just which of the three types enumerated by Mr. Silver would lend itself most readily and most safely to increase in strength is somewhat debatable, but it would seem that the interlinked type in some form would offer somewhat the best starting point, judging from what can be readily obtained now in strain balls for guy wires.

It is to be hoped that no effort will be spared to do something in this direction before resorting to the use of such a multiplicity of insulator units as the present standard designs would appear to demand.

The writer would personally commend Mr. Silver's judgment in regard to lightning protection in retaining the overhead guard wires and eliminating the lightning arresters. As to something in the nature of choke coil protection against surges of steep wave front, one does not feel so sure. That largely depends on how well built are the high-tension windings of the transformers.

H. B. Dwight: The statement in Mr. Silver's paper that the frequency of 60 cycles should be chosen for an important generating and transmission system, even where there is a large present 25-cycle load, seems a logical and proper conclusion. As stated in the paper, the frequency of 60 cycles is superior to that of 25 cycles for the general supply of energy, and therefore any means which will tend toward the increased use of 60 cycles is a step toward standardization of the right kind.

It may be interesting to note some of the main items of extra expense entailed by choosing 60 cycles for the generating and transmission system where the greater part of the load is 25-cycle. Many residence districts and street lighting loads can be changed to 60 cycles at very small expense. Many 25-cycle transformers can be connected in two parallels and operated at 60 cycles at increased capacity. For power circuits, it would be possible to provide frequency-changer sets for a large part of the total system load.

The cost of the motors of the frequency-changer sets should not be charged to the fact that 60 cycles was chosen, because they take the place of the synchronous condensers which would have been needed for a system of this kind, as stated in Mr. Silver's paper. Although transmission is usually considered easier at 25 cycles than at 60 cycles, the advantage almost entirely disappears when synchronous condensers are used to eliminate voltage variation. Practically the same rating of condensers is needed for a 25-cycle line as for a 60-cycle line, for the same results in a certain case. Although there is less voltage drop to be overcome in the case of the lower frequency, there is less reactance to work on by the condensers.

It might be questioned whether the motor of a frequency-changer set would be as effective for voltage control as a synchronous condenser of the same rating. The answer is in the affirmative if the load power-factor is as low as 80 per cent, the motor of the set being operated at nearly unity power-factor at full load, and at a load power factor at no load, of such a value as not to endanger its stability of operation.

Therefore, if a large part of the load be supplied with 25-cycle energy through frequency-changers, it would not be necessary to scrap the 25-cycle apparatus in use, and yet new installations could be supplied with energy directly from the 60-cycle transformers and so could enjoy the advantages of better choice of motor speeds, better lighting and cheaper and more standard apparatus of all kinds.

Thus, at the time of building a new generating and transmission system, by the additional expense of the generating end of the frequency-changers, and a small duplication of distribution circuits, representing an extremely small percentage of the cost of the system, a 25, 30, or 50-cycle system could be put in the way of gradually becoming a 60-cycle system.

J. F. Peters: There are a number of factors that enter into the design of apparatus suitable for operating on a system as large and as high a voltage as the one proposed by Mr. Silver, that I wish to comment on briefly.

Referring to page 1086 under, Design Features, I note that the author states, "The handling of electrical potentials of 220,000 volts does not appear to involve any disturbing complications or uncertainties. In fact, the manufacturers do not recognize that any serious problem exists." Although that is true to a certain extent, I wish to point out that there are several factors which become of major importance in designs for high voltage, which for low voltage are of no consequence whatever.

In the design of low-voltage apparatus, it is not necessary to take into consideration the electrostatic field, whereas on high-voltage apparatus that becomes the most important factor in the design. Considerable work on the distribution of electrostatic field in transformers and methods of controlling that distribution has been done by Mr. Fortescue, some of the results of his work have been presented before the Institute.

Under high-voltage switching, I wish to call attention to the fact proven by experience that there are more or less high voltages produced by switching. The magnitude of these voltages and their relation to the applied voltage is not definitely known. There is a question in my mind as to the advisability of high-voltage switching on anything as high as 220,000 volts on account of that uncertainty.

Under transformer connections, I am pleased to note that the author has recognized the decided advantages that there are in the star connection. In addition to the advantages that he has stated, I might add that this connection results in transformers being smaller and more efficient on account of the lower voltage that is developed in the windings, consequently less insulation of inactive material.

With reference to limiting size of transformers, referred to on page 1090, from my experience on the design of large transformers, I have found that the mechanical stresses due to short-circuit currents are not serious for high voltage. The short-circuit stresses are not nearly as severe in transformers for high voltage as they are for the same size units on lower voltages. The reason for this is that the density of the leakage field is much less on the former. My experience indicates that the factor that limits the size of transformer units is transportation and handling facilities. Those limits are not only due to overall dimensions, that is railway clearances, but for units of the size suggested by the author, they approach the limit of carrying capacity of even special cars.

On the top of page 1092 the author refers to the relative amount of insulation for the line and grounded ends of transformers. I do not believe that he means to infer that extra insulation is not necessary between turns on the grounded end of the transformer. It is true that the electrical stresses between turns on the grounded end are somewhat less than for the line end, but they are considerably greater than within the body of the winding. I believe that the insulation between turns should be correspondingly reinforced for both ends of the transformer.

F. F. Brand: I am glad to see that Mr. Silver has decided so definitely on the use of grounded "Y" connection. There is no doubt that for high-voltage transmission this connection which preserves a definite voltage relation of the lines to ground and by which the possible voltage oscillations set up by arcing grounds are greatly minimized, is the best connection.

This connection is made almost necessary by the decision to do all of the switching on the high-voltage side, since the tripping out of one phase, either by failure of switches or inability to close and open all three phases at the same time, does not produce a greatly unbalanced electrostatic condition.

The argument that the grounded Y connection increases the hazard due to short circuits, line failures, etc. damaging the apparatus by mechanical stresses, is not of great moment because on such high-voltage systems the reactance of the transformers and of the line must, of necessity, be so high that the current would be limited to a few times normal through any piece of apparatus.

The use of the grounded Y connection would permit appreciable savings in the design of transformers since less insulation is necessary to ground, and it is not necessary to insulate all points of the winding to ground to the maximum extent.

I believe in general that the insulation to ground in the high-voltage transmissions can be reduced, since the probability of over-voltage due to lightning, etc. becomes less and less as the voltage of the system increases.

I believe our standard Institute test of twice the line voltage plus 1000 volts, is unnecessarily high for the higher line voltages irrespective of whether the systems are isolated or grounded and I think that it would be much more logical to adopt a graded test value in which the factor of safety in test would be appreciably higher for the low-voltage units than for the higher-voltage units. Where grounded Y connections are used, the test should, I believe be still further reduced, at least in the higher voltage lines. If both transmitting and receiving neutrals are permanently and effectively grounded, it would appear that we could regard the transmission system essentially as three single-phase systems and could with safety, permit such test values as are ordinarily given for grounded single-phase systems.

I have taken the liberty of drawing up suggested curves of test voltage for both isolated and grounded systems which give a graded test depending on the line voltage. See Figs. 3 and 4.

The conclusion that the high test values now applied are not necessary for high-voltage apparatus is borne out by the fact that there are a number of installations in operation of old transformers in which the test value was only 1.5 times the line voltage although I believe that some, if not all of these systems are operated grounded Y, and, further, it has not been the practise in the past to insulate step-down transformers with a rated voltage lower than rated voltage of the step-up transformer to the same value, the insulation being based entirely on the rated voltage. It would appear that all apparatus on a given system or circuit should be insulated to the same value. As a matter of fact, under light load conditions the step-down transformers are frequently subjected to an even higher voltage than the step-up machines, and since these step-down transformers have withstood such conditions without failure, it would appear unnecessary to insulate a step-up machine to such a high value as is our present practise.

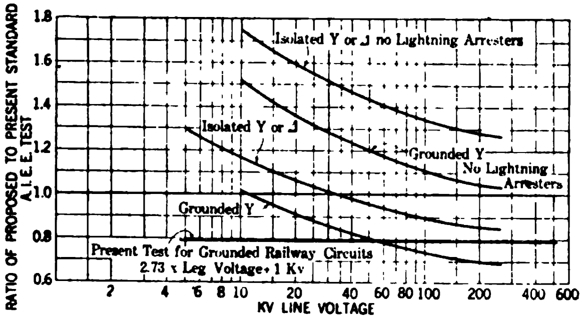


FIG. 3

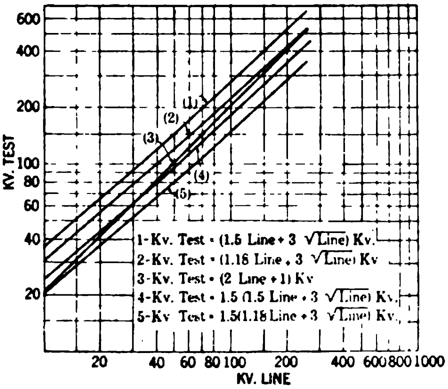


FIG. 4—KV. TEST

1. Ungrounded circuits, no lightning arresters.
2. Grounded Y circuits, no lightning arresters.
3. Standard A. I. E. E. Test.
4. Proposed Standard Test Ungrounded Circuit.
5. Proposed Standard Test Grounded Y Circuit

With Mr. Silver's conclusion to omit the use of protective equipment, I am not so much in sympathy. Lightning arrester equipment in general protects the apparatus against excessive voltages to ground. It is not as truly selective to frequency to the extent which we desire but it should be borne in mind that lacking any protective equipment to discharge over voltages to ground, the apparatus will be subjected to surges originating or applied to the windings at a higher voltage than if the proper protective equipment were connected.

With proper protective equipment, failures to ground on high-voltage apparatus are extremely rare as pointed out previously, such failures as do occur usually being internal failures between coils or turns caused by local high voltages. Undoubtedly without protective equipment, the apparatus would have to be insulated not only to withstand greater voltage to ground but against higher local voltages which occur due to oscillations or waves of steep front.

I do not feel that it would be safe to use apparatus without protective equipment unless the insulation was increased something in the order of 50 per cent over that otherwise required.

On the curves attached I have shown values which would be derived if the test values suggested were increased 50 per cent to take care of the cases without protective equipment.

C. F. Harding: Although Mr. Peek has pointed out in the discussion of Mr. Silver's paper that the corona loss on such a line, where the amount of energy transferred is large, will be rather a small item, yet Mr. Silver has given considerable attention to that subject and I want to mention briefly a point which I think is worthy of consideration in connection with that loss. As I understand it, Mr. Silver has based his calculations upon the formula developed by Mr. Peek, presented to the Institute a number of years ago. If I remember rightly, that formula was derived from test voltages extending up in the neighborhood of 140 or 150 kv. The speaker presented a paper about seven years ago, on the subject of corona loss on transmission lines which checked very closely the Peek data up to about 150 kv., but departed therefrom materially between 150 and 180, or 190-kv., seeming to indicate that above 150-kv. values, it might be desirable to introduce new constants into the formulas. These calculations in the neighborhood of 220 kv. may therefore be slightly in error, and it is hoped that in the near future some new data will be available in that range.

Also, very little has been done in connection with corona to ground. It was found in some investigations with which I have been familiar, that where the lower wire is in fairly close proximity to the ground, as must necessarily be the case with these long spans and wide spacings with the 9-ft. strings of the insulators, there is a relatively large loss between the lower wire and ground, due to corona. It may be necessary, therefore, in some of the future designs, even to go to the extreme measure of putting each conductor on a separate relatively light tower line, separating the conductors by means of separate towers.

So much has been said about the different types of conductors, copper, copper-clad and reinforced aluminum, it would seem well worth while in the future to study the possibilities of steel tubing, making the diameter of the conductor relatively large and keeping the tensile strength high, the resistivity of the metal used being of relatively small importance.

With regard to the question of bushings, although those have been mentioned in connection with transformers, little has been said about roof bushings or wall bushings, used out of doors. It probably will be necessary to use bushings with a much lower factor of safety upon such lines than those which are being used upon the 140 and 110-kv. lines. In connection with the new 600,000-volt laboratory installation at Purdue University, with which we hope to make some tests upon experimental lines and insulators in the very near future, we have had difficulty in getting our lines through the building with anything available at present in the way of bushings, even with a great reduction in the factor of safety. Of course, Mr. Silver's paper anticipates new designs available in the future, but I think the factor of safety will have to be cut down materially.

F. W. Peek: Regarding Mr. Harding's comments, my 1910 corona loss measurements were made up to almost 250 kv. The voltage range under discussion was thus actually covered. Very little would be gained by placing the conductors on single towers and separating them a considerable distance. The loss occurs due to high dielectric flux density at the surface of the conductor. The flux density depends upon the surface, or diameter of the wire, and the capacity. The capacity is not greatly reduced by increasing the spacing. The greatest reduction in flux density or stress can be made by increasing the diameter of the conductor.

I do not think that the bushing problem is serious. We have used bushings up to about 750 kv. without difficulty.

J. A. Koontz: High-tension power transmission at 220 kv. has been a problem confronting the western engineers for several years. While to date it is not a reality, I firmly believe that lines will be operating at such a voltage before many years. In fact, had it not been for the war and consequent difficulties in financing large hydroelectric developments in the past two years, I believe that such a system would now be operating on the west coast.

Mr. Silver's statement is quite true that such high voltage has a limited field and can be used economically only where large blocks of power must be transmitted over long distances. The trend of voltage increase in the past ten years has shown that with increased voltage it has been possible to get increased reliability. This is due largely to raising the factor of safety all along the line. Following this trend, and by carefully working out all the details, I believe the same high service standard could be incorporated in a 220-kv. transmission system and still show economy with improved reliability.

Some of the real problems to be solved where there may still be some doubt are those of corona, insulation and proper oil and disconnecting switches. There is one question concerning corona which I would like to ask Mr. Silver, and that is, if any tests have been made on large conductors in order of 1 in. in diameter to see if 0.87 is the proper conductor irregularity factor, as from some preliminary tests made at Stanford University on a 1 in. 37-strand aluminum conductor it would appear that 0.72 would be nearer the proper value than 0.87.

On pages 1049 to 1052, Figs. 3 to 6, are shown corona loss curves which I do not believe would represent operating conditions under good practise, as it does not seem to me logical to hold the generator voltage constant and permit of receiver voltage variations, but rather, hold a constant receiver voltage and increase the generator voltage when necessary to take care of maximum load conditions. In this manner, the customer will not be subjected to a line drop of both transmission and feeders, and the corona loss will be reduced.

Fortunately, the Pacific slope weather conditions are such that mechanical problems are greatly reduced, as in most cases we do not have to design more than 25 per cent of our line for sleet loading, and even where sleet is encountered we do not have to take care of such heavy load conditions as would be necessary in the Eastern climate.

I notice in the tower design that Mr. Silver has left only a 4-ft. minimum clearance from towers when insulators are deflected to their maximum condition. This does not seem to me ample, as from experience on high-voltage lines on this coast, flash-overs seem to be the principal service interruptions on high-voltage lines, and to eliminate this, it would seem necessary to maintain ample clearance, in fact, experience would indicate that four feet would be a minimum for 150,000 volts, as bringing the conductor close to the tower, intensifies the electrostatic field, and unless special precautions are taken the conductor may fire with corona near the steel tower members, which would tend to aggravate any flash-over conditions. This is a point which seems to me should be given serious consideration in any high-voltage line design.

The insulator problem is certainly very important and the exact method of taking care of same is one which will probably present as difficult a problem as any encountered. With a long string of insulators I believe grading of some sort will be essential so that insulator units in the long string have more nearly the same voltage impressed across each one.

The question of automatic operation on 220-kv. lines, should be given very serious thought before attempting same, as at the present most of the long high-tension lines are seldom interrupted by high-voltage switching. When such large blocks of power are essential with power problems of this magnitude, I believe the generating stations and substations should be given greater flexibility, as the failure of a piece of equipment would necessitate taking out such a large block of power that I think it would be difficult in general to pick up this power on the stand-by steam stations in order that there might be no serious interruptions to service, or, if the auxiliary steam stations were kept under condition to always take care ot such a shift in load, it would mean unnecessary fuel expense. I have thought of the possibility of attempting to group the transformers and line as a unit, but have always been afraid that this would present operating difficulties regarding proper flexibility.

The question of omitting all arrester equipment, I believe is wise, as with very high voltage the insulation strength is such that the troubles that the ordinary lightning arrester would take care of are of little consequence, and I believe greater safety would result in omitting, rather than installing, any of the present arrester equipment.

H. G. MacDonald: The modern tendency toward large concentration of power has necessitated a revision of design and radical departures from previously well-established precedents. The forms of construction which had proved adequate during the period of small or moderate powers are entirely insufficient for modern requirements. Mechanical structures, form and location of contact elements, and speed of operation have all undergone modifications.

Best modern practise tends toward a construction built to withstand considerable internal pressures, as guarantees call for several repeated operations in quick succession in an attempt to locate and clear short circuits. It is well demonstrated that the maximum need of heavy construction is not due to the shocks of the actual circuit breaking operation, but to the formation of excessive amounts of gas, deterioration and disturbance of oil due to repeated operations, and the ultimate explosion of this gas. No device which has for its object simply the confining of the stresses due to circuit interruption without considering the incident stresses from the attendant phenomena will produce a breaker which can be considered safe.

Modern breakers of moderate interrupting ability will usually have elliptical tanks with bracing across the bottom, steel tops held by tie bolts secured to the bottom bracing, entirely enclosed operating mechanism, leaving a comparatively clear slightly crowned top, light moving elements permitting of high and rapid acceleration, and contacts placed well below the surface of the oil, and so shaped as to reduce voltage discharge and to deflect the arc and the attendant gases away from any insulating surfaces. Adequate vents will be located in such a manner as to facilitate the rapid escape of the gases by the creation of a draft of air through the chamber above the oil. The elliptical tank gives the maximum strength for a given space and in connection with the overhung steel top and tie rods, makes a construction sufficiently rigid for any service except the very heaviest where a round tank construction will be used.

A prime consideration in successful circuit interrupting is speed of opening. As the time during which the arc persists determines the amount of contact material vaporized, the amount of oil broken up, the amount of gas liberated, and consequently the pressure formed, it is obvious that quick opening is highly essential. The simple means of accomplishing this end is by highly accelerating the moving element. Where this element is light, this will be readily accomplished, but when the moving element becomes heavy, and a long travel is necessary to obtain the proper break distance, some auxiliary means must be used to quicken the contact separation. Quick break contacts have been added to certain designs of breaker by means of which rapid separation of the arcing contacts is obtained without unduly accelerating the entire moving member. Another means of introducing quickly into the circuit the necessary insulating distance is by using quadruple break contacts instead of the ordinary double break. By producing all four of these breaks simultaneously, it is obvious that twice the separation is obtained within a given time as compared to the double break. As a maximum resort, quadruple quick break contacts afford the most rapid separation obtainable under present designs. The quadruple break, when used in connection with the round tank construction accomplishes in a single circuit breaker what would require two sets of pole units in series in the case of the oblong flat-sided tank, and at no sacrifice of insulation clearances, and in combination with the maximum mechanical strength.

A circuit breaker for 220,000-volt service will inherently possess a large interrupting ability. The insulation requirements will very largely determine the physical proportions and if a consistent design from a purely mechanical standpoint is worked out, a structure of very considerable strength will result. An oil tank to retain the volume of oil required, and with fittings and covers sufficiently rigid to carry the operating means and the contact elements, will be of no mean proportions. By comparatively small additional reinforcement, and by the use of suitable material at vital points, a structure capable of interrupting very large amounts of power will be produced. As so much stress is being laid on continuity of service under all contingencies, the breaker design should be such as to insure this to the greatest possible extent. The only safe course is to make the whole breaker structure sufficiently strong to care for the maximum stresses which might occur under any conditions which might reasonably be anticipated. No existing method of ventilation will dispose of the gas with absolute certainty that no explosion will occur in the main body of the breaker. The explosion chamber used on certain designs of breaker does not eliminate the formation of gas. This gas will presently pass into the air chamber above the oil and must be disposed of. The arrangement of details, and the movement of the breaker parts present the possibility of the arc which originates in the top portion of this explosion cylinder (the cylinder being practically tight at the top) following the moving contact in its downward travel, expanding as it goes, and expelling all the oil before it. The cylinder is now filled with incandescent metallic vapor forming a conducting path for the full length of the cylinder. As the moving contact leaves the cylinder by a very small distance, the possibility of the formation of a large amount of gas and serious damage to the breaker seems to be not very remote. If, on the other hand, the breaker structure entire is made with a view to withstanding abnormal pressures, and the design provides for a rapid contact separation to a sufficient distance so as to preclude the possibility of the arc holding through the distance, the maximum safety and assurance of continuity of service is afforded.

L. B. Chubbuck: Referring particularly to the description on pages 1093 and 1094 of proposed 220-kv. oil circuit breakers, I note Mr. Silver's suggestion of breakers with rupturing capacity of 3,000,000 to 4,000,000 arc kv-a. We have furnished the Ontario Hydroelectric Power Company with a number of large 110-kv. breakers which have been subjected to very heavy short circuits and believe our experience may be of interest. Three Niagara stations are interconnected to give a combined generator capacity of nearly 200,000-kv-a. and while bus reactors are used, the main H. T. breakers have successfully handled short circuits up to nearly 1,000,000 arc kv-a. These breakers are of very heavy round tank construction with motor fans on each tank to prevent the collection of an explosive gas mixture in the expansion chamber.

We have found that older design breakers are not capable of handling such service, and on even much lighter service will smoke badly after one or two operations. As a result of serious trouble with such breakers used on too heavy service, the necessity for a large factor of safety in heavy capacity circuit breaker design cannot be emphasized too strongly.

On these proposed high-capacity, high-voltage power systems, out-door breakers are recommended, also as much sectionalizing, and as little switching at the generating station as possible.

J. N. Mahoney: Commenting on the problem of oil circuit breakers for high-voltage high-duty service, the cylindrical form of tank of the boiler drum type has particular advantages. This form is inherently adapted for the use of four breaks in series per pole or per phase.

When properly designed this form of tank will withstand explosion of hydrogen or hydro-carbon gas and air in the space above the oil. It is true however, that good design also includes provision for removing, displacing or neutralizing such gases and preventing their slow accumulation to any considerable pressure as an explosive mixture.

Because of the inherent possibilities, the tank design should be adequate to at least withstand an explosion of a mixture of hydrogen and air at atmospheric pressure with a reasonable factor of safety.

There should be no difficulty in meeting the insulation requirements with present well tried methods and materials. The only uncertainty is in predicting the probable voltage surges to be met when such a large system is disturbed by unusual conditions.

E. B. Meyer: The fundamental consideration underlying Mr. Silver's paper on 220-kv. power transmission is the economic utilization of our fuel and water power resources in a manner calculated to conserve them to a maximum extent.

During the period of the war the need of conservation, coupled with the necessity for a maximum useful expenditure of power, made this subject one of the very considerable import. The cessation of hostilities, however, has relieved the pressure somewhat, but the question of the practicability of high-voltage trunk lines is still one which will claim the attention of engineers and capitalists.

It is not my intention to go into the consideration of the technical features of the 220-kv. transmission project, believing that these problems can be solved as they are encountered.

The author points out that such a trunk line as he has in mind must be operated at a very high load factor, the loading per circuit being 100,000 kv. or higher.

With a transmission line having such a large capacity, continuity of service becomes at once the primary requisite and it is, therefore, necessary to take into consideration the fact that spare circuits must be erected to insure the service.

The cost of this insurance to the service, in the form of spare circuits, increases the fixed charges, per unit of energy delivered very materially, and it is doubtful whether a system operating less than three or four lines, delivering in the neighborhood of a half-million kv-a., could bear this expense and still prove to be an attractive financial investment.

The generation and distribution of an amount of power of the magnitude indicated as necessary to the financial success of such an enterprise would require exceedingly large expenditures of money and would have to be preceded by a thorough and radical reorganization of the engineering policies as at present contemplated by the central station industry.

In arguing for the erection of the large generating stations located in the coal fields, there are two basic considerations:

First: Savings in freight charges with the consequent release of railroad equipment for other uses.

Second: The opportunity of utilizing low grade fuel which would not be worth transporting.

There is some doubt as to the feasibility of carrying out a project of this nature, except on an exceedingly large scale, as the differential in favor of a plant at the distant energy source is quite small.

The erection of a plant in the coal mining district for instance would require that the following factors be very carefully investigated in arriving at comparative construction and operating costs:

1. Availability of cooling water and possible high cost of erecting dams, cooling ponds, etc.
2. Increased cost of boiler plant to provide grate and furnace space sufficient to burn low grade fuel.
3. Cost of transmission line right-of-way and cost of constructing and maintaining the line.
4. Line losses and transformer losses.
5. Labor costs and housing facilities for construction and operating force.

C. E. Howell: A study of Mr. Silver's paper appears to definitely bring out at least two apparent difficulties to be met in the construction and operation of the super-power transmission systems of the near future. These seem to me to be a matter of human inertia and commercial conservation rather than lack of ideas. These two points are:

First, the low mechanical strength of any high-voltage insulator now on the market;

Second, lack of methods of protecting a large portion of a transmission system from the effects of line trouble on a small section of it.

The practical application of any insulators on the market at the present time probably would necessitate the use of the usual disk with cap and pin, or the insulator known as the "Hewlett Disk." No other insulators have had sufficient application to eradicate apparent defects. These two types have an ultimate mechanical strength so low that it would undoubtedly be necessary to limit their ultimate load to approximately 3000 pounds per string of insulator units. To withstand the mechanical loads which will be impressed on insulators on high-voltage lines of the future, it would be necessary to use a large number of strings of insulators if the above types are employed. This would necessitate large expenditures for insulators, hardware, etc., as well as increasing the dimensions of the supporting structures and therefore their cost. To even imagine replacing a defective disk in a six string tension assembly with 17 disks per string on a line similar to the one described by the author of this paper, causes one to pause.

As a constructive criticism, it is suggested that the present types of insulator shapes be discarded for the moment and suspension units using comparatively large amounts of porcelain in compression to permit greater working loads per string of units, be employed. Small diameter insulators for low-voltage distribution lines and for guy anchors have been constructed on this principle with resulting mechanical strengths of comparatively enormous values, and it is believed that the same ideas may be extended to include the development of insulators for use on high-voltage large capacity lines. It is to be hoped that better ideas than this may be forthcoming soon, but the above suggestion should lead to a departure from the present practise of insulator construction and perhaps to an insulator worth perfecting.

The great transmission systems of the future will require better systems of protection from line trouble than those now employed. As systems and interconnected systems become larger and larger, difficulties of minimizing effects of scattered short circuits, etc., on the whole net work will multiply. The possibilities of obtaining switching equipment to successfully interrupt short circuits on individual circuits consisting perhaps of less than 5 per cent of a system, are good, but means for preventing this short or ground from demoralizing the remaining 95 per cent of a larger system are less easily perfected. Sooner or later service will demand some method of nearly instantaneously reducing the value of a short circuit to a reasonable figure (thus not effecting the remainder of the system), and later disconnecting the circuits in trouble. Most certainly a million kilowatt system with 220 kv. or even higher kv. transmission lines will not be permitted to lose say 800,000 kw. of its load because some small portion of the system is subjected to a "bump." The present-day method of interrupting a portion of the system in trouble and at the same time effecting the operation of the remainder for perhaps an hour or more, is analogous to applying brakes to a high speed train in such a manner that the tracks become unservicable, thus disrupting the schedule of the remainder of the railroad indefinitely, although undoubtedly stopping the train in question.

It is suggested as a means of accomplishing the results which it will soon be necessary to obtain in operation, that switching equipment be so designed, constructed and arranged that it will be possible to nearly instantaneously insert in a circuit in difficulty resistance or reactance, or a combination thereof, before interrupting such circuits. This would relieve the remaining portion of the system of some of the effects of the short circuit. This scheme will, undoubtedly, be recognized as the idea which was intended to be employed in the reactance type of oil switch which was placed on the market several years ago, and which has had a more or less successful career. It is believed that either resistance or reactance, or a combination thereof, may be employed to perform the function suggested above, but that it will be necessary to liberally construct the switching equipment employing this feature, in order that mechanical difficulties may be prevented. This may necessitate employing two switches, or two separate switching elements in one switch, or a switch containing two separate elements and the reactance or resistance.

There are undoubtedly other and better ways of solving this problem but whatever they may be, it is to be hoped that better means of switching large capacity systems, than now employed, may soon be available.

T. B. Parker: In the design of steel towers for transmission lines, it has been found necessary to fit each type of tower to definite conditions, which shall as nearly as possible represent the requirements of actual operation. As in the paper under discussion, it is usual to specify three sets of design loadings; (1) a vertical load, representing the dead weight of structures and conductor and the effect of vertical angles; (2) a transverse, horizontal load, representing the direct over-turning effect of wind and horizontal angles; and, (3) a torsional load to provide for the effect of broken wires and unbalanced loading.

In view of the large savings that can be made by small changes in detail, it is desirable, when possible, to subject each tower type to actual tests. It is best to make test loads identical with design loads, and, for the sake of simplicity, to consider each set of loads as separately applied. This means that a tower will be designed for the maximum stress produced by any one of the three non-simultaneous sets of loadings.

Line towers, however, are normally subjected to loads which are not purely vertical or horizontal, but a combination of both. It is therefore necessary to know the effect of many different load combinations upon each type of tower, before choosing the proper type for each location. To determine these effects requires much study, while their neglect may result in danger to the line, or in loss of economy.

It is evident that separate design and test loads do not lend themselves to convenient use in line design, while combined test loads would needlessly complicate the procedure and make comparisons difficult. The solution would appear to be the use of separate design and test loads, with the addition of combined loads, each made up of definite proportions of the original vertical and horizontal loadings. Thus, a tower designed to carry non-simultaneously certain vertical and horizontal loads might also be required to support three-quarters of the horizontal load, together with one-half of the vertical. Two or three such specified combinations suffice to definitely establish the characteristics of a tower and to clearly define its limitations.

This method has the advantage of retaining the separate, non-simultaneous test loads, but extends the design work to cover mutually consistent combinations. Possibilities of overload are avoided, and greater economy secured by taking advantage of the maximum strength of towers under all conditions.

J. B. Crane: The paper presented by Mr. Silver is very important at this time as it points out one way in which large systems can be interconnected to secure the maximum advantages of diversified loads.

On page 1087 the author shows a typical diagram and it is suggested that it might be possible to cutout the 220-kv. bus at the main substation and to use the 66-kv. oil switches on the low-tension side of the transformers. This is, of course, on the assumption that the substations will be operated at 66 kv. or 110 kv. and that further transformation will be necessary in case it is decided to furnish low-tension power from the substations.

The use of 220 kv. for transmission presents some very interesting operating problems and on account of the present high cost of building transmission lines it is suggested that a careful study should be made to replace insulators with current on the line in order to obviate the expense of building and maintaining duplicate transmission lines for any single service.

The use of fifteen insulators in one string should allow a factor of safety large enough for the proper testing and renewal of defective insulators. The writer has had some experience with operation at 110 kv. where we have been using seven insulators on tension and eight insulators on strains or semi-strains. The practise is to test the insulators once a year (this will probably be shortened to once in six months) and where three defective insulators are found in one string the trouble is immediately reported and the line gang replaces same as soon as possible. Where less than three defective insulators are found the matter is reported on the regular daily report and the insulators are changed at the earliest suitable time. There was one case recorded where the line was operating with four defective insulators in one string leaving only three good insulators for protection of the line.

It is suggested that it would be possible to put two clamps in series on each wire and make suitable suspension hangings above each of these clamps so that an entire new string of insulators could be placed on a tower to take care of any defective strings and suitable fittings could be made for fastening the insulator assembly to the clamps and to the tower so that same could be done without interrupting the service.

The writer believes that an experimental line one-half mile long should be built and operated at 220 kv. as soon as possible in order to work out some of these operating details and to ascertain just what kind of trouble would be likely to be encountered in practise.

A. E. Silver: Several speakers have mentioned the interlinking type of insulator as being especially suitable for 220-kv. service. However, due consideration must be given to the difficulties of mechanical assembly under the heavy loading requirements of 220-kv. construction, which with present designs of this type of insulator would be even more serious than with the standard cap and pin type.

Mr. Hanker's point as to the possibility of over excitation of generators by the line charging currents, resulting in abnormal voltages on the transmission line, is well taken. In specific cases careful consideration should be given to eliminating danger from this source.

Referring to Mr. Summerhayes' point regarding Figs. 1 and 2, the titles are misleading, although it is noted in the foot notes accompanying these figures that generator voltages of 225 kv. and 170 kv. and corresponding receiver voltages of 200 kv. and 150 kv. are used in the comparison.

It seems to me that the essential thing now is not to endeavor to draw any definite conclusions but to come to a thorough realization that the problem of developing 220-kv. transmission is definitely confronting us and to present clearly to the engineering profession the conditions of the problem and then to all work together to bring out the best possible solution.

Presented at the 35th Annual Convention of The American Institute of Electrical Engineers, Lake Placid Club, N. Y., June 27, 1919.

THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS II

The Effect of Lightning Voltages on Arrester Gaps, Insulators and Bushings on Transmission Lines

BY F. W. PEEK, JR.

Abstract of Paper

This paper treats of some of the practical applications resulting from an investigation of the effect of lightning voltages on insulators, bushings and protective gaps.

There is a great difference in the relative lightning spark-over voltages of various gaps as well as a great difference in the settings imposed by operating conditions. Both of these factors must be considered in comparing the relative protective values.

A gap must be set so that the normal line voltage does not cause it to spark-over. Gaps are generally used out of doors. Rain lowers the 60-cycle spark-over voltage of all uncovered gaps and thus imposes a greatly increased setting and decreased protective value since the lightning spark-over voltage is not changed by rain.

The covered sphere gives the maximum protection. The protective value is constant under all conditions.

The sphere-horn, having electrodes of points, horns and spheres, gives very good protection over the whole range of frequency and wave front. The spheres discharge the very steep waves, the horns the moderate ones, and the points continuous high-frequency waves, of slanting front and static.

The protective value of selective gaps varies with the wave front. Its protective value is a minimum for very steep wave fronts and for waves of slanting front. Over a certain range its protective value is very good.

The relative protective values of various gaps for steep and slanting wave fronts and high frequency are shown graphically in Figs. 14, 15, 16 and 17. The relative protective values are approximately independent of the point on the 60-cycle wave at which the discharge occurs.

Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

Bushings and insulators with equal 60-cycle spark-over voltages may have entirely different lightning spark-over voltages. A bushing should be designed for a high lightning spark-over voltage.

The lightning wet spark-over voltage of a bushing or insulator is the same as the dry spark-over voltage.

IN 1915 I presented a paper covering an extensive investigation on the effects of transient or lightning voltages on air, oil and solid insulations, line insulators and the discharge voltages of various gaps.[1] An exact study was made possible by the development of the "impulse generator," which was also described in the paper referred to above, and as a result some very important fundamental relations were discovered.

It is the purpose of this paper to treat in more detail a few of the important practical applications which have been made of these relations within the last few years. The present discussion will be concerned principally with lightning arrester gaps, bushings and line insulators. It is hoped in particular, to make clear the advantages and disadvantages from a strictly practical standpoint of the various arrester gaps.

The fundamental relations referred to above, bearing on the present discussion, will be briefly reviewed:

When a 60-cycle voltage is slowly applied to a gap and gradually increased, spark-over will occur at some definite voltage. This is the minimum voltage that will cause sufficient ionization for the gap to discharge and it requires a relatively long time.

Lightning voltages, or voltages of relatively steep wave front start at zero or line voltage and increase at the very rapid rate of millions or billions of volts per second. When such voltages are applied across a gap or insulator, spark-over does not occur at the instant the minimum or 60-cycle voltage is reached, as considerable time is required at this voltage. When this voltage is reached the spark begins to form but is only completed after the rapidly rising voltage has reached some higher value. The "slower" the gap the higher the voltage will rise. In a uniform field, break-down takes place over a relatively short path, everywhere, at the same time. In the case of a non-uniform field represented, for instance, by the needle gap, corona forms around the electrodes before spark-over. A vast amount of air must be ionized. The condition is equivalent to putting the corona or arc resistance in series with an ever increasing capacity represented by the unbroken dielectric. Time is thus required to bring all of the space between the electrodes up to the break-down gradient and during this time, the lightning voltage rises higher and higher.

To summarize: (1) Two gaps or insulators with equal 60-cycle spark-over voltages may have entirely different lightning or impulse spark over voltages because of the time lag.

---

1. "The Effect of Transient Voltages on Dielectrics," F. W. Peek, Jr., A. I. E. E., Vol. XXXIV, 1915, page 1857.

"Lightning," *General Electric Review*, July 1916.

(2) The time lag is the greatest in a non-uniform field or for electrodes where corona precedes spark-over; it is minimum for a uniform field.

(3) The time lag for any given electrodes and spacing is not constant, but depends upon the steepness of the wave or the rate at which the voltage is applied. The spark-over voltage increases and the time lag decreases with increasing steepness of wave front.

(4) Lightning or impulse spark-over voltages, unlike 60-cycle spark-over voltages, are not appreciably lowered by rain.

The above discussion means, of course, that certain gaps and insulators which have equal 60-cycle spark-over voltages may have entirely different lightning spark-over voltages. The ratio between the impulse and 60 cycle spark-over voltage was termed the impulse ratio. When there is no time lag the impulse ratio is unity; the greater the time lag, the higher the impulse ratio. Under certain conditions selective gaps may have an *apparent* impulse ratio of less than unity.

It is very important to utilize these principles in design; protective gaps should have an impulse ratio of unity or low lightning spark-over voltage, while insulators and insulation should have a high impulse ratio or high lightning spark-over and puncture voltage.

The practical application of these principles to various protective gaps will first be discussed.

PROTECTIVE GAPS

General. The lower the voltage at which a given arrester gap can be set the greater is its protective value. In practise, the setting must be such that the gap does not discharge under any normal operating condition. The 60-cycle spark-over voltage of a gap is very much decreased if the electrodes become wet. The decrease in voltage due to moisture differs greatly with the shape of electrodes. It is minimum for points and maximum for plane surfaces. The 60-cycle spark-over voltage of a gap may be affected by other surface conditions, but by far the greatest effect is that caused by moisture. See Table I. Practically all high-voltage arrester gaps are installed out of doors. These gaps must, therefore, be set so that the line voltage does not cause spark-over during a rain storm. This means that with any gap with "fast" electrodes the setting must be approximately doubled and the protective value thus reduced.

The wet and dry 60-cycle spark-over voltages of 6.25 cm. spheres is given in Fig. 1. That the lightning spark-over voltage is not appreciably changed by rain is shown in Fig. 2. In comparing the relative protective value of lightning arrester gaps it is, thus important to make the comparison on equal wet 60-cycle spark-over voltages or by the setting imposed by the operating conditions.

TABLE I

SPHERE GAP

The Approximate Effect of Rain, Ice, Dust, etc. on the 60-Cycle Spark-over Voltage of Sphere Gap.

Foreign material on sphere surface.	Voltage per cent of normal
Thin coating of dust	98
Coating of oil	100
Heavy coating of oil and sand	75 - 90
Thin coating of ice	75 - 90
Thick coating of ice	75 - 80
Surface oxidized	100
Ordinary pitting	90 - 100
Rain 0.2 in. precipitation per min. Polished spheres	40 - 50
Rain 0.2 in. precipitation per min. Pitted spheres	40 - 50

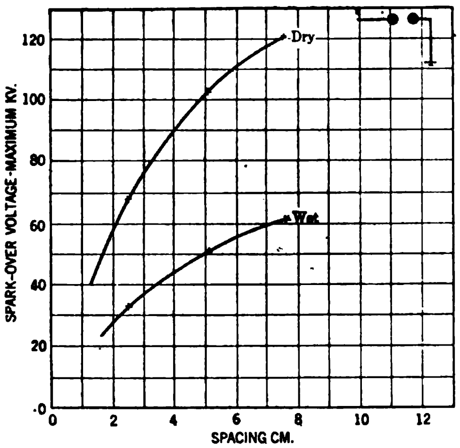


FIG. 1—SPHERES—WET AND DRY SPARK-OVER VOLTAGES

6.25 cm. spheres—06-cycles—one sphere grounded—0.2 in. rain—data Table II

Operating conditions other than rain or inherent properties of the arrester proper may make it necessary to increase the setting of certain types of gaps and not of others. Rain is, however, the chief factor in non-selective gaps. As an example of the effect of rain on the setting, assume a 66,000-volt line with grounded neutral. The voltage to ground is

$$\frac{66{,}000}{1.73} = 38{,}000$$

The arrester gap must be set at about 25 per cent above this or 47 kv. wet. Referring to Fig. 2, if the gap is protected from the weather the lightning spark-over voltage is 47 kv., wet or dry; if the gap is not protected from the weather the dry 60-cycle spark-over voltage must be 94 kv. in order to make the wet 60-cycle spark-over voltage 47 kv. and the apparatus is thus subjected to double the stress which would obtain if a covered gap were used. This follows because the lightning spark-over voltage approximately corresponds to the dry setting. There may be no gain in protection with a gap discharging at very low lightning voltages if in practise it must be set at a wide spacing to prevent line voltages from continually causing it to spark over.

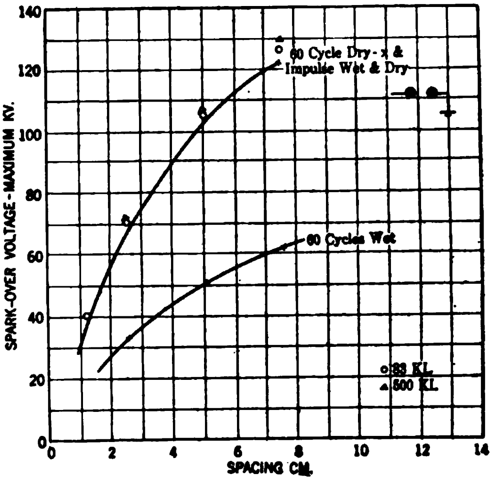


FIG. 2—SPHERE—WET AND DRY SPARK-OVER VOLTAGES

6.25 cm. spheres—60 cycles and impulses—0.2 in. rain—one sphere grounded—data Table II

The horn gap is not affected by the weather to as great an extent as the sphere gap. If a sphere and a horn are adjusted for equal wet 60-cycle spark-over voltages, the dry 60-cycle spark-over voltage of the horn will be lower. For low-frequency surges the horn would thus discharge at a lower voltage. For steep wave front lightning voltages however, the lag of the horn, which may easily have an impulse ratio of 2, will cause it to give inferior protection. See Fig. 3.

THE SPHERE GAP—THE SPHERE HORN

The sphere gap has an impulse ratio of unity. It thus offers equal protection for all sorts of transient voltages, and is without time lag when set at not greater than diameter spacing. When exposed to the weather, however, the setting must be high enough so that the line voltage will not spark-over during rain. See Figs. 1 and 2.

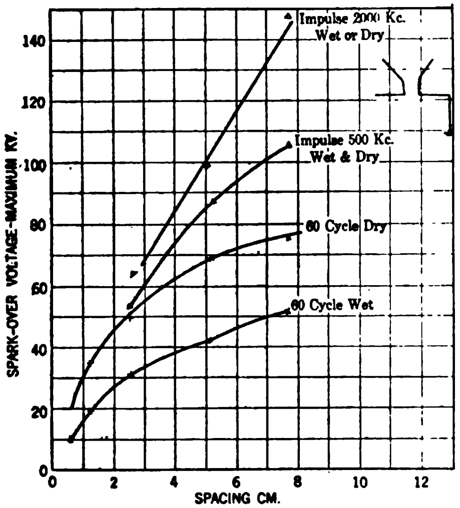


FIG. 3—HORNS—WET AND DRY SPARK-OVER VOLTAGES
60-cycle and impulse—wet and dry—data Table IV

In the practical gap the sphere and horn were combined; the horn being used to assist in breaking the dynamic arc and for the gain in discharging low-frequency surges due to the smaller difference between the wet and dry spark-over voltages. The difference between the wet and dry spark-over voltage of points is less than with the horn. A point is sometimes added to further increase the protection at low-frequency surges.

This gap has proved very successful in its several years of practical use, very greatly increasing the protective value of arresters. See Fig. 4. Take for comparison equal wet settings of 50 kv. at 2000 kilocycles the spark-over voltage of the horn is 135, the sphere 100. For impulses below 500 kilocycles, the spark-over voltage of the horn is lower than the sphere. Thus, when a sphere horn is used the discharge takes place across the sphere for steep wave fronts and across the horn for low-frequency surges. The gain due to the sphere is greater at higher voltages and steeper wave fronts. The

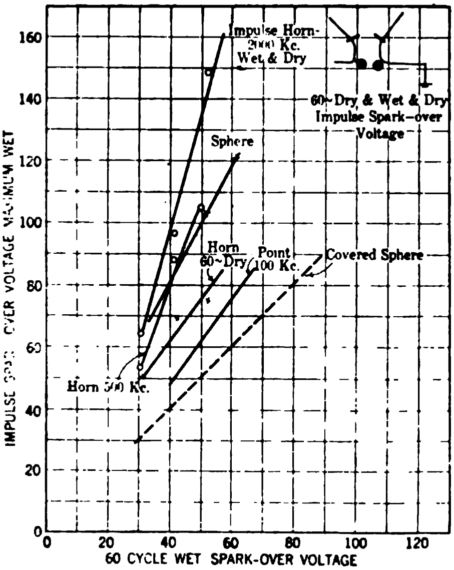


FIG. 4—SPHERE—HORN

Relative protective values of the component parts—data Tables II, IV, and V

covered gap, shown by the dotted line is superior at all wave fronts.

The Covered Sphere. If a sphere gap is covered and shielded from the weather its protective value is greatly increased since the setting imposed by the condition that the normal line voltage must not discharge over the gap is cut in half. Such a gap, therefore, discharges lightning voltages of half the value of the uncovered sphere. This gap gives the highest degree of protection. It is not possible to use it with all types of arresters since a horn is often necessary to assist in breaking the dynamic arc. See Fig. 5. A properly designed hemisphere may also be used in this type of gap.

A gap not appreciably affected by the weather and still providing an arc breaking horn may be built as shown in Fig. 6. The way this accomplishes the desired results will be described later.

Since the gap requires two spheres in series, it is necessary to determine if such an arrangement has appreciable time lag and, therefore, high lightning-discharge voltages. Two 6.25-cm. spheres connected in series are shown in Fig. 7. If gap *A* is set at approximately 25 kv. and gap *B* at approximately 75 kv. the 60-cycle spark is not (25 + 75) = 100 kv. but is 75 kv.; the lightning or impulse spark-over is 91 kv. See Fig. 7.

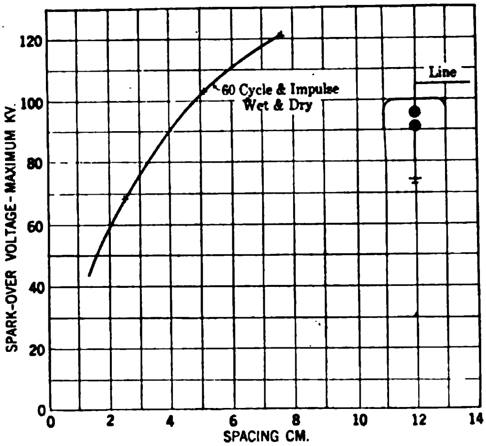


Fig. 5—Spheres—Wet and Dry Spark-Over Voltages

6.25 cm. spheres—covered gap—60 cycle and impulse—wet and dry—one sphere grounded—data Table V

The two gaps break down at 75 kv., 60 cycles, instead of the sum of the two or 100 kv. because the applied voltage does not divide evenly between them. The voltage reaches 25 kv. across the low-voltage gap and breaks that gap down before it reaches 75 kv. on the high-voltage gap. All of the stress is thus transferred to the high voltage gap and it breaks down as soon as a total voltage of 75 kv. is reached. If, now, capacities are adjusted across *A* and *B* so that the voltage divides in proportion to their relative break down voltages both gaps will break down simultaneously. The break down voltage will be equal to their sum. This is called the "balanced" gap. When two sphere gaps each without appreciable lag are placed in series it is found that unless the gaps are "balanced" there is considerable lag. This would be expected because one gap breaks down first and puts resistance in series with the capacity of the other similar to the corona in the needle gap. Balancing the gaps causes simultaneous spark-over and there is no appreciable lag. See Fig. 7.

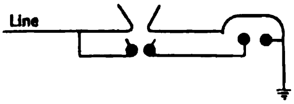


Fig. 6

If two gaps are placed in series as in Fig. 6 and balanced by properly adjusting their relative capacities there will be no appreciable lag. The rain affects only the outside gap. For example, if the outside gap is set at 10 kv. and the inside gap at 50 kv. the outside gap may be reduced to 5 kv. by rain. If balanced wet, the total wet spark-over is 55 kv. while the dry spark-over voltage is about 60 kv. This gap is thus without appreciable lag and not appreciably affected by rain. The only object of the outside gap is, of course, to transfer the dynamic arc to the horn where it rises and breaks.

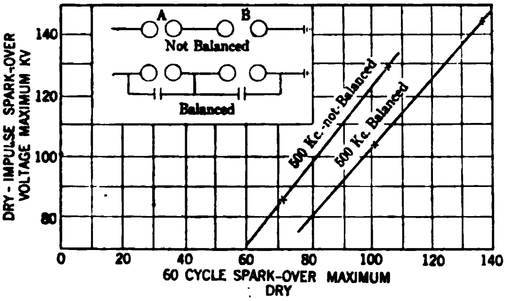


Fig. 7
Data Table III

The impulse and 60-cycle characteristics of this gap are shown in Fig. 8.

The advantage of the sphere gap is that it gives equal protection under voltages of all frequencies and wave front and is practically without lag.

Selective Gaps. Various forms of selective gaps have been proposed from time to time. Probably the most interesting and important of these is that investigated by Mr. Allcutt and shown in Fig. 9.[2] In this gap the division of voltage is not greatly affected at 60 cycles by the auxiliary electrode. The auxiliary electrode is held at mid-potential because it is connected at the mid-point between two equal condensers. The capacity current is too small at 60-cycles to cause any appreciable "drop" across the resistance. If the condenser circuit were opened on one side, the gap on that side would break down at about half voltage. This is exactly what happens under impulse.

For steep wave fronts the resistance has the effect of opening the condenser circuit on that side. See Fig. 10. The gap on that side breaks down. The voltage does not immediately disappear across the arc. The gap has lag for the same reason as the double unbalanced gap discussed above. Whether it is above or below the 60-cycle setting depends upon the impulse. The effect is similar to that which would result from a needle gap which could be set at, for instance, 100 kv. for 60-cycle operation and instantly and automatically reduced

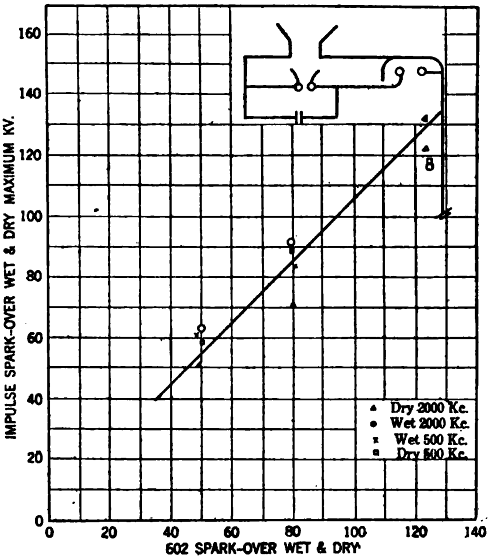


FIG. 8—WET AND DRY SPARK-OVER COVERED DOUBLE GAP BALANCED
Data Table VI

---

2. "Lightning Arrester Spark Gaps," C. T. Allcutt, TRANS. A. I. E. E., 1918, Vol. XXXVII, Part II, p. 855.

to a 50 kv., 60-cycle setting whenever an impulse came on the line. For moderately steep wave fronts the spark-over voltage would be greater than 50 and less than 100 kv.; but for very steep wave fronts, the impulse ratio of the 50 kv. gap would be greater than (2), or the spark-over voltage would be greater than 100 kv. The impulse ratio of the selective gap is always greater than unity; the *"apparent" impulse ratio* is greater or less depending upon the steepness of the wave front. The reason this distinction is made is discussed elsewhere.[3] The apparent impulse ratio should be used in comparing protective values. This characteristic for the selective gap is shown in Fig. 11 and compared with a sphere gap for the same dry 60-cycle settings. The sphere gap spark-over voltage is practically constant for all wave fronts. The spark-over voltages are the same for 60 cycles. At moderate wave fronts the selective gap has about 5 to 20 per cent lower spark-over voltage than for spheres, while for steeper wave fronts the voltage is higher on the selective gap. The protective value of a gap, as already pointed out, depends not only on its lightning discharge voltage for a given 60-cycle setting, but also upon the setting which is imposed upon it by operating conditions. Fig. 11 shows the relative protective values of spheres and

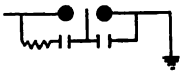


FIG. 9

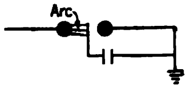


FIG. 10

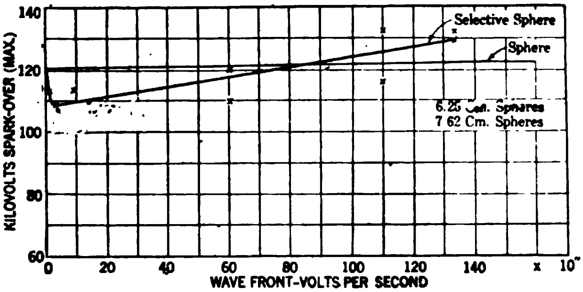


FIG. 11—VARIATION OF SPARK-OVER VOLTAGE WITH WAVE FRONT—SPHERE AND SELECTIVE SPHERE—

Data Table VII

---

[3] "Lightning Arrester Spark Gaps," C. T. Allcutt, TRANS. A. I. E. E., 1918, Vol. XXXVII, Part II, p. 855.

selective spheres assuming equal dry 60-cycle settings are possible. The settings must be such that the line voltage does not frequently spark-over and cause the destruction of the energy absorbing device under certain operating conditions. The effect of rain makes it necessary to set a non-shielded selective gap at about double the voltage that would be necessary in the protected gap. See Fig. 12.

Other forms of selective gaps have been devised and it is possible to extend the selective principle to a number of gaps in series, theoretically (neglecting lag) making it possible to discharge an impulse at a small fraction of line voltage. Such a gap would of course necessitate high initial setting and give very little protection against lightning impulses.

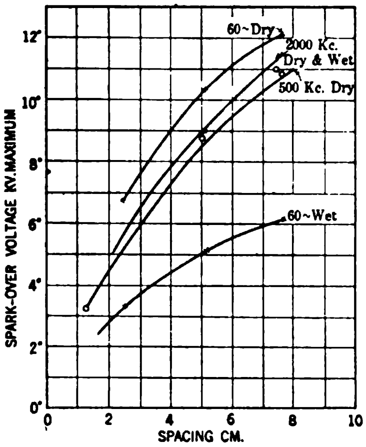


FIG. 12—SELECTIVE SPHERE

60-cycle and impulse—wet and dry—Data Table VIII

The selective principle may also be readily applied to covered gaps if it is deemed advisable.

RELATIVE PROTECTIVE VALUES OF THE GAPS ALREADY DISCUSSED

The following comparison of the different gaps is the result of extensive research. The tests were made with the impulse generator. The methods of conducting the tests, the precautions, accuracy, etc. are the same as discussed in the former paper.[4] For convenience the connection diagrams are shown in Fig. 13—(a) is used when the impulse only is applied to the gap; (b) when the impulse is superposed on the 60-cycle wave.

There are three cases which require consideration.[5]

Case A. Where the impulse occurs at the zero point of the 60-cycle wave and is thus not affected thereby.

Case B. Where the impulse occurs at the maximum of the 60-cycle wave and is additive.

Case C. Where the impulse occurs at the maximum of the 60-cycle voltage wave, but in the opposite direction.

Case C is naturally the most dangerous case. Case A is equivalent to applying the impulse without 60-cycle voltage.

In making impulse tests it is found that there is a certain minimum impulse voltage that will spark-over the gap only occasionally and that the voltage must be increased to cause spark-over at every application. This difference between minimum spark-over and spark-over at every application may be inappreciable for some gaps and quite appreciable for others. This is discussed in my former paper. It is, thus, obviously necessary to record the method of making the tests. The data for all of the curves with description of method of making measurements will be found in the tables. Unless otherwise stated, the impulse voltage was increased until the gap sparked once in five applications. "Frequency" is used for convenience only. It means, unless otherwise stated, that an impulse approximating a single half sine wave of the frequency stated has been used. By applying a super-voltage or a voltage higher than the discharge voltage of the gap the steepness of

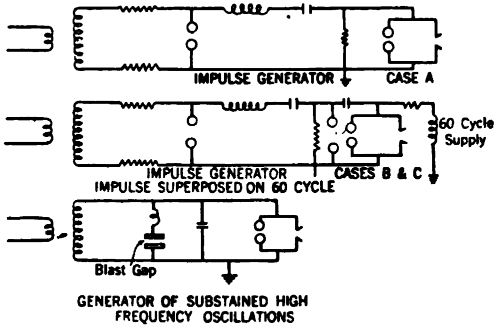


FIG. 13—CONNECTIONS USED IN MAKING TESTS

---

4. "The Effect of Transient Voltages on Dielectrics," A. I. E. E., F. W. Peek, Jr., 1915.

5. "Lightning Arrester Spark Gaps," C. T. Allcutt, A. I. E. E., May 1918. Discussion, F. W. Peek, Jr., A. I. E. E., June 1918.

the wave or rate of application is increased. For instance a super-voltage at 100 kilocycles might be steeper than a lower voltage at 2000 kilocycles. In making measurements by adjusting two gaps in parallel for equal spark-over, as is done in the case of super voltages, it is important to arrange the gaps symmetrically with equal length of leads; otherwise inconsistent results will be obtained. With steep waves and unsymmetrical leads it is possible to short-circuit one gap and still obtain sparks on the other.

The over-voltages that cause insulation failures in practise may be divided into three classes:

1. Gradual increase of voltage on the line due to static or low frequency surges.
2. Very high frequency oscillations of voltages generally too low for any gap arrester to discharge, but which may cause very high internal voltages in apparatus.
3. The form of voltage with which we are principally concerned—lightning voltages of very steep wave fronts where the voltage across the apparatus increases from normal to a very high value in perhaps a millionth of a second.

Condition. (1) is readily taken care of by any gap and need not be further discussed; (2) is of some interest but is a condition generally not taken care of by a gap arrester—some results of tests will be given however; (3) is the steep wave front condition that represents lightning proper and with which we are mostly concerned.

Relative Protective Value of the Horn, Sphere-horn, Selective Sphere Gap and Covered Gap.

Impulse Voltages of Steep Wave Front. The spark-over voltages of various types of gaps are plotted with equal wet 60-cycle settings in Fig. 14. Values are plotted for both wet and dry electrodes. The wave applied was a single half-cycle of a 2000 kilocycle wave with a 340-kv. maximum; that is, at super-voltage. The rate of application of voltage of the wave front was thus about 70×10^{11} volts per second. I believe that waves steeper than this occur on lines in practise. In fact, I first noted that there was a difference between the 60-cycle and lightning spark-over voltages of various electrodes by the existence of such waves on an operating line. The bushings on the line always "protected" the lightning arrester horns although the horns had a lower 60-cycle spark-over voltage. By measuring the impulse spark-over voltages of the bushing and the arrester gap in the laboratory it was found that the bushing protected the horn for a wave front at which the impulse ratio of the horn was over (2); this corresponds to a steeper wave than the one under immediate discussion.

It will be noted that the covered gaps give by far the best protection under this condition. For example, when all the gaps are set on the line at 100 kv., lightning voltage discharges respectively at 100 kv. on the covered gap, 115 kv. on the balanced covered gap, 225 kv. on the sphere of the sphere horn, 225 kv. on the selective sphere, and 320 kv. on a horn.

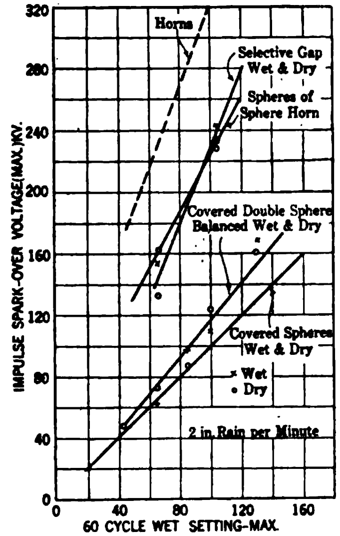


FIG. 14—RELATIVE PROTECTIVE VALUES OF HORNS, SPHERE-HORNS, SELECTIVE SPHERES. COVERED SPHERES—STEEP WAVE FRONTS

2000 kilocycles—340 kv.—impulse—data Tables IX, X and XI

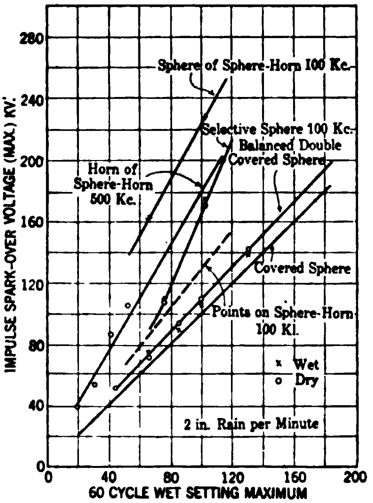


FIG. 15—RELATIVE PROTECTIVE VALUES OF SPHERE-HORN, SELECTIVE SPHERES, COVERED SPHERE—MODERATE WAVE FRONTS

Impulse—single half-cycle 100 kilocycles—non-grounded—data Tables IX, X and XI

Moderate Wave Fronts. A similar comparison is given in Fig. 15 for moderate wave fronts. The impulses being single half cycles of 100 kilocycle waves, the average fronts ranging from 0.5 to 1. $\times 10^{11}$ volts per second.

It will be noted that here, also, the covered spheres give the best protection. For example, at a 100-kv. line setting the impulse spark-over voltages are respectively 100 kv. for the covered sphere, 110 kv. for the balanced covered sphere, 170 kv. for the selective sphere, 178 kv. for a horn or the horn of the sphere horn, 130 kv. for points of the sphere-horn, and 222 kv. for the sphere. If these data are compared with that in Fig. 14 the value of the sphere horn combination is well illuslustrated. For the steep wave fronts the sphere affords the better protection, while for the moderate waves the horn affords the better protection and a still greater gain is made by adding points. This comes about, of course, due to the difference between the wet and dry setting.

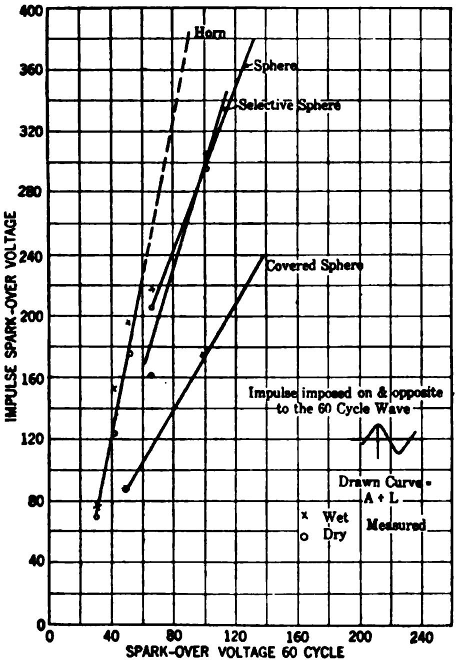


FIG. 16—RELATIVE PROTECTIVE VALUES OF VARIOUS GAPS

Impulses imposed on and opposite to the maximum of the 60-cycle wave—data Table XII

Ratio for Comparing the Relative Protective Value of Various Gaps. From the above discussion it is readily seen that in order to compare the relative protective value of various gaps two factors must be considered.

1. The increased 60-cycle setting imposed by operating conditions to prevent the gap from continuously discharging *due to rain or harmless surges.* Let the ratio of the actual operating setting to the normal setting be called α where the normal setting is the setting that just prevents the line voltage from arcing over under ideal conditions.

2. The impulse ratio (or apparent impulse ratio for the selective gap) for the wave under consideration. Let the impulse ratio be called β

The relative protective value of two gaps is then $\frac{\alpha_1 \beta_1}{\alpha_2 \beta_2}$

For example:—a gap must be set at 50 kv. (max.) to prevent the 60-cycle line voltage from causing it to spark-over under ideal conditions. The relative protective values of a horn and a covered sphere for the 2000 kilocycle wave are obtained as follows from Figs. 3 and 5.

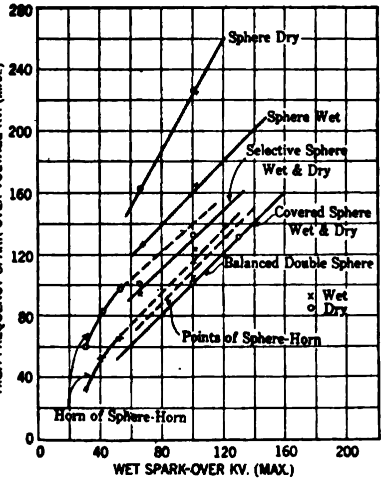


Fig. 17—Relative Discharge Values Spheres, Selective Spheres, Covered Spheres,—50,000 Cycle Sustained Oscillation—
Data Table XIII

Horn	Covered Sphere
$\alpha_1 = 75/50 = 1.50$	$\alpha_2 = 50/50 = 1$
$\beta_1 = 133/75 = 1.77$	$\beta_2 = 50/50 = 1$
$\alpha_1 \beta_1 = 1.50 \times 1.77 = 2.65$	$\alpha_2 \beta_2 = 1$
$\alpha_1 \beta_1 / \alpha_2 \beta_2 = 2.65$	

The horn permits the lightning voltage to rise to 2.65 times the value of the voltages permitted by the covered sphere.

Combination of Lightning and 60-cycle Voltages. The lightning spark-over voltage is a minimum when it occurs at the maximum of the 60-cycle wave and in an additive direction (case B). The lightning voltage is a maximum when it occurs at the maximum of the 60-cycle wave but in the opposite direction (Case C). The relative effects are approximately the same for all of the types of gaps discussed. If the lightning voltages for case A, Case B, Case C and the 60-cycle line voltage are called A, B, C and L respectively the lightning spark-over voltages are approximately:

Case A. A
Case B. $A - L$
Case C. $A + L$

Data for Case C are plotted in Fig. 16.

High Frequency Oscillations. The effect of sustained high frequency oscillations not very highly dampened is shown in Fig. 17. For connections see Fig. 13 (c). It is probably very rarely that oscillations with such a low damping factor occur on a transmission line. The arcing ground condition is more nearly approximated by a series of the impulses discussed above. Note that the horn and points give good protection for sustained oscillations.

LINE INSULATORS, BUSHINGS AND INSULATION

General. Line insulators and bushings should have a high impulse ratio or lightning arc-over voltage. The bushing mentioned above as protecting the horn had a low impulse ratio. The 60-cycle and lightning spark-over voltages were nearly equal. The horn would have given protection in this case if the impulse ratio had been higher. Bushings are now designed with a high impulse ratio.

The 60-cycle spark-over voltage of a bushing or insulator is often very appreciably lowered by rain. It is fortunate, however, that the lightning spark-over voltage is not appreciably changed by rain.

The data below were taken on different lengths of strings of Hewlett disk insulators. The impulse was a single half-cycle of a 200-kilocycle wave, or of very moderate wave front.

SPARK-OVER VOLTAGES—DISK INSULATORS
(One side Grounded—Dry)

No. of units	60-cycle spark-over	Impulse spark-over 200 kc.	Impulse ratio	String efficiency 60-cycle	String efficiency impulse 200 kc.
1	80	85	1.06	....	....
2	142	167	1.18	0.87	0.98
3	204	262	1.28	0.85	0.99
4	261	345	1.36	0.81	1.01
5	317	410	1.30	0.79	0.97
6	368	....	....	0.77	....

$$\text{String efficiency} = \frac{\text{Measured spark-over of string}}{\text{Number of units in string} \times \text{spark-over voltage of one unit}}$$

It is interesting to note that even with this moderately steep wave front the lightning spark-over voltage is approximately the product of the spark-over voltage of a single unit and the number of units in a string. The impulse voltage increases with increasing steepness of wave front.

The wet impulse spark-over voltage is approximately the same as the dry.

Impulse ratios of three or more have been obtained on bushings. More complete data on line insulators have been published elsewhere.[6]

CONCLUSIONS

1. There is a great difference in the relative lightning spark-over voltages of different gaps as well as a great difference in the settings imposed by operating conditions. Both of these factors must be considered in comparing the relative protective values. It is shown that if the lightning spark-over factor is represented by the impulse ratio β, and the setting factor by α, the relative protective value of two gaps are inversely as $\frac{\alpha_1 \beta_1}{\alpha_2 \beta_2}$.

2. β is due to time lag; α to the fact that the gap must be set so that the normal line voltage will not cause it to spark-over. Rain lowers the 60-cycle spark-over voltage of all gaps, and thus affects the ratio α. Rain does not lower the lightning spark-over voltage.

3. All uncovered gaps require a high setting factor, because rain lowers the spark-over value at 60 cycles. The effect is much less for points than it is for spheres.

4. The covered sphere gives better protection than any uncovered gap, because both α and β are low. The protective value is constant under all conditions.

5. The sphere-horn, having electrodes of points, horns, and spheres, gives very good protection over the whole range of frequency or wave front, due to the different values of α and β for its various electrodes. The spheres discharge the very steep waves, the horns the moderate ones, and the points continuous high frequency, waves of very slanting front and static.

6. The protective value of the selective gaps, as the name

---

[6] "Factors Determining the Safe Spark-over voltage of Insulators and Bushings for High-Voltage Transmission Lines," F. W. Peek, Jr., *General Electric Review*, June 1916.

implies, varies with the wave front. Its protective value is a minimum for very steep wave fronts, and for waves of slanting front. Over a certain range its protective value is very good.

7. The relative protective values of various gaps are shown in Figs. 14, 15 and 16.

8. The relative protective values of various gaps are approximately independent of the point on the 60-cycle wave at which the discharge occurs, as shown in Figs. 14, 15 and 16.

9. Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

10. Bushings and insulators with equal 60-cycle spark-over voltages may have entirely different lightning spark-over voltages. A bushing or insulator should be designed for a high impulse ratio or lightning spark-over voltage.

11. The lightning wet spark-over voltage of a bushing or insulator is the same as the dry spark-over voltage.

APPENDIX

TABLE II

SPHERE GAP

Wet and dry 60-cycle and impulse spark-over.
6.25-cm. spheres—One grounded 2 in. rain per minute.

Spacing		Spark-over voltage (maximum)			
Inch	Cm.	60-cycle and impulse dry	60-cycle wet	Impulse 83 kc. wet	Impulse 500 kc. wet
0.5	1.27			40	
1.	2.54	68	33	71	72
2.	5.08	103	51	105	106
3.	7.62	121	61	126	130
CASE B Impulse assisting and at maximum of 60-cyce wave.					
		Impulse dry 500 kc.			
1	2.54	52			
2	5.08	60.5			
3	7.62	86			
CASE C Impulse opposing and at maximum of the 60-cycle wave.					
1	2.54	108			
2	5.08	148			
3	7.62	160			

No parallel gap—one discharge in five applications 60-cycle voltage ½ dry discharge voltage of the gap in cases B and C.

TABLE III
60-CYCLE AND IMPULSE SPARK-OVER OF TWO GAPS IN SERIES.

6.25-cm. spheres.
One sphere grounded.

Spacing gap				60 cycle spark-over				Impulse spark-over	Impulse ratio
A		B		A	B	Sum A and B	Measured A and B		
Inch	Cm.	Inch	Cm.					500 kc.	
		Gaps not balanced for simultaneous spark-over							
0.45	1.19	1.02	2.59	35	68	103	72	86	1.20
0.45	1.19	2.15	5.46	33.7	107	140	106	129	1.21
		Gaps balanced for simultaneous spark-over							
0.45	1.19	1.02	2.59	35	67	102	101	103	1.02
0.45	1.19	2.15	5.46	33.7	107	140	136	144	1.06

Impulse voltages applied and increased in values until one spark-over occurred in five applications. No. 60-cycle voltage on the gap.

TABLE IV
HORNS

Wet and dry 60-cycle and impulse spark-over
Voltage of horns (3/8 in.)
0.2 in. rain—one horn grounded.

Spacing		Spark-over voltage—maximum					
		Dry			Wet		
Inch	Cm.	60 cycle	500 kc.	2000 kc.	60 cycle	500 kc.	2000 kc.
0.25	0.63	19	....	....	9.2	....	....
0.5	1.27	35	39.7	44.5	19.7	....	....
1	2.54	50	53	63.5	31.0	....	45
2	5.08	69	88	96.5	41.5	....	99
3	7.62	74.5	105	148	52	117	148

No parallel gap.—One discharge in five applications. 2000 kc. (b) wave Table XIV.

TABLE V
COVERED SPHERES
Wet and Dry—60-cycle and Impulse Spark-over Voltage
6.25-cm. spheres—One sphere grounded.

Spacing		Spark-over Voltage—Maximum					
Inches	Cm.	60 cycles dry	500 kc. dry	2000 kc. dry	60 cycles wet	500 kc. wet	2000 kc. wet
Case A—No. 60-cycle voltage on the gap.							
0.50	1.27	....	....	....	....	....	....
1.00	2.54	68	68	68	68	68	68
2.0	5.08	103	103	103	103	103	103
3.0	7.62	121	121	121	121	121	121
Case B—Impulse assisting and at maximum of the 60 cycle voltage.							
1	2.54	....	52	....	....	....	....
2	5.08	....	60.5	....	....	....	....
3	7.62	....	86	....	....	....	....
Case C—Impulse opposed to and at maximum of the 60-cycle voltage.							
1	2.54	....	108	....	....	....	....
2	5.08	....	148	....	....	....	....
3	7.62	....	160	....	....	....	....

No parallel gap—One discharge in five applications—60 cycle voltage ½ dry discharge voltage of the gap applied in cases B and C. 2000 kc (b) wave Table XIV.

TABLE VI
Double Covered Sphere Gap—Balanced
Balanced Wet
(0.2 in. Rain—6.25-cm. spheres—One sphere grounded)

Total 60-cycle spark over max.	Gap setting Kv.max. dry		Impulse spark over total kv. max.	
	A	B	Wet	Wet
			500 kc.	2000 kc.
50	14	42	59	63
79	23.5	71	89	91
124	35	105	118	117
			Dry	Dry
48	14	42	61	51
80	23.5	71	83	71
123	35	105	132	122

Average measured voltages. 2000 kc. (b) wave Table XIV.

TABLE VII

SELECTIVE SPHERE-HORN

(6.25-cm. Spheres with Auxiliary Electrode). One side grounded.

Gap setting		60-cycle spark-over kv max.	60-cycle spark-over ½ gap kv. max.	Impulse spark-over total gap	Apparent impulse ratio	Impulse ratio
In.	Cm.					
Single half cycle 83 kilocycles						
½	1.27	38	23	28	0.74	1.21
2	5.08	105	67	81	0.77	1.20
3	7.62	121	88	113	0.94	1.29
Single half cycle 500 kilocycles						
½	1.27	38	23	32	0.84	1.40
2	5.08	105	67	87	0.83	1.30
3	7.62	121	88	109	0.90	1.24
Single half cycle 2000 kilocycles (a) wave						
½	1.27	38	23	34	0.90	1.48
3	7.62	121	88	114	0.94	1.30
				Impulse spark-over total gap	Apparent impulse ratio	Wave front volts sec.
3	7.62	121	88	113	0.94	0.4×10^{11}
3	7.62	121	88	109	0.90	2.1×10^{11}
3	7.62	121	88	114	0.94	9.1×10^{11}
3	7.62	121	88	110–120*	.91–1.00	60×10^{11}
3	7.62	121	88	106–133*	0.88–1.10	110×10^{11}
3	7.62	121	88	130–132*	1.07–1.10	133×10^{11}

Above data for Case A—Impulse applied at zero of 60-cycle wave.
(*Range of equal spark-over.—Super Voltages, (b) wave Table XIV.)

TABLE VIII
SELECTIVE SPHERES

Wet and Dry 60-cycle and Impulse spark-over 6.25-cm sphere—One Grounded
0.2 in. Rain per minute

Case A							
Spacing		60-cycle spark-over Kv. max.		Impulse spark-over kv. max.			
Inches	Cm.	Dry	Wet	Dry		Wet	
				500 kc.	2000 kc.	500 kc.	2000 kc.
0.5	1.27	....	....	32	34	....	....
1	2.54	68	33	49	....	41	48.5
2	5.08	103	51	87	....	74	89
3	7.62	121	61	109	114	104	113
Case B—Impulse assisting and at the maximum of the 60-cycle wave.							
1	2.54	....	....	54	....	....	....
2	5.08	....	....	54	....	....	....
3	7.62	....	....	67	....	....	....
Case C—Impulse opposing and at max. of 60-cycle wave.							
1	2.54	....	....	97	....	....	....
2	5.08	....	....	115	....	....	....
3	7.62	....	....	147	....	....	....

Applied 60-cycle voltage ½ dry 60-cycle voltage of the gap and impulse one discharge in five applications. (2000 kc (a) wave Table XIV.)

TABLE IX
SELECTIVE SPHERES

12.5-cm. Spheres—Non-Grounded Impulses
Case—A

Setting		60-cycles spark-over Kv. (max.)		Impulse		Spark-over	
Inches	Cm.			2000 kc. 340 kv. (max.)		100 kc. single-half cycle.	
		Dry	Wet	Dry	Wet	Dry	Wet
2.80	7.10	162	66	133	153	107	109
5.34	13.5	227	102	233	242	170	174

Super Voltage Impulses measured at equal spark-over. 2000 kc. (b) wave.

TABLE X
COVERED DOUBLE SPHERE GAP—BALANCED
7.6-cm. spheres—Non-grounded

Total 60-cycle spark-over kv. (max.)		Setting—60-cycles kv. (max.)				Impulse spark-over kv. max.			
						2000 kc. 340 kv.		100 kc.	
Dry	Wet	Outside dry	Outside wet	Inside dry	Sum wet	Dry	Wet	Dry	Wet
....	130	63.0	34	101	136	160	168	142	138
....	100	51.0	25.3	75	100.2	123	109	109	105
....	85	38.3	19.8	61	80.8	85.5	96.7	93	88
....	65.5	29.6	....	49.6	....	71	62	71	73
....	44	14.1	....	31.1	....	46.5	46.5	50.5	....

Super-voltage impulses measured at equal spark-over. (2000 kc (b) wave Table XIV.)

TABLE XI
SPHERES—IMPULSES
12.5-cm. Spheres—Non-grounded

Spacing		Spark-over kv. max.		Impulse Spark-over voltage kv. max.			
Inch	Cm.	60-cycle	60-cycle	(2000 kc.) 340 kv. max.		100 kc.	
		Dry	Wet	Dry	Wet	Dry	Wet
2.80	7.10	162	66	164	....	162	....
5.34	13.5	227	102	227	227	....	....

Super-voltage impulses measured at equal spark-over.
2000 kc (b) wave Table XIV.

TABLE XII

IMPULSE SPARK-OVER VOLTAGES WHEN THE 60-CYCLE LINE VOLTAGE IS ON THE GAP.—CASE C.

Spacing		60-cycle spark-over		Applied 60-cycle	Impulse spark-over kv. max. 2000 kc.					
Inches	Cm.	Kv. max.		Kv.	Case A No. 60-cycle		Sum A & L.		Case C measured impulse	
		Dry	Wet	L.	Dry	Wet	Dry	Wet	Dry	Wet
Selective spheres 12.5-cm.										
2.80	7.10	162	66	50	138	135	188	185	160	161
5.34	13.5	227	102	76.5	227	....	303	303	296	305
Spheres 12.5 cm.										
2.80	7.10	162	66	50	....	164	214	....	204	217
5.34	13.5	227	102	76.5	227	227	303	303	305	305
Covered Spheres 6.25-cm.										
0.70	1.8	50	50	37.5	50	50	87.5	87.5	87.5	87.5
1.58	4.0	100	100	75	100	100	175	175	175	175
Horns										
1	2.54	51	30.7	23	58.5	60	81.5	83	69	76
2	5.08	40.6	67.5	30.5	105	110	135.5	140.5	124	153
3	7.62	52	75	39	145	150	184	189	175	196

Case C—Impulse applied opposed to the maximum of the 60-cycle wave.
Applied 60-cycle kv. 75 per cent of wet 60-cycle spark-over.
Gaps not grounded. One discharge in five impulses.
2000 kc. (b) wave Table XIV.

TABLE XIII
SPARK-OVER VOLTAGES OF GAPS FOR SUSTAINED HIGH FREQUENCY OSCILLATIONS.

Gap-setting		Frequency of oscillations—Spark-over kv. max.									
Inches	Cm.	19,600		50,000		100,000		200,000		160,000 from high freq. coil	
		Dry	Wet	Dry	Wet	Dry	Wet	Dry	Wet	Dry.	Wet
Selective Spheres 12.5-cm.											
2.80	7.10	112	105	100	95	77	90	77	92	108	125
5.34	13.5	200	143	131	123	126	127	120	122	170	186
Spheres 12.5 cm.											
2.80	7.10	162	106	162	126	163	106	162	118	163	....
5.34	13.5	227	173	227	164	12.27	173	227	169	227	....
Double gap—Balanced—Covered											
Setting Kv.-60 cycles Wet											
	100	....	98	....	105	....	102	93	100	112	112
	130	....	107	....	126	....	128	132	124	147	150
Covered Spheres											
....	50	50	50	....	....	....	50	50	50	50	50
....	100	100	100	....	....	....	100	100	100	100	100
Inches	Cm.	**Horns**									
1	2.54	....	....	58	33.5	....	....	....	....	....	....
2	5.08	....	....	82	54	....	....	....	....	....	....
3	7.62	....	....	97	67	....	....	....	....	....	....
Points											
1	2.54	....	....	28.5	28.5	....	....	....	....	....	....
2	5.08	....	....	46	46	....	....	....	....	....	....
3	7.62	....	....	64	64	....	....	....	....	....	....
4	10.2	....	....	74	74	....	....	....	....	....	....
5	12.7	....	....	88	88	....	....	....	....	....	....

TABLE XIV
IMPULSE GENERATOR CONSTANTS

Impulse	Wave	Capacity farads	Inductance henrys	Resistance
83	Single Half Cycle—Sine	1.25×10^{-9}	3.73×10^{-3}	1000
100	Single Half Cycle—Sine	1.25×10^{-9}	3.73×10^{-3}	2000
500	Single Half Cycle—Sine	0.62×10^{-9}	0.28×10^{-3}	750
2000	Single half Sine (a)	0.625×10^{-9}	3.34×10^{-5}	450
2300	(b) wave	1.25×10^{-9}	8.7×10^{-5}	3000

DISCUSSION ON "THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS II" (PEEK), LAKE PLACID, N. Y., JUNE 27, 1919.

Wm. A. Del Mar: Mr. Peek showed a series of curves giving the spark-over voltages for different gaps. I want to ask him whether it may be deduced from these curves that the Institute standard spark-gaps are correct with either dry or wet spark balls for voltages having a frequency of one million or more cycles per second.

F. W. Peek: Replying to Mr. Del Mar, the Institute standard sphere gap curves read correctly for either 60-cycle or transient voltages when the electrodes are dry. When the sphere surfaces are wet the gap sparks over at about half this value at 60 cycles, but for transients of the order of a single half cycle of a one-mlliion-cycle sine wave the spark over voltage is practically as given on the standard curves. There would, of course, be considerable error for continuously applied undamped high-frequency voltages.

C. T. Allcutt: Perhaps the most serious fault with Mr. Peek's paper is the presentation of a large amount of experimental data on discharge gaps, without giving any description whatever of the actual gap structures tested by him. He gives a diagrammatic representation of his ingenious balanced double-sphere gap and describes the theory of its operation, but his description of the gap structure itself is limited to the figures regarding the size of spheres employed. Whether the gap tested was a laboratory model or a commercially practicable gap such as might be applied to an electrolytic arrester has been left to the imagination of the reader. It is impossible for other experimenters to check Mr. Peek's results without further information concerning his apparatus.

Even more objectionable is his failure to describe the structure used by him in testing the type of selective gap now known commercially as the impulse gap. I wish particularly to call attention to the data given in Table VII. These data were given by Mr. Peek at the Atlantic City Convention last June, in his discussion of my paper on "Lightning Arrester Spark Gaps."[1] These data, which apparently contradicted my own, were presented by Mr. Peek without any description being given of the gap structure used in his experiments, the natural inference being that he had used a gap structure built exactly according to the description given in my paper. The fact that he presented these apparently contradictory data without giving sufficient information to enable the reason for the discrepancy to be pointed out was called to his attention in the closure to that discussion.[2] It is unfortunate that Mr. Peek has not

---

[1] C. T. Allcutt, "Lightning Arrester Spark Gaps,", TRANSACTIONS, A. I. E. E. Volume XXXVII Part II, p. 833. F. W. Peek, Discussion, TRANSACTIONS A. I. E. E., Vol. XXXVII. Part II, p. 855.

[2] C. T. Allcutt, Discussion TRANS. A. I. E. E., Vol. XXXVII. Part II, p. 867.

taken advantage of this opportunity to correct the omission. Following are a few of the facts that should have been included in his paper and without which his data are of little value.

(1) Nature and capacity of condensers employed.

(2) Nature and resistance of unbalancing resistors employed.

(3) Size, shape and position of auxiliary electrode.

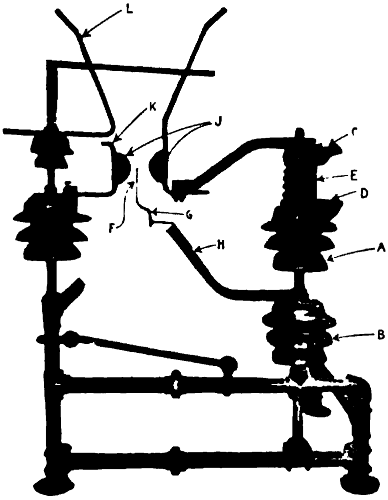
(4) Position of the various conducting elements of the gap structure which determine the capacity between the various electrodes.

The same objection applies to the data given in Table IX, although in this case there is no direct implication that the gap tested was substantially the same as the one described by me last June. In fact, some of the data given in Table IX check fairly well with some results of tests that have been made in our laboratory on the standard commercial impulse gap structure so it is entirely possible that some of Mr. Peek's tests were made on such a gap. The difference existing between Mr. Peek's results and those given in the curves accompanying this discussion may then be due either to a gap structure differing materially from the one used in my tests, or, to improper adjustment of the gap electrodes and the fact that Mr. Peek's tests were made with non-grounded impulses. It is obvious that under service conditions one of the electrodes is at ground potential until after the discharge across the gap has taken place, so any tests made should be with one electrode grounded if service conditions are to be properly simulated.

The use of non-grounded impulses is also open to criticism in connection with the tests on the balanced, double-sphere gap recorded in Table X, although in this case it is not believed that the test results would be much changed by the use of grounded impulses.

As I do not believe that Mr. Peek's data on the selective gap in any way represent the characteristics that are obtained in practise with the commercial form of impulse gap, I am presenting here curves showing the discharge characteristics of a standard 44-kv. impulse gap. For comparison with Mr. Peek's data I have plotted the curves showing the impulse discharge voltage as a function of the 60-cycle discharge voltage under rain conditions. I heartily endorse Mr. Peek's contention that this is the only proper basis of comparison for gaps used out of doors. For some time past in our laboratory we have been comparing protective gaps on the basis of what we have termed "protection factor" instead of on the basis of "impulse ratio." Protection factor is defined tentatively as the ratio of the impulse discharge voltage to the 60-cycle discharge voltage under rain conditions. For in-door gaps the protection factor is the same as the impulse ratio. Mr. Peek's quantity is proportional to the protection factor.

Fig. 1 is a photograph of the gap tested. Fig. 2 is a schematic diagram of the electrical circuit of the gap. The capaci-



[ALLCUTT]

FIG. 1—44-Kv. IMPULSE GAP.

tance for maintaining the auxiliary electrode at the proper potential is furnished by the pin type insulators (*a*) and (*b*). These insulators have an electrostatic capacity of about 2×10^{-11} farads. Due to the effect of capacity to ground, the auxiliary electrode is not maintained at a potential half way between the potentials of the two main electrodes by the condensers (*a*) and (*b*). It has been found by experiment that the proper position of the auxiliary electrode is such that its distance from the sphere horn "*m*" is six-tenths of the gap length. In this position it does not disturb the field when a 60-cycle voltage is applied across the gap. The unbalancing resistance is furnished by the resistors (*c*) and (*d*). These resistors have about 250,000 ohms resistance each. The resistors are about 12 in. (30 cm.) long and are of a special composition that retains its high resistivity under the most severe conditions of high voltage and high frequency. They are enclosed in porcelain tubes mounted on the porcelain pillar (*e*). The auxiliary electrode (*f*) is a pointed brass rod 0.0808 in. (0.2 cm.) in diameter held in a ⅜ in. (0.95 cm.) diameter brass support member (*g*), which is in turn mounted on an arm (*h*) of 1 in. (2.54 cm.) angle iron. The hemispheres (*j*) mounted on the horns are made of brass and are 12.5 cm. in diameter. In practise the charging resistance of the arrester is connected between the horn members (*k*) and (*l*). For the purpose of test, (*k*) and (*l*) were connected together and grounded to the frame of the gap structure.

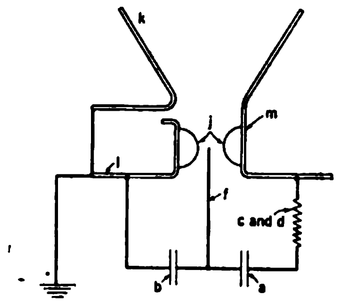


Fig. 2—Diagram of Impulse Gap

The impulse discharge voltage of the gap was determined by direct comparison with a 12.5 cm. sphere gap connected in parallel with it. The high-tension electrodes of the gap under test and of the sphere gap were connected together by a straight lead and the connection to the impulse generator was brought out from the middle point of this lead. The impulse generator itself was connected as shown in Mr. Peek's Fig. 3 (Case A). The following circuit constants were employed:

For 100 kilocycle impulse, $c = 5 \times 10^{-9}$ farads.

$L = 1.25 \times 10^{-3}$ henries and $R = 1000$ ohms.

For 420 kilocycle impulse, $C = 5 \times 10^{-9}$ farads.
$L = 8.3 \times 10^{-5}$ farads, $R = 320$ ohms.
For 3000 kilocycle super-impulse, $C = 5 \times 10^{-9}$ farads.
$L = 8.3 \times 10^{-5}$ henries and $R = 4800$ ohms.

The condenser employed consisted of a stack of one hundred impregnated paper condensers connected in series. Each condenser had a capacity of 0.5 microfarad. Single layer solenoids were used as inductances in the oscillating circuits and water tube resistances were employed. The test methods followed were those outlined in my papers.[3]

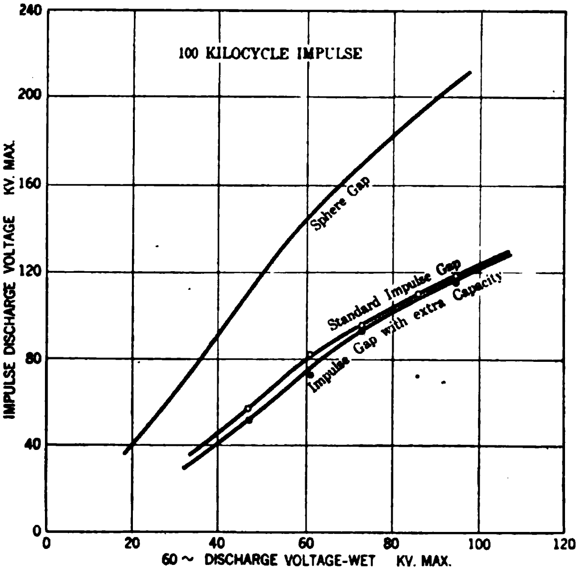


FIG. 3—IMPULSE GAP CHARACTERISTICS

Figs. 3, 4 and 5 give the results of tests made with impulse voltages of different steepness of front. The lowest curve, marked "Impulse gap with extra capacity," gives the results of tests made on the standard gap structure but with additional capacitance connected between the auxiliary electrode and each horn, bringing the total capacity on each side of the auxiliary electrode up to about 10^{-10} farads. This capacity is about the same as that employed in the experimental gap described in my 1918 paper and these results check quite well with the results published in that paper. It will be seen that the improvement due to the additional capacity is slight at 100 kilocycles and 420 kilocycles (Fig. 3 and 4) and, since it is believed that this represents the range of frequencies to be expected in practise,

---

[3] "Lightning Arrester Spark Gaps," C. T. Allcutt, TRANSACTIONS A. I. E. E., Vol. XXXVII.

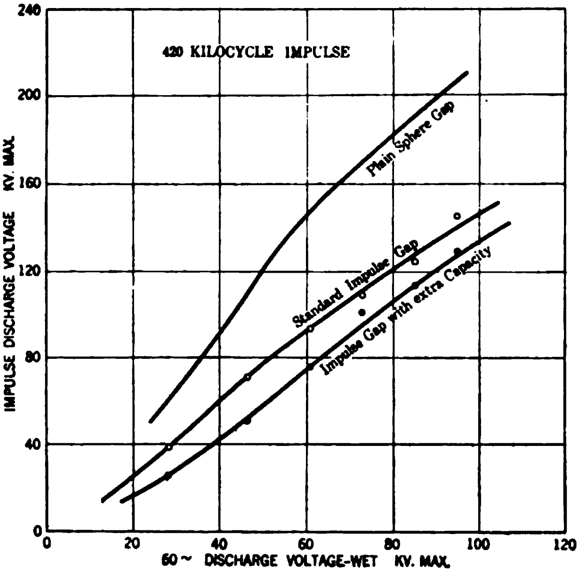


FIG. 4—IMPULSE GAP CHARACTERISTICS

the added expense of the extra capacity was not believed justified in the commercial design.

Fig. 5 is given for the purpose of comparison with the tests

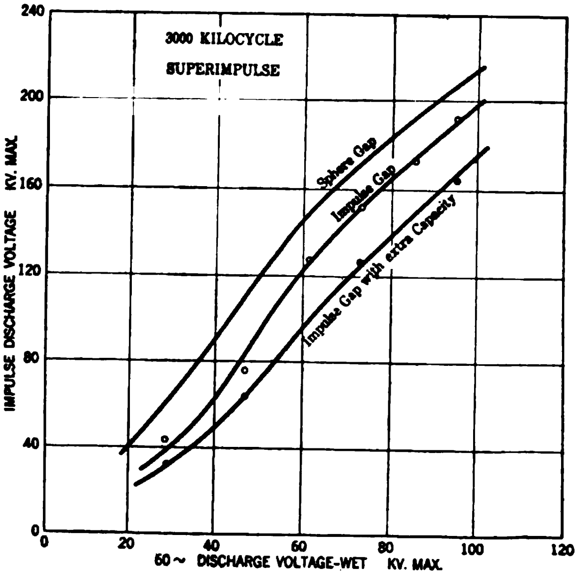


FIG. 5—IMPULSE GAP CHARACTERISTICS

at 2000 kilocycles, upon which Mr. Peek lays such stress. The tests were made with a 213-kv. wave having the same initial steepness as Mr. Peek's 340-kv. 2000-kilocycle wave. I believe this wave to be much steeper than any that can be expected in practise. The initial steepness of Mr. Peek's 2000-kilocycle wave is about 1.2×10^{-13} volts per second. This is equivalent to a 340-kv. sine wave of more than 5000 kilocycles. From the data given by Mr. Steinmetz in his recent paper[4] it may be shown that an infinitely steep wave front of 340-kv. magnitude will be reduced to a maximum steepness of 3.2×10^{-12} volts per second after travelling 30 meters. This is only one-third as steep as the wave used by Mr. Peek. Even if we admit the possibility of starting an infinitely steep wave front in a transmission line, it would have to be started within a few feet of an arrester in order to apply as steep a wave front to the gap as Mr. Peek used in his laboratory tests. For this reason it is not believed that the results shown in Fig. 5 in any way represent a condition liable to occur in practise. Fig. 6 shows the wave fronts used by Mr. Peek and myself. A 2000-kilocycle sine wave front is also plotted for comparison.

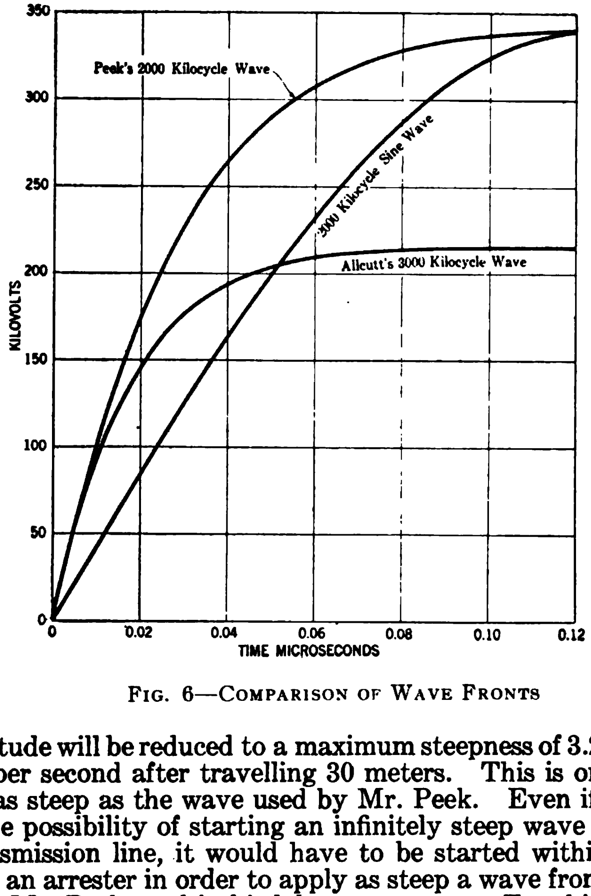


FIG. 6—COMPARISON OF WAVE FRONTS

---

[4] "General Equations of the Electric Circuit," C. P. Steinmetz, TRANS. A. I. E. E., Vol. XXXVIII, p. 191.

Figs. 7 and 8 show the discharge characteristics of the selective gap in comparison with the covered or indoor sphere gap and Mr. Peek's balanced double-sphere gap. Fig. 7 shows a covered or indoor impulse gap as compared with a covered or indoor sphere gap. Fig. 8 gives the characteristics of a double balanced sphere gap in comparison with those of one of a number of forms of protected impulse gaps that are being experimented with. Fig. 9 shows diagrammatically the protected impulse gap tested. The gap structure shown in Fig. 1 was used for the main gap and a 6.25 cm. sphere gap connected between the auxiliary electrode and one of the main electrodes. The sphere gap (g) shunting one-half of the main gap may be protected from the weather. A discharge across (g) is followed immediately by a discharge between the auxiliary electrode (a) and the horn (b). The rising arc quickly transfers from the auxiliary electrode (a) to the horn (c), shorting out the arc across (g) and effectively extinguishing it. This construction has been found to secure a very quick and effective suppression of the arc across (g), this permitting the use of a relatively small shelter.

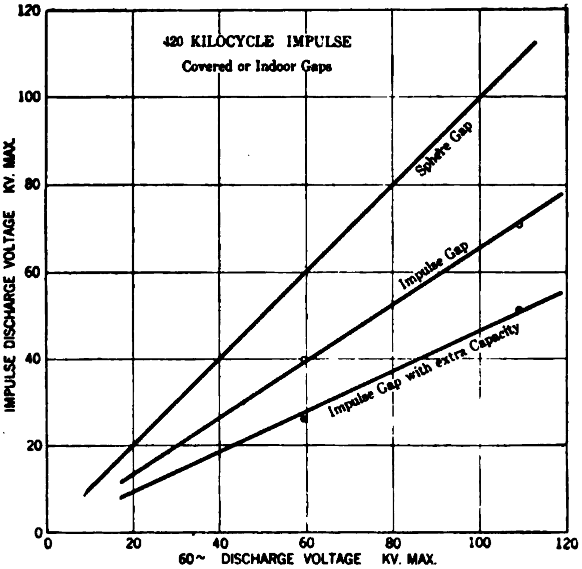


Fig. 7—Covered or Indoor Impulse Gap

In the present state of development, however, there is some doubt as to the efficacy of the protected gap structures proposed. We have found that merely wetting the surfaces of the electrodes of a gap will reduce the discharge voltage to practically as low a value as will actual rainfall. It is obvious that a shelter which merely protects a gap from direct rainfall cannot prevent the formation of moisture on the electrodes in foggy weather, and unless the electrodes can be kept dry under all conditions the purpose of the shelter will be defeated.

In his discussion of the balanced double sphere gap Mr. Peek apparently loses sight of the fact that an *unbalanced* double sphere gap actually has a lower impulse discharge voltage than a balanced double sphere gap with the same gap setting. The higher impulse ratio of unbalanced gap is due to the fact that unbalancing the gap causes a greater reduction in the 60-cycle discharge voltage than in the impulse discharge voltage. This

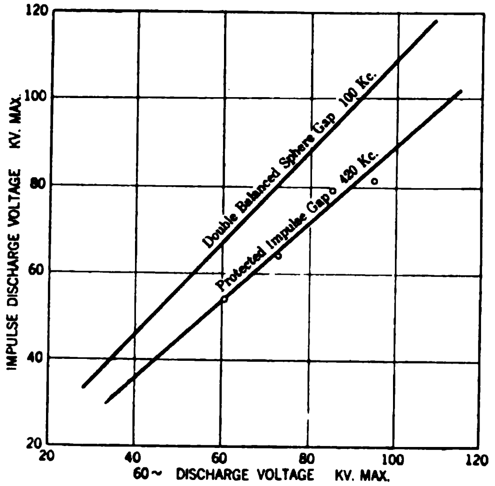


FIG. 8—COMPARISON BETWEEN PROTECTED IMPULSE GAP AND BALANCED DOUBLE SPHERE GAP

may be illustrated by reference to Mr. Peek's Table III. Mr. Peek finds that two gaps in series set at 0.45 cm. and 2.59 cm. respectively, will spark over on 60 cycles at 72 kv., unbalanced, and at 101 kv., balanced. On a 500-kilocycle impulse the same gaps will discharge at 86 kv., unbalanced, and 103 kv., balanced. It is obvious from these figures that the best protection would be obtained by having the gaps unbalanced for impulse voltages and balanced for 60-cycle voltages. This can be accomplished by connecting a high resistance or a reactance in series with one of the condensers used for balancing the voltage across the gaps, *i.e.*, by applying the selective principle of the impulse gap.

Following the line of reasoning given in his discussion at Atlantic City, Mr. Peek again maintains in his present paper that the impulse ratio of the selective gap is an *"apparent" impulse ratio.* We look in vain for a statement of his definition of true impulse ratio as applied to a selective gap. From the figures he gives in Table VII we may infer that Mr. Peek's definition of impulse ratio as applied to such a gap reads something like this: "The impulse ratio of a selective gap is the ratio of the impulse spark-over of the gap to the 60-cycle spark-over of a sphere gap having one-half of the gap setting of the selective gap." What is the reason for this arbitrary definition? Certainly not because the impulse gap may be regarded as consisting of two distinct gaps in series. The balanced double sphere gap also consists of two gaps in series and yet its impulse ratio is computed according to Mr. Peek's original definition of the term (See Table III). Suppose this same balanced double sphere gap were made into a selective gap by the simple expedient mentioned in the preceding paragraph. What would be the definition of its impulse ratio? I believe it will be found simpler to use the term "impulse ratio" in accordance with Mr. Peek's original definition than to attempt to evolve a new definition for every new type of gap that may be devised.

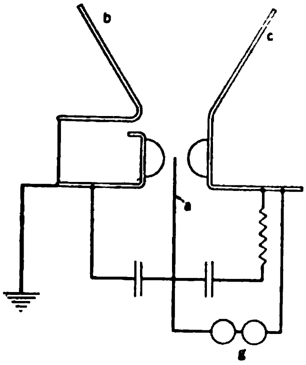


Fig. 9— Diagram of One Form of Protected Impulse Gap

In discussing the theory of operation of the selective gap, Mr. Peek falls into the error of assuming that the discharge voltage from the auxiliary electrode to one of the main electrodes is one-half the discharge voltage of the whole gap. In the selective gaps which I have described and tested the discharge voltage of the half-gap was approximately *one-quarter* the discharge voltage of the whole gap, since the half-gap is, in effect, a needle gap while the whole gap is a sphere gap. Experimental data showing this fact are given in the discussion previously referred to.[5] If the half gap of the selective gap tested by Mr. Peek did have the characteristics that he ascribes to it, (in Table VII, for example), it must have been quite different in design from the one tested by me.

---

[5] C. T. Allcutt, Discussion, Transactions, A. I. E. E., Vol. XXXVII, page 868.

F. W. Peek, Jr.: My object in writing this paper was to put into convenient practical form, spark-over data for various transient and lightning voltages for different types of gaps. The gaps discussed range all the way from the needle gap and the horn to the large sphere. It is pointed out that the protective value of a gap depends not only upon the lightning spark over voltage, but also upon the setting imposed by operating conditions. A ratio is given dependent upon both of these factors. This ratio gives the true relative protective value of a gap and may, therefore, be called the protective ratio. I have also given measured data on the steepness of waves actually occurring on operating transmission lines. Much steeper waves than any used in these tests have been observed on transmission lines. In fact, I first noticed that the lightning spark-over of certain gaps was much higher than the normal spark-over voltage by such waves on a large transmission system.

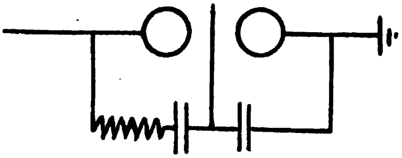


FIG. 10

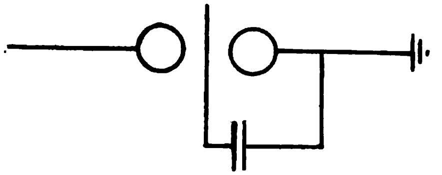


FIG. 11

I called attention to the importance of considering the effect of rain on the spark-over voltage in my discussion at Atlantic City last year. Complete data are now given for the effect of rain, etc. The relative protective values of the various gaps are given in Figs. 14, 15 and 16 in my paper.

Mr. Allcut has asked a number of questions with particular reference to the selective gap, which I will try to answer. I have always tried to give complete data as to methods and constants. Much of this will be found in my 1915 paper, my discussion at Atlantic City, and in the appendix to the present paper. I believe that all of the data necessary to duplicate the work are given.

Various types of condensers were used such as glass plates of very large capacity and high-capacity line insulators. The condensers used in the test had a capacity of the same order as those used by Mr. Allcut.

Water tube resistance was used. Anything above 200,000 ohms gives satisfactory results. The resistance should be as large as possible without giving an appreciable drop due to the 60-cycle capacity current.

The auxiliary electrode was placed between the spheres and set to give maximum 60-cycle spark-over and minimum lightning spark over voltage.

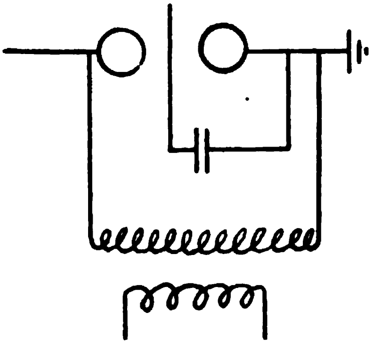


FIG. 12

It is true that careful adjustment of the auxiliary electrode is necessary. It is not, however, especially difficult to make this adjustment and keep it in the laboratory. The gap is also quickly thrown out of adjustment due to leakage by rain on the condensers. I have not found the exact size of the center electrode of the utmost importance. Tests were made on wires varying from 0.035 in. to 0.25 in. The smaller sizes were used on the 6.25-cm. spheres. A small enough electrode was used to give the best results. Some difficulty was had with the very small ones due to vibration caused by the 60-cycle voltages.

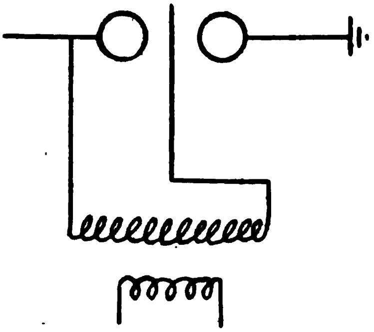


FIG. 13

The sphere part of the gap was always arranged so that distortion did not take place and the spark-over voltage thus corresponded to the standard curves.

Some of the tests were made as a matter of convenience without grounding one sphere. Check tests with one sphere grounded show that the relative protective value is the same for both cases. The method of making tests is always indicated on the data.

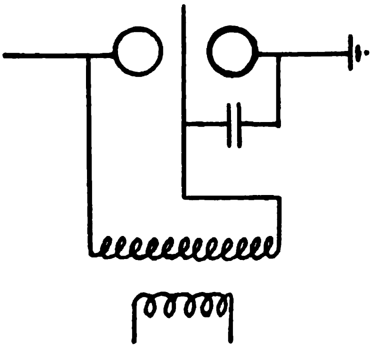


FIG. 14

Regarding Mr. Allcutt's question on the break down voltage of "one-half" the gap, in practise the lightning voltage is between line and ground or, in other words, is applied between the two spheres.

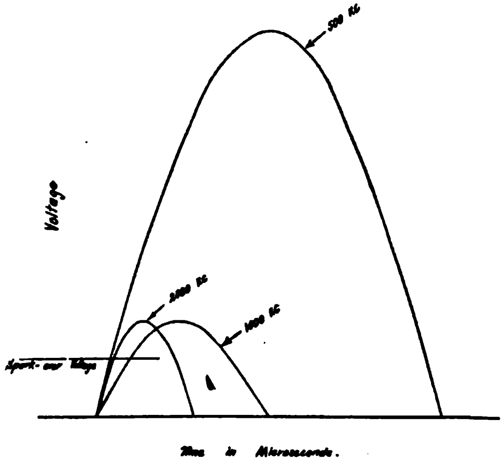


FIG. 15

At operating voltages the auxiliary electrode assumes the proper potential because of the two condensers. When transients are applied the resistance acts as an open circuit so that the potential of the auxiliary electrode is fixed by one condenser as in Fig. 11. In determining the break-down voltage of one-half the gap, therefore the voltage should be applied as in Fig. 12, and not as in Figs. 13 or 14.

The field due to the lightning voltage across the two spheres greatly decreases the stress on the center electrode. The break-down voltage is much higher for Fig. 12. When the test is made as in Fig. 12, the break down is approximately one-half the break down for the total gap; when made as in Fig. 13 it is approximately one-third that of the total gap.

I do not know how Mr. Allcutt's measurements were made, but his figures indicate by either method 13 or 14. The size of the electrode over a considerable range does not materially affect results.

The term frequency may be misleading. It is not the frequency with which we are principally concerned, but the rate or steepness at which the voltage is applied up to the breakdown point. As shown in Fig. 15, a very high voltage—500-kc. wave—may be less steep in effect than a 2000-kc. wave at a lower voltage.

I am interested to note that Mr. Allcutt now checks with me in the fact that the spark-over voltage of the selective gap increases with the steepness of the wave front. For instance, at 80 kv., in his Figs. 3, 4 and 5 the spark-over voltages are 100, 120 and 165 at respectively 100, 420 and 3000 kc. His former tests indicated that the effect of wave front was not material. Our data in general agree qualitatively.

It is of interest to note that the importance of the setting factor may be well illustrated in his Fig. 13. If the needle-gap curve is plotted on this figure, it will be found to give better protection than the impulse gap under this condition, due to the effect of rain. See Fig. 15 in my paper.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Los Angeles, Cal., September 18-20, 1919.

ORDER AND AMPLITUDE OF HARMONICS IN VOLTAGE WAVE FORMS WITH INDICATING INSTRUMENTS

BY LESLIE F. CURTIS

Abstract of Paper

The author presents a method for the determination of the order and percentage of the various components of an alternating wave of e. m. f., using indicating meters and other inexpensive apparatus.

Two examples are given. Oscillograms are included to show interesting phenomena and to check the results of the calculations.

The value of so-called standards for the indication of wave form is questioned.

INTRODUCTION

MANY writers have discussed the relative merits of various combinations of resistance, inductance and capacitance when used as devices for indicating the deviation of alternating wave forms from the sine curve. They have also defined certain terms such as form factor[1], deviation factor, distortion factor[2], etc., applicable to the above devices when used with indicating meters as standards[3]. (They have, however, failed to show any method by which the order and amplitude of the various harmonics might be obtained by indicating meters.) Each of the proposed standards is also open to the objection that the result obtained by its use is entirely dependent upon the order of the harmonics in question and upon the size of the reactances used.

The maximum distortion, which, in practise, might result from the resonance of a certain harmonic in a long transmission line with distributed inductance and capacity, would be produced by such devices in test only by chance. It is, however, only this resonant condition for the higher harmonics that can cause disturbances which can be charged to generator wave

---

1. See references at end of paper.

form. In the absence of oscillograms, it is therefore necessary to produce resonance of the various harmonics generated within the machine before its worst behavior can be predicted.

In making an oscillographic study of the charging currents for condensers of various sizes from the same source of alternating current it was found that the current wave forms varied greatly with the size of the condenser used. The ratio of the apparent reactance to the true reactance of the same condenser when supplied with a sine wave of voltage at the same frequency, as suggested by a previous writer[4], likewise depended upon the size of the condenser. It was also found that the resistance and inductance of the generator circuit had a decided effect on the indications. A method was then developed which gave a solution for the order and amplitude of the harmonics present in a given form from the readings obtained with inexpensive indicating meters and other apparatus.

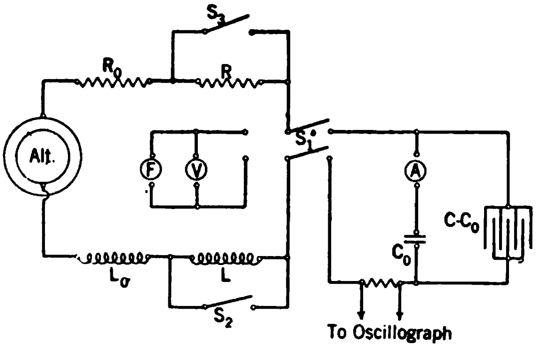


Fig. 1

It is the purpose of this paper to present this solution together with two examples of its use with oscillograms taken to show the various wave forms of current and voltage. The method outlined below gives no indication of the relative phase positions of the components and therefore no accurate indication of the maximum value of the resultant wave. In figuring the maxima of voltage or current waves the relative phase position is important only for the lower harmonics, the third and fifth. Thus for most purposes, the phase position of the components is not required and their order and percentage is a sufficient indication of the character of the wave.

METHOD

The method is that of obtaining resonant conditions for the nth harmonic in the generated voltage by several separate adjustments of inductance and capacitance in series with a certain amount of protective resistance. When resonance has been obtained, a second reading is taken with added series resistance.

The diagram of the connections used is shown in Fig. 1. The alternator supplying the wave to be tested is assumed to have an inductance L_0 and a resistance R_0. Resistance R and inductance L may be added to the circuit at will by means of switches S_2 and S_3. V and F indicate a voltmeter and frequency meter connected to the machine by throwing switch S_1 to the left. On throwing S_1 to the right an adjustable condenser $C - C_0$ is connected in the circuit. In parallel with this is a small fixed condenser C_0, the current in which may be measured by a small hot-wire meter A of negligible resistance. C is therefore the total capacitance.

For a given initial voltage and frequency the resonant points for the harmonics are indicated by a maximum deflection of meter A. If R_0 is not sufficient for protection of the condenser from abnormal voltages at resonance, more resistance may be added and considered as a part of the generator. If L_0 is not large enough to produce resonance of the lower harmonics with the amount of capacitance C available, more inductance may be added and considered as a part of the generator.

The calculation consists of two parts; the determination of the order of the harmonics present and the calculation of their amplitude. Table I gives a summary of the notation used in the calculations.

TABLE I—NOTATION

L	Known inductance.
R	Known resistance.
C_0	Known fixed capacitance.
$C - C_0$	Known adjustable capacitance.
C	Total capacitance.
C_1, etc.	Particular value of C.
ω	Angular velocity of fundamental wave.
n	Order of harmonic.
e_n	Effective nth harmonic in generated voltage.
P_n	Percentage of nth harmonic in terms of fundamental

E	Effective value of resultant generated voltage.
i_n	Effective value of *n*th harmonic in current.
I	Effective value of resultant current.
D	Deflection of hot-wire meter.
K	Calibration constant for A (See Fig. 13).
r_n	Resistance of circuit to *n*th harmonic.
x_n	Reactance of circuit to *n*th harmonic.

ORDER OF THE HARMONICS

The order of the harmonics present may be found from the values of capacitance, C_1 and C_2, necessary to produce resonance with and without the known inductance L in series.

In the first case

$$L_0 n \omega + L n \omega = \frac{1}{C_1 n \omega} \tag{1}$$

in the second

$$L_0 n \omega = \frac{1}{C_2 n \omega} \tag{2}$$

from which

$$n = 1/\omega \sqrt{1/L\,(1/C_1 - 1/C_2)} \tag{3}$$

If the result is not exactly a whole number it may be corrected as it is known that n must be an integer and probably odd. If another harmonic is present, its order may be determined in the same way from new adjustments of capacitance, C_3 and C_4, with and without the inductance L. An alternative method is to continue the curve upon which C_1 or C_2 produced resonance after one of the harmonics has been determined and take one additional reading of capacitance for the resonance of the other harmonic.

In this case

$$L_0 \omega = \frac{1}{C_2 n^2 \omega} \tag{4}$$

and

$$L_0 n' \omega = \frac{n'}{C_2 n^2 \omega} = \frac{1}{C_4 n^1 \omega} \tag{5}$$

from which

$$n' = n \sqrt{C_2/C_4} = n \sqrt{C_1/C_3} \tag{6}$$

AMPLITUDE OF THE HARMONICS

Having found the order of the harmonics present it is easy to determine their amplitude by taking additional readings of the current-squared meter with a known resistance inserted in the circuit and with the capacitance adjusted for the resonance of the harmonic in question.

In order to introduce no errors due to current in circuits in parallel with the capacitance and to necessitate no correction therefor, no voltmeter or other device should be connected when the readings of the hot-wire meter are taken.

For resonance of the *n*th harmonic

$$L_0\, \omega = \frac{1}{C_2\, n^2\, \omega} \tag{4}$$

and

$$i_n = e_n / r_n \tag{7}$$

$$i_1 = \frac{e_1}{\sqrt{r_1^2 + \left(\frac{1}{C_2\, n^2\, \omega} - \frac{1}{C_2\, \omega}\right)^2}} \tag{8}$$

$$i_3 = \frac{e_3}{\sqrt{r_3^2 + \left(\frac{3}{C_2\, n^2\, \omega} - \frac{1}{3\, C_2\, \omega}\right)^2}} \tag{9}$$

It will be noted that the resistance of the circuit is assumed to depend upon the frequency of the harmonic. In practise it is shown that the resistance increases with the frequency. In most practical cases the resistances r_1, r_3, etc., are so small compared to the other terms under the radical that they may be neglected.

Therefore

$$i_1 = \frac{e_1}{\frac{1}{C_2\, n^2\, \omega} - \frac{1}{C_2\, \omega}} = \frac{e_1}{x_1} \tag{10}$$

and

$$i_3 = \frac{e_3}{\frac{3}{C_2\, n^2\, \omega} - \frac{1}{3\, C_2\, \omega}} = \frac{e_1\, P_3}{x_3} \tag{11}$$

But

$$I = \sqrt{i_1^2 + i_3^2 + \ldots\ldots i_n^2 + \ldots\ldots} \tag{12}$$

therefore

$$I = e_1 \sqrt{\frac{1}{x_1^2} + \frac{P_3^2}{x_3^2} + \ldots . \frac{P_n^2}{r_n^2} + \ldots .} \tag{13}$$

The resultant voltage E is

$$\begin{aligned} E &= \sqrt{e_1^2 + e_3^2 + \ldots . e_n^2 + \ldots .} \\ &= e_1 \sqrt{1 + P_3^2 + \ldots . P_n^2 + \ldots .} \end{aligned} \tag{14}$$

and when no harmonic exceeds 10 per cent of the fundamental, e_1 may be replaced by E with less than 1 per cent error.

Now

$$I^2 = \frac{K\,D\,C_2^2}{C_0^2} \tag{15}$$

therefore

$$\frac{K\,D\,C_2^2}{E^2\,C_0^2} = \frac{1}{x_1^2} + \frac{P_3^2}{x_3^2} + \ldots . \frac{P_n^2}{r_n^2} + \ldots . \tag{16}$$

When a resistance R is placed in series a new reading D^1 is obtained and the corresponding equation is

$$\frac{K\,D^1\,C_2^2}{E^2\,C_0^2} = \frac{1}{x_1^2} + \frac{P_3^2}{x_3^2} + \ldots . \frac{P_n^2}{(r_n + R)^2} + \ldots . \tag{17}$$

therefore

$$P_n^2 = r_n^2 \left(\frac{K\,D\,C_2^2}{E^2\,C_0^2} - \frac{1}{x_1^2} - \frac{P_3^2}{x_3^2} - \ldots . \right) \tag{18}$$

and

$$P_n^2 = (r_n + R)^2 \left(\frac{K\,D^1\,C_2^2}{E^2\,C_0^2} - \frac{1}{x_1^2} - \frac{P_3^2}{x_3^2} - \ldots . \right) \tag{19}$$

Now usually all terms except those containing constants of the fundamental and the nth harmonic are negligible, therefore

$$P_n^2 = r_n^2 \left(\frac{K\,D\,C_2^2}{E^2\,C_0^2} - \frac{1}{x_1^2} \right) \tag{20}$$

and

$$P_n^2 = (r_n + R)^2 \left(\frac{K\,D^1\,C_2^2}{E^2\,C_0^2} - \frac{1}{x_1^2} \right) \tag{21}$$

from which numerical values of r_n and P_n may be obtained. If the other terms are appreciable, an approximate solution may be made by equations (**20**) and (**21**) for all of the harmonics and then corrected by using equations (**18**) and (**19**).

It is suggested that if a calibrated condenser is not available, that the adjustments be made for the current maxima and that the corresponding capacitance be measured on a sine wave before changing the condenser setting. If no adjustable condenser is available, a fixed condenser and calibrated inductance may be substituted. The equations should then be modified to use inductance as the abscissa of the resonance curves.

Example 1: The open circuit wave form of a delta-connected alternator, 1200 rev. per min., 220 volts, run at normal speed but at 100 volts, is shown by the oscillogram in Fig. 2. The resonance curves for this machine by the above method without and with the known inductance L in series are shown in Fig. 3.

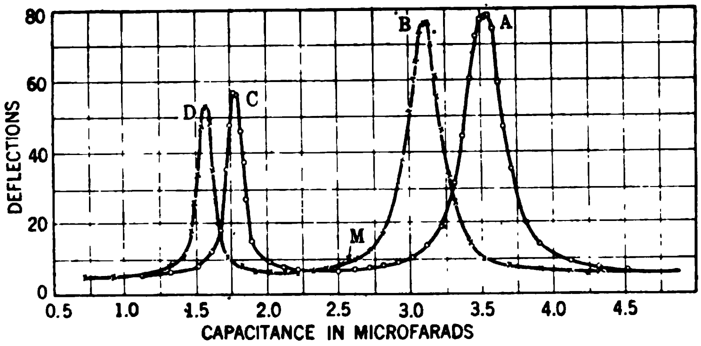


Fig. 3

The inductance L, when measured separately on a 180-cycle wave gave a reactance of 12.6 ohms, and therefore had an inductance of 0.0111 henry. During the test an additional inductance of unknown value was permanently connected in the circuit. C_0 was 0.72 microfarads.

a. Order of the Harmonics:

Maximum points A and B, in Fig. 3, corresponding to 3.53 and 3.11 microfarads, are evidently due to the same harmonic. The equation for the determination of its order is (**3**).

Substitute and $$n = \frac{1}{2\,\pi\,60}\sqrt{\frac{1}{.0111}\left(\frac{10^6}{3.11} - \frac{10^6}{3.53}\right)}$$

from which n equals 4.9 and therefore indicates the fifth harmonic.

Maximum points C and D, at 1.78 and 1.57 microfarads in Fig. 3 are due to the presence of another harmonic. A similar solution to that above gives a value of 6.9 for n and therefore indicates the seventh harmonic.

An alternative solution is, since we know that point A is due to the fifth harmonic,

$$n' = 5\sqrt{\frac{3.53}{1.78}} = 7.0$$

From these solutions it is therefore known that resonance points A and B are due to the fifth, and points C and D, to the seventh harmonic.

b. Amplitude of the Harmonics:

The readings for the determination of the amplitude of the above harmonics are given in Table II. Consider the conditions for point A.

From equation (4),

$$L_0\,\omega = \frac{10^6}{3.53 \times 5^2 \times 2\,\pi\, 60} = 30 \text{ ohms}$$

$$\frac{1}{C_2\,\omega} = \frac{10^6}{3.53 \times 2\,\pi\, 60} = 749 \text{ ohms}$$

$$x_1 = 749 - 30 = 719 \text{ ohms}$$

Substitute in equation (**20**) and

$$P_5^2 = r_5^2\left(\frac{0.01214 \times \overline{3.53}^2}{\overline{100}^2 \times \overline{0.72}^2} - \frac{1}{\overline{719}^2}\right)$$

Likewise for point A^1, when a resistance of 2.66 ohms is placed in series, substitute in equation (**21**) and

Solving

$$P_5^2 = (r_5 + 2.66)^2\left(\frac{0.00713 \times \overline{3.53}^2}{\overline{100}^2 \times \overline{0.72}^2} - \frac{1}{\overline{719}^2}\right)$$

Solve and r_5 equals 7.9 ohms and P_5 equals 4.1 per cent.

Points B and B^1 give similar results, 9.1 ohms and 4.1 per cent.

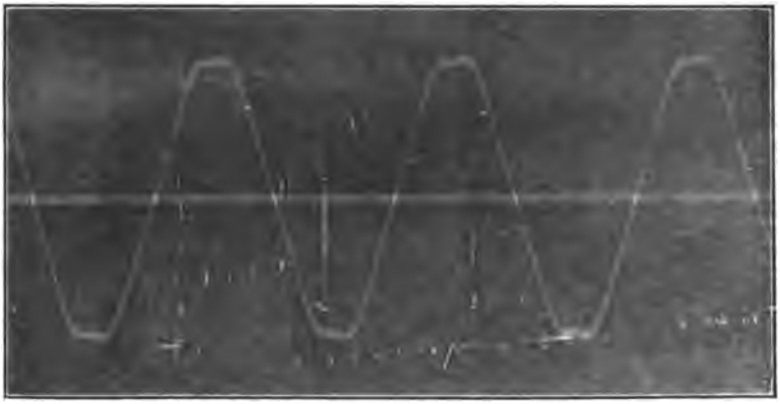
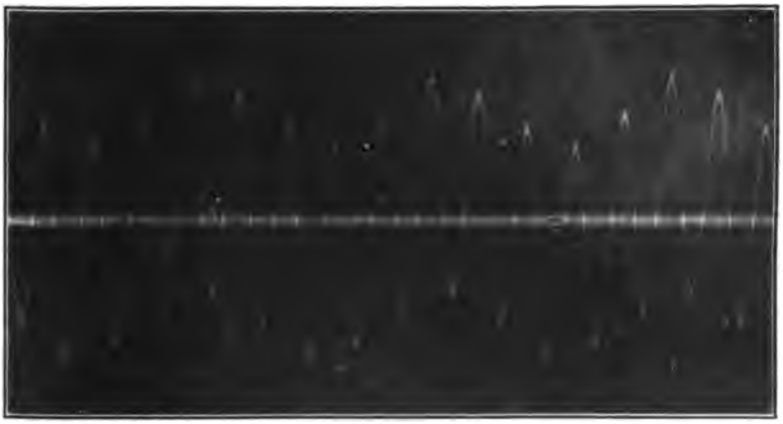


FIG. 2—OPEN CIRCUIT VOLTAGE—EXAMPLE 1 (EFFECTIVE VALUE, 100 VOLTS)



[CURTIS]

FIG. 4—CHARGING CURRENT—POINT A, EXAMPLE 1 (EFFECTIVE VALUE, 0.54 AMPERES)

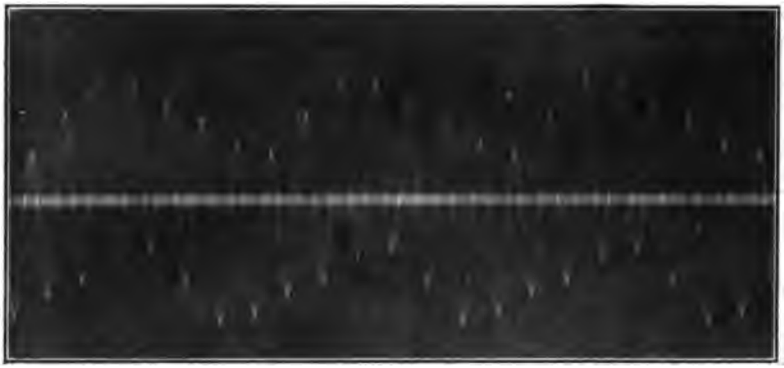
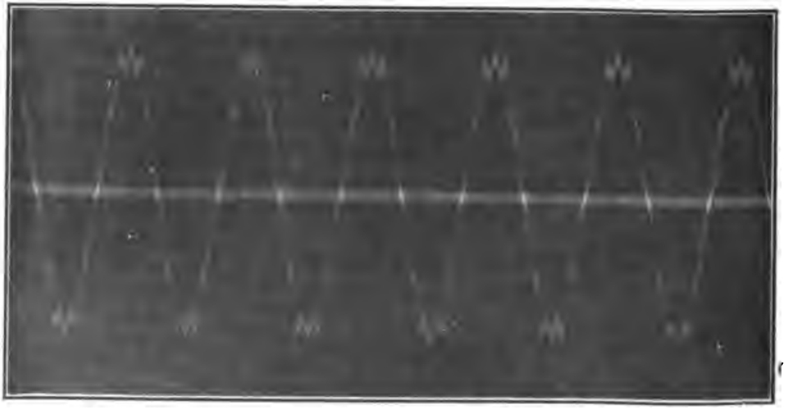


FIG. 5—CHARGING CURRENT—POINT C—EXAMPLE 1 (EFFECTIVE VALUE 0.24 AMPERE)

FIG. 6—CHARGING CURRENT—POINT M—EXAMPLE 1 (EFFECTIVE VALUE, 0.15 AMPERE)



[CURTIS]

FIG. 7—OPEN CIRCUIT VOLTAGE—EXAMPLE 2 (EFFECTIVE VALUE, 100 VOLTS)

Solving in the same way for the constants for the seventh harmonic from the values for points C and C^1 and r_7 equals 9.7 ohms and P_7 equals 1.7 per cent.

TABLE II
CRITICAL VALUES FOR EXAMPLE 1

Point	C	C_0	R	D	KD	n	I
A	3.53	0.72	0.00	78.0	0.01214	5	0.540
A^1	3.53	0.72	2.66	44.0	0.00713	5	0.415
B	3.11	0.72	0.00	76.0	0.01183	5	0.470
B^1	3.11	0.72	2.66	46.0	0.00745	5	0.373
C	1.78	0.72	0.00	57.0	0.0091	7	0.236
C^1	1.78	0.72	2.66	36.0	0.0059	7	0.190
D	1.57	0.72	0.00	53.0	0.00848	7	0.201
D^1	1.57	0.72	2.66	35.0	0.00574	7	0.165
M	2.62	0.72	0.00	10.0	0.0018	5 & 7	0.149

From points D and D^1, the corresponding values are 10.8 ohms and 2.1 per cent.

Oscillograms were taken to check the above results and to show the charging current for the various total capacitances. Fig. 2 is an oscillogram of the no-load voltage of the machine and is the true generated wave form of the machine as no capacitance was connected and the resistance was at least ten times as great as the reactance of the circuit to the seventh harmonic. An analysis of this wave gives 3.5 per cent of the fifth, and 1.6 per cent of the seventh harmonic, no others being appreciable.

Fig. 4 is the wave of charging current (0.540 ampere) for point A and shows the predominance of the fifth harmonic. Fig. 5 is a similar curve of charging current (0.236 ampere) for point C showing the seventh harmonic. Fig. 6 is the wave of charging current (0.149 ampere) for point M corresponding to 2.62 microfarads, Fig. 3 showing the reduction in the amplitude of the harmonics in current and the beats due to both fifth and seventh harmonics.

The resistance of the circuit used was measured with direct current and found to be 5.2 ohms for points A and C, and 6.2 ohms for points B and D. The solutions given above show that the resistance to the higher frequencies was very materially increased.

A summary of all of the above calculated data is presented in Table III.

Example 2: The machine used for this test was a synchronous motor, 1800 rev. per min., 110 volts, run as a generator at 100 volts. The open-circuit wave form of this machine is shown in Fig. 7, which upon analysis gives 3.5 per cent of the fifth, 1.4 per cent of the seventh, 7.0 per cent of the eleventh and 2.5 per cent of the thirteenth harmonics.

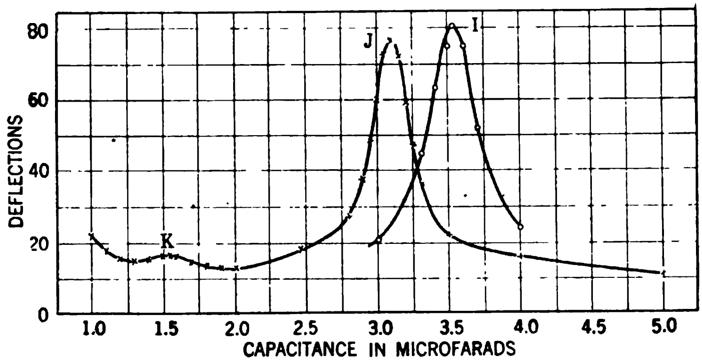


Fig. 8

Unfortunately the machine was fitted with carbon brushes, the resistance of which varied with the current density, so that no check could be made of the above percentages by the method described. It was, however, possible to solve for the order of the harmonics which are indicated in the resonance curves shown in Fig. 8 and Fig. 9. In Fig. 8 a large unknown inductance is connected in the circuit to make possible the resonance of the lower harmonics (points *I*, *J*, *K*) with small capacitances. This was omitted in taking the curves of Fig. 9.

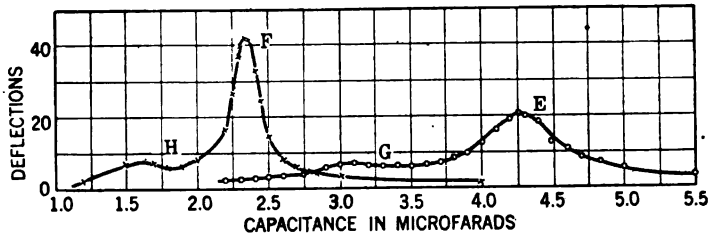


Fig. 9

In Fig. 8 points *I* and *J*, corresponding to 3.52 and 3.10 microfarads, show the resonance of a harmonic without and

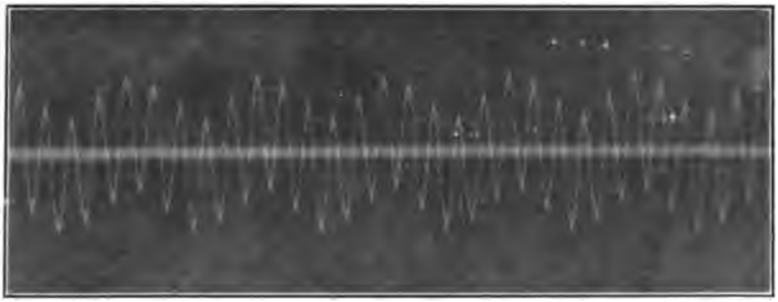


FIG. 10—CHARGING CURRENT-POINT J —EXAMPLE 2 (EFFECTIVE VALUE, 0.34 AMPERE)

FIG. 11—CHARGING CURRENT—POINT F—EXAMPLE 2 (EFFECTIVE VALUE, 0.97 AMPERES)

[CURTIS]

FIG. 12—CHARGING CURRENT—POINT H—EXAMPLE 2 (EFFECTIVE VALUE, 0.31 AMPERES)

with the known inductance L in series. A calculation shows this harmonic to be the fifth. Point K, corresponding to 1.55 microfarads is on the same curve with point J (3.10 microfarads) and therefore indicates the seventh harmonic. An oscillogram of the charging current at point I (0.342 ampere) is shown in Fig. 10. In the above C_0 was 1.0 microfarad.

TABLE III
CALCULATED VALUES FOR EXAMPLE 1

Points	n	r_n	R	Mean P_n	P_n by Oscillograph
A–A^1	5	7.9	5.2	4.1	3.5
B–B^1	5	9.1	6.2		
C–C^1	7	9.7	5.2	1.9	1.6
D–D^1	7	10.8	6.2		
	1			100.0	100.0

TABLE IV
CRITICAL VALUES FOR EXAMPLE 2

Point	C	C_0	D	kD	n	I
E	4.27	0.20	21.0	0.0035	11	1.260
F	2.34	0.20	42.5	0.0069	11	0.971
G	3.10	0.20	7.5	0.0013	13	0.559
H	1.65	0.20	8.0	0.0014	14	0.309
I	3.52	1.00	81.0	0.01255	5	0.394
J	3.10	1.00	77.0	0.1020	5	0.340
K	1.55	1.00	16.5	0.0028	7	0.082

Similar solutions for points E and F, corresponding to 4.27 and 2.34 microfarads, and points G and H, corresponding to 3.10 and 1.65 microfarads, as shown in Fig. 9, give indications of the presence of the eleventh and thirteenth harmonics. An oscillogram of the charging current at point F (0.971 ampere) is shown in Fig. 11 and that for point H (0.309 ampere), in Fig. 12. In the above C was 0.2 microfarads. Fig. 12 is particularly interesting since it shows the beats due to the eleventh and thirteenth harmonics even when the thirteenth is in resonance, that is, the eleventh is not, in this case, negligible. This might also be surmised from the shape of the resonance curve near point H in Fig. 9.

A tabulation of the readings at the critical points in the above example is given as Table IV.

A calibration curve of the hot-wire meter is given in Fig. 13 so that further calculations may be made if desired.

CONCLUSIONS

It is practicable to test for the order and amplitude of the harmonics in any alternating wave of voltage by the use of inexpensive indicating meters, a known inductance and an adjustable condenser without knowing the resistance and inductance of the machine itself.

A small change in the capacitance or inductance of any of the so-called wave form standards may result in a great change in the current in the circuit, making such devices unreliable.

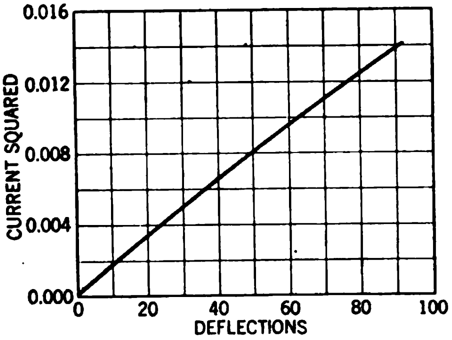


Fig. 13

The resistance of any circuit may change with the frequency of the harmonics present and this fact may introduce an error in making measurements with any of the so-called standards.

REFERENCES

1. *Form Factor and Its Significance*, by F. Bedell, Trans. A. I. E. E., Vol. XXXIV, 1915, pp. 1135–1142.

2. *Distortion Factors*, by F. Bedell, Trans, A. I. E. E., Vol. XXXIV, 1915, pp. 1143–1157, and "*An Analytical and Graphical Solution For Non-Sinusoidal Alternating Currents*, by F. M. Mizushi, Trans. A. I. E. E., Vol. XXXIV, 1915, pp. 1159-1170.

3. *Characteristics of Admittance Type of Wave Form Standards*, by F. Bedell, Trans. A. I. E. E., Vol. XXXV, 1916, pp. 1155-1170.

4. *A Proposed Wave Shape Standard*, by C. M. Davis, Trans. A. I. E. E., Vol. XXXII, 1913, p. 775.

DISCUSSION ON "ORDER AND AMPLITUDE OF HARMONICS IN VOLTAGE WAVE FORMS WITH INDICATING INSTRUMENTS" (CURTIS), LOS ANGELES CAL., SEPTEMBER 18, 1919.

N. S. Diamant: I will call attention to a few points which may be of interest from a theoretical and practical point of view. Considering the latter first, namely, the actual practical value of the method described by Mr. Curtis, I am afraid the method is not quite as useful, convenient and inexpensive as he seems to imply. When you consider the fact that you need *known* inductances, *known* resistances, *known* capacitances, some of which must be adjustable, it will easily be appreciated that the method is not so very simple or convenient; unless it should just happen that a given laboratory is fairly well equipped as to available inductances and capacitances, but does not happen to own an oscillograph. In any case, it would seem that one can safely venture that the practical value of the method is rather limited.

Now coming to the second consideration, namely, the theoretical and academic value of the paper, it is to be greatly regretted that the author does not make any reference whatsoever to the important contributions made on this subject years ago by men like Blondel, Pupin and a great many others. It is discouraging to find that the author gives references at the end of his paper, but really the references he gives have very little to do directly with the subject of the paper and refer more to the *standardization of wave form;* thus when Mr. Curtis states that "*they* have also defined certain terms such as form factor, deviation factor, etc." but "they have failed to show any method by which the *order* and *amplitude* of the *various harmonics* might be obtained by indicating meters," I am afraid he is not quite doing justice to the men who did considerable work along this line before and after the perfection of the oscillograph. The above remarks are not intended in any way to detract from the value of the paper, which I think is considerable from the point of view of an instructor of an electrical laboratory. Before closing may I ask Mr. Curtis what per cent variation he has in mind when he assumes that the resistance of the circuit under consideration depends upon the frequency of the harmonic. In radio work this might be so, but in ordinary electrical work it is not probably very bad practise to assume the resistance as independent of the frequency except in case of very heavy bars. When it is considered that in the method outlined by Mr. Curtis we are expected to use ordinary indicating meters and inductances and capacitances whose value, I suppose, will have to be determined by means of ordinary methods and further when it is considered that if the result which indicates the order of the harmonic does not happen to be an integer, the author just assumes it to be the nearest odd number, I say that when all these approximations are considered as permissible, the effect of frequency on resistance seems to be not a very important or rational refinement to be taken care of.

J. C. Albert: Mr. Curtis' paper presents an apparently simple and practical method for determining the order of harmonics in generator voltage waves, but his failure to obtain the amplitudes of the wave in Example (2) leads to several questions.

The author states that the variable resistance of the carbon brushes introduced unavoidable errors in the calculations of amplitude. It would be interesting to know how much the carbon brushes affect the results, and whether Mr. Curtis has made any experiments to determine how great this error might be. I would also like to ask whether there have been other experiments made on generators not having carbon brushes but having an appreciable eleventh and thirteenth harmonic.

In Example 1 in obtaining point A', a resistance of 2.66 ohms was added in series with the capacity and inductance. I assume this to be the resistance offered to the fifth harmonic, and it would be instructive to know how this is measured.

In deriving equations (**20**) and (**21**) the statement is made that, "Usually all terms except those containing constants of the fundamental and the nth harmonic are negligible," and concludes by stating that, "If the other terms are appreciable, an approximate solution may be made by equations (**20**) and (**21**) for all the harmonics and then corrected by using equations (**18**) and (**19**)."

As shown in the oscillogram Fig. 12, in which the thirteenth harmonic is in resonance, the eleventh has considerable effect. Now in the derivation of formulas (**16**) and (**17**) the resistance component of impedance is assumed negligible in all terms except the nth which is in resonance, and, therefore, has only resistance and no reactance.

It seems from Fig. 12 that in dealing with two successive harmonics of high order, as the eleventh and thirteenth, that even if resonance of the thirteenth is obtained the effect of resistance on the eleventh might be large compared to the effect of reactance on the same, and should enter into the calculation after the first approximation is obtained by equations (**20**) and (**21**) or, in other words, the impedance instead of the reactance only should form the denominator of the $(n—1)$th term of the series in formulas (**18**) and (**19**).

For example, the point marked "E" in Table IV for critical values, the author shows result of the eleventh harmonic obtained with the capacity of 4.47 microfarads. To balance this capacity, assuming the eleventh harmonic on a 60-cycle circuit, an inductance of approximately 0.013 henrys would be required. These values of capacitance and inductance combined show a reactance to the thirteenth harmonic of only a little over eight ohms. While the paper does not state the resistance used in obtaining this point, we find that the resistance in Example (1), point *A*, was 7.9 ohms, so that the reactance might be little greater than the resistance for this particular case.

Another point which might introduce error in the calculations, when two successive harmonics of high order are present, is that the hot-wire ammeter does not necessarily indicate that the *n*th harmonic is in resonance, but indicates that the impedance to the complex wave is a minimum near the point of resonance of the *n*th harmonic, or, as shown by the F. M. Mizushi (authors reference No. 2) that

$\omega = \sqrt{\sigma \div S\,L\,C}$

where σ is the integral distortion factor

and δ is the differential distortion factor

$w - 2\,\pi\,f\,n$ near point of resonance of *n*th harmonic then equation (4) would become.

$$2\pi f L_0 = \frac{\sigma}{2\,\delta\,\pi\,f\,n^2 C_2}$$

This solution for the unknown reactance of the generator in terms of C_2 involves two unknown quantities, the integral and differential distortion factors.

The author states in his conclusion that, "A small change in the capacitance or inductance of any of the so-called wave form standards may result in a great change in the current in the circuit, making such devices unreliable." Is not the same true of his method? Will not the amplitude for the different harmonics vary with the variation of the amount of inductance and capacitance used to obtain their resonance?

Mr. Price: From what little time I have had in reading over this paper here I don't think the method would be very good for anything that was not recurring. That is, for instance, where you take the oscillogram of a switch action, or short-circuit conditions; or something of like nature, you couldn't get conditions like that because you couldn't adjust your condensers or inductances rapidly enough to get that condition. You would have to have a recurring condition.

I would like to ask the author how difficult it is to obtain resonance with his method. How much time it takes. Whether this method could be adapted to taking curves on high-voltage circuits and the possible effect of the current potential transformer on the results obtained.

D. I. Cone: Perhaps the first proposal along this line was that given by Dr. Pupin in the 1894 TRANSACTIONS employing a somewhat similar method to the one presented in this paper. He employed an inductance and a variable condenser to determine by resonance the frequencies present in the voltage drop across a resistance. He placed this resistance either in series to measure current waves or in shunt, to measure potential waves. He used an electrostatic voltmeter to determine the resonance points by indicating the settings of capacitance to give maxima of voltage across the condenser.

The California Joint Committee on Inductive Interference made extensive use of a so-called "resonant shunt" consisting of a large inductance in series with a variable condenser and a telephone receiver. Each harmonic was resonated in turn by setting the capacitance for maximum current of the particular frequency in the telephone receiver. The amount of current for each harmonic was measured by comparing the sound in the telephone receiver to that from an auxiliary source of the same frequency. Knowing the impedances in the circuit, or arranging so that unknown ones were of negligible effect, fairly accurate determinations of frequency and size of harmonics could be made. Considerable apparatus was required for that work and an analysis of harmonics, ranging from the third harmonic up to perhaps the nineteenth, could be run through in about twenty minutes.

That particular method does not take care of the fundamental component at all, which could be obtained with the aid of an indicating meter.

The method given by Mr. Curtis makes use of relatively simple equipment and procedure for determining the more prominent harmonics. However, in the wave analysis of inductive interference work, harmonics of the order of one-half per cent may be important to determine.

The simplicity of the equipment and the method of solution which he employs for amplitude and frequency are the distinct contributions made by his method. He presents it with special reference to the open-circuit voltage of generators. However, it can be applied to the determination of the harmonics of any system and the limitation which he states on page 1183, of his paper, regarding the presence of a voltmeter or other devices, when the readings are taken, does not appear to be necessary; especially if such devices are normally in the circuit.

Now, referring to his introductory paragraph. The last sentence of his first paragraph reads: "Each of the proposed standards is also open to the objection that the result obtained by its use is entirely dependent upon the order of the harmonics in question and upon the size of the reactances used."

In the telephone interference factor meter which was described in the paper beginning page 261 of this volume, by Mr. Osborne, that objection is met in that the impedances within the meter are fairly large and the distinct effort is to sum up all the frequencies in the circuit as they normally exist; so that the objections here stated do not apply to that particular wave factor meter. It is recognized that such a meter has limitations. It serves but one definite purpose, and that is particularly true of the telephone interference factor meter.

The last sentence on the first page of the paper reads: "It is, however, only this resonant condition for the higher harmonics that can cause disturbances which can be charged to generator wave form."

I assume that in making that statement the question of possible damage to the power circuit itself was in the author's mind. Of course when we consider the question of harmonics in reference to interference to the communication circuits, the detrimental effect is felt whether a particular harmonic is in resonance or not. Of course, the detrimental effect is aggravated for harmonics in resonance.

Mr. Curtis points out that it is desirable to resonate the circuit for a given harmonic in order to develop what he calls its "worst behavior." However, he employs, and quite appropriately, a protective resistance to prevent the current in the meter becoming excessive. Hence it is not clear that the setup he employs actually develops the worst condition, although by studying the effect of the variation of the effective resistance in this circuit upon the behavior of the harmonic, one might get an idea of what its "worst behavior" would be.

H. A. Barre: It seems to me that there is in this paper a method of attack on two or three problems I have in mind. One problem is the finding of the cause of flash-over on the insulators of the Big Creek line. These flash-overs occur probably twelve or fifteen times a year. The current consists simply of the arcing over of the insulators and a heavy flow to ground. The effect on the system is not great and I do not think a single case of injury to apparatus has occurred. We seem to be unable to associate these flash-overs with any set of conditions. The line will be operating smoothly when apparently out of a clear sky will come a flash over and current flow to ground.

The attendants at the stations realize that this is the only trouble to be expected on the Big Creek lines, so when trouble arises they assume a flash-over and apply the cure.

This flash-over cure consists in lowering the voltage to a point, determined by experience. The arc goes out and immediately normal voltage is resumed, taking care not to over run. An automatic means of doing this work has been devised but has not proved as successful as the manual method. At the receiving station the operator also assumes a flash-over. In order to keep from boosting the voltage with the condensers, he opens the field thereby assisting the voltage drop. When voltage at receiving station becomes normal again, he assumes trouble is over, and resumes normal operation. In practically all cases he is right.

There is one interesting fact in connection with these flash-overs and that is their occurrence in a limited section of the line, the two middle quarters of the line,—very seldom close to the ends.

If this method of calculation can be applied to the investigation of this phenomena it will be of great assistance to us.

This method might also throw some light on the double frequency effect found at San Fernando. The circumstances were as follows: Test was being made on 36 miles of line, built for 15,000 volts and energized through a bank of star-star transformers with about 4000—25,000 volts in the legs of the wire. It was energized through a separate delta-delta transformer for the purpose of isolating the system. On closing the switch nothing apparently happened at first. After an appreciable time a disturbance occurred in the system, lasted several seconds, then died out. Once in a long while it recurred. The oscillograph showed that their transient current. or whatever it was, was in the form of a flow through the ground connection which, after passing the common point of the star of the transformers, divided into three currents and fed through the line.

The frequency of this current was measured at about 100 cycles, twice the fundamental frequency. The voltage between the cross and the legs of the wire increased to three or four times normal without, of course the voltage between wires increasing.

A practical application of this principle might show what is going on.

Leslie F. Curtis: In answer to Mr. Diamant let me say that the *known* inductances may be wound, the *known* resistances constructed from nichrome wire, and the *known* capacitances arranged from telephone condensers in any electrical repair shop.

It is true that the author was more concerned with the articles referring to *standardization of wave form* than to the work of Drs. Blondel and Pupin because he felt that the standards for wave form suggested within the last few years were faulty and that if the order and amplitude of the harmonic components of a wave are known, other standardization is unnecessary. The statement should have been made that "others have failed to show a *simple and readily applicable* method by which the order and amplitude of the various harmonics might be obtained with *ordinary indicating meters.*"

The variations of the resistance with frequency for the machine tested are noted in Table III of the paper. It is there found that the resistance to the seventh harmonic, that is, seven times the fundamental frequency, or approximately 400 cycles per second, increases from 6.2 ohms to 10.8 ohms. I believe that in an investigation of a circuit of this sort that such a variation of resistance must be taken into account.

The values of the capacitance and inductance used were measured by ordinary indicating instruments from a source of alternating e.m.f. at 60 cycles, which was so arranged as to eliminate most of the harmonics,—in other words, a sine wave of e. m. f. was employed for a simple voltmeter, ammeter and wattmeter measurement.

Mr. Albert asks concerning the calculations for the amplitude in example (2). Even with the variable resistance of the carbon brushes in the circuit, it would have been possible to make calculations for the amplitude if several more readings had been taken—readings with small successive additions of resistance to the circuit. By the use of cross curves and further calculations the amplitude might have been obtained. I omitted such calculations because of their complexity.

Mr. Albert also refers to the effect of the 13th and 11th harmonics in producing an error in the measurement for amplitude. I believe that the paper shows that the order of the harmonic is obtained without an appreciable error, even when the harmonics are of nearly the same order, that is, as he stated, the nth and the $(M \pm 1)$th term. The effect of the resistance in the circuit may then be introduced in the proper equation; though I am not certain that it is justified. The added known resistance is a high resistance wire of such a cross section as to be independent of frequency. Therefore, the 2.66 ohms which Mr. Albert mentions holds for all frequencies.

Mr. Albert also mentions the equations presented by Mr. Mizushi showing that the impedance of the circuit is a minimum at some other frequency than $\frac{1}{n \omega \sqrt{L C}}$. The circuit used, however, is different. The calculation for the maximum current in the meter used in this paper shows that such maximum occurs at a point very close to the assumed resonance point. The current in the complete circuit is not necessarily maximum at the same instant.

Mr. Albert speaks of the variation of amplitude of the harmonics in the circuit with the amount of inductance and capacitance introduced. It is assumed by the author that the generated or initial value of the harmonics in e. m. f. remain constant. The magnitude of the harmonics in the current, of course, depend upon the inductance and capacitance.

Mr. Price asks concerning the time required to make a test of this sort. With the ordinary apparatus at hand in most testing departments, the variable or adjustable condenser is the only piece of apparatus which would offer any difficulty. One may be made from an extra good grade of telephone condensers and when provided with the proper switches for adjustment, a complete test might be made in possibly an hour. The measurements of inductance and resistance can be made by ordinary indicating instruments, they do not offer any particular difficulty.

Mr. Cone notes that the presence of the voltmeter should offer no difficulty. I should explain that the voltmeter indicated in the diagram of connections was used simply to indicate the constancy of voltage on open circuit. The voltmeter might be connected at all times and would offer no difficulty if the readings of the meter were taken only when the resonant circuits were open.

At the time my manuscript was prepared the telephone interference factor had not been suggested. Therefore, I did not intend to include a circuit giving the telephone interference factor among the objectionable standards.

Mr. Cone also notes that I assumed that damage might result, or rather that interference might result only from resonant condition of the harmonics. I intended to indicate that such interference would be a maximum during periods of resonance.

I believe I am unable to answer Mr. Barre's question concerning the trouble—the double frequency effect—at San Fernando. I hope some use may be made of the circuit in solving his problem.

*Presented at the Pacific Coast Convention of
the American Institute of Electrical Engineers,
Los Angeles, Cal., September 18–20, 1919.*

THEORY OF PROBABILITIES APPLIED TO FAILURES OF SUSPENSION INSULATORS

BY L. M. KLAUBER

Abstract of Paper

Nothwithstanding the recent improvements in porcelain insulators, failures are sufficiently common so that allowance must be made for them. A certain factor of safety is required, in the shape of extra insulation, to provide for the electrical unreliability of the insulators themselves aside from conditions of abnormal operating stresses.

There are a wide variety of operating conditions which affect the amount of over-insulation required, and after having found the minimum number of insulators per string required for any given operating conditions the author points out a method of determining the amount of extra insulation desirable from an insurance standpoint according to the law of probabilities. Equations are developed from which the probability of failure for any given case or the ratio between such probabilities for any pair of cases may be determined directly. A numerical example is also given which shows the development of the theory of minimum annual cost for combined mechanical and electrical failures.

AT THE present time there exists a diversity of views as to the primary cause of failure of suspension insulators of the disk type. Extensive investigations are being carried forward to determine the causes of failure in order that such causes may if possible be eliminated in manufacture. Unfortunately, however, while recent types have shown a notable improvement over those produced but a few years ago, failure is still sufficiently common to require allowance therefor in all specifications covering new lines. That is, it is not advisable in any construction to limit the number of insulators in a string to a quantity just sufficient to withstand the line pressure under service conditions, and thus cause failure if one member of the string fails. On the contrary, under present conditions, it is good practise to allow a certain proportion of extra insulators, a factor of safety required by the electrical unreliability of the insulators themselves, rather than by the necessities of abnormal operating stresses.

Obviously, the degree of over-insulation which should be indulged in depends on a variety of factors which cannot be directly expressed in mathematical terms. It is necessary to consider the first cost of the extra insulation and the fixed charges on such cost; also the counter-balancing expense involved in shut-downs, both prearranged and unforeseen. The requirements of continuity of service, and the availability of duplicate lines and stand-by power sources, have an important bearing on design. Also there is the factor of climatic conditions, the occurrence of lightning, temperature ranges, both daily and seasonal, humidity, the frequency of cleansing rains, the presence of salt fogs, wind-carried dust or the fumes of industrial plants. The nature of the line supports must be considered; we must know if the towers are of steel or wood, and if of wood, whether the support ends of the strings are electrically connected together. The transformer connections, whether with grounded or isolated neutral, must be noted. Mechanical stress is of importance. These, and many other conditions all affect greatly the degree of over-insulation necessary or advisable, and show why in some cases a 66-kv. line requires only three units per string, while under other conditions seven may be insufficient.

But having determined the minimum number of units per string required for a given voltage and set of operating conditions, it becomes interesting to investigate the value of additional units, purely from an insurance standpoint; and this investigation may be best performed by calculations of the chances of failure under the laws of probability. The physical condition under which the line operates will of course affect the rate of failure, but these conditions may be grouped and allowed for in the depreciation rate assumed.

In order to permit employment of the simpler laws of probability, it is necessary to assume that the successive failure of units in any string does not increase the liability to individual failure of the remaining members of that string until a minimum is reached which permits complete breakdown. At first glance this appears a most incorrect assumption, since, after the failure of one unit it is evident that there is an increased electrical strain upon the balance. Nevertheless, it is the belief of many investigators that the particular type of electrical failure which is today one of the great difficulties of transmission line operation, is in small degree hastened by increased electrical duty. In fact this opinion seems borne out by the relative frequency of occurrence of multiple failures which is usually found to be in the ratio expected under the laws of probability governing independent events. Furthermore, it has been the experience of some companies that insulator failures in lines out of service or in stocks held in the storeroom, have been in the same proportion as those experienced by units of the same lot in active use. So that while in some cases successive failures of members of the same string may not be considered independent events, in general we will not go far wrong in so considering them.

In consideration of the general case of insulator strings, consisting of any number of identical units, we may utilize the general expression of the probability of independent events happening or not happening:

$$P = p^n + n \,.\, p^{n-1} q$$

$$+ \frac{n\,(n-1)}{1\,.\,2}\, p^{n-2}\, q^2 \,.\,.\,.\,.\,.\,.\,.\,.\, + p^{n-n} q^n \qquad (1)$$

in which p is the probability of an event happening out of a single trial, $q = 1 - p$ is the probability that it will not happen and n is the number of trials under consideration. In this expression the first term is the probability that the event will happen n times consecutively or every trial, the second that it will happen $n - 1$ times out of n, the third $n - 2$ times out of n, etc. The probability of an event happening exactly r times out of n will be represented by the $(n - r + 1)$th term in the above equation, or

$$P_r = \frac{n\,!}{r\,!\,(n-r)\,!}\, p^r\, q^{n-r} \qquad (2)$$

and the probability of an event happening *at least* r times out of n trials is the summation of the terms up to and including the $(n - r + 1)$ th term.

$$\text{Thus:}\quad P_R = p^n + n \,.\, p^{n-1} q \;+\; \frac{n\,(n-1)}{1\,.\,2}\, p^{n-2}\, q^2 \,.\,.\,.\,.\,.$$

$$+ \frac{n\,!}{r\,!\,(n-r)\,!}\, p^r\, q^{n-r} \qquad (3)$$

Translating these equations to terms of suspension insulator failures, we may let n represent the number of units per string.

Likewise assume that if r units fail, a line failure will result, but that if less than r units fail the remaining insulation will be sufficient to "hold" the line and failure will not result. Then if $p =$ the probability of individual insulator failure (electrical) based on experimental data or experience with insulator depreciation, equation (3) gives directly the probability of line failure per string of insulators installed and the total probability of line failure will be found by multiplying the chances per string (P_R) by the number of strings (N) in the line or

$$P_N = N P_R \tag{4}$$

From equations (3) and (4) we may determine directly the probability of failure for any given case, or the ratio L between such probabilities for any pair of cases.

It will be noted that where p is small compared with q, as with depreciations of the order under consideration, only the $(n - r + 1)$ th term is of importance, the preceding terms being of such small value comparatively that to consider them is to assume an accuracy not warranted by the original data. We have, therefore, as a simplified equation of the chances of electrical line failure:

$$P_N = N \frac{n\,!}{r\,!\,(n-r)\,!} p^r q^{n-r} \tag{5}$$

Numerical examples are more easily solved, using this equation in the equivalent form:

$$P_N = N \frac{n\,!}{r\,!\,(n-r)\,!} \frac{(q/p)^{n-r}}{(1/p)^n} \tag{6}$$

as this will usually involve powers of integers rather than fractions.

The application of formula (6) to specific cases will often give interesting results. Fig. 1 indicates the range of example values of P_R plotted for three sets of values of $n - r$ and p Because of the minute values of P_R at the higher values of n the curves are plotted in terms of $\log 1/P_R$. Naturally where the safety factor $\dfrac{n}{n-r}$ is large $1/P_R$ attains values which are of little practical interest.

Fig. 2 is a diagram for the solution of the equation

$$P_R = \frac{n\,!}{r\,!\,(n-r)\,!} p^r q^{n-r}$$

From this diagram values of the probability of failure per string can be quickly determined, with reasonable exactitude. The solution is based on a straight line diagram deduced from this equation in the form:

$$\log 1/P_R = n\,[2 - \log (100 - s)\,]$$

$$+ \; r \log \frac{100 - s}{s} - \log \left[\frac{n\,!}{r\,!\,(n - r)\,!} \right]$$

where $s = 100\,p$ or the depreciation in per cent.

Of equal interest with the probability of failure in any

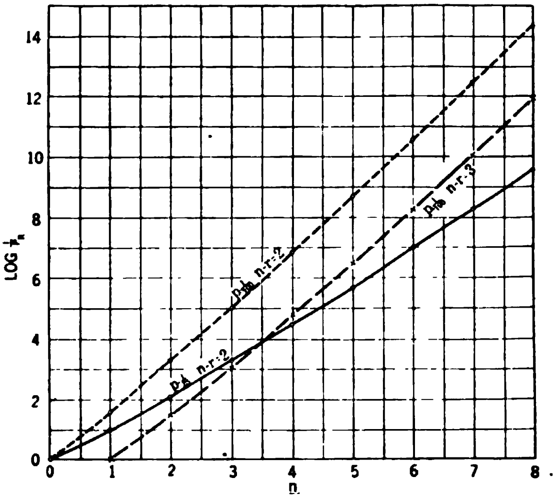


FIG. 1—PROBABILITY OF ELECTRICAL FAILURE PER INSULATOR STRING—EXAMPLE VALUES OF P_R

specific case is the determination of the ratio L between the probabilities of failure in related cases and in particular the decrease in probability of failure (or insurance) derived from the addition of extra insulators. In determining the ratio between probabilities it will be sufficiently accurate to employ the approximate formula (5) and noting that in any assumed case having to do with a specific line, the quantity $n - r$, that is, the number of insulators at which failure will occur is constant, it may be readily shown that

$$L = \frac{n_1\,(n_1 - 1)\,(n_1 - 2)\,.\,.\,.\,(n_2 + 1)}{r_1\,(r_1 - 1)\,(r_1 - 2)\,.\,.\,.\,(r_2 + 1)}\; p^{n_1 - n_2} \qquad (7)$$

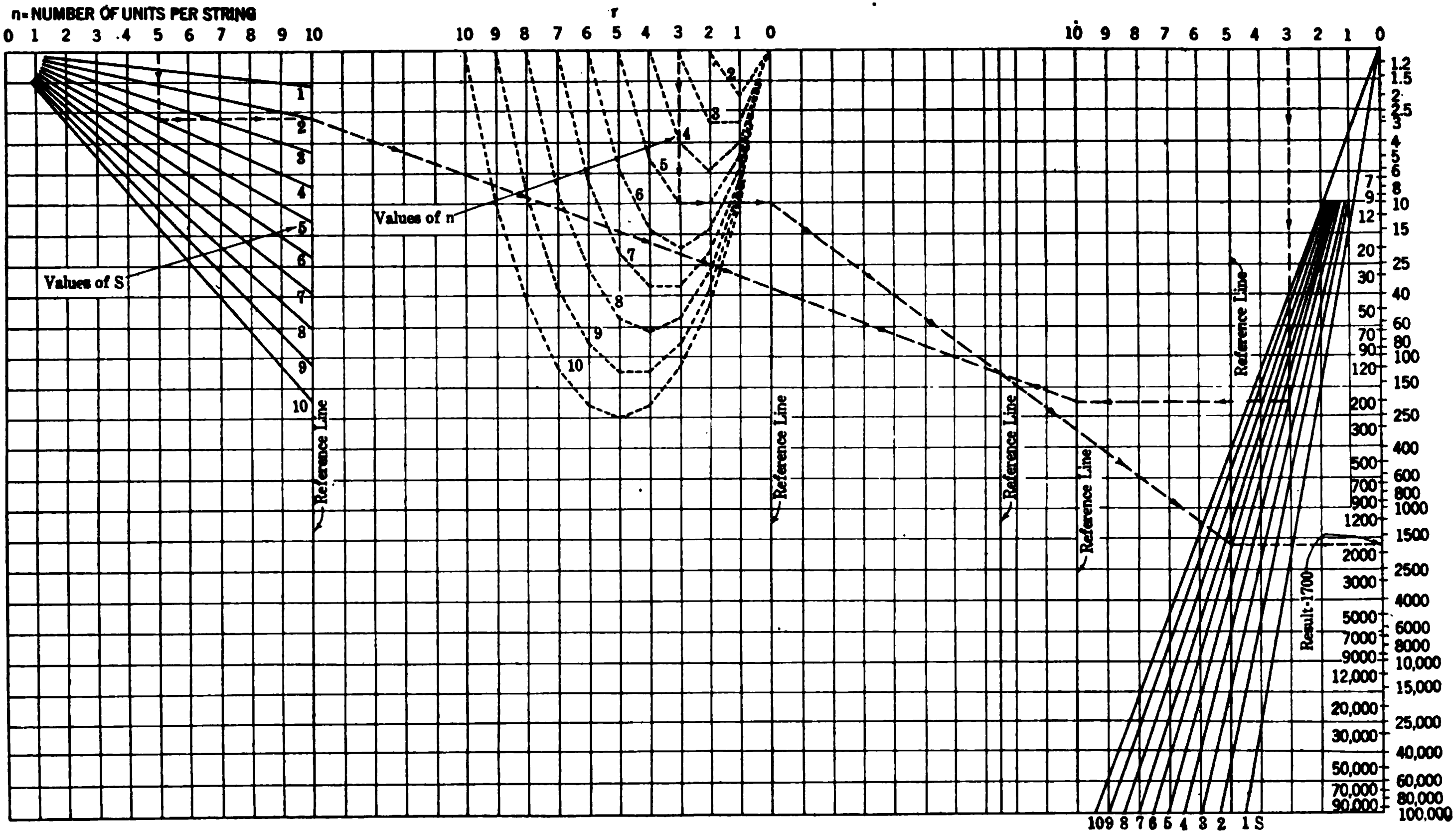


Fig. 2—Diagram for Graphical Solution of $P_R = \frac{n!}{r!\,(n-r)!}\,p^r q^{n-}$ = Probability of Failure per String per Year

n = number of units per string; r = number of units per string that must fail to result in line failure; S = annual depreciation in per cent = $100p$.
Example worked out, $n = 5$, $r = 3$, $S = 4\%$. Result $1/P = 1660$

in which the subscripts 1 and 2 indicate the two conditions assumed as to number of units per string.

It is evident that in most practical cases $n_1 - n_2 = 1$ or 2 since usually it will be desired to determine the reduction in probability of failure secured by increasing a string by one or two insulators. In Table I will be found values of

$$C_L = \frac{\dfrac{n_1\,!}{r_1\,!\,(n_1 - r_1)\,!}}{\dfrac{n_2\,!}{r_2\,!\,(n_2 - r_2)\,!}}$$

for $n_1 - n_2 = 1$ and varying values of $n - r$, and since when $n_1 - n_2 = 1$, $L = C_L\,p$, any problem having to do with the insurance derived by the addition of a single unit may be determined directly from the table.

TABLE I—VALUES OF COEFFICIENT C_L for $n_1 - n_2 = 1$ (1 insulator added to string)

$n - r$ Number at which breakdown occurs	n_2 = Number of insulators in string before addition of one unit									
	1	2	3	4	5	6	7	8	9	10
1	2	1.5	1.33	1.25	1.2	1.17	1.14	1.12	1.11	1.1
2	..	3	2	1.67	1.5	1.4	1.33	1.29	1.25	1.22
3	..	..	4	2.5	2	1.75	1.6	1.5	1.43	1.37
4	..	..	..	5	3	2.33	2	1.8	1.67	1.57
5	..	..	..	..	6	3.5	2.67	2.25	2	1.83
6	..	..	..	..	..	7	4	3	2.5	2.2
7	..	..	..	..	..	..	8	4.5	3.33	2.75
8	..	..	..	..	..	..	..	9	5	3.67
9	..	..	..	..	..	..	..	..	10	5.5
10	..	..	..	..	..	..	..	..	..	11

Figure 3 shows some example curves plotted from the use of Table I.

In addition to these conclusions as to the insurance derived by adding units to strings of insulators in transmission lines, it may be noted with reference to dead-ends in higher voltage distribution lines, that where a single unit is adequate to normal service, but is subject to failure at the rates to be expected amongst suspension units, that is, of an order of magnitude of from one-quarter of one per cent to 4 per cent per annum, the single unit is no longer to be considered as economically sufficient; for the line reliability will then be only the relatively low reliability of the individual units multiplied by the number of units. With two units the probability of failure is greatly reduced and in a general way it may be said that with lines of sufficient voltage so that line failures almost invariably follow the failure of the principal insulating member at any point of support (that is in lines of 10 kv. and up where subsidiary insulators in the form of wood pins, cross-arms or poles will not withstand normal line voltage) it is never economical to use single suspension units as dead-ends. Even insulators of weaker design (electrically) are in such cases to be preferred in pairs to a single unit, provided that one unit of the weaker pair will withstand normal operating stresses in case of failure of the other. For instance, if the rates of depreciation are 4 per cent for the small units and 1 per cent for the large, the ratio of line failures in the two cases will be 6¼ to 1 against the single large unit.

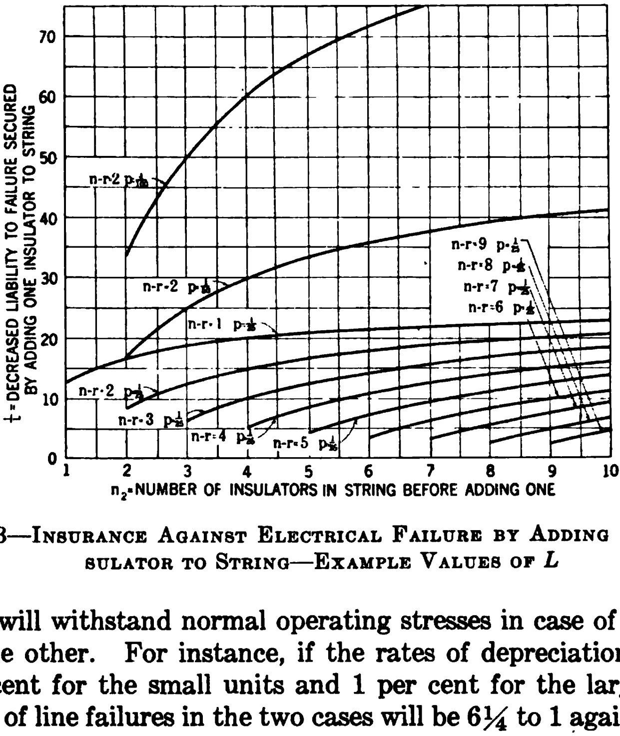


FIG. 3—INSURANCE AGAINST ELECTRICAL FAILURE BY ADDING ONE INSULATOR TO STRING—EXAMPLE VALUES OF L

These considerations likewise show why an insulator string composed of a number of virtually independent units is generally to be preferred to any form of support consisting of a single unit or group of interdependent units.

It may be said that it is difficult to determine $n - r$ or the number of insulators which is one less than will "hold" the line upon which the use of this formula rests. The ordinary 10-in. suspension insulator is usually rated as having a dry flash-over value of 90 kv. and a wet value of 70 kv.; in pairs the corresponding values are respectively 160 kv. and 110 kv. Yet we know from experience that two of these insulators would not stand up on a 66-kv. line under the most favorable conditions for more than a short period. Line surges, weather conditions, etc., all reduce $n - r$, and it is important in choosing a value for $n - r$ in these formulas that the quantity be understood to represent not the number of insulators at which break-down will occur instantly, but rather the number at which break down will occur under normal operating conditions within a short period subsequent to the failure of the $(n - r + 1)$th unit of a string. This break-down may always occur subsequent to surges which are to be periodically expected in any line and if these surges are sufficiently frequent to greatly outnumber the expected string failures, as they will be in almost every practical case, simultaneous failures will not be sufficiently frequent to render the formulas incorrect. The finding of numbers of shattered strings after a total failure is not generally an argument in favor of the occurrence of simultaneous failures, but rather these shattered strings represent previous unexplained momentary failures. Furthermore, the tendency of surges to produce simultaneous failures and thus reduce line outages, is to a certain extent balanced by the original assumption of successive unit failures being entirely independent events, which is not strictly true. In general, it may be said that owing to the difference between test and laboratory conditions, and further because of surges, which tend to give any line a true voltage as regards effects on insulators in excess of its rated voltage, we will not go far wrong in basing $n - r$ on half the wet flash-over rating given by manufacturers to their product. It may be noted from Table I that a variation of one in $n - r$ does not greatly affect the value of L in most practical cases.

As we increase the number of units and thus decrease the chances of electrical failure, we increase the chances of mechanical failure. Mechanical failures are of two general classes,those due to lack of strength or inherent defects in the insulator or its supports, and those due to external causes, such as the malicious use of firearms. Obviously, any mechanical failure means a line failure, particularly at dead-end points where mechanical failures of the first type are most likely to occur.

By mechanical failure is meant the complete breakage of a unit as a means of support. Mechanical failures of the several types which are the initiating causes of electrical break-down (as, for instance, those which follow cyclic temperature changes) but which do not in themselves cause the line to fall, must be considered the equivalent of electrical failures and must be allowed for in the choice of the value of p_e (the probability of electrical failure per unit), since they affect line failures under the same laws. But for true mechanical failures the chance of failure per string is the chance per insulator times the number in the string, or

$$P_m = n\, p_m{}^* \tag{8}$$

or for the whole line

$$P_{Nm} = Nn\, p_m \tag{9}$$

Since electrical and mechanical failures as above defined are independent and exclusive events, the probability of failure of both kinds is equal to the sum of the probabilities of failure of either kind, or

$$P_{Nt} = N\, n\, p_m + N \frac{n\,!}{r\,!\,(n-r)\,!} p_e^r\, q_e^{n-r} \tag{10}$$

This equation may be most easily investigated for the value of n which will give a minimum of annual failures for any set of values of p_m, p_e and r by plotting, particularly as short cuts have been given for determining values of P_{Ne}. But in general we are not as much interested in the value of n which in any case will result in a minimum number of failures. For many values of p_e and p_m this will give impractical or impossible results. What we are interested in is the reduction of cost of service, including all elements of fixed charges, maintenance and depreciation.

Formula (**10**) may be applied to a practical problem in the following manner:

Assume a line containing 1000 insulator strings. Two good insulators in a string will result in a line failure; three will hold the line. It is required to determine whether four or five insulators per string will result in the most economical installation.

Let it be assumed that insulator units in the line as constructed will cost \$3.00 each. The fifth unit is also estimated

---

*The subscripts m and e are used hereafter to distinguish between mechanical and electrical failures.

to cost \$3.00, no additional cross-arm length or tower height being required for clearance owing to the extra 5½ in. of length of string. Fixed charges are taken at 12 per cent, this figure including functional depreciation of insulators, but being exclusive of physical depreciation, which is taken care of under maintenance. The cost of replacing an insulator unit which has failed electrically, but which has not resulted in a line failure, is estimated at \$5.00, it being assumed that the line is gone over with suitable testing apparatus annually, all defective units being replaced at that time. The cost of field testing insulators is taken at 10c. each. The cost of an unexpected line failure, including location of the failure, replacement of the broken units, loss of business and stand-by plant operation, is taken at \$150.00. Naturally, this amount is subject to wide variation, being made up of a number of items, some of which, like the value of continuity of service from a public policy standpoint, cannot be expressed directly in terms of money. In fact it is conceivable that in some cases the cost will be less, but in many cases greatly in excess of the above figure, particularly where service is dependent on a single line and the cost of stand-by energy (if available at all) is considerably in excess of the cost of energy secured over the transmission line. Assume an electrical failure of four per cent per annum, and a mechanical failure (as above defined) of one-fortieth of 1 per cent as an average for units both in suspension and in strain at dead-ends.

We now have the following tabulation of annual costs:

	Quantity			Amount	
	Four units	Five Units	Unit cost	Four units	Five units
Fixed charges	4000	5000	12% of 3.00	\$1,440.00	\$1,800.00
Total electrical failures (4%)	160	200	5.00	800.00	1,000.00
Electrical line failures (P_{Ne})	8.85	0.59	150.00	1,327.50	88.50
Mechanical line failures (P_{Nm})	1	1.25	150.00	150.00	187.50
Insulator testing	4000	5000	0.10	400.00	500.00
	Total annual cost			\$4,117.50	\$3,576.00

It will be noted that all of the above items of cost, with the exception of those due to electrical line failures, vary directly with the number of units in use, and may therefore be grouped. In the above schedule these items total 23.25 per cent of the first cost of the units installed, and the same result would have been secured by taking annual charges at that figure instead of 12 per cent. Summing up the terms of this nature, we have.

$$y = F_c A N n + B N \frac{n\,!}{r\,!\,(n-r)\,!} p_e^r q_e^{n-r} \qquad \textbf{(11)}$$

in which y is the total annual cost, F_c is the per cent covering fixed charges and items of maintenance varying directly with the number of units installed and determined as illustrated in the table above, A is the first cost per unit installed and B the cost per unexpected failure. Differentiating with respect to n and equating to 0 for a minimum we have:

$$F_c A + B \frac{n\,!}{r\,!\,(n-r)\,!} p_e^r q_e^{n-r}$$

$$\left[(n-r)\,!\left(\frac{1}{n} + \frac{1}{n-1} \cdots\cdots\cdots + \frac{1}{n-r+1}\right) + \log_e q_e \right] = 0 \qquad \textbf{(12)}$$

This equation is too involved to be used conveniently in solving for n and it is more practical to determine the value of n which gives a minimum total cost in any case by using Fig. 2 in connection with equation **(11)** and solving by plotting or tabulation. For instance, in the example above discussed we have:

n	$F_c\ A\ N\ n$	$B\ N\ P_R$
3	$2,092.50	$16,595.00
4	2,790.00	1,327.50
5	3,487.50	88.50
6	4,185.00	5.30

A casual inspection shows that $n = 5$ gives a minimum annual cost in this case.

In the utilization of the formulas proposed, it is essential to differentiate between electrical depreciation failures and line condition failures. The first affects the selection of p_e, the second of $n - r$. The first is usually although not invariably characterized by failure by puncture, the latter by failure by flash-over. In some districts, notably along the California coast, climatic and atmospheric conditions are such that any line, regardless of the degree of over-insulation used will ultimately fail by flash-over, if not given continued attention. The gradual accumulation of salt and dust, particularly on the under side of the units, beyond the reach of the infrequent winter rains, will eventually result in flash-over. In other words, as time goes on, if the insulators are not artificially cleaned, r becomes zero and line failures result in great numbers without the occurrence of a single failure of the other type.

It is therefore necessary in considering a problem of this type to note the value which $n - r$ will reach before natural causes, such as a cleansing rain, or some artificial means is used to reduce this quantity to its original value.

On the other hand it should be emphasized that in this discussion the probability p_e of electrical failure is intended to cover those types of failures variously attributed by insulator specialists to porous porcelain, to expansion difficulties induced by cyclic temperature changes or to combinations of the two. In any lot of insulators under a specified set of conditions a certain proportion will become defective, due to these or similar causes, and the balance will remain intact. There is at any given time a reasonably sharp line between the good and the bad; that is to say, between those that will fail when moisture and line conditions are favorable to failure and those that will not. In general, insulators deteriorated in this manner under no circumstances will return to their original quality, although failure may not be instantaneous, and although testing by the several common methods (high-frequency, high-potential and megger) may indicate a variety of degrees of deterioration. But under conditions of incipient failure of the surface leakage type all the individual units will be found uniformly deteriorated (within fairly narrow limits), and in this type of deterioration of condition there is no sharp line between a "good" and a "bad" unit. Furthermore these units may at any time prior to damage by a power arc be returned to their original quality by cleaning. It is evident at once that while failures of the puncture type may be and probably are independent events, failures of the surface leakage type can under no circumstances be so considered. Under flash-over conditions individual units do not act independently.

Although nearly every insulator failure is finally a flash-over there is little practical difficulty in differentiating between the two types of failure. The flash-over due to deteriorated line conditions shows a heavy foreign deposit on the insulator surface. In almost every case the bottom member of the string will be shattered; often the top member also, but rarely an intermediate member. Unbroken units will before cleaning give a zero reading with a megger and a very low flash-over with a high potential test. Subsequent to cleaning, however, unless badly burned by the power arc, they will give as high a megger and flash-over test as when new. Lines subject to difficulties of this kind cannot be tested with a megger in the field even on a clear, dry day, unless each unit is wiped off before test.

In the other type of failure we have the fundamental condition that r units have failed by material depreciation (usually puncture) and the balance $(n - r)$ being unable to hold the line have subsequently flashed-over. The characteristic differentiating this class of failure from the other is the number of punctured units which occur at random throughout the string. Even if subsequently shattered by the power arc they may usually be readily distinguished from units shattered by the other type of failure.

In testing and renewing insulators on the line it is desirable that results be recorded in detail. The distribution of failures between strings, their location in different sections of the same line and the relative frequency of failure of units in corresponding positions in the string, all will be of interest in deducing the causes of failure. It is desirable to know whether bottom, top or intermediate units fail most frequently, and in what ratio; whether units in greater tension at dead-ends fail more frequently than those in suspension; whether units with bolts exposed to the direct rays of the sun fail more frequently than those on the same tower but with bolts toward the north and consequently protected from the sun by the porcelain shells. It is of interest to note whether failures follow hot spells, whether they are decreased by cleansing rains and other effects of the weather. If lines operating under similar conditions contain units differing in manufacture, it is of course important to determine separately the percentage of failures for each type. Also by noting the relative frequency of single and multiple failures, it is possible to determine whether failures follow the laws of probability for purely independent events, as above outlined, or whether successive unit failures are to a certain extent dependent events.

DISCUSSION ON "THEORY OF PROBABILITIES APPLIED TO FAILURES OF SUSPENSION INSULATORS" (KLAUBER), LOS ANGELES, CAL., SEPTEMBER 18, 1919.

W. D. A. Peaslee: It would be a very serious obstruction to the progress of insulator development if the viewpoint expressed by the author were to be adopted in general by the operating engineers charged with the care of high-voltage transmission lines.

In the first place, the author seems to take the mental attitude that the development of the high-tension insulator has reached its ultimate and that more admittedly poor insulator units are to be the solution as against fewer units of a better type with wider margins of safety in the individual unit.

His main contention seems to be expressed in the sentence, "It is not advisable in any construction to limit the number of insulators in a string to a quantity just sufficient to withstand line pressure under service conditions, and thus cause failure if one member of the string fails."

This statement pre-supposes the acceptance of the statement that when a string is *just sufficient* from an engineering standpoint, the failure of one unit of the string will cause the failure of the string. With a properly designed insulator, this is not the case.

In almost all cases, except where excessive salt, dust andf ogs are encountered, the determining factor in selecting the number of units of a string is the voltage stress on the end units. Too few engineers appreciate the actual numerical values of voltage strains on the end units and there is too little published data on the subject. In this connection, the writer will say that a paper is in preparation giving these distributions on strings up to ten units and under various conditions that it is hoped will be of interest to the engineering fraternity.

The end unit of a string must be operated at a voltage stress *below that at which corona appears.* I think all the men who have been studying this subject will agree on this point. If N is the number of units in the string to meet this condition, a string of N-1 units, caused by the failure of one unit will, it is true, probably operate with some slight corona on the end unit of the N-1 string; but with a properly designed unit, this will not be the case, and even under slight corona the unit will operate for an indefinite time if properly designed and made.

It would seem better to work towards the development of such a unit than to accept the present thin porcelain insulator as the ultimate and pay for extra unit insurance that is only a time function anyway.

It will be of interest generally, I think, to state here some progress that has been made, and show briefly what can be realized with an insulator of proper design and quality.

The function of an insulator is three-fold:

1. It must support the line mechanically.

2. It must insulate the line.
3. It must resist all possible temperature changes that may be met in practise, both as to range of temperature and time in which the range is encountered.

To support the line a simple mechanical strength is necessary, and for ordinary cases a unit with an ultimate mechanical strength of 10,000 pounds is considered ample. Every unit should be tested to 5000 pounds before the electrical tests are applied.

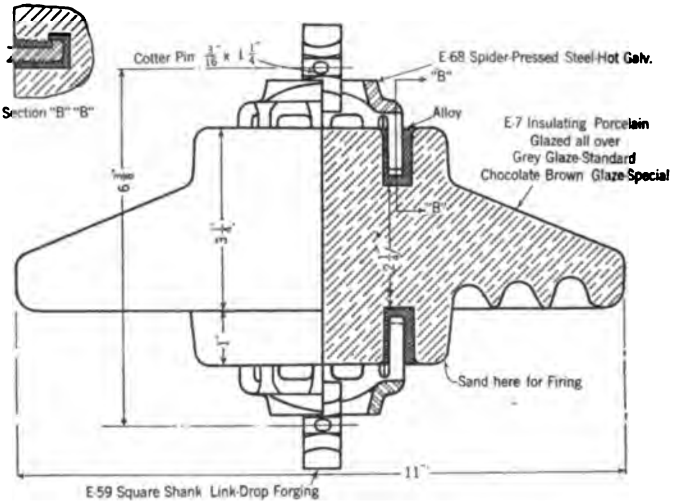


FIG. 1

In the matter of insulating the line, the problem becomes extremely complex. Irregular distribution of the electric stress on units of a string, lightning, transients, dust, etc., etc., are factors that must be taken into consideration. One of the most serious causes of failure of thin porcelain insulators is, the puncturing power of transients of extremely abrupt wave fronts. The writer has presented to the Institute two papers on this subject from a laboratory and operating experience. It may be said that the general answer to this series of factors is an ample thickness of homogeneous porcelain properly fired. A very few years ago the manufacture of such porcelain was admittedly impossible, but that is not the case today, as porcelain of almost any desired thickness can be made absolutely homogeneous and of the highest quality, free from porosity.

The temperature cycle resistance is solved by eliminating the cement cap and pin type of construction with its several materials of different coefficients of cubical expansion in rigid contact and its hydroscopic material for further trouble.

A further electrical requirement is a rational electrical design, avoiding zones of high dielectric flux concentration and producing a unit in which the dry flashover and first appearance of corona are as close together as possible. In a laboratory type, these may be made the same, but commercial considerations of manufacture make this ideal condition impossible of realization in quantity production.

Consider, then, an insulator of the design shown in Fig. 1 and of the following electrical and mechanical characteristics:

Mechanical Strength	10,000 pounds
(Each unit tested to 5,000 — before electrical test)	
Dry flash over 60 cycles	100,000 volts
Wet flash over 60 cycles	50,000 volts
Dry flash over 200,000 cycles	120,000 volts
First leakage corona	45,000 volts
Easily visible corona	60,000 volts
Puncture	250,000 to 300,000 volts

Thickness of porcelain between elecrodes 2¼ inches.

If, now, we use five of these units on 110,000 volts, (a line has been in successful operation for some time under these conditions) the stress on the end unit will be approximately 33,000 volts. This unit then operates with the following safety factors:

Safety Factor

Dry flash over 3.
Wet flash over 1.5.

(If the distribution were the same wet as dry), but as the distribution is better wet than dry, the safety factor as indicated by measurements taken under standard rain test conditions indicate that the safety factor due to reduction of stress on this unit reaches 2.1.

Puncture 7.5 to 9. Due to the very thick porcelain which reduces the flux concentration on the electrodes, the effect of transients is less and the thickness available for resisting them much greater than in the thin porcelain type.

Margin to Corona voltage	12,000 volts	Safety Factor
Dry flash over of string	340,000 volts	2.8
Wet flash over of string	220,000 volts	1.8

The alloy used in fastening the spider legs into the porcelain has the same coefficient of cubical expansion as the porcelain used in the insulator and this, with the spider support, makes an insulator that can be plunged from boiling to freezing water as often as desired without fracture or chipping.

Suppose now that a lightning stroke or rifle bullet cracks a unit. We find ourselves with four units supporting the line and the following conditions, remembering that total destruction of eight spider legs is necessary before the line will drop if a unit fails.

Voltage on end unit, approx.	41,000 volts
	Safety Factor
Dry flash over	2.44
Wet flash over (as discussed before)	1.8
Puncture 6 to 7.5	
Margin to corona	4,000 volts

	Safety Factor
Dry flash over of string 280,000 volts	2.55
Wet flash over of string 185,000 volts	1.68

Under these conditions no corona would be visible though there would be a leakage corona. This string would operate indefinitely but is not the correct number of units for 110,000 volts.

Considerable has been said regarding the danger of using porcelain in tension and its supposedly excessive fatigue when in tension. Research laboratory tests now partially finished, indicate that there is no foundation in fact for these fears, as indeed there is none in logic. The results of these and other tests will be published at an early date.

It would seem then that the basic contention of Mr. Klauber's paper is wrong and that there is relief at present available for the condition he describes. We should look to improvement in insulators rather than to more units of inferior characteristics as a solution for the insulator situation.

I am very glad to see a recognition by one of the operating engineers of the fact that "any line, regardless of the degree of over-insulation used, will ultimately fail by flash over, if not given continued attention" along the California coast regions. This is a fact that I think some engineers have lost sight of in their enthusiasm for adding more units to a string to take care of this creepage condition.

C. O. Poole: Would say that as I view the situation, Mr. Klauber has endeavored, and I think has worked out a principle upon which we can calculate a certain factor of safety upon the ordinary insulators as they are used, and, by varying, it very probably can, with some uncertainty, be applied to different climatic conditions.

This subject of climatic conditions, I think, has a great deal to do naturally with the operation of insulators, as well as with any attempt that may be made to formulate a definite equation that might cover all conditions. As a matter of fact, I think it is impossible to make a definite equation that will cover all conditions with any degree of accuracy. However, I think Mr. Klauber has accomplished one thing and that is the fact that he would eliminate excessive investment for a given insulation on a line. We must, however, acknowledge that with the uncertainties of manufacture and the uncertainty of types of insulators, it throws an element of uncertainty into the whole proposition. While I feel that we will get some good out of the paper, yet I think it is open to betterment in this respect.

J. A. Lighthipe: The insulator game is progressing. Of course, every new insulator manufacturer that comes to the coast has finally developed the only perfect insulator. However, it usually takes some experience to get at the truth of the matter. The insulator game is one we have been watching very closely for the reason that all our service throughout Southern California and middle of California, is depending on the insulators doing their proper work.

Even with the tools that people have contrived to test these insulators, just as Mr. Barre said in the discussion of the preceding paper, out of a clear sky comes a flashover. Sometimes it is serious and interrupts service. I rather agree with Mr. Klauber in his idea. Three insulators will stand up one or two years; perhaps four will stand up six or seven years. That extra three or four years means a great deal to the operating company. What we have been working on as much as anything else is to try and get a careful record of the breakdowns of the insulators and, if possible, find out why they break down in that locality.

We all have our own theories. Perhaps this terrible catastrophe we are experiencing with insulators happened over the oil well sections. We all know that in our oil well sections, especially in the East, there is a constant flow of helium which, is an emanation from radium and, passing our insulators, may ionize them.

I thought my theory just as good as Barre's, but at the same time we are awaiting an insulator which will hold up and we are glad to say they are holding up a little better today, and without doubt a great deal better than those we had a few years ago. The unfortunate part of the test is that it takes two or three years to find out absolutely, whether they are good or bad. Any insulator is good for the first season; most of them last two seasons without any breakdown. But, with all your careful testing, it is expensive to root out the bad insulators. Every improvement that can be made in these insulators which will hold up the line another year means that much reduction in the operating expenses.

J. H. Anderton: From Mr. Barre's remarks regarding insulator flashovers on the Big Creek line, it would appear possible to tie Mr. Klauber's and Mr. Curtis's papers together.

If I am not mistaken, this question of flash-overs on the Big Creek lines was first noticed in 1915. It is perhaps difficult, if not impossible, to definitely explain it. I have, however, a theory which may apply to it as follows:

The length of the Big Creek lines is approximately 241 miles. At 50 cycles a quarter wave length of the fifth harmonic would be 187 miles or 54 miles from one end of the line, and I understand from Mr. Barre that these flash-overs occur at approximately this distance from the end of the line. The transformer connections at both ends of the line suppress the third harmonic and some others; they do not, however, suppress the fifth, which is often found in generators. The current in the fifth harmonic wave may be very small, in fact, such that the line offers no appreciable resistance to it and my theory is that a condition of local resonance is set up in the line due to the fifth harmonic which is, of course, superimposed upon the fundamental voltage and causes the flash-over. There are, of course, innumerable frequencies which could set up resonance conditions in the line such as lightning and similar disturbances, some of which may travel comparatively short distances before causing flash-overs. The recurrence, however, of flash-overs at or near a certain point in the line would usually indicate something inherent in the system.

This question of flash-overs naturally brings up the question of insulators. The selection of suitable insulator string involves the selection of a suitable factor of safety. The line may be so insulated that transient waves of voltage may be carried along the line to the power house and thus destroy apparatus. In general, it would appear preferable to select a factor of safety such as will give a reasonable degree of security against the break down of units in a string but will not prevent the flash over and consequently the dissipation of extreme high-voltage waves which might destroy transformers and generating equipment. In other words, it is preferable to have an occasional string of insulators destroyed than to have apparatus in the generating station destroyed.

H. A. Barre: There is a very pronounced family of harmonics in many systems. There is a very pronounced 11th and 13th and there was a strong third originating with the bank of compensators when the 50 and 60 cycles were tied together. That has been corrected to some extent by interconnection between this bank of compensators and the bank of transformers located in the same station, the compensators being star-star connected and the transformers star-delta. The 3rd harmonic current that circulates in neutralizing, at Eagle Rock, was measured at 1.8 amperes; that 1.8 amperes distributing itself through three large transformers in the ratio of 1000 star and 150,000 volts delta. This made the trouble negligible.

J. B. Fisken: On the last page of Mr. Klauber's paper, he makes this statement: "It is desirable to know whether bottom, top or intermediate units fail most frequently, and in what ratio." We have heard nothing on it and I am sure that I have nothing to offer.

I would like to know, from those who can tell us, something about the potential gradient through the string. That seems to me to have a direct application in the safety effect which is to be obtained by adding units.

There is something I do know about, however, and that is -Mr. Klauber's next point: "Whether units in greater tension at dead-ends fail more frequently than those in suspension."

The Washington Water Power Co. does not have very many suspension units on its system. It has, however, a short line about 28½ miles, a steel tower line with' suspension insulators, operated at 60 kv. Now that line of itself is short and does not introduce many difficulties, but that is not true of the rest of the system, consisting of about 600 miles of 60 kv. line, and it is liable to get any surge that may originate on other parts of the system. The line has been in operation for nine years and the company has yet to find, the first case of failure of the suspension units other than those caused by lightning and the small boy with a rifle.

On the other hand, some two years ago it had a great deal of trouble with the same units used as strains and it was necessary to replace, or rather it was decided to replace all of the strain insulators at that time. These were taken down without the particular position on the line or the particular position in the string of any particular unit being noted and a rough test was run on them, first with the megger and then by direct application of 90 kv. Thirty-three per cent failed under the test. The balance were put back into stock and so far as I know they have been used up and are on the lines today. I think that, perhaps, answers in part this question as to the relative difficulties on string and suspension insulators.

R. W. Shoemaker: We have made quite a number of tests on this very subject. You can take a string of ten insulators 5½-inch spacing, without arcing horns on high voltage, and, commencing with the insulator next the wire, the potential across number one unit will be about 20 per cent, or in other words, twice too much, the insulator third or fourth from the tower will have the least stress. With arcing horns the second insulator from the wire will have the highest stress and not the one next to the wire.

We have a device that is going to be used on our 150,000-volt transmission line now building by which the potential gradient in a string of insulators can be controlled and consists of a metallic disk that may be interposed between any two insulators and in metallic connection with the insulator hardware.

It is possible with this device to make the stress across each insulator in a string exactly equal if such a condition were desirable or necessary.

For example, a disk 12 inches in diameter interposed between the wire and the first insulator reduces the stress on that insulator from 20 per cent to 16 per cent, if the disk is 26 inches in diameter the stress will be 14 per cent. Of course this latter disk is too large for practical purposes, but by using two disks located between the wire and first insulator and between the first and second insulators (counting from the wire)the stress on these insulators may be kept to below 15 per cent.

We have taken a string of insulators and put potential on them until the hardware, without disks, about the wire began to fire, then putting the disks in place the string under the same potential as previously applied would be perfectly quiet even in a dark room.

Mr. Klaüber's paper gives methods for figuring out the probability of certain conditions that may obtain using as basic data the percentage of failure. It seems to me that any results based on this method would be of little value as the "percentage of failure" is an extremely uncertain quantity and varies over a wide range with different types of insulators, conditions of service, etc., so that using this method no prediction of continuity of service could be made for a line without, first, several years' experience with that particular line on which to obtain necessary data.

E. F. Scattergood: In determining generating plant capacity we first figure out the demand or load requirements throughout the year and then give consideration to the question of reserved units, or per cent of reserve plant capacity giving consideration to the importance of the continuity of service as against the cost.

The subject matter of Mr. Klauber's paper relating to the number of units in a line insulator is, it seems to me, a direct parallel; first, an insulator unit, or string of units, has a certain working voltage capacity or rating as we call it, which takes into account the question of not over stressing the insulator under ordinary operating conditions. Then the question arises, shall we add a unit or two as reserve units? Giving reserve strength, rather than capacity in this instance, which may result in the prevention of service interruption by testing the units on the strings along the line every six months or so and replacing bad units before the string fails.

If an insulator deteriorates to the point of worthlessness in the interum, we have a reserve unit there ready to take up the work and avoid interruption of service the same as starting up the extra steam or hydroelectric unit, or extra per cent of plant capacity.

The objection is made that such added insulation throws a higher duty on the station arresters causes a greater menace to station equipment, which is true but the benefits mây far outweigh the objections, which of course, would depend upon the circumstances, the length of the line and the importance of the distributing point at the end of the line.

On a short line the cost of an extra unit or two is quite nominal. The city of Los Angeles, in constructing its power transmission lines from its present large generating plant, 46 miles, by transmission from the city, and from the second large plant now building in approximately the same location, allowed extra insulation for the above purpose. We believe in it, especially in a short line. It adds to the duty of the lightning arrester, no doubt, but it does reduce the interruption of service on the line to a minimum, and that is important.

In considering the paper we must distinguish between insulators, or insulation, necessary to take care of the ordinary conditions of operation, allowing the proper factor of safety, and the proposition of adding one or two units in excess of that. We apply the principle generally in engineering and I see no reason why it should not be applied in line insulation provided we can protect the station equipment from the higher voltages, which are possible on the line through the use of lightning arresters.

L. C. Williams: Insulators, after having been in service for a period of years, will depreciate much more rapidly than those which have been in service for a period of one year. I would like to ask Mr. Klauber if he has taken that point into consideration?

From the manufacturer's standpoint it is quite possible to manufacture suspension insulators which will have much greater electrical and mechanical efficiency than those we have at present but it is also well known that those insulators will be prohibitive in cost from an operating standpoint.

Mr. Peaslee does not mention the unit costs of his insulators. Mr. Klauber takes the unit cost of $3.00 per unit installed on his line. If Mr. Peaslee's insulator being considerably heavier costs considerably more installed on the line, it should be taken into consideration that an economic comparison, even presupposing that four of his units will do the work electrically as against five of the ordinary cap and pin type, could not be accurately made without having the unit cost to base figures on, I cannot make the comparison myself but it is a point worth considering.

Now, as to Mr. Fisken's question of differential. There have been experiments made which give the curve practically the same as shown by Mr. Shoemaker's remarks. In a string of twelve insulators the unit nearest the line wire will take approximately twenty-six per cent of the duty without arcing horns. By adding arcing horns to the lower end of the string alone, that unit will take approximately ten per cent of the duty over a string of twelve and the next unit to the line will take approximately fourteen to sixteen per cent.

I have no data whatsoever on the failures with relation to the position on the string.

Mr. Peaslee mentions a line in successful operation at 100,000 volts with five units in a string of the type he advocates. One of the principal points of Mr. Klauber's paper is climatic conditions which have a decided bearing on his laws of probability. Unquestionably, under certain conditions, that type will operate satisfactorily on 100,000 volts, but on the Pacific Coast experience has shown that puncture value is not necessarily the governing factor, but leakage surface is much more so than the puncture value.

I think it possible the addition of extra units should be taken into consideration as insurance against the leakage over the strings rather than the puncture value of the units in the string.

L. M. Klauber: With reference to Mr. Peaslee's remarks, he would seem to think that I am a pessimist with regard to insulators. Of course, I know from experience and from discussing this matter with the representatives of nearly all manufacturers, that the manufacturers have indeed made great progress in the last few years in their product. If a person is a pessimist I think he is usually a pessimist between two and four o'clock in the morning when the insulators fail and the lines go out. We all know that they do go out. No one hopes more than I do that the manufacturers will make perfect insulators. Probably they are making perfect insulators today, but the last ones we got day before yesterday were not perfect; and we rather fear that when tomorrow comes and the insulators of today have failed we will again be told to overlook the past and think only of the perfection of the present.

With reference to Mr. Anderton's remarks on the subject of over voltage: this paper, being based as it is upon the determination of minimum annual cost, not only specifies what the minimum number of insulators in a string should be but also the maximum. It would seem that dependence on having insulators fail in order to protect station apparatus is a reflection on the manufacturers of lightning arresters. The great difficulty in depending on the insulators is not that they will not fail; they do that readily enough. But it is difficult and expensive to locate a failed string after breakdown and it would seem best to protect against high-voltage strains by other means especially designed for the purpose.

As to the ratio of failures in different members of the string and the effects of unequal stress distribution: our own results and extensive data given us by other companies would tend to show that there is virtually no difference in failures; in spite of the fact that the insulators close to the line are the more highly stressed, they fail no more rapidly than the others. Failure of the type under discussion is not primarily induced by electrical stress and may be almost as frequent amongst units in the storeroom as those on the line.

In reference to a remark made by Mr. Shoemaker as to the cost of an interruption, the particular problem worked out in the paper as an example must be taken as such. I did not mean to indicate that $150.00 would be an average cost of interruption. In using these formulas the values assigned the constants must be based on the particular conditions under which a line is to operate.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Los Angeles, Cal., September 18-20, 1919.

PREDETERMINATION OF SYNCHRONOUS PHASE-MODIFIER PERFORMANCE

BY HUBERT V. CARPENTER

ABSTRACT OF PAPER

The author reviews the method for showing the behavior of transmission lines first given by Perrine and Baum and then shows how it can be used in determining the effect of the use of a synchronous motor operating without load for improving the power factor. The effect of the losses in the motor are shown both in their effect on the line alone and on the line with the step-up and step-down transformers included.

The diagram given shows both the improvement in voltage regulation and the change in power factor due to the phase-modifier for any assumed condition of loading for the transmission line.

The errors of the method are discussed with methods for determining their magnitude, and the advantages of the graphical treatment pointed out.

RAPIDLY growing demands upon our transmission systems for more power and better service have made it imperative in many cases that increased capacity be provided without delay. Increasing the capacity means in most cases improved regulation, but frequently demands also a better efficiency of operation in order that the maximum possible fraction of the output of the generators may be delivered. This is usually thought to demand higher voltages or heavier lines or duplication of lines, all of which require time as well as money.

It is not perhaps fully appreciated that a synchronous motor operating upon the system may, if properly proportioned, improve the efficiency as well as the regulation, thus accomplishing both of the things desired.

While the theory of the synchronous phase-modifier has been fully treated before the Institute it is hoped in the following to make its general effect upon a transmission system a little more easily grasped.

In the consideration of transmission problems the writer feels that far too little use has been made of the remarkable diagram first proposed by Perrine and Baum, (TRANS. A. I. E. E. May 1900.)

The peculiarities of this treatment are that the voltage at the receiver is assumed constant and that at the generator solved for, and that the current is treated throughout as made up of three independent parts, charging current, I_c; load current, $I \cos \phi$; and reactive or wattless component delivered, $I \sin \phi$. The electromotive force required to force each of these components of current through the line is then figured, first for the resistance and then for the reactance of the line, thus giving the following components of e. m. f. required; $\frac{R I_c}{2}$; $\frac{X I_c}{2}$; $R I \cos \phi$; $X I \cos \phi$; $R I \sin \phi$; and $X I \sin \phi$. The charging current values are divided by 2 because I_c distributes itself over the line in a way assumed here to be uniform.

To make up the regulation diagram, Fig. 1, the e. m. f. at the receiver, E, is assumed as the vector of reference and the above values added to get the total voltage needed at the generator.

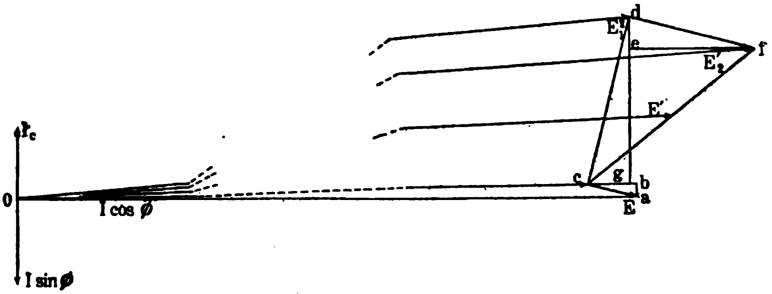


FIG. 1

In Figs. 1 and 2 $a\,b$ is $\frac{R I_c}{2}$, $b\,c$ is $\frac{X I_c}{2}$, $c\,g$ is $R I \cos \phi$, $g\,d$ is $X I \cos \phi$, $d\,e$ is $R I \sin \phi$ and $e\,f$ is $X I \sin \phi$, the last two being shown for 0.8 power factor, lagging. $\frac{R I_c}{2}$ and $\frac{X I_r}{2}$ are plotted first since they are closely constant and assumed to be exactly so here.

The total volts required to force the working component of current through the line is $c\,d$, and is equal to $Z I \cos \phi$. The value of $c\,d$ will therefore vary directly with the kilowatts delivered since E is assumed to be held constant. A scale of kilowatts may be therefore used along $c\,d$ as in Figs. 2 and 3.

An ampere scale may also be added if desired. In the same way a scale of reactive kv-a. may be laid off along *d f* as shown.

The advantage of the method lies in the fact that the entire performance of a line for all conditions of load is shown by one solution, and further that the effect of any change of load is shown directly for any assumed case. The thing really most desired in studying a design for a line is to learn the effect which will be produced by the greatest sudden change in load that may be expected. Questions of this sort are answered directly by this diagram.

Fig. 2 shows that the diagram as there made up is really a chart in which distances along the axis, *c d*, represent power in kilowatts, while distances along the axis, *d f*, represent reactive kv-a. Thus, the point, *d*, Fig. 2, represents 16,000 kw. at unity power factor, and, from the scale below requires 100.8 per cent of the receiver voltage at the sending end. Also point, *j*, shows that an addition of 6000 reactive kv-a., requires the generator voltage to be increased to 106 per cent of *E*. Further, a load of 12,000 kw. at 0.8 power factor, lagging, can be handled without drop in the line if a pure condensance requiring 8250 kv-a. be connected to the system. See points *k* and *k'*.

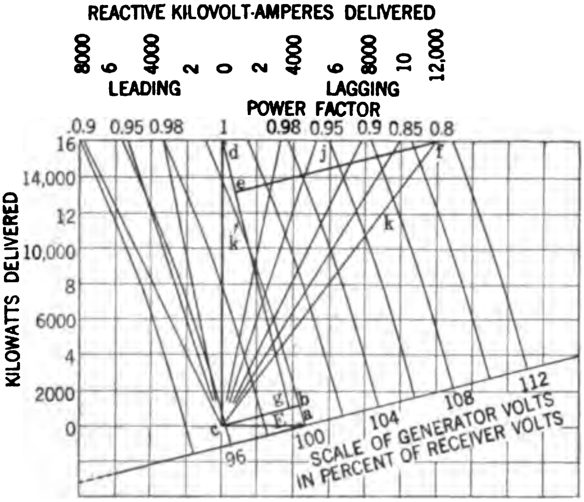


FIG. 2

The time thus spent in rehearsing the workings of the Perrine-Baum diagram is considered well spent for we are now ready to apply it directly to the problem of this paper.

In order to get reasonable accuracy in calculating the effect of an over-excited synchronous motor running on the line without load it is necessary to include the power losses in the machine itself.

To do this in a way best adapted to use with the regulation diagram just shown, the performance of the machine can be plotted in the form shown in Fig. 4. This represents the same current values as determined in the test for the "*V*" curves for the machine except that they are plotted according to their respective power factors and a scale of kw. one way and kv-a. the other used. In other words this plotting shows the active and the reactive components of the kv-a. taken by the machine for any total value, either over or under excited. The slight difference between leading and lagging losses shows the effect of the varying loss in the field windings for different excitations.

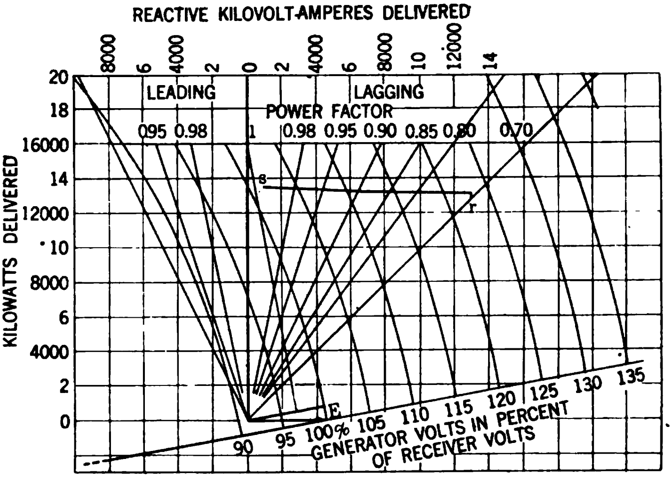


Fig. 3

It will be noted that the data for the machine are now plotted in the same form as is used in the Perrine-Baum chart, that is, with kw. for one axis and reactive kv-a. for the other, so that it can be drawn in directly to proper scale on the chart. This is done in Fig. 3. This figure shows, for example (see points *s* and *r*) that the addition of the phase-modifier working at full capacity of 12,000 kv-a. changes the load from 12,780 kw. at 0.7 power factor lagging, to 13,500 kw. at 0.99+ power factor lagging, and the line drop from 23 per cent to 2 per cent. It is evident that a condenser of one half this size would bring the line drop down to about 12 per cent and the power factor up to 0.88.

A study of the diagram shown and of the properties of the different sizes of line conductors will bring out the fact that a transmission line made up of very large conductors will respond more directly to the corrective action of a synchronous phase-modifier than one using small conductors. That is, if we substitute a conductor of one-half the cross-section of that used in the line figured above the resistance will be doubled but the reactance will be only about 6 per cent greater. This means that with the smaller wire the line, *c d*, of Fig. 1 will be much more inclined toward the horizontal and *d f* toward the vertical. This further tilting of the axes of the diagram makes the effect of a synchronous motor much less, so far as its usefulness in correcting regulation is concerned. It will still be effective in reducing the power lost in the line simply by its effect in reducing the value of the total current flowing in the line. This also applies to cases where the power line is made up of underground cable where the reactance is usually less than the resistance.

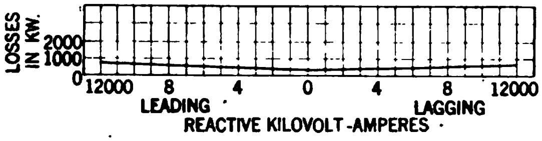


FIG. 4

Owners of small overhead lines may expect the same disappointing results in varying degrees depending on the material of the line, the size, and the spacing.

In all high-voltage systems where the use of phase-modifiers is most advantageous it is necessary to associate step-up and step-down transformers with the line whenever operated, and usually the transformer sets used with any line are the same from day to day. In all such cases there is no need of figuring the line behavior as a separate matter from that of the transformers which are really a part of the line. The resistance of a transformer, if taken over the ordinary range of working temperatures is only slightly variable, and its reactance, depending as it does on the relative positions of the windings and the dimensions of the non-ferrous portions of the leakage paths, is also quite constant and independent of changing load conditions. If then the same transformers are usually used with the line under consideration there is no reason why their resistance and reactance should not be included with the resistance and reactance of the line. Values for the low-tension coils must be changed to high-tension equivalents before adding them in.

In Fig. 3 is given the behavior of the same line as shown in Figs. 1 and 2 with the transformers included. The difference shows the futility of making an exact solution for the performance of a line alone which can never be used without its transformers. It also shows the faulty economy in trying to get a very good regulation in the line when the transformers will prevent it anyway.

Figs. 2 and 3 give the performance of a three-phase line 140 miles long with 300,000-cir. mil. stranded conductors spaced 12 ft., operating at 60 cycles and 110,000 volts between wires.

Fig. 3 includes the transformers which are assumed to handle 16,000 kv-a. at 0.8 power factor and to have a regulation of 1 per cent at unity power factor and 6 per cent at 0.8 power factor lagging.

It will be noted that under most conditions the ratio of X to R will be higher for the transformers than for the line. This may mean that a phase modifier will give good service in improving regulation where a study of the line alone would not make this apparent.

It seems that the reason more use has not been made of this method of studying transmission problems is based on the fact that it is considered inaccurate. The inaccuracy lies in the fact that the static charge which makes up the charging current is assumed to be uniformly distributed along the line, disregarding the variation in voltage, and also in the fact that the charging current is assumed for convenience to be just 90 degrees ahead of the receiver voltage. Inaccuracy is much more feared when the extent of the error is unknown. This difficulty is removed by the exact analytical methods available when needed, any of which may be applied to any one particular condition out of the variety which the chart represents and thus show the error for that point, in fact this is the way to learn to appreciate the graphical method which answers all questions with one solution. As a matter of fact it can be easily shown that the terms calculated in making up the Perrine-Baum diagram are, neglecting line leakage, nothing more or less than the first three terms of the following series which represents the behavior of a line to any required degree of exactness.

$$E' = E + I\,Z + \frac{Y\,Z\,E}{2} + \frac{Y\,Z^2\,I}{6} + \frac{Y^2\,Z^2\,E}{24} + \text{etc.}$$

For any of the lines now in use the error of the method outlined will be found to be well within all useful limits.

Discussion on "Predetermination of Synchronous Phase-Modifier Performance" (Carpenter), Los Angeles, September 20, 1919.

H. B. Dwight: A diagram drawn up to show the particular characteristics of an individual transmission line, as described by Mr. Carpenter is very useful in the design and operation of an electrical transmission system. An additional valuable feature is that the average power factor observed for various fractional loads may be plotted directly on the diagram, thus showing the generator voltage, or the phase-modifier kv-a., required at any load under usual working conditions.

The diagram described in the paper is based on the so-called "split-condenser method" of calculation, in which one-half of the capacitance of the entire line is assumed to be concentrated at the receiver end. It is stated in the paper that the diagram is not used to a greater extent because of the inaccuracy of this method. However, it is not necessary to use the "split condenser," or any approximate method. A diagram which is the same in its essentials as that described in the paper, but based on the complete hyperbolic transmission line theory, has been published by the writer.* This diagram is called a "circle diagram," and the circle corresponds exactly to any of the circles shown in Fig. 2 of Mr. Carpenter's paper, except that accurate transmission line equations are used.

The equations which have been published by the writer for drawing the diagram are as follows:

$$\text{Find } E' + jE'' \doteq E\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2.3.4.} + \text{etc.}\right)$$

$$\text{and } R' + jX' = (R + jX)\ (1 + \frac{YZ}{2.3} + \frac{Y^2Z^2}{2.3.4.5} + \text{etc.})$$

Describe a circle with center (a', b') and radius c',

$$\text{where} \qquad a' = -\frac{E}{1000}\ \frac{E'R' + E''X'}{R'^2 + X'^2}\ \text{kv-a.}$$

$$b' = +\frac{E}{1000}\ \frac{E'X' - E''R'}{R'^2 + X'^2}\ \text{kv-a.}$$

$$\text{and} \qquad c' = +\frac{E}{1000}\ \frac{E_s}{\sqrt{R'^2 + X'^2}}\ \text{kv-a.}$$

where E is the receiver voltage, and E_s the supply voltage.

---

*"The Calculation of Constant-Voltage Transmission Lines," by H. B. Dwight, *The Electric Journal*, Sept., 1914, and "Constant-Voltage Transmission," by H. B. Dwight, pages 78 and 99, John Wiley & Sons, Publishers, 1915.

The equations for a' and b' show that the position of the center is independent of the supply voltage. The equation for c' shows that the radius is directly proportional to the supply voltage. Therefore, after one circle has been drawn corresponding to a certain supply voltage, a series of circles may easily be drawn about the same center, with radii proportional to the supply voltages which the circles represent.

If the first equation is shortened, so as to omit the terms of the series after $\frac{YZ}{2}$, and if the second equation is shortened so that

$$R' + jX' = R + jX$$

the results are exactly the same as those of the "split-condenser" method. The length of some of the lines of the diagram will be changed by one or two per cent. This error is appreciable, and while it may seem a small amount for practical work, there is no reason why this mathematical error should be incorporated in a diagram which is drawn up to show the characteristics of a particular transmission line. It may be said that it is generally admitted that the "split-condenser" method is superseded by the complete hyperbolic theory, and that the small amount of additional work required for the latter method of calculation is worth while in any case where capacitance is worth considering at all.

M. O. Bosler: In the mechanical construction of the energy diagram the distance OE in Fig. 1 when applied to the kv-a. scale of Fig. 2 is $\frac{E^2}{Z} . 10^{-3}$ and the distance of the supply voltage circle from O, OF, is $\frac{E\ E_0}{Z}\ 10^{-3}$. The slope of the voltage line OE, Fig. 1 across the energy diagram Fig. 2 is $\frac{X}{R}$.

By keeping the line impedance Z constant and increasing the ratio of X to R, the power capacity of the line for a given E and E_0 is enormously increased when supplying power at a high or a leading power factor. When a line of this sort gets into trouble, the power factor drops to practically that of the line $\frac{R}{Z}$, and limits the effect of the disturbance by the excessive voltage drop that occurs at such a low power factor.

This point, which can be so clearly illustrated by means of this diagram, should be of importance when operating an inter-connected system of large power capacity.

J. A. Lighthipe: The operation is usually reversed. That is, you get a condenser, or you have a condenser, or you buy as large a one as you can, and then figure back and see how much you can improve the line with this condenser.

We are endeavoring to scatter our condensers as nearly to the center of our load as possible and we have something like five or six condensers scattered into various parts of our system. We have adopted for the small size, about 2000 kw., partly because it can be thrown on a line with a minimum amount of disturbance; in some cases we have put a second one at the same substation.

The method of figuring just what effect your condenser will have is what we really need and want, but to figure out a certain size condenser for a given size transmission line which is loaded constantly throughout the year, why we usually work backwards, that is, get the condenser and then figure out what good it will do. Then, if necessary, we put another condenser a few miles away or sometimes in the same power house.

The old condenser method brings up the early days in the Spokane Coeur d'Alene transmission. There we had one of the first regulators to hold the voltage constant at Spokane but up the other side of Coeur d'Alene in the mining section the regulation was poor. It was finally decided to put a condenser up there with a regulator. This was brought out in the discussions at the Monterey convention.

We are using a great many of them on our system, although most of them are regulated by hand and not by general regulators, except our main substation at Eagle Rock.

I have always figured in the rough that to about every 10,000-h. p. induction motor we should install at least a 2000-kv-a. condenser, as near as we could get to the load center, and it is working out pretty well.

P. M. Downing: My experience has been that synchronous condensers are nearly always afterthoughts. They are put in when voltage conditions force a change. They are limited in capacity and corrective effect and, therefore, it is a case of doing the best with what you have.

They have been generally used on our system in the north central part of the state for a number of years. It is hard to predetermine on that system just exactly what will happen because of a more or less complicated network of lines and the large number of power sources. Where installed however, they are equipped for automatically taking care of voltage and they do it successfully. The growth of the business has been such that they have not been given an opportunity to show just how they would work under ideal conditions.

J. H. Anderton: I note in Mr. Downing's remarks he refers to synchronous phase modifiers as afterthoughts. This may be true to a large extent on some systems, as for instance, the one with which Mr. Downing is identified, since with the gradual growth of such a system and the increased radius of transmission, phase modifiers are installed as required. However, this does not generally apply to phase modifiers. In most instances this apparatus is of such prime necessity as to be the determining factor in the economic possibility of a development. Phase modifiers modify the phase of the total load with respect to the generators without necessarily varying the amplitude of the voltage so that it becomes possible by their use to transmit power any distance with constant voltage at both ends of the line or at any point on a line. Such systems as the Big Creek and Great Western Power Company systems would be economically impossible without their use because of the impossible conditions of voltage regulation.

You are all more or less familiar with the usual computations of a transmission line and I have no doubt you all try to use the simplest methods of obtaining the results. For preliminary work I have found the so-called approximate method quite suitable. This consists of computations showing the various generator voltages required for given loads and constant receiver volts. These are then shown as points on an ordinate of generator voltage, an assumed condenser kv-a. is then taken as abscissa, and for the same receiver volts a second point for the generator voltage obtained; plotted this gives the familiar transmission line diagram. By reading off the diagram a suitable condition of generator and receiver volts with a given kv-a., condenser capacity may be selected. Since reading from a diagram usually involves some inaccuracy, the selection of the kv-a. condenser from these curves is perhaps not permissible. However, the condenser kv-a. may be obtained mathematically for each assumed load by a modification of the generator voltage equation, the mathematics for which does not involve anything beyond the quadratic form.

P. M. Downing: In explanation of my remarks, I might say that the lines I spoke of were those of the Pacific Gas and Electric Company, practically all of which are 150 miles and operated at 60,000 volts. Therefore we do not have the same condition to contend with that would be met on the higher voltage lines.

Leslie F. Curtis: Professor Carpenter is to be thanked for bringing to our attention the fact that a graphical representation of any computation is to be desired. Whether the method developed by Professor Carpenter or that suggested in the previous discussion is used, the graphical method shows the calculator a picture of what is taking place on his line. It, in any case, is useful in checking his computations.

Professor Carpenter also notes that it is of advantage to include the transformers in calculating a line. I think that some of us have not kept this in mind. The calculation of the line involves not only the conductors at the proper spacing, but also the transformers and all of the apparatus connected thereto.

J. F. Wilson: I might refer, briefly, to Mr. Dwight's discussion. Mr. Dwight's transmission line formulas involve only simple arithmetical calculations which will, I think, give practically the same results as does Professor Carpenter's graphical method.

Relative to the suggestion that Mr. Lighthipe made, about working backwards on the proposition, I think that is probably true, but as the application of the synchronous phase-modifier extends and becomes more accepted in its general application, I doubt if that proposition will hold because, as was brought ut here in the other discussions, it will certainly be advantageous, if not necessary, to determine before the line is built and put in operation what effect you are going to get from a given condenser, or rather, what size condensers you are going to require to produce a given per cent regulation, or required power factor.

H. L. Melvin (by letter): Several years ago the writer learned the use of the Perrine-Baum Chart and its adaptation to the predetermination of synchronous phase-modifier performance. Since that time I have had opportunity to use it quite extensively and to check its accuracy. It is sufficiently accurate for all practical purposes for transmission lines now in operation.

The chart is very easily and quickly constructed and once made for a line a vector picture of the performance of the line for any load is given at a glance. The charging current, resistance, reactance and impedance of the line must be determined and if standard cross section paper is used for the kilowatt and reactive kilovolt ampere scales it is necessary to determine the voltage scale. The line drop due to the magnetizing current for the receiver transformers must be calculated, and since the current lags practically 90 degrees, it can be subtracted from the drop caused by the line charging current, giving the vector a c (Fig. 2). It will be noted that if arcs of circles be drawn with *a* (Fig. 2) as a center they will be loci for constant current, kilovolt amperes and I^2R line loss (the loss due to the charging current neglected). On the scale of generator volts actual voltages may be used, or if the step-up transformers are included, low-tension bus voltages or even switchboard voltmeter readings. When superimposing the synchronous condenser characteristic, the point where the decrease in line loss is equal to the losses in the condenser may be determined. The use of synchronous condenser capacity beyond this point for the improvement of voltage regulation will decrease the efficiency of transmission. If a transformer bank is installed for the condenser only (the load current not passing through it) its regulation and losses can be included in the condenser characteristic curve (Fig. 4). When the maximum load change is known the minimum synchronous condenser capacity required, operating lagging and leading, may be determined and also the proper voltage to be held at the generator.

Practically all the information which the chart will give is needed in the analysis of the performance of a transmission line and it is really a remarkable solution that will give the results accurately.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Los Angeles, Cal., September 18-20, 1919.

CALIFORNIA 220,000-VOLT — 1100-MILE — 1,500,000-KW. TRANSMISSION BUS

BY R. W. SORENSEN, H. H. COX AND G. E. ARMSTRONG

ABSTRACT OF PAPER

This paper summarizes the power resources of California and the probable loads to be supplied within the next six or seven years. For the purpose of economically distributing the necessary power and supplying the load, a long high-voltage transmission line is proposed. As this line would interconnect a number of different companies, it assumes the nature of a bus bar. The authors show how the proposed line may link with some of the lines now in service and enumerate the advantages of such interconnection. A comparison is made between the 240-mile Big Creek line now operating at 150,000 volts, 50 cycles, and the operation of this line at 220,000 volts, 60 cycles. Operating data on the Big Creek line are shown to indicate the character of the construction necessary for California conditions. Conclusions are drawn as to the particular features to be observed for successful operation of 220,000-volt lines.

INTRODUCTION

FUELS, particularly oil, must soon be used for isolated power only in places where electric power is not available, as in the propelling of air and ocean craft. In large power systems, especially in the West, the use must be limited to standby service, for peak loads, low water periods, and other emergencies.

POWER RESOURCES

California has available ample hydroelectric power to supply the industrial and agricultural demand for many years.

Small developments aggregating 325,000 kw. have been completed and many others of this type are available. There are also four large projects as indicated in Table I. which can readily be developed to a capacity of 1,500,000 kw. in the near future.

The data for the following tables of resources and loads, of the Northern part of the state, are taken from various reports which have been published and no attempt has been made to verify them.

TABLE I—LARGE POWER RESOURCES

	Now Developed and Under Construction	Proposed Developments 1926	Reasonable Future Development (Not Ultimate Capacity)
	kw.	kw.	kw.
Pitt River	None	200,000	500,000
Feather River	100,000	200,000	300,000
Big Creek	100,000	300,000	500,000
Colorado River	None	None	200,000
Total	200,000	700,000	1,500,000

Total 1926 hydroelectric power development including small projects is 1,025,000 kw.

Load Demand

The best available information indicates a demand in 1926 approximately as shown in Table II.

TABLE II.

1. Sacramento Valley, northern portion	70,000	kw.
2. Truckee River electrification	40,000	"
3. Sacramento Valley, southern portion	125,000	"
4. San Francisco Bay District	250,000	"
5. Fresno District	90,000	"
6. Bakersfield District, including Tehachapi electrification	125,000	"
7. Los Angeles District	300,000	"
8. Barstow and Needles District, including railroad electrification	40,000	"
Making a total of	1,040,000	"

In order to carry this load, approximately 500,000 kw. additional in hydroelectric capacity will be required.

A demand for power such as shown in Table II can be supplied most economically by power developed in large units. Large power units require transmission lines of the highest possible economic voltage.

*It has been shown that, for long transmission, 220,000 volts is economical under conditions which require a much more expensive construction than has proven adequate for the 150,000-volt lines of the Southern California Edison Company.

---

*Silver, *Problems of 220-kv. Power Transmission*, page 1037.

California Transmission Bus

On this basis, a plan as shown on the map, Fig. 1, is proposed. In this plan the interconnection of all the California Power Companies has been assumed, as an economic necessity for its best utilization. Interconnections of limited capacity are not entirely satisfactory because they fail just at the time they are needed most to transfer from one system to another large blocks of power.

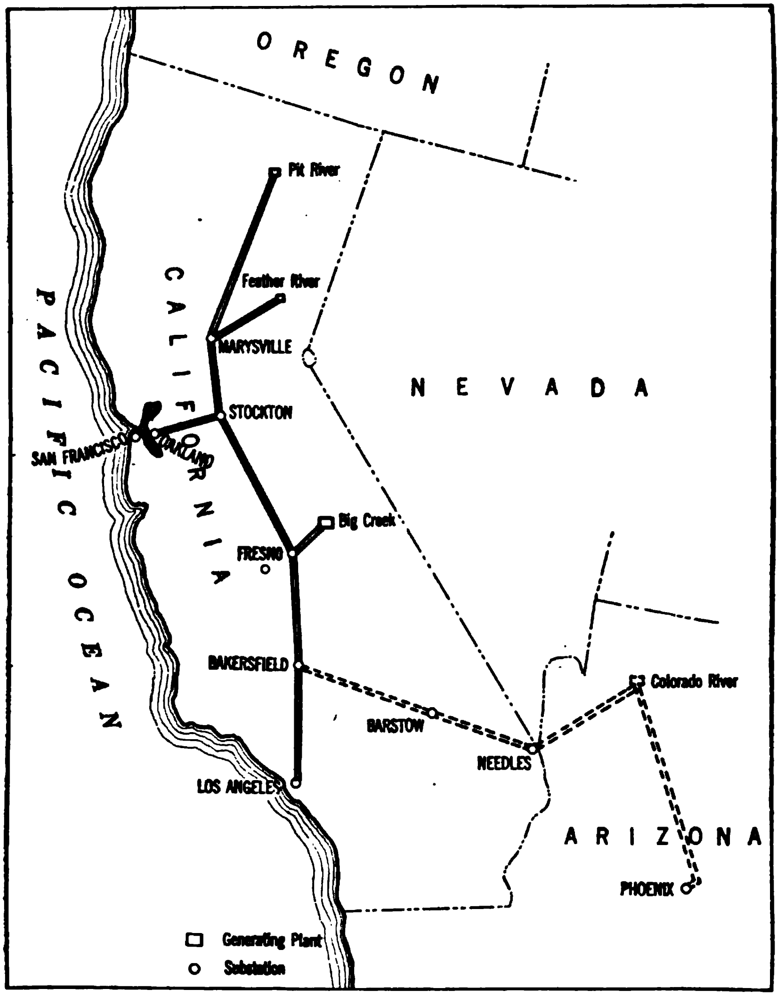


Fig. 1—California 1100-Mile—220 Kilovolt—Transmission Bus

The plan of the proposed scheme involves the construction of a two-circuit transmission system extending from Pitt River to Los Angeles, a distance of 570 miles. Branch lines of like voltage connect the three other power projects and the San Francisco load center to this main line on which the other load centers are located. The main line thus becomes a high-tension bus extending nearly the entire length of the state, hence its name; California Transmission Bus. This arrangement makes possible unlimited interconnection and exchange between all the power companies of the state.

Substations have been located at Marysville, Stockton, San Francisco, Fresno, Bakersfield and Los Angeles. These points are natural load centers and suitable points for connecting with the present power systems. On the Colorado River branch, the construction of which is dependent upon the electrification of the transcontinental railroads, substations would probably be located at Barstow and Needles. The substations divide the lines into sections of suitable length for practical operation, the longest section being 150 miles, as shown in Table III.

TABLE III.

Pitt River to Marysville	150 miles
Feather River to Marysville	60 "
Marysville to Stockton	90 "
Stockton to San Francisco	60 "
Stockton to Fresno	130 "
Big Creek to Fresno	40 "
Fresno to Bakersfield	100 "
Bakersfield to Los Angeles	100 "
Bakersfield to Barstow	110 "
Barstow to Needles	150 "
Needles to Colorado River	100 "
Colorado River to Phoenix	100 "
Pitt River to San Francisco	300 miles
Big Creek to Los Angeles	240 "
Big Creek to San Francisco	230 "

THE TRANSMISSION LINE

The standard frequency, 60 cycles, has been assumed on the basis that the Southern California power systems operating at 50 cycles will ultimately find it advantageous to conform to the A. I. E. E. standard. In the natural growth of the load as shown in Table II, 70 per cent of the 1926 load will be supplied by the 60-cycle systems. Interconnection of such large load centers or power sources through frequency changers, limits the exchange of power, is uneconomical, and increases tremendously the required operating vigilance.

The practicability of the high-voltage line has been well demonstrated by over five years of remarkably successful operation of the 150,000-volt lines of the Southern California Edison Company which, during this period have delivered from the Big Creek power houses over the 240-mile lines to the Los Angeles distribution system, 1,200,000,000 kw-hr. at an average efficiency of 87.5 per cent with a 45 per cent load factor. During this period there have been no interruptions for which the high voltage is responsible, and on the contrary, the system has been free from disturbance and interruption to a greater degree than the lower voltage lines in the same locality.

The present Big Creek lines can be operated at 220,000 volts, 60 cycles, without material change and this is proposed as a link of the transmission bus, and its operation under these conditions will be analyzed and applied to conditions of the proposed system.

Corona. As now operated, at 150,000 volts and 50 cycles, the voltage is only 80 per cent of the lowest critical voltage of any part of the line and there is no corona loss. At 220,000 volts, 60 cycles, corona loss occurs to some extent on the entire line but amounts to but 0.4 per cent of the line capacity during fair weather. With storm conditions over the entire line, and with an assumed reduction of 20 per cent in the critical voltage, the corona loss would be 8 per cent of the line capacity. This loss is not sufficient to make the line inoperative and would occur too rarely to be an economic factor.

Insulation. The Big Creek 150,000-volt lines have nine units in each suspension string and two eleven-unit strings in parallel on dead ends. During the five and one-half years of operation only two insulator string failures have occurred. Both of these were during normal conditions of operation without any apparent cause, other than that of being in a location where the insulators have been found to have a relatively high rate of deterioration.

The Big Creek line towers allow sufficient clearance to permit the lengthening of the nine unit suspension strings to eleven units, and to any desired number of units at dead ends. Table IV. shows safety factors for insulator strings, wet and dry.

The Big Creek line operated at 220,000 volts is at the critical corona voltage and any disturbances resulting in a higher voltage will quickly expend their energy in producing corona loss, which will permit a smaller safety factor to be used.

The curves in Fig. 2, showing arc-over voltage as reproduced from Mr. Silver's paper, *Problems of 220-kv. Power Transmission,* show no practical gain in dry arc-over voltage for strings of more than ten units, and with these facts in view it is proposed that for operating the Big Creek line at 220,000 volts, suspension strings have eleven units and dead end strings 12 units in series. Insulator testing crews have several times reported four and five defective units in a nine-unit suspension string, without any indication of trouble. The only apparent value of a longer string than that proposed would be a decreased probability of sufficient defective units in a string to cause breakdown. Developments of methods of grading insulator units and shielding insulator strings, will in all probability, materially change curves of Fig. 2.

Present day method of insulator testing and maintenance would probably have prevented the two failures which have occurred on the Big Creek lines as previously mentioned. These methods applied to the lines operating at 220,000 volts and the use of the better types of insulators now available will insure successful operation.

TABLE IV. SAFETY FACTORS FOR INSULATOR STRINGS

	Wet arc-over		Dry arc-over	
	9 unit String	11 unit String	9 unit String	11 unit String
(87 kv. to ground) 150 kv........	4.3		4.8	
(127 kv. to ground) 220 kv.......		3.7	3.3	3.4

Charging Current. Long high-voltage lines cannot be operated without synchronous condensers at the receiving station to regulate the voltage, and as a consequence the charging current, even at the standard 60-cycle frequency becomes a factor of no great importance as long as these synchronous condensers are connected to the line.

Without these condensers the line charging current must be furnished entirely by the generators, in which case the generators may become greatly overloaded and at the same time produce a very high voltage over which the operator has no control. To avoid this emergency a transmission line with its generators, transformers, and synchronous condenser must be considered as a unit and as such should be securely coupled together electrically at all times. This has been proved praccal in the case of the Big Creek system in which it is possible to start the 15,000 kv-a. condensers and bring them up to speed with the generators.

Line Capacity. The Big Creek lines as operated at 150,000 volts with 30,000 kv-a. condenser capacity per line at the receiver end are each good for 57,500 kw. at 85 per cent power factor and will have under these conditions a line drop of 11 per cent.

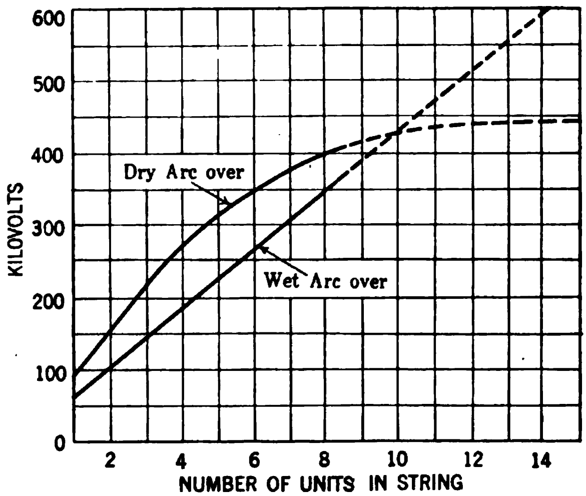


FIG. 2—TYPICAL 60-CYCLE ARC-OVER CHARACTERISTICS OF SUSPENSION INSULATORS

Operated at 220,000 volts these lines should each have a capacity of 125,000 kw. with an equal line drop when provided with the proper condenser capacity, which is approximately 75 per cent of the line capacity in kilowatts.

This is a fair indication of the conditions which will exist in the proposed system, the load centers of which are so distributed as to limit the actual average distance of transmission to about 200 miles. The economic gain in doubling the capacity of lines which cost approximately $6,000,000, the present cost of which would be at least 30 per cent more, would more than offset the cost of all necessary changes, including the adoption of the standard frequency.

Mechanical. The type of construction used on the Big Creek line has proven entirely adequate for California conditions. There have been only three mechanical failures all of which occurred shortly after the line went into service and were all due to defective line hardware. In one case the failure was due to faulty design. This fault was entirely corrected by redesigning the cable clamp so as to grip the steel core independently of the aluminum conductor. The other two were due to individual defects in parts. There have been no tower failures and no tower maintenance whatever has been required. Approximately 20 per cent of the Big Creek line is subject to ice and snow conditions, parts of it reaching altitudes of 5000 feet. Similar conditions exist over practically the entire proposed 220,000-volt system.

Operation. The most interesting feature of the operation of the Big Creek system is its reliability, which has been equal to that of steam plants of similar capacity located near load centers. Flashovers have caused only momentary interruptions and have in no case resulted in damage such as to prevent immediate resumption of service. During the greater part of the time the power has been carried over a single line for a large part of the distance.

The operating history of the Big Creek system discloses no evidence of any trouble due to the high voltage of the system, and in addition has demonstrated that higher voltages may be used with equal or greater reliability. The Big Creek 17,500 kv-a. generators have operated at 60 cycles satisfactorily and delivered full output at this frequency.

High-tension line switching and synchronizing has been carried on consistently throughout the operation of the Big Creek system without trouble, and should be possible on the 220,000-volt system. During times of switching, slight discharges, never followed by any energy current, occur on the arresters. Operating at corona voltage rather than at 80 per cent of the critical voltage it may be possible to absorb these disturbances without arresters.

Complete parallel operation of all lines must be adhered to in the proposed system. Satisfactory protective relay systems for dropping defective sections with little disturbance have been developed for present parallel transmission lines and there appear to be no obstacles to extending these to the higher voltages.

Generators. Curves of Figs. 3 and 4 show generator and line characteristics for 60-cycle, 220,000-volt systems. The full lines are the charging currents in amperes for different lengths of line plotted against per cent normal voltage. The broken lines are generator characteristics of various sizes of generators when connected to condensive loads with no field excitation. The point of intersection of the generator curves with the line

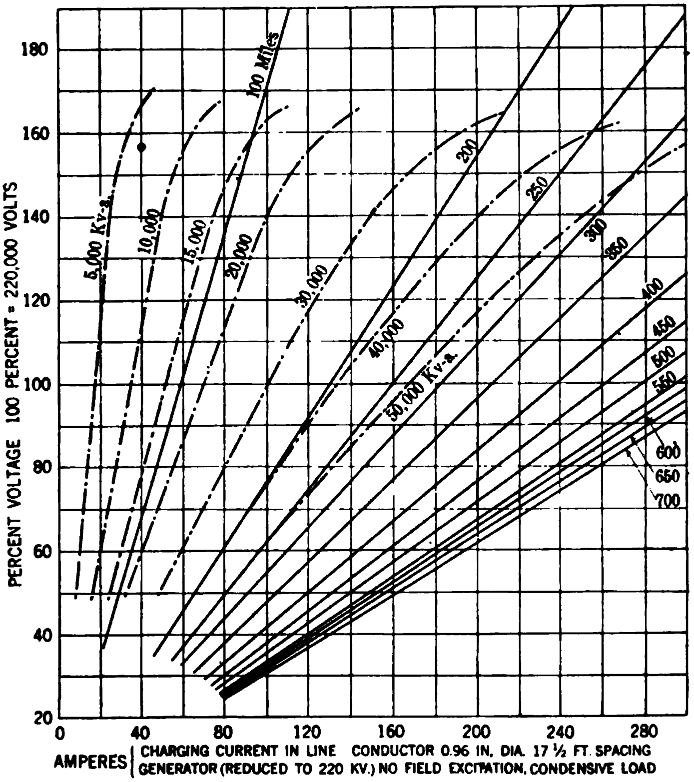


FIG. 3—GENERATOR AND LINE CHARACTERISTICS—60 CYCLES—220,000-VOLTS—GENERATORS WITH SHORT-CIRCUIT RATIO 1.0

charging current curve for any particular length of line determines the voltage to which the generator will build up when connected to that length of line with no field excitation. Fig. 3 is for generators with a short-circuit ratio of 1.0, while Fig. 4 is for those with a ratio of 1.5.

Fig. 3 shows that with 50,000 kv-a. of generating capacity connected to a line of 250 miles, the line can be charged without losing control of the voltage with generators of this design. By having synchronous condensers connected to the line at the receiving station generators of this capacity will bring up any length of line necessary to the successful operation of the proposed system. These curves show that generators for such a system should be designed with the highest short-circuit ratio

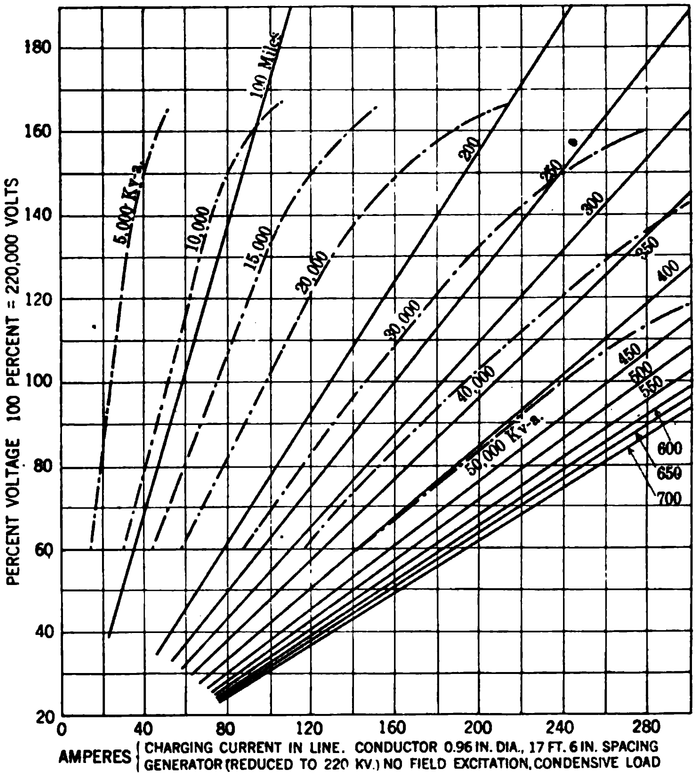


Fig. 4—Generator and Line Characteristics—60 Cycles—220,000 Volts—Generators with Short-Circuit Ratio 1.5

that other conditions will permit in order to reduce to a minimum the tendency to become self exciting.

*Data for these curves were worked out in the laboratory of Throop College of Technology in 1915 and results verified by actual tests on the 17,500 kv-a. generators at Big Creek.

---

*For further data and explanation of generator performance with large condensive loads see Newbury, *Electric Journal*, 1918.

Conclusions. Such a system as proposed is needed immediately; all engineering fundamentals essential to a solution of its problems are well understood and the Big Creek system can be used as a part of the project without material reconstruction.

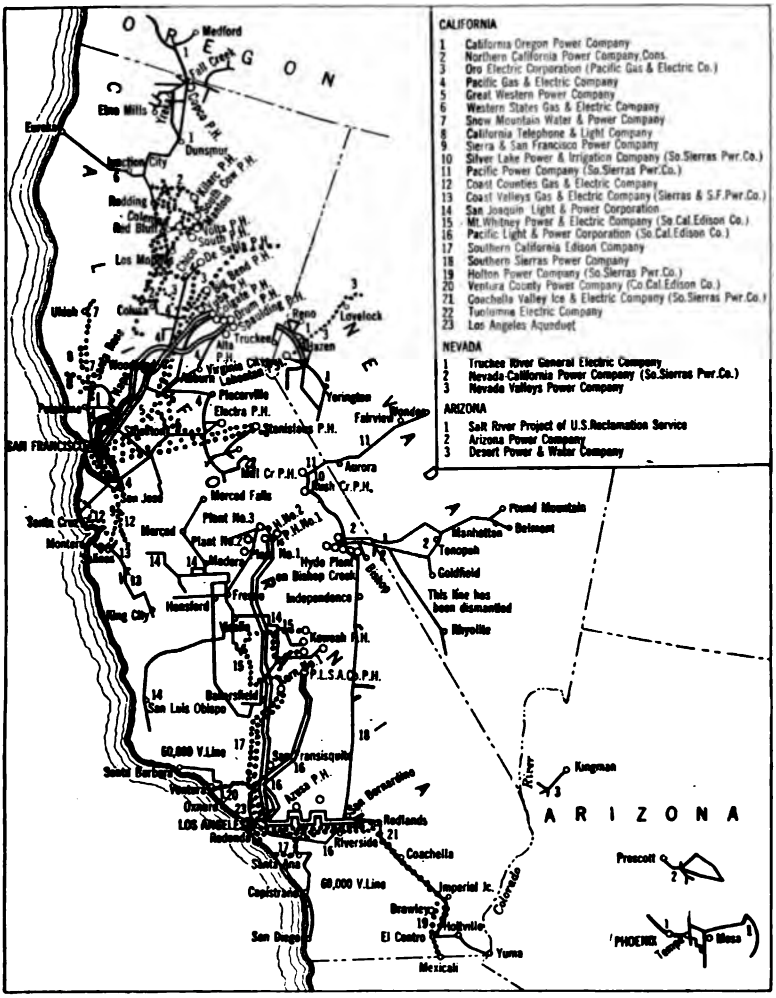


FIG. 5—MAP OF WESTERN TRANSMISSION LINES

To supply this need, arrangements should be made without delay for a complete working out of all details of the proposed system, as otherwise in the future it may be necessary to do a large amount of reconstruction to bring together individually-designed systems, which is never a wholly satisfactory procedure.

COMPARATIVE DATA

	Big Creek lines at 150 kv.	220 kv. lines as proposed by Silver
Aluminum steel cable		
Diameter	0.95 in.	1.036 in.
Circular mils	683,000	808,900
Weight per foot	0.75 lb.	0.94 lb.
Length of average span	750 ft.	800 ft.
Weight of towers without footings		
Suspension	4300 lb.	9000 to 14,000 lbs.
Anchor	6450 lb.	24,000 lb.
Stringing tension at 80 deg. fahr.		
No ice allowance	4740 lb.	
Ice allowance	3130 lb.	
Maximum tension allowed	8500 lb.	17,300 lb.
Insulator strings to carry load		
Suspension	1	2 and 3
Anchor	2	6

For further data regarding the Big Creek line refer to the paper 150,000 *Transmission System* by Woodbury, A. I. E. E. TRANS., 1914.

Map. Fig. 5 shows the existing transmission systems of California, Nevada, and Arizona, varying in voltage up to 150,000 and the names of the operating companies. To avoid confusion these lines are not shown on the map of the proposed system.

The authors wish to express appreciation of suggestions made by Mr. H. A. Barre which led to the preparation of this paper.

DISCUSSION ON "CALIFORNIA 220,000-VOLT—1000-MILE—1,500,000-KW. TRANSMISSION BUS." (SORENSEN, COX, ARMSTRONG), LOS ANGELES, CAL., SEPTEMBER, 20, 1919.

Ralph Bennett: It is interesting to note that this paper can hardly be said to be in advance of actual construction since the lower half of the proposed line is already in operation in the Big Creek Line of the Southern California Edison, while the northern half of the transmission is under contemplation for immediate construction as the Pit River Line of the Pacific and Gas and Electric, and at the full contemplated voltage.

Our gain in reliability in the construction of power transmitting devises is no better illustrated than in the advance between the timid and temporary construction utilized in the latter 90's, and the substantial, permanent and well developed methods now at hand.

Yet it can hardly be said that there has been any considerable advance during this entire period in the abstract technical theory of the art. The change has been in the application of these abstract values to the concrete case of electrical transmission. This is well illustrated in the introduction as a matter of routine equipment of the use of synchronous condensers on these modern lines where the use of such apparatus was considered to be entirely experimental ten years ago.

It is probable that the change in the status of the business from the period of the latter 90's to the present date is greater than the change in its technical features. It is no longer possible to consider the electric lighting plant as a private enterprise. It is today a public utility extending over one or many states, and serving more territory than the average railroad system, and has come under state and Federal regulation more sharp and penetrating than the Government exercises over any other business.

The introduction during the past two years of inter-ties between the various physical properties of the Western power companies has long been foreshadowed. The paper proposes that these ties, now of a temporary and insufficient character, be rendered a permanent portion of the systems. Many companies will be fed by a common net work just as many railroad companies feed across their transfers, loads derived from many sources and intended for terminals as diversified as their origin. Under the ruling under which railroads have operated for a generation they are compelled as a matter of public policy to give equal care to all property entrusted to them regardless of its origin or its destination. They have become common carriers and far from rebelling from this condition they consider it to be an essential portion of their success.

The power business is rapidly developing all of the conditions which have surrounded railroad operation insofar as the interlinking of numerous independent companies is concerned.

It appears logical for conditions eventually so to shape themselves that the power company transmissions will become common carriers, not merely as the result of mutual agreements between existing monopolistic companies, but by action of law. As a result, any producer of power, however small, will be assured that his production will be received on the transmission net work, and that he will receive therefor a price fixed by a state regulating body on a basis of a fair return on his investment.

It is, I think, obvious to anyone acquainted with the present power situation that this would result in the construction of numerous small plants capable of introducing a fair amount of power as an incidental to the handling of water, natural gas, oil, or other power sources.

There are numerous cases within the knowledge of every engineer in this vicinity where considerable blocks of power could be produced on a schedule which would not permit of sale to a local net work, even if such sale were permitted by the railway commission, but where the production cost of the power would be so low that the power could be produced to great advantage if it could be successfully wholesaled to a net work capable of absorbing it regardless of its fluctuations.

Almost every irrigation system in the state possesses drops capable of producing a more or less considerable amount of power.

Numerous desirable combined irrigation and power projects are neglected because it has not been possible in the past to develop a market for the power, although the power in connection with irrigation could be produced at a very low figure.

On the north end of the Pacific coast there are sources of power as a by-product from wood working plants; there are coal mines lying undeveloped because they are not favorably located with reference to a rail haul market; in the oil producing industry vast quantities of oil and natural gas are wasted because they are unsuited to market demands or are in a location such that they cannot be successfully delivered to distant markets, yet they could be sources of large amounts of power which could be readily handled over a transmission bus.

It is unnecessary and perhaps aside from the purpose of the paper to go into detail on these items. The one other matter in the same connection however is of interest. Railroad electrification has been much discussed of recent years but has been to an extent held back by the very considerable problem involved in the extremely irregular use of power which will occur on most of the western roads.

Taken in connection with an ordinary local transmission this makes a load so undesirable as to render the power rate abnormally high.

But in connection with a centralized power system handling one or two million kilowatts these fluctuations would be regularly supplied by the excess capacity of the net work.

The problems of engineering, of finance and of political expediency involved in the changes now taking place in the relations of the power companies to each other, and to the public, are too great to even touch in this discussion.

P. D. Jennings: This paper is especially valuable at this time owing to the rapid advance of fuel prices and the necessity for the conservation of coal and oil for the essential industries that can not avail themselves of hydroelectric central station energy.

I believe, as this paper slightly implies, that this high-tension bus scheme could economically be carried out to include all of the great hydroelectric developments west of the Mississippi River. Of course it is not supposed that there would be an exchange of energy, say, between eastern Montana and southern California, but it is a fact that power plants on this bus line would have the great advantage of the diversity of load demanded, by substations, at the load centers along the line.

The two important considerations to be given, of course, are its operation and financing. It would appear to me that the operation of such a bus net work would require a unified control; and that the load dispatching work would have to be divided up into districts, or regions, which in turn would be responsible to an operating board of control composed of regional operating directors. This method of operation, of course, would require a unit scheme of financing. The probable formation of a holding company whose stock would be purchased and allocated by some equitable plan among all of the great generating systems interested.

Of course such a scheme would probably have many very serious disadvantages, from the individual standpoint of some of the companies involved, as well as some of the Public Service Commissions.

But these difficulties I do not believe would be insurmountable owing to the fact that both the public and the companies would be materially benefited by helping to reduce the rapidly increasing operating costs.

C. O. Poole: To my mind this proposed plan is one of the most important projects presented to the engineering profession from a conservation point of view. I have been, for years, a firm believer in this scheme and can see many advantages that will accrue from it, such as making possible comparatively small hydroelectric developments that will be in reach of the proposed bus, that would otherwise be too remote or without market for the output; or there may be streams suitable for power development that do not have storage facilities to equalize the flow throughout the year, in which event it would necessitate a steam plant in order to carry the load during the non-run-off periods. Under such conditions the investment to make this development might not be justified to supply an individual system, while with the proposed bus such a development might be advantageously made, simply utilizing the output of the stream while the run-off period is on. This same condition might apply also to irrigation systems where there are drops in the canals and water supply streams.

One of the principle advantages of connecting all generating and distributing systems to such a bus would be the improvement of the load factor conditions by diversity of both the generating capacities and the demands of the many diversified conditions of the different systems. For this same reason there would be less necessity for over-installation on water power generating systems, thereby very materially decreasing the spare unit investment and with its consequent less over head expense and reduction in cost for generation. From recent statistics the average yearly load factor of electric systems in the United States is only 30 per cent, and, while California's load factor is materially better than this, it is subject to much improvement.

As pointed out by the authors of the paper, and as theoretically set forth by Mr. Silver's paper in this volume, page 1037, the voltage proposed is not an impossible one to operate, but simply a matter of insulation and clearances. From a transmission point of view I might suggest, however, that the eleven units in the insulator string, as suggested by the authors, might be somewhat inadequate for the insulation of the new line unless insulators of a more reliable nature can be obtained than have been made use of in the past.

On this subject, the experiments cited in Mr. Silver's paper upon the dry arc-over and wet arc-over tests, are of unusual interest and it would seem that the line insulation should be based upon the curve of the wet arc-over, rather than of the dry arc-over conditions, inasmuch as the real test of the insulation is during storm and fog conditions, and, while from the curve mentioned the eleven insulators in the string might approach the flat portion of the dry arc-over curve, yet there is plenty of justification for adding more units in the string when the wet arc-over line on the curve is considered. The operation of such an important line as proposed should carry with it larger factors of safety than an individual system might economically call for, and, therefore, extra insulation in such a system would be, in my estimation, fully justified.

Special interest is attached to the probable conditions of corona loss upon this proposed line. I have been of the opinion for some time that it might be possible so to proportion sections of the transmission system of the higher voltages that the corona loss might be a means of dissipating energy in the event of over-voltages from any cause, and in the present instance it would seem that the line approaches this condition. In the event of an unusual high-voltage surge, energy would be dissipated to such an extent that it would tend to hold down the rise in voltage and, therefore, would be a benefit and protection to the system. I think this a feature that should be very carefully considered and experiments conducted beforehand to determine to what extent the corona loss could be utilized for the purpose mentioned.

Aside from the purely engineering conditions involved in such an undertaking there is another feature, to my mind quite as important as the strictly technical side of the situation, and that is the method of handling the different companies' input to the bus and the output from the bus. With a dozen or more companies taking from and feeding to such a system, many problems are involved that would have to be satisfactorily worked out, both as to the protection of individual systems from overload conditions, and also protection of the bus from service interruption. These features are engineering problems that can doubtless be worked out by a comprehensive system of relay switches. Probably a still more difficult situation is involved in the method of measurement of the current to and from the different companies. This last question also involves the ownership of the transmission bus, as to whether all the distributing companies would have ownership in the bus proportional to the capacities of their plants, or whether each should have ownership in the bus proportional to the kilowatt hours used by the different companies, and each company share its proportion of maintenance and overhead costs of the system, or whether an independent company should own the bus, this independent company being composed of the contributing companies and each of the contributing companies sharing in the ownership of the bus proportional to the sizes of their plants, or as to their proportional use of the bus. These are all much involved questions the solution of which, it seems, should rest jointly between the engineers, the commercial department and the financiers of the institutions involved. Personally I have not, so far, worked out any practical solution to this problem.

To my mind this proposed transmission bus should not be treated as some mythical thing to be considered in the far distant future, but as something that is now needed, and immediate consideration should be given to working out of all details involved in carrying the project to a successful issue.

J. D. Ross: There seems to be no reason why 220,000 volts should not be used commercially, providing the proper synchronous apparatus controls the voltage and providing the insulation has a good factor of safety. The cost of such long lines will, of course, be commercially feasible only where there are large quantities of power to justify the expense. It is probable that before many years this voltage will be considerably exceeded and will be considered moderate.

The city of Seattle is at present embarking on an enterprise on the Skagit River one hundred miles distant from the city. On a 50 per cent load factor, there will ultimately be developed at this point one-half million horse power. On account of this very large quantity of power, it is desired to limit the number of lines as far as possible by using high voltage and for this reason six lines are intended and the voltage will be 160,000. Had the distance been two or three times as great, 220,000 volts might reasonably have been used.

Three of the coast cities besides Seattle own their own electric power systems; Los Angeles, Pasadena and Tacoma.

Some years ago I spoke to the men in charge of these systems of the hope that some day there would be a high-voltage line paralleling the coast which would allow one city to help the other but at that time the voltages possible were not adequate. From Seattle to Los Angeles is about 1500 miles and it is a notable fact that the line at present under discussion is 1100 miles in length. Such a line along the coast would be a benefit to many of the smaller towns especially, and could be participated in by companies and municipalities alike. Of course, probably no city or combination of cities would attempt such a line but the best and largest water powers are rapidly being used up and when it comes later to the proper conservation of all such resources, no doubt the states affected will take a hand and interconnection between states would make such a line an actual fact. One of the great troubles which prevents interconnection of systems at the present time is the dissimilarity of voltage and phase relation and one of the greatest works that the Institute could do would be to bring about a better standardization of voltages.

Leslie F. Curtis: Dr. Magnusson, in the November 15, 1918 issue of the *Journal of Electricity,* considered some of the phases of a similar type of bus for the state of Washington. He selected the state of Washington because he happened to be familiar with conditions there.

The principal points brought out by Dr. Magnusson in his paper are advantages to be obtained by the Federal regulation of such a bus system. I am not prepared to advocate either Federal or private control. I will simply refer you to his paper.

J. B. Fisken: Professor Curtis has brought out the point that I wanted brought out. Dr. Magnusson advocates a Federally owned bus and I think there are numerous objections to such a scheme. There would be difficulty connected with the utilities working on a Federally owned bus.

H. A. Barre: On this general matter of government control, I think the big question, or the big trouble, with government ownership in that sort of thing is that you don't want the umpire to play on either of the teams. If the government would get out of the way and let us do the job, we can do it.

We are coming more and more to recognize the fact that neither the government, or companies, or financiers, or any other element, will have much to say about the development of this project. The development will be according to the natural economic laws which are as unalterable as the multiplication tables. Improper financing, improper engineering and improper relations between the companies and the public, and the public and the organizations of the companies, can gum up the whole game; they can delay it; they can interfere with it, but they can no more stop the building of it than the building of a dam across a river can stop the water for there comes a time when it does run over.

I do not think that this is a good time to sell the United States short. I think there is enough common horse sense in the United States and enough economic pressure for the carrying out of such schemes and we will get the proper answer in spite of all these things that are trying to interfere.

I object to the paper as a whole because I do not like such an arrangement. I do not think we should worry about a 1100 mile bus in California. I think when this thing does come it will come very quickly. It is going to be tied in with the Colorado power for a starter; then south through New Mexico to Albuquerque, Needles, Bakersfield, Mount Shasta, and wind up, as has been said, in Butte.

The great part of the work is done. There are already a great many interconnections. Those interconnections have been the means of a great economic saving in the past two years. Just in our little corner of the job, the San Joaquin Valley would have been absolutely shut down this year if it had not been possible for an arrangement to be made between the Edison, the San Joaquin, the Mt. Whitney, the San Diego and the Southern Sierras Companies. It is not very far from being shut down now as far as the shortage of power is concerned, but the job is still running and the amount of interference to service has been extremely small, that is true in spite of the fact that it was necessary to buy current from San Diego. One of the great stumbling blocks, of course, is the 50-cycle system in the middle of a big 60-cycle territory.

The French government has done a very intelligent thing. They have standardized the whole country at 50 cycles and standardized the voltages in multiples of the square root of 3, as high as anybody wants to go. I do not think anybody would want to go higher than the 110,000 or 120,000 volts at the present time.

That is a line along which we could follow to some advantage throughout the United States through the action of the Institute.

Such a line is not going to be difficult to operate. It will be broken up into regions, as has been suggested, in which a group of plants will supply a group of territories, or a group of loads, and means provided for an interconnection between those groups.

The control of the voltage is one of the most serious troubles and that is going to come through an extension of our work with synchronous condensers, without any question. The real instructive thing that has come out of this interchange is the fact that it has been possible for companies whose interests were to a very considerable extent antagonistic to get together on a broad enough basis to help out each other with their troubles and not try to take all the money the other fellow has when they do it.

L. M. Klauber: I do not know how it should be done, but I hope it will be done soon.

I think that one point that Mr. Barre brought out is of considerable interest; this is that all of these things will be easier to do after they are done. They work themselves out beautifully. A lot of the problems that have been causing us sleepless nights (I refer to problems not of engineering but of operation and management) have really eliminated themselves. We sometimes wake up in the morning and find that the solutions which have bothered us for six months have already been accomplished and that the difficulties which worried us have failed to materialize when the time came.

You probably noticed a brief editorial in a recent copy of the *Journal of Electricity,* wherein it was stated that when the Pacific Fleet came into San Diego Bay, the Crane Valley reservoir of the San Joaquin Light and Power Company, distant many hundreds of miles and three companies from San Diego, had to be drawn on for an extra amount of energy. This, of course, is an interesting proposition. It would be somewhat more interesting, if true, which was not the case. As a matter of fact the shortage on that particular day was due to a defective condenser.

I think that the transmission bus, when the time comes, will work itself out without the difficulties in financing, in operation and in government supervision, which now appear almost insurmountable.

J. A. Lighthipe: I think Mr. Barre is perfectly right when he made the statement that while we are struggling and talking about this proposed bus, we will have it. Nothing is improbable and these great projects simply spring up when the need is there.

The great problem of this business is not only the diversity of the distribution, it is the great diversity of the supply. Our water sheds in California are very erratic and diversified. That problem within itself is a great one.

We have Railroad Commissioners who do not interfere with us. They pat us on the back and tell us to go ahead. As long as the State of California has a Public Service Commission, such as the Railroad Commissioners, with a modern man at the head of it, this matter of public control is not going to bother us.

It is a beautiful thing to talk Socialism and it is a beautiful thing to talk public ownership, but it does not get us anywhere. A man with the nerve will get hold of these projects and bring them to a proper solution. The State of California has reaped the benefit of the pioneers of the power situation and will continue to do so.

The question of voltage is an interesting one. When we had 10,000 volts we were afraid of it; we never dreamed of such a thing in our lives. When Southern California sprung to 33,000 volts even the manufacturers of apparatus shook their heads with uncertainty. I remember a man, in the early days who stated that 50,000 volts would be the ultimate we could hope for. That statement was made in the days when we knew nothing about all these modern troubles and the talk about a 50,000 voltage was thought impossible. However, we kept creeping up until we reached 60,000 volts and then 150,000 volts. We have also learned that the higher the voltage the less trouble we have. That is probably due to the fact that the engineers on the job have worked more thoroughly; that is, we know what we are running into and we build to meet that condition.

The future of the state is so great, where the price of oil used is ever increasing; where every bit of power in this state should be developed; where this power should be used and where we should start a campaign of conservation on what we have in the state.

P. M. Downing: It is very interesting to look at a map and see a 1500-mile bus extending from somewhere in British Columbia or the Northern part of Washington down to the Southern part of California or into Arizona, and from an engineering viewpoint we are able to convince ourselves that the problem is not one impossible of solution. In my opinion the real problem is not one of engineering but rather one of economics. There is no doubt but what in time the generation of power by the use of oil must be superseded almost entirely by water, and yet, at the same time, I do not know of any one particular state or district on this Coast that is going to have a sufficient surplus of hydro power to justify a high-voltage bus such as for transmitting energy from one state to another or from the system of one operating company to the system of another. There is no doubt in my mind but what in a very short time we will see lines operating at voltages as high or even higher than 220,000, but there is an economic limit to the distance that power can be transmitted. There is potential water power all along this coast from Alaska to Mexico that can be developed to meet the local demands. I look for a more general standardization of voltages and frequencies on the systems of the various operating companies and a more general interconnection of the various systems. If these interconnections can be looked upon as forming a bus, I feel that in time we may reasonably expect it to become a reality, but, obviously, interconnections of greater capacity that are necessary to carry the limited amount of energy that will be interchanged, cannot be justified.

When I say that the engineering problems can be more readily overcome than the commercial or economic ones I have in mind, particularly, an arrangement that was made a year or two ago between three of the companies in the northern part of the State of California. Those systems were tied together as a matter of economy and for the utilization of surplus power available.

When that tie-in was proposed it took the engineers but a short time to work out the engineering problems involved. It happened, however, that I also had something to do with drafting the final contract. It took us about three months to get the commercial details worked out. If there had been a larger number of companies involved, the problem would have been a correspondingly more difficult one. The physical or engineering details are simple, and can be taken care of very readily. So I say the whole problem is an economic one. Can any company, or any number of companies, or the State or Federal Government, afford to generate power up in the extreme northern part of this State and transmit it down to San Diego, when there is power nearer San Diego than that in the north? I do not look for a bus of great capacity to be built in the immediate future, except such as might result from the normal interconnection of lines of companies operating in contiguous territory. Negotiations are now under way looking for a connection between the lines of the Pacific Gas & Elec. Co. and those of the San Joaquin Light & Power Corporation. This connection will be made within a comparatively short time and when completed will give a connection from Oregon to San Diego. This connection is of limited capacity but is serving every purpose and in view of the amount of power that is available, a line of greater capacity cannot be justified. Interconnections between the various systems are very desirable and will continue to be made at whatever voltages the systems may operate, but until there is an apparent necessity for a bus I think we may reasonably conclude that the necessary capital to construct one will not be forthcoming.

J. B. Fisken: Mr: Downing has raised a question as to the advisability of this bus line. Personally, I think he is right and that this will solve itself in the interconnection of different systems without any great transfer of power over long distances. I do not ever expect to see the power from Spokane utilized in San Diego.

I see no reason, however, why Spokane power should not relieve some other power that could be transmitted a shorter distance.

This problem, I think, is largely a western one, but perhaps Mr. Stevens can tell us how they do these things in Pennsylvania.

J. F. Stevens. I would have given a great deal if, during the war, such a plan as is proposed in this paper had been in operation in the East, or even if there had been such limited inter-connections as you now have.

It happened that I was connected with one of the Government agencies which had to do with the power situation in Southeastern Pennsylvania. We were confronted with the fact that there was a distinct shortage of power in our district, not only a shortage of coal but also a shortage of generating capacity in our public service power plants. This meant that every pound of coal burned had to be burned at maximum efficiency and every generating plant operated at maximum capacity in order to secure the power necessary for the operation of our war industries.

The natural solution seemed to be a combination of the resources of the power companies operating in the district by tie-in connections which would render available their full generating capacity which then could be distributed to the various industries irrespective of their location. With this in view a conference was called and I found the power companies very willing to discuss the problem but without available funds with which to practically effect a tie-in. They were operating at from 25 to 60 cycles transmitting voltages from two thousand to sixty thousand so that the cost of installing frequency changers and transformers was considerable. The Government was prepared to spend unlimited millions in other directions but, while recognizing that power was essential to production, refused to appropriate any funds for the purpose of tying together the power plants of this district. We found ourselves in the usual position of a war-time bureau with authority to commandeer a tie-in but no authority to expend money for it or to compel the companies to spend money.

As a solution we had introduced into Congress a bill appropriating two hundred millions of dollars for the purpose of constructing power stations located at the mine mouth and erecting transmission lines to serve the district from Washington to New York. This bill failed of passage and, in view of the fact that it would probably have resulted in federal ownership and operation, I am not particularly sorry, for few of us in the East look with favor upon government ownership. The net result, however, has been to stimulate interest in power generation at the mine mouth in stations equipped with large units and transmitting over a very considerable radius. You are fortunate in having available water powers which are almost totally absent in Eastern Pennsylvania but we have as partial compensation large deposits of fuel coal of excellent quality.

Some experiments have been made and the results so far achieved have been exceedingly satisfactory, particularly in localities where there were a considerable number of small public service or municipal plants, and it bids fair to be a solution of the old municipal plant problem. It happens that back in the early days of electric lighting there was quite a mania for municipal ownership of lighting plants and almost all of them have fallen into sorry condition. In order to construct the plants the town would be bonded and no provision made for a sinking fund. They are, therefore, now in the position where they have an obsolete plant with a bond issue against it and no funds for the redemption of the bonds or for the reconstruction of the plant. I have in mind one such plant in Southern Ohio where the municipality was generating at a cost of from nine to twelve cents and selling at four and there are many more like it. They had no money with which to modernize their plant and even if they had the load was not sufficient to bring their generating cost down to a point where they could sell at a profit. The solution was found by establishing a generating plant at a mine not far distant and furnishing power to this town as one of its customers at a price which enabled them to sell to their citizens at a profit by transforming their generating station into a substation.

I firmly believe the time will come when all power in my district will be generated at the mine mouth, the various plants being tied in together on a common bus and distributed to various substations. You on the Pacific Coast have the advantage of water power and I think your plan most admirable. Even the discussion has been typical for when I came in the room the length of the transmission line was given at 570 miles and has now grown to 1500 miles but even the latter figure does not seem unreasonable in view of the existing situation. It is not, to my mind, the question of whether you have local power enough to meet the demands of each locality or whether you could expand your plants so as to secure it. It is the ability to interchange power, supply deficiencies and take care of break downs which is so very valuable. There have been many cases in the East where one large generator would go out in a public service plant which meant a shortage of power and light in that district which shortage could have been cared for had it been possible to tie-in with other power companies operating in the immediate neighborhood.

It does not seem to me that the plan here proposed presents any serious difficulties. The project naturally divides itself into generation, transmission and distribution. The various generating companies sell their power to the transmission companies which in turn would sell to the distributing companies in proportion to their demand. While there are some operative problems yet to be solved the principal problem is one of finance.

Relative to the operating end, I will state that in the East we have quite generally adopted a system of parallel transmission lines. As a rule we do not run two or more circuits on one pole line but mount each circuit on separately located pole lines half a mile or more apart. This practise has tended to save us from lightning troubles and break downs in the transmission circuits.

C. W. Koiner: I have been thinking of this paper along the line of economics. It appeals to me as practically suggesting the interconnections which saved us a great deal during the period of the war. If interconnection and tieing in, whether by as long a bus as this or a shorter one, is good in times of war it certainly ought to be good in time of peace.

The diversity factor which has been touched on by Mr. Lighthipe is the keystone of the light and power business. Without diversity we all would be in some other line of business because there would be no money in the power business under present conditions and circumstances of operation. There would be the gathering up of the crumbs, as it were, of the tieing in the plants that have water during some seasons of the year and none at others, enabling storage to be made, and all the other ways that would tend to mutually help out in times of stress.

I do not think of this paper as applying to the Pacific Coast particularly, but I am thinking of what Mr. Stevens touched on in the East during the war. The fact that the great coal mines lie close to the seaboard, would have made it a wonderful thing to have had the advantage of tying-in during the war. Why haul the coal and cinders and ashes to the seaboard and then cart them out of the city after making the buildings black, creating a lot of dirt producing and inefficient steam plants?

I think this principle was recognized ten or twelve years ago, or longer, but there are a great many things that enter into the problem, one of which has been touched on here and on which, I think, the Chairman invited discussion—government ownership and regulation.

It seems to me that it cannot be accomplished without regulation. Mr. Barre says that they try to rob one another and that they must have an umpire to prevent that kind of thing. The fellow who is able to take advantage, or figure the closest, will get the better of the other, figuratively speaking.

Now, to deny that the government has not the ability to even generate this power and deliver it to the distributor, or regulate it, is to deny that we are capable of efficiently living under democracy. Our government is supposed to be a democracy; therefore, if you cannot solve a problem by and for the people of this character, then we need a boss; we need somebody who will crack the whip over us and make us perform efficient service.

I am not one that believes that we are not capable of doing these things; I know we are. Some of the obstacles are the opposition of those who are entrenched. We may have stock in companies that have the water power cites, and we want more; we can make more money than if we let the government own it and run it for the people. Consequently, we as engineers are working for that side of the house and we naturally are opposed to government ownership and even regulation. But, some of the wiser heads, such as Mr. Phillip Cabot who wrote an excellent paper recently published in the *Electrical World*, recognizes that government regulation is here to stay and that even government ownership is coming on certain things. The water power belongs to all of the people; so does the coal. Somebody happened to see it first and, therefore, the coal is in the hands of private interests, yet the economic life of the country depends upon coal. Now, all monopolies should be controlled, or rather owned by the government.

I am not a Socialist, however; I am not making apologies for I believe in what I say. I believe that the water power of this country must be conserved or passed into the hands of private interests the same as the coal mines, and without regulation and proper control, the people of the state or of the various states, might not get the benefit. We must have regulation for the purpose of interchanging power between the states. It cannot be done without it.

When it came to winning the war we had to work as a government, all pulling together for the same thing. Now, in these problems we have to do the same thing and we might just as well face the problem in that way.

J. F. Stevens: The last speaker has touched upon the question of government ownership and I want to give you one example of the efficiency of government ownership in this line—something with which the Institute was very closely associated.

Just about the time of our entrance into the war, the Institute, seeing the seriousness of the power situation and recognizing that the defense of New York from attack by sea rested in Governors Island, called General Leonard Wood, then in command of the Island, into conference on the subject. We called attention to the fact that the Island was dependent upon the operation of its own power plant, government owned and operated, without which it would be impossible to use the search lights, ammunition hoists or even aim or fire the big guns, and that the failure of the plant, either through the act of an enemy or by accident, would leave New York defenseless from attack.

General Wood acknowledged that our statement of the situation was correct and asked for our solution. We told him we had a large amount of power in and around New York which we were prepared to tie together and deliver to him on Governors Island through submarine cables for use as an auxiliary to or a substitute for the Island plant. His reply was that the government had no appropriation with which to pay for such service. We then offered to raise the money necessary to pay for the tie lines and to buy and lay the cable. Then, to our surprise, he told us there was a regulation which prevented the landing of a cable for any purpose whatsoever on such a government reservation as Governors Island and that it would require an act of Congress to enable the Army to take advantage of our generous offer. Such an act was introduced into Congress but never has been passed. From this you can see the sort of efficiency we may expect from government ownership of utilities.

J. A. Lighthipe: Mr. Koiner said he was not a Socialist. I want to tell him that we all are. I want to say that when we want anything done in a hurry we always have to do it ourselves. I want to tell Mr. Koiner that the money he has accumulated during his lifetime does not belong to him—it belongs to Uncle Sam, and when he dies it goes to the government. Of course, the government does not want to take it all; it allows him some to will to his wife and children and the part the government gets is called inheritance tax.

I suppose Mr Koiner thinks he owns the money his employers pay to him. He does not do anything of the sort. Of course he uses some of it, but the government takes the rest and it is called Income Tax.

Now, this would be a wonderful thing to develop these water resources. It ought to be done as an economic measure, but Uncle Sam has not got any money and how are we going to solve the problem? The government will do this: We will let the individual people invest their money in a Public Utility Corporation, but we are not going to let them run it and bamboozle the country. What we are going to do is to put our own directors in there and tell them how their money is going to be spent; tell them just how much money they can borrow and criticize their method of construction. We are going to tell them how great dividends they are to pay. We call that the Railroad Commission.

That is just what we are running into. If we are going to have any public enterprise in this country, we are not going to have Uncle Sam start it because he is a little too slow. We are going to have the individual enterprising pioneers of this country develop all this power and we are going to have the Railroad Commissioners regulate our rates, etc. This will be done because, although Uncle Sam has not the money to do it, and is never ready to give it to us, he will tell us to put our money in and he will see that it is handled properly and that we get a good interest on the investment. But there will be no watered stock and no high financing. Consequently, every move we make, whether it be the purchase of something; or the installation of a new steam plant; or the installation of various water powers, we make our plans and submit them to the Railroad Commission. They figure out whether we really need this plant, whether the cost would be excessive, and then tell us to go ahead. You then have the endorsement of the Railroad Commission.

Mr. Koiner stated that this thing can not go on without some control. I say we are under control. We cannot make a move in this state without the sanction of our Railroad Commissioners and we are fortunate to have a very liberal set of men at the head of that Commission. We welcome this regulation because we are infinitely better off than some of the other cities that own their plants, because we keep our books in a certain way, as they tell us to; we have a sinking fund, which is lacking in some of our city plants and we are allowed a depreciation which is properly agreed upon. In other words the whole thing is run by the Public Utilities, only we are running it under the direction of the state of California.

Lester S. Ready: I wish to say that I am optimistic regarding the installation of a transmission bus running throughout the State of California. From what we have seen during the last three or four years of the spirit of cooperation by the utilities during the period of the war we cannot help but realize that the problem is not an impossible one.

I believe that if it had been said five or six years ago that the various utilities in Southern California would have stood together during the period of short water supply and not taken advantage of each other the remark would have been considered almost ridiculous. Yet, during the past year the supply of electric power in the southern and central part of California has been divided between the companies, the company with additional supply rendering service to its competitor at reasonable rates. Steam-electric power produced in Los Angeles and San Diego has been used to make possible the supplying of adequate service in the San Joaquin Valley, where a material shortage occurred.

The big supply of hydroelectric power of the state is in the northern section. There is not a great deal of hydroelectric power developed or to be developed, in the south. The Southern California Edison Company has built its lines from Big Creek and there is still a great deal of undeveloped power on that stream which ultimately will be transmitted south of Tehachapi. It is doubtful whether the power supply of the development when fully completed will more than half supply the demands of Southern California, and power from the northern part of the state must be transmitted southward.

Glancing at a map of the state you will see that all of the transmission lines lead southward, and at present the general flow of power is in that direction. There may never be a great deal of power produced on the Pit or Feather Rivers delivered to Los Angeles, but these rivers will relieve the demand upon the plants south of them, so that these in turn may be relieved to supply the demands further south.

From a physical standpoint there are practically only two transmission systems in the state at the present time. From Merced north to the Oregon line there is a great network at present completely interconnected. All companies south of Merced form another great system fully interconnected, and it is not only possible but probable that within six months or a year there will be an interconnection between these two main systems.

The war has done a great deal of good toward the development of interconnections between the utilities in this state and its affect will be a lasting benefit to the state as a result. The district in the south is requiring the full capacity of all the power plants and were it not for interconnection there would be a great shortage of power in the San Joaquin Valley at the present time.

The Railroad Commission is ready to do its part in fostering the interconnections which should be made. It made special efforts during the war to assist in the full conservative use of the power supply of the state and I wish to state that we are very pleased with the way things have been carried out, and the electrical engineers have much to their credit. I am not fully advised of the basis by which Mr. Barre determined the rates which were charged between the companies during the emergency but am certain there has been no profiteering between the companies during this period.

George A. Damon: One great thing that we are doing right here in connection with this proposed high-tension bus line is to establish a "sense of direction." We may not, just at present, be certain of all of the details—or how the enterprise is to be accomplished but I am sure we all feel that we are on the way. The situation is something like the starting of a foot ball game: we all know where the goal is but we are not at all sure of what each play will be to take us across the goal line. We may be quite certain, however, that if we all start in the right direction and stay together by constantly co-operating, eventually, we will accomplish this very desirable advance in the art.

I think it is quite immaterial to the project, at this time, to consider whether the cost is to be financed "individually" or "collectively"—that is by private capital or by public funds. We all love the democracy which we retained at the cost of the great war and recognize that our next problem is to strike an equitable balance between individual interests and the common welfare. My own sense of direction seems to indicate that the economic pressure of the need of this great interconnecting bus will evolve its own system or method of financing and that, as electrical engineers, we need not stumble or hesitate over this difficulty.

From our own technical standpoint the great big important problem which we have not yet discussed is the question of frequency. As we all know we have in use in this part of the country both 50 cycles and 60 cycles. If I am properly informed about 25 per cent of our load is on a 50-cycle system. Under these circumstances we have yet to determine the cost and practicability of combining these two frequencies into one system. We are constantly and rapidly increasing the load on each of the two systems of distribution but here we are seriously considering the tying together of all sources of generation or supply by means of one great interconnecting tie line. Under these circumstances what do we propose to do eventually with these two frequencies? In my opinion this is the real electrical problem to which we should just at this time, address ourselves.

L. M. Klauber: I think that the question of frequency is somewhat like some of these economic problems—it will solve itself over night. The Edison Company has always claimed that 50 cycles is the proper frequency and that operation would be impossible at any other. However, last week when the engineering staff was not watching, the system speeded up to something like 59½ cycles and operated that way for about two minutes; they could as easily do it for two years.

L. J. Corbett: As has been stated, the electrical features involved in the line projected have not been discussed very much. It occurred to me after looking over this paper and listening to the remarks thus far, that some of these points have not been taken up because they are not of extreme importance from an electrical viewpoint. When you have a system as long as the one contemplated, with plants connected in at short distances apart, the actual effect on the system, at San Diego, for instance, due to connecting in the plant at Spokane, would not be of great importance, because the load at San Diego would be taken care of by the plants in the closer vicinity. Only in certain cases where for some reason an entire district would be cut out, would we have even the problem of a long distance transmission line arising. It may be recalled that in the case of a long line there are differences in wave form which result from the transmission line characteristics, but it is my opinion that these would be almost eliminated in such a system as outlined here.

As has been stated by one of the speakers, a 220,000-volt line is not necessary, as an interconnection of the ordinary existing systems, whether 30,000, 50,000, 60,000, 110,000 or 150,000 volts, can be made. The lines can be interconnected just as they are by using proper transformer substations. As for the different frequencies encountered, that feature would present a minor difficulty; it would be an economic question solely, as it would call merely for the additional cost of a frequency changer when connection is made to a plant of a different frequency. It seems to me that everything required can be done with a lower voltage line and with equipment already in use or available.

J. H. Anderton: All previous discussions on this paper have given practically no weight to the engineering problems involved in the design and operation of the proposed line. In fact, it has been stated here that the engineering problems are comparatively simple. This may be quite correct if the comparison be made with the financial side of the problem. I do not believe, however, that we should assume the engineering features as a negligible factor. For instance, I note from the figures given on page 1240 of the paper that the distance between Pit River and Los Angeles is 570 miles; to San Diego it would be approximately 700 miles. These distances are, of course, more or less approximate and while they approach the quarter-wave length of 60-cycle propagation, there will be no danger from this standpoint alone since a practical line has resistance and other constants which prevent the theoretical rise due to the quarter-wave length distance. However, with lines of any length and given constants, a point of highest receiver voltage and maximum charging kv-a. can be found. The converse is also true that with a given length certain constants will give the highest receiver voltage and maximum charging kv-a. With a line of the length under consideration, the use of normal line constants might reasonably give a receiver voltage of from 250 to 400 per cent of the normal generator voltage, assuming that the line were charged from one end. This latter assumption is, of course, impossible due to the kv-a. capacity which will be required to charge the line but indicates in a general way the necessity for a careful determination of all the constants. The complexity of the problem is very apparent from the fact that the line may be operated as a whole or in part and could be charged from various sources.

H. H. Cox: The idea in preparing this paper was that it is now time for some large development in California. California can use 100,000 or 200,000-kw. water power now and relieve the steam plants and shut off the use of fuel. You all know the price of fuel. Fuel means something more to us. In a pound of oil there is a whole lot of other things more valuable than the thermal unit. Therefore, the use of fuel is going to be turned in another direction to make these chemical products, giving us the use of the by-products.

Our confidence in the operation possibilities of the high-voltage line is absolute. We don't think there will be any difficulty at all. In answer to Mr. Anderton, regarding the distance of 750 miles, more or less, this system would never be operated as a transmission from Pit River to Los Angeles. You would have to get the individual sections together. That is, one line from Pit River to San Francisco and another from Big Creek to Los Angeles; then the two would be gotten together and power interchanged. It would be a physical impossibility to build up a line of this distance with generators because the quarter-wave length would act as a short circuit on the line.

In regard to the operation of the line, we state that complete parallel operation of all lines must be adhered to in the proposed system. Satisfactory protective relay systems for dropping defective sections with little disturbances have been developed for present parallel transmission lines and there appear to be no obstacles to extending these to the higher voltages. Such protective relay systems have been in use on 60-kv. systems and are operating satisfactorily.

This transmission bus, as I have said, will probably never transmit power from north to south, but it will be a means of caring for emergencies and transmitting surplus energy to take care of the diversity of the different loads.

The present interconnections have a very limited capacity, as you all know. One company may now have 10,000 kw. to spare and puts in an interconnection of that size. Tomorrow an emergency arises and the other company needs 20,000 kw. The interconnection would not carry it. In the beginning of our paper we assumed the interconnection of all the California power companies on an equitable cost basis. It is now time for a large development of power to take place in California.

G. E. Armstrong: I just overheard, from Mr. Downing, the remark that we will have a job to justify this line. It doesn't look that way to us. In the first place, Mr. Downing said that California had lots of power, I suppose ten years ago that the power we have now looked like quite a bit. When I first came with the Southern California Edison Company we had a peak of 30,000 or 40,000 kw. and we are now running over 175,000 kw. The increase is going on a percentage basis. When on that basis, and the percentage increasing every year, we are going to pick up load pretty fast.

Mr. Downing thinks he won't live to see this line an accomplished fact. I would like to call his attention to one point. For instance, the Southern California Edison Company, if this line isn't put in, is going to need an additional 150,000-volt line to Big Creek very shortly. The two lines we have now are sufficient for about 60,000 kv-a. each. The two Big Creek plants now have a capacity of 60,000 kv-a. The installation of another plant there will bring it up to 100,000 next year and we will need more for the next year. If the voltage is raised to 220,000 shortly, the capacity of each line would be about 125,000. This was the idea that we had in mind as a need of bringing this proposed line in operation.

The San Joaquin Light and Power Co. now needs a transmission line running north and south and having a voltage of at least 110,000; that is what they are figuring on. You can easily see that it would be a waste of money for them to build a line of that capacity when the Big Creek line at 220,000 volts could take care of both the Edison Company and the San Joaquin Light and Power Company.

There is one other point. The longer this proposition is put off, the harder it will be to accomplish it. If it is necessary for the Southern California Edison Company to build another 150,000-volt line to Big Creek, it would not be very favorable to changing to 60 cycles. This same thing would be true of any of the companies in the northern part of the state who find it necessary to extend their lines in the near future. But, if this bus line is considered now and worked out in a reasonable way, I don't see why the next lines could not be built having in view the interconnection of the systems, so they could be operated at 220,000 volts when the time comes.

Presented at a joint meeting of the American Institute of Electrical Engineers and the Institute of Radio Engineers, New York, October 1, 1919.

TRANSOCEANIC RADIO COMMUNICATION

BY E. F. W. ALEXANDERSON

Abstract of Paper

The paper defines the state of the art of today which is the result of developments during the war. Transatlantic radio communication is at present maintained by five first class stations, two in America and three in Europe. These stations operate at wave-lengths between 12,500 and 17,000 meters. The range of wave-lengths suited for such traffic is rather limited, the desirable wave length being included between the limits of 10,000 to 20,000 meters. New developments indicate three methods for increasing the radio traffic without interference between the different messages. These methods are increase of the transmitting speed, closer spacing of wave lengths and directive reception. If these technical possibilities are intelligently used, the author predicts that radio communication will be equal to all demands that will be placed upon it.

The second part of the paper describes the radio transmitting system for the development of which the author is responsible. This system is represented by the naval radio station, New Brunswick, N. J. and comprises new means for generating modulating, and radiating the continuous wave energy. The generator is the high-frequency alternator with which the author's name has become associated. The modulating system is the "magnetic amplifier" which is described in a paper by the author before the Institute of Radio Engineers. The "multiple antenna" system of radiation is described in this paper for the first time. The general theory and figures for the increased radiation efficiency are given. The author also predicts that the multiple antenna will make possible directive radiation on a large scale.

It has already become generally known that a new highway for world traffic has been opened up through the development of transatlantic radio communication. It is now a matter of history that radio was largely used for communication between the United States and armies in Europe and that the Great War was brought to a close by negotiations conducted by radio which led to the armistice. Now, we are ready for an international commerce of unprecedented scope, but lack adequate means for communication.

The recent achievements of radio technique have become common knowledge, and the world has now turned to this new method of communication clamouring that it step in and save the day. This is a condition which places a serious responsibility upon radio engineers. Fortunately, the technique has emerged from the cloud of mystery that used to surround it, and we are in position to treat the problem cooly and scientifically like any other problem in electrical engineering. However, it must not be inferred that the task is any easy one, if the radio technique is to fulfill all the hopes which are placed on it.

It has been demonstrated during the war period that transoceanic communication has become thoroughly reliable, every day in the year, and practically every hour in the day. Thus far, we can say, that the problem is solved. But a second question will be raised: What volume of traffic can be carried by the means at our disposal at the present time, and what is the relation of this radio traffic to the world traffic of to-day and to the world traffic of the future? The facts of the case are briefly the following:

Experience has shown that the wave lengths which are most suited for transoceanic communication lie between 12,000 and 17,000 meters. "This space in the ether" has already been taken up by five first class transmitting stations which, during the war period and up to the present time, have been in continuous service for transatlantic communication. Of these stations, two are in the United States, one in England, one in France and one in Germany. By extending the range of wave lengths down to 10,000 and up to 20,000 meters, and following the same system of intervals there would be room for about seven more stations or a total of twelve first class transmitting stations.

A first class station has such radiating power that its messages can be received in all parts of the world. This is one of the advantages of radio communication; but it implies that if such a station is to be used to full advantage it must have a "right of way" for its wave length over the whole world. Thus if we look at the matter pessimistically, without allowance for the improvements that further engineering developments are likely to bring, it would look as if the capacity of the world for first class radio stations would be about twelve. The rate of transmission at the present time from these stations is about twenty words a minute and it would thus be easy to figure out that the capacity of radio for handling any considerable portion of world communication would be totally inadequate.

The other side of the picture, which the radio engineer of the future must study carefully and closely, should indicate the technical possibilities for improving the situation. The tendency of present day developments points to the following means for expansion of radio traffic:

1. Increase in speed of transmission.
2. Improved selectivity based on the direction of the wave.
3. Improved selectivity making possible closer spacing of wave lengths.

As a basis of discussing the general situation it may be here stated as simple facts:

1. That signals have been transmitted and received at considerably more than 100 words a minute.
2. That signals have been received from Europe while an American high power station within comparatively short distance was radiating on the same wave length.
3. That is has proved practical to separate radio signals differing in wave length considerably less than 1 per cent.

Based on these facts, it is probable that the transmitting speed in the future will average 100 words a minute instead of 20 words a minute.

That the selectivity for direction of the waves will multiply by five the number of stations that may be operated on one wave length; and that the selectivity with reference to wave length will be improved so that the wave lengths of messages will be within 1 per cent of each other, instead of 7 per cent which is the spacing of the stations at present.

These prospects, taken in combination, give us an optimistic picture in which the possible capacity for transoceanic radio traffic of the world is 175 times as great as it is with the practise of to-day.

To claim that the traffic capacity could immediately be increased nearly 200 times, would be an exaggeration, because the different improvements which have been made may partly conflict with the execution of each other if they are to be used simultaneously. This optimistic picture is therefore to be regarded as a goal—perhaps never to be reached, but it points the way for almost unlimited possibilities for progress by continued engineering efforts.

In order to avail ourselves of these three improvements simultaneously, the transmitted wave must be a continuous wave which does not "spill over" with harmonics, decrements or variations, into the range of wave length assigned for other communication. When wave lengths are spaced 1 per cent apart, each wave must be sufficiently pure so as to have no objectionable components outside the limits of one-half of one per cent. While rules to this effect should be rigidly enforced it must be appreciated that there are certain fundamental limitations.

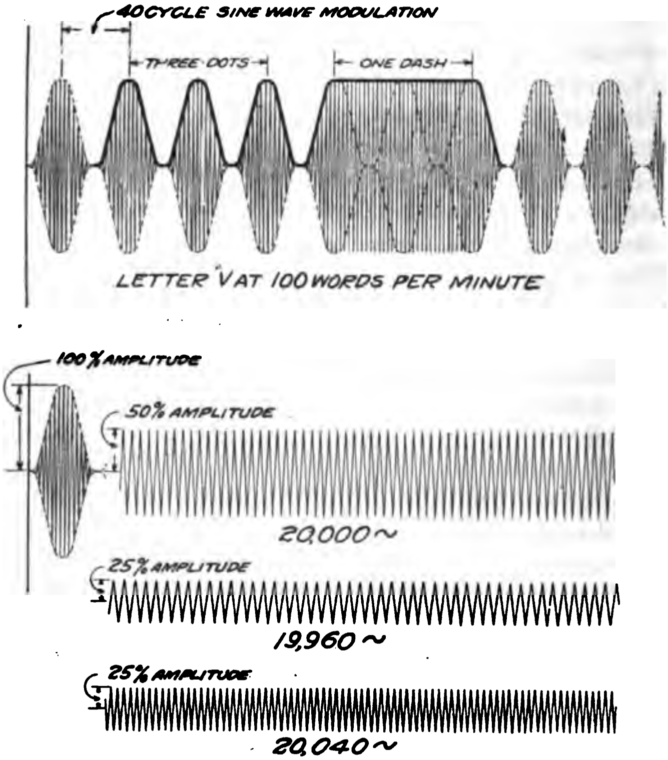


Fig. 1—Method of Resolving Sine Wave Modulation into Three Continuous Waves

High Speed Signaling

Modern transoceanic radio signaling is conducted by means of continuous waves. It must be appreciated, however, that signaling by an absolutely continuous wave would be impossible, because the making of dots and dashes introduces increments and decrements in the radiation. It will be shown that a repetition of increments and decrements can be resolved into a group of closely adjacent continuous waves. This agrees with the well known fact that the tuning of a wave with a decrement is known as a broad tuning. To illustrate this point we may take as a basis a signal at 100 words a minute (5 letters per word). If it is assumed that the increments and decrements in making the dots are not sharp interruptions but a continued variation by sine wave curves as indicated in Fig. 1, it is found by an analysis that such a wave may be resolved into a group of continuous wave components within the limits of 40 cycles above and below the average. This is the theoretical minimum width of the band of wave lengths which are necessary to transmit 100 words per minute. If the dots are defined by sharper interruptions the wave becomes still "broader" and it would not be unreasonable to say that the minimum practical band of wave lengths for 100 words per minute is 100 cycles on each side of the fundamental. This would make possible a spacing of the waves 1 per cent apart when using a wave length of approximately 15,000 meters. Messages at a higher rate of speed will occupy a correspondingly wider "space in ether."

As a conclusion from this analysis it may thus be said that an increase of the speed to 100 words a minute and increase of messages to a spacing 1 per cent apart may be accomplished simultaneously provided that waves are used of such a character and modulation that they contain no radiation except the one needed to accomplish the intended purpose.

DIRECTIVE RECEPTION

The second means for increasing radio traffic consists in improving the selectivity by taking advantage of the direction of the waves. The author's paper printed in the *Proceedings* of the Institute of Radio Engineers, August, 1919, described a receiver referred to as the "Barrage Receiver" which was used to demonstrate directive reception. There are several other types of receiving devices, developed by the United States Navy and other investigators, which accomplish substantially the same purpose. The broad principle underlying all these directional receiving devices is the one discovered by Bellini and Tosi and generally referred to under their names.

In the U. S. Navy's tests of the "Barrage Receiver", it was proven that it is possible to carry on simultaneous two-way communication on exactly the same wave length. If this method of directive reception is carried out consistently in a world system of communication, it may be assumed that transmitting stations operating on the same wave length may be located approximately as shown in the map in Fig. 2—one in Europe, one on the American east coast, one on the American west coast, one in the Far East and one in South America. The American receiving station for European signals in such a system should be located east of the American Atlantic Transmitting Station, and in line with the Pacific Transmitting Station. Thus messages from the two American transmitting stations could both be simultaneously neutralized in the American Receiving station by a "Barrage Receiver," while signals on the same wave length are received from Europe. Interference from the South American station may be neutralized by the use of a double barrage system, while the station in the Far East, though it may not be exactly in line with the two others would be sufficiently near the general direction so that, considering the great distance, it would not cause interference.

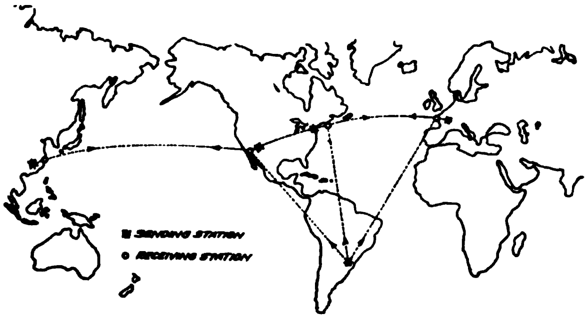
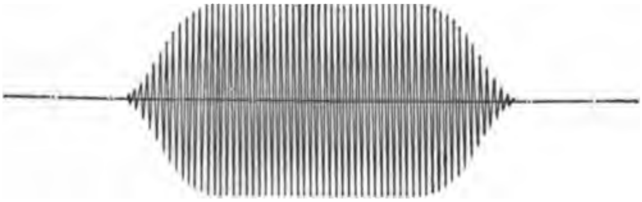


FIG. 2—PROPOSED SIMULTANEOUS RADIO TRANSMISSIONS ON THE SAME WAVE LENGTH

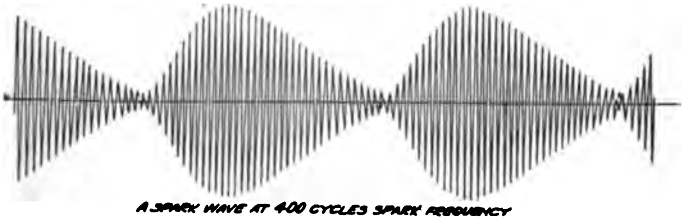
If this communication system is to be duplicated on a number of other wave lengths, the practical conclusion follows that the transmitting stations as well as receiving stations for each district should be grouped in centers, and these centers located relatively so as to make the directive neutralization as effective as possible. The neutralization of several transmitting stations simultaneously may not always work out as first designed. It has been shown by investigations of the Navy Department that the radio waves do not always follow straight lines and not always the same path. However, discrepancies from such origin may again be overcome by further extending the principle of neutralizing waves from several directions simultaneously.

Closer Spacing of Wave Lengths

The method of increasing traffic capacity by closer spacing of wave lengths has great possibilities. It has been shown that the selectivity with reference to wave length can be greatly increased by several successive tunings in either the radio, the audio or some intermediate circuit. It is thus entirely practical to receive either by ear or by photographic records, signals which are considerably less than 100 cycles apart. The theoretical limits for such selectivity in connection with high speed transmission are defined below. As illustrated in Fig. 1, a high speed message is not a single continuous wave, but a band of wave lengths—100 words per minute occupying a space of about 200 cycles. Therefore the same degree of selectivity is not to be expected with a high-speed signal if the interfering radiation is of considerable intensity.



WAVE TRAIN SHOWING ACTUAL NUMBER OF RADIO CYCLES IN A DOT OF THE TELEGRAPH CODE TRANSMITTED AT THE RATE OF 500 WORDS PER MINUTE AT 15,000 METERS WAVE LENGTH.



A SPARK WAVE AT 400 CYCLES SPARK FREQUENCY

Fig. 3

Speaking in terms more familiar to practical radio operators it may be said that high speed telegraph signals assume to some degree the objectionable characteristics of a spark signal. As an illustration of this, Fig. 3 shows the relative decrements in a continuous high-speed telegraph wave and a spark wave, showing that the increments and decrements of the continuous wave signal at 500 words per minute are about equal to a spark wave of 400 sparks per second. These illustrations apply to wave lengths of about 15,000 meters.

THE RADIO TRANSMITTING SYSTEM

Several types of radio transmitting systems are at present in use with a high degree of success. The descriptive matter in this paper will, however, be confined to the system for which the author is responsible as represented by the Naval Radio Station at New Brunswick, N. J.

Generally speaking, any radio transmitting system consists of three essential elements:

1. The generator of radio frequency energy.
2. The modulating system, whereby the energy is controlled so as to produce the dots and dashes of the telegraph code or the modulations of the human voice.
3. The antenna or radiating system.

GENERATING SYSTEM

There are four types of generating systems of high-frequency energy in use at the present time.

1. The spark or impulse generator.
2. The Poulsen-Arc generator.
3. The high-frequency alternator.
4. The vacuum tube oscillator.

The system which will be described, is of the type employing a high-frequency alternator. The installation of New Brunswick contains a 50-kw. alternator shown in Fig. 4 which was operated for sometime for experimental purposes with radio telephony at a wave length of 8000 meters, and later in transatlantic telegraph service at 9300 meters.

A larger equipment which has been in continuous service for the last year consists of a 200 kw. alternator shown in Fig. 5, 6 and 7. Fig. 5 shows the machine partly assembled, the rotor consisting of a solid steel disk. The spaces between the polar projections are filled with non-magnetic material so as to present a smooth surface and thereby reduce air friction to a minimum. The disk runs between the two laminated armatures which are cooled by water pipes as shown in the photograph. The armature winding which consists of wire wound back and forth in straight open slots, is divided in 64

PLATE XLV.
A. I. E. E.
VOL. XXXVIII, 1919

FIG. 4—50-KW. 50,000-CYCLE ALTERNATOR

FIG. 5—200-KW. HIGH-FREQUENCY ALTERNATOR

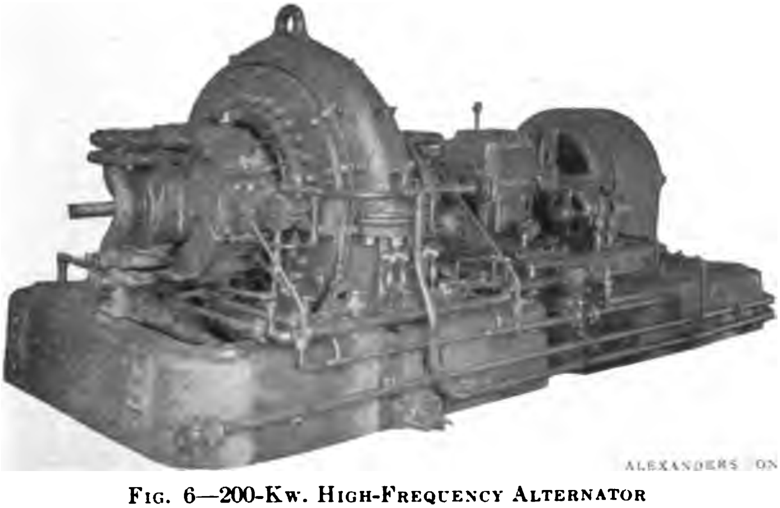


FIG. 6—200-KW. HIGH-FREQUENCY ALTERNATOR

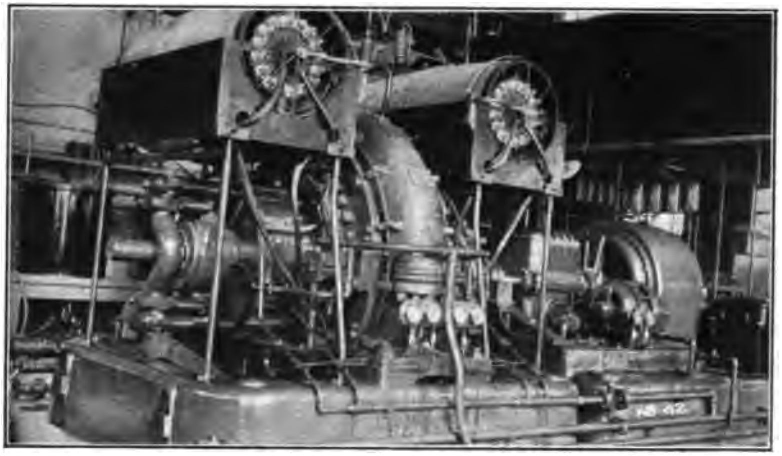


FIG. 7—200-KW. ALTERNATOR SET AS INSTALLED IN THE NAVAL RADIO STATION, NEW BRUNSWICK, NEW JERSEY

[ALEXANDERSON]

FIG. 9—COMBINED CURRENT AND VOLTAGE REGULATORS FOR 'HIGH-FREQUENCY ALTERNATOR CONTROL

sections, each section generating about 100 volts and 30 amperes. The current generated by these 64 windings is collected in the air core transformer mounted on the top of the machine, Fig. 7. This transformer has 64 independent primary windings corresponding to the armature windings. The single secondary winding of the transformer delivers the complete output of the alternator. This collecting transformer is thus to be considered as an integral part of the generating unit and for all purposes of calculation the characteristics of the generating unit, such as electromotive force and current, are given as delivered from this secondary winding. At full output the alternator delivers 100 amperes at an electromotive force of 2000 volts. It can thus be seen that the alternator is designed for a load resistance of 20 ohms. However, the same machine might be adapted for any other load resistance by selecting a different number of turns in the secondary of the collecting transformer. The reason why this particular machine is designed for a high voltage and low current will be given later in discussion of the new type of antenna with which it is used.

The 200-kw. alternator when operated at the New Brunswick wave length of 13,600 meters runs at a speed of 2170 rev. per min. It is driven by an induction motor through a gear of a ratio of 2.97:1 when the high-frequency alternator is used as a source of radiation, the wave length is determined directly by the rotative speed of the machine. Thus, obviously, it is important that the rotative speed should be as nearly absolutely constant as it is possible to make it. An important accessory of the alternator set is therefore the speed regulator. The 50-kw. alternator set shown in Fig. 3 is driven by a direct-current motor whereas, the 200-kw. set is driven by an induction motor of the slip ring type. The 50-kw. set was equipped with a direct-current motor because the problem of speed regulation of that type of motor is somewhat easier. Induction motors were, however, decided upon for the later types because alternating-current power is more easily available in most localities.

SPEED REGULATOR

The speed regulator consists of a speed determining element and a power controlling element. The speed determining element is a resonant high-frequency circuit fed by one of the 64 alternator windings which is set aside for that purpose. The oscillating energy of this high-frequency circuit is associated by magnetic couplings with a rectifying circuit in which the high-frequency energy is changed into direct current. This rectified current in turn, actuates the controlling magnet of a vibrating regulator of the type that is generally used for voltage regulation in power stations. When the driving motor is a direct-current motor it is easy to see how this vibrating regulator may be made to control the speed by regulating the voltage of the power supply to the motor. In order to accomplish the same object with an induction motor some new features have been introduced.

An ordinary induction motor is operated at constant potential. When the motor runs light it draws from the line a magnetizing current which is almost wattless. Thus it operates at a low power factor. When the motor is fully loaded it draws power at a high power factor—a motor of the type used having a power factor of 90 per cent.

When the New Brunswick station was adjusted for operation it was found that a wave length was desired which required the induction motor to work at 19 per cent slip. The rheostat in the secondary of the motor could easily be adjusted so that the motor would deliver the desired power with full load at 19 per cent slip. However, inasmuch, as the output of the alternator varies continually with the making of dots and dashes of the telegraph code, the motor is alternately loaded and not loaded. The tendency would therefore be for the motor to speed up during the intervals. If the potential of the power supply to an induction motor is varied, the motor torque varies by the square of the voltage. It is furthermore easy to show, by the theory of the induction motor, that if a motor consumes power at 90 per cent power factor at full load, and the load is reduced to ¼ by the reduction of voltage to ½, the power factor will remain 90 per cent. In fact it will always consume power at 90 per cent power factor regardless of its load if the voltage supply is adjusted accordingly, so long as the secondary resistance remains constant and the speed remains constant.

Thus it may be said that the standard method of operating an induction motor is at constant potential and variable power factor. The method of operating the driving motor of the radio set may on the other hand be characterized as variable potential and constant power factor.

The problem which thus presented itself was to find means for varying the applied voltage in accordance with action of the speed-determining element, and this has been done in the following way:

Between the motor and the power supply is introduced a choke coil with iron core, the permeability of which can be varied by saturation. The change in permeability is produced by a direct current which is controlled by a vibrating regulator. When the motor carries full load the iron core is saturated so that the choking effect is practically zero. At fractional load the choking effect is automatically adjusted by the regulator so that the motor delivers at all times the power required to hold constant speed. The motor itself operates at all times at its maximum efficiency and power factor, but the power factor of the current drawn from the line varies with the load. Thus when the motor operates at ¼ load the power factor of the line is 45 per cent while the power factor of the motor is 90 per cent. The circuits of the regulator are shown in Fig. 8 and the vibrator regulator in Fig. 9.

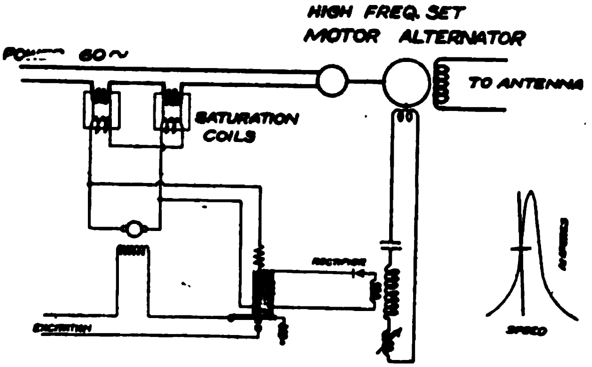


FIG. 8—SPEED REGULATOR ALTERNATING CURRENT DRIVE

MODULATING SYSTEM

The method of controlling high-frequency energy involves an apparatus which has become known as the "magnetic amplifier." This device is described in a paper by the author in the Institute of Radio Engineers, January, 1916, and therefore needs to be referred to only briefly. The magnetic amplifier is a device which is physically of the nature of an oil cooled transformer. The iron core which is made of fine laminations, is designed in such a way that the magnetic permeability of the iron core can be varied by magnetic saturation. By a special combination of tuned circuits as shown in Fig. 10 it has become possible to separate the controlling current from the high-frequency current so that a comparatively weak current of a few amperes controls as many hundreds of amperes in the antenna. When the transmitting station is used for telegraphy the magnetic amplifier is controlled by the telegraph relays which are a part of the wire telegraph system. During the war service, the telegraph key was operated in the centralized operating room of the Naval Communication Department in Washington. When the station is used for telephony the controlling current is an amplified telephone current.

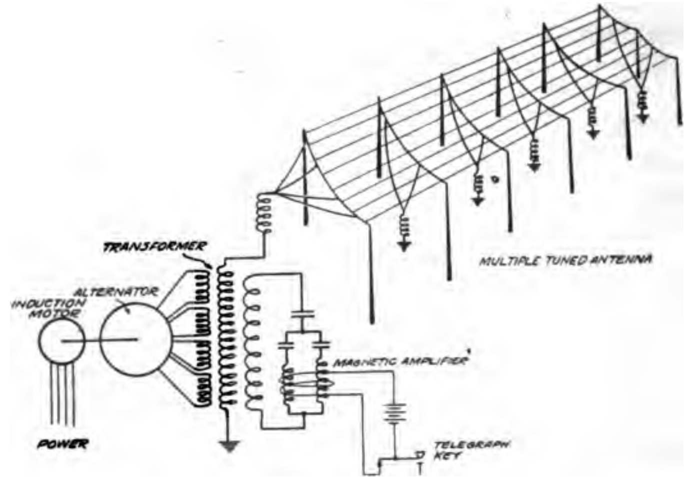
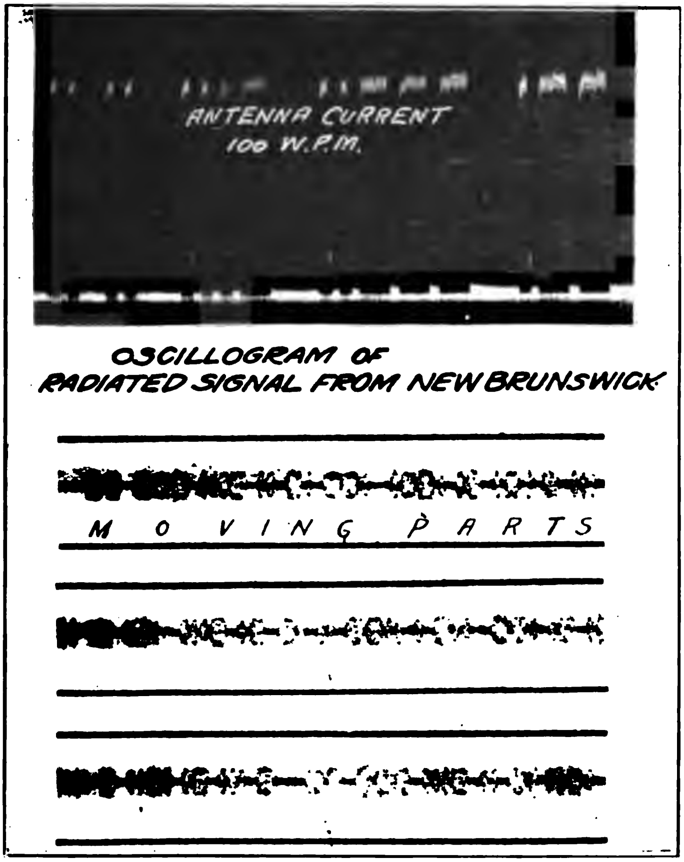


Fig. 10

While the magnetic amplifier has proven to be a very satisfactory and reliable controlling device for ordinary telegraphy its particular advantages are most prominent in high-speed telegraphic transmission and telephonic transmission, on account of its instantaneous magnetic action without any arcing contacts. Fig. 11 shows an oscillogram of radiation at 100 words per minute and a copy of a photographic record of reception at the same speed. Fig. 12 shows the telephone modulation of the antenna current when Secretary Daniels was speaking over the telephone line from Washington, controlling the output from the New Brunswick station, thereby transmitting his voice to President Wilson's ship at sea.

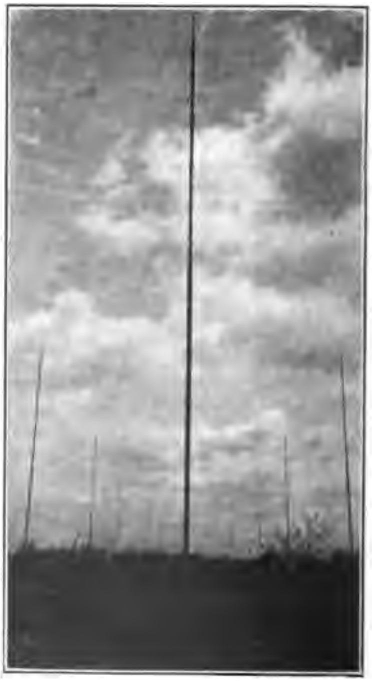


[ALEXANDERSON]

FIG. 11—PHOTOGRAPH OF RECEIVED SIGNAL FROM NEW BRUNSWICK AT 100 WORDS PER MINUTE

FIG. 12

FIG. 14—OUTDOOR TUNING COIL



[ALEXANDERSON]

FIG. 15—PERSPECTIVE OF ANTENNA

THE MULTIPLE ANTENNA

The antenna of the New Brunswick station represents a new departure in the method of radiation. The old antenna structure was originally one of the horizontal Marconi antenna, 5000 ft. (1500 m.) long, 600 ft. (180 m.) wide supported on towers 400 ft. (120 m.) high. The original antenna had a resistance of 3.8 ohms.

The antenna as operated now has a resistance of 0.5 ohms distributed approximately as follows:

Radiation resistance	0.07 ohms
Tuning coils and insulation	0.10
Ground resistance	0.33
Total multiple resistance	0.5 ohms

The reduction in total resistance of the antenna is due to the reduction of the ground resistance. While the old antenna had one tuning coil located in one end, the new antenna has six tuning coils as shown in Fig. 10 and Fig. 13.

THEORY OF THE MULTIPLE ANTENNA

The theory of the multiple antenna can be explained in several ways. Without going into details, an explanation will be presented giving the point of view which has proved most useful for general discussion. For this purpose the multiple antenna may be considered as an aggregate of several antennas of the ordinary vertical type each having its own tuning coil. When regarded in this way, the multiple antenna in New Brunswick is equivalent to six independent radiators placed 1000 ft. apart. The ground resistance of each of the radiators is 2 ohms, the coil resistance 0.6 ohms and the radiation 0.07 ohms making a total resistance of 2.67 ohms. Each of these radiators has an aerial 1000 ft. long and 600 ft. wide, counted at an average height of 300 ft. A total resistance of about 2.67 ohms is, as a matter of fact, the resistance to be expected from an antenna of such dimensions and ordinary design. If one individual antenna such as described is operated with a radiation of 100 amperes, the energy consumption of the antenna would be 26.7 kw., and the radiation efficiency would be 0.07 ohm divided by 2.67 ohm which is 2.6 per cent. If it is desired to increase the radiation from 100 amperes to 600 amperes the energy consumption would be 36 times as great that is, 960 kw. There is, however, another way to produce a radiation equivalent to 600 amperes. If six separate antennas of the dimensions described were built and each operated with a radiation of 100 amperes, each antenna would emit a system of waves proportional to 100 amperes radiation. If all the waves emitted by the six antennas were in phase, the amplitude of the combined wave would be six times as great as the wave emitted by one antenna. Thus the amplitude of the combined wave would be equal to the amplitude of the wave emitted by one antenna when operated at 600 amperes. The energy consumption required for operating one antenna was 26.7 kw., thus it might off-hand be concluded that the energy consumption required for operating the six antennas simultaneously would be 160 kw. This conclusion is not exactly correct because there is an interaction between the radiating effects of the different antennas resulting in a somewhat higher energy consumption. This might be expressed as follows:

The radiation resistance which is 0.07 ohm is common for all six antennas, whereas the ground and coil resistance of 2.6 ohm belongs to the different antennas, individually. Thus the combined circuit of the multiple antenna can be represented by the common radiating resistance of 0.07 ohm connected in series with a group of six multiple resistance of 2.6 ohms each, so that the total current of 600 amperes flows through the 0.07 ohm radiation resistances which represent the ground and coil resistance of the individual antennas. Hence the total energy consumption of the combined antenna is found to be 180 kw. out of which 155 kw. is ground coil and insulation loss and 25 kw. radiation. The radiation efficiency of the multiple antenna is thus 14 per cent against the radiation efficiency of 2.6 per cent for the individual antenna. The above calculation is made for the present operating wave of 13,600 meters. If on the other hand the same calculation is made for the same length of 8000 meters, which has been found more efficient for telephonic transmission, it is found that the radiation efficiency is 30 per cent, the energy being distributed as follows:

Radiation resistance	0.2 ohms
Coil and insulation	0.07
Ground resistance	0.33
Total multiple resistance	0.6 ohms

For the sake of convenience in dealing with the radiation from a multiple antenna it has become the practise to indicate the total radiation from the multiple antenna as the sum of the currents measured in the six ground connections. The equivalent multiple resistance of the antenna is then determined by the equation I^2R = energy consumption, where I is the sum of the currents in the ground connection.

It has been assumed above that each of the individual antennas is operated so that the waves sent out by the same are not only of the same frequency, but exactly of the same phase. It remains to be shown how this is accomplished. Fig. 13 shows the relation between the antenna and the multiple tuning coils and the alternator. As shown by the diagram, the alternator is connected in series with one of the six tuning coils. Arrows on the diagram indicate further how the oscillating currents and the energy currents are distributed. Six independent oscillating circuits are formed, the current in each ground connection corresponding to the charging current of the corresponding section of the aerial. If the antenna had been shock excited so that it continued to oscillate in the way indicated, no current would flow in the horizontal wires between the different sections of the antenna. However, in order to maintain continued oscillations a flow of energy must take place from the alternator which is connected to one of

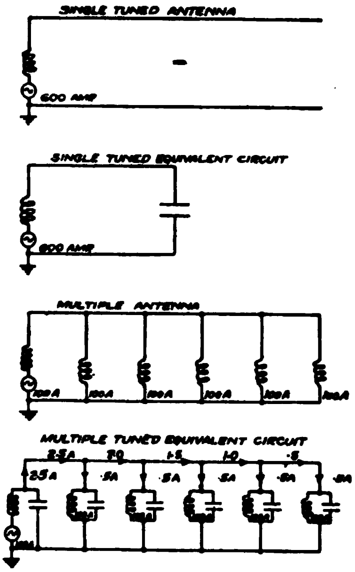


FIG. 13

the six tuning coils. When the antenna is operated with 100 amperes in each of the ground connections, the energy consumption as shown above is 180 kw., that is, each oscillating circuit consumes 30 kw.

What actually takes place is the following: The tuning coil to which the alternator is connected transforms the energy of the alternator into a power supply at a potential of 60,000 volts, and each of the oscillating circuits draws energy from this power supply at that voltage. Thus the energy currents consumed by each oscillating circuit is only 0.5 amperes. It can thus be seen that while the total oscillating current of the antenna is 600 amperes the energy current which flows horizontally from the power source to the multiple oscillating circuits is only a total of 2.5 amperes. In other words, the energy which is delivered by the first tuning coil in the form of 100 amperes at 1800 volts is transformed by the first oscillating circuit and distributed as in a transmission line from which 0.5 amperes at 60,000 volts is drawn in five places. The analogy between the multiple antenna and a high-tension power distribution system is thus apparent.

This point of view is a departure from the conventional theory of radiation; but it must be remembered that there was a time in the development of electric power technique when the introduction of the high-tension multiple distribution system was a radical departure.

DIRECTIVE RADIATION

The multiple antenna as described in its simplest form is adjusted so that the radiation from each of the individual oscillators is in phase. If, however, the antenna dimensions are so chosen that the phase displacement of the traveling wave between the different radiators becomes an essential factor, it is possible to obtain directive radiation. The radiated wave then will not be a simple circular wave but an interference pattern which may be treated like the corresponding phenomena in light and sound waves. Furthermore the phase displacement of the oscillations of the individual radiators may be regulated by tuning. Thus a variety of interference patterns may be created and analysis of these possibilities shows that an efficient unidirectional radiation by such methods should be possible.

Methods for unidirectional radiation have been established through the well known work of Bellini and Tosi. Through the courtesy of Mr. Bouthillon of the French Post Office, results of tests made in France have been placed at the disposal of the author, which show conclusively directive radiation by the Bellini and Tosi antenna.

With the dimensions of antennas used up to the present time, efficient directive radiation has not been practical. It has, however, been proved by various tests that the system of a central power source and a distribution system of energy to a large number of multiple radiators places at our disposal means for constructing radiators of dimensions of one wave length or more. The New Brunswick antenna (1500 meters long) has a minimum wave length of 8000 meters as a single antenna, whereas it can be operated as a multiple antenna at 2000 meters wave length. A detailed analysis of the possibilities of multiple radiation would fall outside the scope of this paper but the author is in position to predict with confidence that directive radiation on a large scale will not only prove practical but that it will be the most efficient method of radiation.

To add directive radiation to the proposed program for increasing the capacity of radio traffic would perhaps be premature until it has been demonstrated on a large scale. However, it deserves mention in order to show that new principles which may be utilized for still greater expansion of the radio technique have not yet been exhausted.

Presented at a joint meeting of the American Institute of Electrical Engineers, and the Institute of Radio Engineers, New York, October 1, 1919.

TELEPHONE REPEATERS

BY BANCROFT GHERARDI AND FRANK B. JEWETT

ABSTRACT OF PAPER

In this paper the authors have endeavored to set forth briefly but clearly the history of the research and development work which has led up to the final production of successful telephone repeaters. The various forms of amplifiers which have been suggested are described and their possibilities and limitations pointed out. The essential properties of repeater networks together with the necessary line conditions for successful repeater operation are described and illustrated. Tandem operation of repeaters is discussed as is also the use of repeaters in four-wire circuits. Illustrations are given showing a few of the more important repeater installations now in regular commercial service in the United States.

INTRODUCTION

DURING the past ten years developments in the design and construction of telephone amplifiers, or telephone repeaters as they are more universally called, and in the art of their application to all forms of telephone circuits have progressed to such an extent as to justify the presentation to the American Institute of Electrical Engineers of a comprehensive paper which shall cover not only the earlier efforts but the present state of development.

As indicated below, the idea of one or more repeaters inserted in a line for the purpose of reinforcing from some local source of energy the weakened current from the sending station is older than the telephone itself. In telephony innumerable attempts by a large number of investigators have gone on continuously almost from the inception of the telephone in an effort to extend the range of telephonic speech through the utilization of energy applied to the telephone line at a point or points between the transmitting and receiving stations. While the net result of all this work is, of course, the state of the art as we now have it, a survey of the earlier developments in the light of present-day knowledge discloses in striking fashion the fact that, unknown to the investigators, the early attempts were destined not to succeed where success was measured by the development of a practical device which would give satisfactory results on a regular two-way telephone circuit.

We know now that the final successful development and application of the telephone repeater had to wait not only for the slow accumulation of comprehensive knowledge concerning all the factors which govern the successful transmission of speech electrically over wires and of the intricate relation of circuits and apparatus to produce desired results with the maze of frequencies involved in speech, but also on developments in physical science which were in themselves quite foreign to the specific realm of telephony. For these reasons the work of the earlier investigators, ingenious though it was, was for years foredoomed to failure by factors over which they had no control and of which, in many cases, they had no knowledge even. In the light of what we now know, much of this early work appears almost impossibly successful and attests to the ingenuity and resourcefulness of the men who conducted it.

Another thing which the final successful development and extensive application of the telephone repeater indicates with striking clarity is the fact that a complete solution of the problem was made possible only by the existence of a great unified engineering and research department, such as that maintained by the Bell System. The elements involved and the multiplicity of detail to be worked out and correlated were far beyond the limitations of any single individual or any limited organization. The best that mathematics, physics, chemistry, engineering, manufacturing and operation could supply were needed for the solution. Further, if the solution were to be obtained in a reasonable time it was essential that the efforts of the experts in all of these fields must be directed in a cooperative attack on the main problem. With the growth of fundamental research groups of highly trained scientists in the engineering department of the Bell System the means for attacking the elusive repeater problem in comprehensive fashion became available and progress toward success became certain and rapid. The loose ends of discrete and desultory researches were gathered in, past and present work scrutinized and the attack directed from a foundation of certain knowledge not theretofore available.

The wonderful success which has attended the work and which has started what bids fair to be a revolution in the entire scheme of telephonic transmission should be a source of gratification to American electrical engineers, to whom more than to any others have been due the other wonderful telephone developments of the past.

In discussing the earlier work on telephone repeaters and that which led up to the first successful device, the authors have not attempted even to enumerate the large number of investigators, much less to allocate credit among them. The names mentioned are those of the men who stand out most conspicuously and the fact that many of them are employees of the Bell System is not remarkable, since it was the largest single agency consistently at work on all problems concerning telephone development. In a complex art which has been vigorously worked for nearly fifty years, it is inevitable that great numbers of patents should have been taken out on all conceivable kinds of devices and systems. Many of these are inherently worthless, while others contain elements of value, although not in themselves disclosing complete workable arrangements. While, therefore, no single invention has solved the repeater problem in its entirety, it is interesting to note that all of the inventions which make possible the successful telephone repeater of today are the work of Americans.

No paper dealing with the development and successful application of telephone repeaters would be complete without mention of the fact that throughout all the later years, during which progress has been certain and rapid, the principal credit for the broad engineering and commercial results obtained is due to Colonel J. J. Carty, past President of this Institute and for many years Chief Engineer and now Vice President of the American Telephone and Telegraph Company. To Colonel Carty more than to any one else is due the credit of having brought about that coordination of all of the elements mentioned above, which was vital to the ultimate success of the undertaking.

HISTORICAL

The foundation of the art of telephony by Alexander Graham Bell followed by almost fifty years that institution of the sister art of telegraphy which was based on the discoveries of Faraday, Ampere, Volta, and Henry. It was inevitable, therefore, that efforts to extend the range of telephonic communication should be influenced by earlier and successful efforts in telegraphic communication. As telegraphic operation was attempted between more widely separated stations it was found that the attenuated currents were insufficient to operate the recording mechanism. To obviate this limitation Morse introduced at the receiving station a local source of energy which supplied current to the recording mechanism but was controlled by the received signals through the medium of an electromagnetic relay.

In this invention was the principle necessary for an indefinite extension of the possible range of communication since the local source might supply current to another stretch of line. It is a curious bit of history that this application was not utilized for several years during which transmission over long distances was accomplished by manual instead of automatic repetition at the intermediate stations.

The delay in this natural development was due to the fact that the relay of Morse, although it was a repeater, was a one-way device since it was actuated only by currents arriving from one of the two telegraph stations on either side of it. It required the use of two such repeater elements and of a circuit arrangement to convert a one-way into a two-way repeater station. Of these circuit arrangements one of the earliest was that of Varley. During the next few years duplex repeaters received considerable attention from inventors. The differentiation, however, which seems so obvious to us today, between the repeater element and the circuit arrangement by which such elements may operate duplex, was infrequently if ever emphasized.

In the extension of the range of telephony by the use of repeaters the duplex feature was even more essential than in telegraphy. The problem was complicated not only by the inherited confusion between element and circuit but also by the greater importance of the third factor in such operation, namely, the lines on either side of the repeating station.

The necessity for distinguishing between these three factors of repeating element, repeater circuit, and lines was early recognized by the engineers of the Bell System. The exact and quantitative relationships between them have been adequately expressed for about a decade. In our discussion of the matter in hand we shall follow, therefore, the logical order which results from this differentiation and treat these factors separately in the above order. Before undertaking their more detailed exposition it may be well to vivify their individuality by the consideration of the concrete case illustrated in Figs. 1 to 5 inclusive.

A principle, which is familiarly known by its direct-current application in the Wheatstone bridge, has proved most fertile in the formation of duplex circuits. Thus in Fig. 1 we have an early method of duplex telegraphy, which is obviously reducible to the simple bridge circuit of Fig. 2. If *a* and *b* are equal then, provided also *R* is equal to *X*, the battery *B* will cause no difference of potential between *m* and *n* and hence no current through the galvanometer in the case of Fig. 2 and no resultant magnetism in the sounder of Fig. 1.

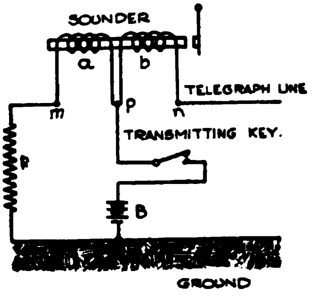


FIG. 1

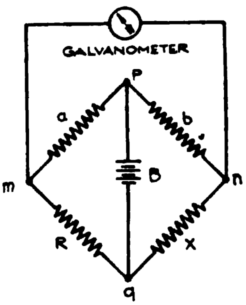


FIG. 2.

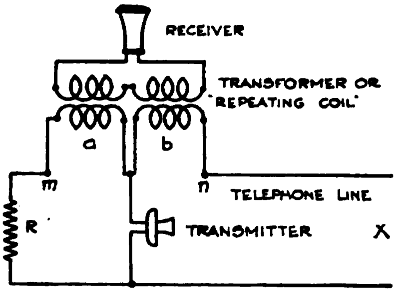


FIG. 3.

The same principle is applicable to the duplex operation of a telephone line as is seen in Fig. 3, where the sounder and the key of Fig. 1 are replaced respectively, by a receiver and a transmitter. If the resistance *R* is not equal to *X*, the impedance of the telephone line, then, the duplex set is not entirely "anti-side tone" since the operation of the transmitter causes some current in the receiver. The amount of current through the receiver and hence the motion of its diaphragm depends both upon the current from the transmitter and upon the amount of unbalance in the bridge which is occasioned by the inequality of *R* and *X*.

If we dispose the transmitter and the receiver as in Fig. 4, allowing any sound from the latter to actuate the transmitter, then we may obtain what is termed a "howling set". The mere jarring of the carbon button due to bringing the instruments into juxtaposition will result in a current from the transmitter and hence, if *R* and *X* are not equal, in some current in the receiver. The consequent motion of the receiver diaphragm causes a new disturbance of the carbon of the transmitter and this in turn causes a new impulse of current through the receiver itself. If the combination of receiver and transmitter is sufficiently sensitive this sequence of events may perpetuate itself and result in a steady tone, the frequency of which is determined by the electrical and mechanical characteristics of the system. The energy for such steady vibrations of the diaphragm is, of course, obtained from the battery associated with the transmitter. (For purposes of simplicity the battery is not shown in Figs. 3 and 4. It may be either a battery associated locally with the transmitter or one at the distant central office.)

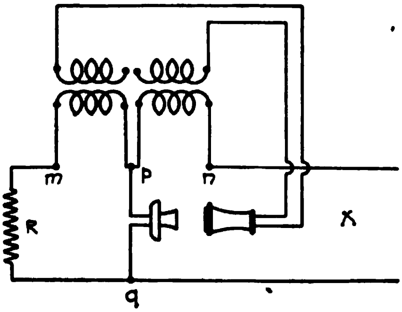


Fig. 4

From this illustration of the "howling set" the importance of line balance may be seen, for the combination of receiver and transmitter constitutes a repeater element. The element may more conveniently be symbolized as in Fig. 5 where the resistance, *R* of Fig. 4, has been replaced by a telephone line. In this last form of the bridge we may recognize the three factors mentioned above. The repeating element has input terminals *1, 2* and output terminals *3, 4*. It is associated with the lines on either side by the bridge circuit *m n q q* in such a manner as to repeat telephone conversations in either direction. Thus voice currents originating at a station on the east line are impressed on the terminals *1, 2* of the element through the action of the transformer. The resulting output from terminals *3, 4* is then transmitted equally to the east, where it is received as side tone by the speaker, and to the west, where it is usefully received by the distant auditor. Further details of this operation will be considered later. We may, however, note that if the unbalance between the lines east and west exceeds a certain magnitude, which depends upon the sensitiveness of the element, then the operation is that of the howling set. Under these conditions the repeater is said to "sing," for it will transmit in both directions a strong steady tone which renders telephonic communication impossible.

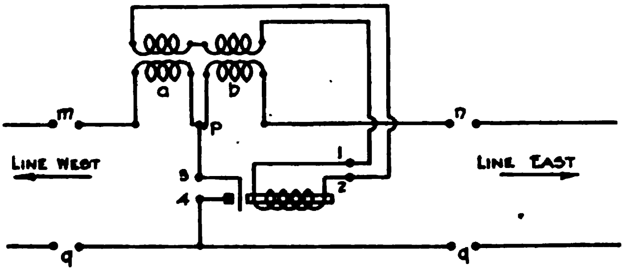


FIG. 5.

The difficulty introduced by this possibility of singing was not serious in the early days of the development of telephone repeaters and only became so with the invention of more sensitive elements. In fact in the very early days the difficulty was to obtain an element of sufficient sensitiveness. What sort of conditions must be met may be illustrated by the fact that the current at the transmitting terminal of a long line is only a matter of 2 or 3 milliamperes. In transmission over the line the current is constantly attenuated so that at the receiving terminal it may be only 3 or 4 hundredths of its original magnitude. Such small currents required greater precision of design than was appreciated by many of the early inventors, although as early as 1888 one of the Bell engineers, E. H. Lyons, had constructed a repeater which would amplify such small currents.

THE REPEATER ELEMENT

Before considering further these early designs we may well list the operating requirements for a successful repeater element. In so doing we shall not be concerned with the physical principles upon which the device operates. To all present intents and purposes we are concerned only with pairs of terminals *1, 2* and *3, 4* of an enclosed and unknown electrical network as illustrated in Fig. 6. If we subject such a hidden element to electrical measurement with alternating current we may obtain, for each frequency which we use, several characteristic ratios. Thus, from the ratio of the voltage impressed upon terminals *1, 2* to the resulting current we obtain the impedance of that circuit and similarly for the circuit of terminals *3, 4*.

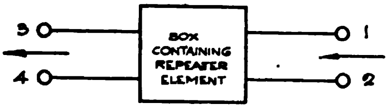


FIG. 6.

We obtain also a mutual impedance by taking the ratio of the voltage produced in the circuit *3, 4* to the current in the circuit *1, 2*. Similarly, there may be measured the mutual impedance in the opposite direction. In case one only of the mutual impedances is zero, that is, that no voltage is produced on one side when a current is flowing in the other, we speak of the device as having a "unilateral mutual impedance." Of the repeaters which we have found best adapted to telephony all are of this unilateral type. The requirements which must be met are identical whether the device is bilateral or unilateral but they may be more conveniently stated in terms of the unilateral type, of which the receiver-transmitter (or mechanical form) shown in Figs. 4 and 5 is a simple illustration. These requirements are:

1. The unilateral mutual impedance must be independent of the amplitude of the impressed current, within the range required for telephone transmission. In other words, there must be a linear relation between the output voltage and the input

current of the repeater, in which case the output is directly proportional to the input. If this condition is not met there will appear in the output one or more harmonics of the impressed alternating voltage.

1a. The input and output impedances of the element must be similarly independent of the impressed voltage or else distortion will occur.

2. The unilateral mutual impedance must be independent of the frequency of the impressed voltage within the range required for the transmission of the human voice. If this condition is not met the various overtones in such a complex sinusoidal current as is initiated by a voice will not be faithfully reproduced in their proper proportions.

2a. The input impedance of the element should be independent of the frequency of the impressed voltage or else it should be the same function of the frequency as is the impedance of the telephone line to which it is to be connected. If this condition is not met a distortion, destructive of good quality, may be introduced. In this case the voltage actuating the repeater is not that which the telephone line would impress upon another similar line but is greater or less by a factor which depends upon the frequency. It is not sufficient that the element repeat accurately its input but it is also requisite that its input circuit receive from the telephone line without distortion.

2b. The output impedance of the element should be similarly independent of the frequency or else properly related to the impedance of a telephone line so that the line may receive without distortion the output of the repeater.

3. The element must give amplification; that is, ignoring the energy sources which may be associated with or be parts of the element, the efficiency as measured by the ratio of output and input energies of telephone currents must be greater than unity. For example in the case of the mechanical element, which was pictured above, the energy is supplied by the battery associated with the carbon button. We are concerned at present only with its efficiency for transferring telephone variations from terminals *1*, *2* to terminals *3*, *4*. How much greater than 100 per cent this efficiency must be will depend, obviously, upon costs and other engineering factors. A repeating element adaptable to our telephone plant may well be expected to give an energy amplification such that telephone currents, which

have been attenuated by about twenty miles of standard cable, will be restored to their original values by its action. In telephone parlance "the element should give a gain of about twenty miles." Since by transmission through twenty miles of standard cable both the current and voltage would be attenuated to about one-eighth, the energy delivered to the repeater circuit would be only one sixty-fourth of the original energy. Of this only one-half reaches the receiving element for the rest is dissipated in the bridge. Hence there must be supplied to the line on the output side of the repeater circuit 128 times as much energy as is received. A glance at the circuit of Fig. 5 shows, however, that the output of the repeater itself is equally supplied to the outgoing and the incoming lines. The element must be capable, therefore, of supplying 256 times as much energy as it receives. In other words, the element should have a telephone efficiency of about twenty-six thousand per cent.

4. The sources of energy associated with the element, and by virtue of which amplification is possible, must have voltages which may safely and conveniently be utilized in the telephone plant. For example, a repeater whose source of energy had a voltage of greater than 250 would be unsatisfactory in a plant which normally employs voltages not greatly exceeding 100.

5. The element must be either insensitive to external electrical disturbances or easily shielded from them so that cross-talk from neighboring telephone circuits or apparatus may not actuate it.

5a. Conversely, the element must not be such as to cross-talk into adjacent circuits or apparatus.

6. The element must be constant in behavior, reliable and of fair life, and not such as to require frequent attention or laboratory conditions of operation. In other words the element must meet the conditions as to reliability of service which apply to other portions of the telephone plant.

7. In size, first cost, cost of energy and of maintenance, the element must conform to the economic conditions of an established plant.

Recognition of these conditions has guided our search for new elements during the past fruitful decade. Even before that time these requirements were recognized although they were not completely formulated. It is surprising how many independent inventors approached us during this time with devices formed by coils and magnets but involving no source of energy, or at least utilizing none, and hence patently incapable of producing amplification. Inherently such devices were static transformers, on the low voltage side of which there might be a greater current but obviously not a greater power than there was expended as input on the high voltage side. In some instances such devices were said to be intended for frequent use along a telephone line at fairly regular intervals. Such effect in improving telephone transmission as they might have would then be explainable by their function as inductances in the line; in other words, by the loading of the line. They thus offered imperfect substitutes for Dr. Pupin's method of loading.

Most of the earlier attempts at the design of telephone repeaters followed the recognized lines of telegraph practise or of electrical power engineering. The life of the telephone, however, has been almost contemporaneous with that of the newer physics which deals with discharges through rarefied gases and which has led to the discovery of the electron and of steady electron streams in vacua. General knowledge of the electronic structure of matter has grown by leaps and bounds in the last few years and coincident with this growth has been our practical development of electronic repeaters.

For purposes of classification, therefore, we may divide repeating elements into three groups depending upon whether their moving parts consist of (a) molecular ions, (b) electrons, or (c) molecular aggregates, that is, ponderable matter in the ordinary sense. The first group we might call "gaseous", the second "electronic," and the third, "electrodynamic." We may also classify according to the means by which the moving parts are actuated, as (a) electromagnetic if the operation is dependent upon a magnetic field which is altered by the received current, and (b) electrostatic if operation is dependent upon an electrical field which is established or altered by the received current.

Electrostatic devices involving moving members of mechanical size are too insensitive to serve as telephone repeaters. In the case of gaseous or electronic devices such a method of control is, however, entirely practicable and, indeed, for electronic systems marvelously effective. Our system of classification may, therefore, be simplified by the omission of one class of mechanical devices. It must, on the other hand, be further complicated by the introduction under gaseous repeaters of a type of device which has no cognate in the other two classes. Such devices are conductors which depend upon the passage of current through themselves to alter their own characteristics. Something of this behavior is familar to all engineers who have worked with carbon arcs and remember their instability. For our purpose we may designate this type of repeater as a "negative resistance device" and postpone the justification of the terminology to later discussion.

The classification of repeater elements in the order in which they will be discussed is, then, as follows:

I. Electrodynamic repeaters
    A. Receiver transmitter type
    B. Generator type
        1. Direct-current generator
        2. Alternating-current generator
        3. Asynchronous generator
II. Electronic Repeaters
    A. Electrostatic
    B. Electromagnetic
III. Gaseous Repeaters
    A. Electrostatic
    B. Electromagnetic
    C. Negative Resistance

I–A. The form of telephone repeater which most naturally suggested itself to the early engineers is the combination of a receiver and a microphone essentially in the manner depicted in connection with our earlier discussion of the problem presented to telephone engineers by the necessity of increasing the range of telephonic communication. In the early years of telephony no other means seemed available and prior to 1903 many unsuccessful attempts were made along this line. The parent Bell organization devoted the efforts of a large number of its best men to this problem, which was also a favorite with inventors outside of its organization. In the minds of the general scientific world further interest had been stimulated by the reported offer of a capitalist to pay one million dollars for a successful invention.

The solution was finally reached by one of the Bell engineers, H. E. Shreeve, who was assigned to the problem in 1903 after an earlier experience in the design of microphones and other telephone devices. Previous inventors had combined receivers and transmitters without recognizing that the normal functions of their diaphragms were those of sound collecting or emitting members and Shreeve omitted these unnecessary features.

Another difficulty in earlier designs had been the "packing" of the carbon of the microphone and one of his first studies had to do with its cause and elimination. He found it to be due to expansion which was occasioned by the heat liberated in the carbon chamber. In his first successful laboratory model a stretched steel strip was used as a connecting link between the receiver and the microphone and the microphone was designed so that the expansion of its parts under the influence of heat did not subject the granular carbon between the electrodes to increased pressure.

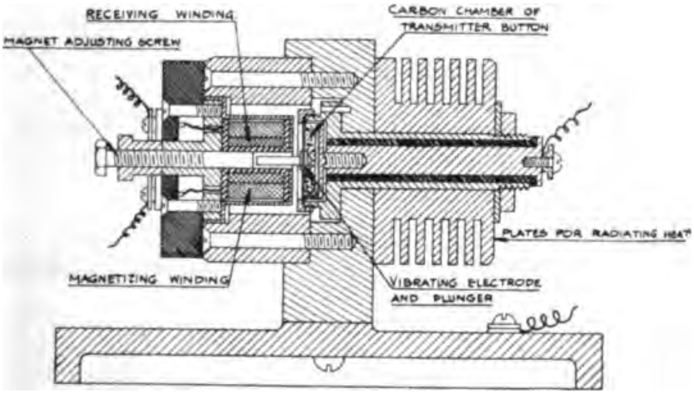


FIG. 7

With this form was obtained our first successful test of repeater operation over a telephone line. The device was tried in 1904 on a circuit between Amesbury, Massachusetts, and Boston and resulted in a greater intelligibility in the transmission of long lists of unrelated words than could be obtained over the circuit without the repeater.

The next step was to obtain more perfect quality reproduction and this was secured by making lighter moving parts and increasing the natural frequency of the moving system. The next model, shown in Fig. 7, was commercially operated on a circuit between New York and Chicago, from August 1904 to February, 1905. In this model the packing of the transmitter element was obviated by the thermostatic action of the vibrating disk, and the natural period of the moving parts was such as to emphasize the important frequencies of the voice. The element was associated with the line by a simple bridge circuit which had been patented by Edison in 1883. The transmission between New York and Chicago was found to be greatly improved.

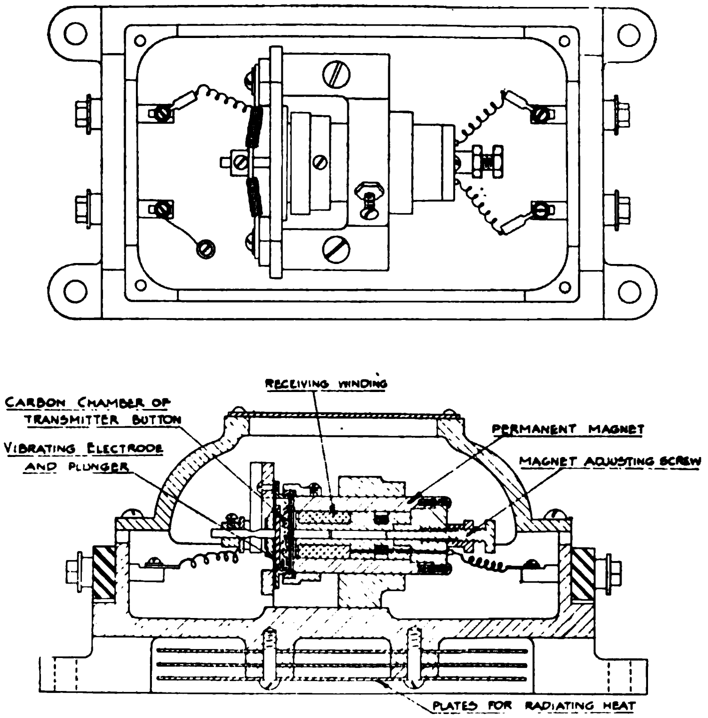


FIG 8

In this installation an additional feature was incorporated to prevent amplification of the so-called "Morse thump," which occurs when a repeatered line is composited and carries superimposed telegraph signals. A repeating coil of low mutual inductance was inserted between the line and the receiving element. This coil was inefficient for currents of low frequency but was relatively efficient for those of voice frequency.

In spite of the success which was obtained with this first repeater installation, the problem was far from being solved. In fact, when an attempt was made to reproduce the model, it was found extremely difficult to obtain vibrating electrodes, which had the necessary thermostatic action in addition to other necessary characteristics. An attempt was, therefore, made to regulate the sensitiveness of the microphone by more positive means and the next model, Fig. 8, had the feature of a thermostat in the form of a zinc strip. This zinc strip was heated by a coil, which was in series with the microphone, and whenever the current through the microphone increased as the result of incipient packing it served to withdraw the rear electrode.

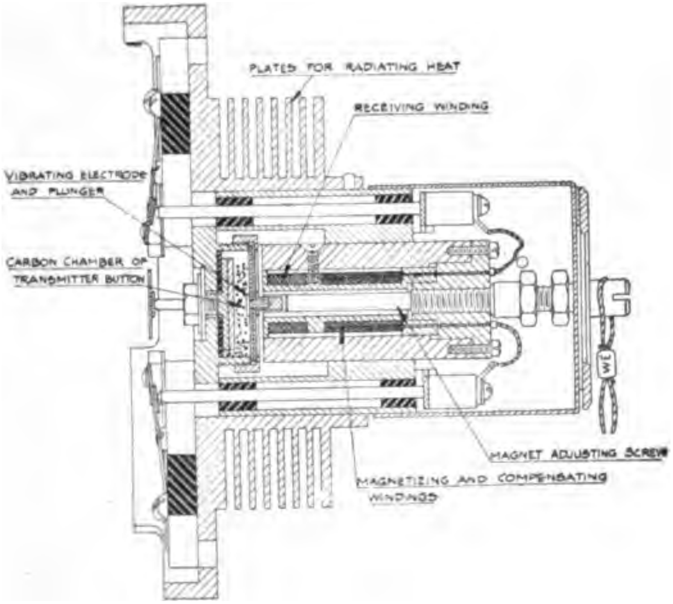


FIG. 9

This repeater, known as No. I-A, was reasonably satisfactory in service until the requirements became more exacting, particularly in the matter of tandem operation with the attendant requirements of improved quality reproduction. An intensive study was therefore undertaken, with the result that the natural frequency of the vibrating system was raised still higher, the weight of the moving parts was still further reduced and a still more sensitive form of regulator adopted.

Without going into two much detail in regard to intermediate steps which were made between the 1-A repeater and the so-called 3-A type, which is the present standard for instruments of this type, it will be sufficient to describe the features of this latest model and to point out the inherent limitations of this class of repeater.

The 3-A repeater, Fig. 9, is of the so-called "cartridge" type and consists of two main parts, the cartridge and the socket. The cartridge contains all of the working parts which are liable to become defective in service and may readily be removed from circuit for the purpose of replacement. It is held in place in the socket by a bayonet catch, and electrical connections are made to its contact posts by springs in the socket. The regulating means which serve to maintain the sensitiveness of the microphone between extremely narrow limits consists of an electro-magnet with differential windings. The initial magnetic field of the receiving element is the resultant field produced by a winding in series with the microphone and an opposing winding in shunt with the microphone. If the steady resistance of the microphone increases, the series winding takes less current and the shunt winding more, or vice versa. These magnetic changes alter the pull on the flexible disk electrode and thus serve to regulate the pressure on the granular carbon.

An inherent defect in this type of instrument is the fact that the sensitiveness falls off rapidly when the input energy is below a certain minimum. At the present time no completely satisfactory explanation for this phenomenon has been advanced. A defect of this character is obviously serious in the operating plant where circuit conditions and volumes of energy at any given point in a circuit fluctuate widely from time to time. Thus at one moment the repeater in a given circuit may be handling the energy from two persons speaking loudly, using telephones very near the terminals of the circuit, while at the next moment it may be called upon to function with the energy which it receives from telephones connected to long circuits, which in turn connect to the circuit with which it is associated. If the persons using the telephones in this latter case have weak voices, if they do not use the telephones properly by talking directly into the transmitter, or if the apparatus

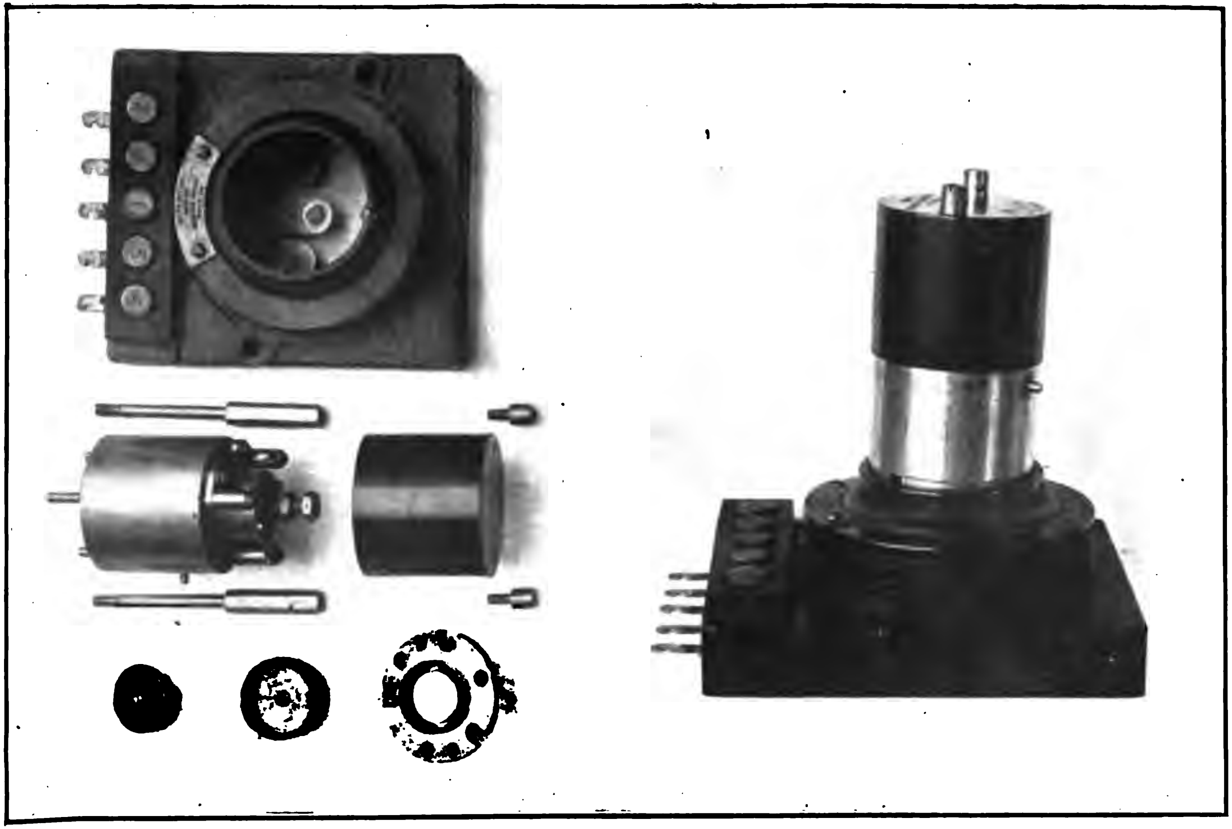


FIG. 9—MECHANICAL TYPE TELEPHONE REPEATER ELEMENT, SHOWING SOCKET, CARTRIDGE AND PRINCIPAL PARTS

GHERARDI AND JEWETT]

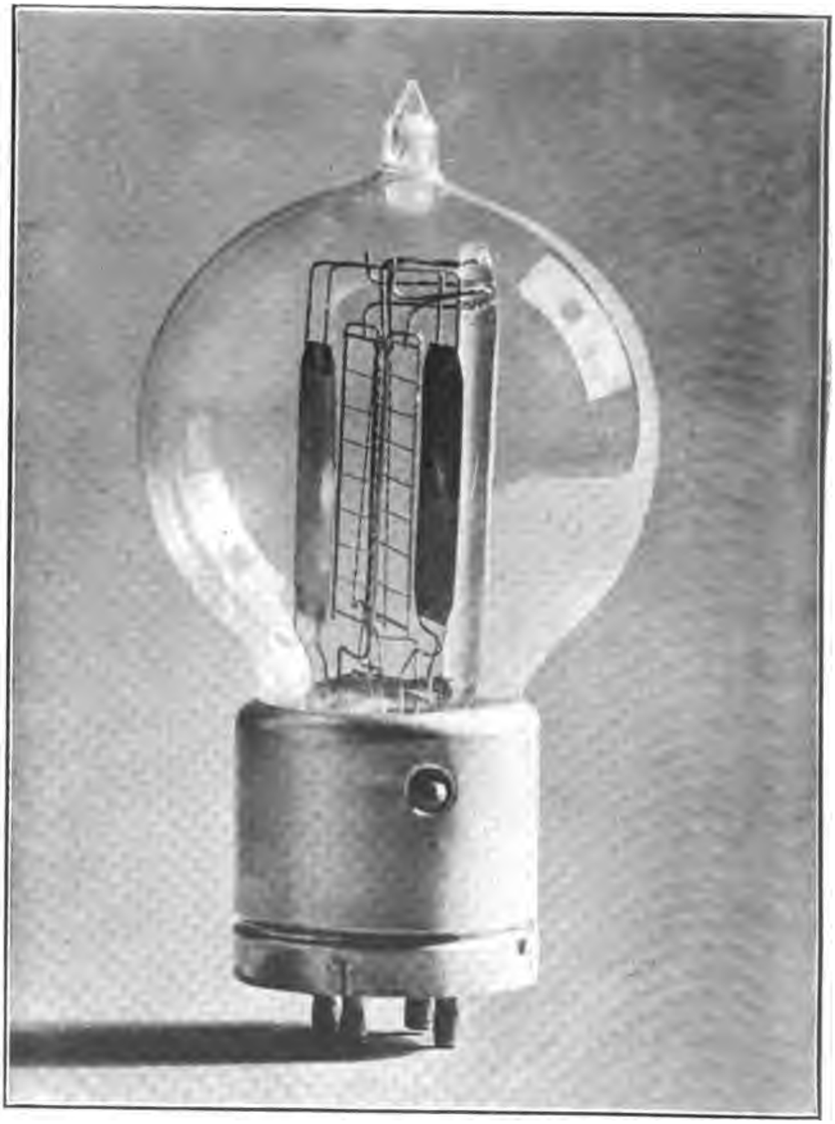


FIG. 13 [GHERARDI AND JEWETT]

or lines themselves are inefficient, it is clear that the energy at the repeater terminals may be many times less than that in the previous case. Further under those conditions maximum amplification is the more to be desired so that a repeater element whose performance fluctuates with the amount of energy which it receives and particularly one whose performance is worse when operating with the smallest energy is subject to a handicap when it comes to applying it generally in the telephone plant.

The Shreeve type of mechanical repeater is nevertheless distinctly serviceable and many units are in commercial service. The amplication is not, however, sufficiently independent either of the amplitude or of the frequency of the input to make them adaptable with best results to use in tandem, although they have been used to advantage on telephone lines the length of which did not require more than three repeater stations.

I–B. The application of the principles involved in direct- and alternating-current generators to the solution of the telephone repeater problem received the attention of some of the best minds in the art during the decade following 1900. During this time the problem was attacked both theoretically and in experimental design by H. S. Warren, M. I. Pupin, and others. The practise of power engineers suggested two typical methods.

(1) A generator of which the armature should constitute the output circuit of the element and the field winding the input circuit. Variations in the field current will result in corresponding variations of the armature voltage of such a machine. In order, however, that there should not be present in the armature voltage variations due to commutation, which would result in extraneous noise in the telephone, it is necessary that such a machine have a high frequency of commutation. If this frequency is well above that of the highest frequency which is essential to the faithful reproduction of the human voice, the noise currents due to commutation may be choked out by inductance or filtered out by a periodic structure like an artificial loaded line. When it is said that frequencies of 2500 cycles per second are required for good quality of telephone speech it is evident that for any practical design of armature and commutation the peripheral speed must approach the allowable mechanical limit.

Various unipolar designs were therefore considered in which no commutation is required. The principle involved may be seen by considering Fig. 10 which shows a form suggested in 1905 by G. A. Campbell. The field coil *M* is solenoidal and carries fixed disks, *w'*, which serve both as a magnetic core and as collectors for the radial e. m. f's. induced in similar disks, *w*, attached to the rotating shaft. In machines of this character where the field current is only of telephonic magnitude the flux is very small and the e. m. f. which may be obtained at any mechanically allowable speed of rotation is much below that requisite for a successful telephone repeater. In addition the eddy current and hysteresis losses are relatively high because of the frequencies involved and the efficiency is correspondingly low. The input impedance, being that of an electromagnet, is not independent of the frequency, as is desired, but instead is such as to discriminate against the higher frequencies of the voice wave.

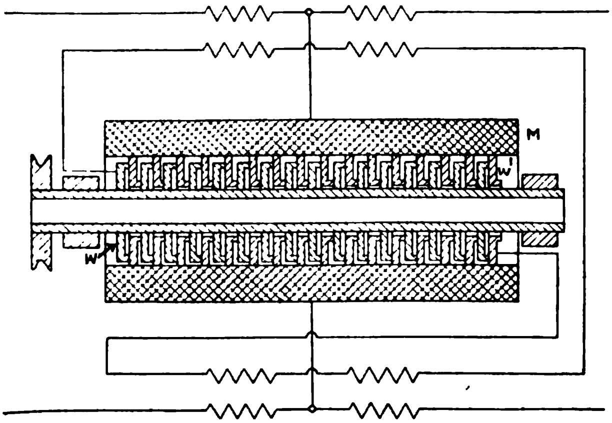


FIG. 10

Even if it were possible to build and operate such high-speed generators as repeaters of feeble currents, the type is deficient in so many of the characteristics required of a successful commercial repeater that the extent of its use is questionable. Now that simpler and practically perfect repeaters are available, it is doubtful whether the limits of physical possibility in this type of apparatus will ever be worked out.

(2) The practical difficulties of eddy current and hysteresis

losses, of limiting speeds and of small input, were also effective in preventing the development of a successful repeater along the lines previously followed in the power engineering of alternating currents.

(3) In 1900, Joseph Lyons disclosed to us his invention, a telephone repeater upon the principle developed and patented in power engineering by Hutin and Leblanc. The principle is perhaps better known as that of the asynchronous generator, which is an induction motor driven above synchronism by a local source of mechanical energy. The conditions under which such a device may serve to supply energy to a transmission line, across which it is connected, are well known to electrical engineers. It was Lyons' idea that the stator should be bridged across a telephone line and, through the interaction of the received current with that induced in the short-circuited rotor, should supply to the line additional energy of the same wave form as it received. Schematically the circuit is as shown in Fig. 11.

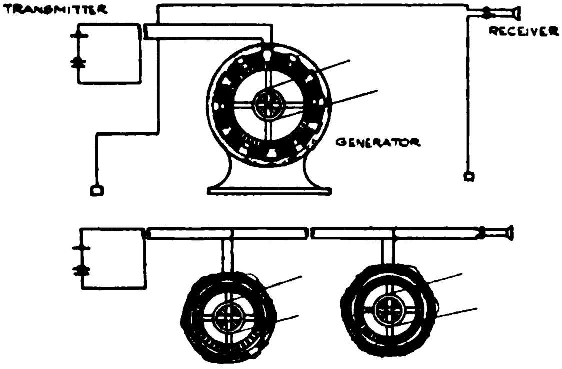


FIG. 11

In order, however, to supply energy at any telephone frequency the speed of the rotor must be at least as large as the quotient of the given frequency and the number of pairs of poles in the stator. Thus in the case of a 12-pole stator a frequency of 2400 cycles per second would require a speed of 400 revolutions per second. Not only are the speed requirements too high for practical operation but the machine would also fail to meet the second requirement for a repeater. The amplification for each frequency would depend upon the negative slip at that frequency and thus the lower frequencies of the voice current would receive more amplification than the higher ones. It was at one time suggested by Dr. Pupin that such distortion might be avoided by using several such asynchronous generators connected at different points of the line and each designed to amplify but a band of frequencies. Even had it been found possible to devise efficient machines to amplify the different groups of frequencies, it is doubtful if the arrangement would ever have proven satisfactory in an operating telephone plant, since at no point along the line between the first and last generator would the current have represented normal speech. This characteristic of the system would, of course, be a serious handicap to proper operation, since it would be almost impossible to tell whether the line were in trouble or not.

That the mechanical limitations of speed, clearance, thickness of lamination, and the other factors familiar to designers of dynamo equipment should have prevented the development of such a generator is understandable when the telephone requirements are stated in terms of power engineering. What was required was a generator which could operate on about one-half of a milliampere of field current. To give an energy amplification of 256 times, its field losses must be less than one per cent of the output. The value of one-half of a milliampere is the effective value of a complex wave form, all the sinusoidal components of which must appear in the output with equal amplification.

II. Electronic repeaters are those in which the moving parts are discrete electrons whose motions are unimpeded by molecular matter. Such devices, therefore, require a vacuum sufficiently high that the motions may be characteristic of free electrons. There must also be a means of supplying free electrons, that is of dislodging them from molecular matter of which they form constituents. The satisfactory method of accomplishing this is to "boil them out" of a piece of metal or metal oxide. Our present electronic repeaters are therefore "thermionic" devices.

The history of thermionics may be said to date from the publication in 1902–1903 of papers by O. W. Richardson in which he advanced a theory of the thermionic emission of electrons, which in all its essentials was accepted by the great majority of the scientific world and may now be regarded as definitely established. According to this theory (which is also a theory of metallic conduction) metals and conductors of electricity contain within their bounding surfaces immense numbers of free electrons which behave like the molecules of a perfect gas. When a metal is heated the kinetic energy of the electrons is increased and the number escaping across the boundary surfaces is likewise increased.

J. J. Thompson outlined the theory in his treatise on conduction of electricity through gases as follows:

"The emission of corpuscles from incandescent metals and carbon is readily explained by the view—for which we find confirmation in many other phenomena—that corpuscles are disseminated through metals and carbon not merely when these are incandescent, but at all temperatures, the corpuscles being so small are able to move freely through the metal and they may thus be supposed to behave like a perfect gas contained in a volume equal to that of the metal. The corpuscles are attracted by the metal so that to enable them to escape into the space surrounding it they must have sufficient kinetic energy to carry them through the layer at its surface where its attraction of the corpuscle is appreciable. If the average kinetic energy of a corpuscle, like that of the molecule of a gas is proportional to the absolute temperature, then, as the temperature increases, more and more of the corpuscles will be able to escape from the metal into the air outside."

In 1904 Wehnelt discovered that the thermionic emission from conductors coated with certain oxides was enormously greater than that of pure metals. Since that date the study of thermionic emission has proceeded most rapidly and the practical applications of the phenomena have more than kept pace during the last eight years.

- Phenomena due to thermionic emission had, of course, been observed for years before a satisfactory theory was advanced. The first of these instances was the discovery by Edison in 1884 of an effect which is now known by his name. He found that if a metallic plate is introduced into the bulb of an incandescent lamp and if the plate is connected to the positive terminal of the filament then a current flows between the plate and the filament as may be observed by a galvanometer inserted as shown in Fig. 12. This conductivity of the space between the plate and filament is unilateral, for if the plate is connected to the other terminal of the battery no current flows. The explanation of the phenomenon resulted from work of Fleming in 1896 and J. J. Thomson in 1899 along lines earlier pursued by Elster and Geitel. Fleming studied the operation of a two-member vacuum tube device having a heated filament and cold plate, and Thomson demonstrated that the mechanism of current conduction between the filament and plate resulted from the emission of negative electrons from the filament.

Through such a tube a current may flow, since electrons may pass and will so do provided the plate is positive with respect to the filament and hence such as to attract the electrons emitted by the latter. The idea of using such a two-element device as a rectifier and hence detector of wireless waves was applied independently in 1905 by Wehnelt and by Fleming.

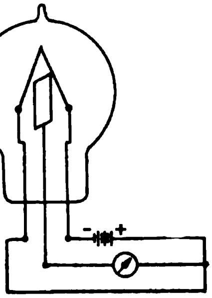


FIG. 12

For the next few years practical progress with thermionic devices was in the hands of the radio-engineers. In his American Institute of Electrical Engineers' paper of 1906 DeForest showed that the discharge between the hot cathode and the plate can be controlled by additional electrodes. In a patent issued the following year he shows inside the bulb such a controlling electrode to which he had given the form of a grid. With his discovery of a means for controlling the electron stream was born the most sensitive of repeating elements. Years of careful technical development were required, however, before the original idea had been perfected to the point where it met fully all the rigid requirements of application in a commercial telephone plant.

To understand the development which was required to adapt the first successful form of audion designed for radio-telegraph receiving to this use, we shall first consider in some detail the structure and physical principles of the device.

The "audion" consists of an evacuated vessel containing three electrodes, from one of which a thermionic emission of electrons is obtainable. Fig. 13 shows the characteristic structure. The filament is heated by the passage of a current as shown diagrammatically in Fig. 14. The other two electrodes are a plate and a grid. If a battery is connected to the filament and plate as in Fig. 14, so as to make the latter positive with respect to the former, then a current will flow in the circuit so formed. The electrons emitted at the filament are drawn across the intervening vacuum by the electrical field which the battery, *B*; in the plate circuit establishes. If an e. m. f. is now applied between the grid and the filament as by the source marked *V* in the figure, the field between plate and filament is altered and the current in the plate circuit is correspondingly altered. If the grid is made positive with respect to the filament more electrons are urged across the space between grid and filament. While some of this increase strikes the grid, and thus results in a current in that circuit, by far the greater number continue through the meshes of the grid to the plate. The result is an increased current in the plate circuit. Conversely, if the grid is made negative there results a decrease in the plate current. In this case, however, no current flows in the grid circuit because electrons can be drawn to an electrode only if it is positive with respect to the source of the electrons.

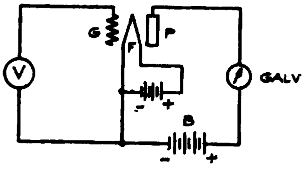


FIG. 14

The characteristic relation between the grid voltage, *V* and the plate current I_B, is that of Fig. 15. If the plate voltage is altered the form of the curve is not altered but the magnitude of the current is changed as illustrated in Fig. 16 which shows a family of such characteristics. It is evident from this figure that the number of negative volts which must be applied to the grid in order to reduce the plate current to zero is always the same fraction of the volts applied in the plate circuit.

Hence it appears that the current in the plate circuit may be altered either by altering the voltage there applied or by a much smaller alteration of the voltage applied to the grid circuit. The device thus gives a voltage amplification.

As long as the grid is kept negative no current can flow in that circuit and any alterations in its voltage are unaccompanied by any current variation and hence are entirely wattless. Such variations are accompanied by current variations in the plate circuit and result in an energy expenditure in that circuit. The telephone efficiency is thus seen to be practically infinite since an energy output may be obtained by a wattless variation of the input voltage.

To understand why the characteristic curve of Fig. 15 flattens out as the grid is made more positive we need only to remember that in the passage of a current through the plate-filament circuit two physical phenomena are involved. The first of these is the thermionic emission of electrons at the filament and the second is their enforced and directed transit between filament and plate. The emission takes place whether or not a voltage is impressed on the plate-filament circuit, and depends only upon the filament temperature. The process is much like evaporation in a closed vessel and soon reaches a statistical equilibrium where electrons are leaving and return to the filament at the same rate, that is number per second. Impressing a positive potential on the plate disturbs the equilibrium for it withdraws electrons from the tube. When this voltage is small it withdraws but a relatively small number per second; that is the current is small as in the lower portion of the characteristic. As the voltage is increased the current

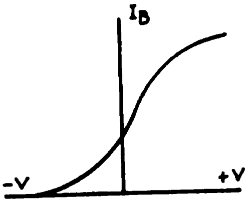


FIG. 15.

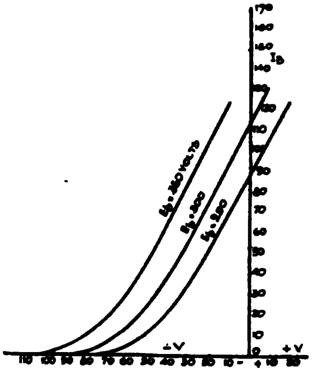


Fig. 16

increases but reaches a steady maximum value for a voltage sufficient to draw across the interspace as many electrons per second as the filament can emit at the operating temperature.

Although the number of electrons per second which pass between filament and plate and hence the current is so limited, the velocities with which the individual electrons move will continue to increase with the voltage. If there is a residual gas in the tube its molecules will be ionized by their collisions with the electrons provided the velocities of the latter are sufficiently high to disrupt the electronic systems of which these molecules are formed. Such ionization, however, increases the number of electrical carriers and hence the charge per second which may be transferred. The presence of gas in a tube is in general, therefore, evidenced by such a characteristic as the full line of Fig. 17.

The voltages involved in the detection of radio signals are much smaller than those met with in telephony so that no serious inconvenience arises from the presence of residual gas. As a consequence, in the production of his early successful audions for radio reception DeForest did not find it necessary to employ all of the known means for producing the highest possible vacuum.

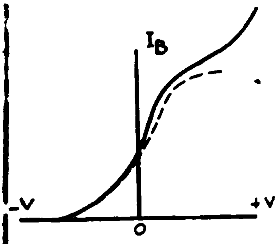


FIG. 17

In dealing with the very much larger energies which obtain under the conditions of telephonic repeater action, it is necessary to adopt methods of manufacture which will insure the removal to a high degree of all gaseous material in the tube, including that which may be occluded in the walls of the tube or the enclosed metallic parts. When such evacuation is accomplished all of the advantages of a pure electron discharge are retained, even for the higher voltages and currents which have to be dealt with in a telephone repeater. In other words, for a given plate voltage there is obtained a wider range of voltage which may be impressed upon the grid without departing from the true thermionic character displayed in Fig. 15.

In addition to securing the necessary conditions in this respect, a large amount of research work has been done by the engineers of the Bell System in producing suitable forms and proportions of tubes to give the best results with the various types of repeater circuits and in devising a type of filament which will insure a maximum of life and a minimum of power consumption for the amplifying tube and at the same time insure a uniform operating characteristic throughout the life of the tube. Further than this, the work has resulted in ability to produce in quantity tubes of exactly the same characteristics, a feature which is of the utmost importance in telephone repeater operation but which is of minor importance in radio work. Non-uniformity in the tubes would very seriously handicap the commercial operation of this type of amplifier since it would require the readjustment of apparatus each time it might become necessary to replace a worn-out or damaged tube.

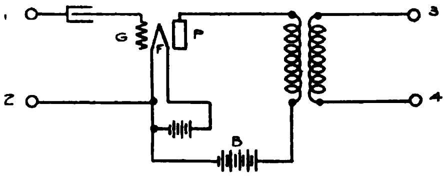


FIG. 18

Not only was the original form of audion deficient as a telephone repeater from the standpoint of its energy capabilities, but the conditions of its use in radio reception were inherently different from those which obtained in a telephone repeater circuit. We may express this fact by saying that for radio detection we make use of the ability of a device to distort a sinusoid, which is impressed upon it, while for telephonic purposes we require the very minimum of distortion.

Associated with the audion in DeForest's use was a condenser in the grid circuit as shown in Fig. 18. The input terminals were *1* and *2* of that figure and the output terminals were *3* and *4* of the repeating coil. If employed as the amplifier in a telephone repeater, the repeating coil serves to impress on the telephone line to which the device is connected the variations in the plate circuit current from the battery *B* which may be occasioned by any alterations in the potential applied to the input circuit. The condenser, however, serves as an electron trap if a sinusoidal e. m. f. is impressed on terminals *1*, *2*. Thus imagine that the first half wave of the sinusoid makes the grid negative with respect to the filament. This polarity tends to force electrons from the grid to the filament, but, since electrons are not emitted by the grid, no transfer takes place. Making the grid negative does, however, reduce the current through the plate circuit as we saw in connection with Fig. 15. Now, the succeeding half wave tends to force electrons in the opposite direction, that is from filament to grid, which is a possible direction of transfer in this unilateral circuit. The individual electrons so transferred to the grid cannot pass through the condenser and complete the journey back to the filament although others from the opposite plate of the condenser may be induced to make the rest of the journey. The result is that each positive half wave increases the number of electrons on the grid and its adjoining condenser plate and thus results in a further increase in negative potential. But the more negative the grid becomes the smaller is the current in the plate circuit and the lower the point on the characteristic curve about which we are producing sinusoidal variations.

Obviously, such an action is very satisfactory for detecting the presence in the input circuit of a train of sinusoidal waves of small amplitude since the effect is cumulative. Equally obviously, however, such action may absolutely prevent the passage through the system of a voice actuated train of waves since a word or two may make the grid so negative as to reduce practically to zero the plate current upon the variations of which the transmission of the wave depends.

The correction of the circuit arrangement by the elimination of this condenser, which aided distortion, was but a detail in the broader research which we have made on the application of thermionic devices. This work involved a study of appropriate circuits in which these tubes could be used, of thermionics, of methods of producing high vacua, of gages for indicating the degree of vacuum obtained, a study of various types of filament to secure efficiency and uniformity, and of the physical relations in three-element vacuum tubes of the geometrical form and separation of the electrodes. Some of these researches as for example those of Van der Bijl, W. Wilson, and others have already been published.

The net result of these researches has been our ability to design and construct tubes adapted to any of the purposes of the art of communication. In the final form in which we use a vacuum tube as a telephone repeater we obtain a characteristic relation between input voltage and output current which is free from distortion as far as we can recognize such effects. This is obtained partly by the design of the tube and partly by the proper adaption to it of its associated circuit.

In the first place we arrange that the repeater shall have a constant and finite input impedance by bridging between grid and filament a high resistance. To this resistance current is supplied by a step-up transformer as indicated in Fig. 19. The voltage effective in the grid-filament circuit is then the drop across the resistance. In the second place we generally arrange that the grid shall not be positive at any time in the cycle of the impressed voltage. This is accomplished by inserting a battery, *C*, in the grid circuit. In this way the tube acts as a device for amplifying the impressed voltage and introduces no distortion due to the unilateral conductivity of the grid circuit.

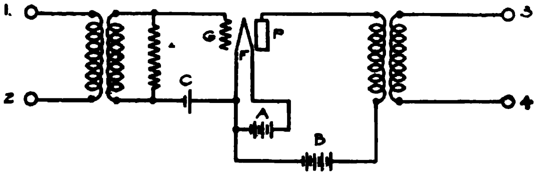


Fig. 19

The characteristic relation between grid voltage and plate current which is shown in the curves of Fig. 16 is obviously not that of a perfect repeater since there is not a linear relation between input voltage and output current. The voltages indicated on these curves are those of the battery in the plate circuit. The condition under which they were obtained is that of no external resistance in this battery circuit so that the voltages are really those which are effective between the plate and filament within the tube. The values of the current in these plots are due to the combined actions of this effective voltage and of the voltage impressed upon the grid circuit. For each of these curves the effective plate voltage is constant and the curve gives the relation between the input voltage and the output current. If, however, there is external resistance in the plate circuit, then the effective voltage between the plate and the filament will not be that of the battery, but will be less by the amount of the *IR* drop in the external circuit. An increase in the grid voltage will not, then produce as large an increase in plate current because such an increase as it would produce if the plate voltage was constant is partially neutralized by the decrease in the effective plate voltage which the *IR* drop occasions. The result is that by properly proportioning the external resistance of the plate-filament circuit we may obtain a characteristic relation between grid voltage and plate current which is essentially linear as shown by the full line curve of Fig. 20.

The requirements for a telephone repeater stated above are thus fully met in our construction of the audion and in the circuit arrangement for its operation. In later portions of this paper we shall meet illustrations of its perfection in the report of experiments and commercial operation.

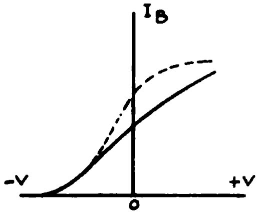


FIG. 20

In the case of the audion the control of the electron stream is electrostatic. It is possible, of course, to control such a stream by electromagnetic means since the stream is really a current. Such a control, is, however, inherently less efficient and sensitive than that of the voltage actuated device described. We shall, therefore, give no detailed discussion of such methods but will pass at once to the discussion of devices in which the presence of gas molecules results in an ionization and the consequent presence in the tube of positive carriers of molecular size as well as negative electrons.

III. The distinction between electronic and gaseous repeating elements is not a matter of the presence or absence of gas since an absolute vacuum is unobtainable. It is rather a matter of whether or not gas is an irrelevant and undesirable element or an indispensable one. In the vacuum tube of DeForest, gas was not indispensable as we have seen and such gas as was residual limited the range of voltages, throughout which the tube had the desired characteristics when used as a repeater.

III-A. In the repeating element of von Lieben which was invented in Europe somewhat later than the time the audion was produced here, gas is an essential constituent. This device has a Wehnelt cathode, that is a filament coated with oxides of the alkaline earths from which electrons are thermionically emitted. The function of the cathode is not, however, to provide all the carriers as in the case of the filament of the audion but rather to provide means for ionizing the gas present in the tube.

The tube is a three-element device and has in addition to the thermionic cathode, an anode of aluminum (corresponding to the plate of the audion), and an intermediate sieve-like electrode in the form of a perforated aluminum disk.

The function of the sieve is to control the discharge which takes place between the anode and the cathode, in case a potential is properly applied. The stream of electrons which proceeds from the cathode to the anode is impeded by the presence of gaseous molecules. The consequent collisions will result in the ionization of some of the gas molecules if the velocity is sufficiently high and this velocity depends upon the potential which is applied to the circuit. The ionization results in the presence of more electrons which are available for increasing the current and also in the presence of positive ions of molecular size which are similarly available but move in the opposite direction. The introduction of a potential between the filament and sieve permits of an external control of the motions of the electrons and positive ions, attracting the former and repelling the latter or vice versa.

Small changes in the potential applied to the sieve may result in relatively large changes in the ionization and hence in the current in the anode circuit. The device may, therefore, be used to give amplification but for reasons which will appear presently it is not adapted to telephonic work.

Since the tube contains carriers of both kinds a stream of one kind or the other will inevitably flow to the sieve and thus here will be a current in that circuit which will depend for its direction upon the polarity of the applied voltage and for its amount upon the conductivity of the gaseous space within the tube. If the cathode and sieve are used as the input circuit, corresponding to the filament-grid circuit of the audion, then it is evident that the impedance of this input circuit will vary with the impressed voltage. Such a variation will result in a serious distortion and is not in conformity with our requirements for a repeating element.

The presence of both kinds of carriers makes impossible the simple expedient which is so valuable in the case of the audion where the unilateral conductivity of the grid-filament circuit is utilized. Further the pressure of the gas within the tubes is subject to fluctuations which it is almost impossible to control. This results in large variation in amplification. In comparison with the electronic device the von Lieben tube is seriously deficient as a telephone repeater, and the electrostatic control of gaseous devices has not been found capable of as efficient and simple application to telephonic problems as is the electrostatic control of electronic devices.

III-B. The electromagnetic control of gaseous devices has likewise been investigated for telephonic purposes. The oppositely moving streams of positive and negative carriers will be deflected in the same direction by an electro-magnetic field. Provided that the motion of such a discharge stream may be made to alter properly the conductivity of either the circuit of the discharge itself or of some auxiliary circuit; a repeating element may be formed.

Some of the early work on such systems is that of Hewitt whose experiments on mercury vapor arcs are well known. The method which he proposed to employ may be seen from Fig. 21, where is shown a tube containing mercury vapor through which a current is passing from the battery *B*. A receiver is connected in series with the battery and the arc. The arc stream itself is controlled by the electromagnet which is excited by the transmitter. This arrangement, which is essentially a very insensitive unilateral repeating device, has never, so far as is known, been made to function as a telephone amplifier. In addition to its inefficiency, this arrangement is undesirable because of noise, distortion and variable amplification.

A later step in the development of an electromagnetically controlled mercury arc repeater was taken by Dr. H. D. Arnold. He utilized an auxiliary circuit for the output and so disposed it that its conductivity was altered in conformity to the motion of the arc stream. In the general idea of using a mercury arc stream and influencing it magnetically, Arnold of course, follows Hewitt.

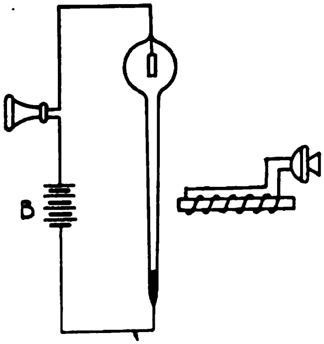


FIG. 21

The auxiliary circuit devised by Arnold is at right angles to the arc stream and includes three electrodes, one about the center of the stream and the other two approximately equally spaced on opposite sides as shown by a cross section of the tube in Fig. 22, the main arc stream being perpendicular to the plane of the diagram. The electromagnet is in the plane

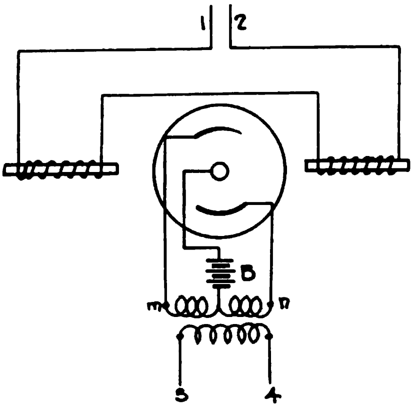


FIG. 22

of these electrodes and thus serves to deflect the main arc stream toward one or the other of the side electrodes depending upon the polarity of the magnet. The ions in the main arc stream are available carriers for current between these electrodes and the central electrode if a potential is maintained between them as by the battery, *B*.

In the normal condition, when the arc is undeflected, similar portions of the arc stream are included between the center electrode and each of the side electrodes. The conductivity of the two paths thus offered to the potential of the battery are then equal and there is no difference of potential between the terminals *m*, *n* of the coil, to the center tap of which the battery is connected. In so far as the conductivities of these two branches of this bridge circuit tend to vary as the result of variations in the ionizations of the arc stream no difference of potential will be produced between the terminals of the coil, since both conducting branches will tend to vary similarly due to a common cause. Fortuitous variations in the arc stream are, therefore, not repeated in the external circuit and the difficulty of arc noise is practically eliminated. When, however, the arc stream is deflected by the voice actuated electromagnet, the conductivities of the two branches are oppositely changed and a resultant current circulates.

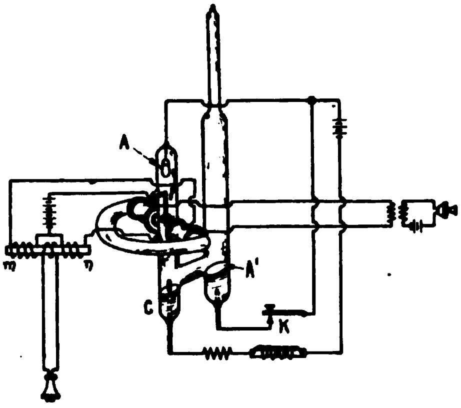


FIG. 23

The device as originally described for patent is shown in Fig. 23. An auxiliary anode, formed by a mercury electrode shown at *A'* is used for starting the arc stream. In this form the arc is struck by tilting the tube and allowing the mercury of the anode A' to run into contact with that of the main cathode C. As the tube is restored to its vertical position an arc is formed between A' and C, provided that the key K, has been closed. Opening the circuit at K allows the arc to form between C and A.

This gaseous device is capable of good amplification and is fairly free from distortion except such as may be introduced by the fact that impedance of the electromagnet may not be identical with that of a telephone line. It was tried out experimentally on telephone lines but has never been used for any length of time and then only under special engineering supervision. For example, units were installed on the important long distance circuits between New York and San Francisco but were never used commercially.

III-C. The third type of gaseous device which we have investigated as a possible repeater is the negative resistance type. A typical voltage-current characteristic for an arc stream is that of Fig. 24, where the ordinates give the impressed voltages and the abscissas the resulting current.

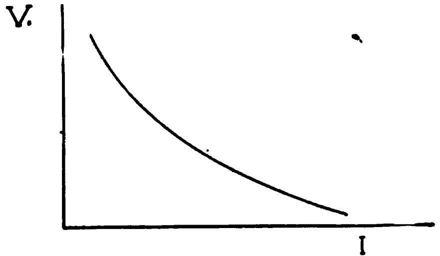


FIG. 24

For an ordinary resistance the slope of the relation of voltage and current is positive in the usual sense, since for any point on the curve dV/dI is positive, and represents a slope of less than 90 degrees measured counter-clockwise from the axis of I. For the arc, however, with its falling characteristic, the slope is negative. The resistance which the arc offers to a change in its condition is thus to be considered negative.

If an arc is connected in series with an ohmic resistance and a source of potential, as shown in Fig. 25, which is essentially that suggested at one time by Hewitt, who dealt with a mercury arc in an enclosed tube, the arc, owing to its falling characteristic, has the effect of making the total resistance in the circuit appear to be less when small current variations exist in the circuit than when steady conditions exist.

The reason for this is that any small increase in current is accompanied by a decrease in the voltage across the arc itself, thus leaving a still greater voltage across the ohmic resistance than would be obtained if the circuit consisted merely of elements obeying Ohm's law. If the source, *V*, is a transmitter and if there is in the circuit also a receiver, then it follows that the energy received by the latter may, under proper conditions, be greater than that delivered to the circuit by the transmitter. In other words there is a possible telephonic amplification.

The magnitude of the amplification with the Hewitt device obviously depends upon the relative magnitudes of the negative resistance and the external ohmic resistance. The external resistance may in part be that of a telephone line and for that reason this method has been considered as a means of producing telephonic amplification. The negative resistance of the arc can never do more than remove some of the effects of the external resistance. If it could do more, of course, the circuit would "sing." We have found in practise that the system can never increase the loudness of telephone speech unless the line resistance itself is sufficient to stabilize the arc. In practise the arc would have to be connected to various telephone lines, each having different characteristics, and in order that an arc which will amplify, when connected to a high resistance line, shall not "sing" when connected to a line of lower resistance, a certain amount of extra external resistance has to be added to the arc circuit, which resistance will be different for each different type

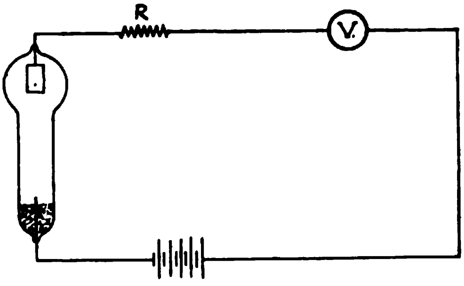


FIG. 25

of line. For practical telephony, therefore, this negative resistance device is not of service.

Even if the possible amplification were sufficient under actual conditions there would still remain the fact that the dynamic characteristic of the arc varies with the frequency of the impressed telephonic waves, giving prohibitive distortion and variable amplification. Further, in common with the other types of gaseous discharge apparatus which we have considered above, the negative resistance device would be too unsteady for practical use.

The two devices which we have found practicable under the exacting requirements of commercial telephone service are the receiver-transmitter element and the three electrode vacuum tube or audion. The electromagnetically controlled arc stream with auxiliary electrodes, which, as indicated above, has been used by us to a limited extent experimentally, and while possessing many of the characteristics of a commerical telephone repeater, is not in its present state of development adapted to general telephonic use. Of the three types, mechanical, electronic, and gaseous the electronic device with its thermionic emission is the most nearly perfect. In fact the amount by which it fails to meet all the requirements for a perfect repeater is so small as to be negligible except under the most rigorous conditions.

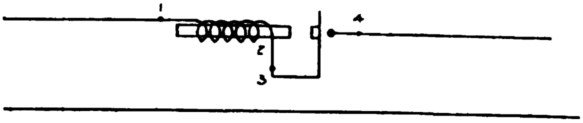


FIG. 26

The Repeater Circuit

The electrical network by means of which one or more repeater elements may be associated with the telephone lines is conveniently known as the "repeater circuit," as explained above. In two-way operation, such as telephony requires, there may be distinguished three general types of such circuits, of which the bridge or balanced circuit, described earlier in this paper, is the more usual type.

The other types do not depend upon balance for their successful operation. One of these is commonly called the "booster." Fig. 26 shows one form of series booster, in which the input and output of the element are connected in series with each other and with the line. A form of shunt booster is similarly indicated in Fig. 27. A large number of other forms of booster circuit are possible. Negative resistance devices are by their nature adapted to the booster scheme of connection.

In the case of booster circuits there is, for operation without singing, a limiting relationship between the amplification for which the element is adjusted and the impedance of the line in which it is connected. Part or all of the output currents from the element must flow through the input circuit of the element itself, and thus the amplification of the incoming telephonic current depends upon the "building up" of currents in the circuit by the interaction of input and output. It is obvious that if the lines do not offer a sufficient load such a circuit with ts element may act as an oscillation generator, that is, there may be singing.

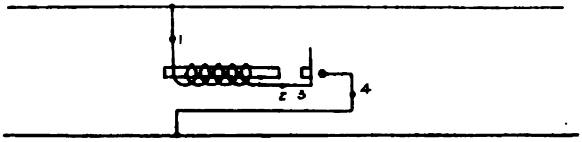


FIG. 27

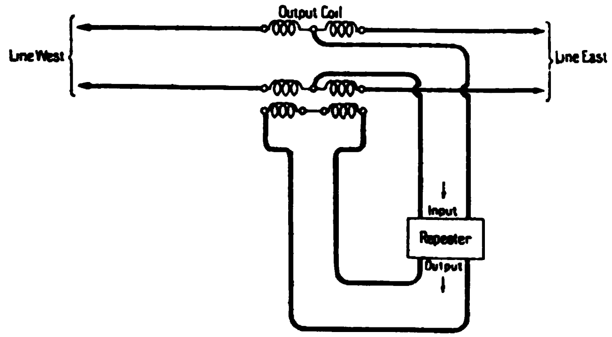


FIG. 28

The third type of repeater circuit is somewhat analogous to the single line repeater of telegraph practise. Incoming voice currents operate sensitive relays which switch the terminals of the element according to the direction from which the voice currents originate so that the circuit is equivalent at any instant to a one-way repeater circuit.

These types of circuit, which do not depend upon balance, have in general been found not so desirable for practical use as the bridge type of duplex circuit and will not be discussed in further detail.

Of the bridge type we may distinguish two classes depending upon whether a single repeater element is used to amplify transmission in the two directions or whether separate elements are used for the opposite directions. The typical circuit shown in Fig. 28 is obviously of the first class. Such circuits we may call "21-type," indicating two-way, one-element circuits. The other class, which we shall discuss a little later, is generally known as the "22-type", since it is a two-way, two-element circuit.

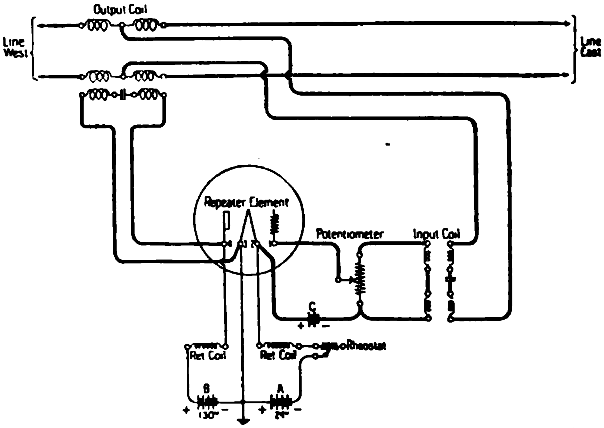


Fig. 29

The input and output terminals of the element in the 21 type circuit, Fig. 28, may obviously be interchanged without altering the physical operation of the circuit, in which case the element transmits to the two lines in parallel instead of in series. The choice between these two connections is influenced largely by the ease with which, through the medium of repeating coils, the input and output impedances of the element may be made similar to the impedances of the network to which they are connected.

A practical form of the 21 circuit is shown in Fig. 29 with its associated vacuum tube amplifier. The incoming energy is supplied through an input repeating coil, between which and the telephone lines is interposed a "filter," not shown in the figure. This filter, of which many satisfactory types have been invented by Dr. G. A. Campbell, is a physically periodic structure, consisting of similar networks in series, which is so designed as to discriminate against the passage of sinusoidal currents of frequencies above or below the necessary telephone range and thus prevents the element from receiving currents which arise from the simultaneous use of the metallic circuits of the telephone line for telegraph currents or other signal currents. The potentiometer is introduced to regulate the gain occasioned by the element. Retardation coils and condensers serve to prevent cross-talk into adjacent telephone circuits and to permit the use of a common battery to supply all the repeater sets at any repeater station. A rheostat is also introduced, as shown, to control the heating of the filament and thus its thermionic emission.

Before proceeding to describe the 22 type circuit we may well restate the physical operation of the 21 type in terms somewhat different from those used in our introductory discussion of the circuit. Let us follow the telephone current, assuming for the moment that it is traveling from west to east. The current that comes to the repeater from the west is partly transmitted through the circuit to the line east but the larger part of it is divided between the input and output circuits of the telephone repeater element. The energy which flows into the output circuit is merely dissipated. The energy which flows into the input circuit is amplified by the repeater element and the amplified current passes to the "output coil." At this point if the line west and the line east are similar in characteristics, the energy divides, half going to the east and half to the west. The half that goes west, that is, towards the direction from which the transmission came, serves no useful purpose. The half that goes east (combined with the small amount of energy that passed directly through the repeater) may be many times larger than the original energy coming from line west and is the useful amplified transmission.

When the lines west and east are identical in characteristics, it is evident by symmetry that there is no voltage set up across the input circuit, and there is, therefore, no possibility of a "singing" condition no matter how large amplifications are employed. This circuit therefore permits two-way transmission over similar lines.

In case, however, the line west and the line east are different in their impedance characteristics for any part of the telephone frequency range, then for such frequencies there will be voltages set up across the input circuit due to current flowing from the output circuit and a tendency towards singing and distortion. The conditions which hold when such unbalances exist will be discussed later.

The duplex bridge circuit, as we saw from our earliest discussion, is applicable as a terminal circuit whereby two-way transmission may be had over a single pair of wires. (See Fig. 3). In such operation the telephone line is balanced by the inclusion in the opposite arm of the bridge of an artificial line of similar impedance-frequency characteristic. At a repeater station we may, therefore, terminate both incoming lines by bridge circuits, balance each line by an artificial line, and operate both lines as a through circuit for two-way conversations provided we connect the input branch of each bridge to the output branch of the other. In connecting the input terminals of one bridge circuit to the output terminals of the other bridge circuit we may introduce a one-way amplifier as shown in Fig. 30.

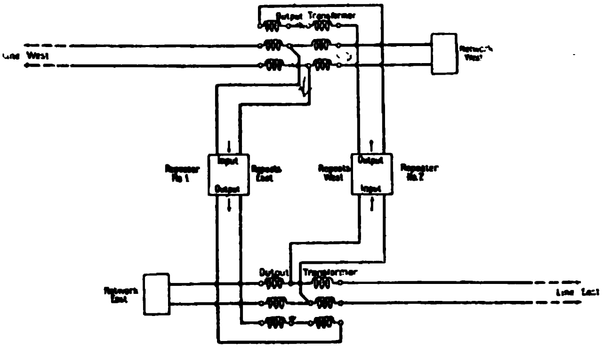


FIG. 30

The resulting circuit is the 22-type defined above, although its evolution did not follow the line which we have just followed for simplicity of discussion. It will be noted that this circuit makes use of a separate repeater element for transmission west to east from that used for transmission east to west, that it requires two "output transformers," and that each line, instead of being balanced by the line in the opposite direction, is balanced by an artificial line indicated as "network west" or "network east." Assuming again the transmission to be coming from the west, we trace the currents through the repeater circuit as follows: A small part of the energy goes through into the network west; the largest part, however, is divided between the input circuit of repeater No. 1 and the output circuit of repeater No. 2. That part which enters the output circuit of repeater No. 2 is dissipated without doing useful work. The part reaching the input of repeater No. 1 is amplified and the amplified energy flows through the output transformer connected to line east. The "network east" is made to have as nearly as possible the same impedance as the actual line for the full range of telephone frequencies. If the balance were perfect, there would be no voltage whatever set up across the input circuit of repeater No. 2, and therefore no possibility of setting up a singing condition whatever values of amplification were used.

Actually, however the balance can never be absolutely perfect, so that there will be a certain voltage set up across this input. This will, in turn, be amplified by repeater No. 2. These amplified currents will then flow through the output transformer connected to the line west. If the "network west" has exactly the same impedance as the line west, then the energy will equally divide between the two and there will be no voltage set up across the input circuit of repeater No. 1. This condition will then prevent any possibility of a singing condition being set up. Again, however, there will be in any practical case some unbalance so that a small voltage will be set up across this input circuit. If the two unbalances involved (*i. e.* line east against "network east" and line west against "network west") are sufficiently large, it is evident that the telephone currents have a closed path through the two repeaters and through these unbalances so that a singing condition may be set up.

This circuit is, however, very much more stable than the 21-type circuit. It will be noted that if either of the networks exactly balances its line, there may be any degree whatever of unbalance between the other network and its line without permitting singing. In general, the fact that the singing condition requires two unbalances simultaneously leads to a more stable condition.

There is another property of the 22-type circuit which is of great importance in connection with tandem operation of repeaters, that is, when two or more repeaters are used at different points in a line. As pointed out, transmission coming from the line west and reaching a 21-type repeater, as in Fig. 28, will result in an amplified current being sent out towards line east; but it will also result in an equally large current being sent back into line west. In the case of the 22-type, however, if the transmission comes in from line west and the balance between network east and line east is good, there will be very little energy sent back into line west. As will be pointed out in more detail below, under "Tandem Operation of Repeaters," this feature has a large advantage as the energy sent back into line west would react on any other repeater which was in circuit in that direction.

A 22-type circuit with vacuum tube repeater elements connected into it is shown in Fig. 31.

In connection with the presentation of the 22-type circuit it is interesting to note that this circuit, which is today the best known means of producing two-way telephone repeater operation with one-way amplifying devices, is the invention of Mr. W. L. Richards, one of the engineers of the Bell System, who devised and patented the arrangement in 1895—years before successful telephone repeaters were available for utilization in telephone systems.

BALANCING OF LINES

As pointed out above, the 21-type circuit depends for its stable operation on the balancing of the telephone line east against the telephone line west, and the 22-type circuit depends on the balancing of each of the telephone lines against an artificial line or "network." "Balancing" requires that the two lines or the line and its artificial line which are balanced against each other shall have closely the same impedance for the whole range of telephone frequencies.

Assuming that we have repeater elements which can give us sufficient amplification without appreciably affecting quality it is evident that the amount of amplification which we can get in any practical case depends on the degree to which this balance condition can be carried. This in turn, as will be more evident after reading the further discussion, depends on line conditions. It is then the line conditions, rather than the repeater element or its circuits which finally limit in every practical case the amplifications which can be obtained with a repeater.

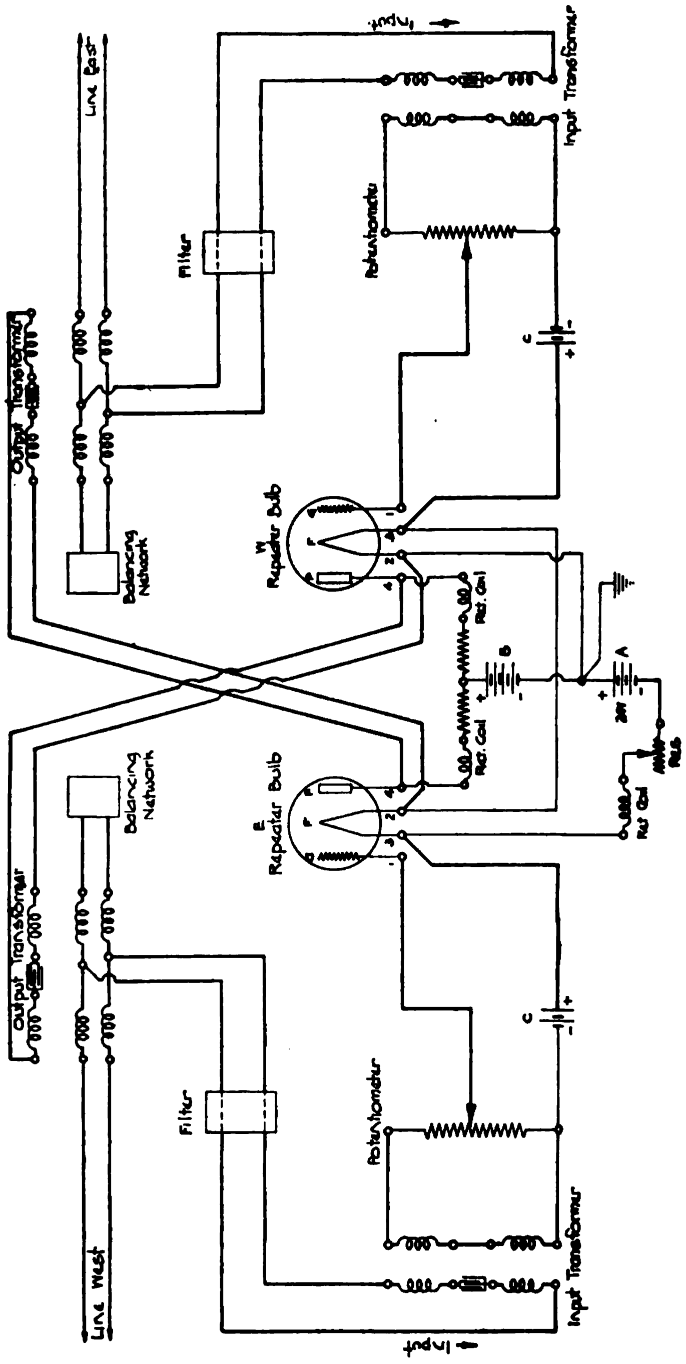

Line East
Input
Input Transformer
Potentiometer
Filter
Output Transformer
C
Balancing Network
W Repeater Bulb
Ret. Coil
B
A
20V
Rheo
E Repeater Bulb
Balancing Network
Output Transformer
Filter
C
Potentiometer
Line West
Input Transformer
Input

Fig. 31

Let us consider, therefore, what the impedance of a telephone circuit looks like as seen from the repeater terminals.

TELEPHONE LINE IMPEDANCE

If we take a very long No. 8 B. w. g. open wire circuit, which is the largest size of copper in general use in the telephone art, and suppose this circuit to be absolutely uniform in its characteristics, then its impedance, that is, the ratio of the voltage applied across it to the current entering it will be as shown by Curves 1 and 2 in Fig. 32, in which curve 1 gives the resistance and curve 2 the negative reactance of the impedance.

For such a line we can make up a very simple form of network which has approximately the same impedance and can be used for balancing in a repeater circuit. Curves No. 3 and 4 of Fig. 32 indicate the impedance of a network consisting merely of a condenser in series with resistance. It will be noted that even this simple circuit is a very good approximation of the impedance of a long line except at low frequencies. Curves Nos. 5 and 6 show the impedance of a somewhat more complicated network, as indicated on the diagram, which gives a very good approximation over the whole telephone frequency range.

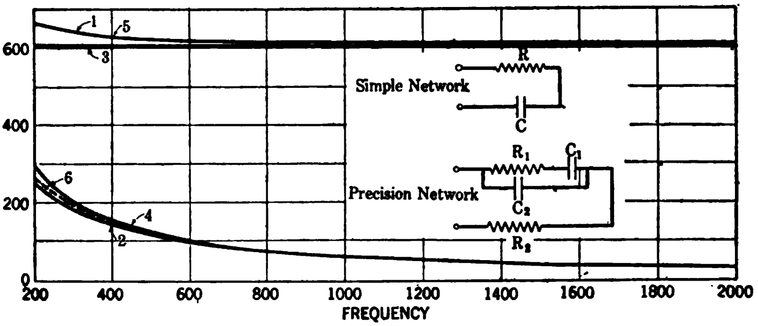


FIG. 32—NON-LOADED NO. 8 GAGE OPEN WIRE LINE

In a practical case every line has slight irregularities which affect somewhat its impedance. An impedance curve for an actual line which is, however, comparatively regular, is shown by curves 1 and 2 of Fig. 33. The measurements on this line were made with the end of it closed through an impedance similar to that of a long length of the line itself. Curves 5 and 6 are copied in from Fig. 32 for comparison.

It is difficult, however, to keep a long open wire circuit free from short stretches of cable. Fig. 34 shows both components of the impedance of a No. 8 gage circuit having a total length of about 165 miles with the distant end closed as before through an impedance equal to that of a long length of the line itself, but with 1¼ miles of underground cable at a point 105 miles from the end at which the measurements were made.

It would evidently be more difficult to make a network having the same impedance as this. It can be done, however, fairly simply by building an artificial line representing the 105 miles from the sending end to the place where the cable is located, putting in series an artificial line representing the cable, and putting beyond the artificial line an impedance network simulating the impedance of a long line, which might be in this case merely a resistance in series with a condenser.

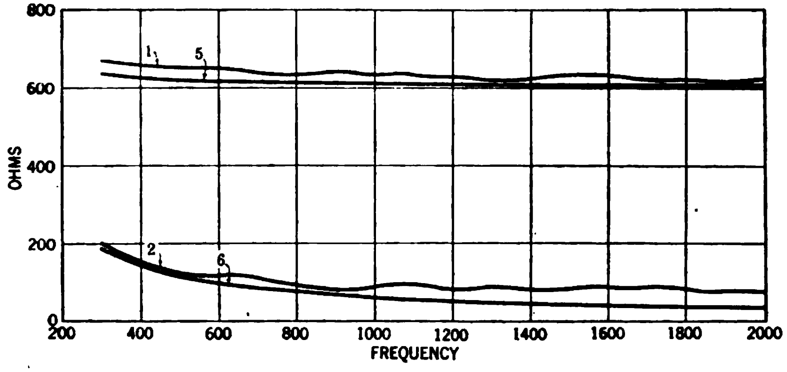


FIG. 33—NON-LOADED NO. 8 GAGE OPEN WIRE LINE

If a line has many irregularities, however, the setting up of a balancing network, while always theoretically possible, would become very expensive and inconvenient. A fundamental requirement, then, in obtaining good repeater action on a line is the maintenance of a high degree of uniformity in the electrical characteristics of the line.

In this particular case the irregularity caused by the cable can, to a considerable extent, be overcome by inserting in the cable one or more loading coils to make the impedance of the cable approximately that of the open wire line. In this actual case a single coil was cut into the cable near its mid-point, and the measured impedance after this had been done is indicated in Fig. 35. In this figure the curves after loading are shown in heavy lines and the old curves are indicated in lighter lines for comparison.

In the case of loaded lines the problem of maintaining sufficient regularity in the electrical characteristics to give a satisfactory impedance curve becomes considerably more difficult, due to variations in the inductances of the loading coils and in the distances between these coils. To illustrate this a number of impedance curves of loaded circuits will be given showing the effects of their irregularities on the impedance characteristics. In these curves, for simplicity, only the resistance component will be shown, as the reactive component is considerably smaller and goes through about the same irregularities as does the resistance curve.

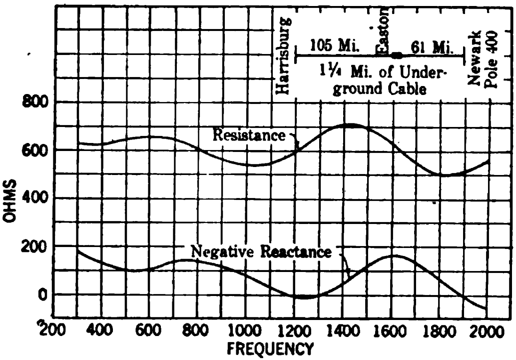


FIG. 34—HARRISBURG TO POLE 400 NEWARK, WIRES 23/24 COIL OUT AT EASTON

Fig. 36 shows an impedance characteristic of an open wire line loaded at approximately 8-mile intervals with loading coils having approximately ¼ henry each. This circuit would give commercial service from a transmission standpoint. It will be noted, however, that its impedance characteristic is extremely irregular, and it would be difficult to obtain gains from a telephone repeater operated in conjunction with it.

Fig. 37 shows the resistance of a circuit of similar constants representing a less extreme condition of irregularity. This would have been considered, before the advent of repeaters, a reasonably good circuit so far as irregularity is concerned. These irregularities were due largely to comparatively small variations in spacing of loading coils and in the inductances of the loading coils. Loading coils have to be placed on a telephone circuit at particular points in the transposition layout of the circuit in order to prevent crosstalk between it and other circuits on the same lead. It is difficult, however, exactly to space transpositions in view of the variations in the route taken by a line, lengths of cable, junction points with other lines, etc.

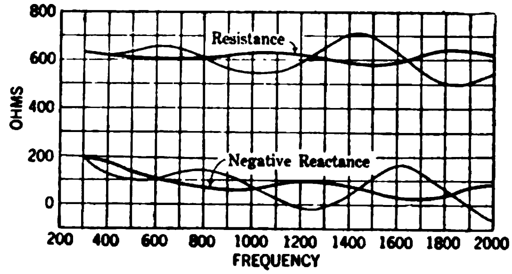


Fig. 35—Harrisburg to Pole 400 Newark, Wires 23/24 Coil in at Easton

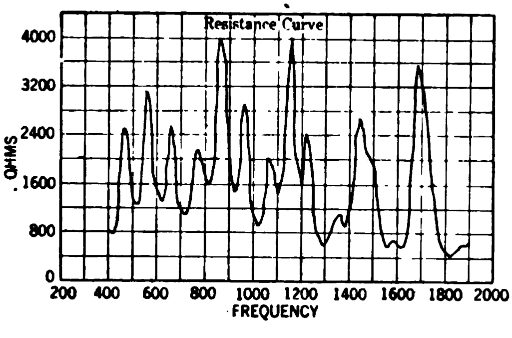


Fig. 36—Section of Transcontinental Circuit Pittsburgh to New York before "Clearing Up"

This led in the past to variations in the lengths between the successive loading points. Aerial loading coils are exposed to current surges due to lightning, and with some of the types of coils which have been put into use this has led to variations in their inductances. For any given frequency it may be that the effect of a large number of such irregularities are cumulative in raising the impedance of the circuit for that frequency, whereas with another frequency only slightly removed from the first, the effect of the irregularities may be to reduce the impedance. The result then with such impedance is indicated in Fig. 37 where the variation was due not to a few large irregularities, but to a large number of small irregularities.

This particular circuit was one of those entering into the transcontinental line, so that it was very carefully gone over, retransposed where necessary, the loading coils replaced by

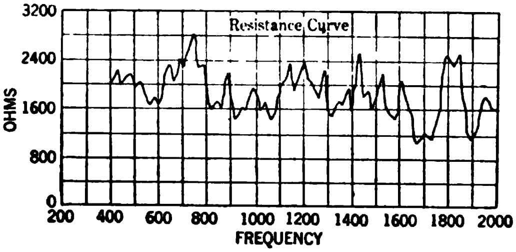


Fig. 37—Section of Transcontinental Circuit Pittsburgh to Chicago before "Clearing Up"

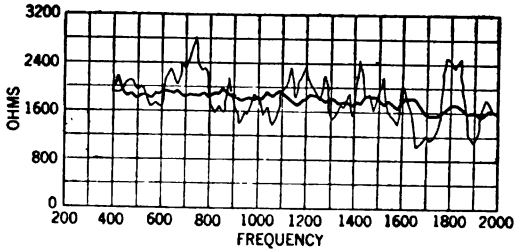


Fig. 38—Section of Transcontinental Circuit—Pittsburgh to Chicago after "Clearing Up"

others having a particularly high degree of stability and uniformity and the spacings made very uniform. The result of this work is indicated in Fig. 38 in which the dark line shows the impedance after this work had been done and the light line is a repetition of Fig. 37 for comparison. The circuit will now permit of large gains with repeaters.

In the cases where a large variation in impedance is due to one or more large irregularities, the impedance curves often offer a very interesting and convenient method of determining the point at which irregularity exists. For example: Fig. 39 shows the impedance of a circuit as measured from one end, the large wave-like variation being due to defective apparatus at some point in the circuit. By measuring the frequency range between successive "humps" on this "wave" it is possible to determine the distance from the sending end to the point of irregularity. In practical work many cases have arisen where omitted or defective coils or other irregularities have been located in position by this method.

In maintaining this uniformity of the characteristics of the line, one of the largest difficulties is caring for short lengths of cable which may occur in the line at river crossings, in going through towns, etc. It has sometimes been difficult to explain to those outside of the telephone field, but who are acquainted with the fact that by loading and telephone repeaters we can talk through long lengths of cable, why it is that we have so much difficulty with short lengths of cable which come into our open wire circuits, and why we are averse to cutting such short lengths into important long distance lines at villages and at crossings of railroads, power lines, etc. Considering, however, that a 500-ft. length of cable has as much capacity as three-quarters of a mile of open wire, it is evident that even a short length of cable has a very large effect on the operation of any

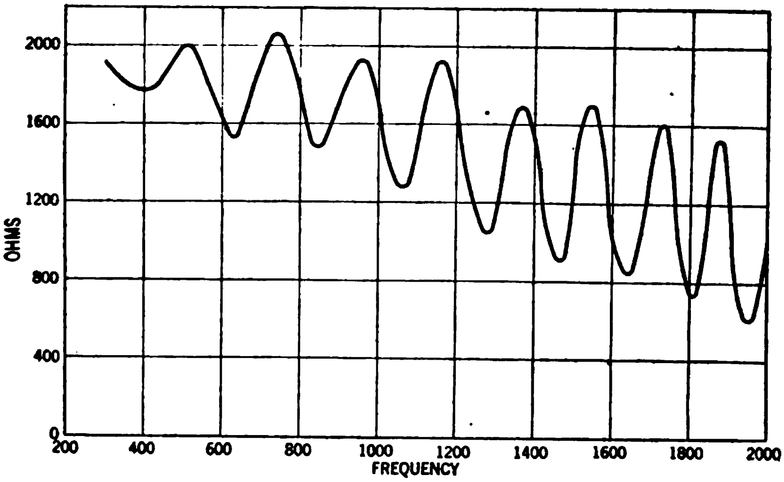


Fig. 39—Loaded No. 8 Gage Open Wire Line

Note: This irregularity was caused by defective apparatus located at a distance of about 15 sections from terminals at which measurements were made.

repeater on the line. In laying out a circuit, such short lengths of cable can be partially corrected for in the spacing of the loading coils. Changes made after a line is laid out are of course much more difficult to take care of.

In cable circuits the problem of maintaining uniformity is similar to that of open wire lines, requiring very accurate spacing of the loading coils and requiring loading coils of very uniform characteristics and of high electrical stability.

BALANCING ARTIFICIAL LINES

Assuming, then, in any case, that the lines have been made as uniform electrically as is economical, there is the problem of constructing artificial lines (or networks, as they are generally called) which shall have within a desired degree the same impedance as the actual line. It should be noted that these artificial lines do not need to have transmission characteristics corresponding to the actual lines, but need to have only the same impedance characteristics.

A very simple form is that indicated above for use in balancing a non-loaded open wire line, consisting of a single resistance in series with a single capacity. As already noted, a better approximation for a non-loaded open wire line can be made by adding another resistance and condenser as shown in Fig. 32.

For loaded lines, it is necessary to use a somewhat more complex artificial line, although here again the basic form is comparatively simple. Fig. 40 shows in light lines for comparison an impedance curve much like that of Fig. 38, and in heavier lines, the impedance of an artificial line consisting of two condensers, a resistance and two inductance coils arranged as indicated by the diagram in this figure. For completeness both the resistance and reactance of the impedance are shown in the figure.

In the case of a loaded line the impedance depends very markedly on the place in the loading section from which the circuit is measured. In practise lines are generally terminated either at mid-section or at mid-coil, since these points are symmetrical points in the loading and make for maximum flexibility in connecting circuits together. Most of the figures in this paper were taken at mid-coil.

It is evidently necessary that the balancing line should balance not only the actual line itself, but any apparatus between the line and the point at which the repeater is applied, so that all coils, composite sets or other devices used in series with the line must be, with the necessary precision, balanced in the artificial lines by corresponding coils, condensers, etc.

TANDEM OPERATION OF REPEATERS

The above discussion has referred particularly to the operation of a single repeater in a circuit. Where several repeaters are used in tandem at different points in a circuit, the same considerations are involved. In addition there is the further consideration that not only can circulating currents be set up involving each repeater by itself, and which may lead to "howling," or to distortion in quality, but such circulating currents may follow paths involving any two or more of the repeaters operating in conjunction. As noted above, the 22 type circuit involving artificial balancing lines has a large advantage over the 21 type for tandem operation, due to the fact that transmission entering it from either direction does not cause a large amount of energy to be thrown back in the same direction, as is the case with the 21 type circuit. There is thus very much less interaction between the repeaters with the 22 type.

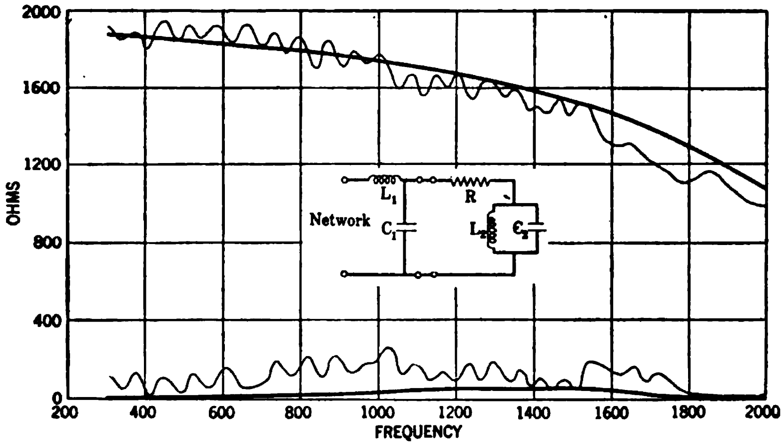


FIG. 40—LOADED NO. 8 GAGE OPEN WIRE LINE ½ COIL TERMINATION

Effective tandem operation of repeaters requires, therefore, a high degree of electrical uniformity, and also requires that the reflection effects between the repeaters and their connected lines shall be reduced as much as possible, so that each repeater brings in as little irregularity to its adjacent repeaters as possible.

THE FOUR-WIRE CIRCUIT

In the above we have considered the application of repeaters in ordinary circuits in which the same pair of wires is used for transmission east as is used for transmission west. After the above discussion regarding balance, it is evident that there would be considerable advantage in transmission if separate pairs of wires were used for transmission east and for transmission west. A circuit of this type, designated as a "four-wire circuit," is shown diagrammatically in Fig. 41.

The term "four-wire" does not mean necessarily that four wires are used, but that different circuits are used for transmission east and transmission west. Each of these circuits may comprise a pair of wires, or may be a phantom circuit, etc.

In a circuit of this kind there will be one-way repeaters in each of the two branches. The two branches must be brought together at the ends for connection to two-wire lines, or to the subscribers' loops. There is, therefore, a possibility for circulating currents to be set up flowing over the two sides of the circuit in series. In order to minimize such circulating currents, artificial balancing lines are used at the points where the four wire connects to the two-wire in a manner similar to the arrangement of the 22-type circuit. It should be noticed that if the total amplification of both branches of the four-wire in series is less than the total transmission equivalent, the circuit cannot "sing." If the balance at the ends is good, the gains may be even greater than the transmission losses without producing singing. As in the case of two-wire operation, however, large distortion may be brought into the circuit by the

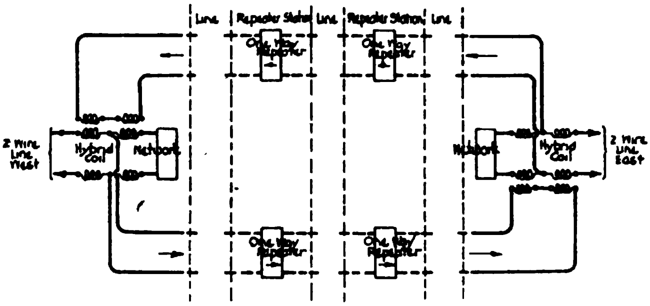
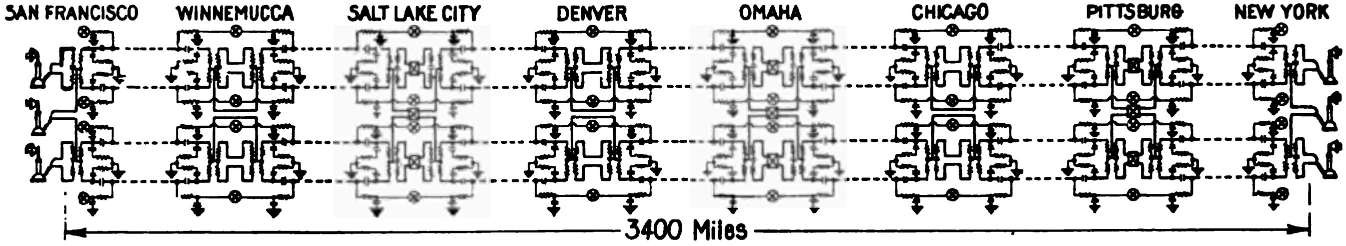


FIG. 41.

NEW YORK – SAN FRANCISCO CIRCUIT
When Set Up for Through Telephone and Telegraph Circuits
Simplified Diagram



This circuit provides for three simultaneous telephone conversations between New York and San Francisco. At the same time it provides for four telegraph messages in each direction.

Telephone repeaters indicated by ▣
Telegraph apparatus " " ⊗

Line wire No. 8 B.W.G. copper
Inductance of side circuit loading coils 0.25 henry
" " phantom " " " 0.15 "
Spacing of loading coils 8 miles

Line equivalent = 56 miles of standard 19ga. cable
Apparatus loss = 7 " " " " " "
Total equivalent = 63 " " " " " "
Total repeater gain = 42 " " " " " "
Resultant equivalent = 21 " " " " " "

FIGURE 42

circulating currents even when the amplifications and balance are not such as to produce singing.

Circuits of this kind evidently use twice as many wires as ordinary circuits. They permit, however, the use of smaller wires. It is a question of economics then, as to whether a circuit of this kind proves in in any practical case. For long distance cable operation, and for other special cases, there are conditions where we find that these four-wire circuits have a field of use.

METHOD OF APPLYING REPEATERS

In the practical operation of repeaters there are two methods of connecting them into lines which have come into general use. These are:

(a) "Through line repeaters."

(b) "Cord circuit repeaters."

The through line repeater is one which is associated with a particular line and connected directly into the line. The operators, therefore, have no control over it. It is generally set to give a definite amplification, depending on the characteristics of the line.

The cord circuit repeater is connected into a cord circuit under the control of an operator. The two ends of the circuit generally appear before the operator as two plugs which she may connect into any pair of circuits terminating before her which are in proper shape for repeater operation. The amplification given by the repeater is generally under her control, the amplification in each case depending on the pair of circuits which is connected together.

Where the circuits which may be used with a cord circuit repeater vary considerably in characteristics, so that the same artificial balancing lines will not work satisfactorily with all of them, various arrangements are used so that proper balancing lines will be connected into the circuit corresponding to the actual lines to which the circuit is connected. The commonest arrangement for doing this consists in having a jack in the switchboard immediately below each jack to which a toll line connects. This extra jack has connected to it an artificial line corresponding to the actual line. The repeater circuit then terminates in double plugs so arranged as to pick up both the actual and artificial lines simultaneously.

PRACTICAL RESULTS OBTAINED

As already noted, the amount of gain which may be obtained in any practical case now depends almost entirely on the degree of electrical "uniformity" which it is economical to maintain in the telephone circuits to which the repeaters are connected, since we can obtain within practical limits as large amplifications in the repeater elements as we desire.

We have already indicated the large effect which this electrical uniformity requirement has had on the design of loading coils, and on practises with regard to their use. About every feature of open wire and cable construction and office equipment which affects the electrical characteristics of the circuits has been in some way affected by this repeater requirement. Not only in the design and construction, but also in the daily operation and maintenance of circuits with repeaters, this requirement has to be kept constantly in mind.

At the present time more than 1000 telephone repeaters are in service in the plant. We now will describe a number of typical repeater circuits and will indicate the results which are being obtained.

TRANSCONTINENTAL LINE

The circuit over which transmission is given between the cities along the eastern coast and the Pacific coast cities is shown diagrammatically in Fig. 42. Originally there were three repeaters used in this circuit, located at Pittsburgh, Omaha and Salt Lake City. Since that date, largely for flexibility in switching traffic, the number has been increased to six located at Pittsburgh, Chicago, Omaha, Denver, Salt Lake City and Winnemucca. The total gains were only slightly increased by the change. The circuit consists of No. 8 B. w. g. copper conductors loaded at eight-mile intervals with coils of one-fourth henry each. This gives a line equivalent of 56 miles of standard 19 gage cable. There is a further transmission loss due to composite and other apparatus of about 7 miles, the total equivalent being, therefore, about 63 miles. This is reduced by repeaters to a resultant equivalent of 21 miles, which is, therefore, about one-third of the equivalent without repeaters.

The drawing shows the important points on the line, indicates the positions of telephone and telegraph repeaters and indicates the compositing apparatus used in separating the telegraph and telephone currents.

BOSTON-WASHINGTON CABLE

The largest use of repeaters on cables has been in connection with the circuits between the various cities which are served through the underground cable route extending from Boston through Providence, Hartford, New Haven, New York, Philadelphia and Baltimore to Washington. The use of repeaters on small gage cables placed in this subway became of the greatest importance during the war in caring for the tremendously increased requirements for telephone facilities between these large cities along the Atlantic seaboard.

Most of these telephone repeaters are located in the large cities along the route of this cable. At two places, however, Princeton, N. J., and Elkton, Md., the repeater requirements were such as to justify the putting up of buildings especially for repeater operation. Fig. 43 shows an illustration of the building which was put up at Princeton. There are now about 200 repeaters working at Princeton. The ultimate capacity of the building will be over 500 repeaters.

Fig. 44 shows a typical group of repeater racks, each rack carrying two complete repeaters. Fig. 45 shows at the left, front and back views of some of these repeater racks. At the right it shows the face of a test board from which the different repeaters and associated circuits may be tested. Fig. 46 shows the power board where the operating currents and potentials applied to the repeaters are measured.

The repeaters shown in the above photographs are of the vacuum tube type. Fig. 47 shows a front view of an installation of the mechanical type repeaters at Providence, R. I. Fig. 48 shows a rear view of the repeater racks.

The longest cable circuits set up along this route are those from Boston to Washington, a total distance of about 455 miles. A typical layout for a circuit of this kind consists of loaded conductors, No. 10 B. & S. gage from Boston to New York, No. 13 gage from New York to Philadelphia and No. 10 gage from Philadelphia to Washington. This circuit has an equivalent of about 30 miles of No. 19 gage standard cable, which by means of repeaters at Hartford and Philadelphia is reduced to about 14.5 miles. It should be noted here that No. 10 gage conductors, are somewhat larger conductors than would now be placed for this business, and are being used because they were available, having been placed before the use of repeaters had been developed to their present effectiveness.

FIG. 43

[GHERARDI AND JEWETT[

PLATE LII
A. I. E. E.
VOL. XXXVIII, 1919

Fig. 44

[GHERARDI AND JEWETT]

PLATE LIII.
A. I. E. E.
VOL. XXXVIII, 1919

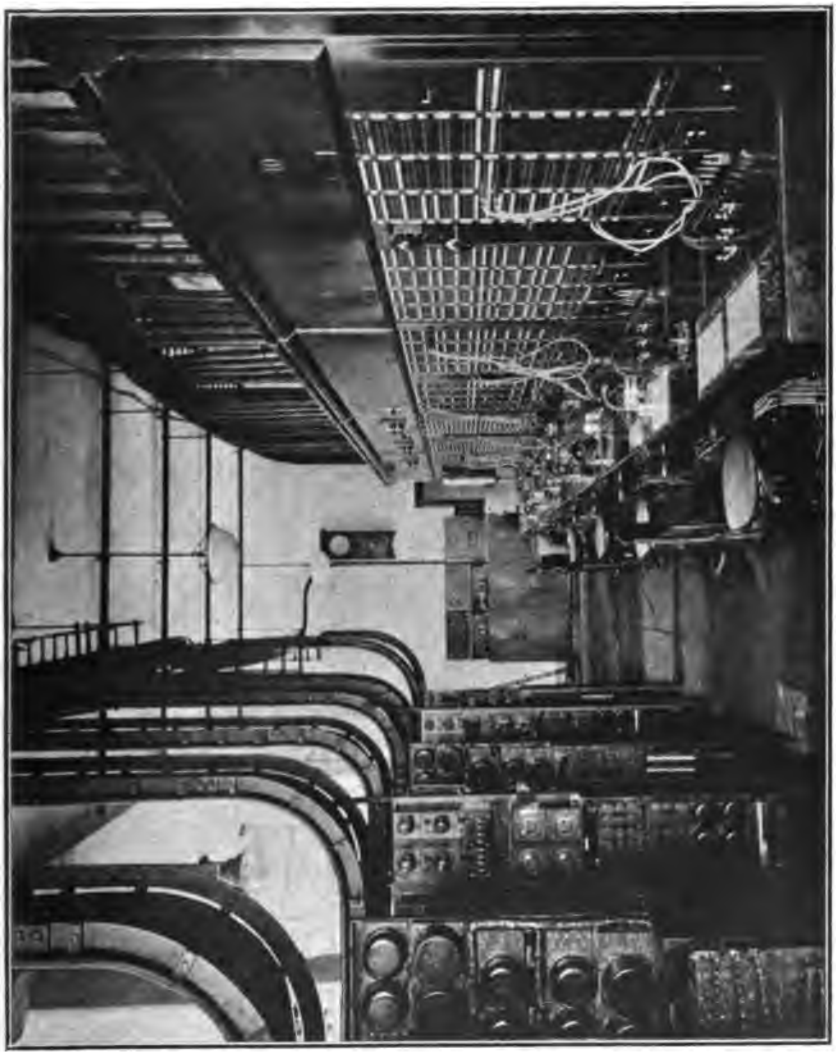


FIG. 45

[GHERARDI AND JEWETT]

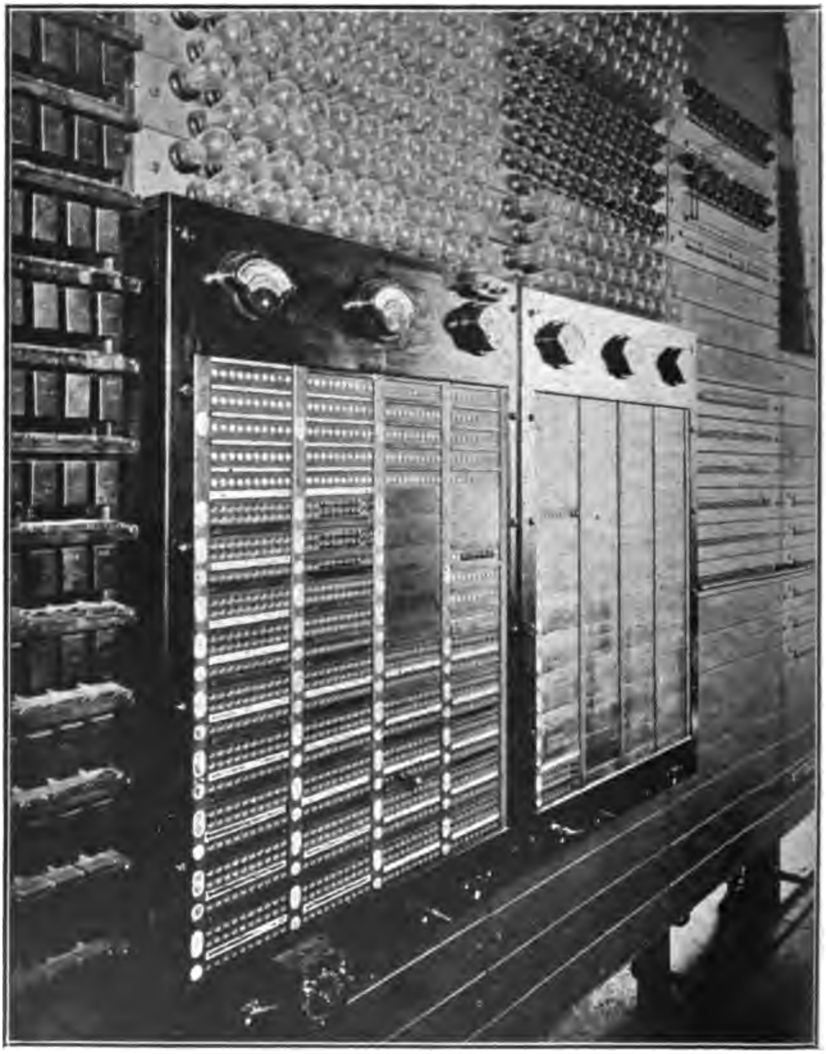


FIG. 46 [GHERARDI AND JEWETT]

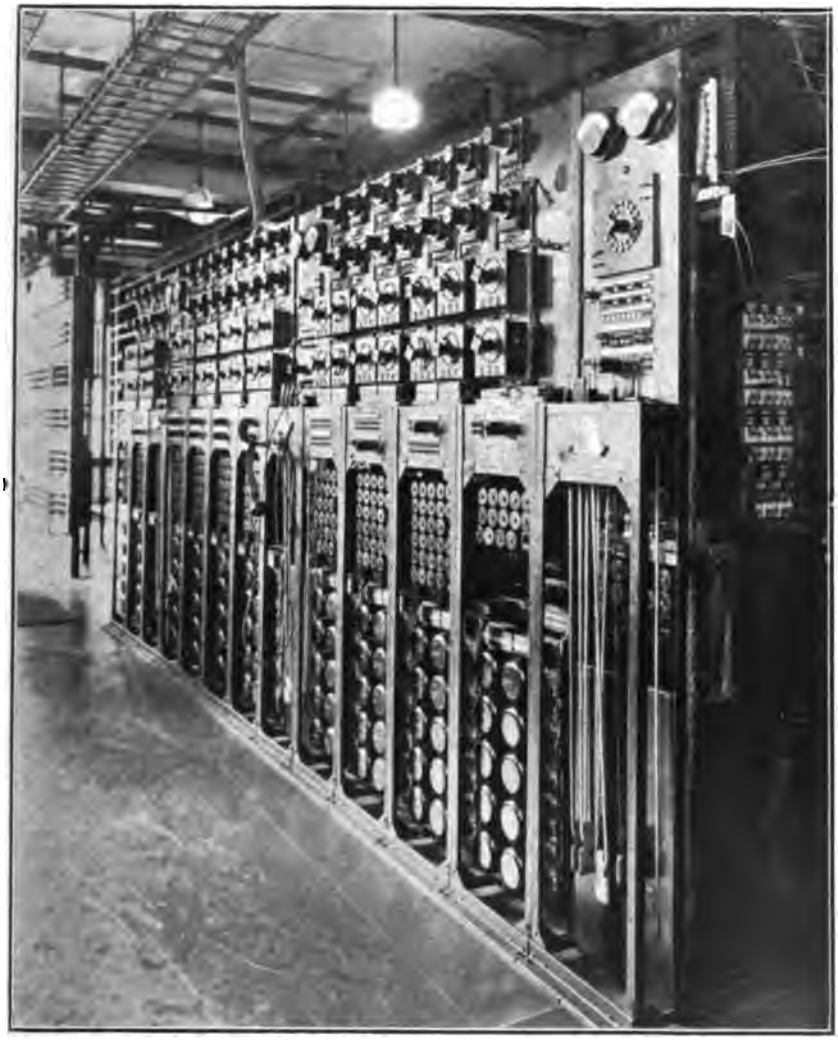


Fig. 47 [GHERARDI AND JEWETT]

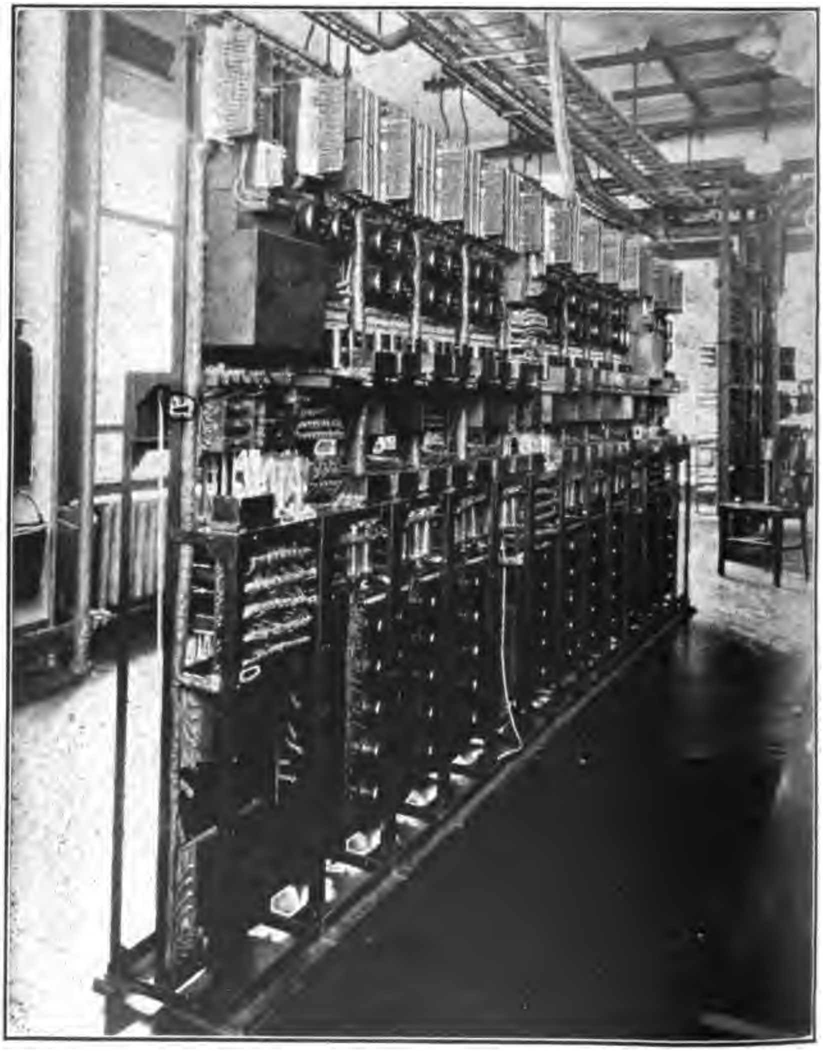


FIG. 48 [GHERARDI AND JEWETT]

The longest circuits electrically, that is, those having the greatest attenuation, which have been operated with repeaters along this route, are New York-Washington circuits, consisting of No. 19 gage cable loaded with coils of about one-fourth henry inductance, spaced at intervals of about 1.16 miles. The distance is about 225 miles. This gives a line equivalent of about 62 miles standard cable. By the use of four repeaters at Princeton, Philadelphia, Elkton, and Baltimore, the equivalent of this circuit is reduced to about 13 miles of standard cable.

The above circuits make use of the 22-type repeaters. Where there is but one repeater in a circuit, however, the 21-type gives, under many conditions, satisfactory service. For example, between New York and Philadelphia, a distance of about 90 miles, a group of circuits is being worked on No. 19 gage conductors, with a single 21-type repeater at Princeton. The circuit without repeaters has an equivalent of about 24 miles of standard No. 19 gage cable. The repeater reduces this to about 14 miles.

SUMMARY

In the foregoing pages we have endeavored to present a short history of telephone repeater development, a statement of the factors which control their successful operation, an idea of the possibilities and limitations of different types of repeaters and repeater circuits, and three or four typical examples of the present commercial line combinations in which telephone repeaters are vital links in the every-day telephone service of the public. These, we believe, will give the members of the Institute a fair picture of the state of the art as it now exists, both technically and commercially.

In concluding the paper it seems only necessary to add a word with regard to the further influence which the telephone repeater is likely to have on telephonic communication in the future.

As indicated in the earlier parts of the paper, the net result of the vast amount of work already done has been to produce amplifying devices capable of taking energy from a local source or sources and under the control of enfeebled telephonic currents from a distant transmitting station deliver back into the telephone line currents which are faithful reproductions of these enfeebled control currents but with many times their energy. Furthermore, the work already done has developed the methods which permit of utilizing the amplifying properties of these repeaters in the various kinds of telephone lines which go to make up the modern telephone network and methods of constructing and maintaining lines to secure maximum efficiency and economy from the use of repeaters. The tools thus at our disposal are of such a character as to permit of our forecasting the changes which their use is likely to produce in the evolutionary growth of the telephone plant. When combined with the benefits to be derived from the loading of cable circuits, the use of the practically perfect repeaters now available puts an entirely different aspect on many of the problems which have confronted the telephone engineer in the past. Not only will there be the opportunity to utilize smaller gage wires than has hitherto been possible but the economical field for underground and overhead toll cables will be greatly extended into the region which it has heretofore been possible to serve only with some form of open wire construction. Viewed from the standpoint of continuity of service alone, this is an advantage of almost inestimable value. Further, the range of reliable and commercially possible long distance telephony will be very greatly extended.

Numerous fields which have in the past been fruitful regions of experimentation have likewise been made obsolete by the development of the telephone repeater. Most noteworthy of these is that looking to the production of powerful or so-called loud speaking transmitters. Many investigators have in the past devoted much effort to the production of this type of equipment and for many years a large group of telephone engineers, notably those in Europe, looked upon the production of a practical loud speaking transmitter as a key to the solution of long distance telephony. Comprehensive studies by the engineers of the Bell System early convinced us that the solution did not lie in this direction and the success of the telephone repeater is a vindication of the adequacy of these early studies and of the consistent line of development which followed them.

In conclusion, the authors wish to express their appreciation to Mr. E. H. Colpitts, Assistant Chief Engineer of the Western Electric Company, Incorporated; Mr. O. B. Blackwell, Transmission Development Engineer of the American Telephone and Telegraph Company; Mr. A. B. Clark, of the American Telephone and Telegraph Company, and Mr. H. E.

Shreeve and Mr. John Mills of the Western Electric Company Incorporated, for the very great help they have rendered in the selection from a mass of information of the data for this paper and for their assistance in its preparation.

September 18, 1919

Presented at a joint meeting of the American Institute of Electrical Engineers and the Institute of Radio Engineers, New York, October 1, 1919.

PRINCIPLES OF RADIO TRANSMISSION AND RECEPTION WITH ANTENNA AND COIL AERIALS

BY J. H. DELLINGER

Abstract of Paper

Coil aerials are coming to replace the large antennas in radio work. The advantage of the coil aerial as a direction finder, interference preventer, reducer of strays and submarine aerial, make it important to know how effective such an aerial is as a transmitting and receiving device in comparison with the ordinary antenna. In this article the mathematical theory is presented and, as a result, the answer to this question is obtained. Experiments have verified the conclusions reached, and the formulas which are obtained are a valuable aid in the design of an aerial to fit any kind of radio station.

A great many questions and hazy ideas on the behavior of radio waves are cleared up by the study which was made and here presented.

It is found that the coil aerial is particularly desirable for communication on short wave lengths. A coil aerial is as powerful as an antenna only when its dimensions approach those of the antenna. For other reasons, however, a small coil aerial is in many cases as effective as a large antenna.

It is shown that an advantageous type of radio aerial is a condenser consisting of two large metal plates. This type of aerial has many of the advantages of the coil aerial.

The fundamental principles of design of aerials are given in this paper. On the basis of this work the actual functioning of any type of radio aerial can be determined either from measurements made upon the aerials or from actual transmission experiments.

The investigation has opened up a large and most interesting field for further research and progress in the utilization of radio waves.

I. INTRODUCTION

IN a radio transmitting or receiving set, either the condenser or the inductance coil is made of large dimensions. It is then called the aerial, and effects the transfer of power between the radio circuits and the ether. The coil aerial has the inherent advantage of serving as a direction finder and interference preventer, but is less effective quantitatively as a transmitting or receiving device than the condenser type of aerial, commonly called the antenna. Both kinds of aerial are very simple in construction, consisting merely of one or more wires. An antenna consists of a wire or set of wires connected in parallel and constituting one plate of a condenser, the other plate being the ground beneath. The coil aerial is one or more turns of wire constituting a simple coil or loop. When an antenna is used its circuit is completed, in general, by placing an inductance coil in series with it and the ground; and when a coil aerial is used its circuit is completed by connecting a condenser across its terminals. The typical connections are shown in Figs. 1 and 2.

The antenna is used when it is desired to communicate over as great a range as possible or reduce the power of the apparatus as much as possible. The coil is used when directional properties are particularly important. The coil radiates and receives electric waves better in the direction of its plane than in the direction of its axis, whereas the performance of the antenna is much more nearly independent of direction of the waves. By arranging a coil so that it can rotate it makes an excellent direction finder. When thus used on a ship or an airplane, a coil aerial is sometimes called a radio compass. It has also been called a radio goniometer. By turning it so that its axis is parallel to the direction of propagation of the waves from some particular station, that wave is not received while waves from other directions are received. The coil may thus serve as an interference preventer. It is possible to attain some slight reduction of the effects of strays, commonly called static, by using combinations of coil aerials. Submarine communication is more successful with coil aerials than with antennas because the coil can be protected from the short-circuiting effect of the water while an antenna can not. The numerous advantages of the coil aerial make it highly im-

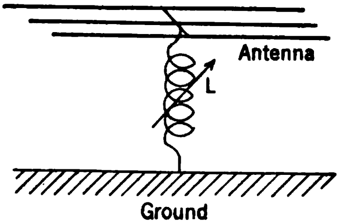


FIG. 1—SIMPLE ANTENNA CIRCUIT

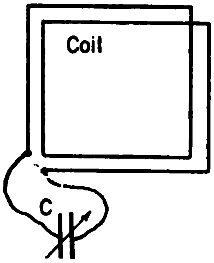


FIG. 2—SIMPLE COIL AERIAL CIRCUIT

portant to know the relative sensitiveness or power of transmission of the device in comparison with the antenna. This publication provides the answer to this question and sets forth the theory of radiation and reception and the action of antenna and coil aerials. The relative effectiveness of any coil and antenna is given by formula (**32**) to (**36**) in Sec. IV 1 below. The uses of the coil as a direction finder, interference preventer, reducer of strays, and submarine aerial, are not treated in this article.

The most important question considered is the practical one: How far can communication be maintained by the use of any specified antennas or coil aerials. Formulas are developed by which the current received in an antenna or coil is calculated in terms of the current in a transmitting antenna or coil, resistance of receiving aerial circuit, the distance, wave length, and dimensions of the aerials. The formulas have been found to be useful in the design of aerials and in the selection of an aerial for a particular kind of communication. They were worked out before there was any experimental information available to answer the question of the comparative quantitative value of the two kinds of aerials. Not much information on this has been obtained from experiment even yet (1919), but such experiments as have been made have substantiated the formulas. The work described in this paper was done in 1916 and 1917. The results were given in "Radio Transmission Formulas," a confidential paper of July, 1917, which was circulated in the Signal Corps and Navy. Publication was withheld during the war at the request of the Signal Corps. The formulas have also been given by the writer of the present paper on page 234 of "The Principles Underlying Radio Communication," 1918, Signal Corps Pamphlet No. 40, a book which can be purchased from the Superintendent of Documents, Washington, D. C.

Historical. The coil aerial and the condenser aerial (antenna) both date back to the first experimenter with the electric waves that make radio telegraphy possible. H. Hertz in 1888 used an open oscillator, which was the forerunner of the antenna, as his transmitting apparatus. For receiving he used a circle of wire, which was the first loop or coil aerial, and observed its directional properties.

The possibility of a loop or coil aerial as a transmitting device was discussed from the theoretical standpoint by G. F. Fitzgerald and later by J. A. Fleming, in *Electrician*, 59, pp. 936, 976, 1016; 1907. Fleming derived expressions for the radiated fields, using a curious theory in which the four sides of the coil were replaced by Hertzian doublets.

The use of a large loop or single-turn coil as an aerial in practical radio communication was described by G. Pickard in Proceedings of the Wireless Institute of America, 1, May 1, 1909. He discussed its properties both as a radiation and receiving aerial. He described its use as a direction finder, stating that he had determined directions with it to better than 1 degree.

In spite of this work and proposals by others, the antenna was used almost exclusively as the transmitting and receiving device until 1913. The use of the coil aerial received a great impetus by the publication of an article by F. Braun in *Jahrbuch der drahtlosen Telegraphie und Telephonie*, 8, p. 1, 1914; on the Use of Closed Circuits in Place of Open in Radio Telegraphy. He discussed the advantages of a coil aerial as a receiver and transmitter, both from the theoretical and the experimental standpoint.

Since 1913 there has been a great deal of development work done on coil aerials and they were extensively used in the war. The development of the coil aerial as a practical direction finder and receiving device was begun at the Bureau of Standards in 1915. Using electron tubes as the detecting apparatus, transatlantic signals were received on a coil inside a room. Experiments with the coil as a transmitting device were carried out at the Bureau in 1917. Among the very few published treatments of development and use of the coil aerial are those in Bucher's text-book, "Practical Wireless Telegraphy" 1917, p. 256; and "Radio Direction-Finding Apparatus" by A. S. Blatterman, *Electrical World*, 73, p. 464; 1919. Most of the descriptions to date have been confidential reports of the military services of various countries.

The theoretical discussions by Fleming and Braun are cumbersome and needlessly complicated and the results are not well adapted to practical use. The present paper presents an original treatment that is relatively very simple but none the less exact and leads to conclusions that apply directly to practical work. This paper also points out a number of misconceptions that have existed, and endeavors to clear up some of the controversial points on the radiation of waves and the functioning of aerials.

II. DERIVATIONS OF THEORETICAL FORMULAS

1. RADIATION FROM AN ANTENNA

Formula (8) below, giving the radiated magnetic field at a distance from an antenna, is a well-known formula. It has been given by various writers, and is the only one presented in this paper that requires any deep consideration of fundamental electromagnetic theory. The result is in fact implicit in Maxwell's classical treatise, "Electricity and Magnetism". The derivation given here is much more direct and brief than the others the author has seen, and is given only for that reason. The derivations of formula (10) and following ones are still simpler, and will be of more interest to most readers.

The units used in this paper are international electric units, the ordinary electric units based on the ohm, ampere, centimeter, and second. (See paper by the author on "International System of Electric and Magnetic Units", Scientific Paper of the Bureau of Standards No. 292). The unit of magnetic field intensity is the gilbert per cm., often called the cgs. unit. The only exception to the use of units of the international system is in certain of the practical formulas where lengths are expressed in meters or miles where so stated.

In the following discussion is calculated the magnetic field intensity produced by a flat-top antenna, having electric current of uniform value throughout the length of the vertical portion. Most antennas in practise approximate closely this condition.

The symbols used are:

i = instantaneous current
I_0 = maximum value of current
I = effective value of current
H_t = instantaneous value of magnetic field intensity
H_0 = maximum value of magnetic field intensity
H = effective value of magnetic field intensity
h = height of aerial
d = distance from sending aerial
ω = 2π times frequency of the current
t = time
λ = wave length
c = velocity of electric waves = 3×10^{10} cm. per second
Subscripts s = sending, r = receiving, a = antenna, c = coil.

In Fig. 3 the upper heavy line represents the flat top of the antenna, and the lower heavy line the grounding area. Suppose a current is flowing, having the instantaneous value i in the vertical portion. The magnetic field intensity at any point due to a varying current is different from that due to a steady current. Consequently the field cannot be calculated in the same way that the magnetic field intensity of a straight wire is ordinarily calculated. When the current is varying, the magnetic field intensity is calculated by the aid of a quantity called the vector potential in such a way that the variation with time is taken into account. The instantaneous value of the vector potential of current in the vertical conductor at a distance d in a plane perpendicular to the conductor, is

$$A = \frac{[i]\,h}{d} \tag{1}$$

where $[i]$ indicates that for any time t the value of i is taken for the instant $(t - d/c)$.

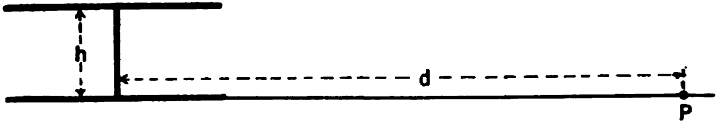


FIG. 3—CALCULATION OF MAGNETIC FIELD AT A DISTANCE FROM AN ANTENNA

Suppose the current in the antenna is a sine-wave alternating current,

$$i = I_0 \sin \omega t \tag{2}$$

$$\therefore\ [i] = I_0 \sin \omega\,(t - d/c)$$

$$A = \frac{h\,[i]}{d} = \frac{h\,I_0}{d} \sin \omega\,(t - d/c) \tag{3}$$

The magnetic field intensity is calculated from the vector potential by the general relation $H_t = 0.1$ curl A, which for this simple case of a straight conductor becomes

$$H_t = \frac{1}{10}\frac{\partial A}{\partial d} \tag{4}$$

the direction of H_t being perpendicular to the plane of h and d. From equation (3),

$$H_t = -\frac{h\,\omega\,I_0}{10\,c\,d} \cos \omega\,(t - d/c) - \frac{h\,I_0}{10\,d^2} \sin \omega\,(t - d/c) \tag{5}$$

This equation gives the magnetic field intensity at any point P at a distance d from the antenna. The second term represents the ordinary induction field associated with the current, while the first term is the radiation field. At a considerable distance the second term is negligible because the second power of d occurs in the denominator. The first term then represents the magnetic field radiated from an antenna at the distance d from the antenna. The distance d is measured along the earth's surface, because the waves follow the curvature of the earth's surface instead of proceeding straight out into space. For a considerable distance from the antenna, the maximum value of the magnetic field intensity during a cycle is therefore

$$H_0 = \frac{h\,\omega\,I_0}{10\,c\,d}$$

Expressing in terms of effective values,

$$H = \frac{h\,\omega\,I}{10\,c\,d} \qquad (6)$$

Henceforth H means the radiated field unless it is specifically stated to be the total field. The last equation may be expressed in terms of wave length instead of ω by the relation

$$\frac{\omega}{c} = \frac{2\,\pi}{\lambda} \qquad (7)$$

$$\therefore H = \frac{2\,\pi}{10}\;\frac{h\,I}{\lambda\,d}$$

Using the subscript s to indicate that it is the sending rather than the receiving antenna which is considered,

$$H = \frac{2\,\pi}{10}\;\frac{h_s\,I_s}{\lambda\,d} \qquad (8)$$

This derivation follows the conceptions presented in the early pages of Lorentz, "The Theory of Electrons". It is equivalent to Hertz's intricate proof, but is more direct. The way in which the result is expressed here accords more closely with the physical ideas and with actual practise, being expressed in terms of current rather than electric charge, since it is current that is actually measured in an antenna and the current furthermore is generally uniform in the vertical portion of the antenna.

Formula (8) gives the radiated magnetic field from a sending antenna at a distance d along the earth's surface. The units are the gilbert per cm. for H, the ampere for I, and the centimeter for all lengths, as previously stated.

Undamped alternating current in the antenna was assumed. The same result, however, is obtained if the current is damped. At very great distances from the sending aerial, the magnetic field is less than that calculated by formula (8), because of absorption of the power of the wave as it travels along. This may be taken into account by multiplying the right-hand member of (8) by a correction factor F_1. The value of this factor for daytime transmission over the ocean, derived from the experiments of L. W. Austin, Scientific Paper of the *Bureau of Standards No.* 159; 1911, is

$$F_1 = \epsilon^{-0.000047\, d/\sqrt{\lambda}} \qquad (9)$$

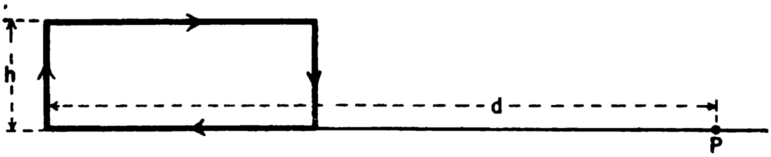


FIG. 4—CALCULATION OF MAGNETIC FIELD RADIATED FROM A COIL

for d and λ both in meters. This correction ordinarily needs to be applied only when the distance is greater than 100 kilometers.

2. RADIATION FROM A COIL

It was formerly the belief that a coil could not radiate, because the current up one side of the coil (Fig. 4) produces a field equal and opposite to that down the other side of the coil. This is erroneous because the two equal fields are not exactly opposite. The phase between the two departs from 180 deg. because of the finite time required for the field to be propagated from one side of the coil to the other. It is only along the axis of the coil that the calculated radiation is zero. The actual resultant field radiated from the coil may be deduced in either of two very simple ways, both of which are interesting from the physical standpoint. The first deals with the instantaneous values of the magnetic field, and the second with the effective values.

The following additional symbols are used:

l = horizontal length of coil aerial.
N = number of turns of wire of coil aerial.
θ = phase angle between values of field intensity a distance l apart in the wave.

First Deduction. Consider a rectangular coil of height h and horizontal length l. The magnetic field at a point P in the d direction is the resultant of the fields arising from current in the two vertical sides of the coil, the horizontal sides contributing nothing. The magnetic field at P due to any one of the vertical wires of the coil is calculated from equation (5) above. Neglecting the second terms, because d is large, the instantaneous values of the magnetic field (Fig. 4) at the distance d and $(d - l)$ respectively from the two vertical sides are

$$H_d = -\frac{h\,N\,\omega\,I_0}{10\,c\,d} \cos\omega\,(t - d/c)$$

$$H_{d-l} = +\frac{h\,N\,\omega\,I_0}{10\,c\,(d-l)} \cos\omega\left(t - \frac{d-l}{c}\right)$$

The resultant field H_t is the algebraic sum of these two, which becomes since $(d - l)$ is very nearly d when d is large,

$$H_t = -\frac{h\,N\,\omega\,I_0}{10\,c\,d}\,2\sin\omega\left(t - \frac{d - l/2}{c}\right)\sin\frac{\omega\,l}{2\,c}$$

The effective value of the resultant field is

$$H = \frac{2}{10}\,\frac{h\,N\,\omega\,I}{c\,d}\,\sin\frac{\omega\,l}{2\,c}$$

Using the relations

$$\frac{\omega}{c} = \frac{2\,\pi}{\lambda}$$

and $\sin\frac{\omega\,l}{2\,c} = \frac{\omega\,l}{2\,c}$,

the latter holding when the angle is small, *i. e.*, l small compared with the wave length,

$$H = \frac{4\,\pi^2}{10}\,\frac{h.\,l.\,N.\,I.}{\lambda^2\,d} \tag{10}$$

This is the radiated magnetic field from a sending coil aerial at a distance *d* along the earth's surface, the direction of *d* being in the plane of the coil. The units are international units as stated under equation (8). The deduction assumes that the ground below the coil is not so good a conductor as to form an image of the coil. Thus the formula applies to a radiating coil in an airplane as well as to one at a ground or ship station.

The formula applies for either damped or undamped current I_s in the sending antenna. For very great distances the right-hand side of the formula must be multiplied by the distance correction factor F_1 given in (9), the same as for a radiating antenna.

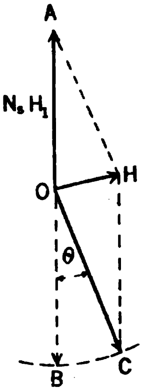


Fig. 5 — Phase Relations of Magnetic Fields Radiated from a Coil

Second Deduction. The radiated magnetic field due to one of the sides of the coil is $N_s H_1$, and from formula (8),

$$H_1 = \frac{2\pi}{10} \frac{h_s I_s}{\lambda d} \qquad (11)$$

If the two vertical sides of the coil coincide, their magnetic fields would be equal and opposite, as shown by the lines *O A* and *O B*, Fig. 5. But since the two vertical sides are separated by the distance *l*, at any instant the field at *P* (Fig. 4) from the left side of the coil has traveled a distance *l* farther than the field from the right side. If then $N_s H_1$ is the field at *P* due to the right side, the field at *P* due to the left side is shifted in phase from the position *O B* to *O C* in Fig. 5, where the angle θ between them is the phase angle between the values of the field a distance *l* apart in the wave.

The distance *l* is the same fraction of the wave length that the angle θ is of a complete cycle, 2π. That is,

$$\frac{\theta}{2\pi} = l_s/\lambda$$

or

$$\theta = 2\pi\ l/\lambda \qquad (12)$$

The resultant of *O A* and *O C* is their vector sum,

$$H = N_s H_1 \sqrt{2\,(1 - \cos\theta)} \qquad (13)$$

When θ is small, *i. e.*, l small compared with the wave length,

$$\frac{H}{N_s H_1} = \sin\theta = \theta$$

$$\therefore H = N_s H_1 \theta \qquad (14)$$

Thus the radiated magnetic field from a coil is equal to the field from one side of the coil multiplied by the phase angle θ corresponding to the distance l between the sides of the coil.

From (**14**) and (**11**),

$$H = \frac{2\,\pi}{10}\,\frac{h_s N_s I_s}{\lambda\, d}\,\theta \qquad (15)$$

This equation, together with (**12**), gives identically formula (**10**) obtained by the first deduction.

It was assumed in these deductions that the current was uniform throughout the coil. If the distributed capacity of the coil is appreciable the current in the coil will be different at different points. Thus the current in the middle may be greater than at the ends. This also may give rise to radiation from the coil, but is an entirely separate phenomenon from the phase angle between the two sides of the coil which has been discussed. This question of distributed capacity requires consideration particularly when coils are used having dimensions comparable with the wave length. The phenomenon is discussed further under "Antenna Effect" in Sec. VI 3 below.

3. RECEIVED CURRENT IN AN ANTENNA

The current flowing in the receiving aerial circuit when the field intensity of the wave traversing the aerial is known can be calculated in several ways. An electromagnetic wave in space has both an electric and a magnetic field intensity which are at right angles to each other and to the direction of propagation of the wave. The two field intensities are related to each other by

$$\mathcal{E} = 300.\,H \qquad (16)$$

where $\mathcal{E}$ is in volts per cm. and H in gilberts per cm.

The following additional symbols are used in this and the following section:

$\mathcal{E}$ = electric field intensity
E = electromotive force in receiving aerial
R = resistance of receiving aerial circuit
ϕ = magnetic flux

First Deduction. The electromotive force, and thence the current produced in an antenna may be calculated from the principle that relative motion of a magnetic field and a conductor create an electromotive force in the conductor whose value is

$$E = 10^{-8} h_r Hc \tag{17}$$

when the directions of the field, the motion, and the conductor are mutually perpendicular, h_r being the length of the conductor and c the velocity of the relative motion. This then gives the e. m. f. in an antenna of height h_r, produced by electromagnetic wave having magnetic field intensity H and travelling with the velocity of c.

In ordinary practise, the reactance in series with the antenna is varied to produce resonance to the frequency of the incoming wave, so that

$$I_r = \frac{E}{R} \tag{18}$$

Inserting for c its value, 3×10^{10} in equation (**17**).

$$I_r = 300 \cdot \frac{h_r H}{R} \tag{19}$$

This is the current in amperes received in a flat-top antenna using the centimeter as the unit of length, with resistance of circuit in ohms, and the magnetic field intensity in gilberts per cm.

The received current is less than that given by the formula if the wave is damped, since an undamped alternating field was assumed in the discussion. For a damped field the e.m.f. acting on the aerial is similarly damped and equation (**18**) does not hold. Correct results are obtained by multiplying the right-hand side of formula (**19**) by the correction factor F_2 obtained as follows:

If the magnetic field intensity and hence the e. m. f. has the decrement δ', the effective current is not I_r, defined by (**18**), but another value which we shall call I_ρ. The value of I_ρ may be found by the aid of the generalized definition of decrement given in the author's paper, "The Measurement of Radio-Frequency Resistance, Phase Difference, and Decrement", *Proc. I. R. E.*, 7, p. 27; Feb. 1919.

For decrements smaller than about 0.2, the logarithmic decrement is one-half the ratio of the average energy dissipated per cycle to the average energy associated with the current at the maximum of the cycle.

Taking the average power as E^2/R, the average energy dissipated per cycle $= \frac{E^2}{fR}$. The average energy associated with the current at the maximum of each cycle $= L\,I_p^2$. The energy-ratio definition of decrement just given applies to the sum of the decrements acting, *viz.*, the decrement δ' of the e. m. f. and the decrement δ of the aerial circuit. The value of δ is $\frac{R}{2fL}$. Applying the decrement definition

$$\delta' + \delta = \frac{\frac{E^2}{fR}}{2\,L\,I_p^2}$$

$$= \frac{E^2\,R}{2fL\,R^3\,I_p^2}$$

$$= \frac{E^2\,\delta}{R^2\,I_p^2}$$

$$I_p^2 = \frac{E^2}{R^2}\,\frac{\delta}{\delta' + \delta}$$

From the relation, $E^2/R^2 = I_r^2$,

$$I_p = I_r\sqrt{\frac{\delta}{\delta' + \delta}} = I_r\sqrt{\frac{1}{1 + \frac{600.\,\mathrm{L}\,\delta'}{R\,\lambda}}}$$

where L is in microhenries and λ is in meters.

This reduces to $I_p = I_r$ when δ' is small compared with δ. Thus in the particular case of an undamped wave, where $\delta' = 0$, no correction is needed.

Correct results are obtained from equation (**19**) for any damped wave by multiplying its right-hand member by the correction factor F_2, given by

$$F_2 = \sqrt{\frac{1}{1 + \frac{600\ .\,L\,\delta'}{R\,\lambda}}} \qquad (20)$$

where L is the inductance of the receiving aerial circuit in microhenries and λ is wave length in meters, and δ' is the logarithmic decrement of the damped wave that is being received.

Second Deduction. The same formula may be derived from entirely independent consideration of the electric field associated with the wave. The e. m. f. between two points in space is the product of the distance between them by the electric field intensity along the line joining them. Thus the e. m. f. produced in a flat-top antenna is $\mathcal{E}$ times the height, the direction of $\mathcal{E}$ being assumed to be vertical.

$$E = h_r \mathcal{E} \tag{21}$$

Inserting the value of $\mathcal{E}$ from (**16**) and dividing by the resistance,

$$I_r = 300 \cdot \frac{h_r H}{R} \tag{22}$$

This is identically the same formula obtained above from consideration of the magnetic field.

4. RECEIVED CURRENT IN A COIL

The current in a receiving coil aerial can be calculated in a number of different ways, all very simple and all giving the same result. The first conception which will be presented is simply that an e. m. f. is produced in the circuit by the time variation of magnetic flux through it.

The other modes of calculation involve the phase angle between the two vertical sides of the coil. The e. m. fs. acting in the two vertical sides are exactly equal and oppose each other in producing a current around the circuit when the plane of the coil is perpendicular to the direction of propagation of the wave. When the coil is turned in any other direction, however, the e. m. fs. in the two sides are not exactly opposite in phase because of the difference in time required for the field to be propagated to one side of the coil and to the other. The e. m. f. can be calculated either from the electric or the magnetic field, as in the discussion of received current in an antenna. The resultant e. m. f. can be found either from the algebraic sum of the instantaneous e. m. fs. in the two vertical sides or the vector sum of the effective e. m. fs. These two methods are used in the second and third deductions respectively, below.

The phase angle between the two sides of the coil is a very different thing from the phenomenon caused by the distributed capacity of the coil. It is assumed in the deductions given here that the current is uniform in all parts of the coil, which is not true when the distributed capacity is appreciable. Such capacity is large in coils of dimensions comparable with the wave length, and in such cases consideration must be given to the separate and additional phenomenon of distributed capacity.

First Deduction. Assuming that the dimensions of the coil are small compared with the wave length, the magnetic field intensity is practically uniform throughout the coil. When the plane of the coil is parallel to the direction of propagation of the wave, the e. m. f. induced in the coil is

$$E = 10^{-8}\,\omega\,\phi$$

$$\text{Now, } \phi = \mu\, h_r\, l_r\, N_r\, H$$

Since the permeability $\mu = 1$, and $I_r = E/R$ because in ordinary practise the condenser in series with the coil is adjusted to produce resonance with the frequency of the incoming wave,

$$I_r = 10^{-8}\,\frac{\omega\,\phi}{R} = 10^{-8}\,\frac{\omega\, h_r\, l_r\, N_r\, H}{R}$$

$$= 2\,\pi\, c\, 10^{-8}\,\frac{h_r\, l_r\, N_r\, H}{R\,\lambda}$$

$$I_r = 600\,.\,\pi\,\frac{h_r\, l_r\, N_r\, H}{R\,\lambda} \qquad (23)$$

This is the current received in a rectangular coil aerial of N turns, with its plane parallel to the propagation of the wave. The units are international units as stated under formula (**19**). No image is assumed in the ground, so the formula applies not only to a receiving coil at a ground or ship station but also to an airplane direction finder. The heights at which airplanes fly are such that the field of the wave is usually not much different from its value at the ground.

There are two correction factors that may need to be applied to this formula, both of which make the result smaller. If the wave is damped, the right-hand side of the formula should be multiplied by the decrement correction factor F_2, given by (**20**), the same as for a receiving antenna.

When the plane of coil is in some direction other than parallel to the direction of propagation of the wave, the right-hand side of formula (23) must be multiplied by the direction correction factor F_3 given by

$$F_3 = \cos \alpha \tag{24}$$

where α is the angle between the direction of propagation of the wave and the plane of the coil.

Second Deduction. The e. m. f. produced in any one of the vertical wires of the coil is given by either equation (17) or (21) above, deduced from considerations of the action of the magnetic and the electric field intensity, respectively. Each of these equations reduces to

$$E_1 = 300.\, h_r H \tag{25}$$

The instantaneous e. m. f. in either of the two vertical sides of the coil is therefore

$$e' = 300\, h_r N_r H_0 \cos \omega t$$

The instantaneous e. m. f. in the other side of the coil is produced by the magnetic field existing in the wave a distance l away, when the plane of the coil is parallel to the direction of propagation of the wave. This e. m. f., e'', has the same direction in space but the opposite direction as far as producing current around the circuit is concerned.

$$e'' = -300 \;.\, h_r N_r H_0 \cos (t - l/c)$$

The resultant e. m. f. in the circuit is the algebraic sum of these two,

$$e = 300 \;.\, h_r N_r H_0 2 \sin \omega \left(t - \frac{l}{2c} \right) \sin \frac{\omega l}{2c}$$

The effective value of the resultant e. m. f. is

$$E = 600 \;.\, h_r N_r H \sin \frac{\omega l}{2c}$$

Since when the angle is small, *i. e.*, l small compared with the wave length,

$$\sin \frac{\omega l}{2c} = \frac{\omega l}{2c} = \pi \frac{l}{\lambda}$$

$$E = 600 \;.\, \pi \frac{H_r l_r N_r H}{\lambda} \tag{26}$$

Dividing by R, this gives the identical value of I_r obtained in (23) by the first mode of deduction.

Third Deduction. The e. m. f. produced in one of the sides of the coil is $N_r E_1$, where from either equation (17) or (21) above, *i. e.*, from consideration of either the magnetic or the electric field intensity, respectively,

$$E_1 = 300\ .\, h_r H \tag{25}$$

If the two vertical sides of the coil coincided, the e. m. fs. produced in them would be equal and exactly neutralize each other, as shown by the lines $O\,A$ and $O\,B$, Fig. 6. But since the two vertical sides are separated by the distance l, at any instant the field acting on one side of the coil has traveled a distance l farther than that acting on the other side. If then $N_r E_1$ is the e. m. f. in one side of the coil, the e. m. f. in the other side is shifted in phase from the position $O\,B$ to the position $O\,C$ in Fig. 6, where the angle θ between them is the phase angle between the values of the field a distance l apart in the wave.

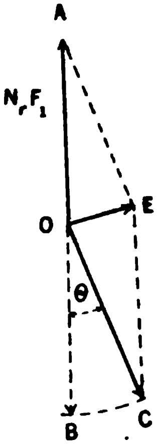


Fig. 6—Phase Relations of Electromotive Forces in Receiving Coil

The distance l is the same fraction of the wave length that the angle θ is of a complete cycle, $2\,\pi$, *i. e.*,

$$\frac{\theta}{2\,\pi} = \frac{l\,r}{\lambda} \tag{27}$$

The resultant of $O\,A$ and $O\,C$ is their vector sum

$$E = N_r E_1 \sqrt{2\,(1 - \cos\theta)} \tag{28}$$

When θ is small, *i. e.*, l small compared with λ

$$\frac{E}{N_r E_1} = \sin\theta = \theta$$

$$\therefore E = N_r E_1\, \theta \tag{29}$$

From (25),

$$E = 300\, h_r N_r H\, \theta \tag{30}$$

Thus the e. m. f. acting in the coil is equal to the e. m. f. in one side of the coil multiplied by the phase angle θ corresponding to the distance l between the sides of the coil.

Equations (27) and (30) combined give (26) and by dividing by R formula (23) is obtained.

III. DISCUSSION OF THEORY OF RADIATION AND RECEPTION

1. Distinction between Induction and Radiation

Certain fallacies which have appeared in text-books and discussions arise from insufficient understanding of the difference between an induction field and a radiation field. Such fallacies are:

a. An "open" circuit can radiate, while a "closed" circuit can not.

b. There is no radiation from a circuit at low frequencies.

c. Induction and radiation are the same phenomenon.

d. The action of an antenna differs from that of a coil aerial in that the former is due to electrostatic fields and the latter to magnetic fields.

These fallacies will now be discussed. Fallacy *c* has led to the supposition that the radiation and reception of electric waves can be taught in terms of transformer action. It should not be difficult to separate the two ideas, for there is a definite and clear distinction between the field due to induction and that due to radiation. The total magnetic field at a distance *d* from a radiating antenna is, from equation (5)

$$H = \frac{2\pi}{10} \frac{h_s I_s}{\lambda d} + \frac{j}{10} \frac{h_s I_s}{d^2} \tag{31}$$

where j indicates that the two terms differ in phase by 90 deg. The first term represents the radiation field and the second term the induction field. The fact that one contains λd in the denominator while the other contains d^2 makes them radically different in nature. This gives the mathematical distinction between induction and radiation. The physical difference is discussed in Sec. 3 below.

The radiation field becomes relatively more important than the induction field as the distance *d* is increased or as the wave length is diminished (*i. e.*, the frequency increased). The question whether radiation or induction predominates in any given case can be settled by calculation from the formula. Thus, the two fields are equal at a distance

$$d = \frac{\lambda}{2\pi}$$

For points closer to the antenna than this the induction field predominates. For points farther away, the radiation field predominates and the induction field falls off rapidly with distance and becomes negligible.

Certain early experiments in wireless signalling used true induction, *e. g.*, the induction telegraphy of Preece and of Dolbear. When higher frequencies were used by later experimenters, signals of appreciable strength were received at distances of several wave lengths. These were genuine radiation signals, now commonly called radio.

2. IS RADIATION LIMITED TO HIGH FREQUENCIES?

The answer to this should be obvious from formulas (**8**) and (**10**). The radiated field does not become zero, no matter how great λ is. For alternating current of any frequency, no matter how low, radiation takes place from the circuit. To be sure, the radiation is greater the higher the frequency, so that high-frequency circuits are better radiators than low-frequency ones, and this is all the basis there is for the mistaken idea that only high-frequency circuits radiate.

This applies to radiation from a coil as well as from an antenna. It has sometimes been stated that a coil will not radiate, the statement being put in the form that only "open" circuits radiate. The statement is doubly faulty since electricity can flow only in closed circuits. The meaning intended by "open" circuit is a circuit containing a condenser of open form, that is, with two plates well separated. There are two misconceptions at the base of the belief that a "closed" circuit or one not containing a condenser would not radiate. In the first place, some have doubtless thought that waves would be started in the ether only by an electrostatic disturbance and thus could not be produced by a metallically closed circuit. Or, supposing it was understood that a magnetic disturbance in the ether would send out a wave just as readily as an electrostatic disturbance, it may have been thought that the radiation from one side of the circuit would be neutralized from that from the opposite side. As has already been shown in this paper, the two disturbances do not exactly neutralize each other, on account of the finite time of propagation from one side of the circuit to the other, and the resultant is what gives rise to the radiation from a metallically closed circuit.

3. EQUIVALENCE OF ELECTROSTATIC AND MAGNETIC FIELDS IN A WAVE

The physical distinctions between radiation and induction are: (a) the latter is fixed in space and the former moves through space with the velocity of light, and (b) in the case of radiation the magnetic field is always accompanied by an electrostatic field of value

$$\mathcal{E} = 300 \; . H \tag{16}$$

and vice versa, whereas in the case of induction there is no fixed relation. It is, of course, true that whenever magnetic induction varies an electrostatic field is produced, and similarly whenever electrostatic induction varies a magnetic field is produced. But it is only in a radiated wave that these variations take place in such a way that one can be calculated from the other by the fixed relation (**16**). When there is a fixed electrostatic field associated with a circuit which does not vary, the magnetic field associated with this electrostatic field is zero, and vice versa.

In a radiated wave, then, the electrostatic and magnetic field are no longer independent phenomena but are strictly equivalent. Indeed, they are but two aspects of the same thing. Perhaps this will be clearer from the analogy of a sound wave. In a mechanical apparatus, elastic action and inertia act independently in various parts of the apparatus. In a sound wave, however, the effects of elastic action and inertia are mutual parts of a single phenomenon, the sound wave.

In considering any effect of the electromagnetic wave, it is equally permissible to consider the electrostatic or the magnetic field associated with the wave. They are equivalent and lead to the same result. This has been amply demonstrated above in this paper. The current received in an antenna, calculated from the electrostatic field, was exactly the same as calculated from the magnetic field. The same agreement was found for the coil aerial. This disposes of the question whether the current produced in an antenna or a coil aerial is caused by the electrostatic or the magnetic field present in the wave, or both.

Complete discussions of electromagnetic waves are given in such treatises as Maxwell, "Electricity and Magnetism", 1873; Jeans, "Electricity and Magnetism", 1907; Lorentz, "The Theory of Electrons", 1909.

4. WHAT RADIATION IS

It has been shown that radiation differs from induction by a definitely calculable amount, that either kind of circuit radiates at any frequency, that there are both an electrostatic and a magnetic field present in every wave, having a constant ratio, and that any effect of the wave may be considered as due either to the electrostatic or the magnetic field of the wave.

Radiation is the moving disturbance of the ether, the energy associated with which does not return to the radiator.

This conception leads to more correct ideas than are current on the mechanism of radiation from an antenna; and permits explanation of the radiation from a coil aerial, which is not covered at all by the usual explanations of radiation in textbooks. Such explanations have led to the impression that the radiation largely depends on the form of the electrostatic lines of force which are present at the edges of the radiator. It might thus be supposed that in a flat-top antenna or a condenser aerial the current in the central portions of the condenser was not effective in causing radiation while only that which spread into the surrounding space from the edges was effective. This appears incorrect. If it were correct, the builders of long flat-top antennas must have wasted a great deal of wire. All of the dielectric current sends a moving disturbance out into the ether. The portion of the energy associated with this disturbance that does not return to the radiator is that connected with the first term of equation (**31**). In this term the total antenna current appears. The radiation is the moving disturbance caused by the whole of the current which the antenna makes flow in the dielectric.

The ordinary treatment of the mechanism of radiation from an antenna is misleading also because it deals with radiation at the fundamental wave length. In practise, antennas are usually loaded. The radiation depends to no degree whatsoever on the value or location of any of the field lines attached to the aerial, but only on the variation of the lines. And *all* the lines when varying give rise to radiation. Thus the stationary field is given by the second term of formula (**31**), the first is the radiation term, and they are independent.

IV. COMPARISON FORMULAS

1. Derivation from Theoretical Formulas

Formulas are here derived to answer the practical question of how far a given coil will send or receive in comparison with a given antenna. The formulas also answer such questions as the length of a coil aerial required to give a particular ratio of performance of coil and antenna.

The ratio of the magnetic field radiated from a current coil to that from an antenna, for a given sending current, distance, wave length, and height, is obtained from equation (**13**). The ratio of the distance from a coil to that from an antenna, at which a given magnetic field is produced, is the same as the ratio of the magnetic field produced by a coil to that produced by an antenna at a given distance. Either ratio is therefore given by the following expression, which assumes the same current I_s, wave length λ, and height h_s, for the coil and the antenna,

$$d_c/d_a = N_s \sqrt{2\,(1 - \cos\theta)}$$

Inserting the value of θ and neglecting the subscript s,

$$d_c/d_a = N \sqrt{2\,(1 - \cos 2\,\pi\, l/\lambda)} \qquad \textbf{(32)}$$

When the length of the coil l is small compared to λ (*i. e.*, for most practical purposes, less than 0.1 λ), this simplifies to

$$d_c/d_a = 6.28\, N\, l/\lambda \qquad \textbf{(33)}$$

This could have been deduced directly from (**8**) and (**10**). The expression is similarly deduced for comparison of the distances obtained with a coil and an antenna of different heights,

$$d_c/d_a = 6.28\, N\, l/\lambda\;\; h_c/h_a \qquad \textbf{(34)}$$

The length of a coil required to give a particular ratio of performance to an antenna is given by solving these formulas for l. From (**32**),

$$l = \frac{\lambda}{2\,\pi} \cos^{-1}\left(1 - \frac{1}{2\,N^2}\left[\frac{d_c}{d_a}\right]^2\right) \qquad \textbf{(35)}$$

When the length of the coil is small compared to λ, the simpler formula suffices,

$$l = 0.16\, \lambda/N\, d_c/d_a \qquad \textbf{(36)}$$

The relative distances at which an antenna or a coil will receive a given wave are given by the same identical expressions that have just been deduced for sending aerials. Thus formula (**32**) may be deduced from (**28**) and (**33**) from (**19**) and (**23**). They give the ratio of the distance from the source at which a given e. m. f. will be produced in a coil aerial to that in an antenna, assuming the same height h_r and wave length λ for the coil and the antenna. They also give the ratio of the e. m. f. produced in a coil to that in an antenna for a given value of magnetic field intensity, or the ratio of currents when the resistances and other quantities are the same in coil and antenna. Equations (**34**), (**35**), and (**36**) similarly hold for receiving as well as sending aerials. For comparison of current in a coil and an antenna of different resistances as well as different heights

$$d_c/d_a = 6.28\, N\, l/\lambda \quad h_c/h_a \quad R_a/R_c \tag{37}$$

The relative distance of transmission between two coil aerials and between two antennas, for a given sending current, is similarly found from equations (**8**), (**10**), (**19**), and (**23**). The ratio of received current for coils and antennas the same distance apart is given by the same formula, which assumes the same sending current I_s and wave length λ for the pair of coils as for the pair of antennas.

$$\frac{d_{cc}}{d_{aa}} = 39.5\, \frac{l_s\, l_r\, N_s\, N_r}{\lambda^2} \quad \frac{(h_s\, h_r)\, c}{(h_s\, h_r)\, a} \quad \frac{R_a}{R_c} \tag{38}$$

All of these formulas assume that the decrement correction factor F_2 is the same for coil and antenna in all cases. If waves of different decrement are used, apply the factor F_2 as stated in connection with (**20**). If the plane of the coil considered is not parallel to the direction of propagation of the wave, apply the factor, cos α, as stated in connection with (**24**).

2. EXAMPLES OF COMPARISON OF COIL AND ANTENNA

What is the length of the coil, either as sender or receiver, equivalent to an antenna of the same height? The answer is given by (**36**). For $d_c/d_a = 1$,

$$l = 0.16\ \lambda/N \tag{39}$$

This is the correct length except for a single-turn coil. Then $N = 1$ the more exact formula (**35**) must be used. This gives, for the equivalent coil,

$$l = 1/6\ \lambda$$

Thus a single-turn coil of length 1/6 the wave length is equivalent to an antenna of the same height. For a coil of 8 turns, however, the length of the coil equivalent to an antenna of the same height is, from (**39**), 0.02 of the wave length.

When the length of the coil is small compared with the wave length, *i. e.*, as already stated, l less than about 0.1 λ, the performance ratio is given by (**33**). For a length greater than 0.1 λ, however, the more accurate formula (**32**) must be used.

Thus, when the length is exactly a sixth of a wave length, from (32),

$$d_c/d_a = N$$

Thus any coil of length 1/6 the wave length is equivalent to an antenna of N times the height of the coil. When the length of the coil is a quarter wave length, similarly

$$d_c/d_a = \sqrt{2}\,N$$

For a coil of length equal to half the wave length,

$$d_c/d_a = 2\,N$$

This is the maximum or best performance for a coil aerial. If the length is increased beyond a half wave length, the performance ratio decreases, and at $l = \lambda$ it is equal to O just the same as for $l = O$.

These values of the performance ratio of a coil aerial are obvious from Fig. 5 or 6.

These comparisons all apply to either transmitting or receiving aerials. They assume, however, in the case of a transmitting antenna or coil, that the same current flows; and, when applied to receiving aerials, that the resistance is the same in either coil or antenna. As a matter of fact, however, it is easy to secure a considerably lower resistance in a coil aerial circuit than in an antenna circuit. This is taken account of by the factor R_a/R_c as in (37) and (38). The difference in current in a transmitting coil and antenna is taken account of by multiplying the right-hand members of (32) and (33) by the ratio of the sending currents I_c/I_a. On this account a coil is sometimes a more effective radiating or receiving device than an antenna of considerably greater dimensions.

The comparison formulas and conclusions drawn from them are subject to the same errors as the transmission formulas, as discussed in Sec. V 2 below.

3. THE CONDENSER AERIAL

Since the dimensions of a coil aerial which would give the same performance as a given antenna are a length equal to $\frac{0.16}{N}$ times the wave length and a height equal to the antenna height, rather large structures are required. For example, a flat-top antenna 30 meters above the ground operating on a 600-meter wave is equivalent to a four-turn coil 24 meters long by 30 meters high. The dimensions of the equivalent coil are thus of the same order as the dimensions of the antenna.

It is possible to escape from the apparent necessity of large structures for effective radio transmission and reception in two ways. First, the coil aerial can easily be made to have a lower resistance than the antennas ordinarily used, and its size reduced in proportion to the reduction of resistance. This is mainly because the condenser used in the coil aerial circuit can be one having practically no resistance while the condenser consisting of antenna and ground has a large resistance. Thus by due attention to the minimizing of resistance in its circuit, the coil aerial may be of small dimensions and yet highly effective. The size may, of course, be reduced also in proportion as the number of turns is increased.

It is equally possible to avoid an aerial of large dimensions without having recourse to a coil aerial. The alternative is to use the antenna principle, but use a special construction of much lower height. At first sight it would appear that this would make a poorer antenna, since the effectiveness is proportional to the height, according to either (8) or (19). And this is true if the antenna is merely lowered a moderate amount. Such lowering increases the capacity only very slightly, not nearly in proportion to the decrease in height. In order to secure an appreciable gain it is necessary to have the height very small and use a special construction to reduce the resistance as much as possible. A good method is to replace the ordinary antenna-ground structure in which the antenna is one plate of a condenser and the ground the other plate, by an aerial consisting of two horizontal metal condenser plates. This may be called a "condenser aerial". The formulas derived for antennas apply to it.

Such an aerial has lower conductor resistance than the ordinary antenna, and since it has greater capacity a small inductance will be used in series with it which will also have smaller resistance and thus reduce the resistance of the circuit. Furthermore, the resistance of an antenna largely arises from the imperfect dielectrics, such as vegetation, buildings, and poor insulators, present in its field (as shown in Scientific Paper of the *Bureau of Standards* No. 269, by J. M. Miller), and the resistance from the grounding wires to ground. These can be eliminated in a condenser aerial. Finally, then, the resistance of the aerial circuit can be reduced to more than compensate for the reduction in height. This will result in a larger current I_r in formula (**19**), or in a larger H in formula (**8**) because of the increase of the sending current.

The advantage of the very low antenna has been observed in the experiments of the Kiebitz and others on so-called earth antennas. It is probable that still greater advantages would be obtained by the condenser aerial as here described. The special construction required to eliminate dielectric loss would involve making the lower plate considerably wider and longer than the upper plate, or else having both plates a considerable distance above the ground, and keeping the space between the plates free from poor dielectrics. An aerial consisting of a pair of metal plates elevated from the ground was used and described by Oliver Lodge in 1897, and again by Lodge and Muirhead in *Proc. Royal Soc.*, 82, p. 227; 1909, who found that it worked best without being grounded. The author is informed that the same sort of an aerial has recently been tried on airplanes, using the upper and lower planes as the condenser plates. Such an aerial would be ideal for airplanes if the space between could be kept free from poor dielectrics. If the plates of the condenser aerial have their length and width approximately equal, the aerial radiates in all directions. If a long narrow condenser is used it would probably be very directional, both as a transmitting and receiving device. Such a condenser might consist of a pair of parallel wires, which would be a considerable improvement on the ground antenna.

An example will make clear how the size of the condenser aerial compares with other aerials. It was found above that antenna 30 meters high was equivalent to a four-turn coil 24 meters long by 30 meters high, both operating on a 600-meter wave and with circuits of the same resistance. For the same wave length and with an inductance of 100 microhenries, in series, the capacity of a condenser aerial would need to be 0.00102 microfarad, which would be given by a pair of square plates 1 meter apart and 10.7 meters on a side. The height is thus reduced in the ratio of about 25 to 1, and the horizontal dimensions 3 to 1 in comparison with the coil aerial.

The aerial can be made as small as desired. If a given coil is to be used in series, the capacity of the aerial is maintained constant by reducing the distance between the plates when the area of the plate is reduced. The author made some interesting experiments with a small condenser aerial as a receiving device, used inside the laboratory with no ground connection. The plates consisted of copper netting. The top plate was 250 cm. square and the distance between them was 15 cm. The signals received, with either a crystal detector or electron tube, were roughly of the same intensity as those received with a simple coil aerial of the type and size ordinarily used as a direction finder.

The indication of absolute direction of propagation of the waves as well as line of propagation which has been developed by French and other workers, using combinations of ordinary antenna and coil aerials was observed in the experiments on the condenser aerial. An inductance coil of rather large dimensions used in series with the condenser acted as a receiving aerial. As this coil was rotated, the signal varied from maximum in one angular position to zero in a position 180 deg. from the first, instead of 90 deg. as occurs when a coil aerial is used independently of any antenna action. Apparently the action of the condenser aerial reinforced that of the coil in one position and neutralized it in the opposite position. When the connections to the coil were interchanged, the effect shifted 180 deg. Reversing the connections of the coil reverses the e. m. f. in the coil, *E*, in Fig. 6, just as a reversal of the direction of the wave would do, whereas the direction of the e. m. f. in the antenna or condenser aerial is unchanged. The reason why the condenser e. m. f. can neutralize the coil e. m. f. is probably that the capacity of the coil introduces different values of reactance to the two e. m. fs. Thus, when the circuit is tuned for one of these e. m. fs. the currents due to the two differ 90 deg. in phase. This phase angle may be shifted 180 deg. by a very slight variation of the reactance of the circuit. Because of this, systems for determining the absolute direction of radio waves require very delicate adjustment.

The ordinary laboratory type of condenser used in radio circuits does not function as a condenser aerial. This is because the interleaving of the plates results in the current in each portion of the dielectric being balanced by the current in a neighboring portion. This is discussed further below in Sec. VI 3 and illustrated in Fig. 17.

V. TRANSMISSION FORMULAS

1. Statement of Formulas

The current received in any aerial may be calculated in terms of the current in any transmitting aerial, either antenna or coil by the following four formulas. They are derived by combining equations (8), (10), (19), and (23). The symbols are as previously given, also stated in the Appendix below.

Antenna to antenna:

$$I_r = \frac{188 \cdot h_s h_r I_s}{R \lambda d} \tag{40}$$

Antenna to coil:

$$I_r = \frac{1184 \cdot h_s h_r l_r N_r I_s}{R \lambda^2 d} \tag{41}$$

Coil to antenna:

$$I_r = \frac{1184 \cdot h_s l_s h_r N_s I_s}{R \lambda^2 d} \tag{42}$$

Coil to coil:

$$I_r = \frac{7450 \cdot h_s l_s h_r l_r N_s N_r I_s}{R \lambda^3 d} \tag{43}$$

Formulas such as (40) have existed heretofore. The formulas here given generalize the antenna-to-antenna formula, so that calculations can be made for any kind of aerials.

The lengths in these formulas may be in any units, provided the same unit is used for all the lengths. The meter is usually the most convenient unit. If the heights and wave length are in meters and the distance d in miles, the four constants in the four formulas become respectively:

0.117
0.736
0.736
4.63

To calculate the distance at which a given current will be received, as when a particular receiving arrangement is specified, the formulas may be stated explicitly for d. I_r and d are interchanged in each formula. For example, the formula for antenna-to-coil (41) becomes

$$d = \frac{1184 \cdot h_s h_r l_r N_r I_s}{R \lambda^2 I_r} \tag{44}$$

All of these transmission formulas are for daytime transmission. Greater values are obtained at night, probably because the waves are reinforced by reflection from ionized layers of the upper atmosphere, which are broken up by sunlight in the daytime. The formulas are all subject to correction factors for distance and for decrement. If the distance is very great (in ordinary cases, over 100 kilometers), the right-hand side of the formula should be multiplied by the correction factor F_1. The value given below for F_1 is for transmission over sea water. Its value for transmission over land would be greater. If damped waves are used, the correction factor F_2 should be similarly applied. Furthermore, if the plane of the receiving coil is not parallel to the direction of propagagtion of the wave, the correction factor F_3 must be similarly applied to formulas (41) and (43) and related formulas such as (44). In formulas (42) and (43) the direction of the wave is taken to be that of the plane of the transmitting coil. The three correction factors are:

$$F_1 = \epsilon^{-0{\cdot}000047\, d/\sqrt{\lambda}} \tag{9}$$

$$F_2 = \sqrt{\frac{1}{1 + \frac{600\ .\ L\,\delta'}{R\,\lambda}}} \tag{20}$$

$$F_3 = \cos\alpha \tag{24}$$

All of the correction factors make the resultant numerical values smaller.

2. DISCUSSION OF TRANSMISSION FORMULAS

The power of wave length in the denominator is different in the several formulas. Thus when a coil aerial is used for both transmitter and receiver the received current is inversely proportional to the cube of the wave length. Thus transmission between coils is better the shorter the wave length. This advantage of coils at short wave lengths applies only for short-distance transmission. When the distance is hundreds or thousands of kilometers, the increased absorption of the waves makes the correction factor F_1 so great that short waves are impractical, so for long distances the comparison favors the antenna rather than the coil. The coil compares most favorably with the antenna, then, for transmission over short distances with very short waves. This is subject to the proviso that current of the same order of magnitude can be gotten into a transmitting coil aerial as into an antenna, or that the resistance of a receiving coil is the same as that of a receiving antenna. Neither of the assumptions is wholly fulfilled, in practise, with the result that the difference of applicability of the two kinds of aerials at long and short wave lengths is less marked. For additional comparisons of antennas and coils and further discussion, see Sec. IV above.

Limitations of Formulas. The formulas can not be expected to give results of great accuracy, certainly not better than a few per cent, because of the ideal conditions assumed in their derivation. Thus it is assumed that no image of the aerial exists in the ground beneath it, that is, the ground is not perfect as a conductor. As a matter of fact, the ground varies greatly in conductivity; and while in most cases the currents induced in the ground below a transmitting or receiving aerial probably have very little effect, these currents may be appreciable in some cases. This is discussed further below under "Height of Aerial." On account of the uncertainty introduced by the ground, the formulas may apply better to airplane aerials than to those on ships or on land.

There are other sources of uncertainty in the application of these formulas. An antenna does not form a flat-plate condenser with the ground of such form that the curving of the field at the edges can be neglected. The simple method of calculating the radiated field is thus in doubt. Similarly, in the case of a radiating coil, the field from the top and bottom of the coil may have some effect at a distance, which has not here been taken into account. It is not certain with how great propriety the earth's surface can be taken to be equivalent to the equatorial plane of the radiating aerial. Frequently radio waves have a wave front that is tilted and not perpendicular to the earth's surface as assumed in the calculation of received current. Furthermore, the formulas assumed uniform current throughout the aerial, which sometimes does not hold because the antenna may have a vertical portion of appreciable capacity or the coil may have rather large distributed capacity. Calculations involving coil aerials are subject to the additional uncertainty arising from the capacity of the coil to ground or the surroundings so that it acts like an antenna as well as a coil. This is discussed under "Antenna Effect" in Sec. VI 3. Another difficulty discussed in the same Sec. is the effects of surrounding objects.

With these departures in the action of the aerials and the behavior of the waves from the conditions assumed, it is impossible to calculate received currents with great accuracy. It is almost surprising that the experimental results check the formulas as closely as shown in Sec. VI 2 below.

Height of Aerial. The value used for h is the length of the vertical side of a coil aerial, the distance from the surface of the ground to the flat top of an antenna, or the vertical distance between the two flat plates of a condenser aerial. In previous discussions it has been assumed that the ground beneath an antenna was a perfect conductor and thus the height of the radiator was twice the value of the h defined here. Experiment however corroborates the view here taken, which assumed that the radiating structure is independent of the earth, the waves becoming attached to the earth soon after leaving the antenna. In the present state of our knowledge the most satisfactory conception is that the radiating structure is the actual structure above the ground level. (Questions of the conductivity of the ground, presence of earth currents, etc., near the radiating aerial, are expressly not considered).

Austin's empirical formula[1] for antenna-to-antenna transmission is equation (40) with a constant twice as great, and quantities h_1 and h_2 used instead of h_s and h_r. These quantities h_1 and h_2 are the "height to the center of capacity" of the transmitting and receiving antenna, respectively. This height is not defined, but its value for any particular antenna is the value that is required to make experimental results fit the formula. Now, as has been stated, such experiments as have been performed agree in general with formula (40). For instance, see the first two examples in Sec. VI 2 below. It must follow since the constant in Austin's formula is twice as great as the constant in (40) that

$$h_1 h_2 = 1/2\, h_s h_r$$

This may be satisfied by various values of h_1 and h_2. One set of values would be

$$h_1 = 0.5\, h_s$$
$$h_2 = 1.0\, h_r$$

Another would be

$$h_1 = 0.707\, h_s$$
$$h_2 = 0.707\, h_r$$

Austin's values for the height of various antennas thus deduced in such a way as to make them fit the experimental values

---

[1] Scientific Paper of the Bureau of Standards No. 226, Equation 5.

observed, do in fact vary from half to full value of the actual antenna heights, and average around 0.7 the actual heights. It is much simpler and more direct to use the formula and the interpretation presented in this paper, bearing in mind that it is subject to the uncertainties introduced by the varying character of the ground.

The idea that the ground is not a good enough conductor to form an image of a transmitting aerial, and that the waves become attached to the ground after leaving the aerial, is in harmony with the ideas of Lodge and Muirhead, already referred to. They found that they got better transmission by using what amounted to a condenser aerial, elevated from ground, with no ground connection. This conception conflicts with the commonly accepted view that Marconi's achievement was the connection of a radiating system to the ground. What then was Marconi's achievement? The best answer to this may be one stated to the author by Prof. A. E. Kennelly, *viz.*, the use of a large radiating system, arranged vertically.

VI. EXPERIMENTAL VERIFICATION OF FORMULAS

1. PRINCIPLES OF MEASUREMENT OF RECEIVED CURRENT AND VOLTAGE

The formulas presented in this paper not only make it possible to calculate approximately the field intensity produced or current received with given aerials, but also give the basis for determining what constants to select for the circuit of a particular aerial to secure the maximum effect. In other words, these formulas furnish the principles of design of aerial circuits. There are a great many points not obvious from mere inspection of the formulas, which are of importance equally in design and in the measurement of received signals. These will now be considered. While this discussion is limited to what takes place in receiving aerials, the same principles and treatment can readily be applied to transmitting aerials.

The received current or voltage can be measured in a number of different ways. It is important to know just what quantity is being considered or measured. Suppose an indicating instrument G, which may be a galvanometer or a telephone receiver, is connected to a rectifying device D in parallel with the condenser of the receiving circuit, as in Fig. 7, where either L is a coil aerial or else C is an antenna or condenser aerial. Does the indication of the instrument measure directly (a) the e.m.f. which the wave causes to act on the circuit $C\,L$, (b) the current in the circuit, or (c) the voltage across the condenser? The answer is, of course, none of these things. The system can, however, conveniently be calibrated in terms of the voltage across the condenser. This voltage V is related to the received current I_r by the relation

$$V = \frac{I_r}{\omega\, C} \tag{45}$$

and since I_r is related to the e. m. f. acting by

$$I_r = E/R, \tag{46}$$

the relation of V to E is

$$V = \frac{E}{R\,\omega\, C} \tag{47}$$

When a detecting apparatus like that of Fig. 7 is used, in which the deflections or signals depend directly on the voltage across the condenser, the results obtained with various receiving circuits will be entirely different from those obtained when the current in the circuit is directly measured, as in Fig. 8. Equations (46) and (47) show at once that the effects of varying the constants of the receiving circuit will be different, depending on whether it is E, I_r, or V that is being measured. These three quantities for a receiving antenna are, from equations (17), (46), and (47), for unit magnetic field intensity,

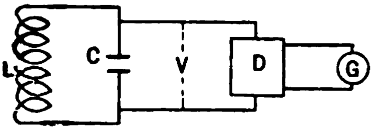


FIG. 7—AERIAL CIRCUIT WITH DETECTING APPARATUS ACROSS THE CONDENSER

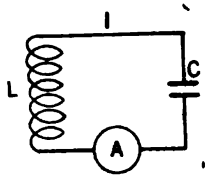


FIG. 8—AERIAL CIRCUIT WITH CURRENT MEASURING INSTRUMENT IN CIRCUIT

$$E_a = 300\ .\ h_r \tag{48}$$

$$I_a = 300\ .\ h_r/R \tag{49}$$

$$V_a = 300\ .\ \frac{h_r}{R\,\omega\, C} \tag{50}$$

The quantity h_r may be called the "e. m. f. reception factor" for an antenna, the e. m. f. in the receiving circuit is proportional to it. Similarly h_r/R may be called the "current reception factor" since it determines the received current. And $\frac{h_r}{R\,\omega\,C}$ or the equivalent $\frac{h_r\,L}{R\,\lambda}$ may be called the "voltage reception factor" of an antenna since it determines the voltage across the antenna.

The most favorable or optimum value of any of the variables that determine the antenna e. m. f., current, or voltage, can be determined either by direct experimental measurement of their values when actually receiving or by calculation from the reception factors. It is desired to learn simply what will produce the maximum E_a, I_a, or V_a. For example, it is obvious that E_a increases indefinitely as h_r increases, but more careful consideration is required to determine what will be the effect on the received current of increasing h_r. The reception factors furnish an alternative to direct reception measurements, requiring instead measurements upon the constants of the aerial circuit.

Coil Aerial Reception Factors. The e. m. f. applied by the passing wave to a coil aerial, the current in the circuit, and the voltage across the condenser are, from equations (**26**), (**46**), and (**47**), for unit magnetic field intensity,

$$E = 600\ .\ \pi\,\frac{a^2\,N}{\lambda} = \frac{1}{10^8}\,\frac{a^2\,N}{\sqrt{C\,L}} \qquad \textbf{(51)}$$

$$I = 600\ .\ \pi\,\frac{a^2\,N}{R\,\lambda} = \frac{1}{10^8}\,\frac{a^2\,N}{R\,\sqrt{C\,L}} \qquad \textbf{(52)}$$

$$V = 36\ .\ \pi^2\,10^{12}\,\frac{a^2\,N\,L}{R\,\lambda^2} = \frac{1}{10^8}\,\frac{a^2\,N}{R\,C} \qquad \textbf{(53)}$$

These equations assume the coil to be square having both height and length $= a$. For a coil that is not square, the formulas apply, replacing a by $\sqrt{h_r\,l_r}$. Two values are given for each reception factor; the first of the two is the more useful, since it is more common to consider the dependence of the reception on λ than on C.

$$\text{E. m. f. reception factor} = \frac{a^2\,N}{\lambda} \qquad \textbf{(51)}$$

$$\text{Current reception factor} = \frac{a^2\,N}{R\,\lambda} \qquad \textbf{(52)}$$

$$\text{Voltage reception factor} = \frac{a^2 N L}{R \lambda^2} \qquad (53)$$

Design of Receiving Coil Aerials. The response produced in a coil aerial circuit may be measured in a great variety of ways. In the first place, it may be considered either from the viewpoint of the e. m. f. acting, the current, or the voltage across the condenser. The first of these, the e. m. f., E, may be determined for any particular case from the e. m. f. reception factor in (**51**). The current I or voltage V may each be determined in four different ways: (a) by direct measurement with a suitable instrument, (b) by measurement of the quantities which make up the appropriate reception factor, (c) by measurement of signal strength in some such device as sketched in Fig. 7, which has been calibrated in I or V, (d) from a "signal intensity reception factor," which can be calculated for any signal measuring device when the law connecting the signal intensity with either I or V is known.

The design of a receiving coil requires knowledge of the dependence of the current or voltage upon the dimensions, etc., of the coil. Measurements made in all of the ways just enumerated give results in agreement with one another, provided due care is given to the avoidance of errors. The sources of error are numerous, as discussed in Sec. 3 below.

While direct measurement of the received current or voltage can be replaced by calculation from the reception factors, the fact remains that the design of an aerial requires experiments. This is because the quantity R in the reception factors cannot be obtained by calculation. It must be obtained by measurement for the particular coil and mode of connections employed.

Measurements upon receiving aerials to determine their constants and the best design for given conditions constitute a most interesting study. In later publications, the results of experiments will be published giving such data for typical coil aerials.

The capacity C in the formulas is the total capacity of the circuit, including the capacity of the coil, L is the pure inductance of the coil, and R is the actual resistance of the circuit. R includes the resistance of the conductors, effective resistance of the condenser and of the coil capacity, effective resistance of the detecting apparatus, and radiation resistance. All of these vary with frequency, and thus measurement of R at the frequency concerned is necessary. On account of the complexity of the quantities entering into the total R, its measurement is no easy matter. The capacity of the coil and other stray capacities may easily vitiate the measurement of R, C, or L. The effect of the detecting apparatus always requires most careful consideration. Even if D in Fig. 7 is an electron tube, it is necessary to consider the resistance which it introduces into the aerial circuit.

Dependence of Received Current and Voltage on Dimensions of Coil and Wave Length. Let R_c = resistance of coil and R_x = resistance external to coil;

$$R = R_x + R_c$$

$$\text{Current reception factor} = \frac{a^2 N}{(R_x + R_c)\,\lambda} \qquad (54)$$

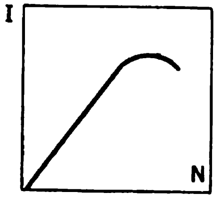


FIG. 9—DEPENDENCE OF RECEIVED CURRENT ON NUMBER OF TURNS

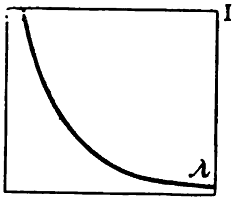


FIG. 10—DEPENDENCE OF RECEIVED CURRENT ON WAVE LENGTH WHEN EXTERNAL RESISTANCE IS LARGE

The variation of received current with number of turns, wave length, and size of coil is readily found by considering the the variation of the quantities in (**54**). In the following discussions the spacing between turns of wire, which affects resistance and inductance, is assumed constant.

a. Varying N, with λ and a constant. When R_x is large compared to R_c, we see from (**54**)

$$I \,\alpha\, N$$

When R_c is large compared to R_x, since $R_c \,\alpha\, N$, roughly,

$$I \,\alpha\, \text{constant}$$

However, R_c increases somewhat faster than proportional to N as N is increased, because of the proximity of the added turns, and hence I decreases somewhat as N increases instead of being strictly constant.

R_s is likely to be large compared with R_c when N is very small, and hence for small N, the variation of I with N will be a straight line, as shown in Fig. 9. As N increases, R_c becomes large compared to R_s and the tendency of I to increase with N is reversed. As a result the curve of I has a maximum. The value of N at this point may be called the "optimum number of turns." Its value will be greater the greater the external resistance.

b. Varying λ with N and a constant. When R_s is large compared to R_c and does not vary appreciably with wave length

$$I \alpha 1/\lambda$$

This variation is shown in Fig. 10. When R_c is large compared to R_s, however, since $R_c \alpha \frac{1}{\sqrt{\lambda}}$, roughly,

$$I \alpha \frac{1}{\sqrt{\lambda}}$$

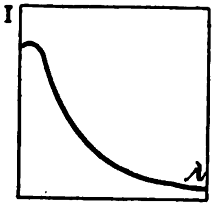


Fig. 11—Dependence of Received Current on Wave Length when Coil Resistance is Large

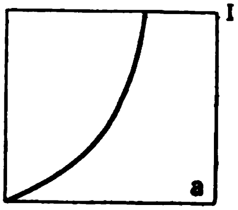


Fig. 12—Dependence of Received Current on Size of Coil when External Resistance is Large

However, the effect of the adjacent turns increases R_c faster than stated as λ is diminished and hence I tends to approach a constant value for short wave lengths, as shown in Fig. 11. These conclusions may, however, be vitiated by the variation of R_s with λ.

c. Varying a, with N and λ constant. When R_s is large compared to R_c,

$$I \alpha a^2$$

This is shown in Fig. 12. When R_c is large compared to R_s, since $R_c \alpha a$,

$$I \alpha a,$$

giving the straight line in Fig. 13. From these two extreme cases it follows that an actual curve is likely to have a form that is a combination of these two, as shown in Fig. 14.

d. Varying a, with λ constant, allowing N to vary in such a way that length of wire is constant. The condition is that $N \alpha 1/a$,

When R_x is large compared to R_c,

$$I \alpha a$$

The curve of I is thus a straight line. When R_c is large compared to R_x, the same conclusion holds but only roughly. R_c increases slightly as a is decreased because of the proximity of the added turns, hence I increases a little faster than proportional to a. This is shown in Fig. 15.

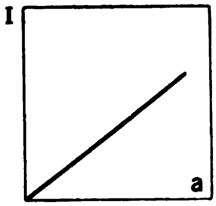


FIG. 13—DEPENDENCE OF RECEIVED CURRENT ON SIZE OF COIL WHEN COIL RESISTANCE IS LARGE

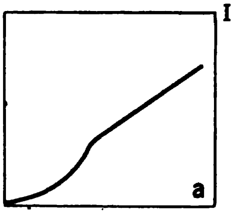


FIG. 14—DEPENDENCE OF RECEIVED CURRENT ON SIZE OF COIL IN TYPICAL CASE

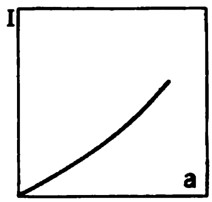


FIG. 15—DEPENDENCE OF RECEIVED CURRENT ON SIZE OF COIL WHEN LENGTH OF WIRE IS KEPT CONSTANT

The voltage reception factor differs from the current reception factor by L/λ.

Thus,

$$V \alpha \text{ Voltage reception factor } = \frac{a^2 N L}{(R_x + R_c)\lambda^2} \qquad (55)$$

It is thus a more complex problem to determine the variation of voltage with N, λ, and a, because the variation of L must be considered in addition. This may be done in each case, just as was done above for current, taking into account the relations:

$$L \alpha N^{2-} \qquad R_c \alpha N^{1+}$$

$$L \alpha \lambda^{0-} \qquad R_c \alpha \frac{1}{\lambda^{\frac{1}{2}+}}$$

$$L \alpha a^{1+} \qquad R_c \alpha a^1$$

where the + or − sign after the exponent indicates that the actual power is slightly greater or less than that given.

The relations obtained for voltage are similar to those obtained for current. There are some characteristic differences as, *e. g.*, with varying N the optimum number of turns comes out greater than for received current. Thus when the detecting apparatus depends essentially on the current the size of the coil should be as large as possible, whereas when it depends essentially on the voltage across the condenser the number of turns should be as large as possible.

On the whole, the received current or voltage or signal intensity is increased by increasing the dimensions (N and a), and by decreasing the wave length. These conclusions are subject to the limitation, discovered by French experimenters, and qualitatively obvious from the known increase of R near the natural wave length λ_0 of a circuit, that poor results are obtained if $\lambda_0 > 1/3\ \lambda$. Thus the dimensions of the coil can not be increased, or the wave length decreased, indefinitely. Beyond the limit mentioned, in fact, better results are obtained by decreasing the dimensions of the coil.

It is an interesting fact that these discussions apply not only to the design of a coil aerial for receiving signals but that they also solve the problem of design of wavemeters. The choice of constants of a wave meter coil for any particular case is settled by the formulas and ideas here presented. The considerations given for received current and voltage apply respectively to the design of wavemeters for measurements upon undamped or damped waves, *i. e.*, to the securing of minimum resistance and minimum decrement respectively.

2. EXAMPLES OF MEASUREMENTS

Measurements having as their object the verification of the transmission formulas were discussed in the preceding section. Any experiments which verify the transmission formulas may also be considered as checking the "theoretical formulas" and "comparison formulas". In fact experimental tests of the transmission formulas are the most rigorous test of the theory presented in this paper. All of the limitations and errors discussed in Sec. V 2 affect the transmission formulas while only a portion of them affect the theoretical and comparison formulas.

The complicated practical conditions of any experiment, the tilting of the wave front, the combination of antenna and coil effects discussed in the next section below, and other uncertainties make close agreement between theory and experiment unlikely. Agreement to 30 per cent should be considered as highly satisfactory verification of the essential correctness of the theory. On account of its field being more definitely localized, experiments with a coil aerial may be expected to yield greater accuracy than experiments with an antenna. The same advantage appertains to a condenser aerial. No quantitative data have been obtained with condenser aerials, to the author's knowledge; such experiments would be very desirable.

Experimental data obtained at the Bureau of Standards on radio transmission and reception are presented below. The agreement between the received current observed and the values calculated from the transmission formulas can be considered as very satisfactory. The author is informed that experiments made by the Signal Corps have led to a similar verification. In some of the Signal Corps experiments it was thought at one time that wide departures from the transmission formulas for coil aerials were observed, the received current for very short waves being much less than the transmission formulas indicated. When, however, the actual values of the resistance at wave lengths used were determined, the agreement was very good. Particular care must be given to proper measurement of the resistance of the receiving aerial circuit.

Antenna to Antenna. Experiments which supply a check on formula (**40**) have been published by Dr. Austin, chief of the Naval Radiotelegraphic Laboratory located at the Bureau of Standards.

For transmission between two antennas located on ships (*Jour. Wash. Acad. Sciences*, 1, p. 275; 1911), $h_s = 29.2$, $h_r = 29.2$, $I_s = 30.$, $R = 25.$, $\lambda = 1000.$, $d = 1000.$ The lengths given in all these examples are in meters. Calculating from (**40**),

$$I_r \text{ calculated } = 0.19$$

$$I_r \text{ observed } = 0.21$$

For the Washington Navy Yard antenna transmitting to an antenna at the Bureau of Standards (*Bull. Bureau of Standards*, 11, p. 74; 1914), $h_s = 36.$, $h_r = 30.$, $I_s = 7.0$, $R = 70.$, $\lambda = 2800.$, $d = 10{,}000.$ Calculating from (**40**) and (**20**)

I_r calculated $= 0.53 \times 10 - 3$
I_r observed $= 0.55 \times 10 - 3$

As already pointed out, the agreement of observed values with the transmission formulas indicates that it is proper to take as the antenna height the actual height from the ground to the flat top.

Antenna to Coil. A number of experiments to check the antenna-to-coil transmission formulas have been made at the Bureau of Standards radio laboratory since early in 1917. The first ones did not involve quantitative measurements but served to give a rough check on the formulas. The calculated value of current was compared with the current as estimated from the loudness of signal in a telephone receiver connected to various types of detecting devices. These signals were interpreted on the assumption that a fairly audible response is given by the currents indicated with the several types of detector.

10^{-2} ampere, thermoelement.
10^{-4} ampere, crystal detector.
10^{-6} ampere, simple audion.
10^{-8} ampere, oscillating audion.

For the Arlington antenna received on a coil aerial at the Bureau of Standards, $h_s = 122.$, $h_r = 4.$, $l_r = 4.$, $N_r = 22.$, $I_s = 102.$, $R = 25.$, $\lambda = 3800.$, $d = 7800.$ The received current calculated from (41) is 0.0018 ampere. The observed signal using crystal detector and telephone, was very loud, thus checking in a qualitative way the result calculated value.

Two similar qualitative experiments were made, transmitting from an antenna at the Bureau of Standards and receiving on a portable coil aerial. In one experiment, $h_s = 36.$, $h_r = l_r = 1.07$, $N_r = 11.$, $I_s = 0.5$, $R = 2.$, $\lambda = 850.$, $d = 16{,}000.$, whence calculated $I_r = 11. \times 10^{-6}$. In the other, $h_s = 12.$, $h_r = l_r = 1.07$, $N_r = 11.$, $I_s = 0.25$, $R = 2$, $\lambda = 600.$, $d = 11{,}000.$, whence calculated $I_r = 5.7 \times 10^{-6}$. In both cases the observed signal was loud with a simple audion, again giving a qualitative check on the formula.

A number of experiments have been made by Dr. Austin during 1918 and 1919, who has kindly placed the results at my disposal. A couple of typical ones will be given. For the Arlington antenna transmitting to a coil aerial at the Bureau of Standards, $h_s = 122.$, $h_r = 1.82$, $l_r = 1.29$, $N_r = 56.$, $I_s = 100.$, $R = 50.$, $\lambda = 6000.$, $d = 7800$.

I_r calculated $= 1.4 \times 10^{-4}$

I_r observed $= 2.1 \times 10^{-4}$

For the same antenna transmitting to a large coil suspended from masts outdoors, $h_s = 122.$, $h_r = 21.6$, $l_r = 24.4$, $N_r = 7.$ $I_s = 100.$, $R = 50.$, $\lambda = 10{,}000.$, $d = 7800.$, $\alpha = 42$ deg. From (41) and (24),

I_r calculated $= 1.0 \times 10^{-3}$

I_r observed $= 1.2 \times 10^{-3}$

A large number of transmission experiments from antenna to coils have been made by the radio laboratory of this Bureau in the early part of 1919. In a typical case, where $h_s = 21.$, $h_r = l_r = 1.44$, $N_r = 8.$, $I_s = 3.$, $R = 7.7$, $\lambda = 700.$, $d = 4800.$, $\delta' = 0.1$, $L = 541.$ microhenries. From (41) and (20),

I_r calculated $= 24. \times 10^{-6}$

I_r observed $= 28. \times 10^{-6}$

The fact that the observed current is larger than the calculated, in these and other cases, is probably due to the antenna effect, discussed in the next section. The coil structure has capacity, which makes it receive the wave by antenna action in addition to the coil action, thus increasing the current actually received.

Coil to Antenna. In an experiment made by Dr. Austin, with a large coil at the Bureau of Standards transmitting to the Arlington antenna, $h_s = 21.6$, $l_s = 24.4$, $h_r = 122.$, $N_s = 4.$,

$N_s = 7.$, $I_s = 1$, $R = 50.$, $\lambda = 2800.$, $d = 7800$, $\alpha = 42$ deg. From formula (42),

I_r calculated $= 1.3 \times 10^{-4}$

I_r observed $= 1.5 \times 10^{-4}$

Coil to Coil. The only data available to the author on the use of the coil aerial for both transmitting and receiving are from experiments made in 1917 by Messrs. Kolster, Willoughby, and Lowell, and these are only qualitative. For transmission from a coil at the Bureau of Standards to a portable coil 40. kilometers away, $h_s = l_s = 3.$, $h_r = l_r = 1.$,

$N_s = 4.$, $N_r = 15.$, $I_s = 10.$, $R = 1.$, $\lambda = 600.$, $d = 40{,}000$. The received current calculated from formula (43) is 4.6×10^{-6}. The observed signal was loud with a simple audion.

For transmission from a coil located at a lighthouse to a coil on a ship 48 kilometers away, $h_s = l_s = 3.05$, $h_r = l_r = 1.22$, $N_s = 3.$, $N_r = 10.$, $I_s = 10.$, $R = 2.$, $\lambda = 550.$, $d = 48{,}000$. The calculated I_r is 1.6×10^{-6}. The observed signal was audible on a simple audion. Comparing with the current sensibility of an audion stated above, it is seen that both of these results furnish a rough check on the formula.

3. DISCUSSION OF EXPERIMENTS

The agreement of the experiments with the theory is highly satisfactory in view of the simple conditions assumed in the theory. The complex practical conditions preclude the likelihood of agreement within a few per cent. The various limitations of the formulas arising from actual experimental conditions are discussed above in Sec. V 2.

One characteristic of the experiments with coil aerials is that the observed value of received current is in every case greater than the calculated value. This strongly suggests that the action of the coil structure involves something additional to the pure action as a coil. This would be expected also from theoretical considerations. The inevitable capacities between parts of the aerial circuit must introduce an action analogous to that of an antenna. When it is borne in mind that the coil action is really a second-order effect in comparison with the action of a system of antenna nature, it appears extremely likely that the stray capacities of any coil aerial circuit would introduce an antenna effect which would have to be considered in addition to the pure coil effect. Besides the reasons thus given by theory and by the excessive values of current observed in experiments with coil aerials, there are two other lines of evidence that the antenna effect is not negligible in coil aerials.

One of these additional lines of evidence is furnished by measurements of current in different parts of a coil aerial or the circuit thereof. If the capacities between parts are appreciable some of the current must leave the conductors and flow off into the dielectric; the current observed with an ammeter must therefore be different in different parts of the circuit. These differences are actually found. The fourth kind of evidence that the antenna effect is appreciable in coil aerials is furnished by considerations of radiation resistance, which will now be discussed. Following that, the antenna effect will be considered in more detail.

Radiation Resistance. It is possible to determine whether in a given system the antenna effect or coil effect predominates by measurements of radiation resistance. The radiation resistance has different values and follows different laws for antenna and coil.

Radiation resistance in general is defined by

$$P = R\,I^2 \qquad (56)$$

in which I is the current in the aerial used as a transmitting device, P is the power radiated, and R the radiation resistance. The study of radiation resistance is an important means of facilitating work on aerials. This may be seen from the simple fact that the magnitude of the radiation resistance gives at once the power radiated, and hence the effectiveness of a transmitting aerial or the range of communication can be judged without making transmission experiments. Field tests are thus in large part replaced by laboratory measurements. In addition to this, it is possible to discriminate between the antenna and coil effects.

The magnitude of the radiation resistance of a flat-top antenna, at wave lengths considerably greater than the fundamental, is given by the well-known expression

$$R_a = (39.7\,h/\lambda)^2 \qquad (57)$$

An approximate expression for the radiation resistance of a coil can be derived very simply, as follows. When a radiated field exists in any part of space, the relation of the power radiated through that portion of space to the magnetic field intensity there existing is

$$P' \alpha\, H^2 \qquad (58)$$

for any given distance from the source, whatever the source may be. The total power radiated is proportional to the integral of P over any surface entirely surrounding the source. This integral will be of the same form for H_c, the field due to a coil, as for H_a, the field to an antenna, except for the effect of the variation of H_c in a plane around the radiating coil, which varies from zero to the value given in (**10**) for any given distance from the source. As a first approximation, this variation may be considered to make the integrated value of H_c one-half as great as it would be if H_c had the value given in (**10**) in all directions around the radiating coil.

$$\therefore \frac{P_c}{P_a} = \frac{1}{2} \frac{H_c^2}{H_a^2} \tag{59}$$

From (**8**) and (**10**), for a given distance from the source and a coil and antenna of same height with same current,

$$\frac{H_c}{H_a} = 2\pi \frac{l N}{\lambda} \tag{60}$$

From (**56**),

$$\frac{R_c}{R_a} = \frac{P_c}{P_a}$$

Hence from (**59**) and (**60**)

$$\frac{R_c}{R_a} = 2\pi^2 \frac{l^2 N^2}{\lambda^2}$$

Inserting the value of R_a from (**57**)

$$R_c = 31{,}100. \frac{h^2 l^2 N^2}{\lambda^4}$$

If the coil is a square one with $h = l = a$,

$$R_c = (13.3\, a/\lambda)^4 N^2 \tag{61}$$

This approximate expression for radiation resistance of a coil gives at once the variation with size, number of turns, and wave length. *E. g.*, for a set of coils of varying size, in which the length of wire is kept constant, $R_c \alpha 1/N^2$. It shows that for a given ratio of size to wave length, $R_c \alpha N^2$. The principal point of interest is that R_c is inversely proportional to the fourth power of wave length.

Since the radiation resistance of an antenna is inversely proportional to the second power of wave length, and that of a coil inversely proportional to the fourth power, the radiation resistance furnishes a means of determining whether a given structure functions as a coil or as an antenna. Rough determinations of radiation resistance which were made upon a particular coil aerial showed a variation of observed radiation resistance inversely as the third power of the wave length, thus verifying the idea that the action is a combination of coil and antenna effect. The observed values however were all higher than the sum of the theoretical R_a and R_c. The measurement of radiation resistance is an extremely difficult operation, and satisfactory methods can not be said to have been developed as yet.

Antenna Effect. Since there are differences of potential between various parts of a coil, acting either as a transmitting or receiving aerial, there must be some dielectric current through the space around the coil and between the coil and ground. It follows that there must be some antenna action, proportional to the amount of this dielectric current and the length of path over which it flows, and this will produce a current additional to that produced by the coil action unless the coil structure happens to have an exact symmetry which causes the antenna effect in each part of the coil to be balanced by an antenna effect in some other part.

Fig. 16 shows the origin of the antenna effect. As in ordinary practise, the leads cause some part of the apparatus to be practically at ground potential, the shield of the condenser is shown connected to ground. An appreciable dielectric current flows from various parts of the conducting circuit to other parts and to ground. Typical paths of this dielectric current are shown by the dotted lines. The line *a b* suggests the dielectric current from the coil structure to ground, the lines *c d* and *e f* the dielectric current between turns of the coil, and the line *g h* the dielectric current between coil and leads. The flow of dielectric current between turns of the coil is in a horizontal direction when the coil is of prismatic form with the turns separated and all of the same area. This part of the antenna effect arises in a receiving coil of this form only when the wave front is more or less tilted from the vertical.

On account of the flow of current off through the dielectric from various parts of the circuit, ammeters placed at different places in the circuit would show different values of current to be flowing. In radio circuits it can not be assumed that the current is the same at all points around the conducting circuit, as was shown by the author in his investigation of high-frequency current measurement described in *Bureau of Standards* S. 206; 1913.

To the extent that these dielectric currents flow, the conductors of the circuit may be considered as an antenna system. Perhaps only the current typified by the line *a b* might be thought of as giving rise to an "antenna" effect, since the others do not flow to ground; still this part of the dielectric current does not differ from the others in nature or effect, and it seems hence advisable to use the suggestive term "antenna effect" to indicate all of the effects arising from the presence of currents in the dielectric.

It might be supposed that the same sort of an effect would be caused by the flow of dielectric current in the condenser of the coil aerial circuit. This is not true ordinarily because a condenser of the laboratory type is used, in which the condenser plates are interleaved. As shown in Fig. 17, the current in one direction in the dielectric is balanced by a current in the opposite direction in the neighboring part of the condenser. This is a non-radiating condenser; and is the analog of a non-inductive coil, which is also non-radiating. A condenser consisting of a single pair of plates would radiate, but is not ordinarily used because it would be much bulkier than the laboratory type of condenser. The condenser consisting of a single pair of plates would be in fact the "condenser aerial," which has been recommended by the author in Sec. IV 3 as worthy of serious consideration in radio practise.

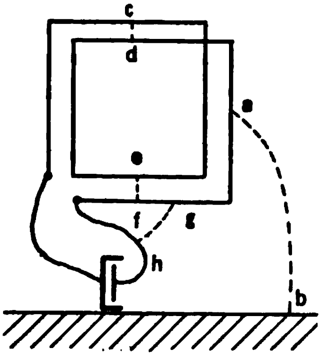


FIG. 16—PATHS OF DIELECTRIC CURRENTS WHICH CAUSE ANTENNA EFFECT IN COIL AERIAL

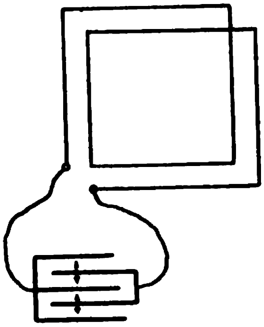


FIG. 17—DIRECTIONS OF FLOW OF DIELECTRIC CURRENTS IN LABORATORY TYPE OF CONDENSER

The effect of the distributed capacities of the aerial circuit must not be confused in any way with the phase angle between the fields existing at the two vertical sides of the coil aerial. The phase angle referred to is the seat of the action of the coil aerial as such. The dielectric currents flowing in the distributed or stray capacities of the circuit, however, give rise to the direct action as an antenna, not depending in any way on the separation between the two vertical sides of the coil. All of these remarks apply both to transmission and reception.

It is rather difficult to determine what fraction of the effect of a coil aerial is due to antenna action and what part to coil action. In many cases, doubtless, the antenna action predominates. It is possible, however, to separate the effects in any particular case by the several different methods. The antenna effect may be calculated, at least for parts of the circuit, by the aid of careful ammeter readings which show what amount of the current has flowed off into the dielectric. The antenna effect may be eliminated, thus leaving only the coil effect, by a carefully arranged system of shields and grounds; or, by a symmetrical arrangement of the coil structure which causes the antenna effect in each part to be balanced by the antenna effect in some other part. The coil effect may be eliminated, on the other hand, in the case of a receiving coil, by taking advantage of the fact that the coil effect depends on the direction of orientation while the antenna effect (at least the major part of it) does not; *i. e.*, by turning the coil so its plane is parallel to the wave front. A method which eliminates the coil effect and retains a part of the antenna effect is to open one of the coil leads, thus leaving the coil aerial connected to the circuit at one point, placing in series with it an inductance coil of very small dimensions but of the same inductance, the circuit being completed by the capacity of the coil aerial to ground.

Effects of Surroundings. Currents are induced in metal and other objects near a transmitting aerial, and sometimes are powerful enough to affect the radiation appreciably. The objects near a receiving aerial have currents produced in them by the passing wave. These currents in nearby objects, which may include the ground, induce e. m. f's. in the receiving aerial. It is to be noted that this effect of neighboring objects is caused by induction, and not radiation from them, which would be comparatively feeble.

The e. m. f. thus induced in a coil aerial from the surroundings is of the same or opposite phase as that caused by the wave. It differs in this respect from the e. m. f. due to the antenna effect discussed above. The antenna effect ordinarily produces an e. m. f. which is 90 deg. out of phase with the coil effects and does not vary with the orientation of the coil. The antenna effect can thus never balance out the coil effect, and it is impossible to reduce the received current to zero no matter how the coil is turned. The e. m. f. induced by the surroundings, however, depends upon the orientation of the coil. This e. m. f. will be reduced to zero by turning the coil at a different orientation from that at which the e. m. f. due to the wave is zero, unless the line between distorting object and the coil aerial is the direction of propagation of the wave. The result of this is that the total e. m. f. is reduced to zero at some orientation other than that obtained when the wave alone acts on the coil aerial. There is thus a distortion in the apparent direction of the wave, caused by objects surrounding the coil aerial.

VII. PRACTICAL CONCLUSIONS

1. Relative Effectiveness of Antennas and Coil Aerials

a. Generally speaking, a coil aerial is as powerful a transmitting or receiving device as an antenna only when its dimensions approach those of the antenna.

b. It is easy to make the resistance of a coil aerial circuit much smaller than the resistance of the ordinary antenna circuit and thus make a small coil as effective as a large antenna. A small aerial as effective as a large antenna can however also be secured by the use of the antenna-like aerial called the condenser aerial. Heeding these principles and using amplifiers in receiving, radio aerials can in the future be much smaller than heretofore.

c. The relative effectiveness of a coil and antenna, in terms of the wave length, number of turns, etc., is given by formula (33) and the related formulas.

d. A coil aerial exhibits antenna action as well as coil action, because of capacities between its parts and surroundings. The antenna action sometimes overbalances the coil action.

e. The advantage of the coil aerial is greatest for short wave lengths. It is consequently likely to be well suited to airplane communication. The increasing advantage of the coil as a transmitting aerial, as the wave length is decreased, is subject to the proviso that the same current can be gotten into a coil as into an antenna. In fact, the whole practical problem is to get as much current as possible into the aerial.

f. The use of coil aerials at both receiving and transmitting ends of the communication is particularly suitable for short waves, since the received current in such a system is inversely proportional to the cube of the wave length.

2. PRINCIPAL FORMULAS

The units used are international electric units, the ordinary electric units based on the ohm, ampere, centimeter, and second; except where otherwise stated. The principal symbols are the following.

SYMBOLS

i = instantaneous current.
I_0 = maximum value of current.
I = effective value of current.
H_t = instantaneous value of magnetic field intensity.
H_0 = maximum value of magnetic field intensity.
H = effective value of magnetic field intensity.
h = height of aerial.
d = distance along earth's surface from sending aerial.
ω = 2π times frequency of the current.
t = time.
λ = wave length.
c = velocity of electric waves = 3×10^{10} cm. per second.
l = horizontal length of coil aerial.
N = number of turns of wire of coil aerial.
a = length of side of square coil.
θ = phase angle between values of field intensity a distance l apart in the wave.
$\mathcal{E}$ = electric field intensity.
E = electromotive force in receiving aerial.
δ' = logarithmic decrement of H or E.
ϕ = magnetic flux.
R = resistance of receiving aerial circuit.
C = capacity of receiving aerial circuit.
L = inductance of receiving aerial circuit.
α = angle between direction of propagation of wave and plane of coil.

Subscripts: s = sending, r = receiving, a = antenna, c = coil.

The following are the principal formulas presented in this paper:

Radiated Magnetic Field Intensity from an Antenna or Condenser Aerial.

$$H = \frac{2\,\pi}{10}\,\frac{h_s\,I_s}{\lambda\,d} \tag{8}$$

Radiated Magnetic Field Intensity from a Coil.

$$H = \frac{4\,\pi^2}{10}\,\frac{h_s\,l_s\,N_s\,I_s}{\lambda^2\,d} \tag{10}$$

Received Current in an Antenna or Condenser Aerial.

$$I_r = 300\,.\,\frac{h_r\,H}{R} \tag{19}$$

Received Current in a Coil.

$$I_r = 600\,.\,\pi\,\frac{h_r\,l_r\,N_r\,H}{R\,\lambda} \tag{23}$$

Distance Correction Factor.

$$F_1 = \epsilon^{-0.000047\,d\,/\sqrt{\lambda}} \tag{9}$$

Decrement Correction Factor.

$$F_2 = \sqrt{\frac{1}{1 + \frac{600\,.\,L\,\delta'}{R\,\lambda}}} \tag{20}$$

Direction Correction Factor.

$$F_3 = \cos\alpha \tag{24}$$

Antenna-to-Antenna Transmission.

$$I_r = \frac{188\,.\,h_s\,h_r\,I_s}{R\,\lambda\,d} \tag{40}$$

Antenna-to-Coil Transmission.

$$I_r = \frac{1184\,.\,h_s\,h_r\,l_r\,N_r\,I_s}{R\,\lambda^2\,d} \tag{41}$$

Coil-to-Antenna Transmission.

$$I_r = \frac{1184\,.\,h_s\,l_s\,h_r\,N_s\,I_s}{R\,\lambda^2\,d} \tag{42}$$

Coil-to-Coil Transmission.

$$I_r = \frac{7450 . h_s l_s h_r l_r N_s N_r I_s}{R \lambda^3 d} \tag{43}$$

The lengths in the four preceding formulas may be in any units. Meters are commonly used. Any of these formulas may be expressed in terms of d. *E. g.,*

Distance at Which a Given Current is Received in a Coil for a Given Transmitting Current in an Antenna.

$$d = \frac{1184 . h_s h_r l_r N_r I_s}{R \lambda^2 I_r} \tag{44}$$

Total Magnetic Field from an Antenna, Including Radiation and Induction.

$$H = \frac{2 \pi}{10} \frac{h_s I_s}{\lambda d} + \frac{j}{10} \frac{h_s I_s}{d^2} \tag{31}$$

Relative Effectiveness of Coil and Antenna, for Same Height and Wave Length.

$$d_c/d_a = N \sqrt{2 (1 - \cos 2 \pi l/\lambda)} \tag{32}$$

Ditto, l small compared to λ.

$$d_c/d_a = 6.28 N l/\lambda \tag{33}$$

Length of Coil Aerial Equivalent to Antenna of the Same Height.

$$l = 0.16 \lambda/N \tag{39}$$

Current in Aerial Circuit.

$$I_r = E/R \tag{46}$$

Voltage across Condenser in Aerial Circuit.

$$V = \frac{E}{R \omega C} \tag{47}$$

Coil Aerial Reception Factors.

$$\text{E. m. f. reception factor} = \frac{a^2 N}{\lambda} \tag{51}$$

$$\text{Current reception factor} = \frac{a^2 N}{R \lambda} \tag{52}$$

$$\text{Voltage reception factor} = \frac{a^2 N L}{R \lambda^2} \tag{53}$$

Radiation Resistance.

$$R_a = (39.7\, h/\lambda)^2 \tag{57}$$

$$R_c = (13.3\, a/\lambda)^4 N^2 \tag{61}$$

3. FUTURE RESEARCH NEEDED

The subject of research on electric waves can be considered as barely begun. The study presented in this article has revealed vast and most interesting problems awaiting solution which can be solved. The functioning of aerials, both in transmitting and receiving, can now be considered as roughly understood. Recent advances in radio measurements and technique open the way to experiments and progress which will bring about far-reaching control of electric waves. A few of the detailed problems which border on the subject matter of this paper and await solution will now be mentioned.

Theoretical Problems.

a. Develop a simple and straightforward derivation of the radiated field from a coil, without consideration of shape of the coil or dealing with the electrostatic field at all.

b. Work up an explanation of the mechanism of radiation that brings out clearly the relation of the radiation to the induction field and shows that all of the dielectric current is effective in causing radiation, which shall take the place of the usual explanation in terms of the snapping off of lines of force.

c. Determine the effects of the phase angle between different parts of the dielectric field in an antenna or condenser aerial, especially the long, low types.

d. Develop methods of measuring radiation resistance.

e. Work out laws of variation of voltage reception factor of coil aerials, and laws of variation of both current and voltage reception factors of antenna and condenser aerials. Similarly, develop accurate and useful transmission factors.

Experimental Problems.

a. Determine the relative effectiveness over a very wide range of sizes, wave lengths, etc., of the various types of aerials. Do this by: (1) direct measurements to verify transmission formulas (2) measurements of the factors that enter into the reception factors, (3) measurements of radiation resistance.

b. Make transmission experiments at very great distances over typical kinds of land, to obtain distance absorption factors.

c. Try out condenser aerials, comparing performance with transmission formulas. Build such aerials with minimum resistance. Demonstrate the non-radiating nature of the laboratory type of condenser, comparing it with condenser aerials.

d. Compare trailing wire, condenser aerial, and coil aerial, on airplane.

e. Find out how directive as transmitting devices coil and condenser aerials and "earth antennas" are; measure magnitude and direction of field at various distances from the aerial, at numerous wave lengths, etc.

f. Determine relative magnitudes of induction and radiation close to transmitting aerials. Determine also directions of fields, to secure complete knowledge of phenomena near radiating systems.

g. Measure currents in ground as well as the fields above the ground, to determine how wave attaches itself to the ground.

h. Study distributed capacities in coil aerial circuit by measuring current at different points in circuit.

i. Determine values of antenna effect, and develop means of controlling or eliminating it by shielding systems, etc.

j. Make quantitative investigation of receiving systems combining antenna and coil aerial. Measure phase of currents. Determine under what circumstances the indication of absolute direction is reversed when the tuning is slightly varied.

k. Determine effects of surrounding objects on currents in transmitting and receiving aerials. Measure magnitude and phase of currents in typical cases.

l. Develop methods of connecting generating apparatus to various types of aerials to get maximum current into the aerial, especially at short wave lengths.

VIII. SUMMARY

The advantages of the coil aerial as a direction finder, interference preventer, reducer of strays, and submarine aerial make it important to know how effective the coil aerial is, in comparison with the ordinary antenna, as a transmitting and receiving device. This article gives the answer. Simple formulas are worked out from fundamental electromagnetic theory, by which the performance of any aerial can be calculated. Experiments have verified the formulas, and show that they are a valuable aid in the choice and design of an aerial to fit any particular requirements.

The principal formulas are of three kinds: theoretical for-

mulas, giving the magnetic field intensity at any distance from either kind of aerial and the current produced by a given field intensity in either kind of aerial; comparison formulas, giving the ratio of performance of antenna and coil aerial under various conditions; and transmission formulas, giving the current in any receiving aerial in terms of the current in the distant transmitting aerial.

The theory and nature of radiation are discussed, and applied to the elucidation of some current fallacies. There has been a vast haziness of ideas on these points. The distinction between induction fields and radiation fields is presented. It is shown that the receiving action in any kind of an aerial may be considered as arising either from the electrostatic or the magnetic field present in the wave. Such questions are discussed as the distinction between "open" and "closed" circuits. It is shown that a metallically closed circuit can radiate, and that radiation takes place at all frequencies, the amount of radiation being greater the higher the frequency.

The ratio of the range of communication obtainable with a coil aerial to that with an antenna is proportional to the number of turns and horizontal length of the coil and inversely as the wave length. The coil aerial is hence particularly suited to communication on short wave lengths. A coil aerial is quantitatively as powerful as an antenna only when its dimensions approach those of the antenna. However, it is easy to make the resistance of a coil aerial circuit much smaller than the resistance of the ordinary antenna circuit and thus make a small coil as effective as a large antenna.

A small aerial as effective as the ordinary antenna may be secured without recourse to the coil principle by using an aerial consisting of a condenser having two large parallel plates, arranged so that the dielectric of the condenser includes no ground. The circuit of such an aerial may be made to have a very low resistance. It appears likely that, with the use of either condenser or coil aerials together with sensitive amplifiers, radio aerials will in the future be much smaller than heretofore. These principles apply with particular advantage to airplane aerials.

A coil aerial usually functions by a combination of the pure coil action and antenna action. The latter arises from the stray capacities and capacities to ground which are inevitably present. The existence of these capacities may be shown by differences in ammeter readings at different points of the circuit. The antenna effect makes the actual received current in experiments with coil aerials larger than the values calculated from the transmission formulas. The observed values are also affected by currents in neighboring objects.

A formula for the radiation resistance of coil aerials is worked out. Comparison of experiment with this formula supplies additional evidence that the coil aerial operates by a combination of antenna and coil effects.

The fundamental principles of design of aerials are given. The various modes of measuring received current and voltage across the condenser are discussed. The relations of these two quantities to the electromotive force acting in the aerial must be carefully observed in calculations or design. Reception factors are derived, to which the received current or voltage are proportional. Experimental data on the functioning of aerials may be secured either from actual transmission experiments or from measurements of the quantities which enter into the reception factor.

This investigation has opened up a large and most interesting field for further research. Progress in the control and utilization of electric waves depends on the investigation of such theoretical and experimental problems as have been suggested in Sec. VII 3 herein.

DISCUSSION ON "PRINCIPLES OF RADIO TRANSMISSION AND RECEPTION WITH ANTENNA AND COIL AERIALS" (DELLINGER), NEW YORK, N. Y., OCTOBER 1, 1919.

F. W. Grover: In the derivation of the formula for the field of an antenna, the current is assumed to be uniform throughout the length of the antenna, and the field of this vertical current at a point *P* is found by using the vector potential. In evaluating the latter, the assumption is made that the point *P* is so far away, compared with the height of the antenna, that *P* is sensibly at the same distance from all points of this vertical portion.

Having obtained the equation (5) for the field due to this vertical current, it is then tacitly assumed that this is the whole field of the antenna. Since it is expressly emphasized in the latter portions (and rightly so, it will be admitted by every one) that the whole displacement current must be reckoned as contributing to the field, some explanation ought to be given as to why the displacement currents of the antenna are not taken into account in this demonstration. Thus far, I have not been able to give a satisfactory explanation, and am here setting forth the difficulties that have occurred to me, in the hope that I may be set right and that perhaps the objections of others may be forestalled.

First, it would seem necessary to make some assumption as to the form of the displacement lines. Here this is difficult, because it is assumed (1) that there is no image of the antenna, and (2) that the current in the vertical portion is uniform. These are of course, strictly speaking, incompatible conditions, since either the displacement currents must return from the flat top to the lower portion of the vertical wire which combats assumption (2) or else they must return by the earth which modifies statement (1). In any case, they must, on the whole, flow in the opposite direction to the current in the vertical portion. To some extent, then, it would seem as though the resultant effect of the antenna would have to be less than that of the vertical portion alone.

Suppose, to fix ideas, that the displacement currents flow vertically. Then, if we consider a filament of displacement current *C D*, it may be regarded as completing the circuit of an equal current in the vertical portion *B A*. Similarly, the symmetrically placed filament *E F* forms a portion of a circuit *B A E F*. If each current in *B A* produce a field *H* at a point *P* in the plane of rectangle, the effect of *B A* totals 2 *H*, which may be represented by the vector *O F* (Fig. 3). *E F* produces a field *H* which is θ degrees different in phase from the vector opposite to *O F*. This may be represented by *O G*, just as in the demonstration for the coil. The currents *A D* and *B E* balance out in their effects at a distant point. In like manner, current *C D* gives a field *H* of phase *O K*. The field due to currents $A\,C$ and $D\,B$ balance out at point P. The resultant must accordingly be $H' = 2\,H - 2\,H \cos\theta$ or

$$\begin{aligned} H' &= 2\,H\,(1-\cos\theta) = 4\,H \sin^2 \theta/2 \\ &= 4\,H\,\theta^2/4 = H\,\theta^2 \text{ when } \theta \text{ is small} \\ &= H\,.\,4\,\pi^2\,(l/\lambda)^2 \end{aligned}$$

which is much smaller than H.

This treatment may be extended to the whole antenna, each filament of displacement current being regarded as completing the circuit of an equal current in the vertical portion. It would seem, then, that the effect of the whole antenna, of the symmetrical construction here shown, must be much less than that of the vertical portion. In fact, whereas the radiation of a coil is a second order effect, that calculated above is a third order effect, being the difference of two coil effects.

Now, actually, the displacement lines of flow will not be straight lines but curved. Each element of a curved path of current may, however, be regarded as equivalent in its magnetic effect to the sum of a vertical and horizontal element (Fig. 4). Summing up for the whole line, the horizontal elements tend to balance out in their effects at a distant point, while the vertical elements sum up to give the effect of a vertical current between the antenna and the ground. In any case, where the antenna is symmetrical about the vertical portion, the above conclusion of a resultant field much smaller than that due to the vertical portion alone seems to be justified.

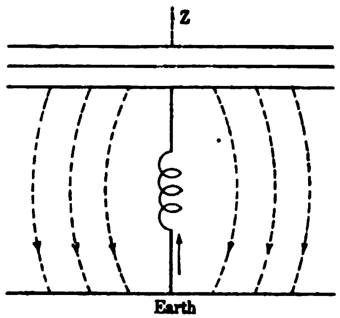


FIG. 1

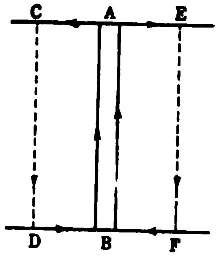


FIG. 2

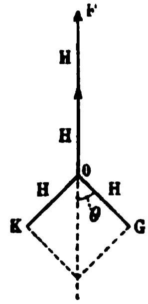


FIG. 3

Likewise the same ideas and conclusions seem to follow for an antenna consisting of a *single vertical wire*. For the displacement lines may be regarded as equivalent to vertical currents opposite in direction to the current in the wire. Any two current filaments symmetrically placed about the vertical wire may be combined with equal currents in the vertical wire and are equivalent to equal rectangles in the same plane as in the previous demonstration. If the point P is in their plane, the equation for the resultant field given above holds. Otherwise the factor cos α has to be applied to this, and the resultant would be very small in every case.

Applying the same ideas to an L antenna, it would seem that it should be equivalent to a coil of width somewhere between zero and the length of the antenna l. That is, such an antenna should be directive like a coil and its field should be a second order effect like that of a coil, *i. e.* less than that due to the vertical portion.

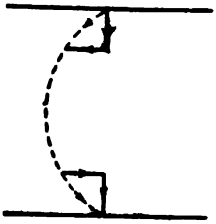


FIG. 4

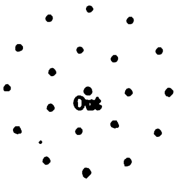


FIG. 5

In the case of the low condenser aerial we are not met by these difficulties, since we may regard the circuit as being completed through a coiled wire of negligible vertical portion and the coil is of such small dimensions that it may be negligible in its "coil effect." The current in the dielectric only would then need to be considered.

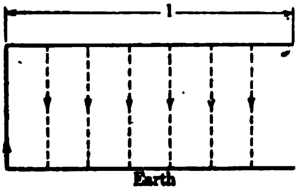


FIG. 6

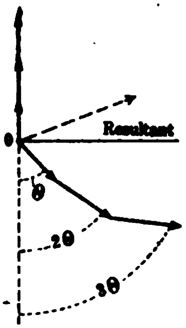


FIG. 7

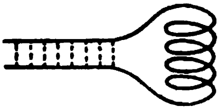


FIG. 8

Recapitulation. To agree, then with the formulas of this paper and with the experimental results, it would seem necessary to consider for coil effect only the current in the metallic portions. The condenser aerial may be treated by considering only the dielectric current. The antenna can be treated by considering either the current in the dielectric or the current in the vertical portion alone. Taken together they nearly cancel out. For unity, it would seem to be the dielectric current which should be chosen as in the condenser aerial. Finally, then, what is the reason why the whole circuit should not be considered?

DISCUSSION OF THE DISTINCTION BETWEEN RADIATION AND INDUCTION

The magnetic field due to a vertical current $i = I_0 \sin \omega t$ is, for a point P at a large distance d from the current,

$$H_t = - \frac{h\,\omega\,I_0}{10\,c\,d} \cos \omega\,(t - d/c) - \frac{h\,I_0}{10\,d^2} \sin \omega\,(t - d/c) \quad (5)$$

$$= - \frac{2\,\pi}{10}\,\frac{h\,I_0}{\lambda\,d} \cos \omega\,(t - d/c) - \frac{h\,I_0}{10\,d^2} \sin \omega\,(t - d/c)$$

The negative signs simply take account of the fact that if the current is along the positive Z axis, and the distance d is measured along the positive X axis, the magnetic field must be along the $-Y$ axis. The first of these terms is called the radiation field and the second the induction field.

The time d/c is the interval which must elapse for a disturbance at the origin to travel the distance d to the point P. That is, action at P at any moment t depends upon the value of the current at a moment d/c seconds previous.

A comparison of the expressions for the two fields shows that the induction field is in phase with the current (excepting for its interval d/c) but that the radiation field is 90 degrees different in phase. Due to the presence of d^2 in the denominator the induction field falls off rapidly with the distance. For small values of λ, the radiation field diminishes not only less rapidly with the distance than the induction field, but for a given distance greater than $\frac{\lambda}{2\,\pi}$, it is greater in amplitude than the induction field.

It is thus easy to see that the "induction field" is the usual field sensibly in phase with the current, which we consider as inducing e. m. f. in nearby circuits. Further, at low frequencies (long wave length) the radiation field is negligible. Thus the discussion in the paper of the mathematical distinction between the two terms is easily seen. The term "induction field" is, as just stated, appropriate, and the first term, which formulates the only portion of the field which is important at radio frequencies and distances is very properly called the "radiation field." Also, it is very important to emphasize the fact that both fields are always present, even though one of them may be entirely negligible. A further point which may be properly emphasized is that, since the time d/c required for the propagation of the field can never be exactly zero, it is never rigorously correct to speak of the magnetic field at a point as being strictly in phase with the current which produces it.

It is not so easy, however, to agree entirely with the statement in the paper of the *physical* difference of the two fields. It would seem to be misleading to speak of the induction field as "fixed in space" (last line of p. 1365). The presence of the quantity d/c in the sine factor of the induction field shows that this field must travel with the velocity of light, as well as the radiation field. At each point, the field oscillates, but for two points separated by a distance λ, the quantity $\frac{\omega d}{c}$ differs by 2π, so that the field is in the same phase at such points,—although the amplitude of oscillation is different.

This propagation of the induction component of the field with the velocity of light is also evident from a consideration of the induction of e. m. f. With regard to this the induction field has no monopoly. The induction of e. m. f. in a conductor by the radiated field is treated in equation (**17**), the usual idea of relative motion of conductor and field being employed, except that here the relative velocity is the velocity of light, a velocity far transcending any velocity attainable in electrical generators. The ordinary induction case, analogous to this, is the induction of e. m. f. in a coil through which the number of magnetic lines is suddenly changed by varying the current in the circuit or in an adjoining circuit. It is easy to show that the usual method of calculating the e. m. f. as equal to the time rate of change of the number of magnetic lines through the coil is entirely equivalent to that derived on the assumption of a relative motion between the coil and the field equal to the velocity of light. The following is a proof of this point.

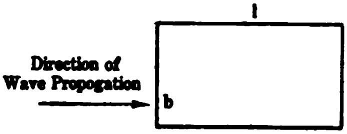


FIG. 9

Consider a rectangular circuit of height b and length l in the direction of propagation of the wave. The induction field has a value $\frac{h I_0}{10 d^2} \sin \omega (t - d/c)$ at points along the nearer side of the circuit, so that the induced e. m. f. in this side is by (**17**) (relative motion conception) $\frac{h b c I_0}{10 d^2} \sin \omega (t - d/c)$. The e. m. f. induced in the further side is likewise

$\dfrac{h\,b\,c\,I_0}{10\,(d+l)^2}\sin\omega\left(t-\dfrac{d+l}{c}\right)$. The resultant e. m. f. in the coil is the difference,

$$E = \frac{h\,b\,c\,I_0}{10}\left[l/d^2\sin\omega\,(t-d/c) - \frac{l}{(d+l)^2}\sin\omega\left(t-\frac{d+l}{c}\right)\right]$$

If l is small compared with d, this gives

$$E = -\frac{h\,b\,c\,I_0}{10\,d^2}\left[2\cos\omega\left(t-\frac{d+l/2}{c}\right)\sin\frac{\omega\,l}{2\,c}\right]$$

or, for l small compared with λ.

$$E = -\frac{h\,b\,\omega\,l\,I_0}{10\,d^2}\cos\omega\left(t-\frac{d+l/2}{c}\right)$$

According to the usual conception, we find the number of lines through the coil, $H\,b\,l$, and differentiate with respect to the time. For H we may, for values of l small compared with λ, consider the average phase as that of the center, and neglect the difference between d^2 and $(d+l/2)^2$.

Thus

$$N = \frac{h\,b\,l\,I_0}{10\,d^2}\sin\omega\left(t-\frac{d+l/2}{c}\right)$$

and

$$E = -\frac{d\,N}{d\,t} = -\frac{h\,b\,l\,\omega\,I_0}{10\,d^2}\cos\omega\left(t-\frac{d+l/2}{c}\right)$$

The two methods are in agreement, and for coils whose dimensions are small in comparison with the wave length, and whose distance from the source is small, we have our usual expression giving an e. m. f. 90 deg. behind the field (current) in phase.

Repeating the demonstration with the radiation field, we have from (**17**)

$$E = \frac{h\,\omega\,b\,I_0}{10\,c\,x}\cdot c\left[\cos\omega\,(t-d/c) - \cos\omega\left(t-\frac{d+l}{c}\right)\right]$$

$$= \frac{2\,h\,\omega\,b\,I_0}{10\,x}\sin\frac{\omega\,l}{2\,c}\sin\omega\left(t-\frac{d+l/2}{c}\right)$$

$$= \frac{\omega^2 h\, b\, l\, I_0}{10\, c\, x} \sin \omega \left(t - \frac{d + l/2}{c} \right)$$

By the second method

$$N = \frac{h\, \omega\, I_0}{10c\, x} \,.\, b\, l \,.\, \cos \omega \left(t - \frac{d + l/2}{c} \right)$$

$$E = - \frac{d N}{d t} = \frac{h\, \omega^2 b\, l\, I_0}{10\, c\, x} \sin \omega \left(t - \frac{d + l/2}{c} \right)$$

In this case also the two methods lead to identical results.

The conclusion, therefore, seems to be certain, that both fields are propagated with the velocity of light, and that both, when they cut a conductor, give rise to an induced e. m. f. which induces it. Since the two fields differ in phase by 90 deg. the e. m. fs., will also. On account of the relatively insignificant magnitude of the radiated field, however, at the usual a-c. frequencies, and the small distances involved in usual apparatus, the e. m. f. induced by the induction field only need be considered in electric generators and transformers.

Another distinction, which is made in the paper, is that accompanying the radiation field, there is propagated an electrostatic field which bears a constant ratio to the magnetic, while with the induction field there is no such constant predetermined relation. From what has already gone before, it would seem that this constant relation between H and E must also be true of the induction field. The induced e. m. f. in a wire of length h has been shown to be equal to $c\, H\, h \,.\, 10^{-8}$ (equation 17) and I have shown above that this equation is true for *both* components of the magnetic field. Now, in the present case, the field is supposed to be uniform along the wire, so that the electrostatic field, which is equal to the e. m. f. per cm., must be $c\, H \,.\, 10^{-8}$ or 300 H, in which H includes both components. The statement is made 1366 that this relation is true only in a radiated wave, but it must not be concluded herefrom that for H *only* the radiation field is to be taken. Both fields are radiated, and the induction field, however small it is, must, strictly speaking, be included in H in the above relation.

As to what is the difference in physical nature between the radiation and induction fields, there remains the point mentioned in the first paragraph of 1367, "The portion of the energy associated with this disturbance that does not return to the radiator is that connected with the first term of equation (31)" (*i. e.* the radiation field). This, of course, means that the energy associated with the induction field, although naturally it flows back and forth, must average zero. That is, the flow through any volume element in one direction over half the cycle is equal to the flow in the opposite direction during the remainder of the cycle.

Now this statement appears plausible, and from the usual a-c. idea of wattless current is what would be expected of the induction field. I have, however, been unable to *prove* it, and the demonstration given below would seem to lead to the idea that a portion of the energy associated with *each field* is returned, and a portion radiated, *i. e.* that there is no difference in nature, respecting net energy flow, between the two components of the field.

In the case of a simple making of a steady current I, energy is flowing out to all points of space during the rise of the current and finally after an infinite time, there is a distribution of energy through space of $\frac{H^2 \mu}{8 \pi}$ per cu. cm. which, since H decreases and becomes zero at infinite distance, is less in remote regions and greater near the circuit, but finite in sum and equal to $1/2\, L\, I^2$.

When the current is broken, this energy all eventually returns to the source. If the current is made to rise and fall periodically, however, it is easy to comprehend that some of the energy sent out would not have a chance to come back before the flow of energy again takes place away from its source, so that some of the energy would be permanently severed from the source. Since this would appear more likely to be the case, the higher the frequency, it is natural to suppose that this energy would be that propagated by the travelling of the *radiation* field rather than that connected with the moving induction field whose amplitude does not involve the wave length. In what follows, the idea of the Poynting vector is introduced to investigate this matter.

The Poynting theorem states that the instantaneous flow of power through any portion of space is equal to $1/4\, \pi$ times the integral of the normal component of the vector product of the electrostatic and magnetic fields, taken over the entire bounding surface of the space in question.

In the present instance, the electrostatic field is along the Z axis, the magnetic field along the $-Y$ axis and the flow of energy along the X axis. The flow of energy through an element of volume $d\,V = dx\, dy\, dz$ is equal to the excess of the amount through one $dy\, dz$ face over that through the opposite face. Since H and E are at right angles, the vector product is their simple product.

Taking a point (x, y, z) the power entering the face $dy\, dz$ is $\frac{E\,H}{4\,\pi}\, dy\, dz$. That over the opposite face is

$$\frac{\left(E + \frac{d\,E}{d\,x}\,d\,x\right)\left(H + \frac{d\,H}{d\,x}\,d\,x\right)}{4\,\pi}\,dy\,dz,$$ so that the net power, is, in the limit, $\frac{1}{4\,\pi}\left(H\,\frac{d\,E}{d\,x} + E\,\frac{d\,H}{d\,x}\right)dy\,dx\,dz.$

Now we have shown that $E = c\,H$ so that the power is

$$\frac{1}{4\,\pi}\left(c\,H\,\frac{d\,H}{d\,x} + c\,H\,\frac{d\,H}{d\,x}\right)d\,V = \frac{2\,c}{4\,\pi}\,H\,\frac{d\,H}{d\,x}\,d\,V$$

If we place $H = H_r + H_i$, the power is

$$\frac{2\,c}{4\,\pi}\,(H_r + H_i)\left(\frac{d\,H_r}{d\,x} + \frac{d\,H_i}{d\,x}\right)d\,V$$

Now

$$H_r = \frac{h\,\omega\,I_0}{10\,c\,x}\cos\omega\,(t - x/c)$$

$$\frac{d\,H_r}{d\,x} = +\frac{h\,\omega^2\,I_0}{10\,c^2\,x}\sin\omega\,(t - x/c) - \frac{h\,\omega\,I_0}{10\,c\,x^3}\cos\omega\,(t - x/c)$$

$$H_i = \frac{h\,I_0}{10\,x^2}\sin\omega\,(t - x/c)$$

$$\frac{d\,H_i}{d\,x} = -\frac{\omega}{c}\,\frac{h\,I_0}{10\,x^2}\cos\omega\,(t - x/c) - \frac{2\,h\,I_0}{10\,x^3}\sin\omega\,(t - x/c)$$

The term $H_r\,\frac{d\,H_r}{d\,x}$ may be regarded as due to the radiation field only, that due to the induction field is $H_i\,\frac{d\,H_i}{d\,x}$, and the remaining terms

$$\left(H_i\,\frac{d\,H_r}{d\,x} + H_r\,\frac{d\,H_i}{d\,x}\right)d\,V$$

may be regarded as the power due to the interaction of the electrostatic field of one component magnetic field with the other component of the magnetic field.

$$H_r \frac{d H_r}{d x} = \frac{h^2 I_0^2}{100\, x^2} (\omega/c)^3 \cos \omega\,(t - x/c) \sin \omega\,(t - x/c) - \frac{h^2 I_0^2}{100\, x^3} (\omega/c)^2 \cos^2 \omega\,(t - x/c)$$

$$H_i \frac{d H_i}{d x} = - \frac{h^2 I_0^2}{100\, x^4} (\omega/c) \sin \omega\,(t - x/c) \cos \omega\,(t - x/c) - \frac{2\, h^2 I_0^2}{100\, x^5} \sin^2 \omega\,(t - x/c)$$

$$H_i \frac{d H_r}{d x} = \frac{h^2 I_0^2}{100\, x^3} \sin^2 \omega\,(t - x/c) - \frac{h^2 I_0^2}{100\, x^4} (\omega/c) \sin \omega\,(t - x/c) \cos \omega\,(t - x/c)$$

$$H_r \frac{d H_i}{d x} = - \frac{h^2 I_0^2}{100\, x^3} (\omega/c)^2 \cos^2 \omega\,(t - x/c) - \frac{2\, h^2 I_0^2}{100\, x^4} (\omega/c) \cos \omega\,(t - x/c) \sin \omega\,(t - x/c)$$

Now the terms in $\cos (t - x/c) \sin (t - x/c)$ integrated over a whole cycle give zero, but the terms in $\cos^2 (t - x/c)$ and $\sin^2 (t - x/c)$ give a finite amount, since the integral of $\sin^2$ or $\cos^2$ over a cycle gives 1/2. It would appear, then, that the induction component as well as the radiation component contributes to the radiation of energy. At large distances and short wave lengths, however, the $H_i \frac{d H_i}{d x}$ term is the smallest of all. On the other hand the $H_r \frac{d H_i}{d x}$ term is equal to the $H_r \frac{d H_r}{d x}$ term.

In conclusion, then, there seems to be no essential difference in the nature of the two components of the magnetic fields, except in the way they vary with the frequency and the distance from the source.

A. Press: In the paper by Dr. Dellinger mention is made of a method which has been referred back to some work of Prof. Lorentz. Strangely enough the same general method was gone over quite independently by myself but I must say that it seems to me that the real value of the method of vector potentials suggested is rather approximative. In the case of a horizontal antenna which was the case considered in my own investigations the difficulty arose in imagining the manner in which the interlinked flux lines or rather the interlinked flux sheets could disentangle themselves from the conductors (or coils).

The true basis after all of estimating the value of H must be the Maxwellian equations of wave propagation. In my later work therefore, which by the way was gone into in my classes at the University of California about a year ago I gave the following derivation:

A round wire is assumed to carry a current

$$i = I_0 \sin p\,t$$

It is required to determine the type of magnetic field that is set up as a function of the wave length of the system. The individual conductors of a two-wire parallel system will be investigated with the origin at the center of a conductor. The required equations of condition are (for polar coordinates)

$$\frac{d B_\theta}{d t} = \frac{d E}{d \gamma}$$

$$\frac{d D_z}{d t} = 1/\gamma \, H_\theta + \frac{d H_\theta}{d \gamma}$$

The above two equations can be made to result in the following

$$1/\gamma^2 . \ \frac{d^2 D_z}{d t^2} = 1/\gamma \ \frac{d D_z}{d \gamma} + \frac{d^2 D_z}{d \gamma^2}$$

$$1/\gamma^2 . \ \frac{d^2 H_\theta}{d t^2} = \frac{d^2 H_\theta}{d \gamma^2} + 1/\gamma \frac{d H_\theta}{d \gamma} - 1/\gamma^2 . H_\theta$$

Writing the simpler of the two equations in the form

$$\frac{d^2 D}{d \gamma^2} + 1/\gamma \ \frac{d D}{d \gamma} = q^2 D$$

With

$$q^2 = 1/\gamma^2 . \frac{d^2}{d t^2}$$

a solution is found to be

$$D_s = -\frac{1}{2\pi\alpha} \cdot \frac{I_0 \cos p\,t}{\gamma} \cdot \frac{J_0\left(\frac{2\pi\gamma}{\lambda}\right)}{J_1\left(\frac{2\pi\alpha}{\lambda}\right)}$$

where α is the radius of the wire and $p = 2\,\pi f$, which leads to the corresponding solution for H which is

$$H_\theta = \frac{I_0}{2\,\pi\,\alpha} \cdot \frac{J_1\left(\frac{2\pi\gamma}{\lambda}\right)}{J_1\left(\frac{2\pi\alpha}{\lambda}\right)} \cdot \sin pt$$

It is seen therefore that there is no out of phase term but rather that a stationary wave is set up. The above solution therefore does not offer any physical difficulties of picturing interlinked flux lines disentangling themselves from the system to produce radiant energy. In reality galvanic effects above described have been found not to produce any actual radiation, in the case of a vertical antenna. The whole problem of the vertical grounded antenna has been treated by the writer. The radiation factors are gone into in detail but from the standpoint of wave propagation wholly.

Presented at a joint meeting of the American Institute of Electrical Engineers and the American Physical Society, Philadelphia, Pa., October 10, 1919

THE VACUUM TUBE AS A GENERATOR OF ALTERNATING-CURRENT POWER

BY JOHN H. MORECROFT AND H. TRAP FRIIS

ABSTRACT OF PAPER

The first part of this article deals with the operation of the tube when separately excited, the variation of power with the amount of excitation, the load impedance, etc., and also gives an analysis of the forms and phases of voltages and currents in the different parts of the circuit.

The second part deals with the efficiency of the tube as a generator; the action is analyzed in detail and the conditions for maximum efficiency deduced, the theoretically deduced conclusions being substantiated by experimental data. Oscillograms are given to show the action of the tube under practically all the conditions which are likely to occur.

THE three-element vacuum tube utilizes the controlling effect of the grid potential on the plate current, that is, the electron flow from the hot filament to the cold (comparatively) plate. It may be used as a detector of high-frequency waves as used in radio communication, or as an amplifier of electrical signals of any frequency or as a generator of alternating-current power of practically any frequency desired, from perhaps one cycle per second to a hundred million cycles per second. This paper deals with its use as a generator; it seems that such a paper is well worth while because of the undoubtedly wide use which will be made of the tube as a convenient source of high-frequency power, especially for laboratory purposes, and radio telegraphy and telephony.

A three-electrode tube, such as the Type P pliotron, will give about 500 watts of high-frequency power at any frequency desired. In addition to the tube itself there is required a small continuous current generator of about 2500 volts and a set of suitable coils and condensers. Its only competitor as a piece of necessary laboratory apparatus is the high-frequency alternator; this is however no true competitor, its comparatively high cost, difficulty of speed control, limitation of frequency, decrease of output at frequencies differing widely from that for which it was built, attention required to maintain it against mechanical failure, make the vacuum tube superior unless several kilowatts of power are required; in this case the Poulsen arc must also be considered. Even then the flexibility of the tube circuit gives it a marked advantage over the machine. For the very high frequencies the tube is the only practical source of power.

Power output of the tube and its variation under different conditions. The relation between plate current, grid potential, and plate potential, is given by the equation

$$I_p = A\,(E_p + \mu_0\,E_g)^x \tag{1}$$

in which E_p = plate potential, referred to filament,
E_g = grid potential,
I_p = plate current,
A = a constant, depending on the size and spacing of the parts of the tube.

The factor, μ_0, is called the theoretical voltage amplification factor of the tube; it really indicates the relative effectiveness of the grid and plate potentials in controlling the plate current. The exponent, x, has been given as 1.5 and 2 by various writers; it is however a variable, not a constant. Its value is nearly 2 for most tubes throughout a large variation in E_p and E_g, but for extreme values of either, x departs widely from this value. Equation (1) is based on the assumption that the plate current is not large enough to draw from the filament all of the electrons emitted from its hot surface, that is, I_p is less than the saturation current of the tube.

The study of the output, phase relations, etc., was carried out with a type P-10 pliotron, rated at 1000—2000 volts on the plate, filament current of 3.65 amperes, and a safe loss on the plate of 250 watts. The connections were as shown in Fig. 1; in reality the grid is placed between the plate and filament but in this diagram, as in the succeeding ones, we have placed it on the opposite side of the filament to gain clarity in the circuit diagrams. The continuous-current generator E_b furnishes about 1000 volts to the plate through a choke coil L_1, this being suitably large. The continuous-current generator E_c serves to maintain the grid at an average negative potential of any desired amount, and the alternator E_g furnishes the exciting potential to the grid. The condenser C_1 must have a comparatively low reactance if a resistance load is being studied; in series with this is the load resistance R, which absorbs the alternating-current power generated.

The alternating-current resistance of the plate circuit of the tube R_p is the factor which determines how large L_1 and C_1 must be; the value of R_p varies somewhat with the magnitude of the exciting voltage E_g. For large values of E_g it is equal to E_b divided by I_b, if the resistance of L_1 is low; if this resistance is appreciable the voltage E_p must be used instead of E_b. For small values of the exciting voltage E_g the value of R_p is lower than this, perhaps one half or one third. The values of L_1 and C_1 must be so chosen that for the frequency used the reactance of L_1 is large compared to R_p and that of C_1 is small compared to R_p. The suitable values of L_1 and C_1 are really determined by the value of the load resistance, R, but as this must be of about the same value as R_p if much power is to be delivered, R_p does indirectly fix their values.

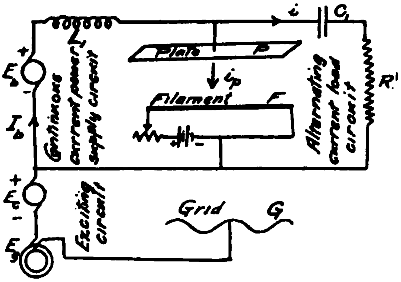


FIG. 1

When the grid potential fluctuates the plate current must vary in magnitude according to the relation given in equation (1); this fluctuating current is equivalent to a continuous current with an alternating current superimposed; the alternating component will practically all flow through the R-C_1 circuit because of the high reactance of the coil L_1.

The first effect studied was the variation of output as the value of the load resistance R, was varied, the excitation being comparatively low; the results are shown in Fig. 2. The curve sheet shows the variation of the various quantities as the value of R was changed; it is seen that a maximum output occurred with R equal to 1000 ohms. For the conditions of this test the value of R_p was 1100 ohms, although the quotient of E_p and I_b gives very nearly 3000 ohms.

In Fig. 3 is shown the behavior of the circuit as the exciting voltage, E_g was varied; the connections were the same as those of Fig. 1. The alternating-current output increases with the square of the exciting voltage up to a certain value of E_g and then increases more slowly. For the lower values of E_g (less than 120 volts) the fluctuating plate current has an approximately sinusoidal form but for the higher values the plate current becomes a series of pulses.

The decrease in I_b in Fig. 2 and increase in Fig. 3 are both due to the fact that the relation between grid and plate potentials and plate current is not linear; if sufficient resistance is put in the load circuit, I_b remains nearly constant as the value of E_g is changed.

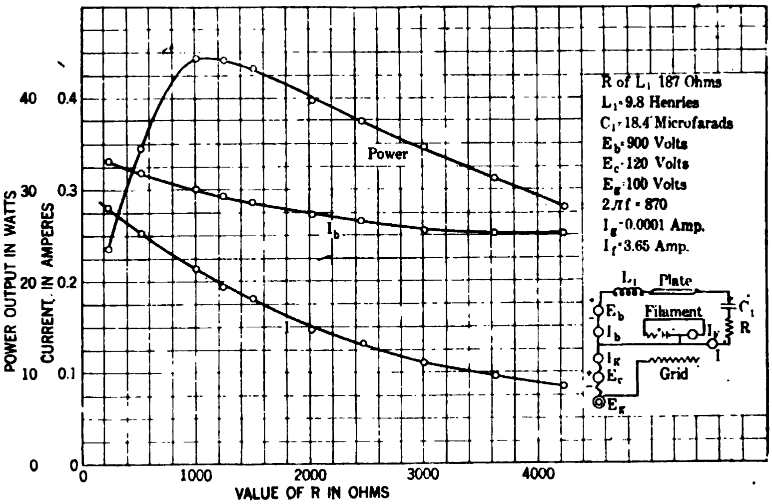


FIG. 2

In Fig. 3 is shown the grid current, as read on a continuous current ammeter; this reached a readable value when the effective value of E_g reached 95 volts. The maximum value of E_g is 134 volts and as E_c was 120 volts it is evident that the grid had to be positive as much as 14 volts before it took an appreciable current. The value of E_g at which I_g becomes readable depends to some extent on the value of the load resistance R.

Safe load of a vacuum tube. The two factors limiting the output of a vacuum tube are the safe filament current and the amount of power which can safely be dissipated from the plate (generally double). The heating of the plate is due to heat radiated from the filament and to the bombardment by the electrons leaving the filament, being accelerated by the high positive potential of the plate, and then being suddenly stopped when impinging on the plate. With a properly evacuated tube the amount of power which can be thus expended on the plate is sufficient to bring it to a dull red heat, this color being judged when the filament is incandescent; if the filament current is quickly reduced to zero the plate has quite a bright cherry red color.

When the tube is not generating any alternating-current power all of the input $E_b I_b$ is used in heating the plate but when alternating current is flowing in the load circuit some of the power supplied to the tube circuit is used up in R and the amount of power used on the plate is equal to $E_b I_b$ minus whatever power is used in the R-C_1 circuit. If the values of E_c and E_g are properly adjusted the amount of power used in R may be equal to or even greater than that used on the plate so the safe value of $E_b I_b$ may be as much as three times the safe plate rating without endangering the tube; if however the tube stops oscillating for any reason the input must be at once reduced or the tube will probably be spoiled.

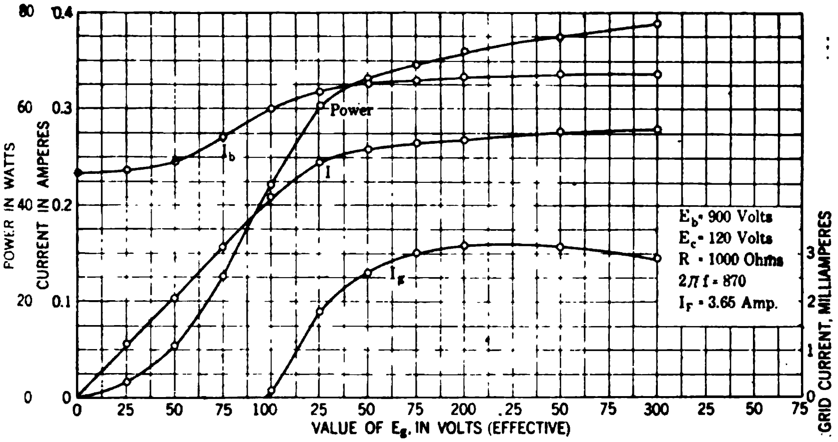


FIG. 3

An approximate idea of the behavior of the tube circuit as the exciting voltage is increased is given in Fig. 4; assuming a constant input, the output and losses follow the variation shown. The efficiency may reach quite high values when suitable adjustments are made. In the small sets used by the Signal Corps the tubes were adjusted for an efficiency of about 30 per cent but it has been found possible to so adjust one of these tubes in the laboratory as to get a fair output with an efficiency better than 70 per cent.

Phases and Forms of Currents and Voltages in a Tube Circuit. If the impedance of the power supply circuit (Fig. 1) is very high compared to that of the load circuit the current I_b is constant, and the equations of the alternating current circuit may be worked out as though the power supply circuit did not exist.

When a voltage E_{mg} sin ωt is impressed on the grid the changes produced in the plate current are the same as though a voltage $\mu_0 E_{mg}$ sin ωt had been introduced into the plate circuit. The changes produced in the plate current circuit by the grid excitation may be calculated on the assumption that the voltage $\mu_0 E_{mg}$ sin ωt was operating in the plate circuit. Thus a voltage E_g on the grid is replaced in our calculations by a suppositious voltage $\mu_0 E_g$ in the plate circuit; the current produced by this voltage can be at once calculated from the impedance of the circuit in which the alternating-current must flow, namely R_p, R, and C_1 in series.

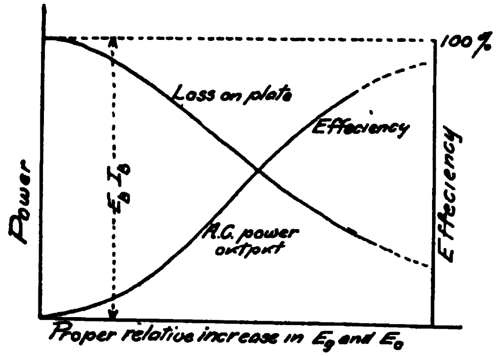
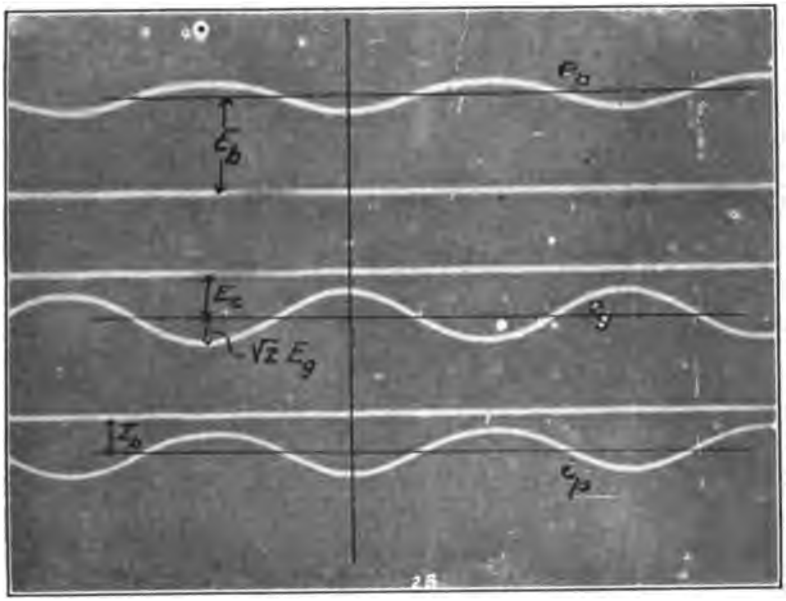


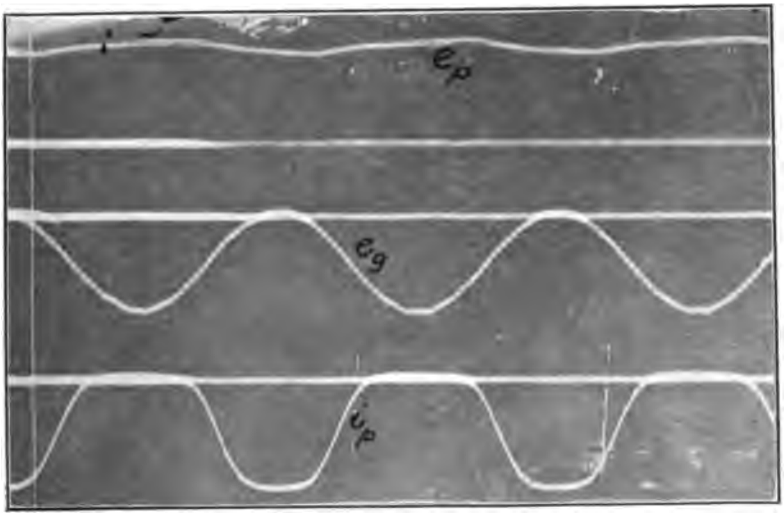
FIG. 4

The vibrators of an oscillograph were introduced in the circuit as shown by Fig. 5 and the polarities so adjusted that currents in the directions indicated by the arrows are positive; when a current is shown on its film below the zero line the



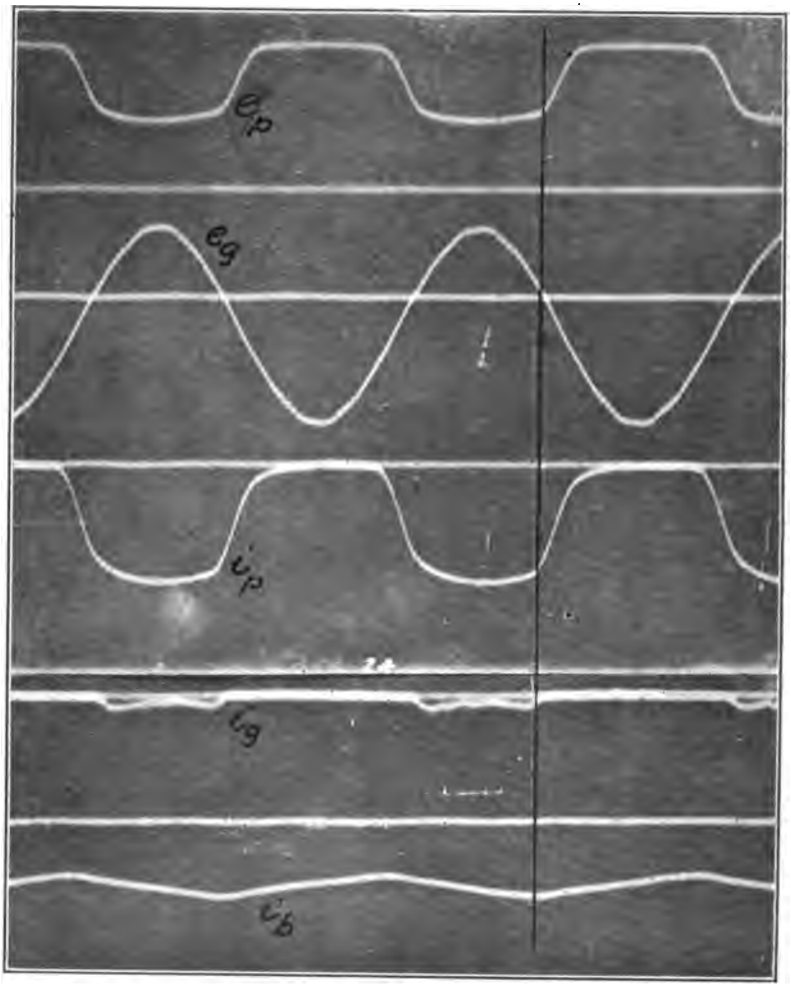
$E_b = 900 \quad I_b = 0.25 \quad E_c = 120 \quad E_g = 50$ FREQUENCY = 140
LOAD $R = 1000$ OHMS $C = 18.4\ \mu f$

FIG. 6



$E_b = 900 \quad I_b = 0.34 \quad E_c = 120 \quad E_g = 100 \quad f = 140$
LOAD $R = 100$ OHMS $C = 18.4\ \mu f$

FIG. 8



[MORECROFT & FRIIS]

$E_b = 900$ $I_b = 0.34$ $E_c = 120$ $E_g = 300$ $f = 140$
LOAD $R = 1000$ OHMS $C = 18.4\,\mu f$
FIG. 7

direction of the current was in the opposite direction to that shown by the arrow. The oscillograph used had only three vibrators; it was sometimes necessary to take several exposures to get all the quantities wanted. This was done by always running the film at the same speed and making one curve (generally i_p) common to all films; it was then possible to so place the films together for reproduction that all quantities were in their correct relative phases.

In the first circuit tested the total resistance in the alternating-current circuit (R_p plus R) was about 4000 ohms and the reactance of the condenser C_1 was 62 ohms; we should expect therefore that the current I_p and voltage E_g would be in phase and moreover if the grid excitation is kept low the alternating component of the plate current, I_p, should be of the same form as the grid voltage. Then as the voltage e_p is determined by the constant voltage E_b and the drop in R, this voltage must also have a sine wave fluctuation in such phase that when the plate current is a maximum the plate voltage is a minimum. Fig. 6 shows the form of plate voltage, plate current, and grid voltage, for the conditions noted below the film. It will be seen that both e_p and i_p are sinusoidal in form and that e_p is 180 degrees out of phase with e_g. The grid current was zero for the conditions given; it may be seen from the film that E_g was not sufficiently large to force the grid to a positive potential.

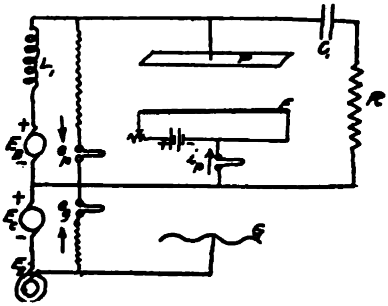


FIG. 5

In Fig. 7 are shown the various quantities for a much greater excitation than was used for Fig. 6. The plate current rises to a value of 0.66 amperes and decreases to zero; the current flowing in the alternating-current circuit must therefore have a maximum value of 0.33 amperes giving an $I\,R$ drop in R equal

330 volts. The fluctuation in e_p as measured in the film is 340 volts. The grid takes current all the time the grid is at positive potential, but it is very small, due to the fact that the plate potential never falls to lower than 660 volts.

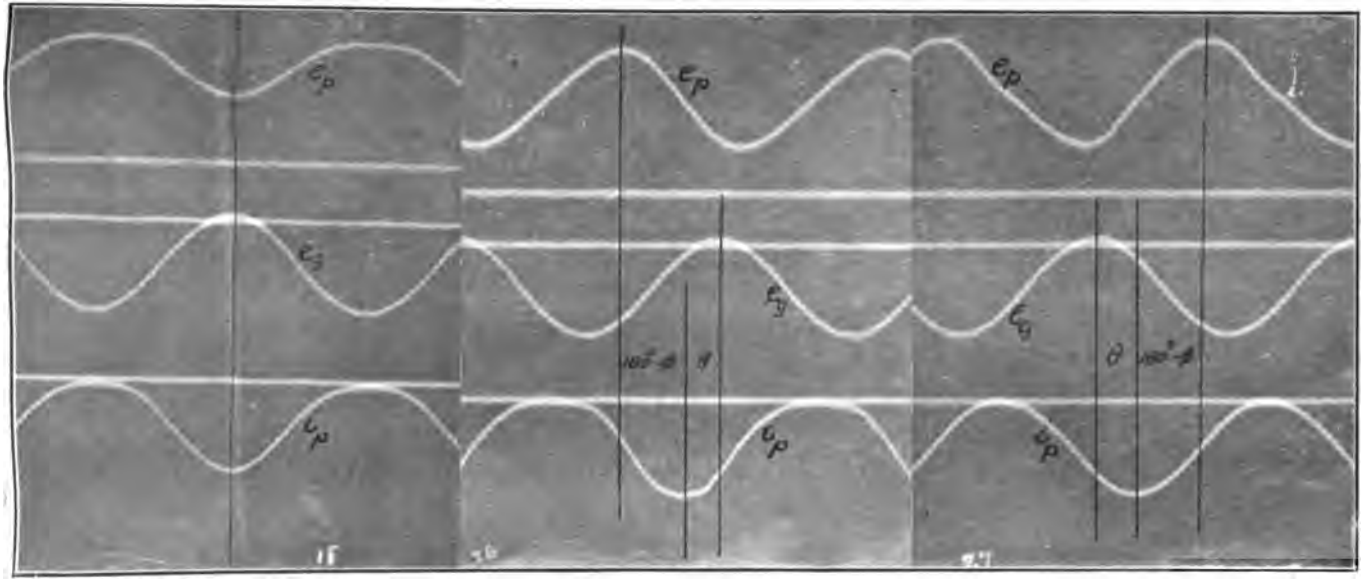
If a greater variation in the plate voltage is desired it can be brought about only by increasing the value of R; if this had been 3000 ohms instead of 1000, the plate potential would have fluctuated throughout a much wider range. As the normal maximum output (assuming sinusoidal variations of voltage and current, is obtained from a tube when both i_p and e_p vary from zero to twice their normal values it is evident that R should have been increased to 3000 ohms to get maximum output. But by referring to Fig. 2 it would seem that 1000 ohms was the proper value of R for maximum output. This discrepancy arises from the fact that the results for Fig. 2 were obtained with low grid excitation while the curves in Fig. 7 were obtained with an excitation three times as large. As previously stated, R_p varies with the excitation, and the maximum output is obtained when the load resistance and tube resistance are the same.

The amount of fluctuation in I_b is small; the value of the alternating component is fixed by the requirement that the reactance drop in L_1, due to it, must be equal to the alternating component of the fluctuating plate voltage. If the fluctuation in e_p were sinusoidal the fluctuation in I_b would also be of that form.

With lower values of load resistance the distortion in plate current occurs even with low excitation; this is shown in Fig. 8 for which R had a value of only 100 ohms. Although there was the same alternating current produced in the output circuit as for the case given in Fig. 7 the fluctuation in plate voltage is very small, due to the very low impedance of the load circuit.

The alternating current of the tube circuit may be analyzed by the laws of the ordinary circuit, in case we are interested only in the sine wave components of the quantities studied. The vector diagrams of the three possible kinds of load circuits are shown in Fig. 9; in these diagrams cos ϕ gives the power factor of the load circuit itself and cos θ gives the power factor of the whole alternating-current circuit, including the resistance of the tube as well as the load circuit resistance.

In Fig. 10 are shown the curves of e_p, e_g, and i_p for circuits corresponding to those assumed in Fig. 9; the three circuits as actually used had the same excitation and nearly the same



[MORECROFT & FRIIS]

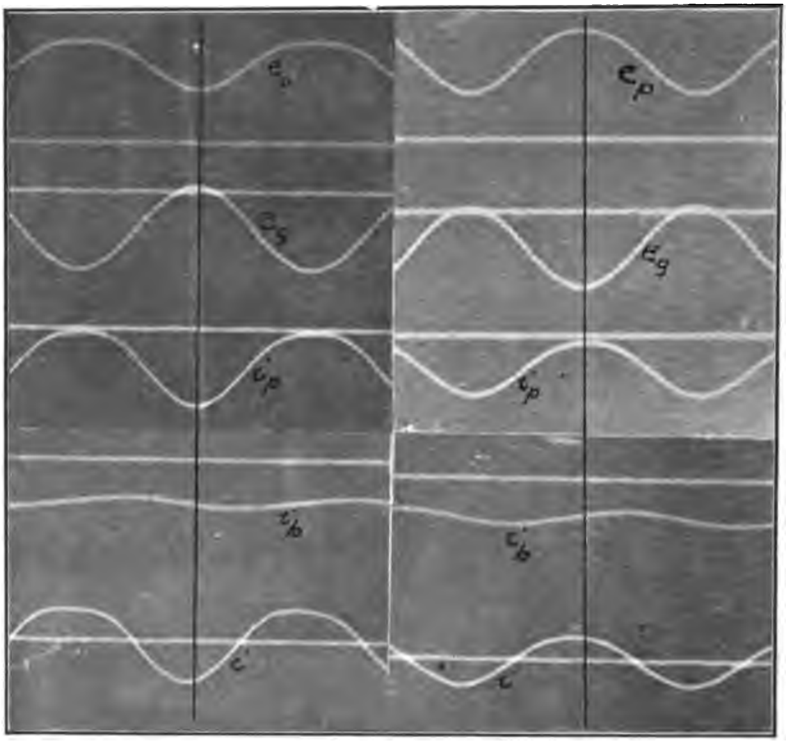
RESISTANCE LOAD
$R = 1000$ OHMS

CAPACITIVE LOAD
$R = O \quad \frac{1}{\omega C} = 1150$ OHMS

INDUCTIVE LOAD
$R = 30$ OHMS $\quad \omega L = 1500$ OHMS

FOR ALL LOADS $E_p = 900 \quad E_c = 120 \quad E_g = 100$

FIG. 10

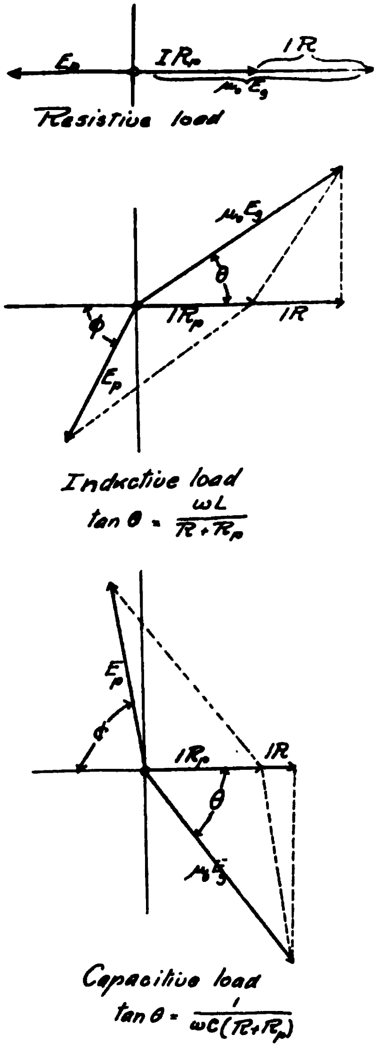


[MORECROFT & FRIIS]

$I_b = 0.295$ $R = 1000$ $I_b = 0.272$ $R = 2010$
$E_b = 900$ $E_c = 120$ $E_g = 100$ $f = 140$

FIG. 11

magnitude of impedance in the load circuit, the first being resistive, the second, capacitive and the third, inductive.



Phase relations in tube circuits

FIG. 9

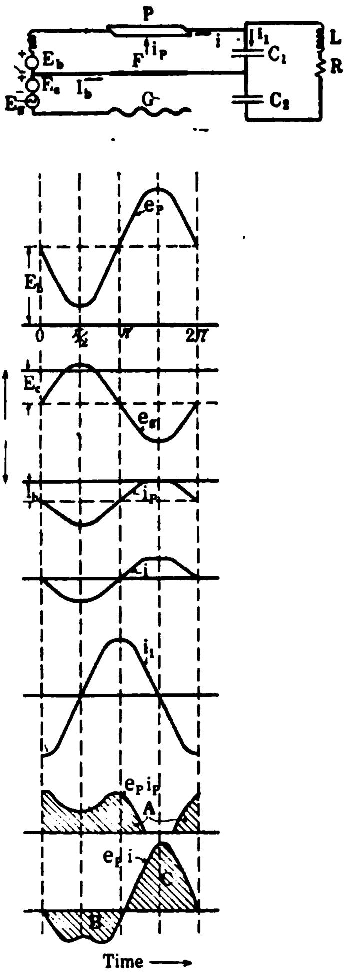


FIG. 13

With resistive load the plate current (alternating component) is in phase with the voltage E_g, with the capacitive load it leads and with the inductive load it lags; the angles ϕ and θ

measured from the film, and the calculated values, agree within the precision of measurement.

There is considerable distortion in all three circuits which could have been nearly eliminated by increasing the impedance of the load circuit to about three times the value used. This would however, have required a corresponding increase in the inductance of L_1 and a suitable coil was not at hand.

The effect of varying the resistance of the output circuit is well shown in Fig. 11; in one case the resistance was 1000 ohms and in the other it was 2000 ohms. It will be seen at once that the distortions produced by the tube can be reduced by sufficiently increasing the impedance of the output circuit. In the case given in Fig. 11 the reactance of the power supply circuit was not increased when the resistance of the output circuit was increased with the result that a much larger fraction of the generated alternating current passed through the supply circuit in the second case than in the first. The greater the amount of alternating-current power used in the power supply circuit, of course, the less efficient is the circuit.

EFFICIENCY OF A TUBE

From the oscillograms given thus far it would appear that the efficiency of a tube generator could not be very high. On the assumption that the plate current and plate voltage both have sinusoidal variations the maximum possible output of the tube would be just half the input; such a high fraction could not be obtained however because the conditions required could not be satisfied. The plate voltage would have to fluctuate between $2\,E_b$ and zero and the plate current between $2\,I_b$ and zero; this latter condition could be satisfied but the first could not be. The plate voltage must not fall below a certain minimum if high efficiency is to be obtained because of the excessive grid current resulting. With the tube we used this minimum was about two hundred volts.

The possible efficiency of 50 per cent mentioned above, it must be remembered, does not consider the amount of power required for heating the filament nor the possible losses in the exciting circuit. If these losses are considered the possible efficiency would be much less, especially in the smaller tubes. We shall now show however that by using peculiar shaped plate currents it is possible to have an efficiency considerably in excess of 50 per cent; in fact the result of this study is that the efficiency of a tube may be pushed possibly as high as 90 per cent by proper design and proper operation, this value neglecting the filament power input.

The importance of getting a high efficiency will be at once appreciated when it is mentioned that a given tube (the one used in this test) has an output of about 200 watts in normal operation whereas if the efficiency could be increased to 90 per cent the safe output would be 2250 watts.

The tests carried out involved an adjustment with separate excitation to find the conditions for maximum output and then transferring the grid connection to a proper point of the circuit to get self excitation, recording for each condition the forms and phases of currents and e. m. fs. The tests were run at low frequency so that oscillograph records might be obtained; the results obtained were duplicated later in a high-frequency run.

Fig. 12 shows the circuit used; simpler ones may be used but the laboratory apparatus at hand was best suited to this one. The diagram also shows where the oscillograph vibrators were introduced and the direction of currents assumed as

FIG. 12

positive; if, on a film, a current is shown below its zero line, it was flowing in the opposite direction to that shown in the diagram. If the frequency of the exciting voltage E_g is chosen the same as the resonant frequency of the load circuit

$$\left(f = \frac{1}{2\pi \sqrt{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}} \right)$$

the impedance of this circuit between the two points M and N, where the tube is attached, will be resistive only, its magnitude being equal to $\frac{1}{\omega^2 C_1^2 R}$ ohms.

The quantities to be considered are shown conventionally in their phases in Fig. 13; the current i_1 which flows in the resonant load circuit may be several times as large as the current i,

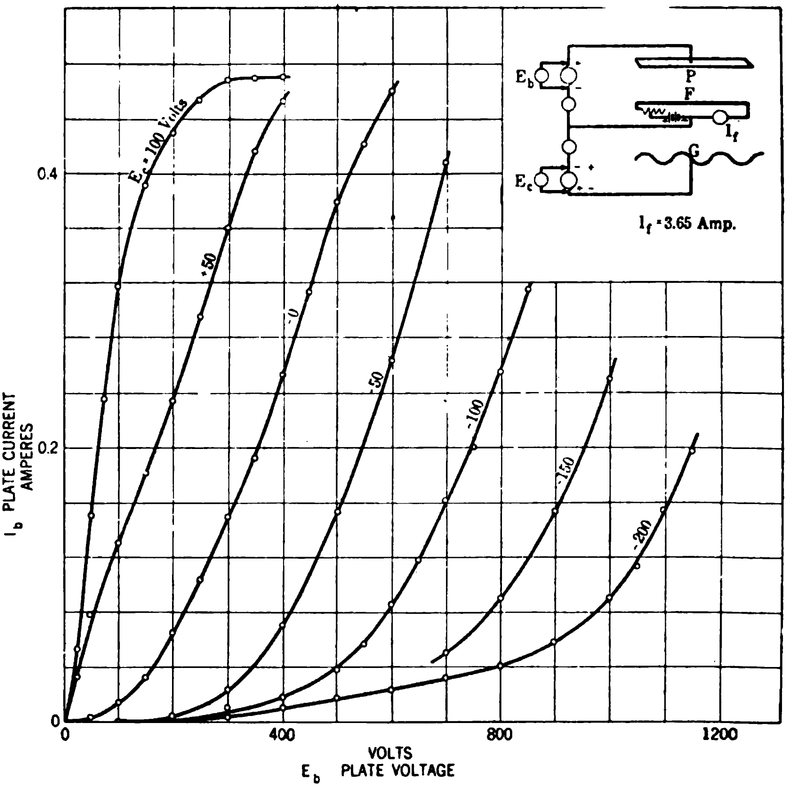


FIG. 14

furnished by the tube. The two important things in this diagram are shown in the lower part of the figure, namely, the curves of $e_p\, i_p$ and of $e_p\, i$. These curves give the power loss on the plate and the power supplied by the tube to the load circuit, respectively. It is at once evident that

$$\text{Energy loss on plate per cycle} = \int_0^{2\pi} e_p\, i_p\, dt = \text{Area } A$$

$$\text{Energy supplied to load circuit} = \int_0^{2\pi} e_p\, i\, dt = \text{Area C} - \text{Area } B$$

It is evidently desirable to make the latter as large as possible and the former as small as possible, if the tube circuit is to operate efficiently. Any ordinary scheme of analysis, using the relation given in equation (1) must fail because ther elation does not hold good for those values of e_p and e_g which are the most important ones in the cycle of operation, namely low e_p with positive e_g, and very high values of e_p with large negative e_g.

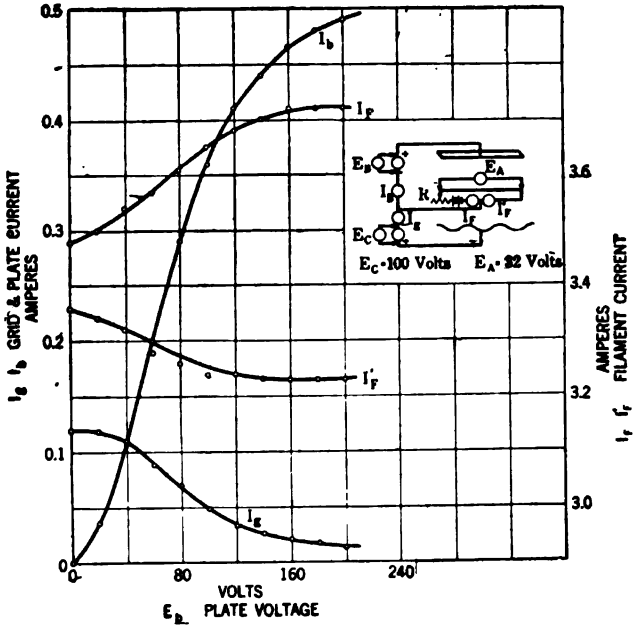


Fig. 15

The ordinary so called static characteristics of the tube used are given in Fig. 14; they are not of much service in predicting the behavior of the tube when the output is forced as high as possible. They did bring out the fact however that the filament ammeter, if a continuous current instrument, does not read correctly the filament current when the tube is generating alternating-current power. The ammeter indicated 3.65 amperes when getting the curves of Fig. 14 and the total emission for such a current is evidently about 0.5 amperes. Now when the tube was oscillating, the filament ammeter reading

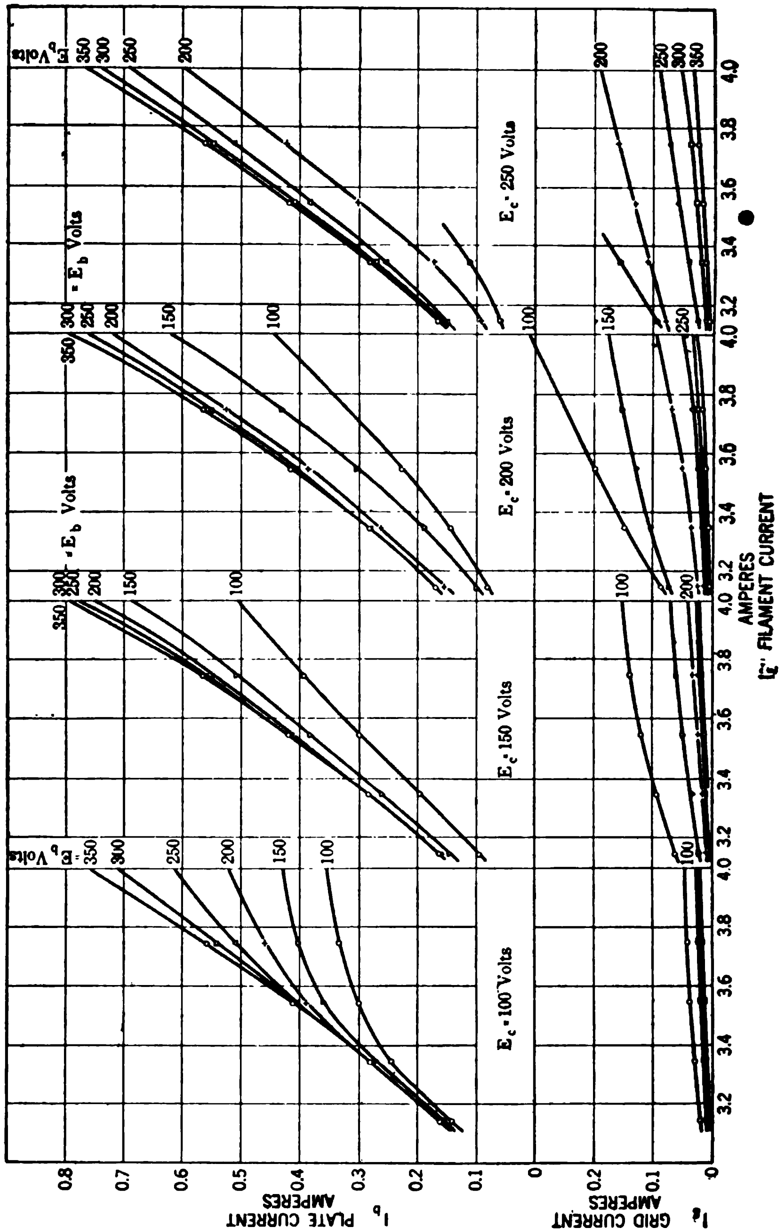

I_b PLATE CURRENT AMPERES
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
I_g GRID CURRENT AMPERES
0.2
0.1
0
I_f FILAMENT CURRENT AMPERES
3.2
3.4
3.6
3.8
4.0
E_c = 100 Volts
E_c = 150 Volts
E_c = 200 Volts
E_c = 250 Volts
E_b Volts
= E_b Volts
350
300
250
200
150
100

FIG. 16

3.65 amperes the total emission was about 0.8 amperes, showing that the filament temperature was much hotter than when not oscillating. Holding the voltage across the filament constant (approximately the condition when the tube is oscillating) the set of curves given in Fig. 15 were obtained. The grid was held at a positive potential of 100 volts and the plate voltage suitably varied. The electron current to the plate increases the filament current at one end and decreases it at the other; the relative increase and decrease will be determined largely by the resistance used in series with the filament battery. It can be seen that even with the larger filament current as great as 3.75 amperes the emission was only 0.5 ampere.

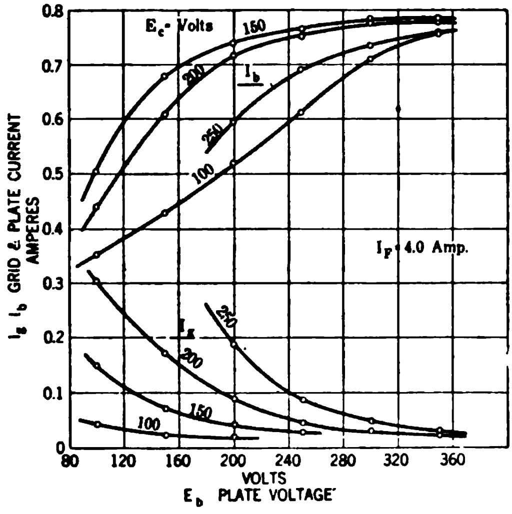


FIG. 17

From some preliminary oscillograph records we knew that in operation the total emission was about 0.8 ampere when the filament ammeter read 3.65 amperes. A brief test showed that the filament current required to give this much emission was 4.00 amperes but this seemed like an excessive current so we got the characteristics required from extrapolation. In Fig. 16 are shown a set of curves showing the variation of plate and grid currents for various filament currents and grid and plate potentials. From this set of curves the results given in Fig. 17 was obtained; as these are important curves they were verified for correctness of form by actually getting them for a lower filament current. These are given in Fig. 18, and are of just the same form as those of Fig. 17.

It is well to point out here that even if we had been able to get the curves of Fig. 17 with a filament current of 4.00 amperes they would not have given the proper values of i_p and i_g for the tube in operation. While getting these static characteristics the plate and grid get very hot, much hotter than when the tube is in operation as a generator. The emission from the filament is fixed by the filament temperature, and this in turn is fixed by the filament current and the temperature of the plate; if this is hotter when getting the static characteristics than when the tube is generating, the value of i_p and i_g obtained would probably be too large.

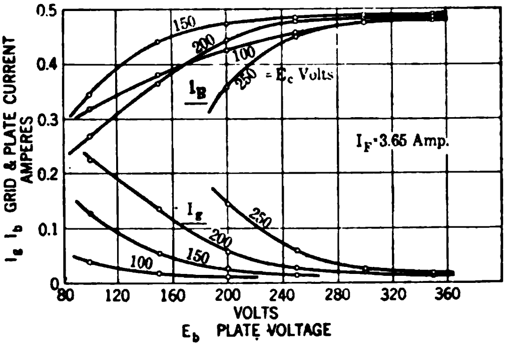
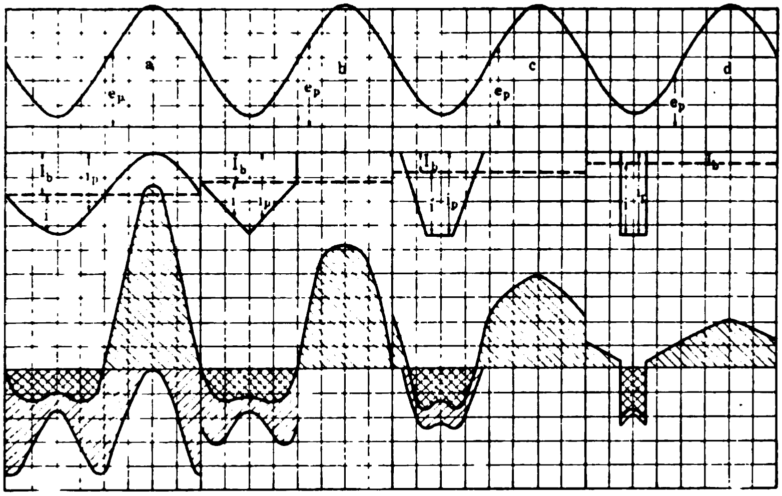


FIG. 18

The curves of Fig. 17, in connection with Fig. 13 enable us at once to give the minimum potential to which the plate should drop and the maximum positive potential for the grid. In order to make the area *A* Fig. 13 small the plate potential, at time $\pi/2$, should be as low as possible. This minimum will be controlled however by the other requirement that the area *C* should be large. If during the time when e_p is low i_p does not have its maximum possible value (saturation current) then the positive alteration of *i* will not be as large as it should be and if this is not large the power input to the load circuit, determined principally by the area of *C*, will be lower than its proper value.

As the average value of *i* must be zero, if its positive loop is to be as large as possible, and the area of *A* to be kept as small as possible the conditions should evidently be so adjusted that at minimum plate potential saturation current should flow, and this flow should last for a short time only. During the rest of the cycle the plate current should be zero.

Fig. 19 shows the calculated losses on the plate and input to the load circuit for four different forms of plate current, the plate voltage having the same form for each. It will be seen that both the losses and the output of the tube are greatest for the sinusoidal plate current, but the efficiency for this condition is only 39 per cent; as the form of plate current approaches a short pulse the efficiency increases, being 82 per cent for the form shown in curve (*d*). The trapezoidal form shown at (*c*) resembles very closely the form we used; the test actually gave about 60 per cent efficiency.



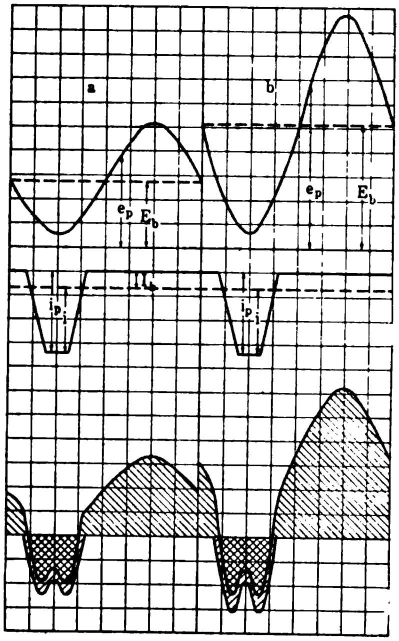
$\int e_p i_p dt$ = 72	$\int e_p i_p dt$ = 35	$\int e_p i_p dt$ = 35	$\int e_p i_p dt$ = 7
$\int e_p i_p dt$ = 47	$\int e_p i_p dt$ = 31	$\int e_p i_p dt$ = 28	$\int e_p i_p dt$ = 23
EFFICIENCY = 39%	EFFICIENCY = 47%	EFFICIENCY = 49%	EFFICIENCY = 77%

FIG. 19

All four curves are drawn with the maximum plate current the same, supposedly the saturation current for the filament current used; by carrying out other constructions it will be evident that any other condition would result in poorer operation.

By referring to Figs. 13 and 17 it may be seen that for the tube we have, the plate potential should not fall lower than 200 volts, that at this time the grid should have a positive potential of 150 volts. With greater or less grid potential, the plate potential being 200 volts, the plate current would be less than saturation value; with less plate potential the current (at time $\pi/2$) would be less than saturation value, and with greater voltage than 200 volts the loss on the plate would be greater than necessary.

It is to be noted that the efficiency will increase for all the cases given in Fig. 19 if the value of the power supply, E_b, is increased, providing that conditions are suitably changed to have the same minimum plate voltage as given in Fig. 19. This is shown by Fig. 20; the two cases given suppose the same form of plate current and same minimum value of plate voltage but in the second the voltage E_b is about twice as large as in the first case. It is seen that the loss on the plate is increased only 25 per cent whereas the input to the load circuit has been more than doubled. The higher the value of E_b the higher is



$\int_0 e_p i_p \, dt = 15$
$\int e_p i \, dt = 34.5$
EFFICIENCY = 70%

$\int e_p i_p \, dt = 19$
$\int e_p i \, dt = 71.5$
EFFICIENCY = 79%

FIG. 20

the efficiency, the limit being fixed by the safe voltage for the tube.

In the tube we used the efficiency did not rise as high as might be expected, due to fact that it took excessively high negative potential on the grid to bring the plate current to zero. The oscillograms showed this effect, so a static characteristic curve was taken to investigate this point; it is shown in Fig. 21. If equation (1) were valid for this tube a negative potential of 260 volts on the grid would have brought the plate current to zero, whereas it took about 1000 volts; although the plate current is small with a grid negative more than 300 volts, this small current has a marked effect on the loss of power on the plate, because of the very high plate voltage during that part of the cycle when this small current is flowing to the plate.

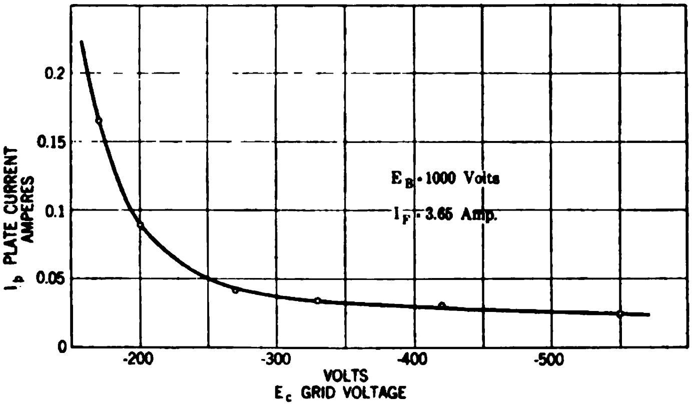


Fig. 21

Experimental proof of foregoing theory. To test the validity of the ideas presented above a series of runs was made with the tube, using the circuit given in Fig. 12 and the results therefrom are shown in Table I. The frequency was kept at the resonant value for the output circuit and each time a set of readings was taken the value of *R* was changed properly to maintain the current in the oscillating circuit constant. This was necessary in order to keep the form of the voltage, e_p, constant as the values of E_c and E_g were varied. While it was not thus pointed out in discussing the current forms of Figs. 19 and 20 the values of E_c and E_g are the factors which bring about the change of current form as the form of e_p is maintained constant. The form of current shown in (a) Fig. 19 was obtained with relatively low E_c and E_g, the value of each of these being increased for the succeeding diagrams of the figure.

TABLE I.

E_b = 1000 volts. $C_1 = 2_\mu F$. $C_2 = 3.91_\mu F$. $\sim$ = 138. $L_1 = 9.8H$. I_f = 3.65 Amp.

Run	E_c volts	E_g effective volts	Input watts	I_2 effective amps.	R ω	Output RI_2^2 watts	$\eta = \frac{\text{Output}}{\text{Input}}$ %
E	120	220	334	0.98	149	143	42.8
	150	220	298	1.00	149	149	50.0
	180	220	241	1.02	119	124	51.5
	210	220	186	1.00	89	89	47.8
	250	220	119	0.96	42	39	32.8
	150	260	302	1.00	149	149	49.3
	180	260	273	1.03	149	158	58.0
	210	260	241	1.99	149	146	60.5
	240	260	197	0.99	119	117	59.5
	270	260	161	0.96	89	82	51.0
	150	300	302	1.00	149	149	49.3
	180	300	283	1.02	149	155	54.8
	210	300	261	1.02	149	155	60.0
	240	300	246	1.00	149	149	60.8
	270	300	212	0.98	134	129	60.8
A	180	340	291	1.01	149	152	52.3
	210	340	278	1.03	149	158	56.8
	240	340	265	1.04	149	161	60.8
	270	340	244	1.01	149	152	62.4
B	300	340	229	0.99	149	146	63.8
	330	340	186	0.99	119	117	63.0
	270	400	260	1.02	149	155	59.7
	300	400	250	1.03	149	158	66.3
	330	400	235	1.02	149	155	66.0
	360	400	222	1.00	149	149	67.3
	390	400	197	0.96	134	124	63.0
	420	400	150	1.02	89	93	61.8
C	450	400	126	0.98	74	71	56.3
D	410	460	228	1.02	149	155	68
	420	500	245	1.05	149	164	67.0
	450	500	237	1.04	149	161	68.0
	480	500	222	1.01	149	152	68.6
	510	500	195	1.04	119	129	66.2
	540	500	176	1.02	104	108	61.5
	570	500	157	1.04	89	96	61.2

In Fig. 22 are shown the efficiency curves for the various runs of Table I and on the curve sheet are given the calculated values of the maximum positive grid potential for that condition in each run which gave maximum efficiency, as indicated at *a*, *b*, *c*, *d*, etc., For the comparatively low value of current in the oscillating circuit which obtained during these tests the form of plate voltage is somewhat different from a sine wave, and the variation of best grid potential may have been due to this cause. It is also possible that the change in efficiency was caused by the change in minimum plate current for the different excitations. The increase in efficiency with increase of E_g and E_c is as would be expected from the analysis given for Fig. 19.

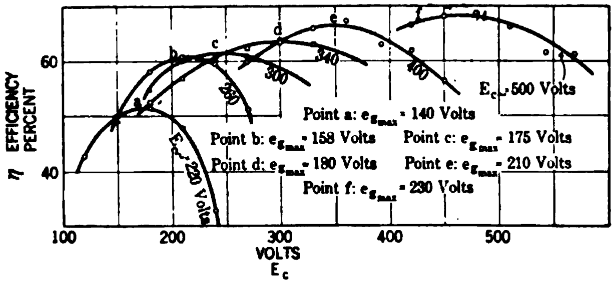


FIG. 22

A series of runs was then carried out (results given in Table II) to study the effect of varying the value of the minimum plate voltage, other conditions remaining the same; this was accomplished by varying R, thus cutting down the value of the oscillating current and hence the variation of voltage across the condenser C_1, Fig. 12. The variation of potential across this condenser, it will be noticed, is what controls the fluctuation of plate voltage.

TABLE II.

E_b = 1000 volts. $C_1 = 2\mu F$. $C_2 = 3.91\mu F$. $\sim = 138$. $L_1 = 9.8H$. I_f = 3.65 Amp.

R_{xn}	e_p min volts	E_c volts	E_g effective volts	Input watts	I_2 effective amps.	R ω	Output $= RI_2^2$ watts	$\eta = \frac{\text{Output}}{\text{Input}}$ watts
A	30	270	300	134	1.12	37	46.5	34.7
	100	270	300	179	1.10	85	103	57.5
	160	270	300	204	1.02	117	122	59.8
	250	270	300	217	0.91	149	123	56.8
B	490	270	300	255	0.60	297	107	42.0

The value of minimum plate voltage can be calculated by subtracting from E_b the I_b resistance drop through L_1 (which was very small for most of our tests) and from this subtracting, the maximum value of the alternating potential drop across C_1. These calculations were made and the results are shown in the curve of Fig. 23; the results verify, better than might be expected, the conclusions reached from theory. With the exception of the first value of e_p (min.) the calculated values agreed with the values measured from the films; the value of 30 was obtained by measurement of the films the calculated value not agreeing very well in this case. Too much reliance cannot be placed on the results of this test however as the exact form of the e_p curve might affect the results considerably; as the value of R was varied the relative magnitudes of i and i_1 (Fig. 12) changed and this would affect the form of e_p.

For various of the runs given in Table I oscillograms were taken of some or all of the quantities involved. For the conditions of run A the curves of e_p, e_g, and i_p are given in Fig. 24. From this film, as from the succeeding ones, the first thing to be noticed is that the grid voltage and plate voltage are just 180 degrees out of phase, showing that the load circuit was resistive only. The maximum positive potential of the grid measures on the film 296 volts and the corresponding value of plate potential measures 220 volts. By reference to the curves of Fig. 17 it may be seen that for these respective voltages a large part of the electron current is drawn to the grid, resulting in the peculiar double humped curve of plate current. The maximum negative grid potential was 650 volts, but even this was not sufficient to make the plate current zero. Its values follow, exactly as can be measured, the values given by the curve of Fig. 21. The slight deformation occurring on the positive alternation of E_g is due to the pulse of current taken by the grid at this part of the cycle; the wave form of the alternator used for E_g was nearly a pure sine wave, as may be seen from some of the other films to be given, in which the grid took no current.

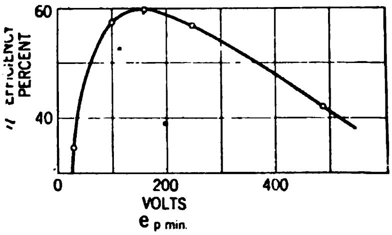
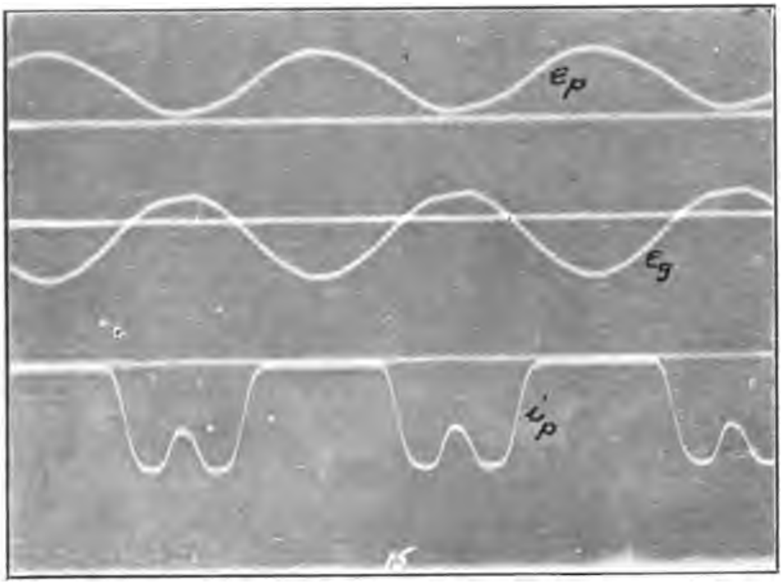
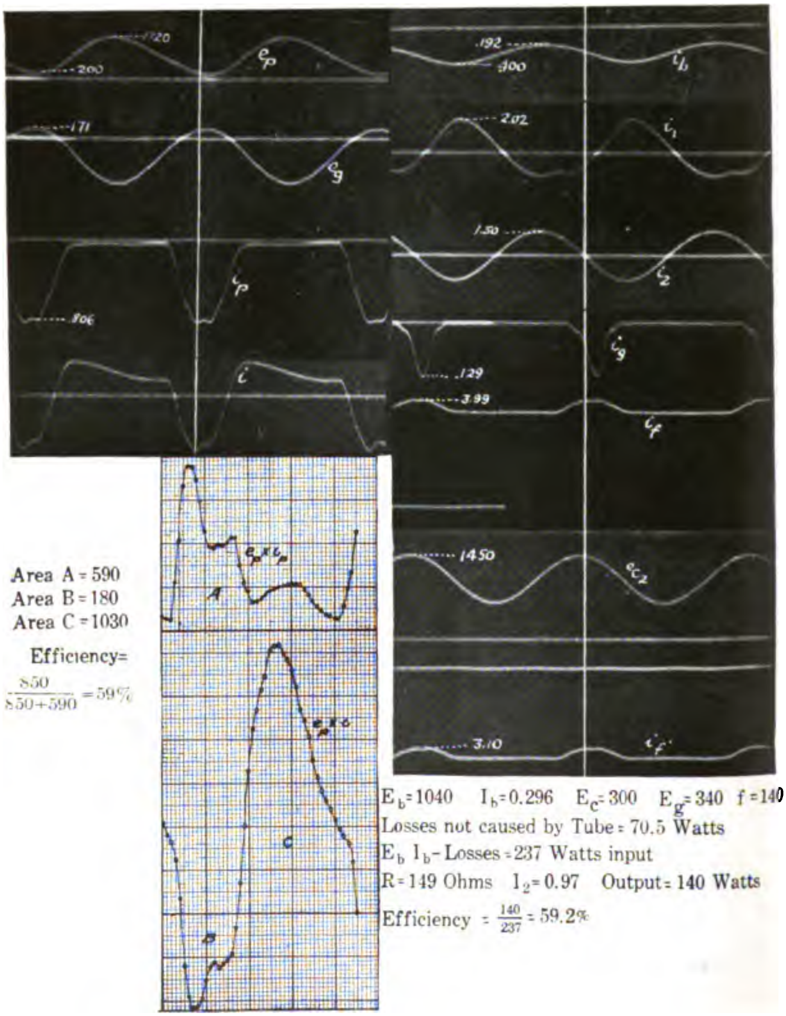


FIG. 23



[MORECROFT & FRIIS]

RUN A TABLE I
FIG. 24



Area A = 590
Area B = 180
Area C = 1030

Efficiency = $\frac{850}{850+590} = 59\%$

$E_b = 1040$ $I_b = 0.296$ $E_c = 300$ $E_g = 340$ $f = 140$
Losses not caused by Tube = 70.5 Watts
$E_b\ I_b$ - Losses = 237 Watts input
$R = 149$ Ohms $I_2 = 0.97$ Output = 140 Watts
Efficiency = $\frac{140}{237} = 59.2\%$

[MORECROFT & FRIIS]

FIG. 25

For run B a set of oscillograms was taken to show all of the quantities involved in the operation of the tube; it required five oscillograph records to get all the quantities wanted. These five films were combined to make the record shown in Fig. 25; in fitting the various films together care was taken to see that they had their proper respective phases. The white line drawn vertically through all the records gives a line of equi-phase.

This set of curves gives the complete story of the circuit and tube. The plate current is very nearly the form shown in Fig. 20, and the plate potential is nearly of the form shown in condition (a) of the same figure. The slight depression in the peak value of i_p is due to the grid taking some current, this depression coinciding in time with the peak of grid current. The form of the positive alternation of the i curve is not like those previously given, due to the fact that it has been assumed that I_p was constant whereas it actually had considerable fluctuation, as shown in the record. If the coil used for L_1 had more inductance this variation in I_b would be diminished; we had only 10 henries with a resistance of 189 ohms, the coil being air core. In practise an iron core coil of greater inductance would be used but we did not want to introduce any other sources of distortion than the tube itself.

The form of current in condenser C_1 differs from that in condenser C_2 because of the effect of i_1, which will practically all flow through C_1 for the circuit as arranged.

The grid current has just the form and magnitude predictable from Fig. 17; the amount of current taken by the grid in this test and the values of E_g and E_c used caused a loss of power on the grid (due to bombardment) of about 10 watts.

The two filament currents i_f and i_f' have forms which might be predicted from curves similar to those given in Fig. 15; in that end of the filament carrying the large current the continuous current ammeter measuring the current indicated only 3.65 amperes whereas the current actually went as high as 3.99 amperes when the plate was taking its maximum current. The exact amount of emission from the filament when the tube is acting as a generator cannot be predicted from the static characteristic; the temperature distribution in the filament which exists in the oscillating condition cannot be duplicated in a static test and it is this temperature distribution which determines the emission.

The drop across the condenser C_2 was taken to see whether or not it had the right magnitude and phase to serve for excitation of the grid when the tube was run self-exciting; the value of C_2 had been adjusted with this point in mind.

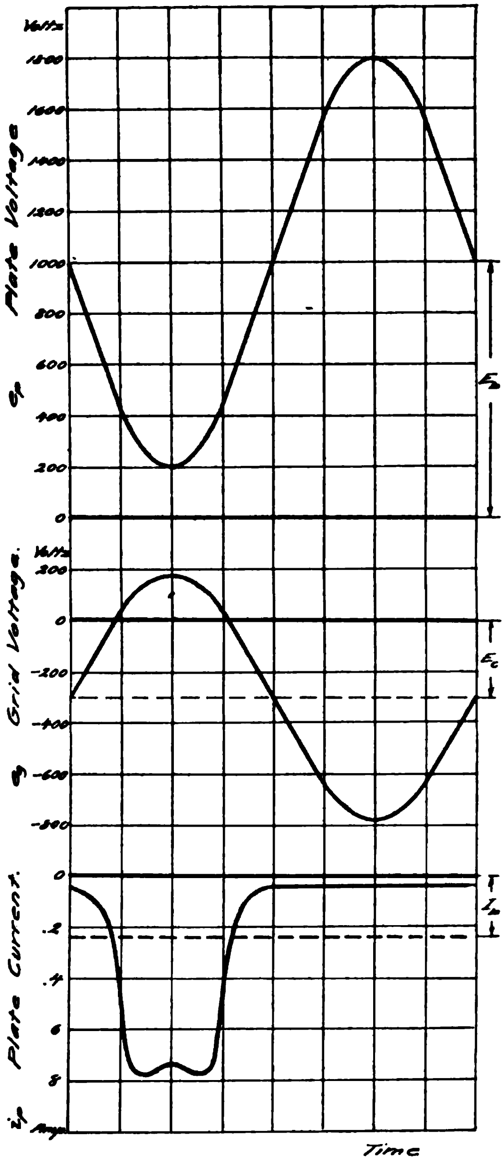


FIG. 26

The scheme of getting the efficiency indicated in Figs. 19 and 20 was tried on this record of e_p, i_p, and i, the power curves of $e_p\,i_p$ and $e_p\,i$ being shown in Fig. 25; the value obtained, 59 per cent, agrees within the precision of the test with that measured by the meters in the test. The value of 63.8 per cent given in Table I was the value obtained when the oscillograph circuits were not connected; the closing of the circuits changed the conditions enough to drop the efficiency to 59.5 per cent.

Fig. 26 shows the form of i_p which is predicted from Fig. 17 after the forms and magnitudes of e_p and e_g have been assumed; this form of i_p is very close to the actual form given in the oscillogram of Fig. 25.

The result of our tests and analysis have then shown that the efficiency of a tube as a generator can be accurately predicted from the three sets of curves given in Figs. 14, 17, and 21 after we have determined, from the curves of Fig. 17, what the best minimum plate potential is and also what the maximum positive potential of the grid should be.

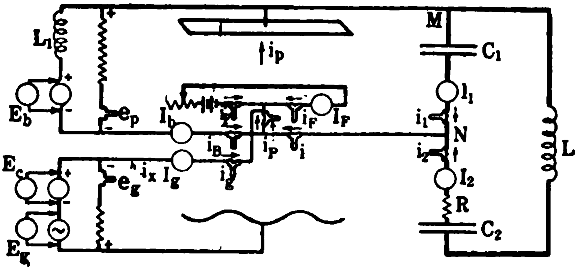
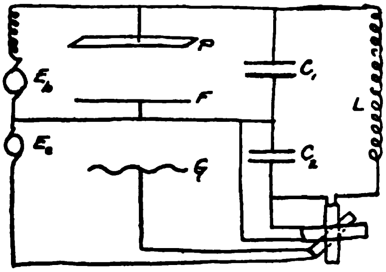


FIG. 27

To get a fair efficiency (60 per cent or better) the value of I_b should not be greater than 25 per cent of the saturation current of the tube; with the efficiency known and the safe radiation of power from the plate being known, the proper value of E_b is fixed.

Self Excited Tube. Using the circuit and constants used in getting the records of Fig. 25 an attempt was made to run the tube self exciting by changing the connections slightly as shown in Fig. 27. The choke coil L_2 serves to prevent the grid from being short-circuited to the filament (for the a-c. excitation) through the machine E_c. The voltage for excitation was obtained from the drop across the condenser C_2, the insulating condenser C_3 being necessary to prevent short-circuiting the machine E_b. With this connection the grid does not get quite as much excitation as shown by the curve e_{c2} in Fig. 25, because an appreciable part of this voltage is used in overcoming the reactance drop in C_3. (In this calculation the capacity of the grid circuit of the tube itself must be considered; in some of the type P tubes this capacity is as high as 200 $\mu \mu f$. when the load circuit has its proper impedance for maximum output.)

The circuit of Fig. 27 refused to act as it did for the separate excitation, giving a small output at a low efficiency; a more careful examination of the record in Fig. 25 gave the reason. The alternating components of e_g and e_p must be exactly 180 degrees out of phase if the maximum output and efficiency are to obtain, as becomes at once evident if the construction of Fig. 19 be carried out for any other than the 180 degree relation. Measurement of the film of Fig. 25 shows e_{c2} to be 33 degrees out of the 180 degree phase with e_p and that much phase displacement is sufficient to completely upset the conclusions so far reached. It was therefore necessary to change the relative phases of e_p and e_{c2}. A possible scheme is conventionally indicated in Fig. 28; a rotating field is produced by proper connection to the load circuit and a rotatable coil placed in this rotating field serves for the grid excitation. We had a simpler scheme at hand so did not try this one.



Grid excitation at adjustable phase

FIG. 28

The difference in phase in the voltages across C_1 and C_2 comes from the effect of the current i, present in C_1 to a greater extent than in C_2. By making the effect of this current small its disturbing effect may be reduced, and this can be done by increasing the values of C_1 and C_2, and decreasing the value of R, the value of L being properly reduced to maintain the same frequency. The increase in capacity will increase the value of the oscillatory current i_1 and as i remains constant its effect on the relative phases of e_{c1} and e_{c2} becomes proportionately less.

The arrangement of apparatus remaining as in Fig. 12 the constants were readjusted for efficient operation and a set of readings was obtained as follows; $E_b = 900$ volts, $E_c = 230$ volts, $E_g = 310$ volts, frequency = 143, $L_1 = 9.8$ henries, $I_b = 0.321$ amperes, $C_1 = 9.2$ microfarads, $C_2 = 18.4$ microfarad. The resistance of the load circuit was 7.80 ohms and the oscillatory current produced was 4.30 amperes giving an alternating-current output of 143 watts. The input to the tube circuit is obtained from the product $E_b I_b$ after certain losses, not chargeable to the tube circuit, have been deducted.

The condensers C_1 and C_2 each consisted of two condensers connected in series because of the high potentials occurring in the circuit. In order to make the two individual condensers divide the voltage E_b equally it is necessary that their insulation resistances be alike, a condition seldom encountered. That condenser having the higher resistance (the better one) will take all of the E_b voltage as well as its share of the alternating voltage of the circuit, resulting in its probable breakdown. To prevent this occurrence leak resistances were used across each of the condensers making up C_1 and C_2, the leaks each being 21,000 ohms making the leak resistance of C_1 and C_2 each 42,000 ohms. Subtracting the I^2R losses in these leaks as well as the $I^2 R$ loss in the choke coil L_1, gives the input to the tube circuit 229 watts; the efficiency was thus 62.7 per cent.

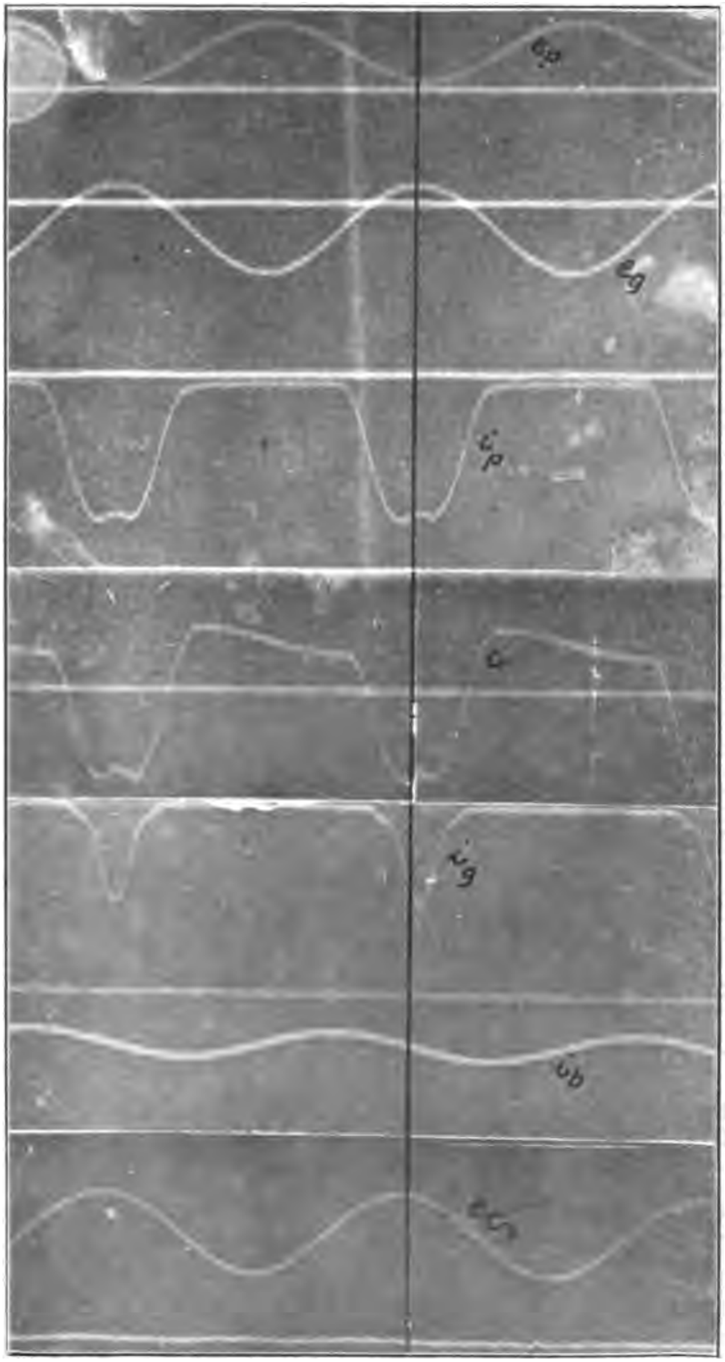
Oscillograms taken of the currents in this circuit are given in Fig. 29. It is evident that the values of E_g and E_c might well have been greater, resulting in a higher efficiency because of the resultant smaller minimum plate current. Although the plate current during the time $A - B$ (Fig. 29) is small, the plate voltage is large and so results in a high unnecessary loss on the plate.

The phase of E_{c_2} is now practically coincident with that of E_g and it should therefore serve as a source of excitation. The circuit did not give as much power however when made self exciting as it should, so the constants were changed slightly to get more power. As finally tested the self-exciting circuit had the constants and performance given herewith: $E_b = 1040$ volts, $I_b = 0.335$ amperes, $C_1 = 7.36$ microfarads, $C_2 = 13.8$ microfarads, $L = 0.201$ henry, $L_1 = 9.8$ henry, $L_2 = 9.0$ henry $E_c = 230$ volts, $R = 8.0$ ohms. The current produced in the oscillating circuit was 4.40 amperes resulting in an efficiency of 57 per cent.

Fig. 30 shows the currents and voltages in this self exciting circuit and it is at once evident why such a low efficiency was obtained; the minimum plate voltage instead of being 200 volts, as it should for this tube, was 300 volts. For this figure the curve of plate current included also the alternating part of the grid current, hence the absence of the depression at the peak value.

The current through the plate current vibrator reversed during part of the cycle, due to the fact that this vibrator carried in addition to the plate and grid currents, an alternating current which resulted from the voltage across the condenser C_2 acting through the reactance of coil L_2 and condenser C_3, Fig. 27. This current is shown as i_x in Fig. 30; when the plate current is corrected by this small amount it is seen that the plate current does not reverse, as we know it cannot with the conditions as they existed in this test.

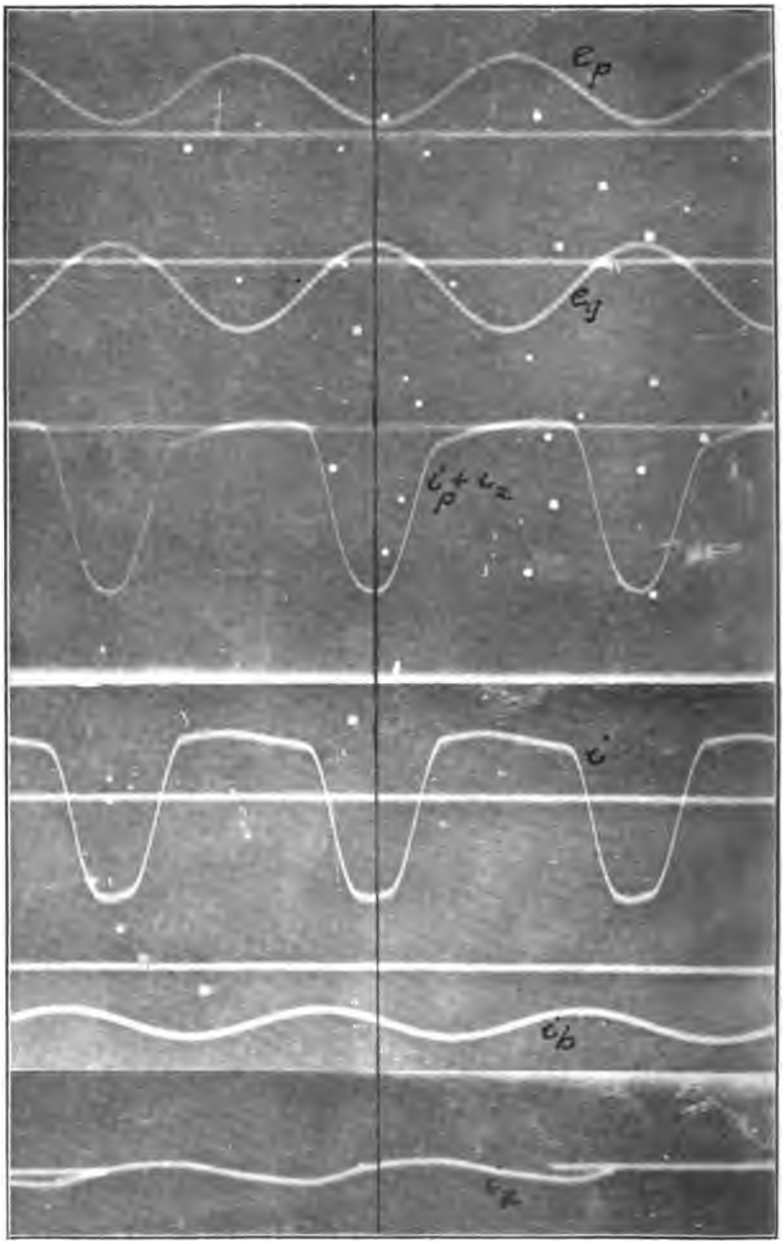
Action of the Tube at High Frequency. It was desired to show that the action of the tube was just the same at high frequency as at the low frequencies used, so a circuit was arranged similar to that of Fig. 27, with smaller values of capacity and inductance. The choke coils L_1 and L_2 used in the previous tests would act as condensers of comparatively low reactance at the high frequency to be used, so they also had to be changed. The constants of the circuit used were: $E_b = 1000$ volts, $I_b = 0.285$ amperes $C_1 = 0.0144$ microfarad, $C_2 = 0.0284$ microfarad frequency = 98,500, $L_1 = 0.023$ henry, $L_2 = 0.016$ henry, $E_c = 240$ volts, $R = 6.16$ ohms (high frequency determination). There were no leaks used with the condensers in this circuit so that the product $E_b I_b$ after subtracting the $I^2 R$ loss on the choke coil L_1, gives the input. It is found to be 284 watts, and as the output to the load circuit was 160 watts the efficiency was 56.2 per cent which is in fair agreement with the results obtained at 166 cycles.



$E_b = 940$ $I_b = 0.310$ $E_c = 230$ $E_g = 310$
$R = 7.80$ $I_2 = 4.30$
INPUT = 229 OUTPUT = 143 EFFICIENCY = 62.5 PER CENT

FIG. 29 [MORECROFT & FRIIS]

PLATE LXIV.
A. I. E. E.
VOL. XXXVIII, 1919



[MORECROFT & FRIIS]

$E_b = 1040 \quad I_b = 0.335 \quad E_c = 230$

$R = 8.00 \quad I_2 = 4.40$

INPUT = 272 OUTPUT = 155 EFFICIENCY = 57 PER CENT

FIG. 30

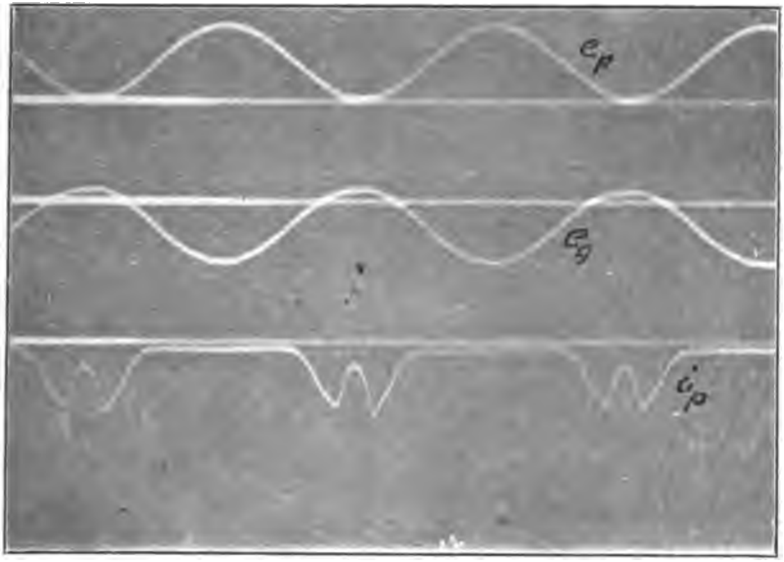
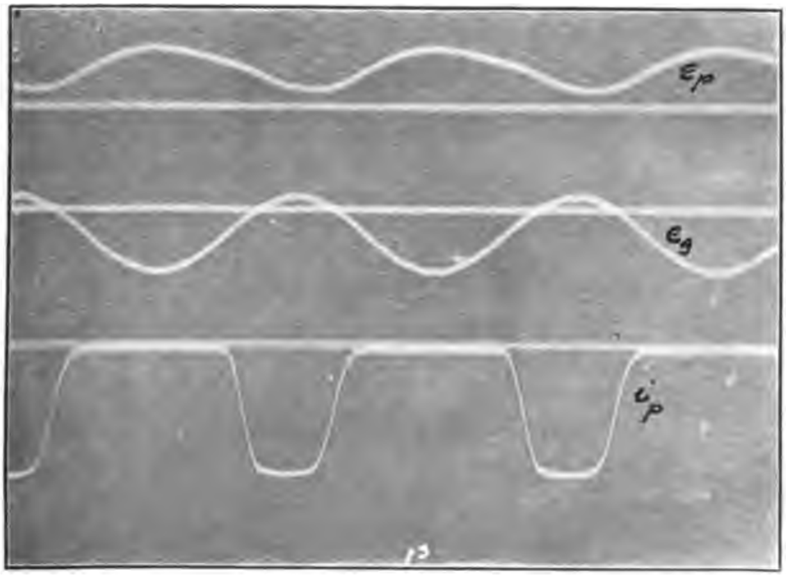


FIG. 31—RUN A TABLE II



[MORECROFT & FRIIS]

FIG. 32—RUN B TABLE II

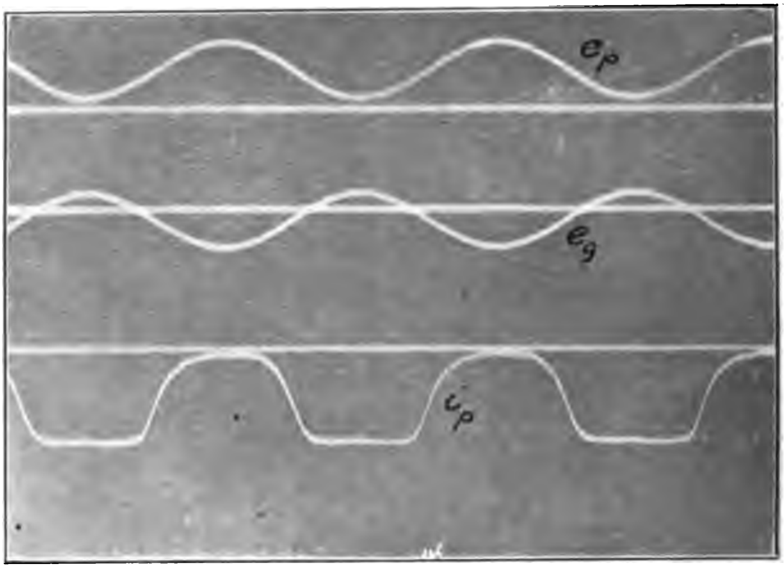


FIG. 33—RUN E TABLE I

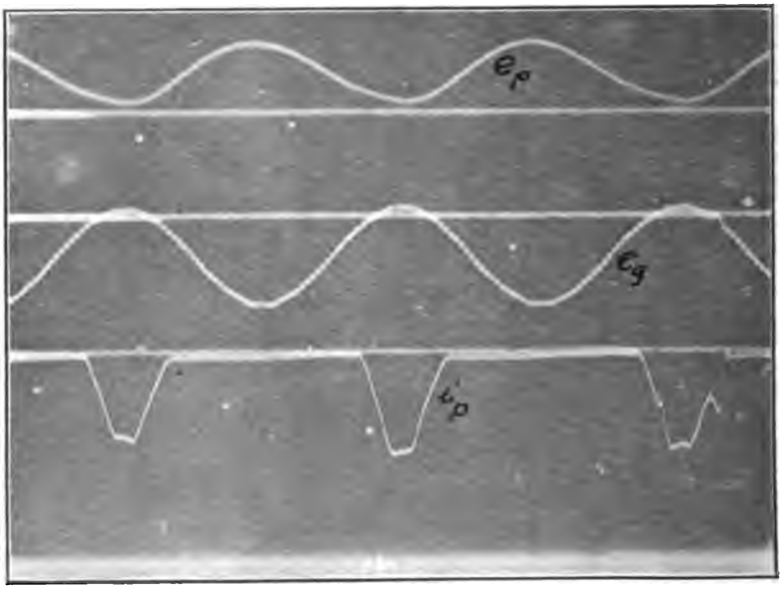
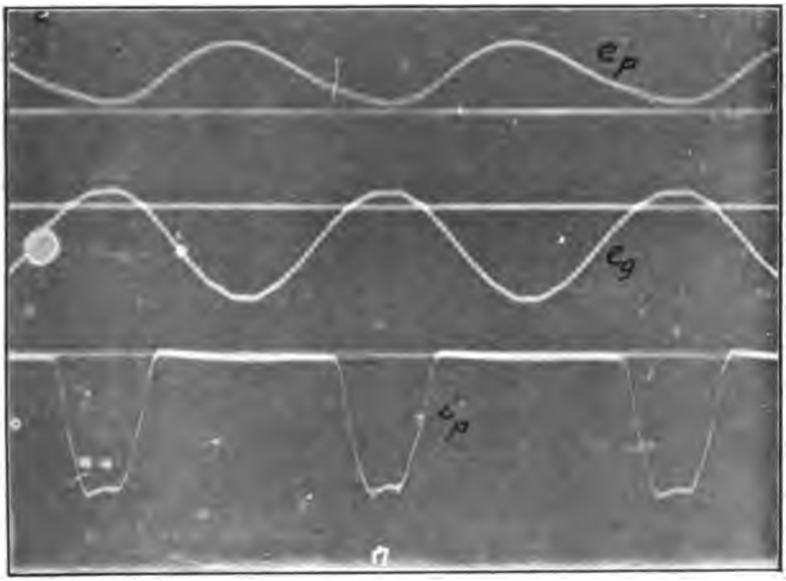


FIG. 34—RUN C TABLE I



[MORECROFT & FRIIS]

FIG. 35—RUN D TABLE I

In Figs. 31 to 35 are shown some special oscillograms of the plate current, plate voltage, and grid voltage, all for the separately excited tube with the circuit shown in Fig. 12; the conditions of the circuit were as noted in Tables I and II.

The conditions obtaining when Fig. 35 was taken show the best adjustments for efficiency which we were able to get with the type P tube; the high efficiency was obtained without unduly decreasing the output. If this form of plate current could be maintained and the value of E_b be increased to 3000 volts the calculated efficiency becomes 85 per cent; this is probably as good as could be done with sine wave shapes of e_p and e_g but it seems as though, by suitably deforming both of them, the efficiency could be considerably increased over this figure.

Tests similar to those described in this paper were carried out using a much smaller tube, that styled by the Signal Corps VT-2. The results obtained with the large tube were duplicated almost exactly in so far as efficiency was concerned. Although the normal adjustments used with these tubes were such that the tube efficiency was about 30 per cent it was found possible to so adjust the values of E_c and E_g that the tube gave an output of 6.3 watts with an efficiency of 70 per cent the voltage used in the plate circuit being the rated value, namely 300 volts. It was found possible to get over 7 watts output with the plate loss considerably lower than its safe rated value; if the plate voltage had been increased to perhaps 400 volts the tube output might have been raised to 10 watts while still having the plate loss within its safe value.

These tests were all carried out with a separately excited tube; with the tube self excited the efficiency was not obtained higher than 61 per cent, with a plate voltage of 300. This run gave an output of 5.6 watts output with a current I_b of 0.0305 amperes; the frequency was 400,000 cycles, the value of R was 53 ohms, the oscillating current 0.325 amperes, C_1 and C_2 being 1360 and 770 micro-micro-farads, respectively. The value of E_c was 40 volts.

With the conditions of a self excited circuit adjusted for the best conditions as previously outlined difficulty may be encountered in starting the circuit to oscillate, a shock of some kind being generally required to start oscillations. Because of this possible difficulty it may be the best practise to run tubes separately excited, using one tube, (so adjusted that it oscillates readily) for exciting others. It may well be that with a high resistance in the oscillating circuit more output can be obtained from two tubes if one only is used as a generator, the other being used as exciter only. Certainly if more than two tubes are to be used it will be well to use one as exciter for supplying the grid voltage for the others.

Somewhat more manipulation is required with the separately excited tube than with the self excited one because two circuits (the resonant circuit of the exciter and the output circuit of the power tubes) must be tuned, but the probable increase in output will make it worth while.

Presented at a joint meeting of the American Institute of Electrical Engineers and the American Physical Society, Philadelphia, Pa., October 10, 1919.

THE POSITIONS OF ATOMS IN METALS

BY A. W. HULL

ABSTRACT OF PAPER

When a narrow beam of X-rays passes through a fine powder of any crystalline material, it produces on a photographic plate placed just behind the powder a pattern of concentric circles. These circles are produced by the reflection of the X-rays from the planes of atoms in the crystal, and their diameters are a measure of the distances between these planes of atoms. By measuring the diameters of the circles the exact positions of the atoms can be determined. The results of this analysis are given for twenty common metals and several salts, with examples and brief description of the method, and a discussion of the results.

THE determination of the exact positions of atoms in solid bodies is the next to last of a series of discoveries, that have made atoms as real as the bricks of which houses are built.

The atom of 20 years ago was the "hypothetical smallest subdivision of matter." The atom of today is a real object of definite shape and size. We know what it is made of. We know its weight in grams. We can see its splash when it impinges on a plate of fluorescent material. We know its exact speed when it flies about as gas. And, lastly, we know its exact position when it forms part of a solid body.

A brief enumeration of these discoveries is a necessary introduction to the following discussion.

First came the discovery of *dancing molecules*. Heat had been considered a substance. The "Kinetic Theory of Gases" proved that it is a condition, *viz.*, the motion of the molecules, which fly about like frenzied bees, bumping against each other and the walls of their enclosure. Through this discovery, all the store of facts and laws about gases can be correlated by the single picture of these dancing molecules. We believe in these dancing molecules as firmly as in the law of gravitation. Whenever we think of gas we see dancing molecules.

The next discovery was J. J. Thomson's *streaming electrons*. Our text-books taught, and some still do, that electricity is not a fluid, though it behaves in many ways like one. Thomson proved that electricity is a fluid, that its atoms are the electrons which constitute the atoms of matter, and that it flows through wires just as water flows through pipes.

Next came the *weighing of the atom.* Faraday showed long ago how to determine the weight of an atom in terms of the charge it carries in electrolysis. There remained, therefore, only the measurement of this "unit charge" *viz.*, the charge of a single electron, by Millikan, to give the exact weight in grams of any atom that can be deposited electrolytically. As soon as the weight of any one atom is known, the weights of all the others can at once be calculated from the known relative atomic weights.

Then came the *counting of individual atoms.* This began with Sir William Crookes' "spinthariscope," and culminated in the beautiful experiments of Rutherford and Geiger, in which they counted one by one the helium atoms (the so-called "α particles") as they emerged from the surface of disintegrating radium; and then allowed them to pass, one by one, into a thin-walled glass tube, until enough had accumulated to form a gas whose pressure could be measured and spectrum analyzed.

These counting experiments led directly to the determination of the *composition of the atom.* J. J. Thomson had proved that every atom contains electrons. Rutherford proved that it also contains a positively charged kernel or nucleus, very small compared to the whole atom, but so dense that it contributes nearly the whole weight of the atom. The hypothetical atom thus became a concrete thing that can be visualized; a tiny, (but large enough to be studied) solar system, with nucleus sun and electron planets. The only respect in which one kind of atom differs from another is the magnitude of the positive charge of the nucleus, which determines how many electrons it can hold in its planetary system, and hence all its physical and chemical properties.

Finally came the discovery, by the Braggs, of the method of determining the positions of the atoms in solid bodies. The beautiful "point lattices" of the crystallographers were hypothetical. They enumerated possibilities, but could not point out the reality. The Bragg measurements of atomic distances give the actual arrangements. They are as accurate and reliable as those of the surveyor or astronomer. The only assumption made is that the arrangement of atoms is a regular one which repeats itself, and this assumption can be checked by experiment. The method consists simply in the measurement, by means of a special "measuring rod" which will be described, of the distance between atoms in three or more different directions. From these measurements a model can be constructed, which can then be checked by further measurements. The model must also agree with known physical properties of the substance, such as density, atomic weight, and crystal habit. A model which contains but one kind of atoms and satisfies all these tests may be regarded as very reliable. The reliability is still further increased by the fact that all the models investigated thus far have turned out to be very simple. In cases where there is more than one kind of atom, *i.e.*, compounds or alloys, an additional factor, *viz.*, the size and shape of the atoms, must be taken account of. There is one type of compound, containing only two kinds of atoms, whose structure is so simple that it cannot be misunderstood. Examples of this type will be included in the following discussion. Compounds containing more than two kinds of atoms have not yet been sufficiently studied to warrant their discussion, but there is every reason to believe that their analysis will be equally simple and reliable.

In the following pages, there will be given, first, a general survey of the results obtained, then a brief description of the method of measurement, and lastly, a more complete discussion of the individual models and some of their properties.

I have referred to the location of atoms as next to last in the series of atomic discoveries. For in order to complete the picture, one more discovery is necessary, *viz.*, the shape and size of the atom. An excellent beginning in this direction has already been made by Langmuir[1] whose theory of atomic structure predicts the shapes and relative sizes of all the atoms, and gives strong chemical evidence in favor of these predictions. The author hopes soon to be able to add the evidence of X-Ray measurements, which will determine not only the shape but the exact size of the atoms, that is, the positions of the electrons in the atoms.

2. *Summary of Results.* The most striking result of these investigations is the extreme simplicity of arrangement of atoms in common metals. Among the metals thus far ex-

---

1. Langmuir, *J. Amer. Chem. Soc.* 41, 868, June, 1919.
2. Hendrick, *J. Chem. Met. Eng.* 21, 3, July, 1919.

amined only three types of atomic arrangement are found, and these are, with one exception, the three simplest geometrical arrangements known. The simplest arrangement of all is not found among metals, but is characteristic of salts, which are composed of equal numbers of positive and negative ions. This type and a fifth type, also very simple, which is characteristic of non-metallic elements, will be included in the discussion for the purpose of comparison.

The most common arrangement in metals is the *face centered cubic* arrangement, shown in Fig. 1. This is also the most important since most of the useful metals,—*e. g.*, aluminium, nickel, cobalt, copper, silver, platinum, gold,—have this arrangement of atoms. Perhaps it would be better to say that those substances are most useful as metals which have this arrangement, since, as will be shown later, their ductility is due largely to this arrangement.

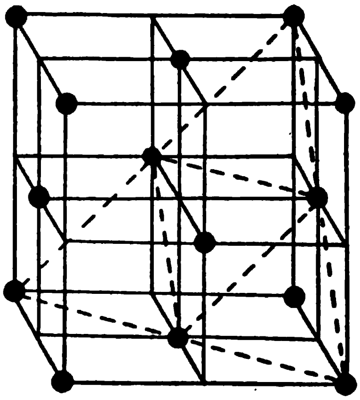


Fig. 1A—FACE CENTERED CUBIC ARRANGEMENT (CUBIC CLOSE PACKING)

The face-centered cubic arrangement is obtained by dividing the space occupied by a single crystal or "grain" of metal up into a system of equal, closely packed cubes (Fig. 1A) and placing an atom at each cube corner and at the center of each cube face. All the atoms in this arrangement, both corner and face atoms, are similarly situated as regards symmetry and relation to neighbors. Each atom is surrounded by twelve others, all equidistant and exactly similarly situated for every atom. It is this high degree of symmetry, combined with the close packing, that makes substances of this type so ductile. This arrangement is known as "cubic close-packing" and is one of the two alternative arrangements that equal hard spheres assume (Fig. 1B) when pressed tightly together, with sufficient shaking to allow them to find their places. This suggests, and the other evidence at hand points to the same conclusion,[2] that the atoms of the substances which have this arrangement are fairly spherical in shape. The necessary shaking corresponds to the temperature required for "annealing." The only difference between the packing of balls and that of atoms of this kind is the ability of the atoms to hold on to each other after they have found their places.

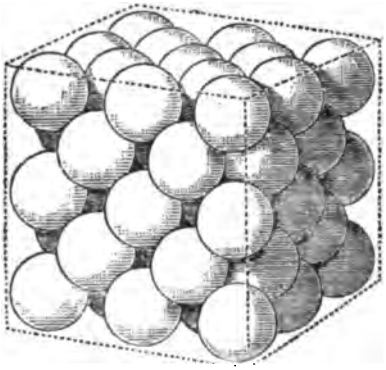


FIG. 1B—FACE CENTERED CUBIC ARRANGEMENT (CUBIC CLOSE PACKING)

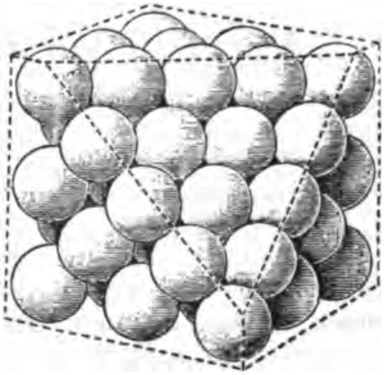


FIG. 1C—FACE CENTERED CUBIC ARRANGEMENT (CUBIC CLOSE PACKING)

There is one important exception to the rule that the most ductile, and therefore the most generally useful, substances are

---

[2] cf. Langmuir, l. c. p. 878.

those whose atoms are in face-centered cubic arrangement, *viz.*, iron. The atoms of iron, and also of chronium, molybdenum, tungsten, and the alkali metals, are in *centered cubic* arrangement (Fig. 2). This arrangement is obtained by dividing the space occupied by a single crystal or grain into equal close-packed cubes, and placing an atom at each cube corner and each cube center. The two sets of atoms, the "corner-atoms" and the "center atoms," are interchangeable, so that if the system of lines in Fig. 2 had started with one of the center atoms, all the corner atoms in Fig. 2 would become center atoms and vice versa. Each atom, whether center or corner atom, is surrounded by eight others in perfect cubic arrangement about it, situated always in the same direction and at the same distances. Hence this arrangement has the same high degree of symmetry as the face-centered cubic arrangement. It is not, however, as closely packed. Smooth, hard spheres cannot be packed in centered cubic arrangement except by the use of constraints, and when so packed are in unstable equilibrium. A slight jar causes them to reassemble in one of the close-packed arrangements. (Fig. 1 and 3) It is evident, then, that the atoms of these elements are either not spherical or that they possess some special forces of attraction localized at cube corners (see Fig. 15).

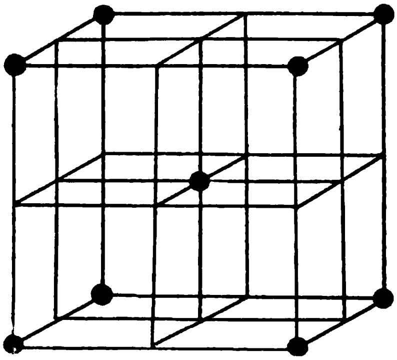


FIG. 3—CENTERED CUBIC ARRANGEMENT

The third type of arrangement found in metals is the *hexagonal close-packed* arrangement (Fig. 3). It is the second of the two alternative arrangements which equal hard spheres assume when closely packed by pressure and shaking. It is less symmetrical than the cubic close packed arrangement, but

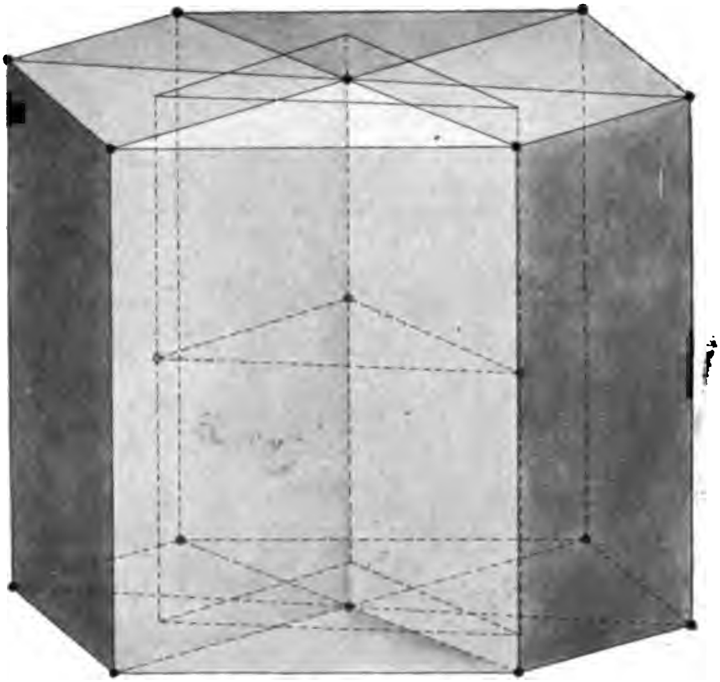


FIG. 2—HEXAGONAL CLOSE-PACKED ARRANGEMENT

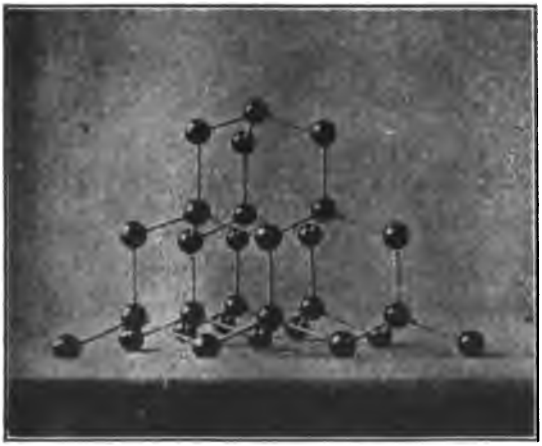


FIG. 5—TETRAHEDRAL ARRANGEMENT

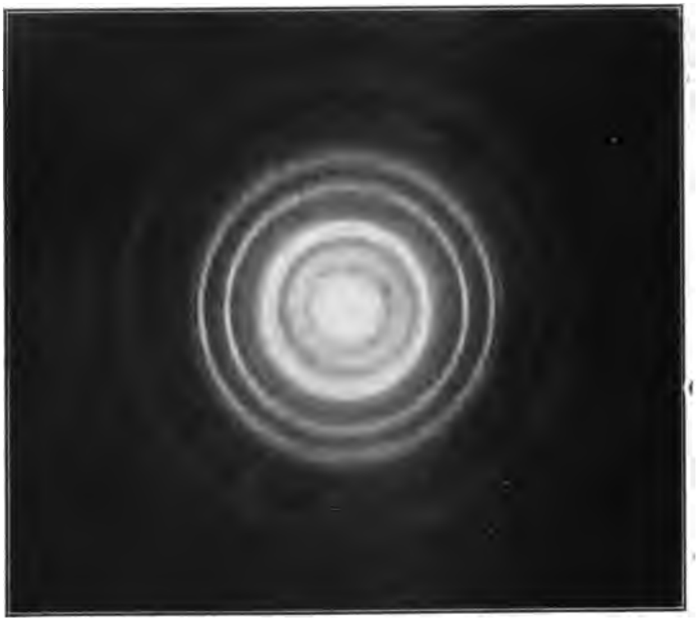


FIG. 7-A—ALUMINUM

[HULL]

FIG. 7-B MOLYBDENUM

FIG. 8—X-RAY SPECTRUM OF TUNGSTEN

equally close packed. The two are closely related, and each can be produced from the other by a simple gliding, as will be shown later. This is the arrangement taken by the atoms of magnesium, zinc, cadmium electrolytic cobalt, and probably to some extent by all cubic close-packed metals when strained (so as to cause gliding.) This arrangement is formed by dividing the space occupied by a single crystal of the substance into a series of equal closely packed right triangular prisms, the bases of which are equilateral triangles, and the altitudes equal to 1.633 times the length of the sides of the triangles. (Fig. 1A.) An atom is located at each prism corner and at half of the prism centers. This arrangement is simpler and more symmetrical than it appears. Each atom is surrounded by twelve others, all equidistant and uniformly spaced about it in dodecahedra arrangement. These dodecahedra are of exactly the same dimensions as in the cubic close-packed arrangement, but are not quite regular, the upper half being rotated 60° from the position corresponding to a regular dodecahedron.

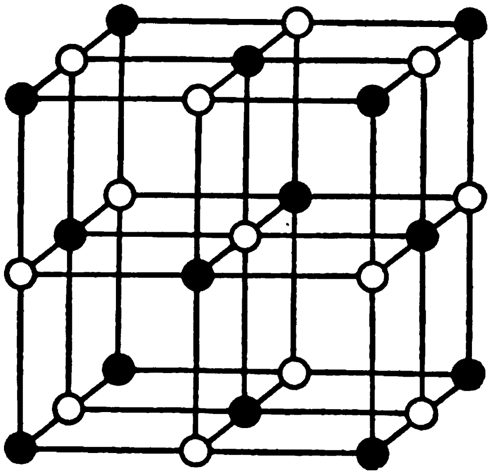


Fig. 4—Simple Cubic Arrangement

There are two other simple types of arrangement, which, though they do not occur among metals, are important for comparison, and their description will make clearer the distinguishing features of metals.

The first is the *simple cubic* arrangement (Fig. 4). It is formed by dividing the space occupied by a crystal into a series of equal, closely-packed cubes, and placing an atom at each cube corner. All the atoms are similarly situated, each being surrounded by six others, all equidistanct, in the direction of the cube faces. In spite of its great simplicity, it has only the same degree of symmetry as the two cubic arrangements already described. It is an extremely "loose packed" arrangement for hard spheres, and would be very unstable. A system of equal cubes, however, if packed in this manner would fill all space.

No elementary solid has yet been found whose atoms arrange themselves in this manner. This fact might be interpreted as evidence that none of the atoms are cubical in shape. There is strong evidence, however, that many of the atoms, especially those of low atomic weight, are approximately cubical in shape[3]. The fact that when these substances crystallize, their atoms do not pack together in simple cubic arrangement, is due rather to the nature of the forces holding them together (see discussion of iron, tungsten, etc., above.) It may be taken as evidence that these forces, in the case of cubical atoms, are not localized at the centers of cube faces, but at cube corners.

The substances which have this simple cubic arrangement of atoms are composed of equal numbers of positive and negative ions. The positive and negative ions alternate in every direction as shown in Figs. 4 and 16, so that each positive ion is completely surrounded by six negative ions and vice versa. The forces holding these atoms together are different from any of those thus far considered. In the cases described above, and in the great majority of compounds, the cohesion is due to the "stray fields" of the atoms. In these *ion compounds* it is due to the electrostatic attraction between the oppositely charged ions. This is stronger than the stray fields, and causes the atoms to pack together as closely as their shape will allow. The fact that they choose to pack in simple cubic arrangement is additional evidence that they are cubic in shape. No ion compounds of this kind, (*i. e.* containing equal numbers of positive and negative ions of approximately the same size) between spherical atoms have yet been examined, but it is to be expected that they will show one of the "close packed" arrangements (face-centered cubic or hexagonal) described above.

The fifth simple type of atomic arrangement is the *tetra-*

---

3. (See Langmuir, l. c. 41, p. 892 ff.)

hedral arrangement. (Fig. 5.) Each atom is surrounded by four others, arranged in a regular tetrahedron about it. It is not as symmetrical as the three cubic arrangements described above (Figs. 1, 2, and 4) for while each atom is at the center of a tetrahedron of neighboring atoms, half of these tetrahedra are positive and half negative; *i. e.*, upside down with respect to the first.

The only substances thus far found with the tetrahedral arrangement of atoms are diamond, silicon, and the "ion compound" $N H_4 Cl$. There is strong chemical evidence that in each of these substances, the unit of structure, (*viz.* the *C* and *Si* atom, and the $N H_4$ ion) is really tetrahedral in shape.

The Measuring Machine. The determination of these atomic arrangements requires the measurement, in as many different directions as possible, of the distance between consecutive *planes of atoms.* The arrangement of atoms, whatever it may be, is assumed to be a regular one which repeats itself throughout the crystal[5]. This assumption can be checked by the result. Through such an arrangement a system of equidistant parallel planes can be drawn in any direction whatever so as to pass through all the atoms. In most directions, these planes will be very close together and sparsely settled with atoms. In a few particular directions, however, they will be far apart and densely populated These are the directions of easy cleavage and gliding. It is these densely populated planes whose distances apart are measured.

The original measuring machine, by which the pioneer measurements were made, was a special form of "spectrometer." It has been simply and charmingly described by its inventors in a book[6] worth reading. The measurements described in this paper were made with a modified form of Bragg machine. The original machine was applicable only to large, perfect crystals, required careful manipulation, and was subject to serious error unless the crystals were very perfect and the number of observations large. The author's modification is free from these errors, requires but one simple observation, and is applicable to all substances which are crystalline, *i. e.*, all in which there is any arrangement to measure.

---

5. (If the arrangement is not regular, there is nothing to be measured. Such "amorphous" solids are very few in number, much fewer than was previously believed.)

6. (W. H. and W. L. Bragg, "X-Rays and Crystal Structure" G. Bell & Sons, London.)

The complete machine is shown in Fig. 6. It consists of a small transformer (or other source of high potential) capable of supplying 1 kw. at about 30,000 peak volts; a Coolidge X-Ray tube, *X*; a thin sheet of properly chosen material, *f*, serving as filter; a pair of slits, s_1 and s_2, in metal sheets, to limit the beam of X-rays; a tiny glass tube, *T*, containing the powdered substance to be measured; and a photographic plate or strip of film bent in arc of circle, *F*. The operation consists in filling the glass tube with a few milligrams of the substance to be analyzed, powdered as finely as possible; "loading" the photographic film holder; exposing over night to X-Rays at 30,000 peak volts and as many milliamperes as the tube will carry safely without watching, (a maximum exposure of 300 milliampere hours); and developing the film.

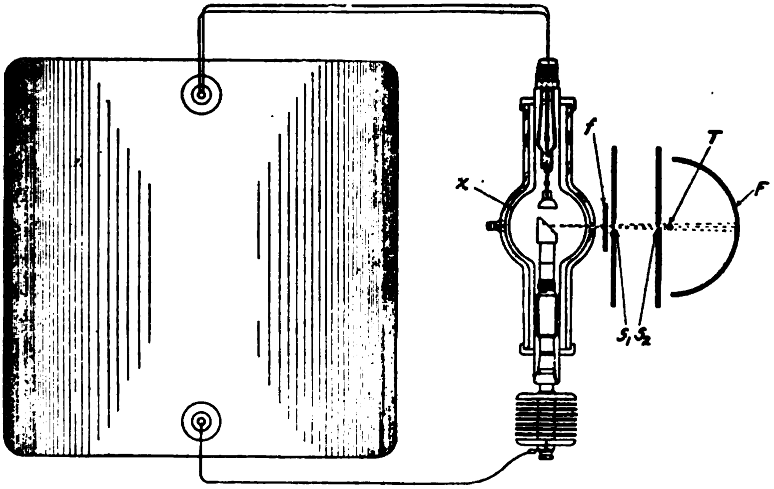


FIG. 6—POWDER PHOTOGRAPH APPARATUS

Typical photographs are shown in Fig. 7 A and 7 B. Fig. 7A is a photograph of aluminium filings, taken with a plate and very short slits, so that the trace of the direct beam in the center of the plate is a circular spot. Fig. 7B is a photograph of molybdenum powder, taken with circular film and slits as shown in Fig. 6[7]. In this case the traces of the circles on the film show as nearly straight lines. The circles and lines are due to the

---

[7] The central line and first "reflected" line in Fig. 7B have been partially absorbed by a "stepped" filter, placed directly in front of the film, for the purpose of measuring the intensity of the lines.

"reflection" of the X-rays by the tiny crystals of molybdenum in the tube, as will be described later. The distances of these circles or lines from the central line on the film are nearly proportional, inversely, to the distances between the planes of atoms, and from them these atomic distances can be easily and quickly calculated. Some examples of calculation are given below.

The Measuring Rod. The measuring rod by which these atomic distances are measured is the wave-length of a particular X-ray.

The possibility of measuring the dimensions of any physical body depends, primarily, upon the possession of a measuring rod of length comparable with the dimensions to be measured. Thus, the discovery and calibration of wave-lengths of visible light opened up a whole new field of measurements, comparable in length with this new measuring rod, such as the thickness of films, imperfection of polished surfaces, displacement of vibrating membranes, increase in length due to thermal expansion, etc. In the same way, the discovery that X-rays are of the same nature as light, and the isolation and calibration of X-ray wave-lengths, opened up a vast new field of measurements of dimensions comparable with the wave-length of X-rays; *viz.*, atomic dimensions.

We are accustomed to think of the measurement of things too large or too small to see and touch as necessarily very rough and approximate. It is somewhat of a surprise, therefore, to note that the only measurements accurate enough to justify the use of eight-place logarithm tables are those of astronomy; that wave-lengths of light are measured to 1 part in 10,000,000; and that the wave-length of X-rays, and by means of it, the distances between atoms, can easily be measured to 1 part in 100,000.

The spectrum of X-rays is exactly like that of visible light, except that the wave-lengths are shorter. It consists (Fig. 8) of bright lines superimposed upon a continuous spectrum. The wave-lengths in the X-ray spectrum depend upon anode material and voltage (Figs. 9 and 10) in exactly the same way that the wave lengths in the visible spectrum depend upon incandescent material and temperature. And just as it is possible to obtain nearly monochromatic yellow light by putting salt in a flame under proper conditions, so by running an X-ray

---

8. (For a more detailed description, see Hull, *Phys. Rev.* 10, 666, 1917.)

tube with proper anode at the right voltage, it is possible to produce a single wave length (line) of such great intensity that practically all the rest of the spectrum can be absorbed by a properly chosen filter, leaving nearly monochromatic X-rays.[8] Fig. 10. It is in this way that the monochromatic X-rays used in these measurements are produced. The measurements described in this paper were made with X-rays from a molybdenum target operated at 28,000 volts constant potential, and the filter was powdered crystal zircon, pressed, with a small amount of organic binder, into a sheet ¼ mm. thick. The resulting spectrum, before and after filtering, is shown in Fig. 10.

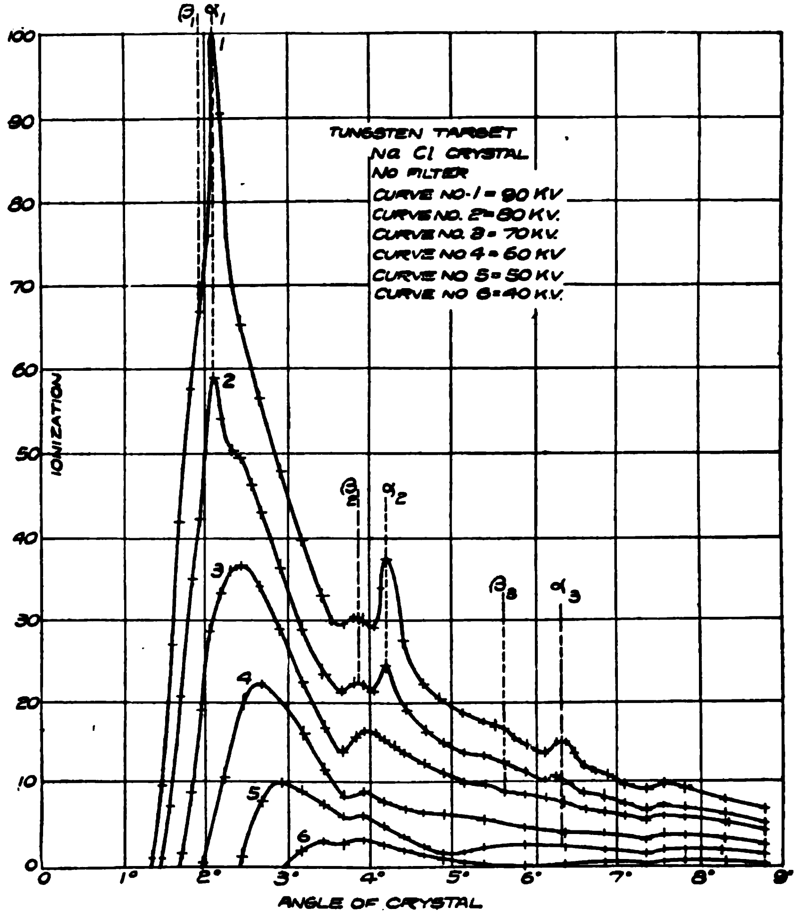


FIG. 9—EFFECT OF VOLTAGE ON X-RAY SPECTRUM OF TUNGSTEN

Calibration of Measuring Rod. The measurement of the wave-length of X-rays in centimeters was part of the pioneer work of the Braggs[9], and was accomplished in the same way as the measurement of visible wave-lengths, *viz.* by the use of a "grating" of known dimensions. It is interlocked with the determination of crystal structure, since the grating used was a crystal, and it was necessary, before using it, to determine its dimensions, *i. e.* the arrangement of atoms in it. The procedure was that of experiment and trial. Preliminary experiments indicated that the atoms of rock salt were in simple cubic arrangement, as shown in Fig. 4. Assuming this to be so, a rock salt crystal was used as a grating to measure tentatively the wave-lengths of X-rays. These wave-lengths were then used for further investigation of the arrangement of atoms in rock salt and other crystals, and were checked and corrected by successive trials.

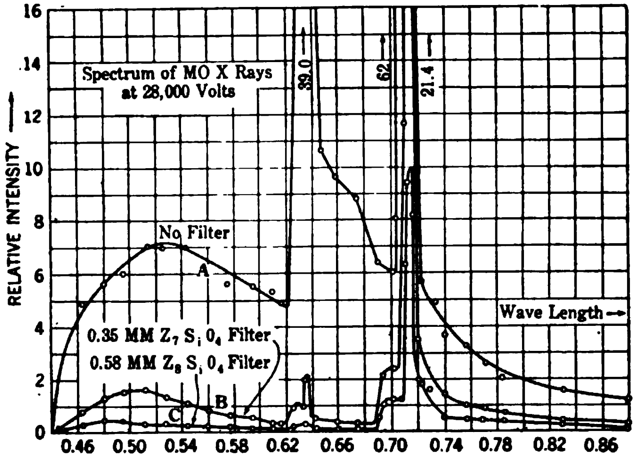


FIG. 10—EFFECT OF FILTERING ON MOLYBDENUM X-RAY SPECTRUM

The method of using the rock-salt grating is shown in Fig. 11. The crystal *C*, with its planes of atoms perpendicular to the paper, is placed in the path of a narrow beam of X-rays. Each plane of atoms acts like a mirror and reflects a small fraction of the rays. The reflection is a maximum when the reflected waves from all the planes (many million, except in the case of very long wave-lengths) are in phase, that is, when each is an

---

[9] X-Rays and Crystal Structure, p. 110).

exact wave-length or some whole number of wave-lengths behind the next. It is easy to show that this is true only when

$$n\,\lambda = 2\,d \sin\theta \qquad (1)$$

where n is an integer, usually 1 or 2, λ the wave-length, d the distance between planes of atoms, and θ the angle of incidence (i in Fig. 11) of the rays on the crystal[11].

The determination of λ requires, therefore, the knowledge of n, d, and θ. θ can be observed, and the orders, n, counted. d must be determined initially from the physical properties of the crystal, *viz.* its density, atomic weights, and arrangement of atoms. This can best be explained by an example:

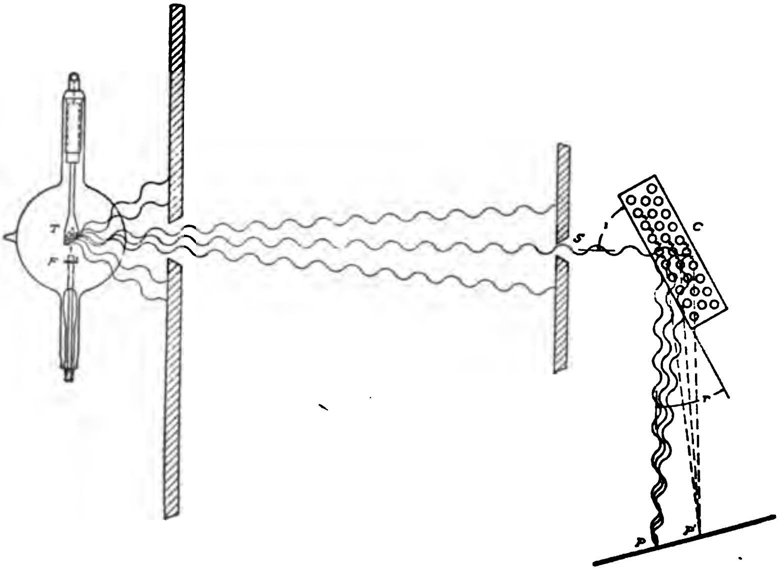
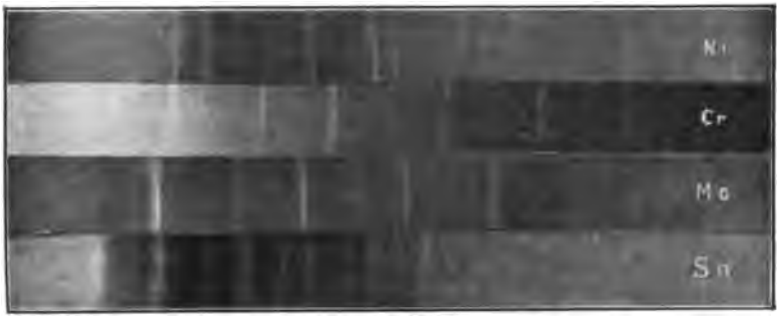


FIG. 11—X-RAY SPECTROMETER

The atoms of rock salt are in simple cubic arrangement, as shown in Fig. 4. There is one atom to each cube, as can be seen by displacing all the atoms in Fig. 4 in the direction of a cube diagonal to the cube centers. Half of the cubes contain sodium atoms and half chlorine atoms. The average weight per cube is the mean of the weights of one sodium and one

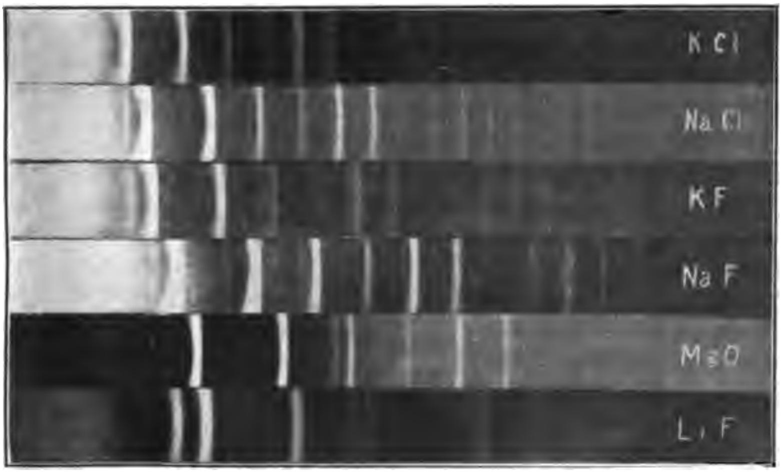
---

10. (It will be observed that in order to measure different wave-lengths the crystal must be rotated. This is the only essential difference between the crystal grating and the ordinary diffraction grating).



[HULL]

FIG. 12—TYPICAL X-RAY POWDER PHOTOGRAPHS OF METAL



[HULL]

FIG. 13—TYPICAL X-RAY POWDER PHOTOGRAPHS OF SALTS

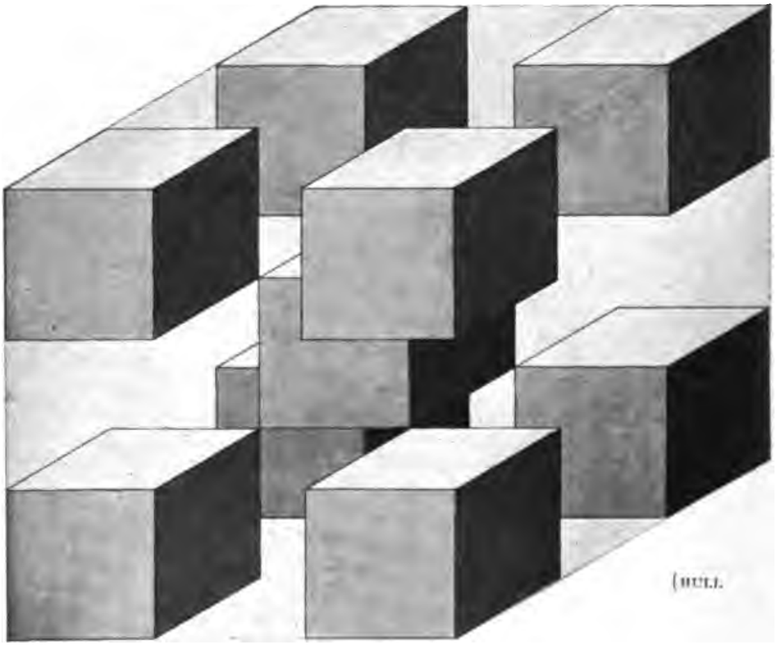


FIG. 15—PACKING OF ATOMS IN CENTERED CUBIC ARRANGEMENT

chlorine atom, *viz.* $1/2\ (38.00 \times 10^{-24} g + 58.50 \times 10^{-24} g) = 48.25 \times 10^{-24}\ g$. Hence the density, which is the weight per unit volume, must be equal to $\frac{48.25 \times 10^{-24}}{d^3}$ where d is the side of one of the small cubes (Fig. 4,) *i. e.*, the distance between planes of atoms parallel to the cube faces. This value of density must be the same as that obtained by measuring and weighing a large crystal, *viz.* 2.174. This gives

$$d = \sqrt[3]{\frac{48.25 \times 10^{-24}}{2.174}} = 2.814 \times 10^{-8} \text{cm.} = 2.814\ \mathring{A}^{12}$$

The wave-length of the "$\alpha_0$doublet" of molybdenum determined in this way is $0.712\ \mathring{A}^{13}$. This is the "measuring rod" with which the following measurements were made.

Interpretation of Powder Photographs. The X-ray wave length thus calibrated can now be used to measure atomic distances. If, in the arrangement shown in Fig. 11, the X-rays are made monochromatic by proper voltage and filtering, then as the crystal is rotated a series of intense reflections will be observed at angles whose sines are in the ratio 1 : 2 : 3 etc., corresponding to successive integral values of n in Eq. 1. If a new face is ground on the crystal at an angle to the first, and exposed to the rays in the same way, another similar series of reflections will be observed, at different angles, corresponding to the different distance (d, Eq. 1) between the planes parallel to this new face. The process of analyzing a crystal consists in observing these reflections from as many faces as possible, and calculating, from Eq. 1, the distance between the planes parallel to them. When a single crystal is used this requires many observations. The work is greatly simplified by using a powder, in which all possible orientations are represented at the same time by one or more of the tiny crystals, and photographing simultaneously the reflections from all these little crystals. This is the method sketched in Fig. 6, and gives patterns of lines like Figs. 7, 12 and 13. It might be expected

---

12. (The Angstrom unit or $\mathring{A}$ ($= 10^{-8}$ cm.) which is the standard unit for expressing wave-lengths of visible light, is even better suited to atomic and X-ray measurements. It will be used or assumed in all the following discussions).

13. (The best measurements of X-ray wave-lengths are those of Siegbahn, *Verh. Deut. Phys. Ges.* 13,300, 1917.

that the number of lines in these patterns would be infinite, since there is an infinite number of different possible planes in any crystal. The reflections from most of these planes, however, come at angles whose sines are greater than 1, (as is evident from equation (**1**) when d is small) and which therefore do not exist.[14]

The analysis of these photographs is very simple in the case of simple substances, like pure metals. It consists in finding, by successive trials, an arrangement of atoms whose planar spacings, beginning with the planes farthest apart and skipping none, exactly fit the observed pattern of lines. The calculation of the planar spacings for all the important planes is not difficult, and with simple substances but few trials are necessary.

The method of calculation can best be shown by an example. Potassium chloride (Fig. 13) gives the simplest pattern of lines yet observed. It consists of 6 regularly spaced lines, then a slight gap, then 7 more lines, (these may not all show in the reproduction), then another gap, etc. The best first guess is that a simple pattern like this corresponds to a simple cubic arrangement. The equation of solid geometry for the distance between planes in a simple cubic arrangement is

$$d = \frac{d_0}{\sqrt{h^2 + k^2 + l^2}} \qquad (\mathbf{2})$$

where h, k, and l are any whole numbers, and d_0 is the side of the cube. The numbers h, k, l, are called the "indices" of the planes. They are the reciprocals of the intercepts of the planes on the X, Y and Z axes respectively.

The largest distance is $\frac{d_0}{\sqrt{1}}$, corresponding to the "indices" 1, 0, 0. This gives the line nearest the center (see equation **1**). The next line is $\frac{d_0}{\sqrt{2}}$, given by the planes whose indices are 1, 1, 0. Then follow $\frac{d_0}{\sqrt{3}}$ (1, 1, 1), $\frac{d_0}{\sqrt{4}}$ (2, 0, 0), $\frac{d_0}{\sqrt{5}}$ (2, 1, 0), and $\frac{d_0}{\sqrt{6}}$ (2, 1, 1). The next line, corresponding to $\frac{d_0}{\sqrt{7}}$,

---

[14] (This means physically that there is no angle of incidence large enough to make the reflection from one plane a whole wave-length behind that from the one in front.)

should be lacking, as there are no three numbers the sum of whose squares is 7. Fig. 13 shows that it is actually lacking. Then follow, in regular order, $\frac{d_0}{\sqrt{8}}$ (2, 2, 0), $\frac{d_0}{\sqrt{9}}$ (2, 2, 1 and 3, 0, 0), $\frac{d_0}{\sqrt{10}}$ (3, 1, 0), $\frac{d_0}{\sqrt{11}}$ (3, 1, 1), $\frac{d_0}{\sqrt{12}}$ (2, 2, 2), $\frac{d_0}{\sqrt{13}}$ (3, 2, 0) and $\frac{d_0}{\sqrt{14}}$ (3, 2, 1). Then should come another gap, corresponding to $\frac{d_0}{\sqrt{15}}$, which cannot be formed from the squares of three integers. These regularities can be better seen in some of the other patterns in Fig. 13, which are exactly similar to that of *K Cl*, except that there are extra lines just in front of the first, third and fifth regular lines due to the difference in weight and size of the two atoms of which these salts are composed. The calculation of the positions of these extra lines is also simple, but will not be given here.

Returning to *K Cl*, the positions of the lines calculated from the above series of inverse square roots are found to coincide exactly with the observed pattern. This proves that the first guess was correct, that the atoms of *K Cl* are in simple cubic arrangement. The result can be independently checked, however, by the density. For the average mass in each of the small cubes, divided by the volume of the cube, must equal the known density of the substance. The value of d_0 calculated from equation 1 is 3.13 Å. The average weight of potassium and chlorine atoms is $\frac{1}{2}(64.5 + 58.5) \times 10^{-24}\,g = 61.5 \times 10^{-24}\,g$. This gives for the density $\frac{61.5 \times 10^{-24}}{(3.13 \times 10^{-8})^3} = 2.01$ which checks with the standard value within experimental error[15].

One more example may be briefly cited, *viz.* molybdenum. Its pattern is shown in Fig. 10. It is found that the series of lines which corresponds to a simple cubic arrangement does not fit this pattern. We therefore try a centered cubic arrange-

---

[15] (These X-ray determinations of density are, in general, much more accurate than the standard ones).

ment. The distance between planes in this arrangement is calculated in the same way as for the simple cubic arrangement, except that half the lines, *viz.* those due to planes the sum of whose indices are odd, are lacking[16]. Hence the relative spacings, beginning with the largest, should be proportional to

$\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{6}}$, etc. The series of lines calculated from these values is found to fit the pattern of Fig. 10 exactly. The length d_0 of the side of the small cube is 3.15 Å. As check, the density of molybdenum should be equal to the weight of two molybdenum atoms divided by the volume of the cube, *i. e.*

$$\frac{318 \times 10^{-24}}{(3.15 \times 10^{-8})^3} = 10.0 \text{ which is the correct value.}$$

DISCUSSION OF RESULTS

The results of the measurements that have been made thus far are summarized in the following table. The list includes most of the metals that are easily obtainable in pure form, a few simple salts, as examples, and the non-metallic elements carbon (diamond) and silicon. The structure of graphite is less simple, and will not be discussed here. It apparently contains a mixture of forms, produced by gliding, like cobalt.

The case of cobalt is exceptional. A finely powdered sample produced by rapid electrolysis showed a mixture of cubic and hexagonal close-packing in nearly equal ratio. After annealing in hydrogen at 600 deg. this sample showed only the cubic form. Another sample, composed of filings from pure cast metal, showed slight traces of hexagonal packing, due presumably to straining. It is probable that the other cubic close-packed metals will behave in a similar manner, but this question has not been studied.

PHYSICAL PROPERTIES

Many of the physical properties of these substances are evident from an inspection of their atomic models. Thus it is clear why metals with face-centered cubic arrangement are soft and ductile. For in the planes parallel to the octahedral faces (the bevelled face in Fig. 1c) the atoms are so closely packed that one plane can slip over the one beneath it without

---

[16] (For details and theory see Hull, *Phys. Rev.* 10,673, 1917).

appreciable elevation, and without the atoms getting away from each other's attractive influence. There are 4 sets of planes parallel to which this easy gliding can take place, *viz.* those parallel to the 4 pairs of octahedral faces, and in each of these planes there are three equally easy directions. There is no direction, therefore, in which a shearing force can be applied to a crystal of this kind which would not be nearly parallel to one of these directions of easy gliding.

TABLE I.

Substance	Arrangement of atoms	Length of side of elementary cube in Ångstroms	Distance between nearest atoms (between centers) in Ångstroms
Aluminum	Face-Centered Cubic	4.05	2.86
Cobalt	(Cubic close-packed)	3.57	2.52
Nickel	“	3.54	2.50
Copper	“	3.60	2.54
Rhodium	“	3.82	2.70
Silver	“	4.06	2.87
Platinum	“	4.02	2.85
Gold	“	4.08	2.88
Lead	“	4.92	3.48
Lithium	Centered Cubic	3.50	3.03
Sodium	“	4.30	3.72
Chromium	“	2.91	2.52
Iron	“	2.86	2.48
Molybdenum	“	3.15	2.73
Tungsten	“	3.15	2.73
Magnesium	Hexagonal (close-packed)	..	3.22
Zinc	“	..	2.84
Cadmium	“	..	3.15
Cobalt	“	..	2.53
Diamond	Tetrahedral	3.56	1.54
Silicon	“	5.43	2.35
Lithium Fluoride	Simple Cubic	2.01	2.01
Sodium Fluoride	“	2.81	2.81
Sodium Chloride	“	2.81	2.81
Potassium Fluoride	“	2.69	2.69
Potassium Chloride	“	3.13	3.13
Potassium Iodide	“	3.55	3.55
Magnesium Oxide	“	2.11	2.11

If the shear takes place in such uniform manner that each plane moves over the one beneath it by the same amount, the original arrangement is reproduced and no strain results. If only the alternate planes move, however, the arrangement becomes hexagonal close packing. This is evident from Figs.

14A and 14B, which give the arrangement of atoms in successive close-packed planes (dotted and heavy lines resp.) in the cubic and hexagonal arrangements respectively. In the cubic arrangement there are three different positions of the planes, and these alternate 1, 2, 3, 1, 2, 3, etc. In the hexagonal arrangement there are only two positions, which alternate, 1, 2, 1, 2, etc.

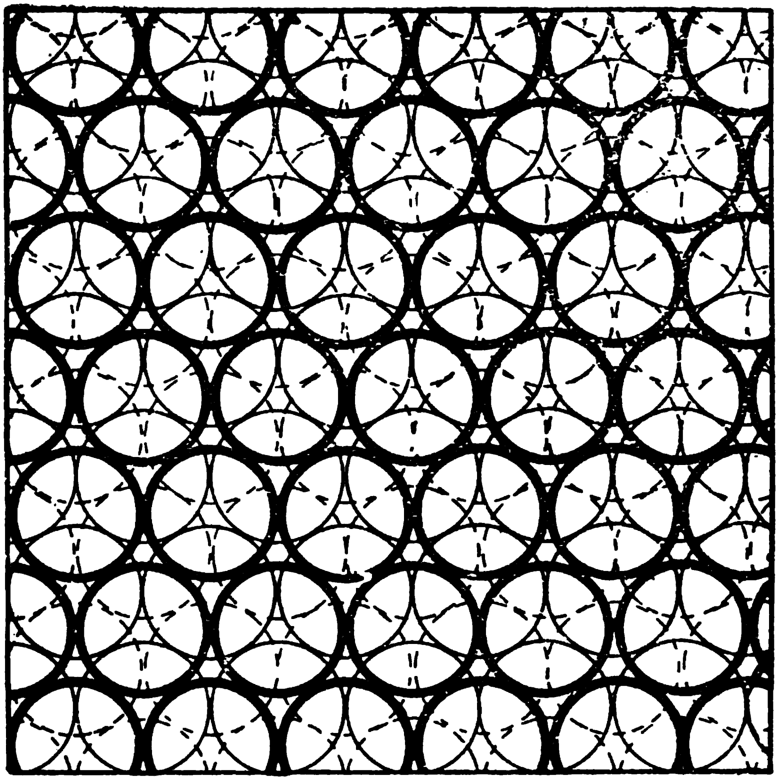


FIG. 14A—ARRANGEMENT OF ATOMS IN SUCCESSIVE PLANES IN CUBIC CLOSE PACKING

In the hexagonal close packed arrangement there is only one set of planes parallel to which this easy gliding can take place. Hence a strained metal becomes less ductile to the extent to which its arrangement becomes hexagonal, and it is evident that mechanical working should produce hardening.

The centered cubic metals should be somewhat less ductile than the face-centered. For the atoms hold on to each other at only 8 points (Fig. 15) instead of 12, and are more likely to move out of each other's influence during gliding.

In the tetrahedral arrangement gliding is impossible. The atoms touch at only 4 points, and in jumping from one stable position to the next would entirely lose hold of each other. This firm interlocking also accounts for the hardness of these substances.

In the case of the simple cubic salts (Fig. 16) gliding is impossible for a different reason. The atoms are held together by electrostatic attraction, each ion being surrounded by six oppositely charged ions. The process of gliding would bring ions of like charge opposite each other, with resulting repulsion and cleavage.

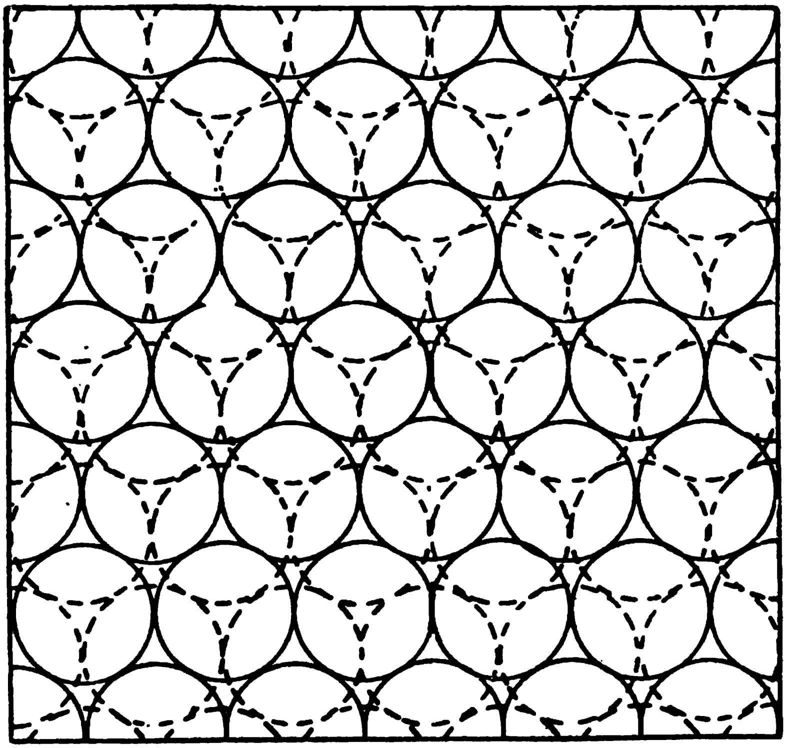


FIG. 14B—ARRANGEMENT OF ATOMS IN SUCCESSIVE PLANES IN HEXAGONAL CLOSE PACKING

The electrical conductivity of metals depends on the ability of electrons to move between the atoms. A discussion of this without a better knowledge of the shape and size of atoms would be premature. It can be seen at once, however, why "ion salts" and crystals like diamond are non-conductors. In each of these arrangements the electrons in the atoms are in complete groups of eight, which is such a stable arrangement[17] that large forces (corresponding to the dielectric strength of the substance) are required to remove them. The atoms of metals, on the other hand, have extra electrons which cannot find places in these stable shells, and are therefore "free" to move from atom to atom.

MAGNETIC PROPERTIES

It is well known that the ferro-magnetism of iron is not a specific property of the iron atom, since iron in solution and in compounds is in general not ferro-magnetic. The ferro-magnetism must depend, not only on the nature of the atoms, but on the way in which they are grouped together. It might have been anticipated, therefore, that the cause of ferro-magnetism was the *centered cubic arrangement* which is characteristic of iron. A glance at Table I shows that this is not the case. Nickel, which is ferro-magnetic, has a face-centered cubic arrangement, like copper. Cobalt is sometimes like copper, sometimes like magnesium. Neither is like iron. Chromium, on the other hand, which is not ferromagnetic, has a centered cubic arrangement like iron. Manganese has not yet been obtained sufficiently pure to determine its arrangement. It is evident, therefore, that while the centered cubic arrangement may be favorable to ferro-magnetism, and may make iron more magnetic than cobalt and nickel, it is not the principal or even an essential factor.

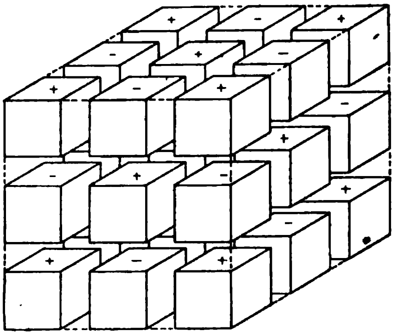


FIG. 16—PACKING OF ATOMS IN SIMPLE CUBIC ARRANGEMENT

---

[17] Langmuir, l. c. p. 873.

Presented at a Joint Meeting of the American Institute of Electrical Engineers and the American Physical Society, Philadelphia, Pa., October 9, 1919.

THE PIEZO ELECTRIC EFFECT IN THE COMPOSITE ROCHELLE SALT CRYSTAL

BY A. MCL. NICOLSON

Abstract of Paper

The piezo-electric effect is an electro-elastic property of certain crystals. It involves the conversion of mechanical into electrical energy, and also the converse effect. The paper presents an exposition of piezo-electricity and related optical and other properties belonging to these crystals. Special reference is made to the comparatively large piezo-electric effects produced by Rochelle salt crystals prepared so as to develop the natural *composite* structure usually associated with substances which have been exposed to shearing stresses, in this case during growth.

Applications of the piezo-electric effect in such crystals to the transmission and reception of sound, are described and demonstrated.

INTRODUCTION

THE expression piezo is derived from the Greek "piezein" signifying "to press." It relates to a variety of solids in the crystalline state, which when subjected to change of stress, become electrically polarized. Piezo-electricity, includes, also, the reciprocal phenomenon, whereby the same crystals dilate or produce stresses when electric charges are applied to certain regions.

Before discussing the piezo-electric effect in these crystals it will be of assistance to consider first the optical activity exhibited by the same class of crystals.

For example, quartz and Rochelle salt are piezo-electrically active for the same reason that they are optically active, viz., on account of their individual structural asymmetry.

OPTICAL PROPERTIES

It has been known for a century that certain[1] substances either in the crystalline state or in solution possess the power of rotating the plane of polarization of polarized light. This property in the case of the crystal[2] as also of the solution[3] was

---

1. Biot. Société Philomathique, Dec. 1815.
2. Herschel, Roy. Soc. 1820.
3. Pasteur, Soc. Chim., Paris. 1860.

shown to be due to structural asymmetry. In crystallography this asymmetry is called hemihedrism and refers either to an asymmetric arrangement of perfectly regular and superposable molecules or to an arrangement of molecules which are themselves asymmetric. For example, quartz and other minerals, so far as we know, do not possess *molecular* asymmetry; yet, on account of the right or left handed spiral or other unsymmetrical *structure* prevailing with respect to a principal axis, the optical plane of polarization is correspondingly rotated. When, however, the mineral structure or crystal is destroyed, the optical activity vanishes. On the other hand, the asymmetry of the active organic crystals and solutions resides in the molecule, and although optical activity is not always strongly manifest in the organic crystal, it is generally strong in the solution.[4] Each asymmetric molecule takes part in the rotation. Inasmuch as the molecule may offer a dextro- or laevo-rotation of light, there must necessarily be an asymmetric configuration or "stereo-isomerism." This is generally described as a molecular structure having that kind of asymmetry that is not superposable with its reflected image, possessing neither a center nor a plane of symmetry. In stereo-chemistry these so-called "enantiomorphic" isomers are of three kinds: right forms, left forms, and a mixture of both types which is optically inactive.

A crystal may be built up of one or other of the two *active* isomers—of dextro or laevo asymmetry. The corresponding hemihedral crystal is identified by certain simple marks—such as a particular face which slopes one way in one isomer, and the opposite way in the other, and by other indications. These two classes of crystal differ only in their geometry—as, for example, the right hand differs from the left hand, or as a *right* asymmetric tetrahedron differs from a *left* asymmetric tetrahedron. Chemically, their composition and properties are identical.[5]

Let us consider some other properties in these optical isomers. The crystal *form* is only one of many ways in which the symmetrical or asymmetrical arrangement of the molecules

---

[4] L. Pasteur, loc. cit., Nernst, Theoretical Chemistry, p. 88, 1911. and J. M. Jaeger: Lectures on the Principle of Symmetry, ch. 7., 1917.

[5] The identity persists so far as they are brought together with bodies which belong to the *symmetrical* class of compounds. But if the isomers are associated with other bodies of the *asymmetrical* class of compounds then their properties differ.

may be manifested. We know that certain physical properties determined by the arrangement of *atoms* and *molecules* within the molecule, according as this is symmetrical or asymmetrical, may disclose themselves in the crystal;[6] as for example, by selective action of solvents on faces, which are attacked in different ways, by unequal development of the faces, by electrical polarization due to temperature and stresses, etc. If we operate by various physical contrivances on certain faces or edges of a *hemihedral* crystal, then we find a *non-uniformity* in the resulting reaction. The etched figures on the prism will be unsymmetrical, or different in character on the basal planes. Changes in temperature and variations in applied stress will develop so-called analogous and antilogous electric poles on the crystal, and impressed electric charges will produce dilatation or deformation.

PIEZO-ELECTRICITY

The same mechanism in asymmetric crystals which operates on polarized light to rotate the plane of polarization also causes a liberation of electricity when the crystal is elastically deformed, or a dilatation when electric potentials are applied. This mechanism is found only in the hemihedral or hemimorphous crystal possessing either the asymmetric arrangement of the *atoms* in the organic molecule such as the tartrates, sugar, camphor, etc., or the asymmetric arrangement of the mineral *molecules* such as quartz, tourmaline, boracite, etc.

H. and P. Curie[7] discovered the effect and proved that it was due directly to stresses, or changes in the applied stresses, rather than to changes in temperature which, under the name of "pyro-electricity," had been known previously as a thermal-electric property of these asymmetric crystals; and the name "piezo[8]-electrique" was applied. Examples of piezo-electric crystals of the mineral type are shown in the accompanying Figs. 1, 2 and 3,—respectively tourmaline, boracite, and quartz. Tourmaline is hemimorphic hemihedral characterized by having uniterminal crystallographic poles, associated with different faces of the crystal, only alternate ones of which are developed. Boracite is a hemihedral tetra-hedron and represents a cube having four alternate corners truncated. It will

---

[6] W. H. and W. L. Bragg, X-Ray and Crystal Structure, p. 146., J. M. Jaeger, loc. cit., p. 256.

[7] Compt. Rend. 91, pp. 294 and 383, 1880.

[8] Hankel, Abh. saechs. Ges. d. Wiss. 12, 1881.

be noted that in both crystals there are so-called "sharp" and "blunt" ends or corners.* If the crystals be pressed between sharp and blunt poles, definite electrification of these poles results. If the pressure be released a reversal of the electric charge takes place.

Fig. 3 shows a well-known application of the piezo-electric effect for measuring small charges. A blade of quartz cut

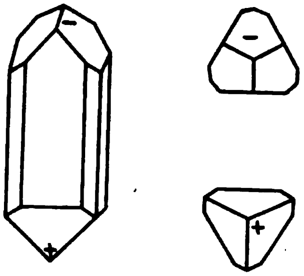


FIG. 1—TOURMALINE

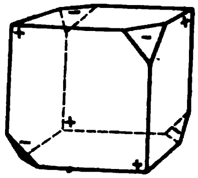


FIG. 2—BORACITE

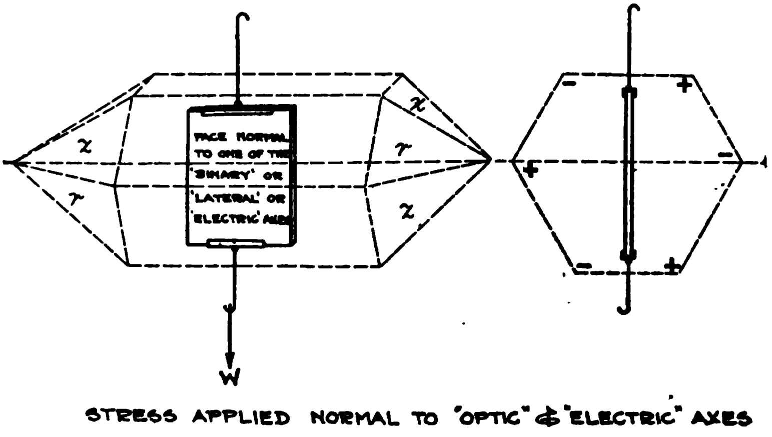


FIG. 3—INDICATING PREPARATION OF THE QUARTZ "PIEZO-ELECTRIQUE"

from the crystal as shown, and coated on its sides with tinfoil, or silvered, liberates electricity, the quantity of which depends directly on the weight which is suspended from it. An electrification, with the object, for example, of balancing a charge on an electroscope, may thus be accurately "weighted out." This method is used in the measurement of ionization currents.

---

*See S. P. Thompson, Elementary Lessons, in Electricity and Magnetism, Lesson VII.

The relation between pressure and électric charge for the quartz "piezo-electrique" as given by Curie is:

$$Q = K \frac{L}{l} P$$

where Q = charge in e. s. u.
K = piezo-electric constant = 0.0677 e. s. u. per kg.
L = length of coating (normal to crystallographic axis)
l = thickness between coatings
P = weight attached in kilograms.

Amongst many mineral and organic crystals investigated by the Curies and others,[9] the crystal of Rochelle salt was found to have the largest piezo-electric constant, approximately, 10 e. s. u. per kg. Apparently no other crystal has yet been found to approach the piezo activity of Rochelle salt, parcularly if the crystal is carefully chosen and specially prepared.

In 1917 the Research Laboratories of the American Telephone and Telegraph and the Western Electric Companies commenced an inquiry into application of the piezo-electric effect. As an outcome of some of the experimental work performed, it was found that Rochelle salt was susceptible of greatly increased piezo-electric activity. An absolute electric charge of 200 e. s. u. per kilogram pressure has been obtained, resulting in potentials as high as 500 volts and alternating currents measurable through a thermocouple. Acoustic tones from a crystal may be heard at a distance of several hundred feet.

Briefly, the increased efficiency is brought about by the following conditions:

1. Selection of particular habit of growth.
2. Desiccation.
3. Development of the crystal into a "composite" polar structure.
4. Application of static compression.
5. Use of electric poles normal to each other.
6. Application of torque.

CRYSTALLOGRAPHY OF "COMPOSITE" ROCHELLE SALT

Rochelle salt[10] crystals are grown from perfect nuclei possessing definite form. The nuclei or seed crystals are immersed

---

[9] See F. Pockels, Winkelmann's Handb. d. Phys., Bd. IV., p. 783.

[10] The chemical formula for sodium potassium tartrate is $NaKC_4H_4O_6 . 4H_2O$.

in a saturated solution[11] of the salt under identical conditions of temperature. The crystals may be grown by the application of temperature gradients to saturated solutions of the salt or by concentration brought about by its evaporation. The former producing a specific type under conditions of rapid cooling, is the method preferred. The crystal may be grown in the mother liquor by suspending from a clean thread, by flotation on mercury, or by being laid on a glass plate. The two last are the methods practised in our laboratory.

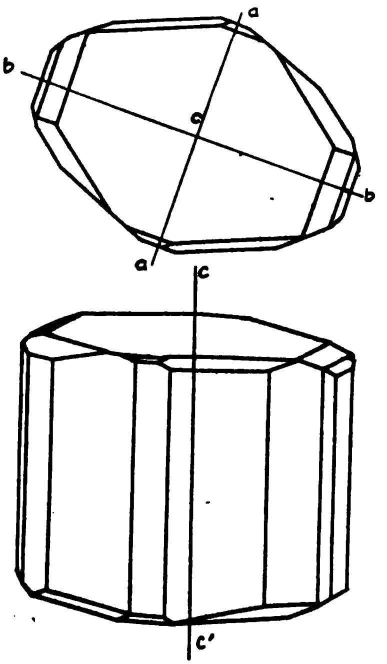


FIG. 4—CYRSTAL GROWN BY SUSPENSION FROM THREAD OR SUPPORTED ON MERCURY

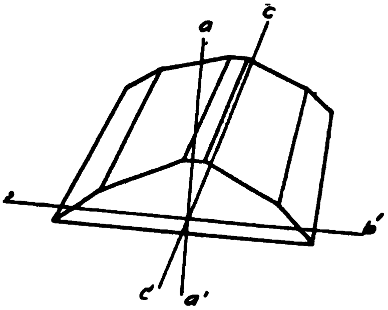


FIG. 5—CRYSTAL GROWN HORIZONTALLY ON A PLATE OR ON MERCURY

Fig. 4 indicates the general appearance of the Rochelle salt[12] crystal. It belongs to the rhombic system is hemihedral and

---

[11] A density of 1.33 at 50 deg. cent. may be conveniently used and the seed crystal, previously warmed to the same temperature as the liquor, should be applied between 38 deg. cent. and 35 deg. cent.

[12] This drawing was made by Prof. H. P. Whitlock of the Amer. Mus. of Nat. Hist., New York.

enantiomorphic. It may be possible to classify crystal suraces into two systems of surfaces or zones which are normal to each other. One system of zones will be parallel with the principal or crystallographic axis *c* and will therefore engirdle the crystal. The other system of zones will comprise the two basal planes parallel with the *a* and *b* axes. In Fig. 4 the zones are shown, respectively, as vertical and horizontal surfaces. In practise, the crystals are grown with the *c* and *b* axes in a horizontal position as is indicated in Fig. 5. This growth forms a particular habit, becoming dominant along the *c* and *b* axes while development along the *a* axis upwards is partially suppressed on account of the supersaturation and consequent gradation in density of the liquor.[13] In obtaining seed crystals, generally from a previous cropping, we select those in which growth along the *b* axis is fully developed (which happens when the seed nucleus grows with its *c* and *b* axes horizontal). These seeds are square, or nearly so, signifying that the growth along the *b* axis is about as great as, or greater than, that along the *c* axis.[14]

On cooling rapidly,[15] the seed crystal will increase in size from a few grams weight to 50 or·500 grams, according to the volume and density of liquor used. An average size of crystal weighs 100 grams and its axial lengths approximate 65mm. x 65mm. x 25mm. The illustration, Fig. 6, shows a group of crystals of the habit displayed in Fig. 5.

Crystals thus rapidly grown develop internal stresses producing strain regions symmetrical with the principal axis. The crystal acquires a composite structure[17] closely related to the surface zones referred to and to the electric poles to be developed as now explained.

At each end of the seed-nucleus and along its *c* axis, there appears a pyramid, not always very pronounced in the nucleus itself, which forms a polar terminal,—see illustration of crystal

---

[13] Lamethérie,—La Genese de la Science des Cristaux, H. Metzger, p. 187, 1918.

[14] The thickness of the crystal along its *a* axis is generally 0.4 *b;* this dimension is a function of the density and head of mother liquor.

[15] Growth occurs principally during the first 12 hours, although it may be continued several days during the condition of supersaturation.

[17] Mineralogists have termed this an "hour-glass" marking. The phenomenon is probably due to shearing strains set up by relatively greater contractions of the outer crystal envelops during cooling.

No. 612 in Fig. 7. The pyramids consist of stratifications normal to the *c* axis and to the rest of the crystal structure, the stratification of which ordinarily is parallel with that axis. The electrical performance of the crystal suggests that the crystal molecules throughout the pyramidal regions during growth are subject to forces which turn them, in planes containing the principal axis, through a right angle. This is indicated by the fact that the crystal, after subsequent treatment tending to render more pronounced these pyramid terminations, develops its electric poles[18] in accordance with the two systems of zones previously described. The poles are accordingly at right angles to each other. This signifies that the pyramids are electrically (+) when the rest of the crystal structure is (−) and vice versa. The effect becomes very pronounced when the crystal is subsequently dried in alcohol and baked in an oven.[19]

POLARITY AND CRYSTAL DRESSING

The raw untreated crystal possesses a large number of local electric poles which are variable in their piezo-electric effect. The desiccated crystal is much stronger[20] in effect and its electric poles are readily found in the case of the composite crystal to reside as described on the crystal surfaces with corresponding signs, respectively, in the "vertical" and "horizontal" zones.

In order to prepare the crystal for piezo-electric use its exposed parts are painted over with special varnish, after which waxed tinfoil electrodes are pressed on the crystal as shown in Fig. 8.

SENSITIVE REGIONS OF CRYSTAL

Referring again to Fig. 8, it will be noticed that both basal planes of the crystal are filed slightly concave in the central or polar regions. The object of this is to render salient the four diagonal "horns" or ends of the basal planes, which are found to be the mechanically sensitive regions of the crystal, that is to say, the extreme outer edges parallel with the *b* axis. For the purpose of obtaining an efficient piezo-electric

---

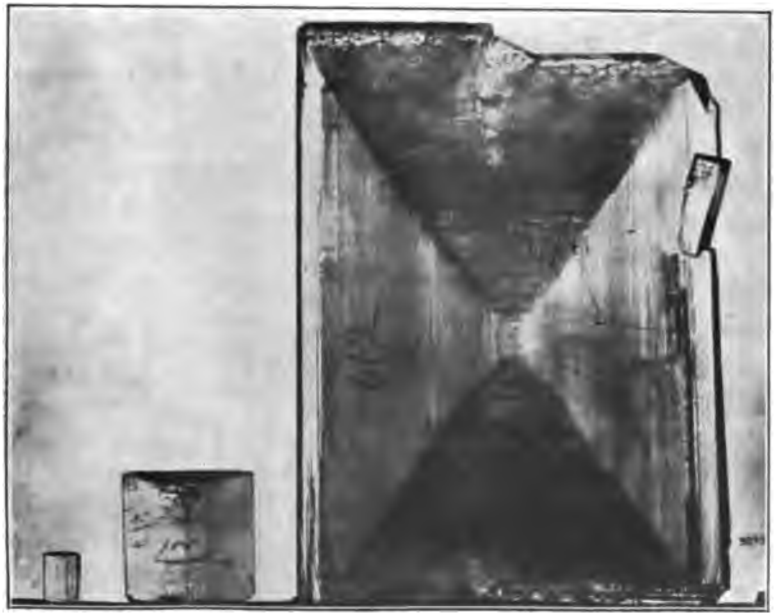
[18] Plus and minus corresponding with the conventional analogous and antilogous poles.

[19] 90 per cent alcohol, 24 hours; 100 per cent alcohol 6 hours, oven at 40 deg. cent. several days. This treatment reduces the weight 3 per cent.

[20] Aging and compression also improved them slowly.

PLATE LXXII.
A. I. E. E.
VOL XXXVIII. 1919

FIG. 6—GROUP OF ROCHELLE SALT CRYSTALS—UNDESICCATED, DESICCATED AND DRESSED



[NICOLSON]

FIG. 7.

FIG. 20 [NICOLSON]

action at least two of these four diagonal corners at one crystallographic pole of the crystal should be perfect and flawless, apart from the composite structure already described.

APPLIED STRESSES

Analysis of the direction of applied stresses to the sensitive regions described has shown that, for the composite structure, a twisting couple about the principal axis excites the greatest electrification of the crystal electrodes. This may be proved in a variety of ways both for small and large piezoelectric effects. One method is to prepare a crystal by dropping its poles into a pool of melted fusible metal,[21] and after the crystal has recovered from a temporary paralysis due to the heating by the metal, connections should be made between the *girdle* pole and the metal poles with high impedance receivers, which will cause sounds produced by rubbing of the metal poles to be audible. It will be found that strokes applied so as to rotate the crystal about its c axis will sound much louder in the receivers than strokes applied in other directions. Similarly the electric charge measured by twisting the crystal

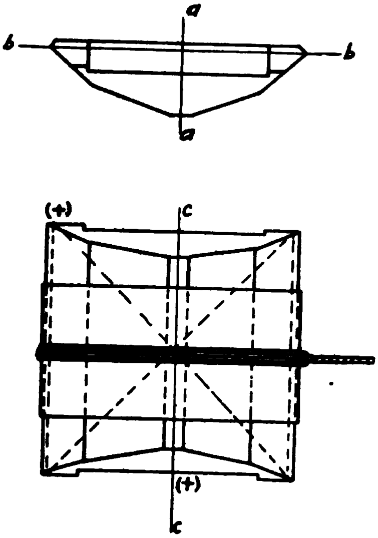


FIG. 8—DRESSING OF CRYSTAL FOR THE GIRDLE ORIENTATION

---

[21] Wood's alloy fuses at 71 deg. cent. and Litowitz's at 55 deg. cent. The crystal should be introduced just before the metal sets into a thin plate.

is much greater than the charges produced by applying axial or diagonal forces, not only for very reduced torques and stresses but for magnitudes of stress approaching saturation in the case of the axial and diagonal forces. In the case of torsion there exists the advantage of the couple, since the force is applied at a given radius from the polar axis.

(*a*) *The Case of a Slab Cut from a Crystal.* It may be mentioned here that a plate cut from a homogeneous[22] crystal, with its electric or *a* axis properly normal to the *c* and *b* axis, is almost insensitive to torsion, while the application of "diagonal" stresses, in accordance with Voigt's[23] crystallographic classification, does not in our practise appear to be as successful even when comparatively clear slabs of crystals are used. This may be due partly to the weakening of the structure in slicing, with consequent introduction of small flaws, and also to the difficulty of preventing in some degree the formation of the composite structure. When the uncut, whole crystal is twisted about its *c* axis so as to bend the *a* and *b* axes, component strains are developed along the principal axis as along the diagonals; but in view of the specific form of crystal and corresponding composite structure (of girdle and pyramids) presented, the resulting stresses set up in the crystal axes are very complex. Obviously the electric field is not as simple as in the case of two parallel plate electrodes.

(*b*) *Axial Compression.* On plotting the relation between electric potential and various loads applied to the composite crystal, so as to bear on the *diagonals* simultaneously, we obtain the approximately quadratic curves shown in Fig. 9. We have, accordingly, found it advantageous to apply permanent static compression and obtain thereby greatly increased effect. In view of the fact that the pressure is applied to the crystal at the limited areas of the top and bottom corners, in the basal planes, the crystallographic axis being vertical, it has been found that over a large range of size of crystal, namely between about 30 and 500 grams, the value of the applied static pressure for maximum change in slope corresponds to an absolute force of approximately 15 kg. At this point in the curve a variation of applied axial force of 1 kg. will produce in a good crystal a difference of potential of 8 volts. Beyond this point, as shown

---

[22] A crystal will be practically isomorphic throughout when permitted to grow at a very slow and uniform rate.

[23] W. Voigt and E. Riecke, Wied. Ann., 45, 1892.

in Fig. 10[24], the curve becomes less steep. The static pressure may be applied permanently to the crystal without apparent fatigue.

For experimental purposes the crystal is inserted in an appliance we call a "spring compressor" which compresses the crystal to the necessary degree and at the same time forms a convenient method for electrically connecting the polar electrodes, previously described. The spring compressors comprise a pair of aluminium disks which are held together by stout steel springs. These are attached by eye-rivets to the lower plate, while thumb screws tighten the springs after a crystal has been inserted. The number of turns of the thumb

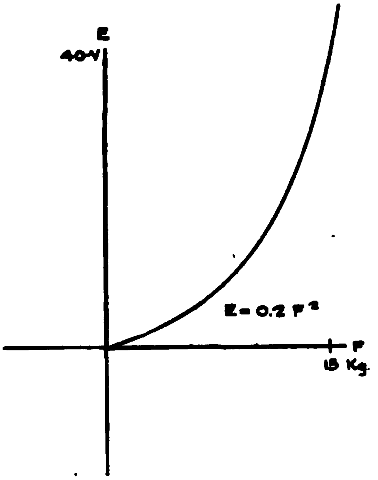


FIG. 9—AXIAL COMPRESSION APPLIED TO "HORNS" OF CRYSTAL (AVERAGE CROSS SECTION 10 CM.^{2})

---

[24] The curves in Fig. 10 show slight hysteresis or fatigue near saturation values of the potential. It will be observed that the deflections at *release* of load are much greater than those for compression, probably for the reason that the action may be effected more rapidly and consequently with less loss of charge. The cycle of operations taken for obtaining the curve is as follows: a load is applied to the crystal so as to compress it; this gives a deflection on the electrometer of a value, say *b*; then the crystal is short-circuited for a moment so as to give zero reading again (shown at *c*). On releasing the load the greater deflection *d* is produced (which is in the opposite sense when the instrument is polarized, as indicated in the upper curve). An adaptation of the Riehle machine was used for large values of compression.

screw may serve conveniently as a gage of the degree of compression after calibration.

Fig. 12 shows the spring compressor with the dressed crystal properly balanced within. Any part of the metal structure will readily convey jars and vibrations to the crystal. If this crystal transmitter be laid on a large sheet of paper and receivers be applied to the electrodes, it is quite easy to hear gentle rubbing of the paper and the tick of a watch—transmitted along the paper to the crystal.

Torsion. As previously described in connection with the fusible metal poles melted on the crystal, it is probable that the compressed crystal receives and responds to components of forces in both axial and twisting senses. In order to analyze the relative motions of the crystal it was simple to reverse the crystal action by applying alternating potentials at different frequencies to the crystal electrodes and discover from the various modes of vibration of the crystal their relative magnitudes. A special microphone was applied to a crystal pole in various positions with respect to the crystallographic axis. The microphone, being in a local battery circuit operating a thermocouple, was actuated by the dilating crystal when the constant alternating potentials of various frequencies were

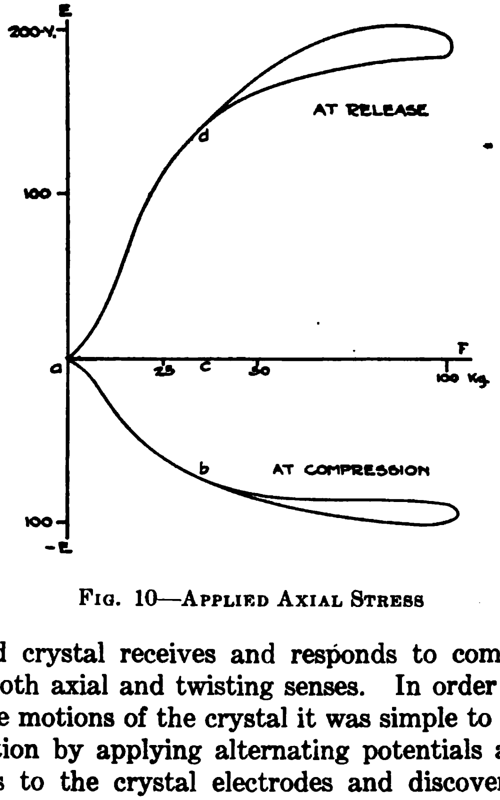


FIG. 10—APPLIED AXIAL STRESS

applied. The resulting measurements showed conclusively that the crystal twisted or moved the microphone button to the greatest degree when the axis of the microphone was attached tangentially to one of the crystal poles. The motions applied to the microphone were much smaller when they were diagonal or axial in character.

The relation between piezo-electric charge and torsion is linear, see Fig. 11, and is given by

$$Q = K F L$$

where K = 100 for several crystals tried
L = lever arm in cm.
F = force in kg.
Q = charge in e.s. units.

The constant for *potential* in volts, produced under similar conditions, is: $K' = 12$.

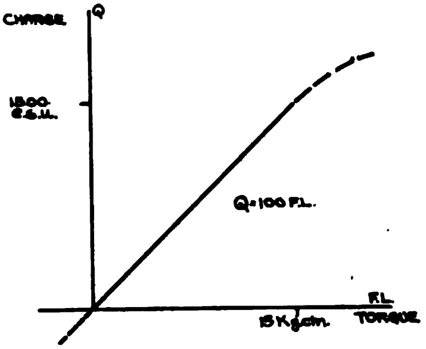


Fig. 11—Electric Charge Obtained when Torsion is Applied to Crystal

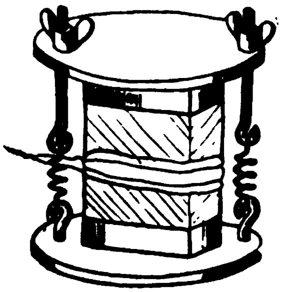


Fig. 12—Assembly

A right-hand torque determines the same electrification that is produced by subjecting the crystal to tension. If steady potentials be applied to the crystal the torsion is given, in one case, by

10^{-5} radian per volt (crystal 7 cm. long)

while expansion parallel with the crystallographic axis (c) is 2×10^{-9} cm. per cm. of length, per volt and is independent of applied pressure.

(*d*) *Alternating Effects.* If we apply rapidly alternating stresses to a crystal it becomes a generator of electrical oscillations, and we may use it as a detector of vibrations of sound, agitating it, for example, by a watch tick, a moving phonograph record, or by speech. If the crystal be tapped, an oscillogram shows that such an impact generates trains of electrical oscillations peculiar to the resonant frequencies of the crystal. Fig. 16 indicates trains of oscillations of 300 and 2000 cycles. A crystal used on a phonograph record will generate several volts, with sufficient power to operate a very large number of telephone[25] receivers.

FIG. 13—PHONOGRAPH TRANSMITTER

FIG. 14—PHONOGRAPH TRANSMITTER

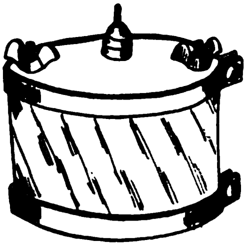


FIG. 15—CRYSTAL TRANSMITTER OR RECEIVER

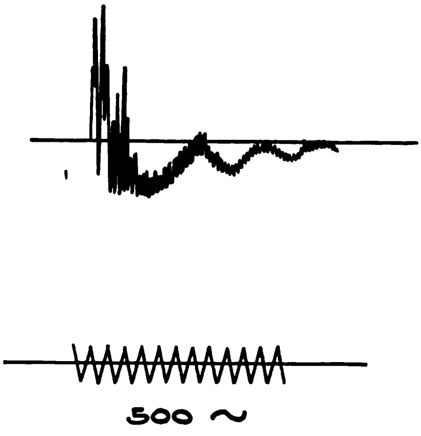


FIG. 16—EFFECT OF TAPPING A CRYSTAL INDICATING IMPACT OSCILLATIONS OF 330 CYCLES AND 2000 CYCLES

APPLICATIONS

Again, to operate effectively, the crystal applied to the phonograph is subjected to torsional vibrations. Figs. 13 and 14 indicate the housings employed. By means of the usual needle,

---

[25] As many as 200 receivers of 12,000 ohms each have been operated from one crystal.

vibrations are imparted to a light plate under the crystal. In Fig. 13 elastic bands are shown compressing the plate against the crystal.

Sound Transmitter and Receiver. Since, with these composite crystals, a given force produces the greater piezo effect when it is applied in such a way as to twist the crystal about its principal axis, and conversely, an applied electrical force produces the maximum mechanical response in the form of twisting motion, it is apparent that whether we are interested in producing electrical or mechanical results from the crystal, the diaphragms should be so attached as to make maximum use of the torsional effect.

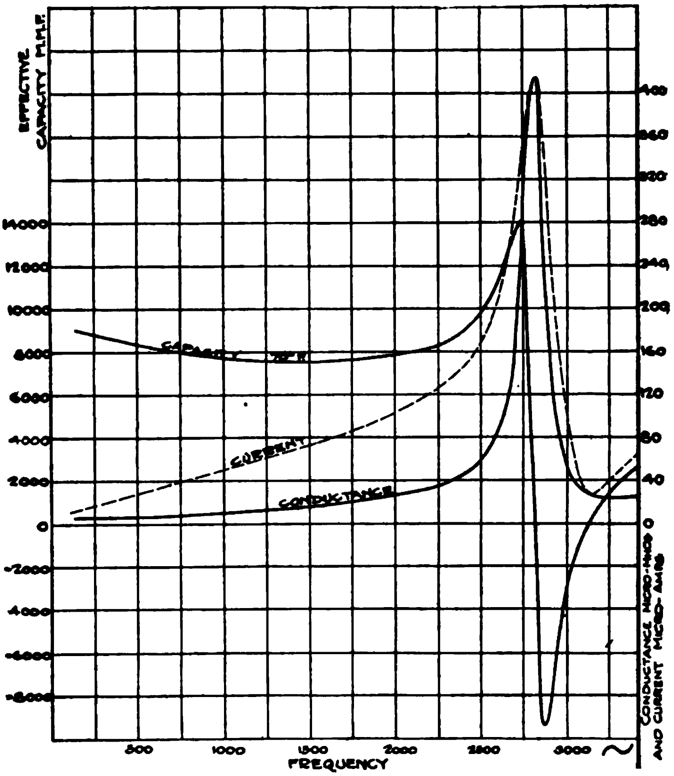


FIG. 17—CAPACITY, CURRENT AND CONDUCTANCE CURVES FOR CRYSTALS

One way is to apply a cylindrical diaphragm so as to surround the crystal, and to attach the cylinder by means of rings to the compressor plates which hold the crystal. By screwing one of the rings over one of the compressor plates the cylindrical diaphragm itself becomes twisted into diagonal corrugations stretched tightly across the crystal poles. This construction is shown in Figs. 15 and 20. The "diaphragm" should be made of a strong light material like gold beaters' skin or bond paper. When aerial tones strike the diaphragm and actuate the crystal as a transmitter, the resulting vibrations reach the crystal body through the poles and corresponding electrical oscillations are generated. Singing against the diaphragm, near resonance (the fundamental may lie between 200 and 600 cycles), will generate about 20 microamperes of current or produce 15 volts on open circuit. A clap of the hands near the transmitter will excite trains of oscillations in an oscillograph similar to the curves shown in Fig. 16. A circuit for the crystal transmitter is indicated in Fig. 18.

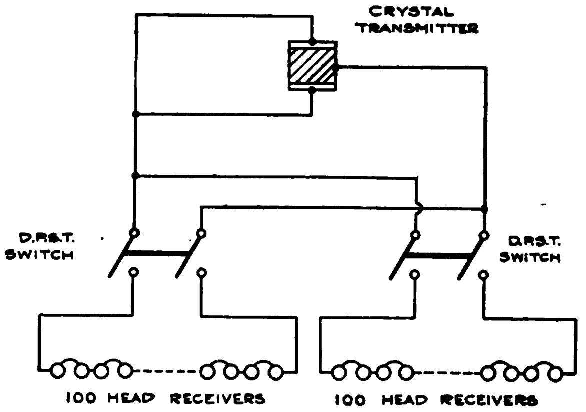


FIG. 18—CIRCUIT FOR CRYSTAL TRANSMITTER

If the crystal receiver, as described, is used with a microphone transmitter and the potential is stepped up to the crystal with a transformer as is shown in Fig. 19, very strong acoustic effects may be obtained as a "loud speaking" crystal receiver.

Crystal Transmitter and Receiver in a Line. With the aid of the vacuum tube amplifier, good transmission of speech and music may be obtained by using the piezo crystal at both ends of a line as sole transmitting and receiving apparatus.

Some Electrical Constants. The crystal may be considered as a leaky condenser having a shunt resistance in excess of 100 megohms. The ohmic resistance is several times greater from the analogous to the antilogous pole than it is in the reverse sense. There is however, no evidence of current rectification.

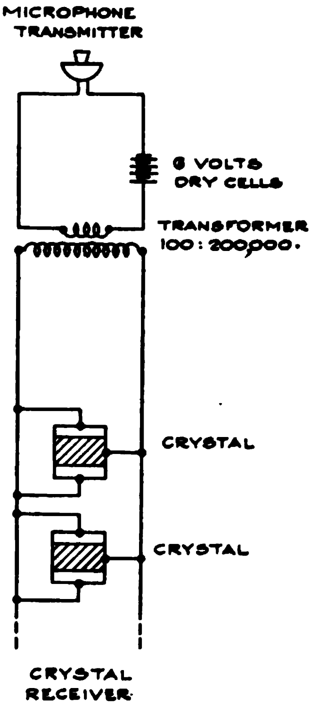


FIG. 19—CIRCUIT FOR CRYSTAL LOUD SPEAKER

The capacity varies with size and may be as great as 10^{-8} F. in a good crystal. It is usually 10^{-9} F. Its impedance at acoustic frequencies varies from 100,000 ohms to 300,000 ohms. Fig. 17 indicates that these properties depend on the frequency of the applied e. m. f. From these curves[26] the so called "motional impedance" of the crystal may be found in the same manner as the motional impedance of an ordinary telephone

---

[26] Made by E. T. Hoch, Physical Lab., Western Electric Co.

receiver[27] which, as is well known, gives the resonance characteristics of the diaphragm. In the case of the crystal the dilatations reach certain maxima where the applied frequency of the alternating potentials (of constant amplitude) coincides with an elastic natural frequency of the crystal. At this frequency, the apparent capacity passes through the greatest change in value—from maximum to zero or negative values, depending on whether the crystal response, as a generator, is in phase or out of phase with the applied potentials. The impedance decreases to a minimum at resonance as indicated by the conductance curve.

CONCLUSION

Curie's law that the electric charge generated by a crystal is, for a given force, independent of the absolute dimensions seems to be well borne out in the variety of crystals discussed in this paper. There is a best shape of crystal which we have already considered.

Very small crystals are generally less effectively rendered "composite" on account of the relative absence of the stresses during shorter growing periods. Hence they are generally less efficient when mounted as described. On the other hand, properly articulated crystals may be grown far larger than is necessary for effective operation. For example, Fig. 7 shows an illustration of a crystal weighing 910 grams. It is difficult however, to prevent cracks and flaws taking place before the large crystal has been effectively cured and hardened.

It has been noticed that the crystals improve with time, particularly when they are first made. Moreover, when they are paralyzed by being baked at too high a temperature they slowly recover. Drying out is probably one cause of this improvement and also realignment or recrystallization of disturbed portions of the crystal molecules.

It may be said that the crystals, after several months' use, or non-use, reach a very steady operating condition in which their activity seems to be permanent, especially for alternating effects where weather conditions have negligible action on sensitive apparatus associated with the crystal. As regards general efficiency and comparison with known apparatus like the carbon transmitter and the electromagnetic receiver, it should be said that the microphone is more sensitive than the

---

[27] A. E. Kennelly, Proc. Amer. Acad. Sc., 1912 p. 114.

crystal transmitter. The microphone with its associated local battery gives out more energy than it receives and hence constitutes an amplifier; while the crystal at present translates only that portion of the energy applied to it which affects its mechanically sensitive regions.

The efficiency of the crystal receiver compares favorably with the electromagnetic receiver for equivalent resonance conditions. It is not improbable that other crystals of organic constitution, *i. e.*, of *molecular* asymmetry, may be found whose piezo electric activity may exceed that of Rochelle salt. So far very little is known about the electrical and mechanical orientations set up in other crystals capable of developing the composite structure described in this paper.

DISCUSSION ON "THE POSITIONS OF ATOMS IN METALS" (HULL)' "PIEZO ELECTRIC EFFECT" (NICOLSON) AND "THE OSCILLATING VACUUM TUBE AS A GENERATOR OF ELECTRIC POWER" (MORECROFT AND FRIIS), PHILADELPHIA, PA., OCTOBER 10, 1919.

John B. Taylor: Referring to "The Arrangement of Atoms in Metals. I do not know how the author bridged the gap between quartz and those things which we know are regular crystals built by regular structure, and common carbon metals of electrical engineering, such as copper, iron, etc. There are crystals in iron, and if it is polished a little it will show the crystals at 1/100 diameters, etc. This is true of the other metals. We also have a copper wire soft drawn without much crystal structure, and so I think the author might leave a little clearer idea in our minds regarding these things, and perhaps some others, if he went over once this gap between the regular structure metals, of which the bona fide crystals are the best examples, and a metal which is annealed and worked with its own crystal structure. I would like to ask if Dr. Hull has any insight into the arrangement of the atoms or whether there is anything you might call arrangement.

Referring to the paper on "Piezo Electric Effect", I would ask if we can be given more numerical information, that is, what are the limits in pressure—the chemical pressure and electrical pressure. Can we supply several hundred thousand volts to one of these crystals and break it up by mechanical force? In general, how far can we go, and what other metals, other than the Rochelle salt may be available for this work? Quartz has been mentioned, but apparently it has some advantages and disadvantages over the Rochelle salt.

Has Dr. Hull, in examining the structure of these metals, examined them under conditions of stress? For instance, if he takes the structure of Rochelle salt and stresses it by mechanical means, which is the same in this case, does he find a different arrangement by the atoms moving up, which can be shown on the photographic film?

A. W. Hull: I will answer Mr. Taylor's question about the relation between metals and substances commonly known as crystalline. In the materials we have investigated so far, the only substance that is not almost completely crystalline is glass. Metals, even ductile metals like copper, seem to be perfectly crystalline. There are cases in which alloying interferes with the perfection of crystal structure. Those have not yet been examined. Except for that, metals are fully as well crystalized and as perfectly crystalline as so-called crystalline substances like quartz. Metals may, however, be distorted by gliding. In the case of quartz, as in rock salt, which I specially mentioned, such gliding is impossible. It always results in cleavage.

John B. Taylor: The structure of metals permits no cleavage plane?

A. W. Hull: Yes, they have a cleavage plane, but the metals cleave much less easily. The planes are able to glide over each other.

John B. Taylor: After the gliding, the arrangement of the atoms still remains the same?

A. W. Hull: They still remain in close contact and the crystal is fairly perfect.

In answer to the second question, very little work has been done in that direction and I have nothing to offer in connection with metals under stress.

A. M. Nicolson: As regards sound emitters of quartz and Rochelle salt, if due consideration is made of the differing modes of vibration and forms of housing, a rough comparison would indicate that Rochelle salt crystals operate more effectively at 500 volts than crystals of quartz at 10,000 volts.

If high alternating potentials be applied to a Rochelle salt crystal it will eventually become warm by dielectric hysteresis, or otherwise, and cease temporarily to exhibit the piezo-electric effect. It may be operated continuously, however, with a total energy dissipation, of about one-half watt. This represents a current of a few milliamperes through a crystal of average capacity of 1000 micro-microfarads.

John B. Taylor: You speak of it as capacity rather than resistance.

A. M. Nicolson: The direct-current resistance of the crystal is very high, in fact, hundreds of megohms when desiccated. Energy dissipated in the crystal dielectric is principally due to hysteresis and mechanical friction. This is represented electrically as *alternating-current resistance,* expressible either as shunt or equivalent series resistance. The magnitudes are respectively of the order of 300,000 ohms and 100,000 ohms.

E. A. Eckhardt: Are clear crystals better than cloudy crystals?

A. M. Nicolson: *Clear* or slowly grown crystals do not generally display the composite structure, or hour-glass marking, which we have made use of. The cloudiness develops on the surfaces during desiccation, otherwise the composite crystal should be quite clear throughout, except where the polar planes of cleavage definitely articulate the crystal into its composite nature. We have found that crystals having a development of this composite structure, with its corresponding orthogonal electric poles, give piezo-electric effects which exceed the effects with the slowly grown crystals offering the usual parallel electric poles and corresponding straight electric field. It should be said, however, that advantage was taken of the fact that the composite crystal could be obtained by rapid growth, while the so-called clear crystals are difficult to develop and entail several weeks' growth under thermostatic control. But such carefully grown crystals as we were able to develop do not seem to offer the same electrical and mechanical qualities which are readily obtainable in the composite type of crystal.

J. P. Wintringham: It might be well to give a few minutes to the essential crystallographic elements. Crystals are divided into classes by the more or less regularity and symmetry of their geometrical forms. All other qualities, mechanical, optical, electric, etc., are governed by the degree of symmetry or regularity. There are only three unit fundamental forms, the cube whose symbol is 100, and its faces are called the faces of a cube or if taken in pairs pinacoids. The three figures refer to the three geometric axes and the one permuted through the three places 100, 010 and 001 indicates three of the faces, minus one gives the other three faces $\bar{1}00$, $0\bar{1}0$ and $00\bar{1}$ or total six of the cube.

The second form is the dodecahedron (110) with 12 sides as the permutation of the symbol shows. Its faces taken in sets parallel to one axis are called prisms of which the most important is the vertical prism.

The third form is the octahedron (111) with eight permutations, one for each side. To the unit fundamental forms may be added four more when some other figure is taken in place of the one. All other of the 1000 or so forms are made by combinations of two or more fundamental forms, or and this is the important point in this connection, part of the faces of a fundamental form may not appear and we may have only half or a quarter of them ½(100), ½(110) &c., the brackets implying a full set of faces.

It is evident that you cannot make a complete figure with only half of the faces of a cube but the octahedron works out all right.

It may be described as a four sided pyramid on top with an inverted four sided pyramid below base to base. The vertical or *c* axis goes from the top apex to the bottom one. Two opposite top faces may be extended together with the two alternate lower faces to make a figure with only four faces, the tetrahedron. The *c* vertical axis remaining in its place goes from the middle of the top edge to the middle of the edge below. It is evident that a great deal of the former symmetry has disappeared. This is just the asymmetry that is to be noted in Mr. Nicolson's Rochelle salt crystals. It may be said to be made up of three pair of pinacoid (or cube) faces with a number of vertical prism faces. None of these is significant. What is significant is a small face just bevelling the top front edge called $o = 1\bar{1}1$ which should appear on the left or another $v = 211$ which would appear on the right. They are the faces of a tetrahedrons or bisphenoids half forms.

I have only found them in his two drawings, Fig. 4, in the top plan the long and narrow faces that are not cut by *a* and *b* axes, in the lower drawing the *c* axis seems to go through one of them.

Crystals are divided into 32 classes of which 20 might show piezo-electric effect.

A. W. Hull: I anticipate it would be perfectly easy to show the results of stress. The relative displacement of atoms will be accurately registered, but as they are exceedingly small it would require very fine measurements to show them.

Irving Langmuir: From the outside dimensions of the change of shape, etc., of crystals under the influence of an electrical field or mechanical stress, you could not calculate the changes in the orbs, the arrangement of the atoms must still be the same, and even without distortion you could calculate them? What about the changes in the distance between the atoms?

J. E. Schrader: Dr. Hull states he has not made any experiments on metals under stress, containing crystals such as mica or quartz. Cannot we get some information about polarization and absorption?

A. W. Hull: I would like to qualify somewhat the answer I gave to that problem. I have not the slightest doubt but that we can get excellent information regarding strains from X-ray analysis. The strains we actually produce are of such magnitude that we can measure them mechanically on the outside of the crystal, and the ease of measuring them inside is greater. By means of the photographs I showed, it is easy to measure the distance between atoms to one part in fifty thousand.

The measurement of elastic coefficients, temperature coefficients of expansion, and densities are extremely easy by this method. The relative ease of the measurement of density in a substance by X-rays as compared with the ordinary method is so remarkable that it is worthy of attention. Density measurements to one-tenth of a per cent can be made in about four hours,—that does not mean four hours of personal work—the tube runs by itself and the analysis requires less than an hour.

D. MacL. Therrell (by letter): Simplified and reduced to its lowest terms the paper by Morecroft and Friis covers the use of the vacuum tube, pliotron, or audion for the purpose of generating an alternating current from a variable direct current.

Some years ago, while conducting a series of experiments in an effort to improve the constants of the telephone transmitter circuit, and avoid the evils of core saturation and transformer or induction coil losses, I discovered a means for deriving an alternating current from a direct variable current, which is identical with the use of the pliotron vacuum tube or audion, as described by Messrs. Morecroft and Friis.

By bridging a low resistance coil of high inductance across the terminals of the variable resistance element, or transmitter electrodes, and placing the load, or primary winding of the induction coil, in series with a condenser of suitable capacity in shunt around these points, or terminals, whose potential varied as a function of the resistance of the transmitter electrodes; I found that an alternating current corresponding exactly with the impressed voice component of the variable direct current was generated in the derived or shunt circuit through the primary coil by the charging and discharging of the condenser in circuit.

The discovery was later used and disclosed by me for the purpose of generating alternating currents in the primary or primaries of a transmitter circuit so that resonance or partial resonance might be obtained for the essential or desired frequencies to be transmitted.

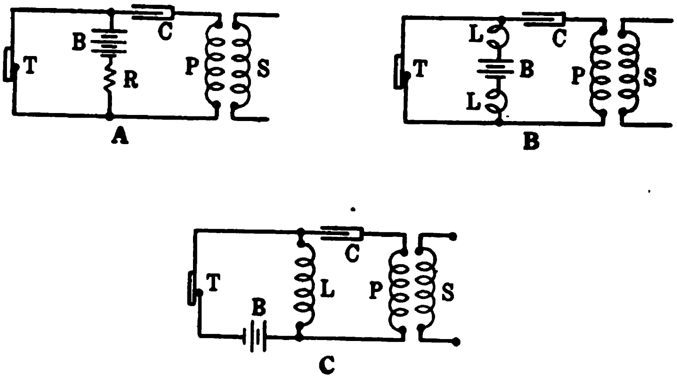


Fig. 1—Therrell "Autostatic Generator"

Both the discovery, of what I term the "Auto Static Generator," and this particular application which permits of tuning for the sinusoidal component of the variable direct current was a distinct and novel step in the art.

The importance of this device was clearly foreseen and fundamentally covered and claimed in U. S. patents issued to the writer for the "Art of Telephony."

This circuit arrangement was also described by me in an article in the *Electrical World*, page 1344, June 30, 1906. It was also described at length in a serial article by J. J. Goodwin, covering my discoveries, in the *American Telephone Journal*, page 67, August 4, 1906.

In the same issue the Editor, Franklin H. Reed, commented editorially in part as follows: "The device by which the primary current is made alternating in character in order that the required condition of resonance may be obtained, represents a distinct and novel step in the art."

In the patents above referred to the circuit coupling or auto static generator is shown under three equivalents as indicated at (*A*) (*B*) and (*C*) of Fig. 1, herewith, which it will be seen is identical with the vacuum tube or audion arrangement of Messrs. Morecroft and Friis as described in connection with Fig. 1 of their paper.

A simplified but exact equivalent of Fig. 1, is herewith shown as Fig. 2, in the usual symbols. This circuit is now seen to be identical with the "Auto Static Generator" arrangement disclosed by me in 1904, as will be most apparent to any one comparing the figures herewith.

The filament and plate in Fig. 2 are equivalent to the microphone in Fig. 1. The alternating-current generator and the grid are equivalent to the acoustical means for varying the resistance of the microphone in Fig. 1. The rest of the circuit is identical in every element and function and detail.

In this connection it is also pointed out that my "Auto Static Generator" circuit is used in the Goldschmidt alternator for wireless and radio telephony as recently disclosed in "The Principles Underlying Radio Communication," issued by the Bureau of Standards, pamphlet No. 40, page 162.

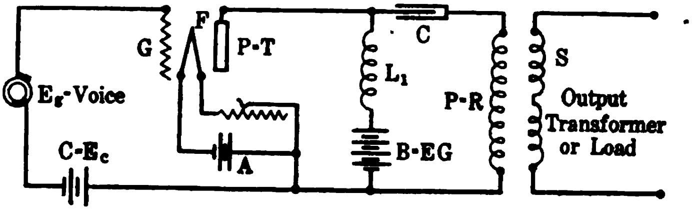


Fig. 2—Audion Adapted to Therrell "Autostatic Generator"

Among the claims granted to me on the "Auto Static" device, in the patents above mentioned are the following:

> "The method of producing an alternating current in the primary coil which consists in placing the primary coil and a capacity in a derived or shunt circuit between points whose potential varies as a function of the transmitter diaphragm, whereby an alternating current is caused to flow through the said primary coil due to charging and discharging of the said capacity as the potential varies across the terminals of the said derived shunt circuit."

Likewise the apparatus case covers:

> "Means for generating alternating currents in a circuit derived from the transmitter circuit and containing the primary coils of a plurality of transformers."

Also the generic claim for:

> "*Means for generating alternating currents in a branch of the transmitter circuit containing the primary coil.*"

Likewise the following generic claim covering the tuning of the transmitter or primary circuit for any frequency to be transmitted:

> "The step in the art of Electrical transmission and reproduction of sound, which consists in making the terminal transmitter circuit, or circuits, wholly or partially resonant for the essential frequencies to be transmitted, thereby increasing the efficiency of transformation and the energy transferred to the secondary circuit."

When it is considered that the vacuum tube, with its input excitation of any kind, is equivalent to the ordinary telephonic transmitter, the scope of these claims is readily obvious and is clearly seen to cover broadly every use of the Audion, Pliotron or vacuum tube for the purpose of generating an alternating current or of preventing a direct current from circulating through the windings of the output transformer.

Of course I am aware that the right to a patent on improvements involving the use of prior devices or inventions, does not necessarily carry with it the right to proceed under such a patent free from domination thereof by patents on the prior devices or inventions.

I am, therefore, now merely calling attention, in the interest of historical accuracy, to the fact that the use of the vacuum tube, pliotron, or audion, or other device, for the purpose of deriving, or generating, an alternating current from a variable direct current, or its equivalent, in telephony, wireless telephony and telegraphy, is an abridgement of and an infringement upon my rights as covered by the generic patents above cited; and this, statement is made without prejudice to any of may statutory rights involved.

John H. Morecroft: The comments of Mr. Therrell on our paper on vacuum tubes adds nothing to the theory or explanations advanced regarding the operation of the device, and is of interest only to those engaged in patent litigation.

The question which Mr. Taylor asked, regarding the possible mechanical strains due to piezo electric forces, was not answered by Dr. Nicolson; I believe he has made no determination of this phase of the question. The piezo electric effect in quartz, which I have investigated to some extent, gives one a very vivid idea of the tremendous strains which the action may call into play.

If a rectangular piece of quartz, in the form of a thin slab, is properly cut from a good crystal, it may be made to actually pull itself into two pieces by these piezo electric forces. The faces of the slab should be parallel to one of the electric axes of the crystal, and the long edges should be perpendicular to the optical axis of the crystal. A typical piece, for example, might be one eighth inch thick, two inches long, and about one inch wide.

Pieces of tinfoil, nearly the size of the faces of the slab, are then shellaced to the two faces, and allowed to dry. If this quartz slab is considered as a rod, its fundamental period of vibration (longitudinal) may be calculated to be about 50,000 cycles per second. That is, if the slab were struck a sharp blow on one end it would sustain longitudinal vibrations of this frequency. The two pieces of tinfoil are connected to a high frequency source of electric power, the frequency of which may be accurately adjusted; a large vacuum tube oscillator is the most suitable source of power. If the frequency of the alternating e. m. f. is now varied until it is exactly the same as the natural period of the slab *the slab will actually pull itself into two or more pieces* after the e. m. f. has been applied for not more than a second or so; in general the slab will break itself into two nearly equal pieces.

It might well be conceived that the rupture of the quartz was simply a case of dielectric breakdown, due to excessive potential gradient through the quartz; that such is not the case is evident when the voltage required for rupture is considered. A piece one eighth inch thick may be ruptured by about three thousand volts; the dielectric strength of the quartz is perhaps ten times this amount. Moreover if the same voltage is impressed on the quartz, at a frequency differing from the critical one by as little as one-half per cent the slab will "stand up" indefinitely.

In general this test is best carried out with the test piece submerged in oil, preferably a light one, such as kerosene. The voltage required in the test will cause corona and leakage if the test is carried out in air, and these two effects tend to heat the quartz and so reduce its dielectric strength.

I have been successful in rupturing pieces as short as one-half inch, the required frequency in this case being of the order of 200,000 cycles per second.

When one considers the high tensile strength of a piece of clear quartz crystal it is evident that these piezo electric actions do bring into play very large mechanical strains indeed.

Presented at a joint meeting of the American Institute of Electrical Engineers and the American Physical Society, Philadelphia, Pa., October 10, 1919.

A REPORT ON ELECTROMAGNETIC INDUCTION[1]

BY S. J. BARNETT

ABSTRACT OF PAPER

This report discusses briefly the chief fundamental results obtained from the days of Faraday to the present time in studying the electromotive forces ordinarily referred to the domain of electromagnetic induction.

Self-induction is first taken up, and the phenomena of self-induction are treated as essentially identical with the phenomena of inertia in dynamics, according to the idea of Maxwell and the idea originally accepted by Faraday. The only recent fundamental progress has been in studying the inertia of free electrons and other ions, and experiments on this subject are referred to.

The motional electromotive force, developed when matter moves in a magnetic field, is next considered, and is derived from Ampère's law on the electron theory. Especial attention is devoted to the motional intensity, and the resulting electric displacement, in insulators, of which nothing has been known until recent years.

The induced electromotive force in fixed conductors and insulators arising from the motion or alteration of other systems is next considered, and is expressed both in terms of magnetic flux and in terms of the general vector potential, which refers the phenomena back to the motion of electrons without the magnetic field as intermediary. The relations between the induced and motional electromotive forces are discussed, as well as the relation of the electric displacement produced in certain cases to the hypothesis of the fixed ether.

The report closes with a treatment of unipolar induction in both so-called open and closed circuits, including brief descriptions of some of the principal experiments, a discussion of the theories involved, and their application to the unipolar generator.

1. IT is unnecessary to emphasize the great importance of this subject alike to the physicist and to the engineer. The world of the physicist is rapidly becoming fundamentally an electrical world, in which a vast part is played by electromagnetic induction; and without electromagnetic induction the world of the electrical engineer could scarcely be considered even to exist.

The general field of electromagnetic induction has become very broad, and it will be necessary to restrict this discussion

---

[1] For references see end of paper.

to fundamental matters, and from among them to select for consideration only electromotive forces and electric displacements, and these only in stationary or quasi-stationary systems. Important related subjects such as the transfer of energy in the electromagnetic field according to Poynting and Heaviside must be excluded. Even the principle of relativity can scarcely be more than mentioned.

To a very remarkable degree our knowledge of the fundamental facts and principles of electromagnetic induction is due to Michael Faraday. Most of the progress made by others has consisted in increasing the precision of his measurements and in developing ideas which originated in less perfect form with him.

2. The principal experiments of Faraday and his contemporaries in the field of electromagnetic induction may conveniently be grouped in four divisions, as follows:

I. Experiments in which the agent producing the magnetic field remains fixed to the earth and the conductor or insulator under examination moves. The electric intensity and electromotive force in the body due to the motion have been called the *motional* intensity and e. m. f. by Heaviside.

II. Experiments in which the body under examination remains fixed to the earth, and the agent producing the magnetic field moves or its electric current changes.[2] The electric intensity and e. m. f. produced under these circumstances will be called the *induced* intensity and e. m. f., although these terms are often applied also to the motional quantities.

According to the Einstein principle of relativity, no discrimination can be made between the motional and induced intensities, as only the relative motion of two bodies can be determined.

III. Experiments on unipolar induction, strictly belonging to groups I and II, but to be considered separately for convenience.[3]

IV. Experiments on self-induction.

3. The last group, on self-induction, will be considered first. When a steady electromotive force is applied to a circuit the current increases from zero until a steady value is reached when the counter e. m. f. of resistance just balances the applied e. m. f. and the electricity therefore moves uniformly. If the applied e. m. f. is made to disappear without otherwise altering the circuit the counter e. m. f. of resistance gradually reduces the current to zero or brings the electricity to rest.

Referring to the phenomenon of self-induction, Faraday says "The first thought that arises in the mind is that the electricity circulates with something like *momentum* or *inertia* in the wire." This idea, however, he soon gave up, but for inadequate reasons, as shown by Maxwell, whose treatment of the subject is based entirely on the idea of inertia. To set electricity into motion an e. m. f. is required just as a force is required to accelerate matter; and to bring electricity in motion to rest an opposing e. m. f. must be applied, just as an opposing force is required to reduce the velocity of a moving body. In strictness it is quite as improper to speak of an "electromotive force of self-induction" as of a "force of inertia" in dynamics. Failure to realize this has given rise to a great amount of confusion in electrical literature.

The fundamental self-induction relation in electric circuits is that of Faraday and Neumann: To alter the current in an electric circuit, and therewith the corresponding magnetic flux, an e. m. f. equal to the rate of increase of the flux must act in the circuit.

While a vast amount of progress has been made in measuring the quantities involved in self-induction and in studying the relations of other phenomena to those of self-induction, it appears that the only really fundamental progress made since the early days has come from studying the behavior of free electrons and other ions accelerated by electric and magnetic fields. Well-known experiments by J. J. Thomson, Kaufmann, Bucherer, Hupka and others have yielded important information on the constitution of the electron and of matter; but it is unnecessary to do more than mention these experiments here.

Along with these experiments should be mentioned the experiments suggested by Maxwell[4] on the electric displacement produced in conductors by their mere acceleration. If a conductor is accelerated, since some of the electrons or other ions are free, an electric displacement would, in general, be expected to result.

Such effects in electrolytes have been observed by Colley,[5] Des Coudres,[6] and others; and after having been looked for in metals by Maxwell, E. F. Nichols,[7] and Lebedew[8] have been observed and carefully measured in a beautiful investigation by Tolman and Stewart.[9] The behavior of the electricity in a circular coil of wire which has been rotating about its axis and is quickly brought to rest as in these experiments, is very similar to that in a coil which has been carrying a steady current when the applied e.m.f. is reduced to zero without altering the resistance.

4. Faraday's experiments of group I. on conductors prove that when a rigid circuit moves in a fixed magnetic field there is produced around it an e. m. f. equal to the rate of diminution of magnetic flux through it, or the net rate at which it cuts across magnetic flux, with proper respect to signs. The two statements are equivalent, since tubes of magnetic induction are always closed. The second form is the more fundamental, as will be evident from what follows.

Similarly, the motional e. m. f. along any part of the circuit, or any line in any conductor moving in a magnetic field, is equal to the rate at which it cuts across magnetic flux.

On the electron and ionic theories of matter this is accounted for by the Ampèrian force

$$F = e\,[v\,H] \qquad (1)^{10}$$

which acts on any electrified particle of the matter with charge e moving with velocity v in a field of strength H. For this gives a motional intensity, or force per unit charge,

$$f = F/e = [v\,H] \qquad (2)$$

The integral of this intensity along any path in the conductor gives the result stated above.

Every experiment made since Faraday's time has confirmed this result. The most precise experiments are probably those which have been made in determining resistances in absolute measure by the method of Lorenz.[11]

In certain cases where different magnetic fields are superposed care must be taken to use for H in (2) the intensity of the field due to the fixed circuits or magnets with reference to which V is determined. An interesting illustration is that of an insulated wire inside a coaxial iron tube moving together with the wire across an (originally) uniform magnetic field due to fixed circuits or magnets. The e. m. f. developed in the wire is precisely the same as if the tube were absent, although the (resultant) field within the tube is greatly weakened by its presence. Since the tube moves with the wire the field due to its magnetization is not effective.

5. If non-conducting matter also is electrically constituted and moves in a magnetic field, each moving electrified constitu-

ent must, as in the case of a conductor, be acted upon by a force in accordance with (**1**), and thus by the motional intensity *f* given by (**2**).

Similarly, if the ether is electrically constituted, and if it moves in a magnetic field, it also will be the seat of a motional intensity given by (**2**).

The motional intensity will act to produce opposite displacements of the positive and negative constitutents of insulators as well as of conductors. This will, in general, give rise to an electric field, and the intensity E of this field will act on all electrical media which the field contains, whether at rest or in motion.

If both the ether and the matter which it permeates move together, as they do on the theory of Hertz,[12] both intensities act on both substances. But if the ether remains fixed, and the matter which it permeates moves through it, as on the theory of Lorentz[13], the intensity f acts on the matter only, while the intensity E acts on both ether and matter.

The motional intensity f, being equal to $[v\,H]$, is entirely independent of the nature of the moving substance provided only it has an electric constitution.

6. Referring to the matter of motional displacement, or electric displacement produced by the motional intensity, and to the displacement produced by the induced intensity, in insulators, Faraday says[14], "I have long thought there must be a particular condition of such bodies corresponding to the state which causes currents in metals and other conductors; and considering that the bodies are insulators one would expect that state to be one of tension. I have by rotating non-conducting bodies near magnetic poles and poles near them, and also by causing powerful electric currents to be suddenly formed and to cease around and about insulators in various directions, endeavored to make some such state sensible, but have not succeeded. Nevertheless, as any such state must be of exceeding low intensity, because of the feeble intensity of the currents which are used to induce it, it may well be that the state may exist, and may be discoverable by some more expert experimentalist, though I have not been able to make it sensible."

7. An imaginary experiment on this subject and unipolar induction combined was described by Poincaré in 1900[15]. But the first actual experiments after Faraday appear to be those of

Blondlot in 1901 on air[16]. In 1902 I began an investigation which included experiments on ebonite, sulphur, and rosin, but the work was not completed until 1908[17]. In the meantime H. A. Wilson[18] published an investigation on ebonite.

8. The essential theory of all these experiments can be given very simply. Consider an insulator in the form of a hollow circular cylinder with dielectric constant K rotating uniformly about its axis in a uniform magnetic field with intensity parallel to this axis. Suppose the inner and outer surfaces of the cylinder covered with conducting armatures of negligible thickness.

Let the number of revolutions per second be denoted by n, the magnetic flux across a right section of the space between the cylinders by φ, the capacity of the cylindrical condenser so far as it depends on the field between the armatures by $K\,C$, the capacity dependent on the field between the outer armature (together with any fixed conductors attached thereto) and the surrounding conductors, connected by a brush to the inner armature, by C'.

Let J denote the dielectric constant of that portion of the dielectric which moves. If the matter alone rotates while the ether remains fixed, $J = K - 1$. If the ether also moves, $J = K$. If the ether rotates alone, $J = 1$. It is clearly immaterial whether the armatures, which cut no magnetic flux, remain fixed or move.

When the cylinder is set into rotation the motional intensity f and motional e. m. f. $\Psi = n\,\varphi$ will act radially, let us say outwardly for definiteness, on the portion of the dielectric which moves. The electric separation produced in this part of the dielectric will give rise to an electric field whose intensity will be directed radially inward within the insulating cylinder and in which there will be a potential difference V directed from the outer armature to the inner armature, and from the outer armature to the surrounding conductors.

The total charge on the outer conductor and its connections, if any, will be zero, since it is completely insulated from the inner conductor and its connections. This charge consists of three parts superposed:

1. A charge $q_1 = -\,n\,\varphi\,J\,C$ (3)
on the inner surface;

2. A charge $q_2 = K\,C\,V$ (4)
on the inner surface;

3. A charge $q_3 = C' V$ (5)
on the outer surface and any conductors connected therewith. Since

$q_1 + q_2 + q_3 = 0$, we get, if $q_2 + q_3 = q_4$,

$$q_4 = (K C + C') V = n \varphi J C = - q_1 \qquad (6)$$

In Wilson's experiments and my own, hollow insulating cylinders were used, and the experimental processes (considerably different in the two investigations) were equivalent to measuring q_4, n, φ, and C and therefrom calculating J, which was found equal to $K - 1$ within the limits of the experimental error.

In Blondlot's experiments a parallel plate condenser (a special case of a part of a hollow cylinder) was used, and a swift stream of air was sent between the plates while they were connected together by a wire at rest, thus making V zero and leaving the motional e. m. f. Ψ to act alone. The connection between the plates was broken when the velocity of the air stream was a maximum, and the quantity q_1 directly measured. It was found to be zero, thus showing that $J = K - 1$, since K for air and ether together is practically unity.

All these investigations thus confirm the theory of Lorentz, according to which the ether is at rest. The results would be identical if there were no ether.

9. The field inside the rotating cylinder is of great interest, but can be discussed here for a few cases only. The solid cylinder rotating about its axis and the parallel plate moving parallel to one surface are particular cases of the hollow cylinder.

(1) First suppose the ether to be at rest and suppose the thin armatures either not existent or else disconnected from all other conductors.

In this case the total electric displacement D at all points of a cylinder of any radius $r \geqq r_1$ (radius of inner surface) $\leqq r_2$ (radius of outer surface) coaxial with the surface of the insulating cylinder is

$$0 = D = \frac{1}{4\pi} \{ (K - 1) f + K E \}$$

$$= \frac{1}{4\pi} \{K E + (K - 1)\, \omega H r\} \qquad (7)$$

where $\omega = 2\pi n$. Hence

$$E = -\frac{K-1}{K} f = -\frac{K-1}{K} \omega H r \qquad (8)$$

There is no intensity of either kind inside the inner cylinder or outside the outer cylinder.

(2) Next suppose the armatures present and connected by a wire at rest. This connection reduces the voltage $\int E\,dr$ to O, but the field intensity E does not vanish except over one cylinder of radius r_0 to be determined. If it did, the charge on the outer armature would be greater than the opposite charge on the inner, and the insulator itself would be internally charged. For f increases proportionally with r, and D would now therefore increase in the same way. The voltage through the insulator being zero, the direction of the field intensity E, always radial, changes in sign at the cylinder of radius r_0. Within this cylinder E has the same direction as f, so as to increase the total displacement; while outside this cylinder it has the opposite direction, so as to decrease the displacement which would exist were f alone acting. In this way the electric flux across all cylinders which can be drawn between the armatures is kept the same and equal to that across the cylinder of radius r_0. At the cylinder of radius r the field intensity is

$$E = -\frac{K-1}{K} \frac{\omega H}{r} (r_0^2 - r^2) \qquad (9)$$

while r_0 is given by the expression[19]

$$\left(\frac{r_2^2 - r_1^2}{2 \log \frac{r_2}{r_1}} \right)^{1/2}$$

(3) If the wire moves with the armatures a field intensity due to the potential difference V from the outer to the inner armature equal to the e. m. f. $n\varphi = \frac{\omega H}{2} (r_2^2 - r_1^2)$ developed in the wire will be superposed on the intensity given by (9), with the result that the displacement within the rotating condenser will be exactly the same as if it contained ether only, at rest, and were charged to the potential difference V[20].

(4)[11] If the ether moves with the matter in case (1), we must substitute K for $K - 1$, and we get in place of (**7**) and (**8**) the relations

$$O = D = \frac{1}{4\pi} K (f + E) \tag{10}$$

and
$$E = -f = -\omega H r \tag{11}$$

Neither matter nor ether is displaced, either within or without the cylinder.

(5)[12] If the ether moves with the matter in case (2), equation (**9**) becomes

$$E = -\frac{\omega H}{r} (r_0^2 - r^2) \tag{12}$$

while r_0 remains unchanged.

(6)[12] If the wire moves with the ether and the matter of case (4) [or if the ether moves with the matter and the wire of case (3)], there is no displacement anywhere, exactly as in case (4), where the wire was absent[21].

10. In the second group of experiments Faraday found that if a conducting circuit was kept fixed and the magnetic flux through it altered in any way by the motion of adjacent circuits or magnets, an e. m. f. was developed around it equal to the rate of decrease of magnetic flux through it, precisely as in the experiments of the first group for the same relative motion of the material systems. In this case the tubes of magnetic induction, which are always closed, must be conceived to move across the circuit, and the intensity induced therein at any point may be conceived as given by the expression

$$E = [H v] \tag{13}$$

where v is the velocity of the tubes. The electromotive force around the circuit is the line integral of this expression.

Furthermore, if the conducting circuit is kept fixed, as before, and the magnetic flux through it changed in any way by the alteration of adjacent magnets or circuits, or currents in these circuits, an e. m. f. is developed and is equal, again, to the rate of decrease of magnetic flux through it. In this case also we may conceive the (closed) tubes of magnetic induction to move across the circuit, producing therein at any point an intensity given by (**13**).

If the circuit is not conducting, but is merely a path drawn in an insulator or in the free ether, the electromotive force around it and the electric intensity at any point of it are given by the same expressions as before. The displacement, the effect of the intensity, of course differs with the conditions.

There is never any essential difficulty in determining the induced e. m. f., or line integral of the induced intensity, around a complete circuit from the Faraday-Neumann law; but the application of equation (**13**) to the determination of the intensity at a point is often impossible. For, except in very special cases, we do not know the velocities of the tubes of induction. Indeed, in certain cases the very conception of the motion of tubes of induction appears to many to be without meaning.

Consistently with experiment, however, the theory has been put in such a form by Maxwell, Clausius, and Lorentz, as to give for the induced intensity at a point, an expression which depends only on the motion of the electrons which produce the magnetic field and the time.

If e_1 denotes the (fixed) charge of a certain electron, r_1 its distance from the point P at which the induced intensity is to be determined, and $\mathbf{v}_1$ its velocity, then at the time t the vector potential at P due to this electron is by definition,

$$A_1 = \frac{e_1 \mathbf{v}_1}{r_1}$$

the (changing) quantities $\mathbf{v}_1$ and r_1 being the velocity and distance at a time earlier than the time t by the interval required for an electric wave to travel from the electron to P. The vector potential A at P due to all the moving electrons is the vector sum of all the minute vector potentials $A_1 = \frac{h_1 \mathbf{v}_1}{r_1}$

$A = \frac{e_2 \mathbf{v}_2}{r_2}$, etc. That is $A = \text{vector} \sum A_1 = \text{vector} \sum \frac{e_1 \mathbf{v}_1}{r_1}$

In terms of this vector potential A the expression for the induced intensity is simply

$$E = -\frac{dA}{dt} \qquad (\mathbf{14})$$

and the electromotive force around the circuit is the line integral of this expression; Even in cases where it is not possible to determine the velocities of the electrons and thus calculate A, it is possible to determine whether A, and thus also $\frac{dA}{dt}$, changes when certain variations in conditions occur, and thus to obtain useful information with respect to E.

11. In many cases the magnetic field in which a body is moving is changing with the time, and we have the motional and induced intensities superposed. Similarly, they are superposed when the body under experimentation and the agent producing the field are both in motion.

If the system referred to as fixed is, instead of being fixed to the earth, fixed to a ship or car in uniform motion, there is no difference in observed results, according to experiments hitherto made. It is understood that the observer and instruments are on board, though this is often a matter of no consequence.

It will be instructive to consider some simple experiments, in part made familiar long ago by Faraday, with a circular cylindrical magnet A and an adjacent coaxial circular coil of wire B with a remote galvanometer connected into the circuit by long twisted leads.

(1) If A is fixed to the earth and B moves along the axis, there will be a motional e. m. f. in B and a certain electric charge q will traverse the galvanometer for a given change of magnetic flux through B. (2) If now B is fixed to the earth, and A is displaced, the relative motion of A and B being the same as before, there will be an induced e. m. f. in B exactly equal to the motional e. m. f. in the previous case, and the same charge q will traverse the galvanometer.

It is of no consequence whether the galvanometer is attached to A, or to B, or to the earth. The indications will be the same, and will appear the same to observers on A, or in B, or on the earth.

Suppose now that the region in which the experiments are performed is permeated by the ether, and for simplicity suppose this fixed relatively to the earth.

When experiment (1) is performed, the only electric intensity and displacement produced will be the motional intensity and displacement in the coil.

When (2) is performed, there will be the same displacement in the coil, due to an induced intensity equal to the previous motional intensity; and at the same time the field about the coil and magnet will be filled with circular lines of induced intensity all coaxial with the magnet and due to the motion of its tubes of magnetic induction. This induced intensity will produce electric displacement in the ether, the lines of displacement also being coaxial with the magnet.

The galvanometer shows no difference between the two cases, but the difference really exists.

If there is no ether, no electric displacement, aside from the current in the coil, will occur in either case.

If an experiment takes place in which relative motion between *A* and *B* occurs in an isolated region, and observers are placed on *A* and *B*, each observer is likely to assume that he is fixed and that the other is moving. The observer on *A* will conclude that there is a motional e. m. f. in *B*; while the observer on *B* will observe the same e. m. f. but will call it *induced.* A third observer, fixed to neither *A* nor *B*, will consider the e. m. f. to be partly induced and partly motional.

12. Faraday, as we have seen, attempted to study the displacement produced in insulators by the induced intensity, but without success. Except for a qualitative experiment of Hertz's[22] made with high-frequency oscillations and lying outside the scope of this discussion, no successful experiments on the displacement in material media appear to have been made. Kuehne[23] has, however, shown that a finite electrified body in a region traversed by moving tubes of magnetic induction is acted upon by a force consistent with equation (**13**).

13. Recently, in an investigation presented to the Physical Society, I tried to find out whether, in a certain special case, the induced intensity produces electric displacement in the ether.[24] But I expressed to the society an uncertainty which has persisted and grown, as to the validity of one of the assumptions underlying the method. The experiment will be briefly considered.

14. Suppose an air condenser, with horizontal plates *A B*, short-circuited by a wire *C* and placed in a uniform magnetic field whose lines of induction are parallel to the plates. Suppose that the condenser is screened (an absolutely essential precaution) by being placed symmetrically within a metallic box whose faces are parallel and perpendicular to the plates, and that the upper plate is connected to the top of the box by a wire *F*.

If the box and condenser move together in the magnetic field parallel to the plates and perpendicular to the lines of induction, the plates will become charged, but the charge on the lower face of each plate will be equal and opposite to the charge on the upper face of the same plate. If during the motion the lower plate is removed and tested for total charge, it will thus be found uncharged.

15. If the box and condenser remain fixed and the magnet is moved, the charges will be exactly as before on the hypothesis that there is no ether. If the ether exists, however, there will be no electric displacement anywhere within the box, and the plates will be without charge on either side. The test of the lower plate for total charge will thus give the same result as before, and the experiment will not discriminate between the two hypotheses.

16. Imagine the magnetic field to be sharply divided into two parts by the plane passing through the upper surface of the lower plate *B*, the complete field throughout the region occupied by the box remaining uniform, but the tubes of induction of the lower part remaining fixed like the condenser, while the tubes of the upper part move when the magnet moves. In the attempt to accomplish this end I used two similar magnets, symmetrically placed with respect to the plate *B*, the lower one fixed to the condenser.

If this arrangement secures the end desired, and the condenser remains fixed while the upper magnet moves, the result will be different according to the hypothesis made with regard to the ether.

If there is no ether the upper surface of the lower plate of the condenser will become charged, there being now no intensity or displacement beneath the lower plate. If the ether exists, the plate will be uncharged.

Experiments made in this way have shown that the lower plate of the condenser does not become charged when the upper magnet moves.

If, however, the assumption made above with reference to the division of the magnetic field and the application of equation (**13**) under these circumstances are not justified, and the principle of superposition holds in such a way that the two magnets independently produce their individual effects, the lower magnet will produce no effect (at the critical time) and the upper magnet will act exactly like the single magnet of Sections 14 and 15 (but with a non-uniform field, which does not affect the theory essentially), producing no (total) charge on the lower plate on either hypothesis. This is the result to be expected from the application of equation (**14**). In this case the experiments simply confirm Section 15.[25]

17. We come now to the subject of unipolar induction.[26] In one of Faraday's experiments a symmetrical magnet rotating uniformly about its axis was touched by a wire *A C B*, including the coil of a galvanometer, at two points *A* and *B*, one near the axis at one end, the other on the cylindrical surface near the center. An e. m. f. was developed in the circuit equal to that which would be developed if the magnet remained fixed and the conductor *A C B* moved, the relative motion being the same as before.

In another experiment a copper disk was mounted coaxially close to one end of the magnet and a galvanometer *C* connected to two points *A* and *B* near the center and edge of the disk. When the disk rotated, an e. m. f. equal to that which would be produced by the same relative motion of *A C B*, was developed as before, whether the magnet rotated or not. But no current traversed the galvanometer when the disk remained fixed.

Similar results are obtained when the magnet is replaced by an electromagnet or by an electric coil wound on a non-magnetic core.

18. Faraday considered his experiments to prove that the tubes of magnetic induction remain fixed and that the magnet and disk move through them, the e.m.f. thus being a pure motional e.m.f. with the disk or the magnet itself as its seat.

It was pointed out by Tolver Preston,[27] however, that the results are equally well explained by the assumption that the tubes of magnetic induction share the rotary motion of the magnet and produce an induced e. m. f. by cutting the conductor *A C B*.

19. Faraday's experiments have been repeated with various modifications a great many times, but the problem of the seat of the e.m.f. was brought no nearer to a solution until the experiments of E. Lecher were published[28] in 1895. Lecher made three beautiful experiments, two of which, at least, are quite difficult to explain except on the assumption that the tubes of magnetic induction remain fixed, or that the seat of the e. m. f. is in the moving part of the circuit.

In one of these experiments, for which alone we have space, two similar electromagnets were mounted coaxially and near together, with the current in both coils in the same direction. A galvanometer was connected in the usual way to two points on one of the magnets. It was found that the current depended entirely on the motion of this magnet, being the same whether the other magnet rotated in either direction or remained at rest. On the assumption that the seat of the e. m. f. is the moving magnet the explanation of this result is quite obvious. Improbable assumptions are necessary to explain it on the other hypothesis. By making sufficient assumptions however, this is possible; and Lecher did not consider his experiments *entirely* conclusive.

20. Many students of physics have thought that this problem could be solved by experiments on so-called open circuits—experiments in which static charges, or forces on bodies with such charges, would be looked for instead of permanent galvanometer currents. The list includes Weber, Tolver Preston, Hertz, Lodge, Lecher, and others. But the theories on which such experiments and proposed experiments have been based appear to be fallacious.

21. As a particular case, consider a long cylindrical air condenser placed in a uniform magnetic field with lines of induction parallel to its axis and short-circuited by disks closing the ends. If this condenser were rotated about its axis it would become charged by the motional e.m.f. in the disks, the charge of the inner armature, the one assumed to be tested, being equal to this e. m. f. multiplied by the capacity.

If, however, the condenser and wire remain fixed, and the agent producing the magnetic field rotates about the axis, the relative motion being the same as before, the result to be expected depends on the hypothesis made with respect to the ether.

First suppose that the ether does not exist. If the tubes of induction do not move, the condenser will remain uncharged. If the tubes of induction move with the agent, an e. m. f. will be induced in the disks producing a potential difference in the opposite direction through the condenser and charging it as when the condenser moved.

If, on the other hand, the ether exists, there will again be no charge in case the tubes of induction remain fixed. If the tubes of induction move, however, there will be superposed on the charge due to the potential difference developed in the corresponding case for no ether an equal and opposite charge due to the e. m. f. induced in the ether between the armatures. There will, as before, be no change on the inner surface of the inner armature. Thus it will remain uncharged, just as if the tubes of induction had remained fixed.

Thus if the armature should become charged in such an experiment it would prove that the ether as an electrical medium does not exist, and that the tubes of induction move.

But if the armature should not become charged, the experiment would prove only that either (1) the ether does not exist and the tubes of induction do not move, or (2) the ether exists.

22. In 1912 I completed an extended series of experiments of this kind, not with the hope (which I originally had) of solving the problem of unipolar induction, but to get what information they might yield on the relations between electromagnetic induction and relative motion.[29]

The experiments show that the condenser does not become charged, the experimental error being a small fraction of the charge the condenser would receive if it rotated while the agent producing the field remained fixed.

This is the first case in electromagnetic induction in which it has been proved that the observed effect does not depend entirely on the relativity of the motion of the essential material systems involved.

23. If the condenser and short-circuiting disks are made to rotate together with the agent producing the field, there will be superposed on the (zero) charge of the condenser in my experiments a charge equal to that it would take if the agent producing the field remained fixed and the condenser rotated. Rotating the agent alone produces no charge; rotating the condenser and disks alone produces a charge. When both are rotated together the effects must be superposed.

This corollary of my experiments has been confirmed in each of two recent investigations; one by Kennard,[30] who had done earlier work in this field; and one by Pegram.[31]

In their experiments the apparatus was essentially similar to my own, but the condenser and coil were both made to rotate. The condenser was found to become charged in accordance with the theory.

In this last rotation experiment we have a positive effect of electromagnetic induction without any relative motion between the essential material parts of the system. In my own rotation experiments we have a zero effect with a positive relative motion. Neither group of experiments, however, is inconsistent with the principle of relativity of Einstein, which relates to translatory motion only, or with the theories of Maxwell and Lorentz.

These experiments do not seem to me to solve the old problem of unipolar induction. As shown above (Section 21) they prove *either* (1) that the ether does not exist and that the tubes of induction do not move, *or* (2) that the ether exists. If we had evidence that the ether does not exist, the experiments would prove that the tubes of induction remain fixed.

24. The *simplest* assumption which will account for all the unipolar phenomena is Faraday's assumption that the tubes of induction are fixed and that the e. m. f. is a pure motional e. m. f. in the moving conductors. The electron theory, moreover, supports this assumption in the simplest case to which it can be applied,[31] viz. that of a thin solenoid carrying a direct current and rotating about its axis. When the solenoid is rotated there is simply superposed on the conduction current density at any point of the solenoid a convection current density of identical nature and zero magnitude, since the positive and negative charges per unit volume of the conductor are equal. And this could not, apparently, affect in any way the state of the magnetic field.

For practical purposes, however, it is of no consequence which assumption is made. On the simpler hypothesis, the calculation of the e. m. f. may be made from equation (**2**); on the other, from equation (**13**); and on either hypothesis, or any combination of the two, it may be calculated from the rate of change of flux through the circuit from the brush A in contact with the cylindrical surface of the magnet, through the galvanometer to the brush B in contact with the center of the end surface of the magnet, from B to a point C fixed on the magnet in the same transverse circle with A, and from C along the arc of the circle to the brush A. With the uniform growth of the arc $A\,C$, which contributes nothing to the e. m. f. and may encircle the cylinder any number of times, the flux through the circuit changes at the same uniform rate on any hypothesis.

25. This arrangement, essentially the unipolar or homopolar generator, in which the rotation takes place about the axis of a symmetrical field and one part of the system cuts the flux

uniformly and always in the same direction, is clearly the only one in which a steady direct current can be electromagnetically produced by rotation without commutation. With rotation about any other axis the e. m. f. will fluctuate or alternate periodically.

Department of Terrestrial Magnetism
Carnegie Institution of Washington.

FOOTNOTES

1. For recent general discussions of electromotive forces of induction see F. Emde, *Elektrotechnik und Maschinenbau* 26, 1908, pp. 997, 1023, 1074, 1119, and 27, 1909, p. 783; also de Baillehache, *Bull. Soc. Int. des Électriciens* 10, 1910, pp. 89, 288.
2. For a brief account of the work of some of the early investigators see Brillouin, *Propagation de l'Électricité*, pp. 159-167, 1904.
3. Much of the work prior to 1895 is described by E. Lecher, *Wied. Ann.* 54, 1895, p. 276.
4. Treatise, §§576, 577.
5. *Wied. Ann.* 17, 1882, p. 55.
6. *Wied. Ann.* 49, 1893, p. 284; 57, 1896, p. 232.
7. *Phys. Zeit.* 7, 1906, p. 640.
8. *Ann. d. Phys.* 39, 1912, p. 840.
9. *Phys. Rev.*, 8, 1916, p. 97; 9, 1917, p. 164.
10. The expression $[v\,H]$ signifies the *vector product* of the vectors v and H. Its magnitude is equal to the product $v\,H \sin\theta$, when θ ($< 180°$) is the angle between the directions of the two vectors; and its direction is the direction in which a right handed screw would be translated if it were rotated through the angle θ from a position in which the slot in its head has the direction of v to that in which it has the direction of H. It is thus perpendicular to both v and H.
11. *Pogg. Ann.* 149, 1873, p. 251. The most recent and precise work on this subject is that of F. E. Smith, *Roy. Soc. Phil. Trans.* A. 214, 1914, p. 27. For some recent experiments on the motional e. m. f. in conductors, consistent, like all others, with equation (2), see C. Hering, TRANS. A. I. E. E. 27, 1908, p. 1341; and A. Blondel, C. R. 159, 1914, p. 674.
12. Electric Waves, p. 241 (*Wied. Ann.* 41, 1890, p. 369). See further section 9, (4), (5) and (6). The existence of $E = -f$, evidently necessary, since f diverges, to keep D ($=0$ in equation (11)) from divergence, and the insulator thus free irom internal charge, appears impossible to account for and thus evidence against justification of the assumption that the ether alone, ($K=1$) or the ether and matter together, ($K>1$) can move. It is thus evidence against the validity of Hertz's theory.
13. *Archives Neerlandaises* 25, 1892, p. 363.
14. Exp. Res. § 1728.
15. *L'Éclairage Électrique* 23, 1900, p. 41.
16. *J. de Phys.*, Jan. 1902.
17. *Phys. Rev.* 27, 1908, p. 425.
18. *Roy. Soc. Phil. Trans.* A., Sept., 1904.

19. Barnett, l. c., pp. 431-433.

20. Attention was first called in print to this result by E. H. Kennard (*Phys. Zeit.*, 13, 1912, p. 1156), though I had previously made use of it.

21. The capacity of the wire is here neglected. The wire itself, of course, becomes charged and there is displacement in the dielectric about it.

22. *Electric Waves*, p. 95.

23. *Phil. Mag.*, Apr. 1910.

24. *Phys. Rev.* 12, 1918, p. 95.

25. As another illustration of the great difficulty involved in getting information about the ether from experiments in electromagnetic induction it is instructive to consider the following: A long circular solenoid with an iron core is traversed by an alternating current, so that tubes of magnetic induction periodically move into and out from the core, producing circular lines of electric intensity centered on the axis. A long narrow, and thin air condenser is mounted near the solenoid with its greatest length and the planes of its armatures parallel to the axis. The armatures are connected by a wire which includes a detecting instrument. If there is no ether, the condenser receives alternating charges on the *inner* surfaces of its armatures; if the ether exists, the condenser receives the same charges, but on the *outer* surfaces of the armatures, the detecting instrument thus indicates the same current on both hypotheses. Essentially the same difficulty would arise in an experiment similar to that of Section 15, but made without an electric screen.

26. In addition to Lecher, l. c., see Barnett, *Phys. Rev.* 35, 1912, p. 323; G. W. O. Howe, *Electrician* 76, 1915, pp. 169, 210, 323, 355; and Pegram, *Phys. Rev.* 10, 1917, p. 591.

27. *Phil. Mag.* 19, 1885, pp. 131 and 215.

28. Lecher, l. c.

29. *Phys. Rev.* 35, 1912, p. 323.

30. *Phil. Mag.*, Feb. 1917.

31 Pegram l. c.

Presented at the 355th meeting of the American Institute of Electrical Engineers, New York, November 14, 1919

PRESENT LIMITS OF SPEED AND POWER OF SINGLE-SHAFT STEAM TURBINES

BY J. F. JOHNSON

ABSTRACT OF PAPER

This paper will be restricted to a discussion of some of the factors which influence limits as applying particularly to turbines of the reaction type. With the employment of high vacua the limit of power will be determined largely by the area obtainable through the last stage.

Limiting factors include:

1. Chosen maximum values of steam speed through the blades in order to keep the leaving losses within permissible limits. For highest efficiency the steam speed should be about 25 per cent greater than the blade speed, but in the last stages it is sometimes made 100 per cent greater as a compromise between efficiency and cost. Similarly the outlet angle of the blades is increased from 20 deg. to 35 deg.
2. Physical characteristics of materials employed and chosen limits to which these may be safely stressed. By varying the form of rotor construction stresses in it may usually be kept within necessary limits up to the point at which limiting stresses in the blades or blade fastenings are reached. For any given rotative speed and blade angle the steam capacity is directly proportional to the stress at the base of the blades regardless of the diameter and blade height.
3. Capacity limits of manufacturing facilities, increased bracing and clearances necessary to insure requisite rigidity and reliability, and capacity loss due to outages for inspection and repair.

Fig. 6 shows maximum capacity at various speeds which are physically possible without exceeding present limits of stresses. It is valuable chiefly as showing the physical relation between speed and capacity with given limiting stress values.

THE recent rapid development of steam turbine-driven electric power-generating units of large capacity has prompted your Society to undertake a study of the present limits of speed and power for such units. This paper will be restricted to a discussion of some of the factors which determine or influence such limits as applying particularly to turbines of the reaction type.

With the employment of high vacua, such as is the present universal practise, the limit of power of a turbine operating at a given speed will be determined largely by the area obtainable through the last stage for the final expansion and passage of the steam prior to its entering the condenser. The significance of this will be apparent when attention is called to the fact that whereas a pound of steam, when entering the first stage, has a volume of less than 2½ cu. ft., when passing through the last stage it has a volume of approximately 395 cu. ft. when expanded to 28½ in. vacuum, and 585 cu. ft. when expanded to 29 in.; a ratio in the latter case of 1 to 234.

Consequently, in any discussion of limits of power, it will be necessary to assume conditions of pressure and superheat of the steam entering the turbine, the vacuum to which the steam is to be expanded in the blading, and the efficiency or rate of steam flow per unit of power. For these conditions 250 lb. gage pressure with 200 deg. fahr. superheat and 29-in. vacuum referred to a barometer of 30 in., and efficiencies as are commonly obtainable with them, will be used.

Limiting factors may be divided into three classes: First; Theoretical, including limiting steam velocities and effect on efficiency of velocity remaining in steam after leaving the last stage, and the area through the blades as affected by blade angle. Second; Physical, including methods of construction, materials, stresses, factor of safety against rupture, reliability factor, and limitations of transportation facilities. Third; Economic, including limits beyond which it may be physically possible, but economically inadvisable, to go, such as effect of size of structure or of character of materials employed on cost, and time required to make inspection and repairs.

THEORETICAL LIMITS

In this class there are but few limitations as affecting capacity at a given speed because with materials of infinite strength and rigidity available it would be possible to build units of infinite capacity; but for a given diameter and blade height the capacity will be limited by chosen maximum values of steam speed through the blades, in order to keep the leaving losses, or available energy in the steam discharged to the condenser, within permissible limits. Throughout the entire turbine, with the exception of the last few stages, steam speeds only about 25 per cent in excess of the corresponding blade speeds are employed in order to secure maximum efficiency. In the latter stages, however, the volumes become so great that a compromise between maximum theoretical efficiency and physical dimensions becomes advisable by increasing the steam speed sometimes to approximately 100 per cent in excess of the blade speed. The steam after being discharged from the last stage, therefore, still contains a small portion of available energy the recovery of which would involve disproportionate expense.

For example, if the pressure drop in the last stage is such as to render available for work thirty heat units which will produce an equivalent velocity of 1225 ft. per sec., and if the blade speed is such that the steam after leaving it still has a velocity of 600 ft. per sec., which is the equivalent of 7.2 B. t. u., this 7.2 B. t. u. will be totally lost, whereas probably 80 per cent of it, (or 6 B. t. u.), might be recovered were it practicable to use an additional stage of proper proportions. This would improve the total efficiency of the turbine approximately 1½ per cent. Higher blade speeds will tend to improve the efficiency by reducing leaving losses, but generally not as effectively as would larger blade areas with lower steam velocities and correspondingly increased number of stages.

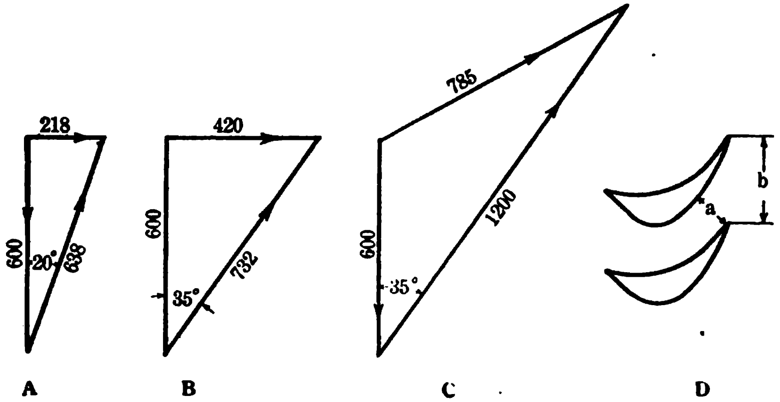


Fig. 1

Having fixed the height of a row of blades, the area of the steam space is dependent upon the angle formed between the center line of the row of blades and the outlet portion of the blade. The smaller this angle is the smaller will be the area and *vice versa*. On the other hand the smaller this angle the higher the efficiency because of the lesser absolute velocity left in the steam discharged to the condenser.

In Fig. 1A is shown a relation between steam speed and blade speed to give highest efficiency. The blade speed is 600 ft. per sec., steam speed 638 ft. per sec., blade angle 20 deg. and steam, speed after leaving blades, 218 ft. per sec. in direction at right angles to direction of rotation. Fig. 1B shows a similar condition giving maximum obtainable efficiency employing a 35 deg. blade angle and Fig. 1C a condition in which the steam speed is 100 per cent greater than the blade speed.

The leaving losses are, in Fig. 1A, 218 ft. per sec., Fig. 1B, 420 ft. per sec., and Fig. 1C, 785 ft. per sec., which is the equivalent of 0.95, 3.5, and 12.3 B. t. u. respectively.

The steam area is ordinarily expressed as a ratio of the perpendicular distance between blades, to the pitch of the blades, as a/b (see Fig. 1D). Highest actual efficiency is obtained by keeping this ratio between 0.25 and 0.3, and this is done in all stages except the last few in high vacuum machines where it is increased to a maximum of 0.5, the equivalent angle being about 35 deg. which includes proper allowance for blade thickness; this ratio having been determined upon as a proper compromise between cost of increased blade height and loss of efficiency due to increased terminal loss.

Some European manufacturers have employed ratios as large as 0.65 and 0.7.

PHYSICAL LIMITS

Chief among the physical factors limiting turbine capacity are the physical characteristics of the material employed and the chosen limits to which these materials may be safely stressed, bearing in mind that either uniformity of quality, or factor of safety sufficient to cover all possible variations, together with inaccuracies in calculation and irregularities of operation, must be provided for. While alloy steels possessing exceptionally high physical characteristics are procurable, their high qualities depend on relatively sensitive metallurgical processes which in the opinion of some engineers cannot as yet be carried out by regular workmen as a manufacturing process with a sufficient degree of reliability to justify their use, and that until this can be done, conservatism demands adherence to the lower strength, lesser sensitive materials. Such materials may, with suitable forms of construction, be safely stressed under the maximum test condition to within a few thousand pounds of their true elastic limits.

When the construction of the rotor is not limited to any one special form, the design may be varied so as to take full advantage of the low speeds in the high and intermediate stages, (where low speeds must be used in order to secure high efficiency), by employing a drum the thickness of which may be varied to keep the stresses within desired limits; while in low pressure stages where the stresses are highest, either disks carried on a shaft, or solid disks suitably held together, may be employed. With the solid disk construction the stresses may be kept within any reasonable limits up to speeds at which the design becomes too massive and expensive.

The steel regularly used by the Westinghouse company for turbine rotors conforms to the following characteristic specifications:

Test rings taken as close as possible to the point of maximum stress must show the following characteristics with standard two-in. specimens:

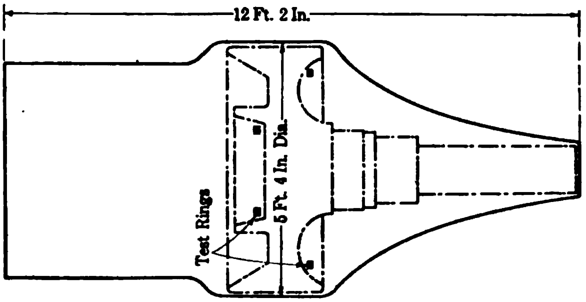


Fig. 2

Tensile strength 65,000 to 70,000 lb. per sq. in.
True elastic limit 22,000 to 25,000 lb. per sq. in.
Elongation 15 per cent to 18 per cent.
Reduction of area 20 per cent to 25 per cent.

The steel must be of best quality, having approximately 0.25 per cent carbon, 0.50 to 0.60 per cent manganese, 0.25 per cent silicon, and not over 0.025 per cent sulphur or of phosphorus.

The material is obtained ordinarily in the form of castings, though occasionally as forgings. The specifications in either case are the same. Especially in the larger sizes the forgings have been difficult to obtain, excessively expensive, and no more uniform or reliable in quality than the castings.

Fig. 2 shows the form of casting for a rotor end. It is cast vertically with the small end down, and after casting, is allowed to cool very slowly in the sand. After removal, the entire upper portion which constitutes the riser, is cut off and the casting then thoroughly annealed by being heated slowly and evenly to a temperature of about 1650 deg. fahr. and allowed to cool very slowly. It is then rough machined to within about ¼ in. of finished surface, after which it is put in a furnace and heated to about 1100 deg. fahr. and allowed to cool slowly to remove any possible internal stresses set up by reason of the metal removed in machining. It is then finish machined and given no further treatment of any kind.

The limit of stress to which this material is subjected is 20,000 lb. per sq. in. when operating at a speed 20 per cent in excess of the normal operating speed. The stress at normal speed is therefore $\frac{20,000}{(1.20)^2} = 13,900$ lb. This stress is 63 per cent of the minimum allowable true elastic limit, about 46 per cent of the yield point and 21½ per cent of the minimum ultimate strength. It is not generally appreciated that should the stress, by reason of defect or excessive overspeed, exceed the true elastic limit, no injury will result other than a slight permanent stretch, together with such blade damage as may result therefrom. In an extreme case of overspeed the rotor drum or solid disk will stretch sufficiently to cause blading to rub to such an extent as to practically insure entirely destroying it, and thus prevent further overspeed, before the ultimate strength and elongation of the material is reached.

If the rotor design can be so modified as to always keep the stresses within necessary limits, then the stress at the base of the blades, or in blade fastenings, determines the maximum capacity obtainable with a given speed.

There exists two interesting relations between the stress at the base of blades, steam passage area through the blades, and rotative speed. *For any given rotative speed and blade angle, the steam capacity or steam area through the blades is directly proportional to the stress at the base of the blades, regardless of the diameter and blade height selected.* This stress can only be modified by unevenly varying the cross sectional area of the blades, such for example, as thickening the blade near the base. *Also for any given stress the area through the blades will vary inversely as the square of the speed, i. e.,* if at a speed of 1800 rev. per min. a given stress and area are obtained, then at 900 rev. per min. the area will be increased four times if the stress is kept constant.

These relations will be readily seen from the following formulas:

$$\text{Area through blades} = \pi \times M\,D \times H \times R.$$

$$\text{If } R \text{ constant} = \text{Constant} \times M\,D \times H$$

$$\text{And stress at base of blades} = \frac{W\,V^2}{G\,r}$$

$$= \text{for steel}\,\frac{0.284 \times H \times \left(\dfrac{\text{rev. per min.} \times M\,D}{229}\right)^2}{32.16 \times \dfrac{M\,D}{24}}$$

$$= \frac{0.284 \times 24 \times M\,D \times H \times (\text{rev. per min.})^2}{32.16 \times (229)^2}$$

If rev. per min. is constant = constant × $M\,D \times H$.

Where H = Blade height in inches.

$M\,D$ = Mean diameter of blades in inches.

R = Ratio of area through blades to total annular area occupied by blades.

0.284 = Weight of blade material in lb. per cu. in.

The area and stress are therefore each equal to a constant times the product of mean diameter and blade height, and when the stress is constant this product will vary inversely as the square of the rev. per min.

The ratio of blade height to rotor diameter is, therefore, not a factor in determining physical limit of capacity, but only in determining efficiency, cost, and to some extent, reliability of the turbine.

Blading used in impulse stages and in low pressure reaction stages in which stresses exceed 15,000 lb. per sq. in. at 20 per cent overspeed is made of a 5 per cent electric furnace nickel steel in which the carbon sulphur and phosphorus are kept very low. It is really a nickel iron having a very fine close structure.

Its physical and chemical characteristics are as follows:

Tensile strength	65,000 lb. per sq. in. minimum
True elastic limit	35,000 lb per sq. in. minimum
Elongation in 2 in.	30 per cent minimum
Reduction of area	60 per cent minimum

Carbon, not over	0.08 per cent
Silicon, not over	0.10 per cent
Phosphorus, not over	0.025 per cent
Sulphur, not over	0.04 per cent
Manganese	0.40 to 0.50 per cent
Nickel	4.5 to 5.5 per cent

This material is annealed by heating to 1425 deg. fahr. and cooled in open air after rolling into sections required for forming into various blade shapes, and is given no further heat treatment.

The maximum stress at 20 per cent overspeed to which this material is subjected is 25,000 lb. The corresponding stress at normal speed is, therefore, 17,350 lb., this being 49 per cent of the true elastic limit, and 26½ per cent of minimum ultimate strength.

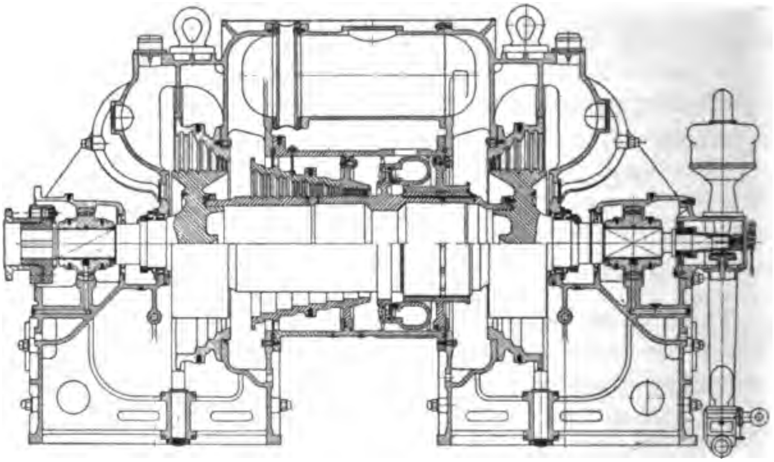


Fig. 5

For the lower stress reaction blading, a copper tin and phosphor bronze is employed, consisting of:

Copper	97 to 98 per cent
Tin	2 to 3 per cent
Phosphorus	0.03 to 0.07 per cent

Satisfactory methods of blade fastening involve no problems unless allowable stresses in the blades are very materially higher than those in the blade carrying element.

Fig. 3 shows a method used where the stress at the base of the blade does not exceed 17,500 lb., and Fig. 4 where stresses between 17,500 lb. and 25,000 lb. are involved. At 25,000 lb.

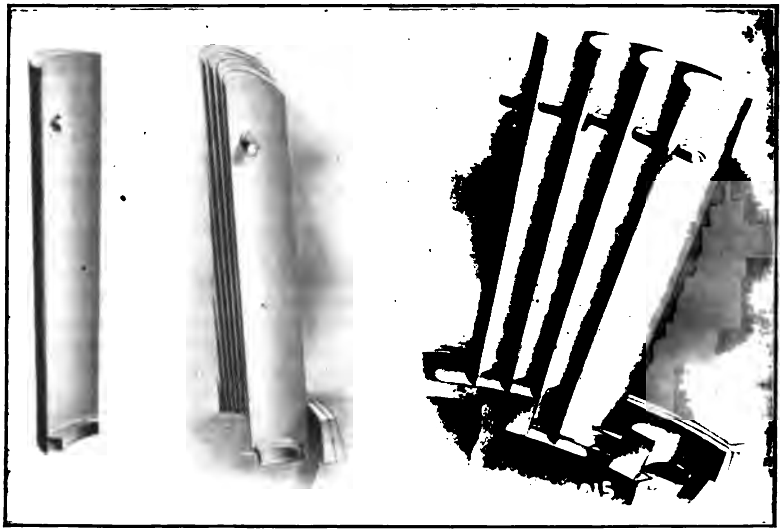


FIG. 3.

FIG. 4

stress at the base of the blade the tensile stress in reduced section of base is 11,800 lb., and the shearing stress in the blade and blade carrying element is 10,000 lb. The compound side wedges are employed to insure an absolutely tight fit of the blade in the groove so as to protect the reduced section of the blade against any stresses other than direct tension.

Increased capacity without decrease of rotative speed or increase of stresses may be obtained by employing multiple low pressure stages. This well known and popular expedient possesses the merit of permitting high-vacuum turbines to be built t speeds and capacities up to approximately the present limits of generator construction, without exceeding moderate diameters blade lengths, and stresses.

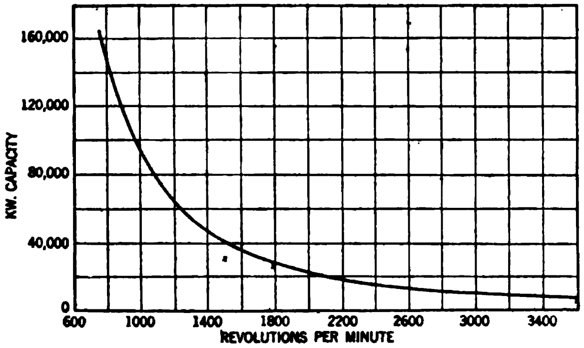


Fig. 6—Limits of Capacity of Steam Turbines with Double-Flow Low-Pressure Stages

Having steel blades of uniform cross-section stressed to 25,000 lb. per sq. in. at base of blades at 20 per cent overspeed and maximum efficiency at 80 per cent of rating—250 lb. steam pressure—200 deg. superheat—29-in. vacuum referred to a 30-in. barometer—with steam velocity through blades of 1225 ft. per sec. at a volume of 585 cu. ft. per lb.

Fig. 5 shows a cross sectional view of a turbine typical of this type.

Fig. 6 is a curve showing approximate maximum capacities at various speeds which are physically possible, employing double flow construction without exceeding the limits of stresses previously given. For equal capacities employing single flow construction the stresses would have to be doubled. The points marked (x) at 3600, 1800 and 1500 rev. per min. represent capacities which have already been built. This curve must not be interpreted as indicating suggested practicable present or ultimate limiting capacities of turbines, but merely as showing a physical relation between speed and capacity with given limiting stress values and operating conditions.

An important limit of size and capacity now being approached is that imposed by transportation facilities. Stationary elements may be readily sectionalized as required and assembled after shipment. Ways may be devised also for partial dismantling of rotor elements, although diameter will be one of the limiting factors, and this cannot be reduced beyond the point of omission of blading. The low-pressure rotors of the 30,000-kw. Interborough Rapid Transit Company turbines were shipped with the last stage blade carrying elements removed.

While the physical dimensions and capacities of turbines are being constantly increased it is essential that the reliability factor be not decreased. The employment of special materials and higher stresses does not usually permit increased capacity or efficiency without a corresponding increase in weight and cost, unless reliability be compromised.

If, having given a satisfactory reliable design of a given capacity employing low stresses, it is proposed to transform it by modification of design and substitution of higher stresses into a unit of larger capacity, greater blade lengths and probably greater blade weights, operating at higher speeds, and involving higher centrifugal forces will be necessary, in order to secure the required area. The rotor structure may possibly be shortened somewhat, but unless its total weight is increased nearly in proportion to the increase in stored energy in the individual blade; the unbalanced effect or disturbance caused by one or more blades breaking (which must be recognized as an inevitable occurrence in any turbine) will be greater, imposing higher stresses in the rotor shaft, bearings, and bearing supports; and a greater factor of strength will be required to withstand these stresses. The greater blade weight and higher speed will also require increasing the mass of the casing in order to prevent the blades from permanently injuring and possibly breaking through it if they should fail. The endurance factor of a turbine when operating under imperfect or abnormal conditions will be higher in proportion to the ratio of stator mass to rotor mass, and of rotor mass to blade mass. The incorporation in the design of a turbine of features which increase the endurance factor will appreciably increase its cost but will also (to a very much greater extent) increase its value to the user.

ECONOMIC LIMITS

In the study of economic limits of turbines of large capacity, consideration must be given to the fact that as yet such units are not required in sufficient quantity to warrant equipping and operating shops for their exclusive manufacture, and that they must, therefore, be produced by largely the same processes and equipment as are used for smaller sizes which are built in greater quantities.

As sizes becomes larger, a greater proportion of special equipment and processes becomes necessary, resulting in increased rates of cost unless accompanied by very material increase in quantity of production. Under present conditions this economic limit of capacity agrees closely with the physical limit of 1500 rev. per min. units.

In the larger low-speed structures the physical proportions become such that using ordinary steel and cast iron, to which we are limited by the metallurgical art, the distortions due to temperature changes and elastic properties of the materials are such that increased clearances and bracing have to be employed, in order to maintain equal reliability and rigidity to a degree which causes the cost per kilowatt for a given efficiency to increase with increasing capacity. Further development of the allied arts and increased demand for larger units wil tend to reduce the influence of this limitation factor.

Another factor tending to limit capacity of single units is the generating capacity loss resulting from suspension of service for inspection or repairs. For example, if a 30,000-kw. unit must be kept out of service ten days for a certain inspection or repair, a 60,000-kw. unit would have to be kept out probably fourteen days for a similar purpose because of the greater time required to handle the larger structure. Therefore, if two 30,000-kw. units were used and each held out of service ten days, the outage loss would be only five-sevenths as great as if a single 60,000-kw. unit were kept out fourteen days.

In order to avoid the limitations or undesirable characteristics just referred to, a number of turbine units of capacities varying from 30,000 to 60,000 kw. have been built in which the turbines have been divided into two or three separate compounded elements, each driving its own generator, and each capable of operating alone on high pressure steam in emergencies.

It is believed that units of this type will continue to be employed for the larger capacities because of the advantages not obtainable in single cylinder types, which will justify their somewhat greater cost.

Presented at the 355th meeting of the American Institute of Electrical Engineers, New York, November 14, 1919.

PRESENT LIMITS OF SPEED AND POWER OF SINGLE SHAFT CURTIS STEAM TURBINES

BY ESKIL BERG

ABSTRACT OF PAPER

This paper starts by showing that the limit of a single-unit turbo generator does not lie in the generator but is confined to the steam turbine, and that the last wheel of the turbine is the limiting feature.

The author therefore takes the last wheel of an 1800-rev. per min. turbine, giving dimension stresses, kind of material used etc., and then designs two turbines, one having 23 stages and the other 13 stages, both machines using this last wheel, and shows that a turbine can be built having its most economical point at 21,000 kw., under steam conditions given in the paper.

Under this condition the last stage absorbs 11.5 per cent of the total adiabatic available energy, and the wheel efficiency is 66.25 per cent. As the load increases, the work done in this stage also increases, so that at 36,000 kw. the energy is practically doubled, but with a sacrifice in efficiency of about 18 per cent, which naturally lowers the efficiency of the turbine as a whole.

A 5000-kw., five-stage, 3600-rev. per min. turbine load curve is also given and discussed, and the author claims that if the construction could be made similar to the large turbine, keeping the same stresses and number of stages, a turbine of this capacity could be built as efficient as the larger machine at 20,000-kw.

THE purpose of this paper is to discuss the limitations of the types of turbines used in the more important work now done by the General Electric Company, and to show the relative results which can be accomplished with such designs under different conditions of load. Different mechanical arrangements will make practicable different characteristics, and the conditions shown only apply to machines of these particular designs. The design of turbines must be governed by many compromises and it would not be possible to give a correct idea as to the whole range of possibilities.

For purposes of illustration, two machines have been selected, one representative of the largest size built for 1800 rev. per min. and one of the largest size now built for 3600 rev. per min. The figures given relate to the turbine alone, and do not include generators. The generators used with such turbines might be designed for various power factors or overload capacities, and would thus be of different efficiencies.

The turbines in question are both of the single-flow type and may be considered representative of capacity limitations of that type, since their construction is that which is adapted to the highest capacity. The single-unit turbine and generator is naturally preferable over the tandem and compound type on account of simplicity, lightness, and efficiency. There are however certain definite limitations in the size for a given speed that these units can be built with material available at the present time. The limitation in the size of a unit for a given speed does not depend upon the generator, but is entirely dependent upon the turbine.

Fig. 1 shows the load curve of the large 1800-rev. per min. machine above mentioned, which is designed to operate with 250 lb. steam pressure, 28.5 inches of vacuum, and dry steam. This turbine has 23 stages, all wheels being of the single-bucket type. The first stage wheel has a pitch diameter of 35 inches, which increases with each successive stage until the last wheel, which has a pitch diameter of 88 inches. This curve shows that for 1800 rev. per min., a turbine can be designed for a vacuum of 28.5 inches, giving its best efficiency at 21,000 kw., this point being marked on the curve by 1, and the water rate at other points being given in proportion to this point. This curve also shows that with a sacrifice of 5 per cent in efficiency, an output of 36,000 kw., can be delivered to the turbine shaft. Above a load of 21,000 kw., live steam is bypassed to the eighth stage shell, the effect of which is shown by the break in the curve.

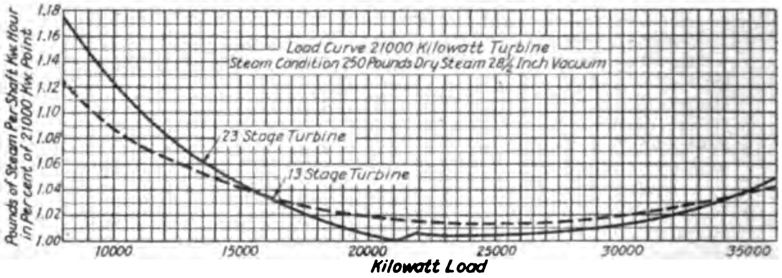


FIG. 1

The dotted line gives the load curve of a similar turbine in which the first eleven stages are replaced by one two-bucket stage. This turbine is designed with multiple valve steam admission. Such a modification may be desirable for the purpose of simplification, or where better efficiency at very light loads is important, as in the case of propulsion of warships.

Fig. 2 gives the vacuum curve of the 1800-rev. per min., 23-stage machine at 21,000-kw. and 36,000-kw. load on the turbine shaft. It will be noticed that the improvement in economy at a load of 21,000 kw., is 2.5 per cent between 28.5 and 29 in., whereas with a load of 36,000 kw., due to congestion of the steam passages, the gain is only 0.55 per cent.

Many large size turbines are now designed for a vacuum of 29 in. The volume of one pound of steam at this vacuum is 652 cu. ft., almost 50 per cent greater than at 28.5 in., and about twice as much as at 28 in., which calls for a corresponding increase in the area of the steam passage in the last stage wheel. If this area is made too small, the steam when leaving the last row of buckets must have a high velocity, which gives a large loss of energy, this loss being in proportion to the square of the velocity. It is therefore important that this area be made as large as possible, and that the exit angle of the blades be made small enough to give good extraction.

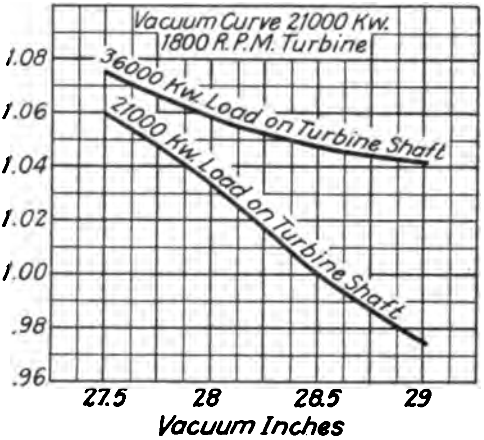


FIG. 2

Efficient action can only be accomplished by using a bucket speed that bears a proper relation to the steam velocity. Consequently to get the largest capacity we must not only use long buckets, but must move them at a very high speed. In order to obtain good bucket action, the buckets should not be more than about one fourth as long as the pitch diameter of the wheel. If they are made longer than this, poor bucket action with consequent loss in efficiency will result, due to the great difference in peripheral speed between the base and the tip of the bucket, the design being made correct for the middle point or pitch line. The flare also becomes excessive, so that the space between the buckets at the tip will be so large that steam can flow between the buckets without doing any work.

The use of a high steam speed in this last stage naturally implies that a relatively large proportion of the total steam energy must be utilized there, and such concentration of work into a single stage is not without its disadvantages, since even if the best relation of velocities is maintained, such a stage doing a large amount of work is naturally less efficient than a stage of similar character doing less work.

Since for the reasons given above, the design of the last stage in such a turbine constitutes the most important limitation, we will consider some details of the design used in this case. Fig. 3 represents the last wheel of the large 23-stage 1800-rev. per min., turbine which has been mentioned. This turbine is designed to operate at 250 lb. steam pressure, 28.5-in. vacuum, and dry steam. The pitch diameter of the wheel is 88 in., length of buckets 22 in. and bucket angles 60 deg. entrance and 40 deg. exit. The wheel is subject to the following stresses at normal speed:

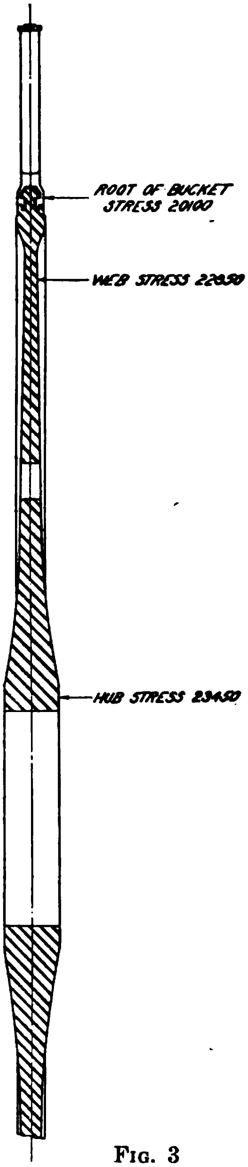


Fig. 3

Stresses in hub, 23,450 lb. per sq. in.
Stresses in web, 22,950 lb. per sq. in.
Stresses in bucket, 20,100 lb. per sq. in.

Elastic limit, (limit of proportionality) of material, 55,000 lb. per sq. in. The material of the wheel and bucket is quenched and tempered 3 per cent nickel steel.

The wheel is stiff enough to avoid vibration effects, and in the absence of such effects the centrifugal strains afford ample factors of safety, even if we assume considerable irregularities and imperfections of metal structure.

CAUSE OF WHEEL BREAKAGES

In some of the first large machines of the type here discussed, very serious trouble has developed through the formation of cracks in the forged wheels. Such cracks have caused wheels to break in three important installations. The cracks which have formed in these wheels have started at holes in the wheel provided either for balancing steam pressures on the two sides of the wheel, or for the attachment of balance weights. The occurrence of these accidents naturally gave rise to much alarm, uncertainty, and difference of opinion as to the cause of the trouble. Calculation showed that the wheels which broke were less stressed than many which were made from weaker metal and had operated for long periods of time. Holes in a centrifugally stressed wheel greatly increase the fibre stress in the vicinity of the hole itself, but such conditions had not caused the formation of cracks in large numbers of wheels in which such localized high stresses existed. Many evidences have now shown that the trouble with these wheels has not resulted from stresses in excess of those which had been previously found to be practicable, but has been caused by fluttering and vibrations of the wheels, which had become possible through the lightness and thinness of their construction. Such vibration gives a periodic character to the stresses normally imposed and so gives rise to the formation of fatigue cracks.

In machines of this type relatively light and narrow buckets have been used, and the wheels have been proportioned with a view to ample centrifugal strength, but with maximum economy of space and weight, consequently these wheels have had much less lateral stiffness than wheels used in turbines of previous types. To overcome such troubles as have developed, it is simply necessary to make the wheels stiffer and to put in holes in parts of the wheel near the hub where a suitable reinforcement of thickness can be provided which both stiffens the wheel and reduces stresses near the holes. Very slight changes of this kind make a great difference in the vibrating characteristics of such wheels, and the proportions used are such that they can easily be brought to the same standards of safety in these respects as have long prevailed in wheels of heavier construction in machines having less numbers of stages.

Experience has shown that in the absence of a tendency to form fatigue cracks through vibration, overspeed in such turbine wheels involves relatively little danger as compared with other types of high-speed machinery. Experimentally and in actual service, wheels have been stretched to a considerable degree of enlargement without the formation of any cracks, and such stretching is a normal condition if the cracks do not exist.

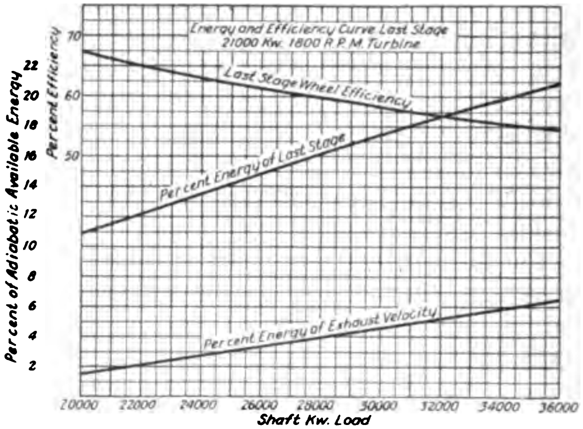


FIG. 4

It has been discovered that such fluttering or vibration of the web of wheels has not only been responsible for the formation of fatigue cracks in the wheels themselves, but has also caused loosening and breakage of buckets. The cure for these difficulties is to use stiffer wheels, and such wheels can carry stiffer buckets, so that the whole structure is incapable of vibration of any amplitude through such forces and periods as arise from the conditions of operation.

Fig. 4 gives the energy and efficiency curves of this last stage. It will be noticed that at the most efficient point, (21,000 kw.), this stage absorbs 11.5 per cent of the total adiabatic available energy, and that the wheel efficiency is 66.25 per cent. The energy represented by the exhaust velocity which is all wasted in the condenser is 1.5 per cent of the total energy. As the load increases on the turbine shaft, the energy in this stage also increases, decreasing its efficiency until at 36,000 kw. the energy in the last stage is 20.9 per cent of the total energy. The wheel efficiency however has been reduced to 54.2 per cent and the energy represented by the exhaust velocity has been increased to 6.4 per cent. This great amount of work in the last stage at such poor efficiency naturally lowers the efficiency of the whole turbine, and in this case the efficiency at 36,000 kw. is 5 per cent lower than at 21,000 kw.

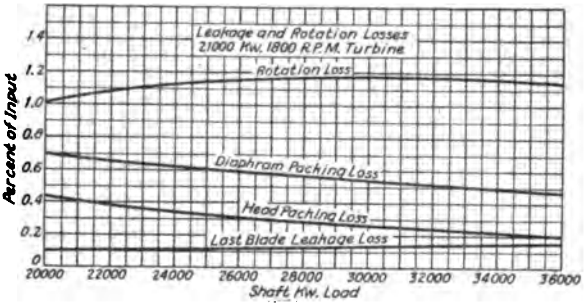


Fig. 5

Fig. 5 shows leakage and rotation losses of the same turbine in per cent of input.

From the above it will be seen that for 1800 rev. per min., a turbine can be designed efficiently for 21,000 kw. which, with a sacrifice of efficiency can deliver 36,000 kw.

Fig. 6 gives a load curve of the smaller 3600-rev. per min. turbine. The water rates are here given in reference to that of the larger machine, the load of 5000 kw. corresponding to that of 20,000 kw. on the 1800-rev. per min. turbine. This turbine has only five stages, one two-bucket wheel in the first stage, the other four stages having single-bucket wheels. The first wheel has a pitch diameter of 35.5 in., and the remaining four wheels a pitch diameter of 51 in. The bucket height of the last wheel is 9.125 in., the turbine being designed for a maximum of 6250 kw.

The reason for such a discrepancy in the number of stages calls for explanation. As the output of a turbine, keeping approximately the same stresses, goes up inversely as the square of the rev. per min., if the same number of stages could be used and clearance and all dimensions proportionately reduced, a 5000-kw. machine at 3600 rev. per min. could be made nearly as efficient as a 20,000-kw. machine at 1800 rev. per min., and developments of smaller multi-stage machines at our Lynn Works have already been made which approximate such possibilities. Constructions however which are practicable on a large scale are not practicable on a small scale, and consequently there are difficulties in getting the space economy in small high-speed machines which would be necessary for accomplishing the result stated. One of the difficulties has lain in the construction of diaphragms, the casting in of nozzle partitions being easy in a large diaphragm and very difficult on a small one. We are working upon types of diaphragms and other parts which may make possible the development of multi-stage high speed machines which afford improved degrees of economy.

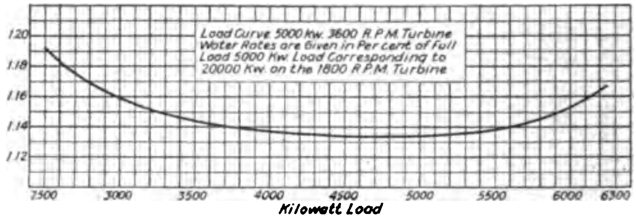


Fig. 6

If a 10,000-kw. turbine is designed for 1800 rev. per min., the only change necessary would be to make the nozzle and bucket heights about half the height of those in the 20,000-kw. unit. This reduction in height of buckets and nozzles would affect the weight, size, and cost of the turbine very little as compared with the 20,000 kw. unit. In regard to economy, the lower bucket heights would reduce the rotation loss somewhat but far from 50 per cent. The diaphragm packing loss, head packing losses, and bearing losses would be practically the same as on the 20,000-kw. unit, so that while a turbine designed for 10,000 kw. would be more economical than the large turbine running at half load, the difference would be small, being only about 6 per cent.

It will be seen from this paper that for a given speed there is one particular size of turbine which can be designed to be most economical as to steam consumption, weight, space, and price per kilowatt. Even if a size smaller than this is required, it would in many instances pay for the central station to install the larger unit, even though it would have to run at reduced load for some time before the station load increased sufficiently to utilize the full capacity.

Presented at the 355th meeting of the American Institute of Electrical Engineers, New York, November 14, 1919.

PRESENT LIMITS OF SPEED AND OUTPUT OF SINGLE-SHAFT TURBO GENERATORS

BY F. D. NEWBURY

ABSTRACT OF PAPER

Output is determined broadly by rotor or stator dimensions. With speeds of 1200 rev. per min. and lower, the stator is the limiting member, while with higher speeds, the rotor is the limiting member.

The most effective rotor diameter is not necessarily the largest diameter. To obtain maximum output at a given speed the rotor proportions must be chosen to properly balance mechanical stresses, rotor ampere turns and flux. American design practise has established 400 ft. per sec. as an upper limit of rotor peripheral speed.

The maximum length of core is determined by such factors as ventilation, bearing temperatures, critical speed and limits to weight imposed by forging and transportation facilities.

Fig. 4 shows present limits to kv-a. rating at speeds from 3600 to 900 rev. per min. These limiting values are given as indicating present boundaries to knowledge and experience, rather than as real physical or other limits that cannot be exceeded.

Mechanical forces due to short-circuit current, and damage caused by armature winding failures, are no greater in the very large generators indicated by Fig. 4 than in present day 20,000 and 30,000 kv-a. units.

No opinion is expressed as to the wisdom of installing very large single-shaft units. If operating engineers desire units of 50,000 to 100,000 kw., there is no question but that such generators can be conservatively designed and constructed.

REDUCED to the simplest terms, maximum output at any speed is attained when slot space is provided for the maximum possible ampere turns (in either stator or rotor), and core cross section is provided for the maximum possible flux. These conditions require the most effective rotor diameter (or stator bore) and the maximum rotor and stator core length. All factors that limit rotor diameter (or stator bore) or core length have a possible bearing on limiting outputs.

The most effective rotor diameter for a given speed is not necessarily the largest diameter. For maximum output, the rotor should have maximum space for winding and maximum tooth and core section for flux. Obviously, these requirements are antagonistic and the actual design is a balance between slot area and tooth and polar area. Again, as the diameter is increased (with a given speed) there is more room for both winding and flux, but with increase in diameter each pound of copper exerts an increasing centrifugal force and the ratio of slot area to tooth cross section must be decreased in order to keep within desired stresses. Beyond a certain peripheral velocity, ampere turns must be decreased, in spite of the increase in available space, and the most effective diameter has been passed. It is seen, therefore, that in order to obtain maximum output at a given speed, the rotor proportions must be chosen to properly balance mechanical stresses, rotor ampere-turns and flux.

Turbo generator design has settled down to one type of rotor, so far as form is concerned. This is the so-called radial slot type, in which the ends of the winding project beyond the core body. This construction is shown in Fig. 1, indicating the slot arrangement. This type of design requires solid rings of very good material for holding these projecting coil ends. The hoop-stress in the coil retaining rings is an important limit to output, and is, in fact, a more important limit than the tooth stress in the main rotor body.

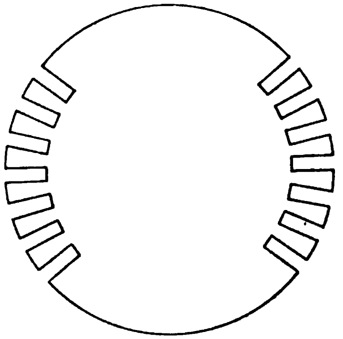


FIG. 1

In a large-diameter low-speed turbo generator (1200 rev. per min. and below) it is generally possible to employ a larger rotor diameter and more rotor ampere turns than can be properly balanced by stator ampere turns. The density of stator ampere turns is limited by the ability to dissipate heat (with permissible temperature differences) and by the permissible concentration of ampere turns in a single slot. Obviously, there is no such rigid limit to weight or depth of copper in a single slot in the stator as is imposed by centrifugal stresses in the rotor, but there is a limit to the depth of stator slots determined by the rapid increase in eddy current losses with deep slots and by the ability to construct and insulate long coils having a very large ratio of depth to width.

Thus, in certain cases the rotor is the limiting member and in other cases the stator is the limiting member. In general, the rotor first reaches its limit in ratings of 1500 rev. per min. and higher speeds, and the stator first reaches its limiting output in ratings of 1200 rev. per min. and lower, considering commercial frequencies.

It is apparent that unless the ratio of rotor ampere turns to stator ampere turns is a fixed design relation, all generators could be designed for the maximum possible output as determined by the stator, and the rotor need never be the limit. As a matter of fact, there is a tendency in this direction, the restraining fact being that as the field is weakened, relatively to the armature, the increase in field current and exciting voltage as the load is increased becomes greater. A reasonable limit to increase in excitation with load is desirable from the standpoint of voltage regulator operation. Regulators can readily handle a range of one to two or one to two-and-a-half, and large generators are proportioned to meet this ratio of no-load to full-load excitation. Voltage regulation has ceased to be a limit to output. These field and armature proportions result in regulations of roughly 25 per cent at 100 per cent power factor, and 40 per cent at 80 per cent power factor. Obviously, such regulations could not be tolerated if regulation were a factor in operation.

A fundamental difficulty in laying down definite limiting outputs is the difficulty in arbitrarily stating limiting stresses. The two principal stresses in the radial slot rotor are the tooth stresses in the main rotor body and the hoop stress in the coil retaining rings. Soft carbon steel is employed for the main rotor body, and a good quality ductile alloy steel (usually chrome-nickel or chrome-vanadium) is used for the coil retaining rings. Turbo generators are designed for a maximum speed 20 per cent above the running speed. At this over-speed, the tooth stresses should be approximately one-fourth the ultimate strength of the carbon steel and the coil-retaining ring stress should be approximately one-third the ultimate strength. This results in working stresses, in both cases, approximately half the yield point. It is important that the material be ductile; carbon steel with proper working, can readily be obtained with 22 per cent elongation and 35 per cent reduction in area and the alloy steel should have 22 per cent elongation and 50 per cent reduction. These figures refer to standard two-inch test pieces under tension.

American design practise has established 400 ft. per sec. as an upper limit of peripheral speed for maximum ampere turns and output for rotational speeds of 1500 rev. per min. and higher. This, of course, assumes existing rotor materials and factors of safety.

It is apparent that having increased the rotor diameter to the most effective value, output will be proportional to the length of the rotor and stator cores, and maximum output will be secured when the length is increased to its limiting value. This limit to length is even more a matter of opinion and judgment than is the limit to rotor or stator diameter. It is determined mainly by cooling air requirements, by bearing proportions, by limits to weight imposed by transportation facilities and the ability to secure forgings of necessary diameter and weight.

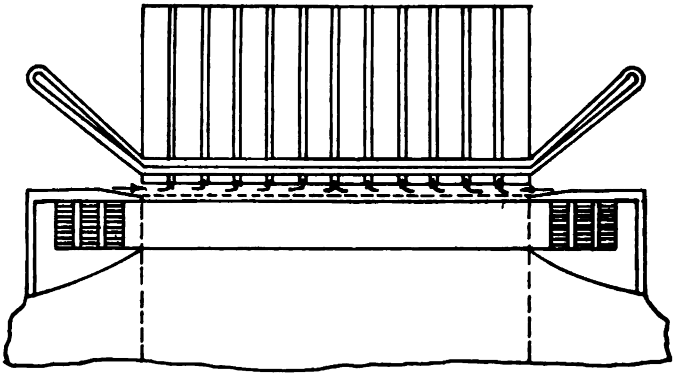


Fig. 2

Ventilation. The generator losses, and consequently the required volume of cooling air increase almost in proportion to the core length. In the simple radial or air gap system of ventilation, shown diagrammatically in Fig. 2, all of the cooling air must pass through the air gap entering the annular openings between stator and rotor at the two ends of the generator. The radial dimension of the airgap is constant with constant rotor diameter and consequently the volume of cooling air can only be increased as the core length is increased by increasing the air pressure.

Also, as core length is increased, the diameter of the shaft extension of the rotor body must be increased and the fan intake becomes restricted. In the axial system of ventilation, illustrated in Fig. 3, the stator ventilation is taken care of independently of the air gap and the requirements as to cooling air becomes less important from the standpoint of limiting output. But with either system of ventilation, designers are already finding it necessary to devise more complicated systems in order to take care of 3600 and 1800 rev. per min. ratings now in prospect.

Rotor Deflection. As the core length is increased, necessitating an increased distance between bearings, the rotor deflection increases. This increases the reversing stress in the shaft material at the journals and reduces the value of critical speed. As the core length is increased, the journal and bearing sizes must be increased in order to keep the shaft stress and the critical speed within desired limits and a limit may be imposed by bearing losses and temperatures.

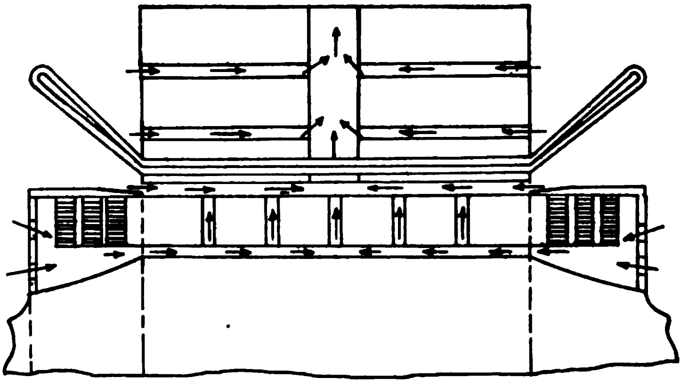


Fig. 3

Winding Temperatures. The limiting ratings given in this paper are based on 150 deg. total rotor winding temperature and from 125 deg. to 150 deg. total stator winding temperatures. It is not probable that ratings will be increased by increasing these temperature limits. Higher temperatures would be of most value in connection with the rotor winding since the rotor limits rating in the two- and four-pole designs commonly used. But temperatures higher than 150 deg. result in relatively little gain in rotor ampere-turns on account of the rapid increase in resistance of the winding. If the rotor winding temperature rise is assumed proportional to the loss, an increase in operating temperature from 150 deg. to 250 deg. (an increase in measured rise from 100 deg. to 200 deg.) results in an increase in ampere turns of only 25 per cent.* Thus doubling the temperature rise and rotor loss results in a gain of only 25 per cent in output.

Temperatures much above 150 deg. in connection with very long rotors are not considered favorably on account of the danger of trouble from "creeping" of the winding caused by linear expansion.

Transportation facilities may impose a limit to size in the case of six- and eight-pole 60-cycle generators. With the larger two-pole, 25-cycle and four-pole, 60-cycle generators now being built, the stators are now too heavy for convenient handling and transportation, and they are assembled in place at the power station. Rotors, from the special nature of their design and the special skill and equipment required for winding and assembling, should be completed at the builder's factory and shipped as a unit. The weight of the complete rotor of a four-pole, 1800-rev. per min. generator of 40,000 kv-a. capacity will be roughly 90,000 pounds. This can be transported without difficulty, but the largest possible 1200-rev. per min. rotor would weigh more than 200,000 pounds, and would require rolling stock and trackage (in some cases) not now available.

Another general limitation to output that applies to the larger diameter rotors is that imposed by the *forging facilities* of the country. At the present time it is not possible to obtain forgings of suitable physical characteristics weighing more than 50 to 60 tons nor much larger than 50 inches (assuming a minimum amount of working down from a 72-in. ingot). This limits the rotor, made from a *single* forging, to an output of roughly 50,000 kv-a. at 1500 rev. per min., and a proportionately decreasing kv-a. at lower speeds, assuming a solid rotor. By adopting the rotor construction involving two or three inch plates and up-set flanged shaft ends, the limiting diameter may be increased sufficiently for the largest 1500- and 1200-rev. per min. outputs, shown by Fig. 4. The design

---

*A discussion of this point is contained in the author's paper, "Rational Temperature Guarantees for large A-C. Generators", TRANSACTIONS for 1916, Part I, Page 1497.

of 1200- and 900-rev. per min. generators for maximum output will be governed by questions of forging and shipping facilities, rather than by more strictly design matters.

Fig. 4 shows in curve form limiting generator capacities at various speeds. At 1500 rev. per min. and higher, the capacity is determined by the rotor and is inversely proportional to the rev. per min. squared. At lower speeds, the capacity is limited by the stator and falls somewhat below the corresponding rotor limiting capacity as indicated by the dotted extension of the rotor curve. This curve is actually based on constant core length, when, as a matter of fact, the length can, with reason, be increased as the diameter is increased. The curve ratings, however, represent maximum lengths of core so far employed, and material extensions in core length involve questions of linear expansion that must be very carefully considered. This limiting capacity curve represents capacities that can be obtained with existing commercial materials and without radical changes in stresses and bearing proportions. The curve does not represent limits that may not be exceeded in the future. It is, more properly, an indication of present boundaries—boundaries that will be extended as our knowledge and experience are increased. It represents also the present judgment of designers, a judgment influenced greatly by the economic and operating advantages of still larger ratings.

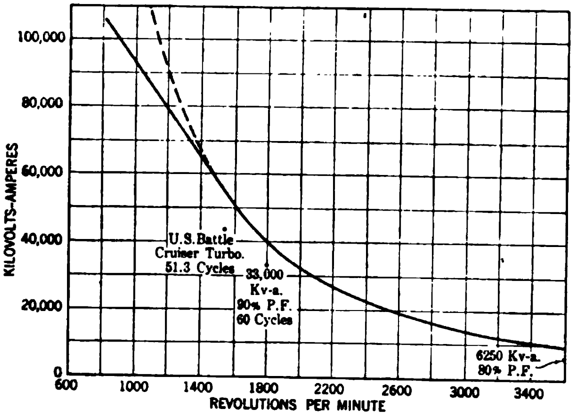


FIG. 4

The capacities shown by the curve are somewhat in advance of accomplished results. Ratings of several turbo generators that are the largest that have been placed in operation or are under construction by the Westinghouse Electric and Manufacturing Company have been added to Fig. 4 with self-explanatory comment.

As previously explained, the limiting capacities given for speeds below 1500 rev. per min. can only be attained by exceeding present transportation facilities if present design types are adhered to.

Incidentally it is interesting to note the advantage in limits gained by the use of 50 cycles as compared with 60 cycles. An increase of nearly 50 per cent in rating is made possible by the 20 per cent decrease in two-pole and four-pole speeds. This is of interest mainly when European and American maximum ratings are being compared.

The bare mention of ratings larger than 50,000 kv-a. raises the question of limits to size of individual generating units imposed by operating considerations, such as the relation between unit and station rating, the extent of the damage in case of winding failures, ability to withstand sudden short circuits and so on.

While the detailed discussion of these questions is beyond the scope of the present paper, some design information affecting operating questions may be of interest.

There is no reason for considering the larger low-speed generators less reliable than the high-speed generators indicated by limiting curve of Fig. 4. As a matter of fact, the lower speed ratings can usually be designed, both in stresses and in electrical factors, with more margin.

Mechanical forces developed by short circuits are determined by the short-circuit ampere turns of the armature winding per inch of armature circumference—and, to a limited extent, by the density of the magnetic field set up by the rotor winding. Both the distribution of ampere turns and the density of the airgap magnetic field are substantially constant for all limiting ratings of a given frequency. While the forces developed in a 25-cycle generator will be greater than in a 60-cycle generator—due mainly to lower reactance and the resulting greater values of ampere turns—all 60-cycle ratings indicated by the limiting curve will have substantially equal forces developed on sudden short circuit. The stresses in the coil ends will be determined by these forces and by those factors determining the rigidity of the winding. Except possibly in the maximum size 1500-rev. per min., 25-cycle generator, with its long coil-end extension, there need be no material difference in the rigidity of the winding. Thus a 50,000-kv-a., 25-cycle, 1500-rev. per min. generator represents the most difficult design from the standpoint of short-circuit stresses. However, such a generator would not differ materially in short-circuit stress conditions from 30,000-kv-a., two-pole generators that have been in successful operation for three years. It can be stated with confidence that the danger of winding failure due to sudden short circuit, with generators of the indicated limiting outputs, will be no greater than 20,000- and 30,000-kv-a. generators that have been placed in operation in large numbers during the past six years.

Another question of interest to those responsible for the operation of large generating units is the extent of damage to winding in case of internal short circuits caused by failure of insulation between turns of the same coil or failure of insulation from copper to ground. Experience with large units now in operation has shown that, in the event of a winding failure that results in the flow of abnormal power current, the chances are that the entire winding will be destroyed and that a hole of considerable size may be burned in the core laminations. Generating units are already of such size that a winding failure usually results in the loss of output from the unit for several months. The results of failures in still larger generators will be of the same degree and will be no more serious except, of course, in that loss in kv-a. output will be greater. In this connection, it is pertinent to point out that the fusing of metal and other local effects of an internal generator failure is a function of station capacity rather than of individual unit capacity. The volume of metal fused at the point of failure is determined largely by the impedance of the generating circuits feeding into the vault. Therefore, the only difference between a failure in one of two 30,000-kv-a. units and a failure in a single 60,000-kv-a. unit is in the impedance of the leads buses, and other connecting circuits between the two 30,000-kv-a. units. The two 30,000-kv-a. unit installation has the obvious advantages that protective reactance may be installed between the units, and the trouble is usually confined to half the station capacity.

The author has not intended to express an opinion as to the wisdom of installing very large single-shaft units. The only purpose has been to point out from the design standpoint the feasibility of certain ratings. Whether it is desirable or even wise to install very large units—above 50,000-kv-a.—will depend very largely on the growth and size of generating stations. When stations double in size—when stations of 300,000 kv-a. and 500,000 kv-a. become typical of American practise—there will undoubtedly be a demand of considerable volume for units of 50,000 kw., 75,000 kw., and 100,000 kw., and if single-shaft units are justified from the turbine standpoint, there is no question but that such generators can be conservatively designed and constructed.

DISCUSSION ON "PRESENT LIMITS OF SPEED AND POWER OF SINGLE SHAFT STEAM TURBINES" (JOHNSON), "PRESENT LIMITS OF SPEED AND POWER OF SINGLE-SHAFT CURTIS STEAM TURBINES" (BERG), AND "PRESENT LIMITS OF SPEED AND OUTPUT OF SINGLE-SHAFT TURBO GENERATORS" (NEWBURY), NEW YORK, N. Y., NOVEMBER 14, 1919.

Philip Torchio: It was a little over twenty-four years ago that The New York Edison Company started the operation of the first steam turbine used in a central station in this country, not a stone's throw from this building, at the West 39th Street steam station. A week ago the same company put in operation a 35,000-kw., 1500-revolution single-flow machine, which is the largest machine designed for that speed. The machine of twenty-four years ago was a 300 h. p. and the present a 50,000 h. p.

A few years after the first experiment on this turbine, built by the French De Laval Company, and operated for a few years with more or less success, the principal American electrical manufacturers started to build steam turbines. The first American built units were about 500 kw., but soon ascended to 5000 and 7500, and in 1906 units of 12,000 were built; in 1908, the largest was 14,000; 1912, 20,000; 1913, 30,000; and in the last year units up to 45,000 kw., 60 cycle, 1200 revolutions, were built.

I have not the statistics available, but probably two-thirds or three-quarters of the central station generating equipment today consists of turbo units. When we consider that the central stations furnish not more than one-sixth of the total power used in the country, aggregating considerably over 100,000,000 h. p., and when we further consider such facts as were brought out by Mr. W. S. Murray, that by central station power generation, of which 93 per cent by steam and 7 per cent by water, savings in the order of $150,000,000 a year and more can be made on the northeastern seaboard section of this country alone, we are forcibly impressed with the importance of steam turbine design.

B. A. Behrend: In 1906 I stated in *Cassier's Magazine* that: "The power station of the future will contain as many 20,000-kw. units as the power station of today contains 5000 kw. units." I venture now, twelve years later, to go on record that the power station of the future will *not* contain the 75,000 and 100,000 kw. units, of which we have heard a great deal in the papers presented this evening.

The course of evolution is never always in one direction. The time must come when the limit is reached, and the downward process must be commenced. I desire to discuss briefly the reasons for the limitations of size from a slightly different angle from that adopted by the speakers.

Beginning with mechanical stresses, as we encounter them in the electrical machinery or in rotating machinery like steam turbines, it must be emphasized that these are not static stresses, but essentially dynamic stresses.

Stresses in bridge members are both static stresses and dynamic stresses, according as the load of the bridge is a static load or a dynamic load. A live load, or a moving load, produces variable or dynamic stresses. In the rotating elements of the steam turbine, and of the electric generator connected thereto, the stresses are usually vibratory stresses changing both in magnitude and in sign. The shaft is bent in such a manner that at one period of the motion it is bent in one direction, and at another period it is bent in the opposite direction, and therefore the material and its molecular structure are subjected to alternating stresses. The same applies to other parts. The ancient rules of the mechanics as they are taught unfortunately even at this day do not apply to the problems we are facing. We must view the theory of elasticity from the point of view of vibratory stresses. Mr. Berg has told us that the disks in the steam turbines may have perfectly safe static stresses, and yet they may be entirely unsafe because of vibration set up in the disk. The maximum stress in a disk, subject to rapid rotation, exists in the center. In any ordinary test bar cut out of a plate, the test bar being shaped as test bars usually are, if a hole is made in it, the maximum stress at the inner surface of this hole is in general twice the ordinary stress which appears in the section. Suppose we call this ordinary stress 10,000 lb., the maximum stress as it appears on the inner surface will be approximately 20,000 lb., but this stress will be a serious stress only if the ductility of the material is small. If the ductility of the material is high, then the stress will be distributed, and the material will flow and rupture will not occur as a result of the initial excess stress caused by the hole in the test bar.

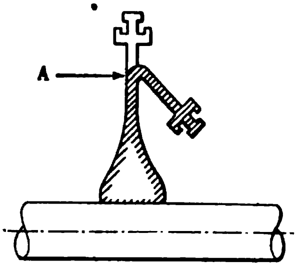


FIG. 1

If two test bars are made of equal and high ductility, one containing a small hole one-thirty-second inch diameter, and the other having no test hole at all, both bars will rupture the same load. The same is not true with bars made of hard steel lacking ductility.

I see little or no mention made of the fact that hard materials must be avoided. The ordinary tempered steel referred to for thin disks is very hard steel, and I doubt if such steel in the form of a disk could be bent flat on itself, into the position Fig. 1, without showing any seams or rupture whatsoever at the point *A*.

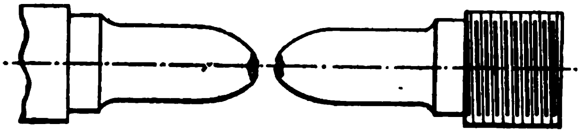
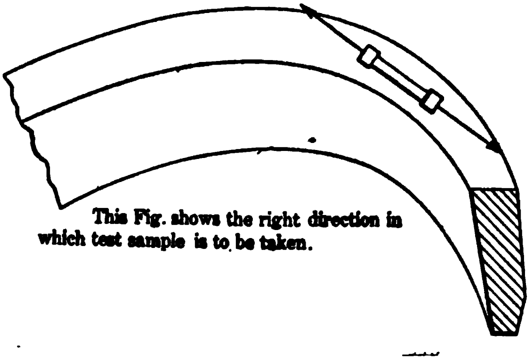


FIG. 2

The elastic limit is, after all, only one criterion of the quality of material. Material for these disks must be ductile and must bend flat on itself, cold, without rupture. If it does show rupture, you may be fairly secure in your assumption that that material is unsuited for the use to which it is put, namely, resistance to stresses reversing in magnitude and sign. All fractures with which I have been familiar have been due to vibratory stresses. I know of no failure of any part of a machine occurring under a pure static stress. There may be such failures, but they are unknown to me.



This Fig. shows the right direction in which test sample is to be taken.

FIG. 3

Reference to the limitation of capacity being due to the turbine only is, in my opinion, incorrect.

The rotating end bells are stressed to such a point that it became necessary to make them as thick as possible so as to reduce these stresses to a point where a satisfactory material can be obtained. The stresses run up to 30,000 lb. per sq. in. under normal conditions, and the ultimate stress of the material is approximately 100,000 lb., with a ductility measured by an elongation of 20 per cent and a reduction of area of 40 per cent in a 2 in. sample. The test bar in breaking must show a fracture like Fig. 2., and making these end bells is no small task, as they are subject to tangential stresses, so that the work of forging has to be done in such a manner that the material is homogeneous tangentially, Fig. 3. A test bar taken radially would show a very greatly reduced "strength" in the sense in which I am using that term, though the elastic limit might be the same, yet the ductility would be less, and therefore the "strength" would be less, so that you see that the term "strength" now means something different from what it meant thirty years ago, as it now includes ductility, whereas twenty years ago it meant nothing but the elastic limit, which is something quite different. These end bells, if thickened too much will make it impossible to put the rotor into the stationary element and therefore a limitation of construction lies in the length of the air gap.

These machines have not been in operation long enough for us to come to a definite conclusion as to the life of the structure. Fatigue may not become apparent in ten years, and yet later fractures may occur, due to fatigue. You may say, as Professor Scott likes to say, "You are pessimistic, as usual." I do not say this is going to happen. I say that vibratory stresses are dangerous, and that we have not had sufficient experience in regard to vibratory stresses to understand their effects thoroughly. It is to be hoped that the societies interested in testing materials will make tests with vibratory stresses, as our mills will not now accept specifications based on vibratory tests.

These remarks tend to show that there are limitations in the design of the electric generator and therefore the assertion made tonight that the limitation of units is entirely a question of the design of the steam turbine cannot be upheld.

To sum up my remarks, we are confronted with great engineering problems the ignoring of which will do us no good, but the bold and courageous facing of which, with all the knowledge that can be brought to bear upon the subject from every possible angle, will lead to success.

The course of evolution, as I said at the outset, cannot always be upward. The size of the machine, as far as cost is concerned, decreases to a certain point, and beyond that point it increases. Another point of importance is the increase in the armature current in machines of 50,000 kv-a. or over, which makes the problem of reducing the heating of the stator coils a difficult one. The loss which is troublesome to the designer is the loss produced in the stator coils as the result of eddy currents. That trouble increases with the increase in the cross sectional area of the coils. It may become necessary to increase the standard voltages now in use in order to obtain smaller conductors in which it is possible to reduce the eddy current losses to such an extent as to make the machines safe as regards coil heating.

W. L. R. Emmet: I have always thought it a strange condition that construction so radically different as the two types of turbines present in these papers should co-exist, but an analysis and examination of these papers, gives a good explanation of it. Both machines described show the evidence of highly scientific study, and the justification for either might be expected to be found in the results which they have produced.

These papers do not give direct comparisons. Mr. Berg's paper simply seeks to show the limitations imposed by rating up of machines. It gives a condition which corresponds to the highest economy obtainable with a machine of that type, and then shows the consequences of modifications of design or of increase of rating. Mr. Johnson's paper shows the general characteristics of limit of design, and gives a very good analysis of all the strains and conditions existing in the turbine, and a statement covering the whole field rather more particularly than Mr. Berg, who simply speaks of two specific turbines.

The inherent differences between the impulse and the reaction turbine might be stated as follows, as I understand them—of course, I understand the impulse turbine very thoroughly, whereas all of the limitations of the reaction turbine I do not understand, although I have studied it to some extent, generally speaking, in certain ways the less work you do in a single operation in a turbine, the more perfect the operation; that is, if you put a small amount of work into a stage of the impulse turbine, you can get rather more perfect nozzle action, and a rather better performance. The Parsons type turbine gives this condition, and in that respect is very good. It has, however, the disadvantage that it is more affected by leakage, and that the action of the stationary parts on account of leakage and other conditions, is presumably less definite, whereas in the impulse turbine, the leakage is very small, and the action of the stationary part, by suitable proportioning and using ample space, can be made very efficient. Thus by a good proportioning of the stationary part, quite high steam velocities may be justified, although in this type, also, the use of high steam velocities and the concentration of a large amount of work in a single stage, is not without its relative disadvantages.

Something has been said concerning these papers relating to limits of capacity, and I am rather inclined to agree with Mr. Behrend that there is no particular reason for building very large turbines; that is, there has been a good deal of demand for very large turbines, but the justification for them is not very apparent to me. With turbines of the type we build, such as described in Mr. Berg's paper, there is a distinct disadvantage in going to large sizes involving low speeds. On account of the great diameter of the wheels, there is a waste of space involved and practical difficulties in diaphragm construction and the physical structure of the stator becomes excessively large and is rather objectionable. The large double-unit machine which has been built by the Westinghouse Company in which the low-pressure element operates at a low speed is a splendid machine, but these machines are quite large and expensive, and there seems to be some question as to whether two entirely independent units, each one complete in itself, may not be about as good a solution of the problem, as far as the user is concerned. The effects of congestion in the low-pressure stages which is a thing which chiefly governs the rating and cost of turbines is a very important matter, but it is extremely hard to get at any numerical comparisons, and people must judge about the relative values from experience or from the guarantees which are offered, and by experience in use.

I can see a certain disadvantage in running very high bucket speeds in a machine of the reaction type, on account of concentration of a large amount of work in a single row of blades, discrepancies in the areas of successive blades, and such things as that in the low-pressure end, but I cannot say, quantitatively, what these limitations may be, so I really do not know very well what the economic limitations are of such turbines as some of those which have been described. In our machines, I do know very well what these limitations are, and Mr. Berg's paper has tried to make the whole matter perfectly clear.

If I have understood or observed the practise of Mr. Parsons himself, he has not gone in for very high bucket speeds in the reaction turbines. The construction now used by the Westinghouse Company from the mechanical standpoint seems to be conservative and good, but what its efficient limitations may be, I do not very well know. I cannot altogether agree with Mr. Behrend's generalizations on the subject of vibrating and static strains. If we could not calculate upon or predict the nature of vibrating strains, we could not build anything because almost everything is subject to motion. However, there are pretty definite laws which govern the question as to whether things do or do not vibrate. One of the essentials of sympathetic vibration is the rather perfect state of elasticity, and many structures which are used in motion, are very far from that perfection of elasticity, which will enable them to vibrate synchronously with any periodic force. As an example of that, I might refer to an armature mounted on a shaft—the shaft may be regarded as an elastic structure loaded; but in point of fact what it is is an elastic structure, the shaft upon which a very inelastic load of material is attached; that is the laminations are upon it, and these laminations make it relatively inelastic. In the same way these rings or end bells described here are loaded by masses of coils giving very high weight, in proportion to the elastic strain on the ring, and also this mass on it is, in itself, giving a condition particularly favorable to dead running, so I would really, without hesitation, put a strain on such rings to the highest possible degree, without any fear of their being broken by vibration.

As to this question of periodic motion, it is the cause in my opinion of the breaking of certain wheels in the General Electric turbines, and some of the possibilities of that situation were beautifully illustrated the other day in Schenectady. We made up out of rubber two or three wheels some rather thick with heavy edges, and some thinner with light edges. These edges were loaded with little staples hooked through them in a manner equivalent to the load imposed by turbine buckets on wheels, and the wheels marked out like a checkerboard. Holes were put in the wheels, they were then revolved on a shaft, and the shaft was made to actuate a high potential spark at one point of its revolution, so that the result of this, when running in the dark, was that this wheel apparently stood still, with an exact and perfect definition of its markings. The dry skin on the surface of the rubber was even visible upon close examination, and as the wheel loaded up, you could see the skin on the surface of the rubber crack, and draw into various forms, indicating the various distribution of strains around the holes, and the lines we drew on the wheels were distorted.

This gave us a beautiful illustration of the nature of centrifugal strains in such a wheel. That is what we did it for, and did not expect to show anything else. But in the case of the lighter wheel when we came to run it up to a given speed, we began to see something else, and that something was this—that the edge of the stationary wheel slowly began to work around like a snake, crawling slowly through the grass, that is, it would have three or four points in it that would go worming around, and as you speeded up the wheel, the snake would travel faster, but not in proportion to the increase in the speed of the wheel, showing the state of elastic distension in this thing had some tendency to equalize itself through a periodic action, which was not an accident.

With the stiffer wheels we had, although they were all very limber, made of rubber, and just a little bit thicker at the edge, we could not produce that effect. We could not run them at a speed so they would stay on the shaft; and at the same time be subject to this motion.

Generally speaking, all tendencies to vibration, and particularly all tendencies to injury through vibration, diminish very rapidly as the object is made stiffer, because the local strain produced by a certain amplitude of vibration is simply a question of the degree of elongation of the fibres on the surface. With small amplitude of motion, the degree of elongation is very minute, and consequently the strain imposed is very little. Movements of such character have unquestionably been the principal cause of the very serious troubles which have recently been experienced with wheels and buckets of General Electric turbines of the newer types.

The lightening of General Electric turbine wheels which has made such trouble possible has been incident to the use of a very light method of bucket arrangement which made the rim of much less weight, and made a great lightening of the web possible from considerations of centrifugal force alone. Lack of stiffness in the rim as well as in the web conduce to this tendency to motion. These rubber wheels showed that, because it was only with the light rim that such motion was produced.

Mr. Berg stated in his paper that the generators can be made up to the limits of the possibilities of the turbine. That applies, of course, to a turbine of a type which he speaks of, but if you made a double-flow turbine of the same speed, you would soon get beyond the capacity of the generator, and I personally do, not think that there is very much practical advantage in trying to build a turbine up to the capacity of the generator in that way. A double flow turbine of the same type could be made of twice the capacity, but it would not be any more efficient, and would be quite complicated and large, and you might find it necessary to have two generators, and probably would, and I think it is better to keep to a reasonable size, and have independent turbine units.

W. J. Foster: Mr. Newbury has very kindly asked me to tell you of the largest generators which have been built by the General Electric Company, and I will begin by telling you that about five years ago there were three 30,000-kw., 25-cycle generators put into operation. These were 25-cycle generators at 1500 rev. per min. Since that time we have put into service several 35,000-kw., 25-cycle generators, and one of approximately 39,000-kv-a. at 90 per cent power factor, now operating for one year.

In the 60-cycle line, we have several 6250-kv-a. 80 per cent power factor, 3600-rev., and one 7500-kv-a., 6000-kw., 80 per cent power factor, that has been in operation about one year, and we are building 3600-rev. per. min. generators up to 9375-kv-a. at 80 per cent power factor.

In the 1800 rev. per min. we have quite a number of 31,250-kv-a., 80 per cent power factor and some 33,333-kv-a. at 90 per cent power factor, and we are building 40,000-kv-a. at a power factor of about 85 per cent.

In the 1200 rev. per min. we have had a 50,000-kv-a. in operation about one year.

Mr. Newbury's curve, which I think is an excellent one, and is scientifically correct, is based apparently on 80 per cent power factor. He makes a statement which agrees with my experience, that the limit is reached in the rotor, and not in the stator, with these high-speed generators. In the curve Mr. Newbury draws, if we remove the power factor restriction, and put it on a unity power factor basis, the capacity can be raised about 20 per cent. In my experience, there is no trouble with such temperatures as Mr. Newbury mentions in at once going to 20 per cent higher kv-a. output on that curve for any given speed. In other words, it has been my experience to design the generators for lower temperatures. They are in the 105 deg. and not in the 150 deg. class, and that is also true in the case of practically all of the 60-cycle generators which have been built by the General Electric Company up to date.

The question as to how lower temperatures may be obtained in the stator is one which Mr. Behrend touched on. I hardly know why he considers the matter of eddy currents so formidable. It may be impossible to design a machine without some eddy currents, but there is no need of having them in anything like such quantities as to imperil the life of the machine or to cause excessively high temperatures.

There is a point with reference to the voltage—when it comes to the very large sizes—the further up we go in capacity, the higher the voltage must necessarily become. If we should build 90,000- or 100,000-kv-a. generators, it would be difficult to build them in less than the highest voltages at which we are now building generators. I think it is possible with 1200 rev. at the present moment, by new equipment, but not making such a radical change as we made six or eight years ago, when we stepped up to 30,000 kv-a., to build 90,000-kv-a. 1200-rev. machines.

In Mr. Newbury's paper reference is made to the ventilation, —axial and radial. I claim that the radial ventilation presents greater possibilities in furnishing larger surfaces for the cooling media to remove heat from the parts where it is generated, from the iron and from the windings. The coils are exposed at short intervals, and thus large areas of them exposed to the cooling air. In addition large surfaces of the core itself are exposed so that heat removal is far more effective than in anything I can conceive of as attainable in axial ventilation. The holes in the core that are put there for the air to pass axially, cannot be right in the teeth, to any great extent. They cannot be very close behind the teeth, except at a considerable loss in efficiency. The area of the walls of the holes must be small in comparison with the area which surrounds the numerous small sections in the radial ventilation.

I remember in one of our 31,250-kv-a., 11,000-volt machines, where a careful analysis was made of temperature drops from the inside of the coils, the temperature drop from inside the coil to the outer surface of the insulation or through the insulation, was between 30 and 31 deg. The drop from the coil surface through the short space of laminations was 4 deg., and from that surface to the air was 12 deg., even at the low rate of flow we had at that time.

A statement was made, I think, in the paper, that in the case of radial ventilation, the area of entrance at the two ends of the machine is limited. That is true. But we are discussing now what has been done, and what is possible, and I wish to point out that several years ago, the first General Electric machine embodying the multiple flow of air in a strictly radially ventilated core was installed. These machines have been tested out with the ventilation both ways—the air passing in simply at the two ends of the machine, and compared with air passing in with multiple paths, and the result has been a decidedly lower temperature with the air in multiple paths. That idea can be carried further, and so the possibilities of running at lower temperatures are many.

I do not see at all that it is necessary to have the stator as a limiting feature in the low-speed machines. If I were drawing a curve for Mr. Newbury, a practise curve, I should be inclined to put the heavy line on the other side of the dotted line, but I agree with him that in the lower speed machines, the chances for lower temperatures in the rotor are better, inasmuch as with the larger diameters it will be possible to ventilate the rotor to a certain extent which is not practicable in very small diameters.

Mr. Newbury makes a reference to periodicity, and he speaks of the advantage of 50 cycles over 60 cycles. I agree with him.

Alexander M. Gray: As to the theoretical limits of single-shaft turbo alternators discussed by Mr. Newbury, such alternators have been built with an output of 45,000 kv-a. at 1800 rev. per min. and one naturally wonders what is the maximum output that may be obtained at this speed.

The rotor diameter is not limited by the disk stress but rather by the hoop stress in the end connection retaining rings. In the case of a 51-in. machine rotating at 1800 rev. per min. the peripheral velocity at 24,000 ft. per min., the stress in a coil supporting ring of this diameter due entirely to its own weight is 15,000 lb. per sq. in., and this ring must also support the end connections.

The maximum rotor excitation. For a given stress in the rotor teeth there is a certain slot depth which gives the largest slot area. The number of ampere conductors that can be placed in this slot is limited by the internal temperature and a considerable portion of the temperature drop is through the rotor slot insulation. This drop can be kept reasonably low by the use of a large number of rotor slots, so that there is a large surface through which the heat may pass. It has been shown elsewhere that with a tooth stress of 14,000 lb. per sq. in. and 100 deg. cent. temperature rise above the entering air, the maximum excitation is about 1000 ampere turns per pole per inch of rotor diameter, or 51,000 ampere turns per pole, for a 51-in. machine.

The maximum stator excitation: In the stator m. m. f. with full-load current is equal to the maximum rotor excitation, then the demagnetizing effect is such that on low power factor the generated e. m. f. is practically zero, and the regulation of the machine exceedingly poor. Machines are now built with the armature ampere turns per pole as large as 0.6 (maximum rotor excitation). In the case of the 51-in. machine, this value is 30,000 ampere turns per pole.

The generated electromotive force can readily be determined if the air gap flux density is known. The flux density is limited by that in the stator teeth and if we assume the stator tooth density to be 100,000 lines per sq. in., and the ratio of tooth to slot equal to 1.1, reasonable value for 13,000-volt insulation, then with a peripheral velocity of 24,000 ft. per min. the voltage induced in the conductor is 1.88 volts per in. of iron, and 65 per cent of this value per inch of machine. This gives along with the figures for armature excitation a value of about 450 kv-a. per in. of iron.

The axial length of rotor. This is limited by the alternating bending stresses in the journal, it being assumed that the bearing holds the journal horizontally. With 19-in. water-cooled bearings the rubbing velocity is 9000 ft. per min., and with such bearings the length of a 51-in. rotor between bearing faces is 310 in., if the bending stress is limited to 5000 lb. per sq. in. This stress, be it noted, is alternating.

With a length between bearing faces of 310 in., the effective rotor length will be about 230 in., and the output about 70,000 kv-a.

Summary. The maximum output of an 1800-rev. per min. single-shaft, turbo-alternator, is about	70,000 kv-a.
The rotor diameter	51 in.
The peripheral velocity of rotor	24,000 ft. per min.
The temperature rise of rotor conductors	100 deg. cent.
The armature m. m. f. at full load equals 0.6 of the maximum rotor excitation.	
The flux density in stator teeth	100,000 lines per sq. in.
The ratio of tooth to slot	1.1
The bearing diameter	19 in.
The rubbing velocity of bearing	9,000 ft. per min.
The axial length of the rotor and stator conductors will be about	240 in.

I do not suggest that the machine such as the above should be built. There will be undoubtedly manufacturing troubles due to the length of coils, and also trouble due to the movement between conductor and insulation, because of the increase of length with increase of temperature. It is possible to reduce the length of the machine and the various factors in the above summary are open to attack.

R. B. Williamson: I desire to discuss Mr. Newbury's paper on "Present Limits of Speed and Output of Single-Shaft Turbo-Generators." The various elements that go to limit the possible output of a turbo generator operating at a given speed have been well brought out by Mr. Newbury in the present paper. In this type of generator the mechanical considerations are equal to if not of greater importance than the electrical features and as the two are more or less antagonistic the design as a whole must be a compromise. In the higher speed machines the maximum possible output is at present limited by the rotor while in large slow-speed generator the stator becomes the limiting factor.

So far as the rotor body is concerned this is made of carbon steel and as pointed out, the peripheral speed of 400 ft. per sec. is as high as it is advisable to run on the higher speed machines. With lower rotational speed the peripheral velocity might, if necessary, be allowed to go higher than this so far as tooth stresses are concerned, because of the lower angular velocity which is of the most importance in determining the centrifugal stresses. It should also be remembered that higher peripheral speeds result in greatly increased skin friction loss. If rotor bodies could be made of alloy steel, the stresses might be pushed higher and larger limiting output thereby secured but this kind of steel when used in large masses is very liable to develop cracks and at present is not reliable for this purpose. Attempts to use this steel for large shafts in other classes of machinery have shown this to be the case. For the coil supporting rings at the ends of the rotor it is necessary to use alloy steel but here the cross-section is comparatively small and the metal is very thoroughly worked during the process of forging and expanding the ring from a solid piece. Test bars from these end rings almost invariably show uniform physical qualities. As indicated in the paper, these end rings may have an influence on the maximum possible output. Referring to Figs. 2 and 3, of the paper it will be noted that the rotor coils project straight out beyond the core. Unless the wedge used for retaining the coils in the rotor slot is made very deep, which is undesirable because of the loss of valuable copper space in the rotor slots, it follows that the outer diameters of the end rings will be greater than the outside diameter of the rotor body as shown in Figs. 1 and 2. The inside diameter of the stator laminations must be slightly larger than the outer diameter of the end rings so that the rotor can be slid into the stator, thus the minimum air gap may be fixed by the end rings which thereby indirectly affect the ampere turns to be supplied by the rotor and the possible limiting output for a given speed.

Referring to the radial system of ventilation, Fig. 2, the amount of air that can be passed through a stator is limited if the air is passed through the air gap because the cross section of the gap is necessarily limited. However, it is entirely practicable to use the radial system by introducing the air at the back of the laminations between the latter and the stator yoke or casing and blowing it radially inwards against the rotor. The air then reverses its direction of flow and passed out radially through the adjacent parts of the stator core. By this means any quantity of air required can be handled readily by providing a suitable number of parallel paths, and the air inlets at the back of the laminations can be made of ample area to keep the air velocities within reasonable limits. This arrangement also has the advantage of introducing cool air into the center of the machine where the parts normally attain highest temperature.

This method of ventilation has worked out successfully in machines having a ratio of stator length to inside diameter of 3 to 1 and the results obtained indicate that much longer stators could be ventilated in this way and still maintain the central portions within the allowable temperature limits.

Referring to the curve shown in Fig. 4 the ouptuts indicated are, as stated by the author, somewhat in advance of anything so far accomplished. For example at 3600 rev. per min. a possible output of 10,000 kv-a. is shown. Machines have been in operation for some time that have carried loads as high as 7500-8000 kv-a. at 3600 rev. per min. in regular commercial service. Of course these generators have long rotors, but we are of the opinion that an ultimate output of 10,000 kv-a. at 3600 rev. per min. could be obtained as indicated by Mr. Newbury without going beyond the limits of present materials provided such output should be considered desirable for the steam end of the unit.

Francis Hodgkinson: The limit of capacity of a steam turbine is exceedingly hard to discuss because of its elasticity. Why should not one of the turbines which have been described have their diameters slightly increased, and be provided with slightly greater length of blades when the turbine would have a corresponding increase in its capacity. To what extent such things may be done, and what should be the limits of stress, is largely determined by the judgment of the designer. This applies to the calculated stresses of centrifugal force, etc., but in addition to these there are the obscure vibratory stresses to which Mr. Behrend refers.

It is difficult to draw definite lines to which materials may be stressed when combined with indefinite vibratory stresses and temperature strains, the result of the inner and outer parts of the revolving limits being at different temperatures on a change of load or operating conditions. To do so is, after all, to determine what risks shall be taken. The designer is sometimes in an unfortunate position, in that commercial pressure is brought upon him to increase speeds and dimensions beyond what he considers best judgment, which at times is hard to resist.

Turbines have increased in capacity of late years in the manner Mr. Torchio has recited, and as far as my association with them has been concerned, I have always felt that every increase of size should be accompanied by an increase in the degree of reliability.

Mr. Behrend, in his discussion, touched on some real truths. Of course, he referred principally to the type of turbine built by competitors of the company I represent, but I venture to say that any of the turbine disks he described would stand the destructive bending tests which Mr. Behrend pointed out as desirable. I believe they would pass such a bending test perfectly well, but such material, ductile and strong though it may be, would still be unable to resist continued vibratory stresses, should they produce strains slightly above the elastic limit in any fibers. I do not say that such vibrations necessarily exist, but should they do so and any fibers be stressed beyond the elastic limit, the end will be disastrous.

The views of engineers have undergone some change of late years in that the physical characteristics of reduction of area is considered one of the most important features, for it is this physical characteristic which, when structures are locally strained beyond the elastic limit, produces a strengthening of that part and permits an evening up of the stresses throughout the structure,

Mr. Emmet referred to a power station in which single units were employed as compared with cross-compound units. I have personally favored these cross-compound units for the reason of simplicity. A turbine, working with high pressures at one end, and high vacuum at the other, calls for skill in arranging the distribution of the metal of the turbine structure so that expansion and contraction shall be uniform and no distortion result. Any simplification of the structure is, therefore, an advantage. By cutting the steam cycle in two, and so similarly dividing the temperature range, there is a direct simplification. Many people regard the cross-compound principle as a redundancy, because there are twice as many bearings. Bearings do not give any trouble. That does not mean anything, but the simplicity of the turbine structure means a great deal. There is a further advantage that different speeds may be employed for each of the elements, selecting that speed which is most appropriate for the steam volumes in the high-and low-pressure elements respectively. For example; for 60-cycle work the high-pressure turbines may operate at 1800 rev. per min., the lows at 1200; similarly for 25-cycle service the high may operate at 1500 rev. per min., the low at 750, although in certain cases two low-pressure turbines have been employed, all three elements operating at 1500 rev. per min. By means of such construction high stresses may be avoided and ordinary commercial materials employed.

B. G. Fernald: In Mr. Johnson's paper the importance of the reliability factor in turbine design is strongly emphasized.

The calculated stresses in his turbine rotor when operating at 20 per cent over normal speed is given at 20,000 lb. per sq. in. or when operating at normal speed at 13,900 lb. per sq. in.

The elastic limit of the material used, as determined by test rings taken from the billet close to the point of maximum stress is from 22,000 to 25,000 lb. per sq. in.

The material used is ordinary carbon steel and is not subjected to any heat treatment other than thorough annealing.

In Mr. Berg's paper the calculated stresses at normal speed in the rotor wheels of his turbine design are given as 23,450 lb. per sq. in. in the hub and 22,950 lb. per sq. in. in the web and he uses a nickel steel disk quenched and tempered and claims an elastic limit of 55,000 lb. per sq. in. No information is given as to the method of obtaining the test specimens or their location with respect to the finished wheel.

Now, on the face of it the element of safety would seem to be greater with the heat treated alloy steel disks of the G. E. design than with the annealed disks of the Westinghouse design, since the margin between the calculated stress and the elastic limit of the material is greater.

The reverse, however, would seem to be the case since Mr. Berg's report mentions several failures of their design and Mr. Johnson reports no failure of the Westinghouse design. From my own knowledge, I would not pretend to offer a definite opinion on the cause of the accident to the disk wheels or to question the accuracy of Mr. Berg's diagnosis of the trouble, but I submit that his explanation does not take into consideration the fact that the failure of the disk occurred where the calculated stresses were lowest but where the unknown internal stresses would probably be highest if a disk of substantially the same form shown in Fig. 3 were quenched by immersion.

I wonder if any one has had such experience as will enable him to tell us that we can heat treat a disk and know when we get through that the disk is free from internal shrinkage stresses?

Recently the Emergency Fleet Corporation placed a contract for eight twin-screw troop ships, each screw being driven by a cross-compound single-reduction geared turbine of 6,000 s. h. p. or 12,000 s. h. p. total for the ship.

The element of reliability was, of course, of extreme importance on a troop ship and overshadowed all other design stresses. However, before the vessels were under construction and the final contracts for the turbine let, the war was over, and it was decided to convert the vessels into passenger steamships.

While reliability was still of the utmost importance and we wanted, if possible, to secure turbine units which would be free from the troubles which had been so prevalent on the smaller cargo vessels, it was necessary that the design take into consideration high economy as well as reliability. It was realized that the turbine designers task in satisfying the dual considerations of reliability and economy would be simplified by using material permitting high unit stresses.

Very little exact information about the character of alloy steel which we would receive from the mills was available to us, but, we did know from general experience during the war that the mills were not making extra good material and it was difficult and unreasonable to attempt to force them to produce the large quantity required and hold them to close limits of quality. Furthermore, our inspection facilities were inadequate to competently handle such an exacting task and our experience had not warranted us in dispensing with all inspection so it was finally decided to limit the designers of the turbines to the use of ordinary commercial carbon steel with no heat treatment other than thorough annealing and all calculated stresses had to be based on and kept well within the elastic limit of such material.

We realized that we were open to criticism for being ultra-conservative, but, we could not secure information warranting us in taking any other course and we were particularly uncertain about the possibility of successfully heat treating alloy steel disks for turbine wheels.

While the information would be too late for use in the connection just mentioned, I would ask Mr. Berg to give us some information about the heat treatment of the disks of his design and the test methods used for determining that the elastic limit mentioned existed in the disk itself. It is recognized, of course, that it is not difficult to obtain by heat treatment an elastic limit of 55,000 lb. in a small bar of uniform section such as is used for test pieces.

B. A. Behrend: As to the points brought up by Mr. Emmet and Mr. Hodgkinson, I may point out that vibratory stresses need not be stresses due to resonance. Vibratory stresses are all stresses which change in direction. If resonance occurs, there will be more rapid destruction in a shorter time interval. Where there is no resonance, the time period is lengthened out before fatigue occurs.

Comfort A. Adams: Mr. Behrend's remarks in regard to the fatigue of metals are very interesting. It may cheer him to know that a very elaborate fatigue phenomena research is now being conducted at the University of Illinois by Prof. H. F. Moore under the auspices of the Engineering Division of the National Research Council. More than $20,000 per year is to be expended for at least two years, of which $15,000 per year is contributed by the Engineering Foundation.

Just a word in regard to the electrical end of this problem. Mr. Newbury pointed out the objection to using high temperatures and the very slight gain in output due to allowing excessive temperatures of the rotor conductors owing to their increased resistance. Is it not possible that the longitudinal expansion at high temperatures and the consequent slipping of the conductors through the insulation would constitute another limitation to temperature for any given length, or a limitation of length at the higher temperatures.

Farley Osgood: I have heard nothing said from the operators' standpoint in this matter of turbine design, the discussion, which has consumed virtually the entire evening, having been confined to the designers' arguments. It is the custom to separate the sheep from the goats, and it seems well that now we do so, in that as the discussion up to date has been held by the sheep, whereas nothing has been heard from the goats, namely, the users, who are in the very large majority, and who are the victims of the enthusiasm of the designers, as we all know. It gives me some consolation to hear less talk of 50,000- and 70,000-kw. units, the authors having confined their discussion to units approximating a 30,000 kw. limit in design.

From the standpoint of the operators there are some very important features which do not seem to have been touched on specifically this evening, and they might very properly be divided into four classes: 1. continuity of service; 2. capacity of unit out of service during periods of repair; 3. investment cost of units out of service or for spare; 4. length of time to make necessary housekeeping repairs to the units.

As to the continuity of service, I believe the designers now have sufficient experience to guarantee a very high percentage of continuous service of a unit if they do not permit their enthusiasm to take them into sizes which lead them into more or less uncertain, and certainly untried, fields of strains in the materials.

As to capacity, I think it is a matter for the decision of the operator, rather than the designer, to elect the percentage of his whole capacity which he can afford to have out of service at the time of regular or emergency repair, and the operators' enthusiasm for improved efficiency from large units should not lead him to a decision which may affect his whole service, when his large units are off the line.

The cost of these large units, as compared with their smaller relatives, should be carefully considered, not so much from the standpoint of their value when running, which question they answer themselves by their performance, but from the standpoint of idle investment when the units have to be off the line.

Finally, and most important of all, is the length of time to care for these large machines, and it seems to be clearly indicated by the papers that such machines require special tools, specially trained men, and usually factory work, when anything but the most minor repairs are made, and this certainly means long delays from every standpoint, so that a machine which can be cared for locally, either by the operating company's men, or local machine shops, has many advantages over a machine requiring specialists and special factory attention, all this from the time-out standpoint. I have said before on this floor that the manufacturers give birth to the children, and the operators raise them and, as most of us know what it means to raise children and take them through the various vicissitudes of early life, I think we can say unhesitatingly that the operating group are much more familiar with the behavior of the units turned out by the designers, than are the designers themselves. I am firmly convinced that the designers do not give enough attention to the units in operation, that they do not spend enough time on the properties of their several customers, to become intimately familiar with facts which should materially assist them in turning out a product which will be more satisfactory to both the maker and the user, and I trust that even in the rush of business, which seems to be upon everybody, the necessity of closer cooperation between the designers and the users will be so appreciated as to bring such a thing about.

I very much regret that there is no time for a detailed discussion from the operators' standpoint, as I notice a number of able operators here, who could add most usefully to the discussion of this most important subject and, in fact, I was not aware that the entire evening was to be given up to the details of design, but I take the liberty of expressing a hope that the Chairman of the meeting at some time may have another meeting on this very important subject, at which the operators and the designers may be brought together and at that time the operators may be permitted to do the shooting instead of the designers, as has been the case this evening.

Eskil Berg: I believe that Mr. Emmet has pretty nearly covered all the questions that were asked. However I would like to point out that the wheel discussed in my paper is a very conservative wheel, the elastic limit being 55,000 lb. which is as low we ever found in actual test. The great majority of wheels test 65,000 lb. or higher. We have, as a matter of fact, many wheels which are 100 inches in diameter running successfully at 1800 rev. per min.

With this wheel given, the best possible design is described, which under the steam conditions stated, is shown to be most economical at about 21,000 kw. It is also shown that by reducing the number of wheels the light load efficiency can be improved by a slight sacrifice of economy at higher loads, in which case the best point would be at about 26,000 kw.

F. D. Newbury: Mr. Foster, in his discussion, advocates radial ventilation in preference to axial ventilation. Up to the present time neither system has reached its limit of development nor become a bar to further increase in rating. It has been necessary, however, in the case of the radial system to employ a four-path design, as described by Mr. Foster, for the larger generators while with the axial system the same conditions have been successfully met with a two-path design, that is, all of the cooling air is introduced from the two ends of the core. The four-path design can be employed with the axial system with no greater complication than in the radial, so that the axial system possesses greater possibilities than does the radial. This is, of course, due to the fact that in the two-path radial system the air-gap annulus is the only air entrance while in the two-path axial system this is supplemented by the parallel axial ducts.

As to the intrinsic merits of the two systems I cannot agree with Mr. Foster that the radial system possesses any material advantage over the axial. The Westinghouse Company has used both systems extensively and has, furthermore, built experimental generators of 5000 kv-a., 3600 rev. per min. and 10,000 kv-a., 1800 rev. per min. with the type of stator ventilation described by Mr. Williamson. These experimental generators were rebuilt one with radial ventilation and one with axial ventilation so that directly comparable results were obtained. Our practise, therefore, is based on practical experience with all three systems of ventilation in use in this country and we can say that each of the three has its advantages and its defects. We prefer the axial system in those cases where the simple two-path radial system cannot be used. Contrary to Mr. Foster's opinion it is possible to obtain ample core cooling surface with axial ducts and the fact that the axial ducts are not actually in the tooth belt is compensated for by the fact that the heat flow is in the direction of the plane of the laminations instead of across the laminations, thereby making use of a conducting path having as much as ten times the heat—conductivity of the transverse path.

Mr. Foster questions the importance of armature coil eddy current losses in these large generators. This is not a serious problem in 25-cycle units but in 60-cycle generators eddy-current losses become an important factor limiting generator ratings. In 30,000 kv-a. 1800 rev. per min. 13,000-volt units designers are now using such large slot sizes—in some cases 6½ and 7 in. deep—that the attempt to increase the stator rating, or to reduce copper temperatures, by increasing the slot depth increases the total copper loss, on account of increase in eddies, in spite of the decrease in I^2R loss and increase in coil surface.

I wish to take issue with Mr. Foster's statement that these large 60-cycle generators of 30,000 and 40,000 kv- a. capacity can be designed with actual copper temperatures within 105 deg. total temperature based on 40 deg. air. This is too large a question to discuss adequately here but it is a very important one and I hope papers on this subject may be presented before the Institute in the near future.

Professor Gray gives certain figures representing limits for 1800 rev. per min. generators. These limiting figures are well in accord with practise except in one important particular. The permissible length between bearings is given as 310 in. based on the reversing stress in the neck of the journal. Critical speed, rather than stress directly, is the most important factor determining the maximum rotor length; with the proportions reached by Professor Gray the first critical speed is about two-thirds of the running speed (in coming up to speed the rotor must pass through its critical speed) and the second critical speed is only a little above the running speed. These are not conditions that would be considered permissible in a large and important unit. The largest 1800 rev. per min. rotors so far built are only half the length arrived at by Professor Gray and the prospect of any considerable increase in core length in one step must be approached with circumspection.

Presented at the 356th Meeting of the American Institute of Electrical Engineers, New York, December 12, 1919.

APPLICABILITY OF AUTOMATIC SWITCHING TO ALL CLASSES OF TELEPHONE SERVICE

BY ARTHUR BESSEY SMITH

Abstract of Paper

This paper is an attempt to place before engineers general information in regard to automatic telephone switching.

The subscriber's requirements are independent of the means used to satisfy them. Automatic switching is uniformly fast and involves reduced mental stress to the user because the passing of the number is positive and waiting time is reduced to the minimum. Viewed by the owner the apparatus has longer life, the service is very acceptable to the public, and but a tithe of the female employees are used.

Automatic switching apparatus has increased greatly in margin of safety. Much progress has been made toward standardization of form and toward best methods of maintenance. A few changes in structure are described and data given to show margins of safety in operation. Present practise regarding party lines and measured service (cash and credit) is stated briefly. The durability of automatic equipment is illustrated by the fact that plants have not yet worn out. Maintenance routines are essential to successful operation. Girls do routing testing with marked success.

Rural telephone lines present problems which have been solved in several ways, influenced by the greater number of telephones per line, the conditions of signalling subscribers, and the inferior insulation often encountered. Code ringing can be retained. Rural automatic service is only a little inferior to city automatic service.

The community automatic exchange serves a small group of subscribers, either isolated or part of a telephone network. The rotary line switch is used because it is simple, reliable, quick-acting, and provides 25 trunks. Eight variable factors in exchange design are presented with a discussion of each.

Toll switching in an automatic exchange gives the toll operator direct dialing to the subscriber, complete control over his line, and periodic ringing. The toll network has also been improved by applying automatic switches to intermediate points, so that the originating toll operator can set up the complete connection herself. The experience of the past twelve years shows that this increases the business-carrying capacity of toll lines at least 50 per cent to 100 per cent. A variety of schemes are available.

The automatic switching of telephone lines is adaptable to all classes of telephone service and offers a flexible means of solving problems.

ENGINEERS and others interested in the building of telephone plants often ask the question, "Will the automatic telephone system meet the needs of this particular case?

Though it may be successful elsewhere, perhaps it can not be adapted to suit the peculiar conditions found here?" To answer such questions as this is the motive underlying the preparation of this paper. Though the writer can not cover every conceivable condition, he believes that it is possible to place on record many facts which will be useful to those who have to do with automatic telephony.

The approach to automatic telephony must be made with an open mind. A highly critical attitude prevents one from appreciating such facts as are observed. The writer formerly regarded automatic switching as desirable but somewhat inflexible. His experience and observation have led him to revise his opinion. Automatic switching can be adapted to meet every need of public and private telephone service, and will do it with marked advantages.

Three kinds of service have already been discussed by the Institute. In 1908, Mr. W. Lee Campbell presented a paper entitled "A Study of Multi-Office Automatic Switchboard Telephone Systems," and again in 1910 a paper on "A Modern Automatic Telephone Apparatus." In 1912 Mr. Gerald Deakin presented "Private Automatic Exchanges in Apartment Houses." In 1910 the writer discussed "The Automatic Telephone in Relation to City Service," including suburban traffic.

The classes of telephone service which are to be treated in this paper are as follows:

The single office exchange, the basis of discussion.
Rural lines.
The Community exchange.
Toll or county line network.
Long distance toll lines.
The comprehensive system, including all kinds of service.

GENERAL CONSIDERATIONS

Before taking up the details of various classes of service, it is well to call attention to a few general factors, such as the requirements of subscribers and of the owners of telephone exchanges and of the characteristics of automatic equipment most generally used.

Subscriber's Requirements. The subscriber's requirements must be stated in terms which are independent of apparatus or methods. Much error will be avoided if we divest our minds of conditions imposed by any one means of rendering service.

The subscriber requires, without unnecessary delay, without undue stress on himself, to be connected to the telephone which he desires (and to no other), to talk to the called subscriber with ease and without interruption or eavesdropping, and to have this service available continuously. Stated concisely the requirements are

1. Speed of connection (and disconnection)
2. Ease of obtaining connection (and disconnection).
3. Accuracy of connection.
4. Voice transmission.
5. Secrecy.
6. Continuous service.

Repeated tests of a formal nature and the general experience of users of both manual and automatic equipment have settled conclusively that the latter is very much superior to the former in speed of connection. But uniformity has value as well as actual speed itself. The average automatic connection is completed in from 4 to 6 seconds, regardless of the time of day or the conditions of business. Subscribers judge the speed of a service somewhat by the average speed, but more especially by the occasions on which they suffer delay. The fact that automatic service is *uniformly* fast, greatly increases its value to the user.

The speed of connection is greatly influenced by previous conditions. If a connection exists, and one of the two subscribers desires immediately to make another call, it can be done most quickly with the automatic telephone. Hanging the receiver upon the hook for one second clears the line and permits the immediate originating of another call. A large amount of telephone business may be transacted in a short time by automatic equipment.

The stress laid upon the subscriber in originating a telephone call is mental rather than physical: It may be divided into two causes, the waiting periods and the transmission of the number of the called telephone.

With the automatic telephone, waiting is reduced to a very low factor. Things begin to happen the instant the receiver is removed from the hook. The subscriber has something to do from the very start, and when he gets through doing it, the connection has been completed and the bell of the called telephone is beginning to ring. The only waiting period is the time of ringing. Even during this time the subscriber's mind is more or less occupied by the sound of the ringing, which he can hear. Or if the called line is busy, he is immediately informed of that fact by the unmistakable "busy tone." Not only does the automatic telephone greatly reduce the waiting time, but to a certain extent it conceals the total time required.

The transmission of a directory number by the dial of the calling device produces less strain on the user than the effective transmission of the same number by the voice. It requires a distinct effort to enunciate the syllables of the various digits clearly enough to prevent mistakes. On the contrary, the sending of the number by a calling device is easy and involves no appreciable stress. For each digit, the finger is inserted in the proper hole, the dial rotated to the finger stop and let go free. The mechanical definiteness of number transmission makes automatic calling easy.

When a subscriber desires to talk to a certain telephone, he wants that telephone and no other. It is a distinct advantage to him to know that he is getting exactly that number. This is afforded by automatic switches to a much greater degree than by human beings. Automatic switches, as now made, have a large factor of safety against error. They are designed and constructed so as to be as permanent in structure as modern materials will permit. Each part is separately adjusted within limits which impose rigorous conditions. The final assembly is again tested under conditions far more severe than those found in service. The chance of getting a wrong number by fault of the apparatus is so small that it can be entirely neglected. This fact is soon discovered by habitual users.

There is nothing inherent to automatic telephone equipment to prevent the very best voice transmission.

Much has been said for and against the value of the secrecy of the automatic telephone. It still remains true that users of the system recognize and value this property. It has been said that the operator at a manual switchboard is too busy to listen to conversations—this is true only during peak loads. Some manual switchboards have "automatic listening", which excludes the operator as soon as her work is done—this prevents her from giving the connection any supervision, and defeats part of the fancied advantage of having a human intermediary. It has been said that the mechanicians employed in automatic switchrooms constitute as much menace to secrecy as the manual operator. This is not so because the man's work is laid out according to a fixed routine. He is not permitted to run about the room according to the dictates of his own judgment. What he shall do, when and how he shall do it are laid out on a schedule. His day is uniformly filled, without peak loads. Secrecy is even more evident in the private automatic exchange, where the personal interest between people is stronger and the danger of eavesdropping greater. These switchboards are without constant attendance; they run for long periods without adjustment or repair.

Absolutely continuous telephone service has come to be the standard. Continuous service is given. But it is well known that manual night service is not as prompt or as good as day service. Automatic switches work just as promptly and accurately at midnight as at noon. The public deserve as good night service as day service, for often the need of a single night call may be more acute than that of many day calls.

Requirements of Operating Company. The requirements of the operating company are determined by viewing the telephone exchange as an investment. It must yield an adequate income to the stockholders. It must be a permanent business. To be stable means that it must render satisfactory service to the subscriber and not be too difficult to manage. These requirements are not all that there are, but they have an important bearing on the business.

1. Earning ability of investment.
2. Permanence of investment.
3. Satisfactory service to subscriber.
4. Ease of management.

Telephone exchanges must pay their way. If automatic exchanges do not pay, there would not be any of them left in existence today, for this movement is twenty-seven years old.

Five exchanges, formerly manual, were converted to automatic. They ranged in size from 2300 lines and 4400 telephones to 38,000 lines and 44,000 telephones. The totals for the five were 47,000 lines and 65,000 telephones. The comparison of expenses is as follows:

	MANUAL	AUTOMATIC
Total operating and maintenance expense per year	$552,600.00	$136,800.00
Average per line per year	11.77	2.92
Average per telephone per year	8.50	2.11

At least ten of the exchanges in the United States are from fourteen to sixteen years old. Based upon the record of the existing plants and the improvements in use today, new exchanges are built every year.

The permanence of an investment depends upon the rapidity with which the equipment is destroyed, the danger of its being rendered obsolete by reason of advances in the art, and the attitude of that court of last appeal, public support.

An apparatus to be a permanent investment as to depreciation, must be capable of being kept in first class running order by ordinary maintenance, without stretching the meaning of the word.

Surely any equipment which in ten years becomes so badly worn that it can not give adequate service, or that the repairs are considered excessive, so that it must be thrown out bodily, can not be considered as being very permanent.

The life of an automatic switchboard is not known. Numerous switchboards now in operation were installed more than 15 years ago, have been in constant service since then, and are still giving the same grade of service. All the moving parts are subject to no other forces than those of the switch itself. Those forces are adequate and are always properly directed every time they act. Those parts which wear are capable of easy renewal and are taken care of by ordinary maintenance.

Although the structure of automatic telephone apparatus has been changing through a considerable period of years, most of the changes are beyond the observance of the subscriber. The oldest equipment gives nearly as quick and satisfactory service as the latest. The telephone instrument has been improved in appearance, but the old ones are still giving a service comparable to that rendered by the new. For this reason, there is little obsolescence to automatic equipment. This being true of old apparatus, it is much more true of the latest apparatus on the market.

Public support is a factor which must receive adequate consideration. Though in the past some have ignored it, the results have been anything but pleasant and have reacted unfavorably upon those who did the wrong. Any business to be classed as permanent, must have with it public support, and the more of it the better.

The public recognizes and supports the company which gives its patrons the benefits of improved service obtained from new devices. They may not clamor for something which is rather hazy or of which they do not know. But let them once experience the benefits due to the new device and their support is assured.

It is a matter of record that the great mass of the public who have habitually used both services, prefer automatic to manual. It is not a doubtful majority—the preference is as nearly unanimous as can be found. In 1911 the Automatic Electric Company sent out 3000 letters to subscribers who had been using both manual and automatic service, and 1869 replies were received of which 91.7 per cent unqualifiedly preferred automatic service. Since then others have investigated the subject with varying results, but always with the ratio in the same direction and very positive.

Other things being equal, people prefer an investment which is easy to manage. One which presents troublesome questions which seem never to come to an end is shunned, because one can never tell when something may upset the financial condition. Any factor which makes the business easier to manage, renders the investment more attractive.

The manual telephone operator is the source of exceedingly vexatious problems. She requires careful handling, adequate accessory conditions, and although her pay is rising from year to year she does not stay long in the business. At best, hers is a nerve trying occupation, and it is no wonder that conditions are as they are. They are inherent.

The automatic telephone switchboard removes about 90 per cent of the female help in central offices. The increase in male help is only about 9 per cent. The duties of the former human operators are performed by machines which require no "managing."

EQUIPMENT CHARACTERISTICS

Automatic apparatus of various kinds has been described so often that a general explanation is unnecessary. The type best known has been the subject of Institute papers, as well as of a few books.

Factor of safety is a thing of great importance. Upon it depend the maintenance expense and the dependability of the service rendered to the public. This is the principle which guided the development of automatic switching. Seemingly small details exert great influence upon the performance of apparatus. Some factors do not impress the casual observer, yet they have great influence upon results.

A brief presentation of some of the improvements which have been made in recent years will bring the record of the subject up to date. It concerns chiefly the redesign of parts of the apparatus and the standardization of the assembly, adjustment, and mode of maintenance.

The automatic switch, Fig. 1, presents a new general appearance. The relays are mounted above, the magnet mechanism in the middle, and the banks and wipers below. The relays and magnets are protected by a cover individual to each switch.

The material of the switch frame has been changed from die-cast metal to cast iron, heavily zinc plated. The die-cast frame was subject to warping. It necessitated rather frequent re-adjustment of the mechanisms mounted on it and sometimes even the chipping down of a lug which interfered with the movement of the wiper-shaft. The cast iron frame is rigid and unchanging. Apparatus mounted on it requires no attention from this cause.

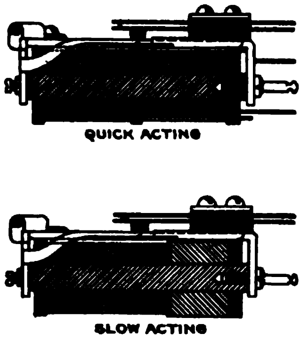


FIG. 2—HORIZONTAL AUTOMATIC RELAYS

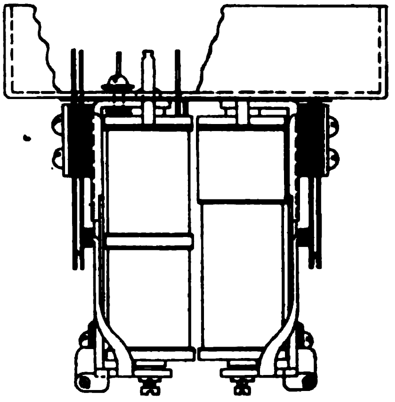


FIG. 3—AUTOMATIC RELAY MOUNTING (TOP VIEW)

The sideswitch has been abolished on nearly all switches and its work performed by relays. The sideswitch as made was the source of considerable trouble due chiefly to the wiper friction. Its elimination has greatly increased the certainty of action of the switch as a whole.

The relay has been completely redesigned (Fig. 2). The pin-pivot has replaced the point-pivot, the insulation between contact springs made permanent, and each relay rendered mechanically independent of all other relays.

FIG. 1A—CONNECTOR SWITCH (FRONT VIEW)

[SMITH]

FIG. 1B—CONNECTOR SWITCH (RIGHT VIEW)

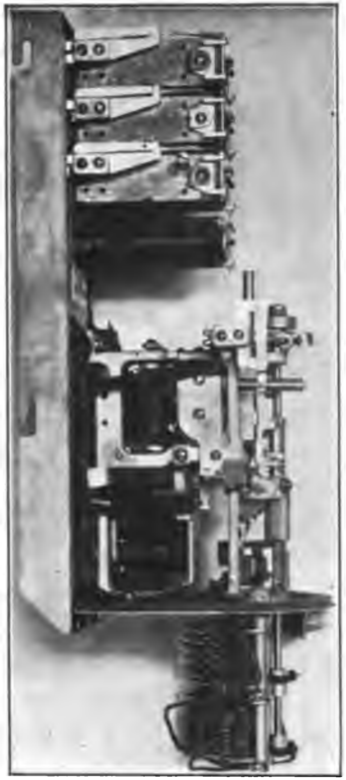


FIG. 1C—CONNECTOR SWITCH (LEFT VIEW)

FIG. 1D—CONNECTOR SWITCH (REAR VIEW UNCOVERED)

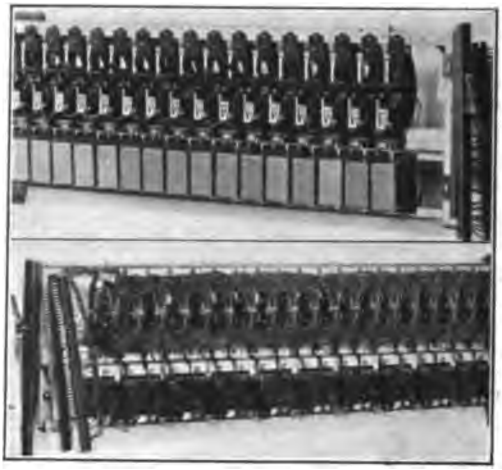
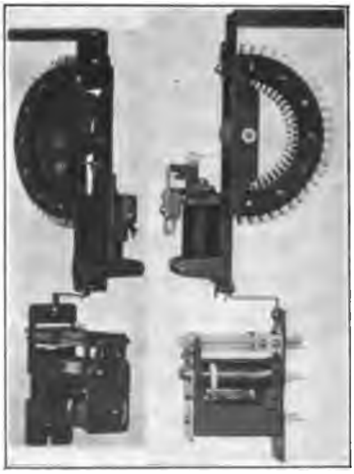


FIG. 7—ROTARY LINE-SWITCHES ON SHELF (FRONT AND REAR)



[SMITH]

FIG. 8—ROTARY LINE-SWITCH (LEFT AND RIGHT VIEWS)

The point-pivot, when adjusted without play or bind was as fine a pivot as could be made. But it was desired to have a pivot which would need no adjusting at all. The plain pin pivot has just enough play to make the armature work freely, and the wear is inappreciable. The point pivot is retained on the line relay of the plunger line switch, where the conditions are not severe.

The relay contact springs are insulated by phenol-fibre, which is a good insulator and very constant. Each assembly of springs is aged by heat treatment, so that it is practically one solid mass. Looseness and uncertainty of spring action have been practically wiped out.

The relays are mounted individually on a base plate Fig. 3. This base plate has openings through which the relay terminals (for coils and springs) extend to the rear. There the wiring is placed, connecting the relays together in a compact but accessible place. A plate covers the wiring so that none of the wiring on this part of the switch is exposed. The few wires which feed the magnets and wipers pass through a single hole in the base plate. The base plate carries also the switch frame, in which the vertical, rotary, and release magnets are mounted.

The adjustment of an automatic switch may be divided into two parts, the relay adjustment and the adjustment of the motor magnets. Every relay is adjusted to a definite armature-stroke with a fixed residual airgap. The amount of contact that each spring makes with its mate is likewise fixed. These distances are measured with thickness gauges placed between the armature and the pole of the magnet.

The contact springs of each relay are given a tension which is measured in terms of the operating and non-operating currents. Thus the spring tension is kept within limits.

The motor magnets (vertical, rotary, and release) are adjusted in a definite sequence to the proper relations to the wiper-shaft. These relations are also expressed in distances, most of them measured by thickness gages.

The final performance of the completed switch is tested by being operated over a line under conditions worse than any imposed by commercial use. The calling device delivers impulses at 14 cycles per second with an impulse ratio of 61 per cent (circuit opened 0.61 of total impulse period.) One test employs a line of zero loop resistance with a 20,000 ohm leak across it. The other test employs a 1200 ohm line without leakage.

The spring tension of the motor magnet armature is adjusted so that perfect operation is secured under these two extremes of line conditions, with the battery voltage between 46 and 49 volts.

The control of automatic switches over the subscriber line is a telegraph problem. The elements involved Fig. 4 are the line circuit, the release relay circuit, the motor magnet circuit and the release magnet circuit. All but the line circuit are wholly within the central office where the conditions are constant. Each element contributes its share toward fixing the line characteristics which are necessary for operating a switch.

The normal operation is as follows: The calling device springs (CD) are opened and closed as many times as there are units in the digit being transmitted. The line relay (LR) follows these impulses. The release relay, (Rel-Ry) having a copper collar remains energized during the series. The series relay pulls up the first time the line relay falls back and remains energized during the series of pulsations to which the magnet (VM) responds by moving the wiper shaft the required number of steps. At the end of the group of impulses, the line relay comes to rest energized, the series relay falls back and causes the local circuit to be changed to suit the next action.

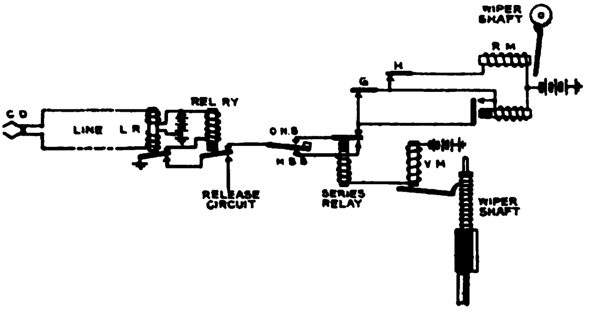


FIG. 4—CONTROLLING CIRCUIT FOR AUTOMATIC SWITCHBOARD

The period of an impulse Fig. 5 is the time from the opening of the line to the next opening of the line. It includes a "break" and a "make." The upper line represents the time that the line current is flowing. It is shown as broken three times, as in sending the digit "3." Each opening of the line produces one pulsation to the magnet. This is represented by a line just below the break in the line current. Owing to the delay of the line relay in obeying the current, there is a little time-lag at the beginning and at the end of each pulsation.

The impulse ratio is the ratio of the time that the line is opened to the total period of the impulse. Applied to the magnet, it is the ratio of the time of current flow to the total period of the impulse. Formerly a fifty per cent (0.5) impulse was used. It has been found that a longer opening of the line circuit gives greater certainty to the selection, so that sixty-one per cent (0.61) is now used. The reasons are discussed below.

The release relay must be held energized continuously, because when it falls back it will cause the switch to release. The greatest break in current which the release relay can stand without letting go is about 77 per cent. (For a group of 10 impulses at 10 per second.) At the beginning of a group of impulses, the release relay is fully energized. While the line relay armature is vibrating, the release relay is kept energized by pulsations fed to it, one pulsation each time the line relay pulls up. If these pulsations are short, the magnetism will not be fully restored each time but will gradually run down. There is always enough magnetism remaining to hold the armature to the end of ten impulses. The aging of a slow acting relay (equipped with a copper collar) always tends to make it hold better.

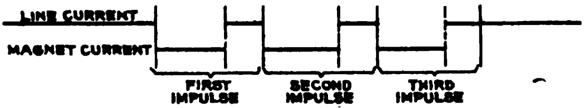


FIG. 5—IMPULSES

The series relay must also remain energized as long as pulsations are being sent to the motor magnet. The shortest pulsation which it can have is about 28 per cent for ten pulsations at 10 per second. The conditions which supply long pulsations to the series relay, give short ones to the release relay. But the short pulsations which each can endure give a large range through which the ratio can vary without causing either relay to cease to function properly.

The vertical magnet requires at least a 27 per cent pulsation and not more than 86 per cent for satisfactory operation under the same conditions. Being in series with the series relay, it gets the same impulse ratio. Since the magnet has considerable work to perform, it can stand a very high ratio.

The rotary magnet of a connector can stand a variation of impulse ratio from 30 per cent to 95 per cent.

The total performance of the automatic switch is judged from the standpoint of the subscriber's line. Ordinarily a switch is adjusted to work over lines of varying length. This is because any telephone may use almost any switch. Sometimes a switch is to be used on one line or trunk only; in this case the line relay can be adjusted to operate on a 2500- or 3000-ohm loop. When adjusted for such a loop it would not work well on a low resistance loop (300 ohms and under). Loop resistance of the subscriber's line does not include the telephone instrument because in an automatic telephone the talking apparatus is cut out during the dialing period.

If the subscriber line is of zero length with no leakance or capacitance, the impulse ratio may vary from 38 per cent to 71 per cent without causing the switch to fail.

The effect of simple line resistance is to lengthen the pulsation delivered to the motor magnet, and shorten the pulsation to the release relay. This results in weakening the release relay so that it may start to fall back near the end of a ten impulse digit and if the line resistance be excessive may even cause premature release. If the loop is 500 ohms, the impulse ratio may vary from 36 per cent to 72 per cent without causing the switch to fail. If the loop is 1000 ohms, the limits are 35 per cent and 68 per cent. The margin for safety is ample.

The practical safe limit for a subscriber's line is a loop resistance of 1000 ohms. This is a greater resistance than is necessary for good common battery transmission, so that any loop which is low enough in resistance for good transmission is good enough for automatic dialing.

The effect of line leakage is to cause the line relay to be slow to fall away and prompt to pull up. This gives the motor magnet and its series relay a shorter pulsation, so that there is less power available to move the switch and a greater likelihood of the series relay not remaining energized throughout the group of pulsations. The practical limit of leakance is about 50,000 ohms for the subscriber's line, measured between wires or from either wire to earth. Since the switches are adjusted to operate perfectly with a leak of 20,000 ohms across the line, the factor of safety is ample.

Distributed capacitance such as found in telephone cable exerts no bad effect upon dialing. A capacitance between the two conductors has a tendency to delay the relay in falling back, and thus to shorten the pulsation delivered to the motor magnet. This partly neutralizes the effect of line resistance. In fact, the capacitance may be increased to seven microfarads without causing the automatic switch to fail.

Lumped capacitance, as found in telephone instruments on a party line, affects the dialing. It is more correct to say that it is the inductance of the bell in connection with the capacitance of the condenser that influences the signals. For this reason the capacitance of condensers used with harmonic bells (5 bells on one party line) is made 0.7 microfarad. With this value at least two more bells could be used without causing any failure of dialing.

In general, line faults have the same effect upon an automatic switchboard that they have upon a manual switchboard. The chief effects are compared in the table following.

	Manual	*Automatic*
Open.	No calls, no indication	No calls, no indication
$+L$ grd.	Line noisy, conversation poor	Line noisy, conversation poor Dialing bad, sometimes impossible
$-L$ grd.	Permanent signal	Permanent signal
sht. ckt.	Permanent signal	Permanent signal
	Crosses between lines.	
$+L$ and $+L$	Cross talk	Cross talk, a second trunk occupied but no trouble
$-L$ and $-L$	Cross talk, two line signals show when one calls	Cross talk, a second trunk is occupied when one calls
$+L$ and $-L$	Permanent signal on one line	Permanent signal on one line

A rotary line switch has been produced which beside giving access to 25 trunks possesses other advantages. It was the equipment furnished by the Automatic Electric Company for the Automatic Telephone Exchange at Orleans, France. Its chief use in America is in the Community Automatic Exchange. It will be described in this paper under that subject.

Periodic ringing is the accepted practise, on both individual and party lines. If the subscriber removes the receiver from the hook, the ringing current is cut off at once, even if it occurs during a time of ringing. The cut-off is quick enough to be complete before the receiver reaches the ear.

Party lines are segregated and served by party line boards, one hundred lines per board.

The preferred practise is to provide a party line board with four or five groups of connectors, each group being supplied with a different frequency of ringing current. Each subscriber line is multipled to all these groups of connectors. The hundreds digit of the call number selects the group of connectors and therefore fixes the frequency which shall be used. The last two digits control the vertical and rotary motions of the chosen connector as usual. Any group of connectors can equally well connect to the desired line, but the bell which rings depends upon which group of connectors is used.

An alternative method is to have but one group of connectors per hundred lines and to equip each connector with a frequency selecting device which is operated by the last digit of the call number. It makes each call number one digit longer, but is somewhat cheaper as to initial cost.

Telephone service for intermittent users (other than regular subscribers) is usually furnished by stations equipped with some form of coin collecting box. For local service, the rate is displayed near or on the telephone instrument together with directions for its use. For toll or long distance service, the user must call for the toll board to receive supervision and information.

The automatic telephone has been successfully equipped with "nickel-first" and "nickel-last" coin-boxes. With the former, the deposited coin is held until the called station answers, then a reversal of current operates a polarized relay in the coin-box which collects the money. With the latter, the user dials the number as usual, the answering of the called station reverses the battery current which operates the polarized relay in the coin box, the latter short-circuits the transmitter and shunts the receiver enough to prevent its use as a transmitter, but still permits hearing that the called station has answered. Depositing the coin clears the transmitter and receiver.

A credit meter is also in use, which records the number of completed connections. It may be attached to the instrument and worked with a push-button (instead of a coin) or may be located apart from the telephone on the subscriber's premises and require no act of the subscriber. An older type of meter is designed to be located in the central office.

If the pay station be used to call a free local number, no current reversal will take place. All free lines are connected to a group of connectors which do not reverse the current when the called station answers.

When the user of a pay station desires an out-of-town connection, he dials the toll recording desk as any local subscriber would do. When the connection is ready, the line operator calls the pay station automatically, requests the payment, and is by sound notified of the deposit of the coins as is the case in manual practise.

Durability. The durability of automatic equipment is attested by the performance of the older plants. A group of ten exchanges scattered over the Union, were installed from May 1903 to December 1904. These exchanges are still (1919) in operation. Their original joint capacity of 19,476 lines has grown to more than 104,000 lines. The original equipment is still in use. During about fifteen years of service, the regular maintenance has been able to keep the apparatus in good shape. Today the cost of maintenance material for the switchboard per line per year is from 7 to 9.5 cents, and for the calling device 2 cents per year.

Maintenance. The proper maintenance of any machine requires that it shall be prevented from deteriorating. It is cheaper from every reasonable standpoint to keep it in good working order than to neglect it until its condition is so bad that it has adversely affected service. Each function should be tested at such intervals as have been found necessary to give good service.

In a public exchange the routines involve inspections occurring daily, weekly, monthly, quarterly, twice a year and yearly. The daily, weekly and monthly routines are merely precautionary inspections, they are not adjustments.

The daily routine includes a rapid inspection of the functions of selectors. Once a week there is a rapid inspection of the line switchboards and of the functions of connector switches. Once a month the alignment of line switch plungers is examined, and the dust cleaned off the switch covers, tops of bays, etc.

Every three months the selector trunks are tested (continuity of bank and terminal wiring, particularly of infrequently used or overflow trunks and switches), wiper cords are examined and replaced if necessary, master banks and wipers are cleaned and inspected, and alarm relays are inspected and adjusted.

Twice a year the contact springs of relays and other devices are inspected for proper movement and tension and brought to standard. The switches are tested for operation under extreme line conditions. Switch shafts and master switches are cleaned and oiled. Banks are cleaned and wipers adjusted and replaced if worn. A rapid inspection is given to the rotary motion of selectors. All equipment is tested for freedom from grounds on frames and supports.

Once a year everything is checked up for conformity to standard and brought to standard. This includes the ratchet mechanism (vertical, rotary, and release) of each selector and connector, the repeaters, the line switch plunger alignment and its bank contacts and the cut-off relays.

The maintenance of an automatic exchange requires the attendance of persons who know what to do and when to do it. The average mentality required is not higher than for a manual plant of the same size.

A single office exchange having 2300 lines and 5500 subscribers telephones requires approximately the following force for maintaining the central office equipment.

One switchman.
One service clerk.
One line tester.
One information and complaint clerk.
One night man (answers toll board in addition.)

A two-office plant having 5400 lines and 10,000 telephones requires the following central office force.

One service chief.
One switchman.
Two apprentices.
Two service clerks.
One tester.
Two information clerks.
Two assistant clerks (also take care of complaints.)

The telephones can be increased to 13,900 without adding to the central office force. A service chief has charge of the switch room and does work on equipment. A switchman makes adjustments and repairs. An apprentice performs routine tests, supervises signals, and answers telephone calls to the switch room; an apprentice may be in training to become a switchman. A service clerk is a girl who performs the same duties as an apprentice. A tester makes tests on local and long distance lines; and in a small plant may act as despatcher, routing the outside troublemen. The duties of information clerk are the same as in a manual plant. The employment of girls to perform routine tests has become a factor in exchange maintenance. They learn easily, are quick and careful in their work, and are proving very satisfactory.

RURAL TELEPHONE LINES

Telephone service on rural lines differs from that in cities and towns in that dwellings are much farther apart, there is usually no common center (except the nearest town) and greater dependence is placed on the telephone.

The requirements of the subscriber are low rental, dependability of service, and the ability to make many calls to those in his own neighborhood. On the other hand, the rural subscriber is usually willing to do more than his city brother. He will gladly push buttons, turn cranks, and do anything else if necessary to get good service.

The operating company must have cheap but substantial lines, with as low an upkeep as possible, and a simple telephone requiring little maintenance.

Manual rural lines are usually operated with local battery transmission. Automatic rural party lines are also in use with local battery and with success. But a special weak current common battery transmitter has been developed which gives as good transmission on a 400 ohm loop as an ordinary transmitter does on zero loop. This has to a large extent made it possible to operate rural lines with common battery, with evident economies.

EQUIPMENT CHARACTERISTICS

The characteristics of the equipment which handles rural lines depend upon the telephone instruments, the method of operation which is desired, and the line conditions which are necessary in this class of business. Insulation can not be maintained at as high a figure as is possible in the city. Line resistance must greatly exceed the usual values. Many more telephones must be put on a single line than is necessary in town. These facts require changes in the apparatus.

Because of the number of bells and condensers (sometimes ten) bridged across the line and the lower insulation which is at times unavoidable, the control circuit and apparatus must have a greater factor of safety than is usual. To secure this a repeater is interposed between the rural line and the regular exchange apparatus.

This repeater has two line relays, Fig. 6, one in each battery lead. The impulsing is done with the relay which is connected to the positive grounded battery terminal. Also, during the time that impulses are coming in, the impedance of the negative line relay is greatly lowered. These two provisions greatly increase the reliability of the selection.

It frequently occurs that rural lines produce so much traffic that it overloads the group of first selectors belonging to the line-switch board (hundred line group). This is relieved either by subdividing the line switches into smaller groups, or by providing each rural line with its own first selector instead of a line switch; in this case the selector performs the functions of repeater as well as of selector. The relative cost of the two plans varies with conditions.

One condition incident to placing so many telephones upon one line is the chance of interference of one subscriber with the dialing of another. This is obviated by providing each telephone with a hook stop. It permits the hook to rise part way, only far enough to connect the receiver across the line (in series with a 2 microfarad condenser.) This allows the subscriber to listen without interfering with dialing which may be going on. If the line is free, the subscriber presses the hook stop; the lever then rises to the full extent of its stroke, connecting up the calling device and completing the talking circuit. Beside preventing interference with dialing, this device permits the rural subscriber to listen as much as he desires—a habit which is firmly established and warmly defended.

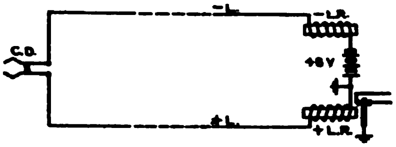


FIG. 6—RURAL LINE CONTROLLING CIRCUIT

The signalling can be either selective or code ringing. The latter is very largely employed by manual exchanges, is understood by the subscribers, and seems to be satisfactory. Selective signalling is better in some respects, but the choice properly lies with those who are closely in touch with local conditions.

To ring ten bells selectively, five bells are bridged from each line wire to earth and rung with five frequencies. Two systems of frequencies are in use. The "multiple harmonic system" employes 16.7, 25, 33.3, 50 and 66.7 cycles per second. The "non-multiple harmonic system" uses the frequencies 20, 30, 42, 54, and 66 cycles per second. This duplication resulted from the historical development of two systems, and there are many installations of both in use today. It is to be hoped that one of the two may be retained as the better, and the manufacture of the other discontinued.

Code ringing permits all the bells to be alike and to be bridged across the line. In order not to interfere with dialing, the bells are wound to 3600 ohms (*approx. 5000* ohms impedance at 10 cycles per second) and the condensers limited to 0.3 microfarad capacitance. Signals may be made of combinations of long and short rings or of two groups of short rings separated by a pause.

Rural lines are served by a line-switch board set aside for this purpose. The special apparatus required is localized to this part of the equipment. If the signalling is selective, each connector has mounted on it a minor switch which picks out the frequency to be used. If code ringing is used, each connector has a code selector and a code switch. The latter groups the code signals, the former picks out the desired signal. The code switch is a single motion (rotary) switch with a 25-point bank. The code selector is the same in form and size as a first or second selector. They involve mechanisms which have been established by experience.

In general, reverting calls are handled by providing special switches leading to reverting call connectors. Each of these connectors is equipped much as a regular rural connector is equipped, but is wired so as to ring back on the originating line. If selective ringing is used, the reverting call connector will send out alternately the frequency of the originating station and of the called station. The total period of repetition is 4 or 5 seconds. To make a reverting call, the subscriber consults a list of the subscribers on his own line and dials the number given for the desired station. He then hangs up his receiver. The alternate ringing begins and continues until the called subscriber removes his receiver from the hook and presses the hook-stop. The originating subscriber, noticing the cessation of his own signal, takes his own receiver, touches the hook-stop and proceeds to converse. Release is accomplished when both subscribers hang up their receivers. If the called station fails to respond, the originating subscriber stops the ringing by removing his receiver pressing the hook-stop, and hanging up the receiver again.

The line requirements for rural lines are as follows: maximum safe loop resistance 1000 ohms, minimum safe insulation resistance, wire to wire or either wire to earth 25,000 ohms. Any commercial telephone cable has low enough capacitance not to affect dialing.

Automatic switching offers to the rural subscriber a telephone service which is vastly superior to manual service and only slightly inferior to city automatic service. He has the same instantaneous, positive, direct calling at night as at noon, and on holidays as on work days. It is inferior to city automatic service only in the loading of the party line with so many telephones. In fact the moving force which hastened the application of automatic switching to rural lines was the pressure exerted by the farmers themselves to be given calling devices.

THE COMMUNITY EXCHANGE

The community exchange, as the term is now used, is a small unattended plant, whose toll switching and miscellaneous calls are handled from a distance and whose apparatus is often designed with the requirements of rural lines primarily in view. The community which it serves may be the farmers of a region, a small village or town, or the small suburb of a town. The traffic is chiefly local though by no means confined to the local telephones.

Miscellaneous services, such as information, complaint, dead numbers, etc., are usually cared for by the nearest public exchange where such service is maintained. Most of these community automatic exchanges are auxiliary to the ordinary type of public exchange. When the C.A.X. subscriber dials the information number, the call is trunked to the information clerk in that exchange, where records are kept for all the territory served.

Toll switching is handled in the same way as the above mentioned miscellaneous services. In fact it may pay to combine on one call number all these things, because one person can care for them all. The toll line operator is provided with a calling device and trunk connections to the community automatic exchange, so that she can complete and control any connection.

The subscribers requirements are the same as for any single office exchange with rural lines. From the standpoint of the operating company, the apparatus must require little attention and the investment must not be too large.

The characteristics of the equipment are influenced by the conditions of the particular installation. Too great rigidity of standardization would result in inefficiency. This will become apparent as we study the matter farther.

The rotary line switch Fig. 7, is the regular equipment for community automatic exchanges. It possesses simplicity of structure, individual control, minimum of motion, and greater trunk group than the plunger type.

The rotary line switch Fig. 8, is a single-motion switch, with a 25-trunk bank set vertically. The motor magnet armature has a pawl which normally locks the wipers on any trunk. The wipers rest where last used. When the magnet energizes, it withdraws the pawl so that it catches the next tooth on the ratchet wheel. When the magnet de-energizes, the armature spring drives the pawl and rotates the wipers to the next trunk contacts, and locks them there. The magnet is self vibrating, like a door bell.

Associated with the switch proper is a line relay and a cut off relay. They are mechanically interlocked so that if the line relay is de-energized, the cut off relay can pull up only part way—just enough to break contacts and clear the line of attachments. If the line relay is energized, the cut off relay can pull up all the way, connecting the subscriber's line to the wipers.

When a call comes through a connector to the subscriber, the connector energizes the cut off relay alone. This clears the line of line relay and ground, but does not connect the line to the wipers. Thus there is a clear pair of wires from the connector to the subscriber's telephone.

When the subscriber originates a call, by lifting the receiver from the hook, the line relay pulls up and causes the trunk to be tested. If the trunk is busy, the magnet rotates the wipers at a speed of 60 to 80 trunks per second until an idle trunk is found. The magnet then stops, the cut off relay pulls up, clears the line, and extends the line to the first selector. During conversation, the cut off relay alone is energized. On release the cut off relay merely falls back—the wipers remain where they are. If the trunk upon which the wipers were resting at first is idle, the cut off relay pulls up completely and without delay. The time required to seize the trunk is only that required for two relays to energize. The maximum time required, if the wipers must pass over 24 trunks, is about half a second.

Most calls take an idle trunk without moving the line switch wipers at all. Tests made on a busy private automatic exchange showed that about 16 per cent to 20 per cent of the calls moved the wipers one or more steps—the rest of the calls required no motion.

Because it possesses no common mechanism, the rotary line switch lends itself admirably to rural exchanges. There is so little to get out of order, and the factor of safety is so large, that it can be relied upon to require very little attention. And if any one part *should* fail, it affects only one subscriber line instead of a group of lines.

The most important variable factors are herewith presented and will be discussed in detail.

1.	Transmission	Local battery or common battery
2.	Line relay	Double wound or ground relay
3.	Line equipment	Lineswitch or selector
4.	Signalling	Selective or code
5.	Signal control	Push-button or periodic
6.	Reverting calls	Push-button, periodic or hand generator
7.	Battery charging means	Motor generator or gasoline engine
8.	Battery charging method	Float or hand start with automatic stop

The tendency is to use common battery transmission as far as possible. The replacement of dry cells is a great and increasing expense. It is to be incurred only if the salvage of present equipment causes it to seem more profitable to retain the local battery, or if the lines are so long that it is necessary for good transmission. As far as automatic operation is concerned, it makes no difference which is used. Most lines are long enough to require a special weak current common battery transmitter. In all cases the polarized receiver is necessary, because the subscriber must be able to listen through a condenser to see if the line is in use.

The line relay (LR-Fig. 6) which handles the impulses should be connected to the positive or grounded battery terminal. Here it has a greater factor of safety than if it has two windings, one being to each battery terminal. If the line conditions are as good as in town, the usual double-wound line relay is fully adequate.

The usual practise is to carry all lines in a community automatic exchange to line switches. All selectors and connectors are usually equipped with the ground line relay circuit, so as to render repeaters unnecessary.

In regions where rural subscribers have been using local battery magneto telephones for years and are accustomed to code signals, the code signal can still be used with automatic. The non-secret nature of the call is by some regarded as a positive advantage. It is difficult for the city-dweller to realize how the farmer uses his telephone. An instance will illustrate one point. One evening Jones repeatedly heard his bell give the signal for the Browns, who lived a quarter of a mile down the road. Knowing that the whole Brown family had gone to town for the evening, he stepped to the telephone, and gave the calling subscriber the information. More congested rural regions seem to require greater secrecy, and the selective signal is favored and sometimes even requested.

The signal may be controlled by the subscriber or by the central office apparatus. The former is called "push-button control" or "push-button ringing." If it be code signalling, the subscriber presses a button on his telephone so as to form the signals. They may be thus made more accurately and easily than by the hand generator, with its revolving armature which must be started and stopped several times in each signal.

To secure push-button ringing, the button is arranged to ground the negative line and open the positive line. The negative line relay remains energized and prevents the connection from releasing. The positive line relay controls the ringing apparatus.

Code ringing was described under the heading "Rural Telephone Lines."

Reverting calls are handled in accordance with the method of signalling used. If local battery magneto telephones are employed, the subscribers ring each other on the same line just as they did with the manual switchboard. If selective signalling is employed, the subscriber dials a special number and receives alternate signals as was described above, (Rural Telephone Lines.) If code signalling is used it may be either periodic or push-button controlled. The latter requires reverting selectors, available to the party lines.

When a rural subscriber desires to call a person on his own line, he dials a special number. He then presses his push-button to form the code signal. This action controls a relay in the reverting selector, which sends ringing current back onto the originating line.

If the code ringing is periodic, reverting calls are handled by reverting connectors which are equipped very much like the regular connectors. The subscriber consults a list of subscribers on his own line (posted on the telephone.) He then dials the special number given for the desired station and hangs up his receiver. The desired code signal is sent back on the line, repeated periodically. When the called stations answer, the ringing is cut off—the originating subscriber then comes in on the line. Both subscribers hanging up release the reverting apparatus.

The charging of the battery is done preferably by a motor generator run by commercial power. The battery bus bars are equipped with a voltage relay, which starts the motor generator whenever the pressure falls to 46 volts and stops it when it reaches 52 volts. A self-closing reverse-current circuit breaker stands between the generator and the battery. When the generator voltage is high enough to be safe, the circuit breaker closes and the charging begins. When the motor has been cut off by the high voltage, the generator voltage dies away until a slight reverse current operates the circuit breaker and cuts off the circuit.

If no commercial power is available, a gasolene unit is installed. This is started by hand as often as necessary. The regular maintainer need not do it, some one who lives near the exchange and is familiar with gasolene engines is employed to set it to going. The stopping is automatic. An ampere-hour meter is included in the battery circuit. It is compensated so as to run slowly when charging, but full speed when discharging. When the meter indicates the necessary ampere-hours have been put into the battery, it opens the ignition circuit of the engine, so that the latter stops. The circuit breaker disconnects the generator from the battery. During the time of charge, the circuit breaker inserts counter e.m.f. cells in the discharge lead to the power board—they prevent undue rise of voltage on the bus bars which would prematurely stop the charging.

The equipment is made up in three general units, the switchboard, the storage battery and the charging machine. The switchboard is made in two or three parts, which are installed side by side so as to form one unit. These parts are the local switchboard, the powerboard and the main distributing frame, and the trunking board. The latter is omitted if the exchange is isolated (no trunks to other exchanges.) All but the trunking board have been fairly well standardized. Because of the many variable conditions to be met in different places, the trunk board, containing selectors and repeaters, is made up in accordance with the needs.

A community automatic exchange needs no special building to house it. It has been installed in a store, in the bedroom of a residence—any convenient place which is available. There are signals to indicate when a fuse blows etc. The conditions of community life and the location of the apparatus are such that someone is always not far away.

This type of exchange has not been in service long enough to secure data of general worth as to the amount of maintenance labor required. In one case a group of three of these exchanges located on a straight line 26½ miles long, together with the telephones belonging to them are maintained by one man. The man is of the same general type as those who maintain ordinary automatic equipment.

The line requirements are the same as those given under "Rural Telephone Lines."

TOLL SWITCHING

Toll service between line and subscriber may be handled in any one of a number of ways. The methods described here are considered the best, as the result of much experience. Person to person calls alone are considered.

The recording toll clerk receives calls from the subscriber over trunks which come from selector banks and terminate on desks. The tickets are made out and passed to the toll line operator in the usual way.

The toll line board is provided with trunks which pass to toll selectors. Over these trunks the toll operator dials the full directory number by using a calling device which is keyed into the circuit. The selectors lead to toll connectors, the latter located in the hundred-line boards, adjacent to the local connectors.

These toll connectors may be used for local service as well.

If all the local connectors are busy, the local traffic will take one or more of the toll connectors. In this case the connector acts like a local connector, having the same current supply relays and condensers. When seized by a toll selector, the local apparatus in the toll connector is cut out, leaving a clear circuit from the last selector through the connector to the calling line. The last selector supplies current through a repeating coil.

The ringing is intermittent, but must be started by the toll operator. When the subscriber answers, the ringing stops automatically. If the subscriber hangs up his receiver, the usual supervision is given, and the toll operator may start the ringing again, if desired.

Intermittent ringing is used for toll calls because it has been found to be the more effective than manual ringing. Where local ringing is periodic and toll calls are rung by hand, much delay results. The subscriber is inclined to think that the manual ring is merely some local call which is very shortly released. Consequently he pays no attention to it. But the periodic signal gets his attention and greatly shortens the time of response to toll as well as to local calls.

The toll connector tests the called line as does any connector. If the line is busy, it sends a busy tone to the operator and also extinguishes the supervisory lamp at the toll board. When the line becomes free, the connector seizes the called line, stops the busy tone, and lights the supervisory lamp at the toll board. When the toll operator sees that the line is free, she starts the ringing by pulling a key.

The release of a toll-to-local connection is controlled by the toll operator. Merely pulling the plug out of the trunk jack causes the switches to be restored to normal.

TOLL SWITCHING BETWEEN EXCHANGES

It is desired to discuss under this heading the general considerations involved and to give some concrete illustrations of how exchanges are linked together.

General Considerations. The linkage of exchanges of any kind into a network requires, beside the toll lines, adequate means for switching these links to each other and to the lines of the subscribers who are to talk to each other. In arranging this switching, thought must be given to the cost of effecting such connections, and the effect of the means on the volume of traffic handled.

The usual manner of handling such calls is to have one or two operators in the originating exchange to connect the originating telephone and the toll line together, another operator at each switching office for toll lines, and one or two operators at the called exchange to connect the toll line to the called telephone. If each exchange has several offices and private branch exchanges, still more operators may be required.

It has been found that the manual switching of toll lines, among themselves and to the subscriber lines, entails considerable loss of time. This is chiefly due to the time required to gain the attention of operators who have other lines to serve beside the one in question. It lengthens the time required to set up a connection, it interferes with supervision, and delays the release of the circuits after both subscribers have hung up their receivers. This is so great a drag on the service that it has led to the establishment of many direct toll lines between important points, so as to do away with intermediate switching. This lowers the time-efficiency of the long direct line, because it can not be used for any intermediate traffic.

For at least twelve years toll lines have been switched by automatic means and experimenters have been at work producing apparatus which is adapted to this service. The first discovery made was that automatic switching increased the capacity of the toll line at least fifty to one hundred per cent. This remarkable increase is due to the speed of connection, the accuracy and promptness of supervision, and the quick clearing of the line after both subscribers have hung up their receivers.

No matter how the switches are controlled, the line must permit manual operation at any time, must operate automatically in both directions if both ends are at automatic exchanges, when seized must give positive indication to all stations concerned and must give supervision of both originating and called subscriber to the one operator who controls the connection.

Uniform operation is highly desirable. This point has received attention in developing automatic switching over toll lines. It is so arranged that if the operator plugs into the jack of an automatic toll line and rings, the call will go through manually as formerly. But if, after plugging into the jack, the operator throws the calling device key, she can put the call through automatically. After dialing the number, the operator rings with the regular ringing key. This operation holds no matter whether she is switching toll lines only, or if she automatically completes the connection to the called subscriber.

A few illustrations will show how automatic switching is applied to various toll traffic services. In order to avoid confusion, the examples will be very simple.

Consider the case of two adjacent exchanges which have free service from one to the other Fig. 9. Let each exchange be represented by four hundred-line-boards, having selectors and local connectors.

The lines (1 and 2) which carry the inter-exchange traffic are equipped at each end with a selector, and are in addition multipled to the banks of local selectors. A repeater is inserted between the selector banks and the lines for the purpose of holding the local switches from releasing, feeding common battery for talking, etc.

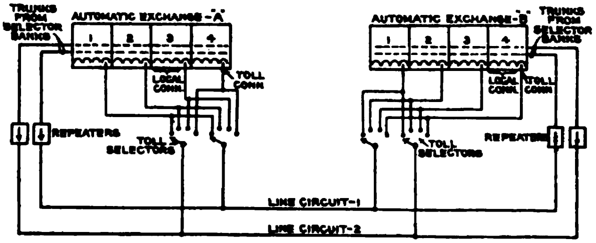


FIG. 9—AUTOMATIC EXCHANGES CONNECTED BY FREE AUTOMATIC TRUNKS

The banks of the toll selectors are run to the jacks of toll connectors, one in each hundred-line-board. These toll connectors have their banks multipled to the banks of the local connectors, so that all subscriber lines can be reached.

When a subscriber in exchange A desires to call one in B, he dials a figure which causes his first selector to seize one of the lines, making it busy on the banks of all other first selectors in both exchanges A and B. He then dials the directory number of the desired telephone, which first operated the toll selector at B and then the toll connector in the hundred-line-board chosen.

If pay service is the rule, the lines can be arranged so that the toll operator at the originating exchange can complete the connection without the aid of the toll operator at the distant exchange, Fig. 10. The toll lines will terminate at each end in toll selectors and jacks and lamps on the toll board. The trunks from local selector banks will also run to the toll board.

When a subscriber in B desires to call one in A, he dials a single figure number which causes his first selector to take an idle trunk to the toll board. The toll operator answers the call in the usual way. She has a calling device which may be associated with any pair of cords. When she plugs into the jack of the toll line, she makes it busy at both ends, showing the busy lamp at exchange A, so that the toll operator there will not try to use the line. The operator at B then dials the call through to the subscriber in A.

The operator at B controls the connection, giving it the usual supervision. When the subscribers are through, she pulls out the switchboard plug, which causes all switches to release.

The identity of the originating station may be verified by having the subscriber hang up his receiver. When the line connection is ready, the operator calls the originating subscriber automatically and permits conversation.

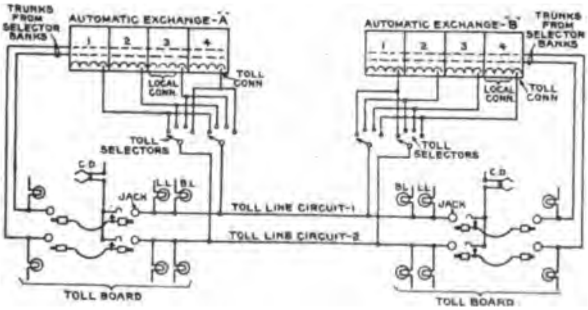


Fig. 10—Automatic Exchanges Connected by Automatic Toll Lines

The apparatus is arranged so that the toll lines can be operated manually without change. If the operator after plugging into the toll line jack rings in the ordinary way, she will cause the customary lamp or drop to be displayed at the other toll board.

If between two such exchanges there is a toll central office, the lines can be interconnected by automatic switches with great saving in time. Consider, Fig. 11, four toll lines meeting at a toll office, which is manually and automatically operated. The former may be necessary for handling traffic to and from local subscribers.

Each toll line will terminate in three places, the toll board multiple, the automatic switch bank multiple, and the jack of the automatic switch belonging to that line.

If a distant exchange seizes a toll line, it is automatically made busy on the banks of the toll selectors and also lights the busy lamps belonging to it on the toll board. The distant exchange then dials the number of the toll line desired. When that line is seized, it is made busy on selector banks and toll board (as was the originating line.)

If the incoming call is for a subscriber in the exchange attached to the toll office, two ways are open for handling it. The distant exchange may ring on the toll line, cause the line signal to show on the toll board at the toll office and use the operator there to complete the connection manually or automatically. The second way is to run trunks from the banks of the toll selectors to the automatic switches of the local exchange and to permit the distant toll operator to dial through to the subscriber and supervise the call.

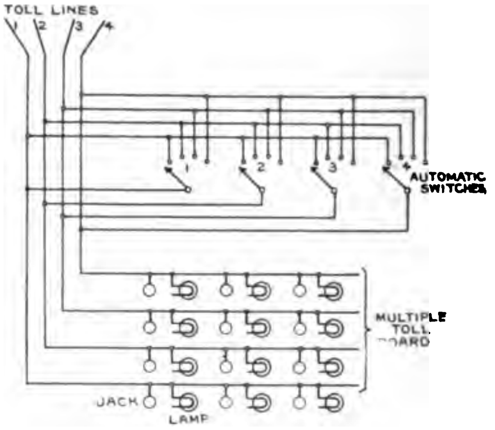


FIG. 11—AUTOMATIC TOLL SWITCHING OFFICE

The application of automatic switching to toll lines in a small net work will illustrate some of the possibilities. For the sake of simplicity, each exchange is limited to 1000 lines.

Let us consider five automatic exchanges Fig. 12. Four of them, A, B, C and D, are located at the corners of an approximate square. Exchange M is near the center of the quadrilateral. Assume further that free service is given between all adjacent exchanges, but that a charge is made for traffic between opposite corners (A and D, B and C). The exchanges are numbered from 1 to 5 as indicated.

The exchanges are connected by lines. To keep the illustration simple, assume that there is only one line between adjacent exchanges, eight lines in all. Any link can easily be increased by adding lines and switches without changing the principles described.

Each exchange has two kinds of first selectors, local and incoming. The local first selectors (illustrated by only one, L) handle all the traffic originated by subscribers in that exchange. One bank level carries the traffic into the local switches to local subscribers. Since this is a small system, to keep the numbering uniform, each exchange has a different local level. In A it is the first level, in B it is the second level, etc. The other levels lead to adjacent exchanges, each according to the number of the exchange. From the second level, a trunk goes through a repeater (R) to the line leading to B, where it terminates on an incoming selector (I–A from A, I–D from D, etc.)

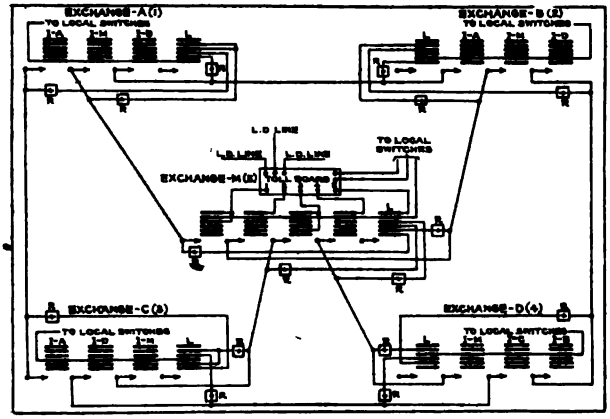


FIG. 12—TYPICAL NETWORK OF SMALL AUTOMATIC EXCHANGES

From the third level a trunk goes through a repeater (R) to the line leading to C, where it terminates in an incoming selector (I-A from A, I-M from M, etc.)

The repeater inserted in the outgoing trunk holds the local switches in the originating exchanges.

The incoming selectors (I–A, I–B, etc.) have the right level multipled to the banks of the local selectors (L) so that they have access through the local switches to the local subscribers.

These free lines are in reality two-way trunks. Each end of a line is attached to an incoming selector for incoming traffic and is multipled to the banks of all the local selectors (L) for outgoing traffic.

Suppose that a subscriber in exchange A calls a subscriber in exchange C, whose local number is 3225. The subscriber in A will first dial the figure "3" which causes his first selector (L) to lift its wipers to the third level and seize the line to C, making it busy at both ends so that no one else can intrude. Then he dials the figure "3" of the local number, which causes the wipers of the incoming selector (I-A) to rise to the third level and to seize an idle trunk to second selectors. The remaining three digits choose the hundreds, tens and units and complete the call.

To handle pay traffic between A and D and between B and C, as well as between the network and the long distance lines to other parts of the country, a toll board is installed at M. All incoming selectors in M have the fifth level multipled into the banks of the local selectors (L) because all the other exchanges are to call M free of charge. But the tenth or "O" level is run separately to the toll board. Any call for long distance by a "C" subscriber will go directly to the toll board, because the directory lists the long distance operator so as to secure that result. "Long Distance" may be listed as "50" or as "00", because the banks of the local selectors (L) at each exchange can be multipled that way.

Trunks from incoming selectors at M and the toll board are kept separate, so that the toll operator knows the origin of each call which comes to her.

If a subscriber at A desires to call some one at D, he is instructed by the directory to call "50" which leads his call to the toll board. The operator makes the charges and completes the call. The operator has lines leading into the local switches through which any exchange is available. If desired, special toll selectors may be set aside for her use. If the identity of the calling station is in question, the operator directs the originating subscriber to hang up. Then she calls the number which he gave, this establishes identity.

If the subscriber at A tries to avoid payment by calling through B or C, he will fail. The incoming selector (I–A) at both these places does not have any connection on its bank for the D exchange. It is impossible to get out of that place. If he tries to dial from A through M, to D, he will run against the same difficulty.

It is possible to multiple the banks of local selectors so that if a subscriber tries to evade payment, his call will be diverted to the toll board. This is done by connecting the level which will be called to the level carrying the trunk to M (see exchange C.)

Suppose that a subscriber in D has business with a subscriber in B, whom he calls frequently free of charge by dialing "22" plus the rest of the number. If that D subscriber moves to C and attempts to call "22" as before, he will find himself answered by the toll operator at M, who will make the charge and complete the call. Level "2" of local selectors in C is multipled to level "5" and the trunk to M. The incoming selector (I–C) in M has its second level multipled to the tenth and thus to the toll board.

LONG DISTANCE TOLL LINES

It seems to the writer that there is no insurmountable obstacle to prevent the extension of the automatic switching of toll lines to include long distance lines of any length. The only limit is that of desirability. Any distance over which telegraph can be worked can be covered by automatic dialing on toll lines. The problem is more difficult than straight telegraphy, for the transmission of the voice must be safeguarded. But as the advantages become more generally known, automatic switching will be extended as far as there is any advantage to be gained. The farther two points are apart, the more line there is involved in each conversation and the greater the need of economizing time.

THE COMPREHENSIVE SYSTEM

The automatic switching of telephone lines of all kinds is available for building up a comprehensive telephone system of large usefulness. It serves equally well the farmer, the small town dweller and the citizen of the metropolis and links them all together. Everywhere that telephone lines are to be joined and disconnected, and where human intelligence is not absolutely indispensible, the automatic switch materially increases the efficiency. Every year people are revising their views on what constitutes a real need for human intelligence, and this revision is not confined to any one industry.

Gradually the use of automatic switching is extending its usefulness and raising the effectiveness and the efficiency of the whole comprehensive system.

SUMMARY

Automatic switches made with reasonable care, to well known and definite limits, give superior telephone service with plenty of margin to spare. Automatic switching is not so easy that any kind of apparatus thrown together in any sort of way will give good service; neither is it so difficult that prohibitively expensive machinery barely secures commercial results.

The ways of connecting automatic switches together so as to form a smoothly operating system are many and varied, and permit adaptation to any class of telephone service. It meets the need for telephony, not as manual switching meets it, but in its own way, which in many particulars is better. This art rests upon a reasonable and scientific foundation.

DISCUSSION ON "APPLICABILITY OF AUTOMATIC SWITCHING TO ALL CLASSES OF TELEPHONE SERVICE (SMITH), NEW YORK, N. Y., DECEMBER 12, 1919.

Selby Haar: I saw an announcement in a New York paper recently that some of the new exchanges which are to be opened by the telephone company in this city will have, at least, in part, some automatic equipment. I wonder if that will be of the type described in the paper tonight.

Lyman F. Morehouse: I think perhaps I can answer the question just asked. There are several types of automatic telephone equipment, and Mr. Smith in his very interesting paper has described one of them, which has attained quite a field of usefulness. The conditions which obtain in New York City, for example, are, however, such as to require the installation of automatic equipment different from that described by Mr. Smith.

Paul S. Clapp: I had the good fortune to be in Central Europe with Mr. Hoover's Commission and assisted in the establishment of communications for that Commission. At Prague we came into rather intimate contact with the Czecho-slovak Director of Public Telegraph, and he said as soon as they could get money in that country, they would build telephone systems. Of course, the labor question is a very important one throughout Europe, and they were particularly interested in anything connected with automatic telephone systems. That statement of the Director of Postal Telegraph has two significances—one, in showing the alertness of European engineers to grasp improvements in American means of communication, and secondly, a realization of the shortness of labor and their desire to introduce machines to do work wherever possible.

Fred L. Baer: (Read by D. McNicol) In these days of disputes between employers and employees when both are so prone to ignore the third party to all such disputes, the long suffering public, it is perhaps of interest to mention that in several instances within the past year automatic exchanges continued to serve the public for periods of a month or more with practically no one in attendance in the central office. While under normal conditions it is necessary to follow maintenance routines, these are largely precautionary methods to anticipate trouble which might ensue from non-standard conditions and many of the routines can be discontinued for a more or less indefinite period in case of an emergency.

In the paper we have learned of many of the advantages of automatic operation under various conditions, but it might be of interest here to describe briefly how the condition of full automatic service, especially in large networks, is approached.

It is not often that the cut-over from manual to automatic operation is effected instantaneously in large networks except where the manual equipment which is to be displaced is of a more or less uniform condition justifying the replacement. In most cases the transition period is likely to be from two years to seven years or more, so that there is a considerable time during which there is a mixed manual and automatic service. Obviously as far as the telephone user is concerned the method of operation should be such that manual subscribers will originate all of their calls as they have been doing and automatic subscribers will originate all of their calls by means of the dial.

Various plans have been followed for affording the mixed service during the transition period, each having advantages peculiar to the particular case. For calls from a manual subscriber to an automatic subscriber the procedure is as follows:

The manual subscriber lifts his receiver and gets the operator who is known as the "A" operator. The "A" operator ascertains the number that is desired and must have means for disposing of the call. Since the subscriber is likely to require service either to a manual or to an automatic number, the "A" operator must be able to take care of either case. The control and supervision of the connection should always remain with the "A" operator. The plans for completing the calls to automatic are,

First. The "A" operator's position can be equipped with a dial and the operator can dial the desired number. Ordinarily the operator will have trunks direct to each office, and it will be necessary for her to dial only the last four digits of the number. The "A" operator's work in handling a call of this kind is practically the same as for handling a manual trunk call by order wire.

Second. The "A" operator can extend the call over a trunk to a sending operator, after having obtained a trunk assignment from the sending operator. The "A" operator's work is the same as in handling a manual trunk call. The second operator's positions, located at one or more convenient points, can be equipped in the following manner:

(a) A dial cut in key for each trunk and a common dial. After learning the number over the order wire and making the proper trunk assignment, the operator uses the dial cut-in key of the proper trunk and dials the number, then restores the dial key to normal. This operator has no supervision except busy guard lamps to prevent double assignment of trunks. When the "A" operator disconnects, the entire connection is cleared. The sending operator with this method of operation should handle from 300 to 350 calls per busy hour.

(b) A special push button sending device, so that when receiving the number over the order wire, this operator sets the call up on an idle sending machine, and when the proper trunk has been seized by the "A" operator, then the machine is started and sends the impulses. This operator has only busy guard lamp supervision and the control of the connection rests with the "A" operator. A second-sending operator with this method of operation should handle from 550 to 600 calls per busy hour.

When only a relatively small portion of a network is to be converted to automatic initially, it may be more practical to use the second plan rather than to equip all "A" positions with dials and instructing the entire staff in the dialling method of operation. Whether the sending operator would be provided with a dial or a special sending device would depend on the length of the transition period, and since the special sending positions are considerably more expensive, than the dial position, and could be justified only on the basis of operating savings. Obviously as the conversion work proceeds, there will be an economical point at which the dials can be placed on the "A" operator's positions.

For calls from automatic to manual the procedure is as follows:

First. The directory listings can differentiate between manual and automatic subscribers. Then the automatic subscriber could be instructed to call a predetermined digit, say "9", for calls to manual subscribers the call would then be routed over a trunk to a nearby manual operator, where it would terminate as a subscriber's line. An "A" operator would answer and complete the connection the same as a manual call.

Second. The directory listings can differentiate between manual and automatic subscribers. Then the automatic subscriber could be instructed to call certain digits to route the calls to the proper manual exchange where the trunks would terminate before an operator who would answer and learn the desired number and the call would be completed by plugging into the multiple. If the trunks are plug-ended and equipped for machine ringing, then the operator can complete about 300 calls during the busy hour.

Third. The directory listings would all be on an automatic basis. Then the automatic subscribers would dial the numbers as listed, and the call would be routed to the proper manual exchange where the last four figures would be displayed before a special operator who could associate the proper trunk with the number indicated, and by plugging into the proper terminal of the multiple complete the connection. An operator with this method of operation should complete from 550 to 600 calls per busy hour.

Fourth. The calling subscriber would proceed as in the previous case, but the call instead of being displayed before an operator, would be set up on selector, and connector switches. The banks of the connector switches would be connected to the corresponding hundreds of the manual multiple, and the connectors would be arranged to function properly with the manual multiple.

When only a relatively small portion of a network is to be converted to automatic initially, it may be more practical to use the first methods, as the conversion proceeds the second method will have its advantages. When a considerable portion of the plant has been put on an automatic basis, then either the third or fourth plans will be more suitable. Which of these two plans is followed would depend largely on the ability to finance the installation of the selector and connector equipment, because the saving in operating labor would be obvious. The fourth plan has many advantages to the operating company, because operating labor can be reduced to a minimum, even calls from one manual office to another manual office can be completed by the "A" operator in the originating office, through the selector and the connector equipment.

It should appear, after describing the various means of affording mixed service during a transition period, that all plans are feasible. A special study is required in each case to determine the plan best suited. The writer, however, feels that the plan which has in view the complete conversion in the shortest time is the one which will eventually be followed, because there are certain advantages which are apparent, but which cannot always be represented in a cost study.

Arthur Bessey Smith: I think it only right, that you should know that although the present automatic switch, to which, I referred, has a bank having only ten contacts on each level yet this does not by any means limit the number of trunks available for communicating between different offices, or trunks between different parts of the same office—all the way from 150 to 200 trunks in a group are available by suitable combinations of the apparatus, and I think I am safe in saying it will not be difficult to increase them to 500 trunks in one group. This has already been done in part.

In regard to the French situation, there is a company in France known as the Thompson-Houston Comany, which has been making some automatic apparatus and is in a position to make more.

I think Mr. Baer's discussion requires no answer from me, because Mr. Baer's is an expert in his own line, and knows whereof he speaks.

Presented at the 356th Meeting of the American Institute of Electrical Engineers, New York, December 12, 1919.

THE SEARCHLIGHT IN THE U. S. NAVY

BY RALPH KELLY

ABSTRACT OF PAPER

The types and uses of searchlights and signaling lights on naval ships are briefly described.

A changed form of 12-inch incandescent searchlight is suggested which will insure the lamp bulb filament always being at the focal point of the mirror and of the correct type for the application.

The present type of low-power searchlight has many faults. These faults may be corrected by supporting the carbons rigidly near the arc, the positive carbon being held at the focal point by a simple automatic control. The best size and material of carbons should be used regardless of the burning ratio.

The introduction of the high-power searchlight revolutionized the application of the searchlight to naval ships. Although great improvements have been made since that time there is still room for a material reduction in the number and complexity of the parts.

It is believed there is also a possibility of a considerable improvement in the electrodes.

The use of the dome glass door enables a searchlight to operate in close proximity to large calibre rifles and makes it possible to successfully build even larger sizes of searchlights than those at present in use.

The star shell has great possibilities, but it is doubtful if it will ever supersede the high-power searchlight.

THERE are three distinct types of searchlights commonly used in the United States Navy:

The incandescent searchlight,
The low-power searchlight,
The high-power searchlight.

These types are used for navigational work, for both daylight and night signaling and for illumination of enemy ships in night engagements. Each type has its field of usefulness in navy work.

The development of the searchlight for naval use was practically dormant for the few years prior to 1915.* In 1915 it was revived by the adoption of the high-intensity light for capital ships after a long series of tests under laboratory and sea-going

---

* "Searchlights" by C. S. McDowell. TRANSACTIONS A. I. E. E. 1915.

conditions. Since that time the development has been so rapid that searchlights purchased in one year became almost entirely obsolete the next year.

The object of this paper is to describe the various types of searchlight equipment at present in use in the navy, their field of application and the paths of future development demonstrated by experience gained during the war. It is hoped that there will be a free discussion along constructive lines of the suggestions for future development of searchlight equipment for naval use.

THE INCANDESCENT SEARCHLIGHT

At present there are four separate forms of the incandescent searchlight used in naval service:

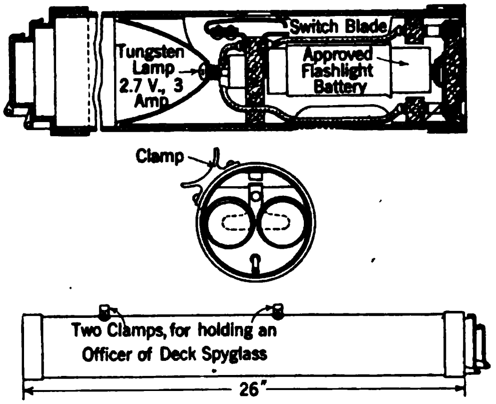


FIG. 1—THE PORTABLE TUBE BLINKER

The portable tube blinker,
The yard-arm blinker,
The Aldis light,
The 12-in. incandescent light.

The two blinkers cannot properly be classed as searchlights as they are used only for signaling purposes. As all sizes of searchlights are largely used for signaling, a description of these lights is included to make the article complete.

(*a*) *The Portable Tube Blinker*, Fig. 1, is extensively used for concealed local signaling at night between ships in column, between neighboring columns of ships or between landing parties on hostile coasts and their supporting ships. It is accordingly designed to throw a feeble beam of light of a very contracted area in order to avoid its being observed at long ranges or by anyone at whom it is not directly aimed.

It consists of a long brass tube with a small flash lamp bulb placed at one end at the focal point of a comparatively large parabolic reflector with a short focal length. The reflector is designed to project practically all the light from the lamp bulb in the form of a small diameter beam to the neighboring ship or station. The inside of the tube is painted black and has a long extension between the lamp bulb and the opening in the tube end. This tends to eliminate all stray light, keeping the beam to a small diameter. This form of signaling light performs its duties well and is universally used in naval service.

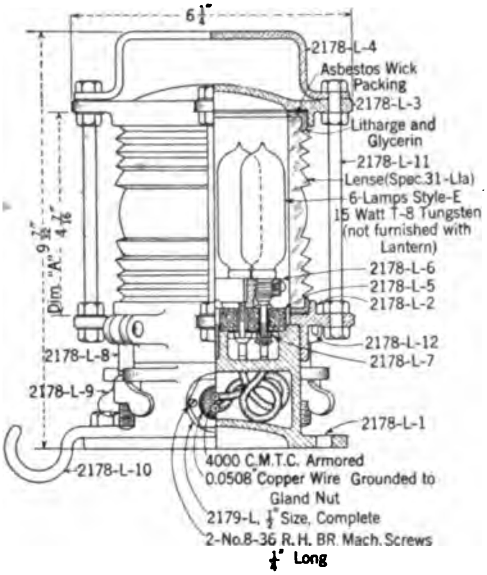


FIG. 2—YARD-ARM BLINKER LIGHT

The only fault that has developed in its use is a minor one which may be easily corrected. When used on a small boat forming an unsteady platform, such as a destroyer or submarine chaser, it is very difficult to keep the blinker directed at the desired receiving station due to the blinker rolling on the rail. A rectangular cross section of blinker or a ball and socket rest on the rail for the present style of blinker would improve the operation.

(*b*) *The Yard-Arm Blinker*, Fig. 2, as its name implies, is a blinker lantern mounted on the yard-arm of a ship and which gives out light that is visible all around the horizon. It is widely used for night signaling in times of peace and in protected waters in times of war. The lantern is designed to contain a high-speed incandescent light with a filament which is placed at the focal point of a Fresnel lens. Signals are sent by blinking the light with a telegraphic key which alternately makes and breaks the circuit. The lamp bulb used in this lantern has been specially developed for Navy signaling work with a concentrated filament that heats and cools very rapidly.

This type of blinker has given general satisfaction and is a valuable addition to the signaling equipment of a ship in view of the large amount of signaling that takes place between naval vessels. In actual practise a circular nest of six commercial tubular lamb bulbs, connected in parallel across the line, are used instead of the correctly designed high speed signal lamp bulb with its filament at the focal point of the lens.

No part of the filaments of any of these lamps coincide with the focal point of the Fresnel lens with the result that the lens disperses all except the direct rays of light to the observer, and could be replaced with a less expensive plane glass cylinder. The nest of lamps with a comparatively long life is used by navy personnel in preference to the short lived high-speed signal lamp, as the blinker lantern is located at the end of a yard arm. Any lamp combination that requires infrequent replacements is for this inaccessible light more popular with the bluejacket than a better short-lived lamp.

The nest of lamps, as far as can be determined, gives entire satisfaction in signaling work, making it possible to replace the comparatively expensive Fresnel lens with an inexpensive plane glass cylinder.

(*c*) *The Aldis Lamp*, Fig. 3, is a projector of British design in which advantageous features are incorporated that make it superior to similar lamps previously used in naval service. While primarily designed for daylight signaling, it has been used successfully as a searchlight and for signaling in airplanes and dirigible ballons.

The projector itself is a light, well balanced, portable affair of aluminum which signals by alternately throwing a searchlight beam on and off an observer. This is accomplished by rocking the mirror back and forth by means of a trigger, the lamp bulb burning continuously. The lamp bulb is manufactured to such close limits that when it is screwed into a special socket in the lamp, its filament is in the focal point of the mirror.

This type of signaling lamp is universally used on ships, shore stations, dirigibles and airships. Its success is due to its high signaling speed, its compactness and portability, but most of all to the fact that, regardless of the personnel handling it, the lamp filament is always at the focal point of the mirror and a long range efficient beam is obtained at all times.

(*d*) *The 12-in. Incandescent Lamp* Fig. 4, is largely used in naval service on all classes of naval vessels for long range daylight signaling in fair weather, and for navigational use at night on small boats. This type of light has not been generally successful in its present form due to the fact that it requires special adjustments and care in service, for successful operation.

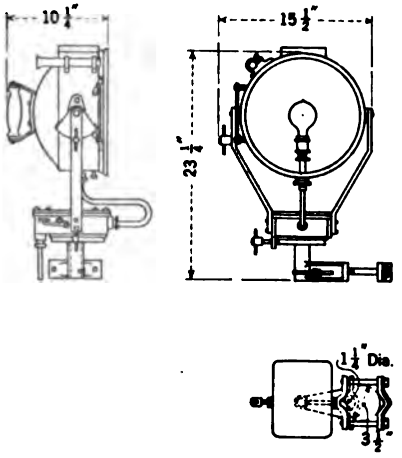


FIG. 4

It has a fixed parabolic mirror with a high-speed signaling lamp bulb which screws into a standard socket. The socket can be adjusted to place the filament of a commercial lamp bulb in the focal point of the mirror. This adjustment is made large to permit the use of several different sized lamp bulbs ranging from 150 to 400 watts. The whole is contained in a cylindrical drum which is commonly made of sheet iron. Signals are sent by blinking the filament of the lamp bulb by a telegraphic type of key.

This lamp has proved successful for navigational work on small craft and particularly so for submarines. A small 9-in. arc lamp was specially developed for submarine use, but its lack of portability, attention required in operation, and necessity for frequent recarboning caused it to give way to the incandescent searchlight. An important factor that entered into this change is that a submarine is frequently submerged without there being sufficient time to dismount the searchlight and stow it below. An incandescent searchlight can be submerged without permanent damage, while an arc lamp is placed out of commission. The small amount of care required for the incandescent type of searchlight, and the comparatively long life of a lamp bulb in comparison with a trim of carbons, makes a correctly designed light of this type ideal for use on small craft with limited personnel.

The faults in the present type of light have been demonstrated by its use in daylight signaling. At the present time the correctly designed Aldis light, with a 4½-in. mirror and 40 watt lamp, is the equal in signaling power to the larger light with its 11-in. mirror and a 150-watt lamp. Considerable adjustment is permitted, both in the horizontal and vertical planes of the 12-in. incandescent light, to allow for variations in filament location in commercial lamps, and to permit the use of various size lamp bulbs. Every time a new lamp bulb is inserted in the projector, the operator should focus the projector for that particular filament. This is rarely, if ever done, and in service in not one lamp that the writer has inspected has the lamp filament been at the focal point of the mirror. In the majority of cases it has been approximately one inch out of focus in each plane. Of course in this condition the mirror is of no value and only the direct rays of light from the filament reach a distant observer.

The high-speed concentrated filament lamp bulb specially designed for this searchlight is almost identical in appearance to a concentrated filament stereoptican type of lamp bulb designed only for searchlight work. Long life being desirable in the latter type of lamp, its filament cools and heats very slowly and it cannot be used at all for signaling work. Due to the similarity in appearance, both types of light are issued indiscriminately for the 12-in. searchlight. The personnel, not realizing that there are two entirely different types of lamps of similar appearance, condemn them both and use a commercial Mazda lamp which happens to fit the same socket, but in which no part of the filament comes any where near the focal point of the mirror.

The design of a new type of light along correct fundamental lines, is contemplated to be arranged so that it cannot be used incorrectly by naval personnel. The same type and size of mirror will be used in a brass non-corrosive drum. The lamp socket will be permanently fixed in such a position that when a 150-watt, high-speed lamp bulb is placed in it, some part of the filament will be in the focal point of the mirror. The ordinary high-speed 150-watt lamp is manufactured with a dimension across the filament of approximately ¼ inch so that in the great majority of cases with such a lamp, some part of the filament will be in the focal point of the mirror, even with ordinary commercial limits of manufacture.

To eliminate the use of lamps other than that specially designed for this class of work, the lamp bulb must be of such design that no other lamp can be used in the socket. One way of accomplishing this object is to make the signaling lamp with a bayonet base. Similar precaution will have to be taken in a 12-in. incandescent searchlight that will be used only for searchlight work to provide only for the use of a concentrated-filament, long-life, stereoptican light.

Attempts have been made to extend the use of the incandescent searchlight to larger sizes such as 18- or 24-in. using larger wattage lamps. To obtain the larger lamp capacity; the filament area must be increased with the result that the intrinsic brilliancy of the filament remains approximately the same and there is practically no increase in light. There is no attempt being made at the present time to increase the size of the incandescent searchlight beyond the 12-in. size.

Low-Power Arc Lights

The number of sizes of low-power arc lamps in navy service at the present time is legion, a 9-in., 12-in., 18-in., 24-in, 30-in., 36-in. and even a few 60-in. are used. The large number of sizes is a natural growth during a long period of years, new sizes having been developed for special applications.

The 9-in. size was developed specially for submarine work and is practically the 12-in. size of lamp with a few necessary modifications to fit it to the smaller drum. It has recently been superseded for submarine use by the 12-in. incandescent searchlight.

The 12-in. size is the most commonly used of all searchlights in naval service. It is used principally for long range daylight signaling on all naval vessels and for navigational lights on small boats. It has a wide field of usefulness and practically every naval ship has at least one of these lights as part of its equipment.

The 18-in. searchlight prior to the present war was used only on merchant vessels. Although the present type is not well constructed, a large number of them were purchased when the United States entered the war as there was a great shortage of searchlight material at that time, and a large number of ships to be equipped.

The 24-in. low-power light has a limited application for navigational work on naval vessels, such as hospital ships, fuel ships and tenders of all types and a certain field for long range daylight signaling.

The 30 and the 36-in. low-power searchlights were placed on battleships and cruisers prior to the development of the high-power searchlight. They are rapidly being replaced by high-intensity lights or being converted into high-intensity lights and will shortly be obsolete in naval service.

It would be of great advantage in the future to use only one size of low-power searchlight, 12 in. or 18. in., developing a searchlight along fundamentally correct lines to give a far greater beam candle power under operating conditions than the present 12-in. light. It is believed that a new 12-in. light can be developed to give as great illumination on the target as the present 24-in. low-power searchlight.

All sizes of low power searchlights in service are essentially the same in design, the mechanism differing only in size. A typical mechanism is shown in Fig. 5. The positive and negative heads are each supported on a separate movable carriage. The carriages are separately geared to a motor or solenoid which feeds the two heads together at a predetermined fixed rate, which is supposed to correspond to the burning away of the carbons. The specifications for purchasing carbons call for carbons that burn at fixed rates which, combined with proper gearing in the lamp mechanism, will theoretically result in the positive crater being in the focal point of the reflector at all times. The arc length is kept fairly constant by a balanced type of feeding mechanism that feeds the negative carbon forwards or backwards when the arc voltage is higher or lower than normal. The present size carbons are of comparatively large diameter, slow burning and with a low current density.

FIG. 3A—COMPLETE ALDIS LAMP EQUIPMENT

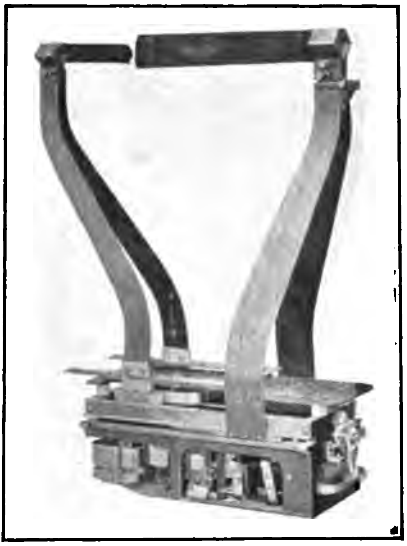
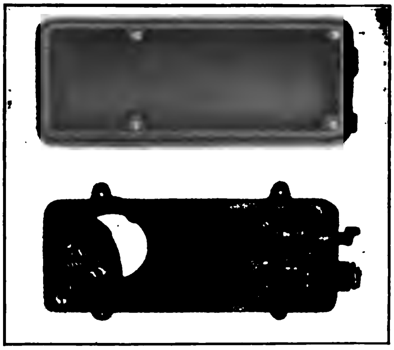


FIG. 5—LAMP MECHANISM FOR A LOW POWER SEARCHLIGHT

FIG. 3B—ALDIS LAMP IN SERVICE



[KELLY]

FIG. 6—THERMOSTAT BOX AND COVER

Fig. 7—Searchlight Lamp with Third Electrode Control

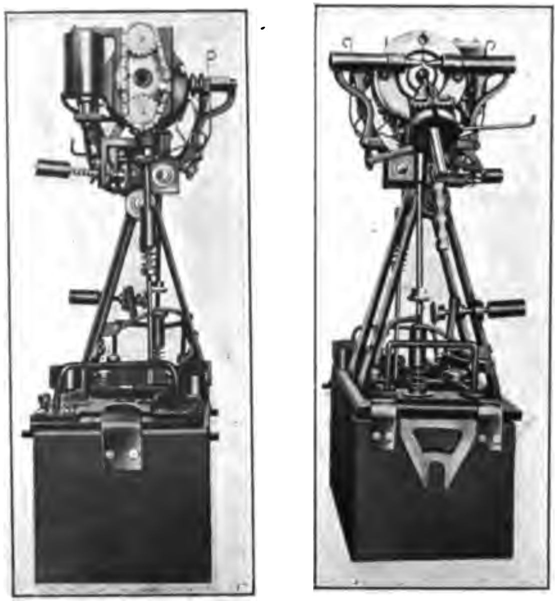


Fig. 9a—Arma 75-Ampere High Intensity Searchlight Lamp Type AA

[KELLY]

Fig. 9b—Arma 24-Inch High Intensity Searchlight Type B

This design of searchlight lamp has many inherent fundamental faults that may be easily corrected. Its principal fault in operation is that rarely if ever is the crater of the positive carbon at the focal point of the reflector. When it is considered that a cross-wise deviation of 1/16 in. of the crater from the focal point of the projector will give a beam of approximately double the width and one quarter the intensity of a standard searchlight beam, it is realized how important it is to maintain the crater of the positive carbon at the focal point of the reflector. Carbons cannot be manufactured commercially to give the exact burning rates specified, but vary from that rate as much as 20 per cent in each direction in extreme cases. Specifying the burning rate of carbons is a distinct handicap to a carbon manufacturer tending to hinder the development of better carbons.

The carbons themselves are of comparatively large diameter producing an arc that has a tendency to wander over the face of the positive crater, resulting in an unsteady beam and a comparatively cool, low-intensity source of light for the searchlight beam. The intensity of a searchlight beam varies with the temperature of the positive crater which makes it desirable to have a carbon current density which will give the greatest temperature at the positive crater. Each grade of carbon has a definite boiling temperature and an increase in current density beyond that necessary to produce that temperature only causes the carbon at the crater to boil away faster with no increase in crater temperature. An ideal condition, therefore, is to use a carbon with the highest boiling point and a current density that will just produce a crater temperature that is equal to the boiling point of the carbon.

The carbon heads themselves are not rigidly supported and have a marked play in all directions. This play is so marked that the rolling of the ship will change the position of the carbons in the searchlight drum. The current is fed into the carbons at their butt ends and flows through the entire length of the carbon to the arc. This tends to heat the carbon throughout its entire length which often causes it to spindle. It also results in a variable arc length, for with a short length carbon the voltage across the arc is materially greater than with new long length carbons.

A correctly designed lamp mechanism for a searchlight should produce the longest arc length possible at which the arc will burn steadily, to reduce to the lowest possible value the shadow cast on the mirror by the negative carbon. The appreciable variation in arc length in the present type of low-power searchlight results in the negative shadow being materially greater during the majority of the burning of a trim of carbons than the best obtainable condition.

A 12-in. low-power searchlight could easily be designed to overcome the faults in the present low-power searchlights. Such a light would not only give a more intense beam than the present 12-in. searchlight under laboratory conditions, but in actual service its more exact mechanism would give a searchlight beam of very materially more power than the present inexact mechanism. The carbon heads in the proposed light would be rigidly fixed in position with absolutely no play nor adjustment, would have a simple automatic control of the positive crater to maintain that crater in the focal point of the reflector at all times, and would have the current fed into the carbons close to the arc. As in the present 12-in. searchlight, the arc length could be controlled by a solenoid feed. A ground glass finder should be provided to enable an operator to check the position of the positive crater.

The automatic control of the positive and the voltage control of the arc results in the carbons being fed independently of their burning rates. Accordingly, if improved carbons with different burning rates are developed they may be used in this lamp with practically no change in the lamp mechanism, a condition which is not possible in the present type of lamp. The carbons would be manufactured with diameters in the neighborhood of one half the present size, which would result in a higher temperature of crater, a closer approach to a point source of light and a smaller negative shadow on the mirror. These carbons should be made of the best combination of materials of the highest boiling temperature that will maintain a steady arc. These searchlights should be equipped with a Venetian blind signaling shutter to permit their being used for daylight signaling as well as for navigational work. Such a light would be suitable for daylight signaling and navigational work on battleships, cruisers, auxiliary ships, destroyers, seagoing tugs, transports; in fact for all naval vessels it would supersede the numerous sizes of low-power searchlights now in naval service.

The change would be of advantage both to the manufacturers and to the navy. The manufacturers will have to develop and keep in stock but one set of tools and patterns and can concentrate on the improvement and development of that one type of searchlight. The navy will benefit by having to keep a stock supply of only one size of low-power searchlight, which will require a stock supply of only one kind of spare parts and accessories like mirrors, lamps, rheostats, switches, shutters, etc.

HIGH-POWER ARC LIGHTS

The invention of the high-intensity arc revolutionized the searchlight for naval use. The 30-in. and 36-in. low-power searchlights previously used had a maximum range of approximately 2000 yards on a dark, clear night. The present 24-in. high-power searchlight has a range of approximately 4000 yards and the 36-in. high-power searchlight has a range of approximately 7500 yards under similar conditions. It is evident that the the low-power lights are useless in gunnery work with modern, broadside defense guns, while the ranges of the high-power lights compare very favorably with those guns.

The high-power searchlight swings to the other extreme from the low-power searchlight in that it is a very complicated exact mechanism with a large number of intricate parts which are designed to operate automatically. Its success depends on the so called high-intensity phenomena obtained by the use of carbons with impregnated flame cores with high current densities, and arranged at an angle to each other. The material of the shell of the carbon is of hard amorphous carbon, the inside being a combination of special mineral salts with the necessary flame producing ingredients. At the beginning the core burns more rapidly than the shell, forming a deep crater which becomes filled with the flaming gases of the core. The combined effect of the high current density of the arc and the angle of the negative arc stream is such that these gases are imprisoned for a short period in the positive crater. These gases are heated to incandescence while imprisoned in the crater, and give a light that is far in excess in intrinsic brilliancy of that given off from the solid crater walls. This increase in intrinsic brilliancy of the source of light for the searchlight results in the superiority in range of the high power searchlight.

The carbon heads are rigidly fixed in the drum with mechanisms included in those heads for feeding the carbons. Contacts placed in the carbon heads, close to the arc, feed current in and out of the arc circuit. The arc length is kept approximately constant by a motor feed that is controlled either by the arc voltage or by arc current and feeds the negative carbon either backwards or forwards to maintain constant arc conditions.

The positive crater is kept at the focal point by automatic devices, either of the thermostatic type or of the third electrode type. Two of the companies manufacturing high power searchlights use a thermostatic control of the positive carbon while the other uses the third electrode control. In the thermostatic device, light from the positive crater is reflected by an optical arrangement of mirrors and lenses to thermostat strips placed in a box on the outside of the searchlight (Fig. 6). When the positive carbon burns back slightly from the focal point of the mirror, the reflected light strikes one of the thermostat strips, and distorts it by the heat. The thermostat strip in distorting closes an electric circuit which causes the positive carbon to feed forward. When it is fed forward to the focal point, the reflected light from the positive crater moves off the thermostat strip which cools instantly and opens up the feeding motor circuit.

This device has worked well in naval service and under the best conditions of operation will maintain the positive crater at the focal point within limits of 1/32 inch, plus or minus. The present thermostat mechanism is mounted on the outside of the drum in a box which is exposed to the weather. In every case in which the writer has inspected searchlights that have seen service, there has been either a considerable amount of water in the box or evidence of water having been recently in the box. Again, the searchlight lamp and the thermostat have a definite relation to each other and an adjustment of one requires a separate and independent adjustment of the other. A thermostat placed inside the searchlight drum and secured to the lamp standard would remedy both of these defects.

The third electrode scheme for controlling the positive crater is more simple and substantial than the thermostat and operates well under service conditions. This device (Fig. 7) consists of a carbon electrode placed above and at right angles to the positive carbon slightly behind the crater. When the positive crater burns back from the focal point, the flame of the arc strikes the third electrode completing an electric circuit of which the conducting flame of the arc forms a part. Current flowing in this circuit causes the positive carbon to feed forward. When the crater is fed to the focal point of the reflector, the flame clears the third electrode and the feeding motor circuit is broken. This type of positive carbon control has the advantage of being placed inside the searchlight drum out of the weather, and of being integral with the positive head requiring no separate adjustments.

The first type of third electrode developed was of carbon which required renewal every three trims of the searchlight carbons. There has since been developed a third electrode which is a copper disk, and which has a very much longer life than the carbons. In operation, a coating forms on this copper disk which protects the copper from burning and prolongs the life of the disk.

The intense heat of the high-intensity crater and the large amount of smoke and fumes emitted from the flame carbons make it imperative to have forced ventilation of the carbon heads and searchlight drum. An early type of high-power searchlight used the comparatively cool vapor of an alcohol flame to cool the carbon tips and the drum was exhausted by a fan connected to a separate motor. The cooling of the high-intensity searchlight was later improved by making hollow lamp standards, jacketing the carbon heads, and forcing air through these chambers and out through an outlet at the top of the searchlight drum, thus doing away with the alcohol flame method of cooling.

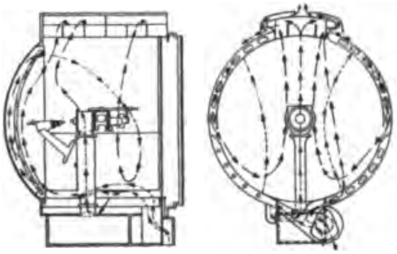


FIG. 8—DIAGRAM OF VENTILATION OF A HIGH POWER SEARCHLIGHT

In the first high-power searchlights great difficulty was experienced due to the large heat absorption and unequal heating of the mirror and a great number were smashed due to these causes. This condition was finally remedied by efficient cooling of both the front and rear of the mirror by a current of air. The ventilating system of a modern high-power searchlight, showing the paths of cooling air around the mirror, is shown in Fig. 8. The heating of searchlight mirrors has been reduced during the past year by the development of a tough white glass of pure materials which has approximately one per cent heat absorption against the five per cent absorption of the grade of glass previously used for mirrors.

Three typical searchlights developed by the three present manufacturers of high-power searchlights are shown in Figs. 9, 10 and 11. An inspection of these illustrations shows the progress in development of these exact well designed mechanisms over the less refined, inexact, low-power searchlights.

The 36-in. high-power searchlight is the largest size used on shipboard for it gives a range comparable to the largest guns and is the largest, at the present time, that will withstand the heavy concussions of the ships gunfire. It is only very recently that the searchlight has been adapted to withstand the concussion of the gunfire of a nearby 14-in. gun. Prior to this time searchlights were equipped with glass front doors made of a number of plate glass strips. This type of front door cut off approximately 25 per cent of the searchlight beam, was not watertight and would smash at the first salvo of the main battery. The success of the ventilation system of the high-power searchlight depends on obtaining air through a duct in the bottom of the drum and forcing it through well defined paths and out through an exhaust outlet at the top of the drum. This scheme of ventilation is made ineffective when the front door is smashed with the result that the arc flickers and the searchlight parts become overheated due to an insufficient supply of cooling air.

The smashing of the front door strips leaves the mirror unprotected and it is an even chance that the mirror will smash at the next salvo from the main battery. It is common practise on shipboard to dismantle searchlights equipped with plate glass front doors before a target practise, and stow them away in a safe place.

This condition has been remedied by the use of a front door made of one solid piece of arch shaped glass similar in shape to the parabolic mirror. This type of door has given excellent service in the fleet and not one of the many dome glass doors that have been installed on navy vessels has been smashed during target practise. This type of door cuts down the searchlight beam no more than the plate glass strips and makes the searchlight entirely self-contained and waterproof.

The trend of future development of the high-power searchlight should be in the simplification and the reduction of the

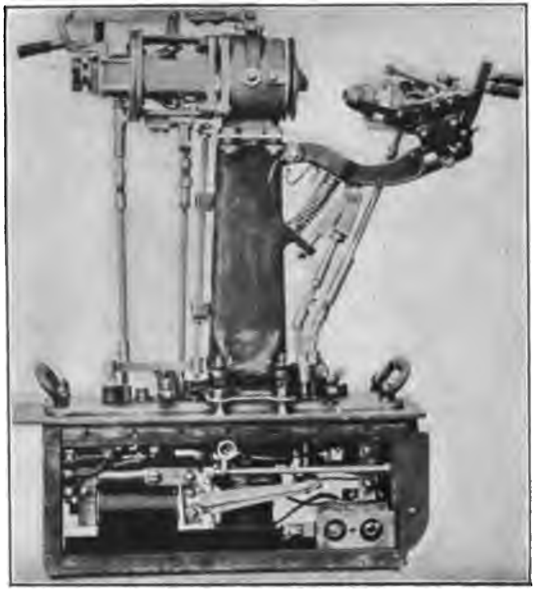
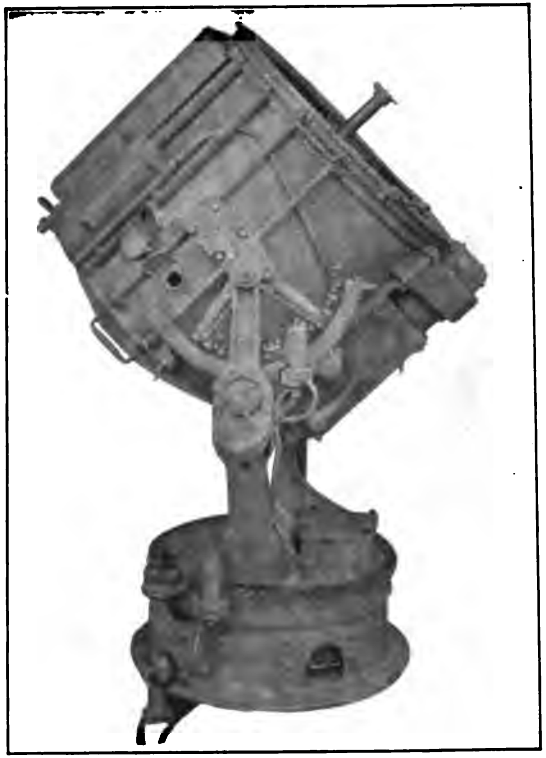


Fig. 10a—Lamp for 24-Inch High Power Searchlight



[KELLY]

Fig. 10b—24-Inch High Power Searchlight

Fig. 11a—Lamp for 24-Inch High Power Searchlight

[KELLY]

Fig. 11b—24-Inch High Power Searchlight

large number of intricate parts on the present searchlights. A searchlight must necessarily be placed in a position exposed to the weather, and will be operated by enlisted personnel. It is a struggle to maintain the present complicated high-power searchlight mechanisms in operating condition under the severe conditions of operation on smaller boats like destroyers. After a few days at sea on a destroyer in rough weather, the present high power searchlights often become inoperative and require a number of hours overhaul to place them in operating condition. Such a searchlight soon loses its popularity with destroyer commanders and they justly complain that the high-power searchlight is more suitable for laboratory use than for destroyer use.

There is still room for a large improvement in the size and material of carbon electrodes for high-power searchlights. At the present time a carbon is used which has a hard carbon shell and a core made of a combination of flaming materials. This core gives off gases which are heated to incandescence in the arc and give the high intrinsic brilliancy light characteristic of the high-power searchlight. It also gives off a very brilliant flame which causes a correspondingly brilliant objectionable foreground illumination and gives off gases that tend to blacken the mirror and the front door.

It is a well established fact that the color and intensity of a searchlight beam is dependent only on the temperature of the source of light. The use of flame-colored salts in the electrode gives a high temperature but also burns with a large brilliant flame. It is believed that a proper selection of carbon materials, perhaps the use of a solid electrode of a very refractory carbon with a small negative carbon, will give the same amount of light that is obtained from the present high-power type of carbons. The flame from a combination like these would be practically transparent with no objectionable foreground illumination, the objectionable gases would be eliminated and the present type of very special carbon electrodes could be done away with.

The star shell for gunnery work is also a factor in the development of high-power searchlights and bids fair to rival the larger high-power searchlights for gunnery work. A searchlight, unless skilfully and carefully manipulated, is more of a debit than a credit in action as it tends to give an enemy a definite point at which to aim. A star shell that satisfactorily illuminates the target, leaving its own ship in darkness, has many decided points of advantage over a large high-power searchlight.

However, even if the star shell gives the results claimed for it, it will be a long time if ever before naval authorities will feel safe in removing the 36-in. high-power searchlight from capital ships. The 24-in. high power searchlight has had such a marked success in long range daylight signaling that it is improbable it will ever be superseded for naval use.

The use of the dome glass door for 36-in. searchlights opens up a field for the use of larger size searchlights, such as the 44-in. and 60-in. sizes for capital ships. It is believed if those larger sizes are ever required for naval service they can be successfully developed.

In closing the writer wishes to express appreciation to Major Howard C. Judson, U. S. M. C. for ideas contributed on carbon development.

DISCUSSION ON "THE SEARCHLIGHT IN THE U. S. NAVY" (KELLY), NEW YORK, N. Y., DECEMBER 12, 1919.

Donald McNicol: Mr. Kelly says that a special type of incandescent lamp is used. Is the novelty of the lamp in the design of the filament or the material of the filament?

G. A. DeGraaf: I ask Mr. Kelly whether the navy has been experimenting with the all-metallic reflector for the searchlight mirror, as the army has been using it. I think it is a glass mirror which has been used in the searchlight which is apt to be destroyed, due to the compression of the heavy gunfire.

M. L. Patterson: Under the heading of "Low power arc lights" the author of this paper has taken up the discussion of 12-in. low power searchlights, as more or less typical of a completed line of low power searchlights. It is fully understood that high power searchlights are superseding the low power searchlight at 24-in. size and above. I do not, however, agree with the author, as stated in paragraph 7, that there should be only one size of low power searchlight, for there is a large field of usefulness for at least two sizes, preferably 12 in. and 18 in.

In paragraph 7 the author states that it is believed that a new 12-in. light can be developed to give as great illumination on the target as the present 24-in. low power searchlight, and I wish to bring out a few facts to show that it is not practicable to accomplish this result. The current and the arc voltage of the two sizes referred to are as follows: 12-in. is 20 amperes and 45 volts across the arc. 24-in. is 50 amperes and 50 volts across the arc.

Since the beam candle power varies as the area of the mirror or as the square of the diameter of the mirror, other conditions being equal, it will readily be seen that for the beam of a 12-in. searchlight to equal that of a 24-in., the intrinsic brilliancy of the crater of the 12-in. must be at least four times that of the present 24-in., or over twelve times the intrinsic brilliancy of the present 12-in. light. I hesitate to quote beam candle figures as data obtained by observers under various atmospheric and operating conditions, show such a wide divergence that any figures on beam candle power, unless accompanied by a statement of the conditions under which they were obtained would be valueless. Nearly all observers, however, agree as to the relative intensity of various sizes of searchlights, and it can, therefore, be stated that the beam candle power of the 24-in. high power searchlight is approximately four times that of a 24-in. low power searchlight. From this it is evident that for the beam of a 12-in. low power searchlight to equal that of a 24-in., it would be necessary to use a 75-ampere high power arc in front of the 12-in. mirror, and it is our experience that it would be utterly impossible to do this as no glass mirror at present manufactured would stand the intense heat of a

75-ampere arc within 6 in. of the mirror, 6 in. being the focus length of the 12-in. mirror.

It should further be noted that an increase in current or an increase in size of the positive crater may not increase the beam candle power to any appreciable extent. It may sound paradoxical, but it is a fact nevertheless that the beam of a 60-in. searchlight is smaller than a beam of the 24-in. at or beyond 500 yards from the searchlight. This is due to the fact that the size of the positive crater of a 24-in. relative to the size of the mirror is greater than in the 60-in. searchlight. This would be true to a greater extent with the 12-in. searchlight, especially if an attempt is made to materially increase the power. The extra candle power would go into beam spread rather than materially increase the foot candle illumination on the target.

It is evident from this paper that the author favors the design of the 12-in. searchlight which will be entirely automatic and have fixed positive heads. If this were carried out, it would be necessary, of course, to have brushes for leading the current to the carbon, which would materially increase the size of the carbon holder. This construction would also require a mechanism for advancing the carbon through the brushes and would require considerable more power than is now required to move the carriages holding the positive and negative columns.

In reference to the current being fed in at the butt ends of the carbons, as referred to in paragraph 11, it has been our experience that there is very little tendency for the carbons of a 12-in. arc to spindle when operating under normal current. The current density in these carbons, as referred to in the last line of paragraph 8, is very low. As regards the variation in arc length, it may be stated that the amount of carbon consumed per trim is from 4 in. to 5 in. and the drop in this length of carbon with the low current density as used in these searchlights will be the only variation in arc voltage (and length). This variation is less than the variation in voltage required to operate the present feeding mechanism. This is, the variation in arc length due to the range in feeding mechanism is greater than the variation due to the drop in the carbons.

The very reason that the 12-in. low power searchlight has had such extensive use, as referred to by the author in paragraph 3, is due to its simplicity and inexpensive design, and while there is no doubt considerable improvement that can be made in the design without materially increasing the complication of the mechanism, it would be a mistake to carry this design to the extent of a completely automatic low intensity mechanism. For automatic control it would require fixed heads, feeding mechanism for advancing the carbons through the heads, a thermostatic or third electric control of positive crater, and ground glass finder, and with all this added complication, many of the advantages of the 12-in. low power searchlight would be lost.

Another point that should be noted is that with the present type of carbon holder, only short length carbon is wasted, whereas if fixed positive heads are used there will be a considerably greater length of wasted carbon per trim.

In paragraph 12, it is stated that it is quite essential to keep the size of the negative shadow at a minimum, and in this connection it should be noted that a change of negative carbon holder to a fixed negative head with feeding mechanism would necessarily increase the size of the negative shadow.

Under the heading of high power arc lights, the writer in the first paragraph has made a comparison in the ranges of the various searchlights, and gives these ranges in a definite number of yards. A statement of range without giving the illumination at the target or the kind of target that would be visible at a certain range, is meaningless.

In paragraph 4, under "high power searchlights," it is stated that two of the companies manufacturing high power searchlights use thermostatic control of the positive carbon, while the other uses the third electrode. In this connection it may be stated that at least one company manufacturing high power searchlights uses both the third electrode control and the thermostatic control. The third electrode feed being used where it is not essential to keep the positive crater in exact focus and the thermostatic feed used where more accurate control of the positive crater is required, as for United States Army and Navy uses.

E. J. Murphy: With reference to the high powered searchlight, I note that the paper* of C. S. McDowell has been quoted and there is no doubt that the type of light described in Mr. McDowell's paper has revolutionized searchlight practise especially for Naval use. Since the advent of this new light, the necessity of using the beam at high angles for defence against hostile air planes made it necessary to abandon the alcohol flame, otherwise the light has remained practically as originally designed. The principles involved have remained unchanged.

I would like to refer to the thermostat control mechanism in Mr. Kelly's paper, I note that he states that this device will hold the positive crater at the focal point of the mirror within one-thirty-second of an inch, plus or minus. He does not state, however, how close the third electrode scheme will hold the positive crater, and it might be interesting to find how accurately this method of control functions.

With reference to the ventilation, I see Fig. 8 shows a pressure system of ventilation. It might be interesting to note that two types of ventilation have been evolved, one the vacuum type, the other the pressure type, and on page 1618 he states that the ventilation will be destroyed in case the front door is

---

*Trans. A. I. E. E., 1915, Vol. I, p. 263.

broken. The company I am connected with has tried out both of these systems, and tests have been made resulting in our discarding the vacuum system on account of certain advantages of the pressure system. The pressure system enabled us to use the fan motor for driving the mechanism of the searchlight itself, thus eliminating any additional motor.

There is another thing which we actually tried out, and that is the effect of wind against the front of the searchlight with a broken door. We noted that with the vacuum system, the arc very frequently was ruptured, making the light practically useless, whereas with the pressure system, the pressure inside of the drum acted against the wind, and the light was operative in a thirty mile wind; under the same conditions the light with the vacuum system was practically useless.

As far as the dome glass door is concerned, I understand there is still some question as to its practicability, on account of refraction of light, etc.

I note that certain illustrations show a thermostat box and the paper mentions that this box is not watertight. This thermostat box is part of the searchlight with which I am familiar. I may say that this searchlight was developed under stress of war conditions, and it was necessary to develop the design in the shortest possible time; we did not provide gaskets for this particular box, but, notwithstanding this omission, I have not found a single case where water rendered these searchlights inoperative. In fact, I am aware that a great many of these searchlights have been at least once across the Atlantic, under heavy weather conditions, and in no case was the searchlight inoperative. Under war conditions, the personnel was not always acquainted with new apparatus such as the thermostat control and had not been previously instructed in the operation of this new searchlight. I have seen reports on at least fourteen of these lights, all of which had been subjected to heavy weather conditions, in no case did any light actually fail in service.

These lights were developed for use on destroyers under stress of war conditions and over 300 were delivered to the Navy before the armistice was signed, we kept ahead of the destroyer program, and this was extremely important, as we all know, on account of the submarine menace.

On the next to the last page of Mr. Kelly's paper there is mention as to whether the searchlights are really useful at sea. There has been some doubt of their value in the past, but reports from Admirals Beatty and Jellicoe, of the British showed that the Germans had superior equipment, and the British avoided battle at night—that is because the Germans had developed tactics that enabled them, when a hostile ship was observed to open the searchlight shutters with the light already directed on the target. This was possible due to highly developed fire control system, which is being now installed in our own ships and has already been adopted by the British

Navy. With this system the light is only shown long enough to fire a single salvo and not long enough to make a target of the vessel. There is at present no doubt that in naval practise, searchlights are absolutely necessary for night fighting.

J. C. Ledbetter: I am very much interested in the high intensity light, and I want to know whether a light of such brilliancy and such marvelous performance could possibly be used in some commercial way. Could not a light of this character be used in stage lighting, or flood lighting, or in some spectacular way, such as street light and advertising lighting in such places as Times Square. It might be that large business concerns would have use for a light of this character.

R. A. Beekman: I wish to point out one feature with regard to the difference between the third electrode and the thermostatic control. The principle object, of course, of both of them is to maintain the crater in the focus of the mirror. The mirror is mounted rigidly in the drum of the searchlight, and hence the logical thing is to have the device, controlling its position, mounted on the drum of the searchlight, which is the form used with the thermostat in the past. It is now proposed to mount the thermostat on the lamp, as is done with the third electrode, and this does not provide a rigid coupling between the position of the positive crater and the focus, as is the case when the thermostat is mounted on the drum.

B. P. Beehler: (Read by R. A. Beekman). I desire to call attention to the following points in Mr. Kelly's paper:

Paragraph 1. The range of a 24-in. high power light is at best 3000 yds.

Paragraph 2, 5th sentence. The current density has no connection whatever in the formation of the gas crater or holding the gas. This feature is entirely controlled by the direction of the arc stream. The high current density of the positive electrode may be maintained without an arc stream or gas ball formation.

Paragraph 4, last sentence. The feeding of the positive carbon in lamp operation is without exception, where automatically controlled, accomplished by independent solenoid or magnet operation.

Paragraph 7, last sentence. The formation of scale or oxidation of metallic third electrode does not increase its life. The prevailing feature permitting of the use of a metal third electrode is the fact that the temperature of flame directly back of the crater is below the melting point of the copper. The metallic electrode or any type of this control has proved unsatisfactory for any arc of over 75 amperes. This 75-ampere arc is not a true gas arc and therefore the maximum temperatures are considerably lower than high current arcs. The use of metallic third electrode has been disapproved by the U. S. Navy for all arcs over 75 amperes.

Paragraph 8, second and third sentence. The object of alcohol in the high power arc is not to cool but to surround the electrodes with an oxygen insulator, thus preventing tapering of electrodes from air burning and enabling the burning of the electrode with its tip far removed from metallic positive head and still maintain the crater at electrode diameter.

The cooling by air, later developed, is merely cooling of the metal, allowing the use of a metal head projecting over the electrode nearly to its tip, which metal insulates the electrode from atmospheric oxygen and thus eliminating the necessity of the alcohol. The air cooling also allows of use of less metal in heads, thus cutting down mirror shadows and beam holes. The high power lamp using alcohol on the electrode had very large and heavy heads, the cooling of which was accomplished by radiation.

Paragraph 11, 4th sentence. The absorption of strip glass in the front door did not exceed 15 per cent and averaged about 10 per cent absorption of light.

Paragraph 13. The dome front doors of the present parabolic and spherical types have proven inefficient. Their disadvantage is that they not only absorb about 15 per cent of light but they actually disperse about 15 per cent of light, throwing this light on the foreground, changing beam formation by virtue of the above fact and blind the observer by making a field of stray light through which he must look to see the target

Paragraph 15. It has been found that the present sizes of electrodes are probably the proper proportions to give best service. A rise in current density, at present about 1.65 amperes per sq. mm. will only add to flame and increase carbon consumption, while reducing the density by increasing the electrode size will cause loss of intensity.

Paragraph 16. It has not yet been established conclusively that the intensity of the high power arc is a factor of temperature. A theory of fluorescence of the gas ball has presented itself indicating that the supposed black body radiation of the arc is not a fact, but that a composite effect of black body radiation and fluorescence tend to give spectrum characteristics which are misleading, and indicate temperatures far in excess of those actually present.

It has been found that the use of solid electrodes will not under any condition of burning give a light equal in intensity or visibility to that of the high power arc. Solid electrodes have a critical temperature at best of 3300 deg. cent. and will not produce the gas ball phenomena as in the special cored electrode. It has been practically impossible to date to operate a solid electrode efficiently at a current density in excess of 0.66 amperes per sq. m.m. whereas the high power positive operates at a current density of 1.65 amperes successfully and efficiently.

Chester Lichtenberg: The Chief of Engineers, who is responsible for Army searchlights, has read Mr. Kelly's paper with considerable interest. He noted that no mention was made of mobile searchlights, although this variety had undergone a remarkable development during 1918 and 1919. He thought that the members of the Institute might be interested in seeing photographic reproductions of some of the more prominent recent developments of mobile searchlight equipment, and he has authorized the presentation of certain representative views.

The mobile Army of the United States was equipped in 1917 with only a few portable searchlights. These were of two varieties. One is shown in Fig. 1. It consists essentially of a 36-in. medium intensity searchlight mounted on an extensible tower, which is in turn mounted on a 3½-ton motor truck. The power unit is a 15-kw., 125-volt gasoline-electric set, mounted on a duplicate 3½-ton truck. Eight of these outfits were part of the engineer train equipments. They were used during the 1917 campaign on the Mexican border. They were not considered sufficiently mobile to accompany some of the army units. As a result, a horse drawn equipment was developed. This consists essentially of a 24 in. high intensity arc searchlight placed on mast or tower extensible to 16 feet above the ground. The searchlight and tower are mounted on a caisson. The energy is supplied by a 5-kw., 65-volt gasoline-electric set mounted on a limber. Eight of these units accompanied the first engineer train which left the United States for France on August 2, 1917.

Experience in France indicated that neither of the mobile searchlight units which formed part of the equipment of the Army at the outbreak of hostilities was suited for operation along the Western Front. The searchlights on masts were particularly unsuited since, if they remained lit more than 30 seconds within the zone of the advance, they were demolished by enemy shells. This was due to the extreme accuracy of the range finding equipment of the German armies and the rapid responsivity and excellent coordination of their artillery fire. A survey station indicated that high-power searchlights on small mobile mounts, which could be quickly emplaced, were desirable. Specifications for these were sent to the United States late in 1917.

The first attempt to meet the specifications of the A. E. F. is shown in Fig. 2. This is essentially the standard 36-in. size seacoast searchlight, with a new design of trunnion arms and with a wheeled carriage. It utilized existing tool equipment so far as this was practicable, at the same time reduced the weight of the searchlight about 50 per cent, and greatly increased its mobility. About 36 of these searchlights were in use in the 1st and 2nd American Field Armies at the time the armistice was signed. They were supplied with power from various kinds of power plants. Some were supplied by French

17-kw. sets of various makes, some by Riker trucks, the engines of which were equipped with 12-kw. generators and some with Mack trucks, the engines of which were equipped with 15-kw. generators.

A. E. F. experience indicated, however, that 36-in. searchlights were not quite powerful enough. New specifications called for 60-in. portable or semi-portable searchlights. To meet this demand, the design shown in Fig. 3 was developed. It consists essentially of the standard seacoast searchlight equipment placed on trailers. On the left is shown a 60-in. stationary type high intensity arc searchlight with its cable reel, rheostat, etc. On the right is shown a stationary 25-kw. 115-volt gasoline-electric set with its radiator, muffler, spare gasoline tanks and auxiliaries. This equipment, although mobile, was quite heavy and could only be moved at relatively low speeds. It was large in size and difficult to entrench. Only a few of these units were ordered to fill in the gap until more mobile equipments could be provided. Meantime large quantities of 60-in. coast defense searchlights, Fig. 4, and their generating equipments were shipped to France and placed in active service. One of these is shown in Fig. 5, with its emplacement and 3-meter sound detecting paraboloid. It is an installation by the 56th U. S. Engineers in the St. Mihiel Sector.

Fig. 6 shows one of the early attempts at making a very light weight, extremely mobile 60-in. high-intensity arc drum-type searchlight. It weighs, complete, about 2500 lb. Before it was completely developed, however, the design had been superseded by the open-type which was suggested in the spring of 1918 by Major R. W. Lewis, the representative in the United States of the A. E. F. Searchlight Regiment, the 56th U. S. Engineers.

The original open-type searchlight, made in accordance with Major Lewis' suggestions is shown in Fig. 7. It was made up at Ellington Field, Texas by a detachment of the 56th U. S. Engineers. They took the mirror and back door from a 36-in. standard Navy searchlight and supported it in rude trunnions and trunnion arms. It was arranged to be rotatable about a horizontal axis only. In front of the mirror was placed the automatic high-intensity arc mechanism, usually placed in the barrel of a 36-in. searchlight. The carbons and the upper part of the mechanism were enclosed in a tin can, which was intended to restrict stray light and to protect the arc from wind. The searchlight was operated for several weeks and proved unusually efficient. It had one very important characteristic. It had a very clearly defined beam, with a minimum of stray light. This was, no doubt, due to the fact that there was no barrel and so there were no reflections from the interior. Beside, it had no front glass so that there was no dispersion.

The open-type searchlight seemed to be such a feasible proposition that negotiations were immediately opened with

FIG. 1—TWO UNIT MOBILE SEARCHLIGHT EQUIPMENT, MOUNTED ON 3½-TON TRUCKS, SEARCHLIGHT BEING ARRANGED FOR ELEVATION BY MEANS OF AN EXTENSIBLE TOWER.

[LICHTENBERG]

FIG. 2—36-IN. BARREL TYPE WHEELED BASE SEARCHLIGHT, SUCH AS USED BY 1ST AND 2ND FIELD ARMIES, A. E. F.

FIG. 3—TRAILER TYPE OF MOBILE SEARCHLIGHT EQUIPMENT, CONSISTING OF 60 IN. BARRELL SEARCHLIGHT AND 25-KW., 125-VOLT GASOLINE-ELECTRIC GENERATING SET.

[LICHTENBERG]

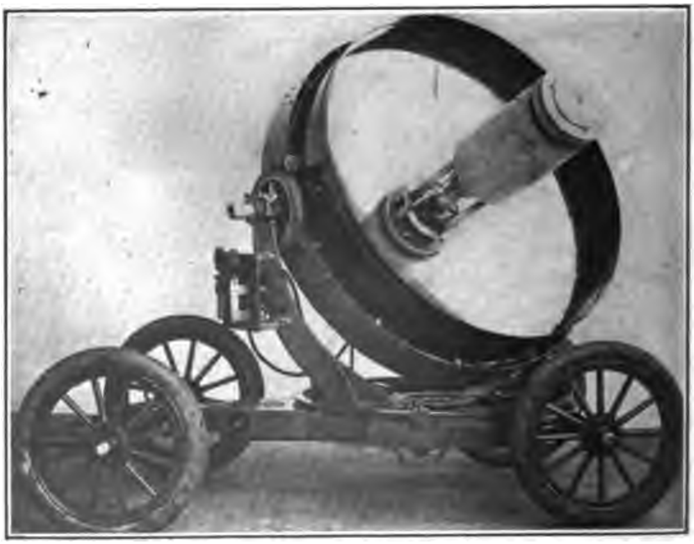
FIG. 4—60 IN. HIGH INTENSITY ARC BARREL TYPE SEARCHLIGHT, DESIGNED FOR COAST DEFENSE INSTALLATION AND USED DURING OPERATIONS OF THE 1ST AND 2ND ARMIES, A. E. F.

FIG. 5—A. E. F. SEARCHLIGHT EMPLACEMENT, SHOWING 60 IN. HIGH INTENSITY ARC, BARREL TYPE SEARCHLIGHT AND THREE-METER PARABOLOID.

[LICHTENBERG]

FIG. 6—60 IN. HIGH INTENSITY ARC, LIGHT WEIGHT, BARREL SEARCHLIGHT ON WHEELED BASE

FIG. 7—ORIGINAL OPEN TYPE SEARCHLIGHT, WITH 36 IN. MIRROR AND 150-AMPERE HIGH INTENSITY ARC, AUTOMATICALLY OPERATED MECHANISM

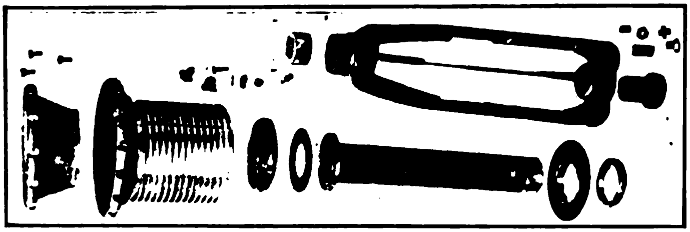


[LICHTENBERG]

FIG. 8—1919 MODEL OF OPEN TYPE SEARCHLIGHT, WITH 60 IN. MIRROR AND 200-AMPERE MEDIUM INTENSITY ARC, HAND FEED MECHANISM

FIG. 9—1919 MODEL OF OPEN TYPE SEARCHLIGHT, WITH 60 IN. MIRROR, SHOWING WHEELS REMOVED AND BASE READY FOR EMPLACING

FIG. 10—MECHANISM FOR 200-AMPERE MEDIUM INTENSITY ARC, AS USED IN OPEN TYPE SEARCHLIGHT



[LICHTENBERG]

FIG. 11—PARTS OF MECHANISM FOR OPERATING 200-AMPERE MEDIUM INTENSITY ARC IN OPEN TYPE SEARCHLIGHT

FIG. 12—VERY LIGHT WEIGHT MODEL OPEN TYPE SEARCHLIGHT, WITH 30 IN. MIRROR AND 150-AMPERE HIGH INTENSITY ARC, HAND FEED MECHANISM

[LICHTENBERG]

FIG. 13—MOBILE SEARCHLIGHT POWER UNIT AND LORRY, WITH COVERS REMOVED, SHOWING GENERATOR, SWITCHBOARD, CABLE REEL AND 36 IN. WHEELED BARREL SEARCHLIGHT

several large searchlight manufacturers to produce commercial designs. Several months' effort was expended and, as a result, a very light weight, 60-in. open-type searchlight was produced. One of the highly developed designs is shown in Fig. 8. It weighs complete 1200 lb. It can be provided with either medium intensity or high intensity arc mechanisms. It has been operated in all kinds of weather and in the ordinary winds where aircraft can travel. It has proven unusually successful. It weighs about one-fifth as much as the old 60-in. searchlights and is just as efficient. Besides, it has no complicated mechanism and is very easily operated by partially skilled personnel. The running gear can be easily demounted from it, as shown in Fig. 9. This makes it a simple matter to emplace. The mechanism also is exceedingly simple. A medium intensity design is shown in Fig. 10. It consists of relatively few parts which are strong, as shown in Fig. 11. The result has been a simplified, very light weight and mobile searchlight, which is easy to move and emplace and which can be operated by a minimum number of partially skilled individuals and yet be just as effective as the very heavy, complicated searchlights previously standardized for certain classes of service. Besides it is easy to manufacture and takes previously standardized mirrors and carbons.

Another form of light weight, open-type searchlight is shown in Fig. 12. It was developed essentially for extremely mobile service where light weight is essential. It weighs 225 lb. as shown. It has a 150-ampere high-intensity arc mechanism, which is hand fed. It has a 30-in. glass mirror. It gives fully as good service up to 14,000-ft. range as do the 60-in. designs. However, it is not nearly so good beyond 14,000 ft. on account of the wide spread of its beam.

The advent of very mobile *searchlights* brought with it the need for mobile searchlight *power units*. One of the early designs is shown in Fig. 1. This was successful but not entirely satisfactory. Another of the early designs is shown in Fig. 3. This unit was not self-propelled and besides could only be moved at low speeds and then not readily. To meet the situation, a standard Engineer Department Mack 5½-ton cargo truck was modified by the addition of certain electrical auxiliaries. A generator was mounted on an extension of the crank shaft. A switchboard was placed under the body. Rheostats were located on the chassis. A cable reel was mounted in the front of the cargo space. A wheeled barrel 36-in. searchlight was placed in the back of the cargo space. The result was a complete and mobile searchlight unit which could be run over practically all ordinary roads at speeds up to 12 miles per hour and which could operate an automatic high intensity arc searchlight practically continuously. It is shown with covers removed in Fig. 13. About thirty of these units were used by the 1st and 2nd Field Armies of the A. E. F. prior to the Armistice.

The development of the open-type searchlight reduced the weight of the searchlight from about 7000 lb. to about 1200 lb. This permitted a very much lighter weight power unit to be employed. The situation was surveyed, and, based on careful studies, the power unit shown in Fig. 14 was developed. It consists essentially of a standard Cadillac ambulance chassis with a 21-kw. G. E. generator, mounted, on the propeller shaft between the transmission and the differential, as shown in Fig. 15. Mounted on this chassis is a special, very light weight body which has ramps for carrying the searchlight, as

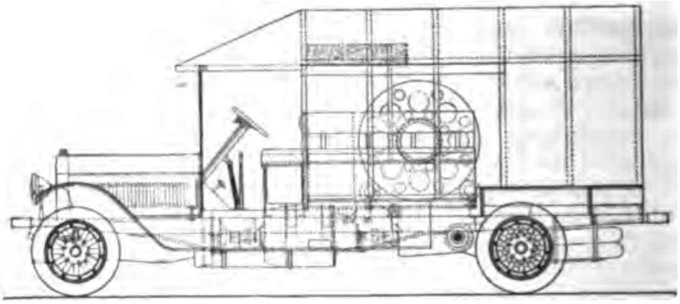


Fig. 18—50-Kw. Size Mobile Searchlight Power Unit and Lorry, Designed for Carrying 60-in. High-Amperage Searchlight, Twelve Men and All Their Auxiliaries

Length overall—21 ft., 10 in.; height—9 ft., 5 in.; width—6 ft., 2 in.; wheel base—177 in.; wheel diameter—36 in.; wheel gauge, front—62 in.; rear—67 in

Note: Side curtains equipped with extra fly of sufficient length to extend 6 feet from side of car at ground line.

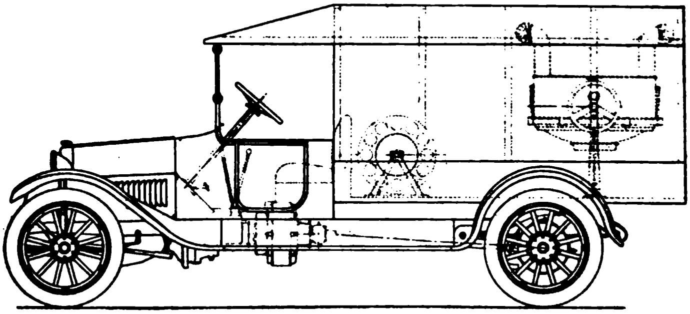


Fig. 19—6.5-Kw. Mobile Searchlight Power Unit, Designed For Carrying 30-in. Open Type Searchlight and Three Men

Length overall—15 ft., 10 in.; height overall—7 ft., 6 in.; width overall—5 ft., 5 in.; wheel base—134 in.; wheel diameter—33 in.; wheel gauge—56 in.

FIG. 14—LATE 1918 DESIGN MOBILE SEARCHLIGHT POWER UNIT AND LORRY, WITH 60 IN. OPEN TYPE SEARCHLIGHT

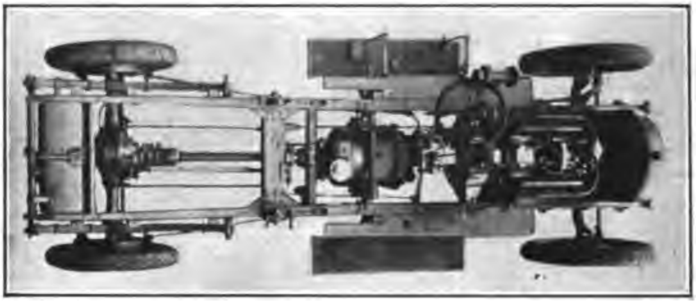


FIG. 15—TOP VIEW OF LATE 1918 DESIGN OF MOBILE SEARCHLIGHT POWER UNIT CHASSIS, SHOWING INSTALLATION OF GENERATOR BETWEEN TRANSMISSION AND DIFFERENTIAL

[LICHTENBERG]

FIG. 16—LATE 1918 DESIGN OF MOBILE SEARCHLIGHT POWER UNIT AND LORRY, WITH COVERS REMOVED, SHOWING SEARCHLIGHT IN POSITION

FIG. 17—LATE 1918 DESIGN MOBILE SEARCHLIGHT POWER UNIT, OPEN TYPE SEARCHLIGHT, PARABOLOID AND 60 IN. BARREL TYPE SEARCHLIGHT, IN OPERATION WITH 2ND FIELD ARMY, A. E. F.

[LICHTENBERG]

FIG. 20—RAILWAY TYPE SEARCHLIGHT EQUIPMENT, WITH POWER UNIT IN BOX CAR AND SEARCHLIGHT MOUNTED ON HINGED TOWER

shown in Fig. 16. The entire equipment was conceived, designed and built in four months and was delivered and in operation with the 2nd American Field Army in October 1918. The complete equipment weighs 9000 lb. It has an intermittent output of 21 kw. It carries a 60-in. open-type searchlight, provided with both medium intensity and high intensity arc mechanisms and a crew of five men, with all the necessary auxiliaries for operation in the field. Another is shown in Fig. 17 operating an open-type searchlight in the 2nd A. E. F. Army sector.

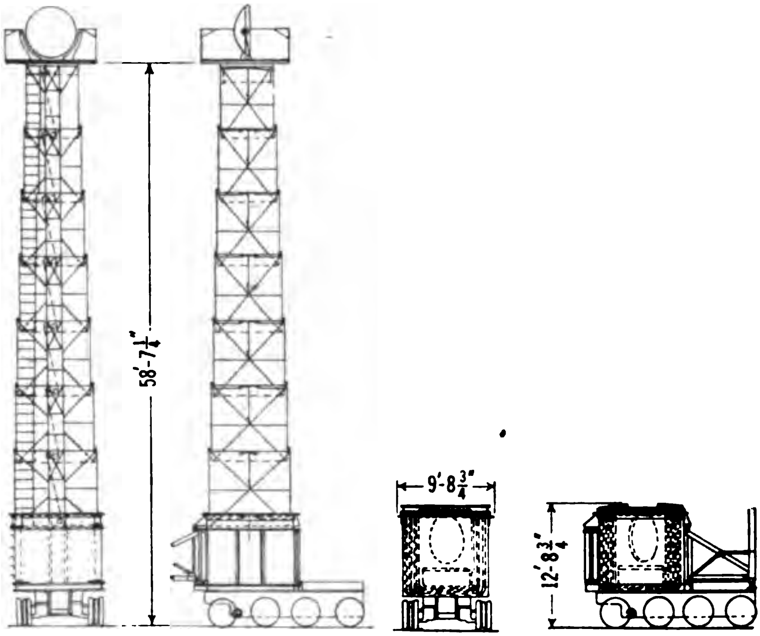


Fig. 21—Mobile Searchlight Tower and Mount, Illustrating Telescoping Type of Extensible Tower

The success of this combined design of mobile power unit and lorry gave rise to a new line of searchlight power units. A 50-kw. design is shown in Fig. 18. It employs a LaFrance Fire Engine chassis with 177-in. wheel base. It has provision for a 60-in. high-amperage searchlight, a crew of 12 men, 500 ft. of cable and all auxiliaries for maintaining this force in the field for one week. Fig. 19 shows a 6.5-kw. unit, employing a Dodge chassis. This carries a 30-in. 90-ampere high-intensity open-type searchlight, with a crew of three to four men.

Mobile searchlight units have been extended to include lights on towers for coast defense purposes. Fig. 20 shows a recently completed design. It consists essentially of two railway cars, which carry a searchlight and its tower and the power plant respectively. The tower, when raised, places the center of the searchlight 33 ft. above the top of the tracks. The searchlight is a 60-in. high-intensity coast defense design similar to the one shown in Fig. 4. It is supplied by power from either one of two standard 25-kw. 115-volt gasoline-electric sets, which are located in the box car. These are similar to the one shown at the right in Fig. 3. The sets are operated in series for supplying power to electric motors located on car axles. The motors are regulated by standard railway grids under the car floors and controllers placed at the car ends. By this means, the unit is made self-propelling at speeds up to 10 miles per hour.

Another form of mobile searchlight tower now in development, is shown in Fig. 21. It consists essentially of a special form of searchlight tower mount, with a telescoping form of tower, on top of which is placed an open type searchlight. The complete equipment weighs twenty tons. It is so arranged as to have the load distributed on a track running over the wheels. This gives an average bearing of about 700 lbs. per sq. ft. It can operate at speeds up to 20 miles per hour on good roads or up to 10 miles per hour over very bad roads. It will go through a swamp where the water is not over 3 ft. deep and will climb a 75 per cent grade.

Ralph Kelly: In answer to Mr. McNicholls question, the special feature of the lamp used for signalling work is that it has a concentrated filament and is filled with hydrogen gas. This gives a high intensity source of light with quick flashing and dying out but with very short life. The lamp used for searchlight work is similar to the signalling light in having a concentrated filament but is filled with nitrogen gas with a slow period of flashing and a long life. I believe the filament temperature of the hydrogen lamp is the highest.

The Navy experimented with metal mirrors a number of years ago but found they did not keep their shape and tarnished very quickly under sea service. The parabolic glass mirror has proven very satisfactory in Naval service with a comparatively few breaks on record that were caused by gunfire.

The army mobile Cadillac searchlight unit represents a remarkable development and among the many types of equipment that have been developed for field work, there are none that can compare with it for mobility and efficiency.

There is only one way to fight enemy planes at night and that is to illuminate them with searchlights and fight them with aircraft. The friendly aircraft have the advantage of darkness while they attack enemy planes illuminated by the searchlights. Anti aircraft guns are of little use except in protecting the searchlight equipment by preventing enemy planes from flying down the beam to a low height where they can bomb the searchlight personnel. I might say that a searchlight must frequently have its position changed to prevent being shelled by enemy artillery or by being used as a land mark for navigation of enemy planes. This is the great reason why a field searchlight should be light and mobile.

Mr. Bassett stated that the searchlight mirrors are fairly well standardized but I don't think the Army Engineer Corps will agree with him on this point. They are still in the midst of a very extensive investigation in the field of metal mirrors in which they have encountered and are still encountering tremendous difficulties.

Mr. Patterson's figures comparing the 12 in. and 24 in. low power searchlight hold good for those lights under laboratory conditions when operated by expert personnel. In actual service the present design of low power light operates very inefficiently as described in the paper. In my opinion there is no question that a well designed low power light, such as the 12 in. can be manufactured to give the same light on the target as the present 24 in. light as it is now inefficiently operated in service. The type and design of that light is a problem for the future but it is generally conceded that their is great room for improvement in the present low power light.

The 12 in. light I had in mind in writing the paper was one designed and built by Major Judson and Lieut. Thompson U. S. M. C. which used carbons of a very refractory material with an arc current of 30 amperes. The positive carbon had a diameter of 3/8 in. while the negative carbon diameter was 1/4 in. which resulted in a far greater intrinsic brilliancy of light source than the present low power light. This light was equipped with a ground glass finder which added materially to its efficiency as it enabled the operator to keep the carbons in focus at all times.

In regard to Mr. Patterson's recommendation that the 12 in. and 18 in. sizes both be retained, it is not only my own opinion but that of Naval officers generally that the Navy standardize on one size of low power searchlight. That size should be either the 12 in. or the 18 in. as determined after service tests.

Commander Beehler's discussion gives the status of the third electrode control of the positive carbon for Naval work, and also answers Mr. Murphy's question on that point.

The data Mr. Murphy gave on ventilation of searchlights are extremely interesting. His company did the pioneer work in the field of searchlight ventilation and had many troublesome obstacles to overcome before they arrived at their present arrangement. This has worked admirably on the 24 in. high power searchlights under the severest of naval service conditions.

As stated in the paper, the thermostat development by Mr. Murphy has proven very successful in service. It will however be of advantage to mount the thermostat on the lamp structure out of the weather and where no special adjustments are required to maintain the positive crater in the focal position.

There was a great deal of question during the war regarding the real value of a searchlight on a naval vessel but there finally developed a strong feeling that well designed searchlight equipment, properly used will prove of immense value to a ship in a night engagement. This feeling was confirmed by both Admiral's Beatty and Jellicoe in their reports on the battle of Jutland.

As Commander Beehler brings out, it is not the high current density of the carbons alone that causes the high intensity phenomena, but the combination of that high current density and the angle of the negative arc stream. The dome glass door, in accordance with his tests, is not as efficient in transmitting light as a clean plate glass strip door. The great point in favor of the dome glass door is that it stands up against gun fire while the present type of plate glass door shatters under gun fire. Until the plate glass strip front door is developed to resist gunfire, it is only feasible to use the less efficient dome glass door for the major characteristic of a front door is that it must withstand gunfire. There is also a field of development in improving the optics of the dome glass door with a view to improving its light transmission efficiency.

The question of size, material, and current density of searchlight carbons is still one in which there are a number of conflicting opinions. If a more refractory material is used for the carbons which gives a greatly reduced tail light, the light source will unquestionably be of lower intrinsic brilliancy than is obtained with the present type of high powered carbons. However it is the light that comes back to the observer from the target that counts and with the absorption of the atmosphere in the nature of the 6th power, it is apparent that there can be a material reduction in the brilliancy of the light source and still effect but very slightly the light reflected back to the observer from the target. Is it then worth while to have a lower brilliancy light source and eliminate the objectionable foreground lighting caused by the tail light of the high power light or is the sacrifice in intrinsic brilliancy of the light source too great for the advantage gained?

Presented at a Section Meeting of the American Institute of Electrical Engineers, San Francisco, Cal., February 7, 1919.

UNIFICATION OF THE MANUAL AND AUTOMATIC TELEPHONE SYSTEMS

BY D. E. WISEMAN

Abstract of Paper

A description is given of the physical consolidation of the Bell manual and the Automatic Electric telephone systems of Los Angeles, Cal., which previously to June 1, 1918 operated as separate systems. While similar consolidations had been made previously they included relatively small volumes of traffic and afforded no engineering precedents as a guide to the consolidation of two systems serving 129,000 stations. The plans for the physical union of the two companies were devised by a joint committee of engineering representatives, and contained a number of novel operating and construction methods which are described.

Under the new system each subscriber has access to every other subscriber in the Los Angeles exchange and to all long-distance lines centering there. Where duplicate services were installed the subscriber was given his choice as to whether he would retain the manual or the automatic system, and about 13,000 duplicates were eliminated, the choice between the two types being equally divided.

IN telephone engineering and the resultant physical and economic accomplishment, June 1, 1918 figures prominently, for on that date the formal union of the Bell manual telephone system operated by The Pacific Telephone and Telegraph Company, and the Automatic Electric system operated by the Home Telephone and Telegraph Company in the City of Los Angeles, was effected under the management of the newly created Southern California Telephone Company, giving to every telephone user in that area a unified and unrestricted exchange telephone service and universal service over toll trunk lines to some eleven million telephones throughout the United States. Consolidations of this character have been made prior to this date but involving relatively small volumes of traffic and simple operating methods so that there were no records of actual performance or established engineering practises to serve as a precedent and guide for determining the effect of and the physical requirements necessitated by the sudden release of two large, distinct and separately bound volumes of traffic into a common channel.

Because of the novel operating and construction methods and the speculative possibilities involved in the important pioneering work of welding these fundamentally different telephone switching systems into a single eighteen million dollar plant serving 129,000 stations, a semi-technical summary of the events has been prepared for general information.

A brief reference to the conditions which brought about the consolidation will be made in order to better understand the problems that confronted the two competing telephone companies and their subscribers. Believing that competition instead of control was the automatic remedy to apply to public utilities, the City of Los Angeles invited telephone competition about sixteen years ago and then struggled along with her business firms and many of her residents paying two telephone bills for a divided and what proved to be an unsatisfactory telephone service. This condition was continued until the year 1916, when the public decided by popular vote to bring about an end to dual telephone service. Negotiations were begun and various proposals were considered by all concerned in an effort to avoid any waste or arbitrary measures. A plan was finally accepted for the organization of a local telephone company, which was to purchase the properties of the existing operating companies and unify the service, continuing with the equipments then in plant and giving the right to the telephone users to determine for themselves whether they would retain their automatic stations or manual stations. War conditions imposed restrictions in the conservation of materials and men for such projects and called for a careful weighing of the expected benefits and expenditures of materials and labor.

With this clear understanding of the requirements, a joint committee of engineering representatives was appointed to determine the methods for the physical joining of the two systems. As a result of their efforts, a fundamental plan, together with preliminary estimates of cost, was submitted and formally approved by the City of Los Angeles, the Railroad Commission of the State of California and by the Attorney General for the Federal Government, and on May 1, 1917, formal authorization was given to proceed with the project.

In order to picture the plants as they existed prior to the consolidation, I shall refer briefly to the physical properties and the operating methods of the two systems. Referring for a moment to Fig. 1, the area served by the duplicate plants was about 200 sq. mi. and lying within the corporate city limits. The open and solid circles show office and wire centers of the Home and Pacific Companies respectively, while the dotted and solid lines mark the areas of the respective districts. The Pacific Company was established first and the opening and location of its offices followed the telephone development of the city. The first office was located in the business area and as the population increased and spread to outlying districts, new offices and districts were established. Each office was located as near as practicable to the center of the wire distributing system as determined by a study of the existing plant and expected growths for 15 or more years hence. As years go by and old offices are outgrown studies very often show an economy in subdividing the original district or changing the boundaries so that this natural process tends to correct any errors in estimates of growths and locations of such expected growths.

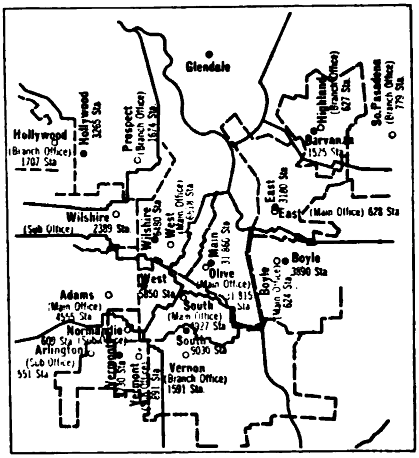


FIG. 1—AREA SERVED BY DUPLICATE PLANTS

May 1, 1917, the Pacific Company's exchange consisted of its standard outside plant and station equipment and nine manually-operated central offices serving about 69,000 stations. Bell equipment was used in units having a capacity of 9600 multiple lines. Telephone connections were established generally by the calling subscriber removing the receiver from the switch hook, causing a light to appear before an answering *A* operator, who upon receiving a request for a particular number cut-in on an order wire to the particular switchboard unit in the district, indicated by the prefix of the number called-for. An operator at the distant switchboard unit, assigned a trunk over this order wire to the calling *A* operator and completed the connection by plugging into the called-for subscriber's multiple. Fig. 2 shows a schematic transmission circuit of a typical connection. The operation of this circuit will be described later and in connection with the unified plant.

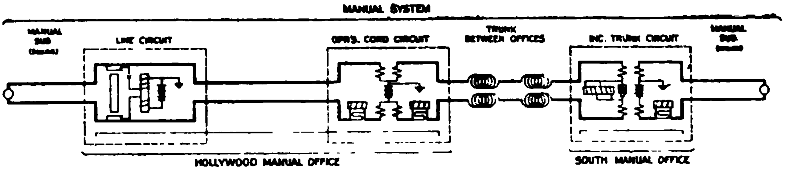


FIG. 2—EQUIPMENT INVOLVED IN A CONNECTION BETWEEN A HOLLYWOOD OFFICE MANUAL SUBSCRIBER AND A SOUTH OFFICE MANUAL SUBSCRIBER

The former Home Company operated an automatic exchange including a parallel and similarly constructed outside plant and fourteen offices serving a total of approximately 60,000 stations. About 35,000 stations were equipped with dials and approximately 25,000 manual stations operated from private branch exchanges and as public pay stations. The lines from the manual stations terminated on a 45-position manual switchboard in the Olive Office. Calls from automatic stations to automatic stations were made by dialing five or six digits, as required, to reach the particular district and individual subscriber. In capacity the former Home Plant was one of the largest and probably the most successfully operated of any automatic system installed in the United States or abroad. Home Company private branch exchange subscribers were reached by dialing the private branch exchange operator, who completed the connection. Calls outgoing from private branch exchange subscribers were trunked to the man-

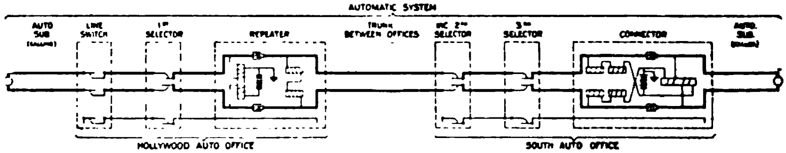


FIG. 3—EQUIPMENT INVOLVED IN A CONNECTION BETWEEN A HOLLYWOOD OFFICE AUTOMATIC SUBSCRIBER AND A SOUTH OFFICE AUTOMATIC SUBSCRIBER

ual transfer board above mentioned, the calls coming in on an automatic traffic distributor which placed each line lamp signal before a non-busy operator. The operator upon taking up these connections would complete the call direct if to another private branch exchange station through the subscriber's multiple or dial the number required, if an automatic station was wanted. Fig. 3 shows a schematic transmission circuit of a typical full automatic connection. The operations and functions will also be described step by step later.

Los Angeles has had the distinction of having more telephones per capita than any other city in the world and is now very close to the top of the list. This general usage is reflected in the number of calls originating in each system, as shown by the records of traffic. The approximate numbers of daily average calls originating in the former Pacific plant and Home plant were in the vicinity of 430,000 and 420,000 respectively.

The engineering problems demanded, therefore, a reasonably close approximation of the volume of existing and added traffic which could be expected to flow between the groups of stations of the various districts of the two plants; a determination of the most direct and economical routing of such calls based on efficient operating methods and the use of existing facilities; and the design and development of an inter-unit trunking plant and switching circuits necessary to maintain the commercial standards for transmission and supervision between the two systems regardless of the mechanical, electrical and operating inequalities. Visual and audible signals peculiar to the separate systems required for supervisory purposes needed to be synchronized or harmonized and extended when necessary so as to afford common usage.

It is obvious that a great many plans and combinations of plans for unification were developed in sufficient detail to determine their relative capital and operating costs and advantages and disadvantages. The plan in principle that was adopted for handling the inter-office traffic between the systems was to operate all existing offices as units of the complete exchange and route the new automatic or manual trunks of each office into the adjacent office of the opposite system, where the connection could be completed by the most direct method and route.

Having referred to the facts that were most vital in shaping the project, I should like to outline the organization and schedule of work involved in the construction program estimated to cost $1,250,000. Every reasonable effort was demanded to bring about a unification of the properties without waste of time, and accordingly a complete schedule for the ordering of materials, manufacturing, assembly and installation thereof was set up after a canvas of probable material deliveries and of the labor situation. It became evident that a period of twelve months would be the shortest time possible to complete the necessary work, keeping in mind that war demands might upset the schedule, and all effort was centered on finishing the project within the minimum time. Specifications and plans were completed for each individual project, numbering all together about 110, and each one was charted with due regard to the materials involved, quantity, desired time of delivery and installation and its relation to the project as a whole. Fig. 4 shows a typical group of projects and illustrates the method of maintaining the schedule for the whole program. As items of material were, of course, duplicated in a great many of the specifications, a master chart was prepared showing the total quantities of each of the thousands of items required at specified dates.

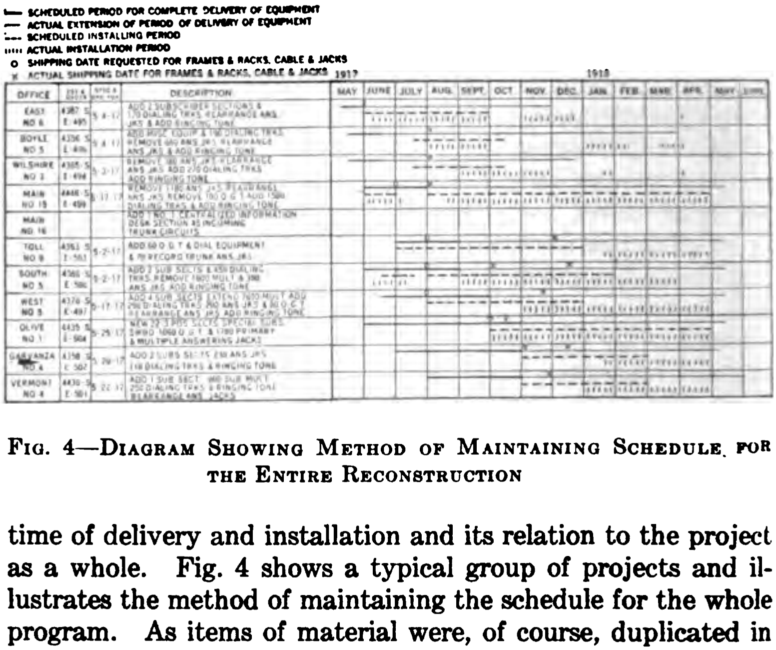


FIG. 4—DIAGRAM SHOWING METHOD OF MAINTAINING SCHEDULE FOR THE ENTIRE RECONSTRUCTION

The largest single project consisted in the design and manufacture of the 66-position special tandem switchboard and associated equipment to be located in Olive Office for the translation and distribution of calls from automatic stations to manual stations in the downtown area. This switchboard, together with the switching circuits were designed in detail by the engineers in the general office of the telephone company, and all of the manufacturing and assembly work was performed in San Francisco. Extensive rearrangements of the Olive office building were necessary to provide space for this equipment and for retiring quarters for the large number of operators required to give the necessary 24-hour continuous service. Fig. 5 shows the floor plan arrangement of this switchboard and associated operating room space.

In the manual offices it was necessary to equip the regular subscriber positions with a dial for sending out the electrical impulses, and to provide outgoing trunks connecting directly with the automatic equipment in the adjacent office. There were approximately 375 subscriber switchboard positions in the manual system in the Los Angeles exchange that required the installation of this special equipment and associated wiring and this proved to be one of the most difficult parts of the work, because such work had to be performed on positions of switchboard that were in continuous operating service. I will refer to Fig. 6 and describe the switching circuit associated with the dials at the operators' positions. Each operator on the subscribers' switchboard is provided with 10 outgoing trunks terminating in the nearest automatic office on first selectors. These outgoing trunks are provided with twin jacks, one above the other, and electrically connected so that the dialing device can be associated with each of the trunk circuits to the distant

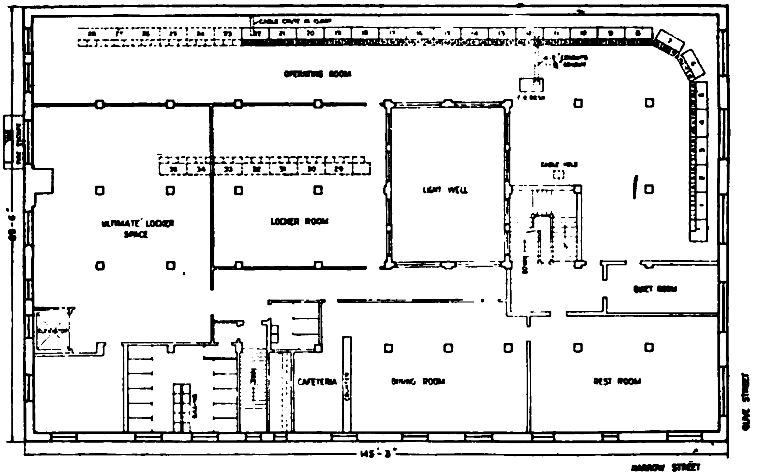


FIG. 5—PLAN OF SECOND FLOOR OLIVE OFFICE, LOS ANGELES, CAL.

office. The plug connecting with the dial is inserted in the lower first jack while a call originated by a manual subscriber for an automatic station is connected to the upper first trunk jack by means of the regular *A* operator's doubled-ended cord circuit. The operator then proceeds to dial the number requested and as soon as this number has been dialed, the operator moves the dial circuit plug to the lower second jack, thereby pre-selecting the trunk circuit for the next call for an automatic station. The operation of the second and third selectors and final connector is the same as described later under "Automatic to Automatic Connections." The circuit is so arranged that the manual *A* operator just referred to receives direct supervision for both the calling manual and the called for automatic station. At the end of the conversation, the lighting of a lamp associated with each end of the *A* cord circuit indicates to the operator that the connection has been finished and all cords are to be cleared from the subscriber's line and trunk jacks. The outgoing dial trunks are used in rotation and the actual number of the trunks connected by such lines to the distant office varies with the traffic requirements.

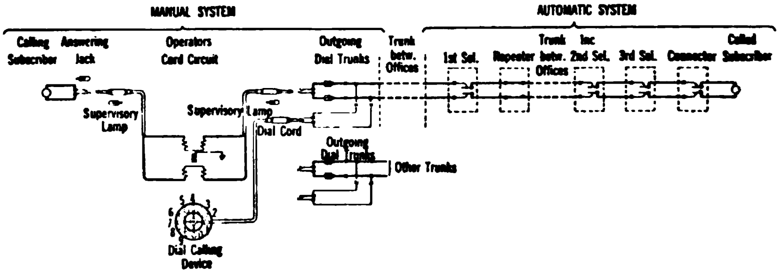


FIG. 6—METHOD OF ESTABLISHING A CONNECTION BY MEANS OF DIAL CALLING DEVICE

Central office telephone installation work has generally been organized for individual projects usually confined to a particular office building. The plan of scheduling the material and utilizing this material to its greatest advantage made it necessary to depart from the regular practises of the installing forces and to provide that all of the work should be treated as one project wherein the men trained for specific work were to be moved from one office to another as materials arrived and thereby facilitating the completion of the work regardless of irregularities in the arrival of materials for a particular office.

This arrangement was one of the important factors in effecting the final completion of the central office work April 30, 1918. The time intervening from the date until the beginning of the delivery of the directory and the formal announcement of consolidation on June 1, 1918 was required in placing thousands of test calls over all combinations of connections to make sure of positive operation. In a typical connection between an automatic and manual station there are 41 relays, and 79 from manual to automatic, having movable parts controlling from one to ten electrical contacts, each of which must function in the proper sequence from the start to the completion of each telephone call. I do not wish to convey an impression that telephone circuits are inherently subject to failure because they are not. The characteristics and operating requirements for each relay, for instance, are known mathematically and what the relay can be depended upon to do for a specific period of time. At regular intervals each type of relay is given its proper current adjustment using measuring instruments designed for that purpose.

The installation of telephone cables and central office equipment of the manual or automatic type required the use of highly trained and skilled labor and the Telephone Company faced the difficulty of obtaining the large number of electricians and mechanics required to hold the schedule and training them for the special work. As it was, considerable overtime became necessary to maintain a working balance between the arrivals of material and the available labor.

Among the larger items of expenditure, and one involving months of study and calculations in voice transmission were the additions and changes necessary in the cable trunking plant in order to maintain commercial standards on all local and long distance connections. These studies included the use of and application of loading coils to the former Home Company cable plant amounting to about 1000 coils and the addition and respacing of many of the coils in the portion of the Pacific Company cable plant. All together about 75,000,000 conductor-feet of various gages of underground telephone cables were ordered and installed to provide new routes and reinforce existing trunk groups required for the consolidated service.

Prior to the consolidation, practically all of the subscribers having private branch exchanges maintained duplicate switchboards and station apparatus. The problem, therefore, of consolidating this type of equipment offered no particular difficulty, as such consolidations could be and were effected by grouping the trunks formerly serving the separate systems on the particular switchboard to be retained, adding thereto the amount of line and trunk facilities desired by the subscriber and eliminating the duplicate switchboard and stations not required. The net effect was to remove about 345 single-position private exchange switchboards from the system. In the case of several of the larger commercial companies, it was necessary to order complete multiple private branch exchange switchboards of the 640-line capacity and about 30 sections of such boards were placed in service as fast as the equipment could be engineered and manufactured. It will be interesting to note that the flow of traffic to and from private exchanges was not greatly disturbed from the existing paths by this plan of consolidating; a principle which was kept well in mind and taken advantage of wherever practicable.

Where individual and party stations were duplicated, both stations were left connected until the new directories were delivered, at which time the subscriber was requested to use the telephone of the particular system that he had made application for and the other station was removed as soon as the construction forces could handle the work. Approximately 13,000 duplicate stations have been removed.

A considerable number of operators were required and for a while it looked very much as if the consolidation would have to be postponed because of the inability to obtain the needed force. Good service depends to a large extent on capable and efficient operators and you can appreciate the difficulty that confronted the operating department in the selection, employ and training of approximately 500 additional operators required for handling the special transfer switchboard installed in the Olive Office building, and for the large number of added positions of switchboard in the various manual offices. A large operating school equipment was hurriedly manufactured and installed and training of operators was started about the first of the year 1918. The schedules also provided for the early installation of dials on the subscriber positions in the manual offices for advance training of the regular operating force. Special observation equipment was designed and furnished for practise work in placing test calls and later for supervision in determining and checking the accuracy of dialing the calls placed by the subscribers. By means of automatic recording devices the numbers called for by subscribers or instructors were compared with the numbers actually dialed and in this way the operating force was gradually brought to an efficient basis by the time the construction work was completed.

Proceeding under the restrictions that each subscriber having duplicate service should determine for himself whether to retain automatic or manual substation equipment, the Commercial Department carried on a vigorous campaign to obtain these subscriber's choices and to arrange accordingly. I do not have the exact figures but I understand that of the 18,680 duplicate stations about 5200 duplicates were retained and the balance were divided evenly between the two types. The expected result of this canvass had been forecasted and was an essential factor in engineering and in construction work.

As a function of the commercial canvass the Directory Department was confronted with the necessity of recasting the entire directory scheme to fit the particular needs of a unified service. A great amount of thought was given to the determination of the most efficient arrangement of listing and numbering subscribers, and while this seems trivial yet a careless directory arrangement reflects on the quality of the telephone service and robs the public of valuable time. Many number changes were involved and a complete relisting of every subscriber's name and number into one alphabetical list introduced great possibilities of errors. (It is a matter of passing interest that 165,000 copies of the directory were issued and distributed in Los Angeles and to other exchanges for long- and short-haul toll traffic and that over 441,000 pounds of paper were required in the printing of the consolidation issue.)

In the present unified plant, local calls are divided into four main groups and are obtained in the following ways:

Calls from Automatic Stations to Automatic Stations are obtained in the same manner as under the former Home management, that is, by the subscriber dialing the number wanted as indicated by the directory listing. Such subscriber sets are provided with the familiar dialing device arranged to transmit from one to ten electrical impulses for each "pull" of the dial. In the automatic system most of the stations are reached by dialing five digits although there are a few thousand six digit numbers. The operation of the first digit of the five digit numbers selects the office district required, the second digit selects the particular thousand, the third digit selects the particular hundred, while the fourth and fifth digits select the tens and unit respectively of the number desired. Referring to Fig. 3, the calling automatic subscriber upon removing the receiver from the switchhook completes an electrical circuit through a Keith line switch, the chief function of which is to direct the simultaneous calls to a minimum number of trunks to first selectors. Each automatic line therefore terminates on a primary line switch having ten paths to as many first selectors, these trunks being multipled to other line switches, depending on the calling rate or traffic load of the particular group of lines. The operation of the first digit of the number wanted causes the mechanism of the first selector to step the brush terminals upward to one of the ten levels arbitrarily connecting to a particular 10,000-line unit. This selector is of the trunk-hunting type, that is, the subscriber, as noted above, having dialed, say, the digit 6, causes six electrical impulses to be transmitted through the selector mechanism through the stepping relay, raising the brushes to the sixth level. The selector then automatically begins a rotary movement and continues until an idle trunk is found. There are ten trunks over which this rotary trunk selection takes place and the traffic is so distributed over selector equipment that with very few exceptions an idle trunk will be found within this group of ten. The connection is then established through what is termed repeater equipment (consisting of coils and relays by the aid of which the calling impulses are repeated from the local to the distant office and talking battery is provided to the calling station) to a particular second selector in the distant office. The dialing of the second digit by the calling subscriber again operates the stepping mechanism to one of the ten levels corresponding to the thousands of the number wanted. Automatic rotary movement takes place, selecting an idle trunk to the third selector where the dialing and selecting of the hundreds is the same as for the first and second selectors. The connection now is established through to the connector which is similar in its operation to selectors with the exception that the tens digit steps the mechanism to the corresponding level while the units digit controls the rotary movement to the particular unit required. The connector also supplies talking battery to the called subscriber and sends out the necessary ringing impulses to call either the individual or party subscriber desired or sends back the busy signal if the line called is in use. The restoring of the receivers to their switchhooks automatically returns all of the connecting equipment to normal position again for use on subsequent connections.

Calls from Automatic Stations to Manual Stations are obtained by operation of an arbitrarily assigned digit, the ninth digit or level being available in this case. Trunks from these first selector ninth levels lead to the nearest manual office and terminate by the aid of the familiar line and cutoff relay on an answering jack with a lamp signal before an *A* operator. Referring to Figs. 7 and 8, the *A* operator upon receiving a line lamp signal, plugs into the corresponding answering jack and requests the subscriber to give the number (and office prefix) wanted as shown by the directory listings. The call is then completed within the office received or is trunked over the manual trunking system to the distant *B* operator in the same manner as described under calls manual to manual. In the downtown district where a considerable amount of traffic is involved, it was found impossible to add to the large number of *A* positions to the various manual units and it was necessary to install a special manual switchboard, shown in Fig. 5 in available space in the Olive office building. This special board contains only the necessary multipled terminating lines, outgoing trunks and key and supervisory equipment necessary for establishing the connections between the automatic and manual system. Automatic calls therefore in this district are completed by the dialing of the digit 9 as before, bringing the line signal in on this special *A* switchboard where such con-

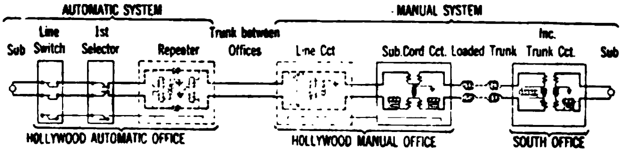


FIG. 7—EQUIPMENT INVOLVED IN A CONNECTION BETWEEN A HOLLYWOOD OFFICE AUTOMATIC SUBSCRIBER AND A SOUTH OFFICE MANUAL SUBSCRIBER

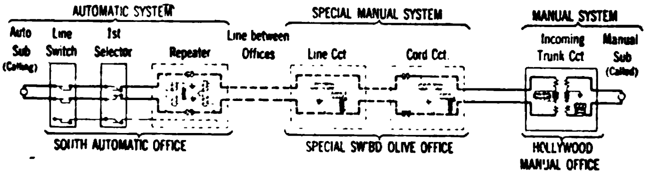


FIG. 8—EQUIPMENT INVOLVED IN A CONNECTION BETWEEN A SOUTH OFFICE AUTOMATIC SUBSCRIBER AND A HOLLYWOOD OFFICE MANUAL SUBSCRIBER

nections are completed as in regular manual practise over trunk lines assigned over order wires.

Calls from Manual Stations to Manual Stations are obtained in the same manner as existed prior to the consolidation, that is, by the agency of the *A* and *B* operators and corresponding manual switchboards. Referring to Fig. 2, the calling manual subscriber upon lifting the receiver from the hook causes a lamp to light before an *A* operator in his district. This line terminates on the ordinary line and cutoff relays, the former operating as soon as the switchhook closes the circuit, causing the line lamp to burn. The *A* operator then plugs into the corresponding line jack with one end of a double-ended cord circuit, this operation causing the cutoff relay to energize and thereby opening the circuit through the line relay and lamp. The operator then throws the listening key and requests the called for number. She then proceeds to complete the connection

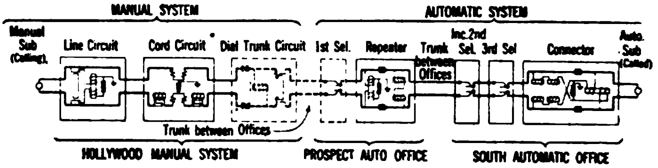


Fig. 9—Equipment Involved in a Connection Between a Hollywood Office Manual Subscriber and a South Office Automatic Subscriber

by plugging either directly into the subscriber multiple or by plugging into a trunk line leading to the distant office required and as assigned by the distant *B* operator over an order wire circuit. The distant trunk operator then plugs into the required subscribers' multiple and automatic ringing proceeds until the subscriber answers. Supervisory lamps associated with both the trunk cord and the double-ended *A* cord furnish the necessary indication to the operators of the establishment of the connection and end of the conversation. When both lamps associated with the double-ended *A* operator's cord relight, the connection is taken down and this operation gives a disconnect signal also to the distant trunk operator.

Calls from Manual Stations to Automatic Stations are obtained by direct trunk circuits. An *A* operator receiving a manual subscriber's request for a particular automatic number, as indicated by the directory listing, proceeds to dial the number in the same manner as described above for an automatic subscriber. Each *A* operator is provided with a group of trunks, as shown by Fig. 6 and described in the first part of this paper, leading to the nearest automatic office and terminating directly on first selectors.

Private Branch Exchange Calls originating from such exchanges of the former Home Company were formerly operated and are still operated on a manual basis. Calls placed by extension stations from these private exchanges are trunked to a manual switchboard in the Olive office building, this switchboard having the familiar subscriber multiple of all the private exchange trunk lines. The incoming private exchange trunk lines, however, are carried through Keith line switches, the function of which is to select a non-busy operator and to place the line lamp signal before that operator. This arrangement is commonly known as the traffic distributor. This pre-selected operator then receives the request for either an automatic or manual number and if for the former, she dials the number as described above for manual *A* operators. If the called for number is for another private exchange station, the operator completes the connection in the multiple similar to a manual-to-manual connection and if the request is for a station in a manual office, it may be obtained by the use of order wires and trunks direct to the office required or over trunk lines to the special Olive office manual *A* board. All connections to and from the former Pacific manual private branch exchange stations are completed in accordance with the above general methods of handling calls to and from manual stations

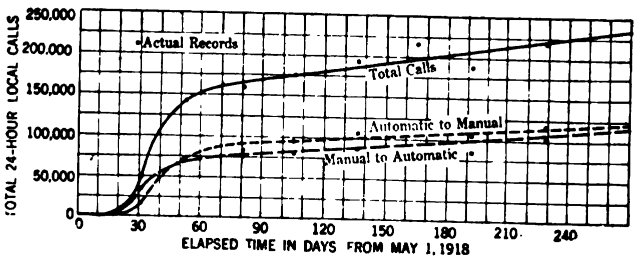


FIG. 10—CURVES SHOWING TENDENCY OF ORIGINATING CALLS BETWEEN FORMER PACIFIC AND HOME STATIONS

Fig. 10 has been prepared to indicate the volumes of traffic expressed in calls that passed between the former automatic and manual stations as at the specified dates. The curve was developed from the following records and immediately after the record of September 13, 1918. (Later records have since been taken and are shown by dots on the original curve.)

	Automatic to Manual	Manual to Automatic	Total calls
May 31, 1918 .... (1 day before formal announcement)	16,308	31,870	48,178
June 21, 1918....................	71,883	70,921	142,804
July 19, 1918....................	83,589	76,556	160,145
August 23, 1918..................	95,059	80,285	175,344
September 13, 1918...............	105,602	88,161	193,763

So far as I know, no definite statement can be made as to the gains accruing to the subscribers at Los Angeles under the consolidated arrangement, because the most important factors are not capable of reduction to equated savings in dollars.

Some of the major factors resulting in direct benefit are:

1. Value to the subscribers resulting from telephone access to each and every subscriber in the Los Angeles exchange and to all long distance lines centering there.
2. Rental savings resulting from the elimination of a large number of duplicate stations and private exchange switchboards.
3. Elimination of the indirect economic loss due to confusion and community service inefficiency of separate telephone systems.

Some of the factors which tend to offset part of the savings are:

1. Added annual charges on the plant and equipment required to provide means for universal service. There was very little elimination of duplicate plant investment primarily because each plant was designed to care for a definite development and volume of traffic. Consolidation obviously does not reduce traffic volume, but because the telephone field is considerably increased to every subscriber, the total volume of traffic in the consolidated plant is substantially increased.
2. Added operating and maintenance costs.

From a careful weighing of these factors it is manifest that the consolidated plant offers substantial savings and benefits over the dual systems. The worth of a telephone system to any community lies, not only in its capacity for effecting good service but that such service shall be universal and available to the maximum possible number of local and distant subscribers.

Presented at a Section Meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., February 11, 1919.

SOME PROBLEMS IN THE OPERATION OF POWER PLANTS IN PARALLEL

BY E. C. STONE

Abstract of Paper

In order to operate two power plants satisfactorily in parallel, the transmission line which ties them together must have sufficient synchronizing power, as well as sufficient carrying capacity. The "synchronizing power" of a line depends upon its resistance and reactance, the bus voltages maintained at its ends, and the maximum kilovolt-amperes it must transmit. The ability of different lines to provide satisfactory parallel operation cannot be measured by any standard which does not take account of all of these factors. Limiting values for "synchronizing power" of lines under various operating conditions are given.

The division of load between two plants in parallel is regulated by steam control; the division of wattless current associated with the load depends upon the voltages generated and may be in proportion to the division of real load, when the difference in voltage at the two busses will vary with the load transmitted, or may be arbitrary so that regardless of the load transmitted, the voltages on the two busses will be maintained constant. The latter plan generally gives better operating conditions on the system as a whole, but creates demands for wattless currents at either or both plants in excess of their normal capacity, and in so doing, involves an additional cost. The excess wattless cross-current so created can be materially reduced by varying the voltage with changes in load transmitted through the use of taps on the line transformers, by inserting additional reactance in the line at light loads, or, when the stations are tied together by several parallel lines, by cutting out one or more lines as the load decreases.

The design of a transmission line involves a consideration of load to be transmitted, voltage, reactance, resistance, losses, and charging current of the line, and of wattless generating capacity at the receiving end of the line. The wattless generating capacity at the end of the line determines how many kilowatts will be transmitted for each ampere of line current, by fixing the power factor of the load transmitted and the voltage at the receiving end of the line. When a line is to be designed for paralleling two plants, it must have sufficient "synchronizing power" to hold the two plants together.

THE advantages to be derived from the operation of all the power plants serving a territory in parallel are many and great. The maximum or peak capacity is increased because of the diversity of the load in different parts of the territory served; small plants with high operating costs are made available for peak service only, without the injurious effects of interruption to service when loading them up and shutting them down; the strain of carrying sharp and sudden peak loads is distributed over the whole generating system; customers in all parts of the system are given the benefits in steady frequency and stability of power supply which come from drawing their power from a source of supply having a capacity vastly greater than their individual requirements.

It is not possible, however, to merely synchronize all the power plants of a system together over any lines that may be available and obtain therefrom all the advantages of parallel operation without experiencing any difficulties. The steam governing devices of the various units at the different plants must be adjusted with respect to each other so as to properly divide the load at all times. The heavy concentration of power in short circuits which results from parallel operation requires that adequate circuit breakers and other protective devices must be installed. The transmission lines used for tying the plants together must have suitable characteristics and sufficient capacity. This paper deals with the latter subject.

When a number of individual machines are operated in parallel in one power plant on a single bus, the impedance in the circuit connecting them is so small as to be quite negligible in its effect on the parallel operation of the machines. When however, two separate power plants are operated in parallel, the transmission line tying them together comes in between the busses, thereby increasing the impedance of the connecting circuit to many times that of a station bus and decreasing its carrying capacity correspondingly. The impedance and carrying capacity of a transmission line thus become very important factors in determining the manner in which the power plants which it ties together will operate in parallel.

The action by which two alternating-current generators connected to a common bus are held in synchronism is commonly understood. If one machine attempts a speed different from that of the other, a difference in phase immediately develops between their voltages, which causes a cross current to flow through the local circuit made up of their armature windings and leads and the bus section between them. Because of the reactance in this circuit, the cross current is of such a phase as to transfer a part of the load carried by the lagging machine over to the leading machine. This in turn causes the former to speed up and the latter to slow down, so that their voltages are brought back into phase and synchronism is maintained.

When two power plants are operated in parallel, the action described above is modified by the insertion of the tie line in the local circuit between generators. The vector diagram for a given set of conditions as to plant voltages and line impedance, is shown in Fig. 1. When the two power plants are exactly in step so that their voltages E and E_2 are in phase, the voltage across the circuit tying them together is e_2, the current I_2 and the power transmitted from station 1 is P_2. The station delivering power to the tie line will be designated

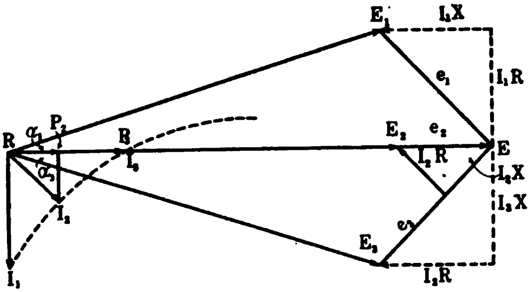


Fig. 1—Vector Diagram

E = Voltage at Station 1.
$E_1 E_2 E_3$ = Voltages at Station 2.
$e_1 e_2 e_3$ = Voltages across tie line.
$P_1 P_2 P_3$ = Power delivered to tie line from Station 1.
R = Resistance of tie line.
X = Reactance of tie line.
Station 1 delivers power to line.
Station 2 receives power from line
$X/R = 1 \quad E_3/E_1 = 0.80$

as station 1 or the transmitting station, and the station receiving power from the tie line will be designated as station 2 or the receiving station.

If station 2 speeds up so as to lead station 1 by the angle a_1, the voltage on the tie line is increased to e_1 and the current to I_1, but since I_1 is at right angles to E, the power transmitted from station 1 to station 2 is zero. Thus station 2, by advancing position from position E_2 to position E_1 has taken from station 1 an additional load equal to P_2. On the other hand, if station 2 lags behind the angle a_3, the current becomes I_3 and power P_3, so that station 2 has then dropped an amount of load equal to $P_3 — P_2$ which amount has been taken up by station 1, loading up the tie line still further until a certain critical angle of lag is reached beyond which the load transmitted over the tie line decreases instead of increases.

The curves of Fig. 2 show how the amount of power transmitted—or load transferred—over tie lines having the same resistance, but different reactances, varies with variation in the phase angle between the voltages of the plants tied together. These curves represent the theoretical transmission of power, on the assumption that the line copper losses are part of the load carried from the plant busses and are divided between

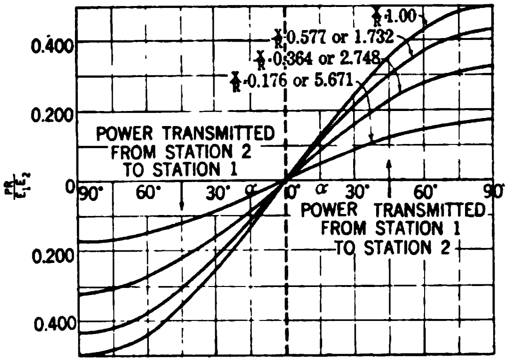


FIG. 2—POWER TRANSMISSION OVER TIE LINE

Resistance and voltages constant.
Line reactance and phase-angle between station voltages variable.
Three-phase circuit.
α = phase angle between E_1 and E_2.
$\beta = \tan^{-1} E/R$

$$\frac{XR}{E_1E_2} = \cos\beta \sin\beta \sin\alpha = \frac{XR}{Z^2} \sin\alpha.$$

the plants in proportion to their bus voltages. This was done in order to show the real transfer of load that takes place between the two plants. These curves, and all other curves in this paper are calculated for three-phase transmission, the values of power being the total power transmitted and the values of voltage, the voltage between line wires of the circuit.

The forces tending to pull paralleled plants out of phase are produced by the variations in the supply of power to the generators and by the variations in the loads drawn from their busses. For example, assuming that the steam governing devices are set so as to make the two plants share the total load equally at all times, one-half of any increase in the load on the bus of one station would have to be carried by the other. That this could be done, the station at whose bus the increase occurred would have to drop behind by a phase angle sufficiently large to permit of the necessary additional amount of power being transmitted over the tie line to meet the new condition. The higher the impedance of the tie line, the greater will be the phase angle required; and if the increase in load takes place suddenly, the lagging plant will at first drop too far behind because of its inertia, and will perform several "vibrations" before coming stable in its new position. In practise, therefore, the phase angle is constantly varying, the degree of variation depending on the magnitude and rapidity of the load fluctuations and on the impedance of the tie line. If it varies too rapidly or over too wide a range, an unstable or "pumping" condition will be created, which will cause swinging loads on the machines and bad voltage fluctuations, and may sufficiently increase the current in the tie line to open the line breakers and separate the plants. To obtain satisfactory parallel operation, therefore, the resistance and reactance of the tie line must be low enough to permit of a sufficiently free exchange of power between plants so that under the most severe conditions of load fluctuations the phase angle between stations will never vary sufficiently to cause harmfully unstable conditions or to open line breakers. In general, practical limitations of carrying capacity of the conductor and of line losses limit the maximum permissible phase angle to from 10 to 30 degrees.

The ability of any line to hold two plants together in satisfactory parallel operation may be called its "Synchronizing Power," which is defined as the change in the amount of power that the line will transmit for each degree change in phase displacement of the voltages of the stations it ties together. Mathematically, this means the rate of change in power transmitted with respect to change in phase angle, and is represented by the "slope" of the curves of Fig. 2. The formula is as follows:

$$Ps = \frac{E_1 E_2}{1000} \times \frac{X}{Z^2} \times \cos a.$$

where

E_1 = voltage at transmitting station.

E_2 = voltage at receiving station.

X = the reactance of the tie line.
Z = the impedance of the tie line.
a = the phase angle between E_1 and E_2.
Ps = synchronizing power in kilowatts.

To be rigidly accurate, the X and Z used above should include the armature windings and bus connections of the generators at the plants, but as these generally have very low impedance compared to the impedance of the line, they can ordinarily be neglected.

On a given line, the greatest change in the amount of power transmitted for a given change in phase angle between the station voltages occurs when the phase angle is zero, while the least change occurs when the angle is 90 degrees. (See Fig. 2). The synchronizing power of a given line is therefore a maximum when the phase angle is zero and zero when the phase angle is 90 degrees. Since the phase angle increases as the load transmitted increases, the synchronizing power decreases as the load transmitted increases.

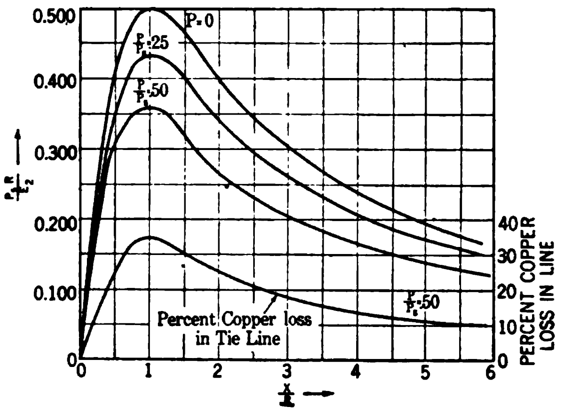


FIG. 3—SYNCHRONIZING POWER UNDER NORMAL OPERATING CONDITIONS

Power factor of load transmitted taken as unity.

$$P_S = E_1E_2\,X/Z^2 \cos \alpha$$

The curves of Fig. 3 show the synchronizing power of lines having the same resistance with varying reactances, when carrying loads of from 0 to one-half the synchronizing power of the line. It will be seen that for a line of given resistance the synchronizing power is greatest when the reactance-resistance ratio of the line is unity and decreases rapidly as this ratio changes from unity value.

While the synchronizing power of a line as defined above may be easily calculated, it is not so easy to determine the amount of synchronizing power that is necessary in a given case to secure satisfactory parallel operation. This will depend on many factors—among them the types of prime movers and their load-speed characteristics, and the character of the load delivered from the busses of the two plants. Reciprocating engine plants will require higher synchronizing power than turbine plants. Loads having rapid and large fluctuations will require more synchronizing power than steady loads. In checking up a number of actual cases, it has been found that under the conditions of fluctuating power load as found in the Pittsburgh district, the least line synchronizing power which will give satisfactory parallel operation of large turbine plants, with the relatively steady load of large systems, is a synchronizing power approximately equal to the capacity of the smaller plant to be paralleled, while with smaller plants, consisting partly of reciprocating engines and partly of turbines and the relatively high load fluctuations of the smaller systems, the tie line to give stable operation must have a synchronizing power equal to not less than 1.5 times the capacity of the smaller plant.

Inasmuch as the synchronizing power of a tie line depends upon a number of factors as explained above, it is obvious that any accurate criterion of the ability of a given line to hold two plants together must take account of all these factors. For example, it is sometimes said that a line having a capacity equal to a certain percentage of that of the smaller plant to be paralleled will give satisfactory operation. Yet two lines may be designed for just this capacity at the same voltage with widely different synchronizing powers. For similar reasons it cannot be assumed that a line which develops not more than a certain copper loss will be satisfactory, for such a statement does not take account of all of the variables.

Synchronizing power of the tie line becomes a limiting factor when the line voltage is low for the distance transmitted. When the value of volts per mile is relatively high, the synchronizing power will be found ample.

Synchronizing power must be given equal consideration whether there are generators or loaded synchronous motors at the receiving end of the line. It is unimportant in connection with synchronous condensers, however, since the energy which they take is always a very small percentage of the maximum power transmitted over the line.

The division of load between two plants in parallel depends of course upon the governor adjustments on the steam end of the prime movers; the division of wattless load associated with the energy load depends upon the voltages generated by the two plants and may be entirely different from the division of real load or energy supply. Thus the plant at one end of a line might supply all of the energy required while the plant at the other end might supply all of the wattless load.

By suitable voltage adjustments the plant which supplies power to the tie line may be made always to supply the wattless component associated with that power—under which condition the difference in voltage at the two busses will vary with the load on the line, being zero at no-load and a maximum when the maximum load is transmitted. Ordinarily, however, this operation produces too wide a range of voltage at the station busses to be satisfactory, and it becomes necessary to regulate the voltages within a closer range. This is possible, since the difference in voltage which is developed between the two ends of a transmission line when power is transmitted over it, depends not only on the amount of power transmitted but also on the amount of wattless energy. Given a certain voltage difference to be maintained between the ends of the line, there is a certain amount of load which with the wattless component associated with it, will just absorb that voltage difference. When the power transmitted is less than this value it becomes necessary in order to maintain the required voltage difference to set up a wattless cross current between the generators of the two stations, which current has the effect of lowering the power factor of the transmitting station and raising the power factor of the receiving station. When the power transmitted is more than the given value, the cross current must be in the opposite phase and has the opposite effect on the power factors of the two stations. The result is that the transmitting station has excess wattless current to take care of when the load on the tie line is light while the receiving station has excess wattless current to take care of when the load on the tie line is heavy. This is provided for by installing generators rated at a sufficiently low power factor or by installing synchronous condensers. At stations where the excessive cross current occurs only at periods of light load, more generators of normal power factor rating may be run at such periods than would be required by the energy load only—a method which is wasteful of coal, but may be more economical than the installation of additional equipment to take care of off-peak wattless current.

Fig. 4 shows how the wattless current in lines of different characteristics must be varied as the energy transmitted varies, in order to maintain constant bus voltages at both stations. Where the curves are below the horizontal zero line the wattless current in the tie line is lagging with respect to the voltage of the transmitting station and leading with respect to that of the receiving station. Where the curves are above the zero line, the wattless current is leading with respect to the voltage of the transmitting station and lagging with respect to the voltage of the receiving station.

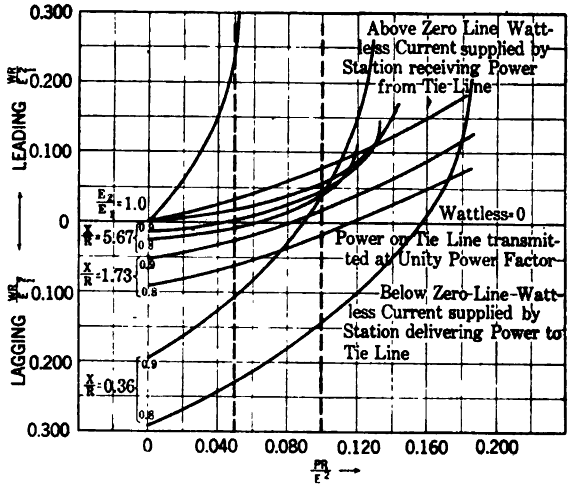


Fig. 4—Wattless Current vs. Power in Tie Line

The important points brought out by the curves of Fig. 4 are:

(1) The variation in wattless current for a given variation in load is very much greater where the reactance-resistance ratio of the line is low than where it is high. This means that the wattless cross current required to maintain constant bus voltages and the excess copper losses and wattless generating capacities arising therefrom increase very rapidly as the reactance-resistance ratio of the line decreases.

(2) When the same voltage is maintained on the busses of

both stations and load is transmitted over the tie line, the wattless cross current which is set up, always leads the voltage of the transmitting station and increases as the load transmitted increases. This means that the receiving station not only must supply all of the wattless current associated with the load received from the other station, but must also supply a certain amount of wattless cross current to the line in order to hold up the bus voltage.

(3) In order that the lagging wattless current associated with a given load as well as the load itself may be transmitted by the tie line, the voltage at the receiving station must be lower than that of the transmitting station.

(4) In view of the conditions shown by Fig. 4, it is necessary when laying out an installation in which constant voltages are to be maintained at the power station busses, to consider the conditions created when the minimum amount of power, as well as when the maximum amount is transmitted over the tie line.

In this connection it is interesting to note that the maximum amount of power which can be transmitted over a given line is independent of the reactance when sufficient wattless current can be generated at the end of the line, and depends only on the resistance of the line. It is that amount of power which equals the square of the transmitting voltage divided by four times the resistance. The voltage at the receiving end to give the maximum power is equal to the impedance divided by twice the resistance, hence is one-half of the transmitting voltage when there is no reactance, is equal to the transmitting voltage when the reactance is 1.732 times the resistance, and is greater than the transmitting voltage for a greater reactance. The very excessive line loss and wattless generating capacity required at the receiving end prohibit ever taking from a line the maximum amount of power that it can transmit.

When it becomes necessary to maintain the same bus voltages at both stations with very light loads transmitted as are maintained at full load, the wattless cross current may develop into an item of considerable cost. For this reason it becomes desirable to reduce it as much as possible. This can be done in two ways as follows:

a. By varying the voltage delivered to the tie line at the transmitting station as the load varies.

b. By varying the impedance of the tie line as the load varies.

The voltage delivered to the tie line is varied through the use of taps on the line transformers, or by means of an inductive regulator or synchronous booster. This method causes little variation in the synchronizing power of the tie lines, but involves considerable expense for special equipment.

The impedance of the tie line is varied through the use of reactance external to the lines or, when the tie circuit consists of several lines in parallel, by varying the number of lines in service. Both of these methods decrease the synchroniznig power as the impedance is increased, but involve little or no cost for special equipment, and will be found of very real value in many cases. It often happens that by a simple switching operation some of the short-circuit limiting reactances at the power stations can be switched into the lines used for paralleling, so that the impedance of these lines can be materially increased at light load periods. Changing the number of lines in service is of course accomplished by the use of the switches regularly installed with the lines, and so requires no special equipment at all.

The design of a line to transmit a given amount of power over a given distance involves a consideration of voltage, resistance, reactance, losses, and charging current of the line, and of wattless generating capacity in the form of synchronous condensers at the receiving end of the line. The selection of the proper voltage involves many factors outside of the scope of this paper, and will not be discussed. To obtain a minimum total cost of line at any given voltage, the annual cost of the copper losses in the line should equal the annual cost of the investment in copper; then, in order to obtain the minimum total cost of transmission, the proper size of synchronous condensershould be installed at the end of the minimum cost line to give the minimum total cost of line and condenser per kilowatt delivered at the end of the line. Thus the cost of copper and copper losses determine the resistance of the line, while the cost of synchronous condenser capacity in relation to the cost of the line determines how many kilowatts will be transmitted for each ampere of line current, by fixing the power factor of the load transmitted, and what is equally important, the voltage at the receiving end of the line. The charging current due to the capacitance of the line produces the effect of approximately the equivalent number of kilovolt-amperes in synchronous condenser capacity located at the middle point of the line and supplied with no additional investment.

When a line is to be designed for paralleling two plants, the minimum as well as the maximum amount of power to be transmitted, and the direction of transmission of each, must be determined from the characteristic curves of the loads supplied from the two busses, from the operating schedules desired of the prime movers at the respective stations, and to a lesser degree, from the load-speed characteristics of the machines and the fluctuating or steady nature of the load. The synchronizing power must also be given due consideration, and if constant bus voltages are to be maintained with a variable load transmitted between stations, the cost of the additional line loss and additional wattless generating capacity required because of the cross current, as well as the cost of the line and synchronous condenser capacity, must be considered in designing the line for minimum total cost of operation.

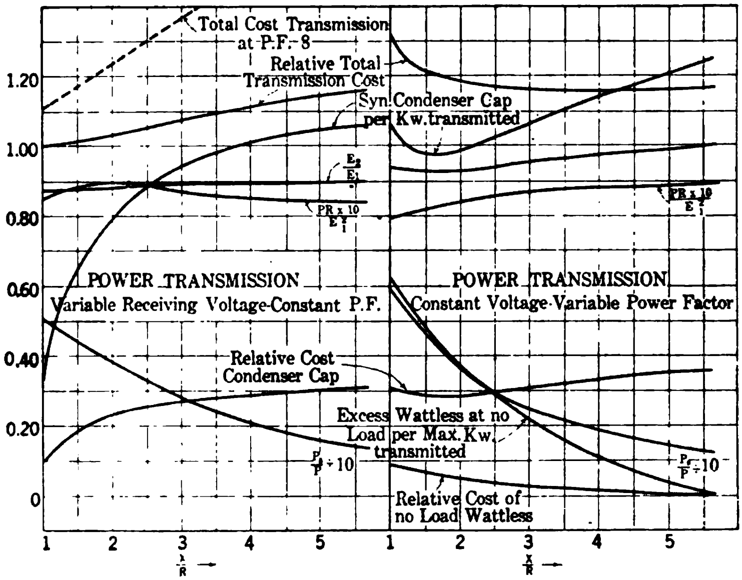


Fig. 5—Power Transmission Data

Transmission voltage 1000 volts per mile.

Cost of copper = Cost of copper loss.

Synchronous condenser capacity and receiving voltage to give minimum total transmission cost.

Power factor of load = 0.8.

Minimum load = 0.

In Fig. 5, curves have been calculated on the basis just outlined to determine the minimum total cost of transmission of a constant amount of power at a fixed voltage per mile, and the synchronizing power of the line resulting therefrom for various values for the ratio of reactance to resistance of the line. The following figures have been assumed as a basis:

Transmission voltage, 1000 volts per mile.

Cost of copper, 22 cents per pound.

Cost of synchronous condenser capacity, $5.00 per kv-a.

Losses of condenser, 0.035 kw. per kv-a. of capacity.

Annual cost of investment, 15 per cent.

Cost of energy, 2 mills per kilowatt-hour.

Power factor of load delivered from busses of both stations, 80 per cent.

Load factor, 50 per cent.

Range of load transmitted, 0 to maximum.

The reactance-resistance ratio of the line is determined chiefly by the frequency, size-of conductors, and design of the step-up and step-down transformers. The inherent reactance of transformers goes up rapidly with their voltage, because of the increased spacing required between their high- and low-tension windings, and low reactance at high voltages is attained only by an abnormal and costly design. The reactance of a given line at given frequency, therefore, can only be reduced by splitting it up into a number of parallel lines having smaller conductors and providing abnormally designed transformers, both of which methods are very costly. On the other hand, the reactance of a line can be readily increased to almost any desired value for a few per cent of the cost of the line by the installation of reactance coils. It is therefore fortunate that good voltage regulation can be attained with high reactances.

High voltage lines are generally designed for large blocks of power, making the combined reactance of the large conductors and of the transformers inherently high. Low voltage lines, with relatively small conductors, and often no transformers, have inherently low reactance.

On the left hand side of the curve sheet, Fig. 5, are shown the conditions where the bus voltage of the receiving station varies with the load. As the reactance-resistance ratio increases from 1 to 5.5, the total cost of transmission increases 16 per cent, showing that high reactance means a slight increase in total cost of transmission, under the minimum cost conditions. The synchronous condenser capacity goes up from 0.33 kv-a. per kilowatt transmitted, corresponding to

90 per cent power factor lagging in the line, to 1.06 kv-a. per kilowatt transmitted, corresponding to 95 per cent power factor leading in the line. Unity power factor obtains when the reactance-resistance ratio is 1.8. The cost of the condenser capacity varies from 9 per cent to 35 per cent of the cost of the line and line losses.

On the right hand side of Fig. 5 are shown conditions when the bus voltages are held constant as the load on the line is varied from zero to the maximum for which the line is designed. The wattless cross current necessary to maintain constant bus voltages introduces an additional element of cost, so that the total cost is higher than before for all values of the reactance-resistance ratio of the line. Furthermore, the total cost increases, instead of decreases, as this ratio decreases. This is because, with a given line resistance, the cross current required to maintain constant voltage increases rapidly as the reactance decreases, as is indicated by the curve showing the wattless cross current required with no power transmitted, which runs from zero for a reactance-resistance ratio of 5.7 up to 0.67 kv-a. per kilowatt of line capacity when the ratio is unity. The minimum attainable cost under these conditions is however very nearly reached when the reactance-resistance ratio is 2.5, so that the only advantage to be gained by a greater proportion of reactance is the raising of the voltage at the receiving end of the line, a feature which is only of importance when power is transmitted in both directions.

Under constant voltage conditions, more synchronous condenser capacity is used, with the result of raising the voltage at the receiving station and cutting out sufficient wattless cross current to more than pay for the additional condenser capacity. The cost of the condenser capacity runs from 30 per cent to 40 per cent of the cost of the line and line losses.

The ratio of synchronizing power to maximum load transmitted over the specified tie line varies from approximately 6, when the reactance-resistance ratio is 1, down to 1.3 when the ratio is 5.7—slightly higher than before because of the higher receiving voltages.

The curve giving values of $\frac{PR \times 10}{E_1^2}$ may be called the design curve. With the transmission voltage E_1 fixed, the resistance R of the line which will transmit a given amount of power

P at minimum cost, or the amount of power *P* which can be transmitted at minimum cost per kilowatt over a line having a given resistance *R* can be obtained immediately from this curve.

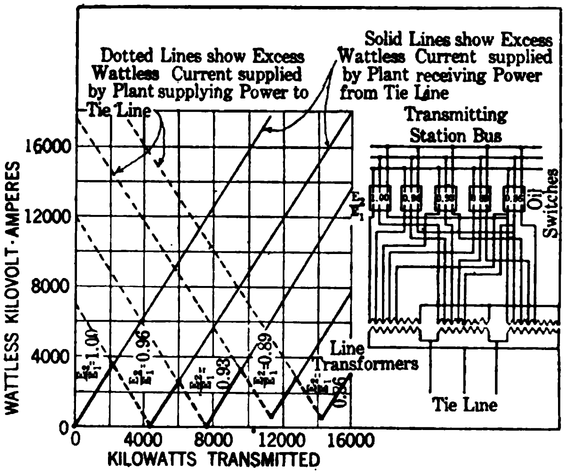
Comparing the curves under the condition of variable or constant bus voltage, it is found that for a reactance-resistance ratio in the line of 5.7, the constant bus voltages are obtained at practically no additional cost. As the ratio decreases, however, the cost of securing constant bus voltages increases, being 8 per cent of the total transmission cost for a ratio of 3, and 31 per cent for a ratio of 1. If the inherent reactance of a specified line is not high enough to give economical operation additional reactance in the form of reactance coils can be inserted at a cost of only a few per cent of the cost of the line. Under the conditions assumed, therefore, constant bus voltages should be obtained for an increase of not more than 10 per cent of the total cost of transmission—which is a very low cost for the advantages to be gained thereby.

The transmission voltage in relation to the distance of transmission has a very direct bearing on the line design problem. In the curves it has been assumed to be 1000 volts per mile of line. If the number of volts per mile is lower, more synchronous condenser capacity will be used and the synchronizing power will be decreased and may become the limiting factor of the design, especially when the power to be transmitted is a small part of the station capacity. In this case, it becomes necessary to use more copper than is required for the load, or to reduce the reactance, or to do both, in order to obtain sufficient synchronizing power.

If the number of volts per mile is greater than 1000, less synchronous condenser capacity will be used and more synchronizing power will be obtained than is indicated in Fig. 5.

In general, it will be found that where constant bus voltages are necessary, the best tie line will be the one which has the greatest reactance consistent with the necessary synchronizing power. Transmission with high reactance, however, necessitates an ample amount of wattless generating capacity at the receiving station, in order to hold up the voltage. Where this is not available, so that a considerable amount of wattless current must be supplied with the load from the transmitting station, high line reactance is impracticable and the cost of maintaining constant bus voltages is greatly increased.

In Figs. 6 and 7 are shown data worked out for specific cases, to determine how the wattless cross current could be reduced. In both cases it was necessary to maintain constant bus voltages at the two plants to be paralleled. In Fig. 6, loads varying from 0 to 15,000 kw. were to be transmitted from one station to the other. In order to keep the excess wattless current—by which is meant the wattless current which either station would be called upon to supply in addition to that associated with the real load it was delivering—below the limit of 4000 kv-a., it was necessary to provide four additional taps on the line transformers and four additional oil switches at the transmitting station to give the voltages as shown. Fig. 7 shows how in a given case, where two lines tie the two plants together, the wattless cross current was reduced at light load periods by switching in a bus limiting reactance already installed, which happened to be just large enough to double the reactance-resistance ratio of the line. Fig. 7 also shows the effect of cutting out one of the two tie lines.



Fir. 6—Effect of Varying Bus Voltages

Transmission on four No. 0 lines in parallel.

Resistance = 0.52 ohn. $X/R = 1.33$

Power factor of load 0.8. Voltage $E_1 = 11{,}000$.

Synchronizing power = 2.25 times the capacity of plant to be tied in.

Fig. 8 is a chart which was worked out to guide the system operators in regulating the cross current between the stations connected by two lines. If two lines are in service and the power factor drops below 45 per cent, one line is immediately cut out. This doubles the kilowatts transmitted by the remaining line, but does not increase its current, so that by cutting out the other line approximately 180 amperes of cross current are eliminated.

In concluding with a brief summary, it must be borne in mind that in designing a tie line to meet any specified condition many variables enter the problem. In general, costs of copper, steel, and coal go up and down together, so that absolute costs are not so important as relative costs of these items. The load factor of the power transmitted affects the overall costs of line losses, and therefore the amount of the copper used. The transmission voltage in relation to the transmission distance has a very direct bearing on the amount of synchronous condenser capacity to be used, and on the synchronizing power of the line. Since the data and curves which have been shown are based upon a series of specific assumptions, which while sufficiently correct under the conditions at hand, might have to be materially changed to meet other conditions, the data presented are of interest in showing the relative weight of the different factors rather than in determining absolute values. It is hoped that in showing the effect of the different variables on the problem as a whole, a guide may be furnished to the

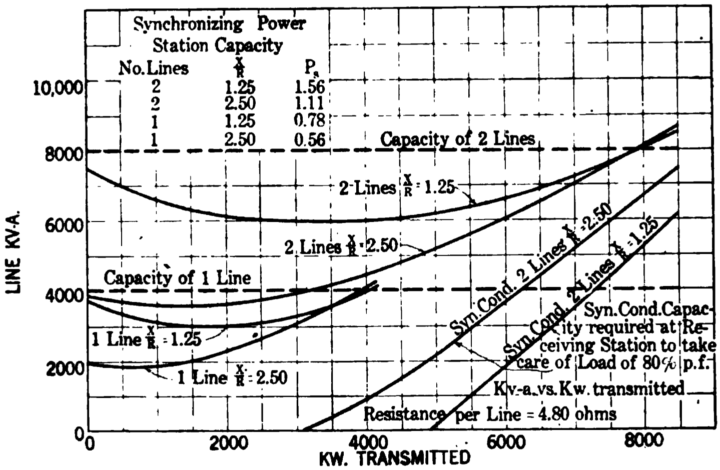


FIG. 7—REGULATION OF WATTLESS CROSS CURRENT BY VARYING THE REACTANCE AND NUMBER OF TIE LINES IN SERVICE

lines which should be followed in working out any specific case. With these limitations in mind, the following points may be emphasized:

In order to operate two power plants satisfactorily in parallel the transmission line which ties them together must have sufficient "synchronizing power," as well as sufficient carrying capacity to carry the maximum load to be transmitted between them. The synchronizing power of a given line depends upon its resistance and reactance, upon the bus voltages maintained at its ends, and upon the maximum kilovolt amperes it must transmit. The ability of different lines to provide satisfactory parallel operation cannot be measured by any standard which does not take account of all of these factors. It has been found from the writer's experience that a synchronizing power about equal to the kilowatt capacity of the smaller plant to be paralleled, for large turbine plants with load fluctuations of relatively small percentage of their capacity, and of 1.5 times the smaller plant capacity for mixed engine and turbine plants with relatively large load fluctuations, is about the least that a line can have to give satisfactory parallel operation.

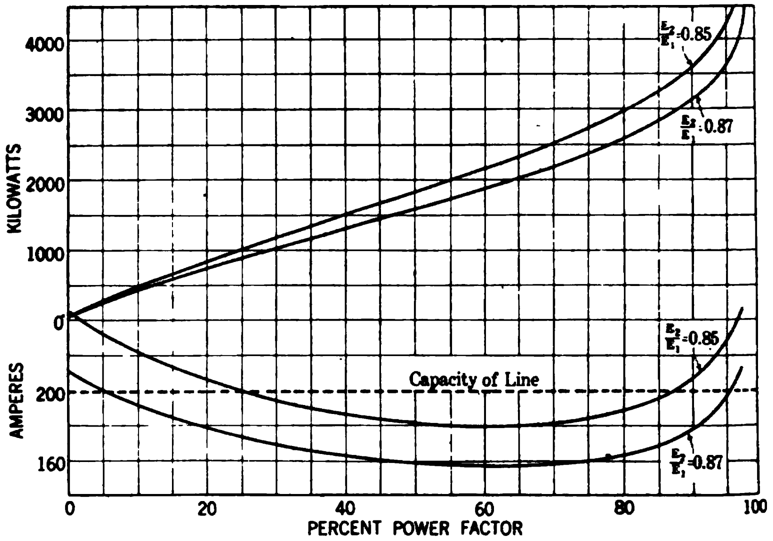


Fig. 8—System Operator's Working Diagram

Performance of one tie line.

$R = 4.8$ ohms. $X = 6.0$ ohms.

When the voltage per mile of transmission is high, line capacity, as determined by load and operating conditions at the two stations, is the limiting factor; when the voltage per mile is low, synchronizing power more often marks the limit, especially when the reactance is high and the load to be transmitted relatively small. The necessity for sufficient line synchronizing power applies equally whether generators or loaded synchronous motors are at the end of the line, but is unimportant in connection with synchronous condensers at the end of the line.

The design of equipment for transmitting a block of power over a given distance at minimum cost must take into account the use of synchronous condenser capacity at the receiving end of the line. The condenser is effective in raising the receiving voltage as well as in controlling the transmission power factor, and materially reduces the amount of copper required in the line. The amount of condenser capacity which may be economically used depends on the power factor of the load, the relation between the cost of line capacity and condenser capacity, and the reactance-resistance ratio of the line. Where this ratio is high, sufficient condenser capacity may be justified to produce a considerable leading component in the tie line current. In a line of given resistance, with the receiving voltage unregulated, the total cost of transmission increases slightly with the increase in reactance.

Where synchronous machines of sufficient capacity, either condensers or generators, are installed at both ends of the line, the bus voltage of both stations can be maintained constant as the load transmitted over the line varies, by the creation of a wattless cross current. This wattless current, however, involves an additional cost in copper loss and in the wattless generating capacity required at one or both stations to take care of it, which cost is small when the reactance-resistance ratio of the line is sufficiently high, but which rapidly increases as the reactance decreases, and for low line reactance becomes very important. This wattless current can be materially reduced under certain conditions, by varying the voltages at the ends of the line as the load varies through the use of transformer taps, by inserting additional reactance in the line at light loads, or, when the stations are tied together by several parallel lines, by cutting out one or more lines as the load decreases.

The increase in the cost of transmission with constant bus voltages caused by this wattless cross current represents the cost of eliminating the regulation of the tie line. The wattless cross current decreases as the reactance-resistance ratio of the line increases, but the synchronizing power also decreases. Hence it will generally be found that the best design of transmission equipment for constant bus voltages is that which makes the reactance of the line as high as is permissible for the synchronizing power required. With such a design, the cost of maintaining constant bus voltages—that is, of eliminating the regulation of the tie line—does not amount to more than a few per cent of the total cost of transmission, which is a very small price to pay for the operating advantages gained. It must be remembered, however, that to obtain constant bus voltages at this slight cost, the installation of sufficient wattless generating capacity at the receiving station is absolutely essential—a feature, unfortunately, that is too often lacking in existing systems.

APPENDIX

In what follows, the formulas are developed which were used in calculating the data presented in the foregoing paper.

Referring to Fig. 9,

Let E or E_1 = Voltage at transmitting station.
E_2 or kE = Voltage at receiving station.
E_0 = Voltage across tie circuit.
$k = E_2/E$
α = Angle between E and E_2
z = Impedance of tie circuit
r = Resistance of tie circuit
x = Reactance of tie circuit
$\beta = \tan^{-1} x/r$
P_1 = Power delivered to tie circuit by transmitting station in watts.
W_1 = Wattless delivered to tie circuit by transmitting station in volt-amperes
$V A_1$ = Total volt amperes delivered to tie circuit by transmitting station
P_2 = Power received from tie circuit by receiving station, in watts
W_2 = Wattless received from tie circuit by receiving station in volt-amperes
$V A_2$ = Total volt-amperes received from tie circuit by receiving station

P_s = Synchronizing power of tie circuit, in watts
I_0 = Current in tie circuit, in amperes
θ_1, θ_2 = Angles between voltages as shown in Fig. 9.

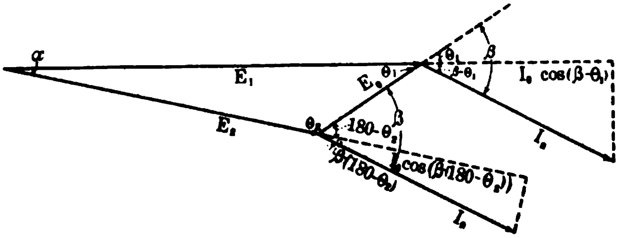


FIG. 9—VOLTAGE AND CURRENT RELATIONS WHEN POWER IS TRANSMITTED OVER TIE LINE

From the triangle of voltages,

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan \frac{1}{2}(\theta_2 + \theta_1)}{\tan \frac{1}{2}(\theta_2 - \theta_1)} = \frac{1 + k}{1 - k} \tag{1}$$

$$\tfrac{1}{2}(\theta_2 - \theta_1) = \tan^{-1}\left(\frac{1 - k}{1 + k} \operatorname{cotan} \frac{\alpha}{2}\right) \tag{2}$$

$$\tfrac{1}{2}(\theta_2 + \theta_1) = \tfrac{1}{2}(180 - \alpha) = 90 - \alpha/2 \tag{3}$$

Adding (2) and (3)

$$\theta_2 = 90 - \alpha/2 + \tan^{-1} \frac{1 - k}{1 + k} \operatorname{cotan} \alpha/2 \tag{4}$$

whence

$$\tan(90 - \theta_2) = \frac{\tan \alpha/2 - \dfrac{1 - k}{1 + k} \operatorname{cotan} \alpha/2}{1 + \dfrac{1 - k}{1 + k} \tan \alpha/2 \operatorname{cotan} \alpha/2}$$

$$= \frac{k - \cos \alpha}{\sin \alpha} \tag{5}$$

Therefore

$$\operatorname{cotan}(180 - \theta_2) = -\tan(90 - \theta_2) = \frac{\cos \alpha - k}{\sin \alpha} \tag{6}$$

Subtracting (2) from (3)

$$\theta_1 = 90 - \frac{\alpha}{2} - \tan^{-1}\left(\frac{1 - k}{1 + k} \operatorname{cotan} \frac{\alpha}{2}\right) \tag{7}$$

whence

$$\tan(90 - \theta_1) = \frac{\tan\dfrac{\alpha}{2} + \dfrac{1-k}{1+k}\operatorname{cotan}\dfrac{\alpha}{2}}{1 - \dfrac{1-k}{1+k}\tan\dfrac{\alpha}{2}\operatorname{cotan}\dfrac{\alpha}{2}}$$

$$= \frac{1 - k\cos\alpha}{k\sin\alpha} \quad (8)$$

Therefore

$$\operatorname{cotan}\theta_1 = \frac{1 - k\cos\alpha}{k\sin\alpha} \quad (9)$$

Again from the voltage triangle,

$$\frac{E_0}{E_2} = \frac{E_0}{k\,E} = \frac{\sin\alpha}{\sin\theta_1} \quad (10)$$

whence

$$E_0 = \frac{k\,E\sin\alpha}{\sin\theta_1} \quad (11)$$

and $I_0 = \dfrac{k\,E}{z} \cdot \dfrac{\sin\alpha}{\sin\theta_1} = k\,E \cdot \dfrac{\cos\beta}{r} \cdot \dfrac{\sin\alpha}{\sin\theta_1}$ (12)

Now

$$P_1 = E \,.\, I_0 \,.\, \cos(\beta - \theta_1) \quad (13)$$

$$= E^2 \,.\, \frac{k\cos\beta}{r} \,.\, \frac{\sin\alpha}{\sin\theta_1} \,.\, \cos(\beta - \theta_1) \quad (14)$$

$$= E^2 \,.\, \frac{k\cos\beta}{r} \,.\, \frac{\sin\alpha}{\sin\theta_1}$$

$$.\,(\cos\beta\cos\theta_1 + \sin\beta\sin\theta_1) \quad (15)$$

$$= E^2 \,.\, \frac{k\cos\beta}{r}\,(\cos\beta\sin\alpha\operatorname{cotan}\theta_1 + \sin\alpha\sin\beta) \quad (16)$$

By substituting value of cotan θ_1 from (9)

$$P_1 = E^2/r\,k\cos\beta$$

$$\left(\sin\alpha\sin\beta + \sin\alpha\cos\beta\left\{\frac{1 - k\cos\alpha}{k\sin\alpha}\right\}\right) \quad (17)$$

$$= E^2/r\,k\cos\beta$$

$$\left(\sin\alpha\sin\beta + \frac{\cos\beta}{k} - \cos\alpha\cos\beta\right) \quad (18)$$

$$= E^2/r\,k\cos^2\beta\,(\sin\alpha\tan\beta + 1/k - \cos\alpha) \quad (19)$$

Again, $W_1 = E \,.\, I_0 \,.\, \sin\,(\beta - \theta_1)$ (20)

$$= E^2 \,.\, \frac{k \cos \beta}{r} \,.\, \frac{\sin \alpha}{\sin \theta_1}$$

$$(\sin \beta \cos \theta_1 - \cos \beta \sin \theta_1) \qquad (21)$$

$$= E^2 \,.\, \frac{k \cos \beta}{r}$$

$$(\sin \alpha \sin \beta \operatorname{cotan} \theta_1 - \sin \alpha \cos \beta) \qquad (22)$$

Substituting value of cotan θ_1 from (9)

$$W_1 = E^2/r\, k \cos \beta$$

$$\left(\sin \alpha \sin \beta \left\{\frac{1 - k \cos \alpha}{k \sin \alpha}\right\} - \sin \alpha \cos \beta\right) \qquad (23)$$

$$= -\, E^2/r\, k \sin \beta \cos \beta$$

$$\left(\frac{\sin \alpha}{\tan \beta} + \cos \alpha - \frac{1}{k}\right) \qquad (24)$$

$$= -\, E^2/r\, k \cos^2 \beta \tan \beta$$

$$\left(\frac{\sin \alpha}{\tan \beta} + \cos \alpha - \frac{1}{k}\right) \qquad (25)$$

$$V A_1 = E \,.\, I_0 = E \,.\, k\, E \,.\, \frac{\cos \beta}{r} \,.\, \frac{\sin \alpha}{\sin \theta_1} \qquad (26)$$

$$= \frac{E^2}{r}\, k \cos \beta\, \frac{\sin \alpha}{(1 + \operatorname{cotan}^2 \alpha)^{½}} \qquad (27)$$

$$= \frac{E^2}{r}\, k \cos \beta \,.\, \sin \alpha \,.\, \frac{(k^2 + 1\ - 2\, k \cos \alpha)^{½}}{k \sin \alpha} \qquad (28)$$

$$V A_1 = E^2/r \cos \beta\, (1 + k^2 - 2\, k \cos \alpha)^{½} \qquad (29)$$

Similarly,

$$P_2 = E_2\, I_2 \cos\,(\beta - \{180 - \theta_2\}\,) \qquad (30)$$

$$W_2 = E_2\, I_2 \sin\,(\beta - \{180 - \theta_2\}\,) \qquad (31)$$

$$V A_2 = E_2 \,.\, I_0 = k\, E \,.\, k\, E \,.\, \frac{\cos \beta}{r} \,.\, \frac{\sin \alpha}{\sin\,(180 - \theta_2)} \qquad (32)$$

From which are obtained by substituting for cotan $(180 - \theta_2)$ its equal $\dfrac{\cos \alpha - k}{\sin \alpha}$ as obtained in (6).

$$P_2 \;=\; E^2/r\, k \cos^2 \beta\, (\sin \alpha \tan \beta + \cos \alpha - k) \qquad (33)$$

$$W_2 \;=\; \frac{E^2}{r} k \cos^2 \beta \tan \beta \left(\frac{\sin \alpha}{\tan \beta} + k - \cos \alpha \right) \qquad (34)$$

$$V A_2 = E^2/r\, k \cos \beta\, (1 + k^2 - 2\, k \cos \alpha)^{1/2} \qquad (35)$$

Copper Loss in tie Circuit. It is obvious that the copper losses in the tie circuit are equal to the power delivered to one end of the tie circuit minus the power received at the other end. That is, (36)

$$I_0^2\, r = P_1 - P_2 = E^2/r\, k \cos^2 \beta\, (\sin \alpha \tan \beta + 1/k - \cos \alpha)$$
$$- E^2/r\, k \cos^2 \beta\, (\sin \alpha \tan \beta + \cos \alpha - k)$$

$$= E^2/r\, k \cos^2 \beta\, (k + 1/k - 2 \cos \alpha) \qquad (37)$$

Synchronizing Power. It may be assumed that the elements of line loss in P_1 and P_2 are variable loads supplied from the busses of transmitting and receiving stations, respectively. On this basis, the power actually transferred from one station to the other for a given phase angle α between their voltages as obtained from (19) to (33) is found to be

$$E^2/r\, k \cos^2 \beta \sin \alpha \tan \beta \qquad (38)$$

$$\text{or } E^2/r\, k \sin \beta \cos \beta \sin \alpha \qquad (39)$$

The synchronizing power P_s, is defined as the change in the amount of power transferred over the tie circuit for one degree change in phase between bus voltages.

Therefore $$P_s = \frac{d}{d\,\alpha} (E^2/r\, k \sin \beta \cos \beta \sin \alpha) \qquad (40)$$

$$= E^2/r\, k \sin \beta \cos \beta \cos \alpha) \qquad (41)$$

which is equivalent to $$\frac{E_1\, E_2}{r} \cdot \frac{x}{z} \cdot \frac{r}{z} \cdot \cos \alpha \qquad (42)$$

$$\text{or } E_1\, E_2 \,.\, x/z^2 \,.\, \cos \alpha \qquad (43)$$

Presented by publication on ✓

SOME NEW FORMULAS FOR REACTANCE COILS

BY H. B. DWIGHT

ABSTRACT OF PAPER

Formulas are presented and derived, which have not been previously published, for mutual inductance of coils with parallel axes, repulsion of coils with parallel axes, and self-inductance of long cylindrical coils.

These formulas apply to practically all cases of reactance coils in common use. They are very convergent and accurate, and will give results to a given degree of accuracy with a minimum amount of labor.

For many engineering problems, precise accuracy is not required, and sets of curves are given from which approximate readings may be taken.

PART I—MUTUAL INDUCTANCE OF COILS WITH PARALLEL AXES

THE common method of mounting reactance coils is with parallel axes, that is, side by side, and it is desirable to have formulas which apply to coils in this position. The values of mutual inductance derived in the following paragraphs are useful for calculating the unbalance in voltage and the means for correcting it, when three coils are placed side by side and connected in a three-phase circuit. The formulas are also useful in the design of radio apparatus. The calculation is very exact for widely separated coils, and it has an accuracy within a small percentage for coils placed as close together as it is usual to mount reactance coils. For usual engineering problems where precise accuracy is not required, it is not necessary to calculate the results, but values may be read from the curves of Fig. 1, thus saving the labor of computation.

A formula for the mutual inductance of two circles formed of one turn of infinitesimally thin wire has been given by S. Butterworth,* for the case when the axes of the circles are parallel, and when the distance between the centers of the circles is somewhat larger than their diameters. In order to obtain a formula suitable for commercial reactance coils of many turns, the author has integrated the above formula four

---

*Scientific Paper No. 320 of the Bureau of Standards, Washington, D. C., June, 1918, Eq. 10 A, and *Philosophical Magazine*, Vol. 31, 1916, page 443.

times over the rectangular section of a cylindrical coil, with the following result:—

M, in henrys,

$$= \frac{2\,\pi^2 a^3 N^2}{0.3937\, b^2\, 10^9} \Bigg[\frac{a}{r}\left(\frac{r}{s} - 1\right)\left(1 + \frac{c^2}{6\,a^2} + \frac{c^4}{144\,a^4}\right)$$

$$+ \frac{a^3}{2\,r^3}\left(1 - \frac{3}{2}\,\frac{s^2}{r^2} + \frac{r^3}{2\,s^3}\right)$$

$$\left(1 + \frac{7}{12}\,\frac{c^2}{a^2} + \frac{13}{240}\,\frac{c^4}{a^4} + \frac{c^6}{960\,a^6}\right)$$

$$- \frac{5}{8}\,\frac{a^5}{r^5}\left(1 - \frac{5\,s^2}{r^2} + \frac{35}{8}\,\frac{s^4}{r^4} - \frac{3}{8}\,\frac{r^5}{s^5}\right)$$

$$\left(1 + \frac{17}{15}\,\frac{c^2}{a^2} + \frac{169}{600}\,\frac{c^4}{a^4} + \frac{41}{2800}\,\frac{c^6}{a^6} + \frac{113}{672{,}000}\,\frac{c^8}{a^8}\right)$$

$$+ \frac{35}{32}\,\frac{a^7}{r^7}\left(1 - \frac{21}{2}\,\frac{s^2}{r^2} + \frac{189}{8}\,\frac{s^4}{r^4} - \frac{231}{16}\,\frac{s^6}{r^6} + \frac{5}{16}\,\frac{r^7}{s^7}\right)$$

$$\Bigg(1 + \frac{155}{84}\,\frac{c^2}{a^2} + \frac{2167}{2520}\,\frac{c^4}{a^4} + \frac{541}{4704}\,\frac{c^6}{a^6}$$

$$+ \frac{2129}{3136 \times 180}\,\frac{c^8}{a^8} + \frac{197}{1960 \times 3456}\,\frac{c^{10}}{a^{10}}\Bigg) + \text{etc.} \Bigg] \qquad (1)$$

where $r^2 = s^2 + b^2$ and $a = \frac{1}{2} d$ = the mean radius of the coil. N is the number of turns in each coil. The two coils are alike and placed as in Fig 1.

The dimensions are assumed to be given in inches; if they are given in centimeters, the factor 0.3937 should be omitted. In measuring the coil, each dimension should be taken as the number of wires or cables in that dimension times their pitch. The actual measured dimension over the copper should not be used, as it is slightly too small.

It will be noted that formula (**1**) is indeterminate when $b = 0$ and therefore $r = s$. For such a case, the following formula should be used:

$$M = \frac{\pi^2 a^4 N^2}{0.3937\, s^3\, 10^9} \Bigg[\left(1 + \frac{c^2}{6\,a^2} + \frac{c^4}{144 a^4}\right)$$

$$+ \frac{9}{4}\,\frac{a^2}{s^2}\left(1 + \frac{7}{12}\,\frac{c^2}{a^2} + \frac{13}{240}\,\frac{c^4}{a^4} + \frac{c^6}{960\,a^6}\right)$$

$$+\frac{375}{64}\frac{a^4}{s^4}\left(1+\frac{17}{15}\frac{c^2}{a^2}+\frac{169}{600}\frac{c^4}{a^4}+\frac{41}{2800}\frac{c^6}{a^6}+\frac{113}{672{,}000}\frac{c^8}{a^8}\right)$$

$$+\frac{8575}{512}\frac{a^6}{s^6}\left(1+\frac{155}{84}\frac{c^2}{a^2}+\frac{2167}{2520}\frac{c^4}{a^4}+\frac{541}{4704}\frac{c^6}{a^6}+\frac{2129}{3136\times 180}\frac{c^8}{a^8}+\frac{197}{1960\times 3456}\frac{c^{10}}{a^{10}}\right)+\text{etc.}\Bigg] \tag{2}$$

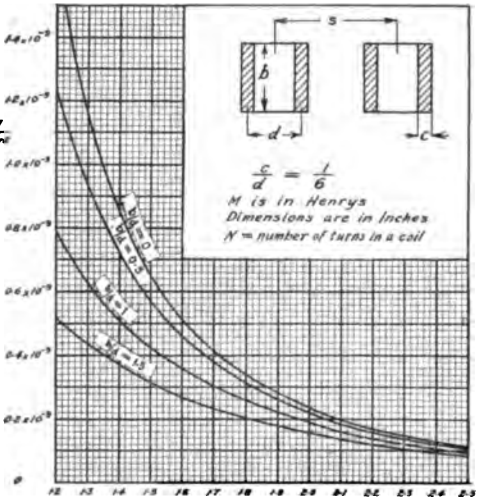


FIG. 1—MUTUAL INDUCTANCE OF REACTANCE COILS WITH PARALLEL AXES

This gives the mutual inductance of two flat disks in the same plane.

An approximate formula for the mutual inductance of two reactance coils with parallel axes has been previously published by the writer.* This was derived by making a certain distortion of one of the coils which, as was stated, tended to make the result slightly too small. The results of this formula for one shape of coil are shown in Curve II, Fig. 2. In order to show that the results were accurate enough for engineering

---

*"Repulsion and Mutual Inductance of Reactors", by H. B. Dwight, *The Electrical World*, p. 1148, June 16, 1917.

work, another formula was derived using a greater distortion which tended to make the results distinctly too large, as shown in Curve III. It is interesting to note that Curve I, which shows the result of the formula now given, lies just a little above Curve II.

The new formula is more convenient to use than the older approximate formula, and gives more accurate results for any given position of the coils. Neither formula is very convergent for s/d less than about 1.2.

The case outlined in Fig. 1, in which the two coils with parallel axes are alike and stand on the same plane, is the simplest for calculation. However, if the coils have parallel axes and are alike, and one stands on a plane a distance e higher than the other, the formula for their mutual inductance is not very complicated. It is as follows:

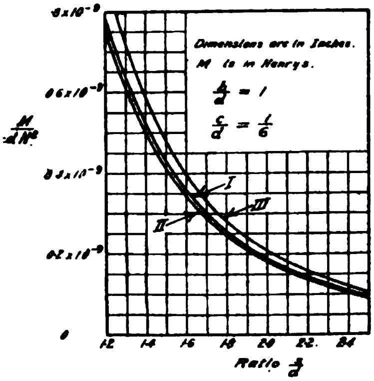


FIG. 2—COMPARISON WITH APPROXIMATE CURVES

M, in henrys,

$$= \frac{\pi^2 N^2}{0.3937\, b^2 \times 10^9} \Bigg[a^4 \left(\frac{1}{p} - \frac{2}{q} + \frac{1}{r} \right)$$

$$\left(1 + \frac{c^2}{6\,a^2} + \frac{c^4}{144\,a^4} \right) - \frac{1}{2}\, a^6 \left\{ \left(\frac{1}{p^3} - \frac{2}{q^3} + \frac{1}{r^3} \right) \right.$$

$$\left. - \frac{3\,s^2}{2} \left(\frac{1}{p^5} - \frac{2}{q^5} + \frac{1}{r^5} \right) \right\}$$

$$\left(1 + \frac{7}{12} \frac{c^2}{a^2} + \frac{13}{240} \frac{c^4}{a^4} + \frac{c^6}{960\,a^6} \right)$$

$$+\frac{5}{8}a^8\left\{\left(\frac{1}{p^5}-\frac{2}{q^5}+\frac{1}{r^5}\right)-5\,s^2\left(\frac{1}{p^7}-\frac{2}{q^7}+\frac{1}{r^7}\right)\right.$$

$$\left.+\frac{35\,s^4}{8}\left(\frac{1}{p^9}-\frac{2}{q^9}+\frac{1}{r^9}\right)\right\}$$

$$\left(1+\frac{17}{15}\frac{c^2}{a^2}+\frac{169}{600}\frac{c^4}{a^4}+\frac{41}{2800}\frac{c^6}{a^6}+\frac{113}{672{,}000}\frac{c^8}{a^8}\right)$$

$$-\frac{35}{32}a^{10}\left\{\left(\frac{1}{p^7}-\frac{2}{q^7}+\frac{1}{r^7}\right)-\frac{21\,s^2}{2}\left(\frac{1}{p^9}-\frac{2}{q^9}+\frac{1}{r^9}\right)\right.$$

$$+\frac{189\,s^4}{8}\left(\frac{1}{p^{11}}-\frac{2}{q^{11}}+\frac{1}{r^{11}}\right)$$

$$\left.-\frac{231\,s^6}{16}\left(\frac{1}{p^{13}}-\frac{2}{q^{13}}+\frac{1}{r^{13}}\right)\right\}$$

$$\left(1+\frac{155}{84}\frac{c^2}{a^2}+\frac{2167}{2520}\frac{c^4}{a^4}+\frac{541}{4704}\frac{c^6}{a^6}+\frac{2129}{3136\times 180}\frac{c^8}{a^8}\right.$$

$$\left.+\frac{197}{1960\times 3456}\frac{c^{10}}{a^{10}}\right)+\text{etc.}\,\Big] \qquad (3)$$

where $p^2 = s^2 + (b - e)^2$

$q^2 = s^2 + e^2$

and $r^2 = s^2 + (b + e)^2$

This formula includes the case of two coaxial coils, when the coils are not near to each other. When the coils are coaxial, the mutual inductance is opposite in sign to the value when the coils are side by side. There is a position, when two coils with parallel axes are diagonally from each other, for which the mutual inductance is zero.

The formula for the mutual inductance of two unequal coils with parallel axes, in any relative position, is similar to the above, but is, of course, more complicated.

Example I. Find the voltage drop due to mutual inductance in each of three coils placed side by side in a row, and carrying three-phase, 60-cycle current, 400 amperes per phase, the data being as follows:—

Mean radius of coils $= a =$ 12.76 inches.
Length of coils $= b =$ 30.87 "
Thickness of winding $= c =$ 4.87 "
Number of turns $= N =$ 114
Distance between centers $=$ 45 inches.

Let the three coils in a row be called *A*, *B* and *C*. For *A* and *B*, $s = 45$ inches.

$$M_{AB} = 0.00142\,(0.0508 + 0.0061 + 0.0007 + 0.00008)$$
$$= 8.2 \times 10^{-6} \text{ henry.}$$

or using Fig. 1, $\dfrac{s}{d} = \dfrac{45}{25.52} = 1.76$

and $\dfrac{b}{d} = \dfrac{30.87}{25.52} = 1.21$

$$M_{AB} = 0.25 \times 10^{-9} \times 114^2 \times 25.52 = 8.3 \times 10^{-6} \text{ henry.}$$

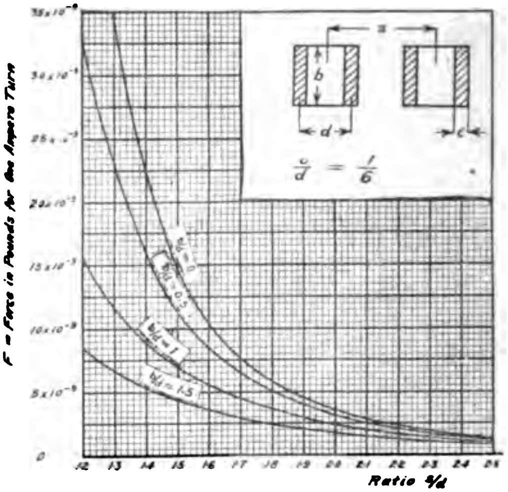


Fig. 3—Mechanical Force Between Reactance Coils with Parallel Axes

Average force in pounds = *F* (from curves) $I_1 I_2 N^2 \cos\theta$ where *N* is the number of turns in each coil and where θ is the phase angle between I_1 and I_2 which are in amperes.

Using a spacing $s = 90$ inches

$$M_{AC} = 1.1 \times 10^{-6} \text{ henry.}$$

Let the currents in the three coils *A*, *B* and *C*, which are 120 deg. apart in phase, be

$$I_A = 400 \text{ amperes}$$
$$I_B = -200 + j\,200\sqrt{3} \text{ amperes}$$
and
$$I_C = -200 - j\,200\sqrt{3} \text{ amperes}$$

The drop in *A* due to mutual inductance is

$$2\pi \times 60j\,(I_B\,M_{AB} + I_C\,M_{AC}) = -9.3 - j\,7.0 \text{ volts.}$$

The drop in B due to mutual inductance is

$2\,\pi \times 60\,j\,(I_A\,M_{AB} + I_C\,M_{BC}) = 10.7 + j\,6.2$ volts.

The drop in C due to mutual inductance is

$2\,\pi \times 60\,j\,(I_A\,M_{AC} + I_B\,M_{BC}) = 10.7 - j\,4.5$ volts.

PART II—REPULSION OF COILS WITH PARALLEL AXES

The formula for calculating the mechanical force exerted between two equal reactance coils placed in the usual way, side by side with parallel axes, may be derived by differentiating the expression for the mutual inductance of the coils in the same position. Curves are given in Fig. 3 which should be more convenient in most cases than the formula, for solving problems.

The differential with respect to s of formula (1) for mutual inductance, using absolute units, is equal to the mechanical force in dynes between the coils when one absampere is flowing in each coil, the currents being in phase. Changing the units, this gives the force in pounds for one ampere turn, which is equal to:

F, in pounds,

$$
\begin{aligned}
= \frac{2\,\pi^2\,a^2}{4.45 \times 10^7\,b^2} \Bigg[& \frac{a^2}{r^2}\left(\frac{r^2}{s^2} - \frac{s}{r}\right)\left(1 + \frac{c^2}{6\,a^2} + \frac{c^4}{144\,a^4}\right) \\
& + \frac{3}{4}\,\frac{a^4}{r^4}\left(\frac{4\,s}{r} - \frac{5\,s^3}{r^3} + \frac{r^4}{s^4}\right) \\
& \qquad \left(1 + \frac{7}{12}\,\frac{c^2}{a^2} + \frac{13}{240}\,\frac{c^4}{a^4} + \frac{c^6}{960\,a^6}\right) \\
& - \frac{75}{64}\,\frac{a^6}{r^6}\left(\frac{8\,s}{r} - \frac{28\,s^3}{r^3} + \frac{21\,s^5}{r^5} - \frac{r^6}{s^6}\right) \\
& \left(1 + \frac{17}{15}\,\frac{c^2}{a^2} + \frac{169}{600}\,\frac{c^4}{a^4} + \frac{41}{2800}\,\frac{c^6}{a^6} + \frac{113}{672{,}000}\,\frac{c^8}{a^8}\right) \\
& + \frac{245}{256}\,\frac{a^8}{r^8}\left(\frac{32\,s}{r} - \frac{216\,s^3}{r^3} + \frac{396\,s^5}{r^5} - \frac{429}{2}\,\frac{s^7}{r^7} + \frac{5}{2}\,\frac{r^8}{s^8}\right) \\
& \left(1 + \frac{155}{84}\,\frac{c^2}{a^2} + \frac{2167}{2520}\,\frac{c^4}{a^4} + \frac{541}{4704}\,\frac{c^6}{a^6} + \frac{2129}{3136 \times 180}\,\frac{c^8}{a^8}\right. \\
& \qquad \left. + \frac{197}{1960 \times 3456}\,\frac{c^{10}}{a^{10}}\right) + \text{etc.} \Bigg] \qquad (4)
\end{aligned}
$$

where $r^2 = s^2 + b^2$ and $a = \frac{1}{2} d$ = the mean radius of the coil. The dimensions may be given in either inches or centimeters, since only ratios of dimensions appear.

It results therefore, that if all the dimensions, including spacing, of a group of coils be increased by a certain ratio, but the currents be left the same, the mechanical force in pounds is not changed at all. However, large coils are generally subject to large forces, since they carry proportionately large currents.

When different alternating currents I_1 and I_2, at a phase angle θ, flow in the two coils, the average force in pounds is

$$F\, I_1\, I_2\, N^2 \cos\theta$$

where I_1 and I_2 are the effective values of the currents in amperes and where N is the number of turns in each coil.

As in the case of the expression for mutual inductance, formula (4) becomes indeterminate when $b = 0$ and therefore $r = s$. In such a case, which is that of two flat disks in the same plane, the following formula may be used:

$$\begin{aligned} F, \text{ in pounds, } = \frac{3\,\pi^2\,a^4}{4.45 \times 10^7\,s^4} \Bigg[& \left(1 + \frac{c^2}{6\,a^2} + \frac{c^4}{144\,a^4}\right) \\ & + \frac{15}{4}\frac{a^2}{s^2}\left(1 + \frac{7}{12}\frac{c^2}{a^2} + \frac{13}{240}\frac{c^4}{a^4} + \frac{c^6}{960\,a^6}\right) \\ & + \frac{875}{64}\frac{a^4}{s^4}\left(1 + \frac{17}{15}\frac{c^2}{a^2} + \frac{169}{600}\frac{c^4}{a^4} + \frac{41}{2800}\frac{c^6}{a^6} \right. \\ & \qquad \left. + \frac{113}{672{,}000}\frac{c^8}{a^8}\right) \\ & + \frac{25{,}725}{512}\frac{a^6}{s^6}\left(1 + \frac{155}{84}\frac{c^2}{a^2} + \frac{2167}{2520}\frac{c^4}{a^4} + \frac{541}{4704}\frac{c^6}{a^6} \right. \\ & \qquad \left. + \frac{2129}{3136 \times 180}\frac{c^8}{a^8} + \frac{197}{1960 \times 3456}\frac{c^{10}}{a^{10}}\right) + \text{etc.} \Bigg] \end{aligned} \tag{5}$$

The mechanical force between two coils side by side with parallel axes is a repulsion when the currents flow in the same direction around both coils, and it is an attraction when the currents are in opposite directions.

The value of force as calculated above is the average force, that is, the effective steady sustained pressure, due to alternating current. It is the same in value as the force due to direct current. At the peak of the alternating-current wave, when the currents are in phase, the force rises to double the above value, and it becomes zero when the momentary current becomes zero.

A comparison is shown in Fig. 4 between calculated values and a test curve.* The approximate data for the test curve are given in example II.

The approximate formula previously published by the author in the *Electrical World* of June 16, 1917, gives results which are slightly less than those of formula (4) which are more accurate. The curves have practically the same relative position as in Fig. 2.

Example II—Find the average mechanical force acting on each of two coils carrying single-phase currents which are in phase, the conditions being as follows:

Mean radius of coils $= a = 12.76$ inches.
Length of coils $= b = 30.87$ "
Thickness of winding $= c = 4.87$ "
Number of turns $= N = 114$
Current in amperes $= I_1 = I_2 = 400$ amperes.

The coils are placed side by side, with the parallel axes 45 inches apart.

$$\text{Force} = 158\,(0.0362 + 0.0065 + 0.0010 + 0.0001) = 6.89 \text{ pounds.}$$

Using the curves of Fig. 3,

$$\frac{b}{d} = \frac{30.87}{25.52} = 1.21, \text{ and } \frac{s}{d} = \frac{45}{25.52} = 1.76$$

$$\text{Force} = 3.3 \times 10^{-9} \times 400^2 \times 114^2 = 6.8 \text{ pounds.}$$

Example III. Find the average mechanical force on each of three coils of the same size and spacing as those in Example II, placed side by side in a row, and carrying three-phase current, 400 amperes per phase. In determining the average mechanical forces due to three-phase currents, the forces due to the products of the instantaneous values of two alternating currents, which are 120 deg. out of phase, are calculated in the same way as the watts due to the products of the instantaneous values of

---

*The test curve was published in an article on "The Mechanical Stresses in Reactance Coils" by W. M. Dann, *The Electric Journal*, page 206, April 1914.

an alternating current and an alternating voltage which are out of phase.

Let the three coils in a row be called *A*, *B*, and *C*. The currents in them are 120 deg. apart, in phase. The average force on *A* caused by I_B is

$$6.89 \cos 120 \text{ deg.} = 3.44 \text{ pounds,}$$

the calculation being similar to Example II.

To find the force on *A* caused by I_c, formula (4) is used, the axial spacing being 90 in. The average force on *A* caused by I_c is

$$0.50 \cos 120 \text{ deg.} = 0.25 \text{ pounds.}$$

This is to be added to the force caused by I_B, making a total average force on *A* of 3.69 pounds, tending to move it toward the center-coil. Coil *C* is also under an average attraction of 3.69 pounds. Coil *B* is attracted by both *A* and *C*, and although there is a momentary force first in one direction and then in the other, the average force on *B* is zero.

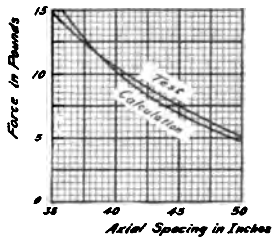


FIG. 4—COMPARISON OF CALCULATION WTIH TEST

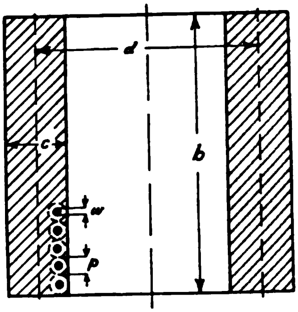


FIG. 5—SECTION OF REACTANCE COIL

Since the above forces increase as the square of the current they attain large values at times of short circuit.

PART III—SELF-INDUCTANCE OF LONG REACTANCE COILS

Reactance coils, as generally manufactured, are cylindrical in shape and have an axial length practically as long as, or longer than, their mean diameter. The radial thickness of the winding is usually considerable. The self-inductance of a thick coil with usual radial thickness of winding, may be as much as 15 per cent less than the self-inductance, L_s, of a very thin solenoid of the same mean diameter. The self-inductance of the above described thick coils may be calculated very precisely by formula (**13**) given below, which may be used to a fair degree of approximation for coils somewhat shorter than their mean diameter. Curves are given in Figs. 6 and 7 from which readings may be taken which will be correct within about one per cent for most shapes of reactance coils.

An alternative method for determining the self-inductance of a cylindrical coil is to obtain the value of L_s from Table XXI, Scientific Paper No. 169 of the Bureau of Standards, and obtain the correction for the thickness of the coils from Dr. E. B. Rosa's equations (91) and (93) and Tables IX and X of the same paper. This method is especially useful for coils just shorter than their mean diameter, since other formulas are then not very convergent.

As indicated in Fig. 5, all the dimensions of a coil should be measured to the pitch lines of the conductors. Thus the length of the coil should be taken as the number of conductors per layer multiplied by the pitch of the conductors. The exact dimensions over the copper are slightly too small and should not be used.

In order to obtain an expression for the self-inductance of a thick cylindrical coil, it is first necessary to know the mutual inductance of two coaxial and concentric thin solenoids of equal length. For long solenoids this has been given in a useful form by T. H. Havelock.*

By using the form in elliptic integrals given in equation (**4**) of Havelock's paper in the *Philosophical Magazine,* and by changing to the dimensions shown in Fig. 5, the expression for mutual inductance becomes:

$$M = \frac{\pi^2 d^2 N^2}{b^2}\Bigg[b - \frac{2\,D}{3\,\pi\,d^2}\left\{ (D^2 + d^2)\,E - (D^2 - d^2)\,F \right\}$$
$$- D \left\{ -\frac{1}{8}\frac{D}{b} + \frac{1}{128}\left(1 + \frac{d^2}{D^2}\right)\frac{D^3}{b^3} \right.$$
$$- \frac{1}{1024}\left(1 + 3\,\frac{d^2}{D^2} + \frac{d^4}{D^4}\right)\frac{D^5}{b^5}$$
$$\left. + \frac{5}{2^{15}}\left(1 + 6\,\frac{d^2}{D^2} + 6\,\frac{d^4}{D^4} + \frac{d^6}{D^6}\right)\frac{D^7}{b^7} - \ldots \right\} \Bigg]$$

abhenrys. (**6**)

***The Philosophical Magazine,* March 1908, Page 340, equation 25: and *Bulletin* of the Bureau of Standards, Vol. 8, No. 1, 1911, Page 56 equation 38.

where d and D are the diameters of the two thin solenoids, where N is the number of turns on each solenoid, and where F and E are complete elliptic integrals of the first and second kinds, to modulus d/D.

The two elliptic integrals can be expanded in terms of the complementary modulus k' as follows:*

$$E = 1 + \frac{1}{2}\, k'^2 \left(\operatorname{logh} \frac{4}{k'} - \frac{1}{1.2} \right) + \frac{1^2\, 3}{2^2\, 4} k'^4 \left(\operatorname{logh} \frac{4}{k'} - \frac{2}{1.2} - \frac{1}{3.4} \right) + \ldots$$

$$F = \operatorname{logh} \frac{4}{k'} + \frac{1^2}{2^2} k'^2 \left(\operatorname{logh} \frac{4}{k'} - \frac{2}{1.2} \right) + \frac{1^2\, 3^2}{2^2\, 4^2}\, k'^4 \left(\operatorname{logh} \frac{4}{k'} - \frac{2}{1.2} - \frac{2}{3.4} \right) + \ldots \qquad (7)$$

where $\quad k' = \sqrt{1 - k^2} = \sqrt{1 - d^2/D^2}$

The expression for mutual inductance can now be integrated twice over the section of the thick coil indicated in Fig. 5, and the following formula for the self-inductance of a long thick coil is obtained:

$$L = L_s + \Delta\, L \qquad (8)$$

where L_s, in henrys,

$$= \frac{\pi^2\, d^2\, N^2}{0.3937 \times 10^9\, b} \left[1 - \frac{4\, d}{3\, \pi\, b} + \frac{1}{8} \frac{d^2}{b^2} - \frac{1}{64} \frac{d^4}{b^4} + \frac{5}{1024} \frac{d^6}{b^6} - \ldots \right] \qquad (9)$$

and where $\Delta\, L$, in henrys,

$$= \frac{\pi^2\, d^2\, N^2}{0.3937 \times 10^9\, b} \left[- \frac{2}{3} \frac{c}{d} + \frac{1}{3} \frac{c^2}{d^2} + \frac{4\, d}{3\, \pi\, b} \left\{ \frac{1}{4} \frac{c^2}{d^2} \left(\operatorname{logh} \frac{4\, d}{c} - \frac{23}{12} \right) - \frac{1}{80} \frac{c^4}{d^4} \left(\operatorname{logh} \frac{4\, d}{c} - \frac{1}{20} \right) + \ldots \right\} \right.$$

---

*[*Bulletin* of the Bureau of Standards, Vol. 8, No. 1, by E. B. Rosa and F. W. Grover, equation 3, page 8.

$$+\frac{c^2}{d^2}\left\{\frac{1}{12}\frac{d^2}{b^2}-\frac{7}{192}\frac{d^4}{b^4}+\frac{17}{768}\frac{d^6}{b^6}\right.$$

$$\left.-\frac{775}{32\times1536}\frac{d^8}{b^8}+\ldots\right\}$$

$$+\frac{c^4}{d^4}\left\{\frac{1}{72}\frac{d^2}{b^2}-\frac{13}{960}\frac{d^4}{b^4}+\frac{169}{7680}\frac{d^6}{b^6}\right.$$

$$\left.-\frac{2167}{32\times2304}\frac{d^8}{b^8}+\ldots\right\}+\ldots$$

$$\left.+\frac{2b}{\pi q d N}\left(\text{logh}\frac{p}{w}+0.14\right)\right] \tag{10}$$

In the above formulas, N is the number of turns of effective conductor in the coil, q is the number of wires or cables in parallel, and the dimensions are in inches. If the dimensions are in centimeters, the factor 0.3937 should be omitted, and if the inductance is to be given in abhenrys instead of henrys the factor 10^9 should be omitted. Note that

$$\begin{aligned}\text{logh}\, m &= \text{the hyperbolic or natural logarithm of } m\\ &= 2.3026 \log_{10} m.\end{aligned}$$

The quantity L_s is the self-inductance of an infinitely thin solenoid of diameter d. Formula (**9**) is the same as (79), Bulletin of the Bureau of Standards, Vol. 8, No. 1 or (21A), Scientific Paper No. 320 of the Bureau of Standards. The general term of the series may be obtained from either of these references. There are other formulas (See 11) which are more convergent than (**9**). The value of L_s may also be accurately and conveniently found from Table XXI Bulletin of the Bureau of Standards, Vol. 8, No. 1.

The quantity ΔL, given by (**10**), is the change in the inductance due to the radial thickness of the winding. Formula (**10**) should not be used for practical problems, because formula (**13**) to be given later, is more convergent, and therefore more convenient and accurate, for any coil with which (**10**) can be used. Formula (**10**) was published by the author in the *Electrical World*, Feb. 9, 1918, page 300. It gives the same result, except for the term $\frac{2167}{32\times2304}\frac{d^8}{b^8}$, as a method published by S. Butterworth in the *Proceedings* of the Physical Society of London, Vol. 27, 1915, page 371.

It is assumed that the coil is wound with round wire or cable. The last term of (**10**) represents the effect of the air space between the wires.

A very convergent formula for the self-inductance of an infinitely thin solenoid which has not been previously published, may be derived directly from (36) or (39), Bulletin of the Bureau of Standards, Vol. 8, No. 1, by making the dimensions of the two solenoids equal, thus obtaining the self-inductance of one solenoid instead of the mutual inductance of two. Both (**36**) and (**39**) give the same result:

$$L_s\text{, in henrys, } = \frac{\pi^2 d^2 N^2}{0.3937 \times 10^9 b} \times \frac{d}{2b}\left[\frac{1}{m} - \frac{8}{3\pi} - \frac{m^3}{8} + \frac{m^5}{16} - \frac{15}{128}m^7 + \frac{21}{128}m^9 - \frac{315}{1024}m^{11} + \frac{297}{512}m^{13} - \frac{39{,}039}{32{,}768}m^{15} + \ldots\right] \qquad \textbf{(11)}$$

where $m^2 = \dfrac{d^2}{d^2 + 4b^2}$

This is much more convergent than (9). It is somewhat more convergent than (20 A), Scientific Paper No. 320 of the Bureau of Standards, since $m^2 = \dfrac{d^2}{d^2 + 4b^2}$ is smaller than $k^2 = \dfrac{d^2}{d^2 + b^2}$. When $b = d$, 9 terms of (11) give six significant figures, but 13 terms of (20 A) are required to give this result.

Formula (11) need not necessarily be used for obtaining the value of L_s, because formulas (76), (77), and (78), Bulletin of the Bureau of Standards, Vol. 8, No. 1, are extremely convergent, and very accurate values of L_s can also be easily obtained from Table XXI of the same Bulletin.

Series in m for thick coils can be derived (see formula 13) which are similar to those in d/b of formula (**10**), but which are much more convergent than they are, and which therefore supersede them for practical calculations. These series in m have not been previously published. The series in m will give

5 or 6 significant figures for the self inductance of a thick cylindrical coil whose length is as short as its diameter, but formula (**10**) is not nearly so accurate for such a short coil. That the two formulas (**10**) and (**13**) are equivalent may be shown by the fact that they give the same results for any problem for which they are both convergent. It is also proved by expanding the terms of one formula by the binomial theorem, and this is found to produce the other formula. The derivation of (**13**) is as follows.

It is first necessary to establish a proposition regarding the mutual inductance of two solenoids, similar to Maxwell's proposition regarding two coils, in paragraph 700 of "Electricity and Magnetism".* In the same manner as in the paragraph referred to, it may be shown that, if M_s is the mutual inductance of the central solenoids of two cylindrical coils of mean radii a_1 and a_2 and of thicknesses of winding c_1 and c_2, then the mutual inductance of the two thick coils is

$$\begin{aligned} M = M_s &+ \frac{c_1^2}{2^2\,\underline{|3}}\,\frac{\partial^2 M_s}{\partial a_1^2} + \frac{c_2^2}{2^2\,\underline{|3}}\,\frac{\partial^2 M_s}{\partial a_2^2} \\ &+ \frac{c_1^4}{2^4\,\underline{|5}}\,\frac{\partial^4 M_s}{\partial a_1^4} + \frac{c_1^2\,c_2^2}{2^4\,\underline{|3}\;\underline{|3}}\,\frac{\partial^4 M_s}{\partial a_1^2\,\partial a_2^2} + \frac{c_2^4}{2^4\,\underline{|5}}\,\frac{\partial^4 M_s}{\partial a_2} \\ &+ \frac{c_1^6}{2^6\,\underline{|7}}\,\frac{\partial^6 M_s}{\partial a_1^6} + \frac{c_1^4\,c_2^2}{2^6\,\underline{|3}\;\underline{|5}}\,\frac{\partial^6 M_s}{\partial a_1^4\,\partial a_2^2} \\ &\qquad + \frac{c_1^2\,c_2^4}{2^6\,\underline{|3}\;\underline{|5}}\,\frac{\partial^6 M_s}{\partial a_1^2\,\partial a_2^4} + \frac{c_2^6}{2^6\,\underline{|7}}\,\frac{\partial^6 M_s}{\partial a_2^6} + \cdots \end{aligned} \tag{12}$$

This formula is most accurate for coils that are far apart, and it does not give good results for coils that are close together. It may be useful for giving extensions to present formulas for mutual inductance of solenoids.

If formula (17), Bulletin of the Bureau of Standards Vol. 8, No. 1, for the mutual inductance of two circles far apart, be integrated to give the mutual inductance of two coaxial solenoids whose centers are far apart measured along the axis, terms are obtained which are of the same form as the second half of formula (38) of the above bulletin, which forms a series

---

*See also Scientific Paper No. 169 of the Bureau of Standards by E. B. Rosa and F. W. Grover, page 33.

in A/l. See formula (3) when $s = 0$. Now proposition (**12**) gives exactly the effect of the thicknesses c_1 and c_2 of the coils for the formula for coils far apart, and it is to be expected that it will give the effect of c_1 and c_2 for the same terms which appear in the last half of (38, Bulletin, Vol. 8, No. 1).

This is found to be the case, and the series in c^2 and c^4 in the last half of (**10**), can be derived by means of (**12**). The latter method is much easier than the method of straight integration of (38, Bulletin Vol. 8, No. 1), since differentiation is in general a simpler process than integration, and fewer and smaller terms are required to be handled.

Now it is noticeable that formula (36) of the Bulletin, Vol. 8, No. 1, contains the first line of (38) of the same bulletin. Therefore the terms in A/r of (36, Vol. 8, No. 1), give the same total result as the terms in A/l of (38, Vol. 8, No. 1). This may be proved definitely by expanding one set of terms by the binomial theorem, and they are found to produce the other set. Therefore, since proposition (**12**) applies to the terms in A/l of (38 Vol. 8, No. 1), it applies to the terms in A/r of (36, Vol. 8, No. 1). A similar statement applies to (39, Vol. 8, No. 1) since that formula is the same as (36, Vol. 8, No. 1) when the two solenoids are of equal length.

The differentiation of (36) or (39) (Vol. 8, No. 1), is not difficult, and then proposition (**12**) can be applied. It is found by expanding some of the terms by the binomial theorem that when $A = a$

$$\frac{\partial^2 M}{\partial A^2} = \frac{\partial^2 M}{\partial a^2}$$

as would be expected, and this produces a certain simplification in the series. Terms from formula (**10**) extended are then included to give the effect of integrating the terms in a/A of (36) or (38) (Vol. 8, No. 1), and the following complete result is obtained:

$$\Delta L, \text{ in henrys} = \frac{\pi^2 d^2 N^2}{0.3937 \times 10^9 b} \Bigg[- \frac{2}{3}\frac{c}{d} + \frac{1}{3}\frac{c^2}{d^2}$$

$$+ \frac{4d}{3\pi b} \Bigg\{ \frac{1}{4}\frac{c^2}{d^2}\left(\text{logh}\frac{4d}{c} - \frac{23}{12}\right) - \frac{1}{80}\frac{c^4}{d^4}$$

$$\left(\operatorname{logh}\frac{4\,d}{c}-\frac{1}{20}\right)-\frac{1}{896}\,\frac{c^6}{d^6}\left(\frac{23}{20}\operatorname{logh}\frac{4\,d}{c}-\frac{4547}{5600}\right)$$

$$+\ldots\Big\}$$

$$+\frac{c^2}{d^2}\times\frac{d}{b}\Big\{\frac{m}{6}-\frac{5}{24}\,m^3+\frac{m^5}{3}-\frac{95}{128}\,m^7+\frac{217}{128}\,m^9$$

$$-\frac{2135}{512}\,m^{11}+\frac{11\times 1961}{2048}\,m^{13}-\frac{13\times 68{,}915}{32{,}768}\,m^{15}$$

$$+\ldots\Big\}$$

$$+\frac{c^4}{d^4}\times\frac{d}{b}\Big\{\frac{m}{36}-\frac{17}{180}\,m^3+\frac{53}{96}\,m^5-\frac{1265}{576}$$

$$m^7+\frac{38{,}857}{4608}\,m^9-\frac{3913}{128}\,m^{11}+\frac{231\times 9551}{20{,}480}\,m^{13}$$

$$-\frac{143\times 10{,}625}{4096}\,m^{15}+\ldots\Big\}$$

$$+\frac{c^6}{d^6}\times\frac{d}{b}\Big\{-\frac{m^3}{120}+\frac{15}{112}\,m^5-\frac{1117}{672}\,m^7+\frac{1183}{96}\,m^9$$

$$-\frac{21\times 3641}{1024}\,m^{11}+\frac{11\times 367{,}621}{10{,}240}\,m^{13}$$

$$-\frac{143\times 109{,}353}{8192}\,m^{15}+\ldots\Big\}+\ldots$$

$$+\frac{2\,b}{\pi\,q\,d\,N}\left(\operatorname{logh}\frac{p}{w}+0.14\right)\Big]\qquad\textbf{(13)}$$

where $m^2=\dfrac{d^2}{d^2+4\,b^2}$

This formula is to be used in conjunction with (**8**). It should be used instead of (**10**) for practical problems, since it is more convenient and accurate. It appears longer than (**10**), but that is merely because more terms have been worked out. In reality, fewer terms of (**13**), than of (**10**), are needed to obtain the self-inductance of a given coil to a certain number of significant figures. The terms which are not needed can easily be neglected. For most coils made, the terms in c^6/d^6 will be found to be practically negligible, even for precision work.

The last term of formula (**13**) represents the effect of the air and insulation space between the conductors. It is assumed that the coil is wound with round wire or cable. If square wire is used, the constant 0.14 in the last term should be omitted. Therefore, for uniform current distribution over the section of the coil, as, for example, with square wire with infinitely thin insulation, the last term becomes zero. For precise work, the constant 0.14, which is an average value, should be changed in accordance with Dr. E. B. Rosa's results for various shapes of coils, given in the Bulletin of the Bureau of Standards, Vol. 3, 1907, page 37, and Vol. 8, No. 1, 1911, page 141.

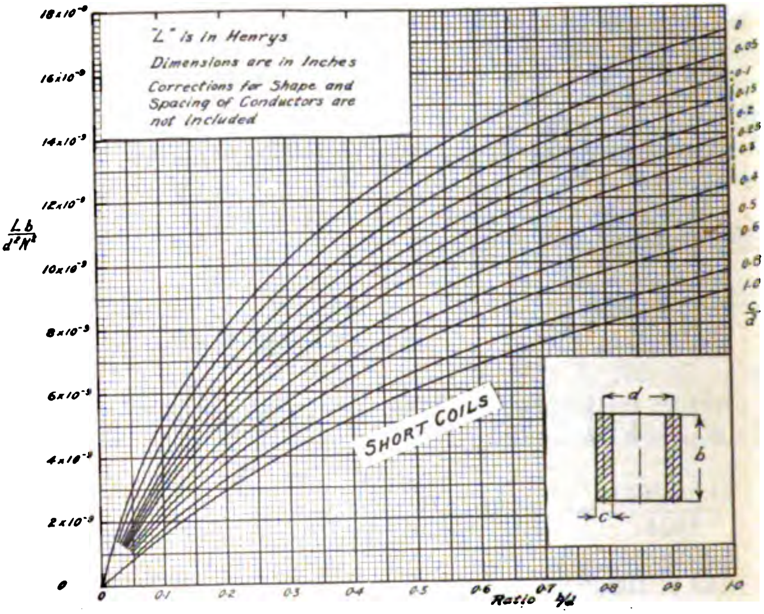


Fig. 6—Self-Inductance of Short Reactance Coils

A set of curves showing the self-inductance of cylindrical coils of various shapes, is given in Figs. 6 and 7. This should be useful for many engineering problems where precise accuracy is not required. The curves can give quicker and more accurate results for many cases than most approximate formulas.

The values of inductance of long coils in which *c* is almost as great as *d*, were calculated by a formula derived by the writer.* The values of inductance of coils considerably shorter than their mean diameter were derived from Tables *C* and *D*, by Prof. T. R. Lyle, Scientific Paper No. 320 of the Bureau of Standards, pages 569 and 570.

Example IV. To find the self-inductance of a coil wound with 1000 turns of round wire in 10 layers of 100 turns each.

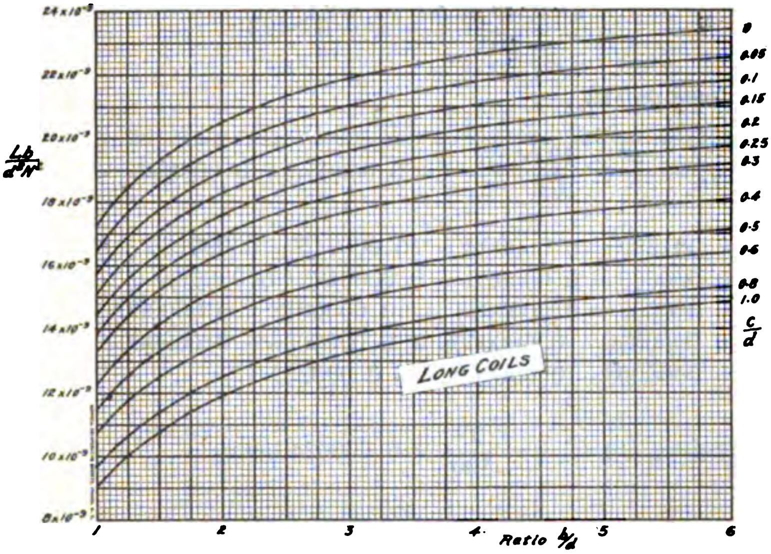


FIG. 7—SELF-INDUCTANCE OF LONG REACTANCE COILS

The diameter of the insulated wire is $p = 0.1$ cm. and the diameter of the bare wire is $w = 0.08$ cm. The mean diameter of the coil is $d = 10$ cm., the length is $b = 10$ cm., and the thickness is $c = 1$ cm. (Example I, Bulletin of the Bureau of Standards Vol. 4, No. 3, page 374.)

By formula (**11**), $m = \frac{1}{\sqrt{5}}$

---

*Formula (5), "Self Inductance of Long Reactance Coils", by H. B. Dwight, *The Electrical World*, Feb. 9, 1918, page 300.

and L_s, in henrys, $= \dfrac{\pi^2 \times 10 \times N^2}{10^9} [1.118034 - 0.424413$

$$- 0.005590 + 0.000559 - 0.000210 + 0.000059$$
$$- 0.000022 + 0.000008 - 0.000003]$$

$$= \frac{\pi^2 \times 10 \times N^2}{10^9} \times 0.688422$$

Table XXI, Scientific Paper No. 169, of the Bureau of Standards, gives

$$\frac{\pi^2 \times 10 \times N^2}{10^9} \times 0.688423$$

By formula (**13**),

$$\Delta L = \frac{\pi^2 \times 10 \times N^2}{10^9} [- 0.066667 + 0.003333$$

$$+ 0.001880 - 0.000002 + \{0.000745 - 0.000186$$
$$+ 0.000060 - 0.000026 + 0.000012 - 0.000006$$
$$+ 0.000003 - 0.000002\} + \{0.0000012 - 0.0000008$$
$$+ 0.0000010 - 0.0000008 + 0.0000006 - 0.0000004$$
$$+ 0.0000003 - 0.0000002\} + 0.000231]$$

Therefore, $\Delta L = - \dfrac{\pi^2}{100} \times 0.060624$ henry.

By (**8**), $L = L_s + \Delta L = \dfrac{\pi^2}{100} \times 0.627798 = 0.061961$ henry.

The value given by (**9**) and (**10**) is 0.06194 henry.

The value given in the Bulletin of the Bureau of Standards, Vol. 4, No. 3, Exam. 1, page 374, is 0.061865 henry, which is less than 1/5 of 1 per cent different from the value given by (**13**). The result given in the above bulletin was obtained by Dr. E. B. Rosa's formula (91) Scientific Paper No. 169, of the Bureau of Standards. This formula has the advantage that it is equally applicable to coils of all lengths, whereas (**13**) is not very convergent for coils which are shorter than their mean diameter. Where (**13**) is convergent, however, it is more accurate than the other formula.

Example V. Find the self-inductance of a coil of square wire with very thin insulation in which

$$\frac{b}{d} = 2, \frac{c}{d} = \frac{1}{10} \text{ and } \frac{\pi^2 N^2 d^3}{8 b^2} = 10^6 \text{ abhenrys.}$$

(Example 9, Scientific Paper No. 320 of the Bureau of Standards, page 559).
By formula (11),

$$L_s = \frac{\pi^2 d^2 N^2}{b} [1.0307764 - 0.2122066 - 0.0004458$$

$$+ 0.0000131 - 0.0000015 + 0.0000001]$$

$$= \frac{\pi^2 d^2 N^2}{b} \times 0.8181357$$

Table XXI, Scientific Paper No. 169 of the Bureau of Standards, gives

$$L_s = \frac{\pi^2 d^2 N^2}{b} \times 0.818136$$

By formula (**13**),

$$\Delta L = \frac{\pi^2 d^2 N^2}{b} [- 0.066667 + 0.003333 + 0.000940$$

$$- 0.000001 + \{0.000202 - 0.000015 + 0.000001\}$$

$$+ \{0.0000003\}]$$

$$\Delta L = - \frac{\pi^2 d^2 N^2}{b} \times 0.062205 \text{ abhenrys.}$$

$\Delta L = - 0.995286$ millihenry.

The value given by formula (29 A), Scientific Paper No. 320 of the Bureau of Standards, is

$\Delta L = - 0.99526$ millihenry.

Example VI. A problem relating to a coil which is more similar to usual current-limiting reactors may be of interest. Find the self-inductance of a coil of mean diameter $d = 50$ inches, wound with 10 layers of cable, 75 turns per layer, two cables in parallel. The diameter of the cable is $w = 0.5$ in., and the distance from center to center of cables is $p = 1$ in. Therefore,

$b = 75$ in, $c = 10$ in., $q = 2$, and $N = 375$.

By formula (**11**),

$$L_s, \text{ in henrys, } = \frac{\pi^2 \times 50^2 \times 375^2}{0.3937 \times 10^9 \times 75} \times \frac{50}{150} [3.1623 - 0.8488 - 0.0040 + 0.0002]$$

$$= \frac{\pi^2 \times 50^2 \times 375^2}{0.3937 \times 10^9 \times 75} \times 0.7699$$

By formula (**13**),

$$\Delta L = \frac{\pi^2 \times 50^2 \times 375^2}{0.3937 \times 10^9 \times 75} \times [-0.1333 + 0.0133 + 0.0031 + \{0.0014 - 0.0002\} + 0.0011]$$

Therefore, $L = \frac{\pi^2 \times 50^2 \times 375^2}{0.3937 \times 10^9 \times 75} \times [0.7699 - 0.1146]$

$$= 0.07700 \text{ henry.}$$

The result, when calculated by (**9**) and (**10**) is the same.

Presented by publication only.

INHERENT LIMITATIONS ON TRANSFORMATIONS POSSIBLE BY STATIONARY APPARATUS

BY JOSEPH SLEPIAN

ABSTRACT OF PAPER

Expressions for electrostatic and electromagnetic energies, Joulian heat dissipation and power are given in complex quantities. The pure imaginary part of the expression for power in a static network is shown to be equal to 2 ω times the difference between the mean electromagnetic energy and mean electrostatic energy. Use is made of this new principle in considering the problems of power-factor correction and phase splitting. It is shown that in general for phase transforming by static apparatus both magnetic and electrostatic storage of energy are necessary, and it is shown how the minimum amounts of each are determined by the load.

The symmetry of the coefficients in the general equations for the steady state in a static network is demonstrated, and it is shown that limitations upon voltage and current transformations follow. The voltage regulation of any phase-splitting arrangement is considered.

CONTINUED failures in attempts to construct networks of stationary apparatus, which shall have certain characteristics, lead to the conviction that such characteristics are unattainable by static means. Examples are power-factor correction without condensers, single-phase to balanced polyphase transformations by condensers only or inductances only, or single-phase to polyphase transformations with inherent regulation for variable load. However convincing such failures may be, it is always satisfying to obtain general demonstrations of the impossibility of attainment of the characteristics desired, especially as the demonstrations generally reveal what characteristics can be obtained.

Considering the requirements of energy storage in the transforming apparatus, the problems mentioned above fall into two classes. In the problem of balanced power-factor correction, the instantaneous power drawn from any one phase is exactly equal to the instantaneous power supplied to the other phases. The instantaneous power supplied to the power-factor correcting apparatus is always zero. Hence there is no *a priori* reason for supposing a relation between the wattless power supplied by the apparatus and the electromagnetic energy stored up in the apparatus. In fact, there is no such relation when rotating machines are considered. The wattless power supplied by a synchronous condenser may be varied over a wide range without changing the kinetic energy or magnetic energy as determined by the flux, which is stored in the machine. For purely static apparatus, however, such a relation does exist as is shown in III. Even when the wattless power it supplies is balanced, a static network must have a minimum of energy storage, and this minimum varies with the magnitude of the wattless load.

In the problem of transformation from single phase to balanced polyphase, there is an obvious requirement for energy storage irrespective of the character of the transforming apparatus. The power supplied by the single-phase line has a double-frequency component and is pulsating. The power drawn by the balanced polyphase load, however, is continuous. Hence it is clear that the transforming apparatus must store up energy while the single-phase power is at its maximum, to be given out again while the single-phase power is at its minimum. This clearly calls for momentary energy storage, and therefore for a minimum mean energy storage dependent on the load. In rotating phase-converters, such as are in general use, the pulsations are in the kinetic energy of the rotor. The mean kinetic energy on the rotor is in general far in excess of the minimum mean energy called for by the nature of the transformation. This is because the heating limitations upon transforming electrical energy into mechanical are what determine the size of the rotor, rather than the energy considerations. If the heating limitations could be removed, the most economical rotating phase converter would be one in which the rotor would come to rest twice per cycle, and where the maximum velocity of the rotor would vary with the load. It is remarkable that networks of static apparatus can be devised which will effect the phase transformation with the minimum amount of energy storage called for above, provided that the polyphase power factor exceeds 0.707. This is shown in section 3. In such networks the condensers are without charge, and the reactors without flux simultaneously twice every cycle.

In all which follows, by static network is meant a network constructed of self and mutual inductors, condensers and resistors.

I. POWER AND ENERGY

Power and energy in an alternating-current circuit are periodic and may be expanded into a Fourier series. If harmonics of current and voltage are absent, these Fourier developments consist only of two terms, a constant and a double-frequency term. The constant may in particular be zero. Thus strictly speaking, it is erroneous to speak of power and energy as double-frequency quantities. Actually they consist respectively of two components, one of zero frequency and one of double frequency. These two components are easily expressed in terms of complex voltage and current.

If $E = e_1 + j\, e_2$, a complex quantity, denotes an alternating e. m. f. in the usual way, the instantaneous e. m. f., e, is given by

$$e = \text{Real part of } (\sqrt{2}\, E\, \epsilon^{j\omega t}) = \sqrt{2}/2\, (E\, \epsilon^{j\omega t} + \hat{E}\, \epsilon^{-j\omega t})$$

where $\hat{E} = e_1 - j\, e_2$ is the complex number conjugate to E, that is having the same real part but with pure imaginary part of opposite sign.

Similarly, if I denotes an alternating current, its instantaneous value i is given by:

$$i = \text{real part of } (\sqrt{2}\, I\, \epsilon^{-j\omega t}) = \sqrt{2}/2\, (I\, \epsilon^{j\omega t} + \hat{I}\, \epsilon^{-j\omega t})$$

The instantaneous power p is given by the product of the instantaneous e. m. f. and the instantaneous current. Thus:

$$\begin{aligned} p &= 1/2\, (E\, \epsilon^{j\omega t} + \hat{E}\, \epsilon^{-j\omega t})\, (I\, \epsilon^{j\omega t} + \hat{I}\, \epsilon^{-j\omega t}) \\ &= 1/2\, (E\, \hat{I} + E\, I\, \epsilon^{2j\omega t} + \hat{E}\, I + \hat{E}\, \hat{I}\, \epsilon^{-2j\omega t}) \\ &= \text{real part of } (E\, \hat{I} + E\, I\, \epsilon^{2j\omega t}) \end{aligned}$$

Thus the instantaneous power consists of a constant component, real part of $E\, \hat{I}$, and a double-frequency component, real part of $E\, I\, \epsilon^{2j\omega t}$.

In the complex plane, the quantity $E\, \hat{I} + E\, I\, \epsilon^{2j\omega t}$ is represented by a point describing synchronously the perimeter of a circle whose center is the point $E\, \hat{I}$, and whose radius is the absolute value of $E\, I$. Since $E\, \hat{I}$ and $E\, I$ have the same absolute values, the circle passes through the origin.

It becomes clear at once from Fig. 1 that when $E\, \hat{I}$ is real and positive, the circle lies entirely to the right of the j axis, so that the power is always positive; when $E\, \hat{I}$ is pure imaginary, the circle has the j axis for a diameter so that the power is positive and negative respectively, half the time.

By the angle of a vector in the complex plane is meant the angle which it makes with the real axis. In Fig. 1, θ is the angle of $E\,\hat{I}$. The angle of the product of two vectors is the sum of the angles of the two vectors respectively. The angle of the conjugate of a vector is the negative of the angle of the vector. Hence, the angle of $E\,\hat{I}$ being the difference between angle of E and angle of I, is the angle of lag of I behind E. Hence:

$$\cos\theta = \text{power factor.}$$

$$\tan\theta = \frac{\text{Pure imaginary part of } E\,\hat{I}}{\text{Real part of } E\,\hat{I}}$$

In any balanced polyphase system, the e. m. fs. in the successive phases are given by E, $a\,E$, $a^2\,E$, ... $a^{n-1}\,E$,

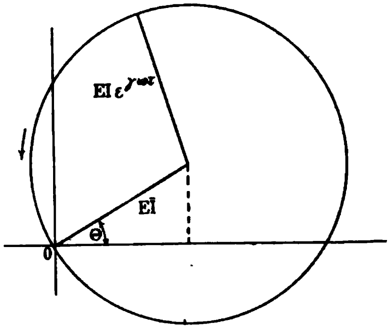


FIG. 1

where a is a primitive complex nth root of unity. If $n = 3$, $a = -\,1/2 + j\,\sqrt{3}/2$.

It is readily seen that a is conjugate to a^{n-1}, a^2 is conjugate to a^{n-2}, etc.

If the phases are carrying balanced currents, they will be given by I, $a\,I$, $a^2\,I$, . . . $a^{n-1}\,I$.

To obtain the power we need the conjugates of the preceding quantities, and since the conjugate of a product is the product of the conjugates we get for the conjugate values, $\hat{I}$, $a^{n-1}\,\hat{I}$, $a^{n-2}\,\hat{I}$, . . . $a\,\hat{I}$.

The instantaneous power in the successive phases is, therefore, real part of $(E\,\hat{I} + E\,I\,\epsilon^{2j\omega t})$, real part of $(a\,E\,a^{n-1}\,\hat{I} + a\,E\,a\,I\,\epsilon^{2j\omega t})$, real part of $(a^2\,E\,a^{n-2}\,\hat{I} + a^2\,E\,a^2\,I\,\epsilon^{2j\omega t})$, etc.

Summing these up and noticing that $1 + a^2 + a^4 + \ldots a^{2(n-1)} = 0$ we get for the total power,

$$p = \text{real part of } (n\,E\hat{I}).$$

In a balanced polyphase system, the instantaneous power input is constant in time.

The instantaneous magnetic energy contained in a system of circuits magnetically coupled is:

$$t_m = 1/2\,(L_{11}\,i_1^2 + L_{22}\,i_2^2 + 2\,M_{12}\,i_1\,i_2 + \ldots)$$

where L_{11}, L_{22}, M_{12} etc. are the coefficients of self and mutual induction of the parts of the system. Substituting

$$i_1 = \sqrt{2}/2\,(I_1\,\epsilon^{j\omega t} + \hat{I}_1\,\epsilon^{-j\omega t}),\ i_2 = \sqrt{2}/2\,(I_2\,\epsilon^{j\omega t} + \hat{I}_2\,\epsilon^{-j\omega t}),\ \text{etc.,}$$

$$\begin{aligned} t_m &= 1/2\,(L_{11}\,I_1\,\hat{I}_1 + L_{22}\,I_2\,\hat{I}_2 + M[I_1\,\hat{I}_2 + \hat{I}_1\,I_2] + \ldots) \\ &\quad + 1/2\,(1/2\,[L_{11}\,I_1^2 + L_{22}\,I_2^2 + 2\,M_{12}\,I_1\,I_2 + \ldots]\,\epsilon^{2j\omega t} \\ &\quad + 1/2\,[L_{11}\,\hat{I}_1^2 + L_{22}\,\hat{I}_2^2 + 2\,M_{12}\,\hat{I}_1\,\hat{I}_2 + \ldots]\,\epsilon^{-2j\omega t}) \\ &= T_m + \text{real part of } 1/2\,[L_{11}\,I_1^2 + L_{22}\,I_2^2 + 2\,M_{12}\,I_1\,I_2 + \ldots]\,\epsilon^{2j\omega t} \end{aligned}$$

$$T_m = 1/2\,(L_{11}\,I_1\,\hat{I}_1 + L_{22}\,I_2\,\hat{I}_2 + M\,[I_1\,\hat{I}_2 + \hat{I}_1\,I_2] + \ldots)$$

therefore represents the mean magnetic energy. It is always real and positive.

The instantaneous electrostatic energy in a group of condensers is $t_e = 1/2\left(\frac{q_1^2}{c_1} + \frac{q_2^2}{c_2} + \ldots.\right)$

$$q_1 = \int i_1\,dt = \int \sqrt{2}/2\,(I_1\,\epsilon^{j\omega t} + \hat{I}_1\,\epsilon^{-j\omega t})\,dt$$

$$= \sqrt{2}/2\left(\frac{I_1}{j\,\omega}\,\epsilon^{j\omega t} + \frac{\hat{I}_1}{-j\,\omega}\,\epsilon^{-j\omega t}\right)$$

Similarly for $q_2\,q_3\ldots$ etc. Substituting in t_e

$$\begin{aligned} t_e &= 1/2\left(\frac{1}{c_1}\,\frac{I_1\,\hat{I}_1}{\omega^2} + \frac{1}{c_2}\,\frac{I_2\,\hat{I}_2}{\omega_2^2} + \ldots.\right) \\ &\quad + 1/2\left(1/2\left[-\,\frac{1}{c_1}\,\frac{I_1^2}{\omega^2} - \frac{1}{c_2}\,\frac{I_2^2}{\omega^2}\ldots\right]\epsilon^{2j\omega t}\right. \\ &\quad \left. + 1/2\left[-\,\frac{1}{c_1}\,\frac{\hat{I}_1^2}{\omega^2} - \frac{1}{c_2}\,\frac{\hat{I}_2^2}{\omega^2}\ldots\right]\epsilon^{-2j\omega t}\right) \\ &= T_e + \text{real part of } 1/2[-\,1/c_1\,\frac{I_1^2}{\omega} - 1c_2\,\frac{I_2^2}{\omega^2} \\ &\qquad \ldots]\,\epsilon^{2j\omega t} \end{aligned}$$

$T_e = 1/2\left(\frac{1}{c_1}\frac{I_1\hat{I}_1}{\omega^2} + \frac{1}{c_2}\frac{I_2\hat{I}_2}{\omega^2} + \dots .\right)$ therefore represents the mean electrostatic energy. It is always real and positive.

The instantaneous rate of heat generation in a system of resistors is $d = r_1 i_1^2 + r_2 i_2^2 + \dots .$ Substituting the equivalent complex quantities:

$$d = (r_1 I_1 \hat{I}_1 + r_2 I_2 \hat{I}_2 + \dots) + 1/2\,(\,[r_1 I_1^2 + r_2 I_2^2 + \dots]\,\epsilon^{2j\omega t} + [r_1 \hat{I}_1^2 + r_2 \hat{I}_2^2 + \dots]\,\epsilon^{-2j\omega t})$$

$$= D + \text{real part of } 1/2\,[r_1 I_1^2 + r_2 I_2^2 + \dots]\,\epsilon^{2j\omega t}$$

$D = r_1 I_1 \hat{I}_1 + r_2 I_2 \hat{I}_2 + \dots .$ gives the mean rate of energy dissipation into heat. It is always real and positive.

II. THE RELATION BETWEEN POWER, ELECTROSTATIC AND MAGNETIC ENERGY AND HEAT GENERATION

Suppose an arbitrary system of inductances, capacities and resistances given, having n terminals. Let $V_1, V_2 \dots V_n$, n complex numbers denote the potentials at these terminals, and $I_1, I_2, \dots I_n$ the currents at the terminals, taken positive when they flow into the system. Let $V_{n+1}, V_{n+2}, \dots$ etc., be the potentials at the nodes or junctions of the branches of the network. Let I_{kl} be the current *from* the node k *to* the node l. Thus $I_{kl} = -I_{lk}$. In what follows we shall include the n terminals among the nodes of the system.

Consider the expression

$\sum_k V_k (\hat{I}_{k1} + \hat{I}_{k2} + \dots .)$. Since by Kirchhoff's laws $I_{k1} + I_{k2} + \dots = 0$ for every internal node, the whole must reduce to $V_1 \hat{I}_1 + V_2 \hat{I}_2 + \dots . V_n \hat{I}_n$

$$\sum_k V_k (\hat{I}_{k1} + \hat{I}_{k2} + \dots .) = V_1 \hat{I}_1 + V_2 \hat{I}_2 + \dots . V_n \hat{I}_n \tag{1}$$

Take any branch a, joining two nodes, k and l. The current in the branch gives two terms in the above expression, namely:

$$V_k \hat{I}_{kl} + V_l \hat{I}_{kl} = V_k \hat{I}_{kl} - V_l \hat{I}_{kl} = (V_k - V_l)\,\hat{I}_{kl}$$
$$= (V_k - V_l)\,\hat{I}_a$$

But $V_k - V_l = r_a I_a + j\,\omega\,(L_{aa} I_a + M_{ab} I_b + M_{ac} I_c + \dots .) + \frac{1}{j\,\omega}\frac{1}{C_a} I_a$

Hence $$(V_k - V_e)\,\hat{I}_a = r_a I_a \hat{I}_a + j\,\omega\,(L_a I_a \hat{I}_a + M_{ab} I_b \hat{I}_a + M_{ac} I_c \hat{I}_a + \dots\dots) - j\,\omega\left(\frac{1}{C_a}\,\frac{I_a \hat{I}_a}{\omega^2}\right)$$

Now write similar equations for the other branches. The left-hand sides will add up to the left-hand side of (1). The right-hand sides will add up to $D + 2\,j\,\omega\,(T_m - T_e)$
Hence:

$$V_1 \hat{I}_1 + V_2 \hat{I}_2 + \dots V_n \hat{I}_n = D + 2j\,\omega\,(T_m - T_e) \qquad (2)$$

Hence the real part of $V_1 \hat{I}_1 + V_2 \hat{I}_2 + \dots\dots V_n \hat{I}_n$ is the Joulian heat generated per second, and the pure imaginary part is $2\,j\,\omega$ times the mean magnetic energy minus the mean electrostatic energy.

III. Application of the Relation, Power Factor Compensation

The principle enunciated in II is frequently useful in determining the possibilities of solution of certain problems, and the minimum amounts of magnetic and electrostatic energy storage necessary.

Let us consider first the problem of power factor correction in a polyphase system by static means. Because of the possibility of introducing arbitrary quadrature e.m.fs. in series with any phase by means of transformers placed across the other phases, many of us have tried to compensate for lagging power factor by means of transformers only. The constant failure of the schemes tried has convinced most of us that the idea is inherently impossible, but it is satisfying to have a definite proof of this fact.

The proposition is to insert a network between a polyphase supply and a lagging load so as to give unity power factor at the generator end. Sum up $V\,\hat{I}$ for all the terminals of the network. For the supply terminals, since $\cos\theta = 1$, $\Sigma\, V\,\hat{I}$ is real; the coefficient of j is zero. For the load terminals since leading currents are supplied *to* the network θ is negative. Hence the coefficient of j in $V\,\hat{I}$ for a load terminal must be negative. Its value is the wattless power supplied to the load through that terminal. Thus $V\,\hat{I}$ summed up for all the terminals must have for the coefficient of j the negative of the wattless power supplied the load. Hence by equation (2)

$$2\,\omega\,(T_e - T_m) = \text{wattless power of load.}$$

Hence in any such network there must be electrostatic storage of energy. In fact the minimum amount possible is $\frac{1}{2\,\omega}$ times the wattless power of the load.

IV. Phase Splitting

The use of condensers and reactances for changing from single phase to polyphase is well known. We shall here determine the minimum amounts of condenser and reactance energy storage necessary for this purpose. The impossibility of supplying a balanced polyphase load of lagging or unity power factor from a single-phase source by means of reactances alone will also be shown here.

Suppose then a network inserted between a single-phase source and a polyphase load such that the polyphase is balanced.

Summing up $V\,\hat{I}$ for the load terminals we get $-\,n\,E_2\,\hat{I}_2$ where E_2 is the polyphase voltage to neutral, and I_2 is the current per phase supplied *to* the load. Summing $V\,\hat{I}$ up for the supply terminals gives $E_1\,\hat{I}_1$ where E_1 is the single-phase voltage and I_1 is the current taken from the supply. Hence for the network,

$$\Sigma\,V\,\hat{I} = E_1\,\hat{I}_1 - n\,E_2\,\hat{I}_2 = D + 2\,j\,\omega\,(T_m - T_e). \qquad (2')$$

The double-frequency component of power supplied by the single-phase source is real part of $E_1\,I_1\,\epsilon^{2j\omega t}$; that withdrawn by the load is zero as shown in II. This double-frequency power must be supplying and removing energy from the magnetic and electrostatic fields in the network. The integral with respect to time of this double frequency component, real part of $\frac{1}{2\,j\,\omega}\,E_1\,I_1\,\epsilon^{2j\omega t}$ must give the double frequency component of the total energy stored. The mean value of all the energy stored added to the double frequency component must never be less than zero as energy can never be negative. Hence we must have

$$T_m + T_e \text{ is not less than absolute value of } \frac{1}{2\,j\,\omega}\,E_1\,I_1$$

$$2\,\omega(T_m + T_e) \text{ is not less than absolute value of } E_1\,I_1.$$

Now absolute value of $E_1\,I_1$ is the same as absolute value of

$E_1 \hat{I}_1$, and this, of course, is not less than the real part of $E_1 \hat{I}_1$. Using the symbol $\geqq$ for "greater than or equals":

$$2\ \omega\ (T_m + T_e) \geqq \text{absolute value of } E_1 \hat{I}_1 \geqq \text{real part of } E_1 I_1 \qquad (3)$$

From equation (2)

$$2\ \omega\ (T_m - T_e) = \text{imaginary part of } E_1 \hat{I}_1 - \text{imaginary part of } n\ E_2 \hat{I}_2 \qquad (4)$$

Subtracting (4) from (3)

$$4\ \omega\ T_e \geqq \text{absolute value of } E_1 \hat{I}_1 - \text{imaginary part of } E_1 \hat{I}_1 + \text{imaginary part of } n\ E_2 \hat{I}_2 \qquad (5)$$

Adding (3) and (4) we get

$$4\ \omega\ T_m \geqq \text{absolute value of } E_1 \hat{I}_1 + \text{imaginary part of } E_1 \hat{I}_1 - \text{imaginary part of } n\ E_2 \hat{I}_2 \qquad (6)$$

In equation (5) the first two terms on the right must clearly combine to give a positive quantity. If the polyphase load has a lagging power factor, imaginary part of $n\ E_2 \hat{I}_2$ will be positive. Hence in this case the right hand side of E will be positive, and not less than imaginary part of $n\ E_2 \hat{I}_2$. Thus with a lagging polyphase load, the minimum condenser energy storage for which a network can be constructed is given by:

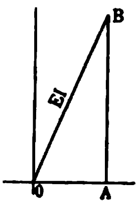


FIG. 2

$$4\ \omega\ T_e = \text{wattless power of load.} \qquad (6')$$

Actually, however, this limit cannot be reached, for to attain it we must make imaginary part of $E_1 \hat{I}_1$ equal absolute value of $E_1 \hat{I}_1$. Since, however, the real part of $E_1 \hat{I}_1$ is determined by the true polyphase power, this equality can be attained only as $E_1 \hat{I}_1$ becomes infinite. (See Fig. 2). Equation (6) shows that at the same time, T_m would have to become infinite.

Similarly for a leading polyphase load, the magnetic energy T_m cannot be less than $\frac{1}{4\ \omega}$ times the wattless polyphase power. To actually attain this value, however, would require zero single-phase power factor and infinite electrostatic energy.

A case of special interest is where the single-phase power factor is unity. In this case the imaginary part of $E_1 \hat{I}_1$ is zero, and the absolute value of $E_1 \hat{I}_1$ equals the true single-phase power. Hence (5) and (6) become:

$4\,\omega\,T_e \geqq$ Single-phase power + polyphase wattless power (lagging). (5″)

$4\,\omega\,T_m \geqq$ Single-phase power − polyphase wattless power (lagging). (6″)

Networks are well known, which effect phase splitting with the minimum of condenser and reactance energy storage indicated above. We now see that it is impossible to do better in this respect.

When the polyphase load has a power factor less than cos 45 deg. then if network losses are negligible, equation (6″) shows that reactance may be dispensed with while still keeping unity power factor on the single phase.

If a phase splitting network employs the minimum amount of energy storage, the total instantaneous energy stored must be zero twice every cycle. When the inductance energy is zero, the inductance voltages are a maximum. When the condenser energy is zero, the condenser voltages are zero. Hence in such a network all the inductance voltages are in phase with each other and in quadrature with the condenser voltages.

IV. THE RECIPROCAL RELATION

Suppose a system of terminal potentials $V_1 V_2 \ldots V_n$ applied to the terminals of a network produces the terminal currents $I_1, I_2 \ldots I_n$. Suppose a different system of terminal potentials $V_1', V_2', \ldots V_n'$ produces the terminal currents $I_1', I_2', \ldots I_n'$. Then

$$V_1 I_1' + V_2 I_2' + \ldots . V_n I_n' = V_1' I_1 + V_2' I_2 + \ldots V_n' I_n \qquad (7)$$

Such reciprocal relations are well known in mathematical physics. The proof as given below follows the lines of the proofs usually given.

Using the notation of II, we have at once:

$$\sum_k V_k (I_{k1}' + I_{k2}' + I_{k3}' + \ldots) = V_1 I_1' + V_2 I_2' + \ldots V_n I_n' \qquad (8)$$

Any branch a joining the nodes k and l contributes two terms to the left hand member of (8), namely: $V_k I_{kl}' + V_l I_{lk}'$.

But

$$V_k I_{kl}' + V_l I_{lk}' = V_k I_{kl}' - V_l I_l' = I_a' (V_k - V_l)$$

$$= I_a' \left[r_a I_a + j\omega (L_{aa} I_a + M_{ab} I_b + \dots) + \frac{1}{j\omega} \frac{I_a}{C_a} \right]$$

$$= r_a I_a I_a' + j\omega (L_{aa} I_a I_a' + M_{ab} I_a' I_b + M_{ac} I_a' I_c + \dots) + \frac{1}{j\omega} \frac{I_a I_a'}{C_a}$$

Summing up for all the branches gives .

$$\Sigma V_k (I_{k1}' + I_{k2}' + \dots) = r_a I_a I_a' + r_b I_b I_b' + \dots\dots + j\omega (L_{aa} I_a I_a' + L_{bb} I_b I_b' + M_{ab} I_a I_b' + M_{ba} I_a' I_b + \dots) + \frac{1}{j\omega} \left(\frac{I_a I_a'}{C_a} + \frac{I_b I_b'}{C_b} + \dots \right) \qquad (9)$$

Similarly, by summing up $\sum_k V_k' (I_{k1} + I_{k2} + \dots)$ first by nodes and then by branches,

$$\Sigma V_k' (I_{k1} + I_{k2} + \dots\dots) = V_1' I_1 + V_2' I_2 + \dots V_n' I_n \qquad (8')$$

$$\sum_k V_k' (I_{k1} + I_{k2} + \dots\dots) = r_a I_a I_a' + r_b I_b I_b' + \dots + j\omega (L_{aa} I_a' I_a + L_{bb} I_b' I_b + M_{ab} I_a' I_b + M_{ba} I_a I_b' + \dots) + \frac{1}{j\omega} \left(\frac{I_a' I_a}{C_a} + \frac{I_b' I_b}{C_b} + \dots \right) \qquad (9')$$

Hence comparing (8) (9) (8′) and (9′) we see that

$$V_1 I_1' + V_2 V_2' + \dots V_n I_n' = V_1' I_1 + V_2' I_2 + \dots V_n' I_n \qquad (7)$$

From equation (7) follows at once the well known reciprocal properties of a network. For example; suppose we supply one volt to the terminal 1 of a network keeping the other terminals at zero voltage. A certain current I_2 will flow at terminal 2. Now suppose we apply one volt at terminal 2 keeping the other terminals at zero potential. A certain current I_1' will flow at terminal 1. Now substituting for V_1, $V_2, \ldots V_n$ the values 1, 0, 0, . . . 0, and for $V_1', V_2', \ldots V_n'$ the values 0, 1, 0. . . . 0, in equation (7) we find:

$$I_1' = I_2$$

That is, a volt placed at terminal 1 produces the same current at terminal 2 as one volt placed at terminal 2 would produce at terminal 1.

In the next section we shall be concerned principally with four terminal networks in which the currents in pairs of terminals are constrained to be equal and opposite. For this special case, putting $n = 4$ in equation (7) and $I_3 = -I_1$, $I_4 = -I_2$, $I_3' = -I_1'$, $I_4' = -I_2'$, we get:

$$(V_1 - V_3)I_1' + (V_2 - V_4)I_2' = (V_1' - V_3')I_1 + (V_2' - V_4')I_2$$

Setting $V_1 - V_3 = E_1$, $V_2 - V_4 = E_2$, $V_1' - V_3' = E_1'$ and $V_2' - V_4' = E_2'$. We find,

$$E_1 I_1' + E_2 I_2' = E_1' I_1 + E_2' I_2 \qquad (7')$$

From this equation we deduce at once this following reciprocal relation. One volt difference of potential placed at one pair of terminals will circulate the same amount of current through a short circuit at the other pair of terminals as one volt difference of potential placed at the second pair will circulate through a short circuit at the first pair.

V. THE GENERAL SINGLE-PHASE TRANSFORMING NETWORK

Any network having four terminals may be used to receive current at some voltage at one pair of terminals and delivering current at some other voltage or phase at the other pair of terminals. We propose to investigate here what are the limitations on such transformations by purely static means.

The solution in detail for the steady state in any network can always be obtained by means of Kirchhoff's Laws generalized for complex quantities. Given the currents I_1 and I_2 in the two pairs of terminals, which we shall now call the primary and secondary terminals, E_1 and E_2 the primary and secondary voltages may be calculated. Now the equations given by Kirchhoff's laws are all linear in the unknowns. The process of solving them only involves linear operations. Hence E_1 and E_2 must be linear functions of I_1 and I_2. Thus:

$$\begin{aligned} E_1 &= a\, I_1 + b\, I_2 \\ E_2 &= b'\, I_1 + c I_2 \qquad (8) \end{aligned}$$

Here the quantities a, b, b', c are complex numbers. However, it is easily seen that they are not independent.

In fact, applying the reciprocal relation (7′) we find:

$$E_1 I_1' + E_2 I_2' = a\, I_1 I_1' + b\, I_2 I_1' + b'\, I_1 I_2' + c\, I_2 I_2'$$

and $E_1' I_1 + E_2' I_2 = a\, I_1 I_1' + b\, I_2' I_1 + b'\, I_1' I_2 + c\, I_2' I_2$

Hence $b\, I_2 I_1' + b'\, I_1 I_2' = b\, I_2' I_1 + b'\, I_1' I_2$

Hence $b\, (I_2 I_1' - I_2' I_1) = b'\, (I_1' I_2 - I_1 I_2')$

Therefore: $b = b'$

Thus the primary and secondary voltages and currents of any network of the kind we are considering always satisfy relations of the form:

$$\begin{aligned} E_1 &= (a_1 + j\, a_2)\, I_1 + (b_1 + j\, b_2)\, I_2 \\ E_2 &= (b_1 + j\, b_2)\, I_2 + (c_1 + j\, c_2)\, I_2 \qquad (8') \end{aligned}$$

What is the significance of the real and pure imaginary parts of the coefficients in (8′). Forming the power expression we have:

$$\begin{aligned} E_1 \hat{I}_1 + E_2 \hat{I}_2 &= [a_1 I_1 \hat{I}_1 + b_1 (I_2 \hat{I}_1 + I_1 \hat{I}_2) + c_1 I_2 \hat{I}_2] \\ &\quad + j[a_2 I_1 \hat{I}_1 + b_2 (I_2 \hat{I}_1 + I_1 \hat{I}_2) + c_2 I_2 \hat{I}_2] \\ &= D + 2 j\, \omega\, (T_m - T_e) \end{aligned}$$

Since the two brackets are real, the first must equal D the rate of generation of Joulian heat. If there are no losses in the network the first bracket must be zero, and the coefficients a, b, c must be pure imaginary.

Since we are interested in the inherent limitations upon the transformation possibilities of an ideal network, that is, one in which the losses are zero, we shall assume in all that follows that a, b, and c are pure imaginary.

$$\begin{aligned} a &= j\, A \\ b &= j\, B \\ c &= j\, C \end{aligned}$$

where $A\ B\ C$ are real. The network equations then are:

$$\begin{aligned} E_1 &= j\, A\, I_1 + j\, B\, I_2 \\ E_2 &= j\, B\, I_1 + j\, C\, I_2 \qquad (8'') \end{aligned}$$

Let us first consider the possibility of a constant potential constant potential transformation. Given E_1 constant, we require E_2 to be independent of the load, I_2.

Eliminating I_1 in (8″) we obtain:

$$E_2 = B/A \, E_1 + \left(\frac{B^2 - A\,C}{j\,A} \right) I_2 \tag{9}$$

E_2 can be independent of I_2 only if $B^2 - A\,C = 0$. This is the condition for perfect coupling. Furthermore, the coefficient of E is real. Hence we conclude, in any constant potential to constant potential transformation without losses, the transformed potential must agree in phase with the primary potential.

Next consider a constant potential constant current transformation. Solving (8″) for I_2 we get:

$$I_2 = -j\,B\,E_1 + \left(\frac{j\,A}{B^2 - A\,C} \right) E_2 \tag{10}$$

I_2 is independent of E_2 only if $A = 0$. Furthermore the coefficient of E_1, is pure imaginary. Hence in any constant potential constant current transformation without losses the secondary current is in quadrature with the primary voltage.

Let us now discuss the problem of determining a network such that for a given secondary load the secondary voltage shall be in quadrature with the primary voltage, and the primary power factor shall have a predetermined value.

The load can be given in terms of its admittance, $y_2 = g_2 - j\,b_2$. The primary power factor can be determined by giving the apparent primary admittance, $I_1 E_1 = y_1 = g_1 - j\,b_1$ Since there are no losses in the network we must have $g_1 = g_2 = g$.

What restrictions do the above conditions impose upon A, B, C? Substituting $I_1 = (g - j\,b_1)\,E_1$, $I_2 = -\,(g - j\,b_2)\,E_2$, and $E_2 = j\,E_1$ in (8″), we get:

$$E_1 = j\,(g - j\,b_1)\,A\,E_1 + j(-g + j\,b_2)\,j\,B\,E_1$$

$$j\,E_1 = j\,(g - j\,b_1)\,B\,E_1 + j\,(-g + j\,b_2)\,j\,C\,E_1$$

Simplifying:

$$1 = b_1 A + g\,b + j\,(g\,A - b_2 B)$$

$$1 = g\,B - b_2 C + j\,(-b_1 B - g\,C)$$

Separating the real and imaginary parts of these two equations, we get:

$$\begin{aligned} 1 &= b_1 A + g B \\ 0 &= g A - b_2 B \\ 1 &= g B - b_2 C \\ 0 &= b_1 B + g C \end{aligned}$$

Solving we find:

$$A = \frac{b_2}{b_1 b_2 + g^2} \qquad B = \frac{g}{b_1 b_2 + g^2} \qquad C = \frac{-b_1}{b_1 b_2 + g^2}$$

Thus we see that A, B, and C are completely determined by the conditions imposed on the network. But A, B, C determine entirely the secondary voltage in terms of the secondary current. Hence we conclude, all networks without losses which effect phase quadrature for a given load with given primary power factor have the same inherent regulation of secondary voltage as the secondary load is varied.

REFERENCES

The relation between wattless power and electrostatic and magnetic energies given in II, was discovered independently by the author. Since this paper was submitted to the Institute the same relation was proved in a different manner by J. B. Pomey, *Revue Generale de l'electricite,* Vol. III, 9, pp. 315–319. March 2, 1918.

The relation has subsequently been generalized by the author to include rotating commutatorless machines, with many interesting conclusions.

Examples of reciprocal relations such as are given in IV may be found in Maxwell, "Electricity and Magnetism", third edition, pp. 105 and 405.

April 15, 1919.

Presented at a joint meeting of the Institution of Electrical Engineers and the American Institute of Electrical Engineers, Calcutta, India, January 2, 1919.

STARTING CONDITIONS OF SYNCHRONOUS MACHINES

BY ALFRED HAY AND F. N. MOWDAWALLA

INTRODUCTION

IN view of the growing popularity and importance of the synchronous motor as one of the standard types of motor available for power distribution purposes, and its increasing use as a means of improving the power factor of a load, no apology seems necessary for a paper containing a detailed study of the behavior of such a motor during what has always been regarded as a somewhat critical period—*viz.*, the period when the motor is being accelerated from rest to synchronous speed. The subject is by no means a new one, and has already been dealt with by several writers, not, however, in a manner sufficiently thorough and exhaustive to make further contributions to it superfluous, and one of the main objects of the present paper is to explain a number of hitherto somewhat obscure points, and to draw attention to others which have not previously been noticed.

REVIEW OF PREVIOUS WORK ON THE SUBJECT

The earliest paper specially devoted to a study of the starting conditions in synchronous machines appears to be one read in 1912 before the American Institute of Electrical Engineers by C. J. Fechheimer.[1] In this the author, after some general introductory remarks, gives an account of experiments made to determine (1) the relation connecting the starting torque with the impressed potential difference under various conditions and (2) the variation of torque, current and power factor with speed, while the rotor is accelerated from rest, a constant potential difference being maintained across the stator terminals. The experimental results are embodied in an interesting series of curves. Among the questions discussed by the author are the desirability or otherwise of keeping the field

---

[1] C. J. Fechheimer, TRANS. A. I. E. E. VOL. XXXI, 1912, p. 529.

circuit open during the acceleration period, and the tendency of certain synchronous machines to run in the neighborhood of half the speed of synchronism. Fechheimer's paper gave rise to a very interesting discussion.

In 1913 E. Rosenberg[2] read a paper before the Institution of Electrical Engineers, on Synchronizing machines in which he considered, among other things, the occurrences during the starting period, and discussed in some detail the behavior of the machine towards the end of the acceleration period, just before it is pulled into synchronism. It may be mentioned that in the discussion on Fechheimer's paper, it was stated by B. G. Lamme that it was "difficult to see just what is going on in the motor at the instant it pulls into synchronism." So far as the authors are aware, Rosenberg's was the first attempt to furnish a detailed explanation of the action in question.

The next contribution to the subject is one by F. D. Newbury, in the form of a paper read before the American Institute of Electrical Engineers in June 1913.[3] The main interest of this paper lies in the oscillographic records which are given of the starting period.

In December 1917 the authors of the present paper published, in the *Journal* of the Indian Institute of Science, an account of some experimental investigations of the occurrences during the starting period of a synchronous machine, and a full theoretical discussion of the type of induction motor whose stator is supplied with polyphase currents, and whose rotor is provided with a single-magnetic-axis winding. As will be seen later the theoretical results obtained explain in a satisfactory manner certain striking peculiarities exhibited by the machine during the starting period.

The most recent addition to the literature of the subject is an article by Theo. Schou in the *Electrical World* of April 6th, 1918 (Vol. 71, p. 714). In this article the author points out that a satisfactory selfstarting synchronous motor should partake of the characteristies of both an induction motor and an alternator, and should present features of design intermediate between these two classes of machines. He accordingly advocates the use of a shorter air gap and longer

---

[2] E. Rosenberg, "Self-synchronizing Machines." *Journal* of the Institution of Electrical Engineers, Vol. 51, p. 62 (1913).

[3] TRANSACTIONS of the American Institute of Electrical Engineers, Vol. 32, p. 1509.

polar arc than are customary in alternators of standard design. He further suggests the use of materials having a pronounced skin effect for the squirrel-cage windings of self-starting synchronous motors[4] and points out the advantages of fractional-pitch windings in reducing the troubles arising from dead points.

TORQUES CONCERNED IN ACCELERATING THE MOTOR AND IN PULLING IT INTO SYNCHRONISM

The self-starting synchronous motor is accelerated and finally pulled into synchronism by the action of a number of torques, differing widely from each other, and the resultant effect of which will largely depend on their relative importance. Cases may arise where, owing to the preponderance of a certain type of component torque, it may be impossible to get the machine to run up to synchronous speed. Again, the torques concerned in the initial acceleration of the rotor are quite distinct from the torque which finally pulls it into synchronism. The authors are of opinion that no really clear understanding of the occurrences during the starting period is attainable without a detailed study of the various torques which act on the rotor during that period. It will accordingly be necessary to consider the nature of the various torques concerned. These torques may be classified as follows:

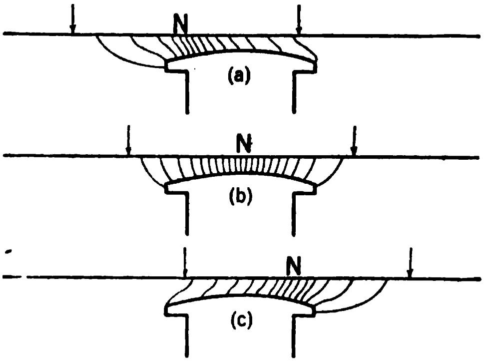


FIG. 1—FLUX DISTRIBUTIONS FOR DIFFERENT RELATIVE POSITIONS OF STATOR FIELD AND MAGNET POLES

---

[4] The utilization of the skin effect in the rotor conductors of induction motors was patented by H. M. Hobart in 1900.

1. Torque due to varying magnetic reluctance (synchronous torque.)
2. Torque due to hysteresis.
3. Torque due to starting squirrel-cage or damping coils if present.
4. Torque due to currents induced in the field winding, if this winding is closed.
5. Torque due to eddy currents.

1. TORQUE DUE TO VARYING MAGNETIC RELUCTANCE

It is a well known general principle of electromagnetism that any electromagnetic system which includes a movable member tends to assume a configuration which corresponds to minimum reluctance and therefore maximum flux. Displacements of the movable member from the position of minimum reluctance call into play forces tending to restore it to that position. The application of this general principle to the special case of a salient pole rotor, which is acted on by the rotating field of the stator, will be easily understood by reference to Fig. 1. In this figure, three different positions of one of the rotor poles are shown relatively to the stator polar surfaces. The center of the stator polar surface is in each case marked with the letter *N*. In Fig. 1B, the relative positions of the rotor and stator polar surfaces correspond to minimum reluctance, and from the symmetry of flux distribution it is immediately obvious that there is no tangential pull on the rotor. In Fig. 1A, the rotor pole is shown displaced from the position of minimum reluctance in one direction, and in Fig. 1C in the opposite direction. If we bear in mind that the dynamical stresses correspond to a tension along the lines of force and a pressure at right angles to them, it is easy to see that in Fig. 1A there is a tangential force acting on the pole from right to left, while in Fig. 1C it acts from left to right. If we now suppose that the stator polar surfaces travel past the pole in a direction from left to right, then the successive positions will be those shown in Fig. 1. For every displacement of the stator polar surface to one side of a rotor pole, there will be an equal displacement to the other side, and the forces corresponding to equal displacements in opposite directions will be equal and opposite. Thus the rotating field will exert an alternating torque on the rotor, and the positive and negative half-waves of this torque will be equal. The frequency of the torque will be equal to twice the supply frequency multiplied by the motor slip, and so long as the motor slips, the mean value of the alternating torque due to varying magnetic reluctance will be zero. The only effect of this torque is to throw the rotor into forced vibrations, having a frequency equal to that of the torque. Owing to the large moment of inertia of the rotor the vibrations will be imperceptible for large values of the slip, *i. e.*, during the greater part of the acceleration period. When, however, the slip has become sufficiently small and in consequence the frequency of the torque sufficiently low, the amplitude of the rotor oscillations will become marked, and will increase with decreasing slip.

Summing up, we see that the torque due to variable magnetic reluctance is for all speeds below synchronism an alternating torque consisting of equal positive and negative half-waves, and hence having a zero mean algebraic value. It is therefore quite inoperative so far as steady acceleration of the rotor is concerned, and only produces equal periodic accelerations and retardations, *i. e.*, it causes oscillations of the rotor. The period of these oscillations is determined by the rotor slip, and steadily increases with decreasing slip. At the same time, the amplitude of the oscillation increases.

The graph of the varying magnetic reluctance torque expressed as a function of the speed is shown in Fig. 2A. For all speeds below synchronism its mean value is zero, while at synchronism it is capable of assuming any positive or negative value between definite limits. Since the speed of synchronism is the only speed at which this torque has a value differing from zero, we may conveniently refer to it as the *synchronous torque.*

2. TORQUE DUE TO HYSTERESIS

In dealing with the torque due to varying magnetic reluctance, we have neglected the effect of hysteresis. It now becomes necessary to take this into account. Owing to hysteresis the rotor will tend to retain more or less strongly the effects of previous magnetizations. Thus referring to Figs. 1A and 1C, if hysteresis were absent, the magnitudes of the torques in these two cases would be equal. Owing, however, to the fact that in the position of the rotating field corresponding to Fig. 1A the magnetization of the rotor is increasing from a lower to a higher value, while in position Fig. 1C it is decreasing from a higher to a lower value, the actual flux in Fig. 1C will be higher than that in Fig. 1A. The same applies to each pair of corresponding or equidistant positions on opposite sides of the position of maximum flux shown

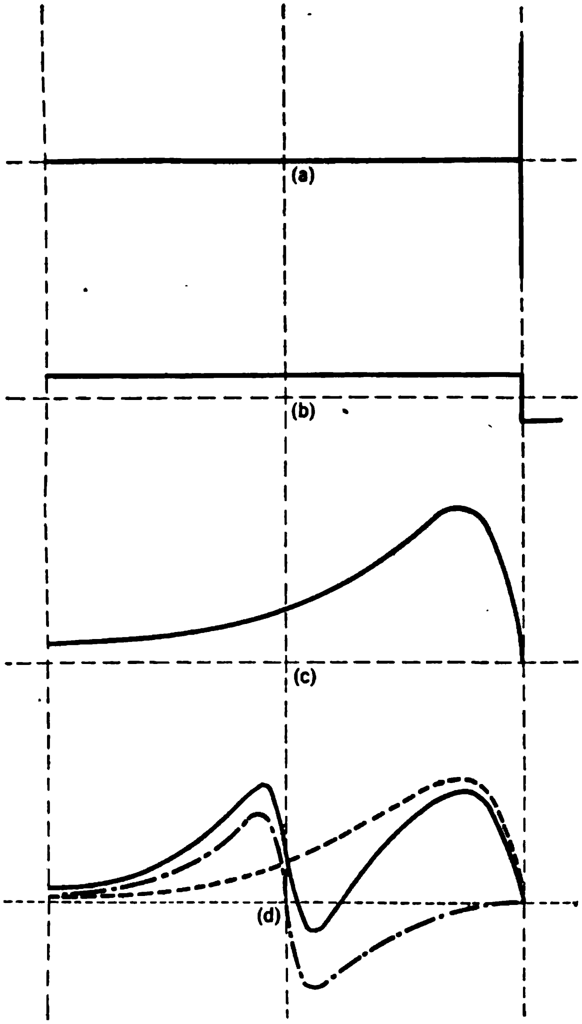


FIG. 2—TORQUE-SPEED CURVES

in Fig. 1B. For any such pair of positions, the driving torque is greater than the retarding torque. The effect of hysteresis is thus seen to be the production of a disparity between the positive and negative half-waves of the varying magnetic reluctance torque, the positive half-waves being uniformly larger than the negative ones. This effect is equivalent to raising the magnetic reluctance torque waves above the axis of time, *i. e.*, to the addition of a steady driving torque to the alternating magnetic reluctance torque. The torque due to hysteresis is thus seen to be a steady driving torque and is instrumental in producing acceleration of the rotor. It is to be noted that, assuming the flux per pole to remain constant during the acceleration period, the hysteresis torque has the same value for all speeds below synchronism. If the speed were made to pass through synchronism to higher vaules, the hysteresis torque would undergo reversal at synchronism.

The graph of the hysteresis torque as a function of the speed is shown in Fig. 2B.

3. TORQUE DUE TO STARTING SQUIRREL-CAGE OR DAMPING COILS IF PRESENT

Little need be said about this torque, which may be called the induction motor torque, as everybody is familiar with the relation connecting the torque and speed of an induction motor. The squirrel-cage of a self-starting synchronous machine forms the rotor winding of an induction motor whose stator windings are represented by the armature; the relation connecting torque and speed will be of precisely the same nature as in an induction motor.

The graph of this torque as a function of the speed is of the well-known form shown in Fig. 2C.

The method of varying the torque-speed curve of an induction motor by the introduction of resistance into the rotor is also well known. The effect of introducing resistance is to cause a shearing of the torque speed curves backwards towards the origin, the maximum torque remaining unaffected in value, but occurring at a lower speed. If a very powerful torque is necessary at starting, it is advisable to use a high-resistance squirrel-cage. On the other hand, with such a squirrel cage the speed to which the motor finally settles down corresponds to a large slip, and this, as will be seen later on, makes it more difficult to pull the rotor into synchronism. The ideal arrangement would be one in which the squirrel-cage resistance at starting is such as to give maximum torque, the resistance then automatically decreasing with increase of speed in such a manner that at each speed maximum torque is maintained, until finally the lowest possible resistance is reached, corresponding to a very small slip. The use of the skin effect in conductors for automatically decreasing the rotor resistance with increasing speed has recently been proposed by Theo. Schou.[5]

4. TORQUE DUE TO CURRENTS INDUCED IN FIELD WINDING, IF THIS WINDING IS CLOSED

If the field winding be closed, the currents induced in it will give rise to a torque, and a careful study of the nature of this torque is essential to a clear understanding of the occurrences during the starting period. The armature of the machine may again be regarded as forming the stator winding of an induction motor, of which the rotor winding is represented by the field coils. There is, however, this very important difference between the starting squirrel-cage and the field winding: the currents induced in the squirrel-cage are capable, according to their distribution in space, of giving rise to a field whose magnetic axes may occupy any positions whatsoever relatively to the center lines of the field poles; but the currents induced in the field winding can only produce a field whose magnetic axes are coincident with the center lines of the field poles. This is conveniently expressed by saying that the field winding is a "single magnetic axis" winding; because it can only produce a field having a single definite set of magnetic axes, namely, those corresponding to the center lines of the salient poles. Now a motor having a polyphase stator, but a single-phase or single-magnetic-axis rotor, exhibits certain striking peculiarities which differentiate it sharply from a motor in which both stator and rotor windings are polyphase. The earliest reference to this type of motor which the authors have been able to find occurs in a paper by H. Gorges[6]. Since such motors are not ordinarily used in practise, their characteristics do not seem to be very generally known, and have only occasionally been referred to. A complete analytical theory of this type of motor is given in the Appendix. The torque-speed curve of such a motor is shown in Fig. 2D, and its most striking characteristic is the torque reversal which occurs over

---

[5] Loc. cit.

[6] H. Gorges: "Ueber Drehstrommotoren mit verminderter Tourenzahl, *Elektrotechnische Zeitschrift*, Vol. 17, p. 517 (1896.)

a certain range of speed in the neighborhood of half-synchronism. The analytical theory of this motor, is somewhat complicated, but the following general explanation based on a paper published in 1898 by F. Eichberg[7] may be useful. The currents induced in the single-phase rotor winding by the rotating field of the stator give rise to a field which, relatively to the rotor core, is a simple alternating or oscillating field. By a well-known transformation this oscillating field may be replaced by two equal and (relatively to the rotor core) oppositely rotating fields, the crest value of each rotating field being half the maximum crest value of the oscillating field. Now if the slip of the motor be s and if its speed of synchronism be denoted by n, the frequency of the rotor currents will be $s\,f$, where f is the frequency of supply, and the speed of its component rotating fields relatively to the rotor core will be $s\,n$, the speed of the rotor in space being $(1 - s)\,n$. Regarding the direction of rotation of the rotor as positive, the speed of one of the rotating fields relatively to the rotor core is $+\,s\,n$, while that of the other is $-\,s\,n$. Hence the speeds of the rotor rotating fields *in space* are $s\,n + (1 - s)\,n = n$, and $-\,s\,n + (1 - s)\,n = (1 - 2\,s)\,n$. The interaction between the stator field, and the first rotating component of the rotor field, whose speed n in space is the same as that of the stator field, gives rise to a torque in every respect similar to that of an ordinary induction motor with polyphase windings on both stator and rotor. The second rotating component of the rotor field, whose speed in space is $(1 - 2\,s)\,n$, is clearly incapable of reacting with the stator field in such a manner as to give rise to a resultant torque; for, owing to the difference of speed, the relative position of the fields is constantly changing, periodically passing through a succession of cycles during each of which the average algebraic value of the torque is zero. Although incapable of torque production by interaction with the stator field, the second rotating component of the rotor field is capable of giving rise to a torque by different kind of action. In sweeping across the stator conductors, it induces in them e.m.fs. of frequency $(1 - 2\,s)\,f$, and these produce currents of the same frequency in the stator windings and the circuit external to them (represented by the mains and every-

---

[7] F. Eichberg, *Zeitschrift fur Elektrotechnik* (Wein), Vol. 16, p. 578 (1898).

thing connected across them, generators, motors, lamps, etc.). Since the total impedance external to the stator windings is extremely small in comparison with that of the windings themselves, the result, so far as the currents of frequency $(1 - 2s)f$ are concerned, is nearly the same as if the stator windings were short-circuited. The currents give rise to a rotating field whose speed $(1 - 2s)n$ in space is the same as that of the inducing rotor field, and the interaction of these two fields whose relative space position is invariable, results in the production of a torque. To fix ideas, we may think of the second component of the rotor field as produced by a polyphase winding on the rotor supplied with suitable polyphase currents, having a frequency sf, and of the rotating field due to this as inducing currents of frequency $(1 - 2s)f$ in the stator windings. The arrangement would then be equivalent to a polyphase motor whose primary is represented by the rotor and whose secondary is represented by the stator. The slip of this imaginary motor would be $(1 - 2s)$ and so long as s is less than ½, the slip and torque would be positive. Zero slip would occur at $s = ½$, *i. e.*, at half the speed of synchronism. Beyond this point the slip and torque would assume negative values.

The resultant torque of the motor would be obtained by taking the algebraic sum of the torques due to the two oppositely rotating components of the oscillating rotor field. This torque is represented by the full line curve in Fig. 2D, the dotted and chain-dotted curves corresponding to the component torques due to the two rotating components of the rotor field.

If we suppose that the torque arising from the current in the field winding is large in comparison with the other torques acting on the rotor, so that the dominant effect is that due to the field winding, then it is evident that the torque reversal which occurs at half-synchronous speed will tend to make the machine run in the neighborhood of that speed, and it will then be impossible to run the machine up to full synchronism.

In the discussions which have taken place regarding the tendency of the machine to settle down to a speed in the neighborhood of half-synchronism, erroneous views have frequently been expressed. Some engineers appear to hold the opinion that the machine *locks* into exact half-synchronism. As we have seen, the speed to which it settles down, although near half-synchronism, is not definite, and may, according to the special circumstances of each case, be anywhere in the neighborhood of that speed. It is no more correct to say that the machine locks into half-synchronism, than it would be to say that an ordinary induction motor locks into full synchronism.

5. Torque Due to Eddy Currents

If the field structure is laminated throughout, the torque due to eddy-currents will be insignificant. The case is otherwise, however, with solid field poles in which large eddy-currents may arise. If we were to imagine the rotor replaced by a solid cylinder of conducting material, then the rotating field of the stator would give rise to rotating eddy-current sheets in the conducting cylinder, and the axes of such current sheets[8] would follow the axes of the rotating stator field. There would in this case be perfect freedom of motion of the axes of the current sheets relatively to the rotor, and this condition is closely approximated to in an ordinary squirrel-cage winding. If we next suppose that the conducting cylinder is cut up into a number of sectors by radial barriers of insulating material, then the freedom of motion of the axes of the current sheets relatively to the rotor would be largely destroyed, and these axes could only swing through an angular distance not exceeding the angular width of a sector. Now this is approximately the case corresponding to a salient pole rotor with solid poles. In such a rotor, owing to the restriction imposed on the free development of eddy currents by the relatively large spaces between the field poles, the axes of the eddy currents can only travel through a relatively short distance. If the axes could not travel at all, the arrangement would be identical with that of a rotor having a single magnetic axis winding; while if the axes could travel with perfect freedom, it would be identical with that of a squirrel-cage rotor. Hence we see that the torque due to eddy currents in the solid field cores will partake partly of the nature of the torque due to a single-magnetic-axis rotor winding, and partly of that due to an ordinary polyphase rotor. The single-magnetic-axis effect is, however, in many cases found to predominate, and considerable difficulty may then be exper-

---

[8] By the axes of the current sheets are meant the lines along which the current density is zero or the lines with which all the individual current filaments are linked.

ienced in getting the rotor to pass well beyond half-synchronous speed.

PULLING INTO SYNCHRONISM

Of the various torques concerned in accelerating the rotor from rest to synchronism, three, namely the induction motor, the single-magnetic-axis and the eddy current torque, are functions of the speed, and may hence conveniently be referred to as the speed torques. Let us suppose that by the action of the speed and hysteresis torques the rotor has been brought to a speed not far removed from synchronism. Since in the neighborhood of synchronism, the speed torques rapidly decrease with decreasing slip—as shown in Fig. 2—and assume zero values at synchronism, it is evident that these torques would never be able to bring the rotor up to full synchronism; and the hysteresis torque is generally much too weak to effect this. The rotor is finally pulled into synchronism by the varying magnetic reluctance torque, and is maintained at synchronous speed by the same torque, all the other torques vanishing at that speed. As already explained, the varying magnetic reluctance torque may for this reason be conveniently termed the synchronous torque, and we shall in what follows refer to it as such.

We have already seen that the synchronous torque is an alternating torque having a zero mean value for all speeds other than that of synchronism, and is thus incapable of exerting any steady driving or accelerating effect so long as the speed of the rotor is below synchronism. The frequency of the synchronous torque is given by $2\,s\,f.$, and the forced oscillations of the rotor to which the synchronous torque gives rise have the same frequency as the torque itself. Now the rotor speed oscillations call into play a further alternating or oscillating torque, owing to the fact that the speed torques change with the speed of the rotor. The speed torques may for small values of the slip be taken to be proportional to the slip, and hence their changes to be proportional to the changes in the speed. The effect is the same as if we were to substitute for the fluctuating speed torques a constant torque equal to the sum of the mean values of the speed torques, together with an oscillating torque whose amplitude is proportional to that of the speed fluctuations.

For the sake of simplicity we shall assume the speed fluctuations to obey the simple harmonic law. They may then be graphically represented in a vector diagram by the projections on the vertical axis of the vector $O\,V$ in Fig. 3, this vector rotating at $s\,f$, revolutions per second. The instantaneous projection of $O\,V$ gives the difference between the instantaneous speed and the mean speed. Since the oscillating component of the speed torques may, as we have seen, in the neighborhood of synchronism be taken to be proportional at every instant to the difference between the instantaneous and the mean speed, and since increase of speed produces decrease of speed torques, it is evident that the oscillating or fluctuating component of the speed torques may be represented by a vector $O\,F$ in direct phase opposition to $O\,V$. Next, if we assume that the alternating synchronous torque is also a simple harmonic function of the time, then the resultant of the synchronous torque and the oscillating component of the speed torques will give us the alternating torque which gives rise to the periodic accelerations and retardations of the rotor. The phase of this resultant torque is easily determined; for since its zero value must occur at the instant of maximum speed, it is evident that the vector $O\,R$, which represents the resultant torque, must be 90 deg. ahead of $O\,V$, as shown in Fig. 3. Lastly, the synchronous torque vector $O\,S$ is obtained by subtracting from the resultant accelerating torque $O\,R$ the oscillating component $O\,F$ of the speed torques. The angular velocity of all the vectors in the diagram of Fig. 3 is directly proportional to the slip, being, in fact, equal to $4\,\pi\,s\,f$.

Let ω denote the excess of the instantaneous rotor speed over the mean speed (corresponding to the vertical projection of $O\,V$ in Fig. 3), and let y stand for the instantaneous resultant accelerating torque (vertical projection of $O\,R$). Then, if K is the moment of inertia of the rotor,

$$y = K\frac{d\,\omega}{d\,t},$$

or

$$d\,\omega = 1/K\,y\,d\,t$$

and hence, taking as the origin of time the instant at which ω is zero,

$$\omega = 1/K \int y\,d\,t,$$

from which it is seen that the amplitude of the speed fluctuation $O\,V$ is proportional to the time-integral over a quarter-period of the resultant accelerating torque; or, since the period of this torque varies inversely as the slip, $O\,V$ is proportional to $\frac{O\,R}{s}$

By means of the vector diagram of Fig. 3 we can easily show that the amplitude of the speed fluctuations must increase with decreasing rotor slip. For, assuming the diagram to represent the conditions prevailing at a given mean speed, if the mean speed increases, $O\,R$ must decrease; for if it were to remain constant, then owing to the increase of its period due to the decrease of slip, its time-integral over a quarter-period would be increased, and $O\,V$, which is proportional to this time integral, would increase. This again would cause $O\,F$ ($S\,R$), which is proportional to $O\,V$, to increase; but since the length $O\,S$ is constant, an increase of $S\,R$ could only be brought about by a decrease of $O\,R$ (as shown by the dotted lines $O\,S'$ and $R'\,S'$ in the figure). It follows *a fortiori* that $O\,R$ could not increase with increase of mean speed.

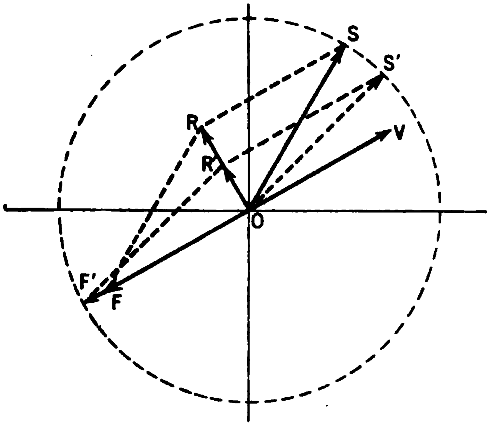


Fig. 3—Vector Diagram of Speed Fluctuations and Torques

We thus see that as the mean speed of the rotor gradually increases, the vector $O\,V$ undergoes steady elongation, the vector $O\,F = S\,R$ a similar steady elongation ($O\,F$ is proportional to $O\,V$) and the vector $O\,R$ a steady contraction. The vector $O\,S$ remains fixed in magnitude, but gradually approaches $O\,V$. At the same time the angular velocity of all the vectors in the diagram steadily decreases. If we were to consider the actual paths traced out by the extremities of the vectors during the last few cycles preceding synchronism, we should find that V traces out a spiral path opening outwards, F a similar path, R a spiral path contracting inwards, while S continues to move in its original circular path. Just before synchronism is reached, $O\,V$ is moving with extreme slowness and $O\,S$ is only very slightly in advance of it. As $O\,V$, having passed through the horizontal position, moves into the first quadrant, its projection gradually increases until the value of this projection when added to the mean rotor speed gives the speed of synchronism. At this instant all the torques have disappeared with the exception of the synchronous torque.

It is clear that at the instant when synchronism is first reached the synchronous torque cannot be less than the total torque resisting the motion. For, if such were the case, then balance of the total driving and resisting torques must have taken place at some instant preceding synchronism, and such balance would have prevented any further increase of speed, *i. e.*, it would have prevented the rotor from reaching synchronism. Hence at the instant when synchronism is reached, the synchronous torque must either equal or exceed the total resisting torque. In the first case, the rotor will steadily maintain synchronous speed. In the second, further acceleration will take place, and the rotor will settle down to the steady speed of synchronism only after a number of oscillations, the final position which it takes up relatively to the stator poles being such that the synchronous torque arising from the displacement of the magnetic axes of the stator and rotor is exactly equal to the total resisting torque. Whether the rotor comes up to synchronous speed quietly without oscillations, or whether such oscillations take place before it finally settles down to the steady speed of synchronism, the running will correspond to stable conditions. For in either case a momentary increase of speed results in decrease of driving torque, and a momentary decrease of speed in increase of driving torque. The momentary changes in the driving torque which arise during speed fluctuations, are due partly to changes in the synchronous torque, which tend to check such fluctuations, and partly to the reappearance of the speed and hysteresis torques, which have a similar effect.

OPEN VERSUS CLOSED FIELD WINDINGS AT STARTING

The advisability or otherwise of closing the field windings at starting has been repeatedly discussed. The danger of breaking down the insulation by the high voltage induced in the field windings when the stator circuits are first connected to the mains must be taken into account. Although this danger is entirely avoided by short-circuiting the windings before the stator is connected to the mains, there is no doubt that, from the point of view of initial torque and rapidity of starting, it is inadvisable to have the field circuit closed. The effect of closing field windings is similar to that of reducing the resistance of the squirrel-cage or eddy current path,—a procedure which is well-known to lower the initial torque. Again, as the neighborhood of half-synchronism is approached, the powerful single-axis rotor effect may seriously affect the acceleration of the rotor, and may frequently entirely prevent the machine from attaining any speed greatly exceeding that of half-synchronism. In order therefore to increase the acceleration of the rotor during the early stages of the starting process, the field should be kept open; any risk of breaking down the insulation may be guarded against by the use of a suitable field break-up switch.

Now although it is advisable to keep the field circuit open during the initial stages of the starting operation it by no means follows that it would be equally advantageous to keep it open until the machine has been pulled into synchronism. The slip with which the rotor ultimately tends to run under the action of the speed torques will depend on the resistance of the circuits in which the currents giving rise to the speed torques circulate. By lowering this resistance the torque will be momentarily raised and the speed increased. Now closing the field circuit would be equivalent to such reduction of resistance, so that the short-circuiting of the field during the final stages of the starting operation will cause the mean rotor speed to approach more closely to the speed of synchronism than would otherwise be the case. There is thus a distinct advantage in closing the field circuit during the final stages of the starting operation, after the rotor speed has reached a value not differing greatly from synchronism.

Cases may in fact arise where a machine with its field open might refuse to pull into synchronism, but could be made to do so by closing the field circuit. This conclusion has been verified experimentally. A certain machine was started with its field open, the rotor potential difference being so low that the rotor settled down to a speed below synchronism, and refused to pull into synchronism. The moment, however, that the field circuit was closed, the rotor locked into synchronism.

OSCILLATIONS IN STATOR CURRENT DURING THE PERIOD IMMEDIATELY PRECEDING SYNCHRONISM

It is well known that as the speed of synchronism is approached violent fluctuations in the stator current gradually become noticeable. These are indicated in Fig. 5 of Rosenberg's paper, and are easily accounted for. So long as the speed is below synchronism, the field poles are slipping past the stator poles, and periodic fluctuations are taking place in the reluctance accompanied by corresponding fluctuations of reactance which throw the stator current into oscillations. The frequency of these oscillations being $2\,s\,f$ (since the reluctance returns to the same instantaneous value after the pole has moved through a distance equal to the pole-pitch) they are not noticeable at low speeds, and only become apparent when the slip has become sufficiently small.

SOME EXPERIMENTAL RESULTS

(*a*) *Relations Connecting Stator Potential Difference with Stator Current, Stator Input and Power-Factor, Rotor Speed and Field E. M. F. when Field is Open-Circuited.* The experiments embodied in the series of curves given were carried out on a four-pole, five-kw. three-phase converter designed for a continuous current voltage of 100–130 volts at a speed of 750 rev. per min. This machine had laminated main poles, was fitted with commutating poles, but had no special starting devices. While the experiments about to be described were being carried out, the brushes were entirely removed from the commutator. Before each set of readings the machine was allowed to run light for a sufficiently long time to get the bearings into a steady state.

In the first set of experiments, the results of which are exhibited graphically in Figs. 4 and 5, a number of gradually increasing potential differences were applied to the rotor slip rings, and after the speed corresponding to any given potential difference had settled down to a constant value, readings of the speed, current, power, etc., were taken when the potential difference had been raised sufficiently to enable the machine to lock into synchronism, it was still further increased, and then a second series of readings, corresponding to decreasing values of the potential difference was obtained. In the illustrations both the ascending and descending branches of the various curves are given.

Fig. 4 shows the relations connecting speed and field e. m. f. with stator potential difference. Below a potential difference of about 15 volts across the slip rings the machine would not run at all. The speed then gradually increased with the potential difference, the increase becoming much slower beyond a certain point, and at a slip ring-potential difference of about 45 volts the machine was able to lock into synchronism. During the descending set of readings, synchronism was maintained down to a voltage of about 35 volts. Below this point the speeds obtained with given voltages were found to be uniformly higher than those corresponding to the ascending branch of the curve. Since, as shown by Fig. 5, the power supplied to the machine was found to be lower for decreasing values of the potential difference. in spite of the higher value of the speed, it is to be inferred that for decreasing values of the potential difference the resisting torque was uniformly less. This would indicate a decrease in the frictional resistances, probably due to the temperature of the bearings being higher during the descending set of readings than during the ascending set.

The changes in the field e. m. f. are related to those in the speed. The field e. m. f. may be regarded as proportional to the product of two factors, namely the maximum flux per pole and the slip. At first the field e. m. f. rises with increase of potential difference, indicating that the increase of flux is more important than the decrease of slip. Beyond a certain point the decrease of slip is more important than the increase of flux, and the field e. m. f. begins to decrease. It does not vanish at synchronism, indicating that there is either swaying or pulsation of the flux which enters the main poles. Since for descending values of the potential difference the speed is uniformly higher and hence the slip lower than for ascending values, we should expect the field e. m. f.

to be uniformly lower in the former case, and the curve of field e. m. f. shows that such is the case.

Fig. 5 shows the relations connecting stator current, stator power and power-factor with potential difference. The difference between the ascending and descending branches of the power or input curve has already been referred to. It must be remembered that when the machine is not running synchronously, its behavior is similar to that of an induction motor. Hence, owing to the lower resisting torque during the descending set of readings we should expect a smaller current and also a lower power-factor (as is at once evident from consideration of the circle diagram) than during the ascending set; and the curves of Fig. 5 fully confirm this.

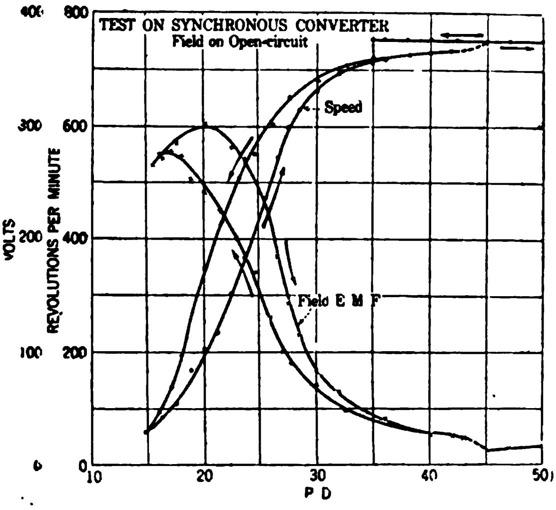


FIG. 4.—CURVES CONNECTING SPEED AND FIELD E.M.F. WITH STATOR P.D. WHEN FIELD IS OPEN-CIRCUITED

(*b*) *Relations Connecting Stator Potential Difference with Speed and Field Current, when the Field Circuit is Closed.* Figs. 6 and 7 give the connection between potential difference and speed when the field circuit is closed through various resistances, and in Fig. 6 the curve corresponding to the field on open-circuit, previously shown in Fig. 4, is repeated for the sake of comparison.

When the field was on dead short-circuit, the machine refused to run up to anything like synchronous speed, and seemed to approach asymptotically a speed somewhat above half-synchronism.[9] The explanation of this fact has already been given (reference may be made in this connection to Fig. 2D).

The curves of Fig. 7 show that by the introduction of a suitable amount of resistance into the field circuit the tendency of the machine to settle down to a speed in the neighborhood of half-synchronism may be overcome, and that the machine may be made to lock into synchronism. This result may be explained as follows. Considering the complete torque-speed curve of an induction machine over the entire range of slip, positive and negative, we may regard the point of zero slip as dividing this curve into two branches, one of which corresponds to positive values of the slip, and the other to negative values. If we now suppose resistance to be introduced into the rotor circuit, then as is well known, the result is to produce a shearing of the two branches of the torque speed curve in opposite directions from the point of zero slip. Referring now to the torque speed curves of Fig. 2D, it must be noticed that the point of zero slip for the chain-dotted curve

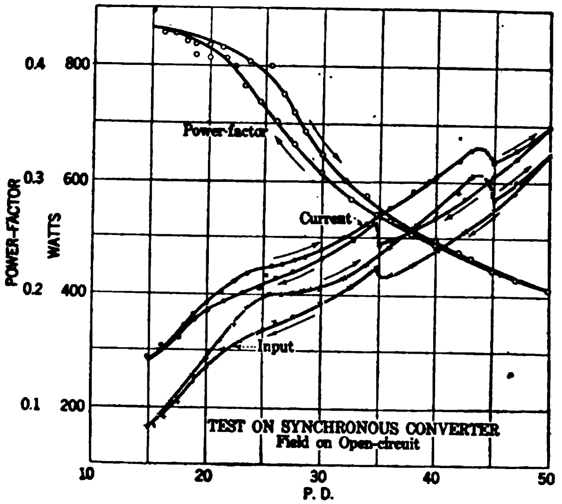


FIG. 5—CURVES CONNECTING STATOR INPUT, CURRENT AND POWER FACTOR WITH STATOR P.D. WHEN FIELD IS OPEN-CIRCUITED

---

[9] Incidentally the fact that the machine reached a speed in excess of half-synchronism definitely disposes of the erroneous view previously referred to that the machine tends to lock into exact half-synchronism.

corresponds to half-synchronism, while for the dotted curve the point of zero slip is at full synchronism. From this it follows that the introduction of resistance will in the region between half and full synchronism cause a shearing of the dotted and chain-dotted curves in opposite directions, the dotted curve being sheared from left to right, while the chain-dotted one is sheared from right to left. It is easy to see that this will cause a rise of the minimum in the resultant curve (the full line curve of Fig. 2D), and if the resistance remains sufficiently large the minimum resultant torque will assume a positive value, so that the driving torque will be positive over the entire range of speed from zero to synchronism. The shearing of the dotted and chain-dotted curves in opposite directions in the region between half and full synchronism is, however, only one of the causes concerned in suppressing the negative portion of the resultant torque-speed curve, and besides this there is another cause. The dotted curve is the torque-speed curve of an induction motor whose stator is supplied at constant potential difference and frequency; whereas the chain-dotted curve is the curve of an imaginary induction motor whose stator is supplied at variable

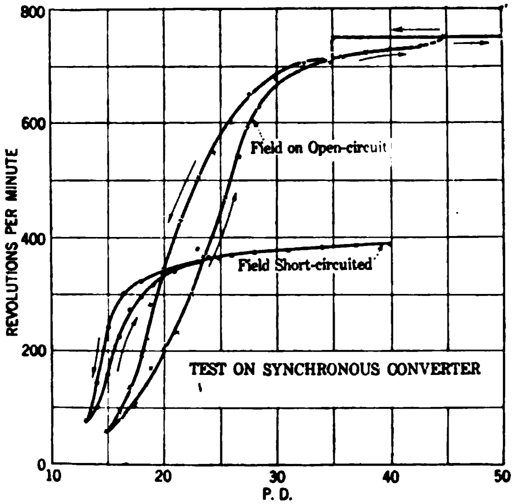


Fig. 6—Curves Connecting Speed with P.D. when Field Is(a) Open-Circuited, and (b) Short-Circuited

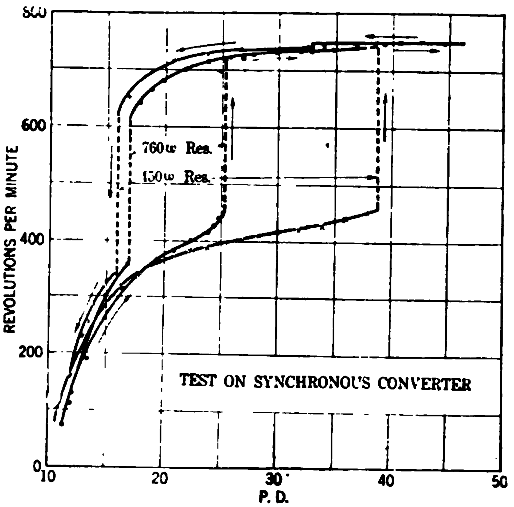


Fig. 7—Curves Connecting Speed with P.D. when Field is Closed through External Resistance

potential difference and variable frequency, the potential difference being proportional to the frequency. Now the introduction of resistance into the field windings is equiv-

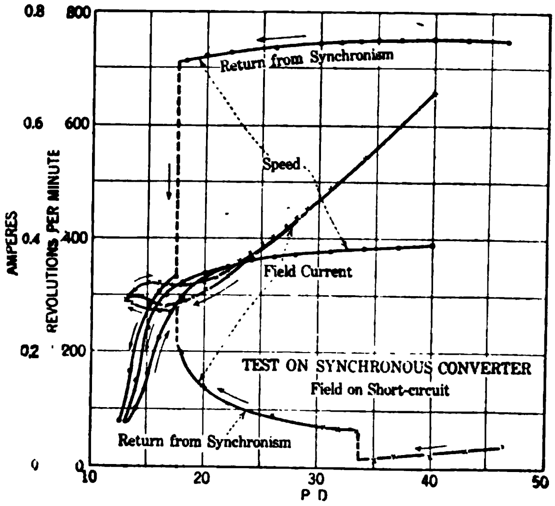


Fig. 8—Curves Connecting Speed and Field Current with P. D. when Field is Short-Circuited

alent to the introduction of resistance into the primary winding of the imaginary motor (since the primary winding of this imaginary motor is represented by the field winding) and is thus equivalent to a reduction of the potential difference across its terminals. This will result in a reduction of all the ordinates of the chain-dotted curve. While therefore, the introduction of resistance into the field circuit results in a simple shearing of the dotted curves from right to left unaccompanied by any change in the values of the ordinates, the effect on the chain-dotted curve is a two-fold one, namely a shearing from left to right accompanied by a shrinkage of the ordinates. This shrinkage of the ordinates will further help to suppress the negative portion of the resultant curve.

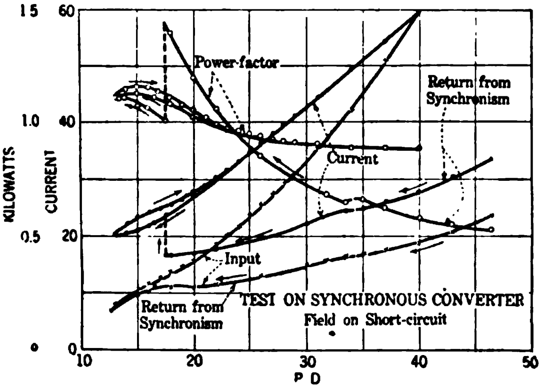


FIG. 9—CURVES CONNECTING STATOR INPUT, CURRENT AND POWER FACTOR WITH P.D. WHEN FIELD IS SHORT-CIRCUITED

It will be noticed that in the curves of Fig. 6 which refer to the open-circuit and short-circuit conditions of the field, there are no discontinuities in the speed curves (except that which occurs at the instant of breaking from synchronism in the case of the open-circuit curve); whereas the curves of Fig. 7 show two well-marked discontinuities (one on each of the curves), in addition to the discontinuities at break from synchronism. These discontinuities are readily accounted for by considering the shape of the resultant or full-line torque-speed curve of Fig. 2D. In the cases to which Fig. 7 refers the resultant torque-speed curve lies, as already explained, wholly above the axis of speed, all its ordinates being positive; but the curve has two maxima separated by a minimum. It is the existence of this minimum which causes the discontinuities in the speed curves. Stability of running can only be secured by working on a portion of the torque-speed curve which has a downward slope from left to right. With increasing potential difference and speed the point on the (varying) torque-speed curve corresponding to the stable running condition for the given potential difference gets displaced further and further to the right, until finally it reaches the minimum point

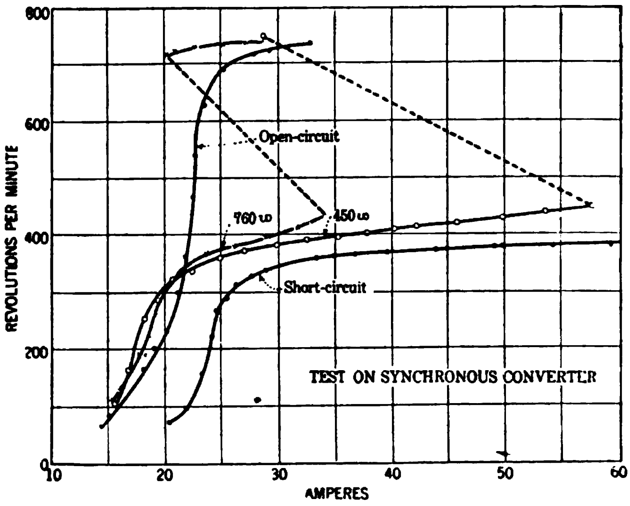


FIG. 10—CURVES CONNECTING SPEED WITH STATOR CURRENT UNDER VARIOUS CONDITIONS

on the curve. An increase of potential difference beyond the value corresponding to this minimum point results in a passage into the unstable region which lies between the minimum and the second maximum, and no stable running is possible in this region. It is only after the speed has passed beyond the second maximum of the torque-speed curve that stability can again be reached. The point where the discontinuity occurs along the ascending branch gives approximately the speed corresponding to minimum torque; while the discontinuity on the descending branch marks approximately the second maximum of torque. The first maximum of torque would correspond roughly to the lowest speed at which the machine will run.

In Fig. 8 are shown the relations connecting speed and potential difference, and field current and potential difference, with the field on dead short circuit. Each curve is shown as having three branches. Two of these correspond to the values obtained by first increasing the speed to a certain value and then decreasing it. The third branch, marked "Return from synchronism" was obtained by first open-circuiting the field and raising the potential difference to a value sufficient to enable the machine to lock into synchronism, then short-circuiting the field and taking a set of readings while the potential difference was being decreased. The first two branches of the speed curve are identical with those shown in Fig. 6. The relations connecting current and power factor with potential difference are given by Fig. 9,

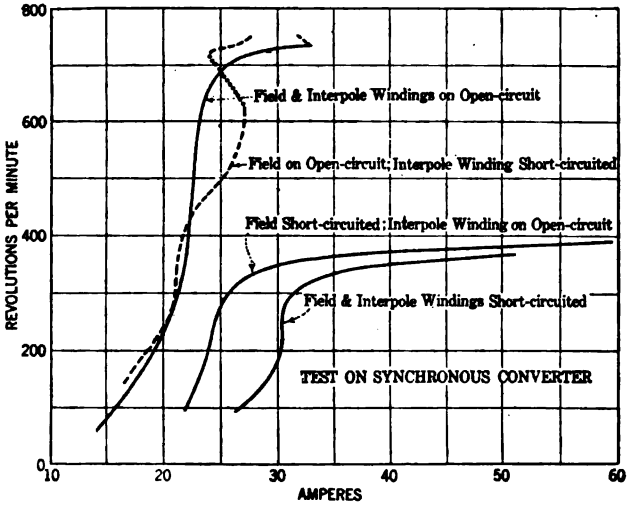


FIG. 11—CURVES CONNECTING SPEED WITH STATOR CURRENT UNDER VARIOUS CONDITIONS

Returning to Fig. 6, it will be seen that the machine starts with a lower potential difference when the field is short-circuited than when it is on open circuit; and this might at first sight appear to contradict the statement previously made regarding the advantage of starting with the field circuit open. Such, however, is not the case; for the real basis of comparison is not the potential difference applied to the stator, but the *current* taken by it. The relation connecting speed with current for the various arrangements tried is shown in Fig. 10, and it will be seen at once that the machine starts up with a considerably lower current when the field is open-circuited. If the field is closed through a resistance, there will be a certain value of this resistance which gives the best initial results, but even with this value the current during the intermediate stages rises to a higher value than when the field is open-circuited. On the other hand, it will be noticed that with the field closed through a resistance high enough to allow of the machine being pulled into synchronism, a higher value of speed is reached with a given current than with the field on open circuit.

As the machine experimented on was provided with interpoles, it was thought desirable to try the effect of short-circuiting the interpole winding. In Fig. 11 are given four curves, two of which are for the sake of comparison reproduced from Fig. 10. It will be seen that the worst results are obtained with both main and inter-pole fields short-circuited, and that the short-circuiting of the interpole windings alone does not produce any very marked effect, and is not sufficient to prevent the machine from running up to synchronism.

Summary of Conclusions Reached

1. During the initial stages of the starting period the field should be kept open. If the induced voltage exceeds the limit of safety, a field break-up switch should be provided.

Closing the field circuit not only largely increases the current during the initial stages of the starting period, but may entirely prevent the machine from running up to synchronous speed. This is due to the single-magnetic-axis effect of the field winding.

2. If the field is kept closed and the machine only reaches a speed in the neighborhood of half-synchronism, there is no tendency to lock into exact half-synchronism.

3. There is a distinct advantage in short-circuiting the field after the field has reached a value not differing greatly from synchronism. This will greatly facilitate the final locking into synchronism.

APPENDIX

ANALYTICAL THEORY OF POLYPHASE INDUCTION MOTOR HAVING SINGLE-PHASE WINDING ON THE ROTOR

The type of motor under consideration is one whose stator is provided with a polyphase winding, but whose rotor carries only a single-phase or single-magnetic-axis winding. We may speak of the stator winding as the *primary* and of the rotor winding as the *secondary* winding of the machine. We may further suppose that the rotor carries a supplementary single-phase winding whose magnetic axes are coincident with those of the secondary winding and which we may conveniently speak of as the *tertiary* winding. The secondary winding may be taken to correspond to the solid masses of the field poles of a synchronous machine in which eddy currents are induced by the rotating field; while the *tertiary* winding may be taken to represent the field winding of the synchronous machine.

Owing to the extreme complexity of the problem, it becomes necessary to make the following simplifying assumptions:—

(1) We shall suppose that the stator winding of the machine is so arranged that each phase when traversed by a current gives rise to a simple sine distribution of magnetic flux in space.

(2) We shall neglect the effect of variations in the permeability.

(3) The time-variations of certain of the quantities concerned will be supposed to follow the sine law.

It is obvious that during the rotation of the machine the mutual inductance of the secondary with any one phase of the primary winding will undergo periodic fluctuations, and from supposition (1) it follows—as can be easily shown—that such fluctuations will obey the sine law if the speed of rotation is constant. Let M_0 denote the maximum value of the mutual inductance of the secondary and a certain phase of the primary winding. Then if m_a, m_b and m_c denote the instantaneous mutual inductances of the three phases with the secondary at time t we may write, since the space displacement of the three oscillating magnetic flux waves due to the armature current is 120 electrical degrees,

$$m_a = M_o [(1 - S) wt + \alpha]$$

$$m_b = M_0 \sin \left[(1 - S) w t + \alpha + \frac{2 \pi}{3} \right]$$

$$m_c = M_0 \sin\left[(1 - S)\, w\, t + \alpha + \frac{4\,\pi}{3} \right]$$

where $w = 2\,\pi \times$ frequency of armature current, and s = slip of rotor $= \left(\frac{\text{synchronous speed} - \text{rotor speed}}{\text{synchronous speed}}\right)$.

Let the currents in the three primary phases be represented by

$$i_a = I_m \sin w\, t$$

$$i_b = I_m \sin\left(w\, t + \frac{2\,\pi}{3} \right)$$

$$i_c = I_m \sin\left(w\, t + \frac{4\,\pi}{3} \right)$$

If we provisionally assume that both the secondary and tertiary circuits are open, so that they are incapable of reacting on the primary circuits, then in order to maintain the currents i_a, i_b and i_c in the three phases of the primary we must provide impressed e. m. f's. which are given by

$$e_a = Z_1 I_m \sin\,(w\, t + \theta_1)$$

$$e_b = Z_1 I_m \sin\left(w\, t + \frac{2\,\pi}{3} + \theta_1 \right)$$

$$e_c = Z_1 I_m \sin\left(w\, t + \frac{4\,\pi}{3} + \theta_1 \right)$$

In the above expressions, Z_1 denotes the equivalent impedance of each phase when the three windings are supplied with currents differing 120 deg. in phase. If r_1 = resistance of each phase, and L_1 = true self-inductance of each phase (*i. e.*, flux linkage with phase when unit current is flowing through it and when remaining two phases are devoid of current), then

$$Z_1 = \sqrt{r_1^2 + (3/2\; w\, L_1)^2}$$

The fact that the equivalent reactance of a phase is 3/2 times its true reactance is due to mutual inductance between phases, and is a consequence of the assumption that the flux distribution in space due to any one phase follows the sine law.

The angle θ_1 in the above expressions is such that $\tan \theta_1 = \dfrac{3/2\; w\, L_1}{r_1}$.

The total instantaneous flux linked with the secondary due to the primary currents i_a, i_b and i_c is

$$\Phi = m_a\, i_a + m_b\, i_b + m_c\, i_c = 3/2\, M_0\, I_m \cos\,(S\,w\,t - \alpha)$$

This flux gives rise to an e. m. f. in the secondary of amount

$$e_2 = -\frac{d\,\Phi_2}{dt} = \frac{3}{2}\, S\, w\, M_0\, I_m \sin\,(S\,w\,t - \alpha)$$

We shall now assume that the secondary is closed, and that the e. m. f. e_2 is allowed to produce a current in it.

Let r_2, L_2 be the resistance and self-inductance respectively of the secondary. Its impedance is then

$$Z_2 = \sqrt{r_2^2 + (s\,w\,L_2)^2}$$

and the e. m. f. e_2 gives rise to a secondary current

$$i_2 = \frac{3}{2\,z_2}\, s\, w\, M_0\, I_m \sin\,(s\,w\,t - \alpha - \theta_2)$$

where
$$\tan\,\theta_2 = \frac{s\,w\,L_2}{r_2}$$

For the sake of simplicity, we shall put

$$k_2 = \frac{3/2\; s\,w\,M_0}{z_2}$$

so that we may write

$$i/2 = k/2\, I_m \sin\,(s\,w\,t - \alpha - \theta_2)$$

The secondary current reacts on the inducing primary circuits, and gives rise to magnetic fluxes linked with them. The flux linkage with the first phase, whose instantaneous mutual inductance with the secondary is m_a, is given by $m_a\, i_2$, and the e. m. f. to which this flux gives rise is

$$-\frac{d}{dt}\,(m_1\, i_2) =$$

$$-\frac{d}{d\,t}\,\{M_0 \sin\,[\,(1 - s)\,w\,t + \alpha]\, k_2\, I_m \sin\,(s\,w\,t - \alpha - \theta_2)\,\} =$$

$$(1/2 - s)\,w\,M_0\,k_2\,I_m\,\sin\,[\,(1 - 2\,s)\,w\,t + 2\,\alpha + \theta_2]$$
$$- 1/2\,w\,M_0\,k_2\,I_m \sin\,(w\,t - \theta_2)$$

Similar expressions (with suitable changes of time-phase) hold good for the remaining two phases.

It will be noticed that each of these e. m. fs. consists of two components of different frequency. If we wished to maintain the original currents i_a, i_b and i_c unaltered, then in addition to the original impressed e. m. fs., e_a, e_b and e_c we should have to provide e. m. fs., equal and opposite in phase to those induced in the primaries by the secondary current i_2. Now since these latter e. m. fs. consist of two components, one [such as

$$- 1/2\, w\, M_0\, k_2\, I_m \sin\, (w\, t - \theta_2)$$

in the first phase of the primary of fundamental frequency], and the other [such as $(1/2 - s)\, w\, M_0\, k_2\, I_m \sin\, [\,(1 - 2\, s)\, w\, t + 2\, \alpha + \theta_2]$ in the first phase] of a frequency which is a function of the slip, the source of impressed e. m. f. would have to be capable of a continuous variation of wave-shape in order that the currents i_a, i_b and i_c might remain unaltered. It is needless to point out that such special continuous variation of wave-shape could not be secured in practise. We shall therefore make an assumption which is much more likely to conform to actual conditions. We shall suppose that the source of impressed e. m. f. continues to supply a pure sine wave, but that the excitation of this source is varied so that it not only provides the original e. m. fs. e_a, e_b and e_c but, in addition, any other sine-wave components of *fundamental frequency* which may be necessary to balance fundamental frequency e. m. fs [such as the e. m. f. $- 1/2\, w\, M_0\, k_2\, I_m \sin\, (w\, t - \theta_2)$ in the first phase] induced in the primaries by the secondary. The remaining components in the induced e. m. fs. whose frequency differs from the fundamental, and which are not balanced by corresponding components in the impressed e. m. f. waves, give rise to additional primary currents whose magnitude we proceed to determine.

For the sake of simplicity, we shall assume the impedance of the source of impressed e. m. f. to be negligible in comparison with the impedance of the primaries of the machine under consideration, so that the total impedance of the circuits on which the induced e. m. fs. of frequency other than the fundamental act will be represented by the impedance of the primaries. The equivalent reactance of each phase of the primary corresponding to e. m. fs. such as $(1/2 - s)\, w\, M_0\, k_2\, I_m \sin\, [\,(1 - 2\, s)\, w\, t + 2\, \alpha + \theta_2]$ in the first phase, is $3/2\, (1 - 2\, s)\, w\, L_1$.

Hence the corresponding current in the first phase is given by

$$i_a{}' = \frac{(1/2 - s)\, w\, M_0\, k_2\, I_m}{\sqrt{r_1{}^2 + [3/2\, (1 - 2\, s)\, w\, L_1]^2}}\, \sin\, [\,(1 - 2\, s)\, w\, t + 2\, \alpha + \theta_2 - \theta_3]$$

If for the sake of simplicity we put

$$k_1 = \frac{(1/2 - s)\,w\,M_0}{z_3}$$

then

$$i_a' = k_1\,k_2\,I_m \sin\left[\,(1 - 2\,s)\,w\,t + 2\,\alpha + \theta_2 - \theta_3\right]$$

Similar expressions hold good, with suitable changes of time-phase, for the corresponding currents in the remaining two phases.

Before proceeding further, it will be convenient to collect all the results so far obtained.

In the first phase of the primary, we have the following components of impressed e. m. f.:—

$$e_a = z_1\,I_m \sin(w\,t + \theta_1) \qquad (1)$$

and

$$e_a' = 1/2\,w\,M_0\,k_2\,I_m \sin(w\,t - \theta_2) \qquad (2)$$

where

$$z_1 = \sqrt{r_1^2 + (3/2\,w\,L_1)^2} \qquad (3)$$

$$\tan\theta_1 = \frac{3/2\,w\,L_1}{r_1} \qquad (4)$$

$$k_2 = \frac{3/2\,s\,w\,M_0}{z_2} \qquad (5)$$

$$z_2 = \sqrt{r_2^2 + (s\,w\,L_2)^2} \qquad (6)$$

$$\tan\theta_2 = \frac{s\,w\,L_2}{r_2} \qquad (7)$$

Next, in the first phase of the primary we have the current components

$$i_a = I_m \sin w\,t \qquad (8)$$

and

$$i_a' = k_1\,k_2\,I_m \sin\left[\,(1 - 2\,s)\,w\,t + 2\,\alpha + \theta_2 - \theta_3\right] \qquad (9)$$

where

$$k_1 = \frac{(1/2 - s)\,w\,M_0}{z_3} \qquad (10)$$

$$z_3 = \sqrt{r_1^2 + [3/2\,(1 - 2\,s)\,w\,L_1]^2} \qquad (11)$$

$$\tan\theta_3 = \frac{3/2\,(1 - 2\,s)\,w\,L_1}{r_1} \qquad (12)$$

Again, considering the secondary, we have in the induced e. m. f.

$$e_2 = 3/2\,s\,w\,M_0\,I_m \sin(s\,w\,t - \alpha) \qquad (13)$$

which gives rise to the secondary current

$$i_2 = k_2 I_m \sin (s w t - \alpha - \theta_2) \qquad (14)$$

We may now proceed with our investigation.

In addition to the e. m. f. e_2 in the secondary, which is due to the original currents i_a, i_b and i_c in the primaries, and which produces the current i_2, we now have another e. m. f., due to currents of the type i_a', in the primaries. This second e. m. f., as in the case of e_2, may be shown to be

$$e_2' = 3/2\, s w M_0 k_1 k_2 I_m \sin (s w t - \alpha - \theta_2 + \theta_3) \qquad (15)$$

If we were to provide an e. m. f. in the secondary equal and opposite to e_2' this latter e. m. f. would be neutralized, and the only currents in the primaries and the secondary would be those already considered and given by (8), (9) and (14).

Since, however, no such neutralization actually takes place, the e. m. f. e_2' is free to act, and gives rise to a current in the secondary.

$$i_2' = k_1 k_2^2 I_m \sin (s w t - 2\,\theta_2 + \theta_3) \qquad (16)$$

This current, in turn, reacts on the primary, and, proceeding as before, we can show that it induces in the first phase of the primary an e. m. f. given by

$$(1/2 - s)\, w M_0 k_1 k_2^2 I_m \sin [\,(1 - 2\,s)\, w t + 2\,\alpha + 2\,\theta_2 - \theta_3]$$
$$- 1/2\, w M_0 k_1 k_2^2 I_m \sin (w t - 2\,\theta_2 + \theta_3)$$

This e. m. f. is seen to consist of two terms of different frequency. We shall assume, as before, that the term of fundamental frequency is balanced by an equal and opposite component

$$e_a'' = 1/2\, w M_0 k_1 k_2^2 I_m \sin (w t - 2\,\theta_2 + \theta_3) \qquad (17)$$

in the impressed e. m. f. wave of the primary, while the other term, being unbalanced, gives rise to a current in the first phase of the primary

$$i_a'' = k_1^2 k_2^2 I_m \sin [\,(1 - 2\,s)\, w t + 2\,\alpha + 2\,\theta_2 - 2\,\theta_3] \qquad (18)$$

Again, the current i_1'', like i_1 and i_1', induces an e. m. f. in the secondary, given by

$$e_2'' = 3/2\, s w M_0 k_1^2 k_2^3 I_m \sin (s w t - \alpha - 2\,\theta_2 + 2\,\theta_3) \qquad (19)$$

The e. m. f. e_2'' gives rise to a secondary current

$$i_2'' = k_1^2 k_2^3 I_m \sin (s w t - \alpha - 3\,\theta_2 + 2\,\theta_3) \qquad (20)$$

The current in its turn, reacts on the primary, inducing in the first phase an e. m. f.

$$(1/2 - s)\, w M_0 k_1^2 k_2^3 I_m \sin [(1 - 2\,s)\, w t + 2\,\alpha + 3\,\theta_2 - 2\,\theta_3]$$
$$- 1/2\, w M_0 k_1^2 k_2^3 I_m \sin (w t - 3\,\theta_2 + 2\,\theta_3),$$

the fundamental frequency component of which we shall assume, as before, to be balanced by a component in the impressed e. m. f. equal to

$$e_a''' = 1/2\, w\, M_0\, k_1^2\, k_2^3\, I_m \sin(w\,t - 3\,\theta_2 + 3\,\theta_3) \qquad (21)$$

while the other component is left free to produce in the first phase of the primary a current

$$i_a''' = k_1^3\, k_2^3\, I_m \sin[\,(1 - 2\,s)\,w\,t + 2\,\alpha + 3\,\theta_2 - 3\,\theta_3] \qquad (22)$$

Proceeding in this way, we ultimately obtain, for each of the quantities under consideration—primary impressed e. m. f., primary current, secondary induced e. m. f., and secondary current—an infinite series of terms.

For the sake of simplicity, we shall put

$$k = k_1\, k/2 \qquad (23)$$

and

$$\theta = \theta_2 - \theta_3 \qquad (24)$$

Considering first the primary impressed e. m. f. the first phase we see from (1), (2), (17) and (21), that this is given by

$$z_1\, I_m \sin(w\,t - \theta_1) + 1/2\, w\, M_0\, k_2\, I_m \sin(w\,t - \theta_2) + k \sin(w\,t - \theta_2 - \theta) + k^2 \sin(w\,t - \theta_2 - 2\,\theta) + k_3 \sin(w\,t - \theta_2 - 3\,\theta) +]\,\ldots\ldots \qquad (25)$$

Similar expressions, with suitable time-phase differences, hold good for the impressed e. m. fs., in the other two phases.

Taking next the primary current in the first phase, and using (8), (9), (18) and (22), we find that this current is given by the infinite series.

$$I_m \sin w\,t + k\,I_m\, \{\sin[\,(1 - 2\,s)\,w\,t + 2\,\alpha + \theta] + k \sin[\,(1 - 2s)\,w\,t + 2\cdot\alpha + 2\,\theta] + k^2 \sin[\,(1 - 2\,s)\,w\,t + 2\,\alpha + 3\,\theta] + \} \qquad (26)$$

Similarly, using (13), (15) and (19), we find for the secondary induced e. m. f.

$$3/2\, s\, w\, M_0\, I_m\, [\sin(s\,w\,t - \alpha) + k \sin(s\,w\,t - \alpha - \theta) + k \sin(s\,w\,t - \alpha - 2\,\theta) +] \qquad (27)$$

Lastly, using (14), (16) and (20), we obtain the secondary current.

$$k_2\, I_m\, [\sin(s\,w\,t - \alpha - \theta_2) + k \sin(s\,w\,t - \alpha - \theta_2 - \theta) + k_2 \sin(s\,w\,t - \alpha - \theta_2 - 2\,\theta) +] \qquad (28)$$

Each of the four expressions (25), (26), (27) and (28) involves an infinite series of the type.

$$\Sigma = \sin \varphi + k \sin (\varphi \pm \theta) + k^2 \sin (\varphi \pm 2\,\theta) + k^3 \sin (\varphi \pm 3\,\theta) +$$

This series may be summed as follows:

Putting $\sin \varphi = \dfrac{1}{2j}\,(e^{j\varphi} - e^{j\varphi})$

and $\sin (\varphi \pm n\,\theta) = \dfrac{1}{2j}\,[e_1^{j}\,(\varphi + n\,\theta) - e^{-j}\,(\varphi \pm n\,\theta)]$

where $j^2 = -1$, we may write

$$\Sigma = \frac{1}{2j}\,\{e^{j\varphi}\,[1 + k\,e^{\pm j\theta} + k^2\,e^{\pm 2j\theta} + k^3\,e^{\pm 3j\theta} + \ldots .] - e^{\pm j\theta}$$

$$[1 + k\,e^{\pm j\theta} + k^2\,e^{\mp 2j\theta} +]\,\}$$

$$= \frac{1}{2j}\left(\frac{e^{j\varphi}}{1 - k\,e^{\pm j\theta}} - \frac{e^{-j\varphi}}{1 - k\,e^{\mp j\theta}}\right)_1$$

which, after a number of transformations, may be reduced to the simple form

$$\Sigma = \frac{\sin \varphi - k \sin (\varphi \mp \theta)}{(1 - k \cos \theta)^2 + (k \sin \theta)^2}$$

If we put

$$a = \sqrt{(1 - k \cos \theta)^2 + (k \sin \theta)^2} \qquad (29)$$

and

$$\tan \beta = \frac{k \sin \theta}{1 - k \cos \theta} \qquad (30)$$

then, after a further transformation, we may write

$$\Sigma = 1/a \sin (\varphi \pm \beta) \qquad (31)$$

Primary P. D. Applying this result to (25), we find for the primary impressed e. m. f. in the first phase.

$$z_1\,I_m \sin (w\,t + \theta_1) + \frac{1/2\,w\,M_0\,k_2\,I_m}{a} \sin (w\,t - \theta_2 - \beta)$$

This may be thrown into the form

$$E \sin (w\,t + \varphi) \qquad (32)$$

where

$$E = I_m \left\{ \left[r_1 + \frac{1/2\,w\,M_0\,k_2}{a} \cos (\theta_2 + \beta) \right]^2 + \left[3/2\,w\,L_1\,\frac{1/2\,w\,M_0\,k_2}{2} \sin (\theta_2 + \beta) \right]^2 \right\}^{\frac{1}{2}} \qquad (33)$$

and

$$\tan \psi = \frac{3/2\, w\, L_1 - \dfrac{1/2\, w\, M_0\, k_2}{a} \sin (\theta_2 + \beta)}{r_1 + \dfrac{1/2\, w\, M_0\, k_2}{a} \cos (\theta_2 + \beta)} \qquad (34)$$

Primary Current. Similarly, using (31) in (26), we obtain for the primary current in the first phase

$$I_m \{\sin w\, t + k/a \sin [\,(1 - 2\, s)\, w\, t + 2\, \alpha + \theta + \beta]\,\} \qquad (35)$$

It will be noticed that, with a pure sine wave of impressed e. m. f., the primary current wave is a distorted one.

Secondary Current. Proceeding in a similar manner with (27) and (28), we obtain the following expressions for the secondary induced e. m. f. and secondary current:—

Secondary induced e. m. f.

$$= I_m \frac{3/2\, s\, w\, M_0}{a} \sin (s\, w\, t + \alpha - \beta) \qquad (36)$$

Secondary current

$$= I_m\, k_2/a \sin (s\, w\, t - \alpha - \theta_2 - \beta) \qquad (37)$$

Both the above waves are pure sine waves.

Tertiary E. M. F. We next proceed to determine the e. m. f. induced in the open tertiary circuit. This e. m. f. is the resultant of the two e. m. fs. induced, by the primary and secondary currents. Let m_a', m_b' and m_c' denote the instantaneous mutual inductances between the tertiary and the three phases of the primary. Then we may write

$$m_a' = M_1 \sin [\,(1 - s)\, w\, t + \alpha]$$

$$m_b' = M_1 \sin \left[(1 - s)\, w\, t + \alpha + \frac{2\, \pi}{3} \right]$$

$$m_c' = M_1 \sin \left[(1 - s)\, w\, t + \alpha + \frac{4\, \pi}{3} \right]$$

The total instantaneous flux linkage with the tertiary due to the joint action of the currents in the three phases of the primary is

$$M_1 I_m \sin [\,(1 - s)\, w\, t + \alpha]\, \{\sin w\, t + k/a \sin [\,(1 - 2\, s)\, w\, t$$

$$+ 2\, \alpha + \beta]\,\} + M_1 I_m \sin \left[(1 - s)\, w\, t + \alpha + \frac{2\, \pi}{3} \right]$$

$$\left\{ \sin\left(w\,t + \frac{2\pi}{3}\right) + k/a \sin\left[(1-2\,s)w\,t + \frac{2\pi}{3} + 2\,\alpha + \beta\right]\right\} + M_1 I_m \sin\left[(1-s)\,w\,t + \alpha + \frac{4\pi}{3}\right]$$

$$\left\{ \sin\left(w\,t + \frac{4\pi}{3}\right) + k/a \sin\left[(1-2\,s)\,w\,t + \frac{4\pi}{3} + 2\,\alpha + \beta\right]\right\}$$

This, after simple transformations, is reducible to the form

$$3/2\, M_1 I_m \cos(s\,w\,t - \alpha) + 3/2\, k/a\, M_1 I_m \cos(s\,w\,t - \alpha - \beta)_1$$

and the tertiary e. m. f. due to this flux is

$$3/2\, s\,w\, M_1 I_m \sin(s\,w\,t - \alpha) + 3/2\, s\,w\, M_1 I_m\, k/a \sin(s\,w\,t - \alpha - \beta)$$

Next, if we denote by M_2 the mutual inductance between the tertiary and the secondary, the flux linkage with the tertiary due to the current in the secondary is

$$M_2 I_m k_2/a \sin(s\,w\,t - \alpha - \theta_2 - \beta)$$

and the tertiary e. m. f. due to it is

$$-\,s\,w\, M_2 I_m k_2/a \cos(s\,w\,t - \alpha - \theta_2 - \beta)$$

Thus the total tertiary e. m. f. is given by

$$s\,w\,I_m \{3/2\, M_1 \sin(s\,w\,t - \alpha) + 3/2\, M_1\, k/a \sin(s\,w\,t - \alpha - \beta) - M_2 k_2/a \cos(s\,w\,t - \alpha - \theta_2 - \beta)\}$$

This may be exhibited in the form

$$e_3 = E_3 \sin(s\,w\,t - \alpha - j) \qquad (38)$$

where

$$E_3 = I_m\, s\, w \{ [3/2\, M_1 (1 + k/a \cos\beta) - M_2 k_2/a \sin(\theta_2 + \beta)]^2 + [3/2\, M_1\, k/a \sin\beta + M_2 k_2/a \cos(\theta_2 + \beta)]^2 \}^{1/2} \qquad (39)$$

and

$$\tan j = \frac{1/a\, [3/2\, k\, M_1 \sin\beta + k_2 M_2 \cos(\theta_2 + \beta)]}{3/2\, M_1 (1 + k/a \cos\beta) - M_2 k_2/a \sin(\theta_2 + \beta)} \qquad (40)$$

Primary Power. We shall next consider the power impressed on the primary. Since the variable frequency component in the primary current wave (**35**) is incapable of contributing to the power, the only effective component is that of fundamental frequency, and this yields an amount of power in the first phase given by

$$1/2\, E\, I_m \cos\varphi$$

so that the total primary power in all three phases is

$$3/2\, E\, I_m \cos\varphi \tag{41}$$

Power Factor. The r. m. s. value of the primary current (35) being

$$\frac{I_m}{\sqrt{2}} \sqrt{1 + (k/a)^2}$$

the volt-amperes per phase are given by

$$1/2\, E\, I_m \sqrt{1 + (k/a)^2}$$

so that the power factor of the primary is

$$\frac{\cos\varphi}{\sqrt{1 + (k/a)^2}} \tag{42}$$

Mechanical Power. We shall now consider the total *mechanical power* transmitted to the rotor of the machine. If the losses due to hysteresis and eddy-currents are neglected, the mechanical power is easily obtained by subtracting the primary and secondary copper losses from the total primary power. A consideration of the mechanical power is, it is needless to point out, of very great interest in connection with the self-starting qualities of the machine.

Torque. The value of the torque exerted by the rotor is easily determined from the mechanical power by dividing the latter by the angular velocity.

Up to the present, we have supposed the tertiary to be on open circuit. If now we imagine it to be closed, tertiary currents will appear which will react on both primaries and secondary. The same general principle as that already used might be employed for investigating the reactions of the various circuits on each other. But if we are concerned only with the general nature of the results obtained, it is simpler to suppose the secondary and tertiary circuits replaced by a single equivalent secondary. Thus the general law of variation of the tertiary current will be the same as that of the secondary, which we have already investigated.

The method used in the above investigation is, it will be observed identical in principle with Lord Kelvin's method of electrical images as applied to problems in electrostatics. It may possibly be found to be of great use in connection with certain types of alternating-current problems.

We shall apply the above theory to the study of a particular case which corresponds approximately to a certain synchronous machine for which experimental results are available. The following numerical values (at a frequency of 12.5) have been assumed for the various constants:—

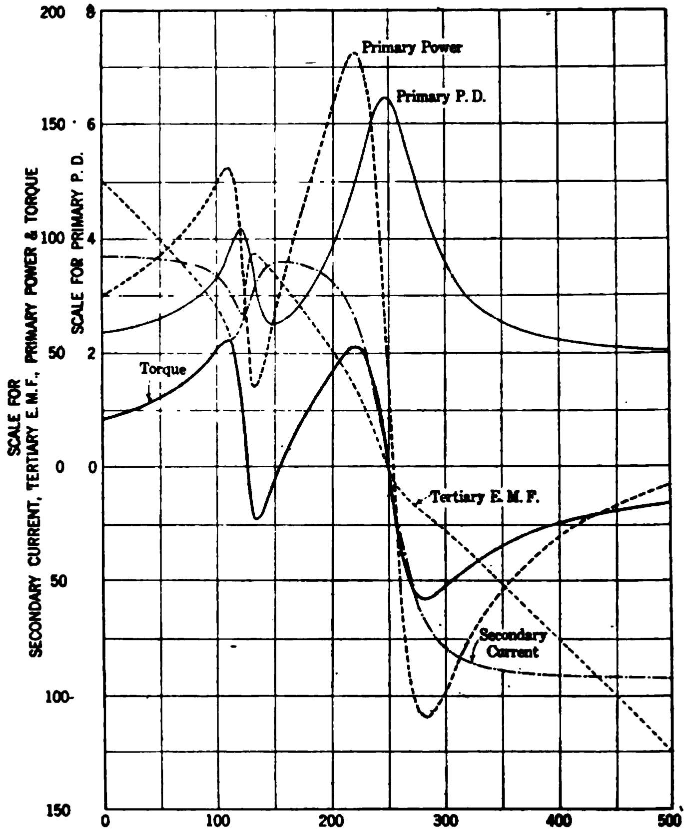


FIG. 12—CURVES CONNECTING PRIMARY P. D., PRIMARY POWER, SECONDARY CURRENT, TERTIARY E. M. F. AND TORQUE WITH SPEED WHEN THE PRIMARY CURRENT IS MAINTAINED AT A CONSTANT VALUE

$r_1 = 0.006 \quad r_2 = 0.004 \quad w\,M_0 = .043 \quad w\,M_1 = 4.2$

$w\,L_1 = 0.0513 \quad w\,L_2 = 0.046 \quad I_m = 48.3 \quad w\,M_2 = 2.3$

By the aid of the formulas established above, values of the primary p. d., primary current, secondary current, tertiary e. m. f., primary power, mechanical power, torque, power factor and phase angle were calculated for various values of s

and a constant value of 48.3 for I_m. Since all the variables involving I_m are directly proportional to it, we can immediately find their values for any other value of I_m. Accordingly, we can find the values corresponding to those values of I_m for

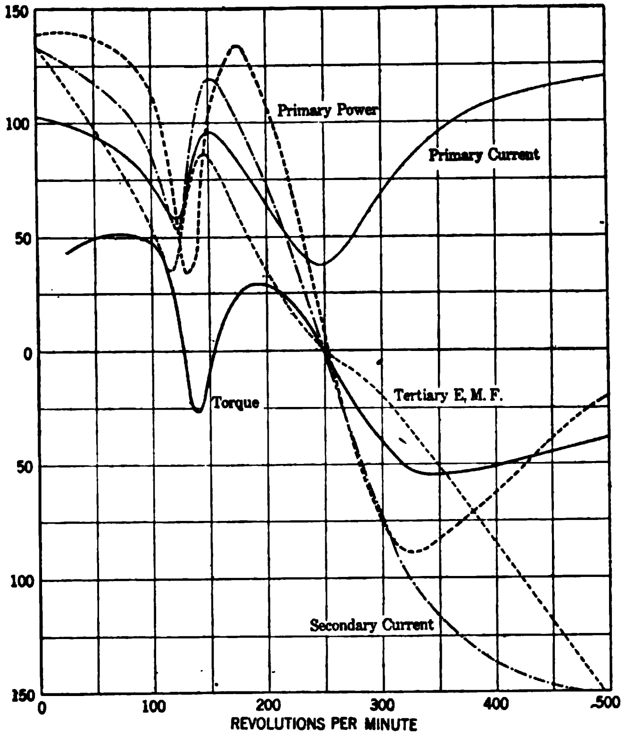


FIG. 13—CURVES CONNECTING PRIMARY CURRENT, PRIMARY POWER, SECONDARY CURRENT, TERTIARY E. M. F. AND TORQUE WITH SPEED WHEN THE PRIMARY P. D. IS MAINTAINED CONSTANT

For primary current curve multiply vertical scale readings by 2
" " power " " " " " " 10
" secondary current " " " " " " 3
" tertiary e. m. f. " " " " " " 4
" torque " " " " " " 10

various values of which yield a constant value for the primary current. This will give us the variations of the quantities with s when the *primary current is maintained constant.* The values so calculated are given in Table I, and some of them are

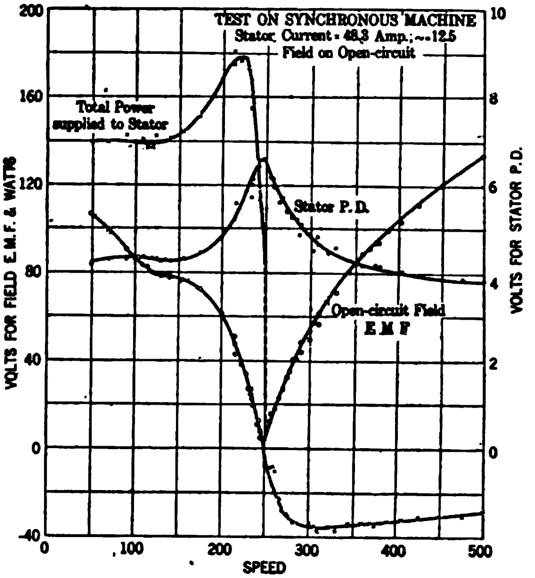


FIG. 14—STATOR P. D., STATOR POWER AND ROTOR E. M. F. CURVES CORRESPONDING TO A CONSTANT STATOR CURRENT OF 48.3 AMPERES AT A FREQUENCY OF 12.5

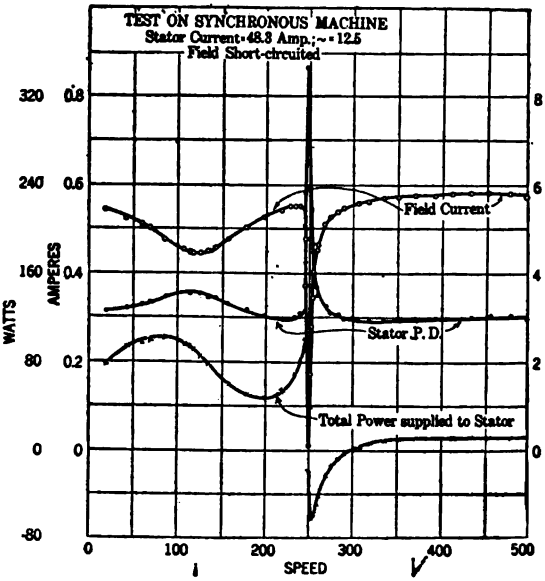


FIG. 15—STATOR P. D., STATOR POWER AND ROTOR CURRENT CURVES CORRESPONDING TO A CONSTANT STATOR CURRENT OF 48.3 AMPERES AT A FREQUENCY OF 12.5

plotted in Fig. 12. Similarly, we may obtain the values of the variables for different values of s when the values of I_m are so chosen as to yield a constant value of the primary p. d. We thus obtain the results given in Table II, which corresponds to a *constant primary line P. D.* of volts, and from which the curves of Fig. 13 have been plotted.

The theoretical curves of Figs. 12 and 13 may be compared with the experimental curves shown in Figs. 14, 15 and 16. It will be noticed that the general shapes of the curves correspond in the two cases, and that the peculiarities of shape are satisfactorily accounted for by theory. The experimental curves, however, do not exhibit such very pronounced peaks and valleys as do the theoretical ones. In this connection it must be remembered that the theoretical curves are based on a number of assumptions which are not quite realized in practise. Among others, the assumption has been made that the primary p. d. retains its pure sine wave form, and remains unaffected by the variable frequency component of the primary current. In actual experiments, the generator was a machine of smaller output than the synchronous motor, and its e. m. f. would certainly suffer distortion.

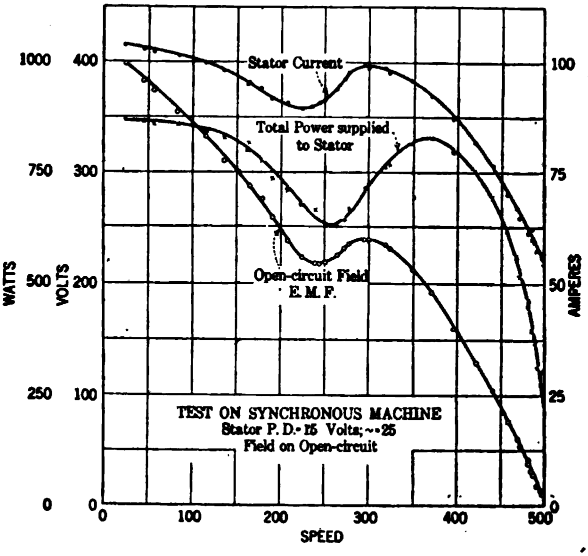


FIG. 16—STATOR CURRENT, STATOR POWER AND ROTOR E. M. F. CURVES CORRESPONDING TO A CONSTANT VALUE OF THE STATOR P. D. = 15 VOLTS AT A FREQUENCY OF 25

TABLE I.

$r_1 = 0.006$; $w L_1 = 0.0513$; $r_2 = 0.004$; $w L_2 = 0.046$; $w M_0 = 0.043$; $w M_1 = 4.2$; $w M_2 = 2.3$; frequency = 12.5.

Primary Current Maintained Constant at 48.3 amperes.

Speed 7 p. m.	Primary terminal P. D.	Secondary current.	Tertiary e. m. f.	Power supplied to primary.	Power factor.	Phase angle.*	Mechanical power	Torque, in 107 c. g. s. units.
0	2.34	92.2	125	75.9	0.388	67°26′	0	
25	2.42	91.8	113	82.3	0.405	66° 5′	6.6	25.05
50	2.60	91.0	100	92.5	0.426	64°47′	17.4	33.2
75	2.78	89.2	86.8	103.7	0.446	63°31′	29.8	37.9
100	3.29	84.3	70.6	124	0.451	63°11′	53.8	51.4
105	3.47	81.9	65.7	128	0.441	63°49′	59.4	54.0
110	3.70	78.5	60.2	131	0.422	65° 3′	64.1	55.6
115	3.96	73.7	55.8	127	0.384	67°24′	63.6	52.8
120	4.16	68.3	59.9	108.5	0.312	71°49′	47.8	38.0
122.5	4.18	66.7	67.9	95.9	0.275	74° 4′	36.1	28.2
125	4.10	66.7	7.77	77.6	0.226	76°55′	17.8	13.6
127.5	3.92	68.8	86.1	59.7	0.182	79°30′	− 1.2	− 9.3
130	3.67	72.3	91.2	45.7	0.149	81°26′	− 17.2	−12.6
135	3.13	80.0	93.4	35.4	0.135	82°14′	− 32.1	−22.7
140	2.75	85.2	91.2	41.1	0.179	79°42′	− 29.9	−20.4
145	2.56	87.9	88.1	52.0	0.243	75°57′	− 20.9	−13.8
150	2.50	89.8	85.1	63.4	0.303	72°22′	− 10.4	− 6.6
162.5	2.61	89.8	77.3	88.1	0.403	66°14′	13.8	8.1
175	2.86	88.7	69.2	110	0.457	62°98′	36.1	19.7
187.5	3.23	86.4	60.9	131	0.483	61° 5′	58.8	29.9
200	3.71	82.3	52.4	153	0.492	60°31′	83.5	39.9
212.5	4.36	74.9	43.1	173	0.474	61°42′	108.3	48.7
220	4.85	67.9	36.7	181	0.455	63°33′	120.3	52.2
225	5.21	61.3	31.9	181	0.515	65°30′	123.5	52.4
230	5.58	53.0	26.4	174	0.373	68° 6′	120.8	50.1
235	5.93	42.6	20.3	157	0.317	71°31′	108.2	44.0
240	6.23	29.9	13.5	129	0.248	75°39′	83.6	33.3
245	6.42	15.5	6.5	89.2	0.166	80°26′	46.2	18.0
250	6.46	0	0	42.1	0.078	85°32′	0	0
255	6.33	15.3	5.5	− 5.2	−0.0098	90°34′	− 48.1	−18.0
260	6.08	29.2	9.7	− 45.6	−0.0897	95° 9′	− 91.0	−33.4
265	5.74	41.2	12.9	− 75.6	−0.157	99° 3′	−124.4	−44.8
270	5.37	51.0	15.4	− 94.7	−0.211	102°10′	−147.1	−52.0
275	5.00	58.8	17.7	−105	−0.251	104°34′	−161.0	−55.9
280	4.66	65.0	19.8	−109	−0.281	106°18′	−168.2	−57.4
287.5	4.20	71.8	22.9	−108	−0.308	107°58′	−171.0	−56.8
300	3.62	79.2	28.3	− 98.3	−0.325	108°57′	−165.4	−52.6
325	2.82	86.1	39.8	− 71.3	−0.303	107°37′	−242.9	−42.0
350	2.55	89.2	51.5	− 54.7	−0.256	104°50′	−128.5	−35.0
400	2.21	91.4	76.1	− 30.9	−0.167	99°36′	−106.4	−25.4
450	2.09	92.3	100.5	− 17.1	−0.098	95°38′	− 93.2	−19.8
500	2.03	92.7	124.8	− 8.4	−0.0496	92°51′	− 84.8	−16.2

*Angle of lag of primary current behind primary p. d.

TABLE II.

$r_1 = 0.006$; $w L_1 = 0.0513$; $r_2 = 0.004$; $w L_2 = 0.046$; $w M_v = 0.043$; $w M_1 = 4.2$; $w M_2 = 2.3$; frequency $= 12.5$.

Primary P. D. Maintained Constant at 10 Volts.

Speed, 2 p. m.	Primary current	Secondary current	Tertiary e. m. f.	Power supplied to primary.	Power factor.	Phase angle.*	Mechanical power.	Torque in 107 c. g. s. units.
0	206	394	534	1385	0.388	67°26′	0	
25	199	378.5	465	1400	0.405	66° 5′	112	426
50	186	350.5	386	1370	0.426	64°47′	258	493
75	174	321	312	1340	0.446	63°31′	386	492
100	147	256	214	1146	0.451	63°11′	496	474
105	139	236	189	1060	0.441	63°49′	492	448
110	130.4	212	163	952	0.422	65° 3′	467	405
115	122	186	141	812	0.384	67°24′	405	337
120	116	164	144	625	0.312	71°49′	276	219
122.5	115.6	160	163	550	0.275	74° 4′	207	161
125	118	163	190	462	0.226	76°55′	106	81
127.5	123	176	220	389	0.182	79°30′	− 8	−
130	132	197	249	340	0.149	81°26′	− 128	− 93.9
135	154	255	298	361	0.135	82°14′	− 327	−231.5
140	176	310	332	545	0.179	79°42′	− 395	−270
145	189	343	344	793	0.243	75°57′	− 319	−210
150	193	357	340	1013	0.303	72°22′	− 166	−106
162.5	185	344	296	1290	0.403	66°14′	203	119
175	169	310	241	1335	0.457	62°48′	440	240
187.5	150	267.5	189	1250	0.483	61° 5′	563	287
200	130	222	141	1110	0.492	60°31′	608	290
212.5	111	172	99	910.5	0.474	61°42′	571	256
220	99.6	140	76	769	0.445	63°33′	512	222
225	92.8	118	61	666	0.415	65°30′	456	194
230	86.5	95	47.4	559	0.373	68° 6′	388	161
235	81.4	71.7	34	447	0.317	71°31′	307	125
240	77.5	48.1	21.7	332	0.248	75°39′	215	85.6
245	75.2	24.1	10.2	217	0.166	80°26′	112	43.8
250	74.8	0	0	101	0.078	85°32′	0	0
255	76.3	24.1	8.7	− 13	−0.0098	90°34′	− 120	− 45
260	79.4	48	16	−123	−0.0897	95° 9′	− 246	− 90.4
265	84.1	71.7	22.5	−229	−0.157	99° 3′	− 377	−136
270	89.9	96.9	28.7	−328	−0.211	102°10′	− 509	−180
275	96.5	118	35.3	−420	−0.251	104°34′	− 644	−223
280	104	135.5	42.4	−504	−0.281	106°18′	− 775	−264
287.5	115	171	54.5	−614	−0.308	107°58′	− 969	−322
300	133.5	219	78.2	−750	−0.325	108°57′	−1260	−402
325	171	306	141	−898	−0.303	107°37′	−1800	−529
350	189	349	202	−838	−0.256	104°50′	−1970	−537
400	218	413	344	−630	−0.167	99°36′	−2170	−518
450	231	442	481	−393	−0.098	95°38′	−2140	−454
500	238	456	614	−204	−0.050	92°51′	−2055	−392

*Angle of lag of primary current behind primary p. d.

DISCUSSION ON "STARTING CONDITIONS OF SYNCHRONOUS MACHINES (HAY AND MOWDAWALLA), CALCUTTA, INDIA, JANUARY 2, 1919.

Quentin Graham: In considering the various torques which act upon the rotor during the starting and synchronizing periods the authors have confined themselves to those torques which are active when the field winding is either open-circuited or short-circuited. They have not considered the conditions which exist when the field is excited before synchronism is reached. In this case there is another torque to be taken into account which is the predominant one during the period of pulling into synchronism.

When the motor is running in synchronism its torque, due to the interaction between the ampere turns of the field winding and those of the armature winding, depends upon the position of the poles relative to the revolving m. m. f. of the armature. The torque rises from zero to a maximum as the relative shift of the rotor passes through one half a pole pitch. This is the limit of stable operation as a synchronous motor. With a further relative shift of the rotor the torque falls off to zero and then goes through a negative half cycle. The complete cycle extends over two pole pitches. Now if we have the machine running as an induction motor with a small slip and then excite the field winding the rotor will be subjected to this cycle of torque every time it slips one pair of poles. The result is that the rotor is made to oscillate about its mean position at slip frequency. It will be noticed that the frequency of oscillation of an open-circuited or short-circuited rotor, due to the non-uniformity of the magnetic circuit, is twice slip frequency as has been brought out in the paper.

The diagram and the analysis given by Messrs. Hay and Mowdawalla for the case of a rotor oscillating at twice slip frequency is applicable to the case of the excited rotor which oscillates at slip frequency. It has been shown by Rosenberg, in the article to which reference is made in the present paper, that the torque due to the excited field is usually large compared to the torque component which is dependent upon the oscillating speed for its existence. Or, referring to Fig. 3, of the present paper, the vector *O S* should be large in comparison with the vector *O F* if the diagram is to be used to represent conditions when the field is excited. This would justify the neglecting of the torque represented by the vector *O F* in calculating the pulling-in ability of a machine as has been done by Rosenberg.

The pulling into synchronism with a fully excited field is of particular interest since it is this condition which nearly always exists in practical work. The motor is brought up to within a few percent of synchronous speed as an induction motor and if it is carrying an appreciable load it will not be able to pull into step of its own accord. The field must then be excited in order to cause the rotor to come up to synchronous speed. From the analysis which the authors have given it can be seen that if the motor does not pull in to step during the first half cycle of positive or accelerating torque it will not pull in at all. That is, during the following half cycle of torque it will be retarded an equal amount and will therefore continue to oscillate in speed but will not reach synchronous speed.

It may be deduced also that the certainty of pulling in to step depends upon the position of the rotor poles relative to the stator m. m. f. wave at the instant that the field is excited. If the rotor is in such a position that it must pass through the whole of the accelerating half cycle of torque before entering the zone of retarding torque the likelihood of its reaching synchronous speed is greater than if the field were excited a little later so that only a part of the accelerating wave is passed through before retardation commences. It is somewhat analogous to the rise in flux in a transformer that is switched on to the line, the maximum flux reached being dependent on the point of the voltage wave at which the switch is closed. Experiments have proved that the pulling-in ability of a motor does vary considerably with the point on the slip cycle at which the field is closed.

The conclusion which the authors reach that the field should be kept open-circuited during the early part of the starting period is open to question. While it is true that the starting torque is higher with open-circuited than with short-circuited field, as shown by theory and confirmed by experiment, the difference between the two is not very great in motors having well designed starting windings. Furthermore the complicated construction which would result from applying field break-up switches to revolving field synchronous motors would more than offset the gain in starting torque. Closing the field circuit during starting has been found to be the most practicable method.

REPORT OF THE BOARD OF DIRECTORS

FOR THE FISCAL YEAR ENDING APRIL 30, 1919

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Thirty-fifth Annual Report, for the fiscal year ending April 30, 1919. A general balance sheet showing the condition of the Institute's finances on April 30, 1919, together with other detailed financial statements, is included herein.

Directors' Meetings.—The Board of Directors held ten regular meetings during the year; seven of these were held in New York, one in Atlantic City in June, one in Philadelphia in December, and one in Boston in March.

The practise has been continued of printing from month to month in Section I of the Institute PROCEEDINGS information regarding the more important activities of the Institute which have been under consideration of the Board of Directors, the committees, and the various officers.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 17, 1918. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1918, was presented as published in full in the June 1918 issue of the PROCEEDINGS. The Tellers Committee presented its report upon the election of officers for the year beginning August 1, 1918.

Directly following the business meeting came the ceremony of the presentation of the Edison Medal to Colonel John J. Carty.

Annual Convention.—The Thirty-fourth Annual Convention was held at Atlantic City on June 26 to 28, 1918. This was strictly a business meeting at which 15 technical papers were presented at five sessions. Several meetings of the Section Delegates were held. Major General Black delivered an address on "Engineers and the War."

October Meeting.—The 342nd Institute meeting, scheduled to be held at Philadelphia, October 11 and 12, 1918, was cancelled on account of the influenza epidemic. Presentation of the technical papers was postponed until the meeting of December 13, 1918.

New York Meeting.—The 343rd Institute meeting was held in New York on November 8, 1918 under auspices of the Industrial and Domestic Power Committee.

Toronto Meeting.—The 344th Institute meeting was held in Toronto, Ontario on November 22 and 23, 1918. Three technical papers were presented on power installations and developments in Ontario and Quebec.

Philadelphia Meeting.—The 345th Institute meeting was held in Philadelphia on December 13, 1918. Two technical papers were presented and two addresses on Research.

New York Meeting.—The 346th Institute meeting was held in New York on January 10, 1919. The evening was devoted to an address by Major General Squier on aeronautics.

Mid-Winter Convention.—The Seventh Mid-Winter Convention was held in New York on February 19 to 21, 1919. Attendance exceeded 1300. Four technical sessions were held and fifteen papers presented. The first session was held jointly with the A. I. M. E. A complete report of this convention was published in the March 1919 PROCEEDINGS.

Boston Meeting.—The 348th Institute meeting was held at Boston on March 14, 1919. Morning, afternoon and evening sessions were held at which three technical papers and several addresses were presented on widely divergent topics.

New York Meeting.—The 349th Institute meeting was a joint meeting with the Illuminating Engineering Society held at New York on April 11, 1919. One paper was presented on the subject of Lighting Codes.

Meetings and Papers Committee.—The Meetings and Papers Committee has held meetings every month at the time and place of the regular meetings of the Institute held during the past year as referred to above. At these Committee meetings the programs of the Institute meetings were planned and the various papers offered for presentation and the policy of the Committee were discussed. The policy of the Committee has been to continue the practise of holding some meetings outside of New York; one was held in Toronto in November, one in Philadelphia in December, and one in Boston in March.

At the beginning of the fiscal year the Committee found that owing to the war activities of the membership it was very difficult to get good papers, but with the cessation of war activities a considerable number of desirable papers were offered. After completing plans for all the meetings through and including November 1919, a considerable number of papers is still available. In fact there is material available for some Section meetings, which the Committee would like to encourage in order that certain papers well worthy of presentation may be presented at an earlier date than would be the case if they had to wait for the regular Institute meetings.

A Pacific Coast Convention, to be held in Los Angeles in September 1919, has been authorized.

Technical Activities.—The Board of Directors' Committee on Technical Activities was appointed for the purpose of co-relating the work of the various Technical Committees and to bring about the adoption of programs of work for the Committees to undertake, which might prove of advantage to the Institute. In the past the Technical Committees have been mainly concerned with securing papers for meetings and with few exceptions there have been no other activities. It has been realized for a long time that there were other fields that the Technical Committees could work in to the advantage of the Institute. Some committees had already undertaken programs mainly with the object of preparing statements of existing successful practise for the guidance of people interested in various industries, but in the past it has not always been possible to carry out such programs, as there was no system for insuring the continuity of such action. The Board's Committee has communicated with the Chairmen of Technical Committees during the year and there has been an increase in the amount of work undertaken by these committees. The Industrial Power Committee has had for sometime a large program which is being worked out, and the other committees have started work along somewhat similar lines, such as the Iron and Steel Committee, which is preparing to summarize the successful practise of the Iron and Steel Industry with regard to electrical installations; the Marine Committee which has been engaged in preparing proposed installation rules for which there is at present a great need, and others of the Committees are engaged on similar useful programs which will be presented to the Institute either in the form of papers or reports.

The Annual Reports of the sixteen technical committees of the Institute will be presented at the Annual Convention of the Institute in June 1919, and will probably be printed in the July or August issues of the PROCEEDINGS.

Editing Committee.—The Editing Committee has jurisdiction over the publication of the annual TRANSACTIONS and has been engaged with Volume XXXVII covering the year 1918. This volume will be published in two parts, Part I is completed and Part II is now in the bindery. The volume contains approximately 2000 pages and 87 plates of half-tone illustrations, comprising papers and discussions presented at Institute meetings during the year.

Sections Committee.—The year ending May 1st witnessed the crest of the wave of American activity in the great war. It is then not surprising that it should include the slight resulting depression in attendance at the meetings of the various sections of the Institute indicated in the table below.

Whether drawn from the lessons of the hour regarding the value of co-operation or led by some other inspiration, the tendency in section affairs during the year has been that of continued development in the direction of affiliation with the sections of other technical societies. This has resulted not only in the highly desirable objective of greater co-operation and unity among engineers, but has also placed the engineer in higher standing with his community by his greater participation in civic and municipal affairs in those centers where such affiliation has been undertaken.

The Sections Committee, encouraged by the well established policy of the Institute which has long aimed to develop this co-operation, appointed at their conference at Atlantic City last June, a committee to study existing methods and formulate suggestions for the guidance of all sections so located as to make such affiliation profitable.

This committee, now working in conjunction with the general committee on the development of Institute activities, should be able to tender a helpful report at the meeting of the sections delegates next month. The sections at Atlanta, Chicago, Milwaukee, St. Louis, Spokane, Indianapolis, Philadelphia, Portland, Ore., and San Francisco, now well established in this plan of affiliation should find numerous additions to their ranks as time progresses.

The activities of the Student Branches have continued to be materially modified by the war conditions, notably by the organization of the Students Army Training Corps. A considerable number of the Branches have been inactive during the greater portion of the year, but since January 1, many have reorganized and are now active; all will probably resume normal activity by next Fall.

Two new Branches were authorized during the year at the University of Pennsylvania and Drexel Institute.

	For Fiscal Year Ending						
	May 1 1913	May 1 1914	May 1 1915	May 1 1916	May 1 1917	May 1 1918	May 1 1919
SECTIONS							
Number of Sections.........	29	30	31	32	32	34	34
Number of Section meetings held....................	244	233	246	251	265	245	217
Total Attendance..........	22,825	22,626	23,507	28,553	31,299	34,614	25,837
BRANCHES							
Number of Branches.......	47	47	52	54	59	59	61
Number of Branch meetings held....................	357	306	328	360	368	268	156
Attendance................	11,808	11,617	12,712	15,166	16,107	10,683	6,441

Standards Committee.—The Standards Committee has held during the year six regular monthly meetings, one special meeting, and a joint meeting with the U. S. National *Reorganization of Work.*

In order to expedite the work of the Committee, the entire system of subcommittees has been reorganized so that all detail consideration of changes in standards and other technical affairs is now referred in each case to a specific sub-committee. Action by the Standards Committee is taken only after receiving recommendations from a subcommittee. A large amount of detail discussion has thus been eliminated. There are at present twenty-four of these sub-committees.

Revision of the General Form of the Standardization Rules.—A large share of the time of the Standards Committee during the past year has been devoted to a consideration of changes in the form and arrangement of the Standardization Rules. This matter has been in the hands of the Subcommittee on the ***Form but not the Substance*** for some time past and a general system of revision by which the Standardization Rules are divided into chapters was reported and has been adopted.

Changes in Substance of the Present Rules.—It is hoped to complete the full text of this revision of the form of the Standardization Rules this year. A few minor changes of substance will be included.

A revised chapter covering Lighting and Illumination and based on the latest rules of the Illuminating Engineering Society has been recommended by the Subcommittee on Lighting and Illumination and substituted for the present chapter.

Cooperation with the General Committee on Standardization of Electrical Equipment and Supplies of the War Department.—At the request of Major N. J. Neall, Chairman of the above Committee, that the Standards Committee should cooperate with the General Committee of the War Department in the preparation of specifications, the Standards Committee voted to render all possible assistance through its subcommittees. Dr. P. G. Agnew was appointed a special subcommittee to further this work which has been carried on throughout the year. Very effective assistance has been rendered the War Department Committee.

Joint Meeting with U. S. National Committee of the I. E. C.—At a joint meeting of the U. S. National Committee of the I. E. C. and the Standards Committee it was decided to recommend the early reappointment of the subcommittee of the I. E. C. on Rating, to meet preferably in Paris. Professor C. A. Adams, Mr. H. M. Hobart and C. O. Mailloux will represent the Institute. These representatives while abroad will also take up with the Rating Panel of the British Engineering Standards Association the actions taken and suggestions made by the Standards Committee.

American Engineering Standards Committee.—The consideration of suggested changes in the Constitution and Rules of Procedure made it necessary to continue the organization committee until May 1918, when a revised draft of the Constitution and Rules of Procedure was agreed upon and sent to the five appointing societies (A. S. C. E., A. I. M. M. E., A. S. M. E., A. I. E. E., and A. S. T. M.) for their approval. This approval was received from all of the societies, and representatives appointed by each met on October 19th, 1918, and organized the American Engineering Standards Committee.

Soon after organization it became evident that the Committee must be enlarged in order to secure the cooperation of several important organizations engaged in standardization work. This fact was emphasized by the action of a conference held in Washington, January 15, 1919, to consider the method of developing and approving Industrial Safety Codes. At this conference it developed that the principal organizations would not cooperate with this Committee unless they had direct representation on it. Such representation is not possible under the present Constitution, so after careful consideration the Committee has decided that an increase in its membership is desirable. It will therefore be necessary to make further changes in the Constitution. Such a revision has been drafted and approved by the Committee and will be submitted to the five Founder

Societies for their approval during the month of May 1919. It is expected that this revision will enable the Committee to begin active work in the near future.

Public Policy Committee.—The Public Policy Committee, during the administrative year about to close, has held itself in readiness to take up any matters which might be referred to it by the Institute authorities, but has not initiated any recommendations. The reason for the inactivity of the Committee is that for the last two years the several matters which would have naturally been acted upon by it have been considered by Engineering Council.

Committee on Safety Codes.—The Committee on Safety Codes has continued to represent the Institute on the Electrical Committee of the National Fire Protection Association, and representatives attended the meeting of the N. F. P. A. Committee in New York in March, this being the intermediate meeting, at which was discussed the various features of the Electrical Code, in preparation for the regular biennial meeting in 1920, at which time changes in the Code are officially made.

The Committee was represented at the conference in Washington with the Bureau of Standards, for the purpose of discussing matters relating to the American Engineering Standards Committee, and the desirability of the writing of an Industrial Safety Code.

The Committee has been active during the year on routine matters of a code nature, with the N. E. L. A.

The first part of the fiscal year the Committee was more or less inactive, due to war conditions, but since the signing of the armistice interassociation matters have revived, and almost all the activities of the Committee occurred in the last quarter of the year.

Board of Examiners.—The Board of Examiners during the year held sixteen meetings, averaging about two and one-half hours each. It considered and referred to the Board of Directors a total of 2111 applications for admission or transfer to the higher grades.

Some applicants for admission were objected to on ethical grounds. Although the number of such cases was small, considerable time was required for the necessary investigations; adverse action was taken in a few such cases. The Board continued the practise of requiring questionable applicants to furnish proof of loyalty to the ideals of the United States.

The result of the Board's work for the year is given in the following tabulated statement:

Applications for Admission

Recommended for grade of Associate	1213	
Not recommended	9	1222
Recommended for grade of Member	87	
Not recommended for admission to this grade	23	110
Recommended for grade of Fellow	4	
Not recommended for admission to this grade	1	5
Recommended for enrolment as Students		659

APPLICATIONS FOR TRANSFER

Recommended for grade of Member....................	81	
Not recommended for transfer to this grade............	19	100
Recommended for grade of Fellow.....................	12	
Not recommended for transfer to this grade............	3	15
Total number of applications considered...............		2111
Applications reconsidered............................		6
Total..		2117

Membership.—Sixteen hundred and ninety-five applications for membership in the Institute have been received this year, which is by far the largest number ever received in any one year. The nearest approach to this was for the year ending May 1, 1918, when 1235 applications were received.

A printed list of reasons why a man should become a member of the Institute was printed for distribution to desirable candidates. Several thousand of these were called for by the Sections.

The Membership Committee feels that new members should be obtained at an increasing rate to keep pace with the industry.

The 10,000 mark has been passed, the membership of the Institute as of May 1, 1919, being 10,352. This shows a net increase of 1070 members since May 1, 1918, approximately twice the net increase of the previous year, which was 572.

During the five years ending May 1, 1917, the net increase was only 1251; the total applications received in these five years, 2440. These figures emphasize the greatly increased activity of the past two years.

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1918.	**6**	**464**	**1332**	**7480**	**9282**
Additions:					
Transferred..............	....	**24**	**88**	....	....
New Members Qualified...		**6**	**84**	**1221**	....
Reinstated..............	....	**4**	**7**	**55**	....
Deductions:					
Died...................	....	**4**	**7**	**66**	....
Resigned...............	....	**1**	**4**	**77**	....
Transferred.............	....	....	**18**	**94**	....
Dropped...............	....	**4**	**15**	**229**	....
Membership, April 30, 1919..	**6**	**489**	**1467**	**8290**	**10252**

Net increase in membership during the year.................................... 970

Deaths.—The following deaths have occurred during the year:

Fellows—A. O. Benecke, M. H. Collbohm, W. L. Hooper, C. R. McKay.

Members—George E. Claflin, *F. J. Duffy, Charles H. Hile, W. J. Jenks, Etna Kuhlman, Frank I. Porter, O. O. Rider.

Associates—E. G. Anderson, H. F. Anderson, Arthur S. Andrews, Samuel Avis, G. R. Baker, *Remson Bishop, John Bottomley, Ralph E. Brown, W. B. Burbeck, *E. W. Caldwell, J. J. Campbell, Eric Carlson, *Lucian Carr, *W. W. Crawford, E. W. Currier, Harry L. Curtis, *R. F. Day, Edward D. Donald, *John J. Donnohue, Hugh H. Emery, E. B. Fahnestock, I. Fujioka, John D. Gaboury, W. K. Greenwood, John Hanover, Vernon E. Hess, W. Lesniewski, Edward Lineberry, Claudius B. Little, *George Mac Indoe, J. A. MacQueen, Glen W. Merrill, *Charles J. Moore, A. F. Moray, *N. I. Moultrop, H. P. Myers, Archie Oakes, B. A. Ordonez, C. H. Parmly, D. H. Plank, James M. Poyner, H. E. Randall, Jr., O. B. Reynolds, E. M. Rhett, Arnold Ruegg, Owen Samsel, Charles P. Seeger, S. V. Setti, *D. A. Shanks, C. F. Shipman, Charles F. Sise, *J. C. M. Small, Herbert E. Smith, Alan Smout, C. W. Steiner, John F. Stevens, S. Sugiyana, T. R. Taltavall, Ralph E. Thomas, *A. R. Thompson, Stuart Thomson, L. J. Vogel, H. Webber, F. S. Wheeler, Richard M. Wilson, Minor I. Woodward, N. R. Work.

Total deaths, 77. *Died in U. S. Service.

Employment Service.—An employment service has been maintained for many years at Institute headquarters for the purpose of assisting members in obtaining positions. The services of the Institute consist principally in acting as a medium for bringing together the employer and the employee. No charge is made.

The engineering service bulletin is published each month in the Institute PROCEEDINGS and it has served to place many men in positions of responsibility, both in this country and abroad. The bulletin is subdivided into two parts: one containing announcements of vacancies, and the other containing lists of men available, with condensed records of their experience. The value of this service to the membership will continue to increase from year to year as men of engineering and executive responsibility realize the opportunity afforded by it to reach technical men whom it would be difficult to find through other channels.

Demobilization greatly increased the number of men available, while the number of positions open has greatly decreased. The service has been extended without charge to non-members who are or have been in the service of the Army or Navy. An effort has been made to keep those men on the available file in closer touch with positions open than heretofore possible when dependence was alone placed on the Service Bulletin of the PROCEEDINGS, by issuing to them when deemed advisable, a list of the opportunities on file.

The Institute's employment service is coordinated with that of other organizations, through the medium of the Engineering Societies Employment Bureau, organized under the auspices of the Engineering Council and conducted under the direction of the secretaries of the national societies of Civil, Mining, Mechanical, and Electrical Engineers.

Development of the Institute.—In October 1918 the Board of Directors authorized the appointment of a Committee on Development to consider suggested or proposed modifications in the activities and methods of work of the Institute, including the relations of the Institute to other organizations. This action was taken with a view to making a survey of, and adjusting, the work of the Institute, having in mind the rapidly changing social, economic, and industrial conditions.

Each Section of the Institute was requested to designate a member of this Committee; and these representatives of Sections, together with seven additional members appointed by the President, constitute the Committee, which has been actively at work for several months. The membership was urged to submit suggestions and most of the Sections have held at least one meeting for the purpose of discussing the subject. On account of the wide geographical distribution of the membership, no meetings of the entire Committee have been held, but the representatives of the Sections have made reports to an Executive Committee of seven members of the Development Committee. This Executive Committee has held two meetings, at which numerous suggestions from the membership throughout the country have been considered. The principal features of these suggestions representing the consensus of the views expressed, are being compiled by the Executive Committee for further consideration by the members of the general Committee; and the whole subject will be thoroughly considered and discussed in conferences to be held under the auspices of the Development Committee at the Annual Convention of the Institute in June, after which it is expected that a report, embodying definite recommendations, will be presented to the Board of Directors.

War Activities.—The Institute continued during the past year in active cooperation with the various government departments and agencies engaged in war work as listed in the Report of the Board of Directors for 1918.

The work in furtherance of Allied victory in which the Institute aided during the period from May 1918 to the signing of the armistice included cooperation in obtaining personnel for many branches of service, as follows: candidates for the 4th series of Officers Training Schools, for Submarine Service, for the Coast Artillery Corps and the Forest Service; specially trained men for work in the Bureau of Mines and to act as instructors in the various army training schools; for the U. S. Navy Steam Engineering School at Stevens Institute and for commissions in the Ordnance Dept.

The compilation and correction of the Institute's Honor Roll has been continued until from all information at hand it shows a total of 1417 members have served in the uniformed forces of the United States and the Allies of which number 21 died in service. Practically all of the 1417 members voluntarily entered the service and it is particularly gratifying to note the large proportion of officers, as given in the summary. A large proportion of these officers received their commissions, or were inducted into the service as a result of calls made upon the Institute by various departments of the Army and Navy.

SUMMARY

U. S. ARMY		U. S. NAVY	
Major Generals	7	Rear Admirals	1
Brigadier Generals	5	Captains	1
Colonels	9	Commanders	6
Lieut. Colonels	23	Lieut. Commanders	9
Majors	110	Lieutenants	45
Captains	219	Lieutenants (j. g.)	72
First Lieutenants	199	Ensigns	75
Second Lieutenants	162	Miscellaneous	75
Sergeants	34		
Corporals	17	Total	284
Miscellaneous	267	U. S. MARINE CORPS	
		Lieutenants	3
Total	1052	Private	1
		Total	4

ALLIED FORCES

British Army	63	Italian Army	1
British Navy	6	Japanese Navy	1
French Army	6		
		Total	77

Total on Honor Roll..............1417

U. S. National Committee, International Electrotechnical Commission.—The U. S. National Committee of the I. E. C. held a joint meeting with the Standards Committee in Philadelphia, December 12, 1918, at which resolutions were adopted favoring the resumption of I. E. C. activities, and recommending the early reappointment of a subcommittee of the I. E. C. on Rating, to meet preferably in Paris, for the purpose of considering certain communications from the French Committee, particularly in regard to I. E. C. rules on Temperature Elevations of Machinery.

At this meeting, the printing of the I. E. C. Rating Rules as an appendix to the 1918 edition of the A. I. E. E. Standardization Rules, was suggested and authorized.

The following documents have been received from the General Secretary and distributed among the members of the U. S. National Committee:

(1) The Annual Report of the Honorary Secretary (Colonel Crompton) for the year 1918.

(2) A communication from the French Committee regarding a term for the unit of "Reactive Power."

(3) A list of Statutes of the I. E. C.

(4) A communication from the Dutch Committee in regard to an international standard of comparison of aluminum.

The U. S. National Committee will be represented by Messrs. C. A. Adams, H. M. Hobart, and C. O. Mailloux at a meeting of the I. E. C. Special Committee on Rating, at Paris, May 5-7, 1919.

It is believed that the conventions of I. E. C., interrupted for more than four years by the war, will now be resumed.

American Engineers Go To France.—At the invitation of the French Society of Civil Engineers and a committee of the French Engineers' Congress, and with the approval of the French government, a delegation of American engineers was appointed to study with French engineers problems involved in the industrial rehabilitation of France; such as, utilization of commercial ports, the development of navigable waterways, the development of water power, improvement of road systems, and many other questions of development.

Immediately on receipt of the invitation a delegation was organized representing the American national societies. The Executive Committee of the Institute selected Lewis B. Stillwell as the A. I. E. E. representative. The complete delegation, which sailed from New York on December 5, 1918, was, as follows: Messrs. James F. Case, Chairman, Geo. W. Fuller, A. M. Hunt, Nelson P. Lewis, Charles T. Main, E. Gybbon Spilsbury, Lewis B. Stillwell, George F. Swain, and George W. Tillson.

On the evening of February 10, 1919 a joint meeting of the four Founder Societies was held in the Engineering Societies Building, New York, at which the delegates presented informal reports, abstracts of which were published in the PROCEEDINGS for March, 1919.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical, and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the national societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the April 1919 PROCEEDINGS.

Library.—The library of the Institute is combined with the libraries of the national societies of civil, mining, and mechanical engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc.

An abstract of the annual report of the Engineering Societies Library covering the calendar year 1918, was published in the April 1919 PROCEEDINGS.

Engineering Council.—The Engineering Council represents the result of an organized effort inaugurated in the latter part of 1916 by the national societies of Civil, Mining, Mechanical, and Electrical Engineers, to establish an instrument through which united action could be brought about upon matters of common interest to engineers and which would serve as a connecting medium between the engineer and the public welfare in matters of interest to the engineering profession. The first meeting of the Council was held on June 27, 1917.

For details regarding the field, aims, and activities of the Council, members are referred to the numerous statements published from time to time in the monthly Institute PROCEEDINGS. A resume of the annual report covering the activities of the Council for the year ending February 13, 1919, was published in the April 1919 PROCEEDINGS.

Engineering Foundation.—The Engineering Foundation is a fund established by the United Engineering Society on January 27, 1915, through the generosity of Mr. Ambrose Swasey, of Cleveland.

The purpose of the Engineering Foundation is the advancement of the engineering arts and sciences in all their branches to the greatest good of the engineering profession and the good of mankind, which it is proposed to accomplish largely through the promotion of engineering research.

The Engineering Foundation is administered by a board upon which the Institute and other national engineering societies are represented. The annual report of the Engineering Foundation was also published in the April 1919 PROCEEDINGS.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and in addition has appointed representatives upon a number of new organizations, some of the more recent being the Engineering Division of the National Research Council, and the American Bureau of Welding.

Edison Medal.—The Edison Medal for 1917, which had been awarded to Col. John J. Carty "for his work in the science and art of telephone engineering" was presented to Col. Carty with appropriate ceremonies at the Annual Meeting of the Institute held in New York on May 17, 1918.

The Edison Medal for 1918, has been awarded to Benjamin G. Lamme "for invention and development of electrical machinery," and the presentation will take place at the Annual Meeting of the Institute which will be held in New York on May 16, 1919.

John Fritz Medal.—The John Fritz Medal Board of Award which is composed of representatives of the national societies of Civil, Mining, Mechanical and Electrical Engineers awarded the 1919 medal to General George W. Goethals, the builder of the Panama Canal. The presentation will take place at the Engineering Societies Building in New York on May 22, 1919.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report is included herein.

NEW YORK
CHICAGO
DETROIT
SAINT LOUIS
BOSTON
CLEVELAND
BALTIMORE
PITTSBURGH

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
CABLE ADDRESS "HASKSELLS"
30 BROAD STREET
NEW YORK

SAN FRANCISCO
LOS ANGELES
NEW ORLEANS
SEATTLE
DENVER
ATLANTA
WATERTOWN
LONDON

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE OF AUDIT

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1919, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly sets forth the financial condition of the Institute on April 30, 1919, that the Statement of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS,
Certified Public Accountants.

NEW YORK,
May 13, 1919.

AMERICAN INSTITUTE OF

EXHIBIT A.

GENERAL BALANCE SHEET

ASSETS

REAL ESTATE:			
One-fourth Interest in United Engineering Society's Real Estate, 25 to 33 West 39th Street:			
Land and Building		$472,500.00	
Real Estate Equipment		14,292.79	
Total Real Estate			$486,792.79
EQUIPMENT:			
Library—Volumes and Fixtures		$ 39,847.55	
Works of Art, Paintings, etc.		3,001.35	
Office Furniture and Fixtures		12,428.68	
Total		$ 55,277.58	
Less Reserve for Depreciation		10,840.47	
Remainder—Equipment			44,437.11
INVESTMENTS:			
Bonds—City of Wilmington, Delaware, 4½%, 1934, Par Value $15,000.00		$ 15,782.05	
United States Third Liberty Loan, 4¼% Bonds		10,000.00	
Total Investments			25,782.05
WORKING ASSETS:			
Publications Entitled "Transactions," etc.		$ 12,599.00	
Paper and Cover Paper		2,162.05	
Paper for Volume 37 and Advertising		336.69	
Badges		1,166.74	
Total Working Assets			16,264.48
CURRENT ASSETS:			
Cash		$ 14,916.68	
Accounts Receivable:			
Members for Past Dues		9,728.15	
Advertisers		678.80	
Miscellaneous Sales		388 09	
Miscellaneous Printing and Subscriptions		1,034.00	
Accrued Interest on Investments		109.38	
Accrued Interest on Bank Balances		284.59	
Total Current Assets			27,139.69
FUNDS:			
Life Membership Fund			
Cash	$ 438.67		
Chicago, Burlington & Quincy Railroad Company Bonds, 4%, 1958, Par Value $5,000.00	4,868.75		
Accrued Interest	33.33	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 137.61		
New York City Bonds, 4½%, 1957, Par Value $2,000.00	2,242.33		
New York Telephone Company Bond, 4½%, 1939, Par Value $1,000.00	878.75		
Accrued Interest	67.50	3,326.19	
MAILLOUX FUND:			
Cash	$ 142.35		
New York Telephone Company Bond, 4½%, 1939	1,000.00		
Accrued Interest	22.50	1,164.85	
Midwinter Convention Fund—Cash		135.89	
Total Funds			9,967.68
DUES PAID IN ADVANCE—INTERNATIONAL ELECTROTECHNICAL COMMISSION, LONDON, ENGLAND			250.00
Total			$610,633.80

ELECTRICAL ENGINEERS.

APRIL 30, 1919.

LIABILITIES

CURRENT LIABILITIES:		
Accounts Payable—Subject to Approval by the Finance Committee	$ 6,362.87	
Due United Engineering Society, Account Building Addition, Including Accrued Interest	7,625.75	
Dues Received in Advance	2,805.45	
Entrance Fees and Dues Advanced by Applicants for Membership	455.50	
Total Current Liabilities		$ 17,249.57
FUND RESERVES:		
Life Membership Fund	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund	3,326.19	
Mailloux Fund	1,164.85	
Midwinter Convention Fund	135.89	
Total Fund Reserves		9,967.68
SURPLUS: Per Exhibit "B"		583,416.55
Total		$610,633

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

STATEMENT OF INCOME AND PROFIT AND LOSS.

FOR THE YEAR ENDED APRIL 30, 1919.

EXHIBIT B.

REVENUE:			
Entrance Fees		$ 7,225.00	
Dues		102,908.43	
Students Dues		4,044.00	
Transfer Fees		1,060.00	
Advertising		10,641.33	
Subscriptions		3,144.92	
Sales of "Transactions," etc.		2,954.80	
Badges Sold	$3,285.75		
Less Cost	2,387.83	897.92	
Interest on Investments		1,089.61	
Interest on Bank Balances		936.40	
Exchange		41.91	
Total			$134,944.32
EXPENSES:			
Meetings and Papers Committee:			
Salaries		$ 5,940.00	
Binding and Mailing "Proceedings"		5,887.20	
Printing "Proceedings"		8,947.43	
Engraving "Proceedings"		2,549.13	
Paper and Cover Paper		8,342.44	
Envelopes		882.18	
Stationery and Miscellaneous Printing		378.53	
General Expense		348.03	
Meetings		6,115.33	
Total			$ 39,390.27
EDITING COMMITTEE:			
Volume No. 34		$ 56.19	
Volume No. 35		48.63	
Volume No. 36		4,721.88	
Volume No. 37		7,062.51	
Total		$ 11,889.21	
Add Decrease in Inventory:			
April 30, 1918	$14,049.00		
April 30, 1919	12,599.00	1,450.00	13,339.21
EXECUTIVE DEPARTMENT:			
Salaries		$ 19,805.00	
General Expense		2,097.24	
Express		371.17	
Postage		2,479.30	
Advertising		3,371.90	
Stationery and Miscellaneous Printing		4,019.83	
Year Book and Catalogue		4,597.61	
Total			36,742.05
FORWARD			$ 89,471.53
TOTAL REVENUE—(Forward)			$134,944.32

Total Revenue—(Forward)		$134,944.32
Expenses—(Forward)		$ 89,471.53
Sections Committee:		
Sections' Meeting	$ 6,230.07	
Branch Meetings	81.11	
Delegates Convention Expense	3,286.44	
Salaries, New York Office	2,340.00	
Stationery and Printing, New York Office	768.46	
Total		12,706.08
United Engineering Society:		
Assessment	$ 4,800.00	
Library Committee	4,000.00	
Engineering Council	3,733.36	
Total		12,533.36
General:		
Membership Committee	$ 1,647.34	
Finance Committee	150.00	
Standards Committee	1,005.49	
Code Committee	30.00	
American Engineering Standards Committee	600.00	
Committee on Institute Development	71.00	
Annual Dues—International Electrotechnical Commission	250.00	
President's Special Appropriation	331.72	
Honorary Secretary	4,000.00	
John Fritz Medal Award	55.89	
European Standards Committees—Conferences	1,500.00	
Interest on United Engineering Society Building Addition Loan	556.95	
Miscellaneous Printing	252.73	
Membership Classification	771.75	
Amortization of Premium on City of Wilmington, Delaware, 4½% Bonds of 1934	52.14	11,275.01
Total		$125,985.98
Add:		
Increase in Accounts Payable—Subject to Approval by the Finance Committee, Expenses Undistributed at:		
May 1, 1918	$ 6,304.01	
April 30, 1919	6,362.87	58.86
Total Expenses		$126,044.84
Net Revenue		$ 8,899.48
Profit and Loss Credits:		
Proceeds from Sale of Old Electrotypes	$ 440.30	
Refund of Payment to Pan American Engineering Society	15.00	
Total		455.30
Gross Surplus for the Year		$ 9,354.78
Profit and Loss Charges:		
Uncollectible Dues Written Off	$ 1,985.00	
Dues of Members in Military Service and in Countries Affected by the War, Written Off	1,685.50	
Provision for Depreciation of Furniture and Fixtures	1,489.35	
Adjustment of Inventory of April 30, 1918, of Library Volumes and Fixtures	184.00	
Total		5,343.85
Net Surplus for the Year		$ 4,010.93
Surplus, May 1, 1918		579,405.62
Surplus, April 30, 1919		$583,416.55

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Statement of Cash Receipts and Disbursements for Designated Purposes, for the Year Ended April 30, 1919.

Exhibit C.

Receipts:	
Life Membership Fund	$216.68
International Electrical Congress of St. Louis Library Fund—Interest and Royalties	94.50
Mailloux Fund—Interest	45.00
Midwinter Convention Fund—Interest	6.54
Total	$362.72
Disbursements:	
Life Membership Fund	216.68
Midwinter Convention Fund	34 23
Mailloux Fund	70.00
International Electrical Congress, San Francisco, 1915—refund of subscriptions	40.50
Total	$361.41

RECEIPTS AND DISBURSEMENTS PER MEMBER.

During each fiscal year for the past eight years.

Year ending April 30	1912	1913	1914	1915	1916	1917	1918	1919
Membership, April 30, each year	7459	7654	7876	8054	8212	8710	9282	10352
Receipts per Member	$13.19	$13.45	$14.08	$14.06	$13.62	$13.30	$13.17	13.05
Disbursements per Member	12.44	15.57	12.86	13.54	13.74	12.75	11.99	12.79
Credit Balance per Member	$ .75	*$2.12	$1.22	$ .52	*$ .12	$ .55	$1.18	$ .26

*Deficit.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON, *Secretary.*

New York, May 16, 1919.

SYNOPTICAL AND TOPICAL

INDEX

OF

A. I. E. E. TRANSACTIONS

Vol. XXXVIII, Parts I and II

The main headings under which these synopses are classified were arrived at by a careful study of all the papers contributed since the organization of the Institute.

The method of making this classification may be called the automatic method, since it is created by sorting the papers themselves into groups and then naming the groups.

Many papers fall naturally into several different groups and in such cases they are inserted under as many different heads as it is thought they rightfully belong.

The classified synopses are designed for those searching for comprehensive information on any given topic, while the subject index is intended for those looking up specific and definite data or information.

MAIN SECTIONS OF SYNOPTICAL INDEX

2. GENERAL THEORY

THE GENERAL EQUATIONS OF THE ELECTRIC CIRCUIT—III. VARIATION OF CONSTANTS, r, L, C AND g, AND ITS EFFECTS

C. P. Steinmetz **Vol. xxxviii—1919, pp. 191-260**

In the usual theory of transients the assumption is made, that resistance, inductance, capacity and conductance are constant. This however, is not correct, and as the result thereof, it was not possible to theoretically investigate, and numerically calculate the dissipation of high-frequency disturbances, the flattening of the wave fronts of impulses, the rounding off of steep waves, etc., with the time and the distance of travel, and therefrom to determine the distance, to which the danger from such disturbances extends, and to investigate the conditions of line construction, which limit the danger zone of such phenomena to the smallest local extent.

In the following, two of the foremost causes of change of the line constants with the equivalent frequency are investigated, the unequal current distribution in the conductor, and the electric radiation from the conductor.

Equations of the line constants as function of the equivalent frequency are derived, and applications thereof made to a few problems.

It is shown that in high-frequency conduction the section of the conductor is of little importance, but the circumference is the determining factor, except at very high frequencies, where size, shape, and material—within certain limits— becomes of secondary importance.

The conditions, which cause a flattening of steep wave fronts and impulses, and a rounding of irregular waves, are investigated.

For convenience, the theoretical part has been separated and placed in an appendix, giving in the text the discussion, with numerous tables derived from the theoretical equations, and curves illustrating these tables.

Discussion incorporated with that of paper by H. S. Osborne on "Review of Work of Sub-Committee on Wave Shape Standard of the Standards Committee."

THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS

J. R. Carson **Vol. xxxviii--1919, pp. 345-427**

The purpose of this research was to make a broad theoretical study of transient phenomena with a view to developing methods of calculation directly applicable to engineering problems. The investigation starts with the problem of formulating the current in an electrical network or transmission system in response to a suddenly applied e. m. f. of arbitrary form. A simple formula is derived which expresses this current in terms of two independent functions: one, the applied e. m. f. expressed as a time function, and the other a characteristic function of the constants and connections of the system, this latter being termed the "indicial admittance" of the system. A systematic investigation of methods for solving and computing the indicial admittance follows, in the course of which original solutions for transmission and artificial lines are derived and a new method involving integral equations is developed.

Discussion incorporated with that of paper by W. W. Crawford on 'Telephone Circuits with Zero Mutual Induction."

ORDER AND AMPLITUDE OF HARMONICS IN VOLTAGE WAVE FORMS WITH INDICATING INSTRUMENTS

Leslie F. Curtis **Vol. xxxviii—1919, pp. 1179-1190**

The author presents a method for the determination of the order and percentage of the various components of an alternating wave of e. m. f., using indicating meters and other inexpensive apparatus.

Two examples are given. Oscillograms are included to show interesting phenomena and to check the results of the calculations.

The value of so-called standards for the indication of wave form is questioned.

Discussion, pages 1191-1198, by Messrs. N. S. Diamant, J. C. Albert, D. J. Cone, H. A. Barre and L. F. Curtis.

A general discussion on determination of harmonics.

PREDETERMINATION OF SYNCHRONOUS PHASE-MODIFIER PERFORMANCE

Hubert V. Carpenter **Vol. xxxviii—1919, pp. 1223-1229**

The author reviews the method for showing the behavior of transmission lines first given by Perrine and Baum and then shows how it can be used in determining the effect of the use of a synchronous motor operating without load for improving the power factor.

The diagram given shows both the improvement in voltage regulation and the change in power factor due to the phase-modifier for any assumed condition of loading for the transmission line.

The errors of the method are discussed with methods for determining their magnitude, and the advantages of the graphical treatment pointed out.

Discussion, pages 1230-1235, by Messrs. H. B. Dwight, M. O. Bosler, J. A. Lighthipe, P. M. Downing, J. H. Anderton, L. F. Curtis, J. F. Wilson and H. L. Melvin.

Discussion of various methods of determination including use of circle diagram.

A REPORT ON ELECTROMAGNETIC INDUCTION

S. J. Barnett **Vol. xxxviii—1919, pp. 1495-1518**

This report discusses briefly the chief fundamental results obtained from the days of Faraday to the present time in studying the electromotive forces ordinarily referred to the domain of electromagnetic induction.

Self-induction is first taken up.

The motional electromotive force, developed when matter movse in a magnetic field, is next considered and is derived from Ampere's law on the electron theory.

The induced electromotive force in fixed conductors and insulators arising from the motion or alteration of other systems is next considered.

The report closes with a treatment of unipolar induction in both so-called open and closed circuits, including brief descriptions of some of the principal experiments, a discussion of the theories involved, and their application to the unipolar generator.

No discussion of this paper.

SOME NEW FORMULAS FOR REACTANCE COILS

H. B. Dwight **Vol. xxxviii—1919, pp. 1675-1696**

Formulas are presented and derived, which have not been previously published, for mutual inductance of coils with parallel axes, repulsion of coils with parallel axes, and self-inductance of long cylindrical coils.

These formulas apply to practically all cases of reactance coils in common use. They are very convergent and accurate, and will give results to a given degree of accuracy with a minimum amount of labor.

Sets of curves are given from which approximate readings may be taken.

No discussion.

INHERENT LIMITATIONS ON TRANSFORMATIONS POSSIBLE BY STATIONARY APPARATUS

Joseph Slepian **Vol. xxxviii—1919, pp. 1697-1711**

Expressions for electrostatic and electromagnetic energies, Joulian heat dissipation and power are given in complex quantities. The pure imaginary part of the expression for power in a static network is shown to be equal to 2 ω times the difference between the mean electromagnetic energy and mean electrostatic energy. Use is made of this new principle in considering the problems of power-factor correction and phase splitting. It is shown that in general for phase transforming by static apparatus both magnetic and electrostatic storage of energy are necessary, and it is shown how the minimum amounts of each are determined by the load.

The symmetry of the coefficients in the general equations for the steady state in a static network is demonstrated, and it is shown that limitations upon voltage and current transformations follow. The voltage regulation of any phase-splitting arrangement is considered.

No discussion.

STARTING CONDITIONS OF SYNCHRONOUS MACHINES

A. Hay and F. N. Mowdawalla **Vol. xxxviii—1919, pp. 1713-1755**

A detailed study of the behavior of the synchronous motor during what has always been regarded as a critical period—*viz.* the period when the motor is being accelerated from rest to synchronous speed.

Discussion, **pages 1756-1757, by Q. Graham.**

3. UNITS MEASUREMENTS AND INSTRUMENTS

THE ABSOLUTE MEASUREMENT OF THE INTENSITY OF SOUND

Arthur G. Webster **Vol. xxxviii—1919, pp. 701-712**

This paper includes a description of a series of acoustical researches extending over a period of twenty-eight years. The properties of vibrating bodies and the subject of elastic hysteresis are discussed. Two fundamentally important instruments for the absolute measurement of sound have been developed and the theory given. The first is the standard of sound, called the phone, which is capable of reproducing at any time a sound of the simplest character and which permits the output of sound to be measured in watts of energy. The second is an instrument called the phonometer for measuring a sound in absolute measure. This instrument is now practically as sensitive as the human ear. The determination of the space distribution of sound and of the effect of disturbing bodies, and the measurement of the reflecting coefficient of surfaces have been accomplished. The phonotrope is a third instrument designed and used to find the direction of a source of sound, for example a fog signal.

Discussion, (including that of paper by A. E. Kennelly and H. Nukiyama), pages 713-723, by Messrs. J. B. Taylor, T. E. Shea, B. A. Behrend, R. L. Jones, A. G. Webster, H. S. Osborne, A. Press and A. E. Kennelly.

A general discussion including the algebraic theory of the receiver and the application of the circle diagram.

UTILIZING THE TIME CHARACTERISTICS OF ALTERNATING CURRENT

Henry E. Warren **Vol. xxxviii—1919, pp. 767-781**

By the invention of a very small, simple and reliable self-starting synchronous motor, in conjunction with convenient means for regulating the average frequency of an alternating circuit, there has been developed a new field of usefulness for electric power. It is now feasible to drive all kinds of timing devices such as clocks, graphic instrument movements, time recorders, etc., directly from the lighting circuits. Remarkable accuracy is obtained, and the amount of care is minimized by the elimination of winding and regulating.

No discussion.

THE DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE

R. W. Atkinson **Vol. xxxviii—1919,pp. 971-1016**

The data given pertain particularly to the field of three-conductor three-phase cables when supplied with three-phase voltage, and are primarily the solution by physical measurements of some of the geometric problems of the three-conductor three-phase cable.

Data are given so that it is possible, from electrical measurements on three-conductor cable, to determine certain specific quantities as permittivity, resistivity, etc. of the dielectric of three-conductor cables in the same way as can readily be done for single-conductor cables, from geometric considerations.

Also, there is shown the potential and stress distribution in a three-conductor cable.

Discussion (including that of paper by W. S. Clark and G. B. Shanklin), pages 1017-1036, by Messrs. H. W. Fisher, C. W. Davis, W. A. Del Mar, D. W. Roper, P. Torchio, H. L. Wallau, W. R. Atkinson, E. B. Meyer and G. B. Shanklin.

A general discussion with particular emphasis on comparison of stresses in various types of cables.

ORDER AND AMPLITUDE OF HARMONICS IN VOLTAGE WAVE FORMS WITH INDICATING INSTRUMENTS

Leslie F. Curtis **Vol. xxxviii—1919, pp. 1179-1190**

The author presents a method for the determination of the order and percentage of the various components of an alternating wave of e. m. f., using indicating meters and other inexpensive apparatus.

Two examples are given. Oscillograms are included to show interesting phenomena and to check the results of the calculations.

The value of so-called standards for the indication of wave form is questioned.

Discussion, pages 1191-1198, by Messrs. N. S. Diamant, J. C. Albert, D. J. Cone, H. A. Barre and L. F. Curtis.

A general discussion on determination of harmonics.

4. INSULATION AND DIELECTRIC PHENOMENA

IONIZATION OF OCCLUDED GASES IN HIGH-TENSION INSULATION

G. B. Shanklin and J. J. Matson **Vol. xxxviii—1919, pp. 489-536**

This paper deals with the ionization of occluded gases in solid insulation from the standpoint that these gas spaces are the weakest part of an insulation design and should receive first consideration. The stress at which ionization starts in different types of built-up insulation, such as used in cables and coils, is measured and from these measurements a safe working stress determined.

Discussion incorporated with that of paper by F. Dubsky on "The Dielectric Strength of Air Films Entrapped in Solid Insulation and A Practical Application to the Problem for Alternator Coils and Cables."

THE DIELECTRIC STRENGTH OF AIR FILMS ENTRAPPED IN SOLID INSULATION AND A PRACTICAL APPLICATION OF THE PROBLEM FOR ALTERNATOR COILS AND CABLES

F. Dubsky **Vol. xxxviii—1919, pp. 537-558**

An experimental investigation of the strength of air films of various thicknesses between glass plates. In the arrangement used, the breakdown of the air or the starting point of corona could be readily observed.

Tests were also made of the dielectric strength of air films between other solid insulations.

It was found that the dielectric strength of air films between insulations was practically the same as the dielectric strength of air films between conductors.

The second part of the paper is devoted to the practical application of the data to the design of armature coils and cables, and several specific examples are given.

Discussion (including that of paper by G.B. Shanklin and J. J. Matson), pages 559-575, by Messrs. H. G. Reist, L. W. Chubb, J. B. Whitehead, C. A. Adams, W. J. Foster, H. W. Fisher, R. E. Argersinger, C. F. Harding, R. W. Atkinson, D. Du Bois, F. W. Peek, Jr., E. E. F. Creighton, J. J. Matson, W. I. Middleton, E. W. Davis and F. Dubsky.

A general discussion.

HIGH-TENSION SINGLE-CONDUCTOR CABLE FOR POLYPHASE SYSTEMS

W. S. Clark and G. B. Shanklin **Vol. xxxviii—1919, pp. 917-969**

In this paper the dielectric, inductive and general line characteristics of three-conductor and single-conductor cable are compared. The advantages and disadvantages of each type of cable are brought out in a way that will aid in deciding the merits of individual problems.

Discussion incorporated with that of paper by R. W. Atkinson on "The Dielectric Field in an Electric Power Cable."

THE DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE

R. W. Atkinson **Vol. xxxviii—1919, pp. 971-1016**

The data given pertain particularly to the field of three-conductor three-phase cables when supplied with three-phase voltage, and are primarily the solution by physical measurements of some of the geometric problems of the three-conductor three-phase cable.

Data are given so that it is possible, from electrical measurements on three-conductor cable, to determine certain specific quantities as permittivity, resistivity, etc. of the dielectric of three-conductor cables in the same way as can readily be done for single-conductor cables, from geometric considerations.

Also, there is shown the potential and stress distribution in a three-conductor cable.

Discussion (including that of paper by W. S. Clark and G. B. Shanklin), pages 1017-1036, by Messrs. H. W. Fisher, C. W. Davis, W. A. Del Mar, D. W. Roper, P. Torchio, H. L. Wallau, W. R. Atkinson, E. B. Meyer and G. B. Shankin.

A general discussion with particular emphasis on comparison of stresses in various types of cables.

THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS—II

The Effect of Lightning Voltages on Arrester Gaps, Insulators and Bushings on Transm'ssion Lines

F. W. Peek, Jr. **Vol. xxxviii—1919, pp. 1137-1164**

This paper treats of some of the practical applications resulting from an investigation of the effect of lightning voltages on insulators, bushings and protective gaps.

The relative protective values of various gaps for steep and slanting wave fronts and high frequency are shown graphically in Figs. 14, 15, 16 and 17.

Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

Discussion, pages 1165-1177, by Messrs. W. A. Del Mar, F. W. Peek, Jr. and C. T. Allcutt.

A general discussion.

THEORY OF PROBABILITIES APPLIED TO FAILURES OF SUSPENSION INSULATORS

L. M. Klauber **Vol. xxxviii—1919, pp. 1199-1212**

There are a wide variety of operating conditions which affect the amount of over-insulation required, and after having found the minimum number of insulators per string required for any given operating conditions the author points out a method of determining the amount of extra insulation desirable from an insurance standpoint according to the law of probabilities. Equations are developed from which the probability of failure for any given case or the ratio between such probabilities for any pair of cases may be determined directly. A numerical example is also given which shows the development of the theory of minimum annual cost for combined mechanical and electrical failures.

Discussion, pages 1213-1222, by Messrs. W. D. Peaslee, C. O. Poole, J. A. Lighthipe, J. H. Anderton, H. A. Barre, J. B. Fisken, R. W. Shoemaker, E. F. Scattergood, L. C. Williams and L. M. Klauber.

A general discussion on the practicability of the method described.

5. ELECTRIC CONDUCTORS

THE GENERAL EQUATIONS OF THE ELECTRIC CIRCUIT—III

VARIATION OF CONSTANTS, r, L, C AND g, AND ITS EFFECTS

C. P. Steinmetz **Vol. xxxviii—1919, pp. 191-260**

In the usual theory of transients the assumption is made, that resistance, inductance, capacity and conductance are constant. This however, is not correct, and as the result thereof, it was not possible to theoretically investigate, and numerically calculate the dissipation of high-frequency disturbances, the flattening of the wave fronts of impulses, the rounding off of steep waves, etc., with the time and the distance of travel, and therefrom to determine the distance, to which the danger from such disturbances extends, and to investigate the conditions of line construction, which limit the danger zone of such phenomena to the smallest local extent.

In the following, two of the foremost causes of change of the line constants with the equivalent frequency are investigated, the unequal current distribution in the conductor, and the electric radiation from the conductor.

Equations of the line constants as function of the equivalent frequency are derived, and applications thereof made to a few problems.

It is shown that in high-frequency conduction the section of the conductor is of little importance, but the circumference is the determining factor, except at very high frequencies, where size, shape, and material—within certain limits—becomes of secondary importance.

The conditions, which cause a flattening of steep wave fronts and impulses, and a rounding of irregular waves, are investigated.

For convenience, the theoretical part has been separated and placed in an appendix, giving in the text the discussion, with numerous tables derived from the theoretical equations, and curves illustrating these tables.

Discussion incorporated with that of paper by H. S. Osborne on "Review of Work of Sub-Committee on Wave Shape Standard of the Standards Committee."

HIGH-TENSION SINGLE-CONDUCTOR CABLE FOR POLYPHASE SYSTEMS

W. S. Clark and G. B. Shanklin **Vol. xxxviii—1919, pp. 917-966**

In this paper the dielectric, inductive and general line characteristics of three-conductor and single-conductor cable are compared. The advantages and disadvantages of each type of cable are brought out in a way that will aid in deciding the merits of individual problems.

Discussion incorporated with that of paper by R. W. Atkinson on "The Dielectric Field in an Electric Power Cable."

THE DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE

R. W. Atkinson **Vol. xxxviii—1919, pp. 971-1016**

The data given pertain particularly to the field of three-conductor three-phase cables when supplied with three-phase voltage, and are primarily the solution by physical measurements of some of the geometric problems of the three-conductor three-phase cable.

Data are given so that it is possible, from electrical measurements on three-conductor cable, to determine certain specific quantities as permittivity, resistivity, etc. of the dielectric of three-conductor cables in the same way as can readily be done for single-conductor cables, from geometric considerations.

Also, there is shown the potential and stress distribution in a three-conductor cable.

Discussion (including that of paper by W. S. Clark and G. B. Shanklin), pages 1017-1036, by Messrs. H. W. Fisher, C. W. Davis, W. A. Del Mar, D. W. Roper, P. Torchio, H. L. Wallau, W. R. Atkinson, E. B. Meyer and G. B. Shanklin.

A general discussion with particular emphasis on comparison of stresses in various types of cables.

8. TRANSFORMERS

ABNORMAL VOLTAGES WITHIN TRANSFORMERS

L. F. Blume and A. Boyajian **Vol. xxxviii—1919, pp. 577-614**

Mathematical analysis is made of a rectangular wave impinging upon a transformer winding and quantitative values deduced of the resulting internal voltage stresses in terms of transformer constants. It is shown that the conclusions also apply in part to abrupt impulses and approximate idea is given of the reaction of transformer to high frequencies. The difference between operating transformer with isolated and grounded neutral is shown. Energy losses are not considered in the mathematics although the manner in which the results are affected is pointed out. Finally, theoretical results are compared with impulse and high frequency tests made in the laboratory.

Discussion, pages 615-620, by Messrs. H. R. Summerhayes, G. Faccioli, C. A. Adams, J. F. Peters, F. Dubsky, F. W. Peek, Jr., A. E. Kennelly and L. F. Blume.

A general discussion.

9. ELECTRICAL MACHINERY AND APPARATUS

TRANSMISSION LINE RELAY PROTECTION

H. R. Woodrow, D. W. Roper, O. C. Traver and P. Macgahan
Vol. xxxviii—1919, pp. 795-826

The Protective Devices Committee made an analysis of the transmission line protective relay situation throughout the country and prepared a summary of the experiences of the operating companies with this protective apparatus. Questionnaires were sent out to sixty-one operating companies asking for their experience and present practise.

Replies were received from 32 and from these a general analysis was made of the practise of relay protection for transmission lines.

The Protective Devices Committee undertook the work of standardizing the nomenclature which it is hoped will be adopted by all parties.

Discussion incorporated with that of paper by W. E. Richards on "Grounded Neutral Transmission Line."

PREDETERMINATION OF SYNCHRONOUS PHASE-MODIFIER PERFORMANCE

Hubert V. Carpenter **Vol. xxxviii—1919, pp. 1223-1229**

The author reviews the method for showing the behavior of transmission lines first given by Perrine and Baum and then shows how it can be used in determining the effect of the use of a synchronous motor operating without load for improving the power factor.

The diagram given shows both the improvement in voltage regulation and the change in power factor due to the phase-modifier for any assumed condition of loading for the transmission line.

The errors of the method are discussed with methods for determining their magnitude, and the advantages of the graphical treatment pointed out.

Discussion, pages 1230-1235, by Messrs. H. B. Dwight, M. O. Bosler, J. A. Lighthipe, P. M. Downing, J. H. Anderton, L. F. Curtis, J. F. Wilson and H. L. Melvin.

Discussion of various methods of determination including use of circle diagram.

THE VACUUM TUBE AS A GENERATOR OF ALTERNATING-CURRENT POWER

John H. Morecroft and H. Trap Friis **Vol. xxxviii—1919, pp. 1415-1444**

The first part of this article deals with the operation of the tube when separately excited, the variation of power with the amount of excitation, the load impedance, etc., and also gives an analysis of the forms and phases of voltages and currents in the different parts of the circuit.

The second part deals with the efficiency of the tube as a generator; the action is analyzed in detail and the conditions for maximum efficiency deduced, the theoretically deduced conclusions being substantiated by experimental data. Oscillograms are given to show the action of the tube under practically all the conditions which are likely to occur.

Discussion incorporated with that of paper by A. M. Nicolson on "The Piezo-Electric Effect in the Composite Rochelle Salt Cyrstal."

PRESENT LIMITS OF SPEED AND POWER OF SINGLE-SHAFT STEAM TURBINES

J. F. Johnson **Vol. xxxviii—1919, pp. 1515-1525**

This paper restricted to a discussion of some of the factors which influence limits as applying particularly to turbines of the reaction type. With the employment of high vacua the limit of power will be determined largely by the area obtainable through the last stage.

Fig. 6 shows maximum capacity at various speeds which are physically possible without exceeding present limits of stresses. It is valuable chiefly as showing the physical relation between speed and capacity with given limiting stress values.

Discussion incorporated with that of paper by F. D. Newbury on "Present Limits of Speed and Output of Single-Shaft Turbo Generators."

PRESENT LIMITS OF SPEED AND POWER OF SINGLE-SHAFT CURTIS STEAM TURBINES

Eskil Berg **Vol. xxxviii—1919, pp. 1527-1535**

This paper starts by showing that the limit of a single-unit turbo generator does not lie in the generator but is confined to the steam turbine, and that the last wheel of the turbine is the limiting feature.

The author therefore takes the last wheel of an 1800-rev. per min. turbine, giving dimension stresses, kind of material used etc., and then designs two turbines, one having 23 stages and the other 13 stages.

A 5000-kw., five-stage, 3600-rev. per min. turbine load curve is also given and discussed.

Discussion incorporated with that of paper by F. D. Newbury on "Present Limits of Speed and Output of Single-Shaft Turbo Generators."

PRESENT LIMITS OF SPEED AND OUTPUT OF SINGLE-SHAFT TURBO GENERATORS

F. D. Newbury **Vol. xxxviii—1919, pp. 1537-1546**

Output is determined broadly by rotor or stator dimensions. With speeds of 1200 rev. per min. and lower, the stator is the limiting member, while with higher speeds, the rotor is the limiting member.

The most effective rotor diameter is not necessarily the largest diameter. To obtain maximum output at a given speed the rotor proportions must be chosen to properly balance mechanical stresses, rotor ampere turns and flux. American design practise has established 400 ft. per sec. as an upper limit of rotor peripheral speed.

The maximum length of core is determined by such factors as ventilation, bearing temperatures, critical speed and limits to weight imposed by forging and transportation facilities.

No opinion is expressed as to the wisdom of installing very large single-shaft units.

Discussion, (including that of papers by J. F. Johnson and E. Berg), pages 1547-1565, by Messrs. P. Torchio, B. A. Behrend, W. L. R. Emmet, W. J. Foster, A. M. Gray, R. B. Williamson, F. Hodgkinson, B. G. Fernald, C. A. Adams, Farley Osgood, E. Berg and F. D. Newbury.

A general discussion of factors limiting capacity and reliability with particular emphasis on metal fatigue due to vibration.

STARTING CONDITIONS OF SYNCHRONOUS MACHINES

A. Hay and F. N. Mowdawalla **Vol. xxxviii—1919, pp. 1713-1755**

A detailed study of the behavior of the synchronous motor during what has always been regarded as a critical period—*viz.* the period when the motor is being accelerated from rest to synchronous speed.

Discussion, pages 1756-1757, by Q. Graham.

10. PRIME MOVERS AND STEAM BOILERS

PRESENT LIMITS OF SPEED AND POWER OF SINGLE-SHAFT STEAM TURBINES

J. F. Johnson **Vol. xxxviii—1919, pp. 1515-1525**

This paper restricted to a discussion of some of the factors which influence limits as applying particularly to turbines of the reaction type. With the employment of high vacua the limit of power will be determined largely by the area obtainable through the last stage.

Fig. 6 shows maximum capacity at various speeds which are physically possible without exceeding present limits of stresses. It is valuable chiefly as showing the physical relation between speed and capacity with given limiting stress values.

Discussion incorporated with that of paper by F. D. Newbury on "Present Limits of Speed and Output of Single-Shaft Turbo Generators."

PRESENT LIMITS OF SPEED AND POWER OF SINGLE-SHAFT CURTIS STEAM TURBINES

Eskil Berg **Vol. xxxviii—1919, pp. 1527-1535**

This paper starts by showing that the limit of a single-unit turbo generator does not lie in the generator but is confined to the steam turbine, and that the last wheel of the turbine is the limiting feature.

The author therefore takes the last wheel of an 1800-rev. per min. turbine, giving dimension stresses, kind of material used etc., and then designs two turbines, one having 23 stages and the other 13 stages.

A 5000-kw., five-stage, 3600-rev. per min. turbine load curve is also given and discussed.

Discussion incorporated with that of paper by F. D. Newbury on "Present Limits of Speed and Output of Single-Shaft Turbo Generators."

PRESENT LIMITS OF SPEED AND OUTPUT OF SINGLE-SHAFT TURBO GENERATORS

F. D. Newbury **Vol. xxxviii—1919, pp. 1537-1546**

Output is determined broadly by rotor or stator dimensions. With speeds of 1200 rev. per min. and lower, the stator is the limiting member, while with higher speeds, the rotor is the limiting member.

The most effective rotor diameter is not necessarily the largest diameter. To obtain maximum output at a given speed the rotor proportions must be chosen to properly balance mechanical stresses, rotor ampere turns and flux. American design practise has established 400 ft. per sec. as an upper limit of rotor peripheral speed.

The maximum length of core is determined by such factors as ventilation, bearing temperatures, critical speed and limits to weight imposed by forging and transportation facilities.

No opinion is expressed as to the wisdom of installing very large single-shaft units.

Discussion, (including that of papers by J. F. Johnson and E. Berg), pages 1547-1565, by Messrs. P. Torchio, B. A. Behrend, W. L. R. Emmet, W. J. Foster, A. M. Gray, R. B. Williamson, F. Hodgkinson, B. G. Fernald, C. A. Adams, Farley Osgood, E. Berg and F. D. Newbury.

A general discussion of factors limiting capacity and reliability with particular emphasis on metal fatigue due to vibration.

11. POWER PLANTS AND CENTRAL STATIONS

SOME PROBLEMS IN THE OPERATION OF POWER PLANTS IN PARALLEL

E. C. Stone **Vol. xxxviii—1919, pp. 1651-1674**

In order to operate two power plants satisfactorily in parallel, the transmission line which ties them together must have sufficient synchronizing power, as well as sufficient carrying capacity. The "synchronizing power" of a line depends upon its resistance and reactance, the bus voltages maintained at its ends, and the maximum kilovolt-amperes it must transmit. Limiting values for "synchronizing power" of lines under various operating conditions are given.

The design of a transmission line involves a consideration of load to be transmitted, voltage, reactance, resistance, losses, and charging current of the line, and of wattless generating capacity at the receiving end of the line. The wattless generating capacity at the end of the line determines how many kilowatts will be transmitted for each ampere of line current, by fixing the power factor of the load transmitted and the voltage at the receiving end of the line. When a line is to be designed for paralleling two plants, it must have sufficient "synchronizing power" to hold the two plants together.

No discussion.

12. PARALLEL OPERATION

SOME PROBLEMS IN THE OPERATION OF POWER PLANTS IN PARALLEL

E. C. Stone **Vol. xxxviii—1919, pp. 1651-1674**

In order to operate two power plants satisfactorily in parallel, the transmission line which ties them together must have sufficient synchronizing power, as well as sufficient carrying capacity. The "synchronizing power" of a line depends upon its resistance and reactance, the bus voltages maintained at its ends, and the maximum kilovolt-amperes it must transmit. Limiting values for "synchronizing power" of lines under various operating conditions are given.

The design of a transmission line involves a consideration of load to be transmitted, voltage, reactance, resistance, losses, and charging current of the line, and of wattless generating capacity at the receiving end of the line. The wattless generating capacity at the end of the line determines how many kilowatts will be transmitted for each ampere of line current, by fixing the power factor of the load transmitted and the voltage at the receiving end of the line. When a line is to be designed for paralleling two plants, it must have sufficient "synchronizing power" to hold the two plants together.

No discussion.

13. TRANSMISSION LINES

THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS

J. R. Carson **Vol. xxxviii—1919, pp. 345-427**

The purpose of this research was to make a broad theoretical study of transient phenomena with a view to developing methods of calculation directly applicable to engineering problems. The investigation starts with the problem of formulating the current in an electrical network or transmission system in response to a suddenly applied e. m. f. of arbitrary form. A simple formula is derived which expresses this current in terms of two independent functions: one, the applied e. m. f. expressed as a time function, and the other a characteristic function of the constants and connections of the system, this latter being termed the "indicial admittance" of the system. A systematic investigation of methods for solving and computing the indicial admittance follows, in the course of which original solutions for transmission and artificial lines are derived and a new method involving integral equations is developed.

Discussion incorporated with that of paper by W. W. Crawford on "Telephone Circuits with Zero Mutual Induction."

HIGH-TENSION SINGLE-CONDUCTOR CABLE FOR POLYPHASE SYSTEMS

W. S. Clark and G. B. Shanklin **Vol. xxxviii—1919, pp. 917-969**

In this paper the dielectric, inductive and general line characteristics of three-conductor and single-conductor cable are compared. The advantages and disadvantages of each type of cable are brought out in a way that will aid in deciding the merits of individual problems.

Discussion incorporated with that of paper by R. W. Atkinson on "The Dielectric Field in an Electric Power Cable."

PROBLEMS OF 220-KV. POWER TRANSMISSION

A. E. Silver **Vol. xxxviii—1919, pp. 1037-1100**

Two hundred and twenty kv. is suggested as a logical voltage for high capacity, long-distance transmission, and the important problem-introduced by large concentrations of power, high voltage and high service standards are discussed. The economic and technical considerations underlying design of a 220-kv. system are outlined, and general designs are developed for a typical 220-kv. transmission line.

Discussion, pages 1101-1135, by Messrs. W. S. Murray, W. M. Dann, P. Torchio, S. W. Mauger, R. M. Spurck, J. C. Parker, J. C. Clark, F. C. Hanker, H. R. Summerhayes, R. P. Jackson, H. B. Dwight, J. F. Peters, F. F. Brand, C. F. Harding, F. W. Peek, Jr., J. A. Koontz, H. G. Mac Donald, L. B. Chubbuck, J. N. Mahoney, E. B. Meyer, C. E. Howell, T. B. Parker, J. B. Crane and A. E. Silver.

A general discussion of the super-power problem particularly with reference to the N. Y. and New England section.

THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS—II

The Effect of Lightning Voltages on Arrester Gaps, Insulators and Bushings on Transmission Lines

F. W. Peek, Jr. **Vol. xxxviii—1919, pp. 1137-1164**

This paper treats of some of the practical applications resulting from an investigation of the effect of lightning voltages on insulators, bushings and protective gaps.

The relative protective values of various gaps for steep and slanting wave fronts and high frequency are shown graphically in Figs. 14, 15, 16 and 17.

Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

Discussion, pages 1165-1177, by Messrs. W. A. Del Mar, F. W. Peek, Jr. and C. T. Allcutt.

A general discussion.

CALIFORNIA 220,000-VOLT—1100-MILE—1,500,000-KW. TRANSMISSION BUS

R. W. Sorensen, H. H. Cox and G. E. Armstrong **Vol. xxxviii—1919, pp. 1237-1248**

This paper summarizes the power resources of California and the probable loads to be supplied within the next six or seven years. For the purpose of economically distributing the necessary power and supplying the load, a long high-voltage transmission line is proposed. As this line would interconnect a number of different companies, it assumes the nature of a bus bar. The authors show how the proposed line may link with some of the lines now in service and enumerate the advantages of such interconnection. A comparison is made between the 240-mile Big Creek line now operating at 150,000 volts, 50 cycles, and the operation of this line at 220,000 volts, 60 cycles. Operating data on the Big Creek line are shown to indicate the character of the construction necessary for California conditions. Conclusions are drawn as to the particular features to be observed for successful operation of 220,000-kv. lines.

Discussion, pages 1249-1268, by Messrs. R. Bennett, P. D. Jennings, C. O. Poole, J. D. Ross, L. F. Curtis, J. B. Fisken, H. A. Barre, L. M. Klauber, J. A. Lighthipe, P. M. Downing, J. F. Stevens, C. W. Koiner, L. S. Ready, G. A. Damon, L. J. Corbett, H. H. Cox and G. E. Armstrong.

A general discussion of the advantages and disadvantages of the super-power line.

14. ELECTRIC SERVICE DISTURBANCES AND PROTECTION

REVIEW OF WORK OF SUB-COMMITTEE ON WAVE SHAPE STANDARD OF THE STANDARDS COMMITTEE

H. S. Osborne **Vol. xxxviii—1919, pp. 261-288**

The paper gives a review of the work done during the past three years by a sub-committee of the Standards Committee of the Institute appointed to make recommendations for changes in the Institute's rules regarding the wave shape of alternators. The Sub-Committee last spring made recommendations that for the present the ten per cent deviation rule should be retained without change (except in wording), and that trial use should be made of a supplementary wave shape factor. The new factor, based on the relation between voltage wave shape and interfering effect in telephone circuits, when power and telephone lines parallel each other, is called the "telephone interference factor."

Discussion (including that of paper by C. P. Steinmetz), pages 289-303, by Messrs. C. F. Scott, C. R. Underhill, A. E. Kennelly, C. P. Steinmetz, J. B. Whitehead, W. J. Foster, J. B. Taylor, J. J. Linebaugh, E. E. F. Creighton, L. W. Chubb, C. A. Adams, L. T. Robinson, C. F. Harding and H. S. Osborne.

A general discussion.

THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS

J. R. Carson **Vol. xxxviii—1919, pp. 345-427**

The purpose of this research was to make a broad theoretical study of transient phenomena with a view to developing methods of calculation directly applicable to engineering problems. The investigation starts with the problem of formulating the current in an electrical network or transmission system in response to a suddenly applied e. m. f. of arbitrary form. A simple formula is derived which expresses this current in terms of two independent functions: one, the applied e. m. f. expressed as a time function, and the other a characteristic function of the constants and connections of the system, this latter being termed the "indicial admittance" of the system. A systematic investigation of methods for solving and computing the indicial admittance follows, in the course of which original solutions for transmission and artificial lines are derived and a new method involving integral equations is developed.

Discussion incorporated with that of paper by W. W. Crawford on "Telephone Circuits with Zero Mutual Induction."

TELEPHONE CIRCUITS WITH ZERO MUTUAL INDUCTION

W. W. Crawford **Vol. xxxviii—1919, pp. 429-457**

The paper deals with the reduction of inductive interference in telephone circuits.

Various relative positions of two or more circuits, in which the mutual inductance is zero, and the mutual capacitance unbalance is approximately zero, are discussed. The most important case is that when the two wires of one circuit occupy opposite ends of one diagonal of a square, and the other circuit, the ends of the other diagonal.

Several forms of construction embodying this arrangement, and built largely with standard parts, are illustrated.

Calculations and tentative designs are presented to show that the use of these forms of construction will give greatest refinement of balance against induction from power circuits, and possibly also against cross-talk, increased flexibility in coordinating with the variations in exposure to power circuits, a simplification of the transpositions system, fewer transposition poles and transpositions, and when desired, the realization of a part of these advantages with the lead compressed into less than the normal space.

The cost of initial construction of the proposed forms and the cost of conversion of an existing lead to one of the proposed forms.

Discussion (including that of papers by E. B. Craft and E. H. Colpitts, and J. R. Carson), pages 458-488, by Messrs. G. O. Squier, M. I. Pupin, A. H. Cowles, A. S. Dana, A. Pinto, V. Bush, J. H. Morecroft, A. G. Chapman, H. S. Osborne and J. B. Carson.

A general discussion including a description of the theory of functioning of vacuum tubes.

ABNORMAL VOLTAGES WITHIN TRANSFORMERS

L. F. Blume and A. Boyajian **Vol. xxxviii—1919, pp. 577-614**

Mathematical analysis is made of a rectangular wave impinging upon a transformer winding and quantitative values deduced of the resulting internal voltage stresses in terms of transformer constants. It is shown that the conclusions also apply in part to abrupt impulses and approximate idea is given of the reaction of transformer to high frequencies. The difference between operating transformer with isolated and grounded neutral is shown. Energy losses are not considered in the mathematics although the manner in which the results are affected is pointed out. Finally, theoretical results are compared with impulse and high frequency tests made in the laboratory.

Discussion, pages 615-620, by Messrs. H. R. Summerhayes, G. Faccioli,

C. A. Adams, J. F. Peters, F. Dubsky, F. W. Peek, Jr., A. E. Kennelly and L. F. Blume.

A general discussion.

TRANSMISSION LINE RELAY PROTECTION

H. R. Woodrow, D. W. Roper, O. C. Traver and P. Macgahan

Vol. xxxviii—1919, pp. 795-826

The Protective Devices Committee made an analysis of the transmission line protective relay situation throughout the country and prepared a summary of the experiences of the operating companies with this protective apparatus. Questionnaires were sent out to sixty-one operating companies asking for their experience and present practise.

Replies were received from 32 and from these a general analysis was made of the practise of relay protection for transmission lines.

The Protective Devices Committee undertook the work of standardizing the nomenclature which it is hoped will be adopted by all parties.

Discussion incorporated with that of paper by W. E. Richards on "Grounded Neutral Transmission Line."

GROUNDING THE NEUTRAL OF GENERATING AND TRANSMISSION SYSTEMS

H. R. Woodrow **Vol. xxxviii—1919, pp. 827-831**

Four different high-tension transmission systems in the New York district have been grounded at different times during the past four years, all of which are operating with the grounded neutral at the present time. These systems were all grounded in a different manner and the results obtained from the effect of grounding showed that the troubles have been reduced.

In addition to the grounding, special ground relays have been installed on three of the systems which disconnected the feeder very quickly on a ground and the results show that the addition of this quick acting relay further reduces the stresses on a system.

Discussion incorporated with that of paper by W. E. Richards on "Grounded Neutral Transmission Line."

GROUNDED NEUTRAL TRANSMISSION LINE

W. E. Richards **Vol. xxxviii—1919, pp. 833-838**

The paper describes the conditions on the system in Toledo which was originally delta-connected. Serious trouble was experienced when a short circuit occurred especially with synchronous apparatus. This trouble was overcome by changing the transmission to a Y system with the neutral grounded.

Discussion (including that of papers by Woodrow, Roper Traver and Macgahan and Woodrow), pages 839-869, by Messrs. J. R. Craighead, J. A. Johnson, A. H. Lawton, E. E. F. Creighton, H. T. Conover, O. C. Traver, P. Torchio, D. W. Roper, P. H. Adams, R. N. Conwell, A. W. Copley, C. P. Osborne, P. M. Lincoln, C. P. Steinmetz, E. T. Moore, N. L. Pollard, R. F. Schuchardt, J. C. Parker, H. C. Don Carlos, D. A McKenzie, F. L. Hunt, F. C. Hanker, H. C. Albrecht, E. A. Hester

E. G. Merrick, N. A. Carle, H. R. Woodrow, H. B. Vincent, F. D. Newbury and A. E. Silver.

A general discussion of present practise.

THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS—II

The Effect of Lightning Voltages on Arrester Gaps, Insulators and Bushings on Transmission Lines

F. W. Peek, Jr. **Vol. xxxviii—1919, pp. 1137-1164**

This paper treats of some of the practical applications resulting from an investigation of the effect of lightning voltages on insulators, bushings and protective gaps.

The relative protective values of various gaps for steep and slanting wave fronts and high frequency are shown graphically in Figs. 14, 15, 16 and 17.

Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

Discussion, pages 1165-1177, by Messrs. W. A. Del Mar, F. W. Peek, Jr. and C. T. Allcutt.

A general discussion.

THEORY OF PROBABILITIES APPLIED TO FAILURES OF SUSPENSION INSULATORS

L. M. Klauber **Vol. xxxviii—1919, pp. 1199-1212**

There are a wide variety of operating conditions which affect the amount of over-insulation required, and after having found the minimum number of insulators per string required for any given operating conditions the author points out a method of determining the amount of extra insulation desirable from an insurance standpoint according to the law of probabilities. Equations are developed from which the probability of failure for any given case or the ratio between such probabilities for any pair of cases may be determined directly. A numerical example is also given which shows the development of the theory of minimum annual cost for combined mechanical and electrical failures.

Discussion, pages 1213-1222, by Messrs. W. D. Peaslee, C. O. Poole, J. A. Lighthipe, J. H. Anderton, H. A. Barre, J. B. Fisken, R. W. Shoemaker, E. F. Scattergood, L. C. Williams and L. M. Klauber.

A general discussion on the practicability of the method described.

16. CONTROL, REGULATION AND SWITCHING

PREDETERMINATION OF SYNCHRONOUS PHASE-MODIFIER PERFORMANCE

Hubert V. Carpenter **Vol. xxxviii—1919, pp. 1223-1229**

The author reviews the method for showing the behavior of transmission lines first given by Perrine and Baum and then shows how it can be used in determining the effect of the use of a synchronous motor operating without load for improving the power factor.

The diagram given shows both the improvement in voltage regulation the change in power factor due to the phase-modifier for any assumed dition of loading for the transmission line.

The errors of the method are discussed with methods for determining ir magnitude, and the advantages of the graphical treatment pointed t.

Discussion, pages 1230-1235, by Messrs. H. B. Dwight, M. O. Bosler, A. Lighthipe, P. M. Downing, J. H. Anderton, L. F. Curtis, J. F. Vilson and H. L. Melvin.

Discussion of various methods of determination including use of circle iagram.

STARTING CONDITIONS OF SYNCHRONOUS MACHINES

A. Hay and F. N. Mowdawalla **Vol. xxxviii—1919, pp. 1713-1755**

A detailed study of the behavior of the synchronous motor during what has always been regarded as a critical period—*viz.* the period when the motor is being accelerated from rest to synchronous speed.

Discussion, pages 1756-1757, by Q. Graham.

17. TRACTION

SOME POSSIBILITIES OF STEAM RAILROAD ELECTRIFICATION AS AFFECTING FUTURE POLICIES

Calvert Townley **Vol. xxxviii—1919, pp. 621-627**

Electricity fills every requirement of railroad service, but as it involves a large investment, electrification has proceeded slowly. Electrification has also been retarded because the problem has been largely considered one of replacing the steam locomotive by the electric locomotive whereas in reality the problem is much broader. It really offers a fundamentally different method of train propulsion because the limitations of the steam locomotive disappear and the strictly limited motive power is replaced by one that is practically unlimited, thereby, opening up many possibilities in the methods of railroad operation. A brief review is given of electrified sections of railways showing the advantages which have been realized in both the freight and passenger service.

Discussion, pages 628-649, by Messrs. F. H. Shepard, W. S. Murray, C. Schwartz, J. B. Taylor, N. W. Storer, W. B. Potter, G. F. Sever, C. F. Scott, G. Gibbs, J. Murphy, H. D. Jackson and C. Townley.

Discussion deals chiefly with coal saving incident to electrification particularly that saving involved in the proposed super-power transmission line, etc.

18. LIGHTING AND LAMPS

PRESENT STATUS OF INDUSTRIAL LIGHTING CODES

G. H. Stickney **Vol. xxxviii—1919, pp. 725-742**

Industrial lighting codes have been adopted in four states and in Federal establishments. Similar action is under consideration in several other states and there is prospect of extension throughout the country.

Investigation and experience indicate the need of government regulation of factory lighting. When adopted by industrial commissions under authority granted by Legislatures, the codes become in effect state law. The existing codes correspond in essentials to the Illuminating Engineering Society code, on which they are based.

Discussion, pages 743-765, by Messrs. C. A. Adams, C. E. Clewell, E. B. Rosa, C. B. Auel, M. G. Lloyd, W. Harrison, J. Vogt, W. P. Hurley, W. T. Blackwell, E. G. Perrott, T. F. Foltz, J. R. Cravath and G. H. Stickney.

A general discussion of the code as established in the various States.

THE SEARCHLIGHT IN THE U. S. NAVY

Ralph Kelly **Vol. xxxviii—1919, pp. 1605-1620**

The types and uses of searchlights and signaling lights on naval ships are briefly described.

A changed form of 12-inch incandescent searchlight is suggested.

The introduction of the high-power searchlight revolutionized the application of the searchlight to naval ships.

It is believed there is also a possibility of a considerable improvement in the electrodes.

The use of the dome glass door enables a searchlight to operate in close proximity to large calibre rifles and makes it possible to successfully build even larger sizes of searchlights than those at present in use.

The star shell has great possibilities, but it is doubtful if it will ever supersede the high-power searchlight.

Discussion, pages 1621-1634, by Messrs. D. McNicol, G. A. DeGraaf, M. L. Patterson, E. J. Murphy, J. C. Ledbetter, R. A. Beekman, B. P. Beehler, C. Lichtenberg and R. Kelly.

A general discussion of searchlights including a comprehensive discussion of army practise.

19. ELECTRICITY IN THE ARMY AND NAVY

THE SEARCHLIGHT IN THE U. S. NAVY

Ralph Kelly **Vol. xxxviii—1919, pp. 1605-1620**

The types and uses of searchlights and signaling lights on naval ships are briefly described.

A changed form of 12-inch incandescent searchlight is suggested.

The introduction of the high-power searchlight revolutionized the application of the searchlight to naval ships.

It is believed there is also a possibility of a considerable improvement in the electrodes.

The use of the dome glass door enables a searchlight to operate in close proximity to large calibre rifles and makes it possible to successfully build even larger sizes of searchlights than those at present in use.

The star shell has great possibilities, but it is doubtful if it will ever supersede the high-power searchlight.

Discussion, pages 1621-1634, by Messrs. D. McNicol, G. A. DeGraaf, M. L. Patterson, E. J. Murphy, J. C. Ledbetter, R. A. Beekman, B. P. Beehler, C. Lichtenberg and R. Kelly.

A general discussion of searchlights including a comprehensive discussion of army practise.

20. MISCELLANEOUS APPLICATIONS OF ELECTRICITY

WELDING MILD STEEL

H. M. Hobart **Vol. xxxviii—1919, pp. 63-111**

The Welding Research Sub-Committee, formed in 1918, was a subcommittee of the Metallurgical and Electrical Sections of the Engineering Division of the National Research Council, and the Welding Committee, (under the chairmanship of Professor C. A. Adams) came under the direction of the Emergency Fleet Corporation. The paper is in large part based on the work done by the Welding Research Sub-Committee up to January of this year.

Discussion incorporated with that of paper by O. H. Eschholz on "Fusion in Arc Welding."

WELDING AS A PROCESS IN SHIP CONSTRUCTION

S. V. Goodall **Vol. xxxviii—1919, pp. 113-119**

The paper points to the necessity for a reduction in the cost of shipbuilding, and as riveting is one of the most expensive items of construction the substitution of welding for riveting would decrease the cost of construction considerably. A brief review is given of what has been done in substituting welding for riveting, and to the limited extent to which electric welding has been tried it has been found successful. Lloyd's Register is prepared to classify electrically-welded vessels subject to certain provisions, but shipbuilders have not as yet adopted welding to a large extent for the reason, in the author's opinion, that they know that welds are lacking in uniformity and it is impossible to tell when a joint is good or bad.

Discussion incorporated with that of paper by O. H. Eschholz on "Fusion in Arc Welding."

FUSION IN ARC WELDING

O. H. Eschholz **Vol. xxxviii—1919, pp. 121-129**

This paper calls attention to such characteristics as penetration and overlap, peculiar to this process, which facilitate visual inspection and discusses briefly the effect of arc length, welding procedure, electrode material, arc current and electrode diameter upon these characteristics.

Discussion (including that of papers by H. M. Hobart and G. V. Goodall), pages 130-177, by Messrs. W. H. Hill, H. A. Hornor, C. A. Adams, Capt. Corbett, A. M. Candy, C. J. Holslag, W. L. Merrill, D. C. Alexander, Jr., W. Spraragen, J. C. Armor, S. V. Goodall, R. P. Jackson, F. M. Farmer, W. S. Andrews, J. C. Lincoln, H. Lemp, H. D.

Morton, R. E. Wagner, T. T. Heaton, C. R. Darling, J. Churchward, C. H. Kicklighter, Z. Jeffries, A. S. Kinsey, H. G. Knox, J. Martin and O. H. Eschholz.

A general discussion of electric welding and its practical application.

UTILIZING THE TIME CHARACTERISTICS OF ALTERNATING CURRENT

Henry E. Warren **Vol. xxxviii—1919, pp. 767-781**

By the invention of a very small, simple and reliable self-starting synchronous motor, in conjunction with convenient means for regulating the average frequency of alternating circuit, there has been developed a new field of usefulness for electric power. It is now feasible to drive all kinds of timing devices such as clocks, graphic instrument movements, time recorders, etc., directly from the lighting circuits. Remarkable accuracy is obtained, and the amount of care is minimized by the elimination of winding and regulating.

No discussion.

THE PIEZO-ELECTRIC EFFECT IN THE COMPOSITE ROCHELLE SALT CRYSTAL

A. McL. Nicolson **Vol. xxxviii—1919, pp. 1467-1485**

The piezo-electric effect is an electro-elastic property of certain crystals. It involves the conversion of mechanical into electrical energy, and also the converse effect. The paper presents an exposition of piezo-electricity and related optical and other properties belonging to these crystals.

Applications of the piezo-electric effect in such crystals to the transmission and reception of sound, are described and demonstrated.

Discussion, (including that of papers by A. W. Hull and Morecroft and Friis), pages 1486-1493, by Messrs. J. B. Taylor, A. W. Hull, A. M. Nicolson, E. A. Eckhardt, J. P. Wintringham, Irving Langmuir, J. E. Schroder, D. M. Therrell and J. H. Morecroft.

A general discussion.

21. TELEPHONY AND TELEGRAPHY

RADIO TELEPHONY

E. B. Craft and E. H. Colpitts **Vol. xxxviii—1919, pp. 305-348**

This paper is divided into two parts. The first part describes the development of the art of radio telephony by the American Telephone and Telegraph Company and the Western Electric Company to the accomplishment of trans-atlantic telephony, followed by demonstrations of the use of radio telephony between ships and of methods of connecting radio and wire telephone systems.

The second part is concerned almost entirely with the work of producing radio telephone and allied apparatus for the Army and Navy in the late war.

Discussion incorporated with that of paper by W. W. Crawford on "Telephone Circuits with Zero Mutual Induction."

TELEPHONE CIRCUITS WITH ZERO MUTUAL INDUCTION

W. W. Crawford **Vol. xxxviii—1919, pp. 429-457**

The paper deals with the reduction of inductive interference in telephone circuits.

Various relative positions of two or more circuits, in which the mutual inductance is zero, and the mutual capacitance unbalance is approximately zero, are discussed. The most important case is that when the two wires of one circuit occupy opposite ends of one diagonal of a square, and the other circuit, the ends of the other diagonal.

Several forms of construction embodying this arrangement, and built largely with standard parts, are illustrated.

Calculations and tentative designs are presented to show that the use of these forms of construction will give greatest refinement of balance against induction from power circuits, and possibly also against cross-talk, increased flexibility in coordinating with the variations in exposure to power circuits, a simplification of the transpositions system, fewer transposition poles and transpositions, and when desired, the realization of a part of these advantages with the lead compressed into less than the normal space.

The cost of initial construction of the proposed forms and the cost of conversion of an existing lead to one of the proposed forms.

Discussion (including that of papers by E. B. Craft and E. H. Colpitts, and J. R. Carson), pages 458-488, by Messrs. G. O. Squier, M. I. Pupin, A. H. Cowles, A. S. Dana, A. Pinto, V. Bush, J. H. Morecroft, A. G. Chapman, H. S. Osborne and J. B. Carson.

A general discussion including a description of the theory of functioning of vacuum tubes.

ELECTROMAGNETIC THEORY OF THE TELEPHONE RECEIVER WITH SPECIAL REFERENCE TO MOTIONAL IMPEDANCE

A. E. Kennelly and H. Nukiyama **Vol. xxxviii—1919, pp. 651-699**

The theory of the telephone receiver here offered is based upon the motional impedance circle which has been published in various chapters during the last few years. The new theory, which is stated under definite limitations, is a further development; taking into account the m. m. f. produced by the vibration of the diaphragm in the permanent magnetic field.

Discussion incorporated with that of paper by A. G. Webster on "The Absolute Measurement of the Intensity of Sound."

TRANSOCEANIC RADIO COMMUNICATION

E. F. W. Alexanderson **Vol. xxxviii—1919, pp. 1269-1286**

The paper defines the state of the art of today which is the result of developments during the war. Transoceanic radio communication is at present maintained by five first class stations, two in America and three in Europe. New developments indicate three methods for increasing the radio traffic without interference between the different messages.

Behrend, R. L. Jones, A. G. Webster, H. S. Osborne, A. Press and A. E. Kennelly.

A general discussion including the algebraic theory of the receiver and the application of the circle diagram.

COOPERATION

C. A. Adams **Vol. xxxviii—1919, pp. 783-793**

President's Address.

TECHNICAL COMMITTEE REPORTS

Vol. xxxviii—1919, pp. 871-916

THE VACUUM TUBE AS A GENERATOR OF ALTERNATING-CURRENT POWER

John H. Morecroft and H. Trap Friis **Vol. xxxviii—1919, pp. 1415-1444**

The first part of this article deals with the operation of the tube when separately excited, the variation of power with the amount of excitation, the load impedance, etc., and also gives an analysis of the forms and phases of voltages and currents in the different parts of the circuit.

The second part deals with the efficiency of the tube as a generator; the action is analyzed in detail and the conditions for maximum efficiency deduced, the theoretically deduced conclusions being substantiated by experimental data. Oscillograms are given to show the action of the tube under practically all the conditions which are likely to occur.

Discussion incorporated with that of paper by A. M. Nicolson on "The Piezo-Electric Effect in the Composite Rochelle Salt Crystal."

THE POSITIONS OF ATOMS IN METALS

A. W. Hull **Vol. xxxviii—1919, pp. 1445-1466**

When a narrow beam of X-rays passes through a fine powder of any crystalline material, it produces on a photographic plate placed just behind the powder a pattern of concentric circles. These circles are produced by the reflection of the X-rays from the planes of atoms in the crystal, and their diameters are a measure of the distances between these planes of atoms. By measuring the diameters of the circles the exact positions of the atoms can be determined. The results of this analysis are given for twenty common metals and several salts, with examples and brief description of the method, and a discussion of the results.

Discussion incorporated with that of paper by A. M. Nicolson on "The Piezo-Electric Effect in the Composite Rochelle Salt Crystal."

THE PIEZO-ELECTRIC EFFECT IN THE COPOSITE ROCHELLE SALT CRYSTAL

A. McL. Nicolson **Vol. xxxviii—1919, pp. 1467-1486**

The piezo-electric effect is an electro-elastic property of certain crystals. It involves the conversion of mechanical into electrical energy, and also the converse effect. The paper presents an exposition of piezo-electricity and related optical and other properties belonging to these crystals.

Applications of the piezo-electric effect in such crystals to the transmission and reception of sound, are described and demonstrated.

Discussion, (including that of papers by A. W. Hull and Morecroft and Friis), pages 1486-1493, by Messrs. J. B. Taylor, A. W. Hull, A. M. Nicolson, E. A. Eckhardt, J. P. Wintringham, Irving Langmuir, J. E. Schroder, D. M. Therrell and J. H. Morecroft.

A general discussion.

A REPORT ON ELECTROMAGNETIC INDUCTION

S. J. Barnett **Vol. xxxviii—1919, pp. 1495-1513**

This report discusses briefly the chief fundamental results obtained from the days of Faraday to the present time in studying the electromotive forces ordinarily referred to the domain of electromagnetic induction.

Self-induction is first taken up.

The motional electromotive force, developed when matter moves in a magnetic field, is next considered and is derived from Ampere's law on the electron theory.

The induced electromotive force in fixed conductors and insulators arising from the motion or alteration of other systems is next considered.

The report closes with a treatment of unipolar induction in both so-called open and closed circuits, including brief descriptions of some of the principal experiments, a discussion of the theories involved, and their application to the unipolar generator.

No discussion of this paper.

INHERENT LIMITATIONS ON TRANSFORMATIONS POSSIBLE BY STATIONARY APPARATUS

Joseph Slepian **Vol. xxxviii—1919, pp. 1697-1711**

Expressions for electrostatic and electromagnetic energies, Joulian heat dissipation and power are given in complex quantities. The pure imaginary part of the expression for power in a static network is shown to be equal to 2 ω times the difference between the mean electromagnetic energy and mean electrostatic energy. Use is made of this new principle in considering the problems of power-factor correction and phase splitting. It is shown that in general for phase transforming by static apparatus both magnetic and electrostatic storage of energy are necessary, and it is shown how the minimum amounts of each are determined by the load.

The symmetry of the coefficients in the general equations for the steady state in a static network is demonstrated, and it is shown that limitations upon voltage and current transformations follow. The voltage regulation of any phase-splitting arrangement is considered.

No discussion.

TOPICAL INDEX.

[redacted]