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*Presented at the 336th meeting of the American Institute of Electrical Engineers, Boston January 8, New York January 11, and Chicago January 14, 1918.*

# EFFECTS OF WAR CONDITIONS ON COST AND QUALITY OF ELECTRIC SERVICE

BY LYNN S. GOODMAN AND WILLIAM B. JACKSON

## Abstract of Paper

This paper comprises a consideration of the effects of war conditions as they apply to the electric light and power service of the country, but the principles enunciated relate in their broad application to all kinds of public utility service.

The electric companies are facing a grave situation owing to the Government's needs and requirements, to the abnormal prices of labor, supplies and equipment, to the difficulty of retaining their trained employees, and to the present impossibility of satisfactorily financing extensions.

Owing to these conditions the electric companies may be forced to regularly operate their systems, during the continuance of such conditions, with reduced reserve in capacity of equipment and with partially trained operating forces, with accompanying reduction in efficiency of operation and reliability of service as compared with normal conditions.

It is shown that the increased cost for fuel and other supplies, labor and taxes alone, occasioned by the war conditions, would amount to an increase of more than $116,000,000 over the operating expenses that should have been expected for the electric light and power companies of the country under normal conditions for 1917.

The important advantages which are inherent in the central electric power stations for supplying power for war manufactures and the advisability for the government to make every reasonable endeavor to encourage the development of the central electric stations is pointed out.

To work out the situation to the best interests of the public and the companies will require earnest cooperation between the Government and the companies. Although the situation is one requiring the determination of how the electric companies can best meet the power requirements for war manufactures, yet the way in which the problems involved are worked out will have a bearing upon the cost of electric light and power service not only during the war but for long after its termination.

THE EFFECTS of war conditions in the public utility field are demanding at the present time the most careful consideration of both the operating and the regulatory bodies throughout the country. We have had a preliminary course of training, as it were, both in the experience of our foreign contemporaries and in our own experience prior to and during the entry of the UnitedStatesinto the war. But there is a wide difference between the effect on the electric utility field of this country, with the nation in the status of a neutral with its industries independently meeting the demands of the domestic and foreign markets, compared with the conditions produced by the United States standing as a growing war power, collecting, directing and conserving its vast resources for the prosecution of the conflict.

War conditions are extraordinarily affecting every department of production and distribution of electrical power, and the results not only have to do with the present and immediate future, but the effects are likely to extend far beyond the termination of the war. How far these effects will reach into the future will depend not only upon the length of the war, but to a large extent upon their treatment during its progress, and how resolutely the situation is now faced by engineers, bankers, and government officials. It is therefore important for us to analyze these influences.

This paper deals more particularly with the effect of war conditions upon electric light and power service, but the principles relate in their broad application to every kind of public utility service.

The principal directions in which the effects of war conditions on electric service appear are:

1. In relation to operating:
    - (a) In increased salaries and wages paid for operating.
    - (b) In difficulty of retaining trained operatives, and conversely the need to operate with partially trained forces.
    - (c) In increased cost and difficulty of obtaining fuel and in reduction of its uniformity and quality.
    - (d) In increased cost of other supplies and materials for operation and maintenance.
    - (e) In the need for protecting the properties against enemy agents.
    - (f) In increased taxes.
    - (g) In possible decrease of consumption of electric power by ordinary customers.
    - (h) In possible changes of load factor.
2. In relation to extensions of plant:
    - (a) In the necessity in many cases for quickly caring for large accessions of permanent and temporary business.
    - (b) In increased cost over normal for plant required to care for additional business.

(c) In high cost for money and difficulty of obtaining it at any rate considered reasonable in normal times.

(d) In the difficulty of obtaining equipment in reasonable times of delivery.

Each one of the above factors has a direct bearing upon the cost and quality of electric service and any one of them arising under normal conditions would demand careful consideration and treatment, but when all of these factors arise coincidently and are affecting an industry which has been operating under circumstances requiring for stability and success the condition of steady and predictable markets for its purchases and sales, the results may be not only serious but even destructive if the appropriate measures of relief cannot be obtained.

The effects of the war conditions are being manifested not only in the matter of heavy increases in operating costs but also in the matter of extraordinary increases in cost for new plant required to care for added business. At first thought it might be concluded that the former apply only to the period of the war, but increases in wages for any cause usually result in all or a part of the increases remaining after the cause has been removed, also temporary changes in operating methods are likely to bring about modifications in the permanent methods of operation. The latter may increase the cost per unit of output incident to investment in plant, and therefore affect the cost of service not only for the period of the war but for the life of such plant, unless the increases in plant investment over normal can be amortized through increase in revenue during the period of the war and the period of readjustment which must succeed the war.

The effects of the war conditions have already increased the operating expenses of the electric companies of this country to the extent of over a hundred and sixteen million dollars per year, as hereafter shown. This points to the necessity of readjustment to the new conditions without delay, while at the same time requiring readjustment to abnormal labor conditions.

Any complete study of the effect of the increase in expense for operating wages and salaries on the operations of electric companies, caused by war conditions, is a difficult proceeding because every class of labor is represented, from the unskilled wage worker to the highest class of administrative official, and increases among these classes differ in their proportions, the highest paid men having received the smallest proportional increases, if any;

also because the wages paid for the same kind of labor as well as the increases involved vary throughout the country. Moreover, the growth of companies and accompanying increase in output per unit of labor, improvement in organization and in the science of economic labor application have tended to continually reduce the cost of labor required per unit of output, that would otherwise be required to produce equal or improved results. Although individual salaries and wages have been gradually increasing during past years, the labor cost per unit of service had been decreasing prior to the war. War conditions, however, have greatly affected this situation. The growing scarcity of labor in the ordinary occupations of peace, due to stoppage of immigration, demands for industrial workers for government work and drafts for the National Service, together with the necessity for enabling employees to meet the increased cost of living and the competition for retaining some experienced employees and of obtaining new employees, all combine to increase wage and salary scales. The employment of women has enabled the electric utilities to obtain an additional class of labor at a relatively low rate, but when women take the places of men, more women employees are required than the number of men replaced, and the war conditions tend to exhaust the supply of even this class of labor by offering wider fields of employment. The necessity for female labor to become more nearly self-supporting during war-times further tends to increase the female wage scale.

It is recognized that the labor cost to public utilities will further advance either by increases in the direct wage or by some system of war additions such as are being employed by the other nations at war. This system of war additions is generally recognized as a measure to compensate for the temporary increases in living costs during the war period and is adopted principally in order to facilitate the ultimate return to normal conditions.

It should be recognized that by normal labor conditions for the future, we do not mean the identical conditions existing prior to the war, but refer to the conditions which would normally have been attained had war conditions not intervened.

An analysis of the United States Census statistics shows that the increase in the average wages paid per employee (exclusive of general officers, managers and superintendents) during the ten years from 1902 to 1912 was 11 per cent. The actual increase in wages for like classes of labor may have differed from this figure, since the evolution of electric generation and distribution apparatus has shifted the relation of unskilled and skilled employees, but it plainly shows how abnormal are the sudden great increases in wages which have occurred as a consequence of the war.

During the war period thus far, salaries of officers, managers and general superintendents have in general not greatly increased but increases in wages in the operating departments have ranged from 15 to 50 per cent and we are of the opinion that 25 per cent may be taken as the average increase thus far occasioned by the war.

The total salaries and wages paid to employees of electric companies throughout the United States make up about one-third of the total operating expense, including under the latter heading the cost of power purchased and the renewals and replacement expenses, but not interest on capital debt. Under normal growth from 1912, at the rate indicated by the growth during the previous ten years we find that the salary and wage disbursements of electric companies in the year 1917, had there been no unusual disturbance, should have amounted to $90,000,000 of which one-seventh would have been for general officers, managers and superintendents' salaries and six-sevenths for wages.

From this it is seen that the increase in wages of 25 per cent means an outlay on the part of the electric companies of $19,000,000 for the year.

In the matter of retaining trained operatives, the government's draft for war services and the war scale wages offered in industrial fields both contribute to deplete the trained operating forces of the electric companies. We believe the government will and should, as a matter of good business, not unduly hamper the carrying on of a service so essential to its needs, but many of the finest electrical employees have voluntarily enlisted and the draft has taken a share of trained employees. Also, although the permanent employment in the public utility field tends to make it in that aspect more attractive than more or less temporary employment of war industries, yet many of the best employees of the electric companies have not been able to withstand the temptation of the higher wages obtainable in the latter, associated with the idea of being closer to war activities.

This situation is already being felt not only in increased wage scales, but also in cost for training new operatives, and the continual change in employees will necessarily be reflected in a reduction in economy of operation and in some reduction in the quality of service not only in the supplying of power but also in the accounting departments, and in the new business departments as far as they are maintained. The effects of this condition will become more and more apparent as time progresses, and the utilities must be prepared to meet the conditions of depleted forces and untrained operatives. We are unable to estimate the effects of this condition in terms of dollars and cents, but it is evident that it is a condition that will require no small consideration on the part of those responsible for the operation and service of the public utilities.

The electric utilities meet the largest single item of increased expense in the fuel account. The gross cost of fuel of a given quality is made up of cost at the mine, cost of transportation, and cost of labor for handling; and, due to differences in mining costs, length of haul and purchasing facilities on the part of the utilities, the gross costs as well as the increases have differed materially among the utilities, and throughout the country. In addition to tremendous increases in costs per ton, the utilities are confronted with the necessity of accepting coal of inferior and non-uniform quality, which results not only in lower efficiency due to reduction in heating value but also in lower efficiency incident to operating boiler equipments with continually changing grades of fuel. These conditions are now being keenly felt in the more modern highly efficient steam generating plants.

The cost of fuel has an extremely important bearing upon the total cost of electric service. Estimates based upon the United States Census reports show that this item of expense for all the electric companies in the United States would have reached $50,000,000 for the year 1917 under normal conditions of the country, and would have amounted to about 60 to 65 per cent of the normal generating expense. Definite information as to the amount of increase in fuel cost for the whole country is not available, but from information obtained from various sections of the country, we arrive at the conclusion that on the average the cost per ton of coal to electric companies has increased a little more than 100 per cent on account of war conditions, and that 100 per cent is not far from correct. On this basis the increase of total cost due to the increased price per ton of fuel is $50,000,000. A conservative figure for the increase in tonnage due to lower quality and non-uniformity of grade is 10 per cent, which means an added increase of $10,000,000, making the total increase $60,000,000.

Reliability of service has already been adversely affected in some instances by the fuel situation, and the importance of maintaining continuous service especially in congested territory and in industrial centers producing war materials is obvious. The size of the reserve coal supply required to assure continuity of service is dependent upon the rate of use and the dependability, frequency and regularity of deliveries. Present conditions of coal production and transportation point to the need for more than normal reserve coal supply and its importance we believe fully justifies the abnormal expenditures which have been necessary in the purchase of coal for the purpose of maintaining the supply.

An estimate of the output from steam driven electric central stations which might have been expected for 1917 under normal conditions shows 13,000,000,000 kw-hr., and an average requirement of three pounds of coal per kw-hr. of output, shows the fuel requirements would amount to not over 20,000,000 net tons, which is approximately 3 per cent of the estimated output from the mines for 1917. It is thus seen that a relatively large reserve supply of coal in the hands of every electric company would tie up but a very small part of the coal supply of the country and this supply would be widely distributed over the country and to a certain extent would be in proportion to the populations and industrial importance of the several sections of the country.

The normal cost of materials and supplies other than fuel, used in operation and current maintenance of electric properties, makes up no small proportion of the total annual operating expense, which we estimate as a little over 15 per cent. The percentage increases in the cost of such materials and supplies due to war conditions have been enormous and extremely varied in amount and the determination of the average amount of these increases imposed on electric companies is very complicated, but the indications are that this amounts to as much as 75 per cent. Such an increase in this expense means an increase in expenditures in the neighborhood of $30,000,000 over normal expense for 1917.

The increase in the cost of materials and supplies may be expected to cause operating companies to curtail their repairs and current maintenance to the greatest possible extent but such reduction cannot be large if satisfactory quality of service is to be maintained. In the matter of supplies contemplated for improvements and replacements of plant, however, the difficulty in obtaining equipment in reasonable times of delivery and the abnormal cost of any equipment that may be required naturally points to the advisability, and in many cases the necessity, of retaining present equipment in service until the exigencies of the situation make a change imperative, even though under normal conditions it would be advantageous to make the changes promptly, thus introducing an element of reduced economy in operation which under normal conditions would not be present.

The measures which must be taken to protect property from malicious interference by enemy agents comprise the development and maintenance of effective protective structures and lighting systems as well as special policing. The large capacity of individual units of generating equipment and the large capacity of transmission circuits of today increase the necessity for thorough protection owing to the large amount of damage that could be accomplished by an enemy agent if given the opportunity. The government action in restricting the activities of the enemy alien population is an important safeguard in this connection but the necessity for direct protective measures during the continuance of the war adds hundreds of thousands of dollars to the normal expense accounts of individual large electric corporations and runs into the millions in the total cost of service throughout the country. How many millions of dollars it amounts to we are not prepared at this time to say, but we expect the aggregate amounts to at least two or three million dollars.

In the matter of increased taxes we have purely a problem of caring for the increases over those of times prior to the war, and there is no way of predicting how large a factor this expense will become during the progress of the war or how far it will reach into the future. Estimates based upon the United States Census returns indicate that the 1917 taxes paid by electric companies might normally have reached $25,000,000. The proportion of gross revenue required for taxes has apparently been increasing year by year, having been slightly over three per cent in 1902, a little over three and one-half per cent in 1907, and nearly four and one-half per cent in 1912. Taxes on net income made up a very small proportion of the total tax in former years, so that changes in net income had little bearing on the amount of the taxes. This form of income taxation has had growing favor in legislative circles and is freely used by the government in its war tax program. An estimate of the amount of the expense which may be expected to be added to the cost of electric service throughout the country by the operation of the net income tax law is obviously dependent upon the effect of war conditions on net incomes, and the effects of "tax free" bond clauses and taxes on Excess and Undistributed Profits clearly cannot be estimated. We may hazard a guess that the increase over normal expense will lie between $5,000,000 and $10,000,000 for the year 1917.

Summing up the foregoing amounts shows that the extra expenses now imposed on the electric companies on account of war conditions amounts to the immense aggregate per year, as follows:

| | |
|---|---|
| Increased salaries and wages chargeable to operating. | $19,000,000 |
| Increased cost of fuel | 60,000,000 |
| Increased cost of other materials and supplies | 30,000,000 |
| Increased taxes | 7,500,000 |
| | $116,500,000 |

This amounts to a quarter of the normal estimated gross revenue for 1917 of all the electric companies and it wipes out two-thirds of the sum that would have been available for interest, dividends and surplus. It does not include additional expenses caused by the difficulty of retaining trained operatives and the cost of protecting the properties against malicious interference, the magnitude of which we are unable to estimate. It puts the electric companies in a critical position, which is rendered more ominous by the impossibility of foretelling how much larger these extra expenses may become in future months.

It is to be expected that the American people will come to more fully appreciate the need for economies in every direction as the war progresses, and that their consumption of electric power for residential and ordinary commercial lighting will become materially reduced. This effect is now just beginning to be felt by the electric companies. It can result in only a relatively small reduction in the outputs of the electric companies but to that extent it will liberate generating capacity for use in war service and will conserve some fuel. It will, however, liberate relatively little distribution plant which can be used for war demands and will have substantially no effect upon total distribution expenses, so that a very large part of the reduction in gross revenue arising from such economies will appear as a reduction in the net income. As an economic consideration of the war such economies are to be earnestly encouraged, but it is nevertheless true that so far as the electric companies are concerned they mean reduction in net income almost equal to the amounts by which the gross revenues are affected. It is not possible to estimate what the effect on the companies' revenues will be except to say that it apparently will run into the millions.

Before considering further the operating phases of the situation we will take up the effect of war conditions on extensions of plant since the two phases are so closely interrelated.

The effects of the war conditions are being manifested not only in the matter of heavy increases in the operating costs already pointed out, but also in the matter of extraordinary increases in cost for new plant required to care for added business. At first thought it might be concluded that the effects of the latter apply only to the period of the war, but the increased cost of new plant per unit of capacity manifestly affects the cost of service not only for the period of the war but for the life of such plant, unless some measures can be devised to amortize the excess first cost of new plant over the first cost of like plant in normal times.

Many electric companies are now confronted with the necessity of not only caring for their normal growth of business, but have had thrown upon them large demands for power arising from the increasing expansion in the manufacture of material of war, which includes almost every necessity of life, ranging from shipping to textiles and food stuffs, in addition to special war products like arms and ammunition. On this account the electric companies have found themselves obliged to prepare very rapidly to meet increased power demands of several hundred thousand horse power over their normal requirements.

When viewed from every standpoint, it will be seen that the economical central power generating station is the proper medium for the supply of the large power requirements arising on account of the war, since purchased power leaves the manufacturers of munitions and other war material free to devote their energies to the development and operation of their manufacturing plants without diverting any of their energies to the development of power plants or their operation; it provides the greatest possible amount of available power from a pound of fuel; it reduces the total capacity of power generating equipment necessary by taking advantage of the diversity between the power requirements of the various manufacturing establishments; and it provides the greatest possible amount of available power per man occupied in the power generation field and thus diverts the least possible man power from the other needs of the nation. It also makes it possible for the power equipment manufacturers to concentrate upon the production of the larger units of equipment used by central electric stations, thereby making it possible to produce the largest capacity of equipment in the least time. And the centralization of electric power generation equipment, with comprehensive distribution systems. places it in position to most effectively care for the changes in power requirements of industrial establishments to be expected after the termination of the war.

These advantages of the central station power are so large that it is advisable for the Government to use every reasonable means to encourage the central station companies and discourage individual power plants during the war period.

It is to be expected that much of the business arising from the war will prove of a temporary character, so that the electric companies are faced with the double dilemma of finding themselves called upon to provide capacity to care for extraordinary accessions of new business, at costs for the equipment and installation far above normal prices, and this in the face of the fact that it is to be expected that in many cases some of the business will prove of temporary nature without equivalent other demands to take its place after the war.

The electric companies have so far provided for the needs of the situation splendidly, but this has been done largely at the expense of reserve capacity, as the financing of almost no large extensions, if any, has been accomplished since our country become a party in the war. There can be no doubt that the power demands on account of the war have not yet nearly reached their maximum and it will be necessary to devise some way by which it will be possible to care for these demands. This can be accomplished by the adoption and promulgation by the government of certain war measures; one being to make possible the discontinuance or curtailment when advisable particularly at the peak load periods of the electric companies, of power and lighting demands not necessary for prosecution of the war or for the safety and reasonable necessities of the population, and another being for the government to assume the responsibility for seeing that money for extensions required for war service can be obtained on reasonable terms.

The first named measure would also operate in the interests of fuel conservation.

We may show the effect of war prices on the electric light and power business in the aggregate. The United States Census of central stations shows that the total of the revenues received from operation and other sources by all central electric light and power systems (including both hydraulic stations and steam stations) in 1912 was in round figures $302,000,000 and the total operating expenses, including taxes, and renewals and replacement expense, but not including interest on debt, was $184,500,000, leaving a total income of $117,500,000. The reported cost of construction and equipment was $2,176,000,000. Extension of these totals to the present year 1917 shows that under normal growth the total revenues in 1917 would have reached $475,000,000 and the operating expenses, including taxes and renewals and replacement expense would have reached $290,000,000 making the total income before deducting interest on debt, $185,000,000. Estimating the reported cost for construction and equipment would have grown to $3,500,000,000, an increase of 60 per cent in five years, the above income would represent 5.3 per cent of this cost of construction and equipment. If no other factors entered into the problem besides increases in cost of operation, and assuming these increases effective over the whole year, the fuel expense as before pointed out, would increase $60,000,000 for 1917, other supplies $30,000,000, labor expense $19,000,000 and taxes $7,500,000, representing an aggregate increase of operating expenses for these items of $116,500,000. This is an increase of 40 per cent in operating expenses, and it reduces the divisible income to $68,500,000, which amount is equivalent to less than 2 per cent on the cost of construction and equipment. This percentage is still lower in the case of the steam-electric systems of the country taken alone, and additional expenses for training new employees and the lowered efficiency of such employees, the cost of special policing, etc., reduce the amount still farther.

There are certain operating economies and changes which might be adopted by the companies if forced to it by war conditions. The service rendered the customers might be curtailed for example by discontinuance of lamp deliveries and free minor repairs, by less prompt attention to troubles and complaints, thereby reducing the number of "trouble" employees, by bi-monthly or tri-monthly billing and the like; the canvassing and promotion departments might be almost entirely dispensed with, with consequent reduction in meter testing and setting expense; and advertising expense might be eliminated. In the storeroom, increased cost for material points to more than merely salvaging scrap material. It forces the use of old and shopworn supplies and non-standard equipment, and previously used wire and cables. While the use of such equipment may render the service less reliable, it may become an unfortunate necessity.

The current expense accounts may also be reduced by postponement of plant repairs which would normally be made at once; but it is well recognized that the longer repairs are put off the more they cost, and while current expenses may for a time appear lower, the cost in the long run is apt to be increased.

The net results of such economies might amount to as much as 10 to 15 per cent of the normal operating revenue of the electric companies, and every consideration should be given to them, but they are offset by increases in expense which have not been included in the amounts named.

There is a phase of the situation which has not been referred to in the preceding but which in some cases is already being felt, and is likely to be generally felt by electric companies. This is the improvement in load factors arising from putting the less essential industries on an off peak basis and the fact that war industries give an exceptionally long hour demand, owing to the intensive character of war manufacture. This produces two quite distinct results, one being in the direction of reducing the maximum loads which may be carried, particularly on underground cables and to some extent upon generating equipment, and the other being in the direction of decreasing the expense, other things being equal, per kilowatt-hour of total output. How much effect the first result may have it is impossible to predict but it may have marked effect in cases where the load factors become very high, and it is already showing.

A part of the increased cost on account of war conditions has been provided for by many companies by the introduction of coal clauses in contracts for large light and power service. These clauses have taken various forms from the simplest in which the increased (or diminished) charge is figured at a given amount for each dollar of change from a stated cost of coal per ton, to the more elaborate form in which the increased charge is figured upon the relation between the present day cost and duction in economy of operation and in some reduction in the quality of service not only in the supplying of power but also in the accounting departments, and in the new business departments as far as they are maintained. The effects of this condition will become more and more apparent as time progresses, and the utilities must be prepared to meet the conditions of depleted forces and untrained operatives. We are unable to estimate the effects of this condition in terms of dollars and cents, but it is evident that it is a condition that will require no small consideration on the part of those responsible for the operation and service of the public utilities.

The electric utilities meet the largest single item of increased expense in the fuel account. The gross cost of fuel of a given quality is made up of cost at the mine, cost of transportation, and cost of labor for handling; and, due to differences in mining costs, length of haul and purchasing facilities on the part of the utilities, the gross costs as well as the increases have differed materially among the utilities, and throughout the country. In addition to tremendous increases in costs per ton, the utilities are confronted with the necessity of accepting coal of inferior and non-uniform quality, which results not only in lower efficiency due to reduction in heating value but also in lower efficiency incident to operating boiler equipments with continually changing grades of fuel. These conditions are now being keenly felt in the more modern highly efficient steam generating plants.

The cost of fuel has an extremely important bearing upon the total cost of electric service. Estimates based upon the United States Census reports show that this item of expense for all the electric companies in the United States would have reached $50,000,000 for the year 1917 under normal conditions of the country, and would have amounted to about 60 to 65 per cent of the normal generating expense. Definite information as to the amount of increase in fuel cost for the whole country is not available, but from information obtained from various sections of the country, we arrive at the conclusion that on the average the cost per ton of coal to electric companies has increased a little more than 100 per cent on account of war conditions, and that 100 per cent is not far from correct. On this basis the increase of total cost due to the increased price per ton of fuel is $50,000,000. A conservative figure for the increase in tonnage due to lower quality and non-uniformity of grade is 10 per cent, which means an added increase of $10,000,000, making the total increase $60,000,000.

Reliability of service has already been adversely affected in some instances by the fuel situation, and the importance of maintaining continuous service especially in congested territory and in industrial centers producing war materials is obvious. The size of the reserve coal supply required to assure continuity of service is dependent upon the rate of use and the dependability, frequency and regularity of deliveries. Present conditions of coal production and transportation point to the need for more than normal reserve coal supply and its importance we believe fully justifies the abnormal expenditures which have been necessary in the purchase of coal for the purpose of maintaining the supply.

An estimate of the output from steam driven electric central stations which might have been expected for 1917 under normal conditions shows 13,000,000,000 kw-hr., and an average requirement of three pounds of coal per kw-hr. of output, shows the fuel requirements would amount to not over 20,000,000 net tons, which is approximately 3 per cent of the estimated output from the mines for 1917. It is thus seen that a relatively large reserve supply of coal in the hands of every electric company would tie up but a very small part of the coal supply of the country and this supply would be widely distributed over the country and to a certain extent would be in proportion to the populations and industrial importance of the several sections of the country.

The normal cost of materials and supplies other than fuel, used in operation and current maintenance of electric properties, makes up no small proportion of the total annual operating expense, which we estimate as a little over 15 per cent. The percentage increases in the cost of such materials and supplies due to war conditions have been enormous and extremely varied in amount and the determination of the average amount of these increases imposed on electric companies is very complicated, but the indications are that this amounts to as much as 75 per cent. Such an increase in this expense means an increase in expenditures in the neighborhood of $30,000,000 over normal expense for 1917.

The increase in the cost of materials and supplies may be expected to cause operating companies to curtail their repairs and current maintenance to the greatest possible extent but such reduction cannot be large if satisfactory quality of service is to be maintained. In the matter of supplies contemplated for improvements and replacements of plant, however, the difficulty in obtaining equipment in reasonable times of delivery and the coupled with increased cost for supplies and labor increase the expense per unit of output far beyond the point where it may be neutralized by any economic practises of the supplying company. The result has been quite a universal increase in rates, in some cases flat percentage increases of the same amount for light and power, in other cases differing percentage increases for light and power, and in still others increases depending upon changes in cost of fuel. These flat percentage increases have varied from less than 10 per cent to as high as 50 per cent over the rates in effect prior to the war, London rates having been increased 50 per cent according to the London *Electrical Review*.

As to the effect of war conditions in this country on ordinary domestic and commercial light and small power consumption for other than manufacturing purposes, the rate of adding new residence and commercial customers is on the whole falling off due to personal economies of the public and generally fewer extensions of lines by the companies, and to decrease in building activities, and the like. It does not appear that the lighting customers in this country have as yet inaugurated any general lighting economies sufficient to be seriously felt by the companies, and the companies have on the other hand shown a disinclination to increase the rates to small consumers. The drain on our resources will, however, be felt more and more as the war progresses, and it is to be expected that the domestic and commercial customers will decrease their consumption of current.

Government action toward conservation of our resources during the war may result in the adoption of plans such as the "Day Light Saving Plan" now in considerable use in Europe. The effect of such action on the cost of electric service is substantially the same as results from voluntary economy practises, in that the use of current is reduced without full equivalent reductions in expense for generating and distributing plant required or in the expenses connected with operation. While some forms of rates in common use are designed to return, first, as much as practicable of those costs of service not dependent upon the quantity of current used, commercially applicable rates cannot be so nicely balanced as to make the appropriate returns under all conditions. It is obvious that any action directed toward general lighting economy, while conserving our national resources and possibly releasing some plant for uses more essential to carrying on the war, results in higher relative cost per kilowatt-hour of current furnished and this points toward higher rates to the public.

It is evident that increased expense for service arises in every department of the business; in operating labor and supplies and taxes, in protection of the property, and in cost for extensions of plant. And the latter is not only affected by abnormal first cost for equipment and its installation but also by the present difficulty of obtaining money for such purposes at other than exorbitant rates as compared with normal. In the matter of labor expense the cost of service is increased not only on account of increase in wages but by the need for training new employees to the service and the necessity of operating with partially trained employees. These last conditions are likely to be increasingly felt with each new draft of men for the armed forces or other direct government service. The increases in cost of electric service on account of the war conditions are so great that rates for service which were equitable at the beginning of the war, are in some cases now not covering the operating expense, and where companies are being loaded with war business the new business in many cases may become a serious menace to the company, which can only be overcome by taking into account the war conditions in determining the rates to be charged.

Notwithstanding that wages of operating men are higher than ever before, yet the necessity for using many partially trained men in the operation and inspection of electrical systems makes for a lowering of reliability of service and this coupled with the need in some cases to utilize all equipment to its limit with reduced reserve capacity has a further effect in this direction as well as in the direction of reduced operating efficiency. Also it is to be expected that with present conditions added transmission lines will not be installed if the present circuits can possibly carry the loads, even though under ordinary conditions additional circuits would be promptly provided, and like considerations apply to distribution systems with a consequent tendency toward reduced efficiency and a wider range of voltage variations than is now considered good practise. It seems proper that regulatory bodies should take into account these considerations in their requirements for electric service during the period of the war.

It should be recognized that the electric companies while serving the best interests of the public broadly may in some cases be obliged to operate with impaired forces and to utilize their equipment during the war to such an extent that they may be unable to fully maintain present standards of service.

## DISCUSSION ON "EFFECTS OF WAR CONDITIONS ON COST AND QUALITY OF ELECTRIC SERVICE" (GOODMAN AND JACKSON); BOSTON, MASS., JANUARY 8; NEW YORK, N. Y., JANUARY 11 AND CHICAGO, ILL., JANUARY 14, 1918.

### DISCUSSION AT BOSTON

**B. A. Behrend:** The question of conservation of limited natural resources should be viewed in this crisis differently from the way in which we have looked upon it before the war. It is more essential that work be done and results accomplished than that resources be husbanded. It is equally essential that people should realize that the waste of coal for heating purposes in public buildings, hotels and railroad trains during a spell of warm weather takes the coal which should be used during a time of extreme cold. Now is the time to urge the reduction of temperatures in places where human beings aggregate to 65 deg., and orders to this effect should be issued by the Fuel Administration. A man or a woman should be marked as unpatriotic who does not care to comply with so obvious and sane a regulation.

I suggest the drafting of all men below the age of forty-five. If they cannot as yet be used for any government activity, they should be held in readiness, and once drafted and properly "labelled" they should be under the same obligation and responsibility as any soldier. Besides its being unpatriotic to strike or refuse to undertake work assigned or work to be done in the regular course of business, it should be made an offense. Thus, strikes of painters or carpenters or tool makers or moulders should be made impossible by an enrollment of every man under forty-five years of age or even fifty years of age.

The nation and the hammer blows which this nation must deal cannot have the strength of the whole American nation unless every man does his duty. However important economy is at this time or at all times, the increase of production is of even greater importance to the vital issue.

**Mr. Sykes:** I approach this subject from a somewhat different standpoint than Mr. Jackson as my interest has been centered with the consumers of power. During the last few years I have had a good deal of work to do in trying to persuade large industrial plants to use central station power and the progress that has been made in this direction in the last five years has been very marked. I have been particularly interested in power for steel mills where there are very large customers who think nothing of a 5000 or 10,000-kw. load. All their units are large power consumers. Now the situation at the present time from the standpoint of the consumer is a very disturbing one and some of the plants in which I am interested have been very much hindered in their operations due to the irregularity of central station service. Now those of us that are connected with the electrical business understand some of the problems of the central station. However, the customers of central stations very often do not. During the past few years we have been gradually educating the large consumers of power to buy central station energy. We have been told that the one thing that did not go up in price was central station power, whereas coal and everything else went up. Now, we must recognize under present conditions that there must be an increase in price for power. I think large consumers in time will readily pay whatever is a reasonable amount when the central station makes out its case.

From the standpoint of the power consumers, the central stations must run. They do not care how the central station does it, but they must have current to run their mills. It is particularly important at the present time to remember that we are facing interruptions of power all the time, this at a period when we can least afford to have such interruptions. New industrial plants are springing up all over the country. Most of them have not the time to build power stations, nor could they get the equipment. The central station must carry the load necessary to manufacture munitions.

Speaking of maintenance I think we will find before long that electrical manufacturers are going to be in a very serious condition when it comes to supplying repair parts to maintain central station service. Unless some action is taken by the Government or other agency to keep the electrical manufacturers running enabling them to take care of their customers, especially when machinery breaks down, it will not be a question of a few days or weeks, but we will be lucky to resume operation at all. Anybody with electrical manufacturing companies can see this in the offing. In Germany, of course, there are harder conditions than we face for they cannot repair plants. There is no necessity for us to reach that state, but people like electrical manufacturers seem to have all kinds of burdens thrust upon them. Their plants are being turned into munition plants and the electrical situation is pushed into the background. Unless we can hold all organizations together and make parts as they may be required, I think there will be times when very serious shutdowns will occur.

I do not see how load factor is going to trouble us very much, although the load is certainly going to change round. In relation to off peaks. I am from Pittsburg and we do not have much in the way of off peaks there. We are working nights now. We have had our meatless days and our wheatless days, but we have also had powerless days. However, that is a thing which is being corrected with the aid of the United States Government.

I cannot see how central stations can increase their efficiencies at all with the whole industry being rearranged. I can only see decrease in efficiency. All manufacturers are facing a decrease in efficiency. We are losing some of our best men because of the war and we cannot replace these men by poor men and at the same time get efficiency. We will face a comprehensive decrease in efficiency all along the line.

**J. W. Cowles:** I can hardly undertake to speak from the standpoint of generation or even distribution of electric power to-night, but my thoughts naturally turn to that phase of the business which we term "Service." We know that to the central station industries to-day the manufacture and distribution of our products, important as these phases of business are, are yet by no means the only ones and that during the past few years we have been coming to the point rapidly of realizing that it is service which we are selling in the broadest sense, and that there is a great deal to do with our service rendered than merely the product of generation or manufacture. The problems confronting us interest me particularly in the economies that can be introduced and must be introduced in connection with our service to our customers. I am passing by the generation and distribution problems simply because they do not happen to be quite as near to me as service. When it comes to service rendered to 100,000 or more customers the problems are gigantic and to me the problem to-day is how far can we and must we curtail below the standards formerly considered essential and all-important. We are all seeing very marked curtailments in service rendered. Grocery stores not only cease to come around to take our orders, but they have cut their deliveries in two or more. The department stores have done likewise in all directions. How far must we go in that same direction toward reducing our service efforts, all of which cost money and add a very considerable increment to the operating expenses of companies. Those of us who depend on train or electric railway transportation certainly know to their sorrow what the curtailments and decreased service privileges are in every direction. We may not accept all of these curtailments gracefully, as we all doubtless do our share of complaining and growling on our way to and from the office, but at the same time we all appreciate that it is the inevitable and that all of our criticisms against the particular railroad or the particular trolley line that we may be patronizing are unwarranted. We are coming to recognize the fact that we must accept a little more graciously these hardships, and they are nothing less than hardships. So in our own line of business it seems to me that we must accept the inevitable and curtail our service efforts trying to cultivate in the minds of our friends that they must expect less from the electric light companies in the same way that they are expecting less from other industries or concerns with which they have dealings. No one appreciates more than the central station man these problems of to-day and the study that is being constantly given to them certainly bespeak the patience and consideration of all our friends and acquaintances during these most perplexing times.

**C. A. Adams:** There is one aspect of this subject which is immensely interesting to me, namely, its broad aspect. We are dealing with a problem which is not in the ordinary sense technical; it is largely an economical problem. We find more and more that we can no longer be mere engineers in the old sense of that word but that we must be citizens in the broadest sense and take our part in the solution of the big social, economic and political problems, many of which are the direct result of the work of the engineer, and can be best understood by the engineer. This paramount duty is being emphasized by war conditions and should be faced with courage and fair-mindedness. This is the time for men with vision. Who more than the engineer should have this vision?

There are two or three little points in the paper which have caught my attention. The author says, "A conservative figure for the increase in tonnage due to lower quality and non-uniformity of grade is 10 per cent, etc." I think that this is a very considerable under estimate of the percentage increase. Coal as it comes from the mines now is not picked over as it used to be; the percentage of ash has gone up in some cases from 6 per cent to 10 per cent and even to 20 per cent. This means not only an increased tonnage in proportion to the increase in ash, but also lower boiler efficiency and larger boiler capacity. I have heard it stated very recently by central station men that this increase in ash, slate, stone, etc., in the coal and the resulting lower boiler efficiency were sufficient to increase the total consumption by 50 per cent rather than 10 per cent and that the number and capacity of boilers in operation sometimes was greater by about 40 per cent as comppared with the ordinary conditions of a few years ago when the coal was of superior quality. I merely pass these figures along as they do not come from first hand experience.

There is another figure quoted concerning which there is some question in my mind and that is the average of four pounds of coal per kilowatt hour. With our enormous central stations operating at so much higher economy, I should be inclined to put this average at nearer three and one-half pounds.

**W. B. Jackson:** As to the situation in Europe, and how these problems have been met; I cannot speak of them in much detail particularly as to Germany as I find it very difficult indeed to obtain authentic information. In the case of England, the Government has not taken any official stand in the matter, but as I have stated, prices for power have been raised very greatly indeed. Several of the largest companies in London having had their prices raised 50 per cent flat. In Birmingham the prices have been raised 30 per cent flat, with a possible later additional raise in price.

In regard to the isolated plants; it would be rather presumptious to say that all isolated plants are necessarily going to disappear. This question was discussed in the paper, where you will remember we referred to the fact that probably three-quarters of the boilers used in the isolated plants may be considered as not necessary for heating purposes. It is certain that there are a very great number of isolated plants today that have no economic or other proper reason to exist.

### DISCUSSION AT NEW YORK

**F. A. Bryan** (by letter): I think Mr. Goodman covers the subject very thoroughly. Plants operating wholly by water power are not, of course, affected to as great an extent as those operating by fuel alone, while plants operating with water power, with steam reserve, as is our plant, are affected very considerably, although not as much as if we depended entirely on fuel. Our situation is one where we depend both on steam and water power, the steam plant having been until quite recently almost entirely a reserve and being built as an emergency or reserve plant, we did not go into the refinements for cheap operation required in a continuously operated steam plant. The interest on the investment for such a plant over-balanced the cost of occasionally operating the plant to its full capacity. Now we are confronted with this condition. Due to the high cost of fuel, customers who depended partially on our service have increased their demand materially and manufacturers who have not heretofore used our service at all have immediately demanded service. It has placed a load on us far in excess of our anticipations, increasing very materially the amount of energy required of our steam plant and we are, therefore, compelled to use larger quantities of fuel than ever before and perhaps are not in a position to use it as economically as we could had we anticipated and prepared for such demands. I think that you will find this situation true of all combined hydraulic and steam plants. The same argument can be applied to the steam-heating end of the central station business. Where two years ago our New Business Department put forth their greatest effort to have our mains loaded up, we now find that those residing along our mains are not only requesting steam heat, but demanding it and at a time when the price of steam is practically equal to the cost of fuel required to develop it.

The fact that conditions exist as outlined in Mr. Goodman's paper are undebatable. The question with the central station operator today is "What is the remedy?" Business is being presented to him unsolicited in greater quantities than ever before and his plant is being abnormally loaded, while at the same time, his ability to secure additional capacity is almost prohibited both on account of cost and delivery of equipment.

Without this additional equipment, *Service*, such as he has so long boasted of is bound to be impaired and with impaired service, even during war conditions, what is going to be the effect upon his ultimate business? Our position in this matter where large customers are now coming on our lines, is to advise them that in case of any interruptions or lack of capacity on the part of our plant, they must necessarily be the first ones to have their service discontinued but even this, of course, will not remedy the possible interruptions of service which our old and regular customers are bound to have.

Assuming that the central station can purchase additional capacity to take care of increased demand, the cost as compared with costs a year ago, taking into consideration the actual cost of the additional equipment, plus the increased cost of money, is approximately twice the cost under normal conditions. With the State permitting a 7 per cent and 8 per cent return on value under normal conditions, what is going to be the condition of the utility when times return to normal and the value of equipment inventoried is appraised at about one-half of cost. In other words, unless the States change their attitude, the central station will actually lose money on such betterments. In the face of such a possibility, is the central station justified, even if they can get delivery on equipment, in purchasing under existing conditions in order to take care of the increased demands? Furthermore, is a central station, as long as they have auxiliary equipment, under existing rates and costs, justified in loading up this equipment with this abnormal business to the detriment of their service to their regular customers?

I do not see any remedy for the predicament of the average central station today other than the raising of rates to offset the tremendous increase in cost. Nor do I see any legitimate reason why commissions should refuse to increase the rates to offset these abnormal increases and I believe that the public will not protest against a legitimate increase in rates when the central station can show that their costs have so tremendously increased. The increase of rates, as I see it, is the only way in which the public utility can expect to survive the present abnormal conditions.

**R. G. Hudson** (by letter): It is a well established fact that the purchasing power of a dollar is less in time of war than in time of peace. The sale of any commodity during war time at a peace price infers that the commodity costs no more to produce, that the commodity was made before the war, that the commodity is of inferior character, or that the producer is content with a smaller profit.

The cost of producing electric energy is unquestionably greater now than in normal times, none of the electric energy now offered for sale was made before the war and it is obviously impossible to offer an inferior brand of energy. The continuance of the normal rates for electric energy during war time therefore implies that the producer must receive a smaller profit.

In the paper just presented it is estimated that the average return in 1917 on the capital invested in public service plants supplying electric light and power must be less than 2 per cent while in normal times the return would have been over five per cent. The directors of corporations operating plants under average or worse conditions are then confronted with the task of deciding whether the stockholders are not entitled to a greater return upon their investments, how long their plants may be operated under present conditions with any balance of profit and, if the revenue is to be increased, how to accomplish that end.

It is reasonable to presume that our public service commissions must sanction a higher return to investors than can be given to them in some cases under present conditions. The attitude of the federal government is reflected by the recent recommendation of the President that the railroads under government control should receive as compensation the average of their net operating incomes during the past three years. Directors would then seem to be supported by representatives of the public in any effort to make a fair return to the stockholders during the war.

To operate a public service plant with the revenue barely exceeding the operating cost under the present conditions of steadily advancing costs of labor and materials would furthermore be precarious and unwise since public service corporations do not enjoy the privilege accorded the private corporations of accumulating large emergency reserves from current revenue. There is then no alternative in critical cases but to increase the revenue.

In view of the prevailing tendency of the public to economize in all expenditures an increase in rates does not necessarily mean a proportional increase in revenue since it also increases the tendency to use less power. In conjunction with an increase in rates an earnest effort must be made to increase the number of consumers taking off-peak service.

The ideal solution of the problem would be obtained by increasing the number of consumers to the extent that, with each consumer taking less load than usual, the total load on the system would be unchanged and the increased revenue would be proportional to the increased rate. Such a solution takes into account the psychological effect of an increased rate on the consumer in times of enforced economy, the necessity for the conservation of coal, the desirability of maintaining the status quo of the plant, and the problems associated with the reconstruction period after the war. The promotion department was never more important and the field of promotion, while narrowed by an increased rate, must constantly broaden as the condition of the isolated plant becomes more critical.

The author of this paper also raises the distinct and separate question regarding the manner of meeting the threatened shortage of coal and labor. To adjust this situation the public must be instructed openly regarding the existing conditions and the possible consequences. Federal authorities must be convinced of the need for coal and exemption boards of the scarcity of skilled labor. In the absence of governmental regulation skilled laborers who leave their positions to enlist or to take up more remunerative work may only be restrained from such action by building up popular sentiment against it. The watch engineer and the switchboard operator must be made to feel that they serve their country as effectively in their accustomed positions as among the military forces. The opportunity should be given such essential workers to enlist in an industrial Legion of the United States and to wear appropriate insignia indicating their loyalty to the nation.

**Mortimer Freund** (by letter): The central electric generating station industry is so vital a feature of our economic life that its continued welfare is of prime importance during the critical times which now confront us. Our determination to "win the war" can only be realized by placing and maintaining in healthy condition the constituent parts of our social and economic structure. If the central station industry is up against it, to use the vernacular, or approaching this state, it behooves the engineering profession to lose no time in investigating the causes and the remedies and to make sure that the latter are applied without delay.

With this in mind, I do not believe it proper to allow to pass unquestioned the authors' statement that "the economical central power generating station is the proper medium for the supply of large power requirements arising on account of the war", and further that "it is advisable for the Government to use every reasonable means to encourage the central station companies and discourage individual power plants during the war period." Both of these statements are by far too sweeping in character and, as a matter of fact, are only justifiable for such cases, where a careful and disinterested consideration of all the circumstances, will warrant such a conclusion.

It seems to me that for the second statement might better be substituted the following: It is advisable for the Government to encourage all consumers of fuel to use every effort to fulfill their heat, light and power requirements by such means as will utilize fuel most economically and to do away with all existing wastes which are preventable.

There are industrial plants for example, where a great part if not all of the electricity used is virtually a by-product, due to the utilization of exhaust steam from the electric generating units. I have in mind a large industrial plant which up to 1915 operated two separate boiler plants, the output of one of which was utilized almost entirely in pipe coils for drying. The exhaust steam from the electric generating plant was wasted in the non-heating season and only partially used during the heating season. Substitution of purchased electricity had been suggested. Since 1915 the exhaust steam has been utilized in place of live steam, the use of one of the boiler plants has been discontinued and the actual cost of electricity in view of the use of the exhaust steam is far less than the best price which outside service can offer. The action on the part of the management of this mill in undertaking the changes necessary to permit of this, has not only paid them well on the investment made, but has benefited the country to the extent of reducing their fuel consumption. Surely the authors do not recommend that the Government discourage such an individual generating plant, although such a conclusion might be drawn from their statement.

This calls for a pertinent inquiry as to whether or not certain of our central stations are in an unhealthy condition because they are suffering from "indigestion" caused by "biting off more than they can chew." In recent years a number of the central stations have been very energetic in conducting campaigns for the shutting down of private power plants. This procedure in itself is quite commendable, if carried out on a sound basis. But the sale of electricity at ridiculously low and unprofitable rates merely for the purpose of swelling the list of "Private Power Plants Supplanted" to appear in advertisements in the daily press does not appear to be based on sound ecnomic principles. Cases have been recently reported where even industries necessary for the conduct of the war have been hampered due to their having abandoned their private plants for central station service, which is now unable to meet the increased demand.

An engineer, who recommends to his client the installation of a private power plant, where conditions do not so warrant, merely for the sake of increasing his professional fee, is very properly denounced by reputable engineers in a no uncertain manner. A man who installs a private power plant, merely as a show place, where sound engineering does not warrant such a plant, is wasting the natural resources of the country and is to this extent actually an enemy of society. By the same token is the central station management which encourages the shutting down of private plants just for the sake of increasing its own prestige, irrespective of the fact of whether or not real economy is secured (and I include under this term the consideration of continuity of service), no less an enemy of society, because it is a business policy to do so.

I hold no brief for the private power plant. There are unquestionably many private power plants now in operation which should be supplanted by central station service. It has been my own practise to recommend central station service in all cases, excepting those where the installation of a new private power plant or the continuance of an existing plant will show a substantial saving in the cost of operation or other advantages of substantial worth. The private power plant is not an obsolete idea, as some have been trying to tell us. There are cases where it results in the most highly efficient production of heat, light and power possible. Each case should be decided upon its own merits and not on the basis of sentiment. Sweeping generalities are in my opinion unwarranted and misleading.

Central station service and the private power plant should not be in conflict, especially at the present time. Each has its proper sphere and conditions may have arisen or will arise in particular instances to cause one service to supplant the other. It has been and will be to the mutual advantage of many central stations and private power plants to co-operate, by exchange of service at different times of the day or during different seasons of the year. Friendly competition and co-operation should exist between central station management and the management of private power plants, so that maximum efficiency in operation may be maintained and the greatest benefit accrue to the general public. This is a policy in keeping with the spirit of the times and necessary for the conservation of our resources.

**B. J. Arnold:** This whole question, as you all know, is a very serious one. I was talking today with the president of one of our railroads, and he told me that they were within three hours of having to shut down the railroad at one time, on account of a shortage of coal. They fortunately got relief, but they have to shut down sections of the track now in order to keep the service going. I have no doubt that similar conditions exist with other companies in the vicinity of New York and elsewhere, although I do not happen to know the details of those cases. Consequently, it behooves every one of us to do everything we can to conserve what fuel we have or can get, and every manufacturer of electric current I have no doubt is doing his best to make every pound of coal go as for as he can.

One possible way in which the manufacturer of current can help the situation—and incidentally I may say I am not the first man to think of it, but I might emphasize it—would be, with the consent of the public service commissions—because I surmise that consent will have to be obtained—to cut off certain kinds of service during certain periods of the day, as one of the authors has pointed out, and run manufacturing plants, engaged on munitions work, and any other work that can be utilized, during the daylight period, so that the capacity of the plant can be utilized at night for lighting; in other words, distribute the capacity of the plant over the twenty-four hour period by dividing up the classes of work into the kinds of work that can be utilized in the respective periods of the day, and take full advantage of the total possible capacity of the plant.

Some of us who have to do with electric railroads buy power from a different source, and we can buy power from our relay systems, our emergency sources, during certain periods of the day for a much lower price than we can during the peak load, and in that way we can utilize at reasonable cost what surplus power we buy during these light load periods or daylight periods. That principle, applied in a general way to the power producing properties and the power using industries will help out in the general situation.

That is not anything new, but is something we may have to do. The greatest thing we have to consider today is what will we do with our railroads on account of the transportation problems, and I want to emphasize this point. Some of us have been talking, for some time, upon the necessity, or advisability, at least, of Government control of the operation of the steam railroads, and I may say that we thought that applied also to electric railways, but not to such an extent, for the reason that the electric railway companies have shown more of a disposition to cooperate and coordinate their interests than the steam railway companies have; but the great lesson we have to learn from the presidential action which has recently taken place in the Government taking over the operation of the railways is that it creates conditions whereby the steam railway companies are going to learn by a great example the advantages of coordinating their interests and operating their terminals jointly; that is, taking the freight and passenger service from one set of tracks, which are heavily laden, and placing a part of it on others that are less heavily laden, and getting the advantage of utilizing the entire railroad investment of the country to the best advantage, and avoiding additional lines or extensions, where they have heretofore been thought to be necessary, to enable the railroads to compete with each other. That proposition will apply to trunk steam lines everywhere.

We have been laboring in Chicago for years to get the steam railroads which enter that city fully to utilize the main trunk terminals tracks as they now exist rather than to attempt to individually extend or widen them, and we are making some progress. As you know there is another such situation right here in New York. In other cities which we have gone into in the past we have also found this disposition on the part of the railroads, to build side tracks into new industries to get their portion of the business. There are certain cases where the steam railroads have cooperated and agreed on joint tracks in some of the smaller cities, but as a general proposition the steam railway companies endeavor to get their own side tracks into the industry, so as to compete for the business.

I think that is wrong, and I believe that one of the great advantages of governmental control of these roads at the present time is going to be that it will put into operation a system which will bring about the coordinated operation of the railroad terminals, and that we will never go back to the old plan, because when the economies that can be affected by such a system of operation are once recognized, the railroads will take advantage of and continue that method of operation, even when they get the roads back into their own hands.

I do not want to blame the steam railway men, as we know that some of them have attempted to co-operate in the past, to a certain extent, but their conditions, and prejudices and certain laws have worked against them—the Sherman Law had been the principal difficulty in the way—and they have been unable to bring about the shipments of freight as effectively as they could have done, if they had not been handicapped by this law and by their own prejudices, in some instances. It has been necessary to have this governmental step in order to get the problem in the condition it is in today.

I think the same thing that applies to steam railway terminals and steam railway trunk lines can be made to apply more effectively, perhaps, to the production of power than it has been applied in the past, although in the last ten years the tendency has been toward the elimination of the small power producing plant and the placing of the load on the central station, which could produce the power for much less per kilowatt-hour distributed.

There in no question in my mind that where you can utilize the steam not only for heating, but also have some use for that heat, aside from merely heating in the winter time, there is an advantage in having an isolated plant. For instance, in a hotel, where you need steam for cooking all the year round under such conditions as that, the isolated plant, in my judgment, will be superior to the central station for power, that is, it can produce its own heat and electrical energy cheaper than it can buy it from the station. That is the only instance, in my experience, where I found that it would work out in that way, otherwise, it is generally cheaper to buy electrical energy from the central station.

**Philip Torchio:** The problem before us tonight is of national importance. Col. Arnold has just made reference to the parallel of the railroads. I believe that the national investments in public utilities, represent, at least ten per cent, and probably fifteen per cent of the railroads, so that they are of an importance deserving the attention of all people. Incidentally, the production of primary power equals the importance of all railroads, in so far as the consumption of coal of the country is concerned.

It is a strange position in which the public utilities are placed in abnormal times such as we are passing through, probably in normal times it is almost as bad. While the costs of production and operation increase, public utilities are not allowed to adjust their prices accordingly, as all the other commercial industries are allowed to do. So that the public utilities are facing the situation of increased costs, without authority of readjusting the revenue for their service. It is obvious that a situation of this kind cannot exist indefinitely. What the solution should be is hard to outline in a meeting of this kind—I do not know that it is the proper function of engineers to advance schemes to solve financial problems. It is, however, the duty of the engineer, to clearly visualize the economic factors of an engineering problem. It is perhaps one of the saving graces in this war that we seem to crystalize our ideas, to get a clearer insight into broad problems, to get a quicker perception of the true fundamental ethics of social and economic problems, than was ever done before in normal times.

In these days, we can visualize broad economic questions without, perhaps, being accused of doing it for selfish motives—we can visualize the best interests of the country, not only for the period of stress through which we are passing, but also for the future development of the country. To outline this goal is principally the function of engineers. The engineering profession can command a full knowledge of the various details and factors necessary to evaluate and solve these problems.

The power problem is brought home to all of us, especially here in the East, by the lack of fuel. There is an appeal made now that every means possible should be used to economize fuel. All of the advocates of the development of potential water powers are at work advising that the country should promptly put into use the water powers in the country.

Now, it is essential that we really shoud see what the country ought to do. I want to give you a mental picture of the situation in the country as affecting power and the use of coal. The United States in the year 1915 consumed 540,000,000 tons of coal. Of this amount practically one-half, or 270,000,000 tons were used to produce primary power, and the other half was used to produce heat. In addition, there was produced through hydroelectric developments about five per cent of all the primary power used. The 270,000,000 tons of coal generated power at an efficiency of five per cent. The other 270,000,000 tons used for heating gave an efficiency of fifty per cent. Our efforts should be concentrated on bringing about improvements in these efficiencies. In the production of primary power we can eliminate the consumption of coal by using potential water powers. The potential water powers of the country are practically two-thirds in the Mountain and Pacific States, and the other one-third are in the remaining states. If all of the potential water powers of the country east of the western and mountain states were utilized in full, they would replace, it is estimated, about fifty per cent of all the primary power used in the States east of the Western States. It will, therefore, always be necessary, with the ever increasing use of power, that the nation will need, even after all of the water powers are utilized, to generate a considerable bulk of our power from coal, as far as the mind can now penetrate the future of sources of power. Therefore, it would seem that the steam central stations of the nation, east of the western States, are to be a prominent and permanent feature of the national development.

Let us neglect, for the time being, the exploitation of these water powers, and let us confine our study to what we can accomplish in the saving of a part of that 270,000,000 tons of coal that we used in 1915, and probably 350,000,000 that were used last year, for the production of primary power. I said that there was an efficiency of five per cent. If we had today central stations developed on a scale equal to what would be necessary for the development of potential water powers, we would be able today to generate practically four times the energy which was generated with the coal which was used in 1915. Excluding the power that was generated by central stations in 1915, which represented probably 15 per cent of all the primary power used, we could save in the generation of the 85 per cent remaining, three-quarters of the coal used. In addition if we installed systems developed on a unified and large scale to reach out throughout the country to different States, we could by the use of such modern equipment as we would add to our present stations, to a very large extent carry the off peak loads, that are now carried on less efficient old type apparatus.

I estimate that this incidental saving would represent probably one-third of all the coal used by central stations, which will be a clear saving on top of the other large and main saving that I have mentioned before. This is looking into the future, when the coal must be conserved at any cost. This would represent a saving, as I said, of three-quarters of the amount of coal that is being used with old fashioned methods of steam generation.

Now, the nation requires certain products, which cannot be produced except by the use of cheap power, or by importation from other countries. I am referring to materials like aluminum, caustic soda, the nitrates, ferro silicon, electric steel, etc.

There is an immense need in the country for the products of these different industries, which require very large bulk of cheap power in the development of the products. Now, if the country has the water powers, and the country must have these other necessities, it seems to me logical that the water powers should be dedicated to such national uses rather than using the water powers to distribute in small quantities throughout the territories in the field where the steam central station, as I said before, must operate in the future, and will have always to operate. The reason is this,—if you take away from the steam central station the desirable load, that might be put on an economical water power development, the operating cost of that steam central station will increase. That is an economic fact that central station men very clearly recognize, but often engineers at large do not appreciate this fact, that if we transfer the economic load from one station to another, and we keep operating the other station for reserve, we may increase the operating costs with the lesser load so much that we wipe out the economy obtained by the transfer of portion of the load to the other source of supply.

I am not going to take more time in analyzing this problem. I hope that this subject will be more fully discussed; it is of national importance. Nine years ago, in reviewing the future of central stations, I made a forecast, to which one of the leaders of the industry paid the ambiguous compliment of "poetic remarks." The conditions created by this war make it imperative that we visualize the future for our guidance in the present. I am not old enough to indulge in quoting myself, but as the forecast I then made embodies a life study of the power problems and is timely in this connection, I may be permitted to do so:

"In closing this forecast I would say that in the progress of the gradual uplifting of the industrial efficiency of the country, it is the province of the central station companies to furnish to the nation all of the power required by it in all of the commercial industrial and agricultural needs. With thousands of central stations dotting the map of the United States from east to west and from north to south with a closely woven network of power wires connecting the heaviest centers to the most humble hamlets, is it not reasonable to expect that this wonderful organization should ultimately command and control within this territory the generation and distribution of power for everything that lights, or heats, or operates machinery, or propels vehicles or trains?

"To carry out this work successfully, the industry will require professional, operating and business men of broad knowledge, sound views and superior ability, thoroughly qualified to be leaders in their respective fields of activity."

It is hoped that this subject, with the correlated subject of utilization of potential water powers, may be thoroughly threshed out from a broad national point of view in all of its technical and economic details, so that the findings of engineers may be of sure guidance to legislators in their future actions regarding one of the most vital requirements of the country.

**Philander Betts:** It was about three years ago that some of the public utilities began to feel the effects of the war conditions—I refer more particularly to the quality of service that was being furnished rather than the cost. It was not so long after that, however, that some of them began to feel the effect of changes in cost, and something more than a year ago some of the electric light companies in New Jersey filed with the New Jersey Commission proposed increases in rates. These proposed increases in rates were based on increased cost for coal, largely due, not so much to prices paid for the coal itself, but due to the increased cost in getting the coal to the plant. Since that time, conditions have grown worse and have extended to other utilities.

Something more than a year ago a meeting was held in this building of a committee called to consider the advisability of cooperating, or bringing about a system of cooperation, between the owners and operators of all the large power plants in New York City and vicinity. That meeting was not brought about due to any fear at the time of increased cost, but because of the thought that it was necessary to protect against the possibility of interruptions in service. The result of the work of the committee was to determine methods by which each of the companies could obtain service from the other companies, in case of emergency, including full information with regard to available men, materials for repairs, and places where emergency connections could be made between the various systems, and this information and like information was all very carefully made up and placed in the hands of each company.

At about the same time a meeting was held in the Engineers Club of the representatives of all of the large water companies, and the larger municipalities of northern New Jersey to discuss the same subject. In addition to discussing what would have to be done in case one company or one municipality had to call on the others for help, plans were evolved and put into effect last spring, nearly a year ago, providing for police protection. I may say that police protection does not appear, on the face of it, to cost very much, but in the case of at least one water company, which has not spent very much in its ordinary operation, I happen to know, that because of police necessities its operation expenses were very materially increased. That one water company has already been granted an increase in its rates by the Commission. The wisdom of having such co-operation between the water companies was shown in the last two weeks when it became necessary for some of the larger water companies in northern New Jersey to help out the city of Jersey City.

Bringing the matter down a little closer, a number of the companies have found it necessary to file with the New Jersey Commission—I bring this home to New Jersey, a sister State, because these facts have come before us, these are facts which have already affected the people at large—provisions for increases in certain rates, due partly to increased cost for labor, but very largely to the uncertain conditions with regard to coal supply and the growing increases in coal cost. A much more serious condition confronts us, though, and has confronted us in the last few days. A large company in New Jersey—and its representative here can tell you the details—has found it necessary within the past week to cut off probably forty per cent of its power load, due to the fact that it simply could not get the coal at any price under any condition. In cutting off that load, however—we all know that loads are not segregated in a way that makes it possible to cut off certain classes of service—all we can do is to cut out certain localities, without reference to what there may be in that locality, and doing this has caused a cessation in the manufacture of many things needed in the progress of this war.

The situation is more serious even than we thought, in a way. Hundreds of men and women, and young men and young women, have appealed to the Municipal Employment Bureau of Newark for employment, due to the closing down of works where they have been employed. All of those employed in these works that have been shut down, temporarily—we hope—have been turned away by the Municipal Employment Bureau with the statement that it was not wise to merely give them temporary employment in some other place which would probably be affected the same way a few days later. It is rather a pessimistic picture, but it is better, I think, that we face the situation, that we begin to realize what we will have to face.

In one of the leading papers published in this City yesterday, there is an article referring to the necessity for classifying the uses of fuel, in the first place, among other things, was placed the schools, that it was primarily necessary to furnish coal for the schools. The newspaper thought that that was not a proper classification, but it may have been a proper classification for New York City, due to the fact that a great many of the poorer people in the City had no heat at all in their homes, and the only way the children could be comfortable was by keeping them in school. If that is true, then the schools ought to be temporarily arranged so as to accommodate the parents as well as the children during at least a portion of the day. If that is a fair statement of the conditions, then it may be wise to include the schools of New York City in that first class.

But it seems to me the vital thing is to keep things going, and employment is one of the vital things. Employment involves the continuation of the operation of machinery, and therefore it seems to me that the machinery in the shops and factories doing productive work of a certain character must be given the preference. If that is done, then the products that are needed will be forthcoming, and the people themselves will have that employment which is necessary to continue them in their well being.

There is something more which will have to be done, however, at least for some little time, and that is there will have to be a classification with regard to the needs of uses for power. Much has been said about electric signs and things of that kind, but electric signs constitute a small proportion of the load that is worrying the electric power company. There are a great many things for which power is used which will have to be temporarily either laid aside entirely or at least curtailed. This possibly will entail the greatest governmental regulation perhaps, that has ever been attempted, but it will be just as necessary as the imposition of the draft law or as the census which we found was necessary. I think we will have to recognize this, and all who have anything to do with the operation of utility plants, particularly with the business end of them, will have to take systematic steps to acquaint the existing customers and prospective customers of the company with the conditions, with a view to cutting down those uses of energy which interfere with the proper operation of those things which inure to our well being.

**W. S. Gorsuch:** The subject of cost of producing power is a very broad and important one which must be studied from both a financial and economical viewpoint. Especially is this true under the present conditions which make it imperative to carefully analyze and examine power costs, and to look into the possibility of increasing efficiencies.

The increase in cost of labor, material and fuel, also a readjustment of operating conditions, have all had a marked influence upon the cost of producing power. I will briefly discuss in a general way, the magnitude of these effects with particular reference to some of the large first-class power plants.

One of the elements entering into the cost of power which has been materially affected by war conditions is the cost of labor. The authors state that any complete study of the effect of the increase in wages of the electric companies, caused by war conditions, would be a difficult task. The labor situation in the power stations, however, can be more readily analyzed than that of the entire system of any utility.

The increase in wages, reduction in working hours, difficulty in retaining efficient men, training new men, all have been reflected in the cost of producing power. In many instances the utilities have been obliged to employ very inefficient labor at much higher rates than were paid efficient men before war conditions existed. The employment of inefficient labor and the enormous labor turn-over, have resulted in a reduction in economy of operation and an increase in cost of power. This situation has been particularly felt in the boiler-room where the largest portion of the cost of producing power is involved in the handling and burning of coal.

The cost of a large portion of the labor in the power station has increased as much as 55 per cent. Taking into account the direct increase in wages and the increase as a result of a reduction in working hours. I believe that a fair average increase for all labor in the power station is about 30 per cent. That is, considering all the labor charges in power stations of appreciable size, the average increase in cost of operating and maintenance labor over the period just preceding the war, is about 30 per cent.

The problem of reducing working hours is one that the utilities will be confronted with for some time to come. The adoption of a six-day week schedule for every employe in the power stations, necessitates an increase in the number of men employed, and involves a corresponding increase in labor costs for the same output. It would probably be unfair to charge all such increases in cost to war conditions. However, the present conditions are responsible for hastening the adoption of a reduction in working hours.

While the present increases in wages are liable to remain after the return to normal conditions, it is more than probable that we will be in a position to employ a more efficient class of men than at the present time, and that there will be a marked reduction in labor turn-over.

Another item involved in the cost of generating power and which forms a smaller percentage of the total cost of operation and maintenance than labor cost, is Miscellaneous Materials and Supplies. The increase in cost of supplies and material other than coal, for operation and maintenance of the power station, has been very erratic; and it would be a difficult matter to give a reasonable average figure representing the percentage increase. There has been an increase in the cost of many of the repair parts as high as 90 per cent. The effect of such enormous increases in this class of material is to curtail in repairs, where such can be done without interfering with the economy of operation.

By far, the largest and most important item involved in the cost of producing power is that of fuel. The cost of coal like the cost of labor varies so materially throughout the country, that it is difficult to obtain sufficient reliable data to determine an average figure which will apply to all generating plants of appreciable size. Increases as high as 100 per cent and over are on record. Neglecting the quality of coal, I believe that an increase of from 65 to 70 per cent represents a fair average figure for many stations.

The cost of coal has an extremely important bearing upon the cost of power, representing about 70 per cent of the total cost of production of a first-class steam power plant, prior to the enormous increase in cost of coal due to war conditions. For illustration, consider a plant which prior to the war was generating power for 0.40 cents per kilowatt net output, with a coal cost of 0.28 cents per kilowatt hour net output. With an increase of 65 per cent in cost of coal per ton, the cost of coal per kilowatt hour net output will be 0.462 cents, or more than the total cost of operation and maintenance before the increase in cost of coal. On a basis of an increase in coal cost of 65 per cent per ton, and an average increase in total power station labor of 25 per cent and an average increase in miscellaneous material of 50 per cent the cost of coal will constitute 74.3 per cent of the total cost of production, instead of 70 per cent as stated above. The above illustration is on the basis of plant efficiency and B. T. U. per pound of coal remaining the same.

Many utilities have been obliged to accept coal of inferior and non-uniform quality at a considerable increase in cost per ton. The use of such coal results in lower efficiency, and particularly so, when we are operating under conditions of constantly training new and unskilled men who do not realize what it means to burn coal efficiently. The poorer the coal with respect to ash the higher will be the loss of combustible through the grates, which loss forms an appreciable percentage of the yearly waste. This has been of much concern to some utilities who are paying very high prices for coal which contains a high percentage of ash. In the foregoing illustration we considered coal of the same B. T. U. per pound, with an increase in price of 65 per cent per ton. To show the effect on the cost of power with coal costing 65 per cent more per ton and containing less B. T. U. per pound, let us consider two coals, A and B, and apply the "Figure of Merit" (TRANS. A. I. E. E. 1913, page 1648.) which represents the B. T. U. per dollar after correcting for ash, and B. T. U. lost in refuse.

Figure of Merit:

$$= \frac{\text{B.T.U. per ton (moist)} - \left( \frac{2240\% \times \text{ash}}{100} \times \text{B.T.U. per pound of refuse} \right)}{\text{Price per ton, (2240 lb.) (\$)}}$$

| Coal | B. T. U. per pound moist | Price per ton, 2240 lb. $ | Percentage of ash in coal as received. | B. T. U. per pound refuse | Figure of merit |
|---|---|---|---|---|---|
| A | 14,500 | 3.50 | 5 | 3,500 | 9,170,000 |
| B | 13,000 | 5.77 | 10 | 4,000 | 4,890,000 |

From the Figure of Merit of each grade of coal given in the above table, it will be seen that 87.5 per cent more B. T. U. is obtained from coal A. That is, if coal B is used which costs 65 per cent more per ton than coal A, there will be a corresponding increase in cost of coal per kilowatt hour of 87.5 per cent. Apply this increase to the cost of coal and allowing for an increase of 25 per cent in the cost of labor and an average increase in miscellaneous material of 50 per cent the cost of production including operation and maintenance is increased from 0.40

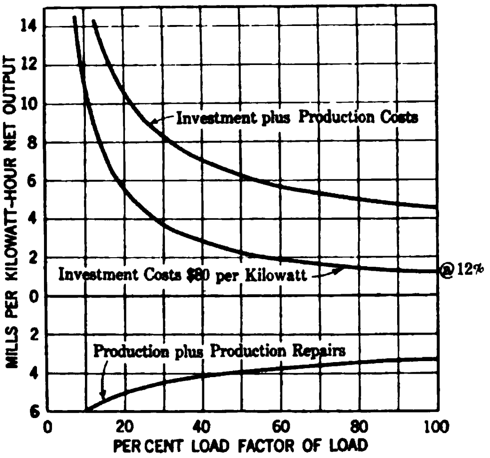

FIG. 1

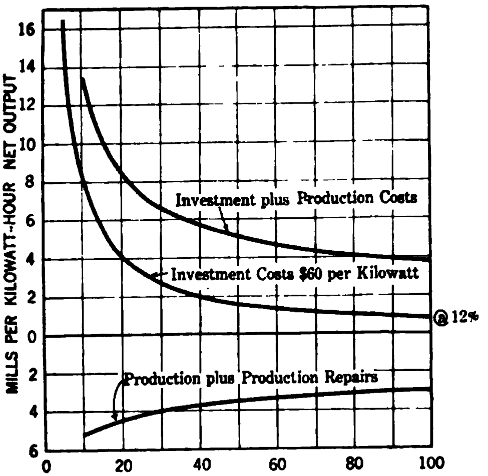

FIG. 2

cents to 0.6845 cents per kilowatt net output, an increase of 71 per cent. The cost of coal per kilowatt net output representing 76.6 per cent of the total cost of production. That is, out of every dollar spent for operation and maintenance cost in the power station, *76.6* cents is spent for coal, the market price of which is beyond the central station manager's control. If to this figure, we add the cost of boiler room labor, neglecting cost of water and other miscellaneous items, we will find that *84* cents out of every dollar is used in the boiler room. This brings out the importance of careful supervision of boiler room operation which becomes more important with the increase in cost of coal, and which, under the conditions cited, calls for an effort to obtain the maximum value from each pound of coal.

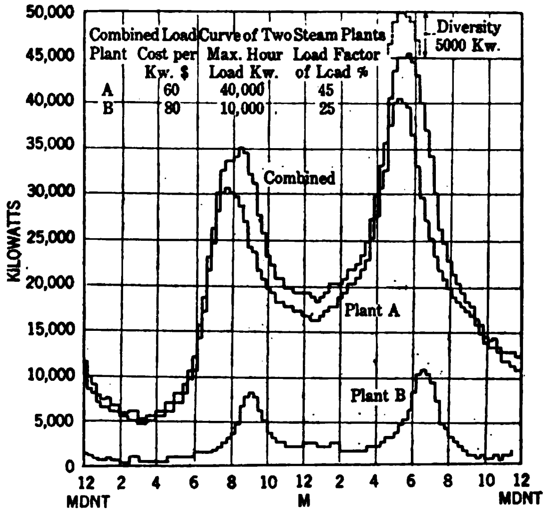

FIG. 3

In view of the above, if any adjustments or increases are made in power rates to cover the cost of coal, they should be made or computed on the basis of the B. t. u. per dollar and not increased cost per ton.

The recent rise in the cost of labor and material has increased operating expenses at a rate much faster than any possible economics resulting from a decided increase in load factor of load. With the same maximum station load, and an increase in load factor of load, there will be a reduction in cost of operation and maintenance and fixed charges. However, the economy effected by operating at a much higher load factor, may not be sufficient to compensate for the very large increase in the cost of coal and labor. This feature is brought out very clearly by referring to the curves in Fig. 1 which are fairly representative of the cost of power for a plant costing $80 per kilowatt before war conditions existed. Consider the plant operating at 40 per cent load factor under normal conditions as to cost of labor and coal. The total cost of power will be 0.700 cents, 0.425 cents of which represents cost of operating and maintenance, and 0.275 cents fixed charges. If now there should be an increase of 50 per cent in the cost of coal per ton, having the same B. t. u. per pound, and an increase in labor of 25 per cent, and at the same time the load conditions readjusted so as to operate at, say, 55 per cent load factor, the total cost of production will be 0.746 cents, 0.546 cents for operation and maintenance, and 0.200 cents for fixed charges. That is, operating at 55 per cent load factor, with an increase in cost of coal and labor of 50 per cent and 25 per cent respectively, the total cost of power per kilowatt hour net output is 6.5 per cent higher than when operating at 40 per cent load factor with no increase in cost of coal or labor. The difference is still greater in a plant costing $60 per kilowatt, as can be seen by referring to Fig. 2. On the other hand, with a higher load factor, the volume of business is increased, and the increase in the cost of coal may be, to a large extent, offset by the sale of power, depending upon the rate charged.

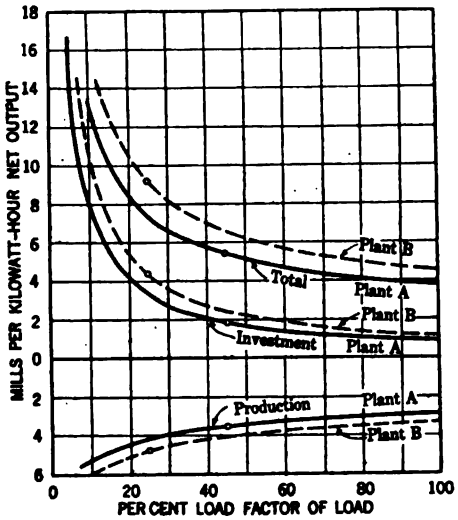

Fig. 4

Much has been said of late about closing small isolated plants and transferring the loads to central stations. Aside from the spirit of patriotism, there is one vital question which concerns the smaller company, and, that is, can the larger power plant generate power cheaper than the smaller plant, taking everything into consideration? The smaller plant will have to carry fixed charges on all its investments whether it generates its own power or purchases power from another source. The question then is, can the central station sell power to the smaller plant at a price equal to or less than the cost of operation and maintenance of the smaller plant?

To illustrate, let us consider two plants, one having a maximum hour load of 40,000 kw. and operating at 45 per cent load factor of load, and a smaller plant having a maximum hour load of 10,000 kw. and a load factor of 25 per cent, the maximum hours occurring at different times. To see what would happen when these two load curves are combined, the smaller has been added to that of the larger plant, as shown in Fig. 3. The corresponding cost of operation and maintenance, also fixed charges per kilowatt hour net output at various load factors are shown in Fig. 4. A study of the curves of both Figures 3 and 4 will show that the result of transferring the load of the smaller to the larger plant as far as the total generating cost is concerned, is that, there is a diversity of power station demands of 5000 kw., and that the reduction in total generating cost per kilowatt hour, practically wipes out the investment cost of the smaller plant. While the illustration represents a special case, it is evident that the problem is purely an economic one, which requires careful analysis and, that the decision must be made on the merits of the individual case.

**H. M. Hobart:** War is such a cruelly sad business that one is reluctant to find any benefits in it. However, as regards this matter of the business of manufacturing and selling electricity, this war has brought about the millenium, or at any rate, has brought about the foundations upon which a substantial millenium can be based. Far seeing engineers, such as Messrs. Ferranti and Merz in England, and Mr. Torchio in America, had long ago explained the situation to us. They had demonstrated to us clearly that if we could have a large enough load of a suitable character the cost of electricity could be halved and halved again, but unless we bring the cost of electricity down to so low a figure as a fraction of a cent, they foretold and figured out clearly, that we cannot hope to get the satisfactory large load. On the other hand, without the large load it is impossible to bring down the cost. What has this tragedy of war done in that respect? It has forced on us the large load of very desirable characteristics.

The authors have, it seems to me, emphasized only one side of the picture. There is a great deal of truth in the side which they portray, and portray so vividly, but there is this other side. The authors do not give even a suggestion that there is this other side, and in the discussion speaker after speaker has echoed their cry of the threatened ruin of the electrical manufacturers. The cost of electricity is based chiefly upon three things: cost of fuel, cost of wages and amount and characteristics of the load. If the cost of fuel is great, it is a handicap; if the cost of wages is great, with relation to the intelligence and activity of those to whom these wages are paid, it is a handicap; but chief of all, for years we have emphasized as the heaviest handicap the smallness of the load. The smallness of the load has been agreed to be the heaviest handicap.

What have the authors done? They have left out the denominator in their paper, and that denominator is the output in kilowatt-hours. We must take into consideration the fuel per kilowatt of output, the wages per kilowatt-hour of output, and the capital and overhead charges per kilowatt-hour of output. The output is going up tremendously. In peace time it would lead us to extravagance. We would buy much more machinery for the generating stations. We now simply cannot. We use to better effect that which we have, and trust to pulling through, and thus reach conditions which enormously decrease the overhead charges per unit of output. The large load also decreases to a slight extent the fuel charge per unit of output, and very largely decreases the wages charge per unit of output. As the years go by, the charge for wages will become less. In every progressive electrical undertaking the managers pride themselves on the increasing utilization of machinery and the decreasing number of men employed per kilowatt-hour of output. The boiler house is an utterly different place today from what it was ten years ago. I consider that all these aspects of the subject should be introduced into the discussion.

There are serious handicaps on which the authors enlarge. But let us turn to England. The authors refer to the situation in London. They say that in an issue of the *London Electrical Review*, the date, of which they do not give, it was stated that there had been a 50 per cent increase in rates. London is an exception—it might well be—there are seventy little stations, equipped years ago, and it is a hard burden on the antiquated machinery in these times. Go to the north of England, the Tyne or the Clyde, or to the Midlands. In undertakings in those districts I should be very much surprised to find that there had been any considerable increase in rates. Any increase in the rates for electricity in these districts will have been very slight if there has been any at all. That is because they are equipped with modern plants. Take the case of the Tyne, dozens of stations all feed into one network, and all sorts of sources of power contribute to a common network. The output is enormously increased by the munitions works that are being put up, and the great activity in shipbuilding and in other lines. For the same reasons an enormous increase in the output is taking place here.

Again, it is only a matter of conjecture, but I think if we were to analyze the output of any large concern manufacturing dynamos or motors, it is very likely we should find three-quarters of the output is for private customers and one-quarter of the output for the electricity supply corporations. It is that ratio which counts, and while I have not here with me any figures to prove the case that the manufacturers of electricity are exactly in clover, I do maintain that both sides of the situation ought to be equally carefully examined before we arrive at a definite conclusion.

**D. C. Jackson:** There is one point in the situation which has not been brought out in the discussion, and which is scarcely touched on in the paper. Many years back it was the habit of people to say that that nation would be the greatest which had the greatest supply of coal, lime and iron ore. Now, that is too materialistic to be convincing, in view of the fact that modern civilization rests upon the principle of Christianity, but there is a good deal of truth in it, after all, when it is analyzed. Civilization of the Twentieth century depends upon those characteristic structures which abbreviate space and annihilate time, that is, on transportation and intercommunication, and these do depend upon the railroads and the means of communication, which in turn distinctly depend upon the supply of coal and iron ore, from which steel may be made. I will not argue that point further, because of lack of time, but I will turn directly to the particular and extraordinary circumstances in which we now rest.

We have taken upon ourselves a laboring oar in the most remarkable and the biggest enterprise the world has seen, and it is one that we have entered upon with the idea firmly fixed in our minds (and doubtless it is right) that it is for the welfare of the world and for the betterment of civilization. Under the circumstances, it is wise to analyze exactly where we are at. War in the old days, at one time, consisted of a man and a horse, with an iron housing, and a couple of armorers to take care of it, but it has become an entirely different matter. For instance, in our Civil War, a very critical situation was met by a single instrument, built in record time, as I remember, three months, and that was Ericsson's Monitor, and one or two other such instances may have provided the solution of the situation in the Civil War. But if we look at the matter today, we will see that a single submarine, or a dozen submarines, will not solve the situation for one group of belligerants, nor will a single flying machine, or perhaps ten thousand flying machines, solve it for the other. The problem has become, apparently, a problem of production on a very rapid and at the same time, economic scale;—rapid, that it may meet the requirements of the situation, and economic, so that it may meet the possibilities of expenditure of funds, I mean in regard to the raising of funds, rather than the mere outlay of them.

That brings us directly to the point of the central station as an important factor in production. There is no question about the necessity of our further expanding our industries, and very greatly expanding them, whether the war goes on for a long time or not, because if we do not expand them we are assured of a long war. The rapidity of expansion of our industries, with the economical production of mechanical power necessary for their welfare is inevitably bound up with the development of the central station to a much larger degree than we are now utilizing in the war, and it is a factor of great importance that our national authorities should take this fully into account, which they have not yet apparently done. It is a part of the duty of electrical engineers in the present juncture to push that particular feature along with the utmost of their enthusiasm and their force.

I will speak of one other point, and that is the shortage of labor. I have been, at a great expenditure of time and mental effort, serving on the District Board which has in Boston the classification of industrial workers with respect to the draft. I have become convinced that there is a very great difficulty in getting satisfactory industrial workers of skill and character who will stay at home and do their work, because such men are full of vigor and full of enthusiasm and are rather likely to desire to go into the great enterprise directly. At the same time a very able and intelligent labor leader said not long ago—"There is no shortage of labor. There are twenty thousand idle hands in the State of Massachusetts." Massachusetts has a population of three million people, and has eight thousand square miles of area. It is not a very great factor in the United States—not so much in those lines, but in some lines it is. I asked him whether he was sure that his statement was correct. He said—"I am sure, but the difficulty is not the number of men, it is the distribution." I told him that I had heard a lot about distribution, that the men were always fifty miles away when you wanted them. He said—"That is not what I mean, but distribution in trades. The large proportion of bricklayers in the State are idle at the present time, but there is a frightful dearth of sheet metal workers." Can not we do something to translate bricklayers, who are skilled men with their hands, and some with their brains, into sheet metal workers. There is a point worthy the consideration of engineers.

**P. H. Bartlett** (by letter): This paper will undoubtedly be regarded as a notable summing up of the present very unsettled and uncomfortable situation of many of the electric light and power properties. The subject has not only been analyzed quite thoroughly from the viewpoint of the problems of cost and operation confronting the industry, but the arguments contain suggestions and possible remedies looking to the solution of some of these problems. Intelligent discussion or contructive criticism is difficult, however, on questions or problems which have arisen so suddenly and more or less unexpectedly and from which the industry is in many instances still struggling to extricate itself. Changes in policies which have become traditional, reorganization of ideas and the alterations in methods and practise necessary to quickly meet the conditions arising in the present emergency have left some of us at least more or less at sea as to the present status. The labor question, however, while very much accentuated at this time, is an ever present one, and a few observations on this subject may be timely.

With the larger operating companies, more perhaps than with the smaller companies, the question of labor turnover has been critical. A recent analysis of figures, complied by several departments of a large operating company, for the year 1917, show, in one instance a turnover of 570 employes out of a total of 547 on the payroll at the beginning of the year or 106 per cent with a total of 498 employes at the end of the year or a net reduction of 49. Naturally all of the positions were not affected, some of them being filled once and others several times, the analysis showing that the positions affected by the labor turnover were 270 out of the total of 547, or a turnover percentage of positions affected of 211 per cent. In analyzing this in more detail, there was one group of 90 highly trained technical men in which the labor turnover during the year was 73 or 81 per cent, leaving a net total at the end of the year of 86 employes; the positions affected by the turnover in this case being 39 or a turnover percentage of positions affected of 187 per cent. In another similar group of 96 trained and semi-trained technical men, the turnover during the year was 96 or just 100 per cent, the number of positions affected by the turnover in this case being higher, namely 56 or 172 per cent. Of the 547 employes in these departments on January 1st, 1917, over 100, or approximately 20 per cent, have entered the military or naval or other service of the government, and of the 498 employes on the payroll at the end of the year, 60 are girls who have replaced men, some of them on what might be called semi-technical work.

While the conditions which have created the present unprecedented demand for labor of all classes have also resulted in a decreased activity in certain other directions, which has tended to reduce to some extent the work to be performed by some of the groups covered by the above figures, and while economics in work in other directions have enabled a certain reduction in some divisions of the force, or rather the necessity of filling certain vacancies has been eliminated, nevertheless, there has been the general and usual increase of from 15 to 20 per cent in essential work which must be accomplished. Considering the more or less great effort to maintain these forces, together with the fact that a large percentage of the existing employes are more or less inexperienced, the results are not such as to encourage optimism.

The causes for such an unusually high percentage of turnover are apparently due to several reasons, some of which have been mentioned. For instance:

First: High remuneration or wages paid by other industries.

Second: Military reasons.

Third: Many of the positions are of such a nature that they are difficult to fill even under normal conditions,

Fourth: The generally unsettled condition and unrest among all classes of labor.

Referring to the first reason, while an certain upward revision of salary must be anticipated, it is obvious that the power company whose rates are regulated by law, and were originally predicated on cost of labor, material and operation in general based on normal trade conditions, is unable to pay salaries equivalent to those of other industries, manufacturers and distributors whose rates can fluctuate with and do reflect every rise in the market, nor can they compete in this respect with the very necessary and essential, but what might be termed mushroom industries, born and fostered under the present national emergency and whose life, in many cases, will be equal only to the life of the emergency and who are in many instances working directly or indirectly on government contracts with a guaranteed overhead and a percentage on the cost. Nor would it appear to be wise to attempt to meet apparently inflated rates of wages, which in the event of a downward trend later on would result in expensive and top-heavy organizations, or the undesirable expedient of reductions to again meet normal conditions. The alternative is that the utility must accept in the rank and file, raw material of little or no experience and begin on a larger scale the more or less laborious process of training men for their particular work. This again adds to the percentage turnover, due to the weeding out of undesirable or unfit material, and also in many instances to the fact that as soon as the employee becomes partially trained along some particular line which makes him valuable in some other industry, there is apt to be another vacancy in the force.

Referring to the second cause mentioned, namely, loss of men due to their entering the military, naval or government service, either by volunteering or draft. This has been in some units quite serious, irrespective of the optimistic view taken by our government officials. It has been stated that about 80 per cent of the industrial and agricultural workers of the country are not subject to call, being either above or below the age limit or on account of dependents and that only 3 per cent have thus far been taken from the industrial or agricultural fields, and while granting this, the fact, remains, as before stated, that over 20 per cent of the trained men, with an average of over four years' service in one department of one power company, have already entered the government service—the majority of them volunteers—and many more will be taken in the next draft unless the industrial claims asked for are recognized. Among the reasons for this high percentage of loss, it may be noted that the industry is young and its great growth has necessitated the employment of young men of high character and ability who can be trained for the particular work in which they are engaged, and these are the character of men who are usually the first to volunteer or who, on account of age, fall within the draft limits. The industry must expect, however, to lose a certain percentage of their forces to the government service and this is not unduly alarming, except perhaps in isolated cases where temporary inconvenience will result, providing that the balance of the forces can be maintained.

The third cause mentioned is merely accentuated under present day conditions. There is probably no type of labor which has to be more carefully selected than employes of public utilities. The work is exacting and responsible, and the duties of quite a large percentage of the total require them to be in more or less close touch with the customer and this applies equally from the office boy who may answer the telephone to the highest class salesman or the maintenance, testing or operating staffs, and when through scarcity of available labor the selection becomes more limited, the turnover is naturally greater as the unfit are weeded out.

The fourth reason, viz., the generally unsettled condition of labor, is particularly accentuated at this time due, no doubt, directly to the war itself and its effects. The wage rates at the present time are probably higher than they have ever been. The laboring class, the organized mechanical trades and in practically every walk of life, except perhaps among the so-called middle class, more money is being earned than ever before and every employee feels the impulse and many are changing their positions or becoming floaters temporarily while the prosperity lasts.

Such a constant hiring, trying out, training or eliminating of new employes results in high cost to the companies which it is difficult to express in dollars and cents, due to error and inefficiency of the new men, to waste of materials, to time and supervision of the older employes necessary in training the new men and to other reasons which cannot be isolated.

Under normal conditions the cost for taking on and training a new employe has been expressed in various companies as varying from $30.00 to $75.00 per man in addition to his salary, and the general inefficiency arising from the introduction of new labor, even when very carefully supervised, is well recognized, but this is, of course, one of the problems that must be met.

The technical employees of the power companies are a highly specialized class and on account of their training and experience are in great demand, and while a reasonable upward revision of salaries from time to time may be necessary, other means must be taken and arguments advanced, as to how these organizations, so vitally necessary at this time, should not only be held together but extended to meet the additional demands that may be made upon them.

The maintenance of these organizations may present more difficulties in those localities where the war industries are center-ing than in others, due to the enormously increased demand for men, and in such cases the results pyramid in their effect, as the greater the demand for labor the greater becomes the demand for, and the use of, power which can in most cases only be supplied by the power company. That the amount of available labor varies considerably in different cities is evidenced by the fact that an advertisement for young men for the electrical industry in one city produced several hundred replies, from which some very good material was secured, whereas an equally attractively worded and spaced advertisement having an equally wide distribution in another city results in only 25 to 30 replies, the results from which were negligible.

In groping for a remedy for these conditions we have found as yet no formula to follow. Experiments which may seem to offer remedies for particular cases must be tried out, and while the situation may appear to be discouraging, especially in view of the responsibilities resting on these organizations, it is well to remember that we are not facing, at least at this time, any such labor shortage as other countries must have met and managed to overcome.

The use of girls and women, in the event of a very acute shortage of man power is, of course, one of the first thoughts to suggest itself and one of our prominent engineers has recently been quoted as saying that he could, providing that the women would undertake it, perform the actual operation of his plants with female labor, with the exception of the rougher work requiring more or less physical endurance. Women have, of course, invaded the field in many power companies in the past year, but probably only in positions which have heretofore been recognized as open to this class of labor, namely, for clerical or other kindred duties. From information obtained from a number of large companies who have not heretofore employed women and who are now employing them in considerable numbers, and bearing out the statement of the paper this evening, it would appear that it requires four girls to replace three men. In one instance, however, it has been found that six girls on semi-technical work, after they became sufficiently experienced, were able to replace eight men. This was probably due to the small number of girls, permitting a very careful selection, and also due to the fact that the work proved irksome to men who were ambitious and who permitted their attention to be distracted by other more interesting work going on around them. This condition would probably not obtain in many cases, but is one instance where female labor is doing more work and doing it more accurately and at less cost than it was done by men.

The use of boys from the senior classes of Manual Training and High Schools, after school hours, on certain limited classes of work has been tried out with some success and further experiments in this direction are in preparation. The importation of colored linemen from the South has also met with some success as has also the hiring of inexperienced younger men and placing them under intensive training for some weeks, but these measures, as with others before mentioned, provide only for specific divisions of the work and do not take care of the trained forces so essential.

One operating engineer with whom the matter was discussed stated:

"The plan which I have tried to carry out in order to safeguard against serious troubles with our labor forces has been, briefly, to keep in closest personal touch with my direct assistants and supervising heads. These men are kept satisfied, their troubles either personal or business, are my troubles and I see that a solution is reached in every case that arises which is mutually satisfactory. In other words, I have tried to stay so close to my assistants and their troubles that there is no possibility of any unrest or dissatisfaction however small in the beginning that is not known by me and corrected before it reaches the stage where it would be past correction. On following this plan you will recognize that it only means giving greater attention to a fundamental principle that should govern all successful departmental work.

I recognize the fact that there is a great labor unrest today which must be reckoned with. The unrest is most apparent in the lower classes of labor and among all classes not capable of thinking clearly for themselves. This spirit, however, will spread in any organization of labor, reaching in many cases men who under ordinary conditions would be immune from such influences. As I view it, a department head must start downward in the scale of labor using every means at his command, giving the maximum of his help, advice and spirit to counteract, as far as possible, the spread of unrest and dissatisfaction which reaches upwards from his lowest scale of labor.

Our efforts thus far have been gratifying and I feel confident that our organization is a unit down to the labor class. This class very frankly we cannot hope to satisfactorily cope with and we are using our efforts merely in replacing the men as rapidly as they leave.

My own feeling in the matter is that I know the job that is ahead during the period of this war, and every man I am able to keep in personal touch with knows what I know—that we've got to see it through as gracefully as our abilities and the means at our command permit."

It is obvious that for the effective carrying on of the work and for the maintenance of the great rank and file of the organizations, reliance must largely be placed in the department heads or in the hearts of the men in immediate supervision. The man who can keep in closest personal touch with his men, will in most cases keep the men close to him, in spite of immediate financial considerations.

The necessity of keeping as nearly intact as possible the supervising forces, who can in turn hold a more or less large percentage of the rank and file and thus provide a nucleus for the training of the new men is of paramount importance and this should extend down to the smallest subdivisions of the organization.

The retaining of these supervising forces of power companies, even down to the minor subdivisions, is not such a difficult problem, as these staffs are usually composed of men who have grown up in the service or have been connected with it for many years, and if they are competent and enjoy the confidence of the management, will remain loyal to the industry, and will not be tempted by the offers of other concerns which may exist today and are gone tomorrow.

The importance of vocational training and the retention of and building up of the personnel of the organizations we are discussing should not be overlooked, and in fact would seem to be of greater importance now than ever, and we have before us the example of our own army and the shipbuilding industry which are possibly the greatest examples of vocational training ever undertaken. It may be argued that vocational training is for peace and not for war times, and it has been argued that to attempt to continue vocational training at this time merely results in the utilizing of the services of urgently required men to train others, who when trained are snatched away by other industries. Vocational training must, however, be continued in one form or another and if this training is confined strictly to the experience required by the men to perform the immediate duties for which they are employed, the results cannot fail to be beneficial.

This discussion has shown that the labor turnover in some power companies at least is very great and an endeavor has been made to point out some of the reasons for it and some possible remedies. How far these remedial measures will hold, in the event of a long continuation of the war, with greatly extended drafts for military and other government purposes, and with the further anticipated drafts on labor for war industries, which must grow in proportion to the military establishment, is problematic. Such a condition would manifestly require drastic readjustments, extraordinary effort and revolutionary changes in the direction of energy and these problems must be met as they arise.

**W. N. Smith:** I want to make one suggestion, prompted by one or two remarks in the early part of the discussion,—that is, the matter of pooling private interests.

We are up against the most fearful emergency civilization has ever known, and drastic readjustments from former practise must undoubtedly be worked out in our country as well as in other countries before the conflict is finished.

In these industrial districts which are served very well in ordinary times by the public service corporations, there are many private corporations owning manufacturing plants of one kind and another, large and small, some very large, which have good sized power plants at their disposal. While this subject is so broad that it can only be touched on lightly in the brief time at my disposal, it does seem to me as though there were a possibility of commandeering for the benefit of the community certain of the private plants, even if only for service during certain hours of the day, for the helping out of the general power situation.

This will have to be done of course through some state or federal agency which would be clothed with broad powers necessary to exercise jurisdiction over a situation involving such large interests.

If a general arrangement like this could be worked out it will also have the effect of enabling an increase of economy of operation in these private manufacturing plants, many of which use very large quanties of coal. Some of them use it very wastefully because of the fact that their commercial profits are not often largely dependent upon the cost of power, that is to say, if the cost of their power is not a very large percentage of the operating expenses, they are apt to ignore those finer economies in the generation and use of steam, which are a matter of every day thought and discussion among central station men and manufacturers of electrical apparatus.

I have been up against this sort of thing myself in a practical way for the last two years and I know how careless the average private plant operator is apt to be about the consumption of his fuel.

In New Jersey or in the Brooklyn district, or for that matter in any large manufacturing city there are some plants which have a good big outfit of boilers, engines and electric generators that could be turned to practical use to help out the community in time of stress, when the public service corporations cannot meet all of the demands upon them, and it seems to me that it is up to us as engineers, and up to the community in general, to consider such a possibility as that of a pooling of generating machinery in private plants, where the same could be used to help out a shortage in the power usually furnished by public service corporations.

Such a proposition as this would be a suitable field for the exercise of jurisdiction by local Public Service Commissions, but in any event there would have to be some Board of Engineers or experts who after taking a census of the available power equipment in a district would go through everything very carefully as regards the availability of private plant equipment and its application as auxiliary to the central station system. In New Jersey for instance, there have been no electric lights on some of the streets recently. There would no doubt be found a considerable number of power plants which might with a little additional apparatus perform that lighting service and turn loose a considerable amount of additional power besides. A great many of the manufacturers power plants already have veteran trained employees in the boiler and engine rooms day and night, so that the element of labor is already provided.

There are many engineering conbinations possible for dealing with such a situation as now confronts us, which is likely to be a continuing, rather than a temporary emergency. I believe that it will be found possible to utilize existing electrical apparatus by emergency methods readily available, without waiting a year for new steam turbines.

**John J. Harold:** I have been interested in problems in connection with this war in three continents,beginning with Africa, then Europe, and finally in this country. The labor phases of the situation, in particular, I have been able to understand as I was Secretary of the Witwatersrand Trades and Labor Council for two years and three months, and my views on that aspect of the subject may be of interest to you.

In order to understand the prices of labor and materials rapidly going into a state of unstable equilibrium, let us go back to what happened in Sweden. Sweden exchanged American goods for German gold until the second embargo fell and they found themselves in possession of German gold which they could not eat. European gold has been coming to this country for the purchase of munitions and for the sake of the "easy" money the producers of food and coal started to make shells and fuses, just in such a way as your bricklayers in Massachusetts could be taught to do sheet metal work. Indirectly they were employed by European belligerents who were consuming American supplies, which they had ceased to produce. It will not be very long before they find out that they cannot eat gold and they will begin to wonder why the purchasing power of the dollar is falling so low.

More than this, prices are going up because of the instability of labor—the man secures employment, works a few days, and then will secure a leave of absence to look for another job. He does not break with his last employer until he lands the new position. The best way to combat this proposition is for employers not to employ any one at least without the knowledge, if not with the consent, of his previous boss. After a year of war England passed the Munitions Act, whereby it was illegal for any of the commandeered factories to employ a mechanic who had left his employment in another munitions works without his employer's consent. Such a man had to remain idle for six weeks, before he could take up another job. To the Irish and Colonials, this did not apply. All the Irishman had to do was to go home to Ireland and reengage there for employment in England. I do not imagine that this principle could be applied to the United States, because although the British Government knew the amount of leeway that an English workman could stand, under similar circumstances the Yankee mechanic would take his yacht down to Florida for the prescribed period.

But what could be done is to grant a bonus of ten per cent on all the wages earned for thirteen consecutive weeks, payable at the end of the fourteenth week. This would tend to stabilize the labor turnover.

Now, we come again to the big question of the load factor. I would suggest that the power companies grant a rebate of, say, ten per cent on the price of all current consumed at all hours of the day other than hours of the peak load. In this way, the manufacturer, who must have power at any price, is able to offset the ten per cent increase in wages that he is obliged to pay for night shift work. Although you will not be able to make the peaks disappear, they will, at least, be rendered less noticeable.

**Philip Torchio:** I would like, before the discussion is closed, to give an answer to Mr. Hobart's question as to the possibility of the cost of power not having really increased. I made some rough figures of the production costs in the last few months of the lighting companies in New York. The production costs, the operating costs alone, increased 95 per cent, and if I include operating costs, plus fixed charges, the total cost increased 50 per cent.

**P. R. Moses** (communicated after adjournment): It is undoubtedly true that costs have increased and are continuing to increase, but these have been largely offset by an increase in revenue, and the writer believes if the public utility will change its policy in connection with private power plants a sufficient increase of revenue, which will not involve additional capital expenditure, can be obtained to offset the increased cost.

A plan as to cooperation between central stations and private power plants has been suggested by me during the past several years, and in a paper presented before the American Society of Mechanical Engineers on April 5th, 1917, it was pointed out how much less capital would be required to take care of peak loads if the private plants were used to supply this demand than if the peak loads were taken care of by public utility plants.

The utilization of localized private plants supplying electricity and exhaust steam for heating is a ready-to-hand means of meeting the increased demand on the central station while at the same time it affords a method of reducing the fuel required for a stated number of kilowatt hours during the winter season, as the electricity obtained from the private plant during this period is as is well known practically a by-product of the heating.

The low cost of the private power plant installation allows taking care of the peak load at barely one-sixth the cost of taking care of this load by increases in the central station including feeders, substations, etc.

There is already available large percentages of spare capacity in existing power plants which by proper cooperation with the central station could be fully utilized, for the purpose of taking care of peak load demands.

I am of the opinion, however, that under present fuel and labor conditions, during such periods of the year when exhaust steam may not be fully utilized and during light load periods, during six or seven months of the year in this latitude, private electrical plants should be shut down, and the current derived from a central source, the public utility company being required to supply this current at a price which will enable the consumer to profitably purchase, and at the same time cover the expense of supply plus a reasonable profit, but without allowance for depreciation on plant or other fixed charges on investment, because supplying the offpeak and summer load as suggested would not require additional investment by the public utility company, except to a very minor degree for meters and connections, and this might be properly charged to the individual consumer, thus avoiding any investment by the public utility company.

It is evident from the published figures of the large public utility plants that such off-peak load could be profitably supplied at from one and one-half to two cents per kilowatt hour in cities like New York, and at a much lower cost where overhead transmission and high voltages are available.

If our public utilities will take up this question in the submitted outline of cooperation with the view to utilizing the private plants to their utmost economic extent, which means to use them during the period when fuel and labor can be doubly used, they would greatly benefit themselves, enormously improve their load factor, increase their revenue, and what is of far greater importance, effect an immense reduction in the amount of fuel burned.

## Discussion at Chicago

**R. F. Schuchardt:** We will all agree, I am sure, that the situation as set forth so ably by Mr. Jackson is of considerable importance to all of us, and to engineers and financiers especially. To a central station man the paper reads very much like a description of the condition in his own company, which merely illustrates that the situation is pretty universal throughout the country. Lest, however, all of you here rush to the stock market to unload your utility securities tomorrow, we should take into consideration that there is also a bright side. Mr. Jackson has pointed out one of the most important of the brighter sides, which is the widespread recognition of the real value to our country of this economic industrial development, the central station, and a general knowledge of the benefits of centralization must react advantageously to the industry.

Also, we are now putting more of our investment to use, or in other words, we are using our reserve to earn money. This might be expressed as living on our fat. We must live on the fat, as we can't get additional financial food, for reasons which Mr. Jackson has pointed out. Also, the public mind is getting somewhat used to everything going up in price or else coming down in quality, and so it may be easier to satisfy the kicker, where kickers are numerous, which of course is not the case in our industry.

For the engineer, however, there seems to me to be a very bright side. It is true there is a good deal more work required of the engineer at this time, but he who at this time doesn't want to do more work than he did before is a pretty sorry sort of a man. In fact, it is a sort of compensation for those of us, who are not privileged to take a more active part in the war, to be able to carry some of the usual burdens of those who do go.

Also the engineer must now use more resourcefulness. Instead of following precedents, he must set new ones. This should arouse renewed enthusiasm and enable him to increase the quality of his work. The engineer, instead of thinking more particularly of kilowatts, must now think more of the relations between those kilowatts and the dollar.

**F. A. Coffin:** An important condition facing all of us is the growing difficulty of financing extensions to meet new customers. We are rapidly approaching a condition where only such new capital expenditures are justified as are directly necessary to the prosecution of the war. It is probable that in addition to the deterring influence of the high cost of obtaining new money we will have strict regulatory action by federal authorities. All of this vitally affects expenditures for line and service extensions which have hitherto been an inseparable part of the process of attaching new business. This necessitates a radical revision of rules with respect to line extensions and requires that during these times new customers will be required to finance at least the difference between normal cost and present abnormal costs in supplying them service.

The second aspect of importance is economy in the conduct of the sales department. I do not share the speaker's pessimism that the war leads to a curtailment of the use of electric service. Early this year we dispensed with all but the nucleus of our sales organization and it is of interest to note that the increased business which we hoped to achieve by the end of the year through the usual sales methods has all come to us unsolicited. The high cost of fuel has no doubt made many industrial enterprises more ready to inquire for electric power supply and a part of the business gained has been secured through the momentum of past sales efforts. But, even discounting the effect of the fuel shortage, there is still a normal increase in the demand for service.

The third aspect to which I desire to direct attention is the question of character of service. The user of electric energy in America receives more in auxiliary service rendered by the Engineering departments, trouble departments, lamp renewal department, etc., for the rates he pays, than the customer in any other country. We have naturally taken great pride in the completeness of the service we furnish, but there is no gainsaying that it imposes a very substantial expense which is probably not justified in these times. The Food Administrators are now recommending extra charge for delivery of parcels from other mercantile institutions and there is no reason why similar modification should not be made in our rules with respect to delivery of lamp renewals, appliances, etc., or that the amount of gratuitous assistance given by engineering departments should not be gauged by the necessities of the situation rather than a mere endeavor to accommodate the customer.

Finally, and by no means the least important duty of the sales department, lies in co-operation with the fuel conservation program of the Government. Lightless nights perform an admirable function of impressing upon all customers the necessity of conservation and no doubt the sum total of little economies effected will help the nation to save coal. But we would be deceiving ourselves as well as the public if we lost sight of the fact that the actual amount of fuel in the power house saved by this method is indeed very small, being a mere fraction of one per cent of the total coal consumed by the central station and an infinitesimal part of the coal utilized by the entire community. There is much to be done besides enforcing and thoroughly co-operating with the rule with respect to lightless nights. Perhaps the most important economy which can be achieved would be the organization by the fuel administrator of a voluntary inspection service for industrial plants, charged with making definite, practical recommendations on the proper insulation of steam pipes, satisfactory maintenance of boiler settings to avoid air leakage, scale elimination in boilers, care in banking and firing boilers, avoidance of general illumination of unoccupied factory space, possible savings effected in shutting down machinery when not required, etc., and the general elimination wherever possible of the non-condensing plant. In such a work the commercial engineering department could do effective work. It is not necessary for me to point out that such assistance by the central station is a labor of patriotism. We are naturally not interested in laboring to make the isolated plant more efficient. Our interest in normal times is to shut it down and sell it service. But the necessities for fuel conservation are so great and pressing and the possibilities in this field are so substantial that no effort should be spared to cooperate to the greatest extent.

**D. W. Roper:** Some years ago in preparing a paper on line transformers for the N. E. L. A., and attempting to justify the use of lightning arresters for the protection of transformers, I had some difficulty, and in order to find a sufficient amount on the correct side of the balance sheet which I drew up for the purpose, I had to go into the question of continuous service and place a value on interruptions to service, that is, on the amount which the company would probably care to spend to prevent an interruption. In doing this I consulted a number of department heads around the company and found a very wide range of opinions, varying from something more than the loss of revenue to 100 times the loss of revenue. That, of course, is a very wide figure, but placing, as I remember it, a value of about fifteen or twenty times the loss of revenue as the value to the company of the prevention of interruption, then the cost of lightning protection for preventing that interruption was warranted.

Since that time I find that the company's estimate of the value of an interruption has undergone some very wide changes with the times. For example, in the most prosperous times we have had, say in the last ten years, the company had been willing to extend their lines to reach the new customer when the estimated revenue from the customer over a period of say two years would cover the investment required for line extension. And at the same time where there was an equivalent amount of business already on the line or a group of old customers the company would be willing to incur a reasonable expenditure for a reserve supply, if that customer or group of customers would warrant it. And in those most prosperous times a large number of such connections were made so as to insure continuity of service.

But nowadays if customers want service from the Edison Company, they either must be on the lines or their revenue must be sufficient in one or two months, instead of two years, to pay for the cost of the extension. That shows immediately how hard they are struggling for new business, if that is the word to use. By the same sign you can gather how much they are doing in the way of providing for reserve connections in the case of the failure of the original supply. That is to say, they are doing nothing along that line at all.

In the most prosperous times, if there was any trouble with your lights or motors the company would, on request, send their trouble man to your premises. If the interruption to your lights was caused by some minor defect in the wiring or in the appliances or the sockets or plugs or switches, the repair man would, if he could do it in a half hour or so, make the repairs for you, sometimes temporarily, but he would make the repairs so as to enable you to get service, and then he would advise the customer what to do in the way of making permanent repairs. Now the company makes a minimum charge of thirty-five cents for sending a repair man to the premises, if the trouble is found to be on your premises. In addition, a further charge is made for any time over the first fifteen minutes. So the initial cost is thirty-five cents for going, and the regular rate for any time used beyond the fifteen minute limit. That applies also to heating appliance repairs in apartment buildings.

There was once a sort of perpetual guarantee on the cords for electric flat irons. The practise now is to charge the customer for the cost of repairs. That is about seventy-five cents, I believe, for the replacing of an electric iron cord. And the same applies to various other devices, toasters, broilers, etc. Those were all repaired if it could be done by the repair man, but not including, of course, the replacing of the heating apparatus.

Lamp bulbs have for a number of years been delivered free.

Of course the ordinance requires that they be furnished free, but it is not required that they be delivered free, and it is probable that within the next few weeks the company will make a charge for the delivery of them similar to that charged for answering a trouble call which I have just described. But there will of course be the option that the customer can get his own lamps free upon applying to one of the numerous delivery stations which the company will establish in various parts of the city.

On the question of station extensions some of you may have noticed the suggestion made by Mr. Budd of the Elevated Railroads to a Commission in Washington some weeks ago. His suggestion was, in effect, that the working hours of various lines of industry be adjusted so they do not all begin and end their working hours at the same time, requiring a very sharp transportation peak at that time, and that if the hours of starting and quitting could be spread over three or four hours in the morning and three or four in the afternoon instead of only one or one and one-half hours, then the peak in labor as well as the peak in power requirements could be very materially reduced.

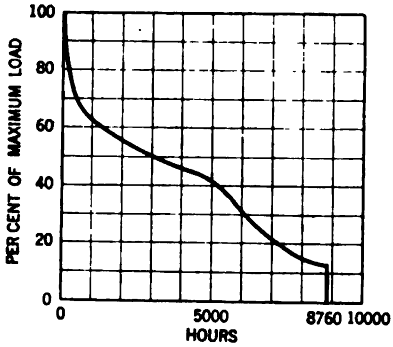

FIG. 5—ANNUAL LOAD CURVE OF COMMONWEALTH EDISON COMPANY.

Some years ago in making studies of the load of the Edison Company I prepared some annual load curves, which were of the form illustrated by Fig. 5. Any point in this curve shows the total number of hours during the year that a particular load has been carried. At that time the upper portion of the curve was investigated in considerable detail and it was found that the upper 25 per cent of the station capacity produced about one-half of 1 per cent of the kilowatt-hour output during the year. From this we can conclude that if Mr. Budd's suggestion could be put into effect so as to spread out the railway peak there would probably be no great difficulty in reducing the peak of the railway load by 25 per cent and if this were done, then the generating station supplying this railway load would probably not need any extensions during the period of the war.

***W. S. Gorsuch** (by letter): For text of this discussion see page 34.

**R. F. Schuchardt**: With relation to the fuel it must be remembered that the generating cost is not the entire cost and that when fuel increases 70 per cent the total cost per kilowatt-hour delivered is not increased by this amount. The fixed charges on the investment are often quite equal to the generating costs; in other words, if the generating stations were to be shut down and all employes were to be dismissed, about one-half of the cost would keep right on.

***F. A. Bryan** (by letter): For text of this discussion see page 22.

***R. G. Hudson** (by letter): For text of this discussion see page 23.

***M. Freund** (by letter): For text of this discussion see page 25.

**C. W. Pen Dell:** We men who have sales work find that our chief duty at present is to ward off those industries that are wanting additional power. We find at present that all of our customers are increasing, some of them slowly and some of them by leaps and bounds. We can hardly keep from supplying a customer who adds a few motors and increases his demand only a small percentage, and yet those small percentages are making very material increases in the demands on some of our lines, and we must watch that class of customers very closely. The larger customer that comes along and wants service for a steel furnace—which they all seem to be thinking of at the present time—asking for from 500 to 1000 kilowatts in capacity can be asked to finance the increase. It has become quite general that customers be required to finance the difference between pre-war costs and present costs.

**R. F. Schuchardt:** The question of financing extensions, is a very interesting one. Most companies require some form of advance deposit either equal to the first two months' bills, as mentioned by Mr. Roper, or to an amount on some other basis as has been described. But the line is only a part of the extension. The point is reached sooner or later when the substation back of the line is loaded to the full extent, or the generating station has no more reserve. Then this curve of Mr. Roper's serves as a new inspiration to the power salesman to go out and try to fill in, not at the peak, but in that big valley of low load. The resultant improvement in load factor, as has been indicated is one of the very beneficial effects which the situation is bringing about for the central station. You are all, of course, acquainted with the fact that in England interconnection of adjoining systems is being promoted by the Government.

**C. A. Keller:** Recently I have had occasion to make a study of increased values of generating stations and substations on account of present high prices and some of the figures probably

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*Discussions presented at both New York, January 11, and Chicago, January 14.

would be of interest to you, particularly since they bear out some of the statements of Mr. Jackson.

| | Percent increase in value today as compared with normal pre-war times. | |
|---|---|---|
| | Individual | Weighted Average |
| ***Generating Stations:*** | | |
| Buildings | 35 | 45 |
| Boiler equipment | 50 | |
| Turbines and auxiliaries | 55 | |
| Electrical | 35 | |
| ***Substations:*** | | |
| Buildings | 35 | 30 |
| Electrical | 20 | |
| Storage Batteries | 50 | |
| ***Line Extensions:*** | | |
| Conduits and tunnels | 30 | 50 |
| Poles, cross arms and pins | 60 | |
| Transmission line conductors | 50 | |
| Overhead distribution conductors | 80 | |
| Underground distribution conductors | 50 | |
| Services | 80 | |
| ***Transformers:*** | 30 | 30 |
| Total Weighted Average | | 43 |

These figures are of course very approximate; the pre-war costs being based upon varying market prices during the fourteen years preceding the war.

**F. H. Bernhard:** But Mr. Jackson has not pointed out a great many simple remedies. He does mention a few in which governmental action might be of advantage, including the possibility of increasing the rates. But in regard to the matter of increasing the rates, while that matter has been taken up in England to the extent of increases ranging from thirty to fifty per cent, a great many American central station companies have, and are doing their utmost to prevent horizontal increases in rates. In the central station business propaganda it has been a very strong argument to show that central station service has been steadily decreasing in cost in comparison with steady increases in costs for living and other expenses, and a desperate effort is being made to continue that downward trend. It is impossible to say whether practically all companies may not have to show an upward price curve before they continue for any length of time. However, Mr. Roper has pointed out quite a few features that may be not only considered but actually carried out, such as the employment of women and disabled soldiers.

This curve of Mr. Roper's is especially interesting, and it seems to me that the entire electrical industry should be actively boosting anything that will help to spread out the peak load, because it must be remembered that it is not the central station industry alone that is going to be benefited by spreading out the peak load, there are the electric railways, the traction lines. We have had in the past week a very forceful example of the importance of the urban transportation lines maintaining means of traffic—some sort of traffic—in the community.

Congestion usually existing during the rush hour periods has been very bad. During last week it was much worse due to heavy snow diverting all traffic to the street car tracks. Isn't it possible, not through merely gentle suggestions, but possibly from a concerted effort through bodies like the Association of Commerce and even through the City Council, to bring about a realization of the importance of staggering hours of employment? There are a great many business houses, factories and department stores closing at 5:30, between that and 6:00. So why can't we spread that time out at least two hours, thus relieving the peak load on all means of transportation.

**R. F. Schuchardt:** I am astonished that more of the gentlemen that have discussed the paper have not mentioned the daylight saving plan. One of the incidental effects of the war is the fact that the economies on which the central station business is built are also being appreciated in their application to other business.

**John W. Mabbs:** The great item in the production of power is fuel. The question up to us is how to solve the fuel problem. The cause for the present excessive price of fuel is given by the coal man as shortage of cars, inability of the railroads to transport the coal and the high price of labor.

The Government has taken over the railroads and by strict, judicious, and energetic business methods the thousands of cars, both loaded and empty, that usually stand for indefinite periods of time on the sidings could be kept moving and would provide ample facilities for transporting and handling all the coal required. This would furnish the cars necessary to run the mines full time or even overtime, instead of half or two-thirds time as at present.

If the excuse of the coal man, of shortage of cars and transportation is eliminated, he comes back with the excuse that he cannot get the necessary labor. As the Government has the right and the power to take over the railroads and draft men to go into the trenches, it also has the right and authority to take over the coal mines, *the country's natural resources*, and draft the men to work in them.

This would put a stop to a most outrageous profiteering in one of the necessities of life, and would not only go far to solve the power question but would be an untold benefit to every man, woman and child in this country.

The natural resources of the country should be conserved and operated for the benefit of all the people, and not for the enrichment of a few.

**A. Honegger:** The existing war conditions abroad and subsequent war emergencies at home have created enormous political and economical changes in our industries, the effect of which as regards public utilities has been eminently portrayed in the paper presented by Mr. William B. Jackson. The facts as they stand have been brought before us in concrete form and complete in most all details, and the results show clearly the urgent necessity for taking measures to meet the unusual conditions.

In the discussion of the paper most speakers have endeavored to seek solutions in proposing means of obtaining economies by improving efficiencies and by charging for incidental services not directly employed in producing and delivering electrical energy. All of these will no doubt have some bearing on the question and will bring some results to the satisfaction of the consumers as well as of the operator.

However, we must be aware of the fact that the operating percentage, the part of the gross revenues which is expended for fuel, labor and supplies, has increased so tremendously that the net has on the average been reduced to a point where the payment of the interest on actual moneys invested has reached an alarmingly close limit. This seems to me the important point brought to our attention by the paper and in part by the discussion.

This condition can only be remedied by an adjustment of rates, by an increase of the units of the gross revenue.

There are existing a number of important institutions, created for the purpose of adjusting rate questions and executive bodies establishing rates for the safeguarding of both the consumer and the operator, the operator referred to being the financiers and managers of public utilities. All of these have made great endeavors in the past to solve the individual problems to the best interest of the parties they represent, but today we are confronted with a far different situation which must be treated from a different standpoint and which must receive a just solution.

Our industries must keep on providing the fixed interest rate on moneys invested, if we shall avoid calamities of national importance and this can only be done by increasing light and power rates. The justice of increasing rates can clearly be illustrated by facts and it is up to the managers to come to the foreground with these facts and approach the institutions having jurisdiction over rates for just increases. Difficulties will no doubt be encountered, but the presentation of facts will surmount these obstacles. The present war has broadened the minds of all clear thinking Americans in a remarkable degree and this will not only help our cause, but should spur those of us most greatly affected to immediate and concerted action. A result must be obtained whereby public utilities will be placed on the basis of all other industries, at least to the extent of producing revenues adequate to paying interest rates on actual moneys invested.

Considerable action has been exhibited in different sections of the country and to my knowledge results have been obtained in certain communities in Iowa and Nebraska, where City Councils and Boards of Trustees have granted increases in rates on the basis of increased costs. A fair way of fixing rates in this newly created period is to have rates established to slide on a scale with increased fuel costs. In the same manner there have been contracts made for selling power in bulk to large consumers and power companies. This will bring public utilities more on the basis of other industries which have the privilege of increasing their selling prices in accordance with rise of raw material prices with the exception that adjustments made and sanctioned by institutions above mentioned will safeguard the acknowledged interests of the consumers prefixedly in case of public utilities.

I suggest that action be taken on a large scale; the facts demand immediate alterations if we want to avoid further losses and amend the present precarious status.

A further point of importance brought to our attention by the paper and discussions is the effect on provisions for improvements and extensions. The funds of the nation are required for the successful carrying out of the war and in this no doubt all public utilities and their fiscal agencies have heartily co-operated with the National Government and will continue to do even more so in the future. This of course, has curtailed the possibilities of larger extensions and new undertakings. It would be unpatriotic to stimulate such undertakings while the funds are needed in the great National Undertaking. Nevertheless there are certain immediate improvements and extensions which have been sanctioned as necessities, and while the ordinary channels for the supply of funds for such purposes have been fairly well throttled, it may be well to look for new sources of supply for provisioning and financing such authorized improvements. These will be found in localities and occupations mostly benefited economically by war activities and it is of interest to certain public utilities to note that recently the consumer in farming communities and also communities with war industries has become an investor in securities. Is it then not logical to seek funds for immediate and necessary improvements by selling securities to your consumers and tie over until conditions become normal; this will no doubt be beneficial and provide a relief to the extent of the possible activities achieved in this direction. It will also add a friend in every consumer who owns your local securities and establish friendly relationship much needed and sought for.

**W. B. Jackson:** You will feel that I have painted a gloomy picture, but I wish to say that I am an optimist with the rest of you, because as expressed in the earlier of my remarks, I have no doubt whatsoever, that this situation will be worked out with fine results from the standpoint of all.

Referring to Mr. Coffin's remarks, I am much interested in what he had to say. In the course of his valuable remarks he says that he does not agree with the statements in the paper relating to the economies in use of current to be expected, but I am inclined to believe that he did not catch the intent of the paper in this, namely, that our residence and ordinary commercial customers will normally and very properly economize in the use of current. I cannot imagine a residence customer today not doing his best to economize in the use of current, as compared with what he would have used two or three years ago; nor of a man using commercial service not making endeavor to economize such as he has never made before.

I have recently seen a concrete case where such economies have been clearly shown in both lighting and small power load. The curve of total current used per annum, taken for the end of each succeeding month, showed a fair increase in total current consumed on account of increase in number of customers, but the use per customer showed a decrease. This was beautifully shown by the fact that the average amount paid per kilowatt hour showed an increase, which meant no small decrease in the amount of current taken per customer.

From Mr. Gorsuch we might gather the impression that the increase in coal cost is the only important factor to be considered but from our analysis of the situation we find that the increase in the cost of coal is only approximately a half of the total increase. It is therefore seen that to obtain any true visualization of the situation as a whole it is necessary to take into consideration all of the items involved, as we have done.

With reference to the reduction of peak loads as compared with the average loads; that is one of the things that we must naturally expect will be done. It would seem that this can only be accomplished through government action, so far as present customers on the lines are concerned, but you can readily see how the valleys will be filled if the non-essential industries are required to keep off of the peaks but permitted to operate during the valleys.

In this paper we have taken the situation as nearly as we can as it stands today, and have pointed out the lines along which the changes are likely to come,—the beneficial changes and the detrimental changes. We have not endeavored to predict far into the future. My hope is that we have accomplished what is needed by such a paper, namely, that we have given you a visualization of the situation, so that you can appreciate how serious it is and how important it is for us to support and help those who are responsible for the correct solution of the problems involved. If we do get behind them, we can be just as confident that these problems will be solved satisfactorily, as we are that we will come out of the war victorious.

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# THE TECHNICAL STORY OF THE FREQUENCIES

BY B. G. LAMME

ABSTRACT OF PAPER

The various frequencies used in alternating-current work in America are first mentioned, and the primary reasons for their introduction are given. This covers, to a certain extent, the merits and demerits of each frequency as then understood, and the reasons each one met certain pressing commercial conditions at the time it was brought out. This is followed by a discussion of various alternating-current applications which were more or less dependent upon frequency.

It is shown that there was an apparent need for two standard frequencies in the region of 60 and 25 cycles, and, further, why 60 and 25 cycles have prevailed. The special fields of application of each one are discussed fully and it is shown why 25 cycles tended to dominate the field.

The persistent developments of the designing engineers gradually overcame the limitations in various types of 60-cycle apparatus so that eventually the 60-cycle system in its application approached more and more closely to the 25-cycle and, in the end, has taken the lead.

The outcome of the battle of the frequencies was determined far more by the conditions in the operating field than by the exploitation of any particular system by designing engineers. As a consequence, the energies of the engineers were directed exclusively toward overcoming the defects and limitations of the systems and not expended in fighting each other.

THE STORY of how and why the various commercial frequencies came into use and then dropped out again, in most cases, is not primarily the story of the frequencies themselves, but of the various uses to which the alternating current has been applied.* In other words, fundamental changes in the application of alternating current have led to radical changes in the frequencies. Some of the applications which have had a determining factor on the frequency of the supply system are as follows; incandescent lighting, transformers, transmission systems, arc lighting, induction motors, synchronous converters, constructional conditions in rotating machinery, and operating conditions. A brief consideration of these items individually,

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*It should be distinctly understood that this paper covers only the story of American development.

from the present viewpoint, indicates that while some of them had, at one time, very considerable influence in determining frequency conditions, yet, in a number of cases, the original reasons have disappeared through improvements and refinements, as will be described later.

At various times the following standard frequencies have been in use in this country, namely, 133⅓, 125, 83⅓, 66 2/3, 60, 50, 40, 30 and 25 cycles per second. These did not appear chronologically in the order given above, and a few odd frequencies in a few special applications are omitted.

In the following, the various frequencies will be considered more or less in the order of their development and basic reasons will be given for their choice, and the writer will endeavor to show why certain of them have persisted, while others have dropped out. It will also be shown why the commercial situation has first tended strongly toward certain frequencies and afterwards swung toward others.

### 133 AND 125 CYCLES

In the earliest alternating work, the whole service consisted of incandescent lighting, and the electric equipment was made up of small high-speed belted single-phase generators and house-to-house distributing transformers. As the transformers were of small capacity and as their design was in a very crude state, it was believed that a relatively high frequency would best meet the transformer conditions. A choice of such an odd frequency as 133⅓ cycles per second, is due to the fact that in those early days (1886 to 1893) frequencies were usually designated in terms of alternations per minute. One of the earliest commercial generating units constructed by the Westinghouse company had a speed of 2000 rev. per min. and had eight poles. This presented a fairly convenient constructional arrangement for the surface-wound type of rotating armature, which was the only one recognized at that time. The speed of 2000 rev. per min., with eight poles, gave 16,000 alternations per minute, or 133⅓ cycles per second according to our present method of designation. Thus the earliest frequency in commercial use in this country was fixed, to a certain extent, by constructional reasons, although the house-to-house transformer problem apparently indicated the need for a relatively high frequency. The Thomson-Houston company adopted a standard frequency of 15,000 alternations per minute, (125 cycles) instead of the Westinghouse 16,000, but the writer does not know why this difference was made. However, the two frequencies were so close together that practically they could be classified as one.

At this time, it should be borne in mind, there were no real transmission problems, no alternating-current arc lighting, no induction motors and the need for uniform rotation of the generators was not recognized. The induction motor, in its earliest stages, came in 1888 and considerable work was done on it in 1889 and 1890, but it required polyphase supply circuits and comparatively low frequency and, therefore, it had no connection whatever with the then standard single-phase, 133⅓ and 125-cycle systems. The synchronous converter was also unheard of (one might say almost undreamed of) at that time.

## 60 CYCLES

In 1889 or 1890 it was beginning to be recognized in this country that some lower frequency than 125 and 133⅓ cycles would be desirable. Also about this time direct-coupled and engine-type alternators were being considered in Europe and it was felt that such construction would eventually come into use in America. It was appreciated that in such case, 133⅓ cycles would present very considerable difficulties compared with some much lower frequency, due to the large number of poles which would be required. For instance, an alternator direct driven by an 80-rev. per min. engine would require 200 poles to give the required frequency and such construction was looked upon as being practically prohibitive. About this time Mr. L. B. Stillwell, then with the Westinghouse company, made a very careful study of this matter of a new frequency, in connection with the possibilities of engine-type generators, and after analyzing a number of cases, it appeared that 7200 alternations per minute (60 cycles per second), was about as high as would be desirable for the various engine speeds then in sight. Transformer constructions and arc lighting were also considered in this analysis. While it was deemed that a somewhat higher frequency might be better for transformers, yet a lower frequency than 60 cycles was considered as possibly better for engine-type generators. A compromise between all the various conditions eventually led to 60 cycles as the best frequency. However, while this frequency originated about 1890, it did not come into use suddenly, for it was impossible to introduce such a radical change in a brief time. Moreover, the direct-coupled or engine-type generator was slow in coming into general use and, therefore, there was not the necessity for the introduction of this low frequency in many of the equipments sold from 1890 to 1892. However, by 1893, 60 cycles became pretty firmly established and was sharing the business with the 133⅓-cycle systems. It should be borne in mind that, at this time, the adoption of this frequency was not considered as a direct means for bringing forward the polyphase induction motor, for the earlier 60-cycle systems, like the 125- and 133⅓-cycle, were all single-phase. Also, it was then thought that the polyphase motor would possibly require a still lower frequency and, moreover, the polyphase system was looked upon as in a class by itself, suitable only for induction motor work. At that time the introduction of polyphase generators for general service was not contemplated. This followed about two or three years later.

In 1890 the Westinghouse company, which had been developing the Tesla polyphase motor, laid aside the work, largely on account of there being no suitable general supply systems for this type of motor. The problem was again revived in 1892, in an experimental way, with a view to bringing out an induction motor which might be applied on standard frequencies such as could be used in commercial supply circuits for lighting and other purposes. It should be understood that at this time such circuits were not in existence but were being contemplated. In 1893, after the polyphase motor had been further developed up to the point where it showed great commercial possibilities, the best means for getting it on the market were carefully considered. It was decided that the best way to promote the induction motor business was to create a demand for it on commercial alternating-current systems. This meant that, in the first place, such systems must be created. Therefore, it was decided to undertake to fill the country with polyphase generating systems, which were primarily to be used for the usual lighting service. It was thought that, with such systems available, the time would soon come when there would be a call for induction motors. In this way experience would be obtained in the construction and operation of polyphase generators and the operating public would not be unduly handicapped in the use of such generators, compared with the older single-phase types.

An early example of this new practise was in the 2000-kw. polyphase generating units used for lighting the Chicago World's Fair in 1893. Here the single-phase type still persisted, as each generator unit was made up of two similar frames placed side by side, but with their single-phase armatures displaced one-half pole pitch from each other so that the combined machine delivered two single-phase currents displaced 90 degrees from each other. It was considered that each circuit could be regulated independently for lighting service, and polyphase motors could be operated from the two circuits. These generators (at that time the largest in this country) were designed in 1892 and were of 60 cycles. These, therefore, indicate the tendency at that time toward lower frequency and polyphase generation, although commercial polyphase motors were not yet on the market.

## 25 CYCLES

At the same time that 60 cycles was selected as a new standard it was recognized that at some future time there would be a place for some much lower frequency, but it was not until two years later that this began to narrow down to any particular frequency. In 1892 the first Niagara electrification, after several years consideration by eminent authorities, had centered on polyphase alternating current as the most desirable system. The engineers of the promoting company had also worked out what they considered the most suitable construction of machine. This involved 5000-h. p. units at 250 revolutions per minute. Prof. George Forbes, one of the engineers of the company had furnished the electrical designs for a machine with an external rotating field and an internal stationary armature. His design used eight poles, thus giving 2000 alternations per minute, or 16⅔ cycles per second. Quite independently of this, the Westinghouse company, in 1892, had been working on the development of synchronous converters, using belted 550-volt d-c. generators with two-phase collector rings added. The tests on these machines had shown the practicability of such conversion and had even proved at this early date, that the converter copper losses were much lower than in the corresponding d-c. generators. Thus it is an interesting fact that the first evidence of this important principle was obtained from a shop test rather than by calculation. The writer, from an analysis of the tests, which were made under his immediate direction, concluded that the armature copper losses must be considerably lower than in the same machine used as a d-c. generator. He also brought the matter to the attention of Mr. R. D. Mershon, then with the Westinghouse company, and the problem was then worked out mathematically by him and the writer, in two quite different ways, but with similar results, showing that the converter did have actually very much reduced copper losses.

As a result of this work of the Westinghouse company on the synchronous converter, it was decided that, to make such machines practicable, some suitable relatively low frequency was required. This appeared to be about 30 cycles. About this time the construction of the Niagara generators was taken up with the Westinghouse company to see whether it would construct these machines according to the designs submitted by the promoting company's engineers. These designs were gone over as carefully as the knowledge of such apparatus, at that time, permitted, and many apparent defects and difficulties were pointed out. The Westinghouse company then proposed, as a substitute, a 16-pole, 250-rev. per min. machine (the speed being definitely fixed at 250 rev. per min.). This gave 33⅓ cycles or as near to the Westinghouse proposed 30-cycle system, as it was possible to get. Then many arguments were brought forward, pro and con, for the two machines and frequencies. Prof. Forbes' preference for 16⅔ cycles was based partly on the possibilities it presented for the construction and operation of commutator type motors, just as with direct-current circuits. The Westinghouse contention was that this frequency was too low for any kind of service except possibly commutator type machines. Tests were made with incandescent lights and it was found that at 33⅓ cycles there was little or no winking of light, while at 16⅔ cycles, the winking was extremely bad. Tables were also made up, showing the limited number of speed combinations at 16⅔ cycles for induction motors, in case such should come into use. This showed how superior the 33⅓ cycles would be as regards such apparatus. It was also brought out that synchronous converters, when such became commercial, would be much better adapted for the higher frequency, as the choice of speeds would be much greater. From the present viewpoint the arguments appear to have been much in favor of the Westinghouse side of the case.

As a consequence of all this discussion the suggestion was advanced by some one, that a 12-pole, 250-revolution machine, (that is, 3000 alternations, or 25 cycles), might meet sufficiently the good qualities of both of the proposed frequencies and would thus be a good compromise. In consequence a 12-pole, 25-cycle machine was worked up by the Westinghouse company and eventually this frequency was adopted for the Niagara generators. Afterwards, while these generators were being constructed it was brought out pretty strongly that the great advantage of this frequency would be in connection with synchronous converter operation, but that it was also extremely well adapted for slow-speed engine-type generators, which were then coming into use. In consequence of the prominence given this frequency it was soon adopted as a standard low frequency, especially in those plants where synchronous converters were expected to form a prominent part of the system.

However, while 60 and 25 cycles came into use, as described above, it must be recognized that they had competitors. For instance, 66⅔ cycles (8000 alternations or one-half of 16,000) was used to a considerable extent by one of the manufacturing companies. Also 50 cycles came into use in certain plants and, to a certain extent, is still retained, but has become the standard high frequency of Europe. Instead of 25 cycles, the Westinghouse company advocated 30 cycles for some of its plants, largely because with the 20 per cent higher speeds permissible with such frequencies, the capacities of induction motors could be correspondingly increased and also incandescent lighting was more satisfactory. However, it was soon recognized that the 66⅔ and 30-cycle variations from the two leading frequencies of 60 and 25 cycles were hardly worth while, and they were gradually dropped, except in plants already installed. A brief attempt was made at a somewhat later period to place 40 cycles upon the market as a substitute for both 25 and 60 cycles. This was done under the impression that 40 cycles would give a universal system for arc and incandescent lighting, transmission, induction motors, synchronous converters and about everything else. This frequency possessed many merits and it was thought, at one time, that it might win out, but apparently the two other frequencies were too well established, and the 40-cycle system eventually lost ground.

The problem of the frequencies finally narrowed down to the two standards, and these two were accepted because it was thought that they covered such entirely different fields of service that neither of them could ever expect to cover the whole. In other words, two standards were required to cover the whole range of service. It was recognized that 25 cycles would not take care of alternating-current arc lighting and that it was questionable for incandescent lighting in general. In other ways, such as suitability for engine-type construction, application to induction motors and synchronous converters and transmission of power to long distances, it met the needs of an ideal system, as then understood. Also, in parallel operation of engine-type alternators, which was one of the serious problems of those days, the 25-cycle machines were unquestionably superior to the 60-cycle ones, due to the lesser displacement of the e. m. f. waves with respect to each other with a given angular variation in the engine speeds. However, although the 25-cycle system presented so many advantages, it could not take care of the lighting business, and, therefore, could not entirely dominate the situation.

As regards 60 cycles, it was felt that this could handle the direct lighting situation in a very satisfactory manner and was possibly better suited for transformers than 25 cycles, although there were differences of opinion in this matter, especially when it came to the larger capacities. It was reasonably well adapted for induction motors in general, but not for very low speeds. In matters of transmission and in the operation of synchronous converters it was thought to be vitally defective.

From the above consideration it would appear that the 25-cycle systems presented the stronger showing as a whole and, therefore, there was a decided tendency toward this frequency, except in those cases where lighting directly from the alternating-current system was considered of prime importance. In those systems, such as many of the Edison companies, where low-voltage three-wire direct current was used from synchronous converters, the tendency was almost solidly toward the 25-cycle system. In those days the central station, which had been committed to the 60-cycle system so deeply that it could not change, was looked upon with commiseration. Sixty-cycle plants were looked upon, to a certain extent, as a necessary evil. In fact, so strong was the tendency toward 25 cycles that in many cases 25-cycle plants were installed for industrial purposes, where 60 cycles would have been better. The 25-cycle synchronous converter development advanced by leaps and bounds and the machines were so good in their operation that it was believed that 60-cycle converters could never be really competitive with them.

On the other hand, in those large plants, which were so "unfortunate" as to have 60 cycles installed, many apparent makeshifts were adopted to meet the various service requirements.

In arc lighting, incandescent lighting, transformers and motors there was no need for makeshifts. However, in conversion to direct current, one of the greatest difficulties appeared. There were many who advocated motor-generators for this purpose, largely because the 60-cycle converter was thought to be impracticable, in spite of the fact that the manufacturing companies were putting them on the market. The 60-cycle converter at that time bore a bad name. It is now recognized that many of the faults of the early 60-cycle synchronous converter operation were not in the converters themselves, but were, to a considerable extent, in the associated apparatus. Low-speed engine-type, 60-cycle generators were not always adapted for operation of synchronous converters. In fact, in numerous cases such generators would not operate in an entirely satisfactory manner in parallel with each other, and yet when it was attempted to operate synchronous converters from these same generators the unsatisfactory results were not blamed upon the generating system but upon defects of the converters themselves. Unfortunately, defects in the generating and transmission systems usually appeared in the converters as sparking and flashing, and such troubles naturally would be credited to defects in the construction of the converters themselves. In fact, in those days, 60-cycle converters were expected to do things which now are considered as absurd. For instance, in one case in the writer's knowledge a 60-cycle synchronous converter was criticized as being a very badly designed piece of apparatus, due to serious flashing at times. Investigation developed that this converter was expected to operate on either one of two independent 60-cycle systems with no rigid frequency relation to each other. The converter in service was thrown from one system to the other indiscriminately, and sometimes it flashed in the transfer and sometimes it did not. The machine was considered to be "no good" because it would not always stand such switching.

At one time the writer stood almost alone in his belief that the 60-cycle synchronous converter presented commercial possibilities sufficient to make it a strong future contender with the 25-cycle machine, provided proper supply conditions were furnished and certain difficulties in the proportions of the converter itself were overcome. One basis for his contention was that in some of the 60-cycle plants, where the generator rotation was quite uniform, the converters were evidently much superior in their operation to other plants, using low-speed engine-type generators with considerable periodic variations. In such plants the hunting tendency of the converters was very greatly reduced, with consequent improvement in sparking and general operation. It was early recognized that hunting was a very harmful condition, both in 60- and 25-cycle synchronous converters, but whereas it was a relatively rare condition in 25-cycle plants it was much more common with 60 cycles. However, the operating public was not particularly concerned whether the trouble was in the generating plant or in the converters themselves, as long as such trouble existed and was not overcome. Very early in the synchronous converter development it was found that hunting would produce sparking or flashing at the commutators of the converters. However, even in those plants where there was no hunting apparent, there was difficulty at times due to flashing, especially with sudden change of load, which resulted in temporary increase in the d-c. voltage. This was a difficulty which was inherent in the converter itself and could not be blamed entirely upon the generating or transmitting conditions, for 25-cycle machines were practically free from this trouble under similar conditions of operation. Investigation developed the fact that this flashing trouble was due largely to unduly high value of the maximum volts between commutator bars. This difficulty was recognized long before it was overcome, simply because certain physical limitations in construction had to be removed. There were two ways in which the maximum volts per bar could be reduced, namely, by increasing the number of commutator bars per pole and by decreasing the ratio of the maximum volts to the average volts per bar, that is, by increasing the ratio of the pole width to the pole pitch, but both of these involved structural limitations in the allowable peripheral speeds of the commutator and the armature core. Here is where a little elementary mathematics comes in. The peripheral speed of the commutator is directly proportional to the distance between adjacent neutral points on the commutator, and the frequency. Therefore, with a given frequency the distance between the adjacent neutral points is directly proportional to the peripheral speed. Thus, with a commutator speed of 4500 ft. per min. which was then considered an upper limit, the distance between adjacent neutral points on a 60-cycle converter is only 7½ in. (19 cm.) This distance is thus fixed mathematically and is independent of the number of poles or revolutions per minute, or anything else, except the peripheral speed and the frequency. With this distance of 7½ in., (19 cm.), about the only choice in commutator bars per pole was 36, giving an average of 16⅔ volts per bar on a 600-volt machine, and nearly 20 volts per bar with momentary increase of voltage to 700, which is not uncommon in railway service.

However, it is not this average voltage which fixes the flashing conditions, but it is the maximum voltage between bars, and this is dependent upon the average voltage and upon the ratio of the pole width to the pole pitch. Here is where one of the serious difficulties came in. The pole pitch is directly dependent upon the peripheral speed of the armature core and the frequency. Therefore, in a 60-cycle machine, if the peripheral speed is fixed, the pole pitch is practically fixed. For example, with an armature peripheral speed of 7200 ft. per min., (which was considered high at that time,) the pole pitch becomes 12 in. (30.48 cm.), regardless of any other considerations. and here was where a most serious difficulty was encountered. If a sufficiently wide neutral zone for commutation was allowed the interpolar space became so wide that there was not enough left for a good pole width. For instance, if the interpolar space was made 6 in. (15.24 cm.) wide, in order to give a sufficiently wide commutating zone to prevent sparking or flashing, due to fringing of the main field, then this left only 6 in. for the pole face. With this relatively narrow pole face the ratio of the maximum volts to the average volts was so high that with the 36 commutator bars per pole the machine was sensitive to arcing between commutator bars, thus resulting in flashing. By widening the pole face this particular difficulty would be lessened or overcome, but with the fixed pole pitch of 12 in. (30.48 cm.) the neutral zone would be so narrowed as to make the machine sensitive to sparking and flashing at the brushes. Thus, no matter which way we turned we encountered trouble. Obviously there were two directions of improvement, namely, by increasing the number of commutator bars, thus reducing the average voltage, and by increasing the pole pitch, thus allowing relatively wider poles with a given interpolar space. These two conditions look simple and easy, but it took several years of experience to attain them. When we have reached apparent physical limitations in a given construction, especially when such limitations are based upon long experience, we have to feel our way quite slowly toward higher limitations. For instance, in the case of the 60-cycle converters we could not boldly jump our peripheral speeds 20 to 25 per cent higher and simply assume that everything was all right. We first had to build apparatus and try it out for a year or so. Troubles, due to peripheral speed, do not always become apparent at once, and thus time tests are necessary. Therefore, while the peripheral speeds of the 60-cycle synchronous converters were actually increased 20 to 25 per cent practically in one jump, yet it took two or three years of experimentation and endurance tests before the manufacturers felt sure enough to adopt the higher speeds on a broad commercial scale. Thus, while the change from the older more sensitive type of 60-cycle converter to the later type occurred commercially within a comparatively short period, yet the actual development covered a much longer period.

Let us see now what an increase of 25 per cent in the peripheral speeds actually meant. As regards the commutator, the number of bars could be increased 25 per cent, that is, from 36 up to 45 per pole, which was comparable with ordinary d-c. generator practise. In the second place, an increase of 25 per cent in the peripheral speed of the armature core meant a 15-in. (38.1-cm.) pole pitch, where 12 in. (30.8 cm.) was used before. Assuming, as before, a 6-in. (15.24-cm.) interpolar space, then the pole face itself became 9 in. (22.8 cm.) in width instead of 6 in. (15.24 cm.) or an improvement of 50 per cent. In fact, this latter improvement was so great that some manufacturers did not consider it necessary to increase the number of commutator bars, although in the Westinghouse machines both steps were made.

The above improvements so modified the 60-cycle converter that it began to approach the 25-cycle machine in its general characteristics. It was still quite expensive compared with the 25-cycle, due to the large number of poles, and its efficiency was considerably lower than its 25-cycle competitor, on account of high iron and windage losses. However, due to the need for such a machine it was gradually making headway, in spite of handicaps in cost and efficiency.

Almost coincident with the initiation of the above improvements in the 60-cycle converter, came another factor which has had much to do with the success of this type of machine. This was the advent of the turbo-generator for general service. As stated before, one of the handicaps of the 60-cycle converter was in the non-uniform rotation of the engine-type generators which were common in the period from 1897 to about 1903 or 1904. But, about this latter date, the turbo-generator was making considerable inroads on the engine-type field and within a relatively short period it so superseded the former type of unit, that it was recognized as the coming standard for large alternating power service. With the turbo-generator came uniform rotation and this at once removed one of the operating difficulties of the 60-cycle converters. However, in the early days of the turbo-generator, 25 cycles still was in the lead and many of the earlier generators were made for this frequency, especially in the larger units. But it was not long before it was recognized that 60 cycles presented considerable advantage in turbo-generator design due to the higher permissible speeds. In the earlier days of turbo-generator work, this was not recognized to any extent, as the speeds of all units were so low that the effect of any speed limitations was not yet encountered. For instance, a 1500-kw., 60-cycle turbo-generator would be made with six poles for 1200 revolutions, while a corresponding 25-cycle unit would be made with two poles for 1500 revolutions. This slightly higher speed at 25 cycles about counterbalanced the difficulties of the two-pole construction compared with the six-pole. However, before long, more experience enabled the 1200 r.p.m., 60-cycle machine to be replaced by 1800 revolutions, and a little later by two pole machines at 3600 revolutions. This, of course, turned the scales very much in the other direction. In larger units, however, the advantage still appeared to be in favor of 25 cycles, but in the course of development, 1500 revolutions was adopted quite generally for 25-cycle work, and this was the limiting speed, as such machines had only two poles, or the smallest number possible with ordinary constructions. On the other hand, for 60 cycles, 1800 revolutions was adopted quite generally for units up to almost the extreme capacities that had been considered, consequently the constructional conditions in the large machines swung in favor of 60 cycles. Therefore, with the coming of the steam turbine and the development of high-speed turbo-generator units, the tendency has been strongly toward 60 cycles. This, with the greater perfection of the 60-cycle converter, had much to do with directing the practise away from the 25 cycles.

However, there were other conditions which tended strongly toward 60 cycles. In the early development of the induction motor, the 25-cycle machines were considerably better than the 60-cycle and possibly little or no more expensive. However, as refinements in design and practise came in, certain important advantages of the 60-cycle began to crop out. For instance, with 25 cycles there is but little choice in speed, for small and moderate size motors. At this frequency a four-pole motor has a synchronous speed of only 750. The only higher speed permissible is 1500 revolutions with two poles, and it so happens that in induction motors the two-pole construction is not materially cheaper than the four pole, consequently the principal advantage in going to 1500 revolutions was only in getting a higher speed where such was necessary for other reasons than first cost. However, in 60 cycles the case is quite different, where a four-pole machine can have a speed of 1800 revolutions, synchronous, a six-pole 1200, an eight-pole 900 and a ten-pole 720 revolutions. In other words, there are four suitable speed combinations where a 25-cycle motor has practically only one. Moreover, with the advance in design it developed that these higher speed 60-cycle motors could be made with nearly as good performance as with the 25-cycle motors of same capacity, and at somewhat less cost. However, leaving out the question of cost, the wider choice of speeds alone would be enough to give the 60-cycle motor a pronounced preference for general service.

However, there is one exception to the above. Where very low-speed motors are required, such as 100 rev. per min., the 60-cycle induction motor is at a considerable disadvantage compared with 25 cycles, or this has been the case in the past. It is partly for this reason that the steel mill industry, through its electrical engineers, adopted 25 cycles as standard some ten or fifteen years ago. At that time, it was considered that in mill work, in general, there would be need for very low-speed motors in very many cases. However, due to first cost, as well as other things, there has been a tendency toward much higher speeds in steel mill work, through the use of gears and otherwise, so that part of this argument has been lost. However, there still remain certain classes of work where direct-connected very low-speed induction motors are desirable and where 25 cycles would appear to have a distinct advantage.

In view of the above considerations, steel mill work has heretofore gone very largely toward 25 cycles, particularly where the mills installed their own power plants. However, in recent years there has been a pronounced tendency toward purchase of power, by steel mills, from central stations, and the previously described tendency of central stations toward 60 cycles has forced the situation somewhat in the steel mills, particularly in those cases where the central power supply company can furnish power at more reasonable rates than the steel mill can produce in its own plant. This, therefore, has meant a tendency toward 60 cycles in steel mill work, even with the handicap of inferior low-speed induction motors. But, on the other hand, remedies have been brought forward even for this condition. The great difficulty in the construction of low-speed, 60-cycle induction motors is in the very large size and cost if constructed for normal power factors, or the very low power factor and poor performance if constructed of dimensions and costs comparable with 25 cycles. In the latter case the extra cost is not entirely eliminated because a low power factor of the primary input implies additional generating capacity, or some means for correcting power factor on the primary system. However, in some cases it is entirely practicable to correct the power factor in the motors themselves by the use of so called "phase advancers" of either the Leblanc or the Kapp type. Such phase advancers are machines connected in the secondary circuits of induction motors and so arranged as to furnish the necessary magnetizing current to the rotor or secondary instead of to the primary. In this way the primary current to the motor will represent largely energy and the power factors can be made equal to, or even much better than in, the corresponding 25-cycle motor; or, in some cases, the conditions may be carried even further so that the motor is purposely designed with a relatively poor power factor, in order to further reduce the size and cost, and the phase advancers are made correspondingly larger. In those cases where the cost of the phase advancer is relatively small compared with the main motor, there may be a considerable saving in the cost of the main motor and then adding part of the saving to the cost of the phase advancer.

One difficulty in the use of phase advancers is found in the variable speeds required in some kinds of mill work. In those cases where flywheels driven by the main motors are desirable to take up violent fluctuations in load, it is necessary to have considerable variations in the speed of the induction motor, in order to bring the stored energy of the flywheel into play. Unfortunately this variable speed in the induction motor is one of the most difficult conditions to take care of with a phase advancer, so that here is a condition where the 60-cycle motor is at a decided disadvantage.

Thus it may be seen from the above that even in the steel mill field, where the induction motor has the most extreme applications, there is quite a strong tendency toward 60 cycles, due to the purchase of power from central supply systems.

There remains one more important element which has had something to do with the tendency toward 60 cycles, namely, the transmission problem. In the earlier days of transmission of alternating current, 25 cycles was considered very superior to 60 cycles due to the better inherent voltage regulation conditions. At one time, it was thought that 60 cycles had a very limited field for transmission work. However, a number of power companies in the far west had installed 60-cycle plants, principally for local service and with the growth of these plants came the necessity for increased distance of transmission through development of water powers. At first it was thought they were badly handicapped by the frequency, but gradually the apparent disadvantages of their systems were overcome and the distances of transmission were extended until it became apparent that they could accomplish practically the same results as with 25 cycles. Part of this result has been obtained by the use of regulating synchronous condensers. It is a curious fact that the possibility of synchronous motors used as condensers for correction of disturbances on transmission systems, has been known for about 25 years, but it is only within quite recent years that they have come into general use as a solution of the transmission problem, and largely in connection with 60-cycle plants. In 1893 the writer applied for a patent on the use of synchronous motors as condensers for controlling the voltage at any point on a transmission system by means of leading or lagging currents in the condenser itself. A broad patent was obtained, but there was no particular use made of it until it had practically expired.

Another improvement came along which still further helped to advance 60 cycles to its present position, namely, the use of commutating poles in synchronous converters. The principal value of commutating poles in the 60-cycle converters, has not been so much in an improvement in commutation over the older types of machines, as in allowing a very considerable reduction in the number of poles with corresponding increase in speed, resulting in reduction in dimensions. As a direct result of this increase in speed the efficiencies of the converters have been increased. If, for instance, the speed of a given 60-cycle converter can be doubled by cutting its number of poles to one-half, while keeping the same pole pitch and the same limiting peripheral speed, then obviously the amount of iron in the armature core is practically halved and, at the same magnetic densities the iron loss is also practically halved. Also with the same peripheral speed and half diameter of armature the windage losses can be decreased materially. Thus the two principal losses in the older converters have been very much reduced. There have also been reductions in the total watts for field excitation, and in other parts, so that, as a whole, the efficiency for a given capacity 60-cycle converter has been brought up quite close to that of the corresponding 25-cycle machine, even when the latter is equipped with commutating poles. This gain of the higher frequency compared with the lower is due to the fact that the lower-frequency machine was much more handicapped in its possibilities of speed increase, and furthermore, the iron losses and windage represented a much smaller proportion of the total losses in the low-frequency machine. This improvement in the efficiency of the 60-cycle converter together with the lower losses in the 60-cycle transformer as compared with the 25-cycle, has brought the 60-cycle equipment almost up to the 25-cycle, so that the difference at present is not of controlling importance. This development has given further impetus toward the acceptance of 60 cycles as a general system.

Formerly a serious competitor with the 60-cycle converter was the 60-cycle motor-generator. This was installed in many cases because it was considered more reliable and more flexible in operation than the synchronous converter. Both of these claims were true to a certain extent. However, with improvements in the synchronous coverter the difference in reliability practically disappeared, but there remained the difference in flexibility. In the motor-generator set, the d-c. voltage could be varied over quite a wide range, while in the older 60-cycle rotaries the d-c. voltage held a rigid relation to the alternating supply voltage. However, with the development and perfection of the synchronous booster type of converter, flexibility in voltage was obtained with relatively small increase in cost and minor loss in economy. This has been the last big step in putting the 60-cycle converter at the front as a conversion apparatus, so that today it stands as the cheapest and most economical method of converting alternating current to direct current. Moreover, while the 25-cycle synchronous converter has apparently reached about its upper limit in speed, there are still possibilities left for the 60-cycle converter.

In line with the above it is of interest to note that for units of 1000 kw. and less, the 60-cycle converter has nearly driven the 25-cycle out of business from the manufacturing standpoint.

For the very large size converters, 25 cycles still has the call, but largely in connection with many of the railway and three-wire systems, which have been installed for many years; that is, the growth of this business is in connection with existing generating systems. However, the 60-cycle converter, in large capacity units, is gaining ground rapidly and it is of interest to note that the largest converters yet built, namely, 5800 kw., are of the 60-cycle type.

One most interesting point may be brought out in connection with the above described "battle of the frequencies", namely, it was fought out in the operating field, and between conditions of service, and not between the manufacturing companies. This is a very good example of how such matters should be handled. Here the engineers of the manufacturing companies were expending their efforts to get all possible out of both frequencies, and consequently development proceeded apace. When 60-cycle frequency seemed to be overshadowed by its 25-cycle competitor, the engineers took a lesson from the latter and proceeded to overcome the shortcomings of the former. It was no innate preference of the designing engineers that has brought the higher frequency to the fore; it was the recognition that it had greater merits as a general system, if its weak points could be sufficiently strengthened; and, therefore, the engineers turned their best efforts toward accomplishing this result.

It must not be assumed, for a moment even, that because 60 cycles appears to be the future frequency in this country, that 25 cycles was a mistake. Decidedly it was not. In reality it formed a most important step toward the present high development of the electric industry. Many things we are now accomplishing with 60 cycles would possibly never have been brought to present perfection, if the success of the corresponding 25-cycle apparatus had not pointed the way. The success of the 25-cycle converter, and the high standard of operation attained, gave ground for belief that practically equal results were obtainable with 60 cycles. Therefore, the 25-cycle frequency served a vast purpose in electrical development; it was a high class pacemaker, and it isn't entirely out-distanced yet.

There has been considerable speculation as to what two standard frequencies would have met the needs of the service in the best manner, and would have resulted in the greatest development in the end. It has been claimed by some, that 50 and 25 cycles would have been better than 60 and 25. In the earlier days possibly the former would have been better, but as a result both standards might have persisted longer. In any case, the general advantages would have been small. In one class of machines, namely, frequency changers, consisting of two alternators coupled together, the 25-50 combination would certainly have been advantageous.

Again it has been questioned whether 30 and 60 cycles would not have been a better choice. This was the original Westinghouse choice of frequencies, but not on account of frequency changers. As stated before, it was felt that 30 cycles could do about all that 25 cycles could, and would give an advantage of 20 per cent higher speed in motors and converters, with correspondingly higher capacities. Also for direct coupled alternators, the two-to-one ratio of frequencies would fit in nicely with engine speeds, in many cases. Possibly, from the present viewpoint, the choice of thirty cycles, would have longer retained the double standard.

Something further may be said regarding the 40-cycle system, brought out by the General Electric Company. This contained many very good features, for the time it was brought out. It was then believed that if the 60-cycle frequency was retained, the double standard was necessary. The 40-cycle system was an attempt to eliminate this double standard. It apparently furnished a better solution than 60 cycles then promised for the synchronous converter problem, and was a fair compromise in about everything else. But it came too late, for the 25-cycle system was too firmly entrenched, and for further development, the designing engineers preferred to expend their energies in seeing what could be accomplished with 60 cycles, as this seemed to present greater possibilities than either 25 or 40, if it could be sufficiently perfected. Thus the 40-cycle system probably missed success due to being just a little too late.

As to 50 cycles, it was stated that this is still in use to a limited extent. Most of the 50-cycle plants in this country are in California. Such plants were started during the nebulous period of the frequencies, and have persisted, to a certain extent, partly because certain 60-cycle apparatus could be easily modified to meet the 50-cycle requirements. Also, as 50 cycles is the standard in many foreign countries to which this country exports equipment, the use of 50 cycles in some home plants has not been unduly burdensome from the manufacturers' standpoint.

In addition to the preceding, there have been certain classes of electric service which have depended upon frequency, but which have not been a determining factor in fixing any particular frequency. Among these may be considered commutating types of a-c. apparatus. The first a-c. commutating motors of any importance, which appeared, were, of course, the 25-cycle, single-phase railway motors. These as a rule have operated from their own generating plants, or from other plants through frequency-converting machinery. One exception in the railway work may be noted in the use of 15 cycles on the Visalia plant in California. There is a pretty well defined opinion among certain engineers experienced in such apparatus that some low frequency, such as 15 cycles, would present very considerable advantages in the use of single-phase railway motors in very heavy service, such as on some of the western mountain roads. Here the problem is to get the largest possible motor capacity on a given locomotive, and the main advantage of the lower frequency would be in allowing a very materially higher capacity within a given space. This does not imply reduced weight or cost compared with the 25 cycles, but simply means greater motor capacity. With the modern, more highly developed, single-phase types of railway motors, it would appear that there may be very considerable possibilities in 15 cycles.

Outside of the railway field, there has been more recently a development of various types of a-c. commutating apparatus, principally in connection with heavy steel mill electrification work. Such apparatus has been largely in the form of three-phase commutating machines and these have been used principally in connection with speed control of large induction motors. As these regulating machines are usually connected in the secondary circuits of induction motors, the frequency supplied is represented by the slip frequency. Consequently where the slip frequency never rises to a large percentage of that of the primary system, such commutating motors are applicable without undue difficulties. Such motors, presumably are better adapted for 25-cycle mill equipments than for 60-cycle, but due to the tendency, already described, for steel mills to go to 60 cycles on purchased power, it has been necessary to build these three-phase commutating motors for the regulation of 60-cycle main motors, in many cases.

There is still another class of service, which has come in recently, where the choice of frequency is of much importance, but where there is no great necessity for adhering to any standard, namely, in heavy ship propulsion by electric motors. As each ship equipment is a complete system in itself, and as it cannot tie up with other systems, there is not any controlling need for maintaining any definite frequency or voltage. Except in similar vessels, there is little chance for duplication in parts, as the various equipments vary so much in size and capacity. In consequence it has been found advisable, at least up to the present time, to design each propulsion equipment for that frequency which best suits the generator and motor speeds, taking into account the various operating conditions and limitations, such as the different running speeds, steaming radius, etc. In consequence, different maufacturers bidding on such equipments may specify different frequencies, depending upon the constructional features of their particular types of apparatus. At the present time with the relatively small amount of experience obtained with the electrical propulsion of ships, it looks as if it would be a considerable handicap to attempt to adopt some standard frequency for all service. Later, with wide experience, it may be possible to adopt some compromise frequency, which will not unduly handicap any of the service.

## CONCLUSION

It has been the writer's intention to show that, as a rule, the choice of frequency has been a matter of most serious consideration, based upon service conditions at the time. Moreover, in view of the wide range of conditions encountered, it is surprising how few frequencies have been seriously considered in this country. Occasion has arisen, times without number, where an obvious solution of a given problem would lie in modification of the frequency to allow the use of apparatus and equipment already designed, but the engineers of the manufacturing organization have steadily held out against such policy, regardless of the apparent need of the moment. The swing of the pendulum from 60 cycles to 25 cycles and back, has covered a period of many years and, therefore, cannot be considered as a fad of the moment, but is the result of well defined tendencies, backed by the best engineering experience available. As a rule no manufacturer has made any particular frequency his "pet," but all have worked to develop each system to its utmost.

## DISCUSSION ON "THE TECHNICAL STORY OF THE FREQUENCIES" (LAMME), WASHINGTON, D. C., JANUARY 18, 1918.

**H. B. Brooks:** Mr. Lamme's instructive paper appeals strongly to those of us who have followed the development of the a-c. art from the beginning.

In addition to the early frequencies of 133 and 125 cycles, I recall that the Slattery a-c. system, made by the Fort Wayne Jenney Electric Light Co., was operated at 140 cycles. That was the day of the "system;" each inventor who designed an alternator felt in duty bound to design a complete system, including generators, transformers, station instruments, switches, cutouts, sockets, and in some cases the lamps also.

Mr. Lamme has not referred to the attempt which was made at an early stage of the art to limit the frequency to a single value, namely, zero. I refer to a rather short but bitter attempt by the advocates of d-c. distribution to have the alternating current outlawed on the ground of its alleged great risk to life.

It may be of interest to glance at the frequency situation abroad. In Italy, which is poor in coal but rich in water power five frequencies are in use, of which 42 cycles is in the lead, with 50 cycles a close second. Around Rome, 46 cycles is used, and there are also some 16 and 25-cycle installations throughout the country. The Italian Electrotechnical Association is planning a campaign to standardize frequencies.

London is probably the worst example of the independent growth of small detached generating systems with no thought of possible future interconnection. Merz and McLellan studied the situation and made a report in 1914 to the London County Council, from which it appeared that there were in the central area of London 41 generating stations representing 31 systems and 8 frequencies. The report advocates the adoption of 50 cycles as a standard frequency for London.

In contrast to the medley of plants and frequencies in London, the cities of Hamburg and Berlin each have electric supply from a single company. The frequency of the Berlin system is 50 cycles. The business in Paris is practically all in the hands of a single company, operating at 25 and 42 cycles.

A single standard frequency would have great advantages. It would permit the interconnection of transmission systems wherever this might be expedient. This question of interconnection is an important one in England at the present time, as the demands for power for war industries have overloaded some systems which are not readily able to tie in with others for mutual assistance.

A standard frequency would benefit the manufacturer by reducing the number of designs, patterns, and stock of generators, transformers, motors and meters. The user would benefit from the consequent reduction of cost, and would be able to move equipment from one locality to another without finding it inapplicable to the new supply conditions.

From the standpoint of export trade, it would be desirable to have an international standard frequency. While the attainment of this ideal appears to be far off, it is a matter which may well be kept in mind by the national engineering societies and the International Electrotechnical Commission, in order that so far as possible the trend of standardization in the different countries may be along converging lines.

I would like to ask Mr. Lamme whether the higher peripheral speeds which have greatly contributed to the success of the 60-cycle converter are made possible by the use of materials of greater tensile strength, or by improved methods of utilizing the materials heretofore available.

**B. G. Lamme:** The improvements have been almost entirely in the design of the machines. We are using practically the same materials that were used in the older machines, but we are proportioning the parts in such a way that they are better sustained, that is, we know more about the stresses and how to take care of them.

**G. S. McCumber:** I have one question that I wanted to ask. Mr. Lamme referred to the chief disadvantage of the 60-cycle system, I assume that he intended increased voltage drop and the lower power factor, but I wanted to know also whether there was any disadvantage due to such other factors as additional difficulties in insulation, corona losses or anything of that kind.

Also one other question. Reference was made to the fact that in heavy ship propulsion a variable frequency system was employed, and I wonder if he would state for us what the standard frequencies or the normal range frequencies are on which that system was based.

**Robert Dalgleish:** Mr. Lamme in speaking of 60-cycle converters brings to my mind the question as an operating engineer—the problem of the 25-cycle converter, which most all of us are now using. The question arises, is the 60-cycle converter in the high voltages, such as 600, in railway operation as successful as it is in the lighting voltages; and is the possibility of the future of the railway turbo-generators—and naturally the converter—to be 60-cycle instead of 25? I can appreciate that the companies who are at present equipped with 25-cycle turbo-generators are naturally going to stick to that. With the question of future installations, where the installation is entirely new, will it be an installation of 60-cycle or 25-cycle turbo-generators?

**Mr. Hunt:** I would like to ask Mr. Lamme what the present limit of size of the 60-cycle rotary converter is, and what are the chief limitations in design that makes that size the limit.

**B. G. Lamme:** Before I answer any of the questions brought up, I want to bring out more clearly what it meant to the manufacturing companies to adhere to a standard frequency. You may not appreciate, unless you have been in the manufacturing business, how easy it is to change the frequency, in many cases.

For, instance suppose a manufacturer has built a 60-cycle, 3300-volt machine at 150 r. p. m. and somebody comes along and wants a 100-r. p. m., 2200-volt machine of somewhat less capacity. The 60-cycle machine already constructed may fit the new conditions in everything except frequency. If a frequency of 40-cycles would be acceptable, then the old machine could be used without any change, but it would introduce a special frequency. Thousands of cases have come up where we could take some existing machine and meet the customer's condition, if it were not for the frequency. I have seen many cases where an existing machine was all right to meet new conditions except that it would have two poles too many or too few, and it looked so easy to make a minor change in the frequency instead of in the machine. In such cases, however, if the customer could not change his speed conditions, the manufacturer would change his design rather than modify the frequency.

There has been an enormous amount of money spent in making such changes, but the engineers of the manufacturing companies feel that they have really spent less money in making them than they would have spent in the end if they had allowed changes in frequency; for every change, no matter how small, means confusion in the end. The engineers of the manufacturing companies have been stubborn on that matter and for a good reason. That is why we have had so little variation in the frequencies in this country.

It has been asked whether the 60-cycle converter in the higher voltages, such as 600 for railway service, is as successful as it is in the lighting voltages. In reply to this I will say that, as the tendency for the last few years has been largely toward 60 cycles in new plants, which may be called upon to handle railway service as well as lighting, it has been necessary to develop the 60-cycle converter for 600-volt railway service up to the point where it compares quite favorably with similar 25-cycle machines. This is indicated by the fact that below 1000-kw. capacities there is but relatively small business in 25-cycle converters; whereas, there is quite a good business in 60 cycles. Above 1000 kw. there is a very strong tendency also toward the 60-cycle converter. In fact, while the 25-cycle rotary converter business is quite large in the units above 1000 kw., yet such machines are sold very largely to 25-cycle plants already installed, or to extensions of such plants.

As to the limit of capacity for which 60-cycle converters can be built, one answer is that the real limit is in the pocketbook of the customer. Such machines can be built of almost any capacity, if there is a demand for them. We could make 10,000-kw., 60-cycle converters today, if anybody wanted them, just as we could make 10,000-kw., 25-cycle machines. The largest 60-cycle converter is of about 5500 kw. compared with 4000 kw. for 25 cycles. Therefore, the 60 cycle is already ahead of the

25 cycle, as far as maximum capacity is concerned; but from the quantitative standpoint the large 25-cycle converters are still ahead because there were so many 25-cycle generating plants installed in the past entirely for railway work through rotary converters.

As to Mr. McCumber's questions regarding the relative disadvantages of voltage drop, insulation difficulties, corona losses, etc., in 60-cycle transmission, I will say that I am not an authority on transmission difficulties, but I understand that all the above troubles were quite serious in the early days, and that much doubt was expressed regarding the possibilities of maintaining satisfactory voltage conditions at the end of a 60-cycle line. Apparently they feared poor regulation more than any other feature. Possibly some of the difficulties in the early times worried the engineers more in the prospect than in the realization, for, apparently, when they came to operate such 60-cycle lines, they did not have as much trouble as was expected and the regulation trouble was overcome in a number of cases by the addition of synchronous condensers.

I remember, not so many years ago, that all of us pitied the Californians, for their adoption of 60 cycles as a general standard, in transmission work. It was thought that they had made a serious mistake, but now they are right in line, although they probably didn't foresee it. They had committed themselves to 60 cycles and they couldn't very well change.

As to the basic frequency used for ship propulsion, as such are variable frequency systems, it may be said that there is no definite frequency except for the highest speed which the apparatus will stand. A number of the battleship equipments have used 35 cycles for the full speed condition with variation from this down to half value or even less. On some of the other vessels as high as 50 and 60 cycles are used. The choice of frequency has been largely dependent upon the motor-speed combinations required.

*Presented at a meeting of the San Francisco Section of the American Institute of Electrical Engineers, San Francisco, Cal., January 25, 1918.*

# CORONA TESTS AT HIGH ALTITUDE

BY B. F. JAKOBSEN

ABSTRACT OF PAPER

This paper describes some corona tests which were made in Peru on a 70-mile transmission line located at an average altitude of 13,300 feet. The voltages range from 40,000 to about 72,000.

The method employed in the tests is described and the calculations are given. The corona losses were found by subtracting the core and copper losses of the transformers and the line copper losses due to the line charging currents from the total loss measured. Core-loss tests were made on the transformers in order to check these against the shop tests. The highest voltage experimented with was 72,300 when a corona loss of 153 kw. was found, or a little over two kw. per mile of line.

The results are compared to "Peek's law" and a wide divergence found, amounting to over 800 per cent at about 66,000 volts and a loss of 58 kw.

Close correspondence is found between these tests and those made by Faccioli on the Shoshone-Leadville line, and the equation of the loss-voltage curve obtained is the same as was established by Faccioli, and shows this relation to be logarithmic and not quadratic for the voltage range investigated.

It is shown that the results are in good agreement with the formulas deduced by Professor Ryan from extensive laboratory tests.

Finally test data are given for a test made during rainy weather and this shows no appreciable difference from tests made during fair weather.

## INTRODUCTION BY FRANK G. BAUM

There has been installed in Peru under the writer's direction a hydroelectric plant having about 120 mi. (193.1 km.) of transmission line, running from 12,000 to 14,000 ft. (3,657.6 to 4,267.2 m.) elevation, with most of the line at about 13,000 ft. (3,962.4 m.).

I considered this an excellent opportunity to make some actual corona tests at high altitude, as these would be of value to the profession. The results are shown in the report of a test on 68.4 mi. (110 km.) of three-phase line by Mr. B. F. Jakobsen, electrical engineer.

The results are encouraging, in that the actual losses are less than expected, and as this fact influences to some extent the size of wire to be chosen, it is seen that the matter is of practical importance to the electrical profession.

Probably there is no line which has more severe lightning conditions than this one. And no doubt the high altitude and consequent light air pressure allows atmospheric discharges that would not occur at lower altitudes for the same cloud potential conditions, for the same reasons that we have corona at high altitude for less voltage than at low altitude.

While not directly related to corona it may be of interest to know that the transmission has been freer from lightning troubles than anticipated. One line about 90 mi. (144.8 km.) long is operated at 44,000 volts, Y-grounded, (this line was formerly delta operated as mentioned in the test report) and another line about 20 mi. (32.1 km.) long is operated 25,000 volts, delta-ungrounded. Aluminum lightning arresters and horn gaps are used. The Y line is more free from lightning troubles than the delta line, but the amount of trouble on either line has been almost negligible.

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AT THE request of Mr. Frank G. Baum of San Francisco some corona tests were made in April 1914 in Peru, South America on a newly constructed transmission line. The tests were made on that part of the line which runs from the power house at Oroya (12,250 ft. or 3,733.8 m. elevation) to the smelter substation at La Fundicion (about 14,000 ft. elevation), see profile Fig. 1.

Several tests were made at frequencies near normal (60 cycles) and as these tests gave practically identical results, only two are given herewith, one for fair weather and one for rainy weather.

A frequency of 61.1 cycles was chosen as this was the highest at which the water-wheels could be operated under control of their governors.

*Power Plant Data.* At the power plant there are three 5000-h.p. generating units, each consisting of two impulse-type waterwheels direct connected to one 60-cycle, 2300-volt, 3-phase star-connected, ungrounded-neutral generator. For each generating unit there is provided three 1000-kv-a. 2300—55,000-volt transformers, connected in delta on both sides. All instrument transformers were in the 2300-volt leads between the generators and the transformers.

*Transmission Line Data.* The line consists of two three-phase circuits of No. 1 B. & S. stranded copper wire and a grounded steel cable ¼ in. (6.3 mm.) diameter, carried on one steel-pole line and arranged as shown in Fig. 2. Each circuit was given two complete transpositions between Oroya and La Fundicion and the distance from Oroya to La Fundicion along the line is 68.4 mi. (110 km.) and the normal operating voltage is 44,000.

The line constants[1] are as follows:

| | |
|---|---|
| Resistance of one wire, 68.4 mi. at 20 deg. cent.... | 46.2 |
| Reactance of one wire, 68.4 mi. at 61.1 cycles....... | 54.6 |
| Impedance of one wire, 68.4 mi.................... | 71.6 |
| Susceptance of one wire, 68.4 mi. and 61.1 cycles.. | $387 \times 10^{-6}$ |

One unit only was used in these tests, but during all the tests about 2500 kw. at a power factor of about 0.6[2] was carried on one of the other units and transmitted to the smelter substation over one of the two circuits.

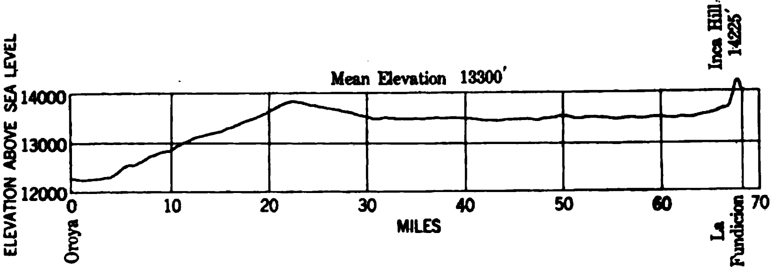

FIG. 1—PROFILE OF LINE, OROYA TO LA FUNDICION

*Transformer Data.* The name plates of the transformers had been lost and the shop-test data could therefore not be applied to each transformer. The resistance of the three high-tension windings in series was measured at 21 deg. cent. with a Wheatstone bridge, and the average taken. The resistance of the low-tension windings was taken as the average of the nine transformers from the shop report, as the Wheatstone bridge was not adaptable for measuring such small resistances. The impedance drop was assumed equal to the average for the nine transformers taken from the shop-test report, as it could not readily be measured with the available equipment. The average im-

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[1] TRANS., A. I. E. E. 1911 Vol. XXX, Part III, p. 2264.

[2] The 1500-h.p. synchronous motor provided at the smelter substation to improve the power factor was not ready for operation when the tests were made.

pedance drop at 60 cycles is 4.105 per cent; assuming 2.105 per cent for the high-tension winding and 2.0 per cent for the low-tension winding, the transformer constants at 21 deg. cent. and 61.1 cycles, with 2300—55,000-volt connection and expressed on the high-tension side, are: High-tension winding:

$$R = 11.45; \quad X = 62.6, \text{ and } Z = 63.6;$$

Low-tension winding:

$$r = 12.5; \quad x = 56.65, \text{ and } z = 57.95$$

*Meters and Instrument Transformers.* The instruments used were standard switchboard instruments furnished with the Oroya switchboard and the current transformers for the ammeter and the indicating and integrating wattmeters were 1200-5 amperes ratio. In order to augment the readings on these instruments, 300-5-ampere current transformers were connected in series with the 1200-5-ampere current transformers on the 2300-volt side and in parallel on the instrument side, thus increasing the readings in proportion of 1 to 5. No power factor indicator was available.

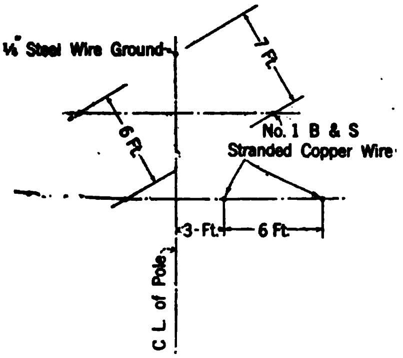

Fig. 2—Arrangement of Wires on Transmission Line

*Method of Tests.* Before starting the corona tests, a test was made to determine the transformer exciting current and the core loss for the range of voltages and frequencies to be employed. In this test the generator current (equals $\sqrt{3}$ times transformer exciting current), generator voltage, kilowatts and frequency were measured.

For the corona tests these same quantities were measured and the corona loss obtained by subtracting the transformer core losses, transformer copper losses and the line copper loss due to the charging current from the total generator output.

*Calculations.* Fig. 3 shows the diagram of connections and Fig. 4 the corresponding vector diagram, wherein:

$E_g$ = generator voltage
$E_t$ = transformer core voltage
$E_L$ = line voltage at generator end
$I_{ch}$ = charging current per wire
$I_{co}$ = corona current (equivalent sine value)
$I_L$ = line current per wire
$I_t$ = transformer exciting current
$I_g$ = generator current

all expressed on the high-tension side and all voltages between wires.

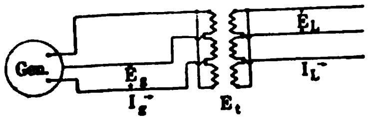

FIG. 3

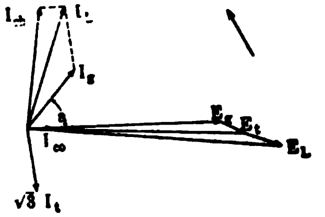

FIG. 4

The current $I_{co}$ due to corona is quite small in all these tests; at 72.3 kv. the corona loss is 153 kw. and the equivalent sine current[3] in phase with the voltage is

$$I_{co} = 153 \div 1.73 \times 72.3 = 1.22 \text{ amperes}$$

The corona current has therefore been entirely neglected in the computations.

Referring to the vector diagram Fig. 4, the line charging current is very nearly 90 deg. out of phase with the voltage[4] and for voltages above 50 kv. the generator current due

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[3] See oscillographs by Bennett, A.I.E.E., TRANS., Vol. 32, p. 1787 and cyclogram by Prof. Ryan, p. 1825 and remarks under 8 on p. 1827, Vol. 32.

[4] At 72.3 kv. the charging current is 16.3 amperes and the line loss for

$$3 \text{ wires } \frac{16.3^2 \times 46.2}{1000} = 12.2 \text{ kw. and the resulting power factor } 0.0975$$

and angle = 84° 20′. For smaller voltages this power factor is still smaller.

to the transformer exciting currents is very nearly 90 deg. out of phase with the generator voltage, see Fig. 7. The generator current during the corona test is also nearly 90 deg. out of phase with the generator voltage (the current leads), except for the higher voltages when the generator current is small, (see Fig. 8) as the line current and $\sqrt{3}$ times the transformer exciting current more nearly balance each other. Since also the transformer reactances are several times larger than the corresponding resistances, and the drop due to the transformer resistances, with the currents involved, is comparatively very small and at right angles to the generator voltage, it follows that the vector relations shown in Fig. 5, may be substituted for those shown in Fig. 4. All this simplifies the calculations very materially without introducing any appreciable errors, as far as current and voltage calculations are concerned.

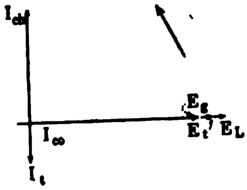

FIG 5

*Formulas. Line.* Neglecting the voltage component due to the charging current and the line resistance, as this component is very nearly 90 deg. out of phase with the line voltage and comparatively very small. Let

$E_r$ = voltage at receiver end of line, when the current at receiver end is zero, then[5]

$$I_{ch} = \frac{E_r}{\sqrt{3}}\, b\left(1 - \frac{b\,x}{6}\right) \text{ and } E_r = \frac{E_L}{1 - \dfrac{b\,x}{2}} = 1.0106\, E_L \qquad (1)$$

$b$ = line susceptance for one wire and voltage to neutral,
$x$ = line reactance for one wire;

Combining:

$$\left.\begin{aligned} I_{ch} &= \frac{E_L \times b}{\sqrt{3}\left(1 - \dfrac{b\,x}{2}\right)}\left(1 - \frac{b\,x}{6}\right) \\ I_{ch} &= 0.226\, E_L \text{ amperes.} \end{aligned}\right\} \qquad (2)$$

*Line Loss Due to Charging Current.* Assuming the charging current per unit length of line to be constant, or in other words,

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[5] Hagood, A. I. E. E., TRANS., Vol. 32, p. 868.

that the current on the line is, at any point, directly proportional to its distance from the receiver end of the line, then;

$i_{ch} = I_{ch}\frac{s}{S}$, where $i_{ch}$ = charging current $s$ miles from the receiver end of the line and

$S$ = total length of line.

$I_{ch}$ = line charging current at generator end.

$$\text{Loss on three wires} = 3\int_0^S I_{ch}^2\frac{s^2}{S^2}\,\frac{r}{S}\,d\,s = I_{ch}^2 \times r \qquad (3)$$

$$= \frac{2.36 \times E_L^2}{1000} \quad \text{kw.}$$

Voltage rise in low-tension transformer windings:

$$E_t - E_g = I_g \times x \div 1.73 = 0.0327\, I_g \qquad (4)$$

Voltage rise in high-tension transformer windings:

$$E_L - E_t = I_{ch} X \div 1.73 = 0.00816\, E_t \qquad (5)$$

In (4) and (5) all voltages are in kilovolts and for leading currents.

Power loss in low-tension transformer windings:

$$\left(\frac{I_g}{\sqrt{3}}\right)^2 \times \frac{3\,r}{1000} = 0.0125\, I_g^2 \quad \text{kw.} \qquad (6)$$

Power loss in high-tension transformer windings:

$$\frac{I_{ch}}{\sqrt{3}} \times \frac{3\,r}{1000} = 0.585\, E_L^2 \div 1000 \quad \text{kw.} \qquad (7)$$

Combining (3) and (7), the loss on the line and in the high-tension windings together is

$$2.945\, E_L^2 \div 1000 \quad \text{kw.} \qquad (8)$$

*Core Loss Test.* The result of this test is plotted in Fig. 7; the relation between the integrating wattmeter readings and the indicating wattmeter readings is plotted in Fig. 6. The indicating wattmeter readings have been corrected as per curve *B* Fig. 6 for use in Fig. 7. The frequency was 61.1 cycles.

***Remarks on Wattmeter Accuracy.*** As already stated two current transformers with different ratios were in series on the 2300-volt side and in parallel on the instrument side. Full rated current on the 1200-5 ampere current transformers is about 50 amperes (on the high-tension side) and 12.5 amperes for the 300-5 ampere current transformers.

At 55 kv. and 61.1 cycles, the core loss for three transformers is (Fig. 7) 24.5 kw.; the exciting current times $\sqrt{3}$ is 2.65 amperes.

Three times the average core loss of the nine transformers at 60 cycles is according to the shop test report 21.94 and the average for the exciting current times $\sqrt{3}$ is 2.75 amperes. The core loss for the nine transformers varies between a minimum of 6.9 kw. and a maximum of 7.81 kw., while $\sqrt{3}$ times the exciting current varies from 2.16 to 3.41 amperes.

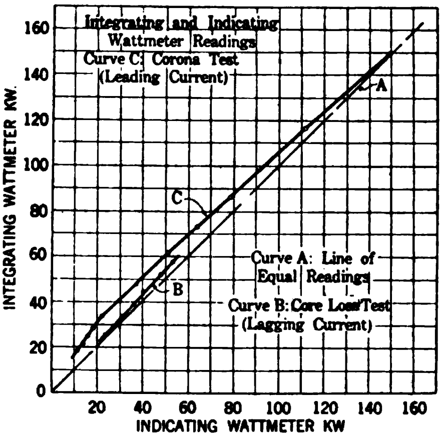

FIG. 6

The influence of the frequency upon the core loss may be estimated roughly, as follows:

$$\left.\begin{aligned} \text{voltage} &= e = c\,f\,B \\ \text{core loss} &= p = k\,f\,B^{3.34} \end{aligned}\right\} \qquad (9)$$

where,

$c$ and $k$ are constants,

$f$ = frequency,

$B$ = magnetic flux per unit area of core, and

the exponent 3.34 is calculated for values between 50 kv. and 60 kv. from Fig. 7, by assuming that the core-loss curve in Fig. 7 follows the formula $p = k\,f\,B^n$.

From (9) it is found, that the core loss at 61.1 cycles is about 96.5 per cent of the core loss at 60 cycles; the core loss as measured was 24.5 kw. at 61.1 cycles and would accordingly have been 25.4 kw. at 60 cycles. The shop tests[6] give for the three transformers with highest core loss only 22.62 kw. or about 11 per cent less than found in the above test.

This discrepancy may be due solely to the instruments and their transformers and in that case would indicate a correction angle of about 40 min. and that the resultant secondary current of the current transformers lead the primary current, *i.e.*, the generator current.[7] In this case, we could expect wattmeter readings about 11 per cent low for the same current and voltage, but a leading current.

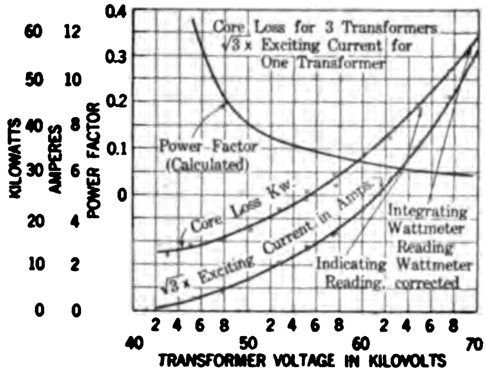

FIG. 7

However, the values for core losses given in the shop reports are for a sine form of the e.m.f. wave. When operating on open circuit, the generator e.m.f. wave is practically a sine curve, but when the transformers are connected to the generator and with the star-delta-delta connection used here, it is likely that a fifth harmonic in excess of the amount demanded by the hysteresis cycle[8] appears in the transformer and generator currents, causing a fifth harmonic to appear in the e.m.f. waves with a corresponding flattening of the voltage crest[9] and a corres-

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[6] The shop reports give (60 cycle, 55,000 volt):—7.07; 7.20; 6.90; 7.39; 7.40; 7.81; 7.30; 7.41 and 7.325 kw. respectively for the nine transformers in the Oroya power plant.

[7] Robinson, A. I. E. E., TRANS., Vol. 28, page 1005.

[8] Janet, Lecons d'electrotechnique générale, 2 Ed., Vol. 2 p. 191.

[9] Curtis, A.I.E.E., TRANS., Vol. 33, p. 1273 and Figs. 10, 11 and 12.

ponding increase of core loss for the same effective voltage. If, as a limiting value, the e.m.f. wave form was rectangular, the core loss would be about $1.11^{3.34}$ or 51 per cent in excess of the core loss for a sine wave with the same effective voltage and same frequency.[10]

TABLE I.—CORONA TEST. 61.1 CYCLES.

"*Corona Test.* April 23rd 1914. Test started at 1.45 p.m. and finished at 2.45 p. m.; temperature at Oroya in shade outside of Power House was 60° fahr. and barometer was 48.3 and 48.2 cm. respectively at 1.45 and 2.45 p. m. Barometer at La Fundicion was 42.15 cm. No temperature was taken at La Fundicion, but may safely be assumed to be the same as at Oroya. Weather at Oroya was very fair with bright sun; at La Fundicion sky was somewhat overcast; no rain at any place along the line, as far as known."

(Values given for high-tension side of transformers.)

| Generator voltage in kv. | Generator current in amperes | Indicating wattmeter kw. | Integrating wattmeter kw. |
|---|---|---|---|
| 40.5 | 9.2 | 11 | 18.7 |
| 43.0 | 9.61 | 14 | 21.6 |
| 45.5 | 10.0 | 16 | |
| 45.3 | 9.97 | 16 | 24.9 |
| 47.8 | 10.24 | 18 | |
| 47.3 | 10.2 | 18 | 28.2 |
| 50.1 | 10.49 | 22 | |
| 50.0 | 10.48 | 22 | 33.5 |
| 52.9 | 10.58 | 29 | |
| 52.8 | 10.58 | 30 | 40.5 |
| 55.1 | 10.5 | 37 | |
| 55.1 | 10.5 | 37 | 47.9 |
| 57.9 | 10.2 | 51 | |
| 60.0 | 9.71 | 64 | 61.0 |
| 62.1 | 9.1 | 78 | 73.2 |
| 64.5 | 7.95 | 108 | 86.5 |
| 66.9 | 6.7 | 140 | |
| 69.3 | 5.1 | 183 | |
| 71.6 | 3.6 | 237 | |
| 69.3 | 5.15 | 177 | |
| 66.9 | 6.98 | 135 | |
| 64.5 | 8.12 | 105 | |
| 62.1 | 9.24 | 77 | |
| 59.7 | 9.81 | 64 | |
| 57.2 | 10.29 | 50 | |
| 55.1 | 10.24 | 41 | |
| 52.6 | 10.58 | 34 | |
| 50.3 | 10.5 | 28 | |
| 64.6 | 7.7 | 111 | 126 |
| 67.2 | 6.31 | 149 | 149 |

The integrating wattmeter readings were calculated from the formula: Kw. = 11,520 $R/T$ where $R$ is the revolutions of aluminum disk for $T$ seconds. The readings were taken for one revolution of disk and the time, varied from 123 to 15.4 seconds and was taken by a stop watch.

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[10] Kapp, Transformatoren für Wechselstrom and Drehstrom, 2 Ed. p. 23.

TABLE II*.—CORONA TEST. 61.1 CYCLES

| | Generator voltmeter-volts | Generator ammeter-amperes | Indicating wattmeter readings-kw. | Integrating wattmeter readings-seconds |
|---|---|---|---|---|
| 1 | 84.8 | 1100 | 55 | 123 |
| 2 | 90.0 | 1150 | 70 | 106 |
| 3 | 95.3 | 1195 | 80 | 92.2 |
| | 94.9 | 1192 | 80 | |
| 4 | 100.0 | 1225 | 90 | 81.5 |
| | 98.9 | 1220 | 90 | |
| 5 | 105.0 | 1254 | 110 | 68.5 |
| | 104.7 | 1252 | 110 | |
| 6 | 110.9 | 1265 | 145 | 56.6 |
| | 110.8 | 1265 | 150 | |
| 7 | 115.5 | 1258 | 185 | 48.0 |
| | 115.4 | 1258 | 185 | |
| 8 | 121.2 | 1220 | 255 | 37.6 |
| 9 | 125.7 | 1163 | 320 | 31.4 |
| 10 | 130.0 | 1038 | 390 | 26.6 |
| 11 | 135.0 | 950 | 540 | |
| 12 | 140.0 | 800 | 700 | |
| 13 | 145.0 | 610 | 915 | |
| 14 | 150.0 | 430 | 1185 | |
| 15 | 145.0 | 615 | 885 | |
| 16 | 140.0 | 834 | 675 | |
| 17 | 135.0 | 970 | 525 | |
| 18 | 130.0 | 1105 | 385 | |
| 19 | 125.0 | 1175 | 320 | |
| 20 | 119.8 | 1230 | 250 | |
| 21 | 115.5 | 1255 | 205 | |
| 22 | 110.3 | 1265 | 170 | |
| 23 | 105.3 | 1257 | 140 | |
| 24 | 135.5 | 920 | 555 | 19.8 |
| 25 | 140.8 | 755 | 745 | 15.4 |

*Measurements as actually taken and from which Table I was derived.

All readings are actual instrument readings, so that the ammeter and wattmeter readings are to be multiplied by 1/5 and the voltmeter reading by 20 in order to obtain actual values.

From Table I, the values plotted in Figs. 8 and 9 were calculated by the formulas given previously.

As a check on the correctness of the measurements, the calculated charging current is shown in Fig. 8 as the straight line and the points are the measured generator currents plus $\sqrt{3}$ times the transformer exciting currents taken from Fig. 7 for the corresponding voltages. The agreement between the measured values and the calculated currents is quite good.

Curve *A* in Fig. 9 is the total loss measured; the integrating-meter readings have been used without correction, but the indicating-wattmeter readings have been corrected according to curve in Fig. 6, where the integrating and indicating-wattmeter readings are plotted against each other.

This was done because the integrating meters could be read with more accuracy and also because it was assumed that the integrating meters would measure the loss more accurately. When the indicating meter readings were accepted as accurate, it was found that the transformer copper and core loss plus the line charging current copper loss (curve B in Fig. 9) exceeded the total loss (curve *A* in Fig. 9 ), for the voltages around 40,000 volts. And this, of course, is impossible.

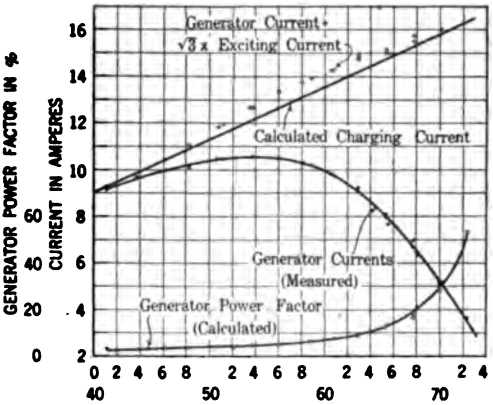

FIG. 8

Curve *B* in Fig. 9 is the sum total of all the losses, except the corona loss, calculated from the test and the given formulas. Curve *C* is the corona loss, the difference between curve *A* and *B*.

*Comparison of Test with Published Formulas.* A.—Peek's Law of Corona, A. I. E. E., Vol. 30, p. 1894, equation (8). $m_0 = 0.87$; $g_0 = 21.1$ kv. eff.; $r = 0.42$ cm.; $s = 183$ cm.;

| Line kv. | Peek's law form. (9) kw. | Curve *C* Fig. 9 kw. | Ratio |
|---|---|---|---|
| 50.5 | 0 | 6.0 | |
| 54 | 22 | 9.6 | 2.29 |
| 58 | 100 | 17.5 | 5.72 |
| 62 | 236 | 31.5 | 7.49 |
| 66 | 428 | 57.5 | 8.31 |
| 70 | 678 | 105.5 | 6.42 |
| 72.3 | 848 | 153.0 | 5.54 |

$b = 45.93$ (mean value); $t = 16.4$ deg. cent.; $\delta = 3.92 \cdot \times 45.93 + (273 + 16.4) = 0.622$; distance = 110 km. $E$ = voltage between wires in kv.; then for this line,

$$P = 1.78\,(E - 50.5)^2 \quad \text{kw.} \qquad (10)$$

The original test data were submitted to Professor Ryan in June 1914 and Fig. 9A was made at that time by Professor Ryan. The close agreement between the two curves suggested further analysis of the losses found on the Oroya line and these are given below.

Referring to Fig. 9A Professor Ryan wrote:

"In Fig. 6, page 342 of the A. I. E. E., 1911, TRANSACTIONS Faccioli gives the corona losses found in fair weather on the 63-mi. transmission from Shoshone to Leadville of the Central Colorado Power Co. at an average elevation of 8500 ft. and a maximum of 12,000 ft. This line has a length of 102 km. while the Oroya-smelter line has a length of 110 km. The Shoshone line developed a corona loss of 51.5 kw. at 95.8 kv.; the Oroya line 55 kw. at 65.6 kv., *i. e.* one-half kilowatt per kilometer in each case. For the purpose of comparison, we may assume that corona was definitely started at this amount of loss, viz.: one-half kilowatt per kilometer of line. This is really about the only value in regard to corona formation that approaches some degree of definiteness when related to voltage, line capacity, barometer and temperature, aside from the fact that the amount of corona loss is largely dependent upon ($E - E'$) rather than $E$, wherein $E'$ is the line voltage that produces a corona loss of one-half kilowatt per kilometer and $E$ is any higher line voltage. It follows therefore, that the corona was started on the Shoshone line at 95.8 − 65.8 = 30 kv. higher than the Oroya line. Other things being equal, the amount of corona loss is due to the amount of voltage that exceeds

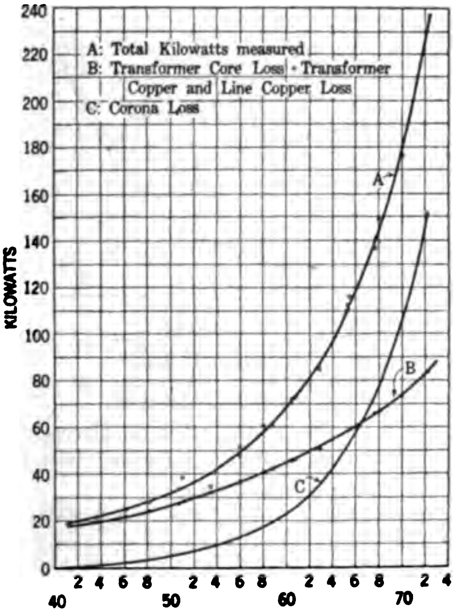

FIG. 9

corona starting voltage. Let us call such voltage that exceeds corona starting voltage, *Corona Voltage*. Then in Fig. 9-*A*:

I. Oroya-smelter transmission corona voltage and corona loss; length 110 km. average elevation 13,300 ft., maximum 13,800 ft.

II. Shoshone-Leadville transmission corona voltage and corona loss; length 102 km.; average elevation 8500 ft.; maximum 12,000 ft.

I have mounted also the corresponding total line voltages. In comparing the two curves, the differences in length of lines and in the elevations must be kept in mind. We know how to allow for the difference in length but we are not sure that we know exactly how to allow for the differences in altitude. Peek came to the conclusion that corona loss is inversely proportional to the density of the atmosphere. Such conclusion is reasonable. The corresponding composite factor for reducing

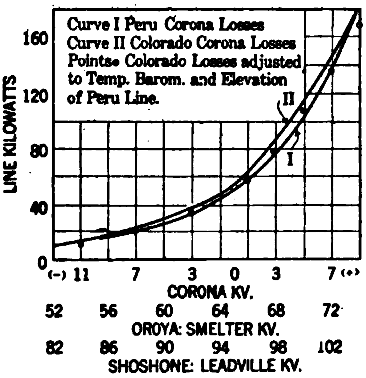

FIG. 9A—CURVE CHART FOR COMPARISON OF OROYA-SMELTER (PERU) AND SHOSHONE-DENVER (COLORADO) TRANSMISSION LINE CORONA LOSSES

| Location of Line | Length km. | Average barom. cm. | Average temp. deg. cent. | Authority |
|---|---|---|---|---|
| Peru | 110 | 45.9 | 16.4 | Jakobsen report Apr. 27, 1914 |
| Colorado | 102 | 52.0 | 5.0 (Assumed) | Faccioli, TRANS. A. I. E. E., 1911 |

the Shoshone-Leadville corona loss to a corona loss corresponding to the Oroya-smelter altitude, barometer and temperature, thus becomes 0.93. In Fig. 9-A, the points enclosed with small circles were located on corresponding ordinates of the Shoshone-Leadville corona loss curve by means of the proportional dividers set at 0.93. You will note that they are in close agreement with the corresponding Oroya-smelter-transmission corona-loss values. Some difference in the forms of the curves must exist because of the differences in the mountain profiles traversed by the corresponding lines. Then, too, the instruments must, inevitably, have been out by small amounts as instruments of the type that had to be employed usually are. Neither cause, however, is a basis for expecting much difference between the two curves."

*Analysis of Oroya-Smelter Corona-Loss Curve.* In Fig. 10 the square root of the corona loss, equal to $\sqrt{p}$, is plotted against the line voltage. As this line is not a straight line, the loss curve is not quadratic. Assuming, however, that the loss curve from 69 kv. upwards is quadratic, we get[11] using the $\Sigma\,\Delta$ method:

$$p = 0.765\,(E - 58.23)^2 \qquad \text{kw.} \quad \textbf{(11)}$$

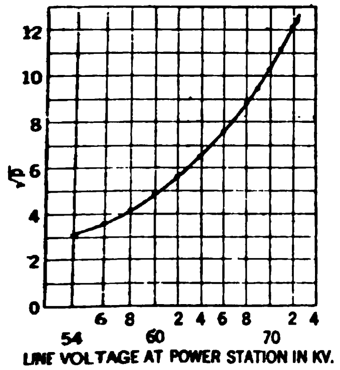

FIG. 10

Plotting the line voltage from 56 kv. upwards against the logarithm of the corona loss from Fig. 9, gives a straight line, and by the $\Sigma\,\Delta$ method is found:

$$\log p = 0.0662\,E - 2.6 \text{ or}$$

$$p = \epsilon^{0.1525\,(E-39.2)} \text{ kw.} \quad \textbf{(12)}$$

where $\epsilon$ = the base of the natural logarithms.

This equation holds good for the whole range and it is the same equation found by Faccioli.[12]

The following shows the close agreement between the test and formula (12) above:

| Line voltage kilovolts | Equation (11) kw. | Curve *C*. Fig. 9 kw. |
|---|---|---|
| 44 | 2.1 | 1.5 |
| 46 | 2.6 | 2.5 |
| 48 | 3.0 | 4.0 |
| 50 | 5.1 | 5.5 |
| 52 | 8.7 | 7.0 |
| 54 | 10.2 | 9.6 |
| 56 | 12.9 | 13.0 |
| 58 | 17.4 | 17.3 |
| 60 | 23.4 | 23.2 |
| 62 | 32.4 | 31.5 |
| 64 | 43.6 | 42.3 |
| 66 | 58.9 | 57.5 |
| 68 | 79.4 | 77.5 |
| 70 | 109.5 | 105.5 |
| 72 | 144.5 | 146.5 |
| 72.3 | 151.5 | 153.0 |

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[11]

| Kilovolts | Fig. 9 kw. | $\sqrt{\text{kw.}}$ |
|---|---|---|
| 69 | 90 | 9.48 |
| 70 | 105.5 | 10.26 |
| 71 | 124 | 11.13 |
| 72.3 | 153 | 12.37 |

[12] $p = 9.5\,\epsilon^{(0.089\,E-9.75)}$. A. I. E. E., TRANS., Vol. XXX p. 349, particularly curve *B* Fig. 10.

B.—Prof. Ryan's formula, A. I. E. E., TRANS., Vol. 30, p. 64.

Equation (3) on p. 69, Vol. 30 gives the voltage at which the loss due to part corona is about equal to the loss due to convection, and this last named loss is given by the equation at the bottom of p. 67, Vol. 30, viz.:

Loss per 1000 ft. of single-phase line

$$= p = 2 \times f^2 \times E^2 \times 10^{-6} \text{ watts} \tag{13}$$

where, $f$ = frequency and $E$-kilovolts between wires. For one wire therefore,

$$P = 4 \times f^2 \left(\frac{E}{2}\right)^2 \times 10^{-6} \text{ watts; and for three wires of} \tag{14}$$

a three-phase circuit,

$$P = 4 \times f^2 \times E^2 \times 10^{-6} \qquad \text{watts} \tag{15}$$

For 61.1 cycles and 68.4 miles of any three-phase line,

$$P = 5.48\, E^2 \qquad \text{watts} \tag{16}$$

The critical corona voltage is, equation (3) p. 69, Vol. 30

$$E_{crit.} = 0.455 \times k \frac{17.9\, b}{459 + t} \; \frac{d + 2\, a}{C} \tag{17}$$

where, $C$ = 1.16 times capacity of 1000 ft. (304.8 m.) of single phase line in microfarads, and $a$ is obtained from Fig. 2 of Prof. Ryan's article. The constant $k$ is a factor selected by judgment guided by the results of practical tests and experience. See Tables I and II on pages 70 and 71 and remarks on p. 68, Vol. 30; select $k$ = 0.75;

In formula (17) above:

$d$ = 0.33 in.; $a$ = 0.069 in.; $b$ = 18.1 in.; $t$ = 61.6 deg. fahr.; $D$ = 72 in. and

$$C = 1.16 \frac{0.00368}{\log \left(\frac{D}{r}\right)} = 0.00161 \text{ microfarad}$$

From (17) above: $E_{crit.}$ = 61.6 kv.

From (16) above: $P$ = 20.8 kw.

The kilovolts in Fig. 9 corresponding to 2 × 20.8 = 41.6 kw. is 63.9 kv., as against the 61.6 found by Prof. Ryan's formula.

Bearing in mind the errors likely to occur particularly on the lower part of the curve, this is certainly as good an approximation as could be expected.

In order to show the influence of the constant *k* of Prof. Ryan's formula, the following table has been computed:

| *k* | *E* form. (15) kv. | 2 × convection loss form. (14) kw. | Corresponding line voltage from Fig. 9 kv. |
|---|---|---|---|
| 0.72 | 59.4 | 38.6 | 63.4 |
| 0.73 | 60.0 | 39.4 | 63.5 |
| 0.74 | 60.9 | 40.5 | 63.7 |
| 0.75 | 61.6 | 41.6 | 63.9 |
| 0.76 | 62.5 | 42.6 | 64.1 |
| 0.77 | 63.5 | 43.1 | 64.2 |
| 0.78 | 64.4 | 45.4 | 64.4 |
| 0.79 | 65.1 | 46.3 | 64.6 |

It is seen from above, that the constant *k* does not largely alter the result and any value between *k* = 0.72 and say *k* = 0.82 will give very good results.

The following test was started at 7.30 p.m. during heavy rain at La Fundicion; light rain, off and on, at Oroya. Probably only local showers along the line. Temperature outside Oroya power house 14.5 deg. cent. at 8.17 p.m.; barometer at Oroya 48.3 cm. and at La Fundicion 42.85, both at 7.30 p.m.

| Generator voltage kv. | Line kv.* | Generator current Fig. 8 amperes | Generator† current amperes | Corrected‡ ind. wattmeter readings kw. | Curve *A* Fig. 9 kw. | Frequency |
|---|---|---|---|---|---|---|
| 58.6 | 59.4 | 10.1 | 10 | 60 | 65 | 60.8 |
| 61.9 | 62.7 | 8.95 | 9.1 | 83.5 | 86 | |
| 64.6 | 65.3 | 8.00 | 8.04 | 110.7 | 110.5 | |
| 66.9 | 67.7 | 7.30 | 6.96 | 138 | 140 | |
| 62.1 | 62.9 | 9.10 | 9.35 | 84.5 | 81 | 61.0 |
| 65.0 | 65.8 | 7.8 | 8.04 | 118 | 115.1 | |
| 67.2 | 68.0 | 6.5 | 6.78 | 148 | 143.5 | |
| 69.5 | 70.2 | 5.0 | 5.4 | 193 | 184 | |
| 71.5 | 72.2 | 3.6 | 4.22 | 237 | 234 | |
| 73.2 | 73.9 | 2.8 | 3.58 | 285 | | |

*By interpolation from other test.

†A smaller scale ammeter used than in the first test.

‡Using Fig. 6.

At 71.8 kv. at Oroya a slight brush discharge was visible at the end of the lightning arrester horns connected to the line; the same was the case at the smelter substation. No visible corona was on the line either at Oroya or at the smelter. The night was quite dark with a new moon behind heavy clouds.

## ACKNOWLEDGMENTS

In conclusion I wish to express my appreciation to Mr. F. G. Baum, Mr. J. P. Jollyman and Professor Harris J. Ryan for suggestions in the preparation of this article.

## DISCUSSION ON "CORONA TESTS AT HIGH ALTITUDE" (JAKOBSEN), SAN FRANCISCO, CAL., JANUARY 25, 1918.

**F. W. Peek, Jr.** (by letter): I wish to call attention to a paper, "Comparison of Calculated and Measured Corona Loss Curves," published in 1915, where the laws of corona from my 1911 paper are discussed and applied to measurements made on practical and experimental lines in different parts of the country. Mr. Faccioli's Shoshone measurements are included in this comparison. I will quote directly from this paper:

"The law of corona takes the general form

$$p = c\,(e - e_0)^2$$

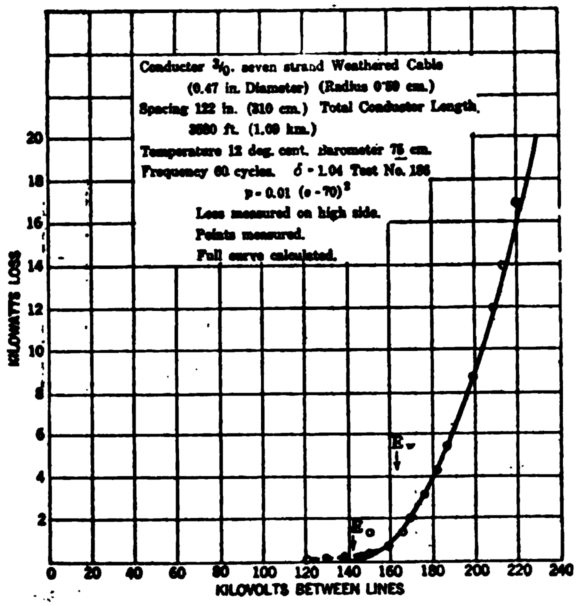

FIG. 1—OUTDOOR SINGLE-PHASE EXPERIMENTAL LINE. (TESTS MADE BY F. W. PEEK, JR., SCHENECTADY, 1910)

which means that under conditions otherwise constant, the loss varies as the square of the applied voltage above a certain critical voltage, $e_0$ or $e_d$. The critical voltage, $e_0$, is called the *disruptive critical voltage*. Visual corona does not start at the *disruptive critical voltage*, but at some higher voltage $e_v$, the visual critical voltage. Both of these voltages have been calculated and marked on the curves. Theoretically, if the conductors were perfectly smooth, no loss should occur until the visual critical voltage, $e_v$, is reached, when the loss should suddenly take a definite value. For $e_v$, and higher voltages, the loss should follow the quadratic law. In practise, due to dirt, points, etc., brushes occur on the conductors at voltages lower than $e_v$. Between $e_v$ and $e_0$, due to these brushes, there is a loss. This loss practically follows the quadratic law for large weathered conductors, where $e_v$ and $e_0$ approach each other in value. For small conductors and especially new conductors with fairly clean surfaces, *the loss between $e_v$ and $e_0$ falls below that of the quadratic law*, as shown in Figs. 2 and 3. If the conductors were highly polished there would be very little loss until the voltage $e_v$ is reached. At this voltage the loss would suddenly take a definite value very nearly equal to that calculated by the quadratic law with $e_v$ as the applied voltage and $e_0$ as the critical voltage in the equation.* In all cases the loss follows the quadratic law above $e_v$. The part of the loss curve between $e_0$ and $e_v$ will thus vary from day to day depending upon the chance condition of the conductor surfaces. Over this unstable section of the curve the loss follows the probability law."

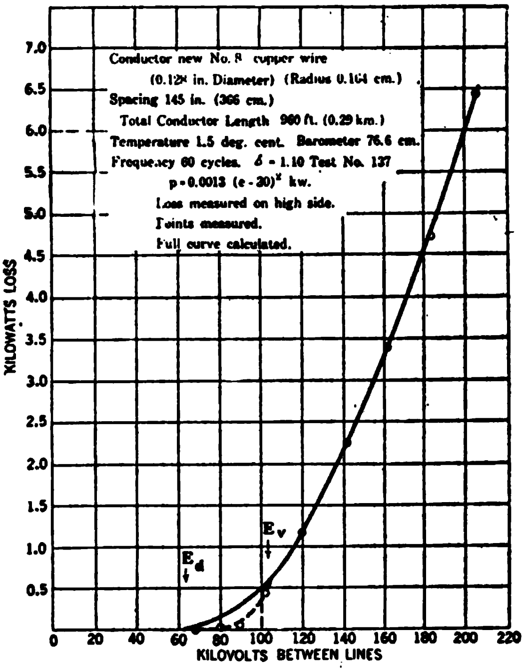

FIG. 2—OUTDOOR SINGLE-PHASE EXPERIMENTAL LINE. (TESTS MADE BY F. W. PEEK, JR., SCHENECTADY, 1910)

$$p = q\,\epsilon^{-h(e-e_0)^2}$$

Excess loss due to irregular irregularities, as spots, points, etc.

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*"$e_0$ or $e_d$ must always be used as the critical voltages in quadratic law."

"It is of *practical importance only to know the limits* of the *loss at this part of the curve*; $e_0$ *should generally be the limit of the voltage on practical lines, as otherwise storm losses become excessive.*"

"Thus, from the considerations above, the quadratic law should be *closely followed in all cases for voltages higher than* $e_v$. For *the part of the curve between* $e_v$ *and* $e_0$ the loss is unstable and dependent upon chance surface conditions, dirt spots, etc. as follows:

1. For large weathered cables, such as are used in practical lines, the quadratic law is closely followed.
2. For new conductors, and especially small ones, the loss at voltages lower than the visual critical voltage, $e_v$, falls below the quadratic law."[1]

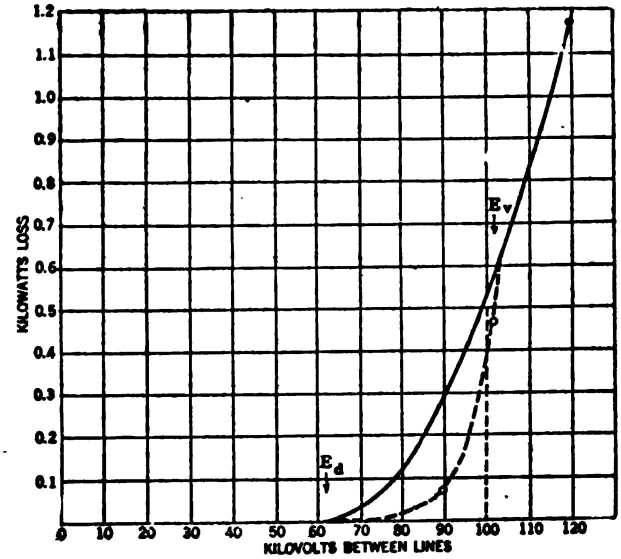

FIG. 3—LOWER PART OF FIG. 2 TO AN ENLARGED SCALE

There are, thus, two critical voltages: the *disruptive critical voltage*, $e_0$, used in the loss equation, which corresponds to a constant gradient of 30 kv. per cm. max., and the *visual critical* voltages, $e_v$, the voltage at which visual corona starts. $e_v$ is always higher than $e_0$. For perfectly polished wires, the loss would occur as shown in Fig. 5, suddenly reaching a definite value at $e_v$ and following the quadratic. Between $e_v$ and $e_0$ the loss is unstable and is caused by spots, points, dirt and other

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[1] "Law of Corona and Dielectric Strength of Air." F. W. Peek, Jr., A. I. E. E. TRANSACTIONS, 1911, page 1892, part III. Also "Law of Corona II". F. W. Peek, Jr., A. I. E. E. 1912, page 1051, part I. "Comparison of Calculated and Measured Corona Loss Curves," F. W. Peek, Jr., A. I. E. E. TRANSACTIONS 1915, page 269, Part I.

irregularities, and not "sub corona", "part corona", etc. Between $e_v$ and $e_0$ the loss may be greater or less than the quadratic, as shown in Figs. 1 to 4. For practical sizes it generally approximates the quadratic over the whole range. This was fully explained in my 1911 paper.[2] In practical lines it is generally not advisable to operate above the $e_0$ voltage. It is thus of great importance to be able to determine this voltage accurately,

In calculating the losses, etc. on the Oroya line I find

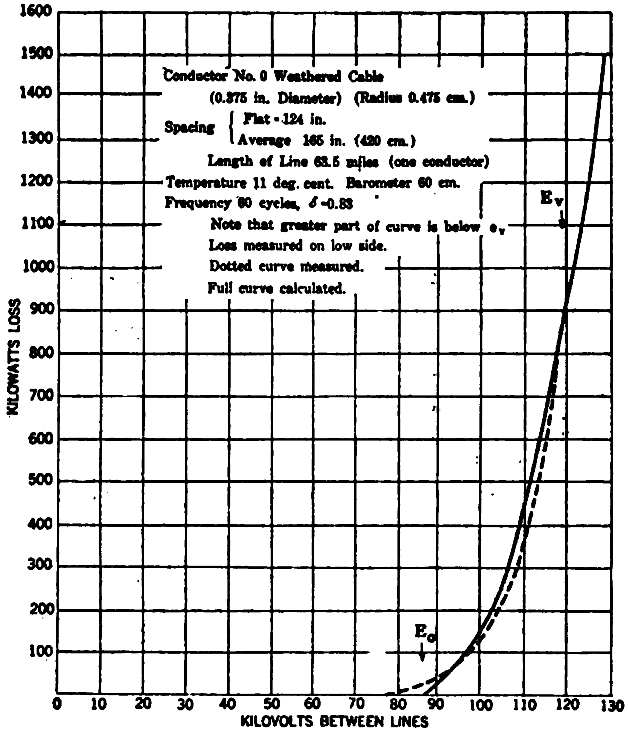

FIG. 4—SHOSHONE—LEADVILLE TRANSMISSION LINE, THREE-PHASE (TESTS MADE BY FACCIOLI.)

$$p = c\,(e - e_0)^2 = c\,(e - 29)^2$$

$$e_0 = \frac{50}{\sqrt{3}} \text{ kv. to neutral.} \quad e_v = \frac{75}{\sqrt{3}} \text{ kv. to neutral.}$$

Or between lines

$E_0$ = 50 kv. *Disruptive critical voltage*

$E_v$ = 75 kv. *Visual critical voltage*

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[2] Also "Dielectric Phenomena in High-Voltage Engineering," page 143, chapter V.

***Mr. Jakobsen's measurements were all made below the visual critical voltage or on the unstable part of the curve where the probability and not the quadratic loss law is followed.*** Mr. Jakobsen does not mention this fact in making comparisons, although he uses these measurements mainly to show that the quadratic is incorrect. His Fig. 9, curve *C*, indicates about 50 kv. for $E_0$, as calculated above. It is quite probable that with a new line the loss for voltages lower than the visual corona voltage would come lower than that indicated by the quadratic law. This is shown by my Fig. 2, where the $E_0 - E_v$ range of voltage is greater than that used by Mr. Jakobsen. With large weathered conductors the loss between $E_v$ and $E_0$ should be as in my Figs. 1 and 4. It is difficult to estimate the accuracy of Mr. Jakobsen's measurements. With the apparatus that he had available, a slight phase displacement caused by the current transformer, potential transformers, etc., at the low power factors would cause very large error . It is quite possible, however, that the loss for the voltage range measured was actually lower than that predicated by the quadratic law.

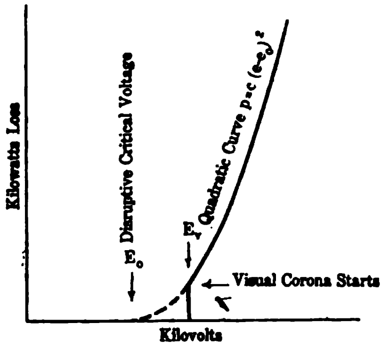

FIG. 5—CORONA LOSS FOR PERFECT CONDUCTORS.

NOTES—The loss for a Perfect Conductor is Shown by a Heavy Line. It is Zero up to $E_v$ When it Suddenly Reaches a Definite Value and Follows the Quadratic Law. For the Actual Loss in Portion Below *E*, See Figs. 1, 2 and 4.

In my Fig. 4 the dotted curve represents the losses measured on the Shoshone-Leadville line by Mr. Faccioli. The full curve is plotted from a direct calculation of the starting voltage and loss by my corona formulas. Contrary to the conclusions reached by Mr. Jakobsen from his Fig. 9A the check is good. As is usual in practical lines with large weathered conductors, the quadratic law approximates the loss even for voltages lower than $e_v$. Fig. 9A does not show that the Oroya and Shoshone losses check. No comparison is actually made by Mr. Jakobsen. The two curves have simply been slid together without any regard as to what the relative starting voltages should be. It only means that the two curves happen to have approximately the same slope for the range plotted. To show the fallacy of this method, I have plotted Fig. 6. The drawn curve is the calculated Shoshone curve. The points are the measured Shoshone and Oroya losses and the calculated Oroya losses, all "good checks", measured by the standards of Mr. Jakobsen's Fig. 9A. It thus does not seem rational to attempt to compare loss curves for different conductors by tying the curves together at some arbitrary loss per mile. It will be found that two curves so tied together at one temperature and pressure will differ widely at some other temperature and pressure.

Mr. Jakobsen's results given in the table on page 105 in connection with equation (11), should naturally be expected. A number of other equations would fit equally well over the limited range. This should be especially expected from an exponential equation. When an equation is written, however, it should be rational. The probability law is the rational equation for an approximation of the excess loss due to irregular irregularities.[3]

The reference to "convection loss" is difficult to understand. Assuming such a loss can be measured in a laboratory on polished wires, on a practical line, with actual local corona loss at points and dirt spots, it would be comparatively negligible. Below $e_v$ the loss in practise can only be that due to irregularities. The accuracy of the measurements at this part of the curve is so doubtful that no attempt should seriously be made at checking a "law". Since a great deal has been said about the visible and non-visible corona loss it may be well to explain. The visual corona voltage, $E_v$, is the voltage at which corona suddenly appears all around the conductor. It is a *very definite* voltage and can be accurately noted, if the wire is viewed in a very dark room. The corona can often not be observed out in the open, however, even on a "dark" night, at much higher voltages. Any appreciable loss below this voltage is due to brushes at roughened and dirty points along the conductor. These brushes are also visible in a dark room but are not readily observed on a line out of doors. All corona loss is thus really due to visible corona or brushes. Below the visual corona voltage the loss is due to the chance brushes at roughened or dirty spots.

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[3] The loss below the voltage $e_v$ can probably be best represented by the sum of two equations, a quadratic giving the loss due to the regular irregularities, and the probability law giving the loss due to the irregular irregularities.

Thus $p\,K\,(e - e^1_0)^2 + \quad q^{-h(e^1_v - e)^2}$

Irregular irregularities may be spots, points, dirt, etc.

Regular irregularities may be the strands in a cable, a weathered surface, a ridge, etc.

For voltages above $e_v$, all irregularities become practically regular and

$$p = c(e - e_0)^2$$

For small conductors $e_0^1$ may be larger than $e_0$. $K$ is smaller than $c$ since it may represent only part of the surface.

The sum of the curves above may be approximated by an exponential.

In the last table in Mr. Jakobsen's paper are given loss measurements made during a storm. These measurements are practically equal to those made in fair weather. This can be explained in only one of two ways. Either the meter arrangement is such that the indication is the same regardless of the actual loss, or the storm was local and confined to a very small portion of the line. There is no doubt whatever that corona loss is greatly increased during rain. This is substantiated by actual measurements and by theory.

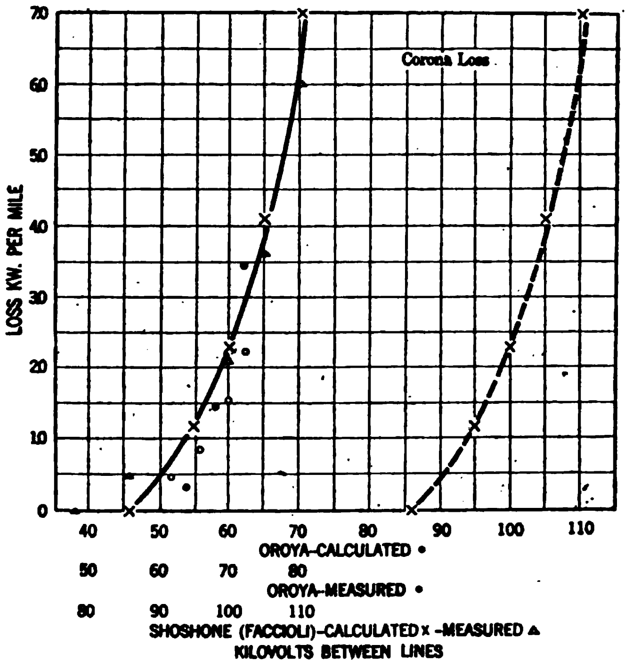

FIG. 6

NOTE—The Four Curves Have Been Moved Together Along the $X$ Axis as in Mr. Jakobsen's Report Fig. 9A to Give the Appearance of a Check—The Dotted Curve is Drawn in to Indicate how This was Done—Actually, no Comparison is Made by This Method.

It would be extremely interesting if Mr. Jakobsen could make measurements on different sizes of conductors at voltages above the visual corona voltage at these high altitudes. This would probably have to be done with a testing transformer on short lengths of line.

I have been very much interested in the data obtained, however, which seem to check the calculated $e_0$. The irregularity loss on this part of the curve may also, for a good surface condition, fall below the quadratic as Mr. Jakobsen has found. I do not agree with Mr. Jakobsen in his interpretation of the data, however.

To Summarize:—

There are two critical voltages which must be considered, the disruptive critical voltage, $e_0$, used in the quadratic law, and the visual critical voltage $e_v$. $e_v$ is always higher than $e_0$. In practise it is generally not desirable to operate above the $e_0$ voltage.

The loss at voltages above $e_v$ always follows the quadratic law.

For voltages lower than $e_v$ there would be no appreciable loss for perfect wires (Fig. 5). In practise there is a loss on this part of the curve due to brushes at irregularities. The rational approximation of these chance losses is the probability law. This loss may be greater or less than that indicated by the quadratic law, depending upon the surface condition. See Figs. 1, 2 and 3. For the larger weathered conductors used in practise, the quadratic law generally gives a good approximation at voltages below $e_v$. This is shown by Figs. 1 and 4. In Fig. 4 a direct calculation of the corona losses on the Shoshone line agrees with Faccioli's measured losses.

All of Mr. Jakobsen's measurements were made at voltages below the visual corona voltage, $e_v$, where the losses are due to irregularities.

Mr. Jakobsen's Fig. 9A does not show agreement with Faccioli's Shoshone loss measurements. The curves have simply been slid together along the $X$ axis without regard to relative starting voltages. It only means that the curves happen to have practically the same slope over the range plotted. Fig. 6 is plotted to show the fallacy of making such a comparison. In this figure the calculated and measured Oroya curves and the calculated and measured Shoshone curves are moved together in the same way, and all show an *apparent* check. Actually, the two Oroya curves do not check; the two Shoshone curves do.

The results in the table on page 105 should be expected. An approximation of this curve should be an exponential. A number of other equations could equally well be made to fit.

Referring to the calculations on page 106, if there is a convection loss, it must be relatively so small compared to the actual corona loss at irregularities, that it is negligible.

The losses during rain given in the table on page 107 practically agree with the fair weather loss. This is contrary to all other measurements and to theory.

In estimating the cost of the loss of power per year due to corona it must be remembered that the loss varies from day to day due to changes in temperature and pressure. A given line with very high losses during the hot summer days may have no loss for the greater part of the year. It is instructive to plot a time-loss curve for a year. The rain losses may be very high during periods of precipitation, but when put on a yearly basis may be a very small part of the total loss. At the part of the curve near the critical voltage, $e_0$, the loss will be considerably affected by surface conditions not under control of the operator, due to dust, fog, sleet, etc. Such conditions can only be estimated. We have found the quadratic law to give the best average estimate of these losses.

**Harris J. Ryan:** Mr. Peek characterizes my comparison of the forms of the Faccioli and Jakobsen high-altitude corona line-loss curves by saying that, "the two curves have simply been slid together without any regard as to what the relative starting voltages should be." Precisely the opposite is the case. The curves were brought together for their form or "law" comparison by allowing for the difference in their corona starting voltages. See text of the present Jakobsen paper, page 103: "For the purpose of comparison we may assume that the corona was definitely started at this amount of loss viz.: one-half kilowatt per kilometer of line." "It follows, therefore, that the corona was started on the Shoshone-Leadville line at 95.8 − 65.8 = 30 kv. higher than the Oroya line." The two curves were therefore given a common origin at the definite initial corona loss of 0.5 kw. per kilometer of line and at the corresponding line voltages of 52 and 82 kv. On Peek's authority the values of the Shoshone line losses were corrected so as to correspond to the values that would have occurred at the barometric pressure and temperature of the Oroya line by using the multiplier 0.93. I know of no more rational proceeding in the circumstances.

In his contribution to this discussion Mr. Peek has stated and restated that the non-visible coronas are controlled by "irregularities" and "irregular irregularities". Ten years ago Foote in his Michigan 110 = kv. transmission studies and practise proposed that a small definite loss due to corona, say 0.5 kw. per mile of transmission line be adopted to determine the corona starting voltage. It seemed to me then* and the results of many studies since cause me to continue to think that Foote's proposal is a good one and should be adopted, viz.: That for practical purposes the corona starting voltage should be taken as the value which causes a loss due to corona of 0.5 kw. per kilometer (0.8 kw. per mile) of transmission line.

Because of the work of Peek and of others we have an abundant knowledge of the visible corona losses. Engineering practise is much concerned with the non-visible losses and knowledge thereof is scanty. Mershon's results obtained in 1905 at Niagara Falls and a few results obtained from tests made on actual transmission lines are all that we have. Much more should be done. The factors involved are many and the work is difficult. The present paper is most welcome. It is to be hoped that more of its kind will follow. The present out-look is that these non-visible corona losses must be handled empirically.

**B. F. Jakobsen:** I shall consider Mr. Peek's remarks under three headings, as follows:

1. Range of test voltage at Oroya.

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*Conductivity of the Open Atmosphere, TRANS. A. I. E. E., Jan. 1911.

2. Accuracy of Oroya tests.
3. Comparison of calculated and measured corona losses.

1. *Range of Test Voltage at Oroya.* The maximum voltage that could be obtained at Oroya by raising the generator voltage, was 72,300 volts. At this voltage no corona was visible on a dark night, either at Oroya at 12,200 ft. (3.72 km.) or at the Smelter at about 14,000 ft. (4.27 km.). The corona loss was about 150 kw. for one circuit or 300 kw. for the two circuits. Mr. Baum states that power is worth ½ cent per kw-hr. and the loss due to corona would therefore amount to $13,000 per year. The copper in both circuits from Oroya to Smelter is 550,000 lb. (249,475 kg.) and it cost about $140,000 in place; therefore we could afford to double the amount of copper rather than accept the corona loss of 150 kw. per circuit. ***The maximum voltage used in the test was therefore considerably above any voltage that could be considered as operating voltage.***

It is the corona loss at voltages below the visual corona voltage that is of importance in transmission line work, since it is important to know how high one may safely go with the line voltage under certain conditions, rather than knowing what the corona loss would be at a voltage several times higher than any possible operating voltage.

2. *Accuracy of Oroya Test.* Faccioli made some tests in order to check the accuracy of his corona tests (see page 347, Vol. 30, Part I) and as these have a bearing on the accuracy of the present test, I shall quote from Faccioli's paper, regarding the Denver-Boulder single-phase test:

> Since the power factor of the readings was very low it was considered advisable to raise this power factor by using an artificial load (a water box) in multiple with the low-tension winding of the step-up transformer which energized the line. The measurements were taken as follows:
>
> The transformer only was connected to the system and readings taken, then the line was connected to the transformer and readings taken at the same voltages as before. The difference between the kilowatts in the two cases gives the total losses of the line. Additional measurements were made, first, with the transformer and auxiliary load; second, with the transformer, auxiliary load and line. The utmost care was taken to keep the auxiliary load constant during these two sets of measurements. The introduction of the auxiliary load proved unnecessary as the results obtained with and without the extra load were the same.

At the maximum voltage employed at Oroya, *i. e.* 72,300 volts, the load on the wattmeters was 1,185 kw. (see Table II of my paper); this instrument load is five times the actual load (Table I and Fig. 9 of my paper) on account of the current transformer connections employed (see "Meters and Instrument Transformers" page 952). The generators are 3750 kw., so that the load on the wattmeters was about 30 per cent of the generator capacity; the power factor on the wattmeters was 0.54 (see Fig. 8 of my paper). The two wattmeters (indicating and integrating) agree at this voltage (see Fig. 6 of my paper), and the measured current agrees closely with the calculated current (see Fig. 8 of my paper.) *There is thus no reason to expect an error of more than a very few per cent at the most, while if Mr. Peek's formula was correct, the error committed would be about 550 per cent.*

3. *Comparison of Calculated and Measured Corona Losses.* In Mr. Peek's paper "Comparison of Calculated and Measured Corona Coss Curve", A.I.E.E. Vol. 34 (1915) Part I, page 269, only a single test made on an actual transmission line is considered, and this is Faccioli's Shoshone-Leadville test, reported in the A. I. E. E. TRANSACTIONS Vol. 30 (1911) Part I, page 337. The profile of this line is shown on page 79 of Vol. 30 Part I. Referring to this particular test, Mr. Faccioli said, page 344:

> Although the Shoshone-Leadville tests were taken on a section of line 63.5 miles (102 km.) long, still the curves of Fig. 6 cannot be used to determine the critical voltage or the law which connects losses to kilovolts because the voltage at the far end of the line is higher than the voltage at the power house and because of the different altitudes through which the transmission line runs. For this reason some experiments were conducted on the Denver-Boulder section of the system.

Fig. 4 of Mr. Peek's paper (Vol. 34, page 269) is reproduced as Fig. 4 in this discussion. This figure shows the close agreement between the loss measured by Faccioli and the Peek formula, but it is quite misleading, because Mr. Peek has used the temperature and the barometer for Shoshone, as given by Mr. Faccioli on page 344, Vol. 30 (1911) Part I, with practically no corrections, although Shoshone lies at only 6000 ft. (1.8 km.) while the maximum point on the line lies at 12,000 ft. and the average for the whole line being about 8500 ft. (2.6 km.). Faccioli gives: temp. = 55 deg. fahr. (12.8 deg. cent.); barom. = 24 in. (61 cm.) while Mr. Peek has used 11 deg. cent. and 60 cm. barometer and Professor Ryan used 5 deg. cent. and 52 cm. barometer in Fig. 9A.

I do not know where Professor Ryan obtained the barometric pressure which he used, but using the following standard formula:

$$H_2 - H_1 = (18{,}400 + 70 \times t_m) \times \log\left(\frac{P_1}{P_2}\right)$$

where $H_2 - H_1$ = difference of altitude in meters,
$P_2$ and $P_1$ = corresponding barometric heights in millimeter of mercury;
$t_m$ = average temperature of air between the two elevations, in centigrade;

with $H_2 - H_1$ = 2500 ft. (763 meters), $P_1$ = 610 mm. (24 in.) and $t_m = (12.8 + 5) : 2 = 9$ deg. cent., I find $P_2$ = 555 mm., instead of the 60 cm. used by Mr. Peek and 52 cm. used by Professor Ryan.

Using the temperature and barometric pressure as given by

Professor Ryan, $\delta = 0.724$ instead of 0.83 as given by Mr. Peek in Fig. 4. With $m_0 = 0.87$ (Peek's law of corona, Vol. 30, Part III, page 1890), $E_0 = 74.2$ kv. between wires, and the loss for three wires at 60 cycles and 63.4 miles (102.5 km.) as per equation (6), page 1894, Vol. 30, Part III, is:

$$P = 0.981\,(E - 74.2)^2\ \text{kw.}$$

This gives the following table for the comparison between the measured Shoshone-Leadville line loss (Faccioli Fig. 6, page 342, Vol. 30, Part I) and Peek's formula:

| Voltage between lines in kv. | Loss by Peek's formula. kw. | Measured loss. kw. | Ratio | Square root of measured loss |
|---|---|---|---|---|
| 85 | 114 | 25 | 4.56 | 5.0 |
| 90 | 246 | 30 | 8.2 | 5.46 |
| 100 | 655 | 115 | 5.7 | 10.72 |
| 110 | 1260 | 360 | 3.5 | 19.0 |
| 120 | 2060 | 1120 | 1.84 | 33.5 |

The square root of the loss is plotted against the voltage between wires in Fig. 7 and this shows a curve exactly like the one found for the Oroya line and plotted in Fig. 10 of my paper, and it shows that the loss measured does not follow the quadratic law, for if it did, this line would be a straight line.

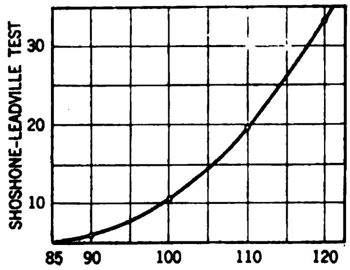

FIG. 7

Since the corona loss is very much affected by barometric pressure and temperature and since these were measured only at one point (the lowest point along the line), this test is decidedly unfit for comparisons.

The increase of voltage towards the open end of the line, as mentioned by Mr. Faccioli, in addition to the higher altitude at the open end of the line, would increase the losses. In comparing a standard formula with these measurements, allowance must be made for this, or the comparison is quite worthless.

Faccioli measured the corona losses by first measuring the transformer core loss + low-tension copper loss and then connecting the transformers to the line and measuring the loss again at the same voltage. The difference between these two losses he took as the corona loss at that voltage, see page 340 near top, Vol. 30, Part I. In this way the high-tension transformer copper loss and the line copper loss, as given by my formula (**8**), was included with the corona losses, and this method would give a corona loss larger than the actual loss.

In order to get an idea of the magnitude of the various losses involved in the corona loss tests at Oroya, Fig. 8 was plotted. Curve *A* represents the core loss of the three transformers; Curve *B* is the high-tension transformer copper loss + the line copper loss (equation (**8**) of my paper), the line copper loss amounting to about 4/5 of the curve *B* loss, equations (**3**) and (**7**). The low-tension copper loss varies as the generator current, Fig. 8 and its maximum value is only 1.4 kw. At 40 kv. it is 1.1 kw. and at 70 kv. it is 0.3 kw. The copper loss in the low-tension transformer windings added to the two curves *A* and *B* in Fig. 8, would give the total loss curve, Curve *B* in Fig. 9 of my paper.

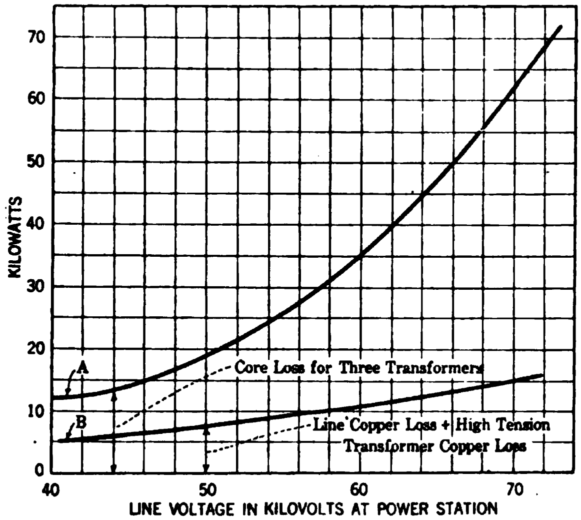

FIG. 8

The Denver-Boulder single-phase line test made by Faccioli and shown in Fig. 9, page 347, Vol. 30, Part I is considered by Mr. Faccioli as being the more accurate test and it would therefore form a better basis for comparisons than the Shoshone-Leadville tests. The constants given by Faccioli are: distance 27.6 miles (44 km.); line No. 1 B & S. diameter = 0.33 in.; spacing = 124 in; barometer 24.4 in. (62 cm.) and temperature 62 deg. fahr. (16.7 deg. cent.). These two last are taken as the average of 24.1 in. and 24.7 in.; and 49 deg. and 74 deg. fahr. as given by Faccioli on page 345, Vol. 30 and no corrections were made for the small differences in barometric pressure and in temperature due to the difference in elevation between the substation at Denver and the average altitude of the line. The profile is shown on page 79, Vol. 30. The average altitude is given by Faccioli as 5300 ft. (on page 344, Vol. 30), but this does not agree with the profile, which is probably correct and which shows about 5900 ft. (1.8 km.).

Taking $m_0 = 0.87$, Peek's formulas give $E_0 = 85.8$ kv. between wires and the loss for the two wires (single-phase):

$$P = 0.199\,(E - 85.8)^2 \text{ kw.}$$

Faccioli derived the following formula for the loss for voltages above 100 kv. (see his Fig. 11 on page 351, Vol. 30):

$$P = 0.036\,(E - 80)^2 \text{ kw.}$$

**Comparison between measured values and calculated values for the Denver-Boulder test by Faccioli and Peek's formula.**

| Voltage between lines in kv. | Loss by Peek's formula. kw. | Measured loss. kw. | Ratio | Square root of measured loss. |
|---|---|---|---|---|
| 90 | 3.5 | 3 | 1.17 | 1.73 |
| 95 | 16.8 | 7 | 2.4 | 2.65 |
| 100 | 40 | 12 | 3.33 | 3.46 |
| 105 | 73 | 19 | 3.84 | 4.36 |
| 110 | 117 | 28 | 4.18 | 5.3 |
| 115 | 170 | 37 | 4.59 | 6.09 |
| 120 | 234 | 48 | 4.86 | 6.94 |

Square root of the kilowatts against the voltage is plotted in Faccioli's Fig. 10. Above 100 kv. the curve between the square root of the loss and the voltage is a straight line.

*It will be seen that in both these cases Peek's formulas give values for the corona loss which are several times higher than the losses found on actual transmission lines for voltages near operating voltages. And this was found also at Oroya, table on page* 102 *of this volume.* That these test voltages were below the visible corona voltage is quite true, but that does not detract from the fact that we are mostly interested in knowing the losses which are likely to occur at or near the actual operating voltages.

It is very important, as Mr. Peek says, to be able to determine $E_0$ (the disruptive critical voltage), but I cannot see why Fig. 9 of my paper shows this to lie at about 50 kv., since the loss starts at about 40 kv.

On long high-tension transmission lines carrying large amounts of power it will most likely be economical to allow a certain percentage loss as corona loss and in order to do this, the corona loss below the visual corona voltage should be investigated on actual transmission lines.

Regarding the losses during rain, which Mr. Peek contends should be considerably larger than the fair weather losses, I had no means of knowing the weather conditions except at the end of the line, as I stated in the paper.

*Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February* 15, 1918.

# RATING AND SELECTION OF OIL CIRCUIT BREAKERS

BY E. M. HEWLETT, J. N. MAHONEY AND G. A. BURNHAM

ABSTRACT OF PAPER

On account of the variable conditions in systems on which circuit breakers are used, it is impossible to give a simple rule which will cover the selection of circuit breakers for all cases. The authors discuss the interpretations of the A. I. E. E. Standardization Rules covering the rating of oil circuit breakers and consider the variable factors which are involved in the selection of circuit breakers for various systems. A method is suggested whereby short-circuit characteristics of various systems can be used for determining the proper selection of oil circuit breakers for average systems. The method does not apply to very large systems or unusual conditions.

THERE IS an increasing demand from engineers and operators for a more uniform statement from the various manufacturers with reference to the rating and recommended selection of electrical protective equipment. It appears that simple concise statements might easily be made that would convey definitely the desired information, but there are so many variables which enter into the selection that a simple statement to cover all cases is impossible.

The object of this paper is, (1) to discuss the interpretations of the A. I. E. E. rules covering the rating of oil circuit breakers, (2) to discuss the factors involved in the proper selection of oil circuit breakers, and (3) to suggest average system short-circuit characteristics which can be used for selecting oil circuit breakers for certain systems.

It is hoped that the interpretation given and the data proposed for oil circuit breaker selection will meet with the approval of those persons interested in this problem or give rise to suggestions leading to improvement.

The subject has received careful consideration by the Standards Committees of the American Institute of Electrical Engineers and several papers on this subject[1] have been presented

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[1] Rating of Oil Circuit Breakers, by E. M. Hewlett, TRANSACTIONS, A. I. E. E., 1916; Rupturing capacities of Oil Circuit Breakers, by S. Q. Hayes, TRANSACTIONS, A. I. E. E., 1916; Rating of Oil Circuit Breakers, by G. A. Burnham, TRANSACTIONS, A. I. E. E., 1913.

to the Institute, all of which have resulted in bringing about a clearer understanding of the various expressions and methods used in connection with the rating of this class of equipment.

Circuit breakers are classified according to their rated pressure, rated current, rated frequency and interrupting capacity.

Systems may be classified according to their normal operating pressure, normal current, normal frequency and current transients.

The *rated pressure* (voltage) of a circuit breaker is the greatest normal pressure in r.m.s. volts between any two wires of any circuit to which the breaker should be connected.

The Standardization Rules of the American Institute of Electrical Engineers require that oil circuit breakers for voltages above 600 volts withstand a dielectric test[2] of 2.25 times rated pressure plus 2000 volts for 60 seconds. Although not stated in the rules we infer that it contemplates a test with the apparatus under dry conditions.

The *normal operating pressure* of a system is the greatest pressure in r.m.s. volts ordinarily maintained between any two conductors.

*The rated current* of a circuit breaker as defined by the A. I. E. E.[3] is "the normal r.m.s. current which it is designed to carry." This rating is covered by the following rule.[4]

"Temperature Tests—Rated current at rated frequency shall be applied continuously until the temperature becomes constant. The maximum temperatures of the various parts shall not exceed the following when the ambient temperature of reference is 40 deg. cent.:

Contacts in air.....................60 deg. cent.
Oil and contacts therein.............70 deg. cent.
Coils (See sections 376-379 incl.)[5]
Other parts (see section 392)[6]

Contacts in air may be subjected to an ultimate temperature at 70 deg. cent. for periods of short duration.

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[2] Rule 755. June 1917, supplement to A.I.E.E., Standardization Rules.

[3] Rule 752, June 1917, Supplement to A.I.E.E. Standardization Rules.

[4] Rule 754, June 1917, Supplement to A.I.E.E. Standardization Rules.

[5] The rules referred to herein, limit the maximum permissible temperature rise of coils to temperatures determined by the insulating materials used.

[6] The rule referred to above reads in part as follows: "All parts of electrical machinery other than those whose temperature affects the temperature of the insulating material may be operated at such temperatures as shall not be injurious in any other respect."

"The Institute recognizes the inherent decrease in capacity of switch and circuit-breaker contacts in air, due to oxidization of the contact surfaces. The rating of air switches and circuit breakers is, therefore, based on sufficient maintenance to keep the temperature within the specified limits."

The *normal current* in a circuit of an electrical system is the rated current in r.m.s. amperes for which that circuit is designed.

The actual current may vary through wide limits from day to day and at different seasons of the year. The upper limit for continuous operation or the rated current is, however, fixed by the capacity of the conductors as determined by the maximum allowable temperature at which the conductors and their insulation may be operated.

The *interrupting* (rupturing) *capacity* of a circuit breaker as specified by the Standardization Rules[8] of the American Institute of Electrical Engineers is:

"——the highest r.m.s. current at normal voltage which the device can interrupt under prescribed conditions at stated intervals a specified number of times."

It is recognized that factors anticipated in the above rule as the "prescribed conditions" may affect the interrupting capacity of the breaker. Such factors are discussed under the section: "Present Interrupting Capacity Rating."

The "stated intervals" and "specified number of times" at a given current and pressure determine the *duty* imposed upon the breaker. The breaker interrupting capacities in r.m.s. amperes published by various manufacturers are based on an assumed duty, *i.e.*, that the breaker will interrupt its rated r.m.s. current two times at a two minute interval and then be in condition to be closed and carry its rated current until it is practicable to inspect it and make necessary adjustments.

The duty, including a statement of the "prescribed conditions" therefore, places a limit on the interrupting capacity of a breaker and any change in duty or prescribed conditions will necessarily affect the rated interrupting capacity.

## PREVIOUS INTERRUPTING CAPACITY RATINGS

It has been the practise in the past to state the interrupting capacity of circuit breakers in terms of the total alternator capacity in kv-a. at a specified reactance. In rating a circuit

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[8] Rule 753, June 1917, Supplement to A. I. E. E. Standardization Rules.

breaker in these terms, consideration was given to the short-circuit characteristics of the machines together with the characteristics of the circuit breakers and relays.

The "arc kv-a." ratings as previously listed were derived by multiplying the interrupting capacities in amperes by an assumed value of pressure. This assumed value was considered as the probable pressure that would be re-established on the bus immediately after the short circuit was cleared or that occurring during the clearing. The "arc kv-a." rating, based on the assumption that the re-established bus pressure will be normal, can be obtained for three-phase circuits by multiplying the interrupting capacity of the breaker in amperes by the normal pressure in volts of the circuit to which it is connected, and by the factor 1.73 divided by 1000.

It is to be noted that the interrupting capacity rating specified by the A. I. E. E. in r.m.s. current anticipates normal pressure to be re-established. Systems having characteristics such that the re-established pressure during short circuit will be higher than normal, will require a larger breaker.

As power stations have increased in capacity, and transmission and distribution net works extended, considerable reactance is introduced between the alternators and the point of short circuit which limits the current appreciably, and the total alternator capacity is no longer a measure of the severity of the short circuit. Service considerations are becoming more severe and larger and more expensive circuit breakers are required. A method of rating circuit breakers that will allow of more accurate selection for a wide range of conditions is desirable.

## PRESENT INTERRUPTING CAPACITY RATING

The rating of a circuit breaker in r.m.s. current interrupted at normal operating pressure simplifies the selection of a proper breaker for a given service condition. Such a rating makes comparative tests possible if a sufficient amount of power is available.

In the A. I. E. E. rule establishing this rating it is qualified by the words "prescribed conditions". It is generally recognized, indicated by test and by the operation of circuit breakers in service, that the power factor or the stored electrostatic and magnetic energy of the system are among the conditions affecting the interrupting capacity at a given r.m.s. current. During the current-opening periods an arc is established, and the current and voltage relations during this period are much more complicated than the simple phase-angle relation covered by the statement of power factor. Furthermore, the arc may be re-established under transient voltage conditions, still further complicating the phenomena. The theoretical and empirical data available on the effect of these conditions on the work done by the breaker are not at present adequate for the authors to prescribe any particular power factor for test, or to suggest any method for general use, to take into account the power factor and energy storage characteristics for all systems. These factors are of special importance when a breaker of relatively small interrupting capacity is connected to a large system in such a way that the breaker may afford the only outlet for the stored energy of the system. Such influences, while extremely difficult to take into account, will not differ widely in average systems. They are taken into account in a general way in the factor of safety employed in the rating of a breaker and their effects need not be considered in ordinary individual problems.

Selection of breakers for unusual conditions, large systems, or those involving large investment, should be checked with the manufacturers.

For average systems, a determination of the r.m.s. current that will flow at the instant the contacts part, irrespective of power factor or circuit conditions, will enable one to select the proper circuit breaker.

## DETERMINATION OF SHORT-CIRCUIT CURRENT

In order to determine the r.m.s. current that the circuit breaker will be required to open, an analysis of short-circuit phenomena on an alternating-current net work is necessary, and it may be desirable to call attention to some of the conditions under which circuit breakers may be called upon to function.

Short-circuiting a system at any point permits an abnormal current to flow immediately in that system. The amount and persistency of this current rush depend upon the characteristics of the synchronous apparatus connected to the system at the time, and upon the impedance in circuit between the synchronous apparatus and the point of short circuit. The value of r.m.s. current which the circuit breaker will be called upon to interrupt, will depend upon the length of time that elapses between the start of short circuit and the parting of the contacts as will be seen from the following analysis of short circuits.

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The greatest transient disturbance of the system which can occur at the point of application of the circuit breaker when the system is short-circuited, governs the selection of a suitable breaker. A diagram showing the current flowing in one phase of the external circuit when a system is short-circuited under ordinary operating conditions, is shown in Fig. 1. In this diagram $O$ is the origin of the co-ordinates and is taken at the instant at which the short circuit occurs. $O X$ is the axis of abscissas and the abscissas represent time. $O Y$ is the axis of ordinates and the ordinates represent current. $C D$ is a curve passing through the maxima of the wave of the total current and $E F$ is a curve passing through the minima. $A B$ is a curve which cuts the vertical everywhere midway between $C D$ and $E F$.

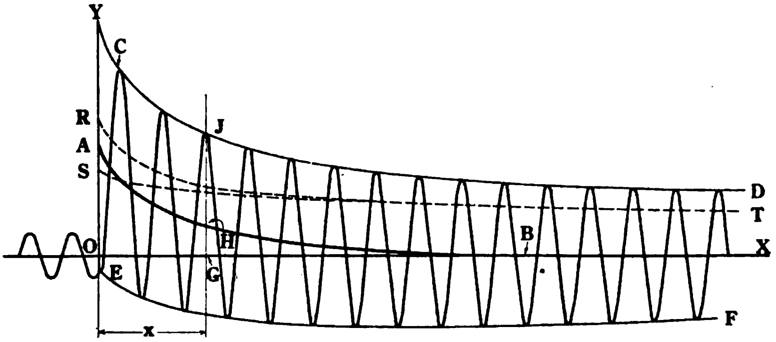

FIG. 1

**Generator characteristics—illustration of behavior of generator current when short circuited from full load at 0.8 power factor**

*The wave of total current* whose crests lie along curve $C$ $D$ and $E$ $F$ and whose ordinates are measured from the axis $O X$ may be regarded as having two components, namely,

1. A direct component
2. An alternating component.

*The direct component* is represented at any time by the ordinate to the curve $A$ $B$ or at the time $x$ by the ordinate $G H$.

*The alternating component* is a wave whose crest value at any time is the difference between the ordinates to the curves $C D$ and $A$ $B$. This difference, at the time $x$, has the value $H$ $J$. The r.m.s. values of this alternating component are shown on curve $S$ $T$. At any instant this component is considered to have the same r.m.s. value as an alternating wave of constant amplitude whose crest value is represented by one half the distance between curves *C D* and *E F* at that instant.

The r.m.s. value of the total current wave under short circuit at any instant is the square root of the sum of the squares of the value of the ***direct component*** and the ***effective*** value of the ***alternating component*** at that instant. It is represented by the curve *R. T.* This r.m.s. value of the total current at the time of parting of the circuit-breaker contacts is used in making circuit-breaker applications.

The r.m.s. current at any point of a system under short-circuit conditions is affected by the following factors.

1. The total kv-a. reactance and transient characteristics of the synchronous machines connected to the system.
2. Number, reactance, resistance, capacitance and arrangement of all circuits over which power can be supplied to the point of short circuit.
3. The kv-a., arrangement, resistance, reactance and capacitance of all reactors and transformers through which power can be supplied to the point of short circuit.
4. Contact resistance at the short circuit.
5. The nature of the short circuit, whether single-phase or multiphase.
6. The kv-a. and power factor of the load being carried at the time of the short circuit.
7. The point of the pressure wave at which the short circuit was established.
8. The use of automatic voltage regulators.

The short-circuit transient for systems may be determined by test, by calculation or, less closely, by assumption. Obviously, the determination by test for all circuits of a large system is expensive and involves considerable time and interruption to service. This will be practicable in but few cases. The determination by calculation is also a matter of considerable labor but is feasible if only the important factors listed above are considered. Practical approximate selection, sufficiently accurate for many cases can be made by using only reactance and an accepted group of time-current decrement curves.

Suggested decrement curves are shown on Figs. 2 and 3 and their ordinates on Table I.

These curves are based on the following assumptions: Transient characteristics for alternators of normal design determined from oscillograph tests: That the effect of capacitance and resistance is neglected: That the contact resistance at short circuit is zero: That the alternator is carrying full load 80 per cent power factor: That the short circuit was established at the point of the pressure wave corresponding to maximum possible instantaneous current: That no automatic voltage regulators are used.

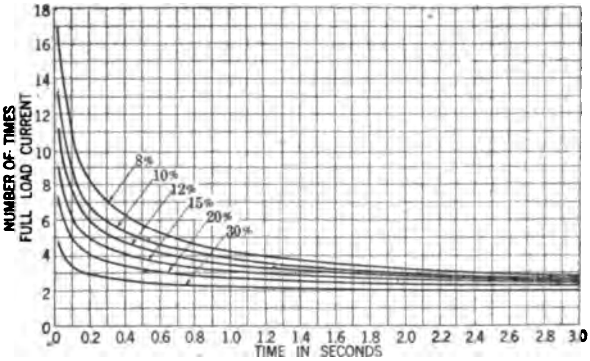

FIG. 2

System short-circuit characteristics—8, 10, 12, 15, 20 and 30 per cent total reactance based on total kv-a. rating of synchronous machines

Time-current curves—r. m. s. current in terms of total full-load current of machines—Initial full load at 0.8 power factor assumed

These curves differ from those that have been usually considered in the past for two reasons: first, r.m.s. values are used instead of peak values; and second, the effect of the increased flux existing under the load condition assumed has been taken into account.

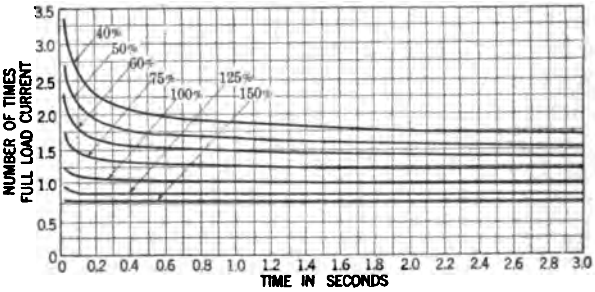

FIG. 3

System short-circuit characteristics—40, 50, 60, 75, 100, 125 and 150 per cent total reactance based on total kv-a. rating of synchronous machines

Time-current curves—r. m. s. current in terms of total full-load current of machines—Initial full load at 0.8 power factor assumed

The effect of using r.m.s. values instead of peak values is to

## TABLE I—SHORT-CIRCUIT CURRENT FACTORS.

| Reactance* | 8% | 10% | 12% | 15% | 20% | 30% | 40% | 50% | 60% | 75% | 100% | 125% | 150% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Elapsed time in seconds from start of short circuit | Current Factors expressed as number of times full-load current.† | | | | | | | | | | | | |
| 0.05 | 13.91 | 11.16 | 9.59 | 7.68 | 6.04 | 4.03 | 3.01 | 2.40 | 2.00 | 1.58 | 1.17 | 0.92 | 0.77 |
| 0.08 | 11.78 | 9.54 | 8.25 | 6.66 | 5.27 | 3.59 | 2.74 | 2.21 | 1.86 | 1.50 | 1.13 | 0.90 | 0.76 |
| 0.10 | 10.94 | 8.89 | 7.68 | 6.23 | 4.97 | 3.41 | 2.63 | 2.13 | 1.81 | 1.46 | 1.11 | 0.89 | 0.76 |
| 0.15 | 9.16 | 7.54 | 6.57 | 5.40 | 4.38 | 3.08 | 2.42 | 2.00 | 1.71 | 1.41 | 1.09 | 0.89 | 0.76 |
| 0.20 | 8.24 | 6.80 | 5.97 | 4.95 | 4.06 | 2.92 | 2.30 | 1.92 | 1.66 | 1.38 | 1.08 | 0.88 | 0.76 |
| 0.25 | 7.55 | 6.28 | 5.54 | 4.63 | 3.82 | 2.79 | 2.23 | 1.87 | 1.63 | 1.36 | 1.07 | 0.88 | 0.76 |
| 0.30 | 7.03 | 5.88 | 5.19 | 4.39 | 3.67 | 2.70 | 2.18 | 1.84 | 1.60 | 1.34 | 1.06 | 0.88 | 0.76 |
| 0.40 | 6.27 | 5.30 | 4.74 | 4.03 | 3.40 | 2.57 | 2.10 | 1.79 | 1.57 | 1.32 | 1.06 | 0.87 | 0.76 |
| 0.50 | 5.74 | 4.91 | 4.40 | 3.80 | 3.23 | 2.48 | 2.04 | 1.75 | 1.54 | 1.31 | 1.05 | 0.87 | 0.76 |
| 0.70 | 4.99 | 4.34 | 3.93 | 3.45 | 2.98 | 2.34 | 1.96 | 1.70 | 1.51 | 1.29 | 1.04 | 0.87 | 0.76 |
| 1.00 | 4.25 | 3.77 | 3.47 | 3.11 | 2.73 | 2.21 | 1.88 | 1.65 | 1.48 | 1.27 | 1.04 | 0.87 | 0.76 |
| 1.50 | 3.63 | 3.31 | 3.08 | 2.82 | 2.53 | 2.10 | 1.81 | 1.61 | 1.45 | 1.25 | 1.03 | 0.87 | 0.76 |
| 2.00 | 3.20 | 2.98 | 2.82 | 2.63 | 2.39 | 2.03 | 1.77 | 1.58 | 1.43 | 1.24 | 1.02 | 0.87 | 0.76 |

*Reactance expressed in per cent based on total kv-a. rating of synchronous machines. This includes both internal reactance of machines and reactance of external circuit reduced to the above basis.

†Rated full-load current based on maximum continuous kv-a. rating of synchronous machines. When the equivalent reactance of line, reactor, transformer, or combination of these expressed in per cent based on the total synchronous machine rating exceeds 150 per cent, the current to be interrupted may be determined directly from the reactance. This is due to the fact that under those conditions the generator reactance and the time of opening of the breaker may be neglected.

appreciably reduce the ratio between short-circuit and rated amperes. For example, assuming 10 per cent reactance and a short-circuit current containing the maximum possible direct component, the ratio of the peak value of the first alternation of the short-circuit current to the peak value of rated current is roughly twenty. Under the same conditions the ratio of the r.m.s. value of the first alternation of the short-circuit current to the r.m.s. value of the sinusoidal rated current is roughly seventeen.

The effect of using the flux at rated voltage and the assumed load instead of at rated voltage and no-load is to increase the short-circuit current by a somewhat less percentage than the alternator reactance percentage. This effect in alternators of low reactance relatively is unimportant but assumes increasing importance as the alternator reactance increases.

The characteristic shapes of the time-current decrement curves have been arrived at by analysis of alternator tests including oscillograph studies of short circuits occurring when the alternators were excited to full voltage and were carrying various loads at various power factors.

In the curves for total reactances up to and including 20 per cent, the reactance is assumed to be wholly within the alternator and for higher values of reactance the alternators were taken at 20 per cent and due allowance made by calculation for the effect of the external reactance. In the latter case, if alternators of other reactance had been assumed the results would have been somewhat different but the error is not large enough to be of practical importance.

The final values of the current *i.e.*, the sustained short-circuit current, have been assumed in accordance with experience and tests and are based on the behavior of machines of normal design.

The study of representative oscillograms of short-circuit tests showed that in most cases the direct component disappeared within 0.5 second and that the transient portion of the alternating component disappeared within 3.0 seconds. These time values have therefore been used in constructing the characteristic curves.

Several alternators with the same reactance and synchronous impedance will not necessarily have the same rate of r.m.s. current decay. This has been considered in constructing the characteristic curves and they may safely be taken as representing the greatest r.m.s. currents that will be given by modern alternators of normal design.

These curves are applicable for selecting circuit breakers for systems as follows

1. Single machines without external reactance.
2. Single machines in combination with external reactance.
3. Multiple machines with no external reactance.
4. Multiple machines in combination with external reactance.

## EXAMPLES

In order to illustrate the use of these curves in making oil circuit-breaker selections, the following examples are given:

*Example* 1. (Arrangement of apparatus shown in Fig. 4.) Alternator rating 5000 kv-a., 2300 volts three-phase, 1250 amperes. Breaker contacts part in 0.25 second after start of short circuit. From table, under 12 per cent reactance, we find that at 0.25 second the current will be 5.54 times normal, therefore the short-circuit current equals 5.54 × 1250 amperes = 6950 amperes.

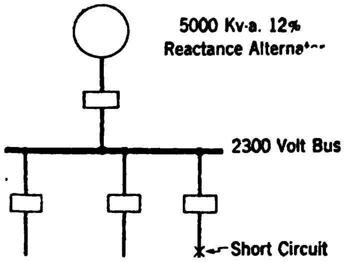

FIG. 4

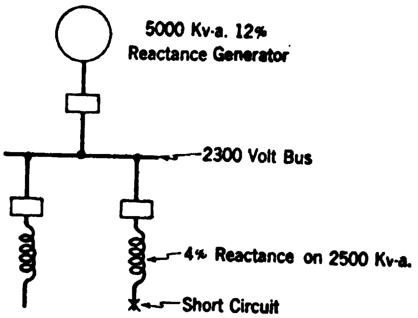

FIG. 5

*Example* 2. (Arrangement of apparatus shown in Fig. 5.) Alternator rating 5000 kv-a., 2300 volts, three-phase, 1250 amperes. Feeder rating 2500 kv-a. Feeder reactance 4 per cent based on 2500 kv-a. Breaker contacts part in 0.25 second after start of short circuit.

Alternator reactance based on 5000 kv-a. = 12 per cent
Feeder " " " " " = 8 per cent
Total " " " " " = 20 per cent

From the table under 20 per cent reactance, we find that at 0.25 second the current will be 3.82 times normal, therefore the short-circuit current equals 3.82 × 1250 amperes = 4780 amperes.

*Example* 3. (Arrangement of apparatus shown in Fig. 6.)

Alternator $A$ rated 2000 kv-a., 2300 volt, three-phase reactance = 8 per cent
Alternator $B$ rated 5000 kv-a., 2300 volt, three-phase reactance = 12 per cent
Alternator $C$ rated 8000 kv-a., 2300 volt, three-phase reactance = 16 per cent

Total alternator kv-a., 15,000
Normal current based on 15,000 kv-a., 2300 volts = 3760 amperes.
Breaker contacts part in 0.4 second after start of short circuit.

Alternator $A$ reactance based on 15,000 kv-a. = 60 per cent
" $B$ " " " " " = 36 per cent
" $C$ " " " " " = 30 per cent

Total reactance at bus = 1/60 + 1/36 + 1/30 = 12.9 per cent.

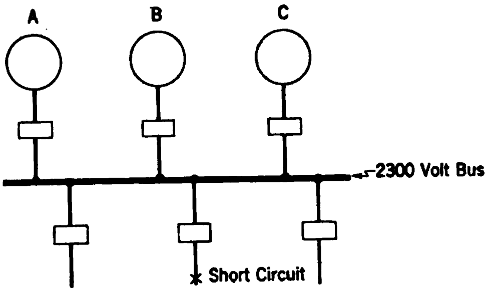

FIG. 6

From the table, interpolating between 12 per cent and 15 per cent reactance, we find that at 0.4 second the current will be 4.53 times normal; therefore, the short-circuit current equals 4.53 × 3760 amperes = 17,030 amperes.

*Example* 4. (Arrangement of apparatus shown in Fig. 7.) Same as example 3, excepting that power is distributed over 2500 kv-a. feeders in which are installed current limiting reactors having a reactance of 3 per cent based on 2500 kv-a.

Breaker contacts part in 0.4 second after start of short circuit.

Total alternator reactance based on 15,000 kv-a. = 12.9 per cent
Feeder " " " " " = 18 per cent

Total " " " " " = 30.9 per cent

From the table using 30 per cent reactance, we find that at 0.4 second the current will be 2.57 times normal: therefore, the short-circuit current equals 2.57 × 3760 amperes = 9650 amperes.

*Example* 5. (Arrangement of apparatus shown in Fig. 8.) Breaker contacts part in 0.1 second after start of short circuit.

Alternators same as for example 3.

Transformer banks each 7500 kv-a.; reactance = 6¼ per cent based on 7500 kv-a.

Lines: 20 miles of 1/0 copper; reactance = 5.5 per cent based on 7500 kv-a. at 60 cycles and 44,000 volts with 4-ft. conductor spacing.

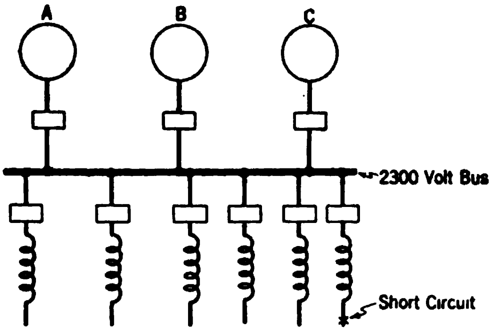

FIG. 7

Total alternator reactance based on 15,000 kv-a. = 12.9 per cent.

| | |
|---|---|
| Parallel reactance of step up transformers based on 15,000 kv-a. | = 6.25 per cent |
| " " " lines based on 15,000 kv-a. | = 5.5 per cent |
| " " " step-down transformers based on 15,000 kv-a. | = 6.25 per cent |
| Total reactance.................... | = 30.9 per cent |

From the table, using 30 per cent reactance, we find that at 0.1 second the current will be 3.41 times normal. The normal current based on 15,000 kv-a., 11,000 volts, three-phase = 788 amperes; therefore, the short-circuit current equals 3.41 × 788 amperes = 2690 amperes.

*Example* 6. (Arrangement of apparatus shown in Fig. 9.) Breaker contacts part in 0.4 second after start of short circuit. Conditions same as for example 5 except that a 475-kv-a., 2300-volt feeder has been added to the low-voltage distribution.

Transformer reactance, 3 per cent based on 475 kv-a.
Total reactance up to 11,000-volt bus
based on 15,000 kv-a. = 30.9 per cent
Feeder transformer reactance
based on 15,000 kv-a. = 94.5 per cent

Total reactance based on 15,000 kv-a. = 125.4 per cent

From the table, using 125 per cent reactance, we find that at 0.4 second the current will be 0.87 times normal. The normal current based on 15,000 kv-a., three-phase 2300 volts = 3760 amperes, therefore the short-circuit current equals 0.87 × 3760 amperes = 3270 amperes.

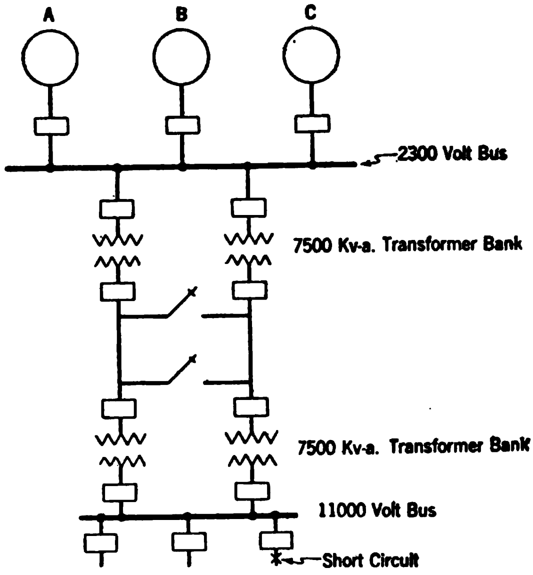

FIG. 8

With reactance of 125 per cent or higher values, the alternator portion of the total reactance becomes of small importance. In example 6 for instance, we have a total reactance of 125.4 per cent. The alternators in this example have a reactance of 12.9 per cent. If the reactance of the external circuit only is considered we have a total reactance of 125.4 per cent − 12.9 per cent = 112.5 per cent.

The short-circuit current on this basis would be

$$\frac{100}{112.5} \times \text{normal, or } 0.890 \times 3760 = 3340 \text{ amperes.}$$

Comparing this value with the 3270 amperes obtained by considering the alternator reactance we have an error of approximately 2 per cent.

If the total reactance is greater than about 125 per cent or 150 per cent, the error will be even less than 2 per cent. For values of external reactance in excess of the table values the alternator reactance may be omitted.

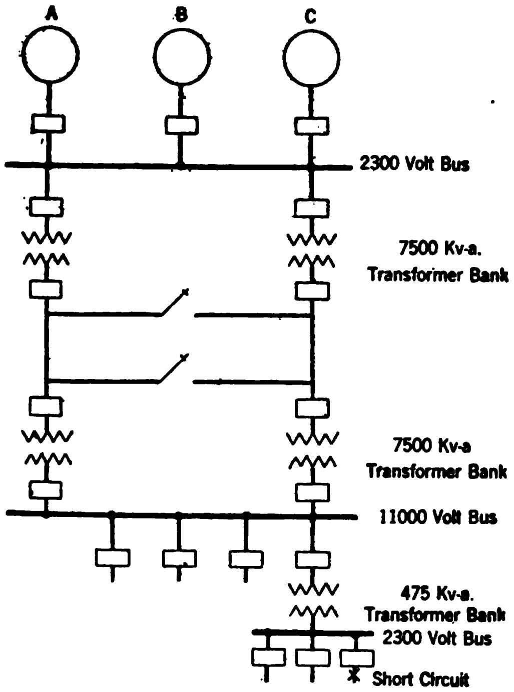

FIG. 9

## PRECAUTIONS

The data given for the selection of oil circuit breakers is applicable only to average systems. Therefore, a short discussion of other factors requiring separate or more detailed attention seems worthy.

Automatic voltage regulators may introduce system transients differing from those which occur on systems not so equipped.

When the alternators are equipped with automatic voltage regulators such regulators will increase the excitation after a short circuit in the endeavor to hold normal voltage on the bus bars. The maximum voltage which can be obtained from the exciters will ordinarily be not more than 50 per cent greater than that required at full load, 80 per cent power factor on the alternators. Under short circuit, the alternator terminal voltage is reduced, hence the resultant flux density in the alternator iron is also reduced. A given increase in excitation, therefore, produces a proportionate increase in current flowing in the short circuit. Hence, as we have assumed the excitation to increase 50 per cent, the sustained short-circuit current will be approximately 50 per cent greater than the sustained current due to full-load 80 per cent power factor excitation.

An appreciable time, however, is required for the excitation to increase to its maximum value. During the first half second the amount of short-circuit current is not affected by the presence of the voltage regulator, but from this time on the current curve is higher, reaching the value at the end of two to three seconds of 50 per cent greater than the current without the regulator.

An exception to the above appears when the external reactance is so high and the short-circuit current so limited that the regulator is able to maintain normal voltage at the generator terminals. In such cases the sustained current may not be increased as much as 50 per cent, but will be limited to the current which will pass through the external reactance with normal voltage impressed upon it.

The interval between the occurrence of the short circuit and the parting of the circuit breaker contacts has, as study of the selection curves will show, an appreciable bearing on the oil circuit breaker that will be selected. The time values usually given for the operation of breakers assume that the breakers have been properly maintained and that their operation will not be impaired by factors resulting from neglect of maintenance or adjustment.

It is believed that the manufacturers of oil circuit breakers should publish information similar to that outlined in this paper to serve not only as a guide in the application of electrical protective devices but also to assist in bringing about a more uniform selection and a better understanding of the various expressions and methods used in connection with the rating of this class of equipment.

## DISCUSSION ON "RATING AND SELECTION OF OIL CIRCUIT BREAKERS (HEWLETT, MAHONEY AND BURNHAM), NEW YORK, FEBRUARY 15, 1918.

**E. M. Hewlett:** What we have been struggling with is to make a "Bench Mark" from which we can all start and work out a common language. In the past the folks who were working on generators, on reactances, on transformers and on the lines outside, had each their own problems and their own equations, which caused much confusion, and this confusion was further added to because they did not all use words in the same way.

Then, as to the quantity and character of amperes used—when we were called upon to interrupt larger capacities as the generator grew larger, we asked the question, what is the current we are to interrupt? We were told by the calculators that it was about three times full-load current,— but we knew we were experiencing currents much larger, that is to say, we felt this even before we had the oscillograph and before we could tell anything from the swing of the ammeter, etc. Finally we obtained an oscillograph and discovered that we had swings that ran up many times,—ten times or so. But then in figuring these amperes, sometimes figured from the zero line, sometimes symmetrically and sometimes unsymmetrically, we found it very difficult to get an agreement, thus the papers presented in Boston a year or so ago started the work of bringing the terms together.

In our work on this paper with the different companies, we have looked over the various generator curves and determined their characteristics, how they average, etc., and have found that it was possible to do a thing that we did not think would be possible, that is, to make a general characteristic curve that was close enough to be within a negligible error. We realized that there may be machines above and below that curve, but consider those as special conditions, and they should be recognized and considered as such.

Now, as to the testing equipments, and the different tests to determine the rupturing capacity, etc., we have been getting larger tests and tests on different plants.

Of course, it is very difficult to get the use of the line and the plant to test 100,000 kw., etc., so that we have only been able to establish a series of tests on smaller capacities, and have had to depend on the tests in operation, incidental short circuits, etc., and the readings taken at that time and observations made that would indicate the limits of switches, so as to give a rating.

We hope later to be able to make tests and to check up and settle definitely all of these different ratings. The work, however, will be carried on as rapidly as possible, and the line extended as capacities are available and as different tests are made available.

The information that we get in reference to the effectiveness of the circuit breakers, etc., is very conflicting. We find that one man will say, "Why, yes, it works fine. It opened the circuit." Then you ask him, "Well, what happened to the circuit-breaker." He will reply, "well, we got the circuit breaker fixed up before we could get the trouble fixed up, but that was all right." Another man will say, "We had a great deal of trouble." Probably he had blown out a half pint of oil and that had bothered him, so that the different viewpoints of the user are not standardized, either.

The kilowatt rating that we formerly used was a great convenience up to a certain point, and we will have now a kilowatt rating to be used on transformers; that is, a transformer limit, a rating that will show the size of transformers that can be used with a given breaker, to limit the capacity and make the breaker of easier application.

If we can get any constructive criticism and help as to what can be done in the way of getting tests, and in standardizing so that we can use the information we have, that is, the fundamental characteristics and r. m. s. current values, we will feel as though quite a step will have been made, and other papers will be brought out from time to time that will finally put the industry in this respect more in the shape of a science than of an art.

**J. N. Mahoney:** In reference to the data suggested, in this paper, as the basis of comparison of "Interrupting capacity" rating between the different manufacturers. Some may question the data, as not being derived from service practise. We state the device obeys Institute Rules regarding "duty" rating by interrupting specified power two times at a two-minute interval. In practise there are all degrees of requirement, ten operations some want, and others are satisfied with a single successful interruption. The natural trend of practise is that requirements are not constant—they change from operation to operation, whereas the condition specified by the Institute rule is necessarily constant, such as a given maximum of amperes. We assume that the usual short-circuit current may be only a fraction of the current interrupting guarantee specified which contemplated maximum conditions. In practise we have found that interrupting the actual maximum two times at a two-minute interval permits of more operations under average conditions. The current is less, maybe one-half, and you get approximately ten operations with little depreciation of the breaker details, as the "short" is out on the line and not at the breaker. I mention this so that it will not be inferred that two operations are the only duties the breaker is good for. It is good for all the operations you want as long as the oil and contacts are in reasonable condition.

**Henry R. Summerhayes:** In the paper there is mentioned the present interrupting capacity rating, and in connection with that rating the words "prescribed conditions" are used. I want to make some comments on "prescribed conditions." They were presumably put in the Institute Rules as to interrupting capacity in order to allow conditions to be described, when we knew enough about the theory of the circuit breaker operations to describe conditions under which tests could be made. The theory is not in a very satisfactory state. Of course, it is generally recognized that the work imposed on the circuit breaker is the kv-a. used up or dissipated in the circuit breaker during the time it is opening, and another element that may appear to have a bearing on that and at first sight a very simple relation is the power factor of the current flowing in the short circuit.

If the current wave and the voltage wave are in phase, 100 per cent power factor, and if according to observations, as circuit breakers ordinarily do, the circuit is opened at the zero of the current wave, the voltage wave is at that time zero, and there is little tendency to establish the arc. If, on the other hand, you have 90 deg. lag., so that the current ceases to flow at the moment when the voltage is at a maximum you apparently have the worst condition, the tendency to re-establish the arc, due to voltage being high.

In the case of most of the short circuits the reactance in the circuit predominates, and the power factor is low, ordinarily varying from zero, or nearly zero to about 20 per cent. With that variation perhaps the voltage wave is fairly flat, and at nearly the maximum voltage, so that ordinary variations of the power factor, theoretically, should not make much difference in the working of the circuit breaker, but if the power factor is very high, then the work is apparently less. That is based on the assumption that the current and voltage during the period of circuit breaker opening follow a sine wave. From this point of view it would appear that the work imposed on a circuit breaker would not vary much for power factors from zero to about 20 per cent, because if the current and voltage during the period of circuit breaker opening follow sine waves, and the current ceases to flow at zero of the current wave, the voltage tending to re-establish the arc will be nearly at the maximum point of its wave, for a variation of power factor from zero to about 20 per cent.

However, the arc in the breaker may set up oscillations in the system superimposing high frequencies on the simple sine waves with reestablished voltages in the circuit breaker higher than the circuit voltage, and with varying phase relation between the voltage and current, so the simple relations no longer exist, and it is difficult to determine by test or in any other way the actual power factor existing at that time.

It is generally assumed that the presence of high reactance and low power factor in the circuit increases the electromagnetic storage of energy which may have to be dissipated in the circuit breaker by these oscillations, and therefore may present a severer condition for the breaker to handle than when the current in the circuit is limited chiefly by the resistance which absorbs the energy of oscillation. According to that view, the observed fact that the circuit breaker has an easier time when the power factor is high may depend on the absorption of the energy oscillations by resistance; that is, the resistance in the circuit predominates rather than being dependent directly on the power factor. The power factor during short circuit may also be affected by the resistance in the arc.

Due to these considerations it would appear, as stated in the paper, that the data available are not sufficient to state how much the interrupting capacity of the breaker is affected by the relative resistance, reactance and capacitance of the circuit. Apparently we have not enough data to prescribe at what power factor the tests should be made.

As the authors point out, the average circuit has parallel feeders and the breaker will seldom be required to dissipate all of the stored energy of the system; in other words, when you have parallel feeders with more or less non-inductive load, it is thought that these act as a resistance, and take up part of the energy. If you set up test conditions, with all of the generators running, and a single feeder, and have the circuit breaker on that feeder, it is a more severe condition than if the breaker opens the same amperes, but with three feeders running in multiple.

It is believed that average systems are generally similar as to the conditions of stored energy, electrostatic and electromagnetic, so that the test conditions are generally more severe than the service conditions, and the curves given in the paper will probably be safe for nearly all conditions.

**W. W. Willard:** The statement is made that the accurate calculation of the currents that will result from short circuit is usually not practicable, because of the dearth of data for the purpose, and also because of the alternator power; and certain suggested diagrammatic curves, shown in Figs. 2 and 3, are offered as a means of determining the excess currents that will result in varying cases from short circuits.

It is worth while to note, however, that under "Precautions," the data given for the selection of oil circuit breakers are applicable only to average systems. We are also reminded that the selection of circuit breakers for unusual conditions, large systems, etc., should be checked with the manufacturers. One of the sets of conditions under which this would seem to be desirable is suggested. This suggestion reminds us that the curve indicated for this purpose should take into account the nature of the short circuit, that is, whether it is single-phase or multiphase. There are several ways in which a short circuit can occur on a system, especially on a polyphase system, and one of these is a case in which all of the legs of the circuit are involved.

One of them is a common case, in which only two legs are involved, and another is the case where a single leg of a circuit is involved in a ground on a system which is Y or star-connected, with ground neutral; or in the case of 3-phase, 4-wire circuits, in which there is a short circuit from one of the phases to the neutral.

The curves proposed in this paper have been drawn in such a way as to include a factor of safety sufficient to provide for the case of a short circuit in which all of the legs of the circuit are involved, and also for the majority of cases where single-phase short circuits occur on polyphase systems, notwithstanding the fact that a single-phase short circuit occurring at the terminals of a polyphase generator may and usually will result in higher values for the sustaining current, that is the current which will exist for a lapse of say, two or three seconds, than would be the case if all the legs of the circuit had been involved in the short.

It is believed also that the curves are drawn high enough to provide for the common case of a ground on one leg of a Y-connected system, with grounded neutral, or of a short circuit from one leg to neutral, unless the neutral is brought out from all of the generators, or all of the generators grounded with current limiting resistance or reactance. If only one of the generators is grounded, and there are several generators, so that the reactance of the grounded generator, figured on the accurate capacity of the generators, is within the limits of the table, such a generator might operate as a ground to a limited reactance. If, however, no such condition exists, and it is desirable to apply a circuit breaker between a generator having a grounded neutral and a generator bus, that is to say, on the generator side of the first transformer bank, the current resulting from a short circuit of that kind will probably be high enough to make it necessary to exercise extra precaution, and to consider that one of the exceptional cases, that will require us to learn the magnitude of such currents from other sources than the curves given in this paper, as the curves given are hardly suitable for that purpose.

**Bassett Jones:** I speak from the point of view of the man who has to advise another man how to invest his money in circuit breakers. The paper presents a very definite, easily understood method of determining the duty that may be imposed on an oil circuit breaker. Except in a very general way, it does not determine a method of selecting any particular variety of breaker. This latter problem has always been shrouded in deep mystery, particularly in view of the fact that so many varieties of breakers have been devised to meet such a multitude of special conditions, that the manufacturers' catalogues and published ratings are not always of use. By this I mean that even when the duty required has been determined by the method outlined in the paper, it does not necessarily follow that the proper or rather the most economical type of breaker can be selected directly from a catalogue or hand book.

A case develops in which with, say, a rated full-load capacity of 12,500 kv-a., 10 per cent reactance in each generator connected to the bus, and with what may seem a reasonable per cent of added reactance in the bus, a short-circuit value of current in a feeder figured in this manner would lead the uninitiated to select a breaker costing, say, $500, when actually a considerably cheaper breaker, with one or another of the multitude of standard reinforcings will meet the conditions. Thus, we find a manufacturer carrying in stock, say, twenty-five different varieties of a single type of circuit breaker, because the consumer has imposed at least this number of operating conditions which the manufacturer must meet if he is to make a sale and keep the business. Each time the manufacturer has sought a new way out of his dilemma by devising a new variety of reinforcing, or a slight change in design which will enable him to get away with the sale. And, withal, he probably takes a chance at that—a chance that will most hurt the consumer if the dice fall the wrong way.

This sort of thing results only in harm to all parties concerned. It works an undoubted hardship on the manufacturer, for in time the cost to him of maintaining so many standards, any one of which may have relatively few sales a year, becomes prohibitive. This means more capital invested, and when we get right down to brass tacks it is the consumer who invests his own capital in the manufacturers' stock room. The term "stock room" here covers a multitude of sins.

In a paper recently presented before the Schenectady Section, I drew attention to the economy both to consumer and manufacturer that could be achieved by "Standardization of Demand" in the special case of the low-tension distributing systems in industrial plants. The same remarks apply in a broader sense to distributing systems of all kinds. The distributing system is to be considered as including everything engaged in carrying energy from the generator to the point of conversion or utilization—switchboards as well as transmission equipment.

The problem of "Standardization of Demand" is obviously up to the consumer and his advisers rather than to the manufacturer.

**Fred C. Hanker:** The rating of circuit breakers has been specified as the highest r. m. s. current that can be interrupted under certain conditions. It is this basis of specifying rupturing capacity which makes it necessary to determine the value of the short-circuit current before the breaker can be properly selected for a specific application.

The authors have given in Figs. 2 and 3 a series of curves for various values of total system reactance. They have, for convenience, tabulated these data.

With this information it will then be possible to properly apply a given breaker, after the method of tripping and the value of feeder reactance in the system is determined.

The elapsed time can be ascertained with accuracy once the method of tripping is adopted and the speed of the breaker is specified. Furthermore, the reactance of the system can be calculated from the characteristics of the apparatus under consideration.

This, then, leaves the short-circuit transients of the system to be determined in one of the ways outlined by the authors, so that the value of the r. m. s. current at the time of opening the breaker can be secured.

Obviously, from the number of factors affecting this characteristic, it is not a simple calculation, and it is always debatable whether all of the variables have been given proper weight or consideration. Even granting the ability to predetermine this characteristic, the best criterion of its accuracy is a check against actual results obtained from oscillograph tests.

The curves as given in Figs. 2 and 3, showing the relative values of r. m. s. current in terms of the total full-load current of machines, and under short-circuit conditions, check, reasonably well, the r. m. s. current values determined from actual oscillograms taken on a-c. generators when short-circuited at full voltage. After the first fraction of a second following short circuit, the actual curves fall slightly below the proposed curves, due to the fact that the oscillograms were taken for short-circuit conditions at no load, and full voltage, whereas the proposed curves are based on full-load conditions, at 80 per cent power factor.

From this check it is the opinion that the curves are satisfactory and represent with safety the short-circuit currents on three-phase short circuits that may be expected from normally designed generators.

The curves that we used in making this check covered a wide variety of apparatus, as they included both high-speed turbo-generators as well as normal speed and low speed waterwheel machines. They do not take into consideration extreme power factor operation, as this has been specified at 80 per cent in the paper. Obviously, for lower power factor operation, where greater excitation is required, the sustained short-circuit values may be somewhat higher. These cases are rather rare, and would be taken care of without difficulty if they are once specified.

Another point is in the sustained value as given in the curves. They cover a certain range in values as given at three seconds. If the machine alone had been considered it might have been preferable to construct the curves on the basis of a given sustained value, but when you take into consideration the external reactance which occurs in all cases, probably above 15 per cent or 20 per cent, and in a large number of cases above 10 per cent and 12 per cent, that sustained value would be different for different conditions.

**Philip Torchio:** The subject of the selection of circuit breakers of large rating is of relatively great importance in but a few instances where the plants have very large capacity, and, as the authors have specified, on such occasion a special study must be made jointly and in cooperation between the manufacturer and the user.

I shall not refer to that type of circuit breaker, except by making a general statement that as far as I know we have not today available a circuit breaker of sufficient capacity to open the short circuits on our larger systems with a short circuit in proximity to the generating stations. In such instances limiting reactances have to be used to confine the "short" within the rupturing capacity of the circuit breakers available.

In the development of the circuit breaker, we have apparently attempted to follow the example of the man who plays his fiddle by main force. We are forcing our circuit breakers to do all the work, and probably we are expecting too much in that direction; still, it is wise to keep on studying the problem, in the hope that some day we will find a circuit breaker that will be of unlimited rupturing capacity.

When it comes, however, to the general use of circuit-breakers —which represents 95 per cent of the circuit-breaker applications—for those cases the paper before us is applicable, and is a step in advance, and should be followed up by further standardization, so that, as some of the previous speakers have already pointed out, the user could more readily get at the values of the ratings of the circuit breakers offered by different manufacturers.

I also want in this connection to give a note of warning with reference to following out the suggestion in the paper that if we select a certain circuit breaker for certain existing conditions, we must remember that the conditions of the supplying electric systems change very materially with the growth of the system. The man who makes a selection must also have a view into the future, and not be guided only by what the conditions are today, or what he expects next year, but he should possibly foresee what the conditions may be in several years.

In my personal experience, I find that many times we are confronted with a proposition that today could be handled with a certain circuit breaker, which is relatively cheap, say costing $300. If we could foresee the conditions which will probably prevail three or four years from now, we would see that that circuit breaker should either be supplemented by a limiting reactance or a larger circuit breaker used. In a great majority of cases you will find that the larger circuit breaker is more economical than the combination of the small circuit breaker and the limiting reactance.

In customers' installations, it may be, however, impracticable to put in the larger circuit breaker—as, having only one circuit breaker on the premises, it would be preferable to avoid the operation of a complicated mechanism. The mechanism that would be perfectly feasible in the substation of an operating company might not be so in an industrial plant or on customers' premises. In that case the limiting reactance, in combination with the small circuit breaker, in many instances, while it may be more expensive in first cost, I believe will be found to be preferable. If you take such point of view, broadly you will standardize the circuit breaker, and you will not have so many circuit breakers to select from.

Referring specifically to the paper, where it is stated—"Although not stated in rules, we infer that it contemplates a test with the apparatus under dry conditions." I imagine that the outlines mean that it was filled with oil, in accordance with the Standardization Rules of the Institute. The wording is not entirely clear to me as to what the dry condition means.

Referring to paragraph, "The breaker interrupting capacities in r. m. s. amperes published by various manufacturers are based on an assumed duty, *i. e.*, that the breaker will interrupt its rated r. m. s. current two times at a two-minute interval and then be in condition to be closed and carry its rated current until it is practicable to inspect it and make necessary adjustments." I suppose it is assumed that also it is in position to open the circuit a third time. That was our understanding—that is, when closed, it would be still in a position to open.

**J. M. Mahoney:** Open mechanically, but not automatically, under the same conditions.

**Philip Torchio:** Quoting again, "Systems having characteristics such that the reestablished pressure during short circuit will be higher than normal, will require a larger breaker," refers to circuit breakers at the ends of long distance transmission lines.

In reference to the dying away of the transient current in the direct component of the current—at least mention might be made—that in a system where they operate without Tirrill regulators, but with rheostatic control in the field, the rheostat resistance in series with the field would tend to accelerate the dying out, because it absorbs more rapidly the energy that is dissipated while in places where the Tirrill regulator is used, with less resistance in the field, probably there would be slower dying out.

In the section referring to the guarantees and heating in a general way we must depend on the manufacturers to arrange the guarantees, but I think there ought to be more co-operation, to get a little more uniformity, so that the user can have clear before him information for the selection of the circuit breakers needed.

**Paul M. Lincoln:** I want to call attention to one paragraph which is particularly interesting to me, it is that referring to the classification of systems, in which the authors say: "Systems may be classified according to their normal operating pressure, normal current, normal frequency and current transients." It is that last item, the "transients" that is the unknown quantity in this classification. It was absolutely unknown to me at the time I entered the operating business some twenty five years ago with the Niagara Falls Power Company. In perfectly good faith, when we came to figure our short-circuit currents at that time, we figured the currents we were going to get on short circuit were the same we got on the short-circuit test made in the test room, and which amounted to two or two and one-half times the normal full-load current. When we got short circuits in practise we found evidence of the fact that the currents were not limited to two or two and one-half times full-load current but were many times that. Such things as the tearing apart of the windings, the breaking of windings, the throwing of cables around the power station, and similar things, gave evidence that the currents were not limited to the two or two and one-half times full-load current that we expected, and it was then that this question of transients began to have a real meaning for us.

Our knowledge of what transients are has, of course, increased in the time between that experience, back in 1896 and 1897, and the present time, but I think even yet there is much we have to learn about the matter of transients.

The authors speak of the use of reactances, and there is a more or less indefinite term used in connection with the rating of reactance. Reactances are usually rated in percentages of the normal voltage, but it is not often stated how much current these reactances are supposed to carry for that given stated percentage of reactance voltage, and the stating of that current is an absolute essential in the rating of the reactance. The percentage stated in terms of the normal carrying capacity of the reactance, or in the normal carrying capacity of the apparatus back of the reactance or in some definite manner is an absolute essential in arriving at the rating of any reactance.

On this question of the rating of circuit breakers, let me say that the manufacturers are at all times giving to the user of these circuit breakers the best they have. Whether or not the best they have is good enough to take care of the job of the user, there is only one way of determining and that is by trial.

As Mr. Hewlett pointed out, it is impossible for the manufacturer to go to the user of the apparatus, particularly if the user is on the order of 100,000 kw., or more, and ask him for the loan of his plant for the purpose of testing circuit breakers.

That is out of the question, because the user of that apparatus is not justified in taking the risk which would be involved in the testing of the circuit breakers under the actual operating conditions. As a consequence, the manufacturer must get his information for designing circuit breakers by the observation of short-circuit conditions that occur, and he must gather that information as it is presented to him from time to time. Consequently the question of just what breaker is necessary, just how big it should be, how much oil to use, and all those things that are necessary to take care of given conditions, particularly when those conditions approach the limit of existing plants, must necessarily depend upon the closest possible cooperation between the manufacturer of the apparatus on the one hand and the user of the apparatus on the other. It is only by that kind of close cooperation between the two that the question can be ultimately worked out.

**R. E. Doherty:** I should like to make some comments that may be useful in applying the curves shown in Figs. 2 and 3. There is a large difference between the sustained short-circuit current of generators which were built 10 or 15 years ago, and those which have been built in more recent years. These curves show sustained values which correspond approximately to the maximum that would be found in machines which have been built during recent years by one of the manufacturing companies, whereas for older machines the curves show values which are more nearly representative of the average. That is, the older machines may have higher sustained short-circuit currents than indicated by the curves, and should therefore be treated as special cases.

It may be of interest that the difference between the old and the more recent generators is largely the result of the introduction of the automatic voltage regulator. Before the advent of the regulator, generators were required to have good inherent regulation, that is, low armature strength, relative to the field strength. That means high sustained current. With regulators, this restriction is removed, and for reasons of economy the armature is made stronger, which means lower sustained current.

**Fred L. Hunt:** It is not understood, that the authors have suggested any more definite means of rating circuit breakers than has already been done by the rules and practises to which they refer. Operating engineers will find all the information and data given in this paper to be most valuable and highly practicable, but it is believed that these engineers desire very much to have a more definite method of rating circuit breakers, presented. Nothing, so far as is known, which is now published by the manufacturers would enable the operating engineer to choose from the published data a circuit breaker suitable for one of the definite conditions outlined in the examples in this paper, assuming that he had the information regarding his own system shown in the examples.

Under present conditions the operating engineer must leave entirely to the manufacturer the question of deciding what circuit breaker is suitable for his particular conditions, without having any means of determining how well he has met the requirements specified, or of comparing the factors of safety which may be allowed by different manufacturers in their recommendations for the same switching requirements.

All will probably agree that the manufacturers are well qualified to recommend the circuit breakers that should be used for any given set of conditions, but if circuit breakers are to be put on anything like the same basis, so far as comparative ratings are concerned, as is now the practise with other electrical apparatus, in order that operating companies may have an independent means of comparing the recommendations made by various manufacturers, and of testing under a fixed set of operating conditions the apparatus produced by various manufacturers, a system of ratings should be established which would be definite with regard to all the conditions which are pointed out in this paper as affecting the rupturing capacity of a switch.

Eight items are mentioned, all of which affect the current flowing to a short circuit at any point in a system. This does not mean, however, that there are necessarily that many different interpretations of a given rating for the rupturing capacity of a circuit breaker.

It is suggested that the manufacturers set down a definite rupturing capacity for a given switch, describing in as much detail as they choose the exact conditions upon which the rating is based, and that they then furnish curves or other data which will show the variation of this rupturing capacity due to—

First: Variation of the re-established bus voltage.

Second: Variation of power factor of the current flowing into the short circuit.

Third: Variation of the ratio between generator reactance and line or system reactance.

Fourth: Variation in the total capacity of synchronous equipment connected to the system.

Fifth: Variation due to nature of the short circuit, whether single-phase, three-phase, or between one phase and ground.

This will then leave to the engineer who applies the circuit breaker the duty of determining the transient characteristics of his synchronous machines, the time setting of his relays, the total impedance of his circuits leading to the point of short circuit, the resistance of the short circuit, the power factor of the load being carried at the time of the short circuit, and the bus voltage to be re-established after the opening of the circuit.

The data suggested above to be supplied by the operating engineer could all be obtained from other sources than from the oil switch designer.

Some such system of rating as that suggested above is required in order that operating engineers may be able to select switches for their circuits in the cases at least of all systems which do not come under the heading of "average systems", and it seems reasonable to ask the manufacturers to supply such information. It also seems desirable that the authors of this paper should define what is meant by an "average system".

The writer has recently had occasion to study switching capacities for a system in which a switch in one substation is apparently required to open a short-circuit current of approximately 40,000 kv-a., whereas a switch in another similar substation not far away is required in the case of a short circuit to open 145,000 kv-a., and at another substation short-circuit conditions may require the opening of 270,000 kv-a. It would be interesting to know whether such a system would be considered an "average system".

**Herbert H. Dewey:** It is gratifying to the transmission engineer, to know that we have reached the point where we are talking about the same kind of amperes. If now we accept the fact that the circuit breaker designers have chosen the ampere-rating in rupturing capacity that their switches are good for, it is up to the power and transmission engineer, the man who makes the practical application of these switches, to determine how much current we have to rupture in a given case.

Some points in connection with determining this current have always been somewhat uncertain. We obtain from the manufacturer the reactance of the generator, transformer, etc., usually in per cent, and we know that a ten per cent reactance generator will give ten times normal current on short circuit. It has never been definitely understood by us all however what this ten times normal current is. One man suggests that it is ten times normal symmetrical, and it may be double that. The paper discusses the different values of current obtained on short circuit and defines the current to be used in determining the duty imposed on an oil circuit breaker as the r. m. s. current at the time the circuit breaker opens. The application engineer must determine the r. m. s. current at the point of short circuit by some suitable method.

On simple systems, which the paper purports to cover, that is an easy proposition. It is merely a question of adding up the reactances in series or multiple, as they occur on the system. When we have obtained the final result we have a reactance of, say, twenty per cent on the rating of the total generating capacity, and by referring to our curve we find that we get so many r. m. s. amperes at a given time.

The next point to determine is how soon the circuit breaker is to open. That will be a pretty definite thing because of its location. We use certain relays that must be selective. We use on the generators, as a rule, non-automatic switches. If we have to choose a generator switch, we consider the time it will take for the operator to open the switch, which means constant value of current. If we have a differential relay, which is expected to open the generator circuit in case of breakdown of the generator itself, that means we have a very quick-operating relay, so that the duty on the circuit breaker would be quite different in the two cases.

Mr. Torchio and Mr. Lincoln pointed out that we are going to have very great increases in the capacity of our systems, and it is increasingly important that we have some definite method of determining the amount of short-circuit current that will flow to a fault. Existing stations have grown away beyond their original capacity as planned, and existing systems have tied in with other systems, so that we are getting enormous blocks of power concentrated within short distances.

The increase in the capacity of circuit breakers necessarily has gone up by leaps and bounds, and we hardly know where it is going to stop. As a matter of fact, it would seem as though there were some possibility of reaching a limit, that is not that of the rupturing capacity of the circuit breaker. It is, of course, possible to build circuit breakers of enormous size, and it is conceivable that we could reach practically unlimited capacity in the circuit breaker, one that may be large enough and strong enough for any situation. It is probable there will not be a demand for this, on account of the damage done at the point where short circuits of these enormous capacities are concentrated. We have ruptured, perhaps, 500,000 kv-a. up to the present time. We have found the bus bars pulled from their supports, the cables damaged, pulled off the wall, and great damage done in other ways outside the circuit breaker itself, so it is probable that these features will be limiting ones, rather than the extreme rupturing capacity of the circuit breaker.

There is another point in answer to Mr. Hunt's question, as to what the scope of these curves is, or, in other words, what is an average system. The authors have put down in this paper examples of what they consider average systems, and you will notice they are all confined to one generating station. When we come to deal with a number of generating stations we get into complications in calculating the current involving the use of Kirchoff's Laws which make the problems very difficult. We have in our department in the General Electric Company in Schenectady a calculation device which will take into account the distribution of current between stations and that makes it very simple to solve a problem of this kind. It is practically the only way it can be done, because with several generating stations the apparent reactance of each line is modified by what is coming in from others, and it is difficult to obtain the direction of flow of current in each individual circuit. By the use of apparatus of this kind, the tables and curves given in the paper would become universal, and any one could calculate to a fair degree of accuracy the short-circuit current on practically any complicated network.

**H. D. James:** I wish to call attention to this statement, "These factors are of special importance when a breaker of relatively small interrupting capacity is connected to a large system in such a way that the breaker may afford the only outlet for the stored energy of the system." It is very proper that this discussion should have been confined largely to breakers applied to large systems that had large capacities behind them. However, a very large class of applications are for the protection of electric motors, and come in a class of smaller breakers. Ninety to 95 per cent of such applications are limited by the distributing transformer, so that the power is limited.

There is danger of trouble, however, in power stations, and other large installations, where the power behind the breaker is large. This is particularly true where the overload protection is included as part of the control equipment. One way of taking care of that is to introduce a time element in the overload capacity of the control equipment or breaker protecting it, so that the feeder breaker will take the load. Unfortunately, the feeder breaker also has a time-element introduced in many cases, so that care must be taken to see that the time-element protecting the motor from overload is longer than the time-element of the breaker back of it, if the power is greater than the circuit breaker capacity of the control equipment.

**N. L. Pollard:** The paper presented contributes a great deal toward clarifying the determination of system rating, but does not assist in the selection of proper oil circuit breakers to meet these conditions, even after determining them.

The sections of the Standardization Rules covering the rating of oil circuit breakers are very indefinite, and do not in any way assist in a selection, unless supplemented by further information furnished by the manufacturer, as recommended in the closing paragraph of the paper under discussion. Such information has been entirely omitted from the paper, although it is a well-known fact that extensive tests covering rupturing capacities of various types of breakers have been made.

This subject is of such vital importance to the profession that further investigation seems advisable, and I would therefore suggest that a committee be appointed, composed of operating as well as manufacturing engineers, who should attack the subject from the standpoint of determining the relative merits of the various design factors. These factors should then be given definite values with regard to the rupturing capacity of the breaker and the results tabulated and embodied in the Standardization Rules.

The maximum rupturing capacity of the oil circuit breaker in amperes at rated voltage, should be stamped on the name-plate, in addition to the normal rating.

**E. G. Merrick:** The typical curves given in the paper are based on three-phase short circuits; the results are sufficiently accurate also for single-phase conditions if instantaneous values are considered but cannot be applied in all cases to sustained current calculations.

If a Y-connected, three-phase alternator with perfect "damper" is short-circuited successively across three terminals, two terminals and one terminal to neutral—the excitation remaining constant—the sustained currents will be approximately in the ratio of 100, 175 and 300. These values are modified more or less depending on the extent to which the single-phase armature reaction is annulled by the damping action of the field, so that for a standard salient-pole machine without amortisseur winding we find that the currents are in the ratio of approximately 100, 130 and 200 and for a salient-pole machine with amortisseur winding, or for a turbo-alternator with solid forged rotor, they are in the ratio of 100, 150 and 250. That is, in the former case, the single-phase short-circuit current will be 1.3 to 2.0 times the three-phase value and in the latter case 1.5 to 2.5 times—the excitation remaining constant.

The single-phase values for short circuits between terminal and neutral can in general be disregarded, in view of the fact that where several machines are operating in parallel, it is customary to solidly ground the neutral of one generator only, or if all machines are grounded, to parallel the individual neutrals through grounding resistances.

If the short circuits occur on the line side of a transformer in series with the alternator, the single-phase values of sustained current, with constant generator excitation, are again different from the three-phase values, the magnitude of the difference depending on the ratio of sustained generator reactance to transformer reactance and also whether the transformer is connected delta-delta or delta-star. These values may again be considerably modified if the transformer is feeding a transmission line whose characteristics are such as to give high values of charging current for the single-phase conditions.

Although the single-phase values of current may be, in certain cases, greater than the three-phase values, it does not necessarily mean that the switch duty has been increased. If the single-phase short-circuit occurs between terminals of generator or transformer, the higher value of current is more or less compensated for by the fact that there are now twice as many active switch contacts in series (charging current neglected).

In view of the usual margin of safety in oil circuit breakers and the fact that they are generally set to operate before sustained conditions are reached, their selection can ordinarily be made on the basis of three-phase values of short-circuit current. The sustained condition is important, however, in certain cases and should be made the subject of a future contribution to this subject.

**P. H. Adams:** Does this paper bring us any nearer to a uniform method of rating oil circuit breakers than we were a year ago? The operating engineer can determine his short-circuit current values, either by test or calculation, but on what basis can he select the oil circuit breaker for rupturing this current? There are on the market today, oil circuit breakers with fanciful ratings, having tin cans for oil tanks. How can we discover the truth in regard to our oil circuit breakers when such conditions exist?

There are ten or more well-known factors entering into the design of an oil circuit breaker, which govern its rupturing capacity. Modesty has apparently prevented the authors from touching this side of the question, therefore I agree with Mr. Pollard, that a committee should be appointed to investigate this matter further and if possible, assign definite values to these design factors from which the rupturing capacity may be determined.

The rupturing capacity in amperes at rated voltage should appear on the name-plate of each oil circuit breaker, and the circuit breaker should rupture this current two times at a one minute interval and then be in condition to be closed and carry its rated load until it is practicable to inspect it and make necessary adjustments, without regard to generator or bus capacity.

If this condition cannot be met, then the Committee should divide oil circuit breakers into several classes, based on generator capacity and clearly define the design factors governing each class.

This is the kind of information the operating engineer desires and is what the title of this paper would lead one to expect to find outlined in it. I thoroughly agree with the closing paragraph of the paper that manufacturers should publish more information on the subject but this will be of little value unless the Institute adopts a standard of comparison. It is disappointing to hear Mr. Hewlett say this is the kind of information we cannot get.

**Ira M. Cushing:** The authors start in by speaking of normal pressure, normal frequency, and all conditions are normal until they begin to talk about the duty of the circuit breaker, and then all conditions become extremely abnormal.

It seems to me the circuit breaker has two, if not possibly three duties to perform. First, it is a device for disconnecting a circuit, or possibly disconnecting the apparatus. That is a very important duty, and in connection with that duty is the second one, namely, the ability to carry normal current. Then there is the third duty, that of opening a circuit under abnormal conditions.

I believe that this paper, to an outsider or possible purchaser of oil circuit breakers, is almost an alarmist paper—he would mentally throw up his hands and say—"I don't dare have one on my system." Therefore, I think this word "duty" should possibly be modified, or there should be a paragraph or two added to the paper, describing what the oil switch or oil circuit breaker should do, besides the opening under abnormal conditions.

Possibly the words "prescribed conditions" are not well chosen. We should use the word "abnormal conditions." You speak of prescribed things as something you expect might happen quite frequently, but when you come to speak of abnormal conditions, then you are looking for something that is different, and that is, of course, what is spoken of in the paper.

At the end of the paper the authors practically nullify the entire thought by admitting that everything that has gone before covers systems that do not have voltage regulators. It is my impression that many systems have voltage regulators, and especially systems that have a large lighting load, and for that reason it would appear to a person, not familiar with the apparatus, that the figures given would not apply at all to the average system.

**J. N. Mahoney:** Mr. Torchio mentioned the probable condition of the apparatus after it performed the duty specified. One of the well know disturbances presented by an oil breaker in operating under near maximum conditions, is to throw some oil. It is self-evident that if the amount of oil thrown is great, it limits the ability to take care of the next successive operation of the breaker. As a matter of fact, that is one of the points that is a controlling feature.

The breaker may open twice, undertaking the maximum duty for which it is guaranteed, and the oil level may be so reduced that if operated a third time you could not expect to get the same standard of performance, as in the first instance. You might operate it at a smaller power duty, and so on down with each successive operation, and in this way you could operate it successfully a large number of times; the first time at the maximum. With a sufficiently small power duty, the breaker might be made to operate indefinitely even in its depreciated condition.

The breaker blowing off a tank, breaking a tank supporting frame, or causing an unusual mechanical destruction of some other kind, is not contemplated as being within the accepted practice, or definition of successful "two-time operation."

The second point in the depreciation, you might say, of operation, is the arcing "tips", or the contacts forming the arcing members. While the conducting contacts may not be injured, the arcing contacts may be consumed considerably. Therefore it is undesirable to operate at the maximum duty any more than a given number of times, until the circuit breakers are inspected and repaired. The device may be closed again, and service maintained for an indefinite period, that is, until the normal inspection period. That contemplates, assuming the breaker is an automatic one, that if it is subjected to two successive maximum operations as stated in the guarantee, the breaker would be made non-automatic after the second operation. It will then be just a conductor, or disconnecting switch, and the automatic protection will be afforded by other means in the circuit. Usually, as Mr. Hewlett remarked, it may be that destruction beyond the breaker, caused by the short circuit gives you all necessary time, even after one operation, to readjust the breaker, before it is necessary or desireable to close it.

The question has been raised by several of the speakers, as to what constitutes an average system. It simply means an average radial system, as practically all general service systems are. Such a system does not have its load in one block, with a single controlling breaker. The difference is not because of any condition of power factor in the system, but the fact, in the single breaker system, there is practically but one outlet for the stored energy, that is the breaker itself. I think that will clarify the situation.

**Philip Torchio:** The point I make I think is rather important: After the second closing, if you require the breaker to do some future interupting work,—do not close it, but leave it there for the normal inspection. Its condition is that it is not immediately capable of opening against its rated powers.

**J. N. Mahoney:** That is the understanding.

**Chester Lichtenberg** (communicated after adjournment): The rating and application of oil circuit breakers requires an intimate knowledge of the phenomena taking place not only in the external electrical circuit, but also in the breaker. It is essential to understand not only how a circuit behaves under transient electrical conditions, but also how the oil circuit breaker performs its energy dissipating and circuit interrupting functions under these conditions.

Consider an oil circuit breaker. It consists essentially of a pair of separable contacts immersed in an oil bath. Suppose this simple breaker to be connected in an a-c. circuit carrying a steady current at a steady pressure and steady power factor. Now assume the breaker contacts to part, and to continue in motion with approximately constant velocity until the circuit shall have been interrupted.

Under these conditions it is found that when the breaker contacts part, an arc is formed. This arc gasifies that portion of the oil in the immediate neighborhood of the contacts, and a spherically shaped globule of gas is formed around them. The pressure in the globule depends on the amount of energy which the breaker is called upon to dissipate, and under extreme conditions it may become as high as several thousand kilograms per square millimeter.

One effect of the pressure in the gas globule is to raise the level of the oil in the bath. It is for this reason that all well designed oil circuit breakers have a so-called "air buffer" between the top of the oil and the oil vessel cover or lid.

As the contacts move further apart, the arc lengthens, and due to internal pressure, the gas globule grows larger. The globule retains its spherical shape, though until the contacts move out of it, or it encounters the side walls of the containing vessel.

When the moving contact passes out of the gas globule, the arc follows, and a path is provided for the relief of the pressure stored in the globule. This causes the globule to burst explosively, and by this action the arc is blown out and oil tended to be ejected from the vessel.

In the interruption of most electrical circuits this is the only action that occurs during a successful operation of the breaker. In the interruption of some circuits, however, there is an added step. The bursting of the first globule, initially surrounding the contacts, is followed by an elongation of the arc through fresh layers of oil, and the formation therein of new gas globules having, however, much less internal pressure than the first. These new globules burst in turn without interrupting the circuit, till finally one of them succeeds in blowing out the arc and interrupting the current flow.

The formation of gas globules, their subsequent enlargement, distortion, and bursting are shown in Fig. 1. This photograph was taken with the successive image camera described in a previously contributed discussion. It is typical of hundreds of photographs which have been taken and studied. The arc images represent exposures made at intervals of about 0.005 second. The first image is at the right end of the lower row. The second, third, fourth, and fifth are adjacent to it on the left in the order named. The sixth is at the right end of the lower but one row. The seventh is at the left of it. The twelfth is at the right end of the third row, and so on. They show quite clearly the different stages in the interruption of a current carrying electrical circuit by an oil circuit breaker.

The bursting of the gas globules when an oil circuit breaker is clearing a circuit, is followed by external evidences of distress. Some of these are as follows: (1) Dull thud; (2) Ejection of oil; (3) Ejection of flame; (4) Straining or bursting of oil vessel. The circuit interruption is also indicated to have taken place explosively by oscillograms contributed in previous discussions. These show that the current is diminished with increased rapidity as the interruption progresses successfully, until the end is approached with great suddenness or abruptness.

This explosive interruption of the circuit has a very important bearing on the application of oil circuit breakers. As I pointed out during a previous discussion, and as is stated in the paper, not only must the characteristics of the circuit to which a breaker is to be connected be known, but also the characteristics of the breaker itself must be known. The explosive action of the breaker introduces an element of wide variablity in the operation of the breaker, and makes it difficult to predetermine its performance with great accuracy.

It is a well known fact that the characteristics of generators, motors, transformers, transmission lines, and other parts of electrical circuits may be determined by test and calculation quite accurately, *i. e.* within 2 or 3 per cent. It is also a fact, though not nearly so well known or recognized, that the performance of an oil circuit breaker cannot be predetermined with an accuracy closer than 15 per cent. This fact requires especial emphasis at this time since many incorrect breaker applications are made because a knowledge of the limits of accuracy of the calculation of systems and breakers is not general.

The limit of accuracy of the predetermination of breaker ratings is an important item at this time also because the paper

FIG. 1 [LICHTENBERG]

under discussion proposes rules for the rating and application of these devices. These rules have been derived from an analysis of a large amount of data accumulated during over twenty years of experience. In this, the proposed circuit breaker application rules follow the precedent established in the development of empirical rules for the calculation of other electrical machines. The great difference is, though that whereas most other electrical machines and devices are calculated for ordinary or steady circuit conditions, the oil circuit breaker interrupting capacity must be computed for extraordinary or transient circuit conditions. Therefore, while apparatus for steady conditions can be built to within 2 cr 3 per cent of the design calculations, oil circuit breakers frequently have interrupting capacities more than 50 per cent outside the design calculations.

One other factor requiring attention is the application of breakers to growing systems. A breaker has a finite capacity, while compared thereto the ultimate capacity of the system may be infinite. The limitation of the breaker, while known, is not always taken into account as the system grows. Finally a point is reached in the expansion where the breakers are quite inadequate and a failure results. The possible future growth of a system should always be considered at the time the breaker equipment is planned, and while it is not recommended that the initial installation of breakers be suited for the ultimate development, it is suggested that the arrangement of units be such that if the system grows beyond the limits of the circuit interrupting equipment initially installed, this may be moved to other parts and replaced by larger breakers with a minimum of expense.

**A. Collins** (communicated after adjournment): It would be of interest to know whether the authors are of the opinion that the factors suggested should be increased if a three-phase generating plant is short-circuited across one pair of phases only, as this may have some bearing on the breaking capacity of switches in which a separate tank is employed for each phase.

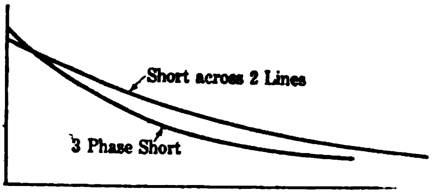

FIG. 2

The curves given are presumably based on three-phase short circuits, and it would be expected that although the short-circuit current across one pair of phases is actually about 13 per cent less than the three-phase short-circuit current, owing to the decreased armature reaction, the resultant field will decay less rapidly, thus producing characteristic curves with a more gradual slope, see Fig. 2.

Under the heading of "Precautions" there are one or two other factors which may tend to increase the stress on the switch which has been selected by the use of the data provided.

It is conceivable that a high resistance fault may be sufficient to operate the protective gear, and energize the trip coil, and that this fault may develop into a dead short circuit at the instant the switch contacts separate. With maximum assymetry, or as the authors describe it, maximum possible direct component in the short-circuit characteristic, the current at the instant of break will be much greater than that given by the figures in Table 1; for example, with 10 per cent reactance the peak value of the short-circuit current would be 20 times the peak value of the full-load current, and the ratio of the r. m. s. values would be approximately of the same order.

In addition there is the doubling effect in transformers and reactance coils, resonance and other phenomena.

It is important that switches, particularly those near to the generating plant should be selected for the most severe conditions that may reasonably be expected, but I should like to hear the authors' opinion as to how far they consider provision should be made, if any, for such contingencies as those just mentioned.

**Charles C. Garrard** (communicated after adjournment): There can be no doubt that, at the present time, the circuit breaker forms a limiting feature in the construction of very large power stations. The application of power limiting reactances has rendered the super-power house possible. Such reactances are, however, costly and, in themselves, undesirable pieces of apparatus. The lower limit of size above which reactances are necessary is fixed to a very great extent by the safe rupturing capacity of the oil circuit breakers; the higher this is, then the greater may be the capacity of the power house without the addition of the reactance coils.

This subject is at the present moment occupying the attention of the standardizing authorities in this country. A conference dealing with it was held recently between Mr. H. M. Hobart representing the American Institute and representatives of the Institution of Electrical Engineers, the British Engineering Standards Committee and the British manufacturers. In addition a committee of the manufacturers themselves has the matter under consideration, and it is hoped that this committee will be able before very long to publish the results of its deliberations. The following remarks, in the meantime, are made in the writers' personal capacity.

In the first place as regards the pressure test I am not quite certain what is meant by "dry conditions;" I presume it is intended that the insulators should be free of moisture but that the oil tanks should be filled to their normal extent with oil. If so I am in agreement, unless of course the circuit breaker is for out-door service in which case the test should be taken under rain-conditions.

I think the limit of temperature rise of contacts in air of 20 deg. cent. (ambient 40 deg. cent.) is too low. For large apparatus especially, it is difficult to work with such a low limit, moreover I can see no reason at all for such a small figure. I think 30 deg. cent. is quite safe. Again the temperature limit of 70 deg. cent. for oil is unduly conservative for large current circuit breakers. This limit might be retained for sizes up to and including 2000 amperes; above this however a temperature rise of the oil of 40 deg. cent. is not unreasonable. The temperature should be measured at the top of the oil.

The most important point of the whole discussion is of course the rupturing capacity. It follows from the authors' arguments that a rating in terms of rupturing current is recommended. In this the authors agree with the German standardizing authorities. The original V. D. E. suggestions for the rating of oil-circuit breakers proposed a kilovolt-ampere rating (see E. T. Z. 16-2-11, 1911, page 171), but this was afterwards changed to an ampere one. There are, however, very considerable advantages in the adoption of a kilovolt-ampere rating. Really, of course, there is no essential difference between this and the authors' system. The ampere rating is simply converted into the kv-a. rating by multiplying by the rated pressure and the factor 1.73 and dividing by 1000. For rapid calculations, however, and for the comparison of circuit breakers on different voltages, the kv-a. rating, in the writer's opinion, is the better one. There can be little doubt that, probably without introducing any error greater than that inherent in the underlying assumptions made by the authors, that the kv-a. rating (rupturing capacity) of a given breaker is independent of the voltage. Thus the same circuit-breaker (if the insulators be suitable) may be used either on a 6000 or a 3000-volt circuit. With the ampere rating it would be necessary to give this one circuit in such circumstances two different ratings for its rupturing capacity which would be likely to cause confusion. Moreover, to the present writer, it seems more logical to rate in terms of kv-a. It is after all the power flowing which is ruptured. It is the absorption of power within the circuit-breaking device over the time taken for the rupture, *i. e.* the total energy absorbed in the oil, which does the damage. The theoretically correct rating would therefore be in kilowatts rupturing capacity; this, however, introduces questions of power factor etc., as to which no exact statement can be made. It seems, therefore, that the kilovolt-ampere rating is the best under the circumstances. Certainly in this country the present tendency is towards this conclusion.

As regards the prescribed duty I am in agreement with the suggestion that the breaker should interrupt its rated rupturing current (or, as I prefer, its rated kv-a.) twice in succession, and then be in condition to be closed and carry its rated current until it is practicable to inspect it and make any necessary adjustments. I do not think however a two-minute interval should be called for between the two operations; it should be allowable that these should follow each other rapidly as this is in accordance with practical operating conditions.

The definition of breaking capacity, therefore, which I would suggest is as follows: "The breaking capacity of an oil circuit breaker should be given in kilovolt amperes based on the rated pressure and the current at the instant of break. The rating of an oil circuit breaker shall be the maximum kilovolt amperes the circuit breaker will break twice in succession, and after this be capable of carrying normal current."

The curves and examples given by the authors as aids in the selection of an oil breaker for any particular duty are most valuable and will be of great service to the user and manufacturer. I am inclined to think however that the method as a whole is too complicated for practical use. It must be remembered that at the present time no generally accepted system whatever is in use. The step from this state of affairs to the very complete statement suggested by the authors is a very large one. A simpler proposal is, I consider, feasible, and while such would constitute a really very great advance over present day practise, it would be more likely to receive general support on account of its greater ease of application.

As an example of what I mean I would point out that the authors' method entails a statement of the time taken by the circuit breaker to open its contacts. This generally under practical conditions, is unknown. Moreover it is a very variable quantity, depending on the electrical and mechanical conditions of the controlling and operating mechanisms, and, in the case of electrically operated breakers, on the voltage of the operating battery. Practical considerations would therefore require an average assumption to be made in nearly all cases.

Seeing therefore that it will nearly always be necessary to make an assumption it seems to me unwise to complicate the issue in an attempt to secure a greater scientific exactitude than the original assumption will allow. I think that the following simple assumptions for example will probably be entirely sufficient.

$$\left.\begin{array}{l}\text{Kilovolt amperes to be broken by}\\ \text{generator circuit breaker}\end{array}\right\} = 0.6\ \frac{(\text{Rated kv-a.} \times 100)}{\text{percentage generator reactance.}}$$

It will be seen that the factor 0.6 has been assumed to allow for the rapid fall of the short-circuit current from its high initial value before the circuit breaker has time to operate. Moreover the doubling effect has been neglected.

For oil circuit breakers at the ends of feeders it will be probably quite sufficient to assume constant busbar voltage in the generating station and calculate the short circuit taking into account the impedance of the feeder. When dealing with feeders I do not think it right to neglect the resistance; this is especially so with underground cables.

I would also like to suggest that the examples given in the paper should be amplified by considering what happens when the oil circuit breaker interrupts a ring main, and also the effect of running synchronous motors.

Applying the simple formula given above to an alternator having 8 per cent total reactance we get kv-a. to be broken = 7.5 times rated kv-a. This value on the authors curves (Fig. 2.) is obtained at 0.25 seconds from the time the short circuit begins. It is obvious that if the time taken by the circuit breaker to operate be shorter than 0.25 seconds, the factor 0.6 is too small. The question really resolves itself to determining what factor represents most closely the general results of experience.

I would like to suggest in conclusion that this simple statement of the usual case should be considered by the American Standardizing Authorities with the object of using it for the usual run of work and reserving the more detailed data of the authors for use on special occasions.

**G. A. Burnham:** Mr. Collins' question as to whether the authors had taken into consideration a single-phase short circuit on three-phase generating plants, was, I believe, fully covered by Mr. Willard's remarks. The authors feel, however, that for average application the curves give a sufficient factor of safety to take care of this condition.

With reference to Mr. Collins's statement wherein a high-resistance fault may be so long developing into a so-called "dead short circuit" that the maximum current might occur at the instant when the switch contacts separated, is one of the factors which must be eliminated in the selection of breakers for average application, or else the breakers would have to be selected on the basis of instantaneous tripping, and this might mean for average conditions, a too conservative application.

Mr. Garrard has contributed an interesting discussion. It is possible that the words "dry conditions" in the absence of any remarks with reference to the dielectric tests for switching equipment for outdoor service might lead one to believe that the dry dielectric test was to be made without oil in the tank. This, however, is not the information which it was desired to convey. It is found necessary to differentiate in the dielectric tests for apparatus used for indoor service and for outdoor service. Apparatus for indoor service is to be tested under dry conditions, that is, the insulators and structure would be dry, but their tanks would be filled with oil to the proper level.

This difficulty will undoubtedly be cleared up in the near future, as there is before the Standards Committee at the present time recommendations for the dilectric test of apparatus to be used indoors and also apparatus for use out of doors.

I do not entirely agree with Mr. Garrard that the kv-a. rating or rupturing capacity of a given breaker is independent of the voltage. There seems to be quite a difference of opinion in America in reference to this point. It is undoubtedly true within reasonable limits, but circuit breakers designed for a given kv-a. interrupting capacity for a given voltage probably would not interrupt the energy in a circuit of equal kv-a. capacity on a very low voltage. It would be necessary, therefore, to give different kv-a. ratings for the same circuit breaker when used on voltages differing widely from the maximum rated voltage.

If all generators had the same reactance and time-current decrement curves, and all transformers had the same reactance, irrespective of their kv-a. capacity, and that it would be assumed that the kv-a. interrupting capacity of a circuit breaker was dependent only on the product of the current and voltage at the instant of the parting of the circuit breaker contacts, the kv-a. basis of interrupting capacity would undoubtedly offer advantages. These assumptions have evidently been the basis of Mr. Garrard's recommendations for determining the kv-a. to be interrupted by generator circuit breakers. It would seem, however, that circuit breakers when called upon to trip within a second to a second and a half after the occurrence of a short circuit would not be applied as conservatively in many cases as would be required, and, on the other hand, in many instances would be very much larger than required.

I fully agree with Mr. Garrard that resistance should not be neglected when dealing with feeders alone; as it has a very modifying effect upon the short-circuit current, particularly as he mentions on underground systems. However, when considering feeder circuits in conjunction with reactors, transformers and the reactance of generators, the effect of resistance as to the value of the short-circuit current is usually relatively small.

Mr. Jones' and Mr. Torchio's remarks concerning standardization are of great interest to the manufacturer. Certainly, if we can arrive at some form of standardization, if only applying to structure, mounting, etc., it will relieve the manufacturers of a great deal of special engineering and hasten the production of standard apparatus.

Standardization is a most important factor. If engineers, in laying out their plans, gave this matter more serious consideration, I can assure you it would be of great economic value, particularly in the moderate sized installation.

In reference to using factors to modify values representing the interrupting capacity of oil circuit breakers where conditions cause widely different re-established voltages, I do not believe it is possible with the data at hand.

I might answer Mr. Hunt and Mr. Pollard on the question of knowing more about the various factors mentioned and the relative effect which they have on the circuit breaker interrupting capacity. At the present time there is no suitable apparatus that would allow one to determine with any degree of accuracy the relative effects of the various factors mentioned. It is a question of a long field experience, or, in other words, careful observation of the particular piece of apparatus under widely varying conditions. The circuit breaker manufacturer would welcome any suggestions that would be of assistance in giving more information pertaining to the action of oil switching equipment. Mr. Pollard and Mr. Hunt also brought up the question that even after the paper had been presented and even after these curves were published by the manufacturers, it would then be practically impossible to make application of them. I think with the general publications as they are today that it is true, to a certain extent, but in the newer publications which all manufacturers are preparing, proper information will be available. There will undoubtedly be listed the minimum tripping time of each type and form of circuit breaker, also the minimum tripping time of various forms of relays that are associated with circuit breakers in order that the total lapse of time may be determined. It will then only be necessary for the customer to determine his relay settings to arrive at the maximum time from the instant of short circuit to the parting of the contacts, and, knowing his reactance, pick out the current value from the transient current curves.

I believe that this will take care of a great deal of the difficulty that we are encountering at present.

Mr. Dewey's and Mr. Willard's remarks cover the question of average systems. It cannot be described in a few words, but I believe these curves, with the exception of the few conditions pointed out, will apply in probably 75 per cent of the applications.

I want to emphasize Mr. Lincoln's statement in reference to cooperation between the user and the manufacturer. The manufacturer tries to get all possible information pertaining to the system to assist the customer in making proper application. A closer cooperation will result in a better understanding of the situation, not only from the standpoint of the customer with reference to the manufacturer, but of the manufacturer in reference to the customer's viewpoint.

The real object of this paper was to state as clearly as we could terms which could be used so that when each of us got through with a problem and assumed the same conditions, we would arrive at the same conclusion. As Mr. Hewlett pointed out, we desire to establish a bench mark to be used by all, and this will allow a more consistent conclusion in the analysis of the various circuit breaker problems.

*Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1918.*

# A NEW STANDARD OF CURRENT AND POTENTIAL

BY CHESTER T. ALLCUTT

ABSTRACT OF PAPER

This paper describes a new secondary standard which is proposed as a substitute for the standard cell in certain classes of d-c. measurements. The device consists of a Wheatstone bridge which will balance for but one value of current.

Various factors affecting the accuracy and permanence of the device are discussed and a number of curves are given showing the characteristics which have been obtained.

THE increasing use of potentiometers in commercial service, especially in connection with the measurements of temperature by means of thermocouples, makes it very desirable to secure a substitute for the standard cell usually required by these instruments. The serious shortage of standard cells caused by the war has emphasized this need. Furthermore, it is being recognized that it is poor economy to use a costly precision standard in a commercial potentiometer which reads to three significant figures at the most.

In order to eliminate the standard cell from the type of thermocouple potentiometer used in measuring the temperature of electrical machines, it has become quite common practise to use a calibrated current-measuring instrument for setting the current. A long-scale galvanometer, in connection with a suitable shunt, is used as a deflection instrument for setting the current in the potentiometer circuit. After setting the current, the same galvanometer is used as a null instrument for balancing the thermocouple e.m.f. against the potentiometer. The chief objection to this practise is the fact that it necessitates the use of a very high-grade galvanometer. A long-scale galvanometer, suitable for use as a deflection instrument, is necessarily much more costly and more delicate than a short-scale instrument of the same sensitivity. For this reason it is desirable to provide a means for setting the current which will not only eliminate the standard cell but will also remove the necessity for a deflection instrument.

## DESCRIPTION OF PROPOSED STANDARD

The new standard which is proposed as a substitute for the standard cell in certain classes of d-c. measurements is essentially a Wheatstone bridge which will balance for but one value of current. Referring to Fig. 1, the device consists of a Wheatstone bridge comprising three constant resistances, *A*, *C* and *D* and a fourth element *B*, whose resistance is a function of the current flowing through it. In practise, the resistance element *B* consists of an evacuated glass bulb which encloses a fine filament of a material having a relatively high temperature coefficient of resistivity. A galvanometer *G* is connected across the bridge and the leads *E* and *F* are connected to an external source of direct current (not shown). For the purpose of illustrating the action of the bridge, let us assume that the resistances *C* and *D* are equal. It is then obvious that the bridge will balance when the resistance of *B* is equal to the resistance of *A*. Fig. 2 shows graphically the relations involved. The bridge will balance

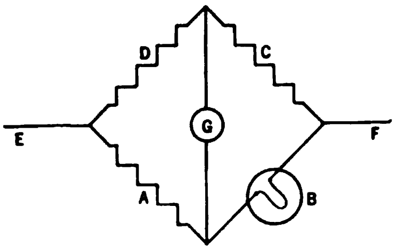

FIG. 1

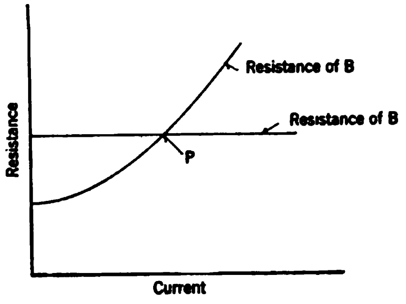

FIG. 2

when the current through the branch *A B* of the bridge is equal to the abscissa of the point *P*. We thus provide a simple null method for setting the current in a circuit at a predetermined value.

## THE VARIABLE RESISTANCE ELEMENT—CONSTRUCTION AND CHARACTERISTICS

The practicability of the device depends on securing a variable resistance having the desired current-resistance characteristics, together with a high degree of permanence. For use with potentiometers, it is necessary to have the bridge balance at a rather low value of current, 20 milliamperes being the usual value for portable potentiometers. The necessary properties of the variable resistance element may be most readily obtained by using the construction already referred to; *i.e.*, a fine filament in a vacuum bulb. It was found possible to obtain resistances in this manner whose value would change very rapidly with small changes in current, even with currents of a few milliamperes. A number of bulbs have been made which would more than double in resistance when heated by the passage of 0.005 ampere. Fig. 3 shows a typical current-resistance curve of such a bulb. It will be seen that for currents of more than two milliamperes the resistance increases very rapidly with the current.

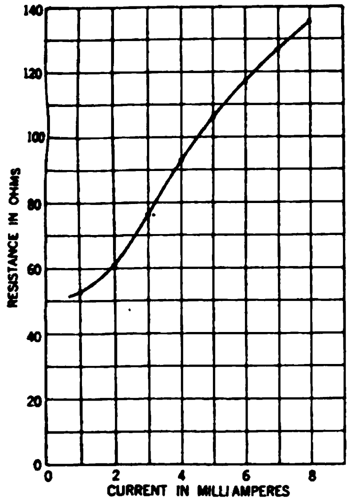

FIG. 3—CURRENT-RESISTANCE CURVE—BULB *G*

Most of the bulbs experimented with by the writer had platinum filaments approximately 0.0005 cm. in diameter and from 1 to 3 cm. in length. Silver coated platinum wire (Wollaston wire) was used, in order to facilitate the handling incident to mounting the filament in place. After mounting the filament, the silver coating was removed by means of nitric acid, leaving the platinum core exposed.

The bulbs were evacuated to from $10^{-4}$ to $10^{-5}$ mm. of mercury. Past experience with high vacua has shown that even with pressures as low as $10^{-5}$ mm., a very high degree of permanence may be obtained. Furthermore, at pressures as low as $10^{-4}$ mm., the change in heat conduction from the filament with changes in pressure is extremely small. In order to secure the permanence of vacuum referred to above, it is necessary to observe the usual precautions, such as heat treatment of the glass, etc., during the process of evacuation.

Another operation necessary for securing the requisite degree of permanence is "seasoning" the filament by passing, for a considerable time, enough current to heat the filament to a bright red heat. Twenty-four hours of seasoning was found to be ample. In every case the filament should be heated to a temperature considerably above the temperature at which it is to operate. Fig. 4 shows the effect of seasoning the filament on the characteristics of a bulb. Curve *A* shows the current-resistance characteristics of a new bulb. Curve *B* is plotted from data taken 24 hours later, while Curve *C* was taken after 24 hours seasoning. Further tests at a later date followed Curve *C* with great exactitude.

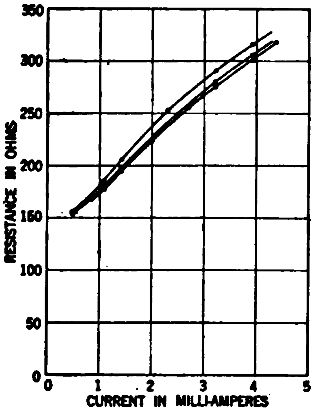

FIG. 4—CURRENT-RESISTANCE CURVES

Curve A—New bulb.
Curve B—24 hours later.
Curve C—After 24 hours' seasoning.

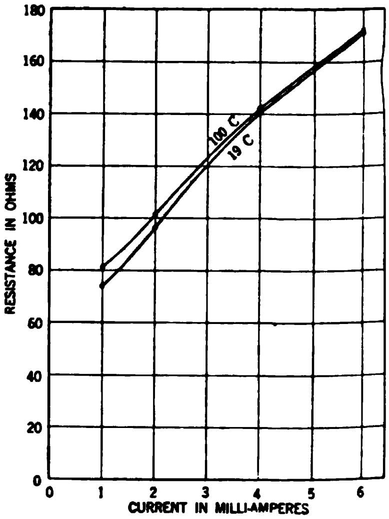

FIG. 5 — CURRENT - RESISTANCE CURVES—BULB *H*—SHOWING EFFECT OF AMBIENT TEMPERATURE

Another factor which might be expected to detract from the permanence of the bulbs is the possibility of a gradual change in the resistance of the filaments due to evaporation. As, in every case, it is proposed to operate the filaments below red heat, no trouble from this source need be feared.

In order to observe the effect of ambient temperature on the current-resistance characteristics of a bulb, the data given in Table I were taken. Fig. 5 presents part of these data graphically. It will be seen that for the larger values of current the effect of ambient temperature is very small. For example, at six milliamperes the temperature coefficient is about 0.007 per cent per deg. cent. change in ambient temperature. For some classes of service this temperature coefficient is negligible. In any case, however, it may be exactly compensated for by giving one of the fixed resistances of the bridge with which the bulb is used a proper temperature coefficient, thus making the current for which the bridge will balance entirely independent of room temperature.

TABLE I—TESTS ON BULB H SHOWING EFFECT OF AMBIENT TEMPERATURES.

| Temp. in oven | Current through bulb | Resistance of bulb |
|---|---|---|
| 19°C. | 0.001 amperes | 74.08 ohms |
| 19 " | 0.002 " | 96.5 " |
| 19 " | 0.004 " | 140.6 " |
| 19 " | 0.006 " | 171.1 " |
| 44°C. | 0.001 amperes | 75.33 ohms |
| 45 " | 0.002 " | 97.65 " |
| 46 " | 0.004 " | 141.05 " |
| 46 " | 0.006 " | 171.3 " |
| 65°C | 0.001 amperes | 77.26 ohms |
| 65 " | 0.002 " | 99.0 " |
| 64 " | 0.004 " | 141.7 " |
| 64 " | 0.006 " | 171.9 " |
| 102°C. | 0.001 amperes | 81.05 ohms |
| 101 " | 0.002 " | 101.5 " |
| 100 " | 0.004 " | 142.0 " |
| 100 " | 0.006 " | 172.1 " |

The degree of permanence possible is shown by the data given in Table II. These data are the results of a series of observations on a bulb, covering a period of several weeks. It will be noted that for the smaller values of current there are differences in the values of the resistance that may be accounted for by differences in room temperature, while for larger values of current the effect of room temperature is apparently negligible.

## CHARACTERISTICS OF NEW STANDARD

Figs. 6 and 7 show the sharpness of balance that can be obtained in a bridge using a bulb of the type described. These curves give the unbalanced voltage of the bridge (*i. e.*, the voltage tending to send current through the galvanometer) as a function of the current flowing through the bridge. The sharpness of balance may be increased by using two bulbs connected in opposite arms of the bridge, but it has been found that a sufficient degree of sensibility may be secured with but one bulb.

TABLE II—RECORD OF TESTS ON BULB G.

| Observation No. | Room temp. | Resistance at different values of current. | | | | |
|---|---|---|---|---|---|---|
| | | 0.001 | 0.002 | 0.004 | 0.006 | 0.008 amperes |
| 1 | 26.5 | 52.6 | 60.7 | 92.6 | 116.9 | 134.9 |
| 2 | 25. | 52.5 | 60.6 | 92.5 | 116.8 | 134.9 |
| 3 | 23.5 | 52.2 | 60.4 | 92.5 | 116.9 | 134.9 |
| 4 | 24. | 52.4 | 60.5 | 92.5 | 116.9 | 134.8 |
| 5 | 25. | 52.5 | 60.7 | 92.7 | 116.9 | 134.9 |
| 6 | 24. | 52.4 | 60.5 | 92.5 | 116.8 | 134.9 |
| 7 | 24.5 | 52.2 | 60.4 | 92.5 | 116.8 | 134.9 |
| 8 | 27. | 52.4 | 60.5 | 92.6 | 116.9 | 134.9 |
| 9 | 27. | 52.5 | 60.6 | 92.6 | 116.9 | 134.9 |

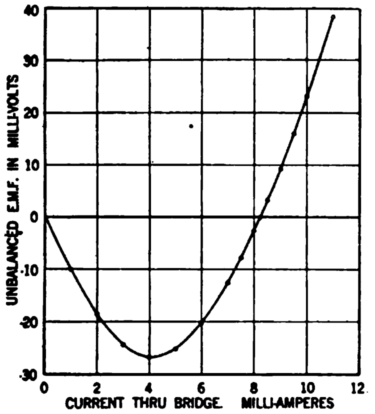

FIG. 6

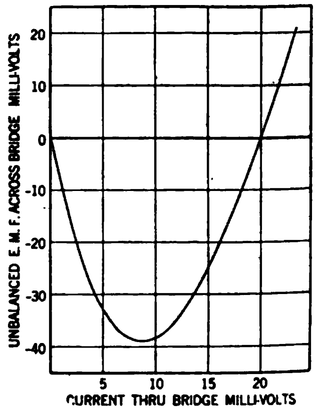

FIG. 7—CHARACTERISTICS OF BRIDGE USING BULB *H*

FIG. 8 [ALLCUTT]

Fig. 8 is from a photograph of the bridge whose characteristics are given in Fig. 7. This illustration also shows a bulb similar to the one used in constructing the bridge. The bridge was designed to balance with a current of 20 milliamperes. In addition, leads were brought out having just one volt potential difference between them when the bridge is balanced. A rheostat is mounted in the same box with the bridge. Fig. 9 is a diagram of connections of the device. In using this standard, a dry cell is connected to the binding posts marked *B* and a galvanometer at *G*. Then the rheostat is adjusted until the galvanometer shows no deflection. When the bridge has been balanced in this manner there is one volt between the terminals marked *E*. The instrument is thus a very simple standard of both current and e.m.f.

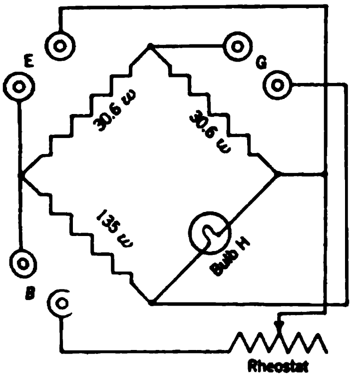

FIG. 9

For use with potentiometers, the current-setting bridge is permanently connected in series with the potentiometer circuit. Means are provided for throwing the galvanometer either across the bridge or in the circuit leading to the unknown e.m.f.

## CONCLUSION

The work done in connection with the development of this new standard has demonstrated that it may be relied upon to maintain an accuracy of ± 0.1 per cent. The sensibility to current changes is ample, as it is very easy to set the current to within 0.1 per cent with the type of galvanometer usually supplied with portable potentiometers. The device may, therefore, be regarded as a suitable substitute for the standard cell for practically all classes of service outside of precision laboratory work

*Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1918.*

# A THERMOELECTRIC STANDARD CELL

BY C. A. HOXIE

## ABSTRACT OF PAPER

This paper considers a means of obtaining a secondary standard of e. m. f. by utilizing the e. m. f. of a thermocouple. The standard thermo cell is fundamentally a standard of current, in that it requires a definite value of current to function properly.

The operation of the cell consists in balancing the potential across a resistance against the thermoelectric e. m. f. of the thermocouple. This requires a definite value of current through a filament which is a source of heat for the thermocouple.

The temperature coefficient has different values, depending upon the temperature of the heating filament. Means are provided for compensating for the temperature coefficient of the cell. The construction of the cell is discussed in detail, particularly the use of gas in the bulb.

A review of the characteristics brings out several advantages of the thermoelectric standard cell. The results of permanency tests on a number of cells are shown. The standard cell has been successfully applied to potentiometers designed for thermocouple work. Further experimental work on this cell is now under way.

MOST engineers are familiar with the uses and limitations of standard cells of the Clark or Weston types. Those who have used either of these cells know that they will not function at freezing or boiling point temperatures, and are easily damaged if an appreciable current is drawn, as by accidental short circuiting.

Though the "Thermal" cell about to be described is not, strictly speaking, a primary standard or source of e. m. f., as is the Clark or Weston cell, it is at least free from the above drawbacks. It may be more properly classed as a secondary standard, its value being determined by comparison with a primary standard. In fact it is just as legitimate to call it a standard of current as of e. m. f., and perhaps a little more so, for the reason that it is simply a combination comprising a resistance, a thermo junction and a heater wire arranged in such a manner that it requires a current of a certain definite value in order for it to function properly. A standard value of e. m. f., however, may be obtained by taking the drop across a suitable resistance placed in the circuit.

To be more explicit, the standard thermal cell in its present form consists of a glass bulb containing a thermo junction *T*, (see Fig. 1) a heating filament *H* which is not in contact with the junction, the balancing resistance *R* and the drop resistance $R_1$. These are mounted in a case with binding post connections on the top. (See Fig. 2.)

### OPERATION

If a battery and galvanometer are connected as shown in Fig. 1, and the current adjusted to a critical value the galvanometer will indicate a balance. The reason for this may be readily understood by reference to Fig. 3. The curve *a* represents the drop of potential across the balancing resistance *R* as a function of the current through the heater *H*. The curve *b* shows variation of the thermal e. m. f. of the junction with the current through the heating filament. These curves indicate that the thermal e. m. f. is approximately proportional to the second power of the current through the heater, while the drop across the resistance *R* varies directly. The point of intersection *C* of these curves is that point at which the thermal e. m. f. equals the drop of potential across the balancing resistance. Therefore by adjusting the current through the heating filament to this critical value, we have a means of obtaining a definite potential drop across the resistance $R_1$. In order to obtain a high value of thermal e. m. f. it is desirable to heat the filament to a high

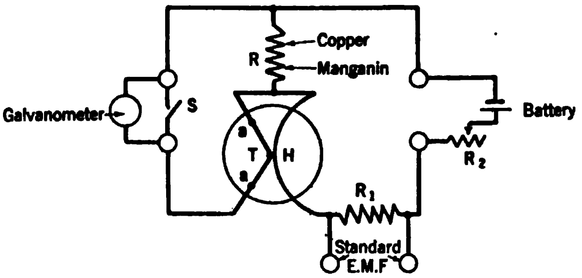

FIG. 1—DIAGRAM OF STANDARD THERMO-CELL CIRCUIT

$T$ = Thermo junction
$H$ = Heating filament
$a$ = Cold end
$R$ = Balancing resistance
$R_1$ = Drop resistance
$R_2$ = Rheostat
$s$ = Galvanometer short-circuiting switch

PLATE III.
A. I. E. E.
VOL. XXXVII, 1918

FIG. 4 [HOXIE]

FIG. 2 [HOXIE]

temperature. The maximum current that can be used without effecting the constancy of the cell, is that which will bring the filament to a dull red heat when viewed in a dark room. Under actual operating conditions the filament current is generally maintained somewhat below this value to prevent damaging the cell due to an accidental increase.

In many of the tests with these cells a lead storage battery having a maximum variation in e. m. f. from 2.4 to 0.8 volts has been employed. By reducing the filament current to about 10 per cent below the reddening point, the cell can be constructed so that 2.4 volts applied to its terminals will not damage the cell. The adjustment to the critical balancing current is then secured by means of a rheostat in the battery circuit.

*Temperature Coefficient.* Since the drop and balancing resistances are constructed of wire which has a negligible temperature coefficient, any change in the potential across the drop resistance must be due to a variation of the thermal e. m. f.

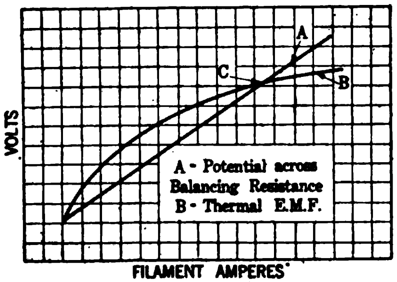

FIG. 3—CURVES SHOWING RELATION BETWEEN POTENTIAL DROP ACROSS BALANCING RESISTANCE AND THE E. M. F. GENERATED AT THE THERMO JUNCTION

It is found that when the cell has been adjusted so that the current necessary for a balance is sufficient to bring the filament to a dull red heat, the cell has no measurable temperature coefficient. If adjusted to any smaller value of current a negative coefficient results. This is due to the fact that up to a certain point, a rise in the surrounding temperature increases the thermal e. m. f. of the couple for a given difference in temperature between the cold and hot ends. Apparently this critical point is reached when the filament attains a dull red heat.

The fundamental cause of the temperature coefficient of the cell is, as previously stated, the positive temperature coefficient of the thermal e. m. f. If a portion of the balancing resistance is constructed with the proper amount of some metal having a positive temperature coefficient, the increase in thermal e. m. f. is balanced by an increased drop across the balancing resistance, hence it becomes unnecessary to vary the filament current. We have therefore compensated for the temperature coefficient of the cell.

## CONSTRUCTION

Considerable experimental work was necessary to determine the proper construction of the various parts of the thermal cell. The results of this work will now be discussed.

*Heating Filament and Couple Wires.* The heating filament and junction wires are kept under tension in order to maintain a constant position with respect to each other. The tension is produced by small spirals located at the base of the bulb. Even with this construction, it is probable that the relation between the heater and the filament will vary, due to contraction and expansion of the metal springs.

At first we would say that any variation in this separation would seriously effect the thermal e. m. f. Experiments show that small differences of separation did not effect the e. m. f. generated at the junction. The explanation of this phenomena is found by considering the transfer of heat from small wires in gases.* It has been found that loss of heat from wires by free convection takes place as if a gaseous film surrounded the wire and the heat was transferred through this film by conduction. It has also been determined that the thickness of the film is independent of the temperature but is dependent upon the diameter of the wire. Evidently if we assume other factors constant, we will obtain a steady transfer of heat from the heating filament to the junction, if we arrange the junction so that it always remains within the gaseous envelope surrounding the heater.

The characteristics of both junction and heater wire must be carefully considered. The heater wire should be of small diameter and of a material having a high resistance and capable of withstanding considerable strain when heated to a cherry red. Tungsten wire 0.07 mm. in diameter was found to meet these requirements and gave the proper temperature with a current of about 40 milliamperes. Experiments indicate that cells constructed with a heater wire of larger diameter do not remain

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*"Convection and Conduction of Heat in Gases" by Irving Langmuir, *Physical Review*, 6-12, vol 34, p. 408.

constant. A heating filament of about 1 cm. in length and having a resistance of approximately 12 ohms has given good results.

The couple wires should have as small diameter as possible in order to decrease the conduction of heat, reduce the lag to a minimum and to permit reaching a maximum temperature. The couple and heater wires are mounted on a single glass stem and after an adjustment of their relative positions has been made, this working unit is placed in the glass bulb.

*Use of Gas in Bulb.* If we produce a high degree of exhaustion in the bulb a low thermal e. m. f. must result. Conversely, if this bulb is filled with gas under pressure, the junction will develop a high thermal e. m. f. the temperature of the filament being the same in both cases. In the first case the transfer of heat by convection is practically eliminated, leaving only the loss of heat by radiation from the small heater wire. This is negligibly small up to a temperature of several hundred degrees. In the second case, the heat is transferred so rapidly by convection that a high current value is required for the heater. Besides affecting the thermal e. m. f., the amount of gas is an important factor in the operation of the cell. Experiments have shown that if the amount of gas exceeds a certain value the cell becomes very sensitive to even slight changes in position. The thermal e. m. f. increases as the junction approaches the point directly above the heating filament.

Nitrogen gas under a pressure of about 2 cm. has given good results. It permits the operation of the cell in any position and a transfer of heat sufficient to produce the required e. m. f. In the construction of the cell, the bulb is first exhausted to about 0.4 microhms. It is then heated to approximately 350 deg. cent. to eliminate moisture, at which time the gas is admitted and the bulb sealed off. A second filament, heated to high temperature for a few minutes is used to eliminate the impurities in the gas.

*Aging of Filament.* After the bulb has been assembled, the current is passed through the filament, raising its temperature to a point considerably beyond the final operating value. This ages the filament and increases the permanency of the cell. (Experiments were made to determine if it were possible to age the heating filament wire before placing in the bulb. This did not seem to produce the desired aging effect, even when heating the filament to a high temperature in gas. Possibly the wire was aged, but the handling necessary to assemble it in the bulb, may have resulted in mechanical strains sufficient to nullify the aging.) Tests are then made to determine the proper values of $R$ and $R_1$, also the amount of positive temperature coefficient metal to be inserted in the balancing resistance.

*Connections Between Bulb and Binding Posts.* Any lag in the temperature of the balancing coil with respect to that of the heater and the couple wires is prevented by spiralled connections, as shown in Fig. 4. By spiralling the connections from the bulb to the binding posts on the top of the cell the heat conducting path is lengthened, so that changes in the surrounding temperature have a uniform effect on all parts of the cell.

## CHARACTERISTICS

The e. m. f. obtainable from this cell is not limited as in other types. The drop resistance may be adjusted so that the potential across it can have any desired value, usually one volt. Any combination of e. m. f. values however, may be obtained by taking taps from the drop resistance.

*Temperature Coefficient.* As previously shown the temperature coefficient may be reduced to zero.

*Effect of Temperature.* The surrounding temperature may vary from values below 0 deg. cent. to over 100 deg. cent. without the slightest injury to the cell. This is due to the fact that no liquid is used in its construction.

*Effect of Short Circuit on Cell.* When these cells are properly constructed they cannot be damaged by short-circuiting any of the external connections.

*Accidental Increase of Filament Current.* An excessive current through the filament may change the standard value of e. m. f. but will not necessarily destroy the usefulness of the cell. This effect will be considered under "Results of Tests."

*Position of Cell.* The position of the cell does not affect the value of the standard e. m. f.

*Strength.* The cells are readily portable and will stand severe shocks without injury. They can be easily shipped if ordinary precautions are taken in packing.

*Accuracy.* Due to the fact that a null method is employed in balancing the cell, the observation error will be a minimum one. The principle sources of error are the temperature coefficient of the cell and thermal e. m. f's. other than the e. m. f. generated at the junction. The error due to thermo e. m. f's. in the galvanometer circuit may be reduced to a minimum by short-circuiting the galvanometer circuit at the standard cell. The galvanometer will then indicate the true zero which must be used in obtaining the balance of the cell. Furthermore, any error in the thermo-junction circuit introduces but half the per cent error in the potential measured across the drop resistance. As previously stated this is due to the fact that the thermal e. m. f. of the junction circuit is proportional to the square of the current through the filament, whereas the potential across the drop resistance is directly proportional to this current. A standard thermo cell that is properly constructed and operated should remain constant to at least 5 parts in 10,000. It is probable that in view of experiments now under way that the constancy of these cells will be greatly improved.

*Permanency Tests.* In Fig. 5 is shown the results of tests on standard thermo cells 1 to 8 inclusive. These tests cover a period of 14 to 23 months and are representative of a considerable number of cells. The following table gives the results of these tests:

| Cell No. | Original e. m. f. | Maximum variation. Parts in 10,000 | Avg. variation from orig. e. m. f. Parts in 10,000 | No. of tests | Period covered by tests. Months |
|---|---|---|---|---|---|
| 1 | 0.9890 | 2.0 | 0.0 | 27 | 23 |
| 2 | 1.0199 | 5.0 | 1.4 | 21 | 14 |
| 3 | 1.1294 | 3.0 | 1.0 | 16 | 15 |
| 4 | 1.0645 | 4.0 | 1.1 | 20 | 16 |
| 5 | 1.2004 | 4.0 | 1.4 | 14 | 14 |
| *6 | 1.0376 | 3.0 | 0.2 | 5 | 4 |
| †6 | 1.0371 | 4.0 | 0.4 | 14 | 11 |
| 7 | 1.1833 | 6.0 | 2.1 | 14 | 12 |
| *8 | 0.9828 | 5.0 | 2.0 | 17 | 15 |
| †8 | 0.9791 | 4.0 | 1.6 | 9 | 7 |

*Before filament temperature was accidently increased to reddening point.
†After filament has been brought up to reddening point.

*Results of Tests.* The curves for cells 6 and 8 indicate the effect of increasing the filament temperature above the reddening point. As previously stated, this produces a permanent increase in the filament resistance. It follows that when the filament resistance is increased, the heater current must be decreased to obtain the new balancing point *C*. See Fig. 3. This decrease in current of course results in a smaller potential across the drop resistance. It is interesting to note, that after the heating filament resistance has changed, the cell continues to function properly except that a lower value of e. m. f. is obtained, assuming of course, that the drop resistance is not changed. In the event of an accident of this nature the standard e. m. f. may be restored to its original value by adjusting the drop resistance.

## CONCLUSION

In conclusion, it is believed that the characteristics of the standard thermo cell justify its use in many measurements in which standard cells are now required. This cell has been successfully used with potentiometers designed for thermocouple work. During the past year several of these potentiometers have been in constant use, and were operated by unskilled labor. During this period no trouble has been experienced that could be charged to the standard cell. Several cells have been burned out through the carelessness of the operator. In every case of a damaged cell it has been found that either the galvanometer or the slide wire or both were destroyed.

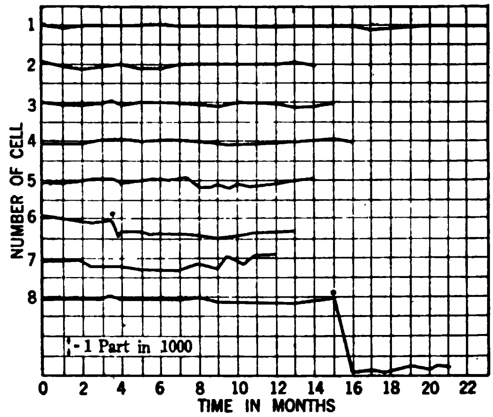

FIG. 5—CURVES SHOWING CONSTANCY OF TYPICAL STANDARD THERMO CELLS

*Filament current accidentally overheated at this point

Very little experimental work has been done during the past year due to conditions imposed by the war. There is still considerable work to be done on the cell some of which is now under way.

## DISCUSSION ON "A NEW STANDARD FOR CURRENT AND POTENTIAL" (ALLCUTT) AND "THE THERMOELECTRIC STANDARD CELL" (HOXIE). NEW YORK, FEBRUARY, 15, 1918.

**Clayton H. Sharp:** These papers indicate the efforts which have been made to satisfy a need in the field of potentiometry. One reason why the need has become acute is indicated by Mr. Allcutt in the beginning of his paper where he says that the standard cells are hard to get, and he also asks—"Why should we use a standard cell which is capable of very high precision in work which does not require such very high precision, and is not capable of such high precision?"

I think one answer to that is that it is not necessary that the cell which is used should be of such high precision in order to have a standard which is suitable for that kind of work.

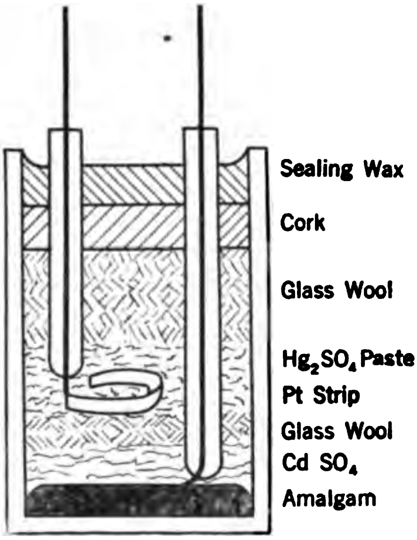

FIG 1.—SIMPLE FORM OF WESTON CELL

Now, to make a Weston standard cell, as a primary standard of electromotive force, requires a very great degree of care and many precautions in the preparation of the chemicals, the assembling of the cells, and in their aging and treatment thereafter. That is for the purpose of making a precision standard. But if you take the materials that go into a Weston standard cell, that is, purified cadmium sulphate, purified mercurous sulphate, and distilled mercury, more or less as they are, and assemble them in a cell, you get something which is not a primary standard, but which gives an electromotive force very close to the electromotive force of the primary standard, which is reasonably constant, which has the low temperature coefficient of the Weston standard cell, and which will serve very well indeed for this grade of potentiometry, and is not subject to the objection of being very expensive or very delicate.

Such cells we have been using for some time in certain classes of our work. They are constructed as shown in the Fig. 1. The amalgam of mercury and cadmium is at the botton of the tube, which may be a vial or something of that kind. There is no glass blowing about it, except to seal the platinum wires in the ends of the glass tubes. On top of the amalgam is a layer of crystals of cadmium sulphate, and on that some glass wool and on top of that, the paste of mercurous sulphate rubbed up with metallic mercury and cadmium sulphate crystals wetted with a saturated solution of cadmium sulphate. The end of the platinum wire which goes through to make the positive terminal is left rather long, and is rolled out or hammered out thin, and then the platinum is amalgamated. That constitutes what is ordinarily the mercury electrode of the cell, and is simply coiled up in the paste. More glass wool is put in and the entire contents are wetted with saturated solution of cadmium sulphate and sealed in with sealing wax on a cork. That cell is not a primary standard, but it is pretty good for low accuracy potentiometer work, and does not cost much. Anybody can make it. That is another way out of this difficulty, not so neat and ingenious as those which have been described, but still worth knowing about.

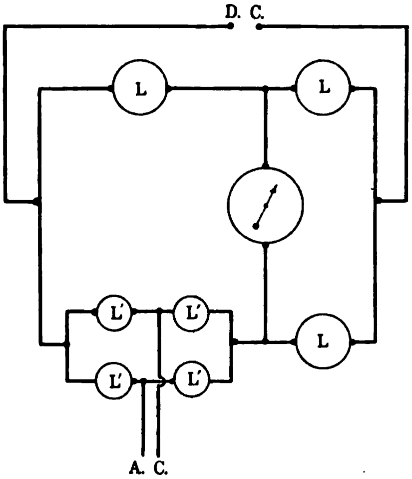

FIG. 2

I was very much interested in Mr. Allcutt's application of what might be called the bolometer principle to produce a standard e.m.f. A good many years ago they used to use a similar arrangement under the name of the Howell bridge, a bridge which consisted of three arms of zero temperature coefficient wire, and the fourth arm made of an untreated carbon filament lamp. The untreated carbon filament having a large negative temperature coefficient, the bridge would be in balance at one value of the current or voltage upon it, and an indicator of that kind was in considerable use in the early days of the industry. More recently, with the advent of tungsten lamps, that idea has been applied in photometry. The current in the photometer lamp is maintained at proper value by taking advantage of the positive coefficient of the tungsten filament, making the lamp one arm of a bridge. The balance can be indicated by a galvanometer or a telephone receiver.

This is something we have tried also—to make an a-c.-d-c. arrangement on this principle. We have made a bridge of seasoned tungsten lamps as shown in Fig. 2. Each of these arms is composed of a single lamp *L*, while the fourth arm is itself an auxiliary Wheatstone bridge made up of the four lamps $L^1$. Direct current passes through the main bridge, while the alternating current is introduced at the two equipotential points on the auxiliary bridge. Variations in the a-c. voltage are reflected with very great sensitiveness in the d-c. measuring instrument. The difficulty with the arrangement is to get a selection of lamps sufficiently constant and so balanced that the bridge is insensitive to changes in room temperature, and insensitive to changes in the direct current which is passing through it. However, it may have its application in certain cases. The sensitiveness of these bridge arrangements is really very remarkable, and I think it is not likely to be appreciated until one does a little experimental work with them.

**F. C. Stockwell:** Mr. Allcutt proposes a substitute for the standard cell, referring presumably to the Weston (cadmium) Secondary Cell. The occasion for using a substitute arises from a serious shortage of standard cells, and their relatively high cost. The device described is an ingenious solution of the problem. It has, however, the inherent characteristic of substitutes in that it is inferior to the original in certain important particulars, viz., simplicity, convenience, accuracy and permanence. "Avoid substitutes" is a good slogan because it is good sense. If standard cells are difficult to obtain from the manufacturer, they are not difficult to set up and they are not expensive.

I had occasion in the latter part of 1914 to set up some cadmium cells at Stevens Institute of Technology for the use of senior students to take the place of the commercial form of secondary standard. Four cells were constructed of the ordinary "H" form. Chemicals of the highest grade (C. P.) were used. The specifications followed were those of the Bureau of Standards, but no attempt was made to carry out the refinements there described for the preparation of the different ingredients. It should be borne in mind that when the greatest care is exercised these cells are reproducible to within 2 or 3 parts in 100,000. As a practical matter chemicals can be used in the form purchased, and with ordinary care the cells are readily set up in a short time. This is not a difficult operation; the simplest apparatus only is required. One of the cells was made from a solution, amalgam, and paste that had not been freshly prepared. Notwithstanding this wholly unscientific procedure, this cell after a few weeks seasoning has been indistinguishable from the others in its characteristics. The cost of the cells for chemicals and glassware was about two dollars each. The materials are readily obtainable now at about 50 per cent increase in price.

These home-made cells have been used for three years for a period of 6 to 8 weeks each year, and they have occasionally been subjected to severe abuse at the hands of students, such as reversal of polarity, and use as a current supply for Wheatstone bridge measurements. The cadmium cell has, however, remarkable recuperative qualities, after a short period recovering its original e.m.f. It may fairly be assumed, I believe, that these cells have operated under conditions which are more severe than would be ordinarily encountered in technical work.

These cells have been calibrated at the beginning of each college year to determine their fitness for further use. The results are shown in the accompanying plot, Fig. 3. It will be noted that their e.m.f's are now about 1/10 per cent lower than when the cells were prepared.

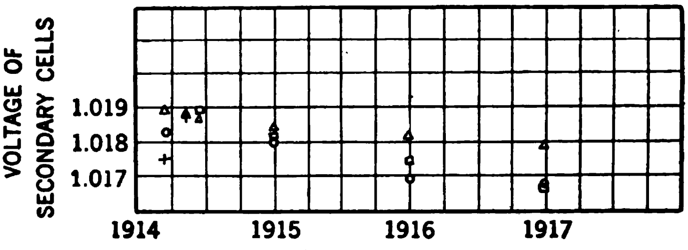

Fig. 3

The experience with the cells which I have described raises the question whether there is a real need for a substitute for the cell as a secondary standard of e.m.f. An authoritative opinion could hardly be based on tests upon so small a number of cells. I believe that it would be very desirable to have recorded the experience of others who have prepared secondary standards in order that this question may be satisfactorily answered.

**W. B. Kouwenhoven:** I think that these two new devices will find a considerable field of usefulness in the university and technical school, and also in certain branches of the profession where it is not desirable to use a small portable standard cell of the Weston type.

I should like to ask Mr. Allcutt what is the effect of overloading his cells, that is, suppose the cell is heated to incandescence by some accidental short circuit? In Mr. Allcutt's aging curves, in connection with Fig. 4, he makes the statement: "Curve *A* shows the current resistance characteristics of a new bulb. Curve *B* is plotted from data taken twenty-four hours later, while Curve *C* was taken after twenty-four hours' seasoning. Further tests at a later date followed Curve *C* with great exactitude." Has he any data showing the effect of overloading on the characteristics of the cells?

**P. G. Agnew:** There is one rather important advantage in both of these types of cells which I think neither author has mentioned, and that is, a very low resistance cell can be readily constructed. In fact, the values given by both authors are lower than that of the unsaturated, and decidedly lower than the saturated Weston cell. That is a point which is of importance in the question of the deflection potentiometer, in which it is desirable to use a pivoted rather than a suspended galvanometer.

**C. T. Allcutt:** Dr. Sharp has brought out the fact, which possibly is not as well known as it should be, that standard cells can be made of a degree of precision suitable to the service in which they are to be used. Mr. Stockwell made a similar remark.

In connection with the question of convenience brought out by Mr. Stockwell, if we consider this current balancing bridge as a unit with the portable potentiometer, we see that the necessary operations in balancing the potentiometer are the same as when balancing against the standard cell, that is, the galvanometer is thrown across the bridge, which is permanently connected in series with the potentiometer bar, and a balance is obtained which sets the current in the potentiometer bar at the desired voltage, and then the galvanometer is thrown off into the circuit in series with the unknown e.m.f., and the measurement is obtained.

In considering the relative cost or difficulty involved in obtaining a bridge of this type and a standard cell, we must consider, in addition to the ordinary type of standard cell, the chemical type; it is necessary in the potentiometer to provide the resistance through which the drop in voltage is obtained, that is compared to the e.m.f. of the standard cell, and that must be charged up to the standard cell in its connection with the portable potentiometer.

For example, if we consider a potentiometer unit, we will find that they comprise a solid wire and a current balancing bridge. That is, the resistance would be manufactured approximately equal. The bulbs can be made with a characteristic within ten per cent. Having given these two resistances, which are comparatively simple, and may be of wire, if necessary, we have then this one resistance which is necessary to adjust in order to fit the whole potentiometer for service. In case a standard cell is used, we have a potentiometer wire and another resistance, and provision for connecting the standard cell across that resistance. The current is varied until the drop through this resistance is equal to the e.m.f of the standard cell.

In the manufacture of the potentiometer, this resistance must be carefully adjusted to give the proper current to balance against the standard cell. So we consider, then, the standard cell with this resistance in comparison with the bridge in the unit.

That cuts down somewhat the relative difficulties involved in the two, that is, the bulb itself becomes the equivalent of the bridge in this case.

Mr. Kouwenhoven has asked two questions; first, concerning the effect of overload. That is, perhaps, best answered with reference to Table II which is a record of a test on one of the bulbs. It is carried up, as you will notice, to currents of 8 milliamperes, at which current the filament was bright red hot, almost as bright as the ordinary carbon filament lamp, and the reproduction of these curves, through a considerable period of time, showed that for short intervals, at least, during the time taken in running the test, the filament could be heated to a bright red heat, without causing any change in its characteristics.

The record of the test given in Table II is not of the same bulb of which I showed the seasoning curves Mr. Kouwenhoven refers to in Fig. 4. I show some further tests in Fig. 6, with great exactitude. I have not the data of this particular bulb. After these other curves were taken it was burned out shortly afterward, so that the test did not cover the full period of time covered with certain of the other bulbs, but if you read these data given in Table II, they are to the same effect, showing the degree of precision with which the bulb will follow the curve after seasoning.

**C. A. Hoxie:** Mr. Kouwenhoven asked about the cold end. Both the cold ends are within the bulb. The copper wire is very short. It extends from the center of the bulb, and about one-quarter inch away— a half millimeter or a little over in each side is the cold end—and it is all contained within the bulb, so it is pretty definite at all times.

**A. C. Campbell:** (communicated after adjournment): Some years ago I described (Proc. Inst. Elec'l Engineers p. p. 10–11, Vol. 30, 1901) an arrangement for measuring current which is identical in principle with the main part of Mr. Hoxie's device. Instead, however, of obtaining a balance for only one definite current, in my apparatus a potential contact connected to the galvanometer could slide along the resistance *R* and thus a considerable range of current could be measured. The practical results which Mr. Hoxie has obtained with his cell are most interesting.

Mr. Allcutt does not seem to know that thermal standards such as he has been working with have been known for many years past. I described the construction and use of such standards in a paper read before the Institution of Electrical Engineers (London) (Proc. I. E. E. p. 12, Vol. 30, 1901), and in the discussion Professor Callendar mentioned that some years earlier he had used similar standards and found that a good degree of accuracy was obtainable. It is interesting to know that the thermal methods are coming to the front once more.

*Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February* 15, 1918.

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# THE CHARACTER OF THE THERMAL STORAGE DEMAND METER

BY P. M. LINCOLN

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ABSTRACT OF PAPER

Following a detailed description of the principle and construction of the thermal storage demand meter the author shows wherein it always indicates what may be called "logarithmic average" rather than "arithmetic average" of power consumption, heretofore indicated by practically all demand meters. The inherent faults of the "arithmetic average" or "block interval" meter are described and examples given demonstrating that the thermal storage meter alone recognizes the true heating effect that fixes size of equipment and therefore cost that should be assessed against the customer.

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THE ADVENT of the thermal storage wattmeter naturally raises a question concerning the character of the quantity that is measured by that device. The object of the following pages is to discuss this question and particularly to analyze the "logarithmic average"—the quantity measured by any thermal storage meter—and compare this quantity with the arithmetical average which is the quantity measured by practically all previously existing types of demand wattmeters.

Let us first consider the fundamental reasons for measuring maximum demand. Briefly stated, the incorporation of maximum demand in a rate for electric service is an attempt to assess upon the user of that service his proper share of the annual cost of the equipment necessary for giving the service. Let us assume a concrete case as an example. Assume, for instance, that the consumption of a given customer is 1000 kw-hr. per year. If this load is taken at a perfectly steady rate throughout the entire year it means a steady consumption of 114 watts continuously. The amount of equipment to supply the load as thus taken is fixed by this continuous load of 114 watts. But now let us assume that our customer insists on taking his entire year's supply in a single day. Instead of equipment to supply 114 watts, we must now provide equipment to supply 41.7

kilowatts; that is, the equipment must be 365 times as large as before. Let us go further and assume that our customer insists on having his entire year's supply in thirty minutes; this would mean an equipment able to deliver 2000 kw. for one-half hour. Obviously, the cost of the equipment for this condition would be enormously greater than that for the supply of 114 watts continuously, and it is only just that the customer that takes his entire year's supply in a day or an hour should pay more for his service than the one who distributes his demands more evenly. Our illustration is, of course, exaggerated, but the exaggeration is one of degree and not one of kind. It is the object of the demand rate to recognize automatically this variation in the equipment necessary with variation in load factor and to assess this equipment cost against the customer. Also, it is the "maximum demand" and not the kilowatt hours of consumption that determines the amount of equipment that must be installed to carry a given customer's load. It is to determine this "maximum demand" that demand meters are used.

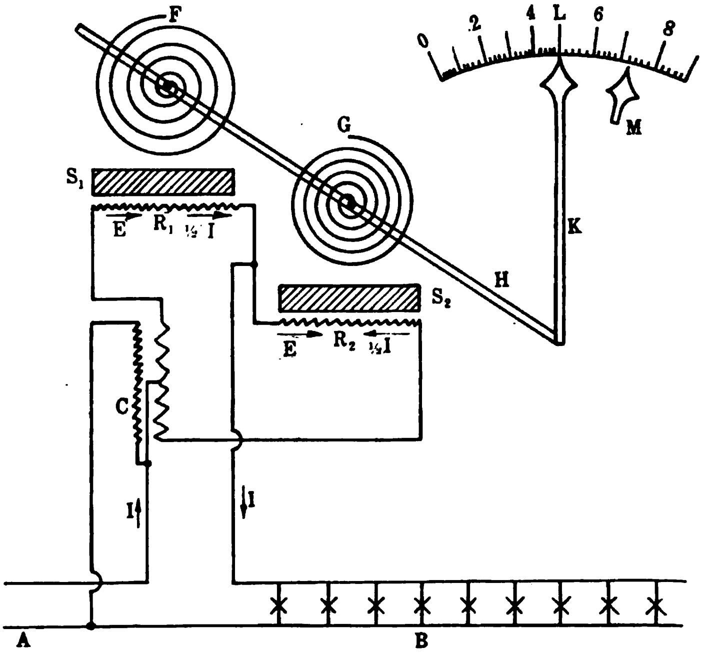

FIG. 1

It may be well at this point to give a brief description of the thermal storage wattmeter as now being built.*

Referring to Fig. 1, *A* is a circuit feeding a load *B*. *C* is a small transformer incorporated within the meter with its primary across the circuit *A*. In series with the secondary of this transformer are two equal resistances $R_1$ and $R_2$. A current is, of course, set up on these resistances that is proportional to the voltage of the circuit *A*. The load current is also caused to circulate through these same resistances in the manner shown in Fig. 1, being taken into the middle of the secondary of the small transformer and being taken out at the connection between resistances $R_1$ and $R_2$. These two currents—one the secondary current, due to the presence of the voltage and the other due to the passage of the load current—are additive in one of these resistances and subtractive in the other, and the difference in the heating effect of the two resultant currents is proportional to the watts of the load *B*.

If we represent the current that passes through the resistance $R_1$ and $R_2$ due to the presence of the voltage by *E*, and the load current therein by *I*, the resultant current in one of these resistances is $E + I$, and in the other $E - I$. The losses are of course, proportional to the squares of these currents and the *differences* in these losses is proportional to the product $E\,I$. This holds true, independent of power factor and wave form, as shown in the paper above referred to.

*F* and *G* represent two spiral springs made from bimetallic strip, attached rigidly to their casings at the outer ends and to a common shaft *H* at their inner ends. These bimetallic springs tend to coil up on an increase in temperature (due to the difference in temperature coefficient of the two metals of which they are composed), but, since the two springs are wound in opposite directions, no movement of the shaft *H* will take place unless there is a *difference* in temperature between *F* and *G*. The shaft *H*, therefore, will not turn with changes in atmospheric temperature or with any other condition that causes both springs to maintain the same temperature, but will respond only to the *difference* in temperature caused by the difference in the losses in resistances $R_1$ and $R_2$. $S_1$ and $S_2$ represent diagram-

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*The complete theory of the thermal storage wattmeter is given in the author's paper read before the American Institute of Electrical Engineers, Oct. 8, 1915, entitled "Rates and Rate Making". (TRANSACTIONS A. I. E. E. Vol. 34, pages 2175 to 2214.)

matically the thermal storage of the cases in which the bimetallic springs $F$ and $G$ are enclosed. Due to this thermal storage, the wattmeter does not respond instantly to a change in load but always indicates the logarithmic average load over the time period immediately preceding the instant of observation, the length of this time period being determined in part by the amount of thermal storage in the cases, shown diagrammatically at $S_1$ and $S_2$. $K$ is a pointer attached to shaft $H$ and traveling over the scale $L$. $M$ is a loose pointer which shows the highest excursion of pointer $K$ since last reset.

Fig. 2 is made from a photograph of a graphic meter of the thermal storage type with the cover removed, showing the working parts. The two cylindrical cases each containing a coiled bimetallic spring may be observed at the top of the instrument. The thermal storage capacity of these cases is so designed that it requires thirty minutes for them to acquire 90 per cent of their final temperature on a steady application of load. The working parts of the indicating meter is a duplicate of that shown in Fig. 2, except for the omission of clock, paper rolls, etc.

A thermal storage meter thus constructed always indicates what may logically be called the "logarithmic average" of the power consumption during the particular time period immediately preceding the instant of observation. Quoting the language of the paper above referred to on this point, the indications of a thermal storage wattmeter "will not be due to the watts passing at that instant, as is the case with the indications of an indicating wattmeter of the usual type, but will be the resultant of all the wattage flow that has passed, each instant of past flow having a value influenced in respect to its time proximity by a logarithmic law. This resultant is not an average in the commonly accepted sense of that word. When we use the word average in its commonly accepted sense, we assume that each instant of time over which the average is taken has equal weight. In the resultant that is obtained by a heat storage meter, each instant of time has not an equal weight, but the influence of each instant decreases with its remoteness in point of time, and the degree by which the watts during any instant influences the total indication is proportional to $e^{-Kt}$, where $e$ is the base of Napierian logarithms, $K$ is an adjustable constant, and $t$ is the time measured backward from the instant of observation. For want of another name, let us call the resultant thus obtained by means of thermal storage the 'logarithmic average.' " The

PLATE IV.
A. I. E. E.
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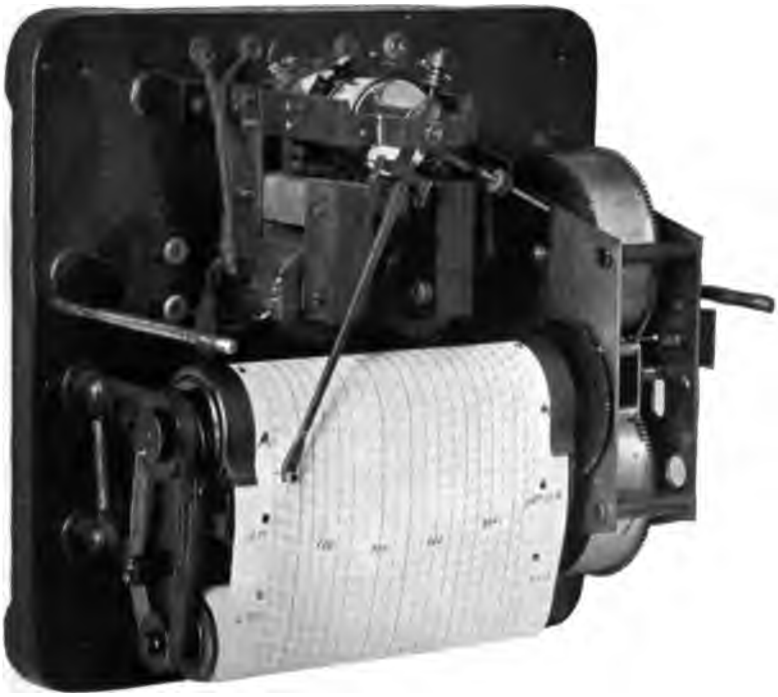

[LINCOLN]

Fig. 2

foregoing quotation indicates the nature of the quantity that is measured by the thermal storage demand meter. In a subsequent paragraph of this paper, a further quotation will be made from my previous paper showing how the "logarithmic average" of a given load may be calculated.

Heretofore, practically all demand meters have indicated in terms of the arithmetical average. This has followed from the fact that the basis of practically all previous demand meters has consisted of a standard watthour meter coupled with some timing device by means of which the integrated value of the load under measurement is obtained over a series of short time intervals. A further mechanism is provided so as to record the highest one of these successive blocks of energy. This type of meter is known as the "block interval" meter.

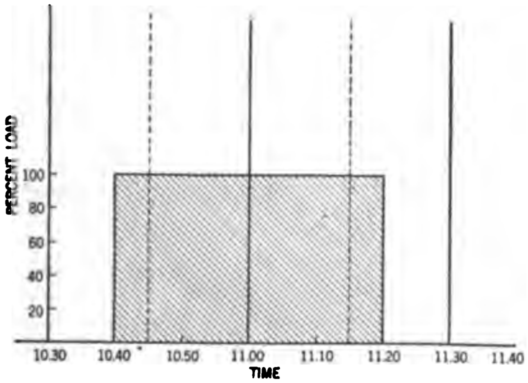

FIG. 3

One of its inherent faults is that it may split an isolated peak of load and therefore become indefinite in its indications. Reference to Fig. 3 will show the reason for this. Suppose we have an isolated block of load that comes on at say 10:40 a.m. and lasts until 11:20 a.m.—such a load for instance as would be involved in the pumping out of a small drydock. Suppose, further that we are using a "block interval" meter with a thirty minute time period to measure the maximum demand of this load. If the time intervals of this meter happened to begin and end on the even half hours—that is, if it integrated the load first from 10:30 to 11:00 and then from 11:00 to 11:30—it is evident that the maximum quantity indicated during any one period would be much less than if the meter periods began and ended on the even quarter hours. It is also evident by inspection that this indefiniteness of indication begins when the duration of the block of load is less than 60 minutes and that when its duration is less than 30 minutes, this "coefficient of indefiniteness"—if we may coin that term—becomes 50 per cent; that is, a load peak of less than thirty minutes duration may be entirely integrated within a single meter period or it may be divided equally between two adjacent periods depending upon the instant of time when these meter periods begin and end. There have been various suggestions of methods to overcome this fault but so far none of these suggestions has borne fruit.

A second and more serious fault of the "block interval" meter is that for isolated blocks of load it does not measure the true value of its heating effect on the equipment that serves the load and therefore does not measure the true value of the duty on that equipment. This matter is further treated in subsequent paragraphs. There has been no suggestion of any method by which the "block interval" meter may overcome this fault and there seems to be no possibility of such a suggestion.

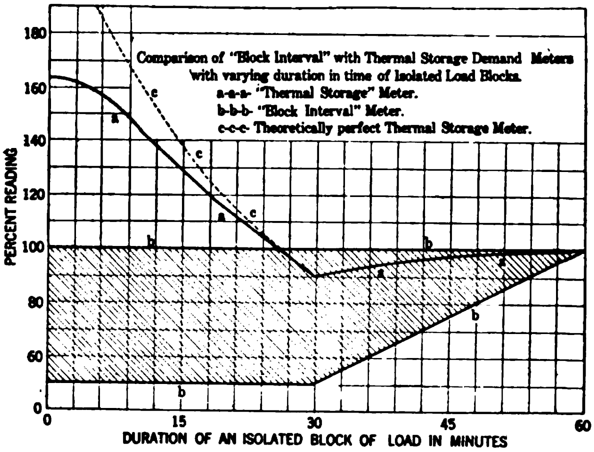

FIG. 4

So long as the loads are steady over long periods of time, the arithmetical (block interval) and the logarithmic (thermal storage) averages are exactly the same. It is only when the duration in time of a block of load begins to come down to the time period of the meter that there is an appreciable difference between the two types. To assume a concrete case again, suppose that service is being sold on the basis of the maximum demand over a 30-minute period. So long as the duration of an isolated block of load exceeds one hour (twice the meter period) the "block interval" and thermal storage meters will give the same results for all practical purposes. Theoretically, for periods of load duration greater than twice the meter period (sixty minutes in our concrete case) the difference between the two types is less than one per cent. For load duration less than 60 minutes, the comparison between the two types of meter is shown in Fig. 4.

The cross hatched area in Fig. 4 indicates what may be called the "area of indefiniteness" of a "block interval" meter of 30-minutes time period; for isolated blocks of load of less than 60 minutes duration, the indications of the "block interval" meter may fall anywhere within this area. On the other hand, the thermal storage meter is perfectly definite in its indication. Each time a given load of given time duration is applied to this type of meter, it gives the same indication.

However, this indication differs from that of the "block interval" meter and the comparison between the two types is given in curve *A A A* in Fig. 4. For a 60-minute block of load, there is a difference between the two types of only one per cent. As the block continues to decrease in time of duration, the thermal storage meter continues to decrease in indication compared to the "block interval" meter (assuming that the "block interval" meter is reading its maximum) until with a 30-minute block of load it reaches 90 per cent. As the time of load duration continues to decrease below 30 minutes, the indications of the thermal storage meter increase until with very short applications of load it indicates about 163 per cent of the maximum of the "block interval" meter and 326 per cent of its minimum.

There will be some one who will at first be constrained to comment adversely on the fact that the thermal storage meter reads higher than the "block interval" meter for all load durations less than about 26 minutes, and that when the load durations are very short, this discrepancy is so large. The action of the thermal storage meter in this respect is, however, entirely defensible. Referring, for instance, to the same concrete example we used above, suppose our customer with a yearly consumption of 1000 kw-hr. insists on taking his entire year's supply in one minute. He would obviously use energy during this one minute at the rate of 60,000 kw. Electrical equipment can, of course, be grossly overloaded for so short a time as one minute, but even

Comparing now the individual tests of a given series and comparing these results with those deduced from purely theoretical considerations, we find that the quantity indicated by the actual meter when subjected to isolated short time blocks of load is not as great as the true logarithmic average. Referring to Fig. 4, for instance, the curve *AAA* and the area *BBB* compare the actual thermal meter with the equivalent "block interval" meter. If the thermal meter had followed a true logarithmic law, the comparison would have been shown by the dotted curve *CCC*

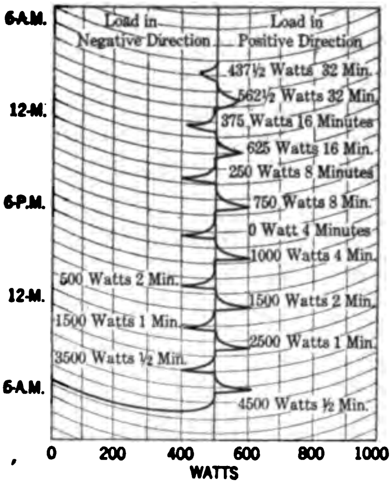

FIG. 6

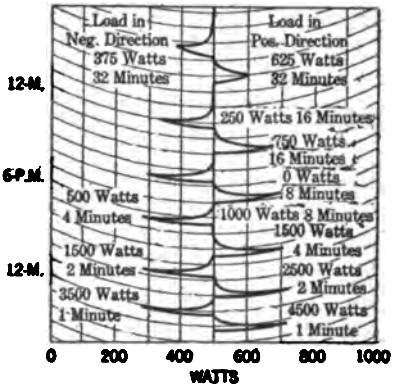

FIG. 7

instead of the solid curve *AAA*. The reason for this and a discussion thereof will be set forth in a later paragraph of this paper.

Figs. 6, 7 and 8 are identical with Fig. 5 insofar as the value of the load fluctuation is concerned, but the fluctuation is made with the meter starting from and returning to the *half* load point instead of the zero load point. In Fig. 6, for instance, there is shown a series of tests that are taken with the application of the following schedule. The meter was operated at 500 watts for a long enough period for the pen to take up the 500 watt position; then, 4500 watts was applied in a positive direction for one-half minute and the load was returned thereupon to this 500-watt point. After the return of the pen to the steady 500-watt position, 3500 watts was applied to the meter in a negative direction for one-half minute, the load then being returned to the 500 watt point. This caused the pen to travel in a negative direction by the same amount as the first application caused it to travel in the positive direction. Also, it will be noted that the travel of the pen in both these tests is the same as in the first test of the first series in Fig. 5. The remainder of the tests in Fig. 6 are according to the following schedule, it being understood that after the application of the scheduled load, the load was in each case returned to the 500-watt point:

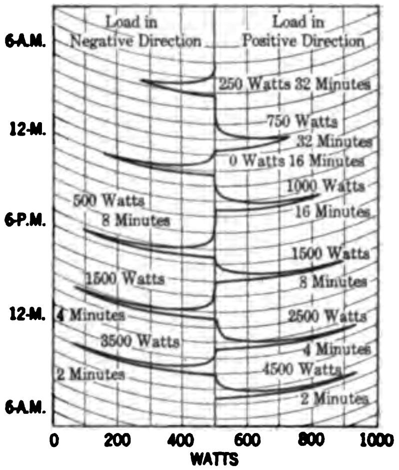

FIG. 8

2500 watts in the positive direction for 1 minute.
1500 " " " negative " " 1 "
1500 " " " positive " " 2 minutes
500 " " " negative " " 2 "
1000 " " " positive " " 4 "
0 " " " " 4 "
750 " " " positive " " 8 "
250 " " " " " " 8 "
625 " " " " " " 16 "
375 " " " " " " 16 "
562.5 " " " " " " 32 "
437.5 " " " " " " 32 "

In other words, this schedule is a repetition of that shown in Fig. 5, except that it goes both ways from the 500-watt point instead of only one way from the zero point. A comparison of the various readings indicates that a given departure in load from the 500-watt point gives exactly the same result as the same degree of departure from the zero point—a result that might have been expected.

Figs. 7 and 8 show the same thing as Fig. 6 except that the loads are respectively twice and four times those in Fig. 6. These results may be compared directly with those in Fig. 5. Fig. 9 shows the result of a load schedule exactly similar to Fig. 5 except that the point of departure is made the full-load or 1000-watt point instead of the zero point as in Fig. 5, or the 500-watt point as in Figs. 6, 7 and 8. Figs. 5, 6, 7, 8 and 9 indicate that a given departure for a given time from the previous steady condition always gives the same result independent of where that previous steady condition has maintained the pen.

In the foregoing tests are given the results of applying isolated blocks of load to the thermal storage meter. The question now naturally arises, suppose the blocks of load are not isolated, but follow each other before the meter has had time to return to zero. The series of tests shown in Figs. 10, 11, 12, 13 and 14 were undertaken to answer this question. Fig. 10, for instance, shows three series of tests. In the first series, (shown at the right hand side of Fig. 10) 1000 watts were put on the meter and kept on for one minute; the load was then thrown completely off for one minute, that is, the time cycle was two minutes long. Both the load and the time of application were then accurately kept on a repetition of this load schedule for about one and a half hours. It will be noted that the meter pen came to a steady value of 500 watts just as if a steady load of 500 watts were applied instead of a full load and zero load during alternat minutes. The second series of tests shown in Fig. 10 is exactly similar to the first, except in point of time of power on and power off, being a two-minute interval instead of one minute; that is, the time cycle was four minutes long. In the third series, the time cycle of load on and load off is ten minutes long, five minutes on and five off. Referring now to Fig. 11, which is a continuation of Fig. 10, this shows three similar series of tests, the difference being that the time cycles are 20 minutes, 30 minutes and 60 minutes, respectively. In each of these power schedules, the average use of power is of course, at the rate of 500 watts. With the two-minute cycle the successive blocks of power blend into each other so that the result is the same as if 500 watts were applied continuously. When time of the cycle is increased to four minutes, the pen responds slightly to successive blocks of load application; the total travel of the pen being perhaps two per cent of the total scale. When the time of the cycle becomes ten minutes, this travel increases to

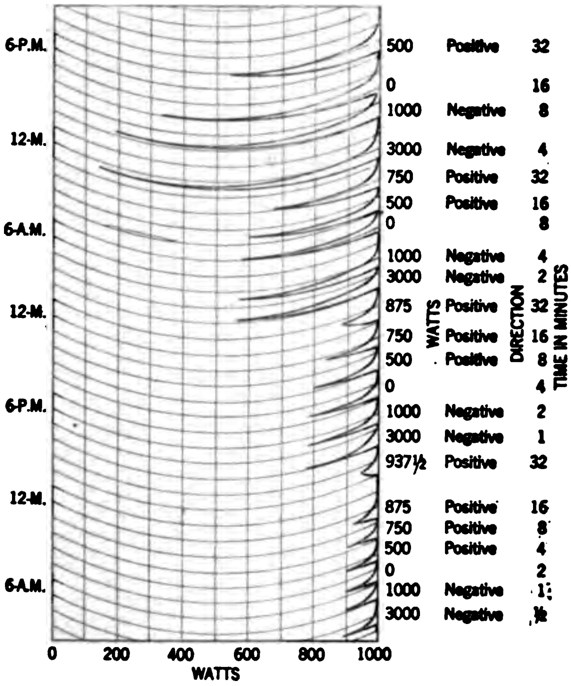

FIG. 9

about ten per cent of total scale. With a 20-minute cycle, this travel becomes about 25 per cent; with a 30 minute cycle about 45 per cent and with a sixty minute cycle, 80 per cent. If we compare the thermal storage meter with the "block interval" meter for these various load conditions, we will note the rather curious fact that for certain time intervals, the "block interval" meter has an "area of indefiniteness," while for others it has not. Figs. 12, 13 and 14 show a number of series of tests on power cycles of the same length as in Figs. 10 and 11—viz., the cycles are of two minutes, four minutes, 10 minutes, 20 minutes, 30 minutes and 60 minutes. However, in Figs. 12, 13 and 14, the power is kept on during one-fourth and three-fourths of the time instead of one-half of the time as in Figs. 10 and 11. The resulting pen traces are highly interesting and instructive. Tabulating all of these results we arrive at the comparison between the two types given in Table I.

TABLE I.

| Duration of cycle minutes | Per cent of time of power on | Meter indications. | | |
|---|---|---|---|---|
| | | Logarithmic | Block interval | |
| | | | Maximum | Minimum |
| 2 | 0.25 | 250 | 250 | 250 |
| 2 | 0.50 | 500 | 500 | 500 |
| 2 | 0.75 | 750 | 750 | 750 |
| 4 | 0.25 | 260 | 267 | 250 |
| 4 | 0.50 | 510 | 533 | 500 |
| 4 | 0.75 | 755 | 767 | 750 |
| 10 | 0.25 | 285 | 250 | 250 |
| 10 | 0.50 | 550 | 500 | 500 |
| 10 | 0.75 | 800 | 750 | 750 |
| 20 | 0.25 | 360 | 333 | 250 |
| 20 | 0.50 | 640 | 667 | 500 |
| 20 | 0.75 | 855 | 833 | 750 |
| 30 | 0.25 | 425 | 250 | 250 |
| 30 | 0.50 | 725 | 500 | 500 |
| 30 | 0.75 | 890 | 750 | 750 |
| 60 | 0.25 | 640 | 500 | 250 |
| 60 | 0.50 | 900 | 1000 | 500 |
| 60 | 0.75 | 980 | 1000 | 750 |

The two minute, ten minute and thirty minute cycles all give an arithmetical average of 250, 500 or 750 watts as the case may be, independent of the point in the cycle where the meter period begins. The other time intervals vary over the limits assigned in Table I, depending on what point in the power cycle the meter period begins.

It is evident from an inspection of Figs. 10, 11, 12, 13 and 14 as well as the foregoing table that the values indicated by the thermal storage meter increase in a definite, logical and consistent manner as the time period between peaks is increased while the "block interval" meter gives results that are indefinite, illogical and inconsistent under the same conditions. In other words, the thermal storage meter recognizes the maximum heating effect of a given load application of any character, while the "block interval" meter does not.

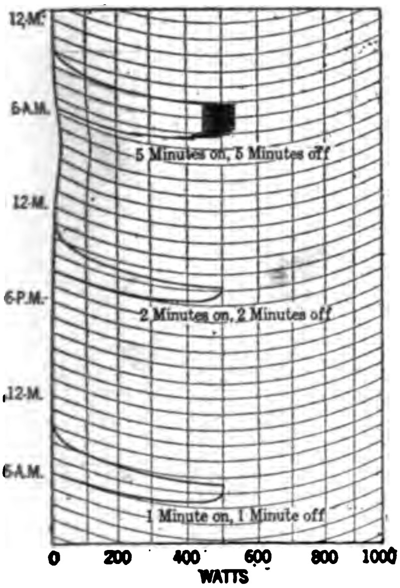

FIG. 10

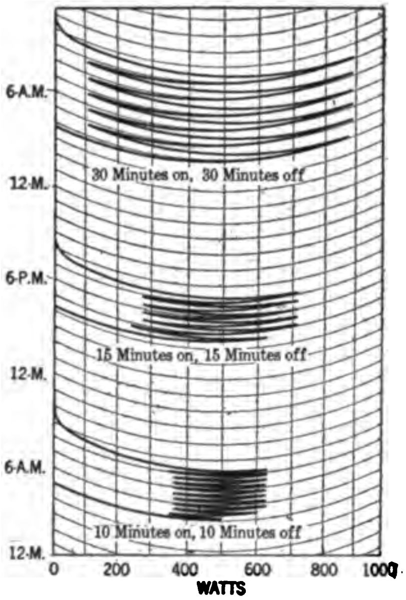

FIG. 11

It may be of interest at this point to make a brief analysis of the action of the thermal demand meter and show the reasons for its departure from indicating a true logarithmic average for isolated short time loads as was referred to in a previous paragraph. Fig. 15 is a reproduction of Fig. 6 taken from the author's A. I. E. E. paper of October 8, 1915, and referred to in a previous paragraph. This shows how the "logarithmic average" of a given load may be calculated from purely theoretical considerations. Quoting from that paper, page 2195: "Suppose we have a load constantly varying with time as indicated by the broken line *C H D E I K F M*. If we apply a thermal storage meter to this load of such characteristics that it requires an hour for it to attain 90 per cent of its final indication, the cooling (or heating) curve of that meter will follow the law indicated by curve *A*. The quantity that will be indicated by such a thermal storage meter at any given instant (for instance, at 12 o'clock in Fig. 6), will be proportional to the cross-hatched area under the broken line *C' H' D' E' I' K' F' M'*. The value of the ordinates of this cross-hatched area at any instant are proportional to the value of the power ordinate at that instant reduced by the ratio of the ordinate of curve *A* at that instant to the maximum ordinate *O G* of curve *A*. If we can just imagine this curve *A* as continually sliding along the power curve, the quantity which it measures will always be proportional to an area that is secured at each instant, just as Fig. 6 shows it at the instant of 12 o'clock.

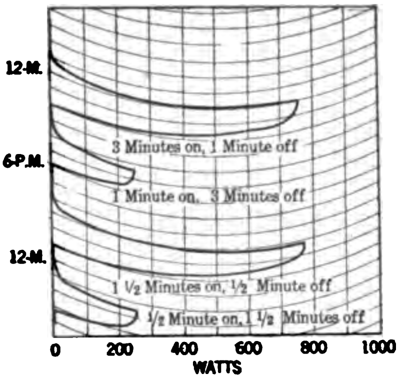

FIG. 12

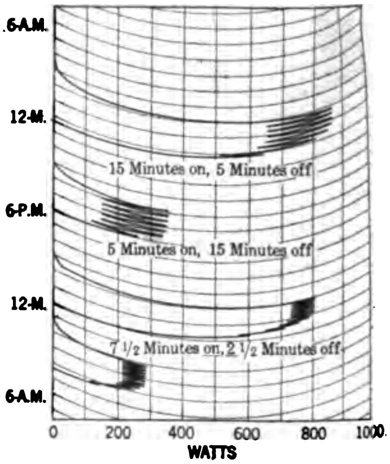

FIG. 13

"If our meter is a ten-minute meter instead of one-hour meter—that is, if it takes only ten minutes to cool down or heat up to within 10 per cent of its final value—the quantity that will be measured will be proportional to the cross-hatched area under the broken line $K'' F'' M''$. In this case, the ordinates of this area are reduced in accordance with the logarithmic curve $B$ instead of $A$; the curve $B$ comes down to 10 per cent of its initial value in ten minutes instead of one hour as is the case of $A$."

The above shows the method of calculating the indications of a theoretically perfect thermal storage meter. For short time applications of load, however, the actual meter is not theoretically perfect. The principal cause for this departure is the fact that diffusion of heat throughtout the mass of a meter element is not instantaneous. Consider, for instance, the application of a one-minute load to such a meter. Test shows that it takes nearly five minutes after the load is thrown off before the pointer reaches its maximum position. The application of this isolated block of load heats one meter element and cools the other from the previous steady condition. This wave of heat change, of course, originates in the resistance, but to effect the meter must pass to the spiral bimetallic springs. That is, the heat must first pass from the resistances to the casings enclosing the springs, then the air inside the casings is heated and this, in turn, heats the bimetallic springs. The heat must get

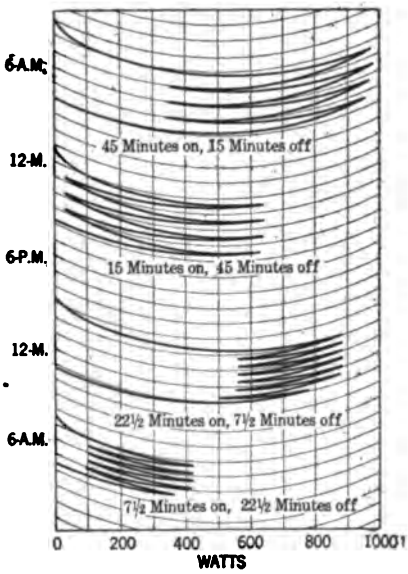

FIG. 14

from the resistances to the bimetallic springs before the meter will respond. This process takes time. During this time some heat that has been put into the resistance escapes from the casing partly by radiation and convection and partly by conduction back through the lead wires. This action leads to no departure from theoretical when the load is steady but only when the load is an isolated block. The amount of this departure can readily be found from the results of the tests shown in Figs. 5, 6, 7, 8 and 9 and the comparison of the actual meter with the theoretically perfect meter is shown by comparing the solid and dotted lines in Fig. 4. This comparison is also shown directly in Fig. 16. It might be noted that such departure as there is from the theoretically perfect meter causes the actual meter to read lower than the theoretical. This is the "safe" position. If any device used in determining a customer's bill, favors the company supplying the service, it can be successfully attacked by the customer. If the contrary is true, it cannot. Hence, the departure noted is on the "safe" direction.

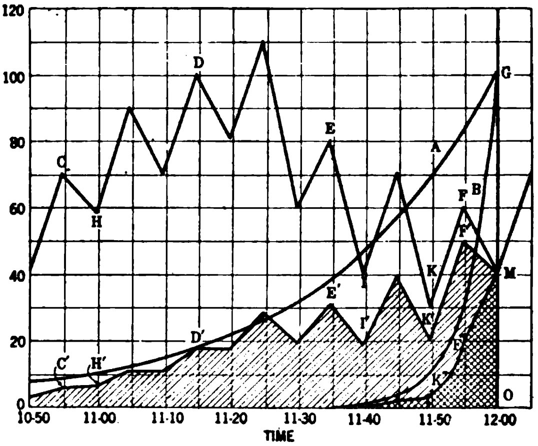

FIG. 15

An objection has been raised to the thermal storage meter in that it reads higher on an increasing load than it does on a decreasing load, the kilowatt hours and the time of application being the same in both cases. The tests shown in Fig. 17 were undertaken to find the value of this discrepancy. In this figure, the first test (beginning at the right hand side) was made by applying 10 per cent load during the first minute, 20 per cent during the second minute, 30 per cent during the third, etc., until ten periods of one minute each had been applied, the last one minute period being 100 per cent load. In the second test, exactly the same load schedule was applied, except that the time of each application was made for two minutes instead of one. In the third test, this time period was increased to five minutes. In the next series of tests, the load was made to decrease with time instead of increase; during the first time interval, 100 per cent load was applied, followed during the next interval by 90 per cent load, then 80 per cent, etc., until during the tenth interval 10

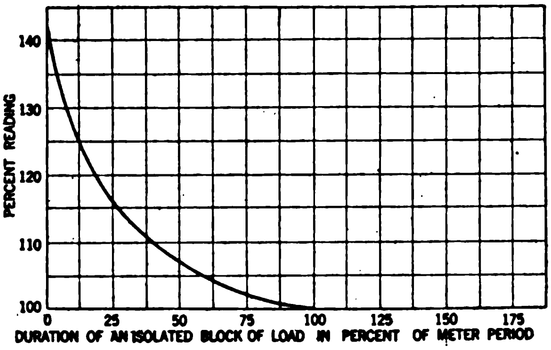

FIG. 16—THEORETICALLY PERFECT THERMAL METER COMPARED TO ACTUAL PERFORMANCE

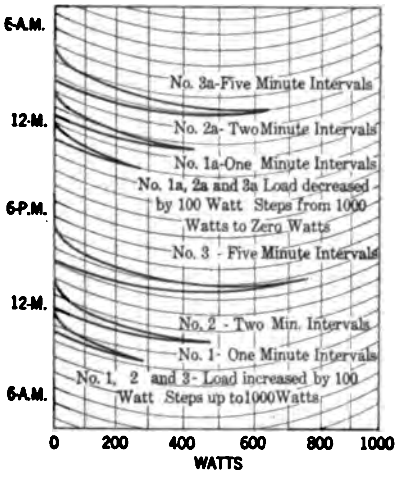

FIG. 17

per cent load was applied. The time intervals were one minute, two minutes and five minutes as in the first series. The comparison of the actual thermal meter, the theoretically perfect logarithmic average meter and the arithmetical average meter for these various load applications is given in the following table. (Percentage is variation from perfect logarithmic meter.)

| Time interval. Minutes | Nature of load | Actual meter | Theoretically perfect meter | Block interval meter | |
|---|---|---|---|---|---|
| | | | | Maximum | Minimum |
| 1 | Increasing | 295 (10.2%) | 328 | 187 (43%) | 93 (71.5%) |
| 1 | Decreasing | 275 (1.8%) | 280 | 187 (33.3%) | 93 (66.7%) |
| 2 | Increasing | 480 (9.4%) | 529 | 373 (29.5%) | 187 (64.7%) |
| 2 | Decreasing | 425 (0%) | 425 | 373 (12.3%) | 187 (56%) |
| 5 | Increasing | 760 (4%) | 792 | 750 (5.2%) | 450 (43.2%) |
| 5 | Decreasing | 630 (0.2%) | 631 | 750 (9.6%) | 450 (28.8%) |

The reason for the difference in indication between increasing and decreasing loads is readily seen by reference to Figs. 18, 19 and 20. In Fig. 18 the method of analysis shown in Fig. 15 is applied to a load increasing by 10 per cent steps. The reading of a theoretically perfect meter on such a load would be proportional to the cross-hatched area in this figure. Fig. 19 shows the same method of analysis applied to a decreasing load. The cross-hatched area in this figure is obviously less than that in Fig. 18. However, the instant chosen in Fig. 19 is not the instant of maximum indication. The instant of maximum indication with a decreasing load occurs before the entire block of load has passed through the meter. The instant shown in Fig. 20 gives a considerably larger cross-hatched area than that in Fig. 19. In other words, with a decreasing load, the maximum indication arrives before the whole load has been put in. Or, to put it in another way, with an increasing load, each increment of load finds the meter already heated by the preceding load and the maximum load is applied to the hottest meter element. With a decreasing load, the maximum load is applied to the coldest meter condition and the maximum temperature arrived at is not as great as with an increasing load. This action is entirely defensible since exactly the same action takes place in the equipment that serves the load. An increasing load heats up transformers, cables, generators, etc., more than does a decreasing load, although the kilowatt-hours and the time of application are exactly the same in each case.

The question may properly be raised as to the proper time period to use in the measurement of maximum demand. At present, the practise of various public service companies varies in this respect over a very large range. One minute is the minimum time duration for maximum demand measurement that the author is aware of and one hour is the maximum. Between these limits a large number of time periods have been proposed and used. So long as it is recognized that equipment cost is the element that dictates the maximum demand portion of a customer's bill, the use of time periods of less than about thirty minutes cannot be justified, since no part of a normal equipment for supplying electric service to a customer has heat storage characteristics that will cause it to arrive at 90per cent of its final temperature in less than thirty minutes and many of the items of such an equipment have a much longer time period. In many cases, short-time periods for maximum demand have been adopted for the purpose of penalizing the customer with a high short-time peak. The thermal demand meter does this automatically and, therefore, there is not the same reason for using short periods when measuring demands with a thermal meter that there is when using the "block interval" meter. For steady loads, it does not matter whether the demand is on a one minute or a

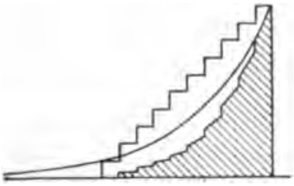

FIG. 18

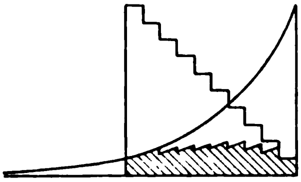

FIG. 19

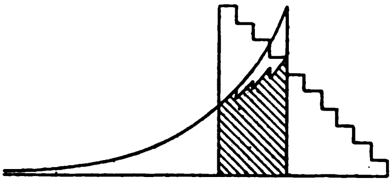

FIG. 20

one hour basis, the result is the same. The average generator, transformer or cable has heat storage characteristics that usually require a time considerably in excess of 30 minutes for them to arrive at 90 per cent of their final temperature when a steady load is applied. A 30-minute meter, therefore, is about the minimum time period for maximum demand that can be justified on the score of assessing equipment costs against the customer and the tendency of the future will undoubtedly be toward longer time periods. The 30-minute thermal demand wattmeter is the first time period to be developed but other time periods will be brought out as occasion requires.

## SUMMARY

1. The cost of electric service is dependent in part upon the cost of the equipment necessary to provide that service.

2. The cost of the necessary equipment to a given customer depends upon his maximum demand and not on his kilowatt-hours of consumption.

3. The thermal storage demand meter gives a perfectly definite indication independent of the character of the load applied, while the "block interval" demand meter becomes highly indefinite on short-time peak-load applications.

4. For short-time peak-load applications the thermal storage demand meter follows a law of the same nature as the heating effect of the load application upon the service equipment, while the "block interval" meter does not.

5. For steady loads the thermal storage and "block interval" demand meters give identical results.

6. The thermal storage demand meter gives a much higher indication for a very short-time peak-load application than does the "block interval" demand meter, thereby making unnecessary the adoption of short demand periods designed to penalize such high peak loads.

## DISCUSSION ON "THE CHARACTER OF THE THERMAL STORAGE DEMAND METER." (LINCOLN), NEW YORK, FEBRUARY 15, 1918.

**C. I. Hall:** A very large part of the justification for the use of a thermal type of demand meter is based upon the idea that the primary rate should be proportioned to the thermal capacity of the equipment required to serve a customer. Since the thermal capacity of the various units of the installation depends very largely, or entirely, upon the current flowing rather than the wattage of the circuit, would it not be very much more logical to employ a thermal ammeter for such measurements than a thermal wattmeter?

It is very interesting to note the modification of the characteristic curve of the thermal storage wattmeter as compared with the curve indicated in Mr. Lincoln's paper on "Rates and Rate Making" of October, 1915. At that time in the discussion there was presented a curve on the characteristics of three different types of demand meters, the first being the Wright demand, the second the logarithmic average as taken from Mr. Lincoln's paper, and the third the characteristic of a thermal ammeter.

The approach of the Lincoln thermal demand meter described at that time to the theoretical logarithmic curve was very close on account of the very rapid dissipation of heat throughout the measuring apparatus.

In connection with the modifications which have been made by the use of a bi-metallic strip where the heat, which is proportional to the wattage, is applied to one end, the deviation from the theoretical logarithmic curve is considerably greater, and by taking the curves as presented in the present paper, it is interesting to note that the curve on the thermal storage wattmeter is now entirely coincident with the curve of the thermal ammeter that was given at that time.

All of the data which have been presented to indicate the differences of indication between the thermal type of demand meter and other types of demand meters are based upon single customer loads. It is, of course, academically interesting to know what these variations are, but no account in this discussion has been taken whatever of the diversity factor. Mr. Ferguson in a paper presented before the Association of Edison Illuminating Companies many years ago stated that the diversity factor was one of the prerogatives of the central station, one of its inalienable rights. In other words, a high diversity factor assists the central station in all of its work and in its rate making.

Therefore, in the discussion of the application of metering devices to actual problems, it is essential to consider the diversity factor as paramount. When so considered, it will be found invariably that any academic differences of individual customer loads are gradually widened out, and that the general effect is the bringing together of the characteristics of various types of meters.

In this connection I believe it would be very interesting to have at some later time a discussion of the measurement of actual load conditions when metered by the various types of demand meters operating upon various characteristic loads.

**F. V. Magalhaes:** It would be interesting to know what the performance of the instrument would be under varying external temperature conditions. Mr. Lincoln dismisses this point with the statement that the operating shaft will not turn with changes in external temperature. There are no actual figures given showing the temperature coefficient of the instrument.

He compares the results obtained by his instrument with one type of commonly used demand meter only and ignores entirely other types of demand meters and in particular one which has been available and in successful operation for several years. I refer to what is known as the Ingalls device. The indications obtained by the Ingalls device permit the demand to be calculated readily and accurately to any minute, and do not confine the demand to predetermined clock intervals.

Mr. Lincoln's comparison between the performance of his instrument and what he calls the Block-interval-type meter is entirely unfair to the Block-interval-type meter, as he shows absolutely no results obtained from the instrument itself on an actual test, but merely states its theoretical or calculated performance on an assumed load. He certainly is well aware that no load in practise follows cyclically the set intervals which can be imposed on a test load. It is a matter of record, as demonstrated by tests extending over long periods, that the block-interval-type meter indicates the actual maximum demand of the circuit to within the instrument's error of registration. It is possible to impose on this type of instrument a short test run with a load definitely controlled within certain time limits, so that the instrument, although it will show with accuracy the maximum demand within its rated interval, will not show the highest figure that may have occurred in the short test period. This is by no means the fair or proper way to test such an instrument, and misleading conclusions should not be drawn from narrow test results so obtained.

Mr. Lincoln is very strongly in favor of standardizing the maximum demand interval for the entire country. This is a laudable desire, but it is very much of a question as to whether his desires in the matter are not colored more by the fact that his instrument is inherently a half-hour instrument rather than by a desire to settle the maximum demand troubles for the entire industry. Mr. Lincoln had some years' experience at Niagara Falls and is undoubtedly familiar with the one-minute and other special maximum demand periods which have in the past and still continue to be effective in that district.

There are other localities where contracts embodying a demand period of one hour have been in effect for years. It is possible to find still other localities where the demand period may be anything between one minute and one hour and the period justified by local conditions which could not be altered merely by a desire to standardize to some other period.

**G. L. Hoxie:** On the first page of Mr. Lincoln's paper he gave the reason for the maximum demand indicator, as follows: "The incorporation of maximum demand in a rate for electric service is an attempt to assess upon the user of that service his proper share of the annual cost of equipment necessary for giving the service."

Now, it seems to be assumed in this paper that the annual cost of the equipment necessary to give the service which a customer uses is a function of the heating of that equipment due to such service. The limiting thing, however, may well be the maximum peak capacity of the generating stations of the company furnishing the current. It is only rarely that maximum possibility of service is determined by the capacity of equipment located outside of the generating or sub-stations. The maximum peak capacity of a generating station is not usually limited by heating. I believe in the case of a modern turbine that the number of kilowatts you can take out depends to a greater extent upon the steam end than upon the electric end, and a similar situation exists with many water power units, not with all, of course.

However, the principal comment, I wish to make is along the line of that of a previous speaker, and is in connection with diversity. Mr. Lincoln has used a very striking illustration of a customer taking one thousand kilowatt hours per year, and, if the service all be taken in one day, he takes current at 365 times the rate, as if his use were spread out over the year, and the inference is that his use of the company's apparatus costs the company 365 times as much as if the service were so spread out.

To carry that same illustration further, let us assume that there are 364 other customers, each taking current on a different day. In that case, so far as the production of the electricity is concerned, there would be no difference between the load of the 365 customers, each taking current on a different day, and one customer taking the same current every day.

It seems to be assumed in the paper that the heating effect shown by the customer's maximum demand meter, taken by itself, is the quantity which should be considered, but as a matter of fact, it seems to me it is the effect upon the generating stations of the company, due to this particular customer's draft of energy, combined as it is with the demands of all the other customers of the company; in other words, the customer's maximum demand, in itself, is not a question with which the generating company is necessarily concerned. The maximum demand of a particular customer *which occurs synchronously with the maximum demand upon the generating stations*, is the one thing which, it seems to me, must be considered, and therefore I would suggest modifying clause No. 2 in Mr. Lincoln's summary so that it will read:

2. The cost of the necessary equipment to a given customer depends upon his demand *coincident with the station maximum demand* and not on his kilowatt hours of consumption.

That of itself eliminates the use of the one-minute peak for the ordinary customer. The one-minute peak of the ordinary customer will be smoothed out long before it reaches the generating station. In the case of a customer like a railway company, which may have a demand which is a large percentage of the total demand of the company, possibly a one minute peak may be an important quantity.

**W. H. Pratt:** There is a question I would like to ask, suggested by the sentence, "On the other hand, the thermal storage meter is perfectly definite in its indication. Each time a given load of given time duration is applied to this type of meter, it gives the same indication." What would be the comparative reading of two instruments, of different capacities, when put on the same load, for instance, an instrument of 1000 kw., and another one of 2000 kw., given a load of, roughly, say 900 kw., or 800 kw. It is, of course, necessary at times to use a device such as is available, but that might not be the device chosen were all capacities available.

It seems to me that there is no serious objection to using the logarithmic law of performance of demand meters for considerable classes of work. It seems to me also that it is a mistake to make a virtue of this logarithmic law, to place a special emphasis upon it, unless the law is pretty closely followed, and I judge from the figures given in this paper that it is not, in fact, that the departure is wide.

**M. G. Lloyd:** The objection which the author has spoken of to this meter in regard to the differences in registration with increasing loads and decreasing loads would not, on the face of it, be any objection if the load under consideration were the only load on the generating station. The fact that this meter very closely reproduces, in the effects within it, what is taking place in the generating apparatus, as regards heating, would make that matter take care of itself. However, in the case of a central station utility, one does not usually have a condition of that kind. The load of a particular customer is simply imposed upon a comparatively large load in the generator, in which case the effect is not reproduced in the generator, and consequently there is some objection to that characteristic in the meter.

Another factor which is to be considered and to which the author has referred, is the time taken for a generator or other apparatus to reach its final temperature, unless his assumptions are that one is starting from a condition of no load, and of course that does not apply to any particular customer's load being thrown on the line; and it is for that reason I think, that the central station does find it important to consider comparatively brief intervals of demand rather than demand covering periods of a half hour or longer.

I should like to ask the author whether this instrument was made up in a form to be indicating rather than recording, in the sense of giving a graphic record.

**A. S. Albright:** We have had the opportunity of making some tests on the indicating form of the type "RH" meter. After various laboratory tests, the "RH" meter was installed on a typical factory load, together with two other demand meters, both of the graphic integrating type, and readings of the three meters were observed weekly for several months. The curves obtained, given in Fig. 1, show the comparative performance of the two types of meters on this installation. The thermal meter shows a higher demand in all cases. The break in the curves is caused by skipped readings.

We have had no experience with the graphic form of Mr. Lincoln's meter.

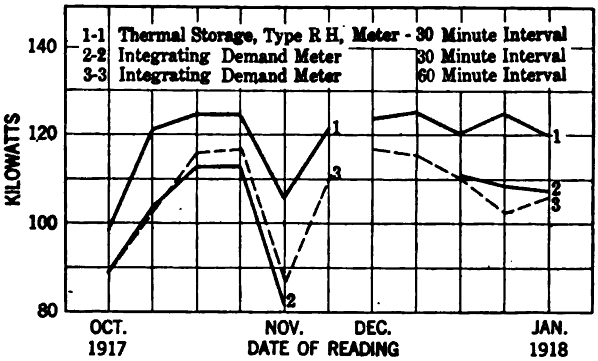

FIG. 1—COMPARISON OF "THERMAL STORAGE" WITH INTEGRATING DEMAND METERS ON A TYPICAL FACTORY LOAD

**H. D. James:** There is another use for this thermal storage meter which I wish to bring out, because I believe it has an important field, and that is in connection with individual pieces of apparatus. I have particularly in mind the motor. Many motors are operated on an intermittent load. The operator is, in many cases, desirous of knowing what is the average heating effect of this load on the motor. If he has an ammeter in front of him, and it varies back and forth, he must make a mental integration of the heat. If, however, we place a time-demand-overload meter on the motor, he does not know what is going to happen until the motor shuts down. If, however, he has a meter which indicates the heat going into the motor—a 30-minute time interval is long enough for most applications—he will have in front of him an ammeter which is giving him a true integration of the heat in his motor, and continually indicating what is going on, and when the needle reaches a predetermined point his motor will shut down, and stay shut down a predetermined time, before he can start it up again.

Such a device is very desirable, and I think it is one of the devices the trade has been working toward for a number of years. We have the thermo-couple in the large generator, but so far it does not seem as though the thermal couple for the small apparatus can be produced commercially. We have a time-element-overload relay, which is commercial, but it has only a short time interval.

If we have a thermal integrating meter which is useful for not only determining the charge the customer should pay for the power he consumes, but also as a protection for his motor, and which will allow the customer to operate the motor close to its maximum, and still keep within the proper heat limit.

**H. L. Wallau** (by letter): The thermal type of demand meter is sure to have a promising future, once its operating characteristics are thoroughly understood.

Its simplicity, reliability, absence of clock troubles, coupled with the fact that on a load of given characteristics it will always duplicate its previous indications, are valuable assets. This was shown by test, and it was also found to be very accurate with variable voltage and at various power factors.

Where a number of such meters are connected in series with a load all should give the same indication. To my knowledge, this is the only meter possessing this very desirable characteristic. Lack of synchronism in clock movements prevents similar results from being obtained with the usual types of demand meters now available. This last feature we have been as yet unable to check by tests because we had but one meter available for testing purposes.

Its accuracy depends upon the brightness of the nickel plated heater casings remaining unimpaired. Just what effect the exposure of this type of meter to fumes and gases in industrial establishments will have, remains to be seen.

The author has discussed its principles, acknowledges that on heavy loads of relatively short duration its registration is above the arithmetical mean, but points out that it does truly follow the heating characteristics of the energy supply devices and therefore that this is a point in its favor.

It is self-evident that if every consumer of a class pays his just share of the cost of delivering current to him, there will be no discrimination in favor of any, and that the ultimate effect must be to allow of rate reductions to the class, as a whole, more rapidly than if some losses must be absorbed due to inequalities in charges favoring short-hour users. Commercial expediency may modify the application of this principle somewhat, but its truth remains unchanged.

As regards the period during which the maximum demand should be measured, considerable difference of opinion is yet manifest. One large company uses an hour period and reads the demand weekly. The average of the four readings is billed as the monthly demand. The object of this is to prevent the undue penalization of a consumer for some abnormal demand which might accidentally be made on a certain day, and which is not typical of the general operating characteristics of his load.

In my judgment the meter with a time period, such that 90 per cent of the load will be indicated in the interval for which the demand is taken under the contract, is preferable to that in which only 63.2 per cent of the load would be indicated in that period.

**P. M. Lincoln:** Mr. Hall refers to the thermal capacity of the equipment, and states that on the basis of thermal capacity that the meter used should be a current meter, an ammeter rather than a wattmeter. That is true within limits. It is not true that apparatus in general has a temperature in proportion to the amperes carried. If all of the heat were due to the passage of the current, that statement would be true, but in electrical apparatus in general, only a part of the heat is due to the passage of the current.

In large turbo-alternators, the armature copper loss amounts to only 10 or 20 per cent of the total, and consequently the heating of that device is not proportioned, by any means, to its ampere capacity. The same is true of other pieces of electrical apparatus which are used, even the cable. The heating of cables depends almost wholly, to be sure, on the current carried, but the next paper indicates that a considerable proportion of the losses which take place in cables is due to the element depending on the voltage, so that an element which depends on current alone is not sufficient to insure the heating of electrical apparatus in general. It must be something which takes in elements other than mere amperes.

I maintain, therefore, that the wattmeter will probably in general come nearer measuring the true capacity of the apparatus than will an ammeter, although the ammeter on exactly the same principles is available, and will undoubtedly have a considerable part in the future of demands as they are measured.

Another thing called attention to is this matter of the time of diffusion, the time it takes for heat to diffuse through the meter element. Mr. Hall's view of that feature and mine seem to be different. I consider the time of diffusion as a distinct handicap. I think that Mr. Hall considers it an advantage, and there is a distinct difference of opinion between us on that particular point. I wish I could produce a device in which the time of diffusion was zero.

It was found that a true logarithmic law would avoid the error shown in Fig. 16 of my paper. It is the time taken for the diffusion of the heat throughout the meter limit which gives rise to departure from the true logarithmic law, and therefore gives rise, in the case of the thermal meter, for analyzing the true logarithmic average as analyzed in Fig. 16.

I wish it were possible, therefore, to avoid the time of diffusion throughout the meter element. I consider that time of diffusion as a distinct disadvantage. Possibly as things go along, we will find means of overcoming it.

Mr. Magalhaes asks concerning the temperature coefficient of the meter. I have made a good many measurements of the temperature coefficient, and the temperature error is so small that it is pretty hard to make an accurate determination of it. In general, I can say that it is about one-tenth or one twentieth of a per cent for each degree of change in centigrade temperature. That is not a very large change, but as I say, it is a difficult thing to measure because it is so small.

Mr. Magalhaes speaks of a device which he calls the Ingalls device. I am not prepared to comment on that, because I am not familiar with the device he mentions.

Mr. Hoxie and several other gentlemen have objected to my conclusions, in that I have taken the cost of the customer's equipment as being proportional to the maximum demand of that customer. Mr. Hoxie and the others who referred to that point spoke specifically of the generator capacity, and if we consider generator capacity only, they are more or less correct in their conclusions. However, it is true that the majority of the cost of electrical equipment for serving the customers is not in the power house. It is out on the line. I think if the cost of equipment for serving customers is analyzed they will find that the cost of equipment at the customers' end amounts to a good many times the cost of equipment in the generating station, and the cost of the equipment at the customer's premises is something that depends on the customer's demand. The diversity factor takes care of that if it goes back into the system, and it is true, as pointed out, that the maximum demand of the customer does not affect the generating capacity to anything like the same extent that it affects the apparatus which is placed at the premises of the customer.

You must remember in this connection that the customers are very many, and that the separate pieces of apparatus to serve them run into thousands of pieces, and the aggregate cost of these pieces in general is considerably higher than the cost of the generating equipment to serve these customers. That consideration, therefore, leads me to quite a different conclusion than if one is considering the generating equipment only.

Mr. Hoxie spoke of my illustration of the customer who insisted on taking his entire year's supply in a single day, and said that if there were three hundred and sixty-five customers, each of whom took his load on one day of the year, that my illustration would not apply, but my illustration does apply, because each of these customers would have to have equipment enough to take care of the whole output of the generating station during that day, which would be practically as much equipment as there is in the generating station, and it would mean practically multiplying the equipment of the customer by 365, which would certainly be a large increase in the total cost of power for that station. I cannot agree with Mr. Hoxie on that point.

Mr. Pratt asked concerning the difference in indication, for instance, of a 1000-watt wattmeter as compared to that of a 2000-watt wattmeter. There is no difference. It does not matter what the capacity is, a 2000-watt wattmeter will indicate the load as much as a 1000-watt wattmeter. There is a difference, however, in the indication of two different times. If we had a 30-watt wattmeter, and compared that with the indications of our demand wattmeter, the indications of these two would not necessarily be the same, and would depend on the character of the load. If the loads are the same, they would give the same results. If the load had high peaks, the meter with the short time element would give higher indications than the one with the longer time element. That depends on the application of a logarithmic law.

Mr. Albright speaks of a number of tests made in Detroit of this device, and states that the indication of the 30-minute thermal demand, as compared with meters of the same kind, and as compared with meters of a different type, taken through a number of weeks, shows that the thermal meter gives a higher demand in all cases, and that is about what one would expect, and would indicate that the particular load he has it on is not a steady load, but one with a number of peaks in it. As I recall it, he applied it to a machine shop load in which the peaks are rather marked, and it was only to be expected that the thermal demand meter would give higher results in the case of such a load than the wattmeter.

Concerning the application of a device of this kind as a motor protection, as spoken of by Mr. James and Mr. Atkinson, that is an entirely practical application, and I look for that application to be developed to a considerable extent in the future.

*Presented at the Sixth Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1918.*

# MEASUREMENT OF POWER LOSS IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES

BY F. M. FARMER

## ABSTRACT OF PAPER

This paper describes the method used at the Electrical Testing Laboratories for measuring the dielectric power losses in 10-foot samples of three-conductor cables with three-phase potential applied to the cable. The difficulties encountered and the methods employed to overcome them are discussed in considerable detail. Typical results are given in the form of data for two specimens of cable, one having a low power loss in the dielectric and one having a high power loss in the dielectric. The data are also presented in the form of curves.

The discussion includes:

(a) The theory of excessive internal dielectric loss as accounting for cable failures at local "hot spots."

(b) The advantages of plotting data with logarithmic scales.

(c) A comparison of results obtained by computation from single-phase measurements with those obtained by direct measurement with three-phase potential.

The conclusions drawn are:

(a) The power loss in the dielectric in a three-conductor cable under actual three-phase conditions can be readily measured in the laboratory with specimens ten feet long.

(b) No special apparatus is necessary for such measurements other than a reflecting high sensitivity wattmeter.

(c) Apparently the power loss in the dielectric cannot in all cases be accurately calculated from data obtained in single-phase tests although it is highly probable that for all practical purposes the discrepancy would not be serious. Further investigation is necessary, however, before final conclusions on this point can be stated.

(d) While the method of determining power losses in the dielectric directly by three-phase measurements involves more complication in preparation and slightly more time in the actual measurements, it has the important advantage that the results are conclusive and not subject to the uncertainty which pertains to results calculated from single-phase measurements.

THE CARRYING capacity of a cable is determined by the temperature at which the dielectric strength becomes dangerously low or at which deterioration takes place at an abnormal rate. If the temperature which the materials in the cable will safely withstand is known, the carrying capacity of the cable is fixed when the hottest part of the cable reaches this limiting temperature.

Until two or three years ago the only factors which had been given serious consideration in determining the temperature of the cable were the $I^2 R$ losses in the conductor and the thermal conductivity of the surrounding media. In other words, it was assumed that the copper loss was the only source of heat that need be considered and that the temperature which would be reached would depend simply upon the amount of heat thus generated and the rate at which this heat was carried away. However, careful investigations of cable failures, for which there was no obvious explanation, seemed to indicate that there were local hot spots which were not due to any outside cause. Attention was then drawn by investigators of insulation problems to the possibility of power losses in the insulation being responsible for abnormal temperature rises. Investigations made by Hochstadter[1], Clark and Shanklin[2], Bang and Louis [3] and others indicated that, under operating conditions which could be considered as normal, these losses might become sufficiently high to raise the temperature of the insulation to the destruction point. The subject of power loss in cables has therefore been attracting much attention and it is particularly pertinent at the present time because of the necessity for operating cables at the maximum possible capacity in order to meet the demand for more power. Furthermore, the amount of the power loss in dielectrics, although small, may in the aggregate be sufficiently large to justify consideration from the economic standpoint when new installations are contemplated at the present prices of materials and labor.

*Object of Paper.* The object of this paper is to describe the methods used in making tests at the Electrical Testing Laboratories for the purpose of determining the power loss in the dielectric of samples of paper-insulated, lead-covered cables 10 feet long, the loss being measured at various voltages and various temperatures. While nothing absolutely new or particularly novel is presented, it is felt that some detailed information in regard to these tests may be of service to engineers who will be called upon to make tests of this character.

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1. M. Hochstadter—"Twisted Cable," *Electrotechnische Zeitschrift,* November 25th, 1915, page 617.

2. W. S. Clark and G. B. Shanklin—*Insulation Characteristics of High-Voltage Cables,* TRANS., A. I. E. E., 1917, page 447.

3. A. F. Bang and H. C. Louis—"The Influences of Dielectric Losses on The Rating of High-Tension Underground Cables," TRANS., A. I. E. E., 1917, page 431.

The power loss in the dielectric of three-conductor cables is probably most frequently determined by making single-phase measurements and then computing the corresponding three-phase loss on the basis of certain assumptions. That is, the power loss in the dielectric between the conductors (conductor loss) and the loss between the conductors and the sheath (belt loss), is measured with single-phase potential at the appropriate Y or delta voltage. The loss with three-phase potential is then calculated on the assumption that these two losses are the same with three-phase potential as with single-phase potential and that their sum is the total loss. Hochstadter[4] pointed out the possibility of this assumption being open to question because of the complex nature of the electrical field in the insulation between the conductors when subjected to three-phase potential.

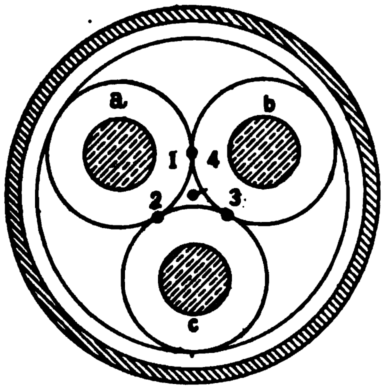

Fig. 1

Referring to Fig. 1 which represents diagrammatically a cross-section of a three-conductor cable, the dots 1, 2 and 3 indicate the points of contact between the various conductor insulations on lines connecting the centers of the three conductors. The dot 4 is the center of the cable and may in three-phase operation, be assumed to be at ground potential. If delta potential, $E$, is applied to the three conductors, $a$, $b$ and $c$, the potential between different points will be as follows:

$$\text{Volts } a-4,\ b-4,\ c-4 = \frac{E}{\sqrt{3}}$$

$$\text{Volts } a-1,\ b-1,\ a-2, \text{ etc.} = \frac{E}{2}$$

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[4] M. Hochstadter—"Twisted Cable," *Electrotechnische Zeitschrift*, November 25th, 1915, page 617.

In other words, points 1, 2 and 3 are not at the same potential and therefore there is a tangential electrical stress in the insulation between the conductors which does not exist when making single-phase measurements. Consequently a power loss might be anticipated in the dielectric in this part of the cable insulation which would not be determined by such a measurement.

That the loss per unit volume of insulation between conductors may be actually higher than elsewhere, is suggested by evidences found by Clark and Shanklin and others in their investigations of cable failures,—that is, the insulation between the conductors was charred although the insulation between the conductors and sheath was apparently uninjured. As it is well known that the power loss in most insulations increases rapidly with temperature, it seemed logical to suspect that the loss per unit volume, and therefore the heat generated in the insulation between conductors, had increased faster than it could be dissipated, with the result that the temperature rose to the destruction point.

Because of the doubt which these considerations cast upon the method of determining the power loss in the dielectric of three-conductor cables from single-phase measurements, it was considered advisable to measure the power loss directly under three-phase conditions. Incidentally, however, single-phase measurements were also made, so that some interesting comparative data were obtained.

*Method Employed.* The method used in these tests was the simple three-wattmeter method for measuring three-phase power. That is, the cable was Y-connected, with the sheath as the neutral, and a wattmeter was connected to each phase in turn. The algebraic sum of the three quantities thus obtained is, of course, the total power dissipated. The high potential which was applied to the cable was obtained with three 15,000-volt, 200-watt, potential transformers connected in Y. The wattmeter was a reflecting dynamometer instrument, the current circuit being connected between the grounded neutral point and the high-tension winding of the transformer in the phase being measured, while the potential circuit was connected to the low-tension winding of a step-down potential transformer connected across the same phase.

The actual measurements were not, however, accomplished as easily as this description might indicate despite the simplicity of the general method. In the first place, the power factor is as low as four or five per cent at moderate temperatures. Consequently, the effect of inductance in the potential circuit of the wattmeter and the phase angle in the step-down potential transformer is very marked and would produce serious errors if not properly taken into account. In the second place, the voltages used are so high and the power being measured so very small (as low as one watt) that losses through paths in parallel with the cable, that is leakage losses to ground from the test circuits, are likely to be relatively large and great care must be taken to eliminate them.

The effect of the inductance in the potential circuit of the wattmeter and the phase angle of the potential transformer was eliminated by the usual method of shunting a variable portion of the series resistance with a condenser as indicated in the diagram in Fig. 2. Thus the resultant equivalent inductance of the circuit is compensated for by the introduction of a proper amount of capacitance. Exact compensation can be determined by the following methods:

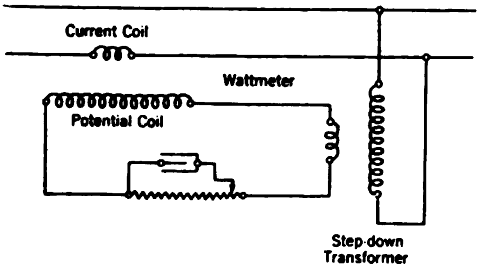

FIG. 2

(a) Measure the inductance of the wattmeter potential circuit and the phase angle of the transformer. Then compute the capacitance and shunt resistance necessary from the relation $L = CR^2$, where $L$ is the equivalent inductance of the circuit in henrys, $C$ the capacitance of the condenser in farads and $R$ the shunt resistance in ohms. This method requires a very careful measurement of the phase angle of the transformers, a measurement which involves either special apparatus or standardized transformers.

(b) Connect a high-voltage air condenser as the load in place of the cable and adjust the resistance which is in shunt with the condenser until the wattmeter shows no deflection. The accuracy of this method depends upon the power factor of the condenser. Normally an air condenser may be assumed to be a perfect condenser and therefore have zero power factor, but at high potentials the corona discharge from the circuits and from the condenser plates may introduce a power component unless great care is taken in the construction and proper means are provided to eliminate this corona discharge. However, when done in the careful manner employed by Shanklin[5] it is probable that this method is the best one for adjusting the compensation.

In the tests discussed in this paper, the compensation was determined by method (a) and checked by method (b), slightly modified. It was not found possible, with the only apparatus available to eliminate leakage entirely from the test circuits and consequently the deflection of the wattmeter could not be reduced to zero. The compensation was therefore so adjusted that connecting various amounts of capacitance to the circuit did not change the deflection. Incidentally, this capacitance was obtained by suspending from the test circuit a number of metal cans about six inches in diameter and 15 inches deep, which were conveniently at hand. Since the adjustment could be made at potentials well below the part where corona discharges would take place from these cans, it was not necessary to take special precautions on this account.

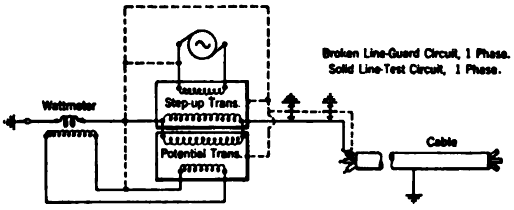

FIG. 3

As previously stated, the cable was tested by applying Y voltage to the conductors, the lead sheath being connected to the neutral and to ground. Obviously, all points of contact with each high-tension circuit provided leakage paths to ground in parallel with the insulation of that particular conductor of the cable, so that the power measured at the terminals of the high-tension winding of the transformer would include the

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[5] G. B. Shanklin, "Compensated Dynamometer Wattmeter Method of Measuring Dielectric Energy Loss," *General Electric Review*, Vol. 19, page 842 (1916).

power expended in these leakage paths. A guard circuit was therefore provided for each of the three phases as indicated by the broken lines in Fig. 3 which shows diagrammatically the arrangement for one phase. This guard circuit consisted of a metallic connection between each of the following:

(a) A piece of tin or tin-foil inserted between ground and each insulator supporting the high-tension circuit, moderate insulation being provided between the metal and ground.

(b) A few turns of fine bare copper wire around the insulation of the conductor about two or three inches from the end (the lead and wrapper insulation of the cable having been cut back several inches in the usual manner to permit separation of the ends of the conductors) which served to pick up any leakage current over the surface of the insulation between the conductor and the lead.

(c) The iron cases of the step-up power transformer and the step-down potential transformer, the transformers being insulated from ground.

(d) The low-tension windings of the transformers. This is necessary because it would be possible for a leakage current to pass from the high-tension winding to the low-tension winding of the transformers as well as to the cases.

(e) A point *between the wattmeter and the grounded end* of the step-up power transformer.

This guard circuit arrangement theoretically eliminates any current from the wattmeter current circuit that does not pass through the insulation of the cable except, of course, that directly through the air due to corona. In these tests, however, the potentials were always below the corona value.

As previously stated, it was not found possible to eliminate leakage currents entirely but it is probable that they were entirely due to insufficient insulation between all parts of the guard circuit, including the circuits to which it is connected, and ground. In fact, there was a partial ground in one phase of the generator which necessitated connecting the low-tension side of the power transformer to ground instead of to the guard circuit. Consequently leakage from the high-tension winding to the low-tension winding of the power transformers could not be properly shunted by the guard circuit.

A special smooth-core generator was used in these tests and oscillograms made under test conditions showed that the wave shape was substantially a sine curve under all conditions.

Suitable switching arrangements were provided so that the current circuit of the wattmeter could be quickly shifted from one phase to another and a similar provision was made for changing the potential circuit from one phase to another. Fig. 4 shows diagrammatically the circuits used in the test and the switching arrangements employed.

*Power Measurements.* The power in each phase was measured as already indicated, the total power to the test circuits and the cable being taken as the algebraic sum of these three measurements. The cable was then disconnected from the high-tension leads, leaving all of the rest of the test circuits exactly the same,

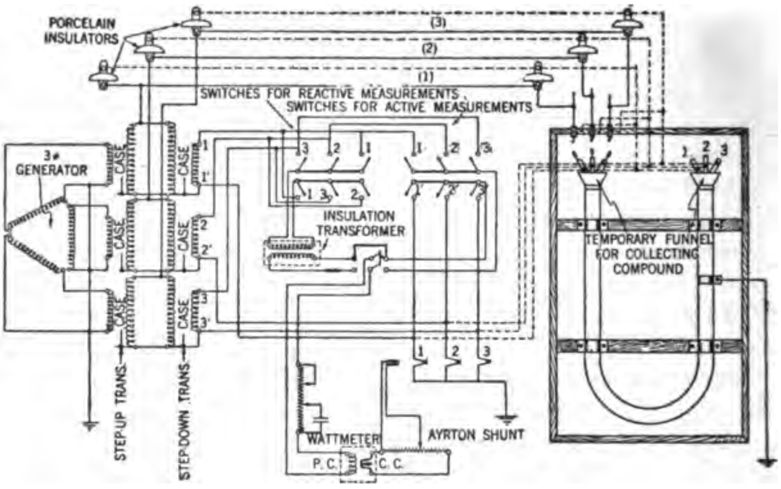

FIG. 4—DIAGRAM OF CONNECTIONS FOR DIELECTRIC POWER LOSS MEASUREMENTS—THREE-CONDUCTOR CABLES WITH THREE-PHASE POTENTIAL

and the measurements repeated. The latter gave the "leakage" loss. The difference between these two quantities was taken as the power loss in the dielectric of the cable.

*Power Factor.* The power factor was obtained by measuring the reactive volt-amperes. This measurement was made by simply shifting the voltage circuit of the wattmeter through 90 degrees by connecting to the proper delta voltage instead of the Y voltage.

*Direct-Current Resistance.* The resistance between the various combinations of conductors and lead sheath was measured in the usual manner with a sensitive reflecting galvanometer at 700 to 800 volts.

*Capacitance and Alternating-Current Resistance.* These quantities are obtained by computation from the above data,—the alternating-current resistance from the power and the capacitance from the reactive volt-amperes.

*Heating of Cable.* The heating of a section of cable 10 feet long to a temperature as high as 125 deg. cent. uniformly throughout its length is not the simple problem that one might expect. The first scheme considered appeared to be a very simple solution until it was tried. A sheet iron pipe about 10 inches in diameter with the cable suitably supported at its center was constructed and low-voltage current circulated through the pipe. It was expected that this scheme would give uniform heating, provide convenient control of the temperature and make the determination of the temperature of the cable a relatively easy matter because of the symmetrical arrangement of heat source with respect to the cable. The first difficulty experienced was the heating at the joints in the pipe due to the greater electrical resistance at these points. Also, there was an excessive cooling at the ends. The pipe was then wrapped with asbestos and a layer of iron wire wound thereon and through this wire current was circulated—additional turns being provided at the ends to compensate for the cooling effect. This scheme was satisfactory insofar as obtaining uniform temperature conditions was concerned, but it was found that at the high temperatures too much compound was being lost from the cable at the ends. It was therefore evident that some provision had to be made to prevent this loss of compound.

The iron pipe was abandoned and a simple oven was constructed consisting of a wood box lined with asbestos of such a size that the cable when bent in the form of a U having about 2.5 feet radius could be mounted in the box in a vertical position. The expansion of the compound was taken care of by soldering a sheet tin funnel to the lead at each end of the cable and into this funnel the compound could expand on heating and flow back again into the cable on cooling. Heat was provided by large incandescent lamps so located by experimental trials as to give the most uniform temperature, two fans being installed to assist in attaining this condition. Switches outside of the box provided control of the amount of heat being supplied and therefore of the temperature.

The temperature of the cable was measured by seven thermocouples soldered to the lead sheath. No difficulty was found in getting these thermocouples to agree within 3 deg. cent. at a temperature of 100 deg. cent. One of the couples was connected to a curve-drawing instrument for the purpose of conveniently showing when a cable had reached constant temperature with any given condition of heating. The final test of thermal equilibrium was the wattmeter indication, the power loss in the dielectric, being very sensitive to temperature. Final loss measurements were not taken until the wattmeter indication had been constant for at least thirty minutes.

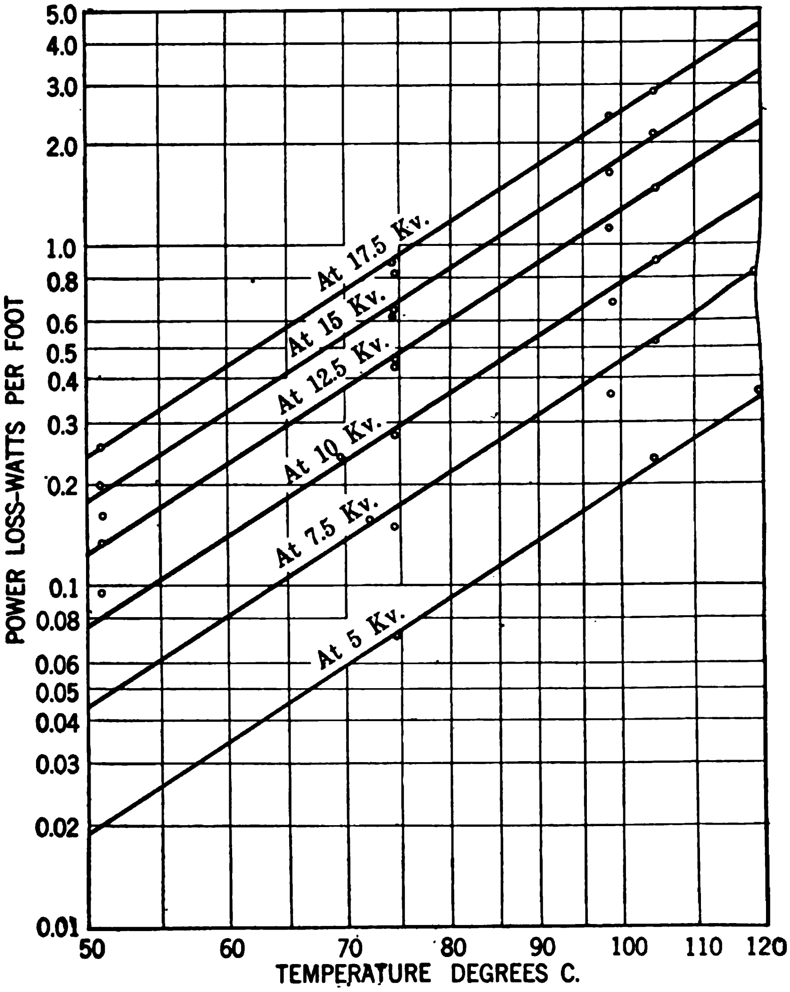

FIG. 5—POWER LOSS VS. TEMPERATURE AT VARIOUS TEMPERATURES—CABLE *A*—(LOGARITHMIC SCALE)

*Typical Data.* While this paper is primarily a discussion of the method of measurement of dielectric losses in cables, it is

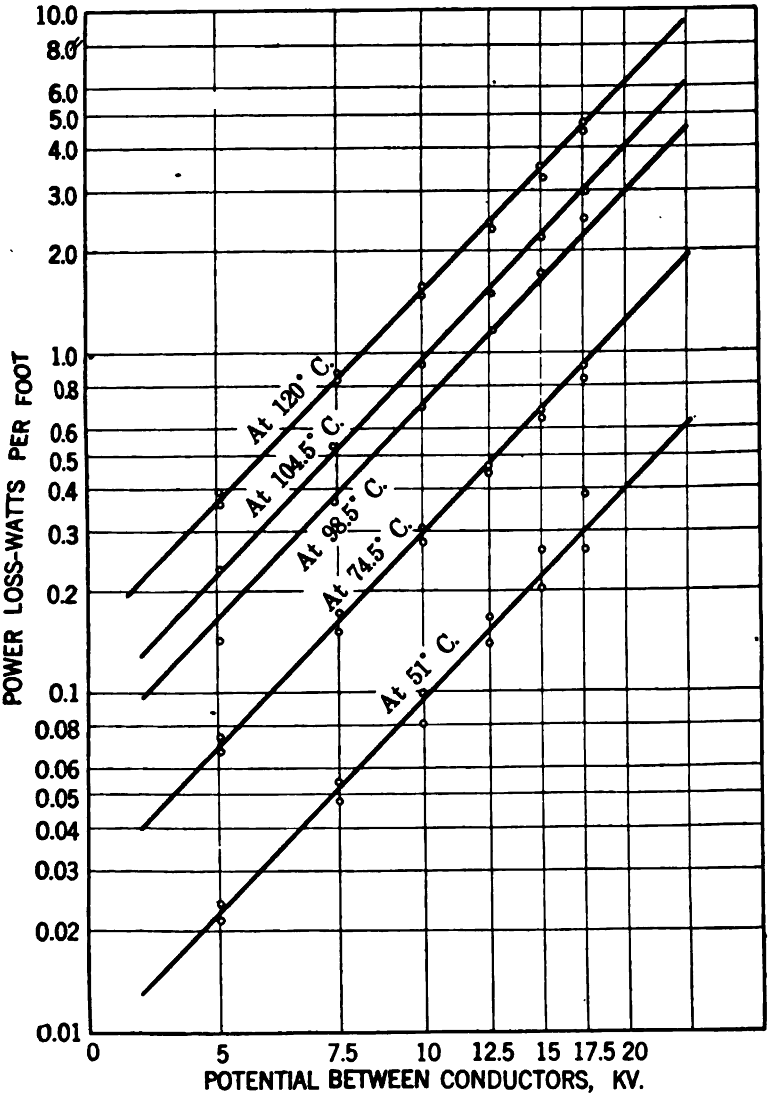

FIG. 6—POWER LOSS VS. VOLTAGE AT VARIOUS TEMPERATURES—CABLE A—(LOGARITHMIC SCALE)

thought that some typical data which have been obtained will be of interest. Table I gives summaries of the power loss in the dielectric and power-factor data for two cables, one having a

TABLE I.

CABLE A, LOW LOSS

(a) *Power loss in dielectric, watts per foot.*

| Temperature deg. cent. | Kilovolts between conductors | | | | | |
|---|---|---|---|---|---|---|
| | 5 | 7.5 | 10 | 12.5 | 15 | 17.5 |
| 50 | 0.02 | 0.045 | 0.076 | 0.125 | 0.18 | 0.24 |
| 60 | 0.035 | 0.082 | 0.14 | 0.23 | 0.33 | 0.44 |
| 70 | 0.058 | 0.14 | 0.235 | 0 385 | 0.55 | 0.74 |
| 80 | 0.092 | 0.215 | 0.37 | 0.60 | 0.83 | 1.15 |
| 90 | 0.135 | 0.32 | 0.55 | 0.90 | 1.25 | 1.70 |
| 100 | 0.19 | 0.45 | 0.78 | 1.28 | 1.80 | 2.45 |
| 110 | 0.27 | 0.63 | 1.08 | 1.75 | 2.50 | 3.40 |
| 120 | 0.35 | 0.84 | 1.40 | 2.35 | 3.30 | 4.40 |

(b) *Average Three-Phase Power Factor**

| Temperature, deg. cent. | Power factor, per cent |
|---|---|
| 50 | 3.5 |
| 60 | 6.5 |
| 70 | 10.5 |
| 80 | 15. |
| 90 | 21.5 |
| 100 | 31. |
| 110 | 41. |
| 120 | 52. |

CABLE B, HIGH LOSS

(a) *Power loss in dielectric, watts per foot.*

| Temperature deg. cent. | Kilovolts between conductors | | | | | |
|---|---|---|---|---|---|---|
| | 5 | 7.5 | 10 | 12.5 | 15 | 17.5 |
| 50 | 0.07 | 0.18 | 0.30 | 0.49 | 0.74 | 1.06 |
| 60 | 0.16 | 0.32 | 0.58 | 0.93 | 1.40 | 2.00 |
| 70 | 0.22 | 0.55 | 0.98 | 1.60 | 2.45 | 4.45 |
| 80 | 0.36 | 0.89 | 1.60 | 2.60 | 3.85 | 5.55 |
| 90 | 0.58 | 1.35 | 2.40 | 3.90 | 5.85 | 8.35 |
| 100 | 0.78 | 1.95 | 3.50 | 5.70 | 8.50 | 12.3 |
| 110 | 1.10 | 2.75 | 4.95 | 8.00 | 12.0 | 17.2 |
| 120 | 1.50 | 3.75 | 6.65 | 10.8 | 16.2 | 23.2 |

(b) *Average Three-Phase Power Factor**

| Temperature, deg. cent. | Power factor, per cent |
|---|---|
| 50 | 10. |
| 60 | 21. |
| 70 | 32.5 |
| 80 | 44. |
| 90 | 55.5 |
| 100 | 66. |
| 110 | 72. |
| 120 | 75. |

*Computed from ratio of total three-phase watts to total three-phase reactive volt-amperes, the figures given being the average for all voltages.

much higher loss than the other. The data as actually obtained on samples 10 feet long are shown graphically in Figs. 5 to 11 inclusive—the first four being curves plotted with logarithmic scales and the last three with arithmetical scales. The figures

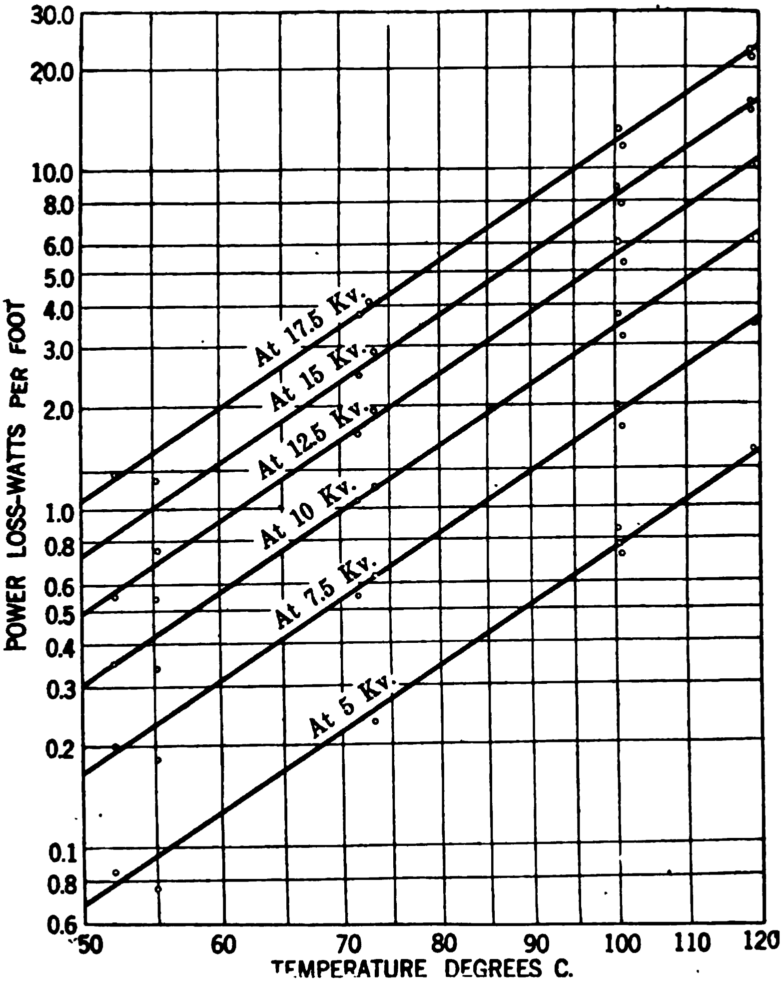

FIG. 7—POWER LOSS VS. TEMPERATURE AT VARIOUS TEMPERATURES—CABLE *B*—(LOGARITHMIC SCALE)

given in the tables were taken from these curves at fixed interval temperatures in order to make comparisons more convenient. It is obviously not possible in tests of this character to adjust the temperatures to exactly predetermined values.

*Use of Logarithmic Scales.* The general relations of the variables in experimental data are quickly comprehended when the

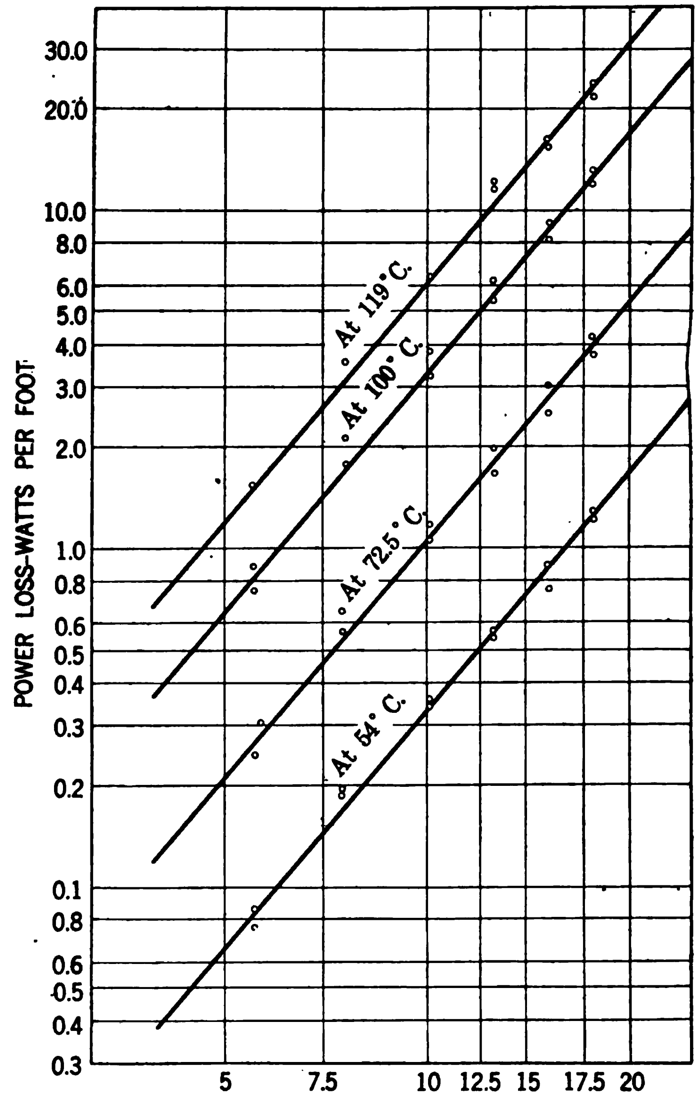

FIG. 8—POWER LOSS VS. VOLTAGE AT VARIOUS TEMPERATURES—CABLE *B*—(LOGARITHMIC SCALE)

data are presented in the shape of curves plotted in the usual way with rectangular co-ordinates and arithmetic scales. But when something more than simply the general relation of the variables is desired, curves plotted with rectangular co-ordinates and logarithmic scales frequently have many advantages. For example, the curves plotted in Figs. 5, 6, 7 and 8 show:

(a) That no serious error was made because the points fall reasonably close to a form of definite curve which is easily drawn, namely a straight line.

(b) That the relation of two of the variables is independent of the third. Thus, in Figs. 5 and 7, the fact that the several lines are parallel shows that a given change in temperature will produce the same per cent change in power loss of the dielectric whether the potential applied to the cable is 5000 volts or 20,000 volts. Similarly, Figs. 6 and 8 show that a given change in voltage will produce the same per cent change in loss whether the temperature is 50 deg. cent. or 100 deg. cent.

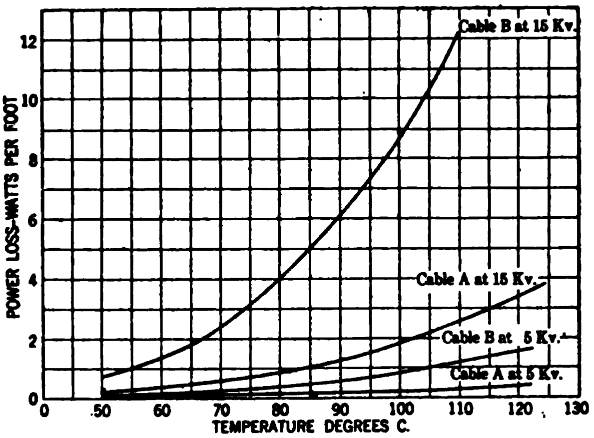

Fig. 9—Average Watts vs. Temperature (Arithmetical Scale)

(c) That the curves being straight lines which make an angle with the axes, the loss varies with some constant power of the voltage and temperature respectively.

(d) That, the relation between the variables may be expressed by the equations:

$$W \text{ (at a given potential)} = k\,T^{n_t} \text{ and}$$

$$W \text{ (at a given temperature)} = k\,E^{n_e} \text{ where}$$

$W$ = power loss of dielectric in watts, $T$ = temperature in degrees centigrade, $E$ = potential in volts, $n_t$ and $n_e$ = exponents and $k$ = constant.

The value of the exponents is readily computed from the relations

$$n_t = \frac{\log \frac{W_1}{W_2}}{\log \frac{T_1}{T_2}} \quad \text{and } n_e = \frac{\log \frac{W_1}{W_2}}{\log \frac{E_1}{E_2}}$$

where $W_1$ and $W_2$ are the losses corresponding to temperatures $T_1$ and $T_2$, or to voltages $E_1$ and $E_2$.

The exponents for cables *A* and *B* referred to in the preceding tables and also for two other cables are given in the following

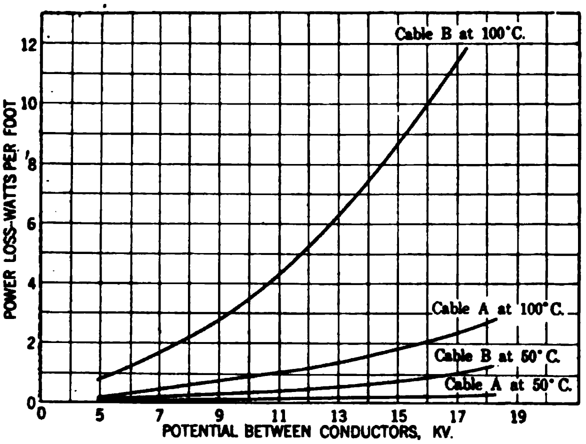

FIG. 10—AVERAGE WATTS VS. VOLTAGE (ARITHMETICAL SCALE)

tabulation. The power loss in the dielectric per foot at 80 deg. cent. and 15,000 volts is given for purposes of comparison.

| Cable | $n_t$ | $n_e$ | $W$ |
|---|---|---|---|
| *A* | 3.36 | 2.07 | 0.85 |
| *B* | 3.54 | 2.34 | 3.85 |
| *C* | 3.77 | 2.01 | 1.76 |
| *D* | 4.26 | 2.16 | 2.62 |

$n_t$ = exponent for temperature change, $n_e$ = exponent for voltage change, $W$ = power loss in the dielectric per foot in watts.

*Three-Phase vs. Single-Phase Tests.* Single-phase measurements were made on a number of specimens in addition to the three-phase measurements. These single-phase measurements included the following, all measured at Y voltage:

(a) $R_1$, $R_2$, $R_3$ = power loss in the dielectric between each conductor and the other two conductors connected to lead sheath, that is referring to Fig. 12, $R_1 = W_1 + W_4 + W_5$, $R_2 = W_2 + W_4 + W_6$ and $R_3 = W_3 + W_5 + W_6$.

(b) $R_4$ = power loss in the dielectric between all three conductors connected together and the lead sheath, that is (Fig. 12) $R_4 = W_1 + W_2 + W_3$.

If it is assumed that the conditions with three-phase potential

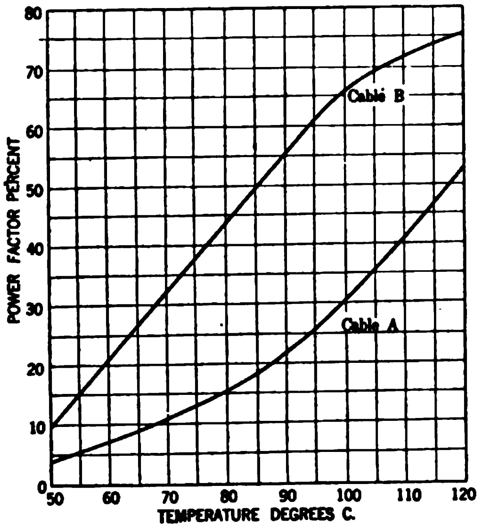

FIG. 11—AVERAGE POWER FACTOR VS. TEMPERATURE

are the same as in these single-phase tests, a value for three-phase loss can be computed from the relation

$$W \text{ (total watts)} = \frac{3\,(R_1 + R_2 + R_3) - R_4}{2}$$

The three-phase loss corresponding to single-phase conditions has been calculated on this basis for a number of cables and the ratio of the measured three-phase loss to computed loss is given in Table II.

These figures do not appear to settle definitely the question of the relation of measured and calculated losses. But if it is assumed that the variations or apparent inconsistencies have no significance and that all of the figures for a given cable can be averaged, the results in the order of the cable designations are 1.015, 0.968, 1.010 and 0.967 respectively. It is questionable, however, if the variations are merely accidental because the data for the calculations were taken from straight line curves plotted on logarithmic paper. Furthermore, in some cases there seems to be a more or less definite change in the ratio with voltage especially in the case of cable *D*.

The most significant thing shown by these figures is that the measured value is very frequently considerably ***lower*** than the calculated value. If the figures are reliable, this indicates that the theory of a greater power loss in the dielectric between conductors with three-phase potential is not substantiated. In any case it is clear that this question should be investigated further although it is probable that, for, all practical purposes, the power loss may be determined from two single-phase measurements without serious error.

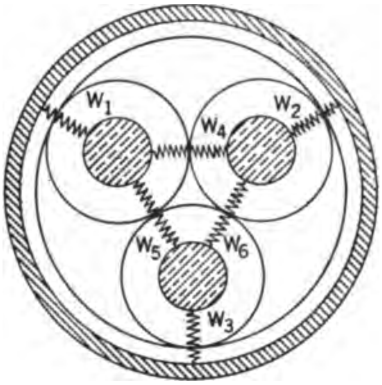

Fig. 12

*Application of Data.* The value of tests of this character may be largely relative because the results do not necessarily apply to operating conditions. In these tests the cable was at a uniform temperature throughout, while under operating conditions the temperature of the dielectric varies from a maximum at the conductor to a minimum at the lead sheath, so that the power loss per unit volume of insulation is higher near the conductor than it is next to the lead. However, Bang and Louis[6] found this gradient to be very small (lead temperature 86 deg., copper temperature 89 deg.) under conditions closely duplicating those in

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[6] loc. cit.

service. Of course if the temperature gradient is as small as these figures indicate, then it may be assumed without serious error that the entire cable is at the temperature of the lead and if this temperature can be measured, the total losses in the insulation can be computed if a laboratory test of a sample of the cable has been made. However, this question of what are the actual temperature conditions in practise is very important and in order to get data under conditions approximating working conditions, tests are now being made with a length of cable 20 feet long installed in a concrete conduit in the laboratory. The dielectric

TABLE II—RATIO OF MEASURED TO CALCULATED THREE-PHASE POWER LOSS IN DIELECTRIC

| Potential between conductors, kilovolts | Temperature, deg. cent. | | | |
|---|---|---|---|---|
| | 50 | 75 | 100 | 120 |
| | | CABLE A | | |
| 5.0 | 0.79 | .... | 1.03 | .... |
| 7.5 | 1.07 | .... | 1.02 | .... |
| 10.0 | 0.91 | .... | 1.03 | .... |
| 12.5 | 1.16 | .... | 1.02 | .... |
| 15.0 | 1.11 | .... | 0.96 | .... |
| 17.5 | 1.11 | .... | 0.98 | .... |
| | | CABLE B | | |
| 5.0 | 1.26 | 0.86 | 0.91 | 0.79 |
| 7.5 | 1.11 | 0.80 | 0.81 | 0.93 |
| 10.0 | 1.12 | 0.87 | 0.88 | 0.92 |
| 12.5 | 1.13 | 0.88 | 0.89 | 0.99 |
| 15.0 | 1.18 | 1.02 | 0.90 | 0.98 |
| 17.5 | 1.11 | 0.93 | 0.97 | 1.03 |
| | | CABLE C | | |
| 5.0 | .... | 1.05 | 1.06 | 0.96 |
| 7.5 | 0.96 | 0.94 | 1.03 | 1.02 |
| 10.0 | 1.03 | 1.00 | 1.09 | 0.95 |
| 12.5 | 0.91 | 0.93 | 1.04 | 0.95 |
| 15.0 | 1.07 | 1.10 | 1.11 | 1.00 |
| 17.5 | .... | .... | .... | .... |
| | | CABLE D | | |
| 5.0 | 0.66 | 0.78 | 0.89 | .... |
| 7.5 | 0.88 | 0.85 | 0.82 | .... |
| 10.0 | 0.98 | 0.89 | 1.02 | .... |
| 12.5 | 1.04 | 0.97 | 0.97 | .... |
| 15.0 | 1.08 | 1.02 | 1.03 | .... |
| 17.5 | 1.19 | 1.15 | 1.18 | .... |

power losses will be measured from time to time while the cable is carrying its rated current and rated voltage until constant thermal conditions have been reached. Incidentally, various temperatures will be measured including the copper by increase in resistance, and the lead by thermocouples. It is hoped that these data together with those already obtained for the same sample under other conditions will allow operation of the installed cable, which this sample represents, at maximum carrying capacity without danger of failures due to hot spots.

## CONCLUSIONS

The following conclusions may be deduced from the preceding discussion:

(a) The power loss in the dielectric of a three-conductor cable under actual three-phase conditions can be readily measured in the laboratory with specimens ten feet long.

(b) No special apparatus is necessary for such measurements other than a reflecting high-sensitivity wattmeter.

(c) Apparently the power loss in the dielectric cannot in all cases be accurately calculated from data obtained in single-phase tests although it is highly probable that for all practical purposes, the discrepancy would not be serious. Further investigation is necessary, however, before final conclusions on this point can be stated.

(d) While the method of determining power losses in the dielectric directly by three-phase measurements involves more complication in preparation and slightly more time in the actual measurements, it has the important advantage that the results are conclusive and not subject to the uncertainty which pertains to results calculated from single-phase measurements.

These measurements have been successfully made up to potentials corresponding to 30,000 volts between conductors. At higher voltages it is probable that corona losses from the test circuits would soon begin to give trouble so that a guard circuit to take care of losses through the air in addition to those directly to ground would have to be provided.

The precision obtained in these measurements was probably not very high, perhaps not over five per cent but this was quite sufficient for the purpose in mind at the time, which was primarily to determine the approximate power loss in the dielectric of the samples at various temperatures. The other data obtained were incidental. It is believed, however, after considerable experience with this method, that by taking particular care in the construction and arrangement of the apparatus, all of the important dielectric data of a cable can be obtained with a relatively high order of precision.

Much of the material in this paper was obtained in connection with a series of tests made for Mr. W. H. Cole, Supt. of Street Engineering, Edison Electric Illuminating Company of Boston, and the author wishes, in closing, to express his particular appreciation of Mr. Cole's courtesy in permitting the use of a portion of the information and data obtained in that investigation.

## DISCUSSION ON "MEASUREMENT OF POWER LOSSES IN DIELECTRICS OF THREE-CONDUCTOR HIGH-TENSION CABLES" (FARMER), NEW YORK, FEBRUARY 15, 1918.

**Henry W. Fisher:** At the top of the second page Mr. Farmer says: "Until two or three years ago the only factors which had been given serious consideration in determining the temperature of cables were the $I^2 R$ losses in the conductor and the thermal conductivity of the surrounding media." I think I will have to differ with Mr. Farmer in that respect, because as far back as the year 1907 I commenced investigations along this line, and at that time there had been papers written abroad on the same subject, and as a result of these investigations, in our manufacture we did take into account the dielectric losses, and undoubtedly other manufacturers have done so, too. For years back we have taken into account the heat due to dielectric loss as well as $I^2 R$ loss when figuring the carrying capacity of high-voltage cables.

As a result of these experiments with saturated paper, cables were constructed in which petroleum products were used in large proportions. Such cables were made as early as 1910, and some of these cables had very low dielectric loss.

About that time, it was found that some of the yellow varnished cambrics had very high dielectric loss. Some had a power factor as high as 20 per cent at ordinary temperatures. That meant an investigation leading to an improvement in the quality of varnished cambric. The above explanation will show that years ago the dielectric properties of cables were greatly improved by the use of a mineral-base compound, although this subject has come prominently before the Institute only within the last year or so.

**R. W. Atkinson:** The method used by Mr. Farmer is very similar to one of the methods we have used in our laboratory, and it is very fortunate for those who may be interested in making these measurements to have the data at hand. There is one thing that a person should be cautioned against, and that is, that not all of the precautions necessary in that sort of work can be described in a short paper. There is one small feature of that kind that undoubtedly was taken care of in Mr. Farmer's measurements, but it might easily lead to serious error if it were forgotten. The Ayrton shunt used in connection with the current coil of the wattmeter makes an exceedingly useful device. It gives a wide range for the instrument, and properly used it is entirely satisfactory. If, however, the coil of the wattmeter has too much inductance, or the Ayrton shunt is inductive (if these are of small magnitude, compensation may be made), serious power factor errors will result.

One difference, not at all fundamental, in the measurements, as we have them made, as compared with Mr. Farmer's measurements, is that we have found it most convenient to use a high non-inductive resistance right across the line in the same place he used his potential transformers. Each method has its own advantages and disadvantages. One disadvantage of the high resistance method is the amount of power that is taken and the fact that a large transformer is needed. If the transformer is available, however—I mean if the transformer furnishing the power for the test is large enough—the method becomes very satisfactory. Phase errors of the resistance, which may not be absolutely non-inductive, can be taken care of in the same way that the transformer errors can be taken care of, and will be so small—at least we have found them so—that they can be easily taken into account.

In regard to logarithmic paper—we have used paper which is logarithmic in one direction and arithmetical in another direction, and plot the temperature in the arithmetical direction. If you wanted to plot the temperature of zero degrees on the logarithmic scale, you would be pretty far off at the left hand, and besides that if even on these particular cables a little lower temperature had been reached, the departure from a straight line would have been very material, but in the case of other cable samples, it will be found that the straight line, using the logarithmic or semi-logarithmic paper, straightens out the curve in every case and is very useful for the reasons suggested.

In the three-phase measurements and three-phase losses, as calculated from single-phase measurements in Table II, I noticed in the first column that the values alternate, low, high; low, high; low. The author tells us these were taken from straight line curves. I wonder why they came out just as they did. We have ourselves found power factors in the case of the three-phase measurements lower than the single-phase. The particular conditions where the three-phase power factors as measured are absolutely different from those calculated from single-phase measurements, are where the power factor or capacity changes with the voltage. If there is a material change in either capacity or power factor with voltage, then no simple theoretical way of calculating three-phase measurements exists. The fundamental assumption in deriving the formula for the calculation falls down when the power factor or the capacity changes with voltage. To illustrate how far off we may get in the case of the single-phase calculation, the calculation from single-phase measurements may give a negative loss three-phase, when the loss is actually very large. We have found an empirical method which gives good results.

**C. W. Davis:** I would like to emphasize particularly the point Mr. Fisher made that dielectric loss tests, particularly on cables, are not new. There has been a tremendous amount of work done along this line for years, and its importance has been well understood by some cable manufacturers, at least both here and abroad, although I happen to know of many cases where attempts have been made to give information on dielectric losses to prospective cable buyers, that it has been looked on as more or less superfluous and decidedly academic.

I think it is rather more nearly in accord with the facts to say that it is only in recent years that people have begun to realize that a few, if not all, cable engineers have been working along the right lines, and have been talking about things that have real significance. Around about 1900 some work along this line was done abroad. A little later, Monash published his very important contribution to this subject, and Fisher, in 1907 in the A. I. E. E. TRANSACTIONS gave the results of some of his work on cables and wires. These tests were called power factor tests, and not energy loss tests, but the terms are practically interchangeable.

Later on a tremendous amount of work was done abroad, particularly in Germany, as well as in this country, and there have been many excellent papers on dielectric loss measurements on cables and condensers in the foreign and domestic technical press and in the bulletins of the U. S. Bureau of Standards. Perhaps the best bibliography on the subject up to 1912 was included as an appendix to a paper presented to the British Institute of Electrical Engineers by Mr. E. H. Payner, in 1912.

Mr. C. E. Skinner, some years ago made use of a high-voltage electrometer for dielectric loss measurements, and while I do not recall that he published any results of dielectric loss measurements in cables, his instruments were entirely suitable for the work; and a similar instrument and a slightly modified method were used by Mr. Rayner for his work.

It is not desirable to limit the methods of test, the main thing is to get results. We have used bridge methods, the dynamometer methods, and the electrometer methods, and have been measuring dielectric loss at high voltage as well as low voltage since 1907.

For five years we have used an electrometer like the one described by Mr. Rayner and Mr. Skinner. We have had four different instruments working at different voltages and working satisfactorily. They are exceedingly simple to work when you know how to work them, but exceedingly difficult when you do not know.

The fact that entirely satisfactory results can be obtained on apparatus of such diverse nature is sufficient evidence in itself that we should not be limited in the methods of measurement. In the electrometer, in particular, you have an exceedingly sensitive piece of apparatus, and while instruments of the sensitivity we have used are doubtless not necessary for regular cable measurements, it is of interest to know that instruments of such sensitivity as will readily measure the losses at from 2 kv. to 20 kv. in condensers and cable specimens ranging from about 0.0005 to 0.001 microfarads have been used daily by us for years with entirely satisfactory results.

Cable specimens 24 in. (60.9 cm.) in length and of such small capacity obviously present considerable difficulties in the way of accurate measurements, greater difficulties than are met with in measuring cable lengths of ten or more feet.

The air condenser, as a method of checking, is a very useful one. We have used air condensers for checking purposes for about four years. Our first air condensers were a series or battery of concentric vertical tubes with guarded ends. These were very satisfactory, except that they took up much room. About three years ago we built plate air condensers, much like those recently described in the paper of Shanklin and Clark, and these have been constantly used for checking since that time. Some of our methods are counter-checked from outside sources. The Westinghouse Company on several occasions measured high-voltage condensers for us by still different methods in order that we might have another check upon our results.

**D. DuBois:** Mr. Farmer makes reference to a paper by Hochstadter in which the electrical stresses in the center of a cable are studied. Hochstadter goes further than Mr. Farmer and states that the stresses between points 1, 2 and 3 in Fig. 1 are 87 per cent of the voltage between conductors. This calculation as Mr. Farmer states is based on the assumption that point 4, in the center of the cable, can be considered fixed at ground potential. I do not believe that this assumption is correct nor do I believe that such high electrical stresses are at any time found between the points in question. In a three-phase cable there is always a neutral plane passing through the cable and this plane rotates with the phase rotation. As it rotates it bends and straightens and at certain times passes through the center point but at other times it moves away towards those conductors of lower potential.

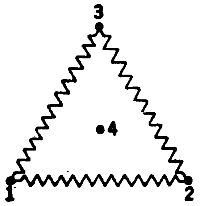

Fig 1.

This shifting of the neutral plane greatly modifies the internal stresses and it is therefore not surprising to see by the Table II that their effect in increasing the losses is not of great importance.

The fact that cables have been found with their central cores badly charred and the other insulation unburned has been explained as due to these central stresses but such charring can be explained very simply.

Any cable, no matter how good the insulation, if hot enough will be burned out by dielectric losses. Mr. Farmer shows that the losses vary with about the 4th power of the temperature and therefore a few degrees difference in temperature make a great difference in the losses. Any cable carrying current is hotter in the center than at any other place. It follows that the center is the first place to reach the condition where heat production exceeds heat loss. When that condition is reached there is a great increase of heat production and a great increase of temperature in the center of the cable. It therefore follows that any cable failing through dielectric loss will first burn at the center.

In cables where dielectric loss is high it is difficult to make tests without the dielectric becoming heated during the tests and in three-phase tests a cable would become slightly hotter in the center. This would cause a greater loss in the center, which might erroneously be considered as due to excessive voltage strains in the center. It may be that in the Table II the definite ratio change in the case of cable *D* could be accounted for by internal heating at the higher voltages during the three-phase tests.

**G. B. Shanklin:** We are glad to see that Mr. Farmer has developed a method of correcting for phase-angle errors by calculation and individual measurement of the different inductances. We have never actually tried this method in our laboratory, looking at it in the light of a correction from the short end of a very long lever. An air condenser that automatically takes care of the combined errors such as inductance angles, corona and line leakage has proved very satisfactory. Mr. Farmer's limit of 30 kv. above which line corona becomes a serious factor, is fully verified by our own experience.

In regard to the method of working up and analyzing dielectric energy loss data—Plotting the values on logarithmic paper and drawing an average straight line through them does not give results of great analytical value. For quantitative or empirical purposes this method does very well but the results have no physical significance. We did not begin to make real progress in studying dielectric energy loss until we reduced watts and current to the more fundamental values, capacity and resistivity per unit length or per unit volume. These fundamentals clearly bring out characteristics that would be otherwise entirely overlooked.

We have wanted for some time to get an accurate comparison between three-phase loss calculated from single-phase readings and measured three-phase loss, and regret that Mr. Farmer did not cover this more in detail. It is possible to get a much higher degree of accuracy with single-phase measurements than with three-phase and we have always felt that this more than compensates for any slight discrepancy between calculated and actual three-phase values.

There should not be any great difference between the calculated and actual three-phase loss in good grade cable. This difference is due to the excess introduced by higher tangential stresses in the central triangle of the cross sections when three-phase voltage is applied. Surface leakage caused by tangential stresses is negligible in good grade insulation. It is only in poor grade material that it is appreciable. The poorer the cable, then, the greater the difference should be between measured and calculated three-phase losses.

I believe that every one feels that the time has come when all cable specifications should contain a clause limiting the dielectric energy loss to some specified value. The Transmission Committee of the Institute has been working towards this end for some time and in the desire to be of help to them I have given the question considerable thought.

Our experience is that any impregnated paper cable having a power factor higher than 35 or 40 per cent at 100 deg. cent. will not withstand reasonably severe operating conditions. This is a simple rule to follow and one not at all difficult of attainment. The power factors and losses are high enough at 100 deg. cent. to largely eliminate those errors that prove such serious factors at lower temperatures. Unless some such simple clause as this is adopted I do not believe energy loss specifications would prove much of a success, for energy loss measurements require more time, care, patience and money than is generally available as a commercial proposition.

**John L. Harper** (Read by Mr. J A. Johnson): So far as I am aware, the method of test proposed and carried out by Mr. Farmer is the only such test made by parties not interested in some particular manufacturer or some particular customer, and Mr. Farmer's laboratory is now the only independent laboratory equipped for this form of work which is impartially open to both manufacturers and purchasers for determining comparative dielectric characteristics of cable.

I have recently divided up an order for cable between five of the large manufacturers of this material, and propose to have samples from each of these manufacturers tested by an independent laboratory for the purpose of determining a comparison between the different makes of cable. In case this independent test were made in Mr. Farmer's laboratory, the condition may possibly arise where a manufacturer would consider that the form of test used by Mr. Farmer did not give true results. With this probability in view, the writer hopes that representatives of the manufacturers may break through the protective crust with which they have surrounded the accomplishments of their respective plants in regard to dielectric conditions and enter into a free discussion upon the merits of the form of test proposed by Mr. Farmer.

The problem of the purchaser and user of cables is to make a cable carry the greatest amount of power possible with the least loss and the greatest safety from interruptions. The two major forms of loss in a cable may be put under the heads of "Copper Losses" and "Dielectric Losses", the copper losses being but to a slight extent under the control of the manufacturer while the dielectric losses are to a greater extent under his control. If it were true that a manufacturer could produce a cable which would operate with practically no dielectric losses, this cable might, and probably would, be an entirely useless thing for the transmission of power, as, in choosing an insulation for the elimination of dielectric losses, some form of insulation might be used that would produce an impracticable and useless cable from other standpoints; therefore, it is the desire of purchasers of cables to know how far the manufacturers can go in the elimination of dielectric losses and at the same time produce a cable satisfactory as to cost, safety and continuity of operation.

Having arrived at this point, it then devolves upon the user to so install and operate his cables that the best results may be obtained.

If, by uniform test, it is established that a decrease in the operating temperature of a cable produces a smaller dielectric loss, the user should either keep his loads below the amount producing high or dangerous temperatures or should remove the heat formed in the cables by artificial means, thus allowing a much greater transmission of power over them than would otherwise be possible.

That the user can greatly increase the capacity of cables by the removal of heat and the consequent decrease of the dielectric losses, is illustrated by an incident in the writer's experience. During the hot season of 1916, a certain conduit was operated with 1,977,000 cir. mils per leg but would carry no more than 22,000 kw. without burnouts from overheating, while this same conduit, with the copper increased to 2,627,000 cir. mils, carried 42,000 kw. without any burnouts from overheating, the temperature being kept down by either submerging or spraying the cables with water. Thus, with the conduit system increased 33 per cent in copper, the load was increased 86 per cent, and without any burnouts at all from overheating the cables.

It is my sincere hope that the presentation and discussion of the method of test proposed by Mr. Farmer may result in the Institute adopting and incorporating in its Standardization Rules some uniform method of test for dielectric losses which could reasonably be used by all manufacturing companies, and which, although it might lack entire perfection for the absolute determination of dielectric losses, would still be sufficiently correct for their determination within commercial requirements.

This would leave the manufacturing companies free to adopt such additional forms of test as they might wish for their own investigation and research, and would, at the same time, give protection to manufacturers accomplishing the best results when in competition with a poorer product tested by a different method which might indicate characteristics better than those which would have been shown by the standard method.

**W. H. Cole:** I am principally interested in the correct interpretation of the results of the tests made under Mr. Farmer's supervision, and what they mean to an operating company. About seventeen samples were included in the work done for the Boston Company during the past year. As a rule not less than two specimens were taken from each manufacturer's product. Practically all the specimens were picked out of our own stock and, therefore, are representative commercial samples.

As to the carrying capacity of a three-phase cable, there seems to be differences of opinion with reference to the existence of so-called tangential pressures and equalizing currents in the dielectric, but if there is anything in the theory that there are equalizing currents resulting from tangential pressures, the carrying capacity of a cable may be limited by the temperature at which the distribution of voltage stresses in the dielectric, become so altered from that obtaining at a lower temperature, as to materially over-stress certain parts of the dielectric. It is possible that this critical temperature in the usual type of three-phase cable construction, may be substantially lower than the governing temperature for actual and permanent deterioration of the insulating material. .

While the correct interpretation of the results heretofore obtained in dielectric loss tests is of great importance, more elaborate investigation is necessary to determine at what temperature a given type of cable construction may be operated before any localized acceleration of dielectric heating takes place, the effects of which are now recognized by many cable engineers. I refer to the cumulative heating of the center of a three-phase cable, generally supposed to be accelerated by the flow of tangential or equalizing currents, apart from the heating due to radial leakage current through the dielectric,

In general it may be said that any reduction of losses in cable dielectrics is of an order of importance equivalent to any progress in the direction of increasing the conductivity of the cable conductors. If the latter improvement were a possibility, it is obvious that no effort would be spared, to secure it.

**L. L. Elden and J. A. Walton** (by letter): The effect of dielectric losses in limiting the carrying capacity of three-conductor cables has received little consideration in cable specifications until recent experience has forced a tardy recognition of their importance.

The data submitted by Mr. Farmer support that of other investigators and should be studied carefully by users of similar cables. In many cases such study will be found to offer a satisfactory solution of many cable failures which have been experienced in the past, and at the same time, indicate methods which may avoid similar occurrences in new installations.

An analysis of the results of all tests of this character reveals one important fact, viz: that the insulating materials (paper and impregnating compounds) used by the various manufacturers of standard three-conductor transmission cables vary widely in dielectric losses at all temperatures and more particularly at the higher temperatures within permissible operating limits.

These results are undoubtedly due to the varying methods of manufacture and to the perfection of manufacturing processes, as much as to the materials themselves. That wide variations exist in the product of a single manufacturer is clearly shown by tests on samples of similar cables manufactured at different periods, sometimes only a few months and sometimes several years apart.

These conditions may be unavoidable, but where the differences are so wide and the losses so excessive in some cables, as to lead to their destruction within a relatively short period of operation under normal conditions, the inference must be drawn that there was something wrong in their manufacture.

It is therefore important in dealing with the effect of dielectric losses, that the constants developed from a single sample of cable be not applied to the product of another manufacturer.

It will be found preferable and more reliable to determine the characteristics of each lot purchased and from such tests determine the loading conditions under which each cable may be operated.

The dynamometer-type wattmeter developed by one of the large manufacturing concerns provides a fairly accurate method of measuring dielectric losses.

If this procedure is followed and routine tests are made for dielectric losses on each cable in service, about twice each year under nearly the same conditions, it will be found possible to keep a careful check on the condition of each cable, and to anticipate failures in many cases, when an increase in dielectric losses is found. This has actually been done in the practise of the company with which the writers are connected, it having been found possible to locate and replace sections of cable operating at higher temperatures than adjacent sections of the same cable before failures have occurred. Examination of the sections removed, revealed serious damage to the insulation from high temperature, due entirely to high dielectric losses caused by deterioration of insulating materials at the existing temperatures.

These conditions are sometimes found to be plainly evident to the touch by comparing the temperature of two sections of a cable each side of a joint, the cable all being made by the same manufacturer.

These losses have been found under test, to vary 1000 per cent between two samples of cable of the same make when excited at operating potential (13,800 volts), but carrying no load. The effect of such high losses on the carrying capacity of a cable may be left to the imagination.

It is further suggested that each cable in service should be confined to the product of a single manufacturer if possible and that repairs or replacements should preferably be made with similar cable or at least with cable possessing characteristics not inferior to the original cable.

From time to time representatives of operating companies, representatives of manufacturers and other investigators have presented data on the carrying capacity of three-conductor cables under various conditions and it is noted that Mr. Farmer is conducting some tests of this character for the purpose of determining the temperature constants of the cables under test.

Here again it is important not to apply the results of such tests on the product of one maker to that of another, as the

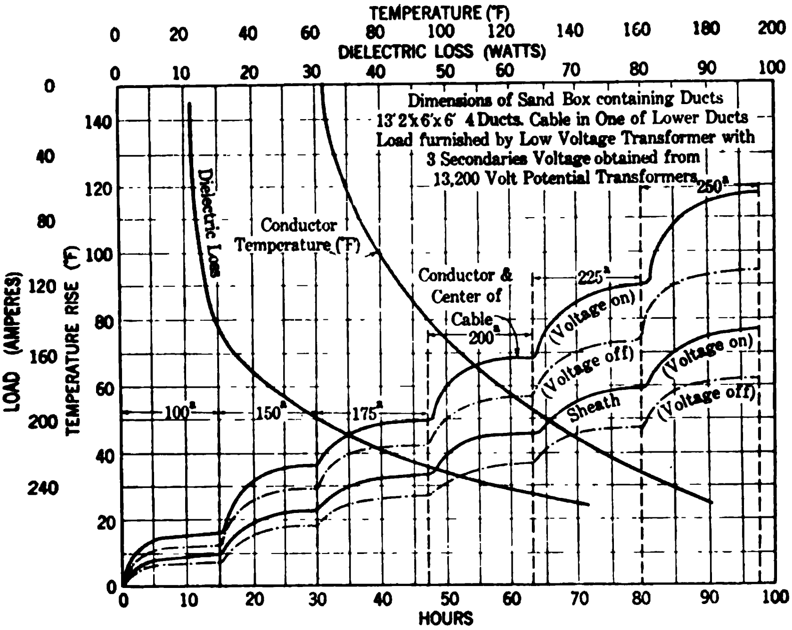

Fig. 2—Curves Showing Temperature Changes and Dielectric Loss in Short Duct System (13 ft.) Under Various Loads
3-Conductor, 4/0 Paper Cable. Length, 33 ft. By resistance coil method.

difference in dielectric losses may be responsible for wide differences in the temperature gradient between the lead sheath and conductors. Variations of fifty per cent on the same thickness of insulation have been noted in this respect.

It is further noted in this connection that although the A. I. E. E. Standardization Rules specify certain limiting temperatures for conductors insulated with paper, the manufacturers of this product are not in accord as to the operating temperatures for which they will guarantee their product, in some cases these limits being materially lower than that permitted by the rule in question.

Under such conditions it seems still more desirable to limit the cable used in each line to the product of a single manufacturer in order that the capacity rating of each line may be determined and based upon the same material.

Recent experience resulting from a study of cable failures has led us to study the temperature rise in 4/0 three-conductor $^7/_{32}$ in. × $^7/_{32}$ in. paper-insulated cables, as used on the system with which we are connected.

The result of this study as far as it has progressed with one manufacturer's cable is shown in Fig. 2.

These tests were conducted to determine the temperature rise at certain predetermined loads. The current at each step was maintained at a constant value until the maximum temperature rise was obtained with that current, after which the current was increased to the next higher quantity, until the entire test was complete.

Two series of tests were made to determine the effect of the dielectric losses in the conductor temperature.

These tests are indicated by the two series of curves marked "Voltage off" and "Voltage on".

"Voltage off" means that the test voltage was approximately 110 volts and "Voltage on" means a test voltage of approximately 13,600 volts.

It is obvious that with such data available and a knowledge of the characteristics of each cable in service, the operating engineer may accurately determine its safe loading capacity for any time of the year by merely observing the sheath temperatures in manholes and ducts at points where maximum air temperatures are known to exist. Such readings are easily taken and will be found to be most valuable if used as suggested.

In making purchases of cable we would recommend careful consideration of the dielectric loss constants at the higher operating temperatures (70 deg.—80 deg. cent.), and particularly with cables for operating pressures of 15,000 to 25,000 volts.

**Philip Torchio:** In line with Mr. Fisher's and Mr. Davis' information on measurement of dielectric losses in cables, I wish to state that I first measured not power factor, but watt loss per foot of cable, on paper and rubber-insulated cables, in 1901, and the results of my investigation were reported in a paper read before the Association of Edison Illuminating Companies at its meeting in 1902. Abstracts from that paper quoted before this Institute last June, pointed out specifically that the dielectric losses in cables were a function of the voltage, the frequency, and of a not yet determined function of the temperature. I believe that those tests were the first investigation ever made of these factors affecting the design and operation of cables.

With the aid of Figs. 3, 4, 5 and 6 I will now make a brief reference to the paper of Mr. Farmer, calling attention to the fact that in measuring losses in dielectric materials we find conflicting results which at times are inexplicable until we analyze further the conditions of the test. I will point out the importance of the shape of the voltage curve by showing results obtained under different conditions of test.

Fig. 3: This is a test, 60-cycle, which shows a ratio of the average voltage to maximum voltage of 14,900 to 20,800. The losses per foot at 100 deg. cent. are 0.8 watts.

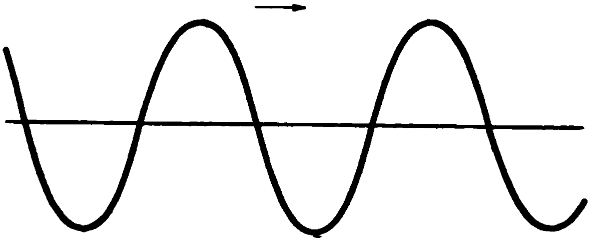

Fig. 3—Effective Voltage = 14910; Maximum Voltage = 20,800; 60 Cycles. Load on Testing Transformer: 8 ft. Single-Conductor Petroleum Cable. Potential Transformer with 700 ohms in secondary Circuit

Fig. 4: This is a curve on 25-cycles, which was very much distorted and shows that the ratio between the average voltage and the maximum voltage was 14,900 to 25,000. The losses were 1.20 watts, at 100 deg. cent. which are higher at 25-cycles than they were at 60-cycles.

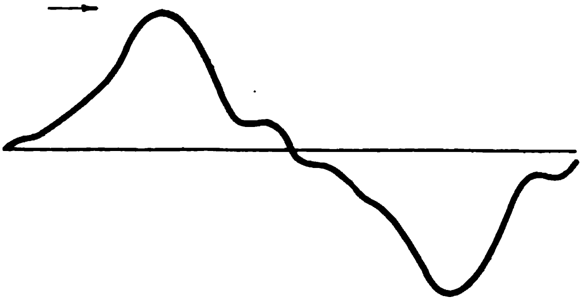

Fig. 4—Effective Voltage = 14,910; Maximum Voltage = 25,400; 25 Cycles. Load on Testing Transformer: 8 ft. Single-Conductor Petroleum Cable. Potential Transformer with 700 ohms in Secondary Circuit

Fig. 5: The next curve is also 25-cycles and by modifying the curve we reduce the losses to 0.68 watts at 100 deg. cent.

Fig. 6: The last curve, which is a 60-cycle curve, showed the average to be 14,921 and with this form at 60-cycles, we reduce the losses to 0.77 watts at 100 deg. cent.

We found that the distortions in the above voltage curves were caused by the transformers and regulators which stepped up and controlled the voltage supplied to the cable. This distortion was dependent upon the current supplied which in turn is dependent upon the voltage and amount of cable connected to the equipment.

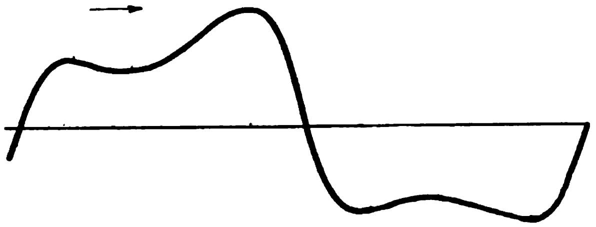

FIG. 5—EFFECTIVE VOLTAGE = 14,910; MAXIMUM VOLTAGE = 21,600; 25 CYCLES. LOAD ON TESTING TRANSFORMER: 8 FT. SINGLE-CONDUCTOR PETROLEUM CABLE. AIR CONDENSER: 95 FT. SINGLE-CONDUCTOR STANOLITE CABLE. POTENTIAL TRANSFORMER WITH 700 OHMS IN SECONDARY CIRCUIT

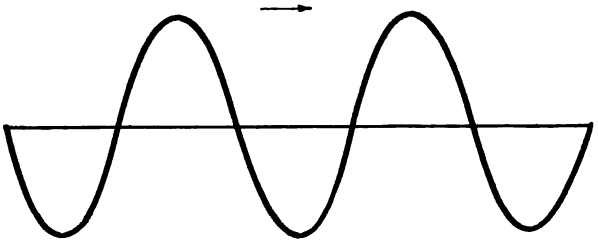

FIG. 6—EFFECTIVE VOLTAGE = 14,910; MAXIMUM VOLTAGE = 21,600; 60 CYCLES. LOAD ON TESTING TRANSFORMER: 8 FT. SINGLE-CONDUCTOR PETROLEUM CABLE. AIR CONDENSER: 95 FT. SINGLE-CONDUCTOR STANOLITE CABLE. POTENTIAL TRANSFORMER WITH 700 OHMS IN SECONDARY CIRCUIT

**F. W. Peek, Jr.:** About two years ago I was asked to investigate the cause of many cable failures on a large system. It was thought that these failures were due to high-frequency disturbances, because the failures were generally accompanied by lightning arrester discharges. An examination of the records showed that practically all of the failures were confined to a single make and type of cable. Everything but the load chart pointed to overheating as the probable cause of the trouble. An examination of some of these cables at practically no load showed them to be quite warm—obviously due to high dielectric loss.

Dielectric loss measurements on sections of these cables showed that the dielectric loss was of the same order as the full-load copper loss. The high-frequency disturbances were caused by the failures and were not the cause of the failures.

An examination of these cables showed them to be very loosely wrapped, and showed that the compound had migrated, leaving parts dry and parts still well impregnated. Such a condition will tend to cause high loss and failure by an unequal distribution of dielectric stress. In 1914, I made some tests to illustrate this, and will give an example here:

Twenty sheets of plain paper were placed between flat electrodes—the "rapidly applied" break-down voltage was 10 kv.

Twenty sheets of the same paper were oiled—the breakdown voltage was 16 kv.

Ten sheets of plain paper, and ten sheets of oiled, making a total of twenty, were tested. The break-down voltage was 8 kv., or less than for the twenty sheets of plain paper.

Of course the nature of the compound is also of great importance. Loss measurements made at the time a cable is purchased will not anticipate deterioration of compound, or migration of compound. These are best anticipated by the chemist, and an examination of the construction of the cable.

Mr. Farmer's three-phase measurements are very interesting and valuable. In making loss measurements at low power factor, great care is necessary to correct carefully for any angular displacement. As Mr. Farmer points out, if the air condenser method of making phase-angle correction is used at high voltages there is danger of error due to loss in the condenser. This is so, because while the loss in the air condenser per unit volume is very low compared to the unit loss in the insulation under test, the volume of the air condenser is so high compared to that of the insulation that the total loss in the air condenser may be high enough to cause serious error. Mr. Farmer made the correction at low voltage to prevent this error.

Referring to Mr. Farmer's Fig. 1: Naturally the points 1, 2, 3 and 4 are not likely to be at the same potential. This fact does not mean break-down or leakage between these points if the cables are properly made and impregnated. There are also many other points, not only on the insulation surface, but also in the space surrounding the insulated cables, not at the same potential. The unit stress is probably generally high at 1. The fact that the surface 1–2 is burned does not mean that there has been leakage over this surface; it may mean break down between 1–4 and 2–4. The values given for these stresses are not necessarily correct.

Referring to the method of heating the cables for test, a very good way is to bell the ends, seal them, and immerse in hot oil.

I have found that the loss in good insulation may be expressed by*:

$$p = afe^2 = bfg^2 \times 10^{-6} \text{ watts per cu. cm.}$$

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*Electrical Characteristics of Solid Insulations", F.W. Peek, Jr. *G. E. Review* 1915, November.

The constants $a$ and $b$ vary with the temperature, thus

$$b = kt^n$$

$$\text{or} \qquad p = k_1 fe^2 t^n = kfg^2 t^n.$$

Where the insulation contains occluded moisture, the loss equation takes the form

$$p = bg^2 (f+c)$$

This often makes it appear that the exponent of voltage is greater than the square. It will be noted that most of Mr. Farmer's values of this exponent are very close to 2.

The method of reducing from single phase to three phase seems complicated and artificial. It would have been very interesting if Mr. Farmer had given curves on the same cables for single phase and three phase.

In selecting cables it is important that in addition to the voltage tests, examination be made of the tightness of wrap, quality of paper, and of the compound. Certain compounds have been found by experience to be undesirable because of probable chemical change, unreliability, and bad temperature characteristics. Unfortunately, poor cables cannot always be reliably anticipated by loss temperature measurements. The loss measurements will, however, be of use to show that the cable is in good condition at the time of purchase. The same result can be more simply accomplished by applying voltage and measuring temperature. Much can, however, be done by examination of paper and compound.

Excessive, long time high-voltage tests should be avoided.

I believe high-voltage direct current offers a good method of making cable tests. I quote from my paper:* "The Effect of High Continuous Voltages on Air, Oil, and Solid Insulations."

"There is considerable difficulty in practise in making a-c. voltage tests on long lengths of cables, due to the size of the apparatus, which is necessarily large on account of charging current. The necessary kilovolt-amperes often amount to several hundred. The wave shape is often distorted by the leading current, the apparatus is difficult to move about, etc. D-c. tests would eliminate these difficulties, as very small apparatus would be required, providing such tests would detect faulty sections, etc. It cannot be said that a given d-c. voltage is equivalent to a given a-c. voltage. The above tests indicate, however, that faulty sections of cable could be equally as well located by d-c. tests as by a-c. tests. In cases of cracks, etc., the air- or compound-filled space would be broken down at the same maximum voltage on direct current or alternating current. In case of a fault due to moisture, the breakdown would apparently take place at about the same voltage, alternating current or direct current. In the case of a cable in good condition there

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*A. I. E. E. TRANS. Vol. XXXV, Part II, p. 800.

would be much less likelihood of injury by direct current than by alternating current of the same maximum voltage. A d-c. voltage equal to the maximum of the a-c. test voltage would, therefore, seem suitable for such tests."

Resistance measurements should be considered.

**M. G. Lloyd:** In regard to the difference between the single-phase and three-phase measurements, I see but three possible causes for such a difference. One has been mentioned by Mr. Atkinson, that due to the fact that parts of the dielectric were stressed differently in the single-phase test from what they were in the three-phase, and with varying power factor in the electric field strength, and with power factor perhaps varying differently in different specimens, we have no definite relation between the two.

Another reason for the difference might be lack of homogeneity in the dielectric material, so that measuring one part of the cable would not give a value proportionate to that which would be obtained if it were all subjected to the same stress.

There is still a third possibility. I think of it in connection with the analogy with magnetic hysteresis, that is, the loss occasioned in magnetic material when it is subjected to a varying magnetic field. Let me recall to your mind that the loss in magnetic material when placed in a rotating magnetic field is different from the loss when it is placed in an alternating magnetic field of the same maximum value. The difference there is generally in this respect,—that in low densities the loss is greater in the rotating field and very low in the alternating magnetic field, but at high densities the loss is lower in the rotating field and greater in the alternating magnetic field. Magnetic hysteresis cannot take place unless the permeability varies with the flux density of the material. Similarly, dielectric loss cannot easily take place without the dielectric constant varying with the strength of the electric field. It seems to me quite possible there may be some analagous effect, that the loss in the dielectric material in the rotating alternating field may be different from that in the simple reversing field, and I suggest that as a thing which may be investigated by those who are experimenting in this subject.

**James R. Craighead:** Mr. Farmer states that he considers the precision of the measurements to be about 5 per cent. The variations of the ratios shown in Table II are decidedly in excess of a 5 per cent limit, while the average test value is within 1 per cent of the calculated. Averaging the values at each voltage shows a curious characteristic by which the ratio goes up with the voltage by a more or less irregular curve, from about 0.92 to about 1.07. Can Mr. Farmer suggest an explanation of this?

**Comfort A. Adams:** In the first place, we do not have a homogenous dielectric. We have different materials—paper, cambric, and impregnated compound. We have some parts more thoroughly impregnated than others. We have some parts that will dry with air space or gas space, and with consequently increased loss. There are different temperatures in different parts of the cross-section; for instance, a much higher temperature in the center than on the outside. This changes the effective a-c. resistance, changes the losses per unit volume, changes the distribution of energy throughout, and accounts, in some part at least, for the great difficulty in comparing different cables.. It is not anything like as simple a matter as you might imagine.

Also, I think there is evidence which will lead us to believe that very minute quantities of moisture make more difference in the cable losses than some other things that we look after carefully.

Finally, this is not only a very important subject, one of the most important in transmission problems, but it is one in the investigation of which the ground has scarcely been scratched, and in which we have an immense amount to learn. We are dealing with problems and analyzing them on the assumption of certain approximate homogeneities, which do not exist, and we have got to get down, way down, to the bottom of things in a degree we have never done before.

**F. M. Farmer:** In regard to the statement in the paper which a number of speakers have taken exception to, where I intimate that nothing has been done on this subject up to two or three years ago, perhaps my wording was not very fortunate. Obviously, what was meant is that nothing has been done which has aroused serious interest among operating engineers until two or three years ago, although the subject has been very active since that time. We all know that there has been a tremendous amount of work done on the general subject of dielectric losses during the last fifteen years, but we also know that very little information has been made available to the profession at large, particularly in connection with underground cables. If the manufacturers have done a great deal of work, as stated by the representatives here, they have until very recently kept the accumulated information to themselves.

As to Mr. Fisher's point in regard to the carrying capacity of cables being limited by the dielectric strength, it is obvious that as the temperature goes up a point will be reached where desstructive action takes place and the cable will break down due to the dielectric strength becoming nil. In other words, if the insulating materials used in the cables did not suffer physical deterioration at high temperatures, the carrying capacity would be limited by the permissible loss in a given installation rather than by the "safe working temperature" as determined by the physical characteristics of the material.

Mr. Atkinson stated that there are a great many points in the way of precautions in tests of this character that have not been mentioned in the paper. Of course it goes without saying that all of the precautions that are usually taken in measurements of this class must be carefully observed. I simply tried to point out some of what might be called the extra precautions which have to be taken care of in this particular class of electrical measurements.

Mr. Davis' point in regard to the desirability of different investigators using different methods is very good. It is not necessary or desirable that everybody use the same method. What is essential is that the results obtained by the different investigators mean the same thing to everybody.

Mr. DuBois' conclusion that there is nothing in the Hochstadter theory may be quite correct. I simply stated this theory as a reason for making comparison between single-phase and three-phase tests. If there is anything in the theory, that is, if there is an extra loss due to tangential stresses, it ought to show up in the ratio between the two quantities. I am convinced that actual tests do not show any such loss of any considerable magnitude and naturally an experiment is usually the ultimate proof. In other words, the results we are getting in these peculiar cable failures are due to the cause which has been discussed here this afternoon, namely that the internal insulation naturally reaches a higher temperature than the rest of the insulation and therefore the loss is increased faster at that point, so that ultimate destruction takes place in the internal insulation long before it does in the outer insulation. In other words, destruction is probably not due to a higher dielectric loss per unit volume produced by tangential stresses.

In reference to Mr. Shanklin's question in regard to the formula which has been given for the three-phase loss calculated from single-phase tests, I think he has forgotten to consider the proper voltage for each single-phase loss. In a single-phase test, the voltage distribution under the application of three-phase potential cannot, of course, be duplicated because in the latter case the belt insulation is subjected to Y voltage while the insulation between conductors is subjected to delta voltage. Therefore, in calculations from single-phase measurements, this should be taken into account by assuming that the loss is proportional to the square of the voltage. Possibly it would have been clearer if the formulas in paragraph A under "Three-phase vs. Single-phase Tests" had been expressed as follows: (for single-phase tests made at Y voltage)

$$R_1 = W_1 + \frac{W_4}{3} + \frac{W_6}{3}, \text{ etc.}$$

Mr. Torchio's data showing the effect of wave form are most interesting. However, it is obvious that measurements of this character should not be undertaken without first investigating this point pretty carefully and being sure that the wave form is substantially a sine curve. It is one of the first precautions to be observed.

In reference to Mr. Craighead's inquiry I have no suggestion to offer. I believe that some of the apparent discrepancies which are most pronounced at the lower temperatures in the first cables tested are due to the temperature not being held for a sufficiently long time and not being measured with sufficient precision. This table was given merely to show that there is no very great difference between the three-phase losses calculated from single-phase tests and those obtained by direct three-phase measurement and is not given with any idea that the ratios obtained are precise. Although for all practical purposes these results show that there is no internal loss, still I feel that this phase of these measurements should be investigated further, knowing as we do now that the measurements could have been made more precisely.

*Lecture before the 6th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 15, 1918.*

# SOME APPLICATIONS OF ELECTROMAGNETIC THEORY TO MATTER

### BY ALBERT C. CREHORE

LAST year you were privileged to listen to an account of recent experimental observations in connection with atomic phenomena, I may say some of the fundamental facts of the physical world, coupled with an account of the way some of these facts are now generally interpreted in terms of "Radiation and Atomic Structure." The interpretation was principally in terms of the current Rutherford-Bohr theory of the atom.

It is the interpretation of physical observations that has engaged the speaker's attention for several years, and it has seemed to your committee that the time is ripe to present to you certain conclusions that have resulted. It goes without saying that no theory has ever yet been proposed that enables us to account for all the facts of observation by steps of logical deduction from certain assumed premises. Nevertheless, there is every reason why this high ideal should ever be kept before us. Unless we aim higher than we expect to attain, we will not accomplish so much as we expect.

I would not have you understand from this that no real progress toward the desired goal has been made. On the contrary, I believe it to be very great. How can we estimate the progress that has been made? Throughout the history of science we have seen one theory giving place to another sometimes in rapid succession. This has had a very depressing effect upon a certain class of people, who do not comprehend the real trend of it. I would impress upon you that this is one of the chief earmarks of progress. We cannot attain the realization of the ultimate goal at once, and, therefore, the only way to approach it is by a series of successive approximations. Each nearer approach involves of necessity some changes in preceding ideas.

It even seems probable that the fundamental hypothesis of atomic structure as composed of electrons, positive and negative, is entirely correct, and that the reason why we are not yet able to account for more of the observed phenomena is because we have not discovered the proper processes to follow in reasoning from the simple to the complex. I suspect that this is the truth of the matter, and, when you comprehend more fully the complexities of most of the processes involved, you will experience no difficulty in agreeing with me.

The discovery of the electron and the isolation of it, so that its properties might be measured, marked a new era in our conception of matter. The fact that each electron always carries an electrical charge of the same amount is most significant. It has led by natural steps to the present electrical theory of matter. It was also most natural to begin to apply to these electrons, as representing electrical charges, the electromagnetic theory that is known to apply to other electrical charges, a theory that had been developed by a generation of men long before the electron was known, as the chief exponent of which the name of Maxwell stands preeminent. This was done, of course, but it was soon found that the existing form of electromagnetic theory was inadequate to cope with single isolated charges moving with considerable velocities through space.

Electromagnetic theory itself, therefore, became the object of much more careful study. The result has been that we have witnessed, even in our day, a gradual evolution of electromagnetic theory. Maxwell would hardly recognize the form in which electromagnetic equations are expressed today. The new thing that has been introduced into the theory is the conception that the effects of a charge moving through the ether are propagated out in all directions from it with a velocity which is characteristic of the ether, the same velocity as that of light. This introduces important differences into the equations, sometimes referred to as "retarded potentials."

Some progress was made by the use of this theory in interpreting atomic phenomena, but there were certain observations, particularly connected with radiation from gases, that simply refused to be brought into line in any way that could be conceived. On the strength of this Max Planck proposed what is now known as his "quantum theory." It stands today as one of the boldest proposals ever suggested in physics. He originally arrived at the theory through a consideration of the radiation of energy from gases, but it has since been shown to have a very general application in all atomic phenomena. The boldness of his proposal may be seen from the following. Finding that electromagnetic theory was inadequate, as he conceived of it, he proposed to lay it aside and not use it. He then gave reasons for his proposal that, whenever energy is radiated anywhere from matter, the energy is transferred in units, or in quanta, so that the total energy transferred is some integral multiple of a fixed minimum of energy, the quantum.

Let it suffice to say that the application of Planck's quantum theory to radiation straightened out the difficulty immediately. Perhaps it would be better to say transferred the difficulty from one place to another, for difficulty there is and will be until the fundamental cause of the necessity for units or quanta of energy is comprehended. However, progress by the use of this new idea was very marked in every direction as regards atomic phenomena, The value of the unit or quantum of energy has now been experimentally determined. It is excessively small, but, when a quantity can be measured at all, its existence is not questioned. Its existence is now regarded as experimentally established.

The present trend of physicists is to generalize this idea, and to say that energy is essentially of an atomic nature, being composed of units in the same way as matter is so composed. This generalization implies that no flow of energy ever takes place that does not go in exact multiples of this unit. It seems to the speaker that there is danger in thus generalizing in advance of definite proof, or, at least, in letting our thoughts be guided as though this general proposition were true. A similar generalization was made without proof some years ago in physics. I refer to the principle of the conservation of matter, which has been published in most textbooks on physics of the older school. We have latterly been obliged to retract this. We do not know now that it is true. We cannot assert, however, that the contrary is true, namely, that matter is being created and destroyed. We simply do not know. We do know that the individual atoms do not remain intact as they were formerly thought to do, and that some of them are actually going to pieces, giving out parts of themselves and losing weight. Whether or not the electrons that they give out always enter into some other piece of matter, and thus make for the ultimate conservation of matter, it is not possible to say. We cannot say that the sum total of electrons in the universe remains constant.

You will easily comprehend that the effect of Planck's theory has been to divert attention from electromagnetic theory, since he was forced to lay it aside. This very act, and the fact that substantial progress was made by so doing, naturally brought electromagnetic theory into disrepute, so far as its application to electrons in atoms is concerned. It is fortunate that he had the courage to do it, because the result has been that we are in possession of valuable information about the atoms, which would otherwise now be unknown.

It must be admitted, however, that it will never be satisfactory to the human intellect to leave the situation in the state we find it today. The solid foundations upon which we must seek to found an atomic hypothesis should in some way be connected with the properties of the electron and its connection with the ether of space. These are fundamental parts of electromagnetic theory, which expresses in mathematical language just this connection between an electrical charge and the ether, that is, between matter and the ether. It even seems possible, nay indeed probable, that some good reason for the existence of a quantum of energy may yet be discovered by the help of electromagnetic theory, as being connected with the very constitution of the atoms themselves. This hope alone is sufficient incentive to urge us to continue the search by the use of this theory, at least to such a point that it can be proved the search is vain, a most difficult undertaking.

You perceive from the above remarks, when taken in connection with the title of this address, "Some applications of electromagnetic theory to matter," the difficult nature of the role that the speaker has assumed. Having come to a decision to apply electromagnetic theory, so far as possible in all strictness to the electrons in the atoms, for the purpose of learning where it gives a good account of itself, and where it requires some modification, I have gradually been led to the conclusion that comparatively little has heretofore been done in this direction, and that the results thus far attained are of so great value that a great deal more should be done.

It is the under dog that has the sympathy of the crowd, especially if he shows sufficient life to put up a plucky fight. During the present phase of the wave of progress in physics, electromagnetic theory, as applied to the atoms is like the under dog. It is on the defensive, and there are strong reasons why those who are striving to save it should have the sympathy of the crowd.

## II. Rutherford-Bohr Atomic Theory

With this introduction you may be better prepared to listen to the following account of some applications of electromagnetic theory to matter. As above stated the Institute had the opportunity last year to hear an address upon topics of absorbing interest to physicists, which included some account of the Rutherford-Bohr theory of the atom. This theory is, no doubt familiar to you, but some of the chief features of it will form the starting point of my remarks tonight. According to their theory an atom is a structure composed in part of a charge of positive electricity, the center of which is the geometrical center of the atom. This charge is supposed to have small dimensions compared with the size of the atom, or even of the negative electron. Around this nucleus one or more negatively charged electrons are supposed to be describing approximately circular orbits, the radii of which are very large as compared with the radius of the electron itself.

There may be several electrons describing the same orbit at the same time. They would then distribute themselves at equal intervals around this orbit on account of the mutual repulsion they have for each other, thus forming a ring of electrons. The atoms may be composed of several such rings all circulating around the same nucleus. Atoms, in the ordinary states of matter, have no resultant charge, and it is supposed that in this case the sum of the negative charges of the electrons is equal to the positive charge of the nucleus, thus making the total zero, when the atom is said to be neutral. Under special circumstances atoms may be caused to lose or to gain one or more electrons than they have when neutral, and then the atom manifests a resultant charge, positive if electrons are lost, negative if gained. It has been observed that such atomic charges are always some exact multiple of the charge on one electron, but it may be either positive or negative.

There are certain remarkable features in this theory introduced by Dr. N. Bohr, relating to the changes which take place in such an atom when it is disturbed by any cause. He finds that the radius of the orbit of the electrons changes from one fixed value to another, there being a series of possible radii; but, when the change begins, it does not stop until it arrives at one of the other possible radii, or, in other words, the radii must change by sudden jumps. This is a direct result of an application of the quantum theory above referred to to the atoms; for, by it, the change in energy must take place in quanta. Bohr has found as a result of his premises that there is a change in the kinetic energy of the electron in passing from one of these fixed orbits to another, and the size of the orbits has been calculated from *electromagnetic theory* by taking this change in the energy always as an exact multiple of the quantum of energy. The atom will then radiate energy when the electrons fall in towards the nucleus from one fixed orbit to another, and they absorb energy when they go outwards from the nucleus.

There are several assumptions that have been introduced in order to arrive at these results. First there is no radiation of energy at all when the orbit remains in one of the series of fixed positions, although the electrons are in rapid motion. Second the ordinary equations of electromagnetic theory apply to the electrons when they are in one of these fixed orbits not radiating energy, but that they do not apply when they are changing over from one radius to another and radiating energy. There have thus come to be recognized several states of the atoms, where the radii are fixed, known as the steady states, and where energy is neither radiated nor absorbed. All other states than these exist only during the radiation or absorption of energy.

## III. Two Great Natural Divisions of Matter

There is thus made a great natural division of all existing physical phenomena. The atoms are either in one of these steady states or they are not. To illustrate, we may consider that most of the matter in the solid crust of the earth is in the steady state, or, more strictly, it would be if it were at the absolute zero of temperature not radiating any heat. It is quite conceivable to think of the temperature of the earth as being lowered to the absolute zero without disturbing in any way the real constitution of the rocks of which the earth is composed, or of the temperature of a crystal being at the absolute zero. As an example of atoms not in the steady state, we may think of substances during the process of undergoing any chemical reactions, where a change of partners takes place with an accompanying transfer of energy among the atoms and molecules. This includes all vital processes in organic substances, where chemical reactions are continually taking place under certain special laws of guidance or causation. This latter class involving energy transfer includes most of the phenomena of the greatest interest to man, but the former class, the steady state, is of the utmost importance even if somewhat more restricted in variety. We owe to it the home in which we live, the solid structure of the earth. It also includes the large class of metals and building materials generally, in which the engineer is particularly interested.

It may be repeated, that, in this first class, the steady states, the Bohr theory, and other theories, assume that the ordinary electromagnetic theory is applicable to the electrons in an atom. The view is taken that it is not applicable to the second class involving energy changes in the Bohr theory, but it might be a better position to take to say that no one has as yet seen just how to apply the theory than that it is not applicable.

If we, therefore, confine our attention to those states of matter where there is no energy transfer, the first class, where the electromagnetic equations are said to apply even by those who indorse in every detail the Bohr theory of the atom, we ought to be able to make a beginning of progress in interpreting some of the physical properties of this class of matter by the correct application of electromagnetic theory. It is time enough to begin a consideration of the other necessarily more complicated class, where there is a transfer of energy, and where the applicability of the electromagnetic equations is at present in doubt, after we have achieved some sort of results in the simpler case. It seems as if this simpler case should naturally precede any consideration of the other, for one reason, because any success in the one is almost sure to shed some light upon the methods of procedure to be adopted in the other.

It is precisely to these steady states of matter, where the applicability of electromagnetic theory is admitted, that the speaker has been endeavoring during the past several years to apply the present form of electromagnetic theory in a more rigid manner than it has been applied heretofore, and it is to a consideration of some of the results of such application that your attention is directed in these brief remarks.

## IV. ELECTROMAGNETIC THEORY

To begin with a few remarks upon the current form of electromagnetic theory seem to be required. Engineers may be disposed to associate the words "electromagnetic theory" with that form of the theory which comes into the subjects of Electricity and Magnetism, as it is taught in the technical schools. There is a very great difference, however, between the general form of the theory and that part of it which is commonly needed for a complete understanding of all of the phenomena usually met with in electricity and magnetism. You will see at once the reason for the difference when you consider that you are always dealing with great numbers of electrons instead of with single isolated ones. An electric current is a continuous stream of electrons often moving in closed curves, which evidently simplifies matters very much, and in every practical case, you will find that a simplified form of the theory is all that you require. For this reason it seems fair to assume that most of you are not very well acquainted with the general form of the theory where single electrons are concerned. The example referred to by the use of the electric current may make the difference clear. Imagine a closed circuit of any shape carrying a steady current. There is a magnetic field set up in the whole region surrounding the circuit, and at each point of this field the magnetic force is a directed quantity, but the point I wish to make is that it is constant at each point of the field. Now imagine all but one of the procession of electrons circulating around the circuit to be removed in some way, so that a single electron instead of a multitude passes around the same circuit. It is evident to anyone that the magnetic force at the point in question cannot be constant as before, but must be pulsating in character with a fundamental period equal to the time of revolution in the circuit. The equations of the general electromagnetic theory must be such that they will give a complete description not only of the variable magnetic force at the point in question with time but of the electric force as well, for, when a magnetic force varies, as it must, it is always accompanied by an electric component, which in such a case as cited will also vary. The very great simplification in the force introduced by having the stream of electrons is, I think, made clear by this example.

The general electromagnetic equations in current use are of too complicated a nature to introduce into a talk of this kind, but I think that some things may be said of them which will serve our purpose. They are necessarily expressed in the language of vector analysis as always dealing with variations of directed quantities in space, and no attempt to put them into other mathematical language that we possess will ever be successful. In fact, until the vector language was developed it was hopeless to cope with such problems. Let us now try to get a definite picture of some of the fundamental facts of electromagnetic theory as applied to just two electrical charges, two electrons if you please. Imagine them isolated from the rest of the physical universe if you can. If each of them is at rest, the ordinary electrostatic force, with which you are familiar, is all that we have to consider, a repulsion in the direct line joining the charges inversely as the square of the distance between them and directly as the product of the charges. But, let them be set into motion. We may now picture a disturbance, sometimes called an electromagnetic wave, as travelling out in all directions with the velocity of light from the charges. If the distance between the charges is considerable, it may be quite a time before the wave from the one reaches the other charge, 8 minutes if one is in the earth and the other on the sun. In 8 minutes the first charge will have moved a considerable distance if its velocity is great. If its velocity were 1/100th of that of light, it would have travelled 1/100th of the distance from the earth to the sun in this time. So, when the wave that left it in its first position actually reaches the second electron, the first electron is not very near to the place that it was. In what direction would the force on the second electron due to the first then be? Evidently not in the original direction it had when they were both at rest. Electromagnetic theory purports to tell us just what the force is at any time on the one charge due to the other, that is the instantaneous force, for it is evidently continually varying.

You will, I think, see from this that the force on the second electron is entirely due to this electromagnetic wave which emanates from the first electron. This wave must travel in a medium, and so electromagnetic theory is founded upon the real existence of an ether fixed in space, and the force upon the second electron is really due to some connection that it has with the ether. One of the fundamental equations in electromagnetic theory expresses in mathematical language this fundamental connection that there is between an electrical charge and the ether. Much more might be added on this topic but, if you are to hear the things I have set out to say, we must pass over this phase of it.

## V. FIRST APPLICATION OF ELECTROMAGNETIC THEORY

It will be understood that electromagnetic theory is perfectly general in that it expresses the force at a single instant of time only and does not prescribe at all how the two charges in question shall move relative to each other. We have been led to believe for many reasons that electrons revolve in approximately circular orbits in the atoms. The Bohr theory above referred to assumes this as the natural stable condition. Granting that this is true, it must be evident to you that the first step in the application of electromagnetic theory to atoms should be the solution of the problem of finding the force that one electron when revolving at a uniform rate in a circular orbit, exerts upon a second electron revolving in a different circular orbit. These orbits must also be fixed in space in perfectly general positions, because the electrons in different atoms are, of course, situated in every possible manner.

It has been the speakers contribution to this subject to solve this problem in its most general form, no terms of any kind having been omitted. It may not, at first, be apparent to you that any solution is required, or rather, perhaps, of what a solution consists, since the general theory gives the force of one of these electrons on the other. But you will see with some further consideration that the equation we want must contain several quantities, the radii of the two orbits in question, the values of the constant velocities of the electrons in the orbits, the location of the position of the center of the second orbit in space with respect to the first, that is the $x$, $y$ and $z$ coordinates in ordinary geometry, and finally the relative positions of the planes of the two orbits, whether parallel or not. All of these quantities must find a place in the final solution, and of course none of them finds a place in the original general equation for the simple reason that it is general, not specifying any orbits or any positions for them. It requires three full pages to print this equation, but events have shown that it contains most important consequences when applied to electrons in atoms. It is some of these that I propose to discuss. The question probably occurs to you how could it have been claimed before that electromagnetic theory had been applied to electrons until the solution of this general equation had been obtained. This is a part of my contention in support of the view that these claims have not been very well founded, and that there remains much of value to be done.

## VI. RINGS OF ELECTRONS

In giving an account of them I shall not follow the order in which the results were obtained, but prefer to review them, as it were, with hindsight instead of foresight. Naturally one of the first things to do with this equation is to apply it to the particular case where the two orbits in question exactly coincide, the two electrons then being in different places in the same orbit, for, in this way we may learn something of the properties of rings of electrons, of which the atoms are supposed to be composed.

A very great simplification is naturally made in the general equation when the two orbits coincide because the coordinates of the center of the second orbit referred to the first are all zero, and the speeds of the two electrons are the same. So the equation reduces to a very manageable form. The *total* force of the second electron on the first may then be conveniently resolved into two components, the one along the radius of the orbit, and the other along the tangent line perpendicular to the one. In general, neither of these components of the force is zero, and, as we make the velocity of motion less and less, the force reduces to the ordinary electrostatic force.

*a—Two Electron Atom.* If there are only two electrons in the ring, it is evident that they must take positions at the extremities of the same diameter of the orbit on opposite sides of the nucleus of the atom, which may now be imagined at the center. For a stable position, both the radial component of the force and the tangential component must vanish, otherwise there would result both a change in the radius of the orbit and an acceleration along the orbit, neither of which conditions represent a steady state. In such an atom, consisting of a nucleus and just two electrons, I have calculated the total radial force on one electron, including that due to the nucleus, that due to the other electron and that due to the centrifugal force of the electron itself. This is shown in the curve *I* on the slide. The equation for the radial force is

$$F_\rho = e^2\left(-\frac{7}{4\,a^2} + \frac{\beta^4}{2\,a^3}\right) + \frac{m_0 v^2}{a}.$$

There are two radii at which the radial force vanishes, the one about $2 \times 10^{-10}$ cm. and the other at $1.85 \times 10^{-8}$ cm. In between these values the force is negative, signifying an attraction of the electron towards the nucleus, and, for all values greater or smaller than this, the force is positive, signifying repulsion. From this it is evident that the larger root corresponds to an unstable position, and the smaller root to the only stable position. This result entails some very radical consequences as regards atomic theory, some of which must be referred to here.

In the first place it makes the order of magnitude of the radius of the orbit of the electrons $10^{-10}$ instead of the much larger order $10^{-8}$ cm. given by the Bohr theory.

In the second place, by omitting the forces due to the ***motion*** of the rings of electrons, the $e^2\ \beta^4/2\,a^3$ term in the equation, and by taking into account the electrostatic forces only, and balancing these against the centrifugal forces as Bohr does, the ordinary conditions for planetary motion as in a solar system obtain, except that there is repulsion instead of attraction between the planets. He has made use of the following theorem which applies quite generally to all such systems "In every system consisting of electrons and positive nuclei, in which the nuclei are at rest and the electrons move in circular orbits with a velocity small compared with the velocity of light, the kinetic energy will be numerically equal to half the potential energy."

It is the use of this theorem that has to be abandoned because of the neglect of the forces due to the motion of the ring. By the help of this theorem Dr. Bohr was led to adopt the equal angular moment of momentum hypothesis for each and every electron in all atoms, which he stated as follows, "in any molecular system consisting of positive nuclei and electrons in which the nuclei are at rest relative to each other and the electrons move in circular orbits, the angular momentum of every electron round the center of its orbit will in the permanent state of the atoms be equal to $h/2\,\pi$, where $h$ is Planck's constant." This theorem is derived directly from the preceding theorem and fails when it fails.

The reason for obtaining such different results by a strict application of electromagnetic theory is apparent. The difficulty at the time Dr. Bohr first published this theory in July 1913 was that there was no available expression for the forces due to the motion of the ring, but now, the general equation referred to above has supplied this. It appears that there is a term in the force $e^2\ \beta^4/2\ a^3$ due entirely to the motion of the ring, that coming from the magnetic component in the general equation, which varies inversely as the third power of the radius, and directly as the fourth power of the velocity in terms of that of light. This term has not been pointed out before. It is a repulsive force and very small at distances anything like $10^{-8}$ cm. but, because it varies as the inverse cube of the distance, it is evident that it rapidly increases the smaller the radius, and must overtake in size any term that varies inversely as the square of the distance, namely the electrostatic forces that Bohr used. It is where the balance between these two kinds of terms is effected that we obtain the stable position of the orbit.

For complete equilibrium, as was stated, the tangential force of one electron upon the other must also vanish. This subject has been discussed in that now classical work on "Electromagnetic Radiation," by G. A. Schott. He has derived an expression for the sum of the forces upon one electron in a ring due to all the others in the ring, but it is not given in a form that is easy to use, involving as it does the use of Bessel's functions, and summations. However, our two results are based upon the same premises, although expressed in different form, and they should agree. They do agree in so far as a comparison has been possible, which gives some assurance that both are correct. Now these tangential forces can never be zero as Schott points out, unless we also take into account the force that the one electron exerts upon itself. The other electrons in the ring exert forces on the one in such a direction as always to accelerate the motion of the one electron, thus doing work upon it. The one electron exerts an exactly equal and opposite force, and thus makes the total tangential force zero. In other words it requires some energy to keep the ring in motion. Strangely enough, however, it requires much less the greater the number of electrons in the ring.

*b—Regulation of Speeds of Rings in Electromagnetic Theory.*

In electromagnetic theory, it is not possible to conceive of a single electron moving alone through the ether, sending out from itself a wave of energy in all directions and still keeping up an undiminishing velocity, unless the electron is itself changing in some way. There must be a source of this supply of energy, to supply the energy radiated, and lost to the electron. Schott has accounted in a beautiful manner for the regulation of the speed of electrons in rings. The regulator acts through the medium of these tangential forces, and Schott has shown that the peculiar nature of these forces is such that the velocity of the electrons in a ring will remain substantially a constant, or subject to very small variation for comparatively great changes in the radius of the orbit. This refers to the actual, not to the angular velocity.

It is not easy in a talk of this kind to carry you along so that the reasons for certain conclusions have much force, partly because there are many related matters that have a direct bearing upon any given one. Suffice it to say here that there are strong reasons to believe that the regulation of the speeds of rings of electrons is accomplished in the manner indicated by electromagnetic theory as pointed out by Schott. The ground is taken here that this loss of energy by radiation is excessively small, too small to be detected by any means as yet at our disposal, and that no observations connected with the quantum theory are in the least affected by its existence.

*c—Velocity of the Ring.*

*d—Line spectra and Rydberg's Constant.*

Let us next consider some of the consequences of the law of radial force which you have seen in the curve. It may now be stated that, in order to get this curve numerically, the velocity of the electron in its orbit had to be known for the different radii, and, in accordance with the above statement, the velocity in the orbit has been taken as approximately constant for all radii between the two roots of the equation. The velocity of an electron in any ring, according to these ideas, may always be taken equal to $p \frac{\pi^2 e^4}{h^2}$, where $p$ is the number of electrons in the ring, $e$ the charge of the electron, and $h$ Planck's constant. The justification for the use of this value is that it leads to precisely the same equations for line-spectra of hydrogen and helium as Bohr gives, which agrees very closely with observed spectra. It may be remembered that the derivation of Rydberg's constant (the constant connected with light spectra) in terms of the charge on the electron and Planck's constant has been one of the most cogent reasons for adhering to the Bohr theory. The expression for Rydberg's constant is

$$K = \frac{2\pi^2 m_0 e^4}{h^3}$$

The force of the argument is that, when the values of the constants on the right determined by physicists in a number of different ways are substituted in this expression, we obtain a number $3.294 \times 10^{15}$. The Rydberg constant, obtained by entirely independent observations on light spectra is found to be $3.290 \times 10^{15}$. Unless this expression were based upon a truth the chances are that no such agreement as this would be found. I will not be able to devote the time here to a demonstration of the following statements, but will merely state that the modifications above discussed lead to precisely the same expression for Rydberg's constant as the above, including also the equations for the line spectra of hydrogen and helium. The new ideas are thus not a step backwards, but, when it is stated that I get in a similar manner to the above an expression in terms of the properties of the electrons for the Newtonian constant of gravitation that agrees just as closely with the value of this constant as determined by the astronomers, it will be seen that this, when added to the Rydberg constant greatly strengthens our position, thus being a step forward.

*e—Energy of Separation of Ring from Nucleus.* We may easily calculate from the force-curve the energy that is required to separate the ring of electrons from the nucleus from one radius to another separating both electrons together. This energy curve is shown as curve II. A minimum point of the energy curve occurs where the force is zero. From this point outwards we have to supply the energy to pull the electrons away from the nucleus and from each other, but beyond the larger root (not seen on the chart) where the force changes sign to a repulsion, the system will do work upon the electrons. Suppose, for example, that we supply from some outside source enough energy to separate the electrons from their normal orbit at $2 \times 10^{-10}$ to $1.85 \times 10^{-8}$ cm. The electrons will arrive there with little or no velocity, all of this energy having been used up, but, after that, the repulsive force of the system begins to act, and it will accelerate the outward speed of the electrons until the fixed amount of energy which corresponds to the separation from $1.85 \times 10^{-8}$ to an infinite distance has all been converted into kinetic energy.

## VII. ELECTRONS EJECTED FROM ATOMS WITH HIGH VELOCITY

It is calculated, in the particular case of the ring of two and a nucleus of two, that the velocity thus acquired is nearly 1/20 of the velocity of light. With a ring of four and with a larger nucleus, the velocity will be greater than this.

In radioactive substances it has been experimentally observed for some years that electrons are projected out from them with velocities that approach very close indeed to that of light, 98 or 99 per cent of it. In the atoms in their normal condition it has never been supposed that the electrons have velocities anything like as great as that of light, and this theory has now given a valid reason why there may be a great increase in the velocitv when electrons are ejected from atoms.

## VIII. X-RAY SPECTRA

In the Bohr theory as has been stated above, there are supposed to be several steady states of the orbits which differ according to the values of an integer, $\tau$. In his theory the radii of these orbits increase with $\tau$, as the squares of the integer, 1, 4, 9, 16, etc., and there is supposed to be a definite change in the kinetic energy of the electron from orbit to orbit. All of these results came about due to the neglect of the forces due to the motion of the electrons. When these are taken into the account, we have no change in kinetic energy from orbit to orbit, but the energy change is of the nature of potential energy, the work done in separating the electrons against the radial forces shown by the curve. It may be stated that we now get a very different series of radii from those of the Bohr theory. The radius of the orbit for all values of $\tau$ from one to infinity never departs very far from the minimum point of the energy curve. We can find these radii corresponding to any value of $\tau$ by drawing a horizontal line across at a height just above the minimum point of the curve equal to the known value of the total energy to separate the electron to an infinite distance from the nucleus. Such a line is drawn in the second slide, which shows this same minimum point of the energy curve to an enlarged scale. Lines are drawn for $\tau = 2, 3$ and $\infty$, $\tau = 1$ corresponds to the minimum point itself. Each line cuts the curve in two points, and each point of intersection corresponds to the proper radius of the orbit for that value of $\tau$. Instead of a single value of the radius as in the Bohr theory there are two possible values.

I fully realize that it is not possible to make the reasoning on this matter appeal to you in the brief time at my disposal, but these results carry such important consequences that I could hardly avoid referring to them. They throw much light upon an experimental observation that has received no good explanation in terms of atomic theory. I refer to the charts of the x-ray spectra of the elements as observed by Moseley. These were shown here last year I believe. Moseley shows a group of four lines close together which he has named the $\alpha$, $\beta$, $\phi$ and $\gamma$ lines in the $L$ series, and in the $K$ series, the $\alpha$ and $\beta$ lines. There has never been any clear understanding why there should be such a grouping of lines in the x-ray spectra. This theory gives a reason for it, and enables us to approximate to the positions of the lines. Bearing in mind that the orbital velocity is constant for varying radii, and that the velocity in a circular orbit is equal to $2 \pi a n$, it is evident that the orbital frequency is inversely proportional to the radius in this theory. So also is the x-ray frequency inversely as the wave length, $\lambda$. If, then, the x-ray frequency is connected with the orbital frequency in a proportional manner, which is very probable, then the x-ray wave lengths should be proportional to the radii of the orbits. A comparison between the observed wave-lengths for the element europium, which happens to be the heaviest element for which Moseley has shown all four lines in the *L*-series, and the radii as estimated for values of $\tau = 1, 2, 3$ and $\infty$ is shown in the next slide. The general agreement is too close to be purely accidental.

In support of this view of the origin of this group of lines is the fact that they all run along in a parallel fashion without any cross-overs or intersections. You can see by the way the spacings are derived from the energy curve that any intersections of the lines in Moseley's chart would be fatal to this view of the matter. There is another interesting matter that this explanation involves. The radii for $\tau = 1$ are slightly larger than for $\tau = \infty$, so that the radii decrease slightly instead of increase for an increase in $\tau$, but the amount of the total change is quite small. That is, we use the values on the left of the minimum point of the energy curve instead of those on the right. In the Bohr theory the radii increase with $\tau$, and as the squares of the integers, as 1, 4, 9, 16 etc.

## IX. IONIZING VOLTAGES

Without attempting here to give any derivation of it, I will merely state that the theory gives a formula of a very simple kind for the voltages that are required to ionize a gas and to start radiation. This is limited as yet to gases having atoms with single rings of electrons like hydrogen and helium. A considerable amount of experimental work has been done recently on hydrogen on account of the interest in it from the standpoint of the Bohr theory. The voltages required the tear electrons off from the nucleus and to drive them entirely away from the atom, thus leaving the atom charged and making what is called on ion of it, can be measured experimentally. The theoretical formula is

$$V = 3.3844 \times \frac{p^2}{\tau},$$

where $p$ is the number of electrons in the ring and $\tau$ an integer. This subject alone is large enough to occupy the whole evening. if it were gone into in detail, so we must content ourselves with saying that the theory gives all of the values approximately for hydrogen that have been observed, including a very recent value between 15 and 16 volts which the Bohr theory gives no account of. It also gives values for helium much nearer to the observed values than the Bohr theory. It seems, however, that helium has not been so exhaustively investigated experimentally as hydrogen in this respect.

## X. THEORY OF CRYSTAL STRUCTURE

Let us now pass to some results of a very different kind that have been derived from the general equation. I refer to the theory of crystal structure. By means of the x-rays you are aware that it has been possible to find the exact location of the centers of the atoms of which a crystal is composed. This method was discovered by Prof. Laue, and first published in a paper by Laue, Friederick, and Knipping. I shall confine these remarks to a few of the simplest forms of crystals belonging to the cubic or isometric system. In all of these the general arrangement of the atoms is in equilateral triangles in a plane. The whole crystal is built up by many separate planes of atoms all similarly arranged in each plane. I have brought here a number of models to help give you an idea of the space lattice arrangement in four of the important classes of crystals belonging to the cubic system. Without models it is not very easy to make much progress in understanding crystal structure. In each one of them you will observe that the fundamental arrangement is the equilateral triangle. The several planes of such triangles are related in several possible ways, which makes the characteristic differences in the resultant space lattice.

*a—Directions of Axes of Atoms in a Crystal.* According to our ideas of atoms each has an individual axis of revolution, being determined by the common plane of the orbits of the electrons revolving around the nucleus. If we fix the attention upon just two atoms whose axes point in different directions, the equations of electromagnetic force show that there is a tendency of the one atom to turn the plane of that of the other until they become parallel to each other before complete equilibrium is established. Now the angle between the directions of the axes of the two orbits is one of the quantities that is contained in the general equation, and unless we know the directions of the axes, we cannot calculate the force by means of that equation.

Fortunately, it has been possible to find the directions of all the axes of the atoms because of the symmetry of the crystals, coupled with the knowledge that each atom in the whole structure tends to turn the direction of the axis of rotation of every other atom. Any one atom feels the turning effect of all the others, and it is only the resultant of them all that determines the position of any given axis. Now, without knowing how much the tendency to turn the axis is, it is evident that if three atoms, which are just alike, are placed at equal distances from the given atom having their axes properly directed, they may annul one another and have no turning effect upon the given atom. Without going into the history of all the troubles that presented themselves before a solution of this question was found, I have a model which shows you an arrangement of the axes of the atoms in one of the equilateral triangular planes, and can prove to your satisfaction that this particular arrangement makes the total turning moment hold each axis in the crystal in just the position indicated by this model. In this model there are only four directions of the axes. One quarter of them all is parallel to one given direction, a second quarter to another, and so for the whole. These four directions are the directions of the medial lines of a regular tetrahedron, or, which is the same thing, the four diagonals of a cube. I have placed colored strings in the models, one string passing through each atom to indicate the direction of its axis in exact accord with the plane model. You will find upon studying the models that each of the parallel planes has atoms with their axes arranged exactly like the plane model. Taking all these planes together you see that every atom has the direction of its axis fixed. But, again, you will observe that each model can be turned in any one of four different positions so that a different set of planes each time makes up the whole crystal. And no matter in which of these four ways you turn the model the relative directions of the axes is precisely the same as in the plane model. In doing this we have really counted each atom four times, and every time the direction of its axis is right. Had some different arrangement of the axes been adopted for the plane model it would not have turned out that you could turn these models in any one of four directions and then found that the individual planes were the same as in the original model. In other words the possible number of arrangements of the axes in any plane is very limited, and it is my belief that this is the only possible arrangement that will satisfy the condition that the turning moments shall be zero.

*b—Order of Magnitude of the Radii of the Orbits.* Having found the directions of the axes, but not before, it is possible to apply the general equation to determine the total force on an atom due to all the rest in the crystal. When the expression for the force thus obtained is equated to zero for the stable equilibrium condition, the only two unknown quantities that remain are properties of the atoms themselves, viz., the sum of the squares of the radii of the orbits of the electrons in the atom in question on the one hand, and the sum of the products of the radius by the velocity for each.

In other words, starting from the known dimensions of the space-lattice forms in certain crystals, and knowing the kind of atoms there are at each point of the lattice, we may, by an application of electromagnetic theory, arrive at some knowledge of the sizes of the orbits of the electrons themselves within the atoms.

Without giving you the process in detail, the results may be briefly stated. The sum of the squares of the radii found in this way show a gradual increase with heavier and heavier atoms according to a definite curve, but the order of magnitude of the radii lies between $10^{-10}$ and $10^{-9}$ cm. This agrees well with the value found in an entirely different way for hydrogen.

*c—Some Kinds of Atoms Show Two Possible Values of Their Radii.* This theory has been applied to twenty different crystals, minerals, containing in all as many separate elements in different combinations. The same element, say chlorine or sulphur, occurs in several of these crystals. It may be regarded as a confirmation of the theory that the same values are obtained for the same kind of atom no matter in which crystal it occurs. This brings out a point of very great interest, because the above statement is not universally true. For example, the element sulphur occurs in four of the crystals, zincblende, $ZnS$, iron pyrites, $FeS_2$, manganblende, $MnS$, and galena, $PbS$. We get the same value for the radii for the sulphur atom from three of these crystals, but a different value in galena, $PbS$. The numerical relation between the sums of the squares of the radii in the two cases is $2^{4/3}$, the sulphur in galena being of smaller radius than the others. In a similar manner several other crystals give different values for the same element, and they always differ by precisely this same factor $2^{4/3}$. The result is very suggestive as indicating that the same atom may have at least two different states, differing in the size of the radii of the orbits, but without affecting the weight. It is possible to see by looking back at the equation for the radial force for the single ring of two electrons, where there were two roots at each of which the radial force is zero, that the more complex equation for atoms with several concentric rings might give more than two roots, and of these there might be more than one stable position for the radii. This is a thing that we might expect from our knowledge of atoms in chemistry. There are different kinds of sulphur atoms, having different valencies, and so with some other atoms. It is most important to observe though that the weights of these atoms are the same. Electromagnetic theory throws much light upon this matter, as we shall presently show.

*d—Rigidity and Bulk—Modulus.* We will conclude the remarks on crystals with the statement that the forces which have been obtained give a very good reason for the rigidity of crystals, and the resistance they offer both to change of shape and change of volume. The estimated value of the bulk modulus is of the same order of magnitude as the experimental values.

## XI. ATOMS AT A GREAT DISTANCE

There is another matter to which the general equation has been applied, which I have left until the last, although it would have been most natural to place it first, as being perhaps the simplest application of the equation. The question is what effect will two atoms have upon each other according to the equation when the distance between them is very large, say one centimeter or more. It may be shown that all of the electrostatic forces between two neutral atoms at a great distance exactly cancel out until we come to terms involving the inverse sixth power of the distance. Each atom has in effect a zero charge at these great distances, and the only part of the forces that we have to calculate arises from the *motion* of the electrons in their orbits. In the final analysis, therefore, to get the whole mechanical force between the atoms, we have to find the average mechanical force that one single electron in the second atom exerts upon another in the first atom. The total force is evidently merely the sum of a number of such expressions, the number depending upon how many different pairs of electrons there happen to be in the two atoms considered. If it happens, as it does, that the force due to any one pair of electrons gives us a quantity that is *not* cancelled out by that due to another pair, we obviously get a real resultant force between the two atoms. We know that such a force exists in fact, the gravitational force, and we should be interested to compare the force given by the equations with the actual known force.

*a—Problem of Two Electrons. Average Force.* In order to solve this problem for two electrons only it was necessary to obtain the average force between the two electrons taken over a very long time, a large number of revolutions. The only way as yet found to do this is to develop the equation into an infinite series of powers of the distance between the centers of the two atoms, and then average the separate terms. The result is that the series contains all powers of the inverse distance between centers, beginning with the first power, and so on. Moreover, the force is not in the direction of the line joining the centers of the two atoms. This is easily understood when it is remembered that the force between the two electrons in the original equation is not in this direction.

*b—Force Resolved Along the Line Joining the Centers of the Orbits.* We are interested, however, in the component of the total force that is in this direction, and hence the total force has been resolved in the direction of the centers. The part at right angles to this does not interest us because it will all be cancelled out by the action of the electrons in other atoms turned in every possible direction.

Now it results in the process of resolving the force along the center line, that the first term of the series, the inverse first power term, cancels out leaving it to begin with the inverse square, and higher powers. Moreover, because the distance between the atoms is assumed to be great, all of the inverse higher powers are evidently negligible, so that the total force varies inversely as the square of the distance between the atoms. The resulting force-equation is of sufficient interest to give in full.

$$F = \frac{1}{2} e_1 e_2 \beta_2^2 \{1 - (-X \sin \alpha + Z \cos \alpha)^2\} r^{-2}. \qquad (1)$$

Here $e$ is the charge on the electron; $\beta_2$ the ratio of the velocity of the second electron, the force of which we are getting, to that of light; $X$ and $Z$ are direction cosines determined by the position of the center of the second orbit; $\alpha$ the angle between the directions of the two axes of rotation, and $r$ the distance between the centers of the orbits.

*c—Simplification of Result in Case of Two Masses of Matter.* Let us apply this equation to two electrons, one in each of two atoms of any gas, liquid or solid, crystals excepted. At any given moment the axes of the two orbits may be turned in any possible directions relative to each other, so that the theory of probabilities must be applied, because it is the force on the average that we require. When this is done, it is shown that the quantity within the brace becomes 2/3, and hence the average force on the first electron due to the second becomes the very simple expression

$$F = \frac{1}{3} e_1 e_2 \beta_2^2 r^{-2}. \qquad (2)$$

This expression is in the form of the law of gravitation in two respects. First, it always gives an attraction, and second, it varies inversely as the square of the distance. Before inquiring into the magnitude of the force, let us apply the equation to the only kind of matter not included above, namely crystals.

*d—Attraction Independent of the Orientation of the Axes of a Crystal.* We have seen that there are but four different directions for all the axes of the atoms in a crystals so far as the theory of crystals has been developed. Would we obtain the same result as before using these four directions only? If not, the theoretical attraction between two crystals would show some differences at the same distance when they are turned about in different directions. The experimental evidence is that the gravitational attraction shows no difference in any direction, and, consequently, if the force is to be identified with that of gravitation, it is necessary that the terms involving the directions of the axes in this parenthesis should give the same result as above even in the case of a crystal. It has been possible to show that they do, and this circumstance is due to the fact that the axes in the crystal are all parallel to the four medial lines of a regular tetrahedron.

*e—A New Geometrical Theorem Obtained from Physical Considerations.* There is an interesting incident in this connection. The equation showed that the following geometrical proposition must be true if the force is to be independent of the orientation of the crystal. "If through any point in space four lines be drawn making equal angles each with any other (that is parallel with the four medial lines of a regular tetrahedron) and, if from a second point in space, at a distance $r$ from the first point, the four perpendiculars be drawn, one to each of the said four lines, then the sum of the squares of these perpendiculars is constant for all points at the same distance from the first point." The quantity within the brace of equation (1) is equal to the square of one of these said perpendicular lines.

The truth or falsity of this proposition was unknown at the time of arriving at the conclusion that it must be true if this is the proper form for the gravitational force. It has since been proved to be true,* and this has added a new geometrical theorem to the list, as it was unknown to the mathematicians to whom it has been submitted. This incident of a mathematical theorem deduced in this physical way helps to show that we may be on the right track in understanding the cause of the force of gravitation.

We see, therefore, that the force tallies in every way with the gravitational force except in the matter of magnitude, which will next be considered.

*f—Magnitude of the Force.* As you have seen above in considering hydrogen, we have a knowledge of the probable speed of the electrons, and there is nothing else in doubt that we need for calculating the force, since the charge on the electron is known. Using these known speeds, the force calculated from this equation comes out about $10^{31}$ times greater than the actual value of the gravitational force. This is an enormous error and cannot be accounted for by assuming any mistake in the velocity of the electrons used. The result is very significant at the same time because the calculation indicates a force $10^{31}$ times greater, ***not less***, than the actual existing attraction. It is a more fortunate result than it would have been had the calculated force come out the smaller of the two because then gravitation might easily be attributed to something else, but, as it stands, we are forced to question the truth of the hypotheses that have entered into the case. What are they? They are really but two. First, that the atoms are structures consisting in part of electrons revolving in circular orbits, or approximately circular orbits as you please. Second, that the modern Lorentz form of electromagnetic equations applies to the case.

We are not yet ready on the strength of any electromagnetic theory to abandon the idea that electrons describe approximately circular orbits within the atoms. It is easier to believe that some modification in the present form of electromagnetic theory is required. Let us look at the expression for the average force upon one electron in a circular orbit due to another a little more attentively. We have already seen that the expression gives a good

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*The proof is due to Prof. F. W. Owens of Cornell University.

account of itself in many ways. It represents always an attraction, and not a repulsion, and obeys the inverse square law. It also shows that, when such forces are summed up for two crystals, the result is independent of the orientation of the crystals. In fact, it has the right look in so many ways that I have been led by natural steps to suppose that some very important factor has been omitted somehow from the Lorentz theory.

*g—Equation of Force Unsymmetrical.* You see by the expression (**2**) that the speed $\beta_1$ of the electron, upon which we are getting the force is not present, indicating that the force does not depend upon the velocity of the first electron. If we should write down the force of the first electron upon the second we would merely put $\beta_1^2$ in place of $\beta_2^2$. Therefore, unless both these velocities were the same, the force of the second on the first would not be the same as the first on the second. This has led me to suspect that the square of the velocity of the first electron should also appear in the force-formula. The order of magnitude of this is $10^{-4}$, so that this alone could not possibly account for the discrepancy of $10^{-31}$ above mentioned. It reduces the discrepancy, however, to $10^{-27}$. Now the mass of the electron itself at slow velocities is $.90 \times 10^{-27}$ grams. If the mass of the electron is also included, equation (**1**) takes the very symmetrical form

$$F = \frac{m_0}{2K} e_1 e_2 \beta_1^2 \beta_2^2 [1 - (-X \sin \alpha + Z \cos \alpha)^2] r^{-2}, \qquad (\mathbf{3})$$

and the average force on one single electron due to another is

$$F = \frac{m_0}{3K} e_1 e_2 \beta_1^2 \beta_2^2 r^{-2}, \qquad (\mathbf{4})$$

where $m_0$ is the mass of the electron at slow velocities, and $K$, the specific inductive capacity of the ether, equal to unity, has been introduced for the sake of keeping the dimensions of the equation right.

*h—When a Factor is Introduced to Make the Equation Symmetrical, the Force-Equation Agrees With the Law of Gravitation.* By means of this equation we may easily show that the magnitude of the attraction agrees to a surprising degree of accuracy with the attraction obtained from the law of gravitation. It may be applied to any two masses of matter equally well, but I will illustrate it by calculating the attraction between two average atoms of hydrogen, because the atomic theory gives the velocity of the electrons in the atom. When the formula is to be applied to two masses of matter containing a large number of electrons, it is only necessary to replace the $\beta_1^2$ by the $\Sigma_1\ \beta^2$, and the $\beta_2^2$ by $\Sigma_2\ \beta^2$, the summations being taken of the velocities squared of all the electrons in each of the two bodies in question. The formula for the total attraction between the two masses then reads

$$F = \frac{m_o}{3K}\, e_1\, e_2\, \Sigma_1\ \beta^2\, \Sigma_2\ \beta^2\, r^{-2}. \tag{5}$$

We have seen above that the value of $\beta^2$ for any single ring of electrons is $p\,\frac{\pi^2\, e^4}{c^2\, h^2}$, and the sum of $\beta^2$ for all the electrons in the ring is $p$ times this. If we are writing the attraction between two atoms of hydrogen, the two summations in (5) are the same, and we get by substitution of the above value

$$F = \frac{m_o}{3K}\, p_H{}^4\, \frac{\pi^4\, e^{10}}{c^4\, h^4}\, r^{-2}, \tag{6}$$

where the $p_H$ denotes the number of electrons in the hydrogen atom.

If this force is to be identified with the gravitational force, it should be numerically equal to the attraction as given by Newton's law, which is usually written

$$F = k\, m\, m'/r^2. \tag{7}$$

Putting $m_H$ in place of $m$ and of $m'$, as denoting the mass of the hydrogen atom, this becomes

$$F = k\, m_H{}^2\, r^{-2}. \tag{8}$$

Equating (6) and (8), as being different expressions for the same force, the $r^{-2}$ cancels out, and, dividing through by $m_H{}^2$, we find a value for $k$

$$k = \frac{p^4}{3}\, \frac{m_o}{m_H{}^2}\, \frac{\pi^4\, e^{10}}{c^4\, h^4}. \tag{9}$$

The $K$ is omitted because it is unity and does not affect the numerical value, and also because it is not at all certain that it should not also appear in the Newtonian equation (7). The value of $k$ as determined by the astronomers is 666.07 × $10^{-10}$. In the right member occur constants that have been independently determined by physicists. The following values are those in current use,

$m_o$ = .90 × $10^{-27}$ grams (From Bucherer's value of $e/cm_o$ and Millikan's value of $e$)
$m_H$ = 1.662 × $10^{-24}$ grams. Millikan's value.
$h$ = 6.547 × $10^{-27}$ Planck's constant. Millikan's value.
$c$ = 2.9987 × $10^{-10}$ cm per sec. = velocity of light.

If with these values we use for $e$, $4.750 \times 10^{-10}$, we get the value of $k$ exactly as determined by the astronomers above given by putting $p = 2$. This value of $e$ is just *one half of one per cent* smaller than Millikan's value of $e$ which is $4.774 \times 10^{-10}$. Values of $e$ determined by other observers and by other methods vary from perhaps 4.65 to 4.81, and it is quite possible that 4.750 is a very accurate value of $e$. In fact, if it is admitted that equation (9) is a true relation, it will be a very accurate way to determine $e$ itself, because the value of $k$ is accurately known, and we would get a value of $e^{10}$ with a fair degree of accuracy, and hence would have $e$ with precision.

The fact that the number of electrons in the ring for the hydrogen atom, $p_H$, must be equal to 2 to satisfy the expression for $k$ numerically is significant as indicating again that hydrogen has two electrons per atom.

The argument for this atomic theory, on account of the numerical agreement with the Newtonian constant, is of the same nature as the argument for the Bohr theory above mentioned, on account of the numerical agreement with the Rydberg constant; but it is doubly cogent, because both these constants are adduced from this theory.

*i—Certain Features of Electromagnetic Theory Brought Into Question.* It is for these good reasons that I have ventured to question the correctness of certain fundamental assumptions in the current form of electromagnetic theory. We are forced to question it or to abandon altogether the idea that the electrons describe circular orbits within the atoms. Of the two alternatives, I venture to think that the proper one has been chosen. As above stated, as we look back over the history of the development of electromagnetic theory from Maxwell's beginning to the present time, a process of gradual evolution has been at work. I had formerly supposed that the commonly accepted electromagnetic theory is complete as we have it, or finished, so to speak, but I have been led to believe that it is yet in its infancy. To avoid being misunderstood some further remarks on this subject seem to be required. The changes in the theory that seem to be demanded are not very great, and will probably not affect most of the applications of the theory. They relate more particularly to the inverse square of the distance terms, which only come into the account in such problems as we have been considering. It may seem impossible to you that any change can be made in the theory, because the equations already express with complete satisfaction all the phenomena in the subjects of electricity and magnetism dealing with gross matter, with which we are acquainted. It was pointed out before that all these things represent very special cases, in fact, the older form of equations of Maxwell's day are just as good for these things. The applications to gross matter are not affected by the modern change in the introduction of the ideas of retarded potentials.

*j—Direct Experimental Verification of Electromagnetic Equations Desirable, But Probably Impossible.* We really need an experimental verification of the general electromagnetic equations. Prof. H. A. Rowland charged the circumference of a wheel and set it revolving at a rapid rate. He succeeded at best in detecting the magnetic force at the center, and showed that the magnitude agreed with the theory. What we really want, however, in order to test the theory is to have the charge at one point only of the wheel, and to measure the instantaneous, not the average, mechanical force upon a second electrical charge in its neighborhood both as to direction and magnitude. Needless to say this has never been accomplished and we may almost say never will be accomplished. We should need a recording apparatus that had no time lag, and that is capable of indicating a force in any direction equally well, and of a very insignificant amount. The conclusion is that we are compelled to resort to indirect methods of testing these equations. It seems that we may legitimately regard the process above described as one of these indirect methods. In such methods, if one assumption fails, we ought not to hesitate to try another, for the original theory itself rests upon assumptions that seem to us correct, but which can only be tested by the indirect method.

## XII. THE RESULTS OBTAINED ABOVE (XI) AGREE WITH THE FORM OF ATOMIC THEORY DEMANDED BY ELECTROMAGNETIC THEORY

The gravitational formula above is so simple, and has so many points in its favor that I have proceeded as though it were true, and have endeavored to find some of the consequences that it implies for atomic theory. We shall next consider some of these and you will note that they are in complete harmony with the conceptions of atomic properties that we have been considering. First, let us take the earth as one of the bodies, and a single atom on the earth's surface as the other body, and write down the force of attraction according to the equation. We get the very simple expression

$$F = k\,\Sigma_A\,\beta^2. \tag{7}$$

where the constant,* $k$, involves the constant charge on the electron, the mass of it, and the summation of the velocities squared for the electrons in the earth, all of which does not vary when we substitute one atom for another.

*a—The Weight of An Atom is Proportional to the Total Kinetic Energy of the Electrons it Contains, Relative to the Center of the Atom.* Now this attraction is merely the atomic weight, and we, therefore, have the suggestion that the atomic weight is merely proportional to the sum of the squares of the velocities of all the electrons in the atom, that is, to the total kinetic energy of the electrons, and it should be added that the velocities must be those relative to the center of the atom. If the whole atom moves, this would add nothing to the weight because the effect of the positive nucleus would exactly annul that of the electrons. But, we know that atoms of the same kind, say hydrogen, have the same weight in a given locality. We may, therefore, conclude that the velocities of the electrons in the atom must always be the same in the different states of these atoms. This is in exact conformity with the theory of the atom above described. The velocity in any ring, according to electromagnetic theory, is a constant quantity dependent upon the number of electrons in the ring. So long as we leave the rings in an atom undisturbed they must always have the same velocity, but, when we change the number in the ring the velocity must suddenly change to a new value, and then the weight will change. We then really have a different atom. It has suddenly changed over to one of a different class.

The atomic theory above outlined also shows that any ring may change its radius without affecting the velocity of the electrons, the kinetic energy remains fixed independent of changes in the radius. The Bohr theory, on the contrary, demands a change in the kinetic energy with any change in the radius. On the one hand we keep a constant weight, and in the Bohr theory the weight must change, if the weight has anything to do with the kinetic energy. But it is a matter of measurement that the weight does not change. I may refer again to those crystals in which the same kind of atoms, sulphur for example, are re-

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*This is not the same constant as the Newtonian constant above.

quired to have different radii in different circumstances, although the weight remains the same. This is in accord with the modified theory.

*b—The Weight of any Ring of Electrons in an Atom is Fixed Independent of its Radius, and Always Contributes the Same Amount Toward the Total Weight of the Atom.* According to these ideas, therefore, a ring of two, three, etc., electrons in any atom must always contribute the same amount to the weight of the atom whether its radius be large or small. Having received this suggestion from the theory, I have endeavored to ascertain whether it might not be possible to find out, from the known atomic weights of the different kinds of atoms, what the probable combinations of rings in such atoms are. The result of this has been more successful than I had reason to hope, knowing the very great irregularities that atomic weights exhibit. In the light of the theory it is possible to write down the approximate weights of any ring of electrons. I say approximate because it is not supposed that the speed is absolutely independent of the radius of the ring, but only so to a first approximation. Taking the speeds as strictly proportional to the number of electrons in the ring, the theory gives the weights of the rings as the squares of the number of electrons per ring, as the numbers, 1, 4, 9, 16, 25, for rings of 1, 2, 3, 4 and 5. Taking hydrogen as a ring of two and atomic weight 1.008, we get the numbers 2.268, 4.032, 6.300, and 9.072 as the weights of rings of 3, 4, 5 and 6 electrons. These are the strictly proportional figures, but I have found by trial that, if the following figures, differing very slightly from the above, are used, namely for a ring of two 1.008, three 2.269, four 3.99975, five 6.2898, and six 9.137, it is possible to get the atomic weights of all the elements from hydrogen to uranium within the probable error of experimental measurement in a large majority of cases. For most of the elements only the first three figures have to be used, corresponding to rings of 2, 3 and 4 electrons. Only a few elements require rings of five, namely the halogens, *Cl*, *Br*, and *I*, and the elements selenium and tellurium. The element iron is the only one based upon rings of six, with the possible exception of glucinum. In other words most of the atomic weights have been accurately obtained by the use of but three numbers, rings of 2, 3 and 4. Of the 70 elements calculated (the rare earths having been omitted) 52 of them fall very near to or within the experimental error of the chemist in measuring the weight, while 18 fall outside of this accuracy. Of these 18, however, the weights of 15 of them come nearer than one part in 1000 to the measured weight, and most of them considerably nearer. Only three of these exceed a difference of one part in 1000. These are *L i*, 7.3 parts in 1000, *K* 1.2 parts, and *Y t* 2.26 parts. The largest difference by considerable margin is in the case of *L i*, which, however, is less than 1 per cent.

The system of rings as built up in this way seems to be based upon rings of four electrons, the number of rings of four increasing steadily with increasing atomic weight, the number of rings of two and of three being few. The cases of iron and the halogens are most interesting, as being built upon a different foundation. Rings of four will not fit these elements.

I would not give you an account of this matter here, were it not for the fact that the weights come out in this way remarkably close when based upon three numbers for the most part, and that these numbers agree so closely with the numbers arrived at by the theory above outlined.

## XIII. THE NUMBER OF ELECTRONS PER ATOM ACCORDING TO THE ABOVE SATISFIES BOTH THOSE WHO HAVE THOUGHT THAT THE NUMBER IS APPROXIMATELY EQUAL TO THE ATOMIC NUMBER, AND THOSE WHO HAVE TAKEN IT AS THE SAME AS THE ATOMIC WEIGHT

There are a few important matters which have been passed over without notice, that may now be mentioned. It has been an object among physicists for some time to discover just how many electrons the different kinds of atoms contain. This table of the elements made in the way just mentioned gives the number of electrons for each atom. Many have come to the conclusion that the number of electrons must be approximately equal to the atomic number, which is roughly one half the atomic weight. Others on different grounds have made the number approximately equal to the atomic weight. This table makes the number approximately the same as the atomic weight, but it will be observed that hydrogen has two and not a single electron. So the table satisfies both views of the matter, while it makes the number nearly the same as the atomic weight, the ratio of the number in an atom to the number in hydrogen is one half the atomic weight.

## XIV. THE NUMBER OF ELECTRONS IN THE HYDROGEN ATOM

You have a right to be surprised that any question can be raised at this time as to the number of electrons in hydrogen. I can find, however, no experimental observation to prove there is but a single electron, and it seems that this view of the matter has been adopted along with the Bohr theory, which requires the single electron.

A few remarks upon this matter seem called for. If the velocities of electrons in rings are regulated as above described through the action of the tangential forces due to the other electrons in the ring upon the one according to electromagnetic theory, then we are deprived of this means of regulation when there is one isolated electron only. We do not have in such a system the perfectly symmetrical dynamic balance that obtains with two electrons.

Then, again, our present view of the matter does not neglect the radiation and loss of energy from the atoms. In electromagnetic theory the relative amounts of loss of energy through radiation for rings of 1, 2, 3, etc., using the speeds as above determined, diminishes very rapidly as the number in the ring increases. The loss from a ring of two is about 4000 times less than from the single electron, and from the ring of three about 40 million times less, and from four nearly a million million times less. For this reason it seems reasonable to suppose that the single isolated electron does not exist in the atoms. I shall not take the time here to answer the obvious objections to admitting loss of energy by radiation on account of the quantum theory, but may assure you that they can be met satisfactorily.

## XI. THE ORDER OF MAGNITUDE OF THE ORBITS OF THE ELECTRONS IN ATOMS

Another matter, which has been passed over relates to the order of magnitude of the orbits of the electrons. There are a number of things that have a very evident bearing upon this question. One of the things we know now with precision is the distance between the centers of the atoms in certain crystals. These distances in different crystals vary from perhaps $10^{-8}$ cm. up to 4 or 5 or possibly 6 times as much, and we may think of the order of magnitude as roughly $10^{-8}$ cm. Now, if the radii or the orbits of electrons in any of the atoms in the crystal approach close to the same order of magnitude, the electrons must necessarily approach each other much nearer than they are from the nucleus to which they belong. Unless we are to assume that such electrons have no mutual influence upon each other, which seems uncalled for, there must occur in crystals an enormous perturbation with an ensuing radiation of energy. This could not possibly be a stable condition. Our exact knowledge of crystals alone ought to be considered a direct proof that the order of magnitude of the orbits of the electrons is much smaller than $10^{-8}$ cm.

Again, the kinetic theory of gases has been cited to show the order of magnitude of the radii. This theory has established the fact that the centers of the atoms in a gas approach each other on the average to within about $10^{-8}$ cm. before being deflected. If the orbits of the electrons were of this order too, they must at times come into very close proximity or even collision, and the result would be that some of the electrons would be separated from the nucleus, thus ionizing the gas under ordinary conditions. This does not occur. We may, I think, regard the kinetic theory as another direct proof that the orbits must be of a much smaller order. These matters, when taken in connection with the fact that the radii come out between $10^{-10}$ and $10^{-9}$ cm., when the forces due to the motion of the electrons is included, which was omitted from the Bohr theory, and with the fact that these radii agree with the values indicated by an application of the theory to crystals, do not leave much doubt as to the true sizes of the orbits of the electrons.

I can picture the thought that is probably occurring to many of you, while listening to this description of a difference of opinion affecting the radii of the orbits between ten and a hundred times. In what condition would astronomers find themselves so long as they were in doubt about the value of the radius of the earth's orbit by a factor of some ten or hundred times? Evidently, until this matter is determined with some precision, all attempts to place atomic theory upon a firm mechanical basis cannot be very successful. It will have to be acknowledged, however, that the physicist does not find any such single simple law applying to atoms at close range as governs the motions of the heavenly bodies, and the reasons why we are not further along are not difficult to understand.

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# THE POLYPHASE SHUNT MOTOR

BY W. C. KORTHALS ALTES

## ABSTRACT OF PAPER

There exists a demand for a reliable, adjustable-speed, alternating-current motor, suitable for operation at a large number of speeds. The neutralized motor with shunt field control is analyzed and it is shown that it is not practical for commercial frequencies on account of the expensive control equipment required. The induction motor with commutator on the secondary side is discussed. It may find some application for the larger outputs; the control is, however, still too complicated to make this type of motor suitable for the smaller machine-tool drives. The induction motor with commutator on the primary side offers the best solution for machine-tool motors. Its theory is discussed in detail, and a complete description is given of the mechanism required to shift the brushes and the new type of armature winding used.

## INTRODUCTION

AS FAR as generation, transmission, distribution and constant-speed motor drives are concerned, the alternating-current system is far superior to the direct-current system.

The development of modern tungsten lamps, has practically done away with arc lamps, so that both systems are equally suitable for lighting.

The development of speed-regulating sets has made it possible to regulate efficiently the speed of induction motors up to several thousand horse power.

The varying speed alternating-current brush-shifting motors, which have recently been put on the market, can be applied to variable speed pumps, blowers, exhausters and certain textile machines.

The single-phase crane motor opens up possibilities in regard to dynamic braking, simplicity of control and operation over a wide range of speed, which cannot be met by the induction motor with resistance control.

The adjustable-speed alternating-current motor makes it possible in plants requiring adjustable-speed motors for machine tool, elevator service, etc., to use the alternating-current system throughout if desired, but due to the fact that the a-c. adjustable-speed motor is more costly than the d-c. adjustable-speed motor, t will be more economical to install machinery for changing over the alternating current to direct current, as long as a large number of motors is involved. However, the general tendency to standardization and centralization increases the number of processes which are carried on at constant speed and call for the simple squirrel-cage induction motor and the use of the a-c. adjustable-speed motor, in cases where only a limited number is required, will make it possible to extend the application of the squirrel-cage motor.

It is not intended to deal in this paper with all the different types of a-c. adjustable-speed motors, which have been proposed by the various engineers that have worked on the problem. The paper will be limited to the types that now seem most important. It is of value to the specialist to know the large number of possible combinations, so that he can use them when the constantly varying conditions make this possible, but engineers in general need interest themselves only in those schemes which are at present suitable for practical application.

Neither is it necessary to discuss the multi-speed induction motor, the speed of which is adjusted by connecting windings wound with different numbers of poles to the line. The field of application and the theory of this motor is well-known. As long as only a few definite speeds are required, it is very satisfactory. However, there are a number of cases for which either a large number of speeds is required, or the desired speeds cannot be obtained with the possible numbers of poles. In those cases, one of the following types of alternating-current adjustable-speed commutator motors can be used. (1) The neutralized polyphase commutator conduction motor, (2) the induction motor with commutator on the secondary side, (3) the induction motor with commutator on the primary side.

## THE NEUTRALIZED POLYPHASE COMMUTATOR CONDUCTION MOTOR

This motor consists of a d-c. armature winding, on the commutator of which is arranged a polyphase system of brushes, connected to a neutralizing winding which neutralizes the magnetomotive force of the armature winding, and is connected to the line. The field winding is connected to the secondary of a transformer, the primary of which is connected to the line. It can be arranged in the same slots as the neutralizing winding, in which case the leakage flux will induce in the field winding a voltage proportional to and in phase with the leakage reactance voltage in the neutralizing winding. The motor field flux in this case lags ninety degrees behind the vector difference of the line voltage and the reactance drop induced in the field winding by the primary leakage flux which is yielded by the ampere-turns of both the field and neutralizing winding. The characteristics of the motor can be determined by considering that the neutralizing winding has no reactance drop, but that a reactance equal to the reactance of the neutralizing winding has been connected between the line and the motor terminals to which both the neutralizing winding and the field winding are connected. The field winding excites a rotating field, so that at synchronous speed the armature voltage and the voltage in the armature coils short-circuited by the brushes is zero, while this voltage increases when running above or below synchronous speed.

The field winding can also be arranged in other slots than those of the neutralizing winding. The field flux in this case lags ninety degrees behind the applied line voltage and the reactance drop of the neutralizing winding should be added to the reactance drop of the armature winding, which means that the difference between the no-load and full-load speed will be greater than before. The neutralized motor with the field winding in the same slots as the neutralizing winding is to the one with the field winding in other slots, as the direct-current shunt motor with resistance in series with the entire motor is to the direct-current shunt motor with resistance in the armature winding. This can be seen from the vector diagrams.

Fig. 1 gives the vector diagram for the neutralized motor with the field winding in the same slots, as the neutralizing winding. $O\,M$ represents the current which flows through both the neutralizing and the armature winding, $N\,O$ the current in the field winding. Assuming that the field winding and neutralizing winding have the same number of turns $w_1$, then the primary leakage flux will be yielded by $N\,M \times w_1$ ampere-turns and the reactance drop $A\,B$ induced by the leakage flux in both the neutralizing and field winding will lag ninety degrees behind $N\,M$. The resistance drop of the field winding, $F\,B$ is opposite to the field current $N\,O$. $F\,O$ is the voltage induced in the field winding by the alternation of the field flux. $AB$, $BF$ and $FO$ balance the applied line voltage $OA$. For the circuit consisting of the neutralizing and armature winding in series, we find that the applied line voltage $OA$ is balanced by the leakage reactance drop $AB$ of the neutralizing winding, the resistance drop $BE$ of the neutralizing winding which is opposite to the current $OM$, the reactance drop $ED$ of the armature winding lagging 90 deg. behind $OM$, the resistance and brush-drop of the armature winding $DC$ opposite to the current $OM$ and the counter e.m.f. $CO$, which is induced by the field flux yielded by the field winding in space quadrature to the phase under consideration, and is proportional to the speed, the number of armature turns and the field flux. Due to the pulsation of the field flux, a voltage $OF$ will be induced in both the neutralizing and the armature winding. If the neutralizing and armature winding have the same number of turns, then the voltage in the armature winding will be neutralized by the voltage in the neutralizing winding and the resultant voltage appearing at the terminals will be zero. This voltage appears, however, when measuring the drop across the brushes and across the terminals of the neutralizing winding individually. The core loss has been neglected in the diagram. It causes the flux to lag behind the ampere turns. The output is proportional to $OC \times OM \times \cos\angle COM$. Fig. 1 has been drawn for operation below synchronous speed.

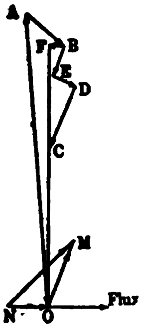

FIG. 1

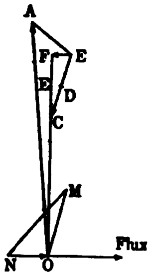

FIG. 2

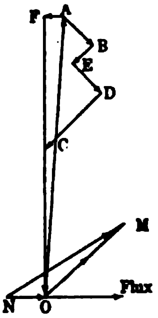

FIG. 3

Fig. 2 gives the same diagram when the motor is running at synchronous speed. It has been drawn under the assumption that the leakage reactance of the armature is proportional to the slip. In that case, the rotor reactance is zero at synchronous speed and the points $D$ and $E$ come together. The voltage applied to the field winding is equal to the vector connecting $A$ with a point $H$ on $O\,F$, so that $O\,H = F\,C$. This vector is not shown in Fig. 2.

Fig. 3 covers the diagram for a motor in which the field winding is located in other slots than those of the neutralizing winding. $A\,B$ lags now 90 deg., behind $OM$ instead of $N\,M$ and the leakage flux yielded by the neutralizing winding does not induce any voltage in the field winding.

Fig. 4 is similar to Fig. 3, except that the motor is running above synchronous speed in which case the rotor reactance appears negative. The voltage to be applied to the field winding has not been shown, but can be found by connecting $A$ with a point $H$ on $F\,O$, so that $F\,H = \frac{n_0}{n}\,O\,C$, if $n$ is the speed of the motor and $n_0$ the synchronous speed.

In case we take the load off the motors covered by Fig. 1, and Fig. 3 the speed will change as $O\,C$ to $O\,F$. Due to the location of the drop $A\,B$, the variation in speed will be larger in the latter, than in the former case. Hence, as far as the variation between the no-load and the full-load speed is concerned, it is better to arrange the field winding in the same slots as the neutralizing winding. However, by locating the field winding in other slots than those of the neutralizing winding, we can build the field winding with definite neutrals like a direct-current motor, so that in the coils short-circuited by the brushes, no voltage is induced by rotation through the main field flux. Moreover, we can locate at these neutral points, windings that excite the "Commutating Flux" inducing in the armature coils short-circuited by the brushes, a voltage which both neutralizes the voltage induced by the alternation of the field flux and furnishes the e. m. f. required to reverse the current in the coils passing the brushes. Furthermore, it is possible to improve the speed-torque characteristics by using a series transformer, the secondary of which is connected to the field circuit and shifts the time phase of the flux depending on the load current drawn from the line. The characteristics of this motor are particularly favorable when running far above synchronous speed. The diagram of Fig. 4 shows how the reactance of the neutralizing winding can be completely compensated by a negative reactance drop of the armature winding, but when predetermining the characteristics, it must be borne in mind that the voltage induced by the commutating pole shifts the point at which the rotor reactance becomes negative to a higher speed, than in a motor which has no commutating poles.[1]

With the most generally used frequency of 60 cycles, it is impossible to take advantage of these favorable operating characteristics above synchronous speed unless we connect the motor to the secondary of an induction motor, in which case we can supply the commutator motor with both a low frequency and a low voltage and use it to control the speed of the induction motor. This has recently been done with great success and has led to a very important development of a-c. commutator motors. A discussion of the various connections and possibilities of this application properly belongs to the subject of speed control of large induction motors and lies outside the scope of this paper.

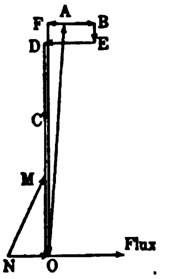

Fig. 4

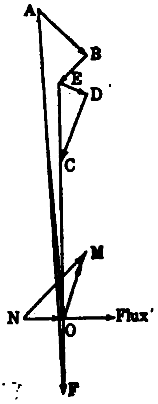

Fig. 5

The motor, with the field winding located in the same slots as the neutralizing winding, is more suitable for operation below synchronous speed and can be used directly on 60-cycle circuits. Instead of building it with a separate field winding, the neutralizing winding can serve at the same time as field winding which only slightly changes the diagram, as can be seen by comparing Fig. 1 with Fig. 5, which has been drawn for a motor in which both field and neutralizing winding have been combined in one. The vector $AB$ is equal to the leakage reactance drop in the neutralizing winding which lags 90 deg. behind $N\,M$, $B\,E$ is the resistance drop of the neutralizing winding, $E\,F$ is the voltage

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[1] See H. Meyer-Delius, *General Electric Review*, 1913, page 976.

induced in the neutralizing winding by the alternation of the field flux, *F E* the voltage induced in the armature winding by the alternation of the field flux, *E D* the reactance drop, *D C* the resistance drop, *C O* the rotation voltage induced in the armature winding. The measured voltage across the neutralizing winding is equal to *A F*, the one across the armature winding reduced to the phase-voltage of the equivalent Y connection *F O*. The resultant of *A F* and *F O* gives the line voltage *A O*. Thus, we see that below synchronous speed, the voltage across the neutralizing winding is higher than the line voltage per phase until *F* and *O* are at the same point which occurs slightly below synchronous speed.

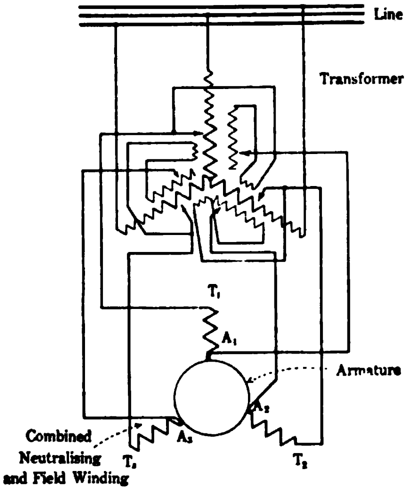

FIG. 6

The diagram of connections of a motor covered by the vector diagram of Fig. 5, has been shown in Fig. 6. Speed regulation can be obtained by connecting the terminals $T_1$, $T_2$, $T_3$ to different taps of the transformer which changes the terminal voltage applied to the motor, or by changing the points to which the armature terminals $A_1$, $A_2$, $A_3$ are connected, which changes the field flux, or by both. The principal disadvantage of this motor is that the transformer has to be designed for its full kv-a. capacity. In order to obtain satisfactory commutation, the armature must be built for a low voltage, which in general makes the ratio of the applied line voltage to the secondary voltage high, so that the size of the transformer is not materially reduced by building it as a compensator. Instead of using the neutralizing winding also as field winding, the armature can serve for this purpose. Fig. 7 gives the diagram for this arrangement. In this case *O M* is the current in the neutralizing winding, *N M* the current in the armature winding. The volt-amperes required for exciting the field is equal to *F O* × *N O* of Fig. 7, instead of *A F* × *N O* of Fig. 5, as is the case when the field current flows through the neutralizing winding. As long as we are running at a lower speed than 50 per cent above synchronous speed, the field can be excited with less volt-amperes by using the armature, instead of the neutralizing winding, as field winding, which results in a small reduction in the size of the regulating transformer and an improvement of the power factor. It is also possible to excite the field by current flowing partly through the field winding and partly through the armature winding, which can be done by impressing voltage of the proper time-phase on both the neutralizing and the armature winding. However, all the above schemes, with the exception of the one where we connect the commutator motor to the secondary of an induction motor, have the disadvantage that a large transformer is required for the main circuit. The size of this transformer can be reduced by using the type of motor, which we will now describe.

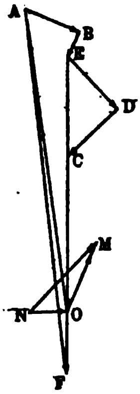

Fig. 7

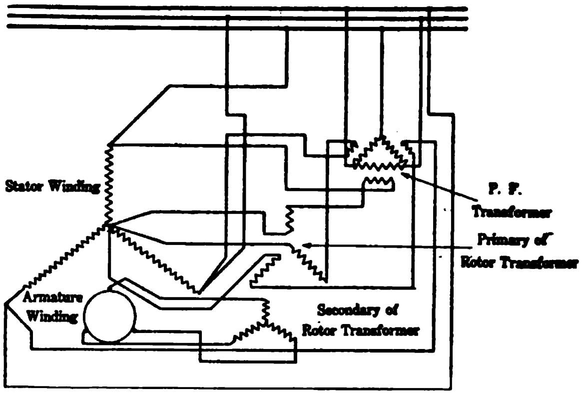

Fig. 8

## Induction Motor with Commutator on the Secondary

In this type of motor, the armature reaction is neutralized by induction instead of by conduction. It is clear that the operation of the motor covered by Fig. 5 and Fig. 6 will not be changed, if we separate the neutralizing winding from the armature winding provided we impress on the neutralizing winding the voltage *A F* and on the armature winding the voltage *F O*. We can then increase the number of turns of the neutralizing winding, so that it can be connected directly across the line while the armature is connected to a rotor-transformer which supplies the voltage *O F*. We can take a brush-shifting series motor, put the brushes in the neutral and connect the primary of the rotor-transformer to different points of the stator winding. In this way, we get a motor which has the same characteristics as a neutralized motor, operated with a constant field flux and a variable voltage applied to the terminals.

Fig. 8 gives the diagram of connections which is suitable for a motor having four poles, or a multiple thereof. Four poles have been connected in series and a tap is brought out at each pole. (The taps have not been shown in the diagram.) This gives four speeds below and four speeds above synchronous speed, in addition to the synchronous speed, and requires thirteen stator terminals. The transformer should be built with a relatively low impedance drop, in order not to spoil the regulation. This can be done as long as only a small variation in speed is required. For instance, if we desire to regulate the speed 25 per cent below and 25 per cent above the synchronous speed, the capacity of the transformer is equal to only 25 per cent of the kilovolt amperes flowing through the motor. The characteristics can be improved by adding a small transformer by means of which the time-phase of the voltage impressed on the primary of the rotor transformer, is shifted.

On small motors, the rotor transformer can be omitted and the stator can be equipped with both a main winding connected to the line and a regulating winding connected to the armature winding. This can be done in various ways.[2]

In all the motors thus far described, the frequency of the rotor currents is reduced to line frequency by means of a commutator connected to the rotor winding, which makes it possible to combine the voltage induced in the rotor with a voltage derived from the line. It is possible to build a satisfactory motor in this way. However, the complicated control which it requires makes it rather unsuitable for the American market. The motor covered by Fig. 8 may find a limited application in special

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[2] See F. Eichberg, *Elektrotechnische Zeitschrift*, 1910, page 749; E. Arnold, *Die Wechselstromtechnik*, Bd. V2.

cases, although too complicated for general application to machine tool work.

The machine tool builders require an a-c. motor which can be installed as easily as a d-c. motor and the speed of which can be changed in a simple manner. In this respect, the next type of motor is more promising.

## THE INDUCTION MOTOR WITH COMMUTATOR ON THE PRIMARY SIDE

Instead of changing over the rotor frequency to the line frequency, we can add to the motor a frequency changer, which makes available at every speed a voltage of the same frequency as the voltage induced in the secondary. This can be done by taking a rotary converter armature, running in a laminated field and driven by the motor of which the speed must be regulated. If we connect the slip rings of the rotary converter to the line and select the number of poles and the speed ratio between the shaft of the motor and frequency changer in the proper way, the frequency of the voltage appearing at the commutator will be proportional to the line frequency times the slip. The ratio of the voltage applied to the slip rings to the one appearing at the commutator is independent of the speed. If we use two movable brush yokes connected to the secondary of the induction motor, by changing the relative positions of these yokes, we can impress on the secondary, voltages of different time-phase and amount and the speed of the induction motor can be regulated, both below and above synchronous speed. If $c$ is the ratio of the number of turns of the winding of the converter to the secondary winding and $n_0$ is the synchronous speed, then the approximate speed range at no load varies between $n_0 (1 - c)$ and $n_0 (1 + c)$.

It is possible to combine the frequency changer and the induction motor, if we locate the induction motor primary winding on the rotor and connect it over slip rings to the line. In that case the frequency changer, which we will denote as regulating winding, does not need to be connected to the line, but its coils can be located on the rotor in the same slots as the primary winding, so that the primary flux induces the voltage in this regulating winding. The secondary is located on the stator and can be built with any number of independent phases, provided each yoke is equipped with the proper arrangement of brushes to correspond to this number of phases. The diagram of connections of a motor of this kind, having three independent secondary phases has been shown in Fig. 9.[3] The brushes $A_1$, $A_2$, $A_3$, are mounted on one yoke and the brushes $B_1$, $B_2$, $B_3$ on another yoke. The motor will operate as an induction motor with short-circuited secondary when the brushes $A$ and $B$ are on the same commutator bar. Both yokes are moved 90 electrical degrees each way from this middle position, the yokes moving in opposite directions. In this manner, regulation both above and below synchronous speed can be obtained. The voltage across every commutator bar is independent of the speed and is proportional to the flux, the frequency and the number of turns in series. If we want to build a motor with the highest possible flux, without exceeding the safe limits for the voltage per bar, we should use a single-turn multiple armature.

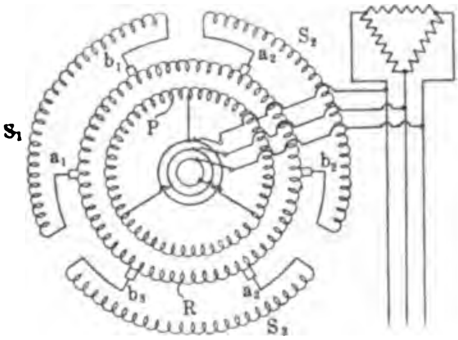

FIG. 9

But even with this winding, the maximum permissible flux is still low, so that this type of motor cannot be built for more than 5 h.p., per pole for 60 cycles and 12 h.p. per pole for 25 cycles, no matter what the speed range is, with the exception that the commutation below synchronous speed is sufficiently better than above, that if we do not make use of the above-synchronous speed range we can use a higher voltage per bar and a larger output per pole. In general, the small output per pole which can be obtained, is no serious draw-back, as most motors for machine tool and elevator work are geared and consequently built for low speed.

On the commutated induction motor with the commutator on the secondary side, the voltage across the commutator bars

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[3] This diagram is the one of the Schrage Patent 1,079,994.

is proportional to the flux, the number of turns and the frequency of slip. Therefore, if we build this motor for a small speed range, we can use a larger field flux and can obtain with 60 cycles and 20 per cent regulation above and below synchronous speed, 30 h.p. per pole and with 25 cycles, 75 h.p. per pole. This is only possible as long as the required starting torque is low, so that we can start the motor with reduced field flux to avoid excessive sparking.

On the commutated induction motor with commutator on the primary side, the voltage across the commutator bars at starting is the same as at running and high starting torque can be developed without any danger to the commutator. This fact makes this motor particularly suitable for reversible operation.

It has been explained above that it is possible to use a larger output per pole, by having the commutator on the secondary, instead of on the primary side. However, as long as we do not exceed the limitations of the output per pole and have 50 per cent regulation both above and below synchronous speed, it is much better, from a commutation standpoint, to use the second scheme. If we should build motors of the same output in both ways, we would naturally use equal field fluxes. This means that the regulating winding of the induction motor with the commutator on the primary side will have one-turn per coil, while the armature winding of the induction motor with the commutator on the secondary side will have two turns per coil. This will give the same sparking voltage at half speed and 50 per cent above synchronous speed for both motors.

It is clear that the commutation conditions, as far as the current commutation is concerned, are much better in the former case than in the latter, as we have a one-turn armature and a lower slot reactance, due to the fact that the regulating winding fills only part of the slots. The current commutation is the limiting feature above synchronous speed and for this reason, the induction motor with commutator on the primary side can be built for a larger speed range above synchronous speed than the one with commutator on the secondary side. This is a great advantage, because the speed-torque characteristics and the maximum output of the motor are much more advantageous above, than below synchronous speed. This partly offsets the limitation as to the maximum output per pole obtainable, as the reduction in speed resulting from the necessity of using a large number of poles, is counteracted to a large extent by the possibility of raising the speed considerably more above synchronism.

The theory of the induction motor with commutator on the primary side can be explained with the aid of the same diagrams as have been given for the other motors. The following derivation of the diagrams made in a different way and some of the equations used in predetermining the characteristics, may be of interest.

In the approximate theory of the induction motor, the exciting current is considered constant[4], and as flowing through a separate exciting circuit. By doing this, a very slight error is made as we neglect the impedance drop of the exciting current, thus figuring with a flux which is slightly larger than the actual flux and we use, too high a value for the exciting current. The approximation simplifies the calculation considerably and will be used in the following. In Fig. 10, we have drawn the approximate vector diagram for the induction motor. We find for the primary circuit that the applied voltage $O H = E_t$ is balanced

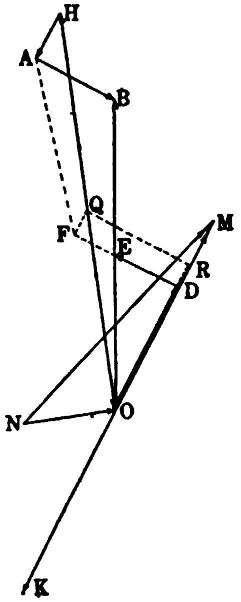

FIG. 10

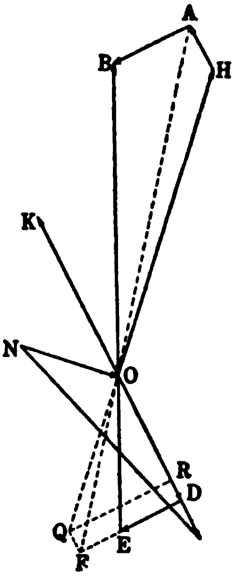

FIG. 11

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[4] See Steinmetz's Alternating Current Phenomena.

by the primary resistance and reactance drop $H A = I_0 r_0$ and $A B = I_0 x_0$ and the voltage $B O = E_0$ induced by the main flux. After having reduced secondary turns to the primary turns, we find that the same voltage $B O = E_1$ is induced in the secondary and that this voltage is balanced by the secondary reactance drop $D E = I_1 s x_1$ the secondary resistance drop $O D = I_1 r_1$, and the counter e.m.f., $E B = (L_1 - s) E_1$. If we draw $Q R \parallel F D$, $E F \parallel B A$ and $Q F \parallel H A$ then $O Q = s E_t$

Let $O Q R = i$ then $\tan i = \dfrac{O R}{Q R} = \dfrac{r_1 + s r_0}{s x_1 + s x_0} = \dfrac{\dfrac{r_1}{s} + r_0}{x_1 + x_0}$

and

$$I_1 = \frac{E_t \sin i}{\dfrac{r_1}{s} + r_0}$$

The torque per phase in synchronous watts equals

$$O B \times I_1 \times \cos \angle B O D = \frac{O D}{s} I_1 = \frac{I_1^2 r_1}{s}$$

The line current $I_t$ is equal to the vector sum of $I_1$ and the exciting current $I_m$, which lags 90 deg. behind $O H$, provided we neglect hysteresis. Thus, the wattless component of the line current is equal to $I_{wt} = I_m + I_1 \cos i$ the watt component $I_1 \sin i$ and the total line current

$$I_t = \sqrt{I_m^2 + I_1^2 + 2 I_m I_1 \cos i}$$

When we drive the induction motor above synchronous speed as an induction generator, the diagram assumes the form of Fig. 11. The counter e.m.f. $E B$ is now larger than the induced voltage $B O$. We have to take into account besides that the rotating field in the rotor has changed its direction, so that the time axis for the rotor rotates counter clockwise, while the time axis for the stator continues to rotate clockwise. For the counter clockwise rotating time axis of the rotor, the secondary reactance drop $D E$ lags 90 deg. behind the current. This secondary reactance drop appears, however, as a negative reactance drop when reduced to the primary circuit in which the time axis rotates clockwise.[5] A complete calculation of an induction motor in accordance with the above method has been given in Table I. It will be noted that this method is the same in principle as the approximate "Steinmetz method."

The induction motor with commutator on the primary side can be calculated in a similar manner. If the primary and the regulating winding are located in the same slots, the leakage between these two windings is so small that it can be neglected. The vector sum of the ampere-turns resulting from the currents in both the primary and the regulating winding, will yield a leakage flux, which induces in the primary winding a voltage $e_s$ and in the regulating winding a voltage $c\ e_s$ if $c$ is the ratio of the number of turns of the regulating and the primary winding.

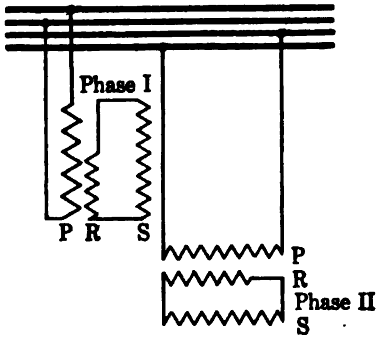

FIG. 12

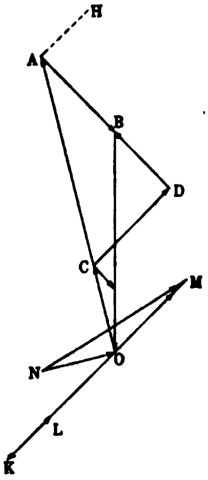

FIG. 13

In order to find how the reactance should be taken into account, we can investigate the conditions at standstill and consider the motor as a quarter-phase stationary transformer having a primary winding, $P$, a regulating winding $R$ and a secondary winding $S$. The windings $P$ and $R$ are fitted in between each other, so as to have only a small leakage between the two. $R$ and $S$ are connected together. Let $P$ and $S$ have the same number of turns, and the ratio of turns of $R$ and $S$ be $c$. If we assume as before that the magnetizing current flows in a separate

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[5] Dr. Rudenberg has called particular attention to this point in the *Elektrotechnische Zeitschrift*, 1910, page 1087.

TABLE I

| | | | | | | |
|---|---|---|---|---|---|---|
| $s$ | 0.01 | 0.0159 | 0.02118 | −0.01 | −0.0159 | −0.02118 |
| $r_1$ | 0.041575 | 0.041575 | 0.041575 | 0.041575 | 0.041575 | 0.041575 |
| $r_0$ | 0.033778 | 0.03378 | 0.03378 | 0.03378 | 0.03378 | 0.03378 |
| $\frac{r_1}{s}$ | 4.1575 | 2.619 | 1.968 | −4.1575 | −2.619 | −1.966 |
| $r_0 + \frac{r_1}{s}$ | 4.19275 | 2.6528 | 2.00178 | −4.1238 | −2.58522 | −1.932 |
| $x_0 + x_1$ | 0.2764 | 0.2764 | 0.2764 | 0.2764 | 0.2764 | 0.2764 |
| $\tan i = \frac{r_0 + \frac{r_1}{s}}{x_0 + x_1}$ | 15.15 | 9.59 | 7.52 | −14.9 | −9.36 | −6.98 |
| $i$ | 86° 13′ | 84° 3′ | 82° 25′ | 273° 83′ | 263° ′53′ | 261° 51′ |
| $\cos i$ | 0.0658 | 0.1036 | 0.1320 | 0.06685 | 0.1067 | 0.1422 |
| $\sin i$ | 0.9978 | 0.9946 | 0.9912 | −0.9977 | −0.9943 | −0.9899 |
| $El$ | 317.5 | 317.5 | 317.5 | 317.5 | 317.5 | 317.5 |
| $I_1 = \frac{El \cos i}{x_0 + x_1}$ | 75.6 | 119.0 | 151.5 | 76.5 | 122.2 | 163.2 |
| or $I_1 = \frac{El \sin i}{r_0 + \frac{r_1}{s}}$ | 75.6 | 118.8 | 157. | 76.8 | 122. | 162.2 |
| $D = \frac{I_1^2 r_1}{s}$ | 23,750 | 36,850 | 48,400 | −24,400 | −39,000 | −51,800 |
| $I_m$ | 59.6 | 59.6 | 59.6 | 59.6 | 59.6 | 59.6 |
| $I_1 \cos i$ | 4.97 | 12.36 | 20.74 | 5.1 | 13.01 | 23.06 |
| $I_{wl} = I_m + I_1 \cos i$ | 64.57 | 71.96 | 80.34 | 64.7 | 72.61 | 82.66 |
| $I_h$ | 4.615 | 4.615 | 4.615 | 4.615 | 4.615 | 4.615 |
| $I_1 \sin i$ | 75.4 | 118.1 | 155.8 | −76.3 | −121.1 | −160.2 |
| $I_w = I_h + I_1 \sin i$ | 80.015 | 122.715 | 160.42 | −71.685 | −116.48 | −155.585 |
| $I_{wl}^2$ | 4,165 | 5,170 | 6,450 | 4,200 | 5,280 | 6,835 |
| $I_w^2$ | 6,400 | 15,010 | 25,750 | 5,150 | 14,700 | 24,190 |
| $(I_w^2 + I_{wl}^2)$ | 10,565 | 20,180 | 32,200 | 9,350 | 19,980 | 31,025 |
| $Il = \sqrt{I_w^2 + I_{wl}^2}$ | 102.8 | 142 | 179.5 | 96.5 | 141.2 | 176.2 |
| $P_1 = D\ (1 - s)$ | 23,500 | 36,250 | 47,400 | −24,600 | −39,600 | −52,950 |
| $F_0$ | 607 | 607 | 607 | 607 | 607 | 607 |
| Mech. Output $= P = P_1 - F_0$ | 22,893 | 35,643 | 46,793 | −25,207 | −40,207 | −53,557 |
| Input $= I_w\, El$ | 25,400 | 38,900 | 50,900 | −22,760 | −36,980 | −49,300 |
| Eff. $= \frac{P}{I_w\, El}$ | 90.2 | 91.7 | 92.0 | 90. | 91.9 | 92.1 |
| P. F. $= \frac{I_w}{Il}$ | 78.0 | 86.3 | 89.5 | 93.7 | 95.2 | 95.7 |
| H. P. $= \frac{3\,P}{746}$ | 92.0 | 143.7 | 188.0 | 102.3 | 161.8 | 215.4 |

circuit, the ampere-turns yielded by $P$ and $R$ must be equal and opposite to the ampere-turns yielded by $S$. Hence, the current flowing in $P$ of Fig. 12 is equal to $(1-c)\ I_1$. The primary leakage flux is yielded by the vector sum of the ampere turns of $P$ and $R$, which is equal and opposite to the secondary ampere turns resulting from $T_1$ flowing through $S$. This primary leakage flux is interlinked with the turns of $P$ and causes the reactance drop $A\ B = T_1\, x_0$, if $x_0$ is equal to the leakage reactance of $P$ considered as the primary of an induction motor. Fig. 13 gives the vector diagram when neglecting the primary resistance drop. In the primary winding $P$, the line voltage $O\ A$ is balanced by the reactance drop $A\ B = I_1\, x_0$ and the induced voltage $B\ O$. The voltage across the regulating winding will be equal to $O\ C = c\ O\ A$ and we find that in the secondary circuit the induced voltage $B\ O$

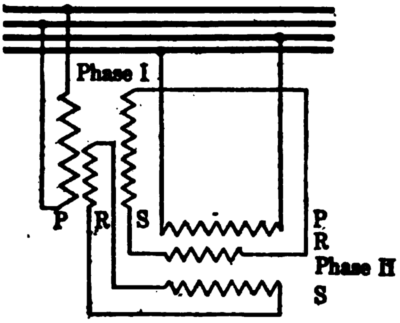

FIG. 14

is balanced by the reactance drop $D\ B$, the resistance drop $C\ D$ and the voltage $O\ C$ across the regulating winding. This voltage is the vector sum of the voltage induced by both the main and the leakage flux. The secondary current $O\ K = I_1$. The primary load current is equal to $-\ (O\ K - K\ L) = -(I_1 - c\ I_1) = O\ M$. We can add the exciting current $N\ O$ at right angles to $O\ A$ and find the line current $I_l = N\ M$. The secondary resistance drop $C\ D$ should include the brush drop and the resistance of both the secondary and the regulating winding. If desired, the resistance drop of the primary load current which is equal to $(1-c)\ I_1\, r_0$ can easily be added, as shown by the dotted line $A\ H$. If $c = 0.5$, the primary load current is equal to $0.5\ I_1$ and the ampere-turns of the primary are equal to those of the regulating winding. In that case, the minimum copper loss will be obtained if we make the copper-cross-sections of both windings equal. When running above synchronous speed, the ampere-turns of the primary winding are equal to the sum of the ampere-turns of both the regulating and the secondary winding, hence in order not to have the primary winding overheat when running above synchronous speed, it is better to make the cross-section of the regulating winding smaller than those of the primary winding. As $A\,C = (1 - c)\,E_l$, we can find the reactance $(x_0 + x_1)$ by an impedance test applying $E_l$ to the slip rings

$$x_0 + x_1 = \frac{E_l\,(1 - c)}{O\,M} \sin \angle\, A\,C\,D$$

or approximately:

$$x_0 + x_1 = \frac{E_l\,(1 - c)^2}{I_l} \sin \angle\, A\,C\,D$$

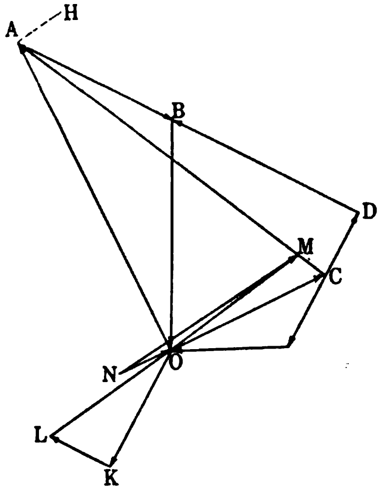

Fig. 15

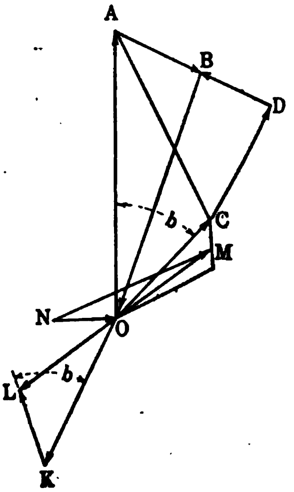

Fig. 16

It is clear that the condition described corresponds to the induction motor with the brushes in the neutral. We can also connect the secondary of phase I to the regulating winding of phase II and the secondary of phase II to the regulating winding of phase I, as has been shown in Fig. 14. The vector diagram of this arrangement has been shown in Fig. 15. In this case, the resultant voltage, which overcomes the impedance drop, is equal to

$$A\,C = E_l\sqrt{1 + c^2}$$

and

$$x_0 + x_1 = \frac{E_l\,(1 + c^2)}{I_l} \sin \angle\, A\,C\,D$$

This corresponds to the commutated induction motor with a brush-shift of 90 deg.

Fig. 16 gives a similar diagram for a brush-shift ($b$) in which case the resultant voltage is equal to

$$A\ C = E_t \sqrt{1 + c^2 - 2\,c \cos b}$$

and

$$x_0 + x_1 = \frac{E_t\,(1 + c^2 - 2\,c \cos b)}{I_t} \sin A\ C\ D.$$

All the above equations for $x_0 + x_1$ have been given without taking the primary resistance loss, the core loss and the magnetizing current into account. This is satisfactory as long as the impedance test is made at greatly reduced voltage. If necessary, these values can be taken into account as follows: Read primary watts ($W$) amp., ($I_t$) volts ($E_t$).
Determine $I_w = I_t \cos \varphi$ and $I_{w1} = I_t$ sine $\varphi$.
Determine at standstill with open secondary $I_m$ for a voltage $E_t$ and subtract $I_m$ from $I_{wt}$ then the secondary current reduced to the primary is equal to

$$I_1 = \frac{\sqrt{(I_{wt} - I_m)^2 + I_w^2}}{\sqrt{1 + c^2 - 2\,c \cos b}}$$

The secondary impedance is

$$z_t = \frac{E_t\,(1 + c^2 - 2\,c \cos b)}{\sqrt{(I_{wt} - I_m)^2 + I_w^2}}$$

Let the core loss with $E_t$ volts applied to the slip rings be $W_c$ watts and the primary resistance loss with a current $I_t$, be $3\ I_t^2\,r_0$ then

$$r_1 = \frac{W - W_c - 3\,I_t^2\,r_0}{3\,[(I_{wt} - I_m)^2 + I_w^2]}\,(1 + c^2 - 2\,c\,s \cos b)$$

and

$$x_0 + x_1 = \sqrt{z_t^2 - r_1^2}$$

A comparison of Fig. 16 and Fig. 13 shows that due to the shift $b$, the secondary current $I_1 = O\,K$ is shifted into the direction of the secondary induced voltage $B\,O$. This means that with the same flux and the same secondary current, we get a higher torque. The possibility of shifting the time-phase of the secondary current by means of the brush-position, can be utilized for obtaining a higher torque per ampere than is possible with the induction motor with resistance in the secondary. The fact that the secondary current at the same time flows through the regulating winding and thereby reduces the primary current, makes this condition still better.

Fig. 17 has also been drawn for a brush-shift $b$, only in this case the motor is running and a counter e.m.f. $E\,B$ is induced in the secondary. The resultant voltage in the secondary is equal to $E\,O = B\,O - E\,B$. If we draw $E\,F \,||\, A\,B$ then

$$\frac{O\,F}{O\,A} = \frac{O\,E}{O\,B} = \frac{s}{1} \text{ or } O\,F = s\,E_t.$$

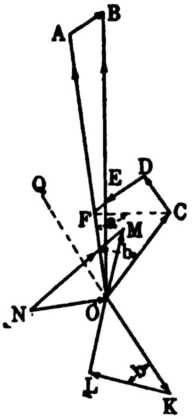

FIG. 17—RUNNING AS MOTOR BELOW SYNCHRONOUS SPEED—BRUSH SHIFT OPPOSITE TO DIRECTION OF ROTATION

Further $$F\,E = s\,A\,B = s \times x_0\,I_1.$$

$$F\,C = \sqrt{F\,O^2 + O\,C^2 - 2\,F\,O \times O\,C \cos \angle F\,O\,C}$$
$$= E_t \sqrt{s^2 + c^2 - 2\,c\,s \cos b}$$

Let $$\angle\, O\,F\,C = a \text{ and } \angle\, C\,F\,D = i$$

then $$\frac{\sin a}{O\,C} = \frac{\sin b}{F\,C} \text{ or } \sin a = \frac{c \sin b}{\sqrt{s^2 + c^2 - 2\,c\,s \cos b}}$$

$$\tan i = \frac{D\,C}{F\,D} = \frac{r_1}{s\,(x_0 + x_1)}.$$

The secondary current is equal to

$$I_1 = \frac{F\,C \sin i}{r_1} = \frac{E_t \sin i \sqrt{s^2 + c^2 - 2\,c\,s \cos b}}{r_1}$$

The torque per phase in synchronous watts is equal to

$$D = B\,O \times O\,K \cos \angle B\,O\,Q = O\,A \times O\,K \cos \angle A\,O\,Q$$

$$\angle A\,O\,Q = \angle O\,F\,C - (90^\circ - \angle C\,F\,D) = a + i - 90.,$$

or

$$\cos \angle A\,O\,Q = \sin (a + i)$$

and

$$D = E_l \times I_1 \sin (a + i)$$

The electrical output is equal to $P_1 = D\,(1 - s)$

The mechanical output ($P$) is equal to the electrical output minus the friction and windage per phase, $P = P_1 -$ friction, windage. The primary current can be calculated by determining separately the component $I_w$ in phase with the applied line voltage and the component $I_{wl}$ which is 90 deg. out of time phase with the line voltage.

If $I_h$ is the core loss current then:

$$I_w = I_h + O\,K \cos \angle Q\,O\,A - L\,K \cos (\angle Q\,O\,A + b)$$
$$I_w = I_h + I_1 \cos (a + i - 90) - c\,I_1 \cos (a + i + b - 90).$$
$$I_w = I_h + I_1\,[\sin (a + i) - c \sin (a + i + b)]$$

If $I_m$ is the wattless component of the magnetizing current then,

$$I_{wl} = I_m + O\,K \sin \angle Q\,O\,A - L\,K \sin (\angle Q\,O\,A + b)$$
$$I_{wl} = I_m + I_1\,[\cos (a + i) - c \cos (a + i + b)]$$

The total line current is $I_l = \sqrt{I_w^2 + I_{wl}^2}$

The power factor $= \dfrac{I_w}{I_l}$

The input $= E_l\,I_w$

The efficiency $= \dfrac{P}{E_l\,I_w}$

When determining the angles from the sine and tangent, some care must be taken to get the angle in the proper quadrant. This can be done readily as long as the possible variation of these angles with the slip is taken into account, with the aid of the diagram. For instance, it follows from Fig. 17 that $F$ moves over $A\,O$ from $A$ to $O$ when the slip changes from 1 to 0.

The angle $CFO = a$ changes from an acute to an obtuse angle. Fig. 18 gives the diagram for the same brush position as used in the diagram of Fig. 17, when the motor is driven above synchronous speed. In case the motor is rotating counter clockwise, the field in the rotor will rotate clockwise. Hence, as long as we operate below synchronous speed, the electrical field will rotate clockwise, in respect to both the stationary secondary winding and the stationary brushes connected to the commutator of the regulating winding. This means that if we want $OC$ to lag behind $OA$, as shown in Fig. 17, we have to shift the brushes in the direction of the rotation of the electrical field, *i.e.* opposite to the mechanical rotation of the rotor. When running above synchronous speed, the electrical field will rotate counter clockwise, with respect to both the stationary secondary winding and the brushes and if we leave the brush position unchanged $OC$ will lead $OA$. When reduced to the primary, for which the time axis rotates clockwise, $OC$ will lag behind $OA$ as in Fig. 17. This has been shown in Fig. 18. When running above synchronous speed, the secondary reactance

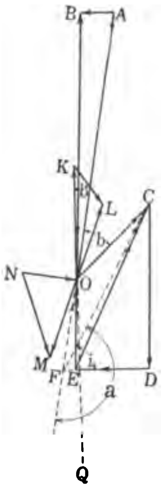

Fig. 18—Driven as Generator Above Synchronous Speed — Brush Shift from Neutral Axis Opposite to Direction of Rotation

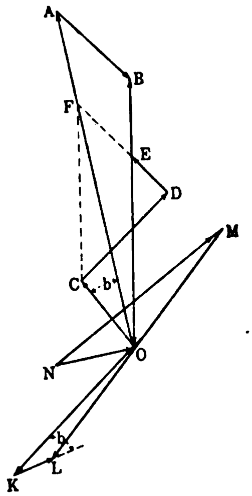

Fig. 19—Running as Motor Below Synchronous Speed — Brush Shift in the Direction of Rotation

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*Since this paper was first published, Mr. J. I. Hull has modified the rules so as to make them apply to the operation in all brush-positions and at all speeds.

drop appears negative when viewed from the primary, as explained for the usual induction generator with short-circuited secondary. Table II gives a complete calculation made in accordance with the above theory. It has been found that the theory is confirmed by test.

Fig. 19 gives the diagram for a motor operating below synchronous speed when the brushes are shifted in the wrong direction, *i.e.*, in the direction of rotation. It will be noted that this results in a low secondary power factor, which is always combined with a reduction in maximum output and larger variation between the no-load and full-load speed.

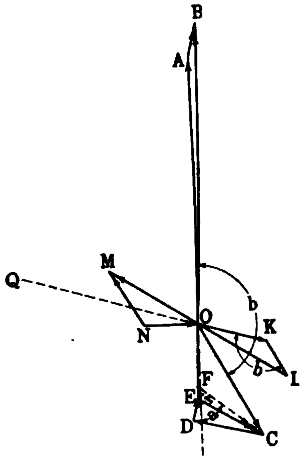

FIG. 20—RUNNING AS MOTOR ABOVE SYNCHRONOUS SPEED—BRUSH SHIFT FROM NEUTRAL AXIS MORE THAN 90 DEG. AND LESS THAN 180 DEG. OPPOSITE TO DIRECTION OF ROTATION. (BRUSH SHIFT IN DIRECTION OF ROTATION FROM REVERSED NEUTRAL AXIS)

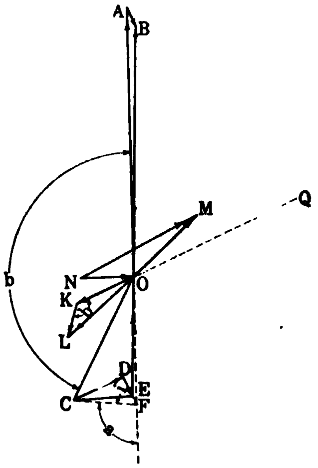

FIG. 21—RUNNING AS MOTOR ABOVE SYNCHRONOUS SPEED—BRUSH SHIFT FROM NEUTRAL AXIS MORE THAN 90 DEG. AND LESS THAN 180 DEG. IN THE DIRECTION OF ROTATION. (BRUSH SHIFT AGAINST ROTATION FROM REVERSED NEUTRAL AXIS)

Fig. 20 gives the diagram for operation above synchronous speed. The counter e.m.f., $E B$ is larger than the induced secondary voltage $B O$ and the voltage $O C$ of the regulating winding is impressed in opposite direction, being shifted over an angle $b$ opposite to the direction of rotation from the neutral position.

Fig. 21 gives the same diagram, only the brushes are shifted in the direction of rotation which causes the secondary current to lead too much and results generally in inferior commutation. Instead of calculating the characteristics by means of simple geometrical equations, circle diagrams can be used.[6] Both the derivation and the application of these diagrams are rather complicated and, therefore, will not be given in this paper.

Fig. 22 gives the tested speed-torque curves of a 440-volt, 3-phase, 1200 rev. per min., 60-cycle brush, shifting motor, built as an induction motor with the commutator on the primary side and on which the brushes are shifted 5 deg. opposite the direction of rotation in the slowest speed position and 5 deg. in the direction of rotation in the highest speed position. This has been done by shifting one yoke faster than the other, while both move in opposite direction, so that one yoke moves through 170 electrical degrees, while the other yoke moves through 190 electrical degrees. This motor has been built for one direction of rotation.

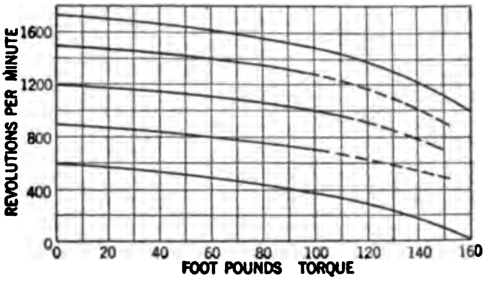

FIG. 22

If the motor has to operate with the same characteristics in both directions of rotation, the brush-shifting mechanism becomes more complicated, as will be understood from Figs. 23 and 24, which give the desired brush-position for maximum and minimum speed in both directions of rotation for a two-pole motor. However, a simple mechanical solution has been found and a model incorporating this scheme is now being tested. A photograph of it is shown in Fig. 25, while the brush-shifting mechanism is represented in Fig. 26. This motor has been built with four-poles and a six-phase secondary, each yoke having 12 studs.

The fast moving yoke *B* on which the brushes *b*1, *b*2, *b*3 have been mounted, is supported by the bearing-housing. The slow moving yoke on which the brushes *a*1, *a*2, *a*3 have been

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6. See O. S. Braystad, *E. T. Z.*, 1903, page 368; E. Arnold, *Die Wechselstromtechnik*, Bd. V2; H. Meyer-Delius, *General Electric Review*, 1914, page 817; H. K. Schrage, *E. T. Z.*, 1914, page 81.

mounted, is supported by the end shield; it is moved by means of a stud which connects it to the disk *C*, which is supported on the bearing housing. The pinions *D* and *E* are keyed to a shaft which can rotate in a bearing supported by the disk *C*. The pinion *D* meshes with a gear *F* which is mounted on the bearing housing and the pinion *E* meshes with a gear *G* which is fastened to yoke *B*. If the gear *F* is held stationary and the disk *C* is turned, both opinions *D* and *E* will rotate. Pinion *E* will drive the gear *G* fastened to yoke *B*. The gear ratio can be selected in such a manner that disk *C* and yoke *A* move in opposite direction to yoke *B* and at a slower rate. The gear *F* is held by a pinion *H* which is keyed to the same shaft as the intermittent gear *I*, while this shaft rotates in a bearing which is rigidly supported from the bearing housing. The intermittent gear *I* is moved through a small angle when it is struck by a pin, which is fastened to the disk *C*. After the pin has moved it, *I* is held stationary by sliding on the outside circumference of *C*. When *I* is moved by the pin, *H* will turn through the same angle and move the gear *F*. By properly selecting the gear ratios, it is possible, when we change from "Slow" in one direction to "Slow" in the other

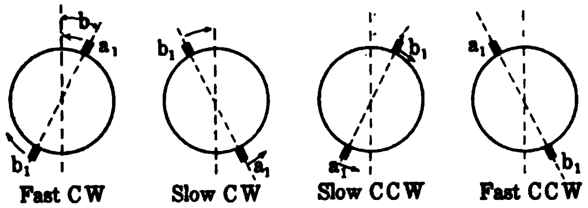

FIG. 23

$b_1$ moves through 360 − 2 *b* deg. *C W*
$a_1$ moves through 360 + 2 b deg. *C C W*. While $b_1$ moves through 2 *b* deg. from slow *C W* to slow *C C W*, $a_1$ should move *C W* through 2 b degrees

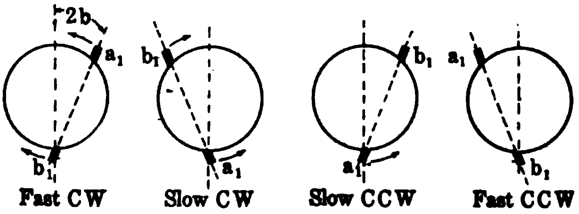

FIG. 24

$b_1$ moves through 360 deg. *C W*
$a_1$ moves through (360 deg. + 4 *b*) *C C W*. While $b_1$ moves through 4 *b* degrees from slow *C W* to slow *C C W*, $a_1$ should stand still

direction, to have yoke *B* remain stationary or move in the same direction as *C*, while the intermittent gear is moved by the pin. The former arrangement gives a change of brush-positions in accordance with Fig. 24, the latter in accordance with Fig. 23. A control switch is connected to the shaft to which *I* is connected, and changes the phase-rotation of the lines to which the collector rings have been connected.

The induction motor with commutator on the primary side can be run single-phase, by opening one lead connecting to the collector rings. If only single-phase is available, it can be started like an ordinary single-phase induction motor by using a split-phase starting device, or as a repulsion motor, in which case a higher torque can be obtained. This connection is shown in Fig. 27, for a motor having a quarter-phase secondary.

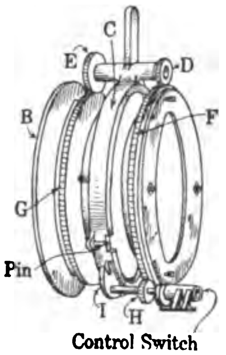

FIG. 26

One phase of the regulating winding is used as the primary of the repulsion motor, a resistance *R* keeps the line current down and the connections are changed from starting to running, by means of a three-pole double-throw switch. If the motor has a three-phase secondary, a four-pole double-throw switch is required.

In its original form, the induction motor with commutator on the primary side is wound with the primary winding in the bottom and the commutated regulating winding in the top of the armature slots. In this way, four coil ends have to be inserted in every slot, which takes up much room for the insulation. Moreover, if the primary winding has to be repaired, it is necessary to remove first part of the coils of the regulating winding. A winding is being developed which is arranged in such a manner

PLATE V.
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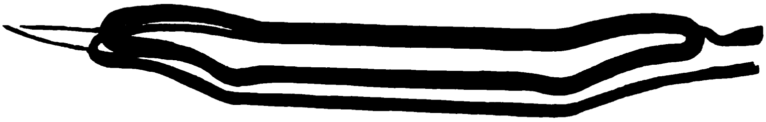
FIG. 25

FIG. 28



[ALTES

FIG. 29

that only two coil ends are located in one slot. If we have an even number of slots and use a coil pitch equal to an odd number, we can arrange the windings in such a way that we get a primary coil in the top of every odd and in the bottom of every even slot, and a coil of the regulating winding in the top of every even slot and in the bottom of every odd slot.

The leads of the primary winding can be brought out on the back end and the leads of the coils of the regulating winding on the commutator end. The insertion of such a winding is no more difficult than the insertion of an ordinary lap winding, it merely being necessary to have two kinds of coils. By doing this, we require half the number of coils and save room in the slots. Fig. 28 shows a photograph of a completed armature and Fig. 29 one coil of the primary and one coil of the regulating winding of a 220-volt, 750-rev. per min., 25-cycle motor with this new type of winding. The coils are inserted into open slots which are closed by a magnetic wedge. It will be noted that the coil of the regulating winding is lighter than the one of the primary winding, and that a slight bend has been added to each end of the lower straight part of the primary coil, in order to make room for the upper half of the next primary coil.

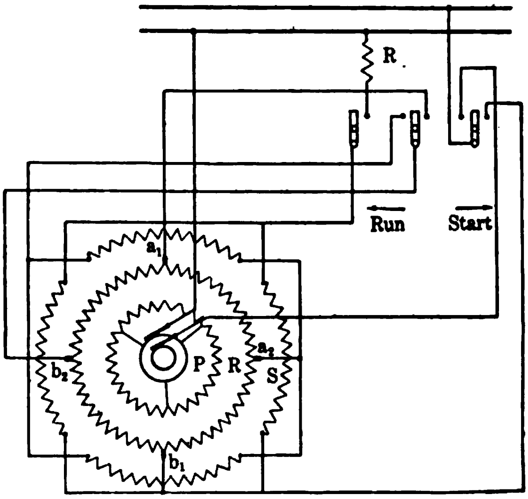


FIG. 27

The regulating winding is a closed commutated winding with one coil per slot. The coils should always be connected in such a manner, that we get a balanced mesh connection without internal short-circuiting current. We can obtain this condition, by making the coil pitch equal to an odd number of slots. This has been shown in Fig. 30, in which we have an odd number of

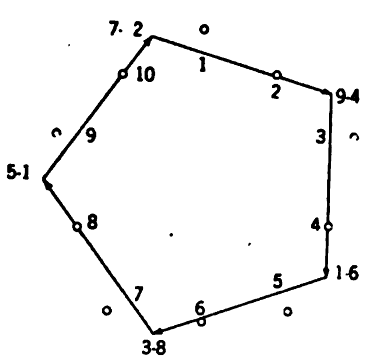


FIG. 30

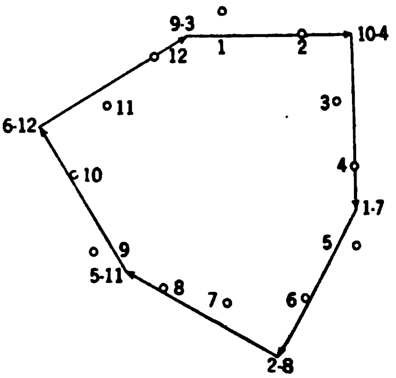


FIG. 31

slots per pole (5), the coils being full pitch slot 1 to 6. The coil ends have been represented by hollow dots. The vectors 1-6, 3-8, 5-10, 7-12, 9-4, represent the values of the induced voltages in the coils 1-6, 3-8, etc., when this armature is

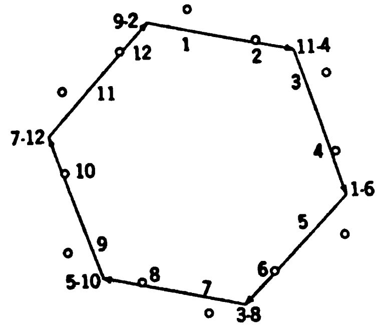


FIG. 32

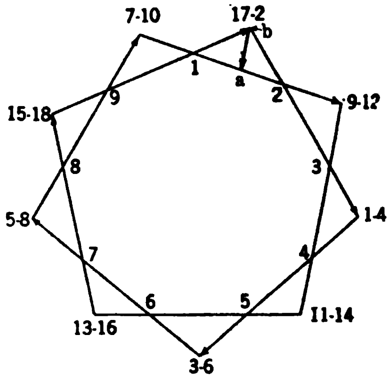


FIG. 33

surrounded by a rotating field. The resultant voltage of all the coils is zero, which means that we will have no circulating current. The same result could have been obtained with a coil pitch 1-4, 1-2, 1-8, 1-10.

In Fig. 31, we have an even number of slots per pole (6) and a coil pitch equal to an even number of slots. This results in a balanced winding, since the coils have been arranged in groups of two, so that we get the same effect as though we had an odd number of slots per pole (3). This arrangement is possible, if the number of slots per pole is equal to an odd number times an integer.

In Fig. 32, we have an even number of slots per pole. This gives also a balanced winding, as long as the coils span an odd number of slots (5). We could also have used a coil pitch 1-4, 1-2, etc.

Fig. 33 gives the vector diagram for a four-pole armature having (18) slots, *i. e.*, 4½ slots per pole. Slot 1 is in the same time-phase as slot 10, slot 2 as slot 11, etc., so that we can use one potential circle. The coil pitch is 1-4. We have 4½ coils for every two poles. Even if we have two commutator bars per coil, we would not get zero voltage between the beginning of coil (1-4) and the middle point of the 5th coil (9-12), but the vector difference would be equal to *a-b*. However, all nine coils together give zero voltage, so that we can use this arrangement either as a four-pole series armature, or as a four-pole multiple armature, without equalizers. If we put on equalizers, a voltage equal to *a-b* would be short-circuited. Neither is it advisable to use bus rings in this case, and we should wind the secondary which connects to the brush-studs, with two separate circuits per phase

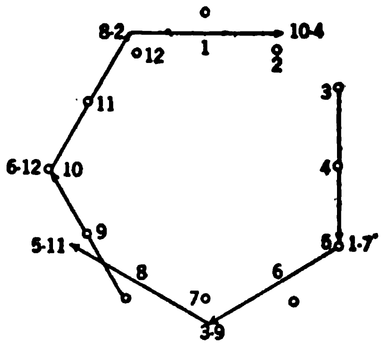


Fig. 34

Fig. 34 is an arrangement which would tend to a large circulating current, and in which we have an even number of slots per pole and a full pitch coil. If we use (10) commutator bars and (2) conductors per slot in all the slots with the exception of slot 5, 11, 6 and 12, in which one conductor per slot should be used, we get a closed polygon and can use this winding.

From Figs. 30, 31, 32, 33 and 34, it follows that the regulating winding can be wound with one coil per slot, as long as certain conditions are fulfilled.

We may now investigate which numbers of slots give a satisfactory primary winding. In order to get a balanced winding without excessive local leakage, it is necessary to have one coil end of the primary winding in every slot. This is possible only when the total number of primary slots is even. The coil ends belonging to the different phases, should be inserted in the same order; otherwise the phase rotation will be reversed on some parts of the circumference. This limits the use of fractional pitch coils for the primary to certain special cases, which will be described below.

The following general rules can be formulated which make it possible to determine whether a certain number of slots is suitable for a primary winding of a given number of poles and phases. (*a*) The total number of slots should be even. (*b*) To get a regular winding, the number of slots per pole, should be divisible by the number of phases $d = \frac{s}{2\,p \times n}$ = integer. (In which $2\,p$ = number of poles, $n$ = number of phases.) (*c*) To get a balanced winding, the total number of slots should be divisible by two times the number of phases $b = \frac{s}{2\,n}$ = integer.

If we want to find whether a satisfactory winding can be obtained, we can proceed as follows:

Determine $e = \frac{s}{2\,p}$ and $b = \frac{s}{2\,n}$

*Case* 1. If $e$ = odd number, make the coil pitch equal to $e$ and connect primary so as to get $b$ slots per phase and an equal number of slots per pole. (For regulating winding, see Fig. 30.)

*Case* 2. If $e$ = even number, see whether $e$ has an odd factor $c$ and if so, make coil pitch equal to $e$ and treat $\frac{e}{c}$ coils like under 1. (For regulating winding, see Fig. 31.)

*Case* 3. If $e$ = even number, determine $\frac{s}{2\,p\,n} = d$ = number of slots per pole per phase. If $d$ is even, then make coil pitch equal to $\frac{s}{2\,p} \pm 1$ and connect as under 1. (For regulating winding, see Fig. 32.) If $d$ is odd, it is impossible to get a balanced winding.

*Case* 4. If $e$ is a mixed number, see whether $\frac{s \times f}{2\,p}$ gives an integer in which $f = p$, or $f < p$ and $\frac{p}{f}$ = integer. This gives an irregular winding, while the regulating winding, if connected as a multiple winding, can have equalizers which span $2\,f$ poles, instead of two-poles. If $p = f$, then we can have no equalizers. (For regulating winding, see Fig. 33.)

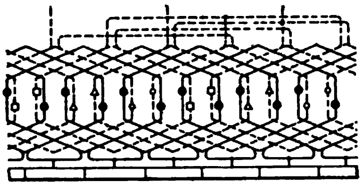


FIG. 35

In all the above cases, we will get a balanced winding only when $b = \frac{s}{2\,n}$ = integer.

A few examples may make it clear how the above rules can be applied.

*Case* 1. The armature shown in Fig. 28 may be mentioned as an example; it has four poles, three phases and sixty slots. $e = \frac{60}{2 \times 2} = 15$; $b = \frac{60}{2 \times 3} = 10$ and $d = \frac{60}{2 \times 2 \times 3} = 5$.

*Case* 2. As an example may be mentioned the 60-slot, 6-pole, 3-phase winding used on the 10-h.p. motor, the speed-torque curves of which are covered by Fig. 25. In this case we have: $e = \frac{60}{2 \times 3} = 10$; $e$ has an odd factor $c = 5$, while $\frac{e}{c} = \frac{10}{5} =$

2. The pitch of the primary coil is equal to 10 slots, $\frac{e}{c} = 2$, coils are treated as a single-coil.

*Case* 3. A 48-slot, single-pole, quarter-phase armature can be wound in this way,

$e = \frac{48}{2 \times 3} = 8$; $d = \frac{48}{2 \times 3 \times 2} = 4$. The coil pitch is equal to $e - 1 = 7$. As another example

Fig. 35 represents the diagram of both the regulating and the primary winding wound with two poles, three-phases and twelve slots. $e = \frac{12}{2} = 6$; $b = \frac{12}{2 \times 3} = 2$, and $d = \frac{12}{2 \times 3} = 2$.

*Case* 4. The scheme covered by this case is suitable for a 60-slot, 8-pole, 3-phase armature.

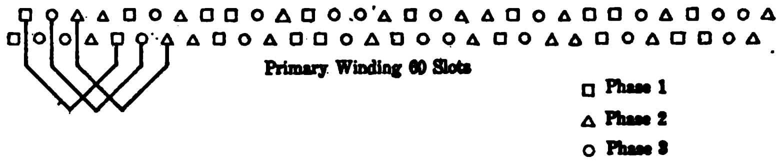


FIG. 36

$e = \frac{60}{2 \times 4} = 7.5$; $d = \frac{60}{2 \times 4 \times 3} = 2.5$, *i. e.*, the winding is irregular, so that we have to use alternately 2 and 3 slots per pole.

$b = \frac{60}{2 \times 3} = 10$, *i. e.*, the phases are balanced.

By making $f = 2$, we get $\frac{s\,f}{2\,p} = \frac{60 \times 2}{8} = 15$.

The equalizers should span $2f = 4$ poles and each phase of the secondary should have two independent circuits. An examination of Fig. 36, which covers the arrangement of the phases of this winding, will show that this arrangement repeats itself every 30 slots, which means that equalizers spanning 30 slots, *i. e.*, four poles will connect slots of equal potential, with respect to the primary winding. From Fig. 33, it follows that these equalizers are also permissible, as far as the regulating winding is concerned.

A 60-slot armature is particularly suitable for this type of winding, as it can be wound with 2, 4, 6, 8, 10 and 12 poles. It is possible to use the same commutator and brush rigging for motors having 4, 6, 8 and 12 poles, by building the secondary with 6, 4, 3, and 2 phases, respectively. This leads to a reduction in the number of required mechanical parts for motors of different frequencies and speeds, which is very advantageous from a manufacturing point of view, and will make this type of motor less costly as soon as there is a sufficient demand for it.

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New York, N. Y., February 16, 1918.*

# THE SECOMOR

## A KINEMATIC DEVICE WHICH IMITATES THE PERFORMANCE OF A SERIES-WOUND POLYPHASE COMMUTATOR MOTOR

BY V. KARAPETOFF

### ABSTRACT OF PAPER

The device consists of four bars of adjustable useful length and with adjustable angles. These bars can be set in a combination to represent the vector diagram of voltages in a motor with any desired constants. By moving the bars to vary the load, complete performance characteristics of the motor can be obtained, including the speed, the torque, the power factor, etc. An additional device called the impedometer permits to take into account the impedance drop in the machine. An adjustable saturation curve made of soft wire is used in connection with the secomor, to enable one to investigate the effect of saturation. A brief graphical theory of the motor precedes the description of the secomor to make its action understandable.

### LIST OF SYMBOLS USED

| | |
|---|---|
| $E$ | total useful voltage (or counter e. m. f.) of the machine. |
| $E_1$ | stator e. m. f. |
| $E_2$ | rotor e. m. f. at standstill. |
| $I$ | current in the machine without the transformer. |
| $I_1$ | stator current with the transformer (Fig. 11). |
| $I_2$ | rotor current with the transformer (Fig. 11) |
| $I_m$ | exciting current of the machine (Fig. 6) |
| $I_0$ | exciting current of the series transformer (Fig. 11). |
| $M$ | exciting m. m. f. of the machine (Fig. 6). |
| $M_1$ | stator m. m. f. |
| $M_2$ | rotor m. m. f. |
| $n_1$ | effective stator turns |
| $n_2$ | effective rotor turns. |
| $P$ | line voltage. |
| $r$ | resistance of the motor circuit |
| $s$ | slip expressed as a fraction. |
| $u$ | ratio $n_2/n_1$ of the effective rotor turns to the effective stator turns. |

$x$ leakage reactance of the machine.
$Z_1$ primary leakage impedance of the series transformer.
$Z_2$ secondary leakage impedance of the series transformer.
$z$ leakage impedance of the machine.
$\alpha$ brush shift angle measured from the opposition setting.
$\alpha'$ same, modified by the transformer magnetizing current, Fig. 12.
$\beta$ angles defined in Fig. 6
$\gamma_s$ an angle defined in Fig. 12.
$\Phi$ useful flux in the air-gap.
$\phi$ phase angle of the machine.

## A. INTRODUCTION

*THE MEANING of the Name Secomor*: An abbreviation of the words "series commutator motor".

*What the Secomor is.* A combination of movable and adjustable bars (Fig. 9) which can be set to represent a vector diagram of voltages, currents, m.m.f's and fluxes in a series-wound polyphase commutator motor with any desired constants.

*The Purposes of the Device.* (1) To enable a designer to select the best electrical constants and to "test" a motor before it has been actually built; (2) To take the place of a complicated circle diagram which does not hold true anyway when the iron is saturated; (3) To do away with an involved analytical theory because it is often difficult to see the effect of separate factors upon the performance characteristics, and because it takes considerable mathematical skill to deduce the equations of the various loci; (4) To add the judgment of the eye and the skill of the hands to the purely mental ability in selecting the constants of a motor for a desired performance, or in judging the characteristics for assumed constants; (5) To enable an investigator or a student to familiarize himself with the motor as if he had one available for tests.

*The performance curves* that the secomor enables one to draw: Current, torque, speed, input, output, efficiency, power factor, magnetizing current. These may be obtained just as easily at a constant applied voltage as at a constant current, or under any variable conditions of service.

*The factors which may be taken into account* and varied at will in the secomor: The ratio of the primary to the secondary ampere turns; the ratio of either one to the exciting m.m.f.; the angle of brush shift; the ohmic drop and the reactive drop;

saturation in iron; variable core-loss and friction; reaction of short-circuited armature coils upon the exciting current.

### B. General Properties of the Motor

A detailed description of the motor, its field of application, its performance characteristics and a complete theory, both graphical and analytical, will be found in E. Arnold's "Die Wechselstromtechnik", (1912) Vol. V, Part 2, Chap. 2. See also the bibliography below.

Diagrams of connections are shown in Figs. 1, 2 and 3. The stator is phase-wound and is similar to that of an induction motor. The rotor is like a d-c. armature with a commutator. With a multiple armature winding the number of brushes per pair of poles is equal to the number of phases; with a two-circuit winding it is considerably less.

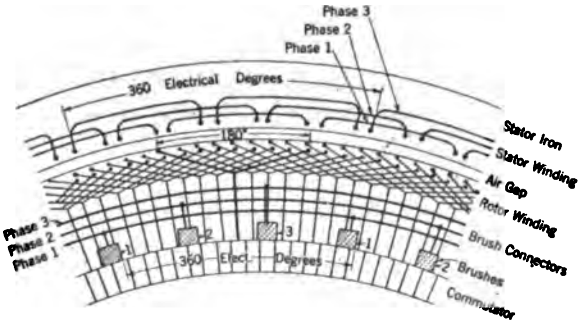

FIG. 1—THE STATOR, THE ARMATURE WINDING AND THE BRUSHES IN A THREE-PHASE COMMUTATOR MOTOR

The rotor is connected in series with the stator, either directly (Fig. 2), or through a current transformer (Fig. 3). In the latter case the stator may be either star or mesh connected.

*Characteristics and the field of application.* The motor has a speed-torque characteristic similar to that of a d-c. or single-phase series motor. As the load increases the speed decreases. At no-load there is a tendency to run away. Roughly speaking, one may consider the motor as a combination of three single-phase series-connected motors built on the same frame. On account of the mutual action between the phases such a view is not quantitatively correct. The motor may be used for crane service and in other applications in which it may replace a series-wound d-c. motor. A peculiar field of its own is in cascade connection with an induction motor, as a counter e.m.f. arrangement for speed control. This is useful in mine-hoist work and in rolling mills; see Arnold, p. 266. A shunt-connected polyphase commutator motor is also suitable for cascade connection.*

*Advantages of a series transformer* (Fig. 3): (a) The stator may be wound for a comparatively high voltage, *e.g.*, 2200 volts; (b) The rotor may be wound for such a value of current as to give the best commutation; (c) When the machine is used as a generator, for example in regenerative braking, there is a possibility of a harmful direct current or of low-frequency alternating currents produced in the armature, due to self-excitation. By saturating the current transformer this tendency is counteracted; see Arnold, p. 64. (d) By properly selecting the saturation and the magnetizing current of the transformer the speed above synchronism may be controlled to some extent, so that the motor does not run away when the load is removed.

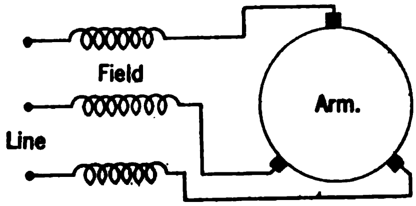

FIG. 2—THREE-PHASE SERIES COMMUTATOR MOTOR—THE ARMATURE SUPPLIED AT THE LINE VOLTAGE

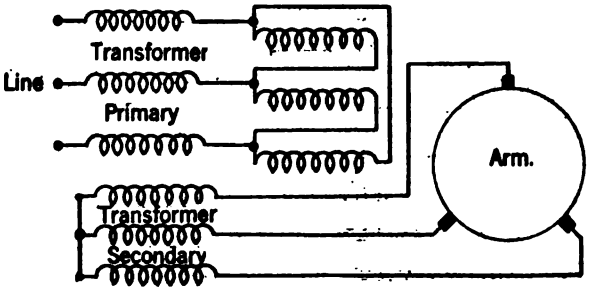

FIG. 3—THREE-PHASE SERIES COMMUTATOR MOTOR—THE ARMATURE SUPPLIED THROUGH A TRANSFORMER

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*See J. D. Wright, *General Electric Review*, 1916, p. 104.

*Four fundamental properties of an armature connected to a polyphase a-c. circuit through a commutator and brushes, Figs.* 1 *and* 2.

(a) The *currents* in the armature conductors are of the same frequency as the line currents, independent of the speed of rotation. Call the armature coils (Fig. 1) between two adjacent brushes "a group". The function of the commutator is to transfer the coils in succession from one group to the next. But a new coil is always substituted in place of the one transferred, so that *the group persists* and forms a steady path for the line currents. This, of course, does not apply to the coils undergoing commutation, in which high-frequency currents are induced.

(b) The *voltages* induced in the armature conductors are always of the same frequency as that of the stator flux which induces them. The relative speed of rotation of the flux and armature influences only the magnitude of the induced voltage but not its frequency. Consider a d-c. machine in which the field current and the flux are periodically varied. The e.m.f. induced between the brushes undergoes corresponding fluctuations. If the field be varied harmonically, the induced e.m.f. will also vary at the same frequency according to a sine law. Speeding up the armature or slowing it down will influence the value of the induced voltage, but its frequency is always that of the flux. With a constant flux the voltage is also constant. Similarly, in an a-c. motor the instantaneous voltage induced in a group of coils between adjacent brushes is proportional to the instantaneous value of the stator flux and to the speed of rotation. The stator flux varies at the impressed frequency, and so does the induced voltage. This simple property of armature currents and voltages makes it possible to use vector diagrams at any speed of rotation. It will be remembered that in an induction motor (without commutator) the frequency of the secondary currents depends upon the speed of the rotor.

(c) The *magnetomotive force* and the *flux* due to the armature currents are nearly the same whether the armature is revolving or is at rest, provided that the armature current is kept constant. We have seen above that the groups of armature coils between the brushes are always stationary, and the currents that flow through these groups are of the same frequency as the line currents. Hence, the *m.m.fs. of the individual groups* are stationary in space at all speeds of the armature. The combined action of several groups of coils produces a revolving field, same as in the stator winding, and this field glides synchronously in the air-gap with respect to the brushes, no matter what the speed of the armature might be. The m.m.f. due to high-frequency currents in the coils undergoing commutation is a disturbing factor, being a function of speed, current, stator flux, etc., but its effect is usually small. For its computation see Arnold, pp. 23 and 56.

(d) The *leakage reactance* of a commutated armature is a function of its speed, unlike that of ordinary a-c. windings. Two kinds of leakage fluxes may be distinguished, (1) those linking with individual armature conductors or with groups of conductors belonging to the same phase, and (2) those linking with conductors belonging to different phases. The first named fluxes (phase leakage) are carried around with the conductors and cause a reactance drop which is independent of the speed of rotation of the armature. The latter fluxes (interlinked leakage) in combination form a true revolving flux which glides synchronously in the air gap whether the armature is standing still or revolving at any speed.

The effect of the interlinked leakage flux depends upon the slip of the armature. At synchronous speed the armature conductors do not cut this flux at all, and the only reactance is that due to the phase fluxes which travel with the conductors. At standstill the interlinked leakage flux exerts its full effect. At speeds above synchronism the armature conductors cut this flux in the opposite direction, so that it partly compensates for the effect of local or phase fluxes. At a certain synchronous speed the total armature reactance becomes zero, and beyond that speed it becomes negative. Whatever the theory of the phenomenon, actual experiments show that the reactance of a commutated armature winding decreases with increasing speed. See Arnold, p. 20.

*Brush shift and the m.m.fs.* The alternating currents in the three stator phases together produce an m.m.f. $M_1$ (Fig. 4) of constant amplitude. This m.m.f. is distributed in space approximately according to a sine law, and it glides synchronously along the air gap. The action is the same as in the stator of an ordinary polyphase induction motor; see for example the theory as treated in the author's "Magnetic Circuit", p. 128. The three rotor currents together produce a similar gliding magnetomotive force $M_2$, whether the armature is revolving or stationary. Its position in space with respect to $M_1$ depends only upon the angle of brush shift and is independent of the value of the current or of the speed of rotation. This is because the m.m.fs. in a commutated armature are the same as if the armature were at rest (see proof above). In a certain position of the brushes called *neutral* the stator and the rotor m.m.fs. are in phase opposition in space (Fig. 4). In this paper the angle of brush shift $\alpha$ is measured from this position. Arnold measures the brush shift angle $\rho$ from the position at which $M_1$ and $M_2$ are

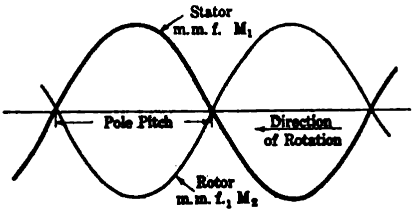

FIG. 4—STATOR AND ROTOR M. M. F'S IN SPACE—THE BRUSHES ARE IN THE NEUTRAL POSITION

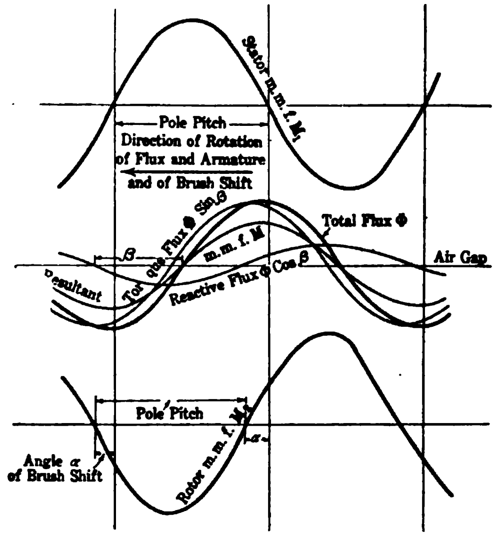

FIG. 5—STATOR AND ROTOR M. M. F'S. AND THE RESULTANT FLUX IN SPACE—THE BRUSHES ARE SHIFTED BY AN ANGLE $\alpha$ IN THE DIRECTION OF ROTATION

in phase coincidence. Hence, his $\rho = 180$ deg. $- \alpha$. In the secomor it is more convenient to deal with an acute angle $\alpha$ than with an obtuse angle $\rho$.

*The Torque.* In the neutral position of the brushes (Fig. 4) the mechanical force between the stator and the rotor is simply a radial repulsion, and the motor develops no torque. Let now the brushes be shifted by an angle $\alpha$ (Fig. 5) in the direction of rotation of both m.m.fs. The resultant m.m.f. $M$ in the air-gap is equal to the sum of $M_1$ and $M_2$, and the flux $\Phi$ which $M$ produces is out of phase with $M_1$ and $M_2$. The relations shown in Fig. 5 are also indicated vectorially in Fig. 6, which is a *space* (not time) diagram. The actual air-gap flux $\Phi$ may be resolved into two space components, one in phase (or phase opposition) with $M_2$, the other in quadrature with it. These components

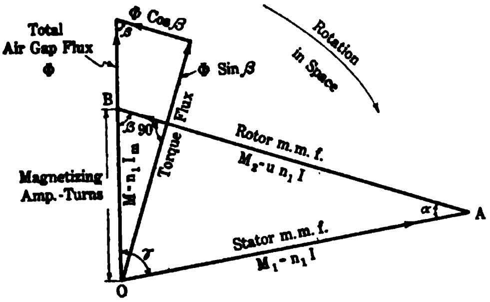

FIG. 6—A VECTOR REPRESENTATION OF THE M. M F'S. AND FLUX SHOWN IN FIG. 5—SPACE DIAGRAM

are marked $\Phi \cos\beta$ and $\Phi \sin\beta$ respectively. The in-phase component $\Phi \cos\beta$ increases or reduces the fictitious flux due to the rotor winding itself, and produces no torque with the armature currents. The useful torque is due entirely to the interaction between the quadrature component $\Phi \sin\beta$ of the air-gap flux and the armature m.m.f. $M_2$. There is a similar relationship in a d-c. machine with the brushes in the neutral, no torque being produced between the armature conductors and the flux of armature reaction. The useful flux must be in space quadrature with that which the armature windings themselves excite.

*Direction of Rotation of the Armature.* By marking the directions of the currents and fluxes in Fig. 5 and applying the familiar right-hand screw rule one finds that the torque is exerted upon the armature conductors in the direction in which the brushes are shifted from the neutral. A simple way to check this fact is to consider the fictitious fluxes due to $M_1$ and $M_2$ acting alone. Since the m.m.fs. are nearly in phase opposition, these fluxes repel each other, and the armature tends to carry its m.m.f. away from $M_1$, by moving to the left. The stationary brushes fix definite groups of armature conductors in space, so that the armature moves away steadily without coming into a position of stable equilibrium. If the brushes were shifted in the opposite direction, the direction of rotation would be reversed. It is of importance to note that the characteristics of the motor are entirely different for the two directions of rotation. In the first case the armature conductors revolve in the direction of the flux, so that the rotor e.m.f. at synchronism becomes zero, and then its sign is reversed. In the second case the rotor e.m.f. increases indefinitely with the speed of rotation. In practise the first combination has been used so far, and is the only one considered in this paper.

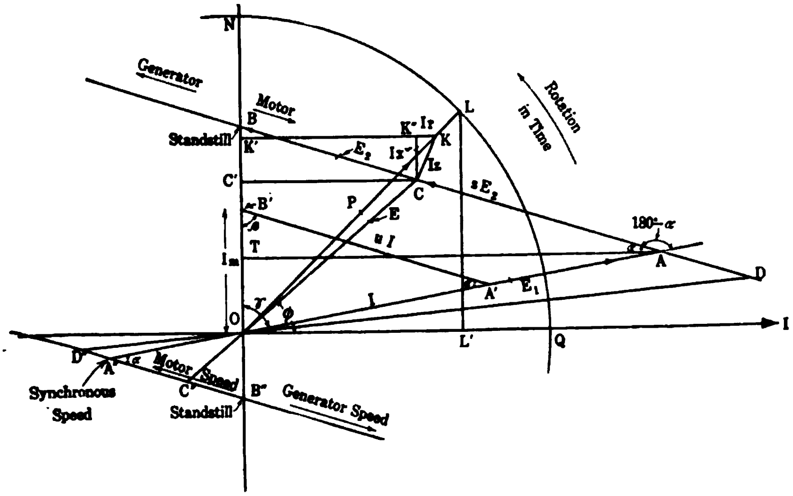

FIG. 7—TIME DIAGRAM OF VOLTAGES AND DATA FOR PERFORMANCE CHARACTERISTICS

### C. TIME DIAGRAM OF E.M.FS.

Consider the motor running at a given current, which is kept constant and let the speed vary, the terminal voltage being adjusted for each speed. The e.m.fs. $M_1$ and $M_2$ are constant and so is the resultant flux $\Phi$, so that Fig. 6 applies at all speeds. The flux $\Phi$ induces certain e.m.fs. in the stator and rotor windings, and these e.m.fs. are shown in Fig. 7.

Fig. 7 is a time diagram, and it shows the applied voltage $O\,K = P$ consumed, in three parts: (1) $O\,A = E_1$ is that part of the applied terminal voltage which is equal and opposite to the e.m.f. induced by flux $\Phi$ in the stator windings. For the sake of brevity $E_1$ is further referred to as the stator voltage. (2) $A\,C = s\,E_2$ is the voltage consumed in the rotor, in opposition to the e.m.f. induced by flux $\Phi$. (3) $C\,K = I z$ is the impedance drop in the windings of the machine and in the brushes.

The voltage induced in the rotor is proportional to per cent slip, being zero at synchronism. Let $A\,B = E_2$ be the voltage which balances that induced in the rotor at standstill. With the brushes in the neutral, $E_2$ leads $E_1$ in time by 180 deg. Shifting the brushes forward by an angle $\alpha$ is equivalent to retarding the armature with respect to the revolving flux by the same angle, so that $E_2$ leads $E_1$ by (180 deg. $- \alpha$). At a slip $s$, the secondary voltage is no more $A\,B$, but is $A\,C = s\,E_2$. The resultant voltage $E = O\,C$ balances the total counter e.m.f. of the machine.

In an ideal motor, without internal resistance or reactance, $O\,C$ is identical with the terminal voltage $P$. As the speed increases from zero to synchronism, with the current kept constant, point $C$ moves along $B\,A$ from $B$ to $A$. Above synchronism the sign of $E_2$ is reversed and point $C$ moves further towards $D$. For points to the left of $B$ the machine acts as a generator.

In an ideal motor no power is consumed at standstill so that the applied voltage $O\,B$ at the speed zero must be in leading quadrature with the horizontal current vector $I$, which is the reference vector. Thus $O\,B$ in the diagram must be vertical. As the speed increases the phase angle between $O\,C$ and $I$ decreases. By continuing $B\,D$ further to the right to its intersection with $I$, a super-synchronous speed is found at which the applied voltage is in phase with the current. Beyond that speed, the motor takes in a leading current.

*The Triangles of Voltages and of m.m.fs. are Similar.* In the construction of the secomor use is made of the fact that the triangles $O\,A\,B$ in Figs. 6 and 7 are similar. The two m.m.fs., $M_1$ and $M_2$, being produced by the same current are displaced *in space* by 180 deg.$-\alpha$, where $\alpha$ is the brush shift angle measured from the opposition point (Fig. 5). Similarly, the two voltages,

$E_1$ and $E_2$, being induced by the same flux $\Phi$, are displaced *in time* by 180 deg.$-\alpha$. The ratio of $M_1$ to $M_2$ is equal to that of the effective numbers of turns in the stator and in the rotor; the ratio of $E_1$ to $E_2$ is equal to the same ratio of turns. Thus, the two triangles have an equal angle between two proportional sides, and therefore are similar. This important relationship permits to incorporate the m.m.f. triangle $O\,A'B'$ in the time diagram (Fig. 7) and to do away with a separate space diagram. To take into account the m.m.f. of the short-circuited coils undergoing commutation a slight correction is necessary; see Arnold, p. 56.

*Leakage Inductance.* Both the stator and the rotor windings are linked with leakage fluxes, and the corresponding reactances cause a voltage drop in time quadrature with the current. The effect of the reactance is shown in Fig. 7 in the usual way, by adding a vector $I\,x$ to the voltage $O\,C$. As is explained above, the total armature reactance depends upon the speed of the machine, so that the vector $I\,x$ consists of two parts, one proportional to $I$ only, and the other proportional to $I$ and to the slip $s$.

*Ohmic Drop, Iron Loss and Friction.* The ohmic drop in the machine is taken into account in the usual way by means of the vector $I\,r$ in phase with $I$. In a variable-speed machine the iron loss is rather a complicated function of both the current and the speed, so that it is hardly feasible to take it into account in a vector diagram. The same is true to some extent of the friction and windage loss. The simplest way is to disregard them in the vector diagram and to correct the obtained values of input and output afterwards. One may also increase the actual ohmic resistance of the machine so as to include the iron loss in the term $I^2r$, and consequently in the vector $I\,r$, but the procedure is of doubtful accuracy and value.

*The Saturation Curve.* In a series-wound machine it is hardly permissible to disregard the effect of saturation in the magnetic circuit upon the performance at large values of current. The starting torque and the characteristics at low speeds are appreciably affected thereby. Let Fig. 8 represent the a-c. saturation curve of the magnetic circuit of the machine. To obtain it experimentally, the brushes are raised from the commutator, and the armature is run at the synchronous speed by some external power. The stator winding is connected to a source of variable a-c. voltage, and a series of readings is taken of the voltage $E_1$ and of the corresponding magnetizing current $I_m$. The values of $E_1$ are corrected for the ohmic drop and those of $I_m$ for the core loss. The corrected values give the curve shown in Fig. 8. This curve is used in connection with the diagram in Fig. 7, and also in connection with the secomor.

### D. Performance Characteristics of the Motor

The brush shift angle $\alpha$ and the ratio $u = M_2/M_1$ of the effective rotor turns to the effective stator turns are the data to begin with. These determine the shape of the triangle $O\,A\,B$ (Fig. 7). Its position with respect to $I$ is also determined since $O\,B$ is perpendicular to $I$. Let $O\,A'B'$ be the m.m.f. triangle transferred from Fig. 6. Then if we make $O\,A'$ equal to $I$ (in magnitude only, but not in phase), $O\,B'$ will represent the magnetizing current $I_m$. This is because in Fig. 6 all the three sides of triangle $O\,A\,B$ have a common factor $n_1$, that is, the number of stator turns.

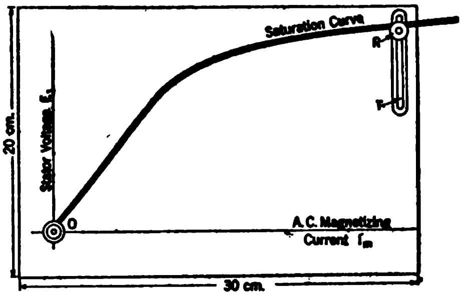

Fig. 8—An Adjustable Saturation Curve Made of Soft Wire

From the saturation curve (Fig. 8) we get the value of $E_1$ corresponding to this magnetizing current and lay it off as $O\,A$. This determines the direction $B\,D$. The rest of the problem is to find such a point $C$ that the impedance triangle $I\,x$, $I\,r$, constructed at it, would give the desired terminal voltage $O\,K = P$ in magnitude. This is purely a geometrical problem which can be solved either analytically or graphically. In the secomor the point $C$ is found mechanically, by means of a few simple trials, shifting one part of the device relatively to another. Herein lies one of the principal advantages of the secomor over the analytical or the graphical method.

*Speed.* In Fig. 7, $B\,C = (1 - s)\,E_2$, so that the speed of the machine $(1 - s) = B\,C/B\,A$, expressed as a fraction of the synchronous speed. Instead of measuring every time two lengths and taking their ratio, it is convenient to draw an arbitrary straight line $A''B''$ parallel to $A\,B$. Produce $B\,O$ and $A\,O$ so as to get points $B''$ and $A''$, and divide $A''B''$ into 100 equal parts, beginning with zero at $B''$. Extend the divisions both ways to cover the operation above the synchronous speed, and also with the machine running backward as a generator. For any operating point, such as $C$, extend $O\,C$, and read per cent speed at $C''$. A proof for this construction follows directly from the similar triangles $O\,A\,B$ and $O\,A''\,B''$.

*Input, Output and Efficiency.* The vector $K\,K'$ is the component of the applied voltage $P$ in phase with the current. Therefore $K\,K'\cdot I$ represents the input into the motor per phase. To it should be added the estimated core loss in the stator. The mechanical output is proportional to the component of the counter-e.m.f., $E$, in phase with the current; hence the output is equal to $C\,C'\cdot I$. The friction loss and the core loss in the armature should be subtracted from this value. The efficiency is found as the ratio of the true output to the true input.

*Power Factor.* The power factor, $\cos\phi$, is equal to the ratio of $K\,K'$ to $O\,K$. A simple way to read it directly is to draw the quadrant $N\,L\,Q$ with $O$ as a center, using 100 convenient divisions, for example 100 mm., as the radius. The intersection $L$ of $O\,K$ with the quadrant determines point $L'$. Radius $O\,Q$ is marked with a uniform scale, zero being at $O$ to 100 at $Q$. Thus, the power factor is read at $L'$ directly in per cent.

*Torque.* In a d-c. series-wound motor the torque is a function of the current and is practically independent of the speed. In the motor under consideration the torque is also independent of the speed, except for the disturbing effect of the armature coils undergoing commutation. The torque depends essentially upon the current and upon the brush shift. With the given $I$ and $\alpha$ the torque is nearly the same at standstill as at any other speed. But at standstill $A\,T$ represents the energy component of both the stator voltage $E_1$ and the rotor voltage $E_2$. The amount of power $A\,T\cdot I$ is absorbed in the stator and an identical amount is returned to the line from the rotor. Thus, $A\,T\cdot I$ is the torque in synchronous watts, corresponding to $I$ at any speed.

Another proof is as follows: In Arnold, on bottom of p. 41 the torque in synchronous watts is expressed as $I^2\,x_a\,u\,\sin\alpha$. Here $x_a = E_1/I_m$ is the so-called exciting reactance of the machine. Multiplying and dividing the foregoing expression by $I_m$ we get, torque $= I^2 x_a u \sin\alpha = E_1 I (u I \sin\alpha / I_m)$. But from the triangle $O A'B'$ (Fig. 7) we have that $u I/\sin\gamma = I_m/\sin\alpha$. Substituting in the preceding equation we find that the torque $= E_1 I \sin\gamma = A\ T \cdot I$.

*Recapitulation.* The motor characteristics (Fig. 7) are essentially determined by the ratio $O A$ to $A B$ of the effective stator and rotor ampere-turns, and by the brush shift angle $\alpha$. The direction of the vector of current $I$ is always along the axis of abscissas, that of the terminal voltage $O K$ approaches $I$ as the speed increases. Select a value of $I$ and plot $O A' = I$, remembering that $O A'$ does not represent the true direction of $I$ in the time diagram, but only its magnitude. Then the following values are obtained:

Magnetizing current $O B' = I_m$

Stator voltage $O A = E_1$. It is found from Fig. 8 as the ordinate corresponding to abscissa $I_m$

Rotor voltage at standstill $A B = E_2$

Rotor voltage at slip $s$, $A C = s E_2$

Total useful voltage at slip $s$, $O C = E = \underset{\cdot}{E_1} + s \underset{\cdot}{E_2}$ (geometric addition)

Reactive drop $C K'' = Ix$

Ohmic drop (which may be increased to cover core loss) $K''K = I r$

Terminal voltage $O K = \underset{\cdot}{P} = \underset{\cdot}{E} + \underset{\cdot}{I} x + \underset{\cdot}{I} r$

Slip $A''C'' = s$, where $A''B'' = 100$ per cent slip

Speed $C''B'' = 1 - s$, where $A''B'' = 100$ per cent synchronous speed.

Input $K K' \cdot I$

Output $C C' \cdot I$

Efficiency $C C'/K K' = 1 - (K K''/K K')$

Power factor $\cos\phi = K K'/O K = O L'$, where $O Q = O L = 100$ per cent

Torque in synchronous watts $A\ T \cdot I$

## E. THE SECOMOR AND ITS USE

The secomor (Fig. 9) is built out of four flat iron bars; these bars represent the principal vectors in Fig. 7. Each bar is provided with a centimeter scale, and they are mounted on an ordinary drafting board. A sheet of cross-section paper is tacked to the board and serves as a universal scale. The four bars and their functions are as follows:

(a) *The stator bar, $OA$,* determines the stator voltage $E_1$ and the current $OA' = I$ (Fig. 7)

(b) *The rotor bar, $DB$,* determines the rotor voltages $E_2$ and $s\,E_2$.

(c) *The speed bar, $D''B''$,* determines the slip $s$ or the speed of the motor.

(d) *The setting bar, $C''L$* determines the point $C$ for which the useful voltage $E$ plus the impedance drop $CK$ gives the terminal voltage $P = OK$.

The setting bar is pivoted at $O$ next to the cross-section paper, the stator bar is pivoted on top of it, and the other two bars slide on top of the stator bar. On each bar one edge is called the "reading" edge; on the pivoted bars the reading edge passes through the geometric center of the pivot $O$.

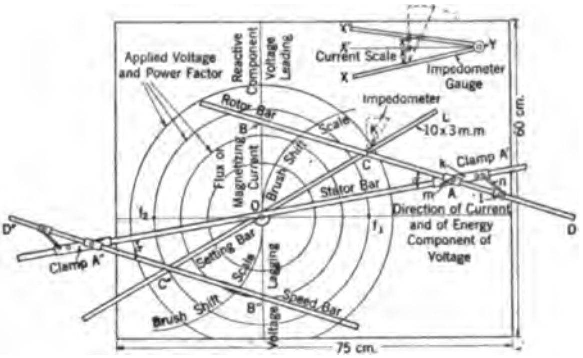

FIG. 9—THE SECOMOR—THE NOTATION CORRESPONDS TO FIG. 7

The clamps $A'$ and $A''$ have to be rather carefully made, to permit of an accurate setting and not to have too much lost motion. The long sleeve $k$ of the clamp slides on the stator bar; the sleeves $l$ and $m$ guide the rotor bar as it slides in them. The sleeve $m$ is pivoted on top of sleeve $k$; $l$ is connected to $k$ by means of a pivoted rod $n$. By tightening the set screw on $k$ the rotor bar may be fixed at any desired point of the stator bar. By loosening the set screws on $l$ and $m$ the angle $\alpha$ can be adjusted, and after the screws have been tightened the angle remains constant. Part of each sleeve is cut out to enable one to see the scale and the reading edges of the bars.

It would be inconvenient to use a protractor for setting the rotor bar and the speed bar at a desired brush-shift angle $\alpha$. Therefore, two large circular scales are provided on the base of the apparatus, with centers on the axis of abscissas, at $f_1$ and $f_2$. These scales are marked in degrees and each is labeled in Fig. 9 "brush shift scale". To use the scale $f_1$ the stator bar is so turned that its reading edge coincides with the axis of abscissas; the clamp $A'$ is shifted until the reading edge of the rotor bar passes through $f_1$. Then the rotor bar may be set at any desired angle. After the set screws on $l$ and $m$ have been tightened the angle remains unchanged when the clamp is shifted. The speed bar must be set at the same angle at $f_2$, and the two bars must always be parallel to each other when the secomor is in use.

The four concentric circles with the centers at $O$ are used for the applied voltage. Any of them can be selected as a locus of point $K$. With some motor characteristics the bars are not long enough to be used with the largest circle and one has to use a smaller one. One of the circles is also used for reading the power factor (arc $N L Q$ in Fig. 7).

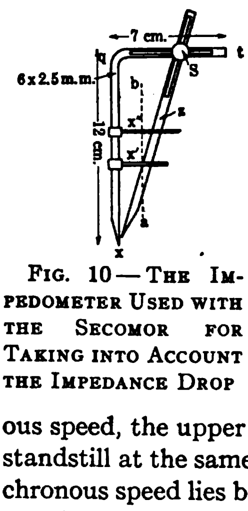

FIG. 10 — THE IMPEDOMETER USED WITH THE SECOMOR FOR TAKING INTO ACCOUNT THE IMPEDANCE DROP

*The Impedance Triangle or the Impedometer* (Fig. 10) is made of steel with two brass indicators, $x'$ and $x''$. The inclined bar $z$ is set for the desired ratio of $r$ to $x$ at synchronism and is fastened with the set screw $S$ and with a clamp at the other end, not shown in the figure. The impedance drop is represented by the inner edges of the three bars. The ower indicator, $x'$, is set for the desired value of $I\,x$ at synchronous speed, the upper indicator $x''$ is set for the value of $I\,x$ for standstill at the same current. The value of $I\,x$ for any sub-synchronous speed lies between $x'$ and $x''$, and for super-synchronous speeds it lies below $x'$. These values can be readily estimated by the eye, the whole correction for the impedance drop being small. Point $K$ (Fig. 7) lies somewhere on $a\,b$ (Fig. 10) but it is not necessary to have this line marked on the impedometer, because the device, when in use is lying on the cross-section paper with $x\,q$ parallel to one of the rulings (Fig. 9), and the eye easily follows the direction $a\,b$.

The two diverging straight edges $X$ and $X''$ (Fig. 9) shown in the upper right-hand corner of the secomor serve as a guage for a quick setting of the impedometer. The horizontal line $X'$ is one of the rulings on the cross-section paper, and serves as a current scale and the locus of the point $x'$ of the impedometer. Bar $X$ is the locus for point $x$, and bar $X''$ is the locus for $x''$. The set screw $Y$ fixes the bars in any desired position.

*The saturation curve* (Fig. 8) used with the secomor, is made out of a piece of ordinary solder wire, and can be readily bent and adjusted by hand to any desired shape. The lower end of the wire is held in a swivel clamp at $O$, and the upper end is clamped at $R$. The clamp $R$ can be loosened and moved up and down in the slot $T$. The whole is mounted on a piece of board; a sheet of cross-section paper tacked to it serves as a universal scale.

*The method of using the secomor* follows directly from Fig. 7, the setting for different constants and for different loads being accomplished by shifting or turning the four bars. The reader can simply follow the "Recapitulation" above. As to the selection of scales for volts and amperes, the simplest method seems to be always to read everything directly in centimeters and to use constants afterwards. When testing an actual machine one usually reads the meters without regard to their constants, and later recomputes the data.

The use of the impedometer requires no particular skill or precision. Having located point $C$ approximately near the chosen circle of applied voltage, the impedometer and the setting bar are shifted to and fro until the electrical condition for the sum of the voltages is fulfilled. This condition depends upon the speed which is simultaneously read at $C''$. Point $K$ must always lie on $a\,b$ (Fig. 10), but its exact position on $a\,b$ is determined by the speed of the machine. At standstill it must lie on the intersection of $a\,b$ with the indicator $x''$, at synchronism it lies on the intersection of $a\,b$ with $x'$. At any other speed $K$ divides $x'x''$ in the same proportion in which $C''$ divides $A''B''$. The length $x'x''$ being small as compared to $O\,C$ or $O\,K$, the judgment of the eye is amply sufficient. An engineer who uses the device regularly will soon find several short cuts which it is not necessary to mention here.

*The Effect of a Series Transformer.* When the armature of the motor is fed through a series transformer (Fig. 3) the vector diagrams (Figs. 6 and 7) become somewhat more involved, and the use of the secomor is accordingly modified.* A series transformer introduces the following new factors:

(a) A magnetizing current and an appreciable saturation in

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*For a detailed graphical and analytical treatment of this case see an article by Dreyfus and Hillebrand in the *Elektrotechnik und Maschinenbau,* Vol. 30 (1912), p. 389.

the transformer core at speeds remote from the synchronous speed, when the armature voltage is high.

(b) A primary and a secondary leakage impedance, due to the transformer windings.

(c) An iron loss in the transformer core.

(d) A new vectorial difference in time phase between the armature and the field currents, and consequently a new space angle between the primary and secondary m. m. f's, not determined by the brush shift alone.

The equivalent electric diagram which takes the series transformer into account is shown in Fig. 11, for one phase only. First, the actual transformer is replaced by a one-to-one transformer, with the corresponding changes in the constants of the

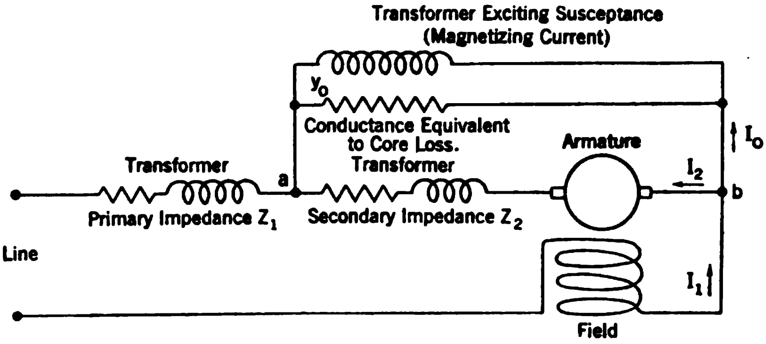

FIG. 11—THE EQUIVALENT DIAGRAM OF CONNECTIONS WHEN THE ARMATURE IS SUPPLIED WITH POWER THROUGH A TRANSFORMER, ACCORDING TO FIG. 3

armature circuit. Then this equivalent transformer is removed and an exciting admittance $\gamma_0$ is shunted around the armature to provide a path for the magnetizing current of the actual transformer and for its core loss. The resistance of the transformer windings and the effect of the magnetic leakage are taken into account by means of two equivalent impedances, $Z_1$ and $Z_2$, in the primary and armature circuits respectively. The combination of $\gamma_0$, $Z_1$ and $Z_2$ is equivalent to the actual transformer in its effect upon the performance of the motor.†

The magnetizing current of the transformer is a function of the voltage between the points *a* and *b* (Fig. 11), and is supposed

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†For the theory of the replacement of a transformer by equivalent mpedances see for example the author's *Electric Circuit*, Chap. XI.

to be given in the form of an a-c. saturation curve similar to the saturation curve of the motor itself, shown in Fig. 8. The voltage $E_{ab}$ between $a$ and $b$ is somewhat different from the e. m. f. $s\ E_2$ induced in the armature, the difference being due to the impedance drop between $a$ and $b$. But it is only at speeds far remote from synchronism that $E_{ab}$, and consequently the magnetizing current of the transformer, are of importance. For such speeds the difference between $E_{ab}$ and $s\ E_2$ may be neglected in the first approximation, and the magnetizing current assumed to be a function of the counter e. m. f. $s\ E_2$ of the machine. A

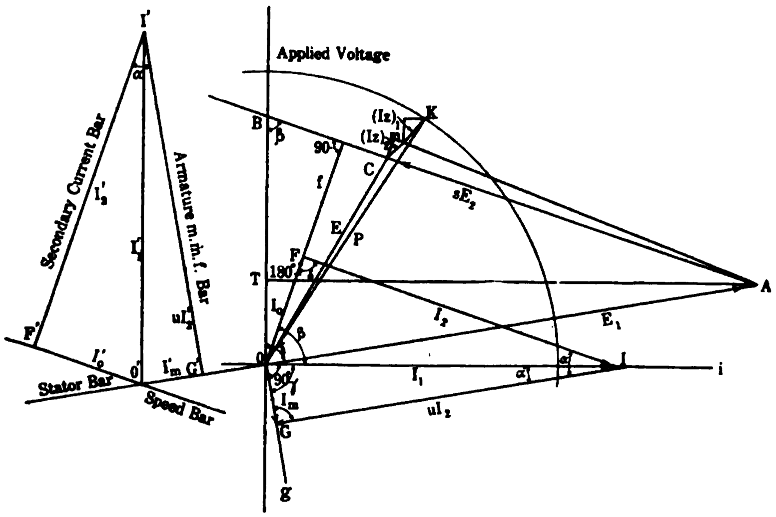

FIG. 12—VECTOR DIAGRAM OF CURRENTS AND VOLTAGES, TAKING INTO ACCOUNT THE MAGNETIZING CURRENT OF THE TRANSFORMER AND ITS IMPEDANCE

correction may be made later, using the approximate values first obtained.

*Vector Diagram with Transformer.* In Fig. 12, let the triangle $O\ A\ B$ represent the same voltages as in Fig. 7, and let $O\ I$ be the vector of primary current $I_1$ which flows through the stator windings. The secondary or armature current $I_2$ differs from $I_1$ by the amount $I_0$ of the magnetizing current in the transformer, as shown in triangle $O\ F\ I$. The transformer is supposed to have a one-to-one ratio. According to the approximate assumption made above, $I_0$ is a function of the armature e. m. f., a-c., and lags behind it by 90 degrees (neglecting the core loss in the transformer). The corresponding corrections will be considered later.

The m. m. f. space triangle which is shown in Fig. 7 in the position $O A' B'$, is shown in Fig. 12 in the position $O I G$. One should think of its three sides as being multiplied by the number $n_1$ of the effective stator turns (Fig. 6). It expresses the fact that the stator ampere-turns $n_1 I_1$ and the armature ampere-turns $n_2 I_2 = u n_1 I_2$ together give the magnetizing ampere-turns $n_1 I_m$ of the machine. The magnetizing current $I_m$ is drawn in lagging quadrature with the stator e. m. f. $E_1$. When the magnetizing current of the transformer $I_0 = 0$ the m. m. f. triangles in Figs. 7 and 12 become identical, except for their position in the diagram and for the fact that in Fig. 12 the vectors are taken to rotate counter-clockwise in space as well as in time. This is done in order to be able to combine the triangles $O F I$ and $O I G$ into one diagram. To convert the triangle $O A' B'$ (Fig. 7) into $O I G$ (Fig. 12), first turn $O A' B'$ in its own plane with $O$ as a center until $A'$ lies on $O I$. Then rotate the whole triangle in space about the new axis $O A'$ by 180 degrees. Point $B'$ will then come into position $G$, and $O B'$ will become perpendicular to $O A$.

The angle between the vectors $F I = I_2$ and $I G = u I_2$ is equal to $180° - \alpha$, same as in Fig. 7, because the vector $F I$ represents $I_2$ as a component of the stator current, while $I G$ represents $I_2$ as the rotor current, and the brush shift of $\alpha$ degrees is measured from the opposition setting. Angle $\alpha'$ in Fig. 12 shows the effect of the magnetizing current in the transformer upon the space phase-angle between the stator and rotor m. m. f's. When $I_0 = 0$, $\alpha' = \alpha$.

Since the directions $O A$ and $A B$ are fixed, the directions $O f$ and $O g$ of the vectors of the two magnetizing currents are also fixed, as well as the direction $O I$. When the load varies, points $G$, $F$, and $I$ move on their respective loci in such a manner that the angle $F I G$ is always equal to $\alpha$. When this condition is fulfilled the ratio of $I G$ to $I F$ remains equal to $u$. This property of the quadrilateral $O F I G$ is of sufficient importance to warrant a proof.

*Lemma*. In a variable quadrilateral such as $O F I G$ (Fig. 12) in which the sum of the two opposite angles remains equal to 180°, and the angles $\alpha$ and $180° - \alpha$ are constant, the ratio of $I G$ to $I F$ remains constant when the three vertices $I$, $F$, $G$, move in an arbitrary manner on the three loci $O i$, $O f$, and $O g$.

*Proof*: If the sum of the angles at $O$ and $I$ is equal to $\pi$, then the sum of the angles at $G$ and $F$ must also be equal to $\pi$, and we have:

$$\frac{I_1}{\sin \delta} = \frac{u I_2}{\sin \gamma} = \frac{I_2}{\sin \beta}$$

from which

$$u = \frac{\sin \gamma}{\sin \beta}$$

With the motion of the three vertices as specified above, the angles $\beta$ and $\gamma$ remain constant, so that $u$ also remains constant, and this proves the theorem.

This value of $u$ is the same as that used in the construction of the voltage triangle $O\,A\,B$, because from this triangle we have

$$\frac{\sin \gamma}{\sin \beta} = \frac{A\,B}{O\,A} = \frac{u E_1}{E_1} = u.$$

*Transformer Attachment.* In the secomor, it would be rather inconvenient to use the quadrilateral of currents $O\,F\,I\,G$ in its true position, because the rods $O\,F$ and $O\,G$ would have to move perpendicularly to $A\,B$ and $O\,A$ respectively. For this reason he quadrilateral is turned by 90 degrees into the position $O'\,G'\,I'\,F'$, in order that the two magnetizing currents, $I'_m$ and $I'_0$ may be represented by certain lengths $O'\,G'$ and $O'\,F'$ on the stator bar and the speed bar. The cross-section paper provides the direction $O'\,I'$, so that only two new members or rods are needed, viz., to represent $I'_2$ and $u\,I'_2$.

The actual construction of the *transformer attachment* is shown in Fig. 13; it is fastened to the secomor board (Fig. 9) in the upper left-hand corner. Two strips of brass are used to represent the sides $I'\,F'$ and $I'\,G'$ of the quadrilateral. They will be referred to as the *secondary current bar* and the *armature m. m. f. bar* respectively. These bars are pivoted together on a setscrew $I'$ and the directions of their inner edges pass through the center of $I'$. The bars may be set at any desired angle by means of the graduated scale $G$ and setscrew $S'$. The position of $I'$ on the board may be varied by moving the iron rod $R$ in the clamp $C$, and by moving the clamp itself along the edge of the board. $S$ is a setscrew which holds $R$ on $C$.

To illustrate the use of the transformer attachment, let us assume that the performance characteristics are desired at a given total current $I_1$ with a given terminal voltage. Let the impedance drop (Fig. 12) be at first neglected so that the counter e. m. f. $O C = E$ is equal to the applied voltage $O K = P$. In the transformer attachment to the left, set the pivot $I'$ at the proper point corresponding to the given length $O' I' = I_1$, and spread the rods $I' F'$ and $I' G'$ to the proper angle $\alpha$. Select a reasonable arbitrary position of point $K$ on the quadrant of applied voltage. This will give some tentative values of the stator voltage $E_1 = O A$ and of the armature voltage $C A = K A = s E_2$. From the saturation curve of the motor (Fig. 8) the magnetizing current $I_m$ is found, and its value is marked on the stator bar as $O' G'$ by bringing the armature m. m. f. bar $I' G'$ to point $G'$. Similarly, the transformer magnetizing current $I_0$ corresponding to $s E_2$ is found from its saturation curve; it must check with the value $O F'$ indicated by the secondary current bar at $F'$. If it does not check, the point $K$ was guessed at wrong and a new trial is necessary. After a few trials, the correct position of $C$ or $K$ is found, and this enables one to find the slip, the power factor, the torque and the output of the machine, in the same manner as without the transformer.

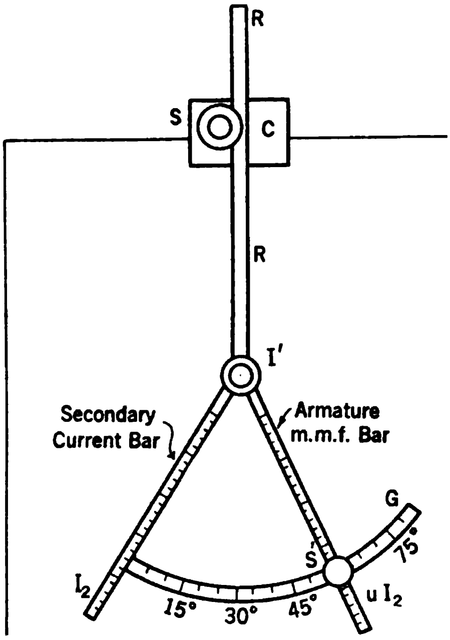

FIG. 13—THE TRANSFORMER ATTACHMENT TO THE SECOMOR

*The Torque.* The expression for the torque is not modified by the magnetizing current of the transformer, if we neglect the core loss. Since $I_0$ is in quadrature with $E_2$, the components of $I_1$ and of $I_2$ in phase with $E_2$ are equal to each other. Thus, either with or without the transformer the true input into the field *at standstill* is equal to the armature output. Both are a measure for the torque in synchronous watts, and the expression $A\,T \cdot I_1$, deduced above, holds true in this case, with the machine running or standing still. In the language of the Vector Analysis, using the dot for the inner or scalar product, we have:

$$I_1 = I_0 + I_2$$

$$(E_1 + E_2) \cdot I_1 = 0$$

Multiplying the first equation by $E_2 \cdot$ we get

$$E_2 \cdot I_1 = E_2 \cdot I_0 + E_2 \cdot I_2$$

But $E_2 \cdot I_0 = 0$, and combining the second and the third equations we get

$$E_1 \cdot I_1 + E_2 \cdot I_2 = 0.$$

This means that the two ideal wattmeter readings are equal to each other, and either one is a measure for the torque in synchronous watts.

*Correction for Impedance.* The impedance drop in the motor and in the transformer (Fig. 11) may be divided into two parts:

(a) That due to current $I_1$ in the stator winding and in the transformer primary;

(b) That due to the current $I_2$ in the motor armature and in the transformer secondary.

Consequently, two impedance triangles are necessary, as is shown in Fig. 12 between points $C$ and $K$. The long side of one is parallel to $I'_1$, that of the other to $I'_2$. In many cases it will be permissible to neglect the difference between $I_1$ and $I_2$ and to use a single impedometer (Fig. 10) for an intermediate value of the current, but if a greater accuracy is required, a double impedometer could be constructed, or a single impedometer applied twice.

We shall assume now that the point $K$ has been tentatively assumed on the quadrant of applied voltage, and then the impedometer applied to find the point $C$. This gives the position of point $A$ and consequently the speed and the magnetizing current $I_m$ of the motor. The magnetizing current $I_0$ of the transformer has been assumed above to be a function of the induced voltage

$C A = s E_2$ in the armature. In reality it is a function of the voltage between points *a* and *b* (Fig. 11). This voltage is equal to $A_m$ (Fig. 12), being a sum of the e. m. f. *A C* and the primary impedance drop *C m*. Thus, a more accurate value of $I_0$ may be taken from the saturation curve. Strictly speaking the direction of the new $I'_0$ in the transformer attachment should be parallel to *A m* and not along the speed bar which is parallel to *A C*. This small correction is best made by the judgment of the eye, without disturbing the bars.

If desired, a correction may also be made for the core loss in the transformer, as is indicated in Fig. 11 by an "exciting conductance". This means the addition to $I_0$ of a component in phase with *A C*, or more accurately, in phase with *A m*. Similarly, the current $I_m$ may be corrected to account for the stator core loss.

It would complicate the mathematical theory of the motor very much if one attempted to take all these refinements into account accurately. A kinematic attachment for this purpose would also be unnecessarily involved for most practical purposes. *The device as described above gives a nearly accurate solution with a simple setting; small corrections are best applied by the judgment of the eye from this setting.* The transformer attachment has a valuable property expressed by the lemma proven above, as long as the sum of its opposite angles is kept equal to 180°. The exact values for the transformer magnetizing current and for the core losses would destroy this valuable property without an adequate advantage in the accuracy of the results achieved.

## CONCLUSION

In presenting the secomor to the electrical profession the author wishes to point out the possibility of predicting the performance of the polyphase series commutator motor by means of a kinematic device. He also hopes to arouse interest in the use of similar kinematic devices for the prediction and analysis of performance of other types of electrical machinery. Besides the secomor, he has designed the "shucomor" or a kinematic device which imitates the performance of a shunt-wound polyphase commutator motor, and as a by-product has obtained a device which gives the performance of the ordinary induction motor. Alternator characteristics can probably be imitated in a similar manner as well as those of single-phase commutating machines.

The usefulness of such devices is not limited to a-c. machinery, but embraces special cases of d-c. machinery as well. Sometime ago the author became interested in the operation of the Entz electromagnetic clutch and transmission used in some gasoline motor cars. He has built a kinematic device which imitates the performance of this ingenious clutch at different engine speeds and with any setting of the regulating resistances. The saturation curves of the two d-c. machines are also incorporated in the device. The device was demonstrated at the A. I. E. E. meeting in Syracuse, N. Y., in May 1917, and has been presented by the author to the Electrical Engineering Department of Syracuse University.

A kinematic device can no more replace human intelligence than a formula or a vector diagram can. But a mechanical device helps and guides an engineer's judgment, and makes it possible to achieve results with less time and trouble. One working on the design of the same type of electrical devices year after year finally attains some proficiency and needs but little outside help. But industrial efficiency demands that younger men do at least routine designing without much previous experience. A mechanical device makes it safer to entrust them with the determination of dimensions, because it enables them readily to check the performance. The effort of more gifted and mature engineers may thus be devoted to new developments and to large important problems, and less of their time need be occupied by the supervision over younger men. A wide use of mechanical devices that imitate the performance of electrical machinery is thus a step in the desired direction.

## BIBLIOGRAPHY

The subject is covered quite thoroughly up to 1910 in E. Arnold's "Die Wechselstromtechnik," Vol. 5, part 2. The theory of both the series and the shunt-wound polyphase commutator motor has been principally studied in Germany, Austria, Switzerland and Sweden, and a few electric manufacturing companies in those countries have put such motors on the market. In France M. Latour has built some 10-pole motors as large as 500 h.p., with the armature connected 12-phase, and with only one stud of brushes per pole, using a two-circuit winding. A brief note on these motors will be found in *La Lumière Electrique*, Vol. 29, p. 289.

A person interested in some phase of the subject should refer to the last volumes of the principal German and French electrical periodicals and search through the back volumes "counter-clockwise," until he has found the particular bit of information sought. For such a search the entry words in the indexes are probably much more valuable than a specific reference to a few articles. An entry word will enable the future

investigator to locate in time articles yet unwritten. The magazines below are arranged in the order of their importance for the particular topic under discussion. The first two periodicals contain by far the largest amount and the best information. Of the American periodicals, *The General Electric Review* so far has been the only one in which original articles on the polyphase commutator motor have been published. *Science Abstracts* (London), part B, Electrical Engineering, is a most useful help in literature search on most any electrical subject.

| NAME OF PERIODICAL | ENTRY WORD IN THE INDEX |
|---|---|
| *Elektrotechnische Zeitschrift* | Elektromotoren |
| *Elektrotechnik und Maschinenbau* | Kollektor, Kommutator, Mehrphasen |
| *Archiv für Elektrotechnik* | Name index, see Glossary below |
| *General Electric Review* | Motors |
| *La Lumière Electrique** | Machines, Moteurs |

GLOSSARY OF GERMAN AND FRENCH TERMS FOR FACILITATING LITERATURE SEARCH

| *English* | *French* | *German* |
|---|---|---|
| machine | machine | Maschine |
| motor | moteur | Motor or Elektromotor |
| single-phase | monophasé | Einphasen or Wechselstrom |
| two-phase | diphasé | Zweiphasen |
| three-phase | triphasé | Drehstrom |
| commutator | collecteur | Kommutator or Kollektor |
| series-wound | série | Reihenschluss or Serien |
| shunt-wound | shunt | Nebenschluss |

*Continued since 1917 as *La Revue Générale de l'Electricité.*

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# COMMUTATION IN ALTERNATING-CURRENT MACHINERY

BY MARIUS C. A. LATOUR

## Abstract of Paper

As is now well known, the inductive reactive effect of a commuted winding in a revolving magnetic field decreases directly with the increase in speed from standstill to synchronism, when its value becomes zero. As first pointed out by the author some years ago, it becomes negative at speeds above synchronism, under which condition the rotor of a motor operates as a capacity.

The author introduces into the discussion of the commutating characteristics of alternating-current commutating motors, his theory that perfect commutation in a continuous-current motor depends substantially on the production of a mean resultant neutral field in the region where commutation is taking place, and shows that the production of a perfect revolving field in a polyphase commutator motor assists in insuring perfect commutation at exact synchronism.

In a single-phase commutator motor a "polyphase" revolving field can be produced at synchronism by utilizing supplementary brushes, short-circuited upon themselves, displaced by 90 electrical space degrees from the main single-phase brushes on the commutator.

As in the case of polyphase motors, the problem of securing perfect commutation at synchronism becomes that of producing a perfect rotating field. It is shown by the author that the use of fractional-pitch windings on the rotor and a sinusoidal distribution of conductors on the stator is of much assistance in this connection.

In a motor built in accordance with the principles set forth, the commutator difficulties are not serious, the overload range is in excess of that of an induction motor, and the machine can act as a condenser on the system.

THE writer has already had the honor of presenting two communications on the use of commutators with alternating currents before the American engineering public, the first being in June, 1903, in co-operation with Mr. A. S. Garfield (see *The Compounding of Self-Excited Alternating Current Generators, for Variation in Load-Factor*, in A. I. E. E. TRANSACTIONS, Vol. XXI, pages 569-577, and discussion, pages 583-585); the other being a paper presented at the St. Louis Congress, 1904, ("*Alternating Current Machines with Gramme Commutators*," published in Transactions of the International Electrical Congress of St. Louis, 1904, Vol. III, pages 149-154).

The purpose of the present paper is to enter more deeply into the theoretical considerations which were outlined by the writer in his earlier publications, and which have since been verified by practical experience. A distinction will be made between the use of commutators for polyphase and for single-phase alternating currents.

## I—COMMUTATOR IN POLYPHASE MACHINERY

Let us consider a direct-current bipolar armature, $A$, Fig. 1, placed in a uniform air gap inside the laminated stator, $B$, with regularly spaced slots and receiving sinusoidal currents of $p$ phases, of frequency equal to $\omega/2\pi$ through $p$ brushes located at points situated $2\pi/p$ in angular distance from each other.

Fig. 1 shows diagrammatically a smooth core armature, on which four brushes, $a$, $b$, $c$, $d$, make contacts through which two-phase (more properly four-phase) currents are supplied. The brushes, $a$, $c$, receive a current $I \sin \omega t$, and the brushes $b$, $d$, receive a current $I \cos \omega t$. Suppose the armature to be at rest and let $L\omega$ be its inductance per phase. If the armature is made to turn at the angular velocity $\omega_1$, measured in the direction of the rotating field developed by the polyphase currents, it will be found that the inductance of the armature becomes $L(\omega - \omega_1)$; consequently, it will vanish at synchronism and, *above synchronism, the arrangement will operate as a capacity* (condenser).

When this property was first made known by the writer, in his patents, in 1900-1901, its correctness was questioned by several prominent electricians. Among these, M. Maurice Leblanc devoted a long article (see *Eclairage Electrique*, October 26th, 1901, pages 113 *et seq*) to the refutation of the writer's conclusions. Many electricians have, since then, accepted as satisfactory the theory which introduces the relative velocity $(\omega - \omega_1)$ of the revolving field developed by the armature with respect to the armature itself. Mr. Leblanc, however, maintained, in his article, that since the current, $i$, in each turn of the armature winding, preserved its variable character, no matter what the velocity $\omega_1$ might be, at any instant $i$ was subject to the same rate of variation, $\frac{di}{dt}$, and consequently the e. m. f. of self-induction between the brushes should persist at all speeds

As was shown by the writer, in a detailed article, published in November, 1901, (see *Eclairage Electrique*, 1901, pages 294 *et seq*), it is quite true that the e. m. f. of self-induction of the sections included in the circuit between the brushes persists independently of the velocity $\omega_1$; but, in consequence of the phenomenon of commutation, there appears between the brushes an e. m. f. of opposite sign which is proportional to the velocity $\omega_1$, and which is due to the mutual induction between the sections which are short-circuited and those which are in the circuit.

In this connection, let us consider Fig. 1. Let $M$ be the coefficient of mutual induction between the sections which are short-circuited by the brushes, $b$, $d$, and the sections that remain in circuit between the brushes $a$, $c$. Let $n$ be the number of sections of the armature. With brushes covering the width of a commutator segment, the duration of commutation for the sections of winding passing under the brushes $b$, $d$, at the velocity $\omega_1$, will be $T = \dfrac{2\pi}{\omega_1 n}$. The e. m. f. $E_M$ developed by mutual induction between the short-circuited sections passing under the brushes $b$, $d$, (where the reversal of a current $\frac{1}{2}\, I \cos \omega t$ is taking place during the time $T$) and the sections which are in circuit between the brushes $a$, $c$, (through which the current $I \sin \omega t$ passes) will be $\dfrac{M}{T}\, I \cos \omega t = \dfrac{Mn}{2\pi}\, \omega_1\, I \cos \omega t$.

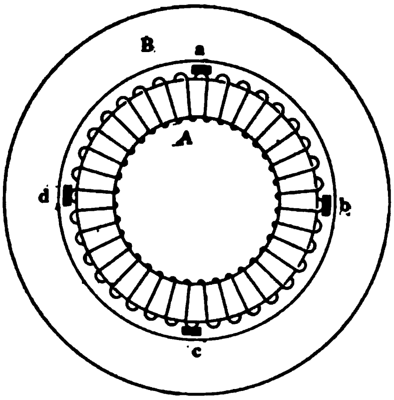

FIG. 1

We first ascertain that $E_M$ is opposed in polarity to the e. m. f. of self-induction $E_L = L\, \omega\, I \cos \omega t$, due to the self-induction of the armature; and we then can show that, in the case where each section of the armature is supposed to produce a sinusoidal flux at the armature periphery, we will have $\frac{Mn}{2\pi} = L$ and consequently $E_L - E_M = L(\omega - \omega_1)$.

It is to be noted that the same expression may be obtained for the e. m. f. ($E_M$) by supposing it to be induced by the rotation of the armature in its own field produced by the current sent through brushes *a*, *c*; but this explanation of the appearance of the e. m. f. $E_M$, though it may always be correct from the mathematical point of view, is not quite so near the physical reality, since, as a matter of fact, the field of the armature itself must follow the armature in its rotation.

The fact that the armature in the arrangement shown in Fig. 1 can operate as a capacity (namely when $\omega_1$ *exceeds* $\omega$) was of particular interest to Mr. Leblanc, who, about that time (in 1902) was endeavoring to find a simple electrical system susceptible of being used in place of the condensers which he was placing in the rotors of induction motors, for the purpose of improving their power-factor. He had even, for that purpose, devised the combination of two single-phase machines provided with commutators. In Mr. Leblanc's arrangement an induction motor *C* was connected in cascade with a simple armature *A*, such as that shown in Fig. 1, running at a speed greatly in excess of synchronism with respect to the frequency of the rotor currents of machine *C* (see Fig. 2). This was an immediate indication of the value of the writer's article of November 23rd, 1901, as Mr. Leblanc himself has since expressly acknowledged, (see *La Lumière Electrique*, July 12th, 1913, page 60, and the *Electricien*, July 25th, 1913, page 658).*

The Swiss firm of Brown-Boveri, utilizing the methods patented by M. Leblanc for improving the power-factor of induction motors, has put on the market, in the last few years, under the name of phase-compensator, armatures with commutators rotating above synchronous speed. Fig. 3, made from a photograph, shows one of these arrangements, designed for a

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*The writer had formally proposed that combination in a letter addressed to *Industrie Electrique* (in 1902) which was not published; but the writer, in any case, considers that M. Leblanc should be credited with the idea of placing condensers or any dynamic apparatus equivalent to them in the rotor of induction motors for the purpose of improving their power-factor. It is by mistake that the arrangement shown in Fig. 2 was, during a certain time, attributed to Mr. Scherbius.

500 h.p. motor, operated by a motor of 0.8 h.p., running at 1000 rev. per min. This outfit relieves the supply mains of the necessity of furnishing 180 kv-a. of reactive current.

With respect to the introduction of a commutator to bring the power-factor of an alternating current system up to unity, the question has been asked whether it would not be possible to raise the power-factor to unity in an alternating current system in some other way than by the introduction of a commutator. The answer to that question is known to be negative.

If we consider a system of non-deformable electric circuits composed of any number $p$ whatever, and at rest, the writer has shown (see *Lumière Electrique*, 1907, page 5) that *whatever may be the complexity of the mutual inductions between the circuits considered by pairs*, the system can only absorb magnetizing power, and cannot supply any. The writer demonstrated, in fact, that the determinant which can be constituted with all the induction coefficients is necessarily positive. A particularly well known case for two circuits is

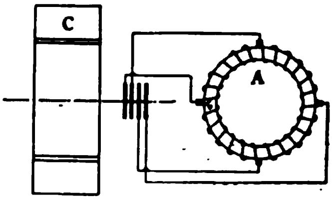


FIG. 2

$$\begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix} > 0$$

Finally, the late Prof. Henry Poincaré—to whose attention the writer had brought the more general statement of the case, to the effect that the appearance and maintenance of any currents whatever in a system of stationary or moving circuits is impossible without the existence either of batteries or permanent magnets, or else of capacities or of means capable of modifying the internal connections of the circuits (such as by commutators) —demonstrated that a theoretical necessity was really involved (see *Lumière Electrique*, March, 1907, page 293).

The possibility of raising the power-factor to unity without the use of a commutator would involve the negation of this necessity, and, therefore, could not exist.

In accordance with what has been said previously, with reference to the expression of the armature inductance in Fig. 1, under the form $L\ (\omega - \omega_1)$, it can be supposed, in order to obtain a first classical approximation, that the flux per phase on the armature-periphery is sinusoidal. But the distribution here considered is a theoretical one, which does not correspond to the real distribution. Let us begin by noting on what fictitious surface this distribution may be defined.

It is the general practise to consider the surface of the air gap, but that is not the point of view which the writer has adopted in his different studies. It has seemed to him that the distribution of flux which it was desirable to determine was that which is related to the geometrical surface containing the axes of the conductors subjected to inductive effects. Reference to Fig. 4, which shows a two-layer drum-winding, shows two cylindrical surfaces, $S'$ and $S''$, which contain the axes of the conductors subjected to induction. We ought, therefore, to consider the flux on the surfaces $S'$ and $S''$. But the writer proposed previously (see ***Lumière Electrique***, January, 1907, page 6) to make an approximation which consists in considering only the midway surface $S$, situated between $S'$ and $S''$, passing through points half-way down across the slots.

FIG. 4

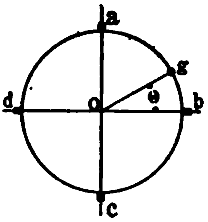


FIG. 5

It will be noted that the magnetic flux referred to comprises the lines of force which other writers consider as constituting local leakage fluxes around the conductors; and that if we extend the surface $S$ between the lateral connections of the armature, it comprises also the external leakage fluxes produced by the heads of the coils. We may imagine that the discontinuity caused by the teeth and slots gives rise to a special supplementary harmonic field. The result is that leakages can be represented by harmonics.

We have already given the name ***magnetic periphery*** to the fictitious surface $S$ of the armature. Diagrammatically, an armature having a commutator will be represented by a circle, whose circumference will be the magnetic periphery (see Fig. 5). It is on this magnetic periphery that the positions defined by the brushes $a$, $b$, $c$, $d$, will be indicated. The position of a radial line to any point, $g$, on the periphery, is defined by the angle $g\,o\,b = \theta$. Let $f(\theta)$ be the periodic function which represents the distribution per phase of the magnetic field normal to the surface $S$.

Simple calculations have already enabled the writer to establish (*see Lumière Electrique*, January, 1907, page 7), that the apparent inductance of the armature in Fig. 1, when supplied with two-phase currents, and when assumed to have an even number of sections, may be expressed at the angular velocity $\omega_1$, as follows:

$$L\left(\omega - \omega_1 \frac{\int_0^{\pi/2} f(\theta)\, d\theta}{\int_0^{\pi/2} f(\theta)\, \theta\, d\theta}\right)$$

It should be carefully noted that, for a sinusoidal distribution, $f(\theta) = a \sin \theta$, the second term reduces to unity, whereas for a triangular distribution $f(\theta) = a\,\theta$, the quotient

$$\frac{\int_0^{\pi/2} f(\theta)\, d\theta}{\int_0^{\pi/2} f(\theta)\, \theta\, d\theta}$$

assumes the value $3/\pi$. In that case, the condition of zero inductance is obtained only at the velocity $\omega_1 = \pi/3\,\omega$.

It is proper to note, however, that a fractional pitch winding enables the velocity at which the inductance vanishes to be brought to a value nearer $\omega$, and that, finally and especially, whatever may be the flux distribution on the magnetic periphery, the approximation to the theoretical expression $L\,(\omega - \omega_1)$ will become closer in proportion as the number of phases employed for supplying the armature in Fig. 1 is increased.

The very important question which remains to be considered in connection with the operation of the armature in Fig. 1 is that of commutation. We may here recall the manner in which that question was approached in the writer's study in 1901 (see *Eclairage Electrique*, page 294). Let $\lambda$ be the full coefficient of self-induction of the sections of armature winding undergoing commutation under the brushes, $a$, $c$, through which is passing the current $I \sin \omega t$. This coefficient ($\lambda$), it should be noted, includes, in the case of windings of the drum type, the effect due to mutual induction between two sections in process of commutation under the brushes $a$, $c$. of opposite polarity. Now, let $T$ represent the time that is consumed in the commutation of one section of winding. The condition which is ***necessary*** and ***sufficient*** in order to obtain perfect or "linear" commutation (which is gone into more fully in the article referred to), is that there should be available, in the winding sections which are in short-circuit, a reversal e. m. f. equal to $\frac{\lambda}{T} I \sin \omega t$.

Now the flux developed along the rectangular axis $b$, $d$ is proportional to $\cos \omega t$. Consequently, the induction resulting from its periodic variation produces an e.m.f. in the sections short-circuited by the brushes $a, c$, which is proportional to $-\frac{d}{d t} \cos \omega t$ and, therefore, to $\sin \omega t$. This e.m.f. is constant for a given current $I \cos \omega t$. There is, therefore, a certain velocity $\omega_1$, for which there is produced, exactly, in the commutated sections, the reversal e.m.f. necessary and sufficient, $\frac{\lambda}{T} I \sin \omega t$, to produce a perfect or linear commutation.

Below that velocity the reversal e.m.f. will be too high and commutating conditions will be produced which are analogous to those existing in a continuous-current dynamo when the brushes are set too far forward in the direction of rotation. Above that velocity, on the other hand, the reversal e.m.f. produced will be too weak, and the commutating conditions produced will be analogous to those which exist in a continuous-current dynamo when the brushes are not set sufficiently far forward in the direction of rotation, or even when they are set in the opposite direction. It will be pointed out later, that precisely this same condition of operation exists (namely,

FIG. 3

FIG. 9

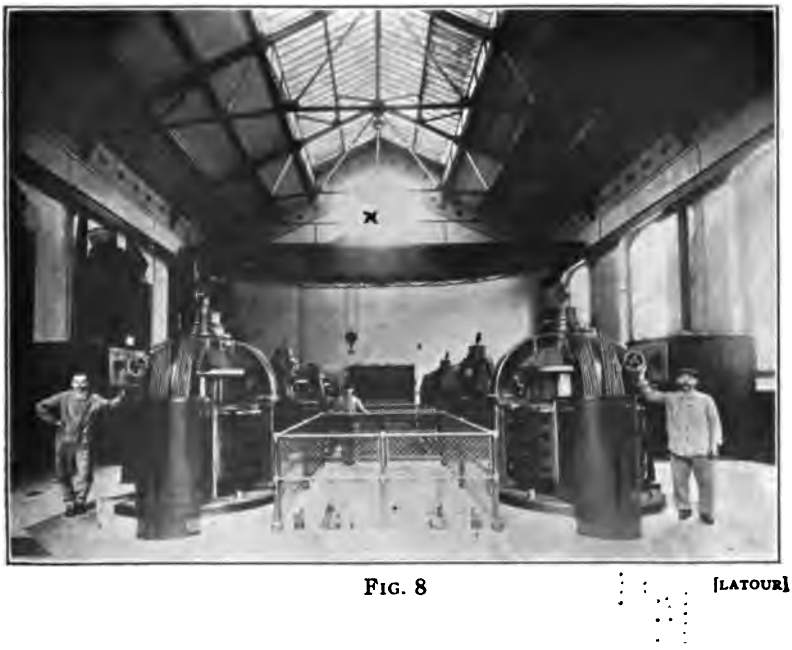


FIG. 8

insufficient reversal e.m.f.) when the armature in Fig. 1 is operating as a capacity, and it will be interesting to note that in the production of reactive current by a commutator we encounter the same commutating difficulties as the reduction of the normal armature reaction or the production of an armature reaction which assists the field excitation in a continuous-current dynamo.

It was shown by the writer (see *Bulletin of the Société Internationale des Electriciens*, June, 1910, page 392) that in the case of an armature placed in a uniform air-gap, inside a laminated stator having evenly spaced slots, the e.m.f. necessary and sufficient to produce the perfect commutation of a current $I$ is $\lambda\, I/T$, no matter what may be the distribution of the flux produced by the armature. This e.m.f. is equal rigorously to the e.m.f. which may be imagined to be induced in the winding sections under short circuit, by reason of the rotation of the armature in the flux developed by the armature as a whole. It should be well understood that the flux under consideration is that measured on the magnetic periphery of the armature, and on the portion of the surface comprised between the axes of the conductors of two contiguous sections in the region where commutation is taking place, as is shown in Fig. 6 (see the cross-hatched portion of the figure). There exists, in fact, a close relationship, in a continuous-current armature, between the coefficient ($\lambda$) of the winding sections under commutation and the flux produced by the entire armature through the shaded surface represented in Fig. 6, that is to say, in the interval comprised between two consecutive winding sections in the region in which commutation is taking place when unit current is passing through the armature. It is, moreover, by virtue of this close relationship that the writer has, since 1902, in the course of numerous controversies (see, for example, E. T. Z.,

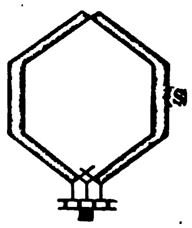


FIG. 6

1906, page 781; *Lumière Electrique*, 1902, page 53; *Electrician*, 1913, pages 105 and 325; *Elektrotechnik und Maschinenbau*, 1913, page 633), defended the very simple point of view according to which the matter of obtaining perfect commutation in a continuous-current dynamo depends substantially on the neutralization of the magnetic field of the armature, or, more properly, on the production of a mean resultant neutral field in the region where commutation is taking place. It is understood that the field is measured on the portion shown shaded in Fig. 6, on a surface $S$ which passes about half-way across the armature slots, and not through a surface situated in the air gap. The exactness of this view seems to be recognized more or less explicitly at the present time.

Having shown that the determination of $\frac{\lambda}{T} I \sin \omega t$ in the case of Fig. 1 amounts to the evaluation of the e.m.f. induced in the short-circuited winding sections by the rotation of the armature in its own field, it is easy to determine, for a given flux distribution, the reversal e.m.f. which is necessary to obtain perfect commutation at the velocity $\omega_1$. This knowledge of the flux distribution enables us to determine the reversal e.m.f. which is available by reason of the variation of the flux which is proportional to $\cos \omega t$, along the direction perpendicular to the axis of the brushes $a, c$. We can then finally determine the difference between these two e.m.f's. and this is what interferes with commutation.

Let $f(\theta)$ be the flux distribution. It will be found by a simple reasoning that in an armature wound with a full pitch winding the e.m.f. under the brushes resulting from an excess or lack of reversal e.m.f. is proportional to

$$\left( \omega - \omega_1 \frac{f(\theta)_{\theta=\pi/2}}{\int^{\pi/2} f(\theta)\, d\theta} \right)$$

If we make the theoretical assumption that $f(\theta) = a \sin \theta$, it will be readily seen that the quotient $\frac{f(\theta)_{\theta=\pi/2}}{\int_0^{\pi/2} f(\theta)\, d\theta}$ reduces to unity, and that perfect commutation is obtained at synchronism for the condition $\omega_1 = \omega$.

If we suppose that $f\ (\theta)$ is of the form $a\ \theta$, which is nearer the actual condition, it will be found that perfect commutation is obtained at the velocity $\omega_1 = \frac{\pi}{4}\ \omega = 0.79\ \omega$. However, the conditions are changed if the number of phases employed for supplying current to the armature in Fig. 1 is increased. With six phases, we find, for the velocity of perfect commutation

$$\omega_1 = \frac{\sqrt{3}}{2}\ \omega = 0.86\ \omega$$

and finally, for twelve phases, we find $\omega_1 = \omega$. By adopting a fractional pitch for the winding of the armature it is possible to approach more rapidly the condition $\omega_1 = \omega$. It is important to note that if the commutation is not perfect at synchronism in all cases, it is because we are not dealing, in general, with a uniform revolving field, in consequence of a non-sinusoidal distribution of flux on the magnetic periphery of the armature. The simple reasoning which the writer followed in a first approximation (1900–1901), and in which he asserted that commutation must always be perfect at synchronism when there is a real revolving field, is exact, for the very reason that the variation of flux in the short-circuited sections vanishes at synchronism.

Everything that can be done to obtain a perfect revolving field (such as increasing the number of phases and using fractional pitch winding) unquestionably assists in insuring perfect commutation at exact synchronism.

When the revolving field produced by a rotor having a commutator is imperfect, the commutation is disturbed at synchronism under conditions equivalent to those wherein it would be disturbed by an external imperfect revolving field produced by the stator surrounding it.

In reality, the increase in the number of phases for the supply of current to the rotor of a commutator machine, as set forth in the writer's German patent No. 145,433 of 1901, is related rather to a specific purpose, and it has less immediate relation to commutation. It may be interesting to note the principal reasons for this.

In the first place, in increasing the number of phases for supplying current to the rotor, the current to be commutated per phase for a given total current in the armature is materially diminished. The frictional surface on the periphery of the commutator remains the same in the two cases, but it is differently distributed. The result of reducing the current per phase is that the resistance of a single line of brushes in the case of a bipolar armature, or of the assemblage of lines of brushes coupled in parallel in the case of a multipolar armature, increases with the number of phases. Now, the increase of this resistance, as is well known, is favorable to good commutation, so long as the machine is not working at a theoretical load corresponding to perfect commutation.

In a load which does not correspond to perfect commutation, there exists, so to speak, in the short-circuited sections, a disturbing e.m.f. which is equal to the difference between the e.m.f. necessary to produce perfect commutation and that which is actually induced; it is this parasitic e.m.f. which is the real cause of the excessive heating and of the sparking at the brushes.

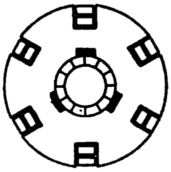


Fig. 7

With reference to sparking, Mr. F. Carter and the writer (see the *Electrical World*, March 31st, 1910, page 804, and *Bulletin de la Société Internationale des Electriciens*, April 6th, 1910, page 276) have advanced the opinion that the effects of a given parasitic e.m.f. increase, other things being equal, with the value of the quotient $l$ of certain determinants composed of the coefficients of self and mutual induction of all the closed circuits in the machine. This quotient $l$ itself has the dimensions of a coefficient of self-induction, and in a continuous current machine it is generally but slightly lower than the coefficient of leakage self-induction of two consecutive winding sections *placed in contiguous slots*. In an armature supplied with polyphase currents, however, the quotient $l$ may fall much below that value.

In this connection reference may be made to Fig. 7, which is the reproduction of a diagram already published by the writer (see A. I. E. E. TRANSACTIONS, 1903, page 583). The diagram represents an armature with a drum winding composed of twelve sections, supplied by three-phase currents. It will be easily seen that, as the width of the brushes exceeds the width of the commutator segments, there is always, at any instant, at least one short-circuited section per slot. Armatures having as few as three slots per pole are seldom used, but if we adopt a higher number of phases, preferably an odd number (7 or 9, for example, as indicated in the writer's A. I. E. E. communication of 1903) the remarks relative to the conditions shown in Fig. 7 can be extended to the case of an armature having a number of slots actually used in current practise. Under those conditions, the quotient $l$ becomes lower than the coefficient of self-induction due to leakage between two sections *placed in the same slots*. The parasitic e.m.f. will then produce practically a simple short-circuit current as if the armature was at rest or revolving slightly. Under these conditions, we will have the sparkless commutation which I named "squirrel-cage commutation."

The three types of a-c. polyphase commutator machines which can be rationally constituted by taking synchronism as the mean working condition, are analogous to continuous-current machines, namely, the shunt machine, the series machine, and the compound-wound machine. The theory of these machines was published by the writer in 1902 (see *Eclairage Electrique*, 1902, pages 50 and 358), and different authors have gone over the subject since in a more detailed manner. When these machines are working above or below synchronism, then, from the point of view of commutation, it is necessary to take into consideration the e.m.f. induced by the *resultant* field of the stator and rotor in the short-circuited winding sections, and to judge the commutation accordingly. If a machine is intended to have the greatest possible range of speed, then it is immediately obvious that it is important to arrange matters in such way that synchronism shall correspond to a speed of perfect operation so far as commutation is concerned. Allowance can then be made for equal values of slip above and below synchronism, and in this way the allowable variation of speed becomes greater. With an absolute slip of 20 per cent, for example, it is possible to obtain speed variations in the ratio of two to three.

When the slip is to be materially increased, the width of the brushes should be reduced as much as possible, in order that the e.m.f. induced between the opposite edges of the brush may be as low as possible; but for mechanical reasons it is scarcely practical to reduce the thickness of the brushes to less than 9 to 10 mm.

This minimum width of brushes being a limitation, it is desirable (for the same e.m.f. developed between the two edges of the brush) to divide this e.m.f. in some way by adopting a type of winding which allows the commutator segments to have a width of half that of the brush (see the writer's article in the *Electrical World* of December 3rd, 1904). As a rule, commutator segments of 4.5 to 5.0 mm. will be used.

With brushes of good quality the e.m.f. allowable between the segments under those conditions will be 1.5 volts. This e.m.f. allows high values of slip to be attained without necessitating a commutator of excessive size.

The writer has devoted much attention in the last ten years to a number of installations in which polyphase machines with commutators have been used. Figs. 8 and 9 represent an installation made about five years ago at the pumping station of the City of Paris. The machines are 500-h.p. motors supplied with three-phase 50-cycle current and operating at speeds ranging between 450 and 700 rev. per min., the synchronous speed being 600 rev. per min. The speed regulation is obtained by shifting the brushes. The rotor is supplied with twelve phases by means of twelve equidistant brush-holders. The total width of the commutator is 40 cm. The operation of the machines is perfect in every respect.

In concluding, the writer believes that the following historical résumé of his part in development of polyphase commutator machines may be of interest.

The English patent to Wilson (No. 18,525, of 1888) describes a polyphase commutator motor having the stator and the rotor connected directly in multiple without any transformer. According to the inventor, a squirrel-cage winding or a short-circuited winding might, if necessary, be placed on the rotor or on the stator or else on both of them at the same time if it were desired to operate the motor at or very near synchronous speed. The writer took up this same mode of connection later (French patent No. 306,229 of 1900) from a more modern point of view. His analysis led him to foresee the importance of lowering as much as possible the e.m.f. in action at the commutator by employing either transformers, a special tension-lowering device, or else sectional windings placed on the stator as means of supplying current to the rotor. In a German patent of 1891 (No. 61,951 to Georges) a series polyphase commutator motor is described. The writer has also introduced in that arrangement a series transformer between the stator and the rotor. Later, the writer adopted the plan of constructing these transformers with an air-gap, in order to improve the characteristics of the motor; and the connections employed are of the star-delta type, as shown in Fig. 10 (German Patent, No. 237,849).

The star-delta connection enables one to regulate the speed in a practical way, within wide limits, by simply shifting the brushes. The star connection is used for starting and for running at lower speeds; the delta connection is used at higher speeds.

Besides the shunt and series connections, mentioned in the two patents just referred to, the writer has also indicated how a compound connection could be used between the stator and the rotor (German patent No. 154,509 of 1901) and has favored the provision of a rotor with a greater number of phases than are obtainable directly from the supply mains (German patent No. 145,433 of 1901). This greater number of phases is obtained by means of a transformer, or else, in the case of a shunt machine, by means of auxiliary windings placed on the stator.

Finally, the writer has also proposed the use of all these kinds of machines as generators, and has called attention to their advantages in regard to their coupling in multiple and their compounding. Before the war, the writer had begun the construction of 100 kv-a. three-phase generators, which were to operate as boosters for mains supplying three-phase currents of 50 cycles. The writer is confident that this question of a-c. commutator generators, *in which, by the way, the commutator is relatively very small,* is a development which is sure to command attention in the near future.

## II—Commutator in Single-Phase Machinery

Let us suppose that the armature in Fig. 1 is supplied with a simple alternating current through two brushes *a*, *b*, placed 180 deg. apart, instead of being supplied with two-phase currents (see Fig. 11). The armature-reactance then retains the same value $L\omega$, whatever may be the angular velocity $\omega_1$. But, if we place, 90 deg. from the brushes *a*, *b*, some supplemental brushes *c*, *d*, connected together by a short-circuit connection (see Fig. 12), the armature-reactance then takes, as a first approximation, the value $L\left(\omega - \frac{\omega_1^2}{\omega}\right)$. It therefore vanishes when synchronism is attained and becomes negative above synchronism.

Thus, when a current $I \sin \omega t$ is sent through the armature by means of the brushes $a$, $b$, there is, in consequence of the arrangement of the short-circuited brushes $c$, $d$, an e. m. f., $- L \frac{\omega_1^2}{\omega} I \cos \omega t$ which is in opposition to the e. m. f. of self-induction $L \omega I \cos \omega t$, and, consequently, balances it at synchronous speed. The result is the same as if two-phase currents were sent into the armature. This phenomenon was discovered by the writer in 1901. This e. m. f., $L \frac{\omega_1^2}{\omega} I$ is produced by the following instrumentality. Let $M$, as before, represent the coefficient of mutual induction between the sections of winding which are in process of commutation under the brushes $a$, $b$, in which there is a reversing current $\frac{I}{2} \sin \omega t$, and the sections in circuit between the brushes $c$, $d$. Let again $n$ be the number of sections of armature winding. At the speed $\omega_1$ the duration $T$ of the short circuit in any section short-circuited by the brushes will be, $T = \frac{2\pi}{\omega_1 n}$. The induced e. m. f. between the brushes $c$, $d$, due to the mutual induction between the sections undergoing commutation under the brushes $a$, $b$, and the sections which are in circuit between the brushes $c$, $d$, will be,

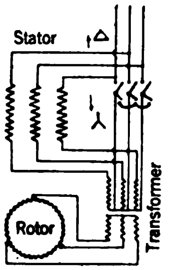


FIG. 10

FIG. 11

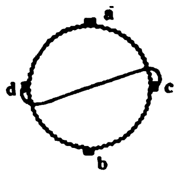


FIG. 12

$$\frac{M I \sin \omega t}{T} = \frac{M}{2\pi} n \omega_1 I \sin \omega t$$

This e. m. f. will produce, in the conductor connecting the brushes *c*, *d*, a short-circuit current which is $\frac{\pi}{2}$ out of phase with respect to that e. m. f. $\frac{M}{2\,\pi}\,\frac{\omega_1}{L\,\omega}\,I\cos\omega\,t$. This current, in phase with cos $\omega\,t$ is, in turn, commutated under the brushes *c*, *d*, and the mutual induction between the armature windings short-circuited by the brushes *c*, *d*, and the windings short-circuited by the brushes *a*, *b*, develops, between the brushes *a*, *b*, an e. m. f.,

$$\frac{M\,n}{2\,\pi}\,\frac{\omega_1}{L\,\omega}\cdot\frac{Mn}{2\,\pi}\,\omega_1\,I\cos\omega\,t=\left(\frac{M\,n}{2\,\pi}\right)^2\frac{1}{L}\,\frac{\omega_1^2}{\omega}\,I\cos\omega t$$

If we suppose the flux to be distributed sinusoidally we find that $\frac{M\,n}{2\,\pi}=L$, as we have already noted; and the e. m. f. induced in consequence of the rotation of the armature becomes equal to $L\,\frac{\omega_1^2}{\omega}\,I\cos\omega\,t$. In reality, the distribution of fluxes on the magnetic periphery is not sinusoidal.

Let $f\,(\theta)$ express the periodic function which represents the actual distribution of flux, and let $r$ represent the resistance of the armature between the brushes. The writer has already shown, by simple calculations (see *l'Eclairage Electrique*, January, 1907, page 811) that the armature-reactance at the velocity $\omega_1$ is, in reality equal to,

$$L\left[\omega-\left(\frac{\int_0^{\pi/2}f\,(\theta)\,d\,\theta}{\int_0^{\pi/2}f\,(\theta)\,\theta\,d\,\theta}\right)^2\frac{1}{1+\frac{r^2}{L^2\,\omega^2}}\,\frac{\omega_1^2}{\omega}\right]$$

On the assumption that the flux has a triangular distribution, $f\,(\theta)=a\,(\theta)$, it will be found that the quotient

$$\frac{\int_0^{\pi/2}f\,(\theta)\,d\,\theta}{\int_0^{\pi/2}f\,(\theta)\,\theta\,d\,\theta}$$

is equal to $\frac{3}{\pi}$ and that the armature-reactance may be expressed as follows:

$$L\left(\omega - \frac{9}{\pi^2}\ \frac{1}{1+\frac{r^2}{L^2\,\omega^2}}\ \frac{\omega_1^2}{\omega}\right)$$

The armature-reactance vanishes, under those conditions, when the velocity attains the value $\omega_1 = \frac{\pi}{3}\sqrt{1+\frac{r^2}{L^2\,\omega^2}}$.

Generally, $\frac{r}{L\,\omega}$ is very small, and it might be said that the reactance vanishes when $\omega_1 = \frac{\pi}{3}\,\omega$, exactly as in the case when the machine is supplied by two-phase currents.

Let us consider now the important question of commutation. The reversal e. m. f. necessary for commutating the current $I \sin \omega t$ under the brushes *a*, *b*, in Fig. 12, is induced by variation of the flux in phase with $\cos \omega t$ which exists along, *c*, *d*.

If we imagine the distribution of flux to be represented by the function $f(\theta)$ it will be seen, from what precedes, that the commutation under the brushes *a*, *b*, requires, in the case of an armature of even (full) winding pitch, a reversal e. m. f. which is proportional to $f(\theta)_{\theta=\pi/2}\ \omega_1$.

On the other hand, it is easily shown, after calculating the current in the short circuit, *c*, *d*, that the e. m. f. induced by the variation of flux along *c*, *d*, in the section short-circuited by the brushes *a*, *b*, is proportional to

$$\frac{\left(\int_0^{\pi/2} f(\theta)\,d\,\theta\right)^2}{\int_0^{\pi/2} f(\theta)\,\theta\,d\,\theta}\ .\ \omega_1$$

The ratio of the two e. m. f. considered, is therefore equal to

$$\frac{\left(\int_0^{\pi/2} f(\theta)\,d\,\theta\right)^2}{f(\theta)_{\theta=\pi/2} \times \int_0^{\pi/2} f(\theta)\,\theta\,d\,\theta}$$

Assuming a sinusoidal flux-distribution ($f(\theta) = a \sin \theta$), it is actually found that this ratio is equal to unity, so that the e. m. f. necessary to insure perfect commutation under the brushes $a$, $b$, is always equal to the e. m. f. actually induced by the field variation along $c$, $d$. But, if we assume that the flux-distribution is triangular $f(\theta) = a\theta$, it will be found that the ratio, above mentioned, becomes equal to $\frac{3}{4}$ simply.

Under those conditions the e. m. f. actually induced in the armature sections which are short-circuited by the brushes $a$, $b$, is not sufficient to insure perfect commutation of the current $I \sin \omega t$, and 25 per cent of the volume of that current will have to be commutated by brush-resistance.

Let us consider the commutation obtained under the brushes $c$, $d$. The e. m. f. necessary to obtain perfect commutation is proportional to the current passing through the short-circuit connection between the brushes $c$, $d$, and proportional to the velocity $\omega_1$.

As the current in the short-circuit connection between the brushes $c$, $d$, is already proportional to the velocity $\omega_1$, the e.m.f. necessary is proportional to $\omega_1^2$; and, taking into account the flux distribution, it will be found to be proportional to

$$\frac{f(\theta)_{\theta=\pi/2} \int_0^{\pi/2} f(\theta)\, d\theta}{\int_0^{\pi/2} f(\theta)\, \theta\, d\theta} \quad \frac{\omega_1^2}{\omega}$$

On the other hand the e.m.f. induced by the variation of flux along $a$, $b$ is proportional to

$$\int_0^{\pi/2} f(\theta)\, d\theta .\ \omega$$

Consequently the e.m.f. causing disturbance in the armature-sections which are short-circuited under the brushes $c$, $d$, which is the result of either excess or insufficiency of the e.m.f. of reversal, will be proportional to

$$\left( \omega - \frac{\omega_1^2}{\omega} \; \frac{f(\theta)_{\theta=\pi/2}}{\int_0^{\pi/2} f(\theta)\, \theta\, d\theta} \right)$$

Assuming a sinusoidal flux distribution $f\ (\theta) = a \sin \theta$, perfect commutation will actually take place when, $\omega_1 = \omega$.

In the case of triangular flux distribution $f\ (\theta) = a\ \theta$, we have

$$\frac{f\ (\theta)_{\theta = \pi/2}}{\int_0^{\pi/2} f\ (\theta)\ \theta\ d\ \theta} = \frac{12}{\pi^2}$$

and perfect commutation will be obtained when $\omega_1 = \frac{\pi}{\sqrt{12}}\ \omega$.

In the case of an armature wound with fractional pitch winding in which each section corresponds to an arc equal to $\pi - \alpha$ instead of a half circumference, $\pi$, if we assume that a winding of this character produces a trapezoidal flux-distribution, the results obtained are as will be described.

With regard to the commutation obtained under the brushes $a$, $b$, the expression of the ratio of the e.m.f. actually induced to the e.m.f. necessary to obtain perfect commutation, will now be as follows:

$$\frac{3 \left(\frac{\pi}{2} - \alpha\right)^2 \left(\frac{\pi}{2} + \alpha\right)^2}{(\pi - \alpha)^3\ (\pi + 2\ \alpha)}$$

A numerical application, such as for instance, when $\alpha = \frac{\pi}{3}$, gives 0.92 for the value of this ratio, which is quite near unity, and, consequently, there is a perceptible improvement as compared with full pitch winding, which gives 0.75 for that ratio. With regard to the commutation obtained under brushes $c$, $d$, the disturbing e.m.f. in the sections which are short-circuited by the brushes $c$, $d$, and which results from either excess or insufficiency of the e.m.f. of reversal, becomes proportional to

$$\left(\omega - \frac{\omega_1^2}{\omega}\ \frac{12}{(\pi - \alpha)\ (\pi + 2\ \alpha)}\right)$$

A numerical application, such as that corresponding to $\alpha = \frac{\pi}{3}$ leads to the conclusion that perfect commutation will be obtained for a velocity $\omega_1 = \frac{\pi}{\sqrt{10.8}}\,\omega$, which is very near that of synchronism.

In addition to the use of armatures with fractional pitch windings, (recommended by the writer in 1905; See *Eclairage Electrique*, page 125) the writer had proposed, in January 1903, the arrangements shown in Figs. 13, 14 and 15, which have a certain analogy with the phase-multiplications previously proposed by the writer for supplying current to the rotor in the case of polyphase machines.

It will be seen at once that, in consequence of the circulation of a current which is 90 deg. out of phase in the short-circuit connections between the brushes, a rotating field will be available at synchronism; and, by analogy with what has previously been said in regard to armatures provided with commutators and supplied by polyphase currents, it may be said that the problem of securing perfect commutation at synchronism comes back to the realization of a perfect rotating field. This result is attained by utilizing, conjointly, the arrangements shown in Figs. 13 or 15 and a suitable fractional pitch winding.

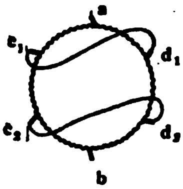


FIG. 13

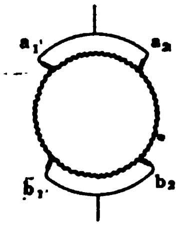


FIG. 14

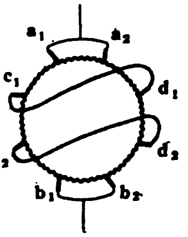


FIG. 15

From a clear understanding of the operation of the armature shown in Fig. 12, it will readily be seen how single-phase commutator machines similar to polyphase commutator machines, may be made with all kinds of windings and connections, shunt, series or compound, having a power factor equal to unity. Figs. 16 and 17, show respectively the shunt and series arrangements.

The machines which have been put on the market are the shunt and series motors, such as for example, those described in U. S. Patent No. 1016021. The writer has devoted attention to the construction of a large number of these motors which have, however, been used more for continuous operation on ordinary power circuits than for electric traction by single-phase current.

Fig. 18 is taken from a photograph of a type of motor of 200 h.p., 600 rev. per min., 50 cycles designed for continuous service, of which a considerable number have been constructed in the last 10 years. The number of sets of brushes and their connections correspond to the arrangements shown in Fig. 15. The speed-regulation, between 400 and 750 rev. per min., is obtained simply by shifting the brushes in accordance with the method of regulation described by the writer in 1903.

In motors which are intended to be regulated by shifting the brushes, it is important to take into account the action of the field produced by the stator, considered by itself, on the commutation of the rotor. In reality, in order that this action may not produce any special disturbance when a given shift is made in the brushes, it is necessary and sufficient that the field produced by the stator, considered by itself, should correspond to a sinusoidal distribution on the magnetic periphery of the rotor. This result will be obtained, with a close degree of approximation, by winding the stator partially on an arc equal to about $\frac{2\pi}{3}$ per pole, and it will be obtained more perfectly in motors of large power by the plan, already mentioned, of arranging for a sinusoidal variation in the number of conductors placed in the slots of the stator, which are supposed to be evenly spaced.

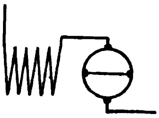


FIG. 16

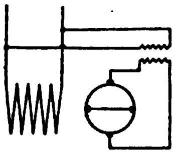


FIG. 17

The term "*compensated repulsion-motor*" was applied, by the writer, to the motor shown in Fig. 17 many years ago, because he appreciated, at that early date, the close relationship which is recognized generally today, between that motor and the repulsion-motor. It is proper, however, to consider the repulsion-motor, not in the primitive form given to it originally by Elihu Thomson, but in the form which it assumes when the stator of an induction motor is adopted in its construction.

FIG. 18

FIG. 21

FIG. 22 [LATOUR]

The writer published his first theory of this form of repulsion-motor (see *E. T. Z.*, June 11, 1903) before entering upon a detailed consideration of the compensated repulsion-motor. The great peculiarity of the repulsion-motor when constructed with the stator of an induction motor, is that a rotating field is produced in that type of motor at synchronism, and that, at that speed, the existence of that rotating field insures perfect commutation. Before the publication, by the writer, of the fundamental theory, the only theory known was that of Steinmetz, which related to the repulsion-motor constructed with pole-pieces, and which consequently did not foresee the formation of a rotating field and its consequences.

Of course, in order to obtain a perfect rotating field, it is necessary to resort to the precautions which haye just been mentioned and discussed with respect to the compensated repulsion- motor, namely, the rotor must have fractional pitch winding and multiple short circuits; and the stator must have a sinusoidal distribution of conductors.

With regard to the motors having a high power-factor, shown diagrammatically in Fig. 16, it is the shunt type of motor which has found most numerous commercial applications. The supply of current to the rotor is effected, as already indicated in the case of polyphase motors, either by a transformer or by an auxiliary winding placed on the stator.

The detailed theory of this motor, published ten years ago by the writer, (see *Eclairage Electrique*, January 1907, page 8) showed the great importance of flux distribution in that motor from the point of view of the torque produced, and also showed that great overload capacity may be given to this motor.

It is interesting to note that this motor can be constructed for very large powers with a commutator of very small dimensions when a heavy torque is not necessary in starting. If proper care has been taken in the design of the winding and in the distribution of the brushes on the commutator, this motor can operate at full load *with extraordinary commutation*. The drawback of a commutator is therefore, not as serious as might be supposed. On the other hand this motor has in its favor the following advantages:

1. It can supply a reactive current to the electrical mains,
2. Its overload capacity is much higher than that of the induction motor.

These advantages are sufficiently real, in the opinion of the writer, to warrant his considering quite practicable an electrical traction system based upon the use of single-phase current of 50 or 60 cycles involving the conversion, on the locomotive itself, of the single-phase current into a continous current by a motor-generator set comprising a commutating shunt motor. By employing high speeds, these motor-generator sets can be made quite light.

The fact that regulating resistances and all accessory apparatus are eliminated, and also the possibility of recovering energy at all speeds without any special complication, render this system more attractive than it might seem to be at first glance. The writer expects to return to this subject later with comparative figures.

With regard to electric traction by the direct use of single-phase alternating current, although that system does not appear to remain in high favor in the United States in recent years, it may be worth while to refer to the two types of single-phase motors to which the writer has given attention.

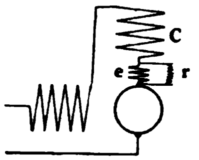


FIG. 19

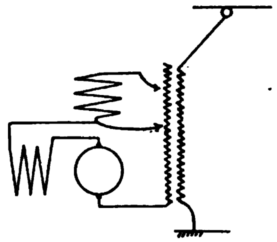


FIG. 20

The electrical world is quite familiar today with the motor having a local commutating field (see Fig. 19) which has, outside of the compensating winding *c*, a local winding *e*, shunted by a resistance *r* in such a way that this local winding produces a field that is out of phase above the sections which are undergoing commutation. With a frequency of 15 or 16 cycles, the loss of power in the resistance *r* can then be rendered negligible. This motor is then, undoubtedly, as the writer has shown (see *E. T. Z.*, Nov., 1912, page 1231) the best variable speed motor so far as commutation is concerned. That motor has been constructed especially in Switzerland by the Maschinenfabrik Oerlikon, but motors of similar type have been constructed in France by M. Perret, in co-operation with the writer.

At a frequency of 25 cycles the introduction of the resistance *r* would occasion too high a loss of energy, and the writer then gives preference to a motor which is designated by him the "elliptical field series motor," which is represented diagrammatically in Fig. 20 (see also U. S. Patent 841257 and *E. T. Z.*, 1906, volume 27, page 89). As the diagram indicates, the arrangement of connections of that motor implies the presence of a transformer *T* for supplying current to the motor. The dephased commutating field is produced by the compensating winding itself. The resemblance of this motor with the motor constructed by Mr. Alexanderson in America, will be readily noted.

Subsequently to the introduction of this type of motor by the writer in German technical publications, under the name of "Series-motor with Elliptical Field" (see article by the writer in *E. T. Z.* 1906, volume 27 page 89) German authors gave it the name of "doppel gepeist" ("double fed"), which seems to have become more or less current to-day in America. Nevertheless, it seems preferable to the writer to retain in the designation of the motor, the peculiarity which gives it good commutation insomuch as good commutation is the matter of utmost importance in commutating motors. It seems proper to the writer also that this designation should suggest a trace of the difficulty which inspired the very conception of the motor itself.

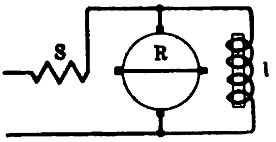


FIG. 23

Figs. 21 and 22 are reproduced from photographs of a 75-h.p. motor for an electric railway of narrow gauge (1 meter) constructed in accordance with the writer's inventions, and of which a considerable number have been in operation for several years. These motors have eight poles and run at 750 rev. per min. with 320 volts. The width of the commutatoris 240 mm. The writer has also constructed, for 25-cycle electric traction lines, some compensated repulsion motors with a reactance coil *l* in parallel with the exciting brushes (see Fig. 23), this arrangement being one proposed by the writer in 1903.

The reactance coil is useful at the time of starting because it allows a greater excess of current in the induced circuit (stator) without requiring an excessive strength of inducing field (rotor), which latter is shunted at low speed. The result is that, ***automatically***, as foreseen by the writer, there is, at the time of starting, a low flux with a heavy current, so that the commutator suffers less. Moreover, the reactance coil limits the hyper-synchronous speed of the motor, which is always to be feared from the point of view of commutation. The limiting hyper-synchronous speed is exactly that for which the reactance of the coil *l* comes into resonance with the capacitance of the rotor. The adjustments are made in such a way that this hyper-synchronous speed cannot exceed a rise of 50 per cent above synchronism. Motors of this type, which have been in practical operation about 10 years under particularly severe conditions, have given entire satisfaction.

It is well known that the drawback of a single-phase traction motor designed for 25 cycles, is the necessity of a very large commutator, owing to the high starting torque required. The volume of the commutator is indeed directly proportional to the frequency of current used, and to the starting torque required (see the writer's article in the ***Electrical World***, Dec. 3, 1904). The commutation may, however, be very good at normal load, and it is the belief of the writer that opinions have been much too pessimistic on that point. At the same time, it is not our intention at this time to defend single-phase traction, which was only an incidental object of this paper.

## Discussion on "The Polyphase Shunt Motor (Altes), "The Secomor—A Kinematic Device Which Imitates the Performance of a Series Wound Polyphase Commutator Motor" (Karapetoff) and "Commutation in Alternating-Current Machinery" (Latour). New York February 16, 1918.

**A. M. Gray:** It might be worth while to spend a few minutes in showing how Prof. Karapetoff's diagram is derived.

The a-c. series commutator motor consists essentially of an induction motor stator and a d-c. armature supplied, in the case of an $n$-phase machine with $n$ sets of brushes per pair of poles. These brushes make the armature equivalent to a stationary $n$-phase armature, because in any one phase, as one coil leaves the coil group another moves in to take its place. Since the stator and rotor are connected in series, two revolving fields are produced, which rotate at synchronous speed no matter what the actual speed of the rotor may be. The connections are such that these fields rotate in the same direction and a torque is produced whose value depends on the strengths of the fields and on the distance between them. If the brushes are moved backwards against the direction of motion of the fields the torque will tend to make the armature rotate in the same direction as the fields.

In discussing the vector diagram of such machines the teacher is liable to get space and time diagrams mixed up together. This I believe can be avoided as follows: In Fig. 1.

$I$ is the current in phase one of both stator and rotor.

$\phi_s$ is the flux that would thread phase one of the stator if the stator m.m.f. were acting alone.

$\phi_r$ is the flux that would thread phase one of the stator if the rotor m. m. f. were acting alone. The rotor revolving field lags that of the stator by $(180 - \alpha)$ electrical degrees.

$\phi$ is the actual flux threading phase one of the stator.

$E_{sb}$ is the e.m.f. generated in phase one of the stator by the resultant flux.

$E_{rb}$ is the e.m.f. generated in phase one of the rotor by the resultant flux. This e.m.f. is proportional to the slip.

$E_s$ is the component of applied voltage to overcome $E_{sb}$.

$E_r$ is the component of applied voltage to overcome $E_{rb}$.

$E_a$ is the applied voltage neglecting resistance and reactance drop.

At standstill the resultant voltage is 90 deg. out of phase with the current, since the output is zero. At half speed the rotor voltage for the same current will have only half the standstill value and the applied voltage will then be as represented by the dotted line. The locus of the applied voltage is the line *a b* for the case of constant-current operation, and this voltage diagram can readily be changed into a current circle diagram by inversion. Such a circle diagram, however, is not very accurate because it does not take account of saturation and change in the rotor reactance. Because of this difficulty Prof. Karapetoff has suggested that the "Secomor", which is nothing more or less than an adjustable voltage diagram, be used. Further discussion of this diagram would be out of place because the various possibilities have already been pointed out in Prof. Karapetoff's paper.

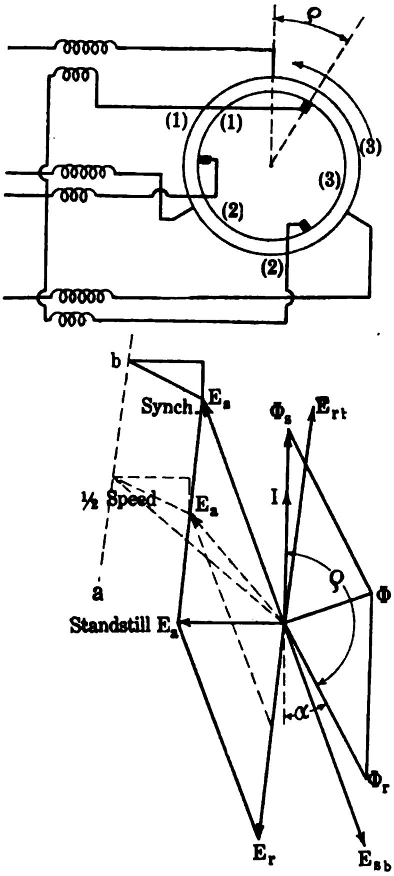


Fig. 1

**Paul M. Lincoln:** Mr. Latour proposes for trunk-line electrification to use a single-phase motor upon the locomotive driving a direct-current generator, which in turn will drive the locomotive. It seems to me if that is a proposition which comes from so distinguished a man as Mr. Latour, it is one which deserves a considerable amount of attention from those who are studying this question of the electrification of railroads.

**Charles F. Scott:** I was struck with one of Prof. Karapetoff's first sentences in which he says: "A combination of movable and adjustable bars which can be set to represent a vector diagram." What is a vector diagram? I would say that a vector diagram is itself a representation of certain physical things, and those physical things are the coils on an armature. If you take the simplest form of an alternator, one having two poles and a simple diametral coil on a rotating armature, and look at the end of the machine, you see a straight line which is the end of a coil which produces a complete cycle during a revolution. The angular position of the coil at a given time determines its relation to the field poles, and therefore represents the phase which the coil is producing. A second coil, placed at right angles gives an electromotive force 90 deg. from the first. Two lines at right angles, representing the physical angle between these coils, can be used in a diagram as a picture of the coils and are called vectors.

Go a step further, and make the length of a line correspond to the number of turns, and it follows that a straight line of proper length and direction (a vector) is simply a physical picture of a coil with a certain number of turns. If two coils are connected in series physically, we simply on the diagram join the lines in series with their proper angular relations and get the resultant. The vector diagram is a simple representation of the physical structure, as to angular position, number of turns and order of connection of the coils. Prof. Karapetoff has reversed the order, and starting from the vector diagram obtains a physical representation by an admirable structure using adjustable bars.

It has been suggested that I take part in the discussion of the single-phase motor as a factor in the railway situation and it has been pointed out now that the single-phase motor has not preempted the whole field of railway work, as some thought it might a number of years ago.

It is easy to look back and comment from the present standpoint. What was the standpoint in 1902, some fifteen years ago when the single-phase motor for railway work was first presented? That subject, by the way, was presented at a meeting of the Institute in the fall of that year. The previous session of the Institute had been a convention at Great Barrington, at which railway motors were considered. There were several railway papers and many pages of discussion. The discussion looked forward to the future of railway work, and the great cry was for a higher transmission voltage, and the great lament was that there was no way known to use such a voltage. Four or five different schemes were presented and discussed, but they were discussed merely as schemes, and nothing seemed practicable.

Then the single-phase system was presented, it was adopted and used on a number of roads, as the one thing which was commercially practicable under a good many of the situations where 600-volt direct-current apparatus was inapplicable.

But great developments have occurred since that time, which was fifteen years ago, and that is in railway history a very long time. If you go back another fifteen years, that is, go back from 1902 to 1887, you will find the electric motor was beginning to prove itself a success. I refer to the Richmond road. This whole development has taken place within thirty years. And then came the single-phase system fifteen years ago, when the operation of railways at high voltage was begun, and in that time have come other developments, the combination of the single-phase transmission and polyphase motors, and the direct -current motor has gone up to voltages which fifteen years ago were presumed to be impossible.

Consequently, in the light of development, the single-phase motor had its place, and is still occupying a place. The single-phase system is doing some of the heaviest work, and while it would be desirable to have one universal system in use everywhere, who knows what that system should be? If it had been adopted fifteen years ago, it would have been one thing; if it were to be chosen today, it might be the same thing or something else and possibly in five or fifteen years from now there may be other things which would make the effect of any final decision at this time a mistake.

**W. C. Korthals Altes:** I think Mr. Latour is pointing in absolutely the right direction so far as polyphase series motors are concerned. He has distinguished himself by opposing the multiplication of phases, and also he has brought out the point about the squirrel-cage commutation.

We have tried such motors, having squirrel-cage commutators, and we can say they operate very satisfactorily. I could show a d-c. motor and an a-c. motor and a three-phase a-c. motor, and put these motors through their stunts. You would see that the d-c. motor will flash and the a-c. motor will not show any sparking, so that the experiments made some years ago would not agree with the results which are obtained. Different conclusions have been reached. There is nothing remarkable about that, because we know the a-c. motor being completely compensated, and the current at all times passing through zero, is in a better condition for commutation. The unfortunate thing is we started out with single-phase motors and not three-phase motors. The three-phase motor is very superior to the single-phase motor, because we have there, as Mr. Latour has pointed out in a very interesting manner, the real sine distribution of the flux, and in all the work we have done we have always tried for that because with those conditions we get satisfactory commutation.

Another motor, which Mr. Latour has referred to, is what he calls the squirrel-cage commutation motor. I must take exception to some features of Mr. Latour's paper with respect to that. In Fig. 7 the author represents a two-pole three-phase commutator motor having six slots and an ordinary drum-wound armature.

Now, if there is a brush which short-circuits two commutator bars, there will be a short-circuited coil in two slots. If there are twelve commutator bars, a brush spanning two commutator bars, no matter in what position the armature is, there will at all times be at least a slot and coil which are short-circuited. You know if you take a direct-current motor with another winding in the links of the field winding, and you suddenly break the field, you will not have a series circuit, because the stored energy of the magnetic field would be equal in the circuit, which is formed.

On a single-phase machine, when the coil passes under the brush, we have to reverse the current. If we have a three-phase armature, we have to pass that current through 120 deg. You see immediately the inductive circuit by shifting through 120 deg. is much reduced on the 6-phase; on the 9-phase it is reduced to 40 deg. By shifting the current through a smaller number of degrees we get a more satisfactory operation, and that is the important thing Mr. Latour brought out. We get ideal current commutation.

We have only built a few motors so far, and I can mention as an example, a 60-cycle, 12-pole, 250-h. p. motor, and it cannot be made to spark. That is the very important work which Mr. Latour has done on three-phase motors.

As far as the single-phase motor is concerned, he has also distinguished himself by his so-called Latour connections. We have built quite a few motors with Latour connections, or something very similar. The two-circuit repulsion motor is not entirely new, but Mr. Latour has combined this with rotor excitation. The single-phase motor, built with many rheostats, as proposed by Mr. Latour, having, moreover, a series transformer to limit the possible voltage which can appear at the brushes, is an ideal motor from a commutation standpoint. But what Mr. Latour has said, in regard to the possibility of building large single-phase motors of high frequency, is absolutely true, although it would be practicable to build them three phase, and in that case they would be less sensitive. That it is absolutely impossible to build large single-phase motors of the commercial type of repulsion motors built in this country is not true. We have no demand for large single-phase motors and so have built none, but we could at any time build the largest. There is no question that motors can be built which operate satisfactorily but whether the motor works well in connection with the railway telephones and transmission and distribution systems, sub-stations, etc., is entirely a railway problem.

Referring to the characteristics of the series motor Prof. Karapetoff has succeeded in taking into account the saturation of the motor itself, which is most important, but of equal if not greater importance, is the saturation characteristic of the transformer.

If we should build a motor in which we had no increase of magnetizing current in the transformer, we would have an absolutely unstable motor. The motor has a characteristic which is very undesirable and the only way of improving that characteristic is to have a transformer which saturates and gives increased stability. I have attempted to accomplish this but have never succeeded because I did not know how to saturate a transformer and obtain that characteristic. If Prof. Karapetoff can show us the way he will do a great thing for the technical engineer.

**H. M. Hobart:** Some fifteen years ago the designers of electric motors were filled with enthusiasm to do their best to improve the single-phase motor. They entertained probably too optimistic an opinion of the possibilities of single-phase motors when applied to railway work. Wholly unintentionally they delayed the progress of railway electrification by perhaps many years. I speak of this, because I think Mr. Altes is wrong in thinking it his function to simply design motors when the railway engineers come to him and ask for them. He knows better than most engineers the difficulties with which he contends in the design of such motors. Notwithstanding these difficulties, by the skill which he has acquired, he designs excellent three-phase commutating motors for certain work. He knows they are heavy, he knows they are more expensive, but he does his best, and when the practical application is of such a nature that, notwithstanding the increased cost and weight, this polyphase motor is the right thing, he gives them an excellent motor. That is entirely right and proper, but if instead of merely designing the motor when he is asked to do so, he were to be less modest, and go to the railway engineers and say—"Look here, there are enormous difficulties in this kind of motor and disadvantages that you may not realize. I see that you are beginning to entertain the thought that they will be suitable for railway work, and with all due deference to your experience in railway work, I will counsel you that they have all sorts of faults. We will, however, do our best to overcome these faults." Were he to do this, he would be serving a useful purpose and helping progress in railway electrification. I felt quite strongly that that was a point that ought to be made, when I heard Mr. Altes state that his conception of his duty was merely to design what he was asked to design.

If we look back fifteen years, I think we can see that if those who had become so enthusiastic over single-phase railway motors had endeavored to curb their enthusiasm, and to be more frank regarding the difficulties, the situation would not have got out of hand to the extent of wasting so many million dollars as we know now were wasted. This assessment of the waste is not an uncorroborated assertion coming from myself alone. We now all realize that. The admission is again today implied quite distinctly in Latour's paper, where we can read between the lines that he would not advocate single-phase equipment of the rolling stock. He would put in a motor generator and use d-c. motors. Mr. Murray some years ago came out with the same admission when he said that by putting a rectifier on his locomotive he would be able to rate up the motors to one and half times the load of which they were capable when operating directly from a single-phase circuit.

Furthermore, I do not think those who did so much in trying to develop the single-phase railway system should feel so disheartened, as I gather they are inclined to feel from some of the remarks which have been made. They did admirably and developed a really substantial working system in several instances. Large values in train mileage have been performed by locomotives and cars operated by single-phase motors, and considering the enormous difficulty with which these single-phase advocates were contending, they did better than could reasonably have been supposed possible. They only produced these relatively satisfactory results by dint of the ablest sort of engineering.

**B. A. Behrend:** Mr. Latour refers most suggestively to the transformation, in the locomotive itself, of single-phase current into direct current. In the powerful electric engines of the Norfolk & Western Railway, single-phase current is being transformed into three-phase current. Thus we note broadly the tendency to utilize single-phase alternating current on the trolley with motors of a type other than single-phase. There are many in this audience here to-day who, sixteen years ago, discussed with great enthusiasm the promise of the application of single-phase currents to railroad work, while others, like Mr. Hobart and myself, showed greater skepticism. These remarks are made, not to deprecate the great credit due those who developed the single-phase railway system, but to point out that there must be reached an agreement among the engineers of the country in regard to the system of electrification, if the great work of operating the trunk lines of the country electrically is to be accomplished with the least opposition and the maximum amount of co-operation.

It is time that such a decision be reached. Almost twenty years ago the turbo-generator began its evolution. Numerous types were developed which have now all been superseded by the radial-slot type, upon which those who started with this type look back with satisfaction. There is likely to be the same result in the field of electrification. It would seem as though it behooved the engineers of the country to understand the issue of common aim and to unite upon one system after weighing with care and understanding the advantages and disadvantages of the available methods of electric propulsion. Thus the directors of our great transportation system can be met by a united front, advocating for each problem in electrification as it may come up the same system, thus infusing confidence into the great problem before us, viz., that of bringing the railroad system of our country up to the utmost demands that may at any time be placed upon it.

**William B. Jackson:** In considering these papers I naturally think of the large practical suggestion that is made by them, which has been referred to by several of the speakers, namely, the work that must still be done, in developing the most effective and satisfactory motor for operation upon a-c. systems.

It has been stated that we are considering merely a railroad problem, that is, the electrification of our trunk line railways. We are doing this and are doing considerably more, for the reason that we need very little vision to see in the not distant future a comprehensive a-c. electric system covering almost every square mile of the now populated territory of the United States. This means the development and regular use of electric current in ways that are now unusual. It means that we need to arrive at the most efficient and most effective a-c. motor for general use, having good speed-regulation characteristics.

There is another phase of the situation which is illustrative of the need for simplicity with flexibility in a-c. motors; the question in the equipment of our large power stations, whether to have alternating current alone for operating auxiliaries or direct current alone, or must we have two systems throughout the power station; one of which to serve constant-current motors and the other to serve variable-speed motors? This seems a very unnecessary situation when we come to the final analysis, since if we need motors operating at both constant speed and variable speeds, motors should and will be developed so as to eliminate even a question as to the installation of a dual system.

It thus seems to me that the problem set out by these papers is a very broad one, taking in the railways as one of the important features, but also involving the general distribution and utilization of power throughout the United States to an extent to which it has not yet been developed but such as our vision readily shows us must be the case in the not distant future.

**V. Karapetoff:** Prof. Gray combines the time diagram of the e.m.fs. with the space diagram of m.m.fs., and this is exactly what I have done in both the device and Fig. 7. Fig. 6, shows only the space diagram, and then I incorporate that diagram into a figure showing the e.m.f. diagram, and I point out that the stator bar is used both for currents and voltages, which is equivalent to saying it is used for the space diagram and time diagram as well. If the time diagram is in the usual counter-clockwise representation, then the space diagram comes in the clockwise representation. While I am not sure that this is always so, I was careful to point the clockwise representation in Fig. 6 and counter-clockwise for the time diagram in Fig. 7.

I am glad to know that at least one manufacturing company in this country has undertaken to further develop the Schrage motor, to which Mr. Altes makes a detailed reference. When I first saw a description of the Schrage motor I was full of hope that that was really the most satisfactory coming type. I could see it, from the principle incorporated in the motor. It is really a combination of an induction motor with the commutator motor incorporated in it, only for the fraction of the output differing from synchronous speed. For that part of the output which corresponds to the synchronous speed we need no commutator, and the ordinary induction motor is satisfactory. What you need is the counter e.m.f. of the proper frequency to be applied at the terminals of the rotor to force the induction motor to operate without $I^2 R$ losses in the resistance at any designed speed, either above or below synchronism, and that is just what the additional circuit in the Scharge motor does.

**W. C. K. Altes:** I find that I did not express myself properly when I spoke about single-phase motors. What I meant to say was that the single-phase motor has low starting torque referring to the generator set, referred to by Mr. Latour. I will leave it to the railway engineers to judge if it is a practical scheme.

We would like to provide a motor which will make it unnecessary in many cases to obtain direct-current, to change from alternating-current to direct-current. It has great advantage in simplicity and efficiency, the only reason why we do propose to build two kinds of motors, a series motor and a shunt motor, instead of building one shunt motor, is that the series motor is so much cheaper than the shunt motor. In cases where we do not require the shunt characteristics, as with fans and pumps and similar applications, we use a series motor, and in cases where we do require the shunt-motor characteristic, as on tools, we will have to use the shunt motor.

# ADDRESS BY PRESIDENT E. W. RICE, JR. AT A. I. E. E. MIDWINTER CONVENTION FEBRUARY, 1918.

MEMBERS of the electrical profession and industry have reason to be pleased with the contributions which they have made for the benefit of the world. While we are glad to think that our science and our industry are fundamentally devoted to the products and conditions of peace, we realize that in the electric light, searchlights, the X-ray, telephones, telegraph, wireless apparatus, electric motors, etc., electricity plays an important part in the grim business of war.

We are in the midst of an extraordinary coal famine, due to causes which it is perhaps undesirable for us to attempt to outline. However, I would like to point out how much worse the situation might have been were it not for the contributions of the electrical engineer; and also how much better our condition might have been if our contributions had been more extensively utilized.

Suppose we assume that the present serious situation is due to a lack of production of coal. It is comforting to consider to what extent conditions surrounding such production have been improved and how the output of our coal mines has been already increased by the use of electrical devices in connection with coal mining—such for example as the electric light, electric coal cutters, electric drills and electric mining and hauling locomotives. I have no figures before me but I think it is a fair assumption that the output of coal mines should have been increased at least 25 per cent on the average by the employment of such electrical devices. If this estimate were cut down to 10 per cent it would still leave a possible increase in the tonnage of coal produced of something like 50,000,000 tons during the past year.

If on the other hand, our situation is not due to a shortage in the production of coal, but rather to the failure of the distributive agencies of the country, which is more probable, it is interesting to see how this difficulty would have been largely removed if the railroads of the country were operated by electricity instead of steam.

Where electricity has been substituted for steam in the operation of railroads, fully 50 per cent increase in available capacity of existing tracks and other facilities has been demonstrated. This increased capacity has been due to a variety of causes, but largely to the increased reliability and capacity, under all conditions of service, of electric locomotives, thus permitting a speeding up of train schedules by some 25 per cent, under average conditions. Of course under the paralyzing conditions which prevail in extremely cold weather, when the steam locomotives practically go out of business, the electric locomotives make an even better showing. It is well known that extreme cold (aside from the physical condition of the traffic rail) does not hinder the operation of the electric locomotive but actually increases its hauling capacity. At a time when the steam locomotive is using up all its energy by radiation from its boiler and engine into the atmosphere, with the result that practically no useful power is available to move the train, the electric locomotive is operating under its most efficient conditions and may even work at a greater load than in warm weather. It may therefore be said that cold weather offers no terrors to an electrified road, but on the contrary, it is a stimulant to better performance instead of a cause of prostration and paralysis.

But this is not all. It is estimated that something like 150,000,000 tons of coal were consumed by the railroads in the year 1917. Now we know from the results obtained from such electrical operation of railroads as we already have in this country that it would be possible to save at least two-thirds of this coal, if electric locomotives were substituted for the present steam locomotives. On this basis there would be a saving of over 100,000,000 tons of coal, in one year.

This is an amount three times as large as the total coal exported from the United States during 1917.

The carrying capacity of our steam roads is also seriously restricted by the movement of coal required for haulage of the trains themselves. It is estimated that fully 16 per cent of the total ton-mileage movement behind the engine drawbar is made up of company coal and coal cars, including in this connection the steam engine tender and its contents. In other words, the useful or revenue carrying capacity of our steam roads could be increased about 10 per cent with existing track facilities by eliminating the entire company coal movement.

I have not mentioned the consumption of oil by the railroads which we are told amounted in 1915 to something like 40,000,000 barrels, nearly 15 per cent of the total oil produced. This fuel is entirely too valuable to be used in a wasteful manner. It is important for many reasons that such a wonderful fuel as oil should be most economically used, if for no other reason than that it will be needed for the ships of our forthcoming merchant marine, for the tractors that till our fields, and the motor trucks that serve as feeders to our railways.

The possible use of water power should also be considered in this connection. It is estimated that there is not less than 25,000,000 h. p. of water power available in the United States, and if this were developed and could be used in driving our railroads, each horse power so used would save at least six pounds of coal per horse power-hour now burned under the boilers of our steam locomotives. It is true that this water power is not uniformly distributed in the districts where the railroad requirements are greatest but the possibilities indicated by the figures are so impressive as to justify careful examination as to the extent to which water power could be so employed and the amount of coal which could be saved by its use. There is no doubt that a very considerable portion of the coal now wastefully used by the railroads could be released to the great and lasting advantage of the country.

The terrors of these "heatless days" will not have been without benefit if they direct the attention of the people and of our law makers to the frightful waste of two of our country's most valuable assets—our potential water power and our wonderful coal reserves. The first, potential water power, is being largely lost because most of it is allowed to run to waste, undeveloped, unused. The second asset, coal, is wasted for exactly the opposite reason. It is being used but in an extravagant and inefficient manner.

Our water falls constitute potential wealth which can only be truly conserved by development and use—millions of horsepower are running to waste every day, which once harnessed for the benefit of mankind become a perpetual source of wealth and prosperity.

While the amount of coal in our country is enormous, it is definitely limited. While Providence has blessed us with a princely amount of potential riches in our coal beds, it is known that there is a finite limit to the amount of coal so stored and when this coal is once exhausted, it is gone forever. It is really terrifying to realize that 25 per cent of the total amount of coal which we are digging from the earth each year is burned to operate our railroads under such inefficient conditions that an average of at least six pounds of coal is required per horse power-hour of work performed.

The same amount of coal burned in a modern central power station would produce an equivalent of three times that amount of power in the motors of an electric locomotive, even including all the losses of generation and transmission from the source of power to the locomotive. Where water power may be utilized, as in our mountainous districts in the West, all of the coal used for steam locomotives can be saved. In the Middle and Eastern states, however, water powers are not sufficient and it will be necessary in a universal scheme of electrification that the locomotives be operated from steam turbine stations, but as I have already stated, the operation of the electrified railroads from steam turbine stations will result in the saving of two-thirds of the coal now employed for equivalent tonnage movement by steam locomotives.

It is therefore not too much to say that if the roads of the country were now electrified that no breakdown of our coal supply, due to failure or distribution, would exist. What this would mean for the comfort of the people and the vigorous prosecution of the war, I will leave for you to imagine.

Of course this picture which I have briefly and inadequately sketched of the great benefits which our country would have received if the roads had been electrified does not improve our present situation and it may be claimed that any discussion of such a subject at this time is of an academic nature. This point of view is, in a sense true, but I think that we can properly take time to consider it because of the effect which it may have upon our future efforts. This picture is not merely an inventors dream but is based upon the solid foundation of actual achievement. We have had enough experience upon which to base a fairly accurate determination of the stupendous advantages and savings which will surely follow the general electrification of the railroads; in fact, I think we can demonstrate that there is no other way known to us by which the railroad problem facing the country can be as quickly and as cheaply solved as by electrification.

The solution of the railroad problem would also "kill two birds with one stone" by solving the fuel problem at the same time.

If it is a fact, as has been stated, that the steam railroads of the country have failed to keep pace with the country's productive capacity—the increased output of manufacturing industries, the extension of agriculture and other demands for transportation —it is obvious that if the country is to go ahead, the railroad transportation problem must be solved and it must be solved at the earliest possible date. It becomes a matter of national importance that the best solution should be reached in the shortest possible time. That solution is best which will give the greatest amount of transportation over existing tracks, in the most reliable manner, and if possible, at the lowest operating cost. We electrical engineers are confident that we can make good our claim that the best solution is to be found in a general electrification of the railroads. That such a solution would be of great advantage to our profession and to our industry is important, although not as important as the great advantage which it would be to our country, freeing it as it would from the present threatened paralysis of business, possibility of untold human suffering and incalculable financial loss. It should give us courage and optimism for the future of our profession to contemplate the service which we may render in this direction, and which it seems to me is immediately at hand. It should arouse in all of us, and particularly in the younger engineers, an enthusiastic confidence in the present and future stability and value of our profession and of the electrical industry. It should satisfy the young engineer that the opportunity for him to render important service is as real and great to-day as it has been in the past for those of us who have seen and participated in the marvelous growth of the industry up to the present time.

We would not be justified in being so confident of the benefits of electrification of railroads if every element in the problem had not been solved in a thoroughly practical manner. The electric generating power stations, operated either by water or by steam turbines, have reached the highest degree of perfection, efficiency and reliability, while the transmission of electricity over long distances, with reliability, has become a commonplace. Electric locomotives capable of hauling the heaviest trains at the highest speeds, up and down the heaviest grades, have been built and found in practical operation to meet every requirement of an exacting service. There is, therefore, no element of uncertainty nothing experimental or problematical, which should cause us to hesitate in pressing our claims upon the attention of the country.

Electrification of railroads has progressed with relative slowness during these many years, waiting upon the development and perfection of all of the processes of generation and transmission and of the perfection of the electric locomotive itself. When all these elements had been perfected, as they now have been for several years, the railroads found themselves without the necessary capital to make the investment.

I realize that the task of electrifying all of the steam railroads of the country is one of tremendous proportions. It would require under the best of conditions many years to complete and demand the expenditure of billions of dollars.

The country, however, has clearly outgrown its railway facilities and it would require, in any event, the expenditure of billions of dollars and many years of time to bring the transportation facilities up to the country's requirements.

It is not necessary that electrification should be universal in order to obtain much of its benefits. It is probable that the most serious limitations of our transportation system, at least in so far as the supply of coal is concerned, is to be found in the mountainous districts and it is precisely in such situations that electrification has demonstrated its greatest value. Electrification of a railroad in a mountainous district will in the worst cases enable double the amount of traffic to be moved over existing tracks and grades.

If a general scheme of electrification were decided upon, the natural procedure would be, therefore, to electrify those portions of the steam railroads which will show the greatest results and give the greatest relief from existing conjestion. Electrification of such sections of the steam railroads would have an immediate and beneficial effect upon the entire transportation system of the country and it is our belief that electrification offers the quickest, best and most efficient solution that is to be obtained.

It may be said that the present is not a propitious time in which to deflect any of the country's money into railroad electrification. I think that in spite of the enormous advantages of which I have spoken, we would be inclined to agree with such a point of view if it were not for the recent unpleasant demonstration of the failure of our railroad transportation systems to meet the demands which have been placed upon them by the industries, aggravated it is true by the war conditions and also by the unkindness of the weather.

After all, the question for the country to decide is whether we dare to limp along with the present conditions of restricted production, due to limited transportation, at a time when the world demands and expects from us the greatest possible increase in our efficiency and total production.

What assurance have we that the present conditions are temporary, and even if they improve as they surely shall with the coming of warm weather, what are we going to do next winter? Of course, even if we should start electrification at once, we could not have all our railroads electrified by next winter but we could have a good start, and as Sherman said about the resumption of specie payments—"The way to resume is to resume," so "The way to electrify is to electrify."

*Presented at the 338th meeting of the American Institute of Electrical Engineers, Cleveland, Ohio, March 8, 1918.*

# DESIGN OF UNDERGROUND DISTRIBUTION FOR ELECTRIC LIGHT AND POWER SYSTEMS

BY G. J. NEWTON

ABSTRACT OF PAPER

In a previous paper presented at the 10th Annual Convention of the Association of Iron and Steel Electrical Engineers the author made some suggestions, based on many years experience in designing underground distribution systems, for the guidance of engineers interested in this class of work.

The previous article treated the subject only in a general manner; the object of this article is to show each step necessary in the design of an underground distribution system, such as is usually required in a medium size city.

No two systems are entirely alike. The operating conditions and municipal requirements are different in every locality, it is impossible, therefore, to make any definite rules that will apply under all conditions.

By assuming the average conditions met with in the smaller cities it is possible to show the fundamental principles of handling this work in a systematic manner. While systems may, and do, differ, still it is possible to design a system for any type of distribution if these principles are followed.

Where costs are given they are based on normal conditions and should not be taken as being the present costs, they are used simply for comparison however and will be of value in that manner only, and if treated as percentage difference will apply under any reasonable conditions.

Owing to lack of space no tables have been printed in this article as they can be found in electrical handbooks.

IN ORDER to show each step in the design of an underground distribution system in the business section of a medium size city, it will be assumed that the present system consists of 110-220-volt single-phase lighting, 220-volt three-phase power and a street lighting system, as this, or some similar arrangement is commonly met with.

In order intelligently to design a system of underground distribution it is necessary to have the following information:

1. The lighting and power load in each building and the most suitable place to terminate the service cables, due regard being given to the present feeding point, so that the new system of

secondary mains can be located to the best advantage with the least changing of the present inside wiring.

2. Location of all street lights that are to be connected to the new system of distribution.
3. Voltage of the lights and motors that are to be supplied from the new system.
4. Location of pipes, sewers, foreign conduit systems and all other sub-surface obstructions in the streets where it is proposed to locate the new conduit system.
5. Kind of pavement on each street.
6. An accurate map of the proposed underground district drawn to a scale that will permit showing the details of property lines, curbs etc.
7. A copy of all city ordinances governing this class of work.
8. Cost of labor, teams, paving, material and equipment required to install the system.

The design of an underground system is not a difficult matter if handled in a systematic manner; there are however, a few general rules that should be adhered to, as far as possible, to get the best results.

1. Design the cable system to give the most economical, efficient and reliable service and then lay out the conduit to serve that arrangement.
2. Try to foresee, and provide facilities for meeting every kind of trouble that is liable to occur. As far as possible provide two sources of supply for every distribution center, particularly those located in important districts.
3. Remember that the ultimate value of the system is not its first cost; an improperly designed and installed system will never give satisfaction and will cost more for changes and additions in the end.
4. Use the best material and experienced workmen; there is no branch of an electrical undertaking where inferior material and poor work will give greater trouble than in an underground system.

Before taking up the actual design it will be well to consider the selection of cables to be used for the various systems as this must be kept in mind while making the design.

*Feeders.* Assuming that the station is equipped with three-phase generators and that the lighting load is balanced on all three phases, it will be advisable to select uniform sizes of cables for the light and power feeders, in order to reduce the amount of emergency cable required and also to standardize the switches and other equipment.

*Primary Feeders* run direct from the station to centers of distribution with few or no taps on them; they are installed in the lower or trunk ducts and are not liable to mechanical injury after being installed. These feeders receive the greatest benefit from the diversity factor, therefore it is not necessary to provide a great amount of reserve capacity as other feeders can be added as the demand increases.

*Secondary Feeders* run from the centers of distribution to the various transformers and are also in the lower ducts; these cables receive less benefit from the diversity factor than the primary feeders therefore they should have a little more reserve capacity.

For the system under consideration both of these classes of cables should be three-conductor cables. They can be either paper or varnished cambric insulated. If paper is used it will be necessary to splice rubber or varnished cambric insulated cable tails on it wherever they enter equipment. Personally I prefer varnished cambric or varnished cloth insulated cables for all distribution work, particularly in the smaller cities where usually the cable department is very limited.

*Secondary Mains.* The secondary mains for the power system will be three-conductor cables as they supply only three-phase power and will have three-conductor services spliced to them.

The secondary mains for the lighting system will have to be large, as that load is single-phase, as far as the secondary distribution is concerned, and next to the service cables they receive the least benefit from the diversity factor.

If three-conductor cable was used there would be a considerable reduction in the carrying capacity over single-conductor cables (25 per cent) also it would not be possible to take more than about two services out of each splice; this would increase the number of service boxes required and add considerably to the cost of the system.

The neutral of the 110-220-volt lighting system should be grounded at every transformer vault, and therefore unless there are some exceptional conditions, such as excessive electrolysis, there is no reason why the neutral can not be a bare copper stranded cable, and by using two single-conductor cables for the "outers" it will be possible to get the greatest benefit from the carrying capacity of the cables installed.

Considering the fact that there is no way of providing any emergency facilities for the secondary mains and that they receive the least benefit from the diversity factor, and are cut at frequent intervals for splicing the services, it is evident that they are the most important link in the distribution system. Trouble on a secondary main is very liable to interrupt service to every consumer spliced to it, and requires considerable time and expense to repair; therefore these cables should be given considerable excess carrying capacity to provide for future growth, and have varnished cloth or cambric insulation. (Except in special cases it is unnecessary to use rubber insulated cable on any part of a distribution system, except to enter equipment as mentioned previously.)

The single-conductor secondary mains will permit four service cables being taken out of each splice if necessary.

It is a good plan when installing secondary mains, particularly when there is a heavy load about the center of a block, to make a loop service of the main, instead of running service cables into the building, as this permits sectionalizing the main in case of trouble and reducing the number of consumers out of service. This arrangement also reduces the time required to locate trouble on a main.

*Service Cables.* The service cables for the power system should be three-conductor, but those for the lighting system should be single-conductor. It is advisable to install three service cables for every consumer where there is any liability of the load being over one kw. and if three cables are taken to all consumers it will be an easy matter to keep the 110-220 volt load balanced without the necessity of opening splices to make changes.

*Laterals.* The lighting and power services should be installed in separate pipes, and in the business district of a city it is a good plan to install two laterals to all consumers when the system is installed. When fibre conduit is used for the laterals, as is generally the case, the cost of installing the additional duct is very small as there is practically no additional excavation or paving required and very little additional concrete.

*Service Boxes.* Service boxes should be about three by four feet, and located at the most convenient points to permit the services entering the buildings, they should be spaced so that the services are not unreasonably long and that not more than four services are taken out of each box.

Keeping the above facts in mind we can now proceed with the design of the system.

## LOAD MAP

Fig. 1 shows the load map. Each block is numbered for future reference and the building lines are shown (it is not necessary to show the individual stores in a building as all of the consumers in a building should be supplied from one service in order to save cable and get the benefit of the diversity factor between consumers). The house numbers should be given but are omitted from this plan to avoid confusion.

The point where the service is to enter the building is shown, and the loads are marked on the plan facing the street from which they are to be supplied. While it is advisable to mark the most desirable location for the laterals still it is frequently necessary to select a less desirable location in order to reduce the amount of conduit or cable; as far as possible however, it is advisable to adopt a uniform method of installing the laterals as this permits standard construction and reduces the cost.

TABLE I

LOAD RECORD BY STREETS

| Street | From | To | Light | Power | Total |
|---|---|---|---|---|---|
| Ave.A. | First St. | Second St. | 88.0 | 98.0 | 186.0 |
| " " | Second " | Third " | 93.0 | 57.0 | 150.0 |
| " " | Third " | Fourth " | 53.0 | 33.0 | 86.0 |
| " " | Fourth " | Fifth " | 110.0 | 41.0 | 151.0 |
| " " | Fifth " | Sixth " | 98.0 | 23.5 | 121.5 |
| Ave. B. | First St. | Second St. | 69.0 | 36.5 | 105.5 |
| " " | Second " | Third " | 93.5 | 45.0 | 138.5 |
| " " | Third " | Fourth " | 197.0 | 102.0 | 299.0 |
| " " | Fourth " | Fifth " | 178.0 | 83.0 | 261.0 |
| " " | Fifth " | Sixth " | 93.0 | 58.0 | 151.0 |
| Ave. C. | First St. | Second St. | 44.0 | 32.0 | 76.0 |
| " " | Second " | Third " | 48.5 | 20.5 | 69.0 |
| " " | Third " | Fourth " | 57.5 | 20.0 | 77.5 |
| " " | Fourth " | Fifth " | 51.0 | 14.5 | 65.5 |
| " " | Fifth " | Sixth " | 40.0 | 32.0 | 72.0 |
| 1st St. | River | Ave. C | 138.5 | 76.0 | 214.5 |
| 2nd " | " | " " | 139.0 | 77.0 | 216.0 |
| 3rd " | " | " " | 120.5 | 62.0 | 182.5 |
| 4th " | " | " " | 214.0 | 62.0 | 276.0 |
| 5th " | " | " " | 128.0 | 31.0 | 159.0 |
| 6th " | " | " " | 109.0 | 25.5 | 134.5 |
| Big Store | | | 65.0 | 150.0 | 215.0 |
| | | | 2227.5 | 1179.5 | 3407.0 |

Streets 12,000 ft., or an average of 284 kw. per 1000 ft.

Having completed the load map the next step is to tabulate it by streets and blocks, as shown in Table 1. Referring to this table, it is seen that the lighting load is 2227.5 kw. and the power load is 1179.5, or a total of 3407 kw. to be supplied as follows:

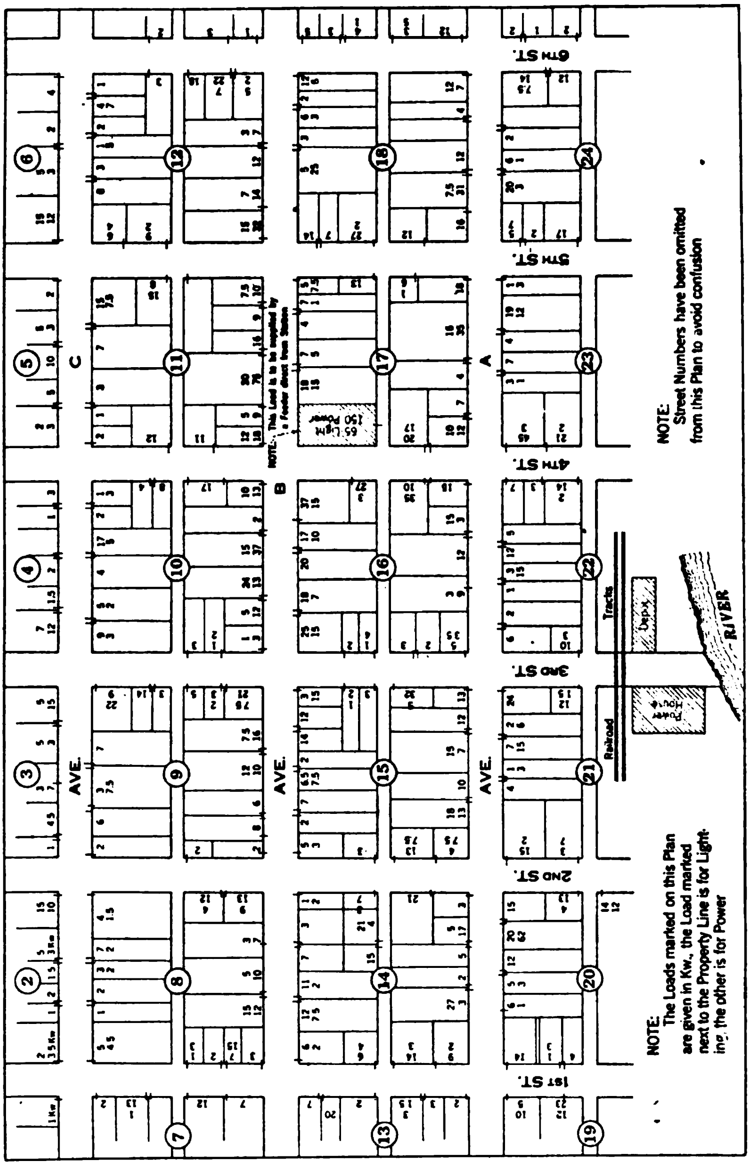


FIG. 1—LOAD MAP

Lighting system a-c. 110-220 volts single-phase three-wire.
Power load a-c. 220 volts, three-phase.
Street light system, a-c. 5.5 amperes, series.

It is always advisable to show the location of the street lights on the load map for, assuming that *all* wires in the district must be placed underground, it is evident that there must be conduit to every lamp and by laying out a rough plan of that system, or circuit, a general idea of the conduit arrangement can be determined. As the location of the street lights is determined by the city authorities it is seldom possible to change their location and by knowing where the conduit must be placed for street lights it will frequently permit a saving in conduit in other localities by changing the secondary arrangement of some of the consumers.

Owing to the fact that conduit systems are very expensive there is a great tendency to select streets having the cheapest pavement in which to install the conduit. While it is desirable to do work as cheaply as possible, still there is more than the first cost to be considered. Where a conduit system is used for feeders or high-tension systems exclusively, and there is no distribution from it, there is no objection to selecting such routes as will permit the conduit being installed as cheaply as possible.

Where a conduit system is installed for distribution to consumers the location should be governed entirely by the best method of reaching the various buildings regardless of the kind of pavement.

### SERIES STREET LIGHT SYSTEM

Fig. 2 shows the series street light system. There are 51 72-volt arc lights and 15 20-volt incandescent lights on the circuit in this district, the approximate voltage of the system being about 4000. As No. 6 6000-volt cable is standard for this class of service it is advisable to use it for all circuits of this system.

As the location of the lamps is determined by the city authorities, and the circuit must reach every lamp, there are only two things to consider; first to use as little cable as possible and second to arrange the circuit so that it will be possible to sectionalize it in case of necessity. The arrangement, shown in Fig. 2, permits sectionalizing the circuit at three points, in case of trouble, without any unnecessary cable being used.

It is doubtful if a circuit as small as this one would require any sectionalizing facilities; however the arrangement is shown here for illustration.

Assuming that the arc lights on the street are to be placed on new iron poles and the incandescent lights in the alleys on wooden poles the best arrangement for the cable would be to use either paper- or varnished-cloth-insulated cable for the main cable and rubber-insulated cable for the pole ends.

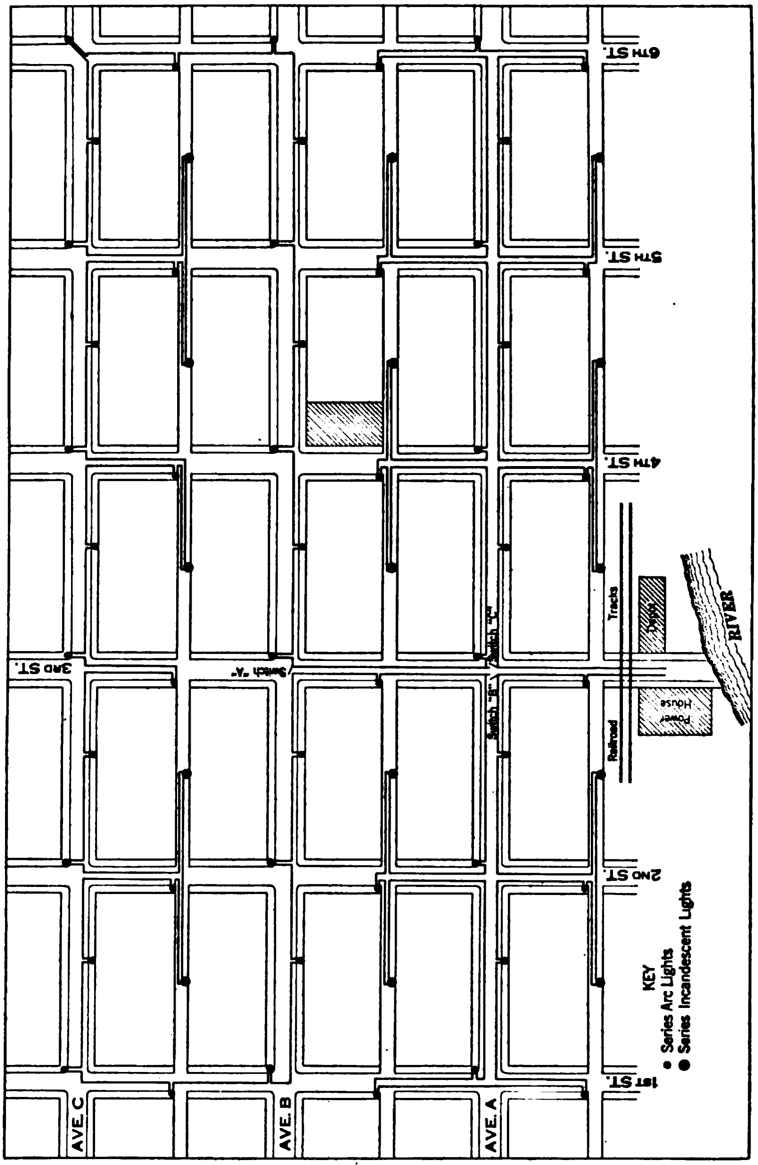


FIG. 2—SERIES STREET LIGHT SYSTEM

Frequently the pole end cable is run from the manhole, or handhole, up the pole to the lamp, this is a mistake, and a better plan is to terminate the pole end cable in series cut-outs mounted in the base of the pole and run insulated wire from this point up to the lamp, this permits the lamps being cut off from the circuit easily in case it is necessary to test the cable for trouble at any time and also saves considerable cable.

Referring to the figure, it is seen that there are many places where two-conductor cable could be used to advantage and in such cases the engineer must decide whether to use all single-conductor cable or two-conductor where possible. The difference in cost will be very small and undoubtedly the single-conductor cable would give the most reliable service, and as two cables can easily be drawn in one duct there is no saving in duct space by using the two-conductor cable.

If two-conductor cable is used it should be made in the round form and not figure "8" or flat, as it is practically impossible to train the flat cable without kinking it, and this will cause damage, particularly with paper-insulated cable as small as this type of cable would be.

Assuming that the cable has to reach the 15 lights in the alleys and that the alleys are paved, and that, with the exception of Block 14, it is unnecessary to run any secondary mains in the alleys, the question may arise as to the advisability of installing steel-armored cable laid directly in the ground for these extensions.

If the alleys are paved there will be very little difference in cost between the two systems, possibly a saving of 25 per cent. This is too small a saving to warrant purchasing and keeping in stock the extra type of cable. The best plan would be to install a single fibre duct to all of the lamp poles in the alleys. Armored cable should never be installed under permanent pavement, particularly in the business section of a city.

In making the estimate for this system it will be assumed that No. 6 varnished-cambric-insulated cable, for 6000 volts, is used, terminating in cutouts in the base of the iron poles and in cutouts mounted on the wooden poles above the lateral pipe.

Owing to the location of the street lights the cable for this system will have to be installed in one of the upper, or distribution, ducts so that it can be taken out at any handhole for future lights that may be installed. If there were other arc circuits, feeding other districts, they would of course be installed in the lower or trunk ducts, such cables could be paper insulated as they would have no taps on them until brought out at the pole from which they were to run aerial.

While a series arc circuit has been shown in this system it is practically certain that in this district the streets, or the principal ones at least, would have an ornamental system, these systems vary so greatly that it was not thought advisable to show one on these plans. If an ornamental system is required in this district it will be found that there is a spare duct in the upper tier of conduit that is available.

While the series arc circuit has been designed first, in this case it does not particularly aid the design of the secondary layout as the load in the district is so heavy and so well distributed along the streets that it is evident that mains will have to be placed along each street; still this method is usually advisable.

## CENTER OF DISTRIBUTION

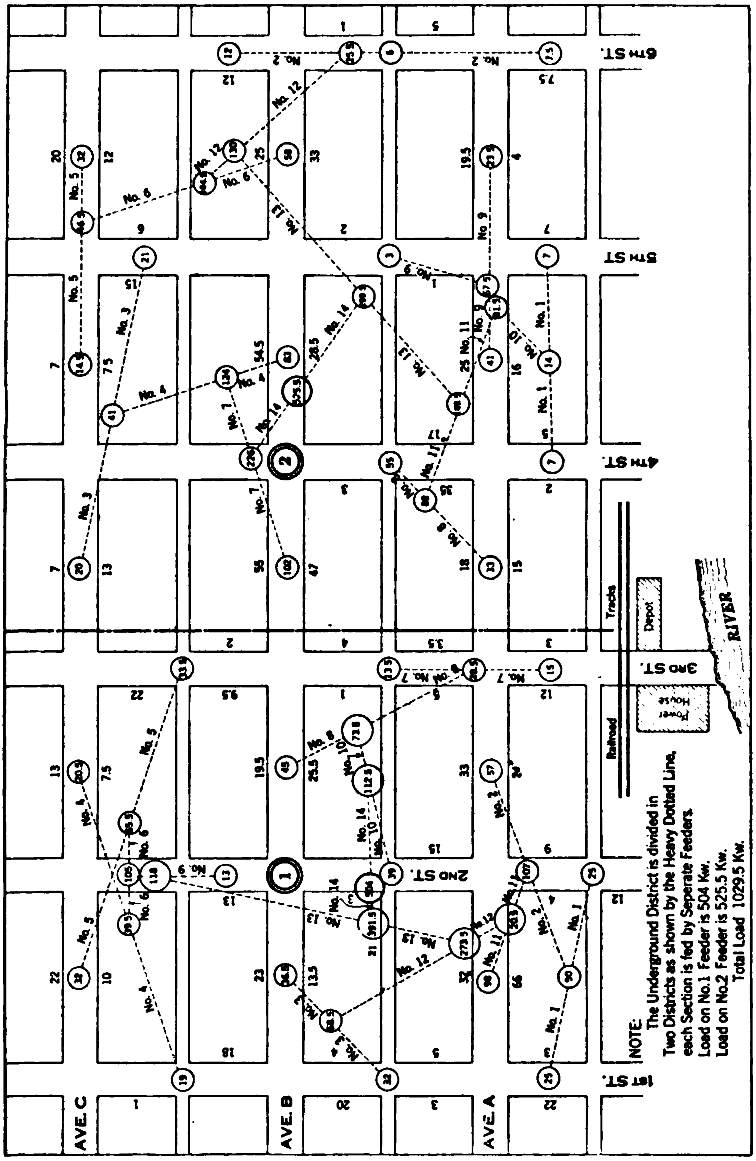
From a distribution point of view, the ideal location for a station would be at the center of the load that it was to supply, but unfortunately such a location is seldom possible or advisable owing to the value of real estate and lack of facilities for receiving and storing fuel. In the case under consideration the power house is already located and the distribution must be designed from that point. It is, therefore, necessary to find where the center of the load is, or if it is too large for one feeder to supply, the load must be divided and separate centers located.

Where the load is large enough to require two or more feeders it should be divided as equally as possible so as to permit using a uniform size of cable for all the feeders. If there is any section where the load is liable to increase rapidly this fact should be considered in locating the center of distribution and assumed to exist, and feeder capacity allowed for it.

Fig. 3 shows the three-phase power arranged for locating the centers of distribution. Ordinarily it is sufficient to assume that the entire load in a block is concentrated at the center of the block, for the purpose of illustration in this case the load has been shown on each side of each street, and the loads are marked facing the streets from which it will be supplied. (This system of marking is carried out on all of the illustrations.)

Referring to Table 1, the power load is given as 1179.5 kw., this includes the 150 kw. that is in the store at 4th Street and Ave. B. which is to be supplied by a separate feeder from the station, leaving 1029.5 kw. (say 1030 kw.) to be supplied by the regular feeders. This is too large a load to supply with one feeder, and as 2/0 three-conductor cable will supply 558 kw.

at 2200 volts, it is advisable to use two feeders of that size and locate two centers of distribution, each having about 515 kw.



NOTE:
The Underground District is divided in Two Districts as shown by the Heavy Dotted Line, each Section is fed by Seperate Feeders.
Load on No. 1 Feeder is 504 Kw.
Load on No. 2 Feeder is 525.5 Kw.
Total Load 1029.5 Kw.

FIG. 3—CENTER OF DISTRIBUTION

or half the total load. Later the actual size of feeders can be calculated, considering the power and diversity factors.

Referring to Fig. 3, the heavy broken line running north and south divides the load as follows: Feeder No. 1, 504 kw., feeder No. 2, 526 kw. (It is always best to try several ways of dividing the load and then select the one giving the best results.)

The loads marked in the small circles in the streets are the sums of the loads in the blocks facing those streets and the circles are located approximately where these centers would be.

The lines joining the various loads are numbered from 1 up to 14, for feeder No. 1, and gives a result of 504 kw., and the large circle marked 504 kw. is the proper point from which to supply this district.

As it is very probable that the transformers for supplying the lighting load will be located at street intersections it is safe to assume this centre of distribution as being at the nearest corner, or Second Street and Avenue B. Center No. 2 is calculated in the same manner.

The order in which the loads were taken is to avoid confusion; the work can be simplified greatly by combining loads of equal size whenever possible.

### 2200-Volt Three-Phase Power Feeders

Fig. 4 shows the power load marked in each block and the two power feeders terminating at the centers of distribution, that were located on Fig. 3. In order to provide for an emergency there is a spare feeder shown laid out in such a manner that the load on either of the regular feeders can be transferred to the emergency feeder in case of necessity. The spare feeder is also arranged to act as an emergency feeder to the large store at 4th Street and Avenue B., should its regular feeder fail.

As far as possible each feeder is run over a separate route. In a small district, such as the one under consideration, separate routes from the station are not warranted and the next best method is to separate the feeders as soon after they leave the station as possible.

Having decided the route of the feeders the next step is to calculate the size of cable to use. The power load is constantly changing and it is impossible to tell what the actual maximum demand will be for any group of consumers and all that can be done is to assume certain conditions as a basis for the calculations.

The maximum demand of a group of consumers is less than the sum of the several maxima, likewise the sum of several groups is less than the sum of their maxima, therefore, in a distribution system the primary feeders will receive the greatest benefit from the diversity factor, the secondary feeders less benefit, and the secondary mains the least benefit.

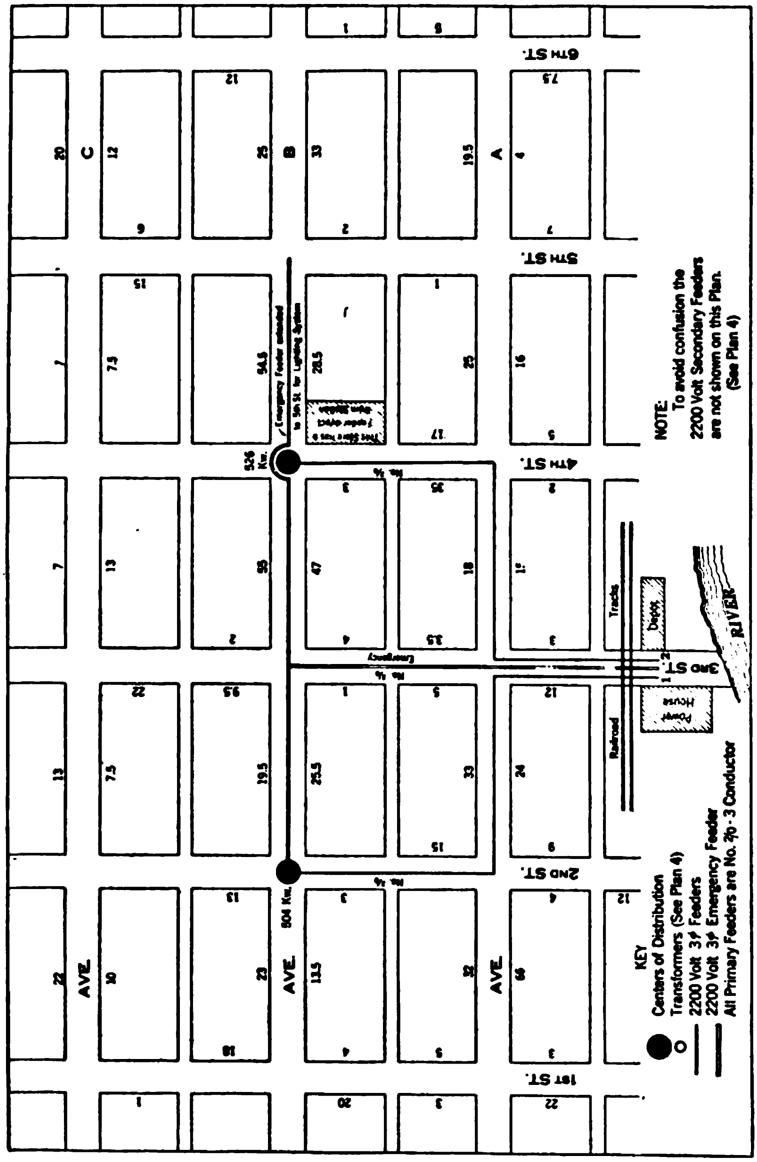


FIG. 4—PRIMARY POWER FEEDERS—2200-VOLT—THREE-PHASE

Assuming that the maximum demand will vary from 60 to 80 per cent of the connected load on various parts of the system, the feeders and mains can be calculated as follows:

Primary feeders, 60 per cent of the connected load.
Secondary feeders, 70 per cent of the connected load.
Secondary mains, 80 per cent of the connected load.

Where definite knowledge of the demand is known other figures should be substituted for those given here.

Referring to Fig. 5, it will be seen that there are 127 consumers having a total connected load of 1030 kw., or an average of 8.1 kw. each. This is rather high for a district such as is under consideration, and it is evident that the demand for power is fairly high in this district. As the system is used exclusively for power the power factor will be about 0.8 and for the feeders, take 60 per cent as the demand factors, as the load on each feeder is practically the same it is only necessary to calculate one feeder.

TABLE II—CARRYING CAPACITY OF MULTI-CONDUCTOR CABLES.
SAFE CURRENT IN AMPERES

| Size | 1-Conductor | 2-Conductor | 3-Conductor |
|---|---|---|---|
| 8 | 45 | 39 | 34 |
| 6 | 64 | 56 | 48 |
| 4 | 91 | 79 | 68 |
| 2 | 125 | 109 | 94 |
| 0 | 168 | 146 | 126 |
| 00 | 195 | 170 | 146 |
| 000 | 225 | 196 | 169 |
| 0000 | 260 | 226 | 195 |
| 250,000 | 293 | 255 | 220 |

Note: This table is based on information contained in the Standard Underground Co's. Handbook, taking the carrying capacity of single-conductor cable as 100 per cent and using 87 per cent for two-conductor cable and 75 per cent for three-conductor cable for continuous operation at a temperature not exceeding 150 deg. fahr.

### EXAMPLE.

Feeder No. 2

| | |
|---|---|
| Connected load.................................... | 526.0 kw. |
| Power factor.................................... | 80 per cent |
| Demand factor.................................... | 60 per cent |

Allow for growth 25 per cent of the connected load.

$$\frac{526 \text{ kw.}}{0.8 \text{ power factor}} = 658 \text{ kw.}$$

658 × 0.6 = 395 kw. present actual load.
526 × 0.25 = 132 " allowance for growth.

527 " total load for which to provide feeder capacity.

$$I = \frac{\text{kw.} \times 1000}{E \times 1.73} = \frac{527 \times 1000}{2200 \times 1.73} = 138 \text{ amperes}$$

Referring to Table II under three-conductor cable, it is seen that 2/0 cable will carry 146 amperes, therefore this is the proper size to use, and this checks the first selection of feeder size in calculating the centre of distribution.

Three-conductor 2/0 cable will deliver 558 kw. at 2200 volts, which is about 6 per cent more than the present connected load on the feeder, or 41 per cent more than the estimated load of 395 kw. Three-conductor 1/0 cable will deliver 481 kw. at 2200 volts and while this would supply the present estimated load, it would leave a very small reserve capacity, particularly as the estimated load is based on assumed conditions.

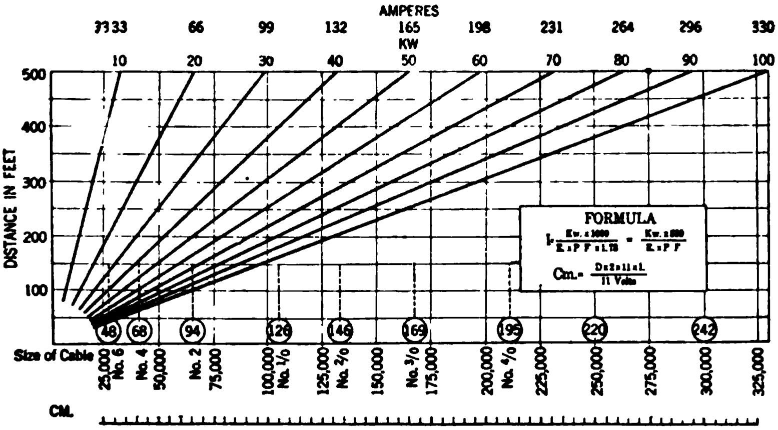


Chart I—Load in Kw.-Three-Phase-220 Volts—80 Per Cent Power Factor—5 Per Cent Lost = 11 Volts

**This chart is for use in determining the size of the three-phase power mains to save calculating each one—in using it care must be taken to see that the cable will carry the amperes that the load requires**

It is in a case like this that the engineer must be guided by his personal knowledge of the conditions, and as cost is frequently the deciding factor we can make an estimate for each size of feeder.

| | |
|---|---|
| Feeder No. 1 | 1000 ft. |
| Feeder No. 2 | 1000 ft. |
| Emergency Feeder | 1400 ft. |
| Total | 3400 ft. |

The following prices are approximately correct for three-conductor, varnished-cambric-insulated, lead-covered cables for 2300 volts working pressure, with copper at 16 cents per lb.

| Size | Price per 1000 ft. |
|---|---|
| No. 4 cable...... | $220.00 |
| “ 2 “........ | 300.00 |
| “ 1/0 “........ | 390.00 |
| “ 2/0 “........ | 450.00 |
| “ 4/0 “........ | 635.00 |

| | |
|---|---|
| No. 2/0 cable 1400 ft. at $450.00........... | $1530.00 |
| “ 1/0 “ 1400 “ “ 390.00........... | 1326.00 |
| | $ 204.00 |

This is a very small amount to save considering that the duct space and cost of installation are practically the same for either cable and the additional security and reserve capacity would justify the use of the 2/0 cable in this case.

Before laying out the secondary feeders it is advisable to design the secondary mains, as the location and size of the transformers will depend on their arrangement.

### 220-Volt Three-Phase Power Mains

Fig. 5 shows the arrangement of the 220-volt three-phase power mains laid out so as to supply all of the power consumers. It will be noticed that some of the mains only extend far enough to supply the load on certain streets and are not continued to the adjacent manhole. This is a good plan to try at first and calculate the size of cable required for each main, and where the cable will be too large the main can then be extended to the next manhole so that it will complete the network and be supplied from each end.

The two feeder districts are kept separate, but where mains from one feeder extend to a manhole of the other district (as at Ave. B. and Third Street) they can be tied together in a suitable junction box so that they are supplied from both ends. The main will form a loop in such cases.

In calculating the size of cable to use for the mains it must be remembered that all of the services are spliced directly to the mains, and the replacement of a main is a very expensive matter, the old cable removed, being in short lengths, is of little value except for junk.

While it is certain that all of the motors will not be operated at the same time still the mains must be of sufficient size to maintain the voltage without excessive drop under the worst condition. Where there is a reasonable prospect that the load will increase it is advisable to install mains large enough to provide for this increase.

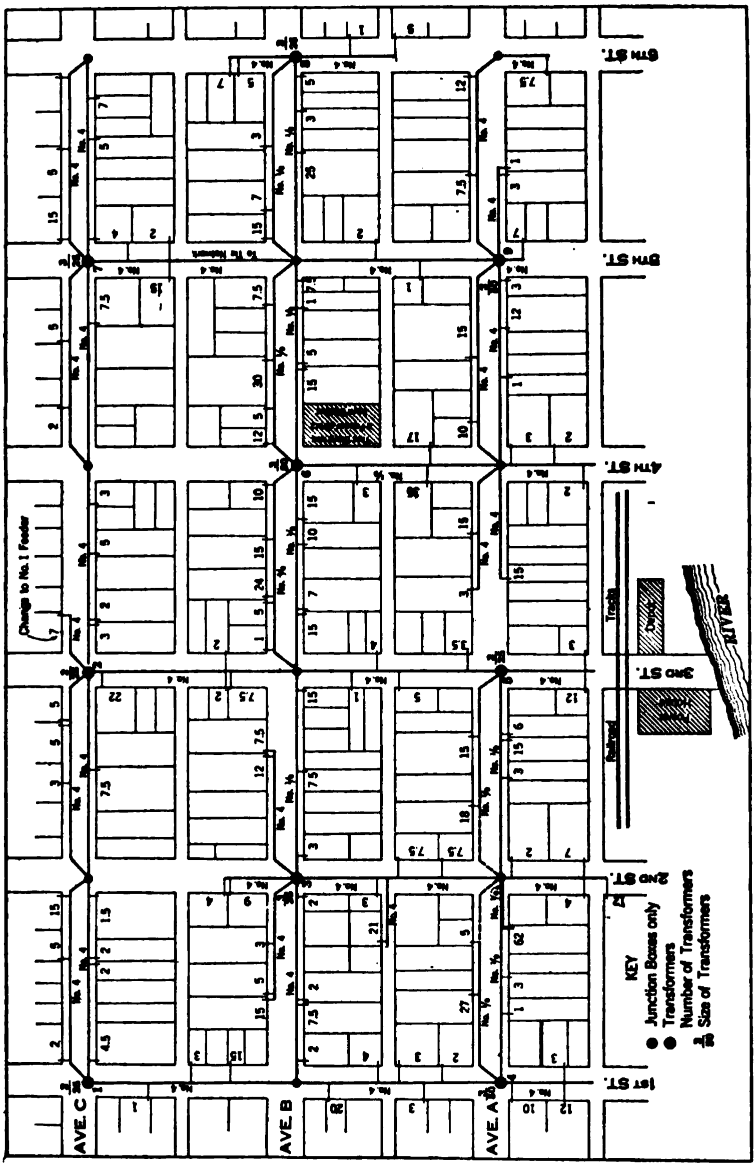


FIG. 5—220-VOLT—THREE-PHASE—POWER MAINS

To illustrate the method of calculating the size of mains, in Fig. 5, take the two blocks on Avenue C, from First to Third

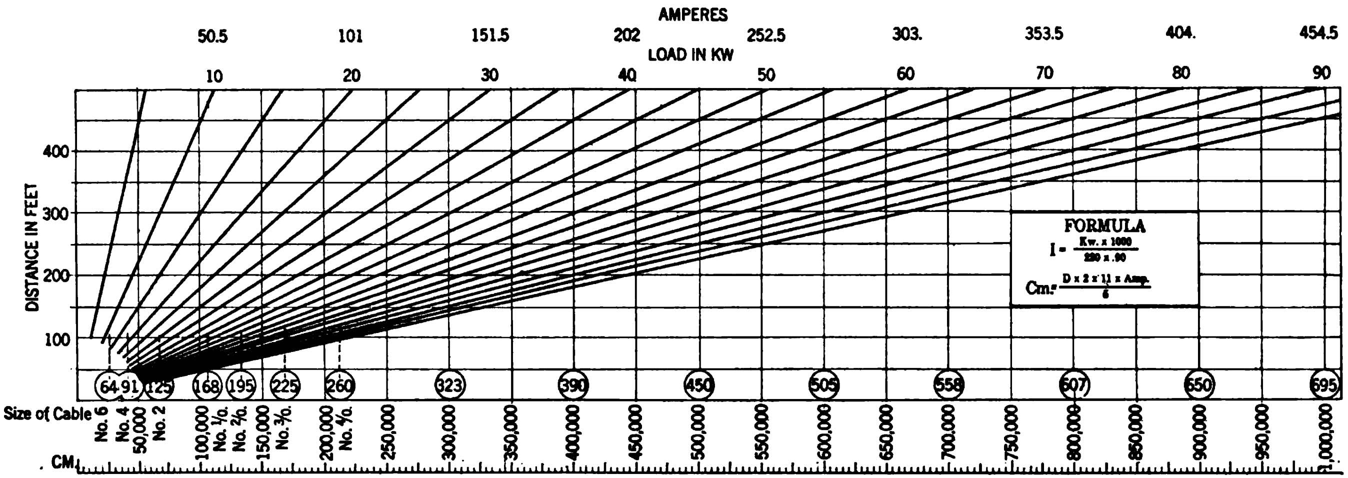


Chart II—220-Volt—Single-Phase Lighting Mains—90 Per Cent Power Factor—2 Per Cent Drop = 5 Volts

**This chart is for use in determining the size of the lighting mains to save calculating each one—care must be used to see that the carrying capacity of the cable given is sufficient for the load.**

**Take the load in kw.—follow the diagonal down to the proper distance and under this point on the scale will be found the size of cable.**

Street, which has mains on both sides of the street and is supplied from transformers at each end.

Allowing 5 per cent drop, 0.8 power factor and assuming that 80 per cent of the motor load is to be operated at the same time, the 0.8 power factor, and the 80 per cent demand factor will, in this case, counterbalance each other and by taking the connected load as the amount to be provided for we will be allowing for these conditions.

First take the load in each block separately and assume that transformer No. 1 supplies the 22 kw. in block No. 1, and that transformer No. 2 supplies the 13 kw. in block No. 2; the center of the 22-kw. load is about 310 ft. from transformer No. 1 and the center of the 13-kw. load is 165 ft. from transformer No. 2.

EXAMPLE

$$I = \frac{\text{kw.} \times 1000}{E \times 1.73} = \frac{\text{kw.} \times 580}{E}$$

$$I = \frac{22 \times 580}{220} = \frac{12{,}760}{220} = 58 \text{ amperes.}$$

And for the size of cable,

$$\text{Cir. Mils.} = \frac{\text{ft.} \times 2 \times 11 \times \text{amperes}}{\text{volts lost}}$$

$$= \frac{310 \times 2 \times 11 \times 58}{11} = 35{,}960.$$

The next larger size is No. 4 or 41,740 cir. mils, and from Table II, under three-conductor cable, it is found that No. 4 will carry 68 amperes so this size will carry the present load but provide little spare capacity for growth. For the present, assume that No. 4 will be used here and proceed in the same manner to calculate the size for the 13-kw. load. The result is 11,220 cir. mils, or between No. 9 and No. 10 wire, but as it is not advisable to use anything smaller than No. 4 for secondary mains it will be assumed that this section will be No. 4 also. So far nothing has been allowed for growth but the mains would extend from each transformer bank far enough to supply all of the loads. This arrangement could be used at first and later when the load demanded the two mains could be tied together with the following result.

The total load in the two blocks will be 22 + 13 = 35 kw. which is assumed to be located at the center, or 400 ft. from each end of the main.

Total load, 35 kw.

Supplied from each end, 17.5 kw. (say 18 kw.).

$$I = \frac{18 \times 580}{220} = 48 \text{ amperes.}$$

$$\text{Cir. mils} = \frac{400 \text{ ft.} \times 2 \times 11 \times 48}{11} = 38{,}400 \text{ (Use No. 4).}$$

This will give a continuous main and would provide for a very considerable increase in load as the entire load is not concentrated at the center of the main, as assumed.

The assumption of 80 per cent demand factor is undoubtedly high, for this class of service and is only used here as an example of the method of calculation. All of the other mains are calculated in the same manner.

The mains should all be calculated and marked in pencil on the plan temporarily, after which it is advisable to select a few standard sizes and use them throughout the system. If the power and lighting cables are all marked temporarily at first it is frequently possible to standardize both systems with a few different sizes of cable.

In calculating the size of mains on Avenue A, from First to Third Street, it was found advisable to extend two of the mains as shown by the dotted lines to permit using No. 1/0 cable for all of the mains in those two blocks and assuming that the total load was concentrated at the manhole at Avenue A and Second Street, which is the worst condition that could occur in this section, the four No. 1/0 mains terminating there are ample to supply it.

The mains as laid out in Fig. 5 are of two sizes, No. 4 and No. 1/0, and these sizes are very suitable as probably with the addition of some three-conductor No. 6 cable, for small services, all of the mains and services can be installed with three sizes of cable.

The consumer service cables will average about 50 ft. each and the loads vary from 1 to 60 kw. The drop in the services should not be more than 1 per cent, or 2 volts, and the full connected load should be provided for to take care of the extra current required for starting the motors.

The services can be divided into three groups as follows:

0 to 10 kw. Use No. 6, three-conductor cable.
11 " 20 " " " 4, " " "
21 " 40 " " " 1/0, " " "

Where the load is over 40 kw. two service laterals can be installed if the operating conditions of the load demand it, and frequently the location of the motors is such that two services are desirable; a case of this kind is shown at Avenue A. and Second Street where two services are required and they are taken from separate mains.

The next operation is to determine the size of the transformers to install at the feeding points. Owing to the fact that the motor load is constantly changing it is a very difficult matter to determine the most economical size of transformers to install, while there are occasional heavy loads still the average load is considerably less.

It is not advisable to have a large variety of sizes as this makes it necessary to keep a large number of transformers on hand for emergency purposes. The best method is to decide on as few sizes as possible and use them in open delta where necessary on small loads.

It will be noticed that the transformers are spaced, and the mains laid out, so that a large increase in load can be taken care of by simply installing transformers at the intermediate manholes without the necessity of changing any of the mains; also at some points the mains can be extended to the next manhole to complete the network.

Following is a summary of the loads to be supplied at the various feeding points. Transformers No. 1 to No. 5 are supplied by feeder No. 1 and transformers No. 6 to No. 9 are supplied by feeder No. 2.

| Location | Connected load | 75 % of load | Number of transformers | Size of trans. | Total capacity |
|---|---|---|---|---|---|
| No. 1 | 51 kw. | 38 kw. | 2 | 25 kw. | 50 kw. |
| " 2 | 61 " | 46 " | 2 | 25 " | 50 " |
| " 3 | 118 " | 88 " | 3 | 25 " | 75 " |
| " 4 | 175 " | 131 " | 3 | 50 " | 150 " |
| " 5 | 105 " | 79 " | 3 | 25 " | 75 " |
| " 6 | 212 " | 159 " | 3 | 50 " | 150 " |
| " 7 | 84 " | 63 " | 3 | 25 " | 75 " |
| " 8 | 76 " | 57 " | 3 | 25 " | 75 " |
| " 9 | 150 " | 112 " | 3 | 50 " | 150 " |
| | | Total transformer capacity.......... | | | 850 kw. |

The total power load was 1030 kw. which is equivalent to 1380 h. p., and as 60 per cent of the connected load was taken as the maximum demand, or 828 h. p. the transformer capacity is a little over one kw. per h. p. which is the usual practise in power installations.

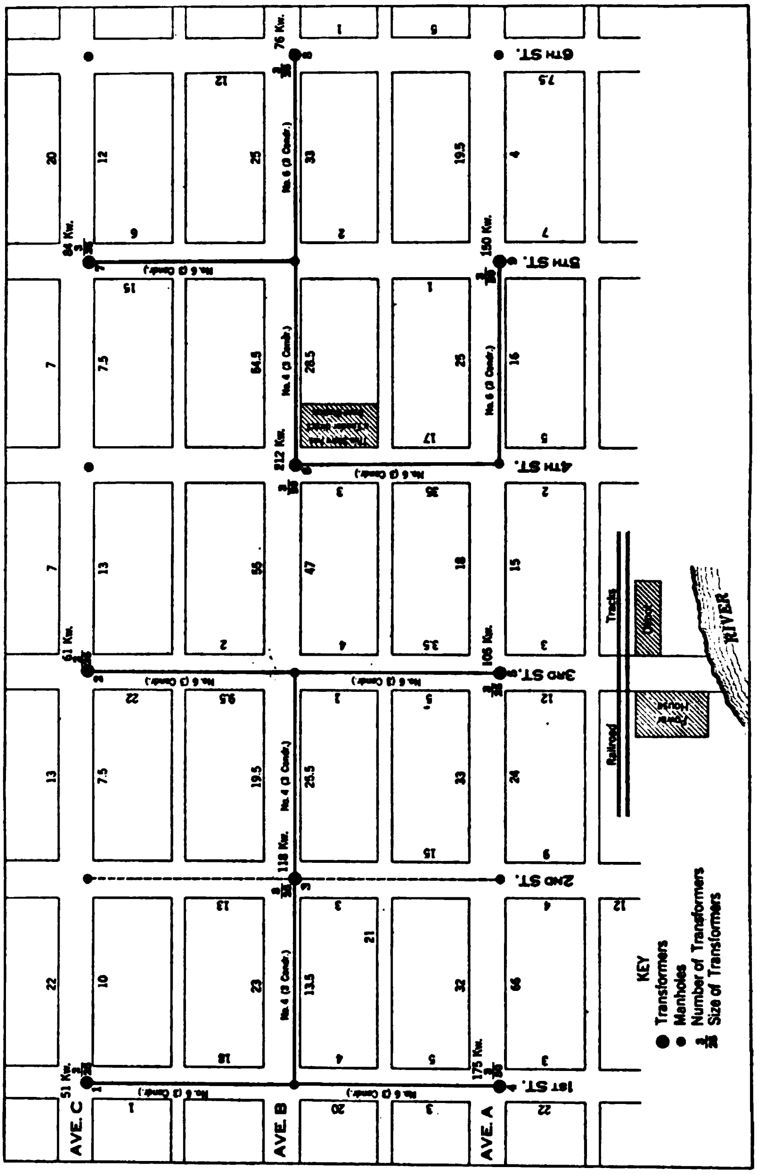


Fig. 6—Secondary Power Feeders—2200-Volt—Three-Phase

At feeding point No. 8, it would no doubt be advisable to start with two 25-kw. transformers in open delta as the load is only 57 kw. but for the purpose of this estimate it was considered best to be on the safe side.

It must be remembered that in designing a system and making an estimate of the cost, considerable allowance must be made for future conditions, as considerable time will undoubtedly elapse between the time that the designs are prepared and the installation of the system. The engineer also must not underestimate the cost.

### 2200-Volt Three-Phase Secondary Power Feeders

Fig. 6 shows the 2200-volt three-phase secondary power feeders that run from the centers of distribution to the various feeding points to supply the transformers and the low-tension network as shown in Fig. 5.

The total connected load and the number and size of transformers is marked at each feeding point. In the original estimate for this system it was assumed that 70 per cent of the connected load would be the maximum demand on these feeders; the cable has been calculated on this basis, and there are only two sizes of cable used in this system.

From the manner in which the cables are installed it is evident that the capacity of the system can be greatly increased by simply running a few sections of No. 6 cable and installing additional transformers at intermediate vaults. This would make it necessary to alter the secondary network so as to maintain a balance but this would be simply a matter of changing connections in the junction boxes.

The largest single load at any point is at Avenue A. and First Street, 175 kw. and as 70 per cent of this, or 122.5 kw., is assumed to be the maximum demand and the three-conductor No. 6 cable will supply this.

On Avenue B., from Fifth to Sixth Street, No. 6 cable is specified, and while this is large enough for the present load it might be a good plan to use No. 4 here as this is on the main feeder that would have to be extended in case the district was enlarged later.

This completes the power system and the next step is to consider the lighting system.

### 2200-Volt Three-Phase Lighting Feeders

Fig. 7 shows the 2200-volt three-phase lighting feeders. By reference to Table No. I the lighting load, exclusive of the big store, at Avenue B. and Fourth Street, is given as 2162 kw. This is the connected load in the district.

The demand of a group of residence consumers will vary from 15 per cent to 30 per cent of the connected load, and the average will probably be between 20 and 25 per cent. In commercial lighting the demand is much higher, as sign and window lights, as well as most of the store lights, are used at the same time. To offset this demand it is seldom that the lights in offices over stores are used when the store demand is at a maximum. The demand for commercial lighting will vary from 40 to 70 per cent of the connected load, depending on the nature of the district and the class of service, some nights the demand being much greater than other nights. The average demand for commercial lighting will be from 50 to 60 per cent of the connected load.

The district that is under consideration is the business district of a small city where practically all of the consumers are in the commercial class, it will therefore be assumed that the demand factor will be 60 per cent of the connected load.

The total connected load is 2162 kw., and 60 per cent is 1297 kw. (say 1300 kw.) and the first thing to decide is the proper size of feeders to use for this load. No. 1/0 will carry 481 kw. at 2200 volts, and it would require three such cables to carry the load and leave about 140 kw. spare capacity. This is about 11 per cent but as it is doubtful if the load could, in practise, be evenly divided between the three feeders this would be a small margin; also if possible it is a good plan to use the same size cables as was used for the power feeders so as to reduce the number of different sizes that must be kept on hand for emergency, and also permit using uniform equipment on the two systems.

The total amount of cable required for this system is about 3800 ft. and it would cost approximately as follows:

| | | | |
|---|---|---|---|
| No. 2/0 | 3800 ft. at | $446.00 per M. | $1694.80 |
| No. 1/0 | 3800 ft. at | 386.00 per M. | 1466.80 |
| | | | $ 228.00 |

The saving in cost, by using No. 1/0 would not be enough to warrant carrying the additional size in stock, and considering that the duct space and cost of installation is the same for both sizes, it is advisable to select No. 2/0 for all primary feeders for both systems.

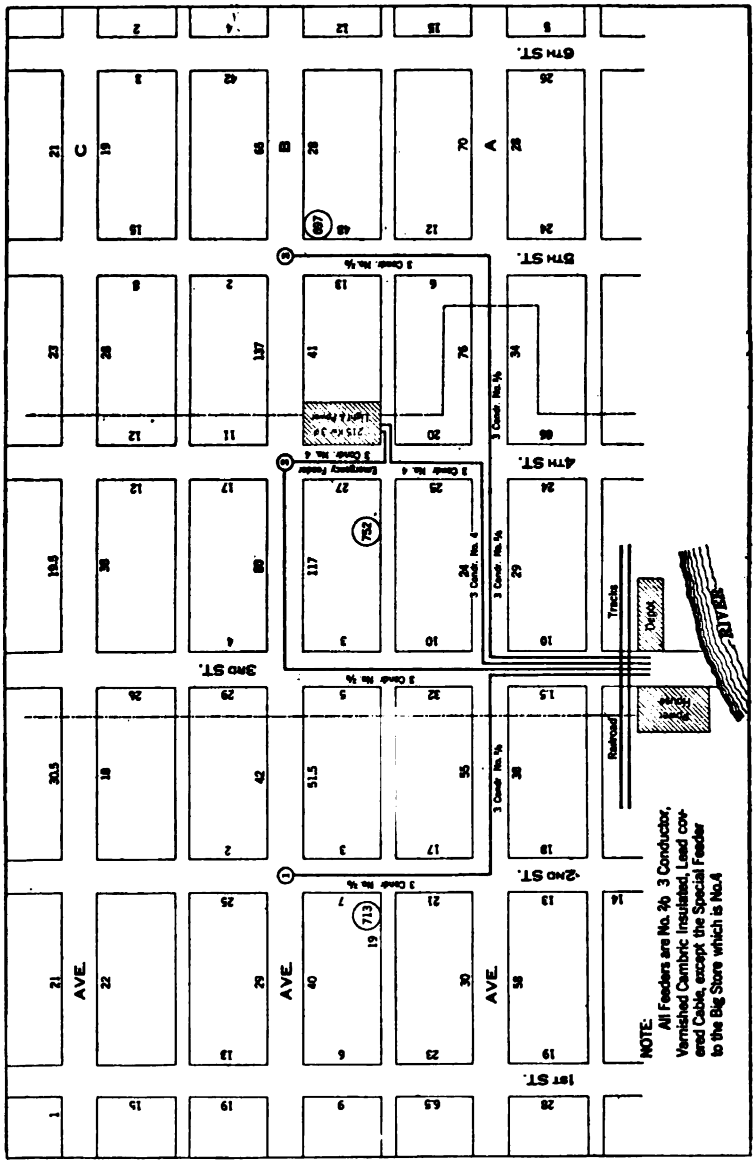


FIG. 7—2200-VOLT THREE-PHASE LIGHTING FEEDERS

There is another point in favor of using No. 2/0 cable for this system, and that is, that the emergency feeder was of this size and by extending it from Fourth to Fifth Street on Avenue B.

(as shown by the dotted line in Fig. 4) it can be used as an emergency feeder for both systems.

By using three No. 2/0 feeders for this system the total feeder capacity will be 3 × 558 kw. or 1674 kw. This is 77 per cent of the connected load, which is rather high, but considering all of the conditions it is the most suitable arrangement.

The total load is 2162 kw., which will give 720 kw. for each feeder, and locating the centers of distribution in the same manner as was done for the power feeders the following result is obtained:

Feeder No. 1, 713 kw.
Feeder No. 2, 752 kw.
Feeder No. 3, 697 kw.

The three centers of distribution are marked approximately where they are calculated by assuming the loads to be concentrated at the points where they are marked. As none of the points are over 200 ft. from street corners, where the power transformers will be located, the same location will answer for this system and one vault will do for both systems. The dotted line divides the district into three sections showing the load to be supplied by each feeder.

The next point to consider is the feeder for the big store, which has to be supplied separately. The total load in the store is 215 kw. 100 kw. power, and 115 kw. lighting. There are three 75-kw. transformers in the store, and as No. 4 is the smallest size of cable used for feeders it is advisable to use that size for this feeder as it will safely carry 260 kw. There should be an emergency feeder of the same size run from the store to the manhole at Fourth Street and Avenue B. for use in case of trouble on the regular feeder.

This completes all of the feeders for the entire distribution system and the next step is to design the 110-220 volt secondary lighting mains.

## 220-VOLT SECONDARY LIGHTING MAINS

The secondary mains for lighting are to be operated on a single-phase, three-wire, 110-220-volt system and the load should be divided as evenly as possible between the three phases. There are many different arrangements of dividing the load and it is advisable to try several methods before deciding which is the most suitable.

In designing this system it is desirable to arrange the circuits so that as few transformers as possible will be required, in order to save space in the vaults, save transformer investment, and to get the benefit of operating as large units as possible.

By arranging the mains so that only one phase is distributed from each vault the number of fuses, switches, junction boxes and other electrical equipment is reduced to a minimum. This not only reduces the cost but saves space which is frequently of vital importance in underground vaults.

The load in this district is fairly high and the blocks are about 350 ft. long, and as practically every building has to be supplied it is evident that by locating the transformers at street intersections they can supply the load in four directions.

Fig. 8 shows the 110-220-volt lighting mains and from which feeder they are supplied. Where mains extend from manhole to manhole they should terminate in junction boxes at each end, fusing the ends from which they are supplied and leaving the fuses out at the other end, to prevent crossing the phases. In case of trouble it is then possible to supply any main from either end by simply changing the direction of supply.

Referring to Table III in the summary of the lighting load it will be seen that the load is fairly well balanced on the feeders and that the load on each phase is nearly equal. This division of the load is, of course, based on the connected load and in the absence of actual data on the operating conditions is the only basis from which to work. After the system is in operation, or rather as the loads are being connected it is possible to alter these first plans so that a better balance can be obtained.

When installing an underground system it is advisable to make rules governing the size of load that is to be supplied by a two wire service. About 1300 watts is as large a load as should be supplied by a 110-volt two-wire service in a business district. All loads in excess of this should have three-wire services. It is advisable to install three service cables to all consumers as this will permit making changes easily when it is necessary to do so to balance the load.

The secondary mains for this system will be two single-conductor cables for the outers and a bare neutral, and the service cables will be three single-conductor cables.

The size of mains was calculated in the usual manner, and in practically every place where 200,000-cir. mil mains are shown on the plan, it will be found that No. 2/0 cable would carry the present load, but in a district such as this one it is not advisable to install any main of less than 200,000 cir. mils.

Where the mains on both sides of a street are supplied from the same phase and are tied together at the next manhole it is

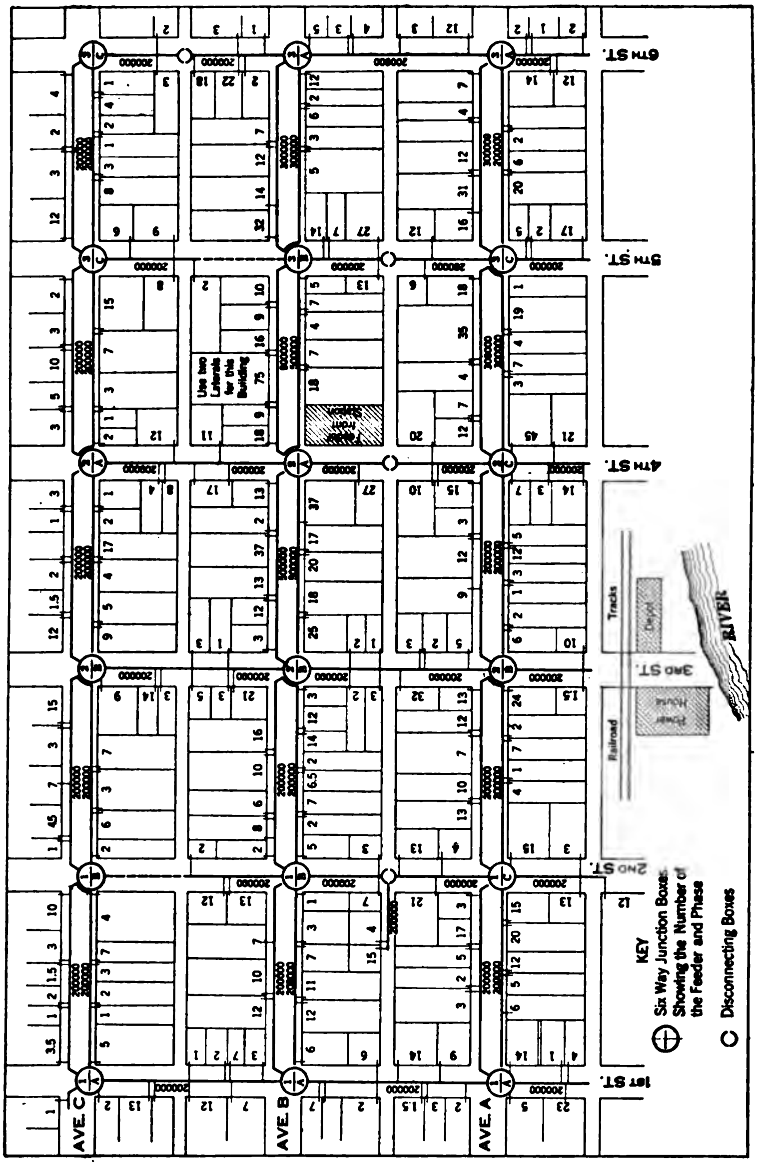


FIG. 8—220-VOLT SECONDARY MAINS

advisable to use the same size cable for both, regardless of the load on each, so that the main can be supplied from either end in case of necessity.

TABLE III—SUMMARY OF LIGHTING LOAD

| Feeder No. 1 Block | Phase A | Phase B | Phase C | |
|---|---|---|---|---|
| No. 1 | 1. | | | |
| 2 | 21. | | | |
| 3 | | 30. | | |
| 7 | 34. | | | |
| 8 | 64. | 25. | | |
| 9 | | 62. | | |
| 13 | 15.5 | | | |
| 14 | 59. | 66. | 21. | |
| 15 | | 54.5 | 72. | |
| 19 | 28. | | | |
| 20 | 19. | | 83. | |
| 21 | | | 56. | |
| | 241.5 | 238. | 232. | |
| Feeder No. 2 | | | | |
| No. 4 | | 19.5 | | |
| 9 | | 55. | | |
| 10 | 67. | 84. | | |
| 11 | 23. | | | |
| 15 | | 37. | | |
| 16 | 144. | 37. | 25. | |
| 17 | | | 96. | |
| 21 | | 1.5 | | |
| 22 | | 10. | 53. | |
| 23 | | | 100. | |
| | 234. | 244. | 274. | |
| Feeder No. 3 | | | | |
| No. 5 | | | 23. | |
| 6 | | | 21. | |
| 11 | | 137. | 38. | |
| 12 | 107. | | 37. | |
| 17 | | 54. | 6. | |
| 18 | 28. | 48. | 82. | |
| 23 | | | | |
| 24 | 54. | | 24. | |
| 6th St. | 36. | | 2. | |
| | 225. | 239. | 233. | |

SUMMARY

| Feeder | Phase A. | Phase B. | Phase C | Total |
|---|---|---|---|---|
| No. 1 | 241.5 | 238. | 232. | 711.5 |
| No. 2 | 234. | 244. | 274. | 752. |
| No. 3 | 225. | 239. | 233. | 697. |
| | 700.5 | 721.0 | 739.0 | 2160.0 |

The phases have been referred to as *A*, *B* and *C* and while this is the common practise a better method is to distinguish the phases by colors, and paint the cables of each phase a different color, this method of marking the phases will prevent mistakes

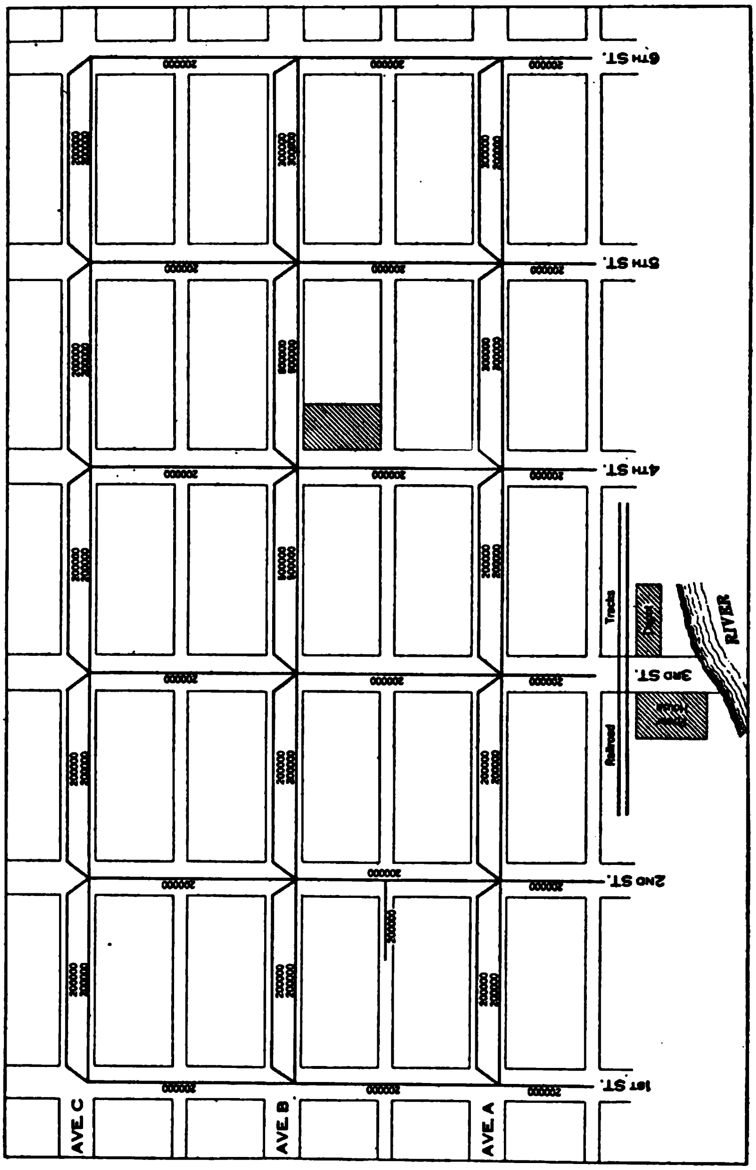


FIG. 9—BARE NEUTRAL MAINS

in making connections as only cables of like color should be connected together.

The bare-neutral network is shown in Fig. 9 and is the same size as the outers wires shown in Fig. 8. Bare cable has been selected for the neutral in this system, but this is a matter that should be decided by the engineer for each particular system.

## 2200-Volt Secondary Lighting Feeders

The cables for this system can be either two- or three-conductor. While the lighting load is single-phase, still it is necessary to divide it between the three phases, therefore, the three-conductor cable has been selected so that all three phases are available at each transformer vault.

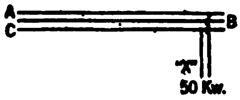


Fig. 10

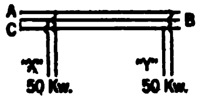


Fig. 11

The following rules for calculating the temperature rise in three-phase feeders supplying one or more single-phase loads were devised by Mr. R. W. Atkinson and are given here with his permission.

*Rule* 1. A single-phase load applied between two of the phases of a three-phase circuit (Fig. 10) is equivalent, insofar as the maximum temperature rise is concerned, to a balanced three-phase load 50 per cent greater.

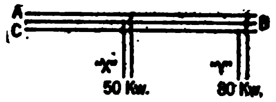


Fig. 12

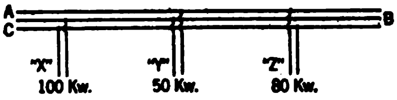


Fig. 13

*Rule* 2. If two single-phase loads, of equal magnitude, are applied, one between phases $A$ and $B$ and one between $A$ and $C$, (Fig. 11) the maximum temperature rise is the same as for a balanced three-phase load 25 per cent greater.

*Rule* 3. If various single-phase loads are applied between the various phases of a three-phase circuit (Fig. 12), the resulting load can be considered to be a combination of the two conditions just mentioned and a balanced three-phase load.

Referring to Fig. 13, and applying these rules the following result is obtained:

| Phase | *A-B* | *B-C.* | *A-C.* | |
|---|---|---|---|---|
| Load.... | 50 kw. | 100 kw. | 80 kw. | = 230 kw. |
| Subtract. | 50 " | 50 " | 50 " | = 150 " three-phase load at *X* |
| | | 50 " | 30 " | |
| Subtract. | | 30 " | 30 " | = 75 " (60 × 1.25) |
| | | 20 " | | = 30 " (20 × 1.50) |
| | | | | 255 kw. |

Fig. 14 shows the secondary lighting feeders using three-conductor cable. The load, size of transformers and the phase from which they are supplied are marked at each feeding point.

Fig. 15 shows two-conductor single-phase feeders supplying this same system and the sizes shown on the plan are based on 70 per cent demand and as this system is used exclusively for lighting the power factor has been taken at 100 per cent. It will be seen that this system requires approximately 2000 ft. more conduit than the system shown in Fig. 14, and the approximate difference in cost of the two cable systems is as follows:

| | | | |
|---|---|---|---|
| No. 6 three-conductor cable | 4800 ft. | at $160.00... | $ 768.00 |
| No. 2 " " " | 1200 " | " 340.00... | 408.00 |
| | | | $1176.00 |
| No. 6 two-conductor cable | 7200 ft. | at $100.00... | $ 720.00 |
| No. 2 " " " | 800 " | " 204.00... | 163.00 |
| | | | $ 883.00 |

This is a difference of $293.00 in favor of the two-conductor cables on the cable alone, but if the additional conduit is considered it will be seen that the three-conductor cable is not only the cheapest but the better system to install.

There is another point to be considered in selecting the three-conductor cable; the sections will be in comparatively long lengths and if replaced by larger cable at any time the old cable can be used in other places, even on the power system, while if two-conductor cable were used it would be available only for the

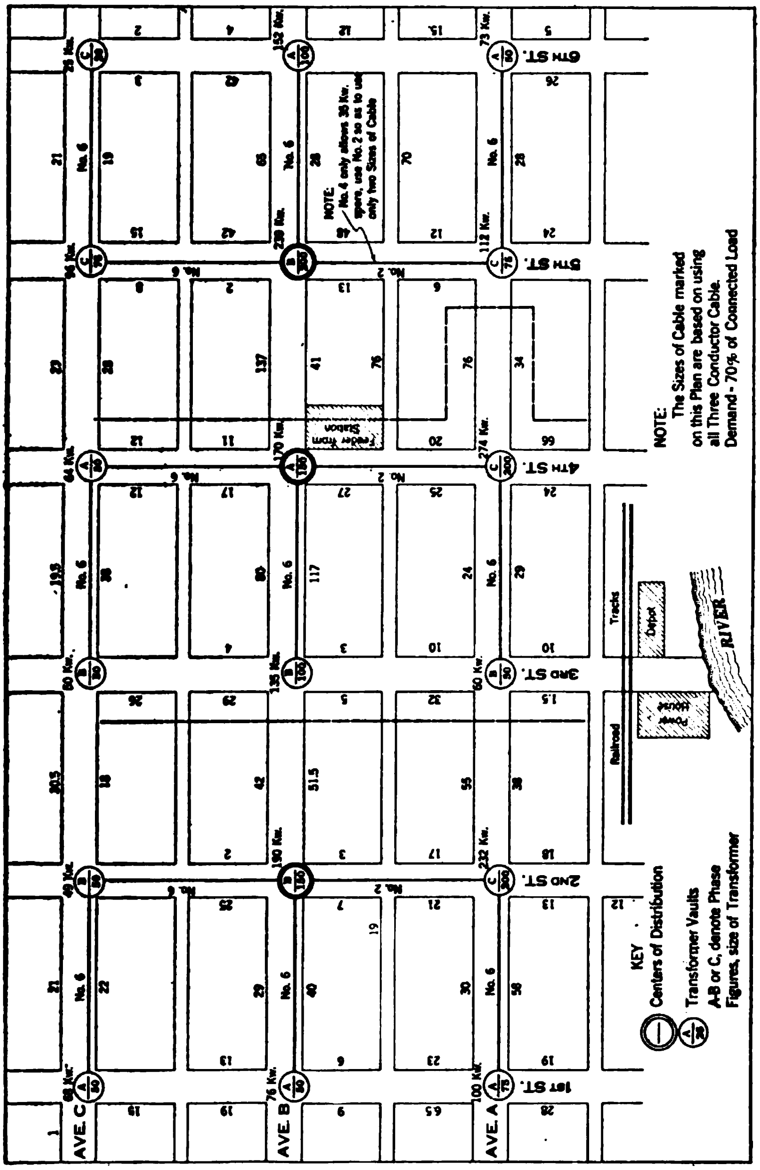


FIG. 14—SECONDARY LIGHTING FEEDERS

lighting system and would be an additional type of cable to keep on hand.

As the three-conductor cable will require less conduit, and be available for use on either system it is advisable to select it for

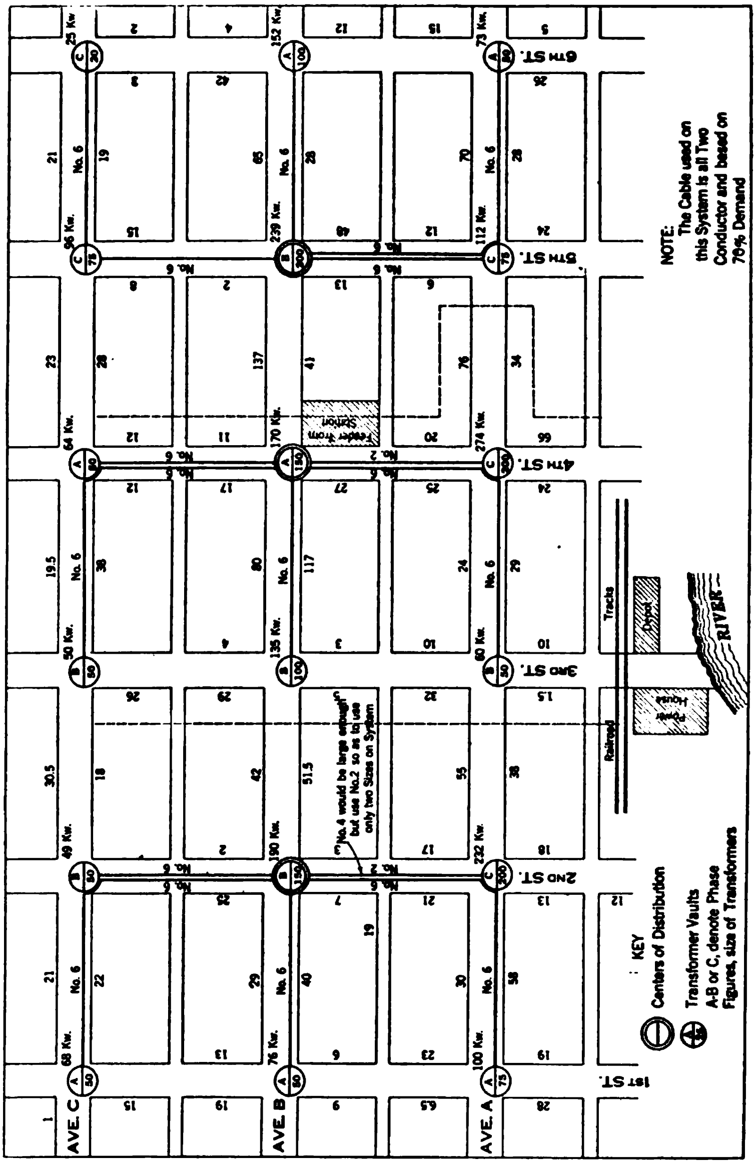


FIG. 15—SECONDARY LIGHTING FEEDERS

these feeders, as the system will be more flexible than if two-conductor cable were used.

The size of transformers is shown at each feeding point and is based on 70 per cent demand factor as follows:

SUMMARY.

| Feeder | Connected load | 70 per cent demand | Transformer capacity |
|---|---|---|---|
| No. 1 | 715 kw. | 500 kw. | 575 kw. |
| " 2 | 753 " | 527 | 600 " |
| " 3 | 697 " | 488 " | 520 " |
| | 2165 " | 1515 " | 1695 " |

The total connected load is 2165 kw. and the transformer capacity is 1695 kw., or 78 per cent. This is not an excessive amount for a commercial district and it is doubtful if a much better arrangement could be made under the conditions assumed in these plans.

It must be remembered that plans, such as these, are made a long time before the work is actually done and are simply for estimating the cost of the system and are subject to final revision when the system is being installed. Owing to the fact that usually the grouping of the consumers on the new underground system will be entirely different from what they were on the overhead system, the demand factor can only be estimated, and as the new system is "cut over" gradually there is ample opportunity to select the size of transformers, balance the loads and get satisfactory results.

## CONDUIT SYSTEM

The first thing necessary in designing a conduit system is to determine the number of ducts to be installed on each street and this can best be done by making a tabulation as shown in Table IV this will show definitely the actual number of ducts required and the proper number of spare ducts can then be added.

The system under consideration is for distribution purposes only and will consist of trunk ducts on the bottom, running from manhole to manhole, and distribution ducts on the top which pass through all of the intermediate handholes or service boxes.

The feeders will all be placed in the lower ducts and the secondary mains for the lighting and power system, and the street light cable will be in the distribution ducts.

Provision must be made for the following cables in the distribution ducts.

1 Three-conductor power main.

2 Single-conductor lighting mains (Outers).

1 Bare neutral for lighting system.

1 Street light cable (Two in some places, but they will be in one duct).

If separate ducts were used for each cable it would require 5 ducts on the top tier, this is a bad arrangement because it would increase the cost of the system, as the width of the trench would be about 29 in. using 3.5 in. conduit. The best arrangement would be to place one of the lighting mains and the neutral in one duct and install the conduit four ducts wide, which would require a trench about 24 in. wide.

By using four ducts for distribution it is possible to lay all of the conduit in multiples of four which is a very good arrangement. On streets having conduit on both sides it is only necessary to lay four ducts on one side, for distribution, keeping the main conduit on the other side of the street.

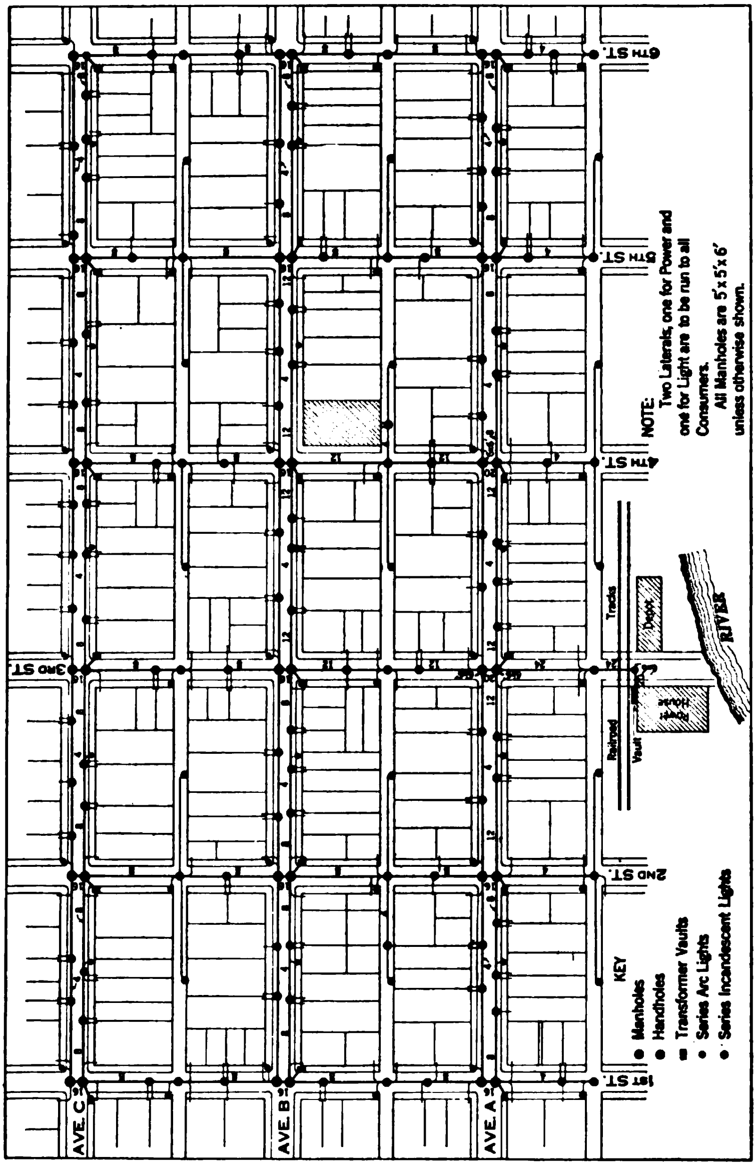
In systems where only three ducts are required for distribution it is sometimes advisable to use multiples of three in laying the conduit and thus make considerable saving in first cost. On all distribution systems the number of ducts wide should be determined by the number of ducts required for the distribution cables.

In laying four ducts they should, if the conditions will permit, be laid two wide and two high, in fact on all conduit systems, except for high-tension cables, it is advisable to have the conduit form as near a square section as possible as this makes a stronger structure. As far as the subsurface conditions will permit it is advisable to use a multiple of the distribution ducts as the number to install in one trench.

The conduit plan must show the point at which all laterals enter the buildings and the location of the street lights so that the service boxes can be located to the best advantage, the exact location, of course, depending on the subsurface conditions.

Fig. 16 shows the conduit arrangement for this system of distribution and is based on the tabulation shown in Table No. IV. To avoid confusion there is only one lateral shown entering each building but there should be two wherever power is to be supplied.

The size and location of the service boxes should be determined by the number of mains passing through them, the number of services that it is permissible to splice on a main at one service box, and the available space for locating them. Long laterals are expensive and require larger service cables, and it should be remembered that to make a neat splice not more than two three-conductor services can be taken from a main in one splice.



NOTE:
Two Laterals, one for Power and one for Light are to be run to all Consumers.
All Manholes are 5'x5'x6' unless otherwise shown.

FIG. 16—CONDUIT SYSTEM

Where the mains are single conductor it is possible to take out four service cables from one splice.

On a system such as is considered here the service boxes should

TABLE IV.—SUMMARY OF CONDUIT

| Side of Street | Street | From | To | Primary Power Feeder | Secondary Power Feeder | Primary Light Feeder | Secondary Light Feeder | Emergency Feeder | Private Feeder | Distribution Ducts | Spare Ducts | Total Duct |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | First St. | Alley No.1 | Ave. A. | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| | " " | Ave. A. | " B. | .... | 1 | .... | .... | .... | .... | 4 | 3 | 8 |
| | " " | " B. | " C. | .... | 1 | .... | .... | .... | .... | 4 | 3 | 8 |
| | Second St. | Alley No.1 | " A. | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| | " " | Ave. A. | " B. | 1 | .... | 1 | 1 | .... | .... | 4 | 1 | 8 |
| | " " | " B. | " C. | .... | .... | .... | 1 | .... | .... | 4 | 3 | 8 |
| | Third St. | Station | " A. | 2 | .... | 3 | .... | 1 | 1 | 4 | 13 | 24 |
| | " " | Ave. A. | " B. | .... | 1 | 1 | .... | 1 | .... | 4 | 5 | 12 |
| | " " | " B. | " C. | .... | 1 | .... | .... | .... | .... | 4 | 3 | 8 |
| | Fourth St. | Alley No.1 | " A. | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| | " " | Ave. A. | " B. | 1 | 1 | .... | 1 | .. | 1 | 4 | 4 | 12 |
| | " " | " B. | " C. | .... | 1 | .... | .... | .... | .... | 4 | 3 | 8 |
| | Fifth St. | Alley No.1 | " A. | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| | " " | Ave. A. | " B. | .... | .... | 1 | 1 | .... | .... | 4 | 2 | 8 |
| | " " | " B. | " C. | .... | 1 | .... | 1 | .... | .... | 4 | 2 | 8 |
| | Sixth St. | Alley No.1 | " A. | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| | " " | Ave. A. | " B. | .... | .... | .... | .... | .... | .... | 4 | 4 | 8 |
| | " " | " B. | " C. | .... | .... | .... | .... | .... | .... | 4 | 4 | 8 |
| S | Ave. A. | First St. | Second St. | .... | .... | .... | 1 | .... | .... | 4 | 3 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 2nd St. | 3rd St. | 1 | .... | 1 | .... | .... | .... | 4 | 6 | 12 |
| N | " A | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 3rd " | 4th " | 1 | .... | 1 | 1 | .... | 1 | 4 | 4 | 12 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 4th " | 5th " | .... | 1 | 1 | .... | .... | .... | 4 | 2 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 5th " | 6th " | .... | .... | .... | 1 | .... | .... | 4 | 3 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | Ave. B. | 1st St. | 2nd St. | .... | 1 | .... | 1 | .... | .... | 4 | 2 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 2nd " | 3rd " | .... | 1 | .... | .... | 1 | .... | 4 | 2 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 3rd " | 4th " | .... | .... | 1 | 1 | 1 | .... | 4 | 5 | 12 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 4th " | 5th " | .... | 1 | .... | .... | 1 | .... | 4 | 6 | 12 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .. | 4 |
| S | . " | 5th " | 6th " | .... | 1 | .... | 1 | .... | .... | 4 | 2 | 8 |
| S | Ave. C. | 1st St. | 2nd St. | .... | .... | .... | 1 | .... | .... | 4 | 3 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 2nd " | 3rd St. | .... | .... | .... | .... | .... | .... | 4 | 4 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 3rd " | 4th " | .... | .... | .... | 1 | .... | .... | 4 | 3 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 4th " | 5th " | .... | .... | .... | .... | .... | .... | 4 | 4 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| S | " " | 5th " | 6th " | .... | .... | .... | 1 | .... | .... | 4 | 3 | 8 |
| N | " " | " " | " " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| | Alley No. 2 | 1st " | 2nd " | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| | Alley No. 2 | 4th " | Big Store | .... | .... | .... | .... | .... | .... | 4 | .... | 4 |
| | Station | To Manhole No. 1. | | 2 | .... | 3 | .... | 1 | 1 | .... | 13 | 20 |

be about 3 ft. by 4 ft. and the depth will depend on the grade of the conduit, as the top tier of ducts must enter the service box. For the purpose of estimating, it may be stated that on a system of this kind that the average depth of service boxes will be from 36 to 40 inches.

Where subway junction boxes and other electrical equipment must be placed in manholes, this fact must be taken into consideration in determining the size of the manholes, but in a system where transformer vaults are built the manholes need only be large enough to permit training the cables properly.

TABLE V—SIZE OF CABLE FOR LATERALS. INDIVIDUAL CONSUMERS LOADS

| For Power | | |
|---|---|---|
| **Load** | **Number of Consumers** | **Size of Cable** |
| 0 to 10 kw. | 92 | 3 Conductor No. 6 |
| 11 " 20 " | 29 | " " " 4 |
| 21 " 40 " | 8 | " " " 1/0 |
| | 129 | |
| **For Lighting** | | |
| 0 to 10 kw. | 168 | 3 Single Conductor No. 6 |
| 11 " 15 " | 48 | " " " " 4 |
| 16 " 35 " | 30 | " " " " 1/0 |
| 36 " 65 " | 4 | " " " " 4/0 |
| | 250 | |

Assume that laterals will average 60 ft. long each and that three-conductor cable is used for all power laterals and three single-conductor cables for the lighting consumers, therefore each lighting lateral will require 180 ft. of cable.

Where electrical equipment is to be installed in manholes it is advisable to build them with square corners as this space is frequently very desirable.

If possible, transformer vaults should be located under the sidewalk (Fig. 17) and on the same side of the street as the main conduit. By locating the vaults under the sidewalk there is less liability of damage from being flooded, also they can be more easily entered in the winter when the ground is covered with ice and snow. All covers to manholes, or vaults, under the sidewalk should be filled with cement to match the color of the walk and also prevent pedestrians from slipping on them.

In many of the old systems, where a large number of ducts were laid in one trench, the manholes were entirely too small to permit the cables being properly trained in them. In a small system, such as this one, large manholes are not required, as it is proposed to locate the transformers in vaults and have only the cables and secondary junction boxes in the manholes.

By locating the transformers and the 2300-volt equipment in the vaults and the low-tension junction boxes in the manholes there is less liability of a burnout damaging the secondary network, also the mains are shorter, and less conduit is required between the vaults and manholes.

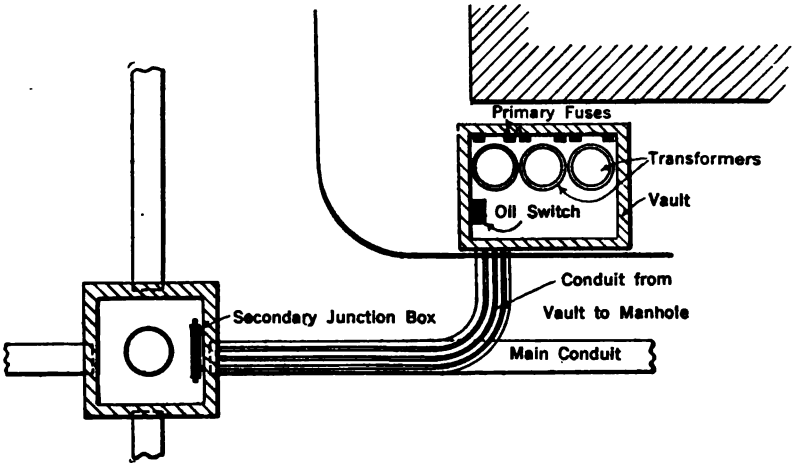


FIG. 17

Fig. 18 shows a typical arrangement of conduit and manholes at a street intersection, each duct between manhole *A* and *B* is numbered to correspond with the ducts leaving manhole *B*.

Owing to the obstructions usually encountered at street intersections, it is generally necessary to build two manholes, but where the grade of the conduit can be maintained across the street it is only necessary to have a service box on one side, deep enough to take the two top tiers of ducts.

If the secondary junction boxes are installed in the vaults it will be necessary to have 18 ducts between the manholes and vaults as follows:

1 Power feeder.
1 Lighting feeder.
1 Emergency feeder.
6 Power mains.
6 Lighting mains.
1 Ornamental street light (may be required).
2 Spare ducts.

While it will not require six ducts each for power and lighting mains in all cases at present, still provision should be made for this number eventually.

In making the estimate for this system the following figures will be taken as being fairly accurate.

All service laterals will consist of two ducts and will average 20 ft. long from the trench to the property line, on streets where there are two lines of conduit, and 30 ft. long on streets having but one line of conduit.

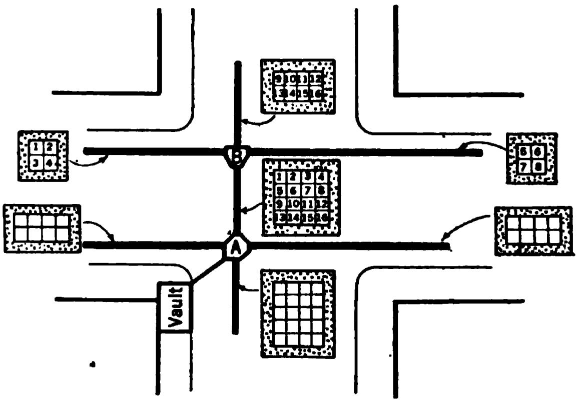


FIG. 18—ARRANGEMENT OF CONDUIT AND MANHOLES AT STREET INTERSECTION

Street light laterals 10 ft. from trench to lamp pole, the lights in the alleys having a single fibre-duct lateral 180 ft. long.

Street crossings and the distance from manholes to vaults are 30 ft. each.

A vault is shown at the station and is frequently required in large systems, but in a system of this size it is very probable that the conduit could enter the basement of the station at the most convenient point for the various systems.

The size of the transformer vaults will depend on the amount of equipment that is to be installed in them. For this estimate it will be assumed that the three vaults at the centers of distribution are 10 by 15 by 8 ft. and all other vaults 10 by 12 by 8 ft. As this district is to be extended later and will probably require larger transformers when the load increases, it is advisable to make the manholes as small as possible and allow sufficient room in the vaults to permit installing the equipment in a neat and workmanlike manner and have sufficient room for operating it safely.

The transformer capacity that can be installed in a vault without providing special ventilating facilities will depend on the operating conditions. For transformer capacities not exceeding 200 kw., under favorable conditions, 3.5 to 4.0 cubic feet of vault space per kw. will permit safe operation; these figures correspond fairly well with the rule for allowing 8 watts transformer losses per square foot of radiating surface in the vault, figuring the sides and ceiling as follows:

Assume four 50-kw. transformers installed in a vault 10 ft. by 10 ft. and 7 ft. high. A 50-kw. subway transformer has 240 watts core loss and 550 watts copper loss, or a total of 790 watts and four such transformers will have 3160 watts loss. A vault 10 by 10 by 7 ft. contains 700 cubic feet and has 380 square feet of radiating surface, counting the walls and ceiling.

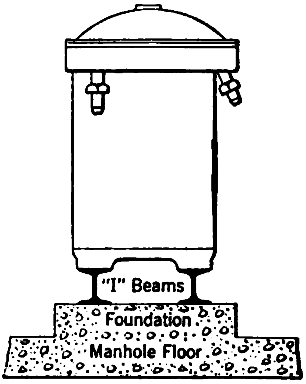


FIG. 19

Allowing 3.5 cubic feet per kw. this would give exactly 700 cubic feet required, which is the size of the assumed vault. Applying the rule of 8 watts loss per ft. of radiating surface the result is 8 × 380 or 3040 watts, which corresponds very closely with the first estimate.

The above losses are based on the assumption that the transformers are fully loaded but in a large distirbution system where the transformers serve both lighting and power loads this is seldom the case as the maximum lighting and power loads rarely occur at the same time, and the overlap of these demands is usually of such short duration that no serious rise in temperature results from this cause.

The above method can be used to calculate the size of vaults for a given transformer capacity not exceeding 200 kw., still it is always necessary to allow sufficient space to properly install and operate the equipment in the vault.

In installing transformers in vaults it is advisable to have them raised above the floor, and if possible mounted on two or three I beams as this permits the air to circulate all around them. (See Fig. 19.) It is a good plan to have a thermometer in each vault and keep a record of the temperature as a guide to the actual conditions. Where this is done the thermometer should be noted immediately on entering the vault before the air has a chance to cool; in taking the temperature of the transformers the thermometer should be placed at the oil level of the transformers.

Having determined the general arrangement of the conduit system the next step is to select the most suitable location in the streets for installing it. In order to plot the location of sewers, pipes and other obstructions it is necessary to prepare a map of each street and alley on a scale of about 20 ft. to the inch.

Records of subsurface conditions are usually far from accurate, but by getting the location of all gates, covers, etc. visible on the surface of the streets it is frequently possible to prepare a very fair plan of the actual conditions. All existing manholes should be entered and measured and they should be laid out accurately on the plan, allowing for the thickness of the walls, as getting the new conduit by existing manholes is frequently one of the most difficult points in conduit construction.

It is a good plan to make two sets of these large scale street maps, one showing the obstructions and the other one to show the new system exactly as it is installed for future reference. These latter drawings should be made on cloth backed drawing paper and can be kept in rolls, and all additions and changes should be entered on them when made, giving the date and all details of the work.

Manholes are built of brick or concrete, except in special cases there is little difference in the cost. In streets where the space is limited and many obstructions are encountered it is cheaper to build brick manholes, but in residential sections, or for high-tension systems, it is frequently cheaper to build concrete manholes when a standard form can be used.

In locating and building manholes every effort should be made to avoid any pipes passing through them, it is frequently possible to change pipes or cut them around the manhole and this should be done if permitted.

The service boxes should be built of brick as owing to their difference in depth a standard form can not be used to advantage.

Vaults, where built under the sidewalk, should be of concrete and have suitable ventilating pipes extending up the side of buildings or adjacent poles. There should be one pipe located low at one end of the vault and one located high at the other end and care should be taken not to locate transformers directly under the ventilating pipes.

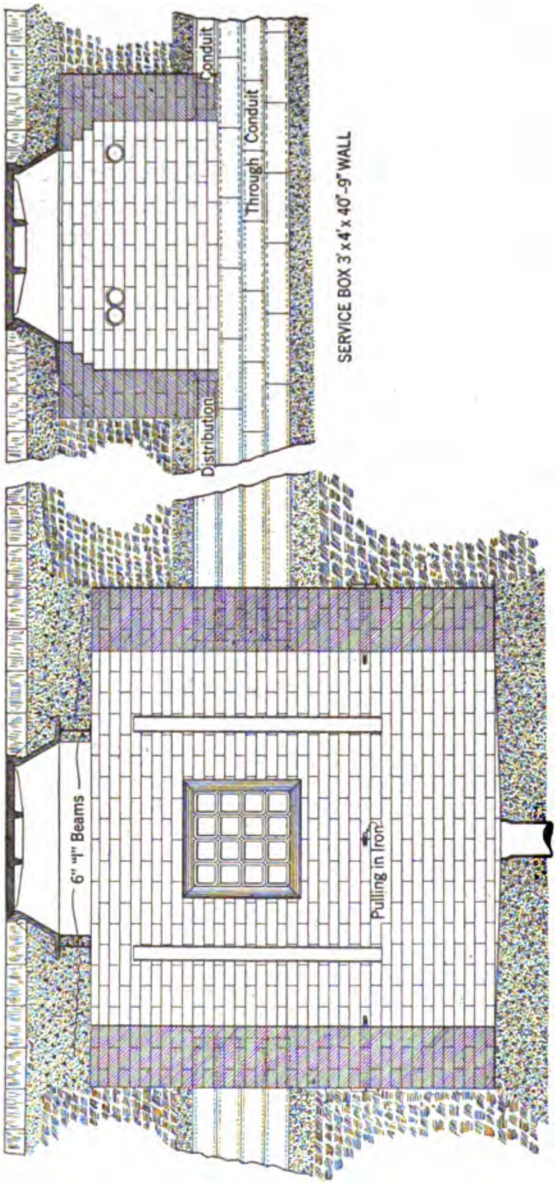


Fig. 20—Manhole and Handhole

Fig. 20 shows the usual arrangement of manholes and vaults.

Owing to the increase in weight of automobile trucks it is very important that all casting be made much heavier than was done formerly. The castings should be inspected at the foundry, or before being placed, to see that the covers set firm and solid in the seat of the casting. No loose or uneven covers should be permitted for no amount of chipping or grinding will make them satisfactory.

Single cover castings are advisable and they should have ventilating holes in the covers for the manholes but the service box castings should be unventilated, Fig. 21 and 22 show castings for manhole and service box respectively.

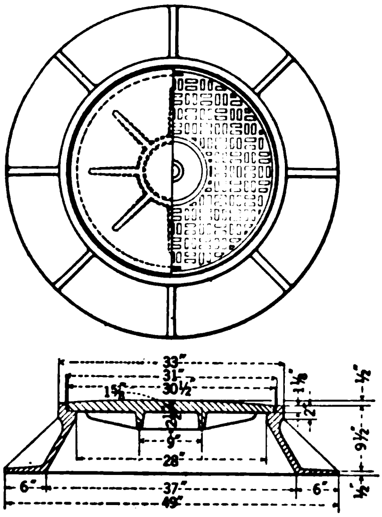


FIG. 21—MANHOLE CASTING

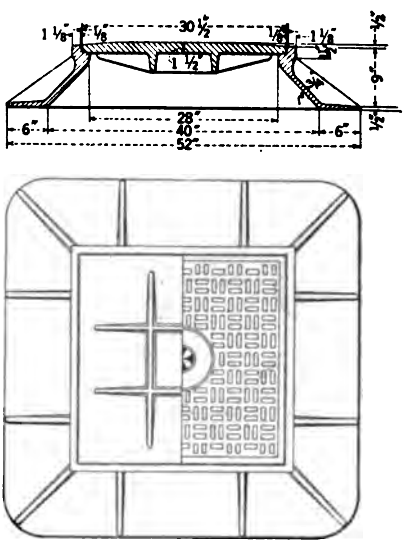


FIG. 22—HANDHOLE CASTING

If possible, it is advisable to locate the conduit about five feet from the curb so as to keep the service box covers out of the gutter and if possible locate the conduit outside of the gas and water main as it will not then be necessary to sandwich the conduit around the service pipes for those systems.

All vaults and manholes having electrical equipment in them should be connected with the sewer and the floors should be sloped in all directions to the sewer outlet.

Cable racks should not be placed until the cable is being installed, except on high-tension systems where the arrangement of the cables will be uniform in all manholes.

When building vaults, or manholes where transformers are to be installed, it is advisable to install a suitable ground connection at the time of construction, a ground cone, or one-in. galvanized pipe should be used if it is not possible to connect to the water pipe system, but whatever kind of ground is decided on care must be taken to make it permanent and reliable, for large vaults it is advisable to install two ground connections.

The conduit as laid out in Fig. 16 is the usual method adopted in systems of this kind. There is, however, another method that can be adopted in some cases which will permit a very material saving in conduit and should be given serious consideration where the geographical arrangement of the city permits.

Take this same district, and referring to Fig. 16, install twelve ducts on each side of 3rd Street from the station to Avenue A., and make the main conduit line east and west on Avenue A.,

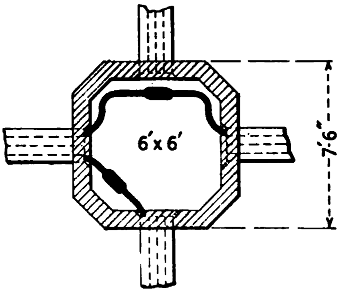


FIG. 23—ORDINARY ARRANGEMENT OF MANHOLE

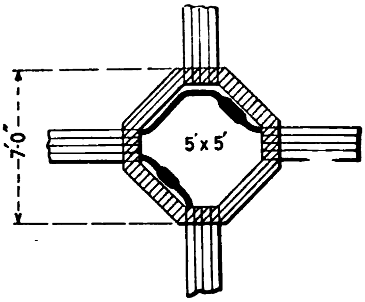


FIG. 24—PROPOSED ARRANGEMENT OF MANHOLE

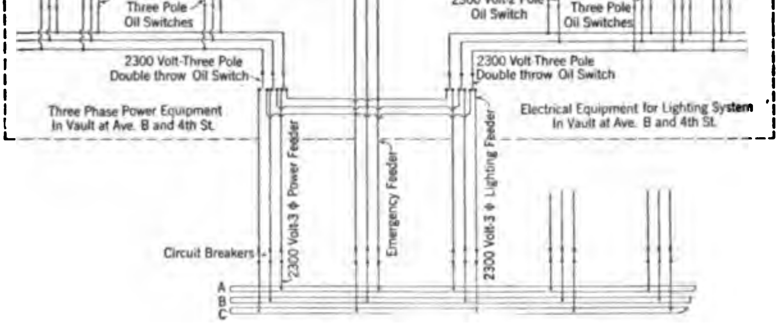
and run all feeders over this route branching them off at the north and south streets, as 1st, 2nd, etc., this would permit reaching every distribution point.

On such streets as Avenue B. and C. it would only be necessary to install four ducts on each side of the street for distribution this would make a saving of approximately 17,000 ft. of conduit on those streets but it would be advisable to add more conduit on Avenue A.

Possibly a modification of this plan would be better, that is, to make the main conduit lines on Avenue A. and on 3rd Street, the saving would probably be about 10,000 ft. If later a substation were to be installed to feed other sections of the city this plan would be very desirable as it is only necessary to have ducts on the avenues sufficient to provide space for the distribution cables and street light system.

## SUBWAY JUNCTION BOXES

When the load in a district is supplied by a low-tension network it is necessary to supply facilities for fusing and disconnecting the mains. When the low-tension network is supplied by low-tension alternating- or direct-current feeders this is a simple matter but where a low-tension network is supplied by subway transformers the conditions are more complicated and additional equipment is necessary to insure reliable service, as both the primary and secondary systems must have protective, sectionalizing and switching facilities. Where the power and lighting load are supplied separately this practically doubles the equipment and space required, and greatly increases the cost of the system.



**2300 Volt-3 phase Station Bus**

FIG. 25—DIAGRAM OF EQUIPMENT IN VAULT

Fig. 25 shows the method of protecting, sectionalizing and switching used on the lighting and power systems; it is a diagram of what would be necessary in one of the vaults, the one chosen for illustration being the vault at Avenue B. and Fourth Street.

It will be seen in case of trouble on one of the regular lighting or power feeders that the load can be switched over to the emergency feeder. Should one of the secondary feeders fail it would cut off the supply to all of the transformers connected to it, but by disconnecting these transformers from the network and connecting this portion of the network to other phases (by putting the fuses in at the several junction boxes) the system could be operated temporarily even though greatly unbalanced.

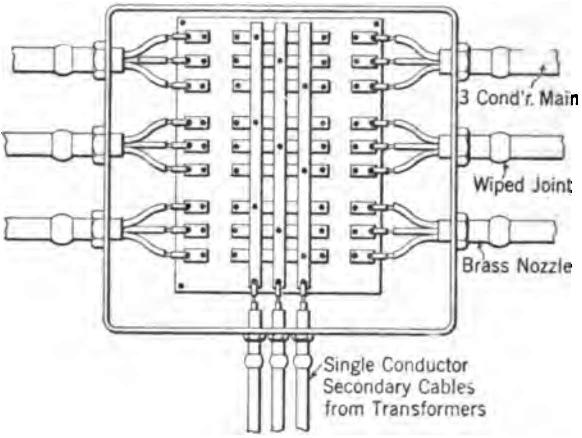


FIG. 26—THREE-PHASE JUNCTION BOX—FOR POWER

If greater security were desired it would be necessary to install "tie feeders" as shown by the dotted line on Second Street, in Fig. 6.

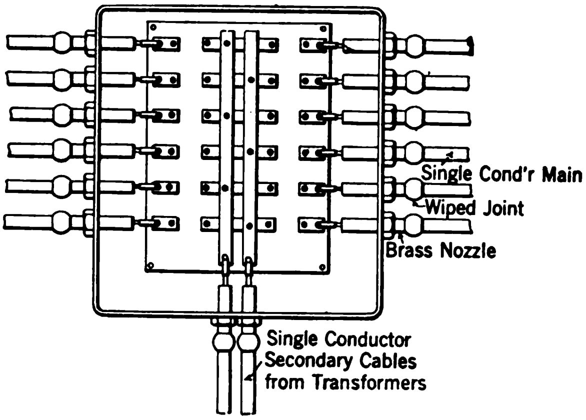


FIG. 27—TWO-WIRE—220-VOLT JUNCTION BOX—FOR LIGHTING

The mains are protected by fuses at each end and in case of trouble on them, or the house services, would be cut off automatically; where there are mains on both sides of the street the number of consumers affected in case of trouble is considerably reduced. The fuses on the mains should be heavy enough to carry at least 50 per cent more current than the normal carrying capacity of the mains, as it is not desirable to have the fuses blow out except in case of serious trouble.

It will be noticed that in Fig. 25 only three fuses are shown on the primary side of the power transformers. In actual practise, where fuses are used on the primary side, it is better to install two fuses for each transformer. There is considerable difference of opinion among engineers as to the advisability of installing any fuses on the primary side of subway transformers as they generally cause more trouble than their protection is worth, but as it is advisable to have some point where the transformers can be easily disconnected from the feeders it is a good plan to install the fuse boxes and use fuses having 100 per cent greater carrying capacity than is required to protect the transformer and then trust to the station circuit breaker to disconnect the feeder in case of serious trouble.

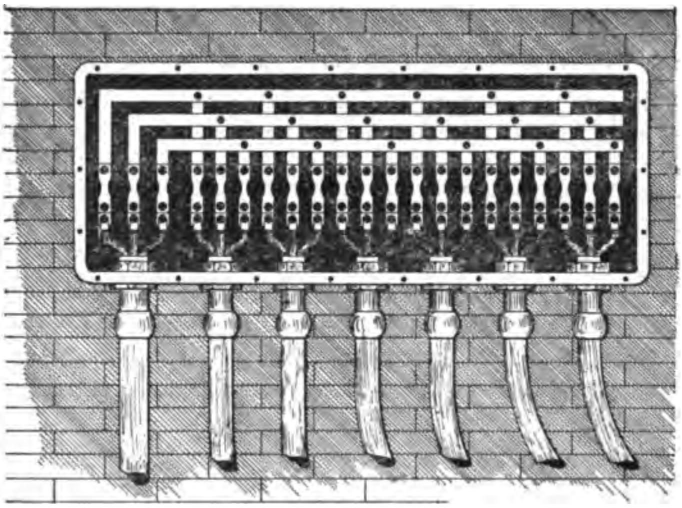


FIG. 28

Assuming that the system is to be installed as shown in Fig. 25, the subway junction boxes shown would provide facilities for disconnecting and fusing the various mains. The junction boxes for the lighting system would be fitted for single-conductor cable and those for the power system for three-conductor cable.

Owing to the fact that there are so many different systems in use there is no standard type of junction box that can be specified, and it is generally necessary to design boxes for each system to meet the conditions. In underground work there is probably no part of the system that is more liable to cause trouble than are the junction boxes; this is not always due to defects in the box but to careless work in installing it or not properly closing the cover.

Fig. 28 shows a type of junction box that is well suited for subway installation, it is placed near the top of the manhole and the cables all enter it from the bottom, it is practically certain that a box of this type will never get flooded as the water would not rise above the conduit unless the entire system got flooded which is very unlikely.

When the fuses on a transformer blow, its load is transferred to the adjacent transformers and they become overloaded and their fuses blow. This is a serious matter as this condition usually occurs at the most inconvenient time, and where additional security is warranted it is advisable to install the "alternating-current network protector."

Having completed the designs for all of the various systems it is only necessary to scale off the amount of material from the plans and tabulate the equipment, as specified in the text or from the plans. Space will not permit giving these various estimates completely but the following samples will show the method to be adopted.

### CONDUIT SYSTEM

| | |
|---|---|
| Conduit in main trench | 139,500 ft. |
| Trench feet | 17,145 " |
| Conduit for laterals in main trench | 4,110 " |
| Conduit for laterals run separate | 10,650 " |
| All conduit used for laterals to be 3-in. fibre. | |
| Manholes | 37 " |
| Handholes | 108 " |
| Vaults 10 by 12 by 7 ft. | 15 " |
| Vaults 10 by 15 by 7 ft. | 3 " |
| Sewer Connections | 18 " |

### THREE-PHASE POWER SYSTEM

| | | | | |
|---|---|---|---|---|
| 2300-volt | three-condr. Cable No. | 2/0 | 3,700 ft. | primary feeders |
| 2300- " | " " " " | 4 | 1,100 " | " " |
| 2300- " | " " " " | 6 | 2,600 " | " " |
| 220- " | " " " " | 1/0 | 4,600 " | secondary mains |
| 220- " | " " " " | 4 | 12,700 " | " " |

| | | | |
|---|---|---|---|
| | 220- " " " " " 6 | 5,520 " | laterals |
| | 220- " " " " " 4 | 1,740 " | " |
| | 220- " " " " " 1/0 | 480 " | " |
| | 25-kw. transformers | 16 | |
| | 50- " " | 9 | |
| | 2300-volt subway fuse boxes | 50 | |
| | 3-pole single-throw oil switches | 6 | |
| | 3-pole double-throw oil switches | 3 | |
| | 6-way junction boxes | 18 | |
| 0 | 30-ampere consumers fuse boxes | 70 | |
| 31 | 60- " " " " | 23 | |
| 61 | 100- " " " " | 27 | |
| 101 | 200- " " " " | 9 | |

The other systems should be handled in the same manner, after which the total cost can be calculated and the specifications prepared.

## DISCUSSION ON "DESIGN OF UNDERGROUND DISTRIBUTION FOR ELECTRIC LIGHT AND POWER SYSTEMS" (NEWTON), CLEVELAND, OHIO, MARCH 8, 1918.

**A. A. Meyer:** The author's paper covers only underground a-c. distribution, but I would like to ask him if he has any data covering d-c. distribution and the relative merits of the two systems. In looking over the distribution systems of some of the largest operating companies we usually find direct current in the down-town or business district where the "load density" is quite high and alternating current in the surrounding territory where the density is appreciably less.

In Detroit we have both d-c. and a-c. distribution systems and the load density has been a good reason for continuing the d-c. system and expanding it to take over the a-c. territory on its border. But of course there have been some other factors besides density determining a change from either one to the other system.

A comparison of load density was made for parts of the Detroit distribution systems. Comparing the load per sq. ft. in the territory served by a d-c. substation with that in a territory served by an a-c. substation, it was found that the load density in the former was about twice as great as in the latter. This comparison includes both power and lighting load. A similar comparison of lighting load only, showed the demand of the d-c. system to be about three times as great as the a-c. system in comparable districts. These ratios are not applicable in all comparisons between d-c. and a-c. systems, but they bring out a point of consideration in designing systems of distribution

You might be interested to hear something about our remote controlled substations and the purpose they serve. These substations are located in our d-c. territory and are automatic in the sense that they have no attendant and are controlled entirely by the operator in a nearby and regular type substation. They serve a territory comparatively remote from the regular substation and are usually connected to the system only during the peak load. There are four such substations with a fifth going in, each having a 500-kw. rotary converter taking energy from a 4600-volt feeder and delivering 250 volts to the feeders tying in with the street mains in the immediate vicinity. Without these remote controlled stations the cost of copper for long and sufficient size feeders from the ordinary substation (with attending operators) would be excessive and considerable beyond the cost of a remote control station with complete equipment. With the growth of a city and the expansion of the d-c. network, some of these substations will probably be discontinued and replaced by the regular type substation. Others, however, will continue for a long time, for they are well adapted to handle large concentrated blocks of power such as required by hotels, large department stores or large shops located in a d-c. network.

**C. W. Rakestraw:** The author seems to have a decided preference throughout his paper for the use of varnished cambric or cloth, and he refers to varnished cambric or cloth all the way through his paper. He seems in taking this position to consider that the cost of installation has nothing to do with it, that it is a question of safety and maintenance, and he goes to varnished cambric immediately and scarcely considers paper. In my opinion the cambric is all right, except on the question of cost, and where any of us are held down to a question of cost, it is practically out of the question for general cable systems.

The author says,—"If three-conductor cable was used there would be a considerable reduction in the carrying capacity over single-conductor cables (25 per cent) also it would not be possible to take more than about two services out of each splice." For that reason, he seems to favor single-conductor cable for his secondary mains. We have found in our practise, that is not so. I know in many cases where we have had more service than that. We have taken four or five services out of one splice. While it is not good practise, it can be done if care is taken in the insulation.

The author speaks of a "bare neutral" all through his system. Have any of the underground men here ever had experience with bare neutrals on their systems? If so, I would like to know how that works out, using the bare neutral, instead of the regular insulated cable.

The author says in carrying the cable into the various appliances—into the manholes, I presume he means—transformers, switches or service boxes, that varnished cambric should be spliced on to the cable ends. I take issue on that. Certainly in going into the oil switches, which he must have to do frequently, it would be unwise to use rubber-covered cable. It will not last. There is no objection to using varnished-cambric cable in that case, whereas if you use ordinary varnished paper it will not serve the purpose.

On our local systems we have never followed that practise, but although it certainly would not do any harm to supply some varnished cambric, it again becomes a matter of cost.

The author says several times that he uses single-conductor cable for his services on all his two-voltage system, his lighting system. I would like very much to ask the author just what the idea is in that, as long as he is taking off alternating-current from his mains, why he would split that into three single-conductor cables, and whether he has any trouble in doing that. There is no objection to using three-conductor cable, and there is certainly some advantage in using it on the score of cost.

The author has given some figures here as to the assumed capacity, that is the assumed capacity of what he calls the primary feeders and secondary main. He says that primary feeders should be figured for 60 per cent of the connected load, and later on he figures the size of the transformers, and arrives at a certain size of transformers which he connects on the system.

I endeavored to check up in the short time at my disposal a couple of the figures presented by the author, and I found that those figures do check fairly well with a similar practise we have here. We have two feeders, practically the same kind of system which the author is endeavoring to portray, that is an a-c. distribution system, in which we have the feeders leading from the substation to the feeding point, and then sub-feeders, or secondary feeders, as he calls them, leading along the street to the transformers and secondary mains, for the distribution of the current. On one of these feeders I find that with a maximum demand, that is an average maximum demand, covering the average of ten or eleven maximum readings, the average kv-a. on this one feeder was 196, and the transformer connected to that was 242. The transformers connected are 123 per cent of the demand on the feeders, and the author in his assumed case has his transformers at about 133 per cent, which seems to check very well with the practise we have found here. That would seem to lead to the assumption, although I have not been able to check that, that the 60 per cent he assumes is quite close to being correct.

In his paper, where the author speaks about transformer connections, he says: "There is considerable difference of opinion among engineers as to the advisability of installing any fuses on the primary side of subway transformers, as they generally cause more trouble than their protection is worth, but as it is advisable to have some point where the transformers can be easily disconnected from the feeders it is a good plan to install the fuse boxes and use fuses having 100 per cent greater carrying capacity than is required to protect the transformer, and then trust to the station circuit breaker to disconnect the feeder in case of serious trouble." On our system we have primary fuses on all transformers. We have adopted a standard bank consisting of one 25-kw. transformer connected 110-220 and a 10-kw. single-phase transformer banked as a teaser, to give three phases on our power work. The light and power are taken from the same bank in that way on this system and that makes a standard bank of 35 kw. We have on the primary side a 15-ampere, 2300-volt fuse, and have very little trouble. Occasionally the primary fuse will blow, but that is generally due to some trouble on the distribution system.

The secondary mains from each transformer are normally connected separately, and there is a box between each set of mains in which the connecting bars are left out to connect up to in order to carry the mains from one bank to the next bank, or to the bank on two sides, in case of trouble in any given bank. That system has been in operation for about four years, and we have had very little trouble, indeed. I think it would be safe to say that since we have put the system into effect we have not had $100 maintenance cost on the system—I mean from added work caused by trouble, and aside from the ordinary inspection.

**H. L. Wallau:** I want to supplement Mr. Rakestraw's remarks in regard to this system by stating that in the TRANSACTIONS of the American Institute of Electrical Engineers, Volume 34, page 746, there is a discussion on this particular system, with a diagram of the connections and type of boxes used, which may be of interest.

**E. Friedlaender:** Operating men in the steel industry now have up the question, whether it wouldn't be advisable to put transmission lines in the plants underground.

The main advantage, probably, would be absolute safety against shocks through coming in contact with the conductors. Another great advantage would be avoiding lightning troubles. Would these benefits justify the additional expense?

There are a few points, which we must not overlook when installing underground systems. We cannot overload underground cables as readily as bare overhead lines. Another point of importance is continuity of service especially when voltages of 22,000 volts or over are used. It would require more time to locate and repair faulty underground than overhead lines.

Another source of trouble in underground systems is the liability of cable trouble in a manhole spreading over the whole system. On overhead lines, shorts or grounds are easily confined to one line, as lines are usually spread far enough apart.

The question of insulation of underground cable is a vital one. Paper is the cheapest and probably is good enough for dry and cool ducts, and lower voltages. Where cables are subjected to heat and moisture, as is often the case in mill work, would not rubber or cambric insulation be better?

Where large currents are handled, great care must be taken to guard against electrolysis.

We cannot overlook the fact that the carrying capacity of underground cable is considerably smaller for the same size cable, than that of overhead cable. The proper construction and ventilation of ducts is therefore of great importance. In many cases in the steel industry, the ground surrounding the ducts is made of cinder, or gravel, and is very porous and dry, and would not carry off the heat.

Should we entirely close up all manholes in our mills, or should we ventilate same? There is danger of gases accumulating in an entirely enclosed manhole, but on the other hand there is danger in the mill of having either hot cinder or metal spilled in the manhole if of the ventilated type.

It probably would be advisable to bring out cables, only, in every other manhole, so that in case of trouble in a manhole, not all cables are affected.

**E. B. Meyer:** The author as noted in his paper undoubtedly favors the use of varnished-cloth or cambric-insulated cable, whereas the writer's experience with large systems of distribution over a period of fifteen years indicates that paper cable can be used almost exclusively for this class of work without fear of giving trouble, the primary feature being, as in all kinds of electrical construction, care in installation.

Service or distribution holes where the space in the street allows should be built on the side of the main conduit so as to permit ample room for the proper racking of cable and subway boxes. In congested streets, however, this plan is not always feasible and under such conditions the hole may be placed on top of the main conduit and sufficient ducts run therein for distribution work. See Fig. 1.

In many systems subway branch boxes are used on the main secondary cable for taking care of service connections to consumers. These boxes add considerably to the cost of installation and it is frequently found necessary to install additional boxes in cases where the number of outlets in the original box installation is not sufficient to take care of the ultimate number of service connections.

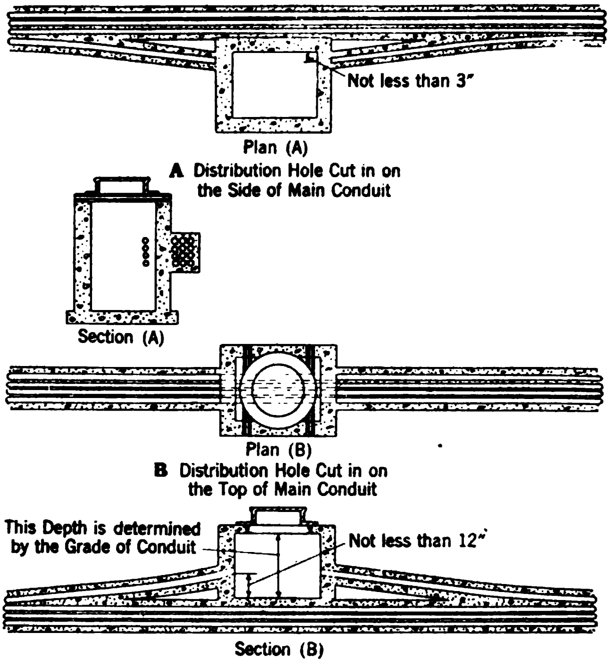
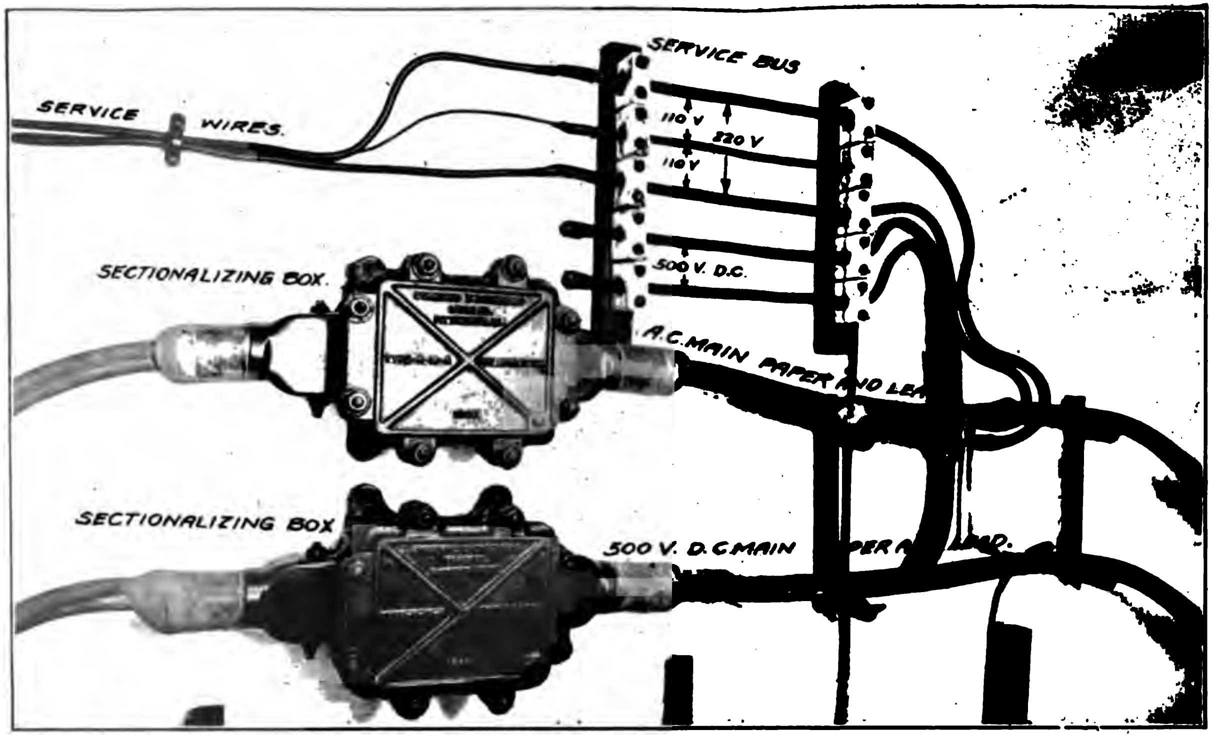


Fig. 1—Methods of Building Distribution Hole in Main Conduit Line

A novel method of taking care of such service connections is by the use of rubber insulated service bus mounted on the wall of the manhole or distribution hole as shown in Fig. 2. In this type of construction the usual splice is made to the main paper-insulated lead-covered cable with a rubber-insu-



[MEYER]

INTERIOR OF MANHOLE SHOWING SECTIONALIZING BOXES ON THE SECONDARY A-C. MAIN AND THE 500-VOLT POWER MAIN. SERVICE BUS MOUNTED ON THE WALL IN THE UPPER CORNER OF THE MANHOLE TO WHICH SERVICES MAY BE TAPPED AND RUN TO THE VARIOUS CONSUMERS

lated lead-covered cable. The lead sheath on the branch cable terminates a short distance below the bus rack. Service connections are made to the bus with rubber-insulated cable covered with weather-proof braid.

Installations of this character have proved very successful and have been in operation on 220-volt 3-wire a-c. systems for a period of over ten years without a failure. The principal advantage with this form of construction is that the service of a lead jointer are not required to make the connections to the service bus and any number of connections up to the capacity of the bus may be installed as the occasion requires. In a system as just described, the secondary mains may be either single- or 2-conductor paper-insulated lead-covered and the neutral of bare tinned copper wire. The lateral connections from the service holes to the customer's premises are usually installed in iron pipes thoroughly coated with asphaltum or other compound to prevent corrosion, the entire system being water-proof from the manhole to the customer.

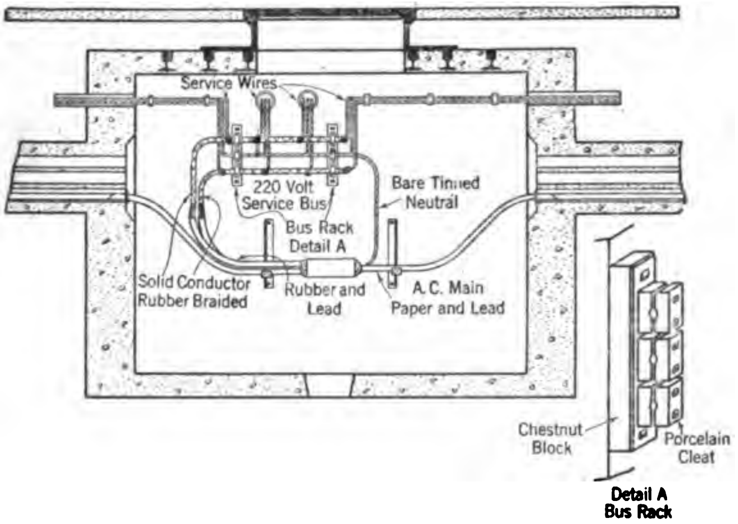


FIG. 2

With reference to series street lighting systems the writer has found No. 8 wire is usually a sufficient size for all classes of street lighting work, a single-conductor rubber-insulated lead-covered cable apparently being best adapted for this particular use as the lead can be removed from the rubber cable making it easy to loop the cable into the bases of the ornamental iron posts along the underground system.

While the author does not specifically state the kind of conduit material recommended, recent practise indicates that fibre conduit has come into very general use for all classes of underground electrical work. In the laying of fibre conduit a concrete base is usually provided. There is also provided a side and top cover of concrete with one-inch concrete separation between the adjacent ducts. Experience has shown that this form of construction resists the heat of an arc from burning cables, preventing trouble communicating from one duct to another. Multiple-tile duct affords the least protection to cables, as it is impossible to prevent the communication of trouble between the ducts at the joints. Multiple duct is, therefore, better adapted for telephone, telegraph and other similar work.

Regarding the several types of manhole heads or covers, the round type seems to be the one more generally used. The use of the retangular type should be avoided as far as possible as in the hands of careless workmen the cover may be dropped into the manhole, causing danger to cables or equipment contained therein.

The author's comments regarding transformer manhole construction and methods of installation seem to be generally accepted as good practise. Transformers should be installed in contact with the bottom of manholes and not blocked up off the bottom unless the transformer case is reliably grounded. Some very serious accidents have been due either directly or indirectly to the result of shocks received from transformer cases placed on wooden blocks in manholes; these accidents being caused primarly by a failure of the transformer or wiring connections whereby high potential was impressed upon the ungrounded transformer case. The neutral connection to the transformer is usually made solid and not brought out through the transformer box. The secondary neutral of the transformer should be a solid copper conductor where it enters the transformer case. If stranded wire is used, water is apt to be syphoned into the transformer when manholes are flooded and special precaution should, therefore, be taken to see that this connection is made water-tight.

The importance of maintaining the oil in underground transformers in perfect condition free from moisture or sediment cannot be too strongly emphasized as the life of the transformers depend on the maintenance of these conditions. Precaution should be taken by operating companies to insure proper installation and operation of the transformer and in addition to an inspection of the oil at least once a year, it is desirable that air pressure be applied to the transformer cases after installation to detect leaks.

For the purpose of checking the load on transformers, the use of a split-core current-testing transformer will be found convenient.

Transformers should be provided with cut-out subway boxes on both the primary and secondary sides if they feed an underground distribution net work. If they feed only isolated sections the cut-out on the secondary side may be omitted. These boxes need not necessarily be fused, as a number of companies believe that fuses give more or less trouble. In most cases the omission of fuses is recommended, the boxes being provided simply as disconnectors in the event of trouble with the transformer. Where solid connections are provided, it is necessary to depend for protection entirely upon the automatic devices in the station.

In general it may be said that an underground system of distribution depends to a large extent upon local conditions and in many cases it will follow the existing plan of the overhead lines.

Mr. Newton's paper, however, has covered the subject in such a way as to assist the central station engineer in laying out a cable system for supplying light and power service.

**Wilfred Sykes:** Mr. Friedlaender has raised some points regarding the type of insulation. It has been my good fortune to be associated with a considerable number of cable experts, in work for the Government where it was necessary to find the best cable that could possibly be designed to carry power from one point to another, the conditions surrounding this installation being such that the cables were likely to be subjected to particularly high temperatures, and also to flooding at times.

This matter was referred to a committee for consideration. On this committee there were representatives of the principal cable manufacturers, and we had a very frank discussion. The various types of insulation were discussed, and everybody came to the conclusion that the best insulation they could get, that is for moderate potentials, around 5000 volts, was varnished cambric.

After we got the insulation around the cable, the question of protecting the cable created a good deal of controversy. Some thought that we ought to have a lead sheath around the cable, and others pointed out that lead sheaths were likely to crack from unknown causes, and that they were pretty sure to crack if there was any vibration, and for that reason another type of moisture protection was advocated and definitely adopted. We know that plain rubber has short life, especially where the temperature is high. Some experiments carried on over a period of years in the New York Edison Company has shown that an insulation made up of reinforced rubber, as it is called, will stand high temperatures, apparently without deterioration, for quite a long time. This reinforced rubber is nothing less than rubberized tapes laid over one another, and held together by cement, and the whole vulcanized. As a moisture protector that is quite effective, but it is not used as an insulator. Laboratory tests made over periods of years, where it has been subjected to temperatures of 100 deg., show it is apparently unchanged. 'The paper-insulated cables will stand a little higher temperature than the cambric-insulated cables without deterioration, but not much. The upper limit of temperature varies somewhat with the voltage to be used. The paper cable was not suitable for this work, because we had to make a number of rather sharp bends in it, and nobody would contend that the paper cable was as good as the cambric cable for that work.

One other point was the question of grounding the sheaths. In one case we had to work with three-phase cable, where the lead sheaths were used, and there was no question of being able to ground the sheath of that cable, and not having any circulating current appreciable. In the case of single-phase cable, with lead sheaths, we made an investigation and found that the circulating current was such that if you grounded the lead sheath you would be likely to burn it off, or you would have such temperatures that the current carrying capacity of the cable was cut down, so that it was not a practicable proposition. The Committee finally came to a decision, that where the cost was not the main consideration, and where the cable would have pretty rough handling, that the varnished-cambric cable was the best type.

**F. M. Hibben:** I would like to ask the members if they have used multiple-conductor arc cable. The author advocates single-conductor cable No. 6, he advocates No. 6 for mechanical reasons only, the current being 5 or 6 amperes. We are contemplating using 4, possibly 6, or a larger number of conductors, perhaps we may reach to ten, all in one sheath for different arc circuits, and I would like to find out if any of the companies have tried anything like that, and with what success. With two-conductor arc cable, you have little chance of one arcing and causing short circuits in adjacent cables. On the other hand, the smaller cost of subway acts as a point in favor of the use of multiple-conductor arc cable.

In one case the author advocated a bare neutral, and in another case he said pull the bare neutral into a duct with a lead-sheath cable. With a slight amount of voltage in the ground from the street railway return current, I think the current from the bare neutral to the rails would eat holes in the lead sheath. In Cleveland we had a d-c. distribution something similar to that, where we pulled out all the bare neutral and put in regular insulated lead cable. We found it impossible to maintain the bare neutral, even when it was in a separate duct—we had more or less electrolytic trouble with it.

**G. J. Newton:** In reply to the questions that were brought out in the discussion of this paper I desire to make the following explanation, after which the various questions will be replied to.

In the first place the paper as presented was very much condensed from a manuscript prepared for book publication, it was not possible, therefore, in the space available to give more than a general outline of the method of proceeding to design a system of underground distribution and a simple system, common to the average cities of moderate size, was selected as being representative of this class of work. It was clearly stated that an a-c. system would be considered as, with the exception of d-c. power, there would be little or none of the d-c. system placed underground in a city of the size considered where a new underground system was installed.

In the second place the conclusions arrived at and stated in the paper do not apply to any particular locality but were intended to represent average conditions and were based on over twenty years experience in designing, building and operating underground systems; most of the questions brought out in the discussion were based on the experience of the various gentlemen in their particular locality. With the conditions assumed in the paper, and the further assumption that the engineer has not had previous experience in this class of work I feel certain that the methods suggested are satisfactory for guidance.

Large existing systems are in charge of competent engineers who are familiar with the most suitable methods and apparatus to use in their particular system, the author had no intention of criticizing their methods but simply desired to give useful information to those engineers who were confronted with this class of work and lacked previous experience in handling it.

Replying to Mr. A. A. Meyer: I believe that both the a-c. and d-c. systems have their particular place in distribution problems but in a place similar to that assumed I doubt if much d-c. distribution would be placed underground, the chances being that the d-c. system would be changed to a-c. in this district with the ultimate intention of doing away with it entirely as the underground district was enlarged.

The methods suggested for designing the a-c. system would apply for d-c. work were it necessary to install that system, as the much less complicated equipment makes that class of design comparatively a very simple matter as this type of service is fairly well standardized in the large cities, and undoubtedly the new system installed in a small place would follow existing practise.

Replying to Mr. C. W. Rakestraw: I stated that "Personally I prefer varnished cambric or cloth insulation in all distribution work, particularly in the smaller cities where usually the cable department is very limited." I see no reason to change my opinion on that statement and refer to Mr. Wilfred Sykes discussion of the paper in which he states that varnished-cambric cable under the conditions assumed would be the best. In preparing the paper I felt that it was proper to suggest what my experience had shown to be the best, if, however, the cost would prevent the use of this class of cable then the engineer must use paper.

In small cities it is foolish to install anything but the best as the system once installed is seldom inspected or taken care of properly and it is better to do a little at a time and do it right and give good reliable service.

I said that if paper cable was used it would be necessary to splice *either* varnished-cambric or rubber tails on it where it entered equipment. Naturally, with the knowledge of modern engineers they would not use rubber insulation in oil-insulated equipment.

The system under discussion was stated to have single-conductor mains for the lighting system as that type of cable gave the greatest capacity for those mains and, therefore, single-conductor services were stipulated. The power system was stated to have three-conductor cable and three-conductor services.

As stated, there is a difference of opinion as to the advisability of installing fuses on the primary side of underground transformers. The fact that fuses are considered advisable on Mr. Rakestraw's systems does not make my statement incorrect.

The author installed a bare neutral on an a-c. distribution system in Peoria, Ill., and as far as I know there has never been any trouble from this neutral, however, at the time that I tested the system it was exceptionally free from electrolysis. In this connection I might state that I have heard of the sheath of the cable being used as the neutral in some three-phase work and would refer you to a paper by Mr. Hood of one of the Canadian companies.

Replying to Mr. Friedlaender: In my paper I pointed out the necessity of providing emergency facilities to take care of every case of trouble that could be reasonably foreseen, I am confident even under the exacting conditions required by a steel mill that with proper study a satisfactory system could be designed and installed, which would fully warrant the cost of the work.

If the entire power and lighting system of the plant were changed to underground or at least the design made with that end in view, it would be possible to provide ample facilities for meeting the most difficult conditions.

Replying to Mr. E. B. Meyers: In the original manuscript it was suggested that service boxes could be built on the side of the main run of conduit but the statement was omitted in the revision for this paper, however, conditions govern this part of the work.

The particular method of taking care of service in Mr. Meyers company is known to the author and is a special case and certainly not suitable for a system in a small city where the cable department is limited.

Undoubtedly fibre conduit can be installed for less money than clay conduit and will give equally good service. This subject was also treated in the original manuscript. Multiple clay duct should not be used on lighting and power systems, either single clay or fiber is better.

By installing the transformers on I beams there is a chance for the air to circulate around and under them and under heavy load they would have more cooling surface. Naturally, these I beams should be connected securely to the transformers and grounded.

Replying to Mr. F. M. Hibben: There is no objection to using No. 8 conductor for series arc light systems and it is quite common practise to have several circuits in the same sheath provided that they all terminate at the same point as is frequently the case where several pass through the underground district in order to supply outlaying sections of the city.

*Presented at the 338th meeting of the American Institute of Electrical Engineers, Cleveland, Ohio, March 8, 1918.*

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# SOME CONSIDERATIONS IN DETERMINING THE CAPACITY OF ROLLING-MILL MOTORS

BY ROBERT F. HAMILTON

ABSTRACT OF PAPER

A consideration in detail of electric drive for rolling mills, including classification of mills and motors, mathematical determinations of energy required for rolling, relation of speed to tonnage, motor capacity and flywheel application.

FOR several years past the attention of engineers in Great Britain has been directed to the electric driving of rolling mills. This is a natural result of the constantly increasing use of electric power for auxiliary apparatus in steel works, and the economies which accompany generation of power in large units. When blast furnace gas may be utilized under boilers for turbo-generators or for driving gas engine units the prospects of electric motor application throughout an entire works are very favorable. The present demand for steel has accelerated the application in that it has caused many manufacturers to scrap existing plant and adopt more modern methods. In a very few months the steel production of the nation has changed from what might have been called a decaying industry into a vital necessity.

## GENERAL CLASSIFICATION

In any rolling-mill electrification the paramount problem is the delivery of a specified tonnage of steel, of which the grade and section are known, at a minimum total cost. Each case must therefore be analyzed carefully and the most suitable equipment chosen. In general, the drives may be classified as follows:

1. Continuous running mills.
    a. Constant-speed motors.
    b. Adjustable-speed motors.
2. Reversing Mills.

Item 1 may include two-high or three-high mills, of nearly all types, *i. e.*, plate mills, sheet mills, cogging or blooming mills, merchant mills, rod mills, etc. By far the greater number is driven by constant-speed motors. Adjustable-speed motors are found where it is desirable to roll a variety of sections in the same mill.

One of the most efficient drives is by means of a high-speed induction motor with ropes or gearing. If speed variation is necessary this may be accomplished in the case of an induction motor by controlling the rotor frequency with a synchronous converter or alternating-current commutating-motor. Instead of the adjustable-speed induction motor it is often found advisable to use adjustable-speed direct-current motors and synchronous converters, assuming of course, that the supply is an alternating-current system. If the first mentioned scheme includes an auxiliary motor on the shaft of the main motor, constant power output may be obtained at the rolls throughout the range of speed. When it is necessary to increase the speed of rolling, the torque required is usually less, which condition is also met by a direct-current motor with shunt-field control. In a few special cases cascade motors or Ward-Leonard motors have been used to drive continuous mills.

The reversing mill is best suited to the work of cogging ingots to billets. The speed of rolling is changed from pass to pass as the billet lengthens and the reversing does away with the lifting tables, which are generally necessary in a three-high mill. Electric motors which are to meet such service must be capable of withstanding large overloads, since no flywheel is interposed between the motor and mill. Ward-Leonard control is used over the range of speed in which the torque is a maximum, and shunt-field control at the top speeds when the torque is reduced. An Ilgner set supplies the energy for the motor.

## SELECTION OF MILL

The type of mill is determined by the tonnage and sections rolled; the temperature at which the rolling takes place; the quality and composition of the product; the general layout of the steel works and the labor conditions existing in the particular district. The latter is emphasized in the consideration of automatic features and skilled attendance.

This discussion presumes that the following facts have been determined:

1. Type of mill including layout.
2. Weight, shape, material and temperature of ingot or billet.

3. Section and tonnage of finished product.
4. Diameter and profile of the rolls.
5. Time required to handle material between passes.

## Energy Required for Rolling

Opinions vary as to the methods employed in calculating the energy required for rolling. Many engineers use a formula which includes the logarithm of the elongation. That is, the torque in any pass is proportional to the logarithm of the elongation of the material during the pass. A basis for this method may be found by plotting a curve of energy versus total elongation as shown in Fig. 1. Such a curve agrees very closely with a logarithmic function.

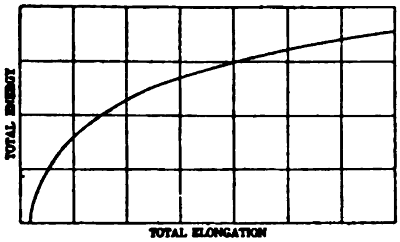

Fig. 1

A second method is based upon the energy per unit volume displaced or upon its reciprocal, the volume displaced per unit energy. Referring to Fig. 2, *a b c d* is considered as the volume of metal displaced in rolling from length $l_1$ to $l_2$, that is, the volume of metal displaced is the product of the reduction in area and the original length.

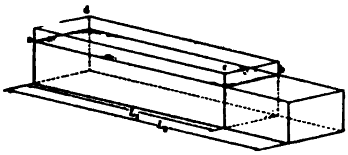

Fig. 2

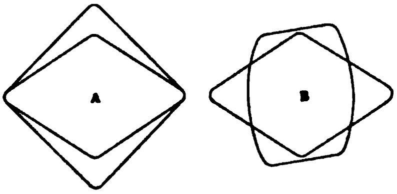

Fig. 3

The method used by the author consists in the application of a certain tangential force at the circumference of the rolls per unit reduction or transposition of sectional area. The following analysis will make this clear:

Referring to Fig. 3, two distinct kinds of rolling are represented by $A$ and $B$ respectively. In case $A$, the reduction in area is the arithmetical difference of the two areas involved, while in case $B$, the transposition in area may be considered equal to that area of the one section which is not included within the outline of the other. Case $A$ is by far the more common in rolling work. A combination of $A$ and $B$ may occur in certain forming passes where there is a slight reduction in area. Case $B$ does not cause an elongation in the material but may be reduced to terms of an equivalent elongation when considering the total energy. This will become apparent in what follows.

Let $l_1$ = length of ingot at start of rolling
$A_1$ = sectional area of ingot at start of rolling.
$l_2$ = length of finished product.
$A_2$ = sectional area of finished product.
$e$ = elongation per pass. $e_t$ = total elongation.
$N$ = number of passes.

Then

$$e_t = \frac{l_2}{l_1} = \frac{A_1}{A_2} \text{ and } e = \sqrt[N]{e_t} = \sqrt[N]{A_1/A_2}$$

assuming for the purpose in hand that there is no increase in density, nor loss of material and that the elongations in each pass are equal, which assumption will do for preliminary considerations.

Now if $K$ be the force per unit reduction in area for any one pass, say the $N_o$th, $R$ the effective radius of the rolls, and $T$ the torque necessary for rolling, $T$ will be given by

$$T = K\,R\,(A_{N_o-1} - A_{N_o}) = K\,R\,A_1\left(\frac{1}{e^{N_o-1}} - \frac{1}{e^{N_o}}\right) \quad (1)$$

Now

$$l_0 = l_1\,e^{N_o}$$

and since energy is the product of force and distance through which the force acts, the energy for the $N_o$th pass is

$$E_o = K\,l_1\,A_1\,(e - 1) \quad (2)$$

and the total energy for rolling is equal to

$$E = (\Sigma\,K)\,[l_1\,A_1\,(e - 1)] \quad (3)$$

since (2) is dependent upon the number of the pass only in so far as $K$ is dependent. If $K_0$ be taken as an average value of $K$ over the conditions of ordinary rolling, $W$ the weight of the ingot which is proportional to $l_1 A_1$, and $E$ the total energy for rolling, then

$$E = N K_0 W (e - 1) \qquad (4)$$

From large numbers of tests the h. p.-sec. per ton $\cdot \frac{E}{W}$, for different values of $e$, have been determined for various sections.

The factor $K_0$ varies with the temperature of rolling and diameter of the rolls as well as with the kind of steel and shape of the section rolled. Within the limits of practise, however, $K_0$ as calculated from the above; *i. e.*

$$K_0 = \frac{E}{W} \times \frac{1}{N(e-1)} \qquad (5)$$

changes very little owing to the first three causes mentioned and rather less than might be expected due to the fourth. For example in a certain reversing-cogging-mill rolling 4-in. by 4-in. billets from three-ton ingots, $K_0$ has a relative value of 3.2 compared with a value of 4.0 for a merchant mill rolling 3-in. channels from 6-in. by 6-in. billets. In the former case 22 passes are employed; in the latter 15. The two examples present totally different conditions of rolling. Of course in finding the total energy consumption per ton, the mill and motor losses must be added to the energy required for rolling.

If $\frac{E}{W}$ represents the units per ton which are being expended to roll steel having a total elongation of $e$, in $N$ passes, then $K_0$ as given in (5) is a measure of the efficiency with which the rolling is being accomplished.

Table I gives a few typical values of $K_0$.

From the above it will be noted that the change in area in any pass is

$$A_1 \left( \frac{1}{e^{N_0 - 1}} - \frac{1}{e^{N_0}} \right)$$

and the length of the entering billet or ingot is given by $l_1 e^{N_0 - 1}$

TABLE I

| Type of Rolling | Original section | Final section | Elongation | No. of passes | h. p.-sec. per ton for rolling | Av. $K_o$ |
|---|---|---|---|---|---|---|
| Ingots to Billets | 20 in. x 20 in. | 6 in. x 6 in. | 11.1 | 15 | 60,500 | $23.2 \times 10^3$ |
| Ingots to Billets | 20 in. x 22 in. | 4 in. x 4 in. | 27.5 | 23 | 85,000 | 23.8 " |
| Billets to Bars | 5 in. x 5 in. | 1.5 in. x 1.5 in. | 11.1 | 12 | 89,000 | 33.4 " |
| Ingots to 70-lb. Rails | 16 in. x 16 in. | 7.2 sq. in. | 35.6 | 23 | 120,500 | 31.2 " |
| Billets to channels | 5 in. x 5 in. | 1¼ in. x ⅜ in. x ¼ in. | 40 | 17 | 164,500 | 39.9 " |
| Billets to *T* bars | 5 in. x 5 in. | 2 in. x 2 in. x ¼ in. | 24.6 | 13 | 141,000 | 38.8 " |
| Sheet Mill | 3 in. x 28 in. | 45-in. sheets,<br>0.11 in. thick. | 27.25* | 17 | 144,000 | 39.5 " |

*For simplicity in this case $e$ has been taken as the ratio of the original to the final thickness of the sheets.

It should be noted that the values of $K_o$ as given have not been reduced to a common temperature but represent typical conditions as found in practise.

Therefore, the volume of metal displaced in the $N_o$th pass is

$$l_1\, e^{N_o-1} A_1 \left(\frac{1}{e^{N_o-1}} - \frac{1}{e^{N_o}}\right) = V\left(1 - \frac{1}{e}\right)$$

where $V$ is the original volume.

This would show that under the assumed conditions the volume displaced per pass (see Fig. 2) is a constant value, and if $K_1$ is the energy per unit volume displaced and $\frac{E}{W}$ the energy per unit mass of the material rolled,

$$K_1 = \frac{E}{W} \times \frac{\rho}{N\left(1 - \frac{1}{e}\right)}$$

where $\rho$ is the average density of the metal.

From the foregoing it follows that

$$\frac{K_o}{K_1} = \frac{1}{\rho\, e} \tag{6}$$

Since $e$ varies slightly for different conditions, equation (6) gives an idea of the discrepancies which will occur between a calculation in which $K_o$ is taken as a given value, and one in which $K_1$ is the factor assumed.

$K_o$ conforms to the method of "force per unit area" while $K_1$ applies to the method of "energy per unit volume displaced" under the same conditions.

With an average value of $e$ determined upon, the number of passes required to roll any given section would be

$$N = \frac{\log A_1 - \log A_2}{\log e}$$

$$= \frac{\log e_t}{\log e}$$

Upon laying out the individual passes it may be necessary to modify $e$ somewhat. Then too, if there is a large temperature drop from start to finish, $K_o$ should vary from pass to pass accordingly. For changes in temperature the writer has derived the following empirical expression, which although not exact is very useful when used within ordinary temperatures for rolling steel.

in which $$K_o = C \log \frac{\theta_m}{\theta}.$$

$\theta_m$ = temperature at which the metal liquefies in deg. cent., generally about 1380 deg.

$\theta$ = temperature of rolling in deg. cent.

For cold rolling the energy required for various materials may be taken as about proportional to the moduli of elasticity of the materials.

As previously pointed out $K_o$ must also be modified to correspond to the shape of the section. A good idea of the values to apply in the individual passes can be obtained from the average value of $K_o$ for rolling different sections.

In this connection it should be remembered that in certain forming passes there will be a transposition or shifting of sectional area without a corresponding volume displacement or elongation and $K_o$ will be altered accordingly. However, what interests the electrical engineer is the aggregate result rather than what may happen in any one pass and this aggregate may be very accurately determined from the average value of $K_o$ for any particular type of section rolled.

Should there be considerable transposition of section $K_o$ as found in the ordinary way will be larger than usual. In such a case transposition may be considered as a certain hypothetical elongation.

With this method all types of rolling may be compared on the same basis.

### RELATION OF MILL SPEED TO TONNAGE

Let $W$ = weight of each ingot in tons

$S$ = tonnage per hour required.

$$\left.\begin{array}{l} t_i = \text{total time of intervals; seconds} \\ t_r = \text{total time of passes; seconds} \end{array}\right\} \text{for one ingot.}$$

Then

$$t_r = \frac{3600\ W}{S} - t_i$$

Now if $l_1$ = original length of ingot
$N_o$ = number of the pass
$d$ = effective diameter of the rolls
$n$ = average rev. per min. during a pass
$t_p$ = time of the pass,

then

$$t_p = \frac{60\, l_1\, e^{N_o}}{\pi\, d\, n}$$

and

$$t_r = \frac{60\, l_1}{\pi\, d\, n}\, [e + e^2 + e^3 + e^4 + \ldots\ldots e^N]$$

from which

$$n = \frac{60\, l_1\, [e^{N+1} - e]}{\pi\, d\, [e - 1] \left[\dfrac{3600\, W}{S} - t_i\right]}$$

$n$ is generally limited by the speed at which the material may be handled. In this case with $n$ known, $S$ is found from

$$S = \frac{3600\, W n}{\dfrac{60}{\pi} \times \dfrac{l_1}{d} \left[\dfrac{e^{N+1} - e}{e - 1}\right] + t_i\, n}$$

or with $n$ and $S$ both known, an insight into the rapidity with which the material must be handled may be found.

Fig. 4 shows a graphic application of the formula which is useful in obtaining an idea of the tonnage which any continuous-running mill may deliver.

The results obtained with a reversing mill are quite different. The calculated torque-time diagram for a typical reversing mill is given in Fig. 5. In this case the accelerating time in the pass is about one-half the time of the pass and the material is discharged from the rolls at top speed. The time of reversing and attaining the "nipping" speed is here included in the time of the interval.

Let $T_p$ = turns in pass.
$n_1$ = nipping speed in pass
$N_o$ = number of the pass
$n$ = Max. speed in pass
$t_p$ = time in pass seconds
$t_a$ = acceleration time in pass.

Now it is good practise to make $n_1 = \frac{1}{3}\, n$ and $t_a = \frac{1}{2}\, t_p$.

Then

$$n = \frac{72\,T}{t_p}$$

$$= \frac{72}{\pi} \times \frac{l_1}{d} \times \frac{e^{No}}{t_p}$$

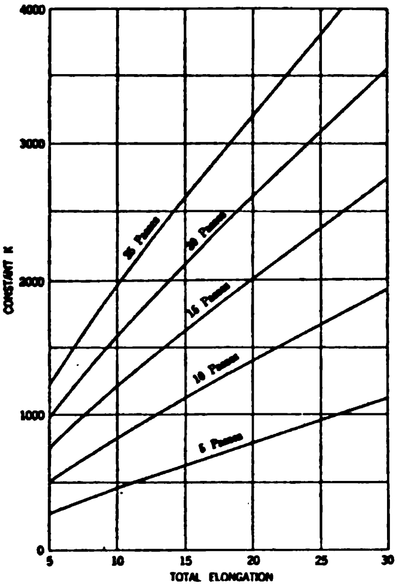

FIG. 4

**NOTE: To find the total time in seconds for billet to pass through the rolls add to the time of the intervals the time $T$, when**

$$T = \frac{L_1}{D} \times \frac{K}{n}$$

$\frac{L_1}{D}$ **the ratio of the original length of the billet to the effective diameter of the rolls.**

$\frac{K}{n}$ **the constant from the curve divided by the average rev. per min. of the rolls.**

Let the time of first pass $= t_1$

Let the time of final pass $= t_f$

Nipping speed in first pass $= n_1$

Nipping speed in final pass $= n_f$

Then

$$t_f = t_1 \times \frac{n_1}{n_f} \times e_s = K_2\,t_1 \text{ where } K_2 = \frac{n_1}{n_f} \times e_s$$

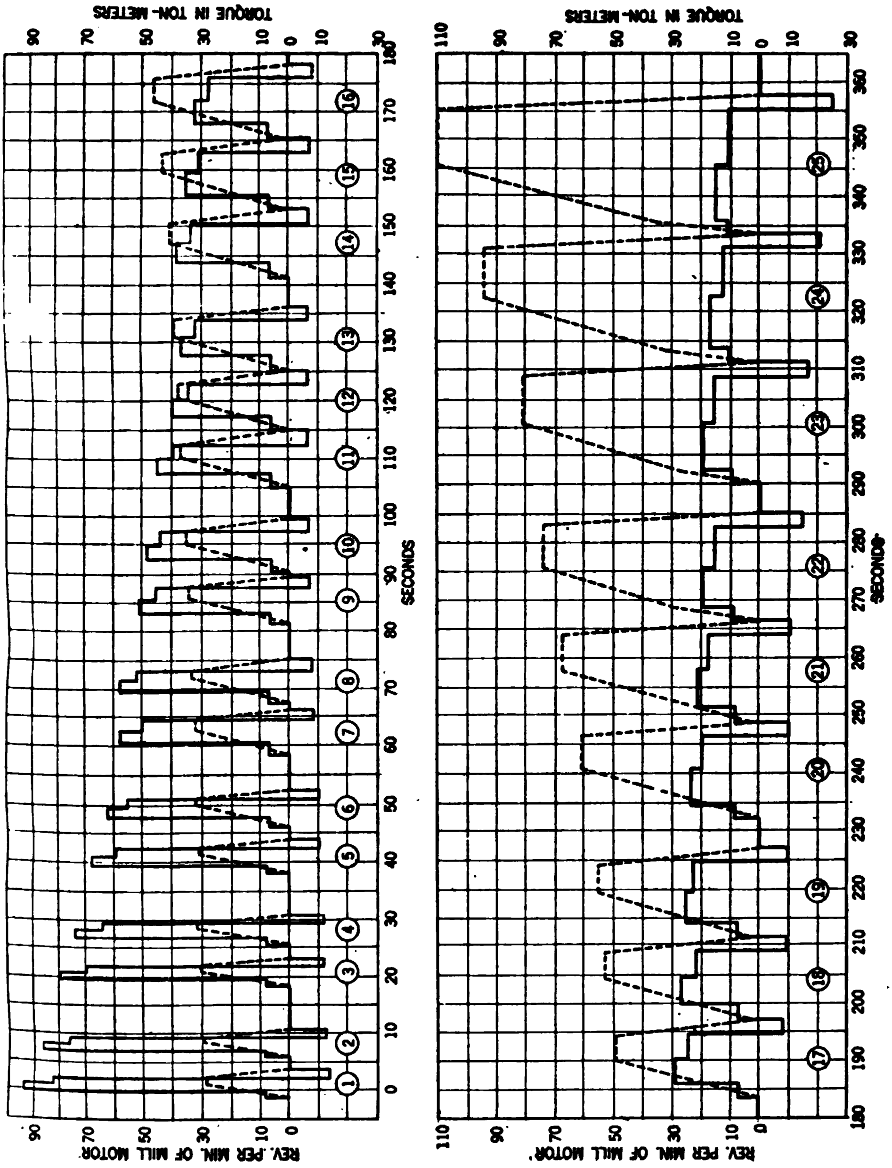
TORQUE IN TON-METERS
REV. PER MIN. OF MILL MOTOR
SECONDS
90
70
50
30
10
0
10
30
110
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
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FIG. 5

and assuming that the times are in geometrical progression the total time of the $N$ passes, $t_r$, is given by

$$t_r = \frac{\sqrt[N-1]{(t_1 K_2)^N} - \sqrt[N-1]{t_1^N}}{\sqrt[N-1]{t_1 K_2} - \sqrt[N-1]{t_1}} = t_1 \left[ \frac{K_2^{\frac{N}{N-1}} - 1}{K_2^{\frac{1}{N-1}} - 1} \right] \quad (10)$$

and

$$t_1 = \frac{t_r (r - 1)}{r^N - 1} \text{ where } r = \sqrt[N-1]{K_2} = \sqrt[N-1]{tf/t_1} \quad (11)$$

$$\text{also } t_{No} = t_1 r^{No-1} \quad (12)$$

## THE DETERMINATION OF THE MOTOR CAPACITY

For a continuous-running mill a good idea of the size of motor necessary may be obtained from the average value of torque, especially where heavy flywheels are employed and the fluctuations above and below this average are not severe. The r. m. s. rating of the motor will always be greater than its average output. This is considered more in detail under flywheel application as will be shown subsequently. In the case of a reversing mill the r. m. s. value of armature current may be taken safely as 10 to 15 per cent greater than that value of current which corresponds to the r. m. s. torque of rolling. This additional current is due to accelerating and retarding torque, friction torque, and to the increase in current per unit torque with field-weakening for the top speeds. This latter should not be over-emphasized, for field weakening takes place at relatively low values of torque.

To find an average value of torque,

From (1)

$$T_{No} = K\,R\,A_1 \left( \frac{1}{e^{No-1}} - \frac{1}{e^{No}} \right)$$

$$= K\,R\,A_1 \left( \frac{e-1}{e^{No}} \right)$$

$$\text{Also } t_p = \frac{60\, l_1\, e^{No}}{\pi\, d\, n} \quad \text{(for continuous mill)}$$

Therefore, torque × time for $N_o$th pass is

$$\frac{60\,K\,R\,A_1\,(e-1)}{\pi\,d\,n}$$

Now $t_i$ = total time of intervals
$t_r$ = total time of passes, $t_\Sigma = t_i + t_r$
$\therefore$ Average torque, $T_a$

$$T_a = \left[\frac{60\,K\,R\,A_1\,(e-1)}{\pi\,d\,n\,t_\Sigma}\right] N$$

Where $N$ is the total number of passes,
and since $d = 2R$

$$T_a = \left[\frac{30\,K_o\,A_1\,(e-1)\,N}{\pi\,n\,t_\Sigma}\right] \tag{13}$$

To this torque is added the average friction torque to find the average torque exerted by the motor.

To find the r. m. s. torque,
Let $K\,R\,A_1\,(e-1) = c_1$
Then

$$T^2 = \frac{c_1^2}{e^{2\,No}}$$

and for a reversing mill,

$$t_{No} = t_1\,r^{No-1}$$

[see (**12**) above.]

Therefore torque-squared-time for the $N_o$th pass is

$$\frac{c_1^2\,t_1}{e^2}\left(\frac{r_o}{e^2}\right)^{No-1}$$

and $\Sigma\,T^2\,t$ for $N$ passes is

$$\frac{\Sigma\,c_1^2\,t_1}{e^2}\left[\frac{\left(\frac{r}{e^2}\right)^N - 1}{\frac{r}{e} - 1}\right]$$

which gives

$$\text{r. m. s. } T = \frac{\Sigma c_1}{Ne} \sqrt{\frac{t_1}{t_2} \left[ \frac{\left(\frac{r}{e^2}\right)^N - 1}{\frac{r}{e^2} - 1} \right]}$$

If torque is in kg-m. then

$$\text{r. m. s. kg-m.} = 38 \frac{d}{l_1} W K_o \frac{e-1}{e} \sqrt{\frac{t_1}{t_2} \left[ \frac{\left(\frac{r}{e^2}\right)^N - 1}{\frac{r}{e^2} - 1} \right]}$$

Where $d =$ diameter of rolls

$l_1$ = original length of ingot.

$W$ = weight of ingot in tons.

$K_o$ = constant for rolling (force per unit reduction.)

$t_1$ = time of 1st pass.

$t_2$ = time of cycle.

$r = \sqrt[N-1]{K_2}$

$e = \sqrt[N]{e_s}$

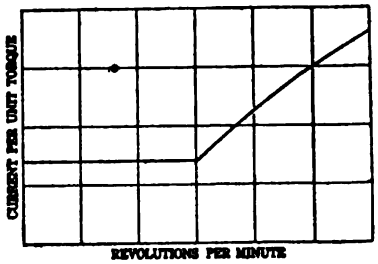

FIG. 6

In applying the above formulas it will be found that the torques in the first passes will be excessive and those in the last smaller than actually occur in practise. When a diagram is constructed the necessary corrections should be applied. As previously stated however, the aggregate result is what is most desired and within ordinary limits that is not greatly affected by what may happen in any one pass.

After torque-time and speed-time curves have been prepared for the mill-motor a current-time curve is drawn. This is determined from a motor characteristic curve which gives current per unit torque versus speed, as shown by Fig. 6. The output of the mill-motor is given for every instant by the product of the torque and speed curves, and to this output are added the losses which are easily determined from the above curves.

In this manner are found the energy diagrams for the generators of the Ilgner sets, as well as the exciter sets. The rating of these machines is obtained by finding the r. m. s. value of current for armature and field circuits and in some cases the average iron

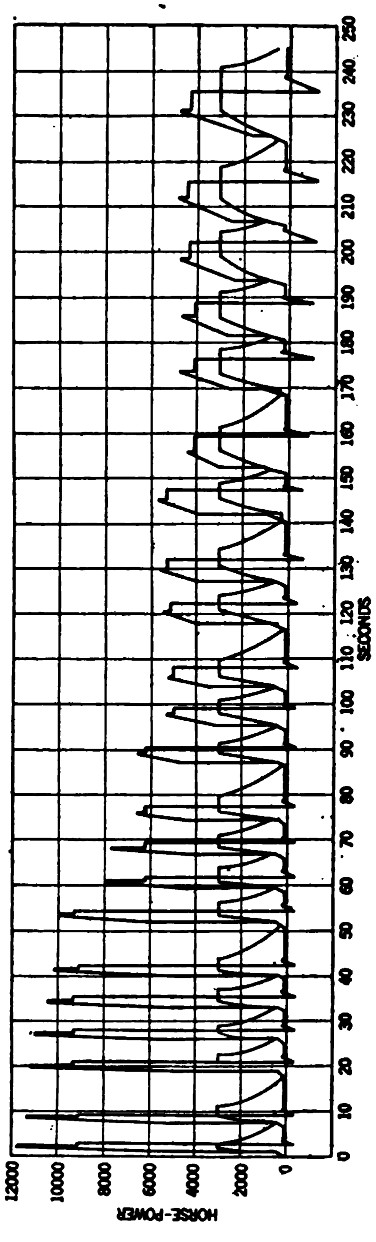

Fig. 7

losses throughout the cycle of operation. A typical "h. p.-time" diagram for a reversing mill is given in Fig. 7.

## FLYWHEEL APPLICATION

After a power-time diagram has been determined upon for a continuous mill or for the Ilgner set of a reversing mill the next step is the calculation of size and weight of wheel. In this it may

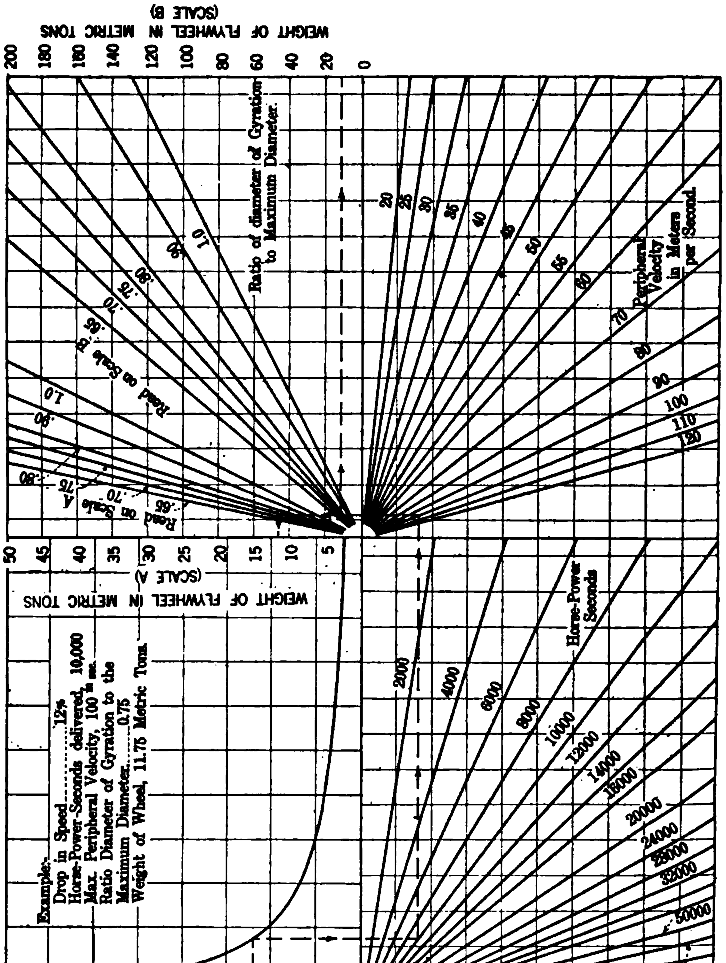

FIG. 8—CHART FOR DETERMINING WEIGHT OF FLY WHEEL

be assumed that all peak loads above the average will be carried by the wheel and that energy will be returned during the intervals in which the load is below the average. The amounts of energy which the flywheel must supply (both positive and negative) are added algebraically one at a time for a complete cycle of retardation and acceleration to the original speed. The largest numerical value is taken as that which the flywheel must supply with a given speed drop.

The necessary weight of wheel may be obtained from

$$W = \frac{C\,E}{K_3^2\,V^2\,(2 - d)\,d} \tag{1}$$

where

$W$ = weight of wheel.
$E$ = energy delivered with total drop in speed.
$K_3$ = ratio of diameter of gyration to maximum diameter
$V$ = maximum peripheral velocity of rim.
$d$ = fractional drop in speed.

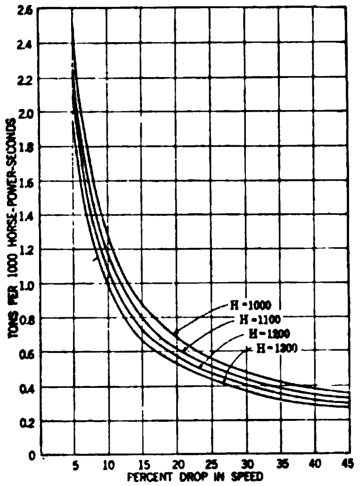

FIG. 9

Thus if $E$ is in horse-power-seconds, $V$ in meters per second and $d$ in per cent drop in speed divided by 100, $W$ will be in metric tons for a value of $C$ equal to 1.491. A graphical application of the formula is shown in Fig. 8, by means of which an idea of fly-wheel weight may readily be obtained.

A very useful way of expressing flywheel efficiency is by the vertical height to which the intrinsic energy of the wheel would lift its weight when running at maximum speed. Fig. 9 gives a ready method of obtaining the weight of the wheel necessary for various designs.

The equation just given shows something which is often not apparent, namely that the speed of the flywheel shaft does not enter into the weight of wheel necessary, provided that the wheel be so designed as to get the maximum allowable stresses in the rim.

At this point it should be stated that there are many other factors which might influence the size of wheel selected. For example if an induction motor with slip-regulator is used, increasing the weight of the wheel means not only an increase in capital outlay, but also in constant losses. To offset this there may be a reduction in size of the motor due to a more constant output; less loss in the slip-regulator and consequently a smaller one. The generators may be designed for a smaller variation in speed and the maximum demand from the line may be reduced. In each problem the total cost of these factors in pounds sterling (or dollars) per annum should be made a minimum. Fig. 10 illustrates the effect upon the motor peak load caused by a change of flywheel weight or the speed characteristics of the motor.

The opinion is often expressed that a mill cannot have too large a wheel. The writer has in mind a merchant mill where half the output of the motor is used continuously to keep an excessively large flywheel running with the result, incidentally, that the full-load output has never been reached.

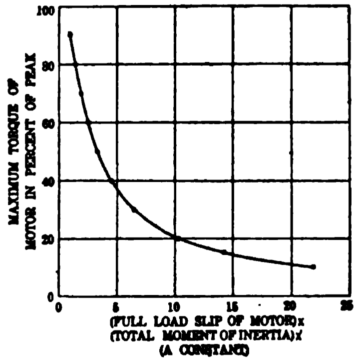

FIG. 10

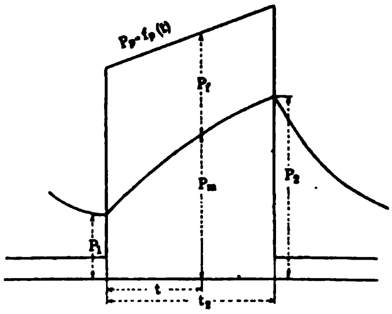

FIG. 11

Referring to Fig. 11

Let $P_p$ = power of the peak load.
$P_m$ = power from the driving motor.
$P_f$ = power from the flywheel.

In general the above quantities can be expressed as follows:

$$P_p = f_p\,(t)$$
$$P_m = f_m\,(n)$$
$$P_f = I n \frac{d\,n}{d\,t}$$

where $t$ stands for time and $n$ for the speed of the flywheel set. The function $f_p$ $(t)$ is obtained from the power-time curve to which the flywheel set is to be applied and $f_m$ $(n)$ from the speed characteristics of the driving motor.

Then the differential equation which shows that the instantaneous rate of transfer of energy to the generators (or to the mill) is equal to the algebraic sum of the power from the motor and that from the flywheel is given by

$$P_p = P_m \pm P_f \tag{2}$$

Now a very common application is that in which $f_p\ (t)$ is a constant value for a given interval of time and in which the torque of the motor is proportional to its drop in speed (or slip in the case of an induction motor), that is

$$P = K\,(n_0 - n)\,n \pm I\,n\,\frac{d\,n}{d\,t} \tag{3}$$

Equation (3) integrated from speeds $n_1$ to $n_2$ corresponding to power outputs of the motor $P_1$ and $P_2$, gives

$$t = \pm \frac{I}{2K}\left[\log\frac{P - P_1}{P - P_2} + \frac{1}{m}\log\left(\frac{1 - \dfrac{2\,n_1 - n_0}{n_0/2\,(m-1) + n_1}}{1 + \dfrac{2\,n_2 - n_0}{n_0/2\,(m-1) + n_2}}\right)\right] \tag{4}$$

in which $m = \sqrt{1 - \dfrac{4\,P}{K\,n_0^2}}$ and $n_0$ is the synchronous speed of the motor.

For all ordinary calculations the second term in the above equation may be taken as equal to the first. Making this assumption, we have

$$P_2 = P - \frac{P - P_1}{e^{\frac{K}{I}\,t_2}} \tag{5}$$

when the flywheel is decelerating, and

$$P_2 = P + \frac{P_1 - P}{e^{\frac{K}{I}\,t_2}} \tag{6}$$

when the wheel is accelerating.

From the differential equation it will be seen that all terms are expressed as power. Therefore in the case of an induction motor $K$ will be given as $K = \frac{P}{n \times s}$, where $P$ is the horse-power output of the motor at speed $n$ and slip $s$ (both in rev. per min.). $I$ is a constant times the moment of inertia of the fly-wheel. Its value may be obtained from $I = \frac{2E}{n^2}$ in which $E$ is the kinetic energy of the wheel in horse-power seconds when the speed is in revolutions per minute.

From this the time constant

$$I/K = 2\left(\frac{E \times s}{n \times P}\right) \qquad (7)$$

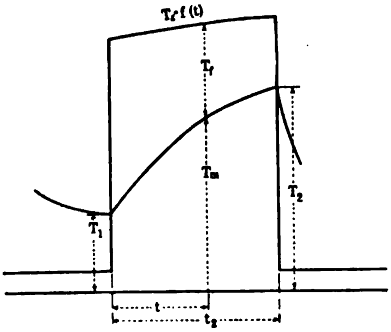

FIG. 12

in which as before $E$ is the kinetic energy of the flywheel at $n$ rev. per min. and $P$ is the power output of the motor with slip $s$ and speed $n$. Where there are heavy peaks of long duration $\frac{I}{K}$ will have a value of many seconds, depending also of course, upon the allowable drop in speed. On the other hand $\frac{I}{K}$ may be only a fraction of a second when the peaks are of short duration and infrequent.

Very often the torque instead of the power which is supplied by the motor and flywheel may be expressed as a function of the time or a constant value. Thus in the example illustrated in Fig. 12

$$c_1 + c_2 t = T_m + T_f$$
$$= (n_0 - n) K' \pm I' \frac{dn}{dt} \qquad (8)$$

or

$$T = c_1 + c_2 t - c_2 \frac{I'}{K'} + \left(T_1 + c_2 \frac{I'}{K'} - c_1\right) e^{-\frac{K'}{I'} t} \qquad (9)$$

or in general if $T_p = f(t)$

$$T = e^{-\frac{K'}{I'} t} \left[ \int \frac{K'}{I'} f(t)\, e^{\frac{K'}{I'} t} \quad dt + K_1 \right] \qquad (10)$$

If $T_p$ is a constant value, then

$$T = T_p - \frac{T_p - T_1}{e^{+\frac{K'}{I'} t}} \qquad (11)$$

when the wheel is accelerating, and

$$T = T_p + \frac{T_1 - T_p}{e^{+\frac{K'}{I'} t}} \qquad (12)$$

when the wheel is decelerating.

The time constant $\frac{I'}{K'}$ in this case corresponds to $\frac{I}{K}$ in the previous discussion. $K'$ is the torque per rev. per min. drop in speed and $I'$ is the torque necessary to accelerate the wheel one rev. per min. per unit time.

In other words $\frac{I'}{K'}$ is the time required for the motor when exerting a constant torque, (assuming that this be possible) to accelerate the wheel over a range of speed which is equal to the drop below synchronous speed necessary to produce this torque in the motor.

When an induction motor is used in conjunction with a slip regulator the current to the motor is limited to a fixed maximum value. Accordingly when the maximum torque of the motor has been reached equation (3) is modified to read

$$P = K'' n \pm I n \frac{dn}{dt} \qquad (13)$$

the solution of which gives

$$t = \frac{I}{K''}(n_1 - n_2) - \frac{I}{(K'')^2} P \times \left( \log \frac{P - K'' n_1}{P - K'' n_2} \right) \qquad (14)$$

$K''$ is the ratio of the h.p. output of the motor to its speed in rev. per min. when the slip regulator comes into action.

If we are working with a torque-time diagram, equation (8) becomes

$$T_m \pm I' \frac{dn}{dt} = c_1 + c_2 t \qquad (15)$$

from which

$$\pm (n_1 - n_2) I' = (c_1 - T_m) t + \tfrac{1}{2} c_2 t^2 \qquad (16)$$

When a flywheel curve has been plotted, the losses in the slip regulator may be obtained, the average value of which determines the size of the regulator. The losses in the driving motor are added to the ordinates which give the output of this motor and from the area of the resulting curve are found the required units per ton. In plotting flywheel curves, or in solving any of the equations given in which exponentials occur, the use of a log-log-slide-rule is recommended.

## DISCUSSION ON "SOME CONSIDERATIONS IN DETERMINING THE CAPACITY OF ROLLING MILL MOTORS, (HAMILTON), CLEVELAND, OHIO, MARCH 8, 1918.

**A. M. Dudley:** One thing that occurs to me in connection with this paper is the effect of the temperature of the material in the roll upon the power required to roll it. It is evident that the layout of some mills allows handling the material more rapidly from the furnace to the finishing pass, and hence operates with minimum power consumption so far as temperature is concerned. There are enough similar items to considerably modify an estimate of the power requirements, as determined in this paper.

Also, the personal characteristics of some operators demand a greater margin of available power than others. Ever since iron has been smelted and forged and worked by man, the processes have been handed down from father to son, and the successful mill application has been regarded as more a question of experience than scientific investigation. Nevertheless, it is entirely possible to make a fair preliminary analysis of the power requirement by some such method as that given by the author, and there is need for further data along the same line. The only work that I can recall on this subject is the one by Dr. Puppe published several years ago.

It is not expected that a new mill will be completely laid out entirely along theoretical lines as to power consumption, but a fair estimate of the total power required is very useful for other problems in the installation. There are two considerations that should be always kept in the foreground in applying electric motors to mill drive. The first of these is that *reliability is the prime consideration*. No chance must be taken of a shut down caused by an overloaded motor. In the nature of the work, a shut down in an iron or steel mill is more serious than in almost any other industry. The cost of power in making steel cannot in any case exceed 25 per cent of the total cost of production, and is lower rather than higher. If this is the case, a loss in all-day efficiency of 2 per cent or 3 per cent, due to the main driving motor being somewhat under loaded, is negligible, if it makes for greater reliability of operation. This consideration alone demands a liberal factor of safety in selecting the motor. The second point is that the electrification of a mill usually increases the output considerably, and the increased output in turn demands more power in the motor to take care of it.

I have in mind a mill laid out for ten thousand tons of steel a month where fifteen thousand tons is regularly rolled, and a peak has been reached where twenty thousand tons were rolled in a single month. The original motor was applied with this possibility in mind and has proven adequate. When orders are in hand and production is rushed, the only limit is the mill itself, and the maximum tonnage that can be crowded through the mill is the load the motor will be called upon to handle. Both of these considerations call for great liberality in the selection of the motor after the power requirements for the nominal tonnage are determined.

**C. A. Menk:** I agree heartily with what Mr. Dudley has just said. About a year ago we were called on, with about two hours' notice, to determine on the size of a motor to do a certain amount of work in a mill which we expected to build in four or five months. The time was short, and we did not have the data at hand which would tell us just the size of the motor we should adopt.

We went to the motor manufacturing companies, in a hurried way, and got their ideas, and those we put with what data we had, it was very little, largely being from steam-driven mills. One man would say—"You can do that" and would show us where we could do the work with a 2000-h.p. motor. Another man would then come along and undertake to show us that it would take a 3000-h.p. motor to do the same work. Then still another would come along, and he would say—"It will take a 4000-h. p. motor to do that work" There must be something lacking in the engineering departments of the manufacturing companies, when they figure on this class of work.

Probably a great deal of the trouble is due to the fact that they do not take into consideration the material you have to roll, the temperatures, etc.

To be safe, we decided we would build a mill that would roll 15,000 tons a month. That was settled on, that was what they wanted. You see, when they made up their minds, that that was what they wanted to roll, we had to roll it. We selected a motor of 4000-h.p. The mill has been operating for about four months, with an output of about 15,000 tons, and then they increased the output of the mill about 25 per cent, so you can see where we would have been if we had taken the 2000-h.p. motor.

I think we should rate these motors liberally, so you can always be safe in doing just about 50 per cent more than you ever expected to, because you will be called on some time to do that extra work. You cannot replace the motor, but you can replace the mill. You can tear out the foundations and go ahead and build in a larger foundation. If you break a couple of brackets made of cast iron, just turn around and make them out of steel. So you see, I think, when you start on the power end, you do not make a mistake in any way, shape or form by putting on plenty of capacity, because you will need it. If four furnaces do not do the work and keep the mill operating fully, build another furnace, so it is strictly up to the driving unit, because that mill is sure to operate, and turn out steel, and if you try to save on the motor you are trying to save on the wrong end.

**N. W. Storer:** I believe it is wise to put in motors amply large for the work, especially if it is to go into steel mill work where so little reliable calculation can be made as to requirements.

There was one point in the remarks of the gentleman who has just spoken, Mr. Menk, in regard to the 4000-h. p. power motor, that I desire to refer to. He says that they increased the output 25 per cent, which is undoubted, but the point is, how much of a load is the motor taking now with that increase? It might only have been taking 2000 h. p. previously, and is now getting that 25 per cent more. It would be interesting to know just how much power is taken by the motor and what factor of safety remains.

**C. A. Menk:** In the steel mill it is a question of doing the work. In a plate mill you heat the slab and bring the slab up to the mill. The first time the slab goes through the mill is when you get big load, and from that time on the peaks go down. At the first big load, or maybe the second pass, or the third pass, the motor may run up to 4500 h. p., but by the time the slab is finished it runs down to 2000, and that is where the large motor comes in, with a good big liberal figure, to allow for these variations.

**Wilfred Sykes:** I wish to take issue with the author on the same line as some of the other speakers. First, on the general classification he says: "In any rolling-mill electrification the paramount problem is the delivery of a specified tonnage of steel, of which the grade and section are known, at a minimum total cost." I have yet to meet the case where we really know the requirements. We always start off with certain ideas as to what is desirable to roll, and these ideas, as a rule, do not match up at all with what we actually get when the installation is made.

Now, the basis of calculating these things is all right. You have got to have a start somewhere, and for many years I have been conducting experiments and tests to determine a basis for calculations of this kind. We started in about nine years ago, and we are still testing and still have a great deal to learn.

After all, these tests give you certain unit values, but you have to use judgment in applying these unit values to your problem. I do not believe it makes a great deal of difference, what particular method of calculation you use, so long as you talk in the same terms all the time.

Now, this question of using logarithmic elongation and all that sort of thing has been discussed quite a lot amongst steel engineers, and I have not yet been able to see that it made any particular difference, so long as you plot your tests in the units that you use.

I do not believe anybody who has been through this business would attempt to use the data gotten from, say, a blooming mill and apply it to a rail mill. The only way I ever found that I could get any results that were at all consistent with actual operating figures was to get tests on a blooming mill, if I was dealing with a blooming mill, and not get tests on some other mill, such as a plate mill or rail mill, and use them as a basis for my calculations on some other mill operation.

Mr. Menk brought out one point very well, and I would like to enlarge upon it a little bit. We start off with some ideas as to what the demand is to be. Take the case of this plate mill, we want to roll 15,000 tons a month. That 15,000 tons a month is absolutely nominal, and not a real figure. What they will roll is all they can put through the mill, if they have the business, and if we are to get anywhere, and successfully apply motors to mill drive, we must study not only the figures given us by the superintendent of the mill, but all other data that are available. The difficulty has been, however, to get steel mill people to put these things down on paper. You can get all kinds of statements based on some plant and the number of tons of steel they roll. We are putting in a better mill than they have and we should get much more, but to get that down to a time study, or something of that kind, is extremely difficult. The electrical engineer has been forced to come in and study this problem.

The only use all this calculation would be is as a guide to judgment. These conditions in the steel mills are not subject to definite schedules, etc., as outlined in a paper of this kind. Take the blooming mill, there the passes are not fixed, you are in the hands of the operator, and the operator can run the mill pretty well any way he wants to, and usually it differs from some other mill, being possibly an advance in the art of mill building—and then you have to face new conditions, and your success depends on the value of your judgment. We find sometimes that judgment in these cases is on a par with a pure guess, but the thing you are up against is they will put through, if they have the business, everything that mill can handle. If the limit of it is in some mechanical part of the mill, that will be strengthened and eventually it will be put back on the motor or engine.

In the development of engines the work has been empirical. They find a certain blooming mill had a 48 in. by 56 in. engine, and the mill stood up, but the engine let go. Next they put in a 55 in. by 60 in. engine, and when they get to a point when everything lets go simultaneously, and the parts cannot be strengthened up any more, then the engine was all right.

That has been going on ever since we began rolling steel, and the mill engineers in this country have reached a point where, if they have a blooming mill, they can pretty well tell what kind of engine it should be, without any calculation at all. They know with a certain size blooming mill it takes a certain size engine to push the metal through. As the art progressed they made the parts larger, and got to a point where the engine would hold together. That, however, is not absolutely true, because in the case of some of the largest engines which were put in, it came to a point where the crank shafts could not be made larger on account of forging facilities.

When we came to motors, starting in that subject anew, we had to go through more or less of that same experience. If we are good engineers, we try to gather all the data we can and utilize the information given us by the steel mills, and are enabled thereby to form a fairly good judgment as to what the equipment should be.

But the point I want to bring out is that the calculations are only the first step. These figures must not be accepted at their face value. The engineer who believes in figures will not get very far. They are a guide to judgment only, and experience must be gathered in different plants to determine the actual size of the motor to be used, and in a case such as Mr. Menk mentioned, where the capacity increased 25 per cent, I am willing to predict when the plant has run another year, it will be another 25 per cent, and it may be five years from now the motor will be the limiting feature. This is our normal experience, and if for instance the tables are slow they will be speeded up and they will change all the small parts, and eventually get to a point where the mill will break or the motor will reach its limit.

That is the condition met with in steel mills.

There is one other condition we meet, and that is when we change over from steam to electric drive we introduce different characteristics in the operation of the mill, which people find out and utilize, so if the motor has more favorable characteristics, which make a larger output possible, they will forget what the engine did and establish a new standard for the motor.

The question of rating has caused a good deal of confusion in the past. We started out, frankly not knowing anything about the subject, and rated the motors 35 deg. continuous, and 25 per cent overload continuous, with 50 deg. temperature rise, and I think what we want to do is to stop fooling ourselves on the question of ratings. We have gained experience in the meantime, and we ought to face the fact that if it takes 1250 h. p. to drive the mill, we ought to have a 1250-h. p. motor to drive it, and not call it 1000-h. p., because it is easier to get through the purchase of a 1000-h. p. motor, because it does not sound so much. We ought to face the situation and try to arrive at what is the right equipment, taking into consideration not only the problem as presented by the builder at the time, but also what it is possible to put through the mill, because it is the business that will actually fix the character of the equipment sooner or later, and not the nominal figures that are so common in many calculations of this kind.

*Presented at the 338th meeting of the American Institute of Electrical Engineers, Cleveland, Ohio, March 8, 1918.*

# SELECTION OF STEEL MILL AUXILIARY MOTORS AND CONTROL AS AFFECTED BY MECHANICAL FEATURES OF THE DRIVE

BY J. D. WRIGHT

ABSTRACT OF PAPER

The author describes manipulators for blooming mills, which consist of side guards and lifting fingers, the former being used to guide the bloom into the proper groove in the rolls while the latter are used for turning the bloom over. The functions, mechanical layout and operation of these manipulators are described, from which conclusions are drawn as to the size and type of motors as well as the type of control best suited for driving these auxiliaries.

USUALLY the first questions which one is likely to ask when considering the proper type of motor and control for any steel mill auxiliary machine, are, "What is its function; what is the mechanical layout, and how does the device operate?" Without a thorough knowledge of the answers to these questions one is unable to make an intelligent recommendation on the application of either motors or control.

When discussing Standardization of motors and control at a meeting of the A. I. and S. E. E. in Pittsburgh about a year ago, Mr. Friedlander, of the Carnegie Steel Co. said, "What we need is more papers descriptive of installations and operation of our mills. In such a way we would gradually get uniform ideas about general requirements at all our mills and thereby drift toward a stable system of standardization." The PROCEEDINGS of the A. I. E. E. and A. I. and S. E. E. contain very few papers describing installations and operation of steel mill auxiliary apparatus and the writer heartily agrees with Mr. Friedlander's remarks regarding the need of such descriptive articles.

It is, therefore, the purpose of this paper briefly to describe a few steel mill auxiliary machines with particular reference to their mechanical details.

## MANIPULATORS FOR BLOOMING MILLS

Manipulators for blooming mills consist of side guards and lifting fingers. The function of the former is to guide the bloom into the proper groove in the rolls while the latter are used for turning the bloom over.

A reversing mill is usually provided with four moving side guards two on each side of the mill, one guard on the side of the mill with the operators pulpit being provided with the lifting fingers. Occasionally a set of lifting fingers is provided on each side of the mill, but as the bloom is usually turned only on the pulpit side, fingers are more commonly provided only on that side.

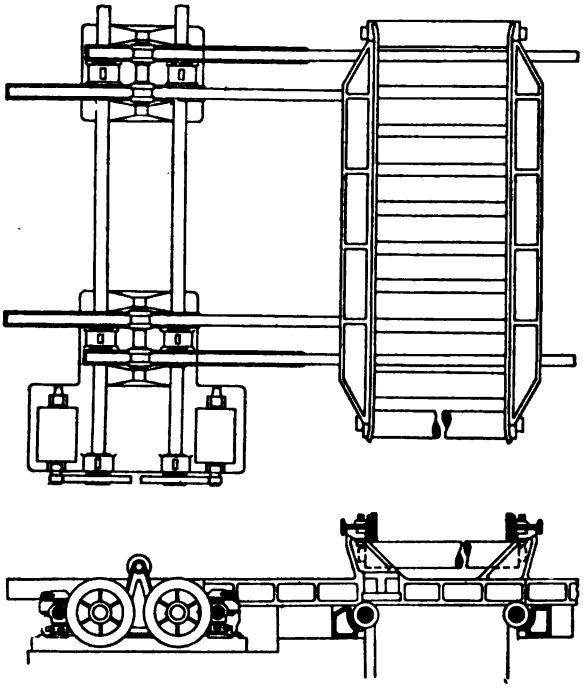

FIG. 2—OPERATING MECHANISM FOR MANIPULATOR SIDE GUARDS

Fig. 1 is a general view of the entering side of a reversing blooming mill equipped with electrically operated manipulators. The two side guards are clearly shown and, on close inspection, the lifting fingers may be seen just inside the left hand guard.

Each side guard on the entering side of the mill is connected

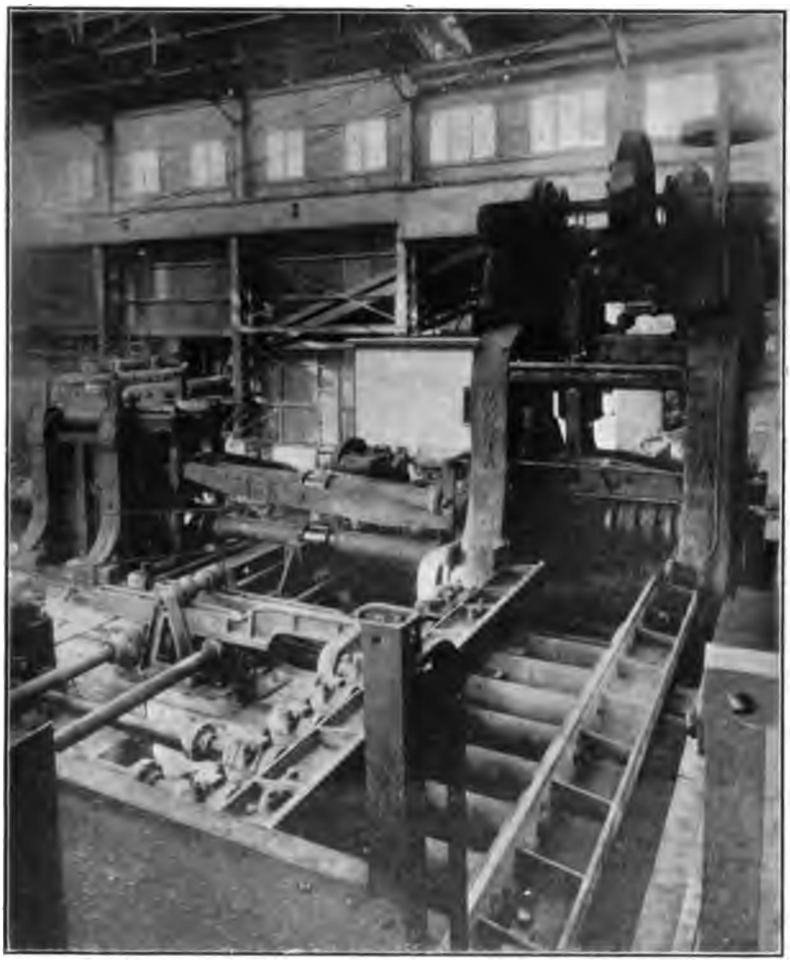

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FIG. 1—BLOOMING MILL WITH ELECTRICALLY OPERATED MANIPULATORS

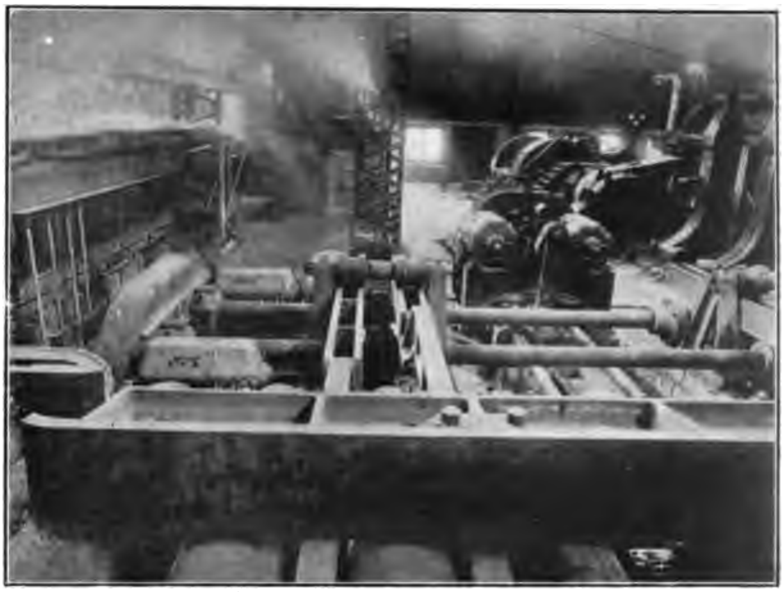

FIG. 3—GENERAL VIEW OF OPERATING MECHANISM FOR MANIPULATOR FINGERS AND SIDE GUARDS

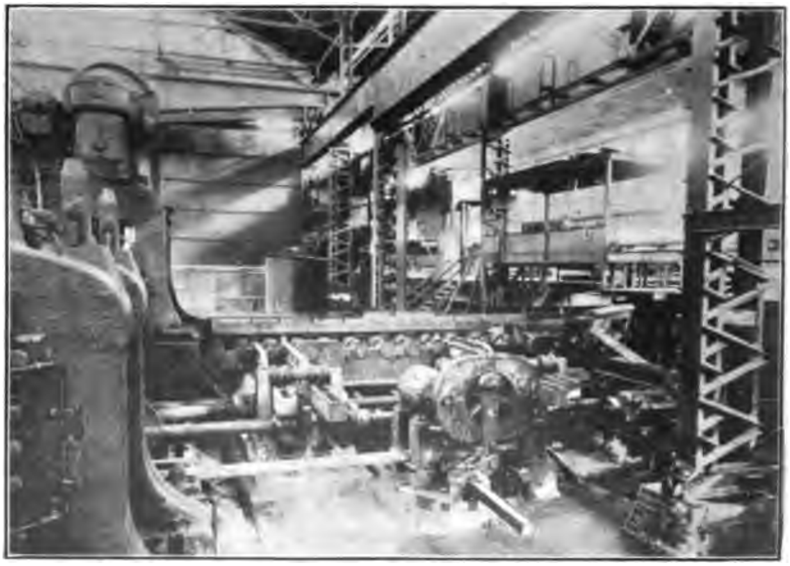

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FIG. 5—VIEW OF BLOOMING MILL WITH ELECTRICALLY OPERATED MANIPULATOR FINGERS AND SIDE GUARDS

by suitable mechanical means to the corresponding guard on the delivery side, so that the right hand and left hand guards on opposite sides of the mill always have the same movement and are always in line.

Fig. 2 shows one method of mechanical connection between the motors and the side guards. Two motors, one on each side of the mill, are geared through slip clutches to a common shaft on which are mounted four pinions which engage with four racks. Two of the racks are attached to one guard on the entering side and the other two to the corresponding guard on the delivery side of the mill.

The remaining two guards on opposite sides of the mill are similarly operated by a second pair of motors. The racks are supported by rollers and mechanical stops are provided to limit their outward movement.

Fig. 3 is a view of the same mill shown in Fig. 1 but gives a more detailed view of the side guard mechanism.

As previously stated, the function of the side guards is to guide the bloom into the proper groove in the rolls. The operation of the guards during the process of rolling an ingot consists, therefore, of many fast, short movements back and forth across the table. Each guard on one side of the mill can move independently of the other and can make a stroke equal to the width of the table. It is possible therefore for the guards when moving inward to come together or against the ingot at full speed. It is the function of the slip clutch to prevent damage as a result of such action should the operator be careless enough to allow it to occur.

The motor equipment for the side guards illustrated in Figs. 1 and 3 consists of four 80-h. p. series motors, two connected in series driving each pair of guards. Satisfactory operation of the guards requires rapid acceleration and retardation, and for such service series motors are most desirable. Some installations, however, use compound wound motors, primarily because dynamic braking for stopping was desired. Furthermore, solenoid brakes are sometimes used to assist the dynamic braking. In the equipment illustrated, neither dynamic braking nor solenoid brakes are used. When a quick stop is desired the motors are plugged. The equipment is therefore extremely simple.

The control for each pair of motors is of course reversible and is arranged so that a bank of resistance is permanently connected in series with the motor armatures. This acts as a buffer and reduces the shock when the guards jam together or against the ingot. A one-point reversible master controller is used for each pair of motors. Limit switches are provided for cutting off power when the side guards are moved to the edge of the table. These limit switches are usually of the track type and are operated by movement of the rack. A geared switch might be used but the track type switch is much simpler and more positive in action.

The lifting fingers, which are used for turning the bloom over, are carried by a frame attached to the two inside racks connected to one of the side guards. They therefore move back and forth with the guard. See Fig. 4. The fingers are located just inside the guard and their lower ends are pinned to a yoke which is suspended from two horizontal shafts by means of two links and levers. The two horizontal shafts, which are supported by bearings in the finger frame, are connected through cranks to a common driving shaft. On this shaft is mounted a large gear to which the two driving motors are geared. It is evident that one-half revolution of the main driving shaft in either direction from the position shown in Fig. 4 will produce a complete up stroke of the lifting fingers. Another one-half revolution would produce a complete down stroke and return the fingers to their original position.

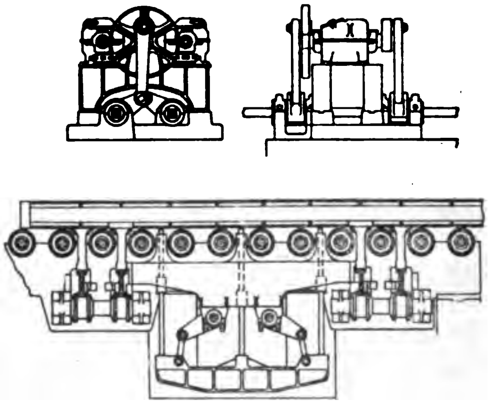

FIG. 4—OPERATING MECHANISM FOR MANIPULATOR FINGERS

The motors may therefore be either reversible or non-reversible. Successful installations of each have been made. When a non-reversing equipment is used, one complete cycle consists of an up and down stroke of the fingers. When a reversing equipment is used forward direction of rotation may produce the up stroke and reverse rotation the down stroke.

The motor equipment illustrated in Figs. 3 and 5 consists of two 50-h. p. series motors connected in series, operating non-reversing. For each throw of the operator's master controller a complete up and down stroke of the fingers is produced. The motors are automatically stopped at the end of the down stroke by a limit switch which cuts off power and completes a dynamic braking circuit.

The dynamic braking connections for a series motor operating non-reversing are comparatively simple. Where reversible control is used, a compound wound motor is preferable if dynamic braking is desired, as the control connections are simplified. Solenoid brakes are required if the equipment is reversible, whereas if non-reversible, no brakes are necessary. It has at times been stated that the operator should be able to control the stroke of the fingers. This of course necessitates the use of reversible control, but the movement of the fingers is so rapid that it very frequently happens that the fingers make a complete up stroke before the operator throws his master controller to the reverse position to lower them.

It has been shown that for the side guards shown diagrammatically in Fig. 2 a reversible equipment must always be provided. The motors may, however, be either series or compound wound. Dynamic braking and solenoid brakes may or may not be used. The operator's master controller may be one-point reversible or multi-point reversible. Track-type limit switches or geared limit switches to limit the movement of the side guards may be used.

For the lifting fingers in Fig. 4 it has been shown that the equipment may be either reversible or non-reversible. The motors may be either series or compound wound. Dynamic braking and solenoid brakes may or may not be used. The operators master controller may be one-point or multi-point reversible or non-reversible. The limit switch may be arranged to stop the motors when the fingers are at the end of their down stroke if the control is non-reversible, and if reversible the limit switch may be used to stop the motors with the fingers in both up and down positions.

There must, however, be one method of operating these devices which is superior to all others. What this method may be can be determined only from actual operating experience, and it is to the steel mill electrical engineers that we must look for the results of such experience The writer sincerely hopes that Mr. Friedlander's suggestion may be followed and that we may have more papers on installations and operation of auxiliary drives.

## DISCUSSION ON "SELECTION OF STEEL MILL AUXILIARY MOTORS AND CONTROL AS AFFECTED BY MECHANICAL FEATURES OF THE DRIVE" (WRIGHT), CLEVELAND, OHIO, MARCH 8, 1918.

**E. Friedlaender:** I prefer using two standard mill-type motors, connected in series for quick reversing work, in place of one special low-speed motor which is hard to get during these times and also quite expensive.

To be able to raise and lower these heavy lifting tables quickly and accurately, it is important that they are properly counterbalanced. Our table operators demand that the controller allows them to stop and reverse machine at will in any position; this in order to gain time and avoid wrecks.

**C. A. Menk:** I was very much interested in Mr. Wright's presentation especially where he showed all the mechanism, with a 150-horse power motor operating the side guards and the lifting arrangement. It all looked to me as if they were going to extremes.

I spoke this afternoon on large mill motors. We have that very same mill equipped and doing that same work, and we have practically the same problems, the tilting, etc., and we do it all hydraulically, using one 40-h. p. motor, and getting all the service we want out of the mill. It looks to me as if in installing all of that lifting mechanism, we are going to extremes in what we are trying to accomplish. I may be wrong, but I would imagine the expense of upkeep would be considerable.

As to the tilting arrangement for unloading the tables, I cannot see any particular advantage in that, especially in connection with the plate, for in turning out plate you are probably rolling on a thousand different orders a week. Now, you have got to sort that product out, so you can make shipment in carload lots and you must pile it up all over the floor until you get a carload. The mill operator may have nine different places where he is going to handle that table. He would have to start right away and assemble and sort out right from those different piles.

I cannot just see the possibility of using nine different places to assemble these plates and get them ready for shipment. It may be done where you roll one class of product for cars, or in case of ship plate, to continually roll that one class of material. But I see every day,one plate rolled for a ship and the next day one rolled for a boiler, and the next plate rolled for a car, and so on. You look over the shop floor and you find it covered continually with small piles of plates, and some of it would be there for a week before you can ship it, and some longer than that. So it seems to me the unloading arrangement might be all right, in some cases, but for a mill where you expect to cater to the trade and furnish everything you get an order for, it would be almost impossible.

Also, in regard to the tilting table, I cannot see why they want to lift that whole table from the one end. There is no reason why that table, as far as I know, cannot be balanced in the middle and could balanced with a very small amount of power. Some of the slides showed the double hinge at one end, if I am not mistaken, the motor at the other end, practically lifting half of the table, whereas if it was balanced in the middle, all you would need to do would be to over balance it to get the desired results.

There are many of these things which I believe can be brought out by a very careful study, and especially in the elimination of excessive mechanism in order to try to accomplish something that you can do with a very small amount of power, even if you have to go back to the motor-driven hydraulic pump.

**T. E. Tynes:** I want to bear out what Mr. Friedlaender mentioned about standard motors in series rather than one special motor of the same speed. I had occasion to look into a problem that required a 200-h. p. motor to drive a table. We found by using two standard-speed motors connected in series that the cost of the two was little more than the cost of the special motor, and we had the benefit of two standard motors in series, for which we had all the necessary spare parts, and were not obliged to buy spares for that drive.

**H. S. Richardson:** I was interested in Mr. Wright's citation of the case of the controller with the manipulator sideguards. In most of the cases I know of, brakes have been used, as there is a great deal of momentum in these side guards, and it seems to me that if brakes are omitted it means that every motion of the side guards has got to be stopped by plugging. That is perfectly safe and is used a great deal, but on account of the very frequent and short moves of these side guards it is putting on the operator additional work that can be eliminated by putting on larger brakes. It does not necessarily complicate the mechanism, except by the addition of the brakes themselves which, of course, are series wound and are set electrically.

Mr. Wright also mentions in his paper a permanent resistance in the side guard motors, which act as a buffer. It seems to me it is preferable to get that result by the use of a torque-limit, or jam relay, unless the mill is laid out and the motor gearing designed with that idea in view. I know of one case where the permanent resistance did not work satisfactorily, because they could not get the torque and the speed. The torque-limit relay, simply as an over-load relay, inserts the resistance permanently in the circuit when the torque exceeds a certain amount.

With regard to the manipulator fingers, Mr. Wright made mention of the fact that they could be either non-reversing or reversing. In the case where they are reversing, the up motion is secured in one direction—the up motion of the fingers—and the down motion in the opposite direction to the motor. It is a very simple matter to get the up and down motion in each direction of the motor, by simply allowing the crank to make a complete return for one up and down motion, and the next cycle is in the opposite direction. In a few cases that has simplified the equipment from the operator's standpoint and it is not at all necessary to use a brake for that reversing operation. In fact, it is much more satisfactory without the brake, providing the finger mechanism is not counter-balanced, but has the natural position of the fingers down, and simply make one complete turn in one direction, and the next turn in the other direction.

Another point in connection with the fingers. There is scarcely any installation, I believe, where it is not sometimes necessary to reverse the finger motor. Very frequently the fingers stick in their bearings, sometimes from expansion by heat or because of shale, and it is desirable to be able to reverse these fingers.

I think the problems that have confronted electrical engineers on the manipulator fingers and side guards, at least in the earlier designs, were made considerably harder by the fact that so little gear reduction has been put between the motor and the load. I know of one case where the travel of the side guard is 72 inches, and the motor made 6.7 revolutions for that complete travel. Also, one of the hardest problems on the side guards is the stopping of the motor, because it is, of course, necessary to clear the pass, and there is generally less than an inch or two, sometimes only a fraction of an inch, clearance between the position where the side guard clears the pass and the final positive stop

If the mechanical engineers would only appreciate these points and give us a little more clearance, it would simplify matters very much. It is a hard thing to get the extra clearance on the side guards, particularly if the side guards have to extend inside the mill housing. I have met a number of cases where, if the electrical engineer had only pointed out to the mechanical engineer what the problem was going to be, he could have very easily had an extra clearance.

**B. G. Beck:** I would like to ask Mr. Menk if he has charge of all of the hydraulic machinery in the winter time? I have charge of that, and it is quite a job. I am a great believer in the idea that if a thing can be done at all it can be done electrically better than any other way.

**G. E. Stoltz:** In reference to the high-speed and low-speed motor, for the various mill auxiliaries—the mill-type motor, as applied, with small diameter and a low inertia, does not leave as much to be gained by going to the low-speed motor as would be expected. In fact not more than 10 or 15 per cent is gained in the current consumed to perform the same operation by installing a special low-speed motor, which is not a duplicate of other motors in the mill, and which, naturally, is harder to handle in case of repairs.

Mr. Wright has mentioned several cases where motors may be used in either parallel or series. I am sorry he did not give the results attained in using the motors according to the two different schemes. It is simply a matter of whether the application is such that the inertia of the motor is a large percentage of the work performed. On a screw-down, the inertia of the motor is a large part, and in order to reduce that part of the load, the motors are placed in series to reduce their speed to one-half. On drives such as main tables or heavy side-guards, this is not the case, and it would be better to place the motors in parallel, if it is desired to obtain the highest rates of acceleration and operation. However, if no effort is made to obtain this quick acceleration, it is naturally easier on the motors to have them placed in series.

I have in mind a case where an effort was made to bring a large mill table up to speed in one and one-quarter seconds, by placing the motors in parallel. A speed of 400 ft. per min. on the main table of a 40-in. mill was obtained in this time with two mill-type motors in parallel. On the side guards a speed of 200 ft. per min. was obtained in one second. We should not overlook the fact that on many applications, the inertia of the motor is rather a small part of the load and a much faster mill will be obtained by placing the motors in parallel.

The question has been brought up as to whether brakes should be placed on side guards. Most of these applications of side guards and screw-downs are inching operations, where it is important to have a quick and accurate stop, particularly when a man is using the side-guard to line a piece up with some particular groove in the mill. This can be obtained to a much greater degree of accuracy by having compound motors and dynamic braking. I think most of these motors are almost always brought to rest before the brake sets, although it is an advantage to have a brake take effect just about the time the motor stops, when the dynamic braking is becoming very weak, so that probably the last operation is obtained by a combination of dynamic braking and shoe brakes.

Another man has mentioned the fact that a jamming relay will be of some advantage on side-guard motion. The operation of a side-guard is so quick that a jamming relay will not have time to act. A better plan is to place permanent resistance in series with the motors, to limit the current to approximately 200 per cent, with the motor at stand-still. With this amount of resistance, the speed of the motor is reduced so that in laying out the mill this should be taken into account.

A number of gentlemen have asked whether or not it is feasible to use reversible control on the fingers. Ordinarily it is not required, but as the last speaker mentioned, very often operators have trouble with the fingers, in which case they want to reverse the fingers, and for that reason reversing control is usually applied. I know of one mill where the operator became so expert in manipulating the fingers that he simply throws on the switch to the forward position, allows the fingers to turn partly over, and turns them back quickly, without allowing them to make a complete cycle.

**J. D. Wright:** Regarding the use of jamming relays on side guards, I believe the installation I showed was equipped with a jamming relay to start with, but the electrical engineer in that particular plant found that unsatisfactory, and put a permanent resistance in series with the motor armature, cutting out the jamming relay, and more satisfactory operation was obtained.

The question of gearing is a very important one which I think electrical engineers overlook. In one particular instance I know of some side guards that were geared for a running speed of 600 ft. per min., which is ten ft. per sec. Now, the total travel of the guard is not more in ordinary cases than 6 ft., so you can see how absurd it is to arrange your gearing for such a high speed as that.

I think what the electrical men ought to do is to get together with the mechanical men more and study these problems from the mechanical standpoint, and in that way the mechanical men can get the benefit of the electrical men's experience in working out the problems satisfactorily.

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# INDUCTIVE EFFECTS OF ALTERNATING-CURRENT RAILROADS ON COMMUNICATION CIRCUITS

BY H. S. WARREN

## ABSTRACT OF PAPER

A brief discussion of inductive interference in general is first given, including reference to the work of the Joint Committee on Inductive Interference in California. Inductive interference due to electrified railroads is then taken up and various possible means for reducing such interference considered. A description is given of four important installations of railroad electrification and the specific means adopted in each case for preventing interference, with the degree of success which has been met with.

## INTRODUCTION

TELEPHONE and telegraph companies in the conduct of their business not only have to maintain their lines and service against ordinary forms of interruption, such as lightning disturbances, mechanical failures, etc., to which all overhead electrical lines are inherently subject, but also, they have to see to it that their lines are protected against interference by electric power lines. Such interference may be due to actual or threatened physical contact between wires of the two systems, to passage of current from one system to the other by leakage, or to the class of disturbances known as "inductive."

One important kind of inductive disturbances on telephone and telegraph lines is that arising from installations of alternating-current railroad electrification, such installations being principally employed in connection with trunk line railroads carrying heavy traffic.

In approaching the subject of interference to communication systems by such electrified railroads, it seems desirable first to consider some aspects of the general subject of inductive interference from power lines, of which interference from electrified railroads is a special case. In this general discussion of the subject it will be convenient to borrow freely from the work of the Joint Committee on Inductive Interference in California.

## HISTORICAL

The induction of voltages in an electric circuit by current changes in a parallel circuit was discovered by Faraday in 1831. The property of self induction of an electric circuit was discovered by Joseph Henry at about the same time. The phenomenon of induced electrostatic charges was known already through various experiments. In 1838 Henry, in the course of his researches, observed that a current was induced in an electric circuit when a Leyden jar was discharged through a parallel circuit. This seems to be the first case on record of electric induction between circuits.

Since the time of Faraday and Henry a stupendous amount of electrical research and experimentation has been conducted and as a result of this and of brilliant theoretical work by Maxwell and others, the fundamental laws of electrostatics and electrodynamics has been very fully worked out. The general equations expressing the laws of induction are, however, not suitable for us in the solution of practical problems. On account of the large number of factors involved, many of which cannot readily be evaluated, it is generally necessary in specific cases to resort to simplifying assumptions and approximations.

As to what actually happens when a voltage is set up in an electric circuit by induction we know very little. It is well to bear in mind that we do not even know what voltage really is, or current, or electricity. We use these terms merely to express certain phenomena and relations found by observation, whereby we are able to derive results of much practical value.

*Characteristics Affecting Disturbances.* Telephonic currents consist of numerous component simple currents varying in frequency from about 200 to 4000 or more periods per second. The signaling currents employed on telegraph lines are also complex, but their most important components are of less than 300 periods per second.

Power circuits are commonly of either 25 or 60 periods per second but, in addition to the fundamental periodicity, harmonics are usually present to a greater or less extent and it is due almost wholly to the harmonics which come within the range of the most important voice wave components, that noise is produced in telephone circuits by induction. By care in designing electric power machinery so as to reduce the proportion of such harmonics, it is probably feasible to avoid much of the noise disturbance to telephone circuits which otherwise would result.

Thus a small expense in eliminating this trouble at its source may obviate a much larger expense later.

*Abnormal Conditions.* It is important to recognize that the inductive effects of power circuits are liable to be greatly magnified at times of abnormal conditions. The violent changes which occur in the electric and magnetic fields surrounding a power circuit, when one of its conductors breaks or becomes grounded, set up in neighboring communication circuits surges which may represent relatively large amounts of power. A parallel which, under normal operating conditions, causes no disturbance may produce very serious interference under abnormal conditions.

*Balanced and Residual Voltages and Currents.* In analyzing inductive phenomena it is advantageous to classify power circuit voltages and currents under two general heads: (1) Balanced voltages and currents, that is, those which are balanced or symmetrical with reference to the earth; (2) residual voltages and currents, that is, those which are wholly unbalanced with reference to the earth. The circuit of the residual voltages and currents is comprised of the metallic circuit conductors as a group, constituting one side, and the earth (including earthed conductors), constituting the other side.

At every instant the algebraic sum of the balanced currents, and likewise of the balanced voltages, is zero. Hence, at any instant the algebraic sum of the currents in the line conductors is the residual current and, similarly, the algebraic sum of the voltages to earth of the line conductors is the residual voltage. For example, in an ordinary railway circuit, consisting of overhead trolley wire and grounded rail return, all the voltage and current are residual.

There is no definite relation between these two classes of voltages and currents and entirely different means usually have to be taken in counteracting their respective induction effects. In general, the residuals cause more inductive disturbance than the balanced components as the residuals are all in phase and their effects are cumulative whereas the induction from the balanced voltages and currents in one conductor is partially neutralized by the induction from balanced components in each other conductor of the circuit. Also the residuals usually contain a larger proportion of harmonics than the balanced components and this is another reason why they are likely to cause more disturbance, particularly to telephone circuits.

*Causes and Preventive Measures for Residuals.* As residual voltages and currents are largely responsible for inductive interference, it is of great importance in the prevention of such interference that they be suppressed or reduced by all reasonable means available.

Star-connected, three-phase transformer banks with the neutral grounded will set up triple harmonic residuals due to variation of permeability of the iron with varying magnetic density and may also cause residuals by reason of inequalities of the transformer impedances. In general, the most effective measure against this triple harmonic effect is the use of a delta-connected secondary or tertiary winding, thus providing a shunt path for the triple harmonic currents. The magnetic density also should be kept as low as practicable. Residuals due to inequalities of impedances in transformer banks can, of course, be eliminated by equalizing these impedances.

Another condition which may produce large residuals is the use of generators with star-connected armature windings. When such a generator with its neutral grounded is connected to a power line, either directly or by auto-transformers, residuals are set up in the line. When such a generator, with its neutral not grounded, is connected to the line through standard transformers, residuals will be impressed on the line if the transformer bank is connected star-star, the line side neutral grounded, and the station side neutral connected to the generator neutral.

Grounding of transformer or generator windings at any points not normally at zero potential, unbalances an electrically connected circuit and thereby causes residual voltages and currents.

Another important cause of residuals, which however may not be so obvious, is the unbalance of a power circuit due to inquality of the capacitances to ground of its several conductors. If the three line conductors of a three-phase circuit are carried throughout in the same relative positions, that is, if the circuit is not transposed, the capacitances to ground will be unequal and a part of what would otherwise be balanced voltage and current becomes residual. These inequalities may be overcome by transposing the power circuits throughout their entire length. To make such transposition effective with respect to interference to telephone circuits it is necessary that the power circuits be transposed at intervals which are short in comparison with the wave lengths of the higher harmonics present. The frequency of transpositions depends somewhat, however, on the inherent unbalance of the conductor configuration. As an indication of the number of transpositions required for a reasonable degree of balance to ground, it may be said that barrels of 6 to 12 miles, the latter applying to a triangular configuration, are usually adequate.

It is evident that the symmetry of a line which has been thoroughly transposed may be destroyed by connecting to it branches or taps which unbalance the capacitances to earth of the line conductors. Of course, if such a tap is grounded, the residuals resulting may be very large.

At times of accident, when a power circuit is in an abnormal condition, residuals of relatively enormous values are liable to be created. These set up correspondingly large induced voltages in parallel communication circuits. This emphasizes the importance of high grade construction and maintenance of power lines involved in parallels so as to minimize the frequency of such occurrences.

*Unbalance of Communication Circuits.* Not only are there these two kinds of disturbing voltages and currents on power circuits, namely, balanced and residual, but each of these components may set up, in a neighboring metallic communication circuit, two different effects; (1) an induced voltage between the two conductors of the communication circuit, which directly tends to cause currents through the signaling instruments; (2) an induced voltage between the conductors of the communication circuit and ground, which by reason of unbalances in the communication circuit indirectly causes currents through the signaling instruments. Theoretically, assuming a telephone circuit and all its connected apparatus absolutely symmetrical, electrically, with respect to earth and always so maintained, voltages induced equally in the two sides of such a circuit would not cause noise in the telephone. In practise it is not possible to attain absolute symmetry although in well constructed and well maintained telephone circuits the degree of balance is very high indeed. The telephone is, however, such a very sensitive instrument that no attainable degree of balance can avoid noise when relatively very high voltages are induced between the telephone wires and ground, as is done in many cases by parallel power circuits. It is therefore essential that induced voltages to ground be limited to values which are permissible on communication circuits so maintained.

*Transpositions Within Parallels.* Interference by induction from balanced currents and voltages can most readily be prevented by means of a coordinated system of transpositions applied to both power and communication circuits within the limits of parallels, the term "parallel" being understood to mean the region within which the two classes of line are in sufficiently close proximity for inductive disturbances to be set up in the communication circuits by the power circuits. It is to be noted that transpositions for this purpose to be applied to power circuits within the limits of parallels, are quite distinct from the transpositions previously referred to as being necessary throughout the entire length of a power circuit in order to equalize the capacitances to ground of the several conductors.

*Principal Factors in Determining Interference.* Before leaving this general part of the subject I will enumerate the principal factors which determine the amount of induction and whether it is sufficient to constitute interference.

1. *The length of the parallel.*

Other things being equal, the longer the parallel, the greater the induced voltage.

2. *The separation of the two classes of lines.*

In general, other things being equal, the less the separation of the power line and communication line, the greater the induced voltage.

3. *Configuration of the power line.*

The investigations of the Joint Committee on Inductive Interference show that the configuration of the power line has an important bearing on inductive effects, the relative merits of different configurations varying with the separation of the power and communication lines, the spacing of the power conductors, and the relative importance of balanced voltages and currents. While it is not possible to draw a simple general rule for determining the most advantageous configuration the differences in particular cases are marked and deserve special attention as oftentimes substantial benefit can be secured in this way at small additional cost. This is particularly true of multiple circuit lines, the resultant induction depending largely on the relative poling of the power circuits.

4. *The magnitudes and fundamental frequency of the normal operating voltages and currents of the power circuits.*

The effect of electric induction, of course, is proportional to the voltage of the power line, and of magnetic induction to the current on the power circuit.

5. *The magnitudes of residual voltages and currents.*

It has already been explained that residual voltages and currents are a principal cause of inductive interference. Hence, while the amount of residuals on metallic circuits is usually small as compared to the balanced components, the inductive effects of the former are liable to preponderate.

6. *The wave shapes of both balanced and residual voltages and currents, involving the magnitudes and frequencies of all harmonics.*

The effect of wave shape on interference, to telephone circuits particularly, is exceedingly important. Wave shapes in practise on different power systems are found to have extremely wide variations. An unfavorable wave shape, *i. e.*, one having a large proportion of high harmonics, may produce a hundred times as much noise as a pure sinusoidal wave of fundamental frequency.

7. *The unbalances of the communication circuits, their magnitude, character and location.*

Such unbalances are caused by inequalities in resistance, inductance, insulation or capacitance to ground. The last mentioned quantity is balanced approximately by transposing the conductors. The other elements enumerated require proper design, construction and maintenance of these lines, whether in open wire or in cable, together with their connected apparatus.

8. *Terminal apparatus of the communication circuits and the distance of such apparatus from the parallel.*

The sensitiveness of the terminal apparatus is, of course, an important factor in determining the allowable amount of induced voltage. Also if the parallel is at a considerable distance from either terminal of the communication circuit, the induced voltages and currents may become considerably attenuated before reaching the receiving instruments.

9. *The voltages and currents of the power circuits under abnormal conditions.*

It has already been stated that the voltages and currents of power circuits under abnormal conditions, which are liable to be largely residual in character, produce the most severe inductive effects. The values of these quantities under abnormal conditions, in relation to corresponding values under normal conditions, vary a great deal on different power systems.

10. *The number of parallels which may effect cumulatively the same communication circuit.*

In many cases the same communication circuit, especially if it be on a long trunk line, may be involved in a considerable number of different parallels. In such cases the induction contributed by each parallel must be sufficiently restricted so that the cumulative results from all will not produce disturbances which cannot be endured.

11. *The importance and character of use of the communication circuit.*

It is obvious that the more important circuits, on which interference is most serious, should be afforded a higher degree of immunity from disturbance than circuits of less importance. Also, of course, the character of the communication circuit as, for example, whether it is a telephone or telegraph circuit, is of fundamental importance in considering the question of inductive interference.

12. *The volume of transmission on the communication circuit.*

In case of a long distance telephone circuit, where the volume of transmission is small, a less amount of extraneously induced current will interfere with receiving than on circuits of less length where there is a large volume of transmission.

13. *The relative cost of preventing interference.*

While in all cases means should be employed which will allow adequate communication service to be given, still it is not expected that complete freedom from inductive disturbances can be attained. Any induced voltage, no matter how small, will generally cause some impairment of service. The amount of induced voltage which it is justifiable to allow, depends to some extent on the difficulty and expense involved in further reducing such voltage. After the foreign voltage has been reduced to an amount which can, if necessary, be tolerated, it becomes simply a problem of balancing the value of further improvement against its cost.

It will be seen that the number of elements affecting inductive interference is quite large. Moreover, some of these elements, as for example, the wave shapes of the power circuit voltages and currents, are not ordinarily known, and have induction-producing values varying enormously in different cases. Hence the difficulty of formulating any simple method of determining in advance whether a given construction will or will not produce interference.

The foregoing discussion, while general in its application, is in many respects concerned with induction from power transmission lines. We will next consider specifically some of the inductive effects of alternating-current railroad installations.

### ALTERNATING-CURRENT RAILROAD ELECTRIFICATION

The reasons why alternating-current railroad electrifications cause large disturbances to neighboring communication lines, principally by electromagnetic induction, will now, I think, be apparent when it is considered that, (1) the railroad trolley current is large, (2) it is all residual current, (3) the railroad circuit, from its nature and use, is more subject to abnormal conditions, such as short circuits, than ordinary power transmission lines.

*Classes of Interference.* Some of the different ways in which disturbances due to alternating-current electrified railroads manifest themselves in the telephone and telegraph plant, may be classified as follows:

1. *Interference with operation.*
    a. Interruption of service
    b. False bell ringing
    c. Noise
    d. Interference with telegraph signals
2. *Physical injury to plant.*
    a. Fire hazard
    b. Magnetization of loading coils
3. *Hazard to employees and to telephone using public.*
    a. Electric shock
    b. Acoustic shock

These various disturbances may be of a most serious nature and telephone and telegraph companies are unable by themselves to cope with the problem of protecting their lines and service against them. In order to make this more clear, we will review briefly some of the fundamental characteristics of telephone service and point out some of the distinctive features of the plant required to make this service possible.

*The Telephone System.* The fundamental electrical problem of telephony is three-fold:

1. The production of an electrical wave which is a faithful copy of the spoken word.

2. The transfer of this wave without appreciable delay over distances which may amount to hundreds or thousands of miles, without excessive change of form by distortion, without the accession of foreign disturbances, and without undue loss of intensity.

3. The production at the receiver of an audible sound wave

which is an adequate counterpart of the electrical wave and, therefore, of the original spoken word.

As speech is carried on telephonically by means of an extremely small amount of energy, it is necessary that a large part of the telephone plant be of a sensitive and delicate construction. This includes the subscribers' sets where occur the delicate transformations from air wave to electrical wave and *vice versa*.

These substation instruments cannot be located at central offices where they would be under the immediate supervision of a trained staff but they must be placed in the subscriber's office, factory, home or wherever they will be most available and convenient for his instant use. There are now over ten million telephone stations in the Bell System. These sensitive nerve ends of the telephone system are distributed throughout the entire country in every conceivable variety of location.

In addition to the delicate substation apparatus, each telephone conversation requires the exclusive use of a connecting circuit. Even though the circuit be hundreds of miles in length it cannot be used for any other telephonic purpose. This exclusive circuit must be low in resistance, capacity and leakage so as not unduly to attenuate the telephone wave. It must be so transposed, balanced and protected that so far as possible it will not pick up electrical disturbances from earth currents telegraph lines or other telephone circuits or itself constitute a source of disturbance to the latter. The network of telephone circuits now comprises more than twenty-two million miles of wires.

In addition to meeting the above basic requirements, the telephone system, in order to realize its potentialities as a utility of the greatest benefit to the public, must include facilities such that at any time on request of a subscriber connections can be made between any two points, without delay or other inconvenience, and the charges for the service must be as low as possible. At present about thirty-two million such telephone connections are made per day in the Bell System.

Prompt, efficient and economical service on the existing scale requires that an immense number of separate circuits be brought together into common central offices and provided with every device and attendance which will facilitate traffic over the system. It requires, for example, that hundreds of wires be crowded into cables, the latest types of which have 2400 conductors within a sheath whose outside diameter is 2 5/8 inches. It requires great congestion of wires and apparatus in switchboards in order that many thousands of lines may be brought within reach of a single operator. It requires elaborate and reliable signaling arrangements to economize time and circuits. It requires uniformity in plant and methods throughout the entire system so as to make possible prompt connection between any two points. While it has been found practicable to devise means for transmitting the required signaling currents over the telephone plant safely, the danger of fires from the currents and voltages employed for signaling has been avoided only by the exercise of extreme care, although these currents and voltages are very small compared with the currents and voltages on power lines.

From this brief consideration of the telephone problem, showing that a large portion of the telephone system is inherently of a delicate nature and susceptible to interference, it is clear that telephone apparatus and circuits would be destroyed if but a small fraction of the powerful currents and voltages used by other electric utilities were permitted to enter into the telephone system.

*Values of Induced Voltage.* In studying the inductive effects of electrified railroads, it has been found advisable to determine approximately the amount of induced voltage in a communication circuit, per mile, per 100 amperes in the trolley, for different horizontal separations between the trolley and the communication circuit, and with different percentages of the trolley current in the rails. For example, it has been determined that, with 60 per cent. rail current (that is, 40 per cent of the trolley current return flowing in the earth as stray current,) the induced voltages per mile, per 100 amperes in the trolley, are in general about 10 volts, 5 volts and 1 volt, at 50 feet, 300 feet and 4000 feet separation respectively. Thus at 50 feet separation, with 1000 amperes in the trolley, a ten mile exposure would result in 1000 volts induced. These are maximum figures in that they are based on the assumption that power is supplied in one direction only. It should be understood that the induction varies considerably in different cases since the induced voltages are affected by all the various conditions which go to determine the course that the stray current takes. Some parallelisms may extend more than ten miles and at times of short-circuit the current may amount to many thousands of amperes, and, in such cases, the induction is liable to be correspondingly more severe unless preventive measures are taken.

The specific effects of these induced voltages will now be touched upon briefly.

*Interruption of Service.* Induced voltages may be high enough to operate the telephone protective devices and, if the current across the protector is sufficient, the line will become permanently grounded and the telephone service interrupted until the protector is restored to normal condition. If the protector is located in a central office, the time required to make repairs is relatively short, but if it is at a subscriber's premises, considerable time may be required for a repair man to reach the station. In cases where the operation of the protector does not actually ground the line, it may lower the insulation resistance, sufficiently to make the line noisy.

It may also sometimes happen that foreign voltage of a value below that required to break down the protector spark gap will yet be sufficient to puncture the insulation of the wiring at some point.

*False Bell Ringing.* Voltages of about 8, 20 and 200 volts, depending somewhat on the prevailing earth potentials, are sufficient to ring ordinary grounded bells, standard biased bells, and (by breaking down protector spark gaps) metallic circuit bells, respectively.

An accidental trolley ground on a 25-cycle single-phase electrification through a thickly settled community may ring scores or even hundreds of subscribers' bells, some of which may be located a mile or more from the railroad. Such false bell ringing is apt to be a source of serious complaint by subscribers, and is particularly annoying when it occurs at an unseasonable hour as, for example, 5 o'clock in the morning.

*Noise.* In order to appreciate the effect of small currents in producing noise in telephone circuits, it must be considered that a very small fraction of a microwatt of power at voice current frequencies will produce an audible sound in a telephone receiver and a few microwatts are sufficient for a telephone conversation in a quiet place. When the current in the telephone receiver caused by induction from outside circuits is large enough to produce an audible sound, it has an important effect on the efficiency of the circuit for transmitting speech, particularly when the circuit is used for talking over long connections so that the energy of the voice currents approaches the minimum which will give a satisfactory conversation. An extraneous sound which is scarcely more than audible to an untrained ear and might be thought to be of negligible consequence, has in reality, the effect of impairing a telephone circuit by a large percentage, or otherwise expressed, of destroying a material part of the circuit's value for service purposes.

The interfering effect of foreign current of a given magnitude depends very greatly, however, upon the frequency. The maximum effect is for current having a frequency of about 1100 cycles per second. At lower frequencies the effect falls off rapidly, and at 25 cycles is probably only about one two-thousandth as great as at 1100 cycles. This fact explains why the inductive interference to telephone circuits from 25-cycle railway systems is not predominantly noise. Twenty-five-cycle current normally has relatively very small components in the telephone-frequency range and the effect of these high-frequency components is damped out much more rapidly than that of the fundamental by separation of telephone and railroad circuits. Noise from such railways is, however, present to some extent, and is liable to become serious under any conditions which produce a bad wave shape in the power circuit.

*Interference with Telegraph Signals.* At 25 cycles, an induced current of one milliampere is liable, under some conditions, to interfere with ordinary Morse transmission, while rapid telegraph systems, printers, etc., are more or less impaired by extraneous currents of any value.

*Fire Hazard.* The use of heavy insulating coverings for wires in telephone switchboards is impossible on account of the necessity of bringing many lines within a limited space. It is not feasible to employ for this purpose such insulation as is considered good practise for electric light and power wires. Thus it is unavoidable that the dielectric strength of the telephone wiring be relatively low.

Investigations of the fire hazard due to foreign voltages impressed on telephone lines indicate that voltages of 200, or even less when backed by considerable power as in the case of induced voltages from alternating-current railways, create a distinct fire hazard.

Although the fire hazard brought about by railroad electrification is due chiefly to the higher voltages induced at times of short circuits on the railroad, it is possible that the repeated electrical stresses of lower voltage, due to normal railroad operation tend to decrease the dielectric strength of the insulation and thereby facilitate breakdown.

*Magnetization of Loading Coils.* Loading coils in very large numbers are now employed in both open wire and cable telephone circuits. These coils are liable to be permanently magnetized by any induced currents which are materially in excess of telegraph currents. While they are magnetized there is a considerable loss of transmission efficiency and it is ordinarily impossible to demagnetize them without removing them from the circuits. Moreover large currents through the loading coils may permanently reduce the permeability of the iron cores and make them unsuitable for use on long toll cable circuits.

*Electric Shock.* At times of short circuits on the railroad and sometimes during switching operations, electrical surges may be set up in the telephone circuits, which are of sufficient intensity to produce electrical shocks to persons at the telephone or working on the circuits at the time. While it is improbable that such shocks will be the cause of serious personal injuries, even minor shocks are objectionable and constitute a basis of complaint as the public expects telephone instruments to be perfectly safe at all times.

*Acoustic Shocks.* Inductive surges such as are capable of producing electric shocks to persons are also liable to cause loud noises in the telephone receivers which may result in acoustic shocks to persons using the telephone at such times. Even the relatively slight clicks which sometimes occur due to battery interruptions may be very annoying to telephone users and acoustic shocks sometimes caused by induced voltages may be much more severe.

*Investigations and Experiments.* Since 1905, when first notified of the intention to install single-phase electrification on a section of the New York, New Haven and Hartford Railroad, the American Telephone and Telegraph Company has done a large amount of work on plans, tests, experiments and studies of various kinds, most of it in conjunction with representatives of railroads and the electrical manufacturing companies, all with the general object of finding means for protecting the telephone and telegraph lines and service against interference from electrification installations. A considerable amount of work has been done in connection with various electrification projects which have not been installed, some of these projects having been abandoned, at least in so far as the specific plans under consideration are concerned, while in other cases the matter is being held in abeyance, awaiting more favorable conditions for undertaking construction.

## MEANS FOR PREVENTING INTERFERENCE FROM ALTERNATING-CURRENT RAILWAY ELECTRIFICATIONS

There are various means which have been proposed, some applicable to the railway system and some applicable to the affected communication systems, for preventing or reducing inductive interference. Some of these means have not been found successful or advantageous in practise while others have proved beneficial in varying degrees. It will be of interest to consider briefly some of these proposals.

*Separation.* The most effective means is to avoid the parallel, wherever practicable, by keeping the communication circuits and electrified railroads sufficiently separated. With the extension of electric traction, and the constantly increasing importance and efficiency of communication circuits, the avoidance of parallels will be increasingly important. However, this first rule for preventing interference, is unfortunately, not one which can be generally adopted in practise. Railroads and communication circuits must serve the same communities and it is necessary that the connecting routes of each be reasonably straight and direct. The field of influence of an alternating-current railroad which uses the running rails as a part of its circuit extends out to a great distance on both sides of the railroad. This makes effective separation from such electrified railroads much more difficult than separation from most other kinds of power transmission circuits.

*Neutralizing Transformers.* Where communication circuits are subject, under normal operating conditions of the railroad, to induced voltages sufficient to interfere with telegraph service, neutralizing transformers can be resorted to and if properly designed and connected into the disturbed circuits, such transformers will effect neutralization of a large part of the induced voltage. The transformers are provided with a plurality of windings, some of which, called primaries, are inserted in certain of the affected conductors which are grounded at or beyond the limits of the parallel, while the remaining, or secondary windings, are connected serially into other of the conductors in which the induced voltages are to be neutralized. Under favorable conditions the remaining, non-neutralized voltage, is only 5 to 10 per cent of the total induced voltage.

For a more complete description and discussion of neutralizing transformers reference is made to an article by Thomas Shaw in the *Electric Journal,* November, 1914.

Neutralizing transformers, however, have serious disadvantages from the telephone company's standpoint. The primary circuits, of which there are ordinarily from one-third to one-half the number of secondary circuits, are practically lost for telegraph purposes although they can be used for telephoning, at somewhat reduced efficiency. The secondary circuits are also reduced somewhat in telephonic transmission efficiency, but not so much so as the primaries.

Neutralizing transformers have served a useful purpose in the early stages of alternating-current railroad electrification where means of restricting the railroad's field of inductive influence were not employed. They continue to have a limited field of usefulness, particularly in making endurable moderate amounts of induced voltages which remain after preventive methods have been applied at the source of disturbance, but they leave the general problem of interference unsolved. They are not applicable to subscribers' lines nor have they been found effective in neutralizing the higher harmonics which cause noise.

*Drainage Coils.* Drainage coils, bridged across a telephone line, with their mid-points connected to ground, provide a low impedance path for currents induced between wires and ground and thus tend to reduce the voltage. Such coils must be exceedingly well balanced or they will themselves constitute a source of unbalance and thus augment noise. Moreover, they increase the susceptibility to noise resulting from irregularities in series resistance or impedance of the telephone circuits. Also they impair telephonic transmission efficiency.

If telegraph service or direct-current signaling is employed on circuits equipped with drainage coils it is necessary to place condensers in series with the coils. The effect of this apparatus on telegraph service is distinctly detrimental.

Drainage coils have not proved to be adapted for general use on commercial systems but are helpful on private telephone circuits of power transmission companies for reducing high electrostatic charges when such private circuits are carried close to high-voltage wires.

*Sectionalization of Telephone Circuit.* An affected telephone circuit may be sectionalized by cutting in repeating coils at one or more points. This may be advantageous in certain cases of exposed rural lines where by placing a repeating coil at each end of a parallel it is possible to change to a metallic circuit through the parallel. It is also sometimes useful on private telephone circuits of power transmission companies as it makes possible the insulation of the telephone sets from the exposed telephone wires. On commercial telephone systems the usefulness of this method is very limited as it introduces large transmission losses, precludes the use of telegraph and brings in difficulties in connection with line signaling.

*Shielding Conductor.* A copper conductor used as a shield may be strung near the disturbed communication wires and grounded at the ends of the parallel. With a conductor of suitable impedance, the current carried by the conductor will have a neutralizing effect on the induced voltages in the near-by communication wires. The action is similar to that of neutralizing transformers but less effective. In case of an aerial cable, the cable sheath itself can be so used instead of a separate copper conductor. The benefits derivable from this method, however, are very limited.

With a view to increasing the neutralizing action of such a conductor, it has been suggested that a part of the railway current could be diverted into it. The quantitative relations involved are such, however, that great difficulties stand in the way of successful application of this scheme on a commercial scale.

*Resonant Circuits.* Combinations of coils and condensers, adjusted to be resonant at the disturbing frequency and connected so as to reduce the disturbing current in the receiving instruments, have been employed to some extent. These afford considerable benefit for low-speed telegraph service such as ordinary hand sending. For higher-speed operation, the benefit obtainable in this way becomes rapidly less. Many modern telegraph systems operate at speeds approaching 25 dots per second which makes it impossible to differentiate in this way between the signaling and disturbing currents.

Similar methods have been suggested for reducing noise but are not usually applicable because the harmonics which cause noise are within the range of frequencies required to give good telephonic quality.

*Balance and Insulation of Telephone Circuits.* It is advantageous to construct and maintain telephone circuits exposed to induction with a high degree of balance and insulation. This includes an adequate transposition system. In all cases of inductive disturbance care should be exercised that these features of the affected lines are properly attended to.

*Use of Relay Sets.* On direct telephone lines the bell is bridged between the two metallic conductors. On two-party selective lines one bell is connected between each side of the circuit and ground. On four-party semi-selective lines two bells are connected between each side of the circuit and ground. On four-party lines, with full selective ringing, the bells are not connected to ground except at times when an operator is ringing on the line and at such times the connection of the bell to ground is established by means of relays. On all these classes of lines both sides of the circuit are grounded at the central office.

It will thus be seen that an induced voltage between the circuit conductors and ground might ring all grounded bells, but under normal circumstances would not ring bells on a direct line or at stations equipped with relay sets. However, if the induced voltage is high enough to operate the telephone protectors, a path to ground is established through the protector, and bells on direct lines or bells at relay set stations may be falsely rung.

*Biasing Bells.* In regions where the induced voltages are not too high, false bell ringing can be obviated by biasing the bells, that is, by stiffening the control springs so that increased voltage is required to ring the bells. Obviously, there are very positive limitations to what can be accomplished in this manner.

*Measures Applicable to Railroads.* The foregoing measures for obviating inductive interference are of a palliative nature and assume a condition of the electrification which produces large inductive effects. Another class of measures for avoiding interference looks to the source of the disturbances, the electrical system of the railroad, and seeks to avoid the conditions which produce large induction. This latter class has, in general, the advantage of benefiting, not one affected circuit only, but all communication circuits within the area affected.

*Double Trolley.* One radical and probably effective method of preventing inductive interference from single-phase railroads would be the use of a double-trolley circuit completely insulated from ground, thus avoiding the use of the running rails as a part of the railway circuit. This method, however, is distinctly unpopular with railway men, mainly for operating reasons, on account of the complexity of the overhead construction particularly in yards, and at sidings and crossovers. Purely from the cost standpoint this method might have advantages in certain cases, where the conditions of exposure are severe and other methods of restricting the earth currents are expensive to apply.

*Frequent Power Supply Stations.* One of the most important methods of interference-prevention is the provision of a sufficient number of substations to supply power to the trolley-rail circuit at frequent intervals. If the substations are near enough together, the amount of stray current and the average length of path of such current can be made small. It is particularly desirable that all sections of electrified railroad which are involved in parallels be supplied with power from both directions rather than by stub end feed.

*Sectionalization of Trolley System.* Considerable advantage may be gained by sectionalizing the trolley, thus decreasing the length of the earth current path as well as reducing the amount of power supplied to a short circuit. However, as each separate section of trolley requires an independent power supply sufficient for its maximum demand, the total transformer capacity required for a given length of electrified road is much increased by any considerable use of trolley sectionalization. Notwithstanding this objection a limited amount of sectionalizing may be used to advantage where the exposure is severe.

*Opposing Polarities.* On railroad lines having two tracks it is possible to connect the two trolleys for opposing polarities, so that the current flowing in the rails and earth is only the difference in the currents of the two trolleys. An instance of this method applied to a direct current railroad is afforded by the City and South London Railway in England which has been so operated for 20 years. As applied to alternating-current railroads of 11,000 volts, this method is considered to have serious operating disadvantages in respect to cross-overs between tracks, and for this reason this plan, which was studied in connection with the revision of the Woodlawn-Stamford electrification of the New Haven Railroad in 1912, was not adopted.

*Balancing Transformers.* This method, which is now employed on the main line electrification of the New Haven Railroad, is of much benefit in reducing stray currents, particularly where power is supplied from both directions. Its use, however, involves a combined transmission-distribution circuit, tied together by the balancing auto-transformers, whereas the general practise in such matters seems to tend toward a separate tranmission line supplying power to the trolley-track circuit through standard transformers.

*Booster Transformers.* Another important method of controlling railroad currents is by the use of booster-transformers placed at frequent intervals along the electrified section. These transformers have a substantially even ratio of transformation, the primary winding being inserted serially in the trolley, and the secondary winding inserted serially in the track circuit. In this way the track current is required to be substantially equal to the trolley current at points where the transformers are located. By placing the transformers near together the leakage of current into the earth between transformers is made small.

A modification of this plan is to install a feeder, electrically connected to the rails at intervals, and insert the secondaries of the booster-transformers in series with this feeder. In this way the current is confined to the feeder instead of the rails. The London, Brighton and South Coast Railway embodies an installation of this type. This arrangement is somewhat more effective perhaps than trolley-track transformers but it involves considerable additional expense for the feeder, which must be of high conductivity. One advantage of the feeder-booster, over the track-booster arrangement, is that successful operation of the former is not so dependent on high grade maintenance of the track bonding.

## SPECIFIC ELECTRIFICATIONS

Having now considered various means which are available, or at least, worthy of being considered, for avoiding or reducing inductive interference to communication systems by alternating-current electrified railroads, we may now direct our attention to some specific electrification installations and see what has actually been done to prevent such interference and with what degree of success. In so doing I will confine my remarks to the salient features of four single-phase installations: (1) New York, New Haven & Hartford Railroad, Woodlawn to Stamford; (2) New York, New Haven & Hartford Railroad, New Canaan Branch; (3) Norfolk and Western Railroad, Bluefield to Vivian, W. Va.; (4) Pennsylvania Railroad, Broad Street to Paoli, Pa.

*Woodlawn-Stamford.* The original electric installation of the New York, New Haven and Hartford Railroad Company between Woodlawn, N. Y. and Stamford, Conn., began operation, in part, in the summer of 1907. This is a section of four-track railroad, about 21 miles in length, and all power was supplied by a generating station at Cos Cob about three miles west of Stamford. To move a train at Woodlawn, the current passed for 18 miles over the trolley wires and paralleling feeders from Cos Cob to the locomotive, the remainder of the circuits from locomotive to Cos Cob, being the running rails and earth. The telephone company's New York-Boston subway is, throughout this section, situated at varying separation averaging about 2000 feet from the railroad, a sufficient distance so that the inductive effect of the trolley current would have been largely neutralized by the inductive effect of the rail current, had these two currents been equal. However due to the long rail path, a large part of the current left the rail and spread into the earth, where its effect in neutralizing the corresponding part of the trolley current was negligible.

After full electric passenger service between Woodlawn and Stamford was inaugurated, the induced 25-cycle voltage on circuits in the New Haven subway, at normal rush hour periods, was as much as 170 volts. On the Shore Line, one of the telephone company's open wire routes between New York and Boston, the corresponding induced voltage was about 300, the higher voltage on the Shore Line being principally accounted for by about a mile and a half of exposure near Greenwich, where the average separation was only about 100 feet. The open wire Shore Line circuits were also affected by noise, which was most intense during periods of train acceleration, the pitch of the noise varying with the speed of the train. The subway circuits being in metal-sheathed, underground cable, and not having any section of very close parallelism, were not made noisy. The Midland Line, another open wire route between New York and Boston, about four miles away from the railroad at the nearest parallel section, sustained corresponding induction of about 40 volts.

Wires of the Western Union Telegraph Company, which were carried on poles located on the railroad right of way, were subjected to much higher voltages. These wires, except a few which were equipped with neutralizing transformers and continued in use by the Railroad Company, were removed to a new pole line which was built a number of miles away.

The conditions as to induction continued substantially as outlined above for a period of four or five years.

Early in 1911 the railroad company made known its intention to extend the electrification to include the Harlem River branch and the New York, Westchester and Boston Railroad. The former is a six-track line used principally for freight, extending about 12 miles from its junction with the main line, near New Rochelle, to the Harlem River. The New York, Westchester and Boston Railroad, which is partly four-track and partly two-track, was constructed principally for suburban service and extends from West Farms at 176th Street, where it forms a junction with the Harlem River branch, to White Plains, a distance of 16 miles. A branch six miles north of West Farms taps the main line of the New Haven Railroad just east of New Rochelle. These two new lines involved the direct connection, to the Western end of the previously electrified section, of additional electrified line to the extent of about 200 miles of single track railroad. Moreover, it was planned that, after the Harlem River branch was electrified, freight trains, as well as passenger trains, on the entire system west of Stamford should be operated electrically.

This proposed large extension of the electrification, with its resulting increase in load, caused considerable apprehension to us of the telephone company. Our estimates of induced voltages under the new conditions, based on the Railroad Company's estimates as to future train loads, indicated over 1500 volts on the Shore Line and nearly 1000 volts on the subway. These values corresponded to maximum normal railroad loads and the induction would have been still greater at times of abnormal conditions. Voltages of this magnitude are far beyond the endurable limit for the telephone company and the matter was taken up by the companies with a view to determining what could be done to ameliorate the situation.

In January 1912, a joint committee of engineers, comprising a representative each of the New Haven Railroad Company, the Western Union Telegraph Company, and the Telephone Company, was formed to study this question. Several different plans for modifying the railroad distribution system were laid before this committee. After six meetings during the ensuing three months, a plan was decided upon and a sub-committee of engineers was designated to work out the details.

This new distribution system involved quite a comprehensive change in the original installation. It was cut over on January 15, 1914 and has been found to bring about a great improvement with respect to induction.

Besides an additional power supply station at West Farms, which in itself effects a considerable improvement, the new distribution system includes the use of 17 balancing auto-transformers of 2000 kv-a. capacity each. These are distributed along the line, in such locations as are most advantageous for supplying power to the trolley circuit, so as to minimize the length of path of current through the rails.

As full accounts of this distribution system have been published I will not undertake an extended description of it here.

The same system has since been used by the Railroad Company in extending the electrification from Stamford to New Haven.

At present, the induced voltages on the through circuits in the subway seldom exceed 30 volts under normal conditions of railroad operation.

The principal interference now incurred (apart from that due to the New Canaan Branch which is discussed elsewhere) is in connection with railroad short circuits in the vicinity of Stamford. Within the section from 2 miles West of Stamford to 5 miles East of Stamford, local subscribers' lines and trunk lines have been affected by false bell ringing, false flashing of line signals, and grounding of protectors, on about twenty different occasions in the past three years, an average of twenty lines being affected on each occasion. It is of interest to note that these troubles are localized within this seven-mile section of the sixty-mile electrification from Woodlawn to New Haven, and also that it is at approximately the middle of this seven-mile section that the New Canaan Branch, referred to immediately below, joins the Main Line.

*New Canaan Branch.* In 1907 the New Canaan Branch of the New Haven Railroad, which previously had been operated at 500 volts direct-current, was reconstructed and its trolley connected directly to the 11,000-volt trolley on the main line near Stamford.

This branch is about six miles long, single track, and the traffic is light so that under normal conditions of operation no interference with telephone or telegraph lines was produced. However, the telephone circuits have been subjected to a great deal of interference from this branch, due to short circuits. Owing to the conditions of power supply, the short-circuit current at New Canaan is about 2500 amperes or ten times the maximum load current. At points nearer the main line the short-circuit current is even greater.

The inductive effects of the New Canaan branch were augmented by the large proportion of earth current, due to the relatively high impedance of the single track. Momentarily voltages as high apparently as 1000 were imposed on trunks between New Canaan and other places and voltages up to 500 on many telephone circuits in the New Canaan exchange. These induced voltages operated protectors, permanently grounded and put out of service many telephone lines and subjected operators to severe acoustic shocks. Due to the recurrence of these surges, the operators in the New Canaan office became so afraid of shocks that the operating efficiency was seriously impaired.

In 1913, after several unusually severe surges from short circuits had been experienced, the matter of finding some means to overcome this interference, which had already been under consideration by the telephone and railroad companies, was taken up with renewed energy. Various plans were proposed and an extended series of experimental tests and measurements were made, as a result of which, means were finally agreed upon as follows: 1. To keep the rails well bonded. 2. To insert a current-limiting reactance in the trolley, near its junction with the main line trolley, so as to restrict the short-circuit current. 3. To install 12 series booster track-trolley transformers at intervals of about ½ mile. 4. Readjust the circuit-breaker, at the junction with the main line, for instantaneous operation.

From the tests and from experience with six booster transformers, it is believed that the above mentioned measures will be effective in preventing this interference although the full installation of transformers and the current limiting reactance has not yet been completed.

It is of interest to note that the balancing transformer plan, although generally giving good results on the main line of the New Haven Railroad, does not afford an effective means for preventing inductive interference under such conditions as exist on the New Canaan Branch.

*Norfolk and Western Railroad Electrification.* This electrified section is between Bluefield and Vivian, West Virginia, a distance of approximately 28 miles. The railroad is double track with numerous yards and sidings and includes some heavy grades. The power house for supplying power to the electrified section is located at Bluestone, about 10.8 miles west of Bluefield. Power is transmitted by duplicate single-phase transmission lines at 44,000 volts to five substations, the distances between which, respectively, beginning at Bluefield, are 8.2, 4.6, 6.6 and 4.8 miles. At these substations the voltage is stepped down to 11,000 for delivery to the trolley-track circuit which is electrically continuous throughout from Bluefield to Vivian.

The original plans for this electrification were taken up with the telephone company who proposed some modifications for the better protection of the paralleling communication circuits. The plans as modified include 23 series booster trolley-track transformers, the average spacings of which are, east of the power house, about a mile and a half, and west of the power house about a mile. Each transformer is 100 kv-a. continuous rating and 400 kv-a. for 2 ½ minutes.

One telephone line paralleling this road has several exposures of about 500 feet separation from the railroad and there are also local circuits of the Bluefield Telephone Company in proximity to the railroad. No trouble has been reported from induction under normal railroad conditions. At times when the electrification wires are down noise has been experienced on some of the above mentioned circuits. Also a telephone trunk circuit which crosses the railroad underground is sometimes thrown out of service when the railroad circuit is in trouble, by reason of the copper block protectors at the crossing becoming grounded. Some trouble has been sustained from corrosion of lead cable sheaths at underground crossings of the railroad in Bluefield but it has not been established whether or not this is due to electrolysis by the alternating railroad currents.

*Broad Street—Paoli.* Plans for electrifying the Pennsylvania Railroad's four-track main line from Broad Street to Paoli were announced early in 1913. The question of interference with the telephone company's lines was taken up with the engineers representing the railroad company and several plans looking to the prevention of interference were considered. As a result of the best information then available it was decided in 1914 to install series booster trolley-track transformers for the purpose of confining the current to the rails.

The power for this electrification is conveyed by a 44,000-volt 25-cycle transmission line to three substations, one each at West Philadelphia, Bryn Mawr and Paoli, and from these substations supplied to the trolley-track circuit at 11,000 volts. Sixteen pairs of trolley-track transformers, each transformer equipping two tracks and having a continuous rating of 80 kv-a. and 600 kv-a. for one minute, are provided, the spacing between transformers being about one mile.

The regular operation of electric passenger trains between Broad Street and Paoli was begun during the latter part of September, 1915. Extended induction tests were made on the section east of Bryn Mawr in April, 1915 and on the entire line in the following August. Further induction tests were made in the summer of 1916.

In July, 1916, the Railroad Company commenced operating synchronous condensers at Radnor and it has been found that, with these condensers in use, the booster transformers are greatly overloaded at times of short circuit. This results in the transformer iron becoming heavily saturated and the magnetizing current, consisting largely of the third harmonic, which under these conditions reaches very high values, necessarily flows through the ground. The current wave is badly distorted as a result of this overloading and the induced voltages during short circuits may actually be higher with the booster-transformers than without them; in fact, it has been found preferable to remove the booster transformers east of Bryn Mawr, and only those from Bryn Mawr west are now regularly in service.

With all booster transformers in service the maximum induced voltages, (peak values)*, during normal operation are about 10 for subscribers' lines and 25 for trunk lines; at times of short circuit, calculations based upon experimental data show that the maximum voltages may exceed 1000 on subscribers' lines and 1200 on trunk lines. With all booster transformers cut out, the corresponding figures are, for normal operation, 50 volts for subscribers' lines and 125 volts for trunks and, at times of short circuit, 225 volts for subscribers' lines and 900 volts for trunks. These figures assume two condensers in service.

This section of railroad follows through a highly developed suburban area where disturbances on telephone lines are extremely undesirable. Unfortunately, a considerable number of cases of bell ringing due to short circuits on the railroad circuit have been experienced. Not all short circuits cause bell ringing however.

During the first three months of electric operation, namely, October, November and December, 1915, the number of short circuits causing bell ringing averaged 10 per month and the bell ringing troubles over 2000 per month. During the following seven months the average number of short circuits causing bell ringing, fell to 4.5 per month and the bell ringing troubles from 735 in January to 57 in July. Since July, 1916, the number of short circuits per month which caused bell ringing have been

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*On account of the-wave shape distortion described above it was found advantageous here to measure peak voltages rather than effective voltages and all values of voltage mentioned in this discussion of the Paoli electrification are peak values.

further reduced and during the year 1917 averaged about 1.5 per month.

The improvement in the bell ringing situation after January 1916 and up to August 1916, was due partly to changes in the railway control circuits and in the operating arrangement of circuit breakers, and partly to the substitution of relay sets for standard party line bells, at subscribers' stations within the regions of heaviest inductive disturbances, and the biasing of bells within the areas of less disturbances. This work required changes in subscribers' apparatus at about 3000 stations.

The synchronous condensers installed in the latter part of July, 1916, increased the maximum voltages impressed on the telephone circuits at times of short circuit on the railroad, so that while the actual number of short circuits causing bell ringing has been reduced since that date, the average number of bell ringing troubles per month has increased. From August 1916 to December 1917 inclusive, there were 22 short circuits causing bell ringing and 1589 bell ringing troubles, an average of 72; whereas for the five months' period preceding the installation of the synchronous condensers, there were 20 short circuits causing bell ringing, and 356 bell ringing troubles, an average of 18, or one-quarter of the corresponding average number in the later period. During the year 1917, 45 per cent of the bells rung were on direct lines and not grounded.

The high induced voltages causing bell ringing have been experienced over the entire electrified portion of the Main Line except within about three miles of Broad Street. It is fortunate that the region of high induced voltages does not extend to Broad Street, otherwise, owing to the density of telephone development within that district the trouble would be exceedingly difficult to cope with.

In a small percentage of cases where bells are rung the induced voltages are sufficient to leave the telephone lines grounded through the protectors, thus interrupting service.

Methods of signaling on call circuit trunks involving use of the ground have had to be abandoned.

Noise tests made on ten trunks which parallel the electrification indicate that the induced currents from the railroad cause a small amount of noise but not enough to constitute interference with service on these comparatively short lines. However, I may mention the fact that in the construction of the new Philadelphia-Reading toll cable, in which the requirements as to freedom from noise are more exacting than in the case of the shorter Main Line trunks, the liability of noise, together with other features of interference, was considered a sufficient reason for changing to a different route in order to avoid exposure to the electrified section of the railroad. The route adopted involves charges of $1000 a year more than the route exposed to the electrification, which otherwise would have been followed.

None of the telephone company's circuits affected by the electrification are now used for telegraphy, hence there has been no interference with this type of service. It is expected, however, that telegraph service over the paralleling toll circuits will be required later and to give this service it will be necessary, unless there is some new development, to employ neutralizing transformers with their attendant disadvantages and limitations.

The high voltages induced on the telephone lines at times of short circuits, also, in the opinion of the telephone company, constitute a considerable fire hazard, although it is fortunately true that no fires have as yet been caused. All terminating trunks at the three directly exposed offices west of Bryn Mawr have been provided with carbon block and heat coil protection. As these trunks are in underground cable, they would not require these protective devices except for the induced voltages. As an additional precaution against fire the telephone company has maintained a special force of night watchmen at several central offices throughout this area where regularly there are no inside men at night, thereby incurring an expense of about $18,000 per year. As a still further precaution, trunks to certain offices west of Malvern were for a time provided with repeating coils at Malvern, but it has recently been necessary to phantom these circuits, which has required the removal of the repeating coils.

A large number of the trunks affected by induction from this electrification are equipped with loading coils. Tests on these coils show that more than 20 per cent of them are magnetized to a greater or less extent but it has not yet been determined whether this trouble has been brought about wholly or in part by induced currents from the railroad circuits or whether it is due to other causes.

Within the area of high induced potentials, telephone subscribers and employees are exposed to the possibility of electric shocks at times of short circuit on the railroad. Fortunately, however, no troubles from such shocks have yet transpired.

Telephone operators and users are also exposed to the possibility of acoustic shocks at times of surges from short circuits, although no serious acoustic shocks, due to this electrification, have been reported.

It will be seen from the foregoing that, notwithstanding all that has been done by the railroad company and the telephone company to reduce interference, there still remain, at times of short circuits, certain hazards of fire and shocks, bell ringing trouble, and other latent interference as described. While this impending interference has not, with the exception of bell ringing, actually materialized into trouble, nevertheless the possibility of such trouble is continually present and the conditions can by no means be regarded as satisfactory.

Looking to further improvement in the situation a number of plans which involve changes in the distribution system of the railroad, have been worked out. The plan which, on the whole, seems entitled to the most favorable regard, involves the installation of additional power supply transformers at Radnor and at a point 16½ miles from Broad Street, and also includes the sectionalizing of the trolley at both these points. This plan further requires the moving of the Berwyn-Malvern aerial telephone cable to a more remote location. By the adoption of this plan it is estimated that the maximum induced voltage at times of short circuits would be brought down to 250. This would largely reduce, but not wholly avoid, the fire and shock hazards, bell ringing on direct lines, and the other evils involving the operation of protectors. The cost of carrying out this plan, including labor and material only, has been estimated at $140,000.

Another plan, involving more extensive changes in the power supply and distribution system to avoid wholly the interference and hazard, would require a much larger expenditure and perhaps would not be warranted by the existing situation.

A different plan would be to install sufficient additional booster-transformers so that they would not become overloaded at times of short circuits. This would probably require placing the boosters one-third to one-half mile apart and would cost from $85,000 to $150,000. This plan has not been worked out in detail, as the railroad company objects to the introduction of insulating joints in the trolley wires at such frequent intervals.

## Conclusion

It may be said in conclusion that means are now known whereby alternating railway currents can be kept sufficiently within control, except under abnormal conditions, to prevent substantial interference to neighboring communication lines, although the application of such means to the extent necessary to produce satisfactory results may involve considerable expense.

Even under abnormal conditions the interference can be greatly reduced by the application of suitable measures, but in some cases there still remains the problem of obtaining a sufficient reduction of interference without incurring a cost which the railroad companies consider excessive.

It is important in each electrification project that the railroad company and the communication companies affected cooperate in determining what interference-preventive measures shall be adopted. Each electrification requires a special study, as the best measures to employ may be quite different in different cases.

I wish to take this opportunity to testify to the broad minded and cordial manner in which the railroad companies and electrical manufacturers concerned have cooperated with us in searching for a satisfactory solution of this problem, a work which, it is probably unnecessary to add, is still in progress.

## DISCUSSION ON "INDUCTIVE EFFECTS OF ALTERNATING CURRENT RAILROADS ON COMMUNICATION CIRCUITS" (WARREN), PHILADELPHIA, PA., APRIL 8, 1918.

**Marius Latour** (communicated after adjournment): It may be of interest to call attention to a very simple system proposed by the writer in 1912, based on resonance, and which allows a Morse telegraphic line to be operated in spite of strong disturbing alternating e. m. fs. This system, which was put in operation on the French Midi (Southern) railway, was new at the time of its introduction there. It comprises three principal features:

1. Connected in parallel with the receiving apparatus, preferably *ahead of the sending* key, is a resonance-circuit capable of eliminating the disturbing current, and operating in combination with a definite inpedance, connected in series with the receiving apparatus, which exaggerates the shunt effect of the resonance-circuit.

2. Connected in series with the telegraphic line is a suitable impedance capable of reducing the disturbing current while allowing the telegraphic signal currents to pass through without deformation.

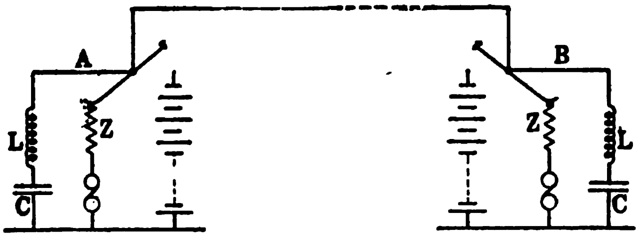

FIG. 1

3. The impedance inserted in the line is distributed all along the line so as to reduce the maximum potential difference between the telegraphic line and the ground.

## I. CONNECTION ON RESONANCE CIRCUIT IN PARALLEL WITH TELEGRAPHIC RECEIVING APPARATUS

The Morse receivers are connected in parallel with the two corresponding stations $A$, $B$, by resonance-circuits $L$, $C$ (Fig. 1) which are tuned to the frequency of the electric railway current, in other words, to the frequency of the interfering current; and an impedance, $Z$, is at the same time connected in series with the receiving apparatus. The resonance-circuits, $L$, $C$, are connected ahead of the sending key. In this manner, the circuit for the interfering current over the line is never opened by the operation of the sending key at either end, because the sending key closes one branch circuit as it opens another, in the manner shown in the diagram; and the conditions remain favorable for the continuous maintenance of resonance.

For the frequency of $16^2/_3$ cycles, which is that quite generally employed for traction by single-phase current in the case of motors of high power, it will be assumed that the inductance is 50 henries and the capacity is 2 microfarads.*

The inductance of the reactance-coil *L*, is constructed with an iron magnetic circuit. It was believed by some that it was not possible to obtain resonance successfully for currents of a few tens of milliamperes with an inductance comprising a magnetic circuit of iron. This opinion is even still to be found expressed in January 1914, in the German Post and Telegraph Annals. But this opinion, while it might have a theoretical basis, is not borne out by practical experience. While it may be true that the permeability of iron, in the case of low, variable, magnetic densities, is a function of the density, it is none the less true that a small air-gap is sufficient to render the total reluctance of the magnetic circuit quite uniform; the irregularities in the permeability can then manifest themselves at most only by the formation of a residual impedance corresponding to the appearance of harmonics which produce no effect on a Morse receiver. With good sheets of silicon steel for the magnetic core, it is easily possible to obtain an inductance of 50 henries, having a small air-gap and which occupies but little space, and whose apparent ohmic resistance, for the frequency of $16^2/_3$, is of the order of a few hundreds of 1 ohm.

The construction of a reactance-coil without an iron core, and having a reactance of 50 henries and such a low ohmic resistance, would be impracticable.

In reality, the frequency of the railway traction current is not absolutely constant, and it is to be expected that the resonance cannot be perfectly tuned. For example, if we suppose the frequency to vary 5 per cent above and 5 per cent below, the normal frequency, or 10 per cent altogether, the impedance *L*, *C*, can attain 500 ohms in reactance or in capacitance, according to whether the frequency is falling or rising with respect to its mean value, and it will be seen that the apparent ohmic resistance of the reactance-coil is not the only thing to be taken into consideration.

It is necessary to exaggerate the effect of the parallel resonance-circuit *L*, *C*, by placing in series with the receiving apparatus an impedance *Z* which increases the total impedance of the portion of the circuit containing the telegraphic receiver. This impedance, *Z*, may be composed either of an inductance or of an ohmic resistance. The advantage of using an ohmic

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*The frequency of $16\frac{2}{3}$ is designated with this precision because one has in mind the process of transformation, by means of motor-generators, by which this frequency can be equally obtained from the higher industrial frequencies, 50 and 25 periods: $16^2/_3 = 50/3 = (2 \times 25)/3$.

resistance is that equivalent and symmetrical effects are obtained when dissonance of the shunt circuit *L*, *C*, is produced, either by an increase, or else by a decrease, of the frequency, in other words, when the shunt circuit *L*, *C*, has either capacitance or else reactance.

## II. CONNECTION OF SUITABLE IMPEDANCE IN SERIES WITH THE TELEGRAPH LINE

Considering the importance of the e. m. f. which causes interference, it is desirable to reduce the strength of the interfering current diverted into the resonance-shunt. This is accomplished by introducing, in the line, an impedance composed of an inductance, *L*, and a resistance, *R*, connected either in parallel, as in Fig. 2, or in series, as in Fig. 3. The resistance portion is, in any case indispensable, in order that the time-constant of the line may not be too high, and to prevent the Morse signals from running together. It is possible to use resonance in a new way to oppose a high impedance to the interfering current. It was for this purpose that Mr. Bethenod and the writer devised, together, an impedance composed of an inductance, *L*, and a capacity, *C*, connected in parallel (Fig. 4).

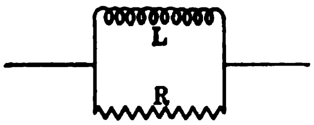

FIG. 2

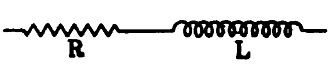

FIG. 3

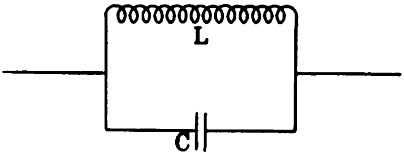

FIG. 4

If this arrangement be regulated so as to produce resonance (for example, by making $L = 50$ henries and $C = 2$ microfarads, for a frequency of $16^2/_3$) an impedance will be obtained which is equal, theoretically, to the reactance of the inductance (5000 ohms) multiplied by the ratio of this reactance to the resistance of the coil (or $500 \times \frac{5000}{500} = 5000$ ohms). Practically, a very high impedance is obtained. The arrangement shown in Fig. 4 is some times called a "stopper". The inductance $L$ is constructed in the same way as that used in the resonance shunts.

When the arrangement is not in resonance, it has a weaker impedance, which is a reactance if the frequency is too low, and a capacitance if the frequency is too high. In order to maintain the time-constant of the line at a proper ratio, it is necessary to put still more resistance, $R$, in series with the "stopper", as shown in Fig. 5.

In the complete arrangement (Fig. 6) comprising resonance-shunts at the stations $A$ and $B$, and a stopper, $D$, between the two stations, the resistance $R$ is already connected in series with the Morse receiver, where it already fulfills the function of the impedance $Z$, shown in Fig. 7. In reality, it will be seen, by reference to Fig. 6 that an interfering current which can close its circuit through line leakage between station $A$ and the point at which the stopper $D$ is located will go through station $A$ without being stopped by the stopper $D$, and that, in like manner, the charging current of the line (due to the fact that the telegraph line and the line parallel thereto which carries the traction current form the two armatures of a condenser that is charged by the potential difference between the traction line and the ground) circulates through station $A$ without being stopped by the stopper $D$. It would seem, therefore, that in order to utilize satisfactorily the properties of the stoppers, each station should directly be protected by a stopper at its point of connection with the line. However, the question

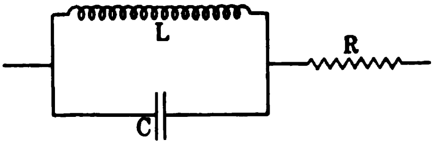

FIG. 5

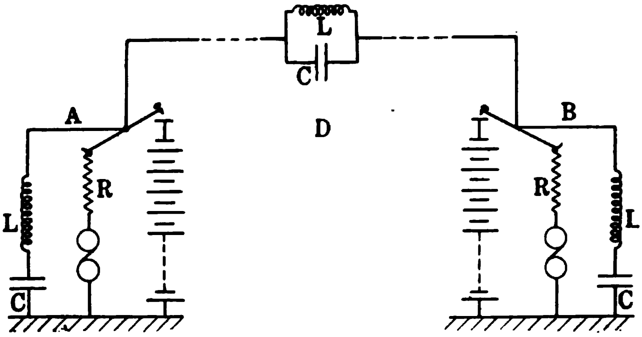

FIG. 6

of introducing a stopper at each station brings up the question of connection stoppers in series.

Now, one of the difficulties in the use of stoppers is that they cannot be connected in series without certain precautions. Let us suppose, for instance, that two stoppers are not tuned exactly alike, as for instance, when one is tuned for the frequency $f_1$ while the other is tuned for a frequency $f_2$, very near to it but still different from it. For an intermediate frequency between $f_1$ and $f_2$, which will be too high for one of them and too low for the other, one of the stoppers will act as a capacity, *i. e.*, it will have capacitance, while the other will act as an inductance, *i. e.*, it will have reactance. It could thus happen that the connecting in series of two stoppers might produce actual resonance in the line and that while expecting to produce an infinitely high impedance a short-circuit might, instead, be produced.

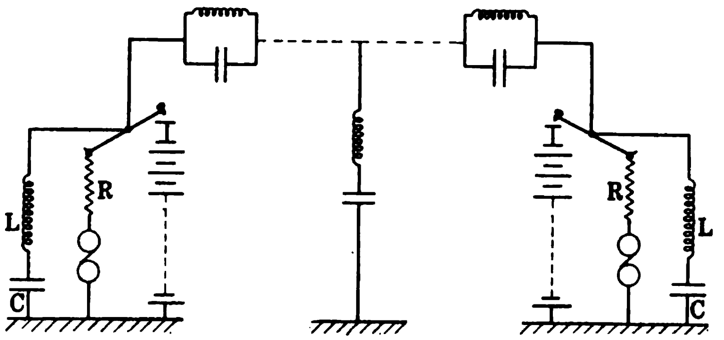

Fig. 7

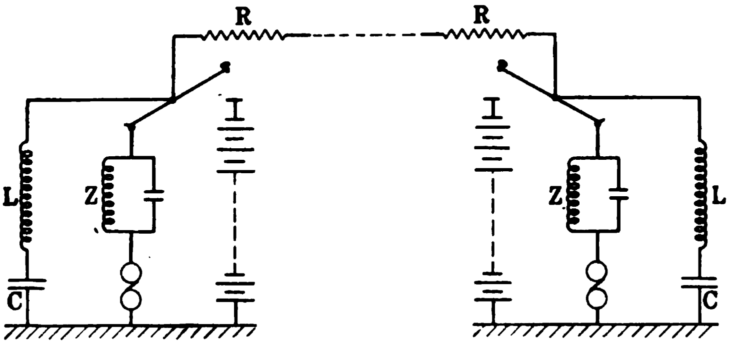

Fig. 8

By a similar process of reasoning it could be shown that it is impossible to connect two resonance shunts in parallel. The connecting in parallel of two resonance-shunts which are not absolutely alike in all respects may cause the unexpected appearance of a shunt-connection of very high impedance. A stopper can be placed at the line connection at each station by placing a resonance-shunt in the middle of the line so that the two stoppers can operate independently of each other, as shown in Fig. 7, which shows a very effective arrangement. The writer has also proposed to give to the impedance *Z*, in Fig. 1, the form of a more or less perfectly tuned stopper. In that case the resistance *R* must be inserted in the line and the arrangement is as shown in Fig. 8. The resistance *R* reduces the strength of the interfering current and it maintains the time-constant of the telegraphic circuit at a value suitable for the transmission of telegraphic signals. The simultaneous use of resonance-shunts and stoppers increases greatly the range of protection obtainable, and it would be a conservative statement to say that the effects of an interfering e. m. f. of the order of 500 volts can be overcome with an e. m. f. of 100 volts in the telegraph line.

## III. DISTRIBUTION OF IMPEDANCE IN THE LINE

It is desirable to find means of reducing the potential difference between the telegraph wires and the ground in order to enable the telegraph wires to be handled without danger in making line repairs. This result is attained by distributing along the entire line the impedance which serves to limit the strength of the interfering current. For example, an impedance of 10,000 ohms would be subdivided into 5 portions of 2000 ohms each. Under these conditions the potential difference induced in the telegraph line is, so to speak, consumed as fast as it is produced, along the line, by the fall of potential caused in the impedance. If the induction produced was uniform along the line and if the impedance were uniformly distributed, the result would be a wire having the same potential with respect to the ground at all points.

In the particular case where the impedance is produced by a series of stoppers placed at different points in the line, use should be made of intermediate dischargers like the resonance-circuits shown in Fig. 7.

The method of protection based purely and simply on the properties of resonance has been used, up to the present time, only on lines working with Morse apparatus. Its efficiency would very likely be questionable for telegraph systems of the Hughes or Baudot type. However, it would not be unreasonable to expect that it could be serviceable with such systems.

*Presented at the 339th meeting of the American Institute of Electrical Engineers, Pittsburgh and New York, April 9 and 12, 1918.*

# NO-LOAD CONDITIONS OF SINGLE-PHASE INDUCTION MOTORS AND PHASE CONVERTERS

BY R. E. HELLMUND

## Abstract of Paper

This paper shows methods and derives formulas for the determination of the fields, the stator and rotor magnetizing currents, and the tertiary voltages for phase converters and single-phase induction motors at no-load. Previous publications on this subject, including text books, are usually rather vague and incomplete, especially with regard to the secondary magnetizing currents and the field forms. Furthermore, numerous conflicting statements are found in previous literature, thus leaving the subject, as a whole, in a rather confusing condition. This paper treats a large number of different cases along similar lines, thereby coordinating and explaining many phenomena previously observed, and it should, therefore, form a desirable basis for further investigations and discussion of this subject matter.

The treatment of all cases is rather uniformly based on the following fundamental considerations. The sum of all e.m.fs. must be zero in both the primary and secondary circuits. With the impressed primary e.m.f. known, this leads to definite conditions governing the primary counter e.m.fs. The same law applied to the secondary circuits gives the condition that the induced voltages must be equal and opposite to the ohmic drops. Having thus certain laws governing the voltages to be induced in the windings, we have at once certain laws governing the fluxes for inducing these voltages. In most cases, it is then found, that only a single definite local distribution of the resultant field satisfies both the conditions for the primary and secondary windings simultaneously; having thus established the required resultant field distribution, we have at once laws for the required resultant distribution of ampere turns around the circumference to bring about such field distributions. Whenever certain portions of the circumference have conductors of either the primary winding or secondary winding alone, the resultant ampere turns found represent at once the ampere turns for the single winding located at this portion. With this fact known, a number of facts regarding the currents can be determined. Whenever both primary and secondary turns coincide at the same portion of the circumference, certain problems arise in determining the distribution of the resultant ampere turns between the two windings. A number of different considerations are used for the various cases to assist in the solution of these problems; all these considerations are, however, based on simple facts.

In order to demonstrate the method by means of the simplest mathematics, two cases of small practical application are given first merely in order to separate the fundamentals from a rather large amount of mathematics which, while necessary, in connection with the more practical cases, are of little value in connection with the understanding of the fundamental principles.

**Starting out from these simple cases, the influences of various factors are taken up in the following cases, one at a time, because a simultaneous consideration of all of them make a clear understanding practically impossible.**

**The paper is arranged so that it can be read to good advantage without going through those mathematical parts marked by vertical rules.**

**By reading the conclusions at the end of the paper, a fair idea of the principal points brought out in the paper can be obtained.**

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## INTRODUCTION

THE EXACT nature of the fields and magnetizing currents of single-phase induction motors, especially of the currents flowing in the rotor, have been the subject of a good deal of speculation in the past. The introduction of the phase converter as a commercial machine has given additional interest to these problems, and has furthermore introduced a number of new problems, as for instance, the determination of the output voltage. The following studies of the no-load conditions in these machines, which form the necessary basis for further studies of the load conditions should, therefore, be of interest to students and designers of such machines.

The fundamental laws and basic considerations used in the paper are very simple and should be easily understood. Also, the mathematics are relatively simple, involving, with very few exceptions, nothing but the solution of several equations with several unknown quantities; the integrals used in a few places are of the simplest kind. Nevertheless the derivations involve by necessity a rather large number of formulas, which will be of interest only to a limited number of readers. An attempt has, therefore, been made to write the paper, so that it can be read to good advantage, without following through the mathematical portions, marked by vertical rules.

The results, given graphically in a large number of figures, and the conclusions reached, should be of general interest. Some of the final conclusions and formulas will enable the designing engineer to predetermine machine performance more accurately than was possible with the previous methods, most of which are based on more or less incorrect assumptions.

Wherever a correct and complete mathematical solution has not been attempted, because it involves an impractical amount of work, the considerations have been carried far enough to indicate plainly to the student and designer how the most favorable design can be obtained and how bad combinations can be avoided.

A number of simple theoretical cases are given first, because it is believed that the demonstration of the methods employed in this paper on such simple cases will be of assistance in the understanding of the more general solutions of the problems given later on.

## CASE No. I

### *Single Primary Coil, Single-Phase Secondary*

The simplest type of a phase converter is shown with its rotor in different positions in Figs. 1a to 1e. It has a single concentrated primary coil *P P'*, a single concentrated secondary or rotor coil *S S'* and a single concentrated output or tertiary coil *T T'*, located on the stator 90 electrical space degrees shifted against the stator coil *P P'*.

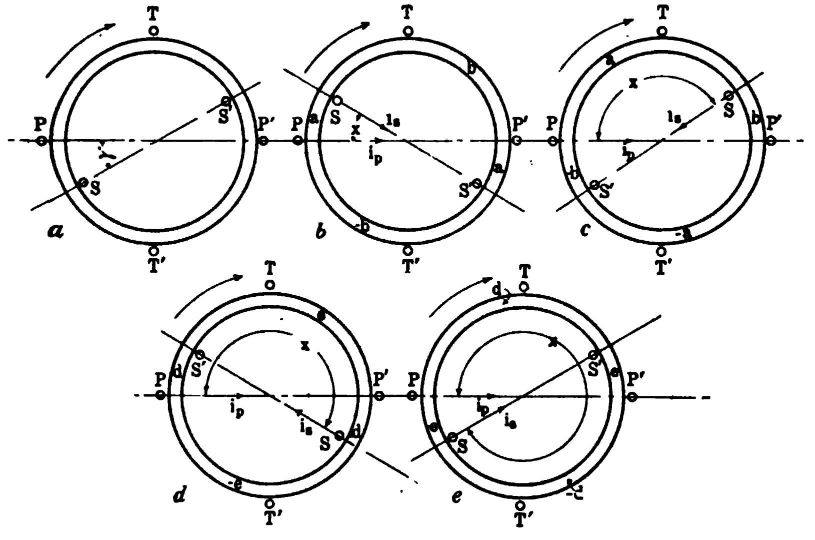

FIG. 1

*The first fundamental condition*, which must be fulfilled in a machine of this kind, is that ***within every closed circuit, the sum of all counter e. m. f's. must at any moment be equal and opposite to the impressed e. m. f.***

Applying this law to the primary coil *P P'*, we know, therefore, that if we impress a sinusoidal voltage wave with instantaneous voltage values $-e_p = - E \sin \alpha$ that the counter e. m. f's. in this winding must be always opposite in direction, but of the same numerical value, namely,

$$e_p = E \sin \alpha \qquad (1)$$

where $E$ is the maximum crest value of the voltage and $\alpha$ the time angle.

The counter e. m. fs. consist of ohmic drops and electromagnetically induced voltages.

The ohmic drops at no-load are usually very small in a-c. machines relative to the inductive voltage and may, therefore, for the present be neglected.

The inductive voltage is always proportional to the rate of change of the total flux interlinking with the coil $P P'$. Since the voltage to be induced is definitely given by equation (1), the flux values within the coil are also definitely given by *the second fundamental conditions, that the rate of change of the flux within a coil must be at any moment proportional to the inductive e. m. f. to be induced.*

This leads to the well known formulas

$$\varphi = \frac{E\, 10^8}{2\, \pi\, f\, n_p} = \frac{0.45\, e\, 10^8}{f\, c_p} \tag{2}$$

and

$$\varphi_i = \varphi \cos \alpha = \frac{E\, 10^8}{2\, \pi\, f\, n_p} \cos \alpha \tag{3}$$

wherein

$e$ = effective primary voltage = $0.707\, E$
$\varphi$ = maximum flux interlinking with coil $P\, P'$
$\varphi_i$ = instantaneous flux interlinking with coil $P\, P'$
$f$ = frequency (cycles per second)
$c_p$ = number of primary conductors
$n_p$ = number of turns in primary coil.

These are the only conditions governing the flux in the primary coil. It is altogether immaterial, with regard to the primary, whether these conditions are fulfilled by a stationary flux fluctuating in size, by a single rotating flux of sinusoidal space distribution, or by a number of fluxes rotating in the same or in opposite directions, or even by a combination of fluctuating and rotating fluxes. It is also immaterial with regard to the primary coil how these fluxes are locally distributed within the coil, whether these fluxes are set up by currents in the primary or the secondary coil, whether by direct currents or alternating currents, or a combination of such currents, as long as the above conditions are fulfilled.

If the secondary and tertiary coils are open circuited and inactive, so that none but the primary coil can furnish magnetomotive forces, the entire magnetization must naturally be furnished by the primary coil $P\,P'$. It is further evident in this case that the flux distribution and magnetic densities within the coil are solely governed by the relative magnetic reluctances of the different paths.

Under the customary assumption of uniform reluctance and neglecting for the present the relatively small magnetic leakage fluxes, the above leads to the well known relations:

The instantaneous gap density is

$$B_i = \frac{p\,\varphi_i}{D\,\pi\,l} = \frac{\varphi_i}{\pi}\,K \qquad (4)$$

if $K = \frac{p}{D\,l}$ and

if the average air gap diameter $= D$

Width of core $= l$

Number of poles $= p$

The instantaneous current value, in the primary coil, is

$$i_p = \frac{B_i}{n_p}\,\frac{l}{K_l} = \frac{\varphi_i\,K}{\pi\,K_l\,n_p} \qquad (5)$$

where $K_l$ is a constant $= \frac{1}{0.315 \times 2g \times G \times S}$

if $g =$ length of air gap on one side; $G =$ the usual gap factor allowing for the slot opening and $S =$ the saturation factor (all dimensions in inches).

From this, the crest value of the current is

$$I_p = \frac{\varphi}{\pi}\,\frac{K}{K_l\,n_p} \qquad (5\text{ A})$$

Let us assume now that the coil $S\,S'$ is short-circuited and revolved at synchronous speed by some external means.

By applying the first fundamental condition given above to this secondary coil, we know that in so far as this coil is short-circuited and has no external voltage impressed, we must have a resultant counter e.m.f. of zero induced within the coil. If there is any current flowing within the coil, it will cause an ohmic drop voltage. Therefore, in order to get a resultant of zero we must have an inductive voltage induced internally, which is equal and opposite to this ohmic drop voltage. The resistance of the coil $S\,S'$ is usually very small so that the ohmic drop is very small and with it also the necessary inductive voltage, as long as the secondary currents are not too large. Under the majority of conditions the required rate of change of the flux interlinking with the coil $S\,S'$ is, therefore, so small as to be negligible, as compared with that required in the primary coil. We are, thus, justified to assume at present for the sake of simplicity, that the flux within the secondary coil remains practically constant. This is, however, theoretically correct only with zero resistance in the secondary.

It is again indifferent with regard to the secondary coil how this condition of practically constant flux within the coil is maintained. We know, however, that if a flux has once been established by some means or other, within the coil, such flux will somehow be maintained practically constant. If the magnetomotive force which has established the flux disappears, or if the secondary coil is moved away from its influence, the flux within the coil will tend to diminish. Any slight decrease of flux will, however, at once induce voltages in the coil which, in turn, cause currents to flow maintaining the flux practically at its original value, if the ohmic drops are zero, or at least negligible.

Let us refer now to Fig. 1a and assume that it happens to represent the rotor position at the time angle $\alpha = 0$, with an angle $\gamma$ between the coil $P\,P'$ and $S\,S'$ as shown. Assuming further synchronous speed, we know that the time $\alpha = \gamma$ will have elapsed, before the secondary coil has moved an angle $\gamma$ and coincides with the primary coil. Now we know from formula (1) that the flux interlinking with the primary coil at the time $\alpha = \gamma$ must be

$$\varphi_{i\gamma} = \varphi \cos \gamma \tag{6}$$

If we neglect the leakage between the coils $P\,P'$ and $S\,S'$ this represents also the flux in the secondary coil at this time.

Since the flux in $S\,S'$ is constant, we know, therefore, that this constant flux of coil $S\,S'$, which rotates with the coil at synchronous speed must be

$$\varphi_s = \varphi \cos \gamma \tag{7}$$

Having now an equation for both the total flux $\varphi_i$ in the primary and the total flux $\varphi_s$ in the secondary coil, we can easily determine how the fluxes around the air gap must be distributed at any time to always satisfy both the equations (1) and (7) for $\varphi_i$ and $\varphi_s$ respectively.

Let us consider for instance a secondary position as shown in Fig. 1b, with an angle $x$ between $P\,P'$ and $S\,S'$. The time angle corresponding to this position is

$$\alpha = x + \gamma \text{ therefore } x = \alpha - \gamma \tag{8}$$

By reference to Fig. 1b, it is further at once evident that the flux between $P$ and $P'$, that is the primary flux $\varphi_i$ must be the sum of the fluxes $\varphi_a$ between $P$ and $S$ and the flux $\varphi_b$ between $S$ and $P'$; similarly, we see that the flux between $S$ and $S'$ that is the secondary flux $\varphi_s$ must be the sum of the flux $\varphi_b$ between $S$ and $P'$ and the flux $\varphi_a'$ between $P'$ and $S'$. Since $\varphi_a'$ is evidently equal to $-\varphi_a$, we have only two unknown quantities which can at once be found from the two conditions just stated. With the fluxes for the different air-gap portions known and the angles for which they apply the densities can be easily found.

For the time $\alpha = \gamma$ to $\alpha = \gamma + \pi$ which it takes the conductor $S$ to travel from $P$ to $P'$, we can, therefore, write the following equations

$$\varphi_i = \varphi_a + \varphi_b \tag{9}$$

$$\varphi_s = -\varphi_a + \varphi_b \tag{10}$$

It follows by addition and subtraction of (9) and (10)

$$\varphi_a = \frac{\varphi_i - \varphi_s}{2}$$

$$= (\cos\alpha - \cos\gamma)\frac{\varphi}{2} \tag{11}$$

$$\varphi_b = \frac{\varphi_i + \varphi_s}{2}$$

$$= (\cos\alpha + \cos\gamma)\frac{\varphi}{2} \tag{12}$$

The densities over the distances $a$ and $b$ of Fig. 1b, are, therefore,

$$B_a = \frac{\varphi_a}{x}K = \frac{\varphi_i - \varphi_s}{2\,x}K$$

$$= \frac{\cos\alpha - \cos\gamma}{x}\,\frac{K}{2}\,\varphi \tag{13}$$

$$B_b = \frac{\varphi_b}{\pi - x}K = \frac{\varphi_i + \varphi_s}{2\,(\pi - x)}K$$

$$= \frac{\cos\alpha + \cos\gamma}{\pi - x}\,\frac{K}{2}\,\varphi \tag{14}$$

Similarly, we find for the time

$$\alpha = \gamma + \pi \text{ to } \alpha = \gamma + 2\pi$$

in connection with Fig. 1D.

$$B_d = \frac{K}{2}\,\varphi\,\frac{\cos\alpha - \cos\gamma}{x - \pi} \tag{15}$$

$$B_e = \frac{K}{2}\,\varphi\,\frac{\cos\alpha + \cos\gamma}{2\pi - x} \tag{16}$$

The field densities thus obtained are shown in full lines in Fig. 2, with regard to their local distribution over the air gap circumference for a number of values of $\alpha$ as indicated, and under the assumption of $\gamma = 0$, which means that the secondary and primary coils are assumed to coincide at the time angle $\alpha = 0$, that is, when the primary flux has its maximum value. The areas enclosed by the full lines represent the total fluxes, as they must exist to fulfill the two fundamental conditions so far stated, for both the primary and secondary coil.

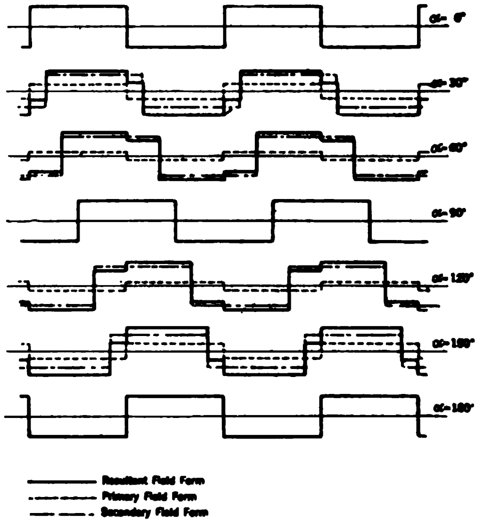

FIG. 2

As previously pointed out, the two conditions of equilibrium could be fulfilled in a number of different ways for each of the windings individually. Since our equations, taking the two windings into account, simultaneously, give only a single solution, however, Fig. 2 represents the only field condition possible under the assumptions made.

For different values of $\gamma$ somewhat different results will be obtained but usually it will be found as in Fig. 2 that the resultant field travels around with the rotor, *i. e.*, we have a rotating field; it will also be noticed that the distribution of the field varies.

As a matter of convenience in theoretical considerations, it is, of course, always possible to substitute a number of component fields for those obtained from the calculations, but the resultant of such field must always conform to our equations.

The determination of the currents flowing in the two windings must be based on *a third fundamental condition*, namely, *that the sum of all ampere turns acting upon a certain part of the gap circumference must be proportional to the magnetic density in such part*, making the usual assumption of uniform magnetic reluctance.

Referring, for instance, again to Fig. 1b, in which certain current directions have been arbitrarily assumed and indicated by arrows, it is at once evident that the density $B_a$ between $P$ and $S$, for instance, must be set proportional to the difference between the ampere turns in $P\,P'$ and those in $S\,S'$. Determining similar relation for other portions, we obtain at least two equations with the two unknown current values $i_p$ and $i_s$, which follow directly from the solution of these equations.

If the secondary coil $S\,S'$ with the turns $n_s$ carries the current $i_s$ in a direction as marked in Fig. 1b, we find the following relations between the currents and densities.

$$B_a = K_l\,(i_p\,n_p - i_s\,n_s) \qquad (17)$$

$$B_b = K_l\,(i_p\,n_p + i_s\,n_s) \qquad (18)$$

By additions and subtraction of (**17**) and (**18**), we get

$$i_p = \frac{B_a + B_b}{2\,K_l\,n_p} = \frac{K\,\varphi}{4\,K_l\,n_p}\left(\frac{\cos\alpha - \cos\gamma}{x} + \frac{\cos\alpha + \cos\gamma}{\pi - x}\right) \qquad (20)$$

$$i_s = \frac{B_b - B_a}{2\,K_l\,n_s} = \frac{K\,\varphi}{4\,K_l\,n_p}\left(\frac{\cos\alpha + \cos\gamma}{\pi - x} - \frac{\cos\alpha - \cos\gamma}{x}\right) \qquad (21)$$

Similarly, we find for the time
$\alpha = \gamma + \pi$ to $\alpha = \gamma + 2\pi$ in connection with Fig. 1d.

$$i_p = \frac{K\varphi}{4 K_l n_p}\left(\frac{\cos\alpha - \cos\gamma}{x - \pi} + \frac{\cos\alpha + \cos\gamma}{2\pi - x}\right) \tag{22}$$

$$i_s = \frac{K\varphi}{4 K_l n_s}\left(\frac{\cos\alpha - \cos\gamma}{x - \pi} - \frac{\cos\alpha + \cos\gamma}{2\pi - x}\right) \tag{23}$$

Fig. 3 shows in light lines the values for $i_p$ (Curve $P$) and $i_s$ (Curve $S$) as a function of the time angle $\alpha$ and as found from (**20**) to (**23**), again for the assumption $\gamma = 0$.

These light curves represent the single and only solution of our equations at any time except at the instant when the secondary and primary coils coincide. At these instances, as for example at the time $\alpha = 0$ with $\gamma = 0$, the above third fundamental condition must be fulfilled as much as at any other time. We have, however, at this instant only the flux $\varphi_a = \varphi_i$, the angle $x$ being 0. Our condition is, therefore, given by

$$B_b = K_l\,(i_p\,n_p + i_s\,n_s)$$

that is the same as in equation (**18**), but there being no value for $B_a$, we have only one equation with the two unknown quantities.

The single equation known for $\alpha = 0$, etc., is satisfied with any values for $i_p$ and $i_s$, as long as

$$i_p\,n_p + i_s\,n_s = \frac{B_a}{K} = \frac{\varphi}{\pi}\,\frac{K}{K_l}$$

or assuming $n_p = n_s$

$$i_p + i_s = \frac{\varphi}{n_p\,\pi}\,\frac{K}{K_l} = i_z \tag{24}$$

This simply means that from our third condition nothing but the sum $i_z$ of $i_p$ and $i_s$ is known, while it is possible to fulfill equation (**24**) for any value of $i_s$ between $-\infty$ and $+\infty$ by assuming $i_p$ to correspond.

The following consideration makes, however, a determination of $i_s$ for the time $\alpha = 0$, $\alpha = \pi$ etc., possible. Assume coil $S\,S'$ coincides with $P\,P'$ at the time $\alpha = 0$ with $\gamma = 0$, *i. e.*, when $\varphi_s = \varphi$. For any other position, the currents in $S\,S'$ tend to maintain this flux, as previously pointed out, by furnishing whatever magnetizing current is required to do so, in addition to the primary current. In order to cause such magnetizing current to flow with even the smallest secondary resistance, an inductive voltage induced by slight changes of the flux $\varphi_s$ is required. While it was permissible to neglect these small changes in our previous calculation of densities and currents, we must not overlook that a small change in the flux is necessary to make $i_s$ flow between the times $\alpha = 0$ and $\alpha = \pi$. We further know that while $S\,S'$ travels from $x = 0$ to $x = \pi$ the primary flux has changed from $\varphi$ to $-\ \varphi$ that is, it has reversed its direction with regard to $P\,P'$. Its direction with regard to $SS'$ which has traveled 180 deg. in the meantime is, however, the same for both $\alpha = 0$ and $\alpha = \pi$. In other words we know that while $\varphi_s$ undergoes slight variations, it must periodically resume in intervals of $\pi$ the same value, which is $\varphi$ in our present case. This, in turn, means that any flux decrease in coil $S\,S'$ must be followed by an increase, further, that for a full time interval $\pi$, the sum of all increases must equal the sum of all decreases. If, therefore, $d\ \varphi_s$ represents the infinitely small flux change during a time $dt$, we know that

$$\Sigma\, d\, \varphi_s = 0$$

Since the rate of change of the flux $\varphi_s$ determines the voltage, we have $\frac{d\ \varphi_s}{dt} = e_s = i_s\, r_s$ or $d\,\varphi_s = i_s\, r_s\, dt$ where $e_s =$ the voltage induced in $S\,S'$ and $r_s$ the resistance. We have further

$$\Sigma\, d\, \varphi_s = r_s\, \Sigma\, i_s\, dt = 0 \qquad (30)$$

between the limits 0 and $\pi$.

The value $\Sigma\, i_s\, dt$ represents the area included by the secondary current curve over the time 0 to $\pi$, the resultant of which must, therefore, be zero, if $r$ has a value different from 0.

From this, we can directly derive our *fourth fundamental condition*, namely, *that the voltages induced and, therefore, the currents flowing in the secondary coil must have an average value of zero, over each complete time period of* $\pi$.

In other words this means that the areas of a curve, including all positive current values must be equal to the area of a curve, including all negative current values.

An inspection of curve $S$, in Fig. 3, reveals that all current values of $i_s$ between $\alpha = 0$ and $\alpha = \pi$ are positive, and it is, therefore, at once evident that the current can be negative only for infinitely short time intervals at $\alpha = 0$, $\alpha = \pi$ etc. This means that the height of the negative current area must be $-\infty$, in order to give with an infinitely small basis an area equal to the corresponding positive current area.

The value of $i_s$ for $\alpha = 0$, etc., having been thus determined, it follows at once from (24) that

$$i_p = \infty + \frac{\varphi}{n_p \pi} \frac{K}{K_t} = \infty$$

The theoretical infinite current values are indicated by heavy vertical lines in Fig. 3. The conclusion to be drawn from the completed current curves is that the primary currents are of line frequency with very marked odd higher harmonics, while the secondary currents are of double line frequency with very marked higher harmonics; the fact that the positive and negative wave shapes are materially different indicates the rather unusual existence of marked even harmonics in the secondary currents.

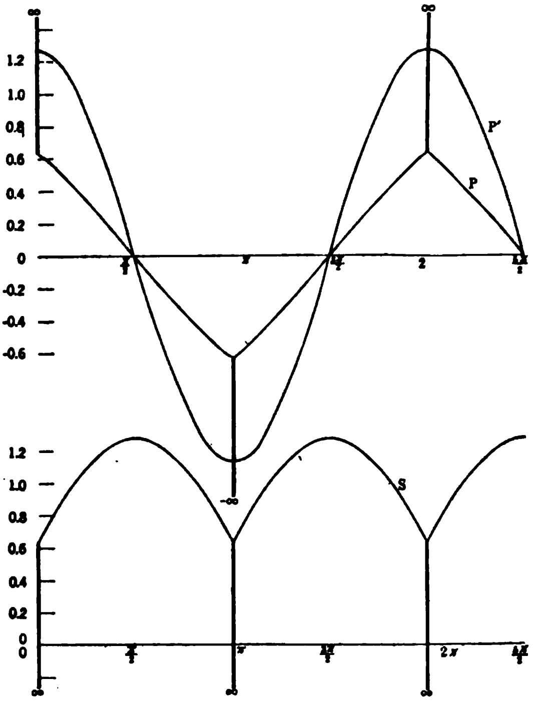

FIG. 3

Fig. 3 also shows in a curve $P'$ the value of $i_p$ as obtained from (5) for the open-circuited secondary. It will be seen that the effective $i_p$ of curve $P$, without the infinitely high extension would be even smaller than that of curve $P'$, which we know to be in contradiction to the facts. This is additional evidence for the necessity of a sudden change of the currents for $\alpha = 0$ and $\alpha = \pi$.

In line with the above, Fig. 2 shows in dotted lines the field components supplied by the stator and the rotor for all cases except for $\alpha = 0$ and $\alpha = \pi$ in which cases these values are $\infty$; in case of $\alpha = \frac{\pi}{2}$ the dotted lines do not appear because the field induced in the primary is zero, and that induced by the secondary coincides with the total field.

For most other values of $\gamma$, similar conditions are obtained, but the coinciding of the coils and, therefore, the infinite current values occur at different values for $\alpha$.

While the current curves derived so far are correct under the assumption made, attention may already be briefly directed towards a number of facts, which have marked modifying effect in actual machines.

In practise, it is, of course, impossible to concentrate the effect of the coil conductors into a point, and, therefore, the passing of the secondary and primary coils is not actually infinitely short. Consequently, we obtain in the secondary, even though the coils are concentrated as much as practicable, certain limited but rather large current values for a rather short time, in place of the infinite negative values for an infinitely short time. Similarly, the additions to the calculated primary current wave are not infinitely large, but will appear as rather sharp corners of limited height extending over a very brief period of time.

Another equally important feature, which makes the existence of infinite current values in actual practise impossible, is the fact that any machine has magnetic leakage fluxes. These fluxes no matter how small they may be will induce very appreciable voltages affecting the working conditions to a marked degree, if the rate of change in such fluxes is large; the latter condition naturally prevails if the currents setting up the leakage fluxes suddenly tend to assume infinite values, as in our case.

While, for these reasons, the previous current curves, as well as some of those given later, do not picture the actual conditions correctly, they are nevertheless very instructive insofar as they indicate the strong tendency towards higher harmonics and secondary phenomena caused thereby. This will be discussed more in detail later on.

The fluxes interlinking with the tertiary phase converter coil $T\,T'$ can easily be determined, since the densities at all portions of the circumference are known, if we neglect again the leakage fluxes. With the fluxes known, the induced tertiary voltages follow directly by finding the rate of change of the tertiary flux $\frac{d\,\varphi_t}{dt}$.

We have

For the time $\alpha = \gamma$ to $\alpha = \gamma + \frac{\pi}{2}$ (Fig. 1b)

$$\varphi_t = B_b \frac{\pi}{2} - B_a x - B_b \left(\frac{\pi}{2} - x\right)$$

$$= (B_b - B_a) \frac{x}{K} \tag{25}$$

For the time $\alpha = \gamma + \frac{\pi}{2}$ to $\alpha = \gamma + \pi$ (Fig. 1c)

$$\varphi_t = B_a \left(x - \frac{\pi}{2}\right) + B_b (\pi - x) - B_a \frac{\pi}{2}$$

$$= \frac{(\pi - x)}{K} (B_b - B_a) \tag{26}$$

For the time $\alpha = \gamma + \pi$ to $\alpha = \gamma + \frac{3}{2}\pi$ (Fig. 1d)

$$\varphi_t = B_e \frac{\pi}{2} - B_d (x - \pi) - B_e \left(\frac{3}{2}\pi - x\right)$$

$$= \frac{(\pi - x)}{K} (B_d - B_e) \tag{27}$$

For the time $\alpha = \gamma + \frac{3}{2}\pi$ to $\alpha = \gamma + 2\pi$ (Fig. 1e)

$$\varphi_t = B_d \left(x - \frac{3}{2}\pi\right) + B_e (2\pi - x) - B_d \frac{\pi}{2}$$

$$= \frac{(2\pi - x)}{K} (B_e - B_d) \tag{28}$$

The voltage of the tertiary coil can be found by finding the value $\frac{d\,\varphi_t}{dt} = e_t$ from these equations, after the previously found values for $B_a$ and $B_d$ in terms of $\alpha$ have been introduced. Thus we obtain for the time

$\alpha = \gamma$ to $\alpha = \gamma + \frac{\pi}{2}$

$$e_t = -\frac{\varphi}{2}\left[\frac{\pi}{(\pi - \alpha + \gamma)^2}(\cos\alpha + \cos\gamma) + \left(1 - \frac{\alpha - \gamma}{\pi - \alpha + \gamma}\right)\sin\alpha\right] \qquad (29)$$

etc.

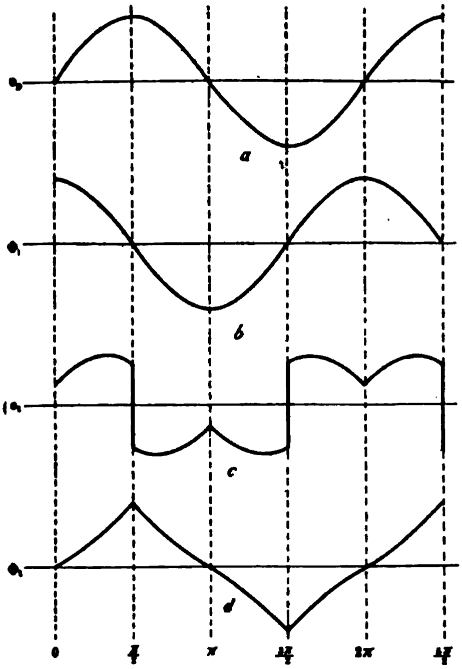

Fig. 4—$\gamma = 0$

Fig. 4 shows the time curves of the primary voltage of the coil $P\,P'$ and the flux $\varphi_i$ interlinking with this coil as found from **(1)** and **(3)**, further the flux $\varphi_t$ interlinking with the tertiary coil $T\,T'$ and the voltage induced in this coil as found from **(25)**, **(26)**, **(27)**, **(28)** and **(29)**.

It will be noted that although the primary voltage and flux follow a sine law, the tertiary flux and voltage are far from such law. The tertiary maximum flux value is the same as that of the primary, but both the average and effective values are lower in case of the tertiary flux.

The case of Fig. 1 for $\gamma = \frac{\pi}{2}$, that is, the case in which coil $S\,S'$ and $P\,P'$ coincide at the time $\alpha = \frac{\pi}{2}$ when the primary flux is 0, is of little interest in connection with the phase-converting problems. It is interesting, however, insofar as we obtain in this case an appreciable tertiary voltage of higher frequency. The fluxes and voltages for this case are shown in Fig. 5. The results obtained are somewhat surprising, since it would appear from a casual consideration that insofar as the resultant secondary flux

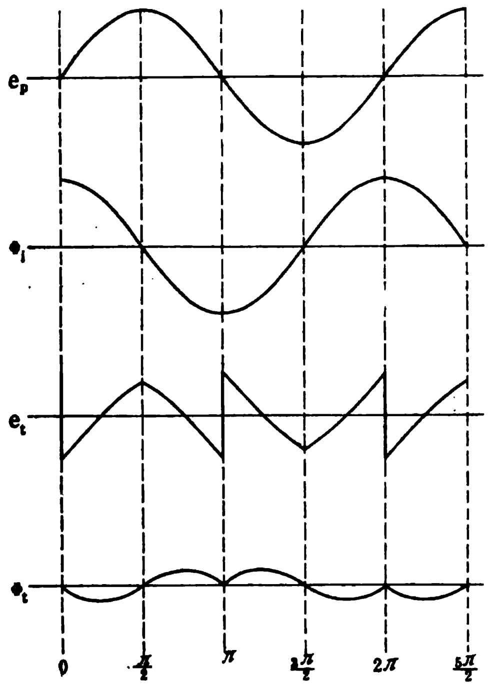

Fig. 5—$\gamma = \frac{}{2}$

$$\varphi_s = \varphi \cos \gamma = \varphi \cos \frac{\pi}{2} = 0$$

is zero, that $\varphi_t$ should be zero.

Closer investigation shows, however, that while $\varphi_s$ is zero, as a whole, it is at times the resultant of two opposing fluxes, which add up to zero inside the coil $S\,S'$, but give certain positive and negative values inside of $T T'$ as shown in Fig. 5d. While these fluxes are relatively small, they are of higher frequency and, therefore, induce an appreciable voltage in $T T'$. We have, therefore, for $\gamma = \frac{\pi}{2}$ a frequency changer instead of phase converter.

While a further study of these phenomena with relation to the resistances and leakage fields would be interesting it has been omitted since it is not related to the subject matter of this paper.

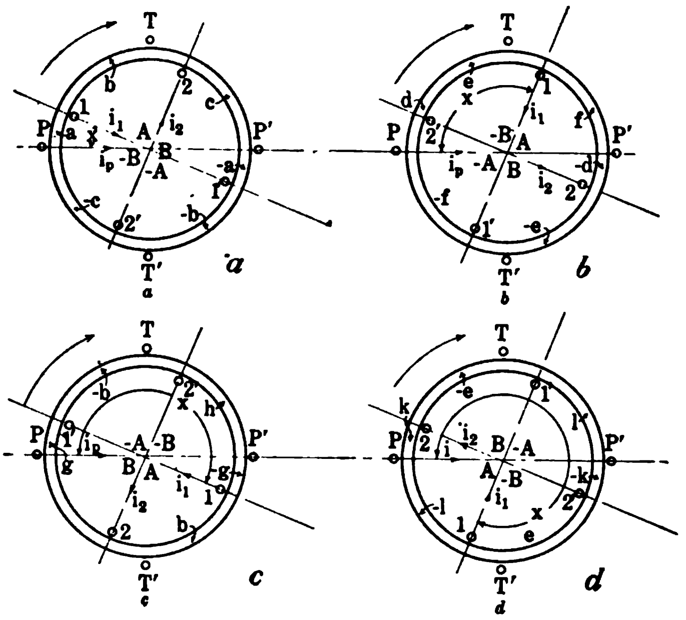

FIG. 6

## CASE NO. 2.

### *Single Primary Coil, Two-Phase Secondary*

Let us now consider a converter with a two-phase secondary with one coil per pole per phase, as shown in Fig. 6. Attached appendix gives the detail calculations carried through along the same lines as in the previous case.

The only difference in the calculations between this and the previous case is that two instead of one secondary coil has to be considered, which leads to three equations with three unknown quantities, instead of two.

Fig. 7 shows for $\gamma = 0$ the currents in the primary and in the two secondary coils. The calculation from the formulas gives again only positive currents for $i_1$, which for reasons previously given, necessitates the infinite negative values, indicated by the heavy vertical lines, and the corresponding infinite positive values of $i_p$ for $\alpha = 0$ and $\alpha = \pi$. The values found for $i_2$ satisfy our fourth fundamental condition, without infinite values

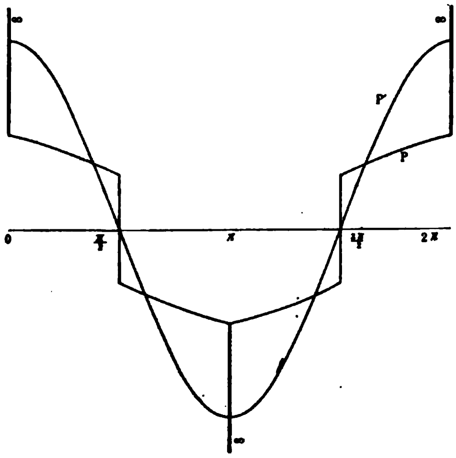

FIG. 7A

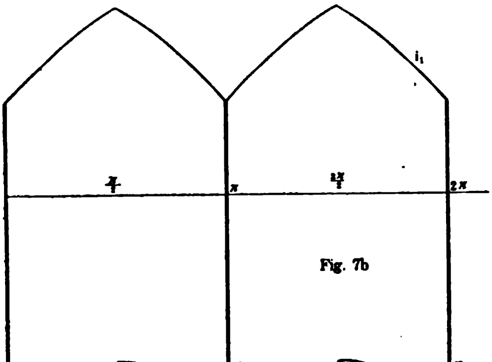

Fig. 7b

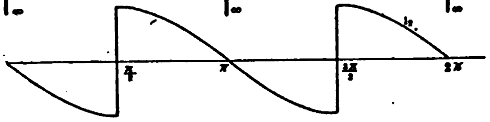

FIG. 7C

Fig. 8 shows the corresponding primary and tertiary fluxes and voltages.

Since we have now a polyphase rotor, the machine will be self-propelling like any single-phase induction motor. If the machine is asynchronous, the friction will cause a very small slip, so that even if the relation $\gamma = 0$ were originally established, the rotor will slip slightly behind synchronous speed, so that the value of $\gamma$ changes gradually, but continuously. Therefore, other values than $\gamma = 0$ are of practical interest.

Fig. 9 has been worked out to give the currents for $\gamma = \frac{\pi}{4}$

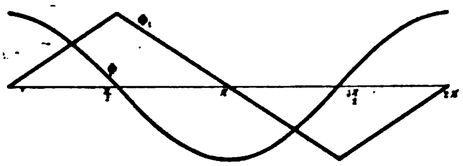

FIG. 8A

that is for the case, where the primary flux reaches a maximum, when the middle of a secondary tooth coincides with the primary coil. As will be seen both secondary currents have now infinite negative values.

Fig. 10 shows the corresponding primary and tertiary fluxes and voltages.

Fig. 11 shows the field forms for various values of $\alpha$ for $\gamma = 0$; while Fig. 12 covers the case of $\gamma = \frac{\pi}{4}$

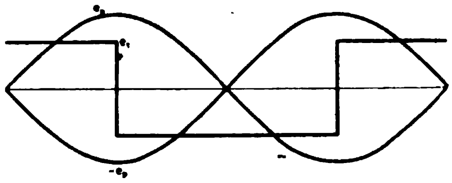

FIG. 8B

$$e_x \text{ effective} = \frac{0.636}{0.707} \text{ or } 0.9\ e_p \text{ effective}$$

As previously mentioned $\gamma$ changes very gradually all the time with a light running asynchronous machine. Since the fields, currents and tertiary voltages change with $\gamma$, as will be seen by a comparison of Figs. 7 and 9, 8 and 10, and 11 and 12 respectively; this means that nearly everything in the machine undergoes a continuous change. Thus, the higher harmonics or current peaks of the primary change their location with regard to the fundamental wave. (Compare Figs. 7A and 9A); the currents in each secondary coil change materially in both size and shape, (Compare Figs. 7B and 9B, also 7C and 9C); also the tertiary voltages change their wave form materially (Compare Figs. 8B and 10B) as well as their peak values; similar conditions apply to the tertiary flux values (Compare Figs. 8A and 10A) interlinking with coil $T\,T'$.

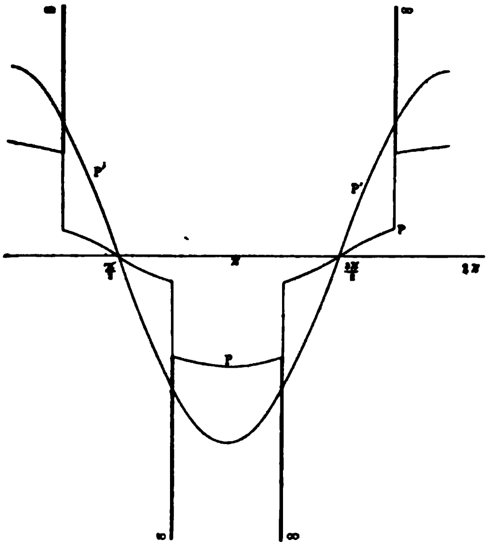

FIG. 9A

The current curves show again general characteristics as in Case No. 1 with regard to fundamental frequency and higher harmonics.

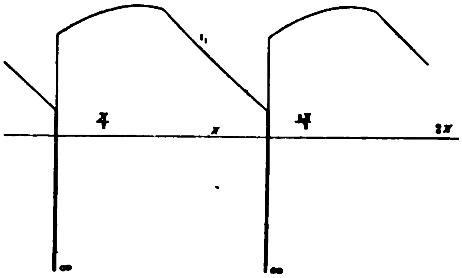

FIG. 9B

The local distribution of the resultant field changes continuously even with constant value of $\gamma$, as indicated in Fig. 11, but changes in the value of $\gamma$, cause further material changes. (Compare Figs. 11 and 12.) Among other things it will be noted for instance that the total area for certain values of $\alpha$ is smaller in Fig. 12 than for similar values of $\alpha$ in Fig. 11. Figs. 11 and 12 show, however, in both cases that the resultant field rotates in the direction of the rotor. These figures also show in dotted lines the fields as furnished by the primary and secondary coils alone.

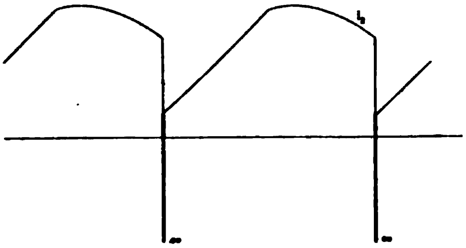

FIG. 9C

The lines $y$ $y$ indicate the center of the resultant field and plainly show its travel from left to right; the center line of the field induced by the primary currents is marked $z$ $z$, and is of course, stationary; the center line of the fields induced by the secondary currents are marked $x$ $x$, and it will be seen that they move in general in a direction opposite to that of the lines $y$ $y$, although their speed is very irregular and reverses for certain intervals of time.

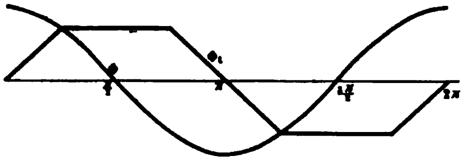

FIG. 10A

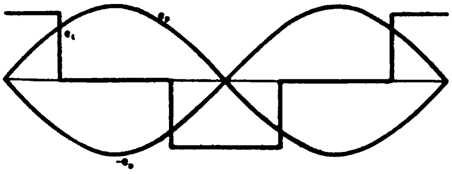

FIG. 10B

$$e_x \text{ effective} = \frac{0.636}{0.707} \text{ or } 0.9\ e_p \text{ eff.}$$

In view of the irregular behavior of nearly everything in the motor, it is rather surprising that exact calculations show the effective value of the tertiary voltage to be unchanged with changing $\gamma$ namely always 90 per cent of the effective primary voltage, although as mentioned before, the wave shape of the tertiary voltage changes continuously with $\gamma$.

### CASE NO. 3

### *Single Primary Coil, Polyphase Secondary*

A case with single concentrated primary and tertiary coils in combination with a polyphase rotor winding with *m* rotor coils as shown in Fig. 13 may now be considered. The rotor position as shown is assumed to correspond to the time angle $\alpha = 0$.

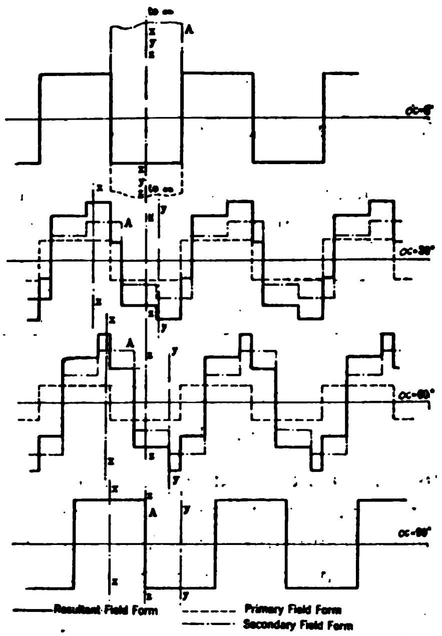

FIG. 11

By finding again, as in the previous cases, the time at which each secondary coil coincides with the primary and by determining the value of the primary flux at this time, we can easily find the constant total fluxes interlinking with each secondary coil.

The secondary coils pass coil $P\ P'$ as follows.

Coil 1 at the time $\gamma$
" 2 " " " $\gamma - \beta$
" 3 " " " $\gamma - 2\beta$
" $n$ " " " $\gamma - (n - 1)\beta$
etc.

Consequently, the constant fluxes interlinking with these coils are

$$\begin{aligned} \varphi_1 &= \varphi \cos \gamma \\ \varphi_2 &= \varphi \cos (\gamma - \beta) \qquad (31) \\ \varphi_n &= \varphi \cos [\gamma - (n-1)\ \beta] \\ \varphi_r &= \varphi \cos [\gamma - (r-1)\ \beta] \\ \varphi_{r+1} &= \varphi \cos (\gamma - r\ \beta) \\ &\text{etc.} \end{aligned}$$

We also have $\gamma = (r-1)\beta$ and $\beta = \dfrac{\pi}{m}$

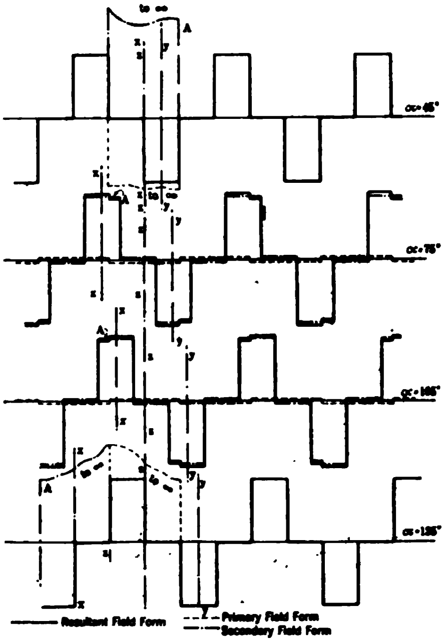

FIG. 12

The determination of the fluxes in the individual teeth and the densities, by the previous method of writing as many equations as there are tooth fluxes, would be, of course, very laborious in the present case with many secondary teeth. The calculations immediately following this prove, however, that the total flux of each secondary tooth can be found as one-half the difference of the total fluxes interlinking with the two adjacent coils, as found above in (**31**). The individual part fluxes of the rotor tooth opposite the primary coil, that is, the fluxes for the angles $u$ and $v$ in Fig. 13 forming part of tooth $R$ can be determined by a similar simple calculation. After all part fluxes are known, the densities follow, of course, directly.

Let us assume for the present a field distribution for the resultant rotor field as shown in Fig. 14, and if we assume further that the flux in tooth $N$ of Fig. 13 is represented by the area

$$\varphi_N = n\, n_1\, p_1\, p$$

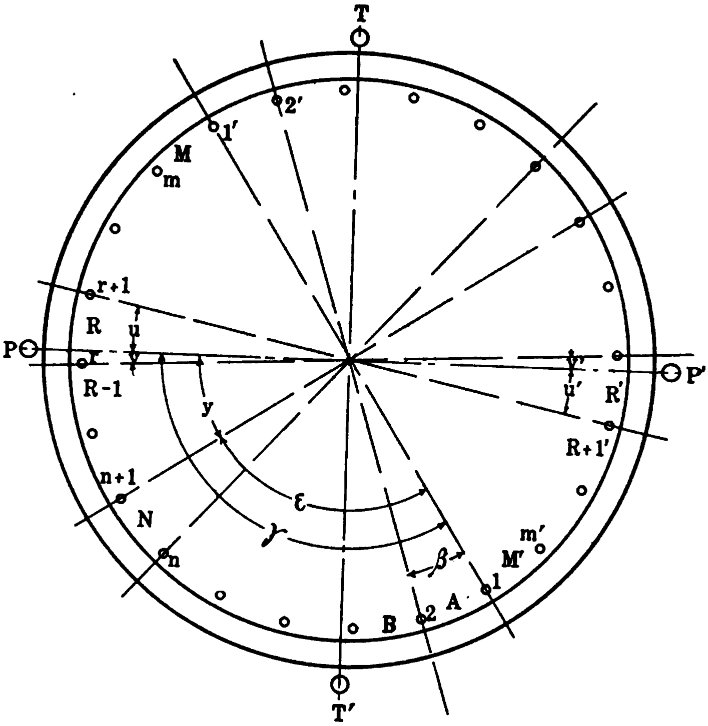

FIG. 13

and that of the opposite tooth $N'$ by

$$\varphi_{N'} = n'\, n_1'\, p_1'\, p'$$

Let further

area $n\, p\, P = Y$
" $P\, n'\, p' = Y'$
" $n_1\, p_1\, P = X$
" $P\, n_1'\, p_1' = X'$

The resultant flux in coil $n$ is then according to Fig. 14

$$\varphi_n = Y - Y' \qquad (32)$$

and that in coil $n + 1$ is

$$\varphi_{n+1} = X - X' \qquad (33)$$

It follows further from the figure that

$$Y = X + \varphi_N$$
$$Y' = X' - \varphi_{N'} \qquad (34)$$

therefore, we can write (32) by introducing these values

$$\varphi_n = X + \varphi_N - X' + \varphi_{N'} \qquad (35)$$

or since obviously opposite teeth carry the same amount of flux and, therefore,

$$\varphi_N = \varphi_{N'}, \text{ we get}$$
$$2\,\varphi_N = \varphi_n - (X - X') = \varphi_n - \varphi_{n+1}$$
$$\varphi_N = \tfrac{1}{2}\,[\varphi_n - \varphi_{n+1}] = \qquad (36)$$
$$\frac{\varphi}{2}\ [\cos(\gamma - (n-1)\ \beta) - \cos(\gamma - n\ \beta)]$$

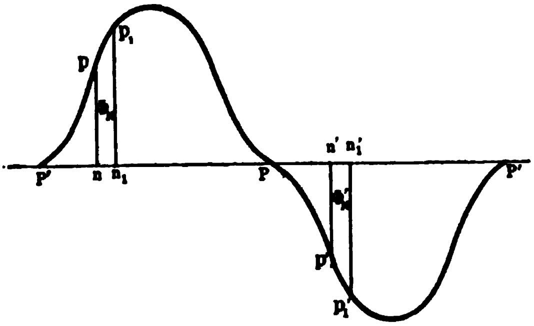

FIG. 14

It appears that the teeth fluxes found from (36) are independent of $\alpha$ and, therefore, constant. They are also uniformly distributed over the entire tooth width in all teeth except in the teeth $R$ and $R'$, which are influenced by the currents in $P$ and $P'$.

It is evident from Fig. 13 that

$$\varphi_R = \varphi_v + \varphi_u \qquad (37)$$

also that

$$-\varphi_i = \varphi_v + \varphi_{R-1} + \ldots\ldots\ldots\ \varphi_B$$
$$+ \varphi_A + \ldots\ldots\ldots\ \varphi_{R+1'} + \varphi_{u'} \qquad (38)$$

Since obviously $\varphi_{u'}' = -\varphi_u$ and since all values in (38) except $\varphi_i$, $\varphi_v$ and $\varphi_{u'}$ have been found to be constant, we can write

$$-\varphi_i = \varphi_v - \varphi_u + K_2 \qquad (39)$$

where $K_2 = \Sigma\,\varphi$ from $\varphi_{R-1}$ to $\varphi_{R+1'}$.

By adding (37) and (39) we find

$$\varphi_v = \tfrac{1}{2} (\varphi_R - \varphi_i - K_2) \qquad (40)$$

and

$$\varphi_u = \tfrac{1}{2} (\varphi_R + \varphi_i + K_2) \qquad (41)$$

We know from (36) that

$$\varphi_R = \frac{\varphi}{2} [\cos (\gamma - (r-1) \beta) - \cos (\gamma - r \beta)]$$

and

$$\varphi_i = \varphi \cos \alpha$$

therefore, we can now determine $\varphi_v$ and $\varphi_u$ in terms of $\alpha$ for the time

$$\alpha = - u \text{ to } \alpha = v.$$

Since $\cos \alpha$ enters the values for $\varphi_v$ and $\varphi_u$, we find that while the sum of the two is constant, each of the two values varies with the time.

For the time $\alpha = v$ to $\alpha = v + \beta$ the tooth $R - 1$ is under the influence of $P\,P'$ and its part fluxes can be similarly determined, etc.

With the tooth fluxes known, the densities follow directly to be

$$B_N = \frac{\varphi_N}{\beta} K$$

etc.

also,

$$B_v = \frac{\varphi_v}{v} K \qquad (42)$$

$$B_u = \frac{\varphi_u}{u} K$$

The step lines of Fig. 15 represent the resultant field distribution for a motor, as per Fig. 13, with $m = 6$, that is, with six secondary slots per pole for various values of $\alpha$ and under the assumption that $\gamma$ = a multiple of $\beta$, which means that one of the secondary coils coincides with $P$ for $\alpha = 0$ and the maximum value of the flux $\varphi_i$.

It is plainly evident again from Fig. 15 that the resultant field travels with the secondary. The sine curves shown in the figure, as well as equation (36), reveal the fact that the sine curve cuts the center of each step. This fact applies to any polyphase rotor, even Case No. 2, although it was not so evident there. In other words, we see that the magnetizing currents in the secondary create as much as possible a resultant rotating field of sinusoidal space distribution, even with the unfavorable rectangular distribution of the primary field assumed in our present case. The resultant field will approach a sinusoidal distribution the closer, the larger the number of secondary teeth and the more the secondary resistance approaches the zero value.

Every slot, primary or secondary, produces a step, unless the current in the slot happens to be zero. It will further be seen that the flux of each secondary tooth is, of course, uniformly distributed over the tooth, except in the tooth opposite to the primary slot; in this case, the ampere conductors in the primary slot form an additional step, without, however, changing the total flux of the particular secondary tooth.

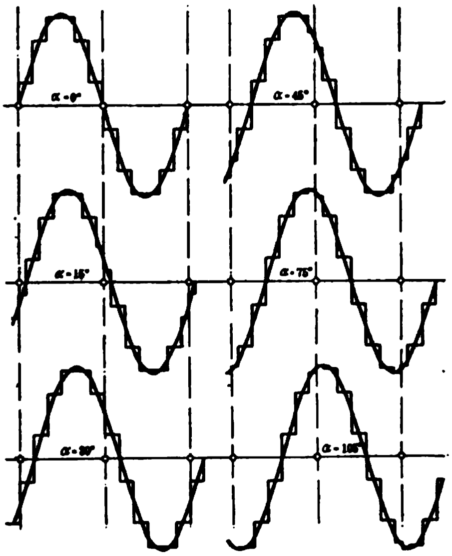

FIG. 15

The currents could again be determined as in the previous cases, by setting the sum of all magnetomotive forces acting upon each tooth or its parts proportional to the known densities. This would, with many coils, lead to an impractical amount of calculations. A much simpler method follows from the simple consideration that the difference between the densities adjacent to each slot is naturally caused by the ampere conductors in the slot.

If we make again the previous assumptions of uniform reluctances, etc., this fact points to the natural conclusion that the difference in the densities at each side of a slot is proportional to the ampere conductors in the slot. This in turn gives, for our present problem, *a fifth fundamental condition to be met* (which now takes the place of the third condition utilized previously), namely, that with densities for all tooth portions being definitely established by other conditions, *the ampere conductors of each slot must at any time be proportional to the difference in magnetic densities at the two sides of the slot.*

Since the densities of all tooth portions are known, this condition offers a very simple way for finding the current in the various coils by simply finding the difference of the tooth densities at each side of the conductors. Definite and fully determined current values can thus be found in all cases except when a rotor and a stator coil coincide, in which case, it is not directly known how the total magnetomotive force distributes between the two coils; this latter exception was discussed in detail under Case No. 1.

In order to obtain the correct mathematical relations, reference may be made to equation (5), from which the ampere turns necessary to obtain a certain density may be found. While equation (5) was written for the single primary coil, the same law applies to any coil with a current $i_n$ turns $N_n$ producing a density $B$. We have, therefore,

$$i_n = \frac{B}{N_n}\,\frac{1}{K_l} \text{ or,}$$

$$B = i_n\, n_n\, K_l$$

Now it is evident that if a single coil produces a density $B$, we will have a density of say $+B$ at one side of the slot and $-B$ at the other side, giving a difference $B_d$ between the densities on both sides of the slot of

$$B_d = B - (-B) = 2\,B = 2\,i_n\,n_n K_l \text{ or,}$$

$$i_n = B_d\,\frac{1}{2\,n_n\,K_l} \qquad (43)$$

With the exception previously stated, we find, therefore,

$$i_1 = (B_{M'} - B_A)\,\frac{1}{2\,n_1\,K_l}$$

$$i_2 = (B_A - B_B)\frac{1}{2\, n_2\, K_l}$$

etc.

$$i_n = (B_{N-1} - B_N)\frac{1}{2\, n_n\, K_l} \qquad (44)$$

$$i_r = (B_{R-1} - B_v)\frac{1}{2\, n_n\, K_l} \qquad (45)$$

$$i_p = (B_v - B_u)\frac{1}{2\, n_p\, K_l} \qquad (46)$$

$$i_{r+1} = (B_u - B_{R+1})\frac{1}{2\, n_{r+1}\, K_l} \qquad (47)$$

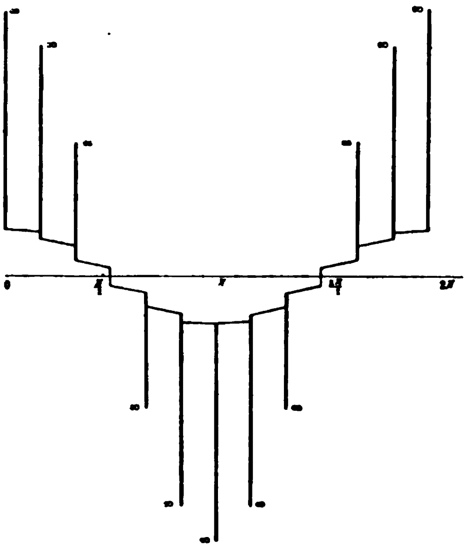

Fig. 16A

Since all values of these equations, except $B_v$, and $B_u$ are constant, it follows that all current except $i_r$, $i_p$ and $i_{r+1}$ are constant for the time $\alpha = -u$ to $\alpha = v$.

For the next following time $\alpha = v$ to $\alpha = v + \beta$ all currents except $i_{r-1}$, $i_p$ and $i_r$ are constant, etc. In other words, it follows that any of the secondary currents are constant, except when either of the adjacent teeth of the coil under consideration passes $P$ or $P'$.

Fig. 16 shows the primary and secondary current curves for the same case as assumed in Fig. 15. In accordance with our fourth fundamental condition, the ∞ negative values are again indicated by the heavy vertical lines in connection with the secondary currents. Each of the – ∞ values reflects into the primary, so that the primary current has a number of those heavy vertical lines shown.

A close study of the secondary currents reveals that each of the currents is uniform for the largest part of the time. The uniform current value is a sine function of the space angle between the particular coil and coil 1.

When the tooth adjacent to the coil under consideration begins to coincide with $P$ or $P'$, the current decreases along part of a sine curve until the coil coincides with $P$ or $P'$, when an infinite negative value is reached for an instant. Subsequently, the current increases again along part of a sine curve, while the other adjacent tooth passes $P$, until the previous constant value is reached again and maintained until the coil approaches the next primary slot.

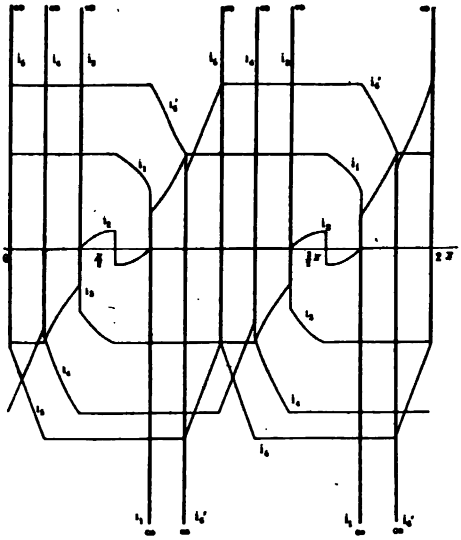

Fig. 16B

These general characteristics apply to all cases considered so far. The constant current value over a certain period does, however, not exist in Figs. 3, 7 and 9, of case 1 and 2, because there is at all times at least one tooth adjacent to each secondary slot coinciding with a primary slot. Therefore, we have in these cases only those parts of the secondary current curves which are infinite or those which follow parts of sine curves. In practise, the infinite values will, of course, again be changed to definite high current peaks as pointed out in Case No. 1.

The determination of the tertiary voltages by means of the previous methods with a number of equations would again be very laborious in the present case. We have found, however, that the density over each secondary tooth is uniform and constant unless it coincides with a primary slot. Since in our case, a secondary tooth passing the tertiary slots $T$ or $T'$ never coincides at the same time with the primary slot $P$ or $P'$, as long as $m \geq 2$, each tooth flux and density remains constant, while passing and cutting the tertiary conductors.

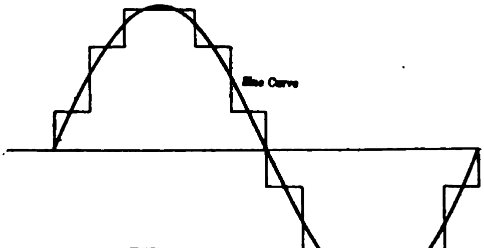

Fig. 17

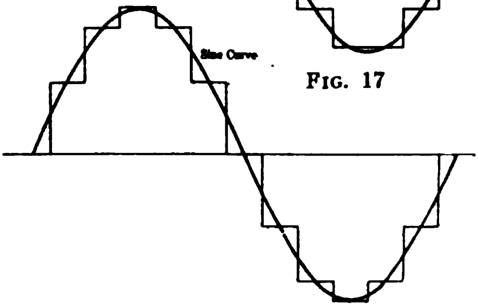

Fig. 18

The determination of the tertiary voltage is, therefore, very simple in connection with the present case, if we apply the theory of cutting magnetic lines in place of figuring with the interlinking fluxes. The tertiary voltage is then simply to be set proportional to the density of the teeth passing the coil sides.

If we determine, therefore, from previous formulas the resultant field form, disregarding the extra step caused temporarily by the primary in each secondary tooth while it passes a primary slot, we have at the same time the tertiary voltage wave. Figs. 17 and 18 show the resultant field and tertiary voltage wave, thus determined for six secondary slots per pole. Fig. 17 applies for $\gamma = 0$, that is, under the assumption that a secondary slot happens to coincide with the primary slot at the time $\alpha = 0$, while Fig. 18 applies for $\gamma = \frac{\beta}{2}$, that is, when the middle of a secondary tooth happens to coincide with the primary slot at the time $\alpha = 0$.

With an asynchronous machine, the small slip will gradually change the values of $\gamma$ and, therefore, the conditions from that of Fig. 17 to that of Fig. 18. This means that the tertiary voltage again changes its shape as in the previous case while the rotor slips against the stator.

With the wave shape of the tertiary voltages at hand, it is comparatively simple to find the effective tertiary voltage values by simply calculating the r.m.s. value of the step curve.

Referring back to Fig. 13, it is evident that all secondary conductors are alike and that nothing is changed if the conductors are numbered, starting with 1 at the conductor just approaching slot $P$, that is, numbering the conductor marked "$r$" with No. 1. This simply means that $\gamma$ is a fraction of $\beta$. Let us now set

$$\gamma = a\,\beta = a\,\frac{\pi}{m}$$

where according to the above, $a$ is a true fraction and $\beta = \frac{\pi}{m}$ follows from the fact that we have $m$ teeth per pole, *i.e.*, for the arc $\pi$. Introducing this in equation (42) for $B_N$, and using equation (36) for $\varphi_N$, we obtain

$$B_N = \frac{\varphi_N\, m}{\pi}\,K = \frac{\varphi\, m\, K}{2\,\pi}$$

$$\left[\cos\left(a - (n+1)\frac{\pi}{m}\right) - \cos\left((a-n)\frac{\pi}{m}\right)\right] \qquad (48)$$

From this it can be shown that all teeth from $A$ to $M$ inclusive, have positive densities for values of $a = 0$ to $a = \frac{1}{2}$. The effective value of the total positive wave can, therefore, be found, by considering the teeth $A$ to $M$, inclusive, while $a = 0$ to $a = \frac{1}{2}$. The effective value of the total positive wave can, therefore, be found by considering the teeth $a$ to $m$, inclusive, while $a = 0$ to $a = \frac{1}{2}$. Similarly, the teeth $B$ to $A'$ have to be considered while $a = \frac{1}{2}$ to $a = 1$.

The effective density value and, therefore, the effective tertiary voltage is proportional to the square root of the mean height of the area, showing the square values of all positive densities. The area of each step of the curve is

$B_N^2 \dfrac{\pi}{m}$, therefore, the total area is

$$\Sigma\, B_N^2 \frac{\pi}{m} \Bigg|_A^M \quad \text{and}$$

$$e_T \text{ effective} \cong \sqrt{\frac{\Sigma\, B_N^2 \frac{\pi}{m}}{\pi}}\;\Bigg|_A^M = \sqrt{\frac{\Sigma\, B_N^2}{m}}\;\Bigg|_A^M \qquad (49)$$

By introducing $B_N$ from (48) into (49) the tertiary voltage follows directly.

It will again be found, as in Case No. 2 that as long as $m \geq 2$, the effective tertiary voltage is constant, and independent of $\gamma$, although the wave shape is continuously changing with $\gamma$. The curve of Fig. 19 shows the tertiary voltages as obtained from (48) and (49) for various numbers $m$ of secondary slots per pole in terms of the primary voltage, assuming equal number of turns in both windings.

The difference between the two effective voltages is caused principally by the difference in field forms, interlinking with the primary winding and that inducing voltages in the tertiary winding. Such differences are usually taken into account under the names of zig-zag and differential leakage. Therefore, we may call the difference between the values of curve 19 and one, the differential plus zig-zag leakage coefficient; its value is indicated by the right-hand scale in Fig. 19. It will be seen that even with the small number of stator slots assumed so far, a relatively small number of rotor slots is sufficient to reduce this leakage coefficient to a very low value. The leakage coefficient thus found and shown in Fig. 19 seems to conform to the following simple equation

$$\sigma = \frac{0.4}{m^2}$$

With our present assumptions, this represents all the so-called zig-zag leakage; the leakage value usually going under the name of differential leakage is zero in our case.

## CASE No. 4

### *Single Primary Coil, Infinite Number of Secondary Phases*

The erratic shape of the primary current curve with infinite theoretical values has made a quantitative calculation of the effective primary current value difficult for practical use in all previous cases. The theoretical case of an infinite number of secondary coils and a single primary coil, in which higher harmonic primary currents cannot exist, lends itself better for this particular purpose and may, therefore, be considered next.

The relations derived for the previous case apply, of course, here if the proper substitutions are made by simply introducing an infinite number of infinitely small secondary teeth into the equations.

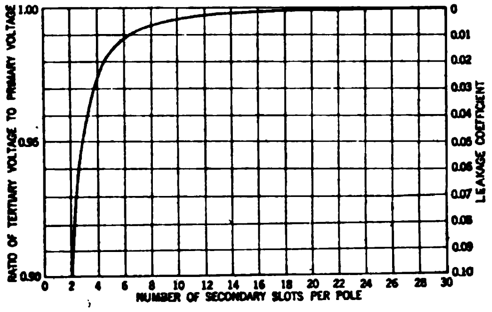

FIG. 19

Take $\beta = d\,\epsilon$

and $n\,\beta = n\,d\,\epsilon = \epsilon$

It follows from (31) by introducing these values

$$\begin{aligned} \varphi_{n-1} &= \varphi \cos(\gamma - \epsilon + 2\,d\,\epsilon) \\ \varphi_n &= \varphi \cos(\gamma - \epsilon + d\,\epsilon) \\ \varphi_{n+1} &= \varphi \cos(\gamma - \epsilon) \end{aligned} \qquad (50)$$

Similarly, it follows from (36)

$$B_N = \frac{\varphi_N}{d\,\epsilon} K$$

$$= \frac{\cos(\gamma - \epsilon + d\,\epsilon) - \cos(\gamma - \epsilon)}{d\,\epsilon} \frac{\varphi}{2} K$$

and $$B_{N-1} = \frac{\varphi_{N-1}}{d\,\epsilon} K$$

$$= \frac{\cos(\gamma - \epsilon + 2\,d\,\epsilon) - \cos(\gamma - \epsilon + d\,\epsilon)}{d\,\epsilon} \frac{\varphi}{2} K$$

or

$$B_N = \frac{\varphi}{2} K \frac{d\,[\cos(\gamma - \epsilon)]}{d\,\epsilon}$$

$$= \frac{\varphi}{2} K \sin(\gamma - \epsilon) \qquad (52)$$

$$B_{N-1} = \frac{\varphi}{2} K \frac{d\,[\cos(\gamma - \epsilon + d\,\epsilon)]}{d\,\epsilon}$$

$$= \frac{\varphi}{2} K \sin(\gamma - \epsilon + d\,\epsilon)$$

It follows directly from (52) that the local distribution of the actual resultant rotor flux follows a sine law around the rotor, so that a constant rotor field with sinusoidal distribution rotates with the rotor. This simply means that the steps of the field form found in the previous case are infinitely small, therefore, giving a smooth sine curve.

The density of the field is zero for $\gamma - \epsilon = 0$, that is, at the conductor 1. With these facts known, we can, therefore, plot the resultant rotor field $\varphi_r$ as shown in Fig. 20 for various values of $\alpha$ under the assumption that $\gamma = 0$.

In determining now the nature of the secondary currents, we proceed again as in the previous case, except that we consider the ampere conductors over an infinitely small angle $d\,\epsilon$ of the secondary in place of the current per secondary coil.

By substitution of the values from (52) and (44), we find the secondary ampere turns $i_n\, n_n$ over any space angle $d\,\epsilon$ for any time, except the moment when the particular angle $d\,\epsilon$ passes the coil $P$, to be as follows:

Considering the ampere turns of an angle $d\,\epsilon$ extending an angle of $\frac{d\,\epsilon}{2}$ to either side of the coil point $n$ we have,

$$i_n\, n_n = \frac{1}{2 K_l} (B_{N-1} - B_N)$$

$$
\begin{aligned}
&= \frac{\varphi K}{4 K_l} [\sin (\gamma - \epsilon + d \epsilon) - \sin (\gamma - \epsilon)] \\
&= \frac{\varphi K}{4 K_l} d [\sin (\gamma - \epsilon)] \qquad (53) \\
&= - \frac{\varphi K d\epsilon}{4 K_l} \cos (\gamma - \epsilon)
\end{aligned}
$$

It follows from (53) that the local ampere-turns distribution around the rotor circumference follows a cosine law, except at the point of the primary coil.

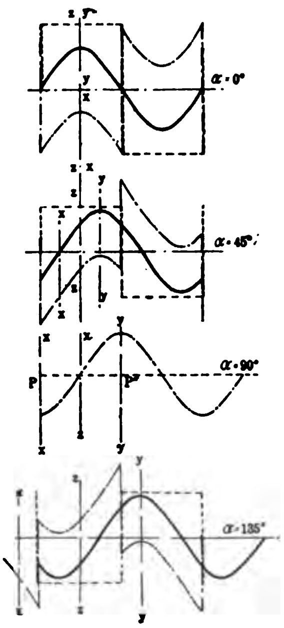

FIG. 20

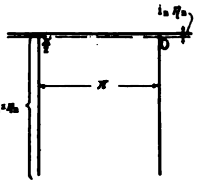

FIG. 21

This is again in line with the previous case, except that the secondary ampere conductors for each infinitely small angle are infinitely small during the periods of constant positive current.

Since the area of the positive wave of the secondary currents has in consequence of this an infinitely small value, we cannot any longer conclude that the negative current values must be infinite, when the secondary coil point passes the primary. On the contrary, we can determine certain definite negative current values.

We know from the previous case that the current in each secondary coil reverses when the particular coil passes the primary coil. The size of the reversed current can be determined from the following consideration. We found that if the flux within a rotor coil decreases after the coil has passed the primary coil, it must increase again to its original value, when it coincides again with the primary coil. In other words, if the flux $\varphi_n$ decreases according to a function $f(\alpha)$ for a certain time, it must increase again according to some function $f_1(\alpha)$ before the time $\pi$ has gone by. Now we know that if the current in the coil during the time of decreasing flux is a function $F(\alpha)$ and another function $F_1(\alpha)$ during the remaining time, the voltage which must be induced in the coil follow functions $r\,F(\alpha)$ and $r\,F_1(\alpha)$.

Therefore, we must have

$$\frac{d\,f(\alpha)}{d\,\alpha} = r\,F(\alpha)$$

and

$$\frac{d\,f_1(\alpha)}{d\,\alpha} = r\,F_1(\alpha) \qquad (54)$$

and since we know that the sum of all $d\,f(\alpha)$ must be equal to the sum of all $d\,f_1(\alpha)$ we can write

$$r\int F(\alpha)\,d\,\alpha = r\int F_1(\alpha)\,d\,\alpha \qquad (55)$$

where the first integral is to be taken for the limits between which the current is positive and the second integral for the limits between which the current is negative. In our case, we know the current to be $i_n$, that is, a constant and of uniform direction during the time $(\pi - d\,\alpha)$ and of opposite direction during $d\,\alpha$, we have therefore,

$$\int_0^{\pi-d\alpha} F(\alpha)\,d\,\alpha = \int_0^{\pi-d\alpha} i_n\,d\,\alpha = i_n(\pi - d\,\alpha)$$

and if the current $i_{nz}$ in opposite direction is assumed to be constant during the infinitely short time $d\,\alpha$, we have

$$\int_{\pi-d\alpha}^{\pi} F_1(\alpha)\,d\,\alpha = \int_{\pi-d\alpha}^{\pi} i_{nz}\,d\,\alpha, \text{ therefore,}$$

$$i_n(\pi - d\,\alpha) = -\,i_{nz}\,d\,\alpha$$

or,

$$i_{nz} = -i_n \frac{\pi - d\alpha}{d\alpha} \tag{56}$$

$$i_{nz} = \frac{\pi - d\alpha}{d\alpha} \frac{\varphi}{4} \frac{K}{K_l} d\epsilon \cos(\gamma - \epsilon) \tag{57}$$

Since both $d\alpha$ and $d\epsilon$ are indefinitely small, it follows that $i_{nz}$ must not any longer be infinite as in the previous cases.

It will be seen from (57) that the values for the negative secondary ampere conductors $i_{nz} n_n$ are proportional to cos $(\gamma - \epsilon)$. The time curve of the current conductors in any of the secondary coils appears, therefore, as shown in Fig. 21, if we imagine the value $i_n n_n$ infinitely small.

The determination of the primary current values follows again from the condition that the known resultant field must be proportional to the sum of the magnetizing effect of the primary ampere turns and the just determined secondary ampere turns.

With uniformly distributed secondary conductors, it is again immaterial which coil point is numbered 1, and we may, therefore, make $\gamma = 0$, which merely means that coil point 1 happens to coincide with $P$ when $\alpha = 0$. It further means that any coil $n$ passes $P$ at the time $\alpha = -\epsilon$. This follows directly from Fig. 13.

A coil point with $\epsilon = -\frac{\pi}{2}$ therefore, passes $P$ at the time $\frac{\pi}{2}$. It follows also from (53) and (57) that both the positive and negative currents in this conductor are zero. Reference to Fig. 20 for $\alpha = 90$ deg. $= \frac{\pi}{2}$ further reveals that at that time the resultant flux in the axis $Z\,Z$ of the primary coil is zero, and that the sine-shape flux curve has its maximum at the primary coil point $P\,P'$. This, in turn, means that there is no change in flux density at these points (the differential coefficient of a cosine curve being zero at its maximum value), and, therefore, no resultant ampere conductors acting. It follows directly that under this condition and with the secondary coil points at $P$ and $P'$ carrying zero current, the primary current for $\alpha = \frac{\pi}{2}$

must also be zero. We know further that all other secondary coil points carry at this moment positive currents as per equation (53), and evidently the resultant field at the time $\frac{\pi}{2} = \alpha$ must be induced by these currents alone. Since the field is constant, we know, therefore, that the positive secondary currents alone are just sufficient to induce the necessary resultant field. From this, we can at once derive our *sixth fundamental condition* applying only to the present case, namely, that *the sum of the primary ampere conductors and the negative ampere conductors of the secondary coil points coinciding with the primary coil point must be zero at all times.*

This leads to the following calculation, resulting in a formula for the primary magnetizing current.

With $\gamma = 0$, each secondary coil point $n$ passes the primary coil at the time $\alpha = -\epsilon$ and it follows therefore from (57) and the condition just stated, that the primary ampere turns at the time $\alpha$ must be

$$i_p\, n_p = i_{nz}\, n_n = \frac{\pi - d\,\alpha}{d\,\alpha}\;\frac{\varphi}{4}\;\frac{K}{K_l}\,d\,\epsilon\cos(-\epsilon) \tag{58}$$

where $-\epsilon = \alpha$

In this equation, we may neglect the infinitely small $d\,\alpha$ as compared to $\pi$ and let $\pi - d\,\alpha = \pi$. We may further let $d\,\epsilon = d\,\alpha$, both being infinitely small and obtain

$$i_p\, n_p = \frac{\pi}{4}\,\varphi\,\frac{K}{K_l}\cos\alpha \tag{59}$$

We find the crest value of the current

$$I_p = \frac{\pi}{4}\;\frac{\varphi}{n_p}\;\frac{K}{K_l} \tag{60}$$

It follows from (59) that the primary current follows a cosine law, as is to be expected with an impressed sinusoidal voltage and a rotating field with sinusoidal distribution.

We know from (5a) the value with open secondary, and, therefore, find the ratio of the open to the closed secondary magnetizing current in the primary to be

$$\frac{\frac{\pi}{4}}{\frac{1}{\pi}} = \frac{\pi^2}{4} = 2.45 \text{ for the present case.}$$

Knowing the flux set up by the primary at each instant, namely,

$$\varphi_p = \left(\frac{\pi}{2}\right)^2 \varphi \cos \alpha$$

and the resultant flux $\varphi_s$, the flux set up by the secondary currents, being the difference of the two, can now be easily plotted for the various values of $\alpha$ as in Fig. 20. It will be seen that the flux furnished by the secondary has a rather peculiar space distribution, being the difference between a sinusoidal and rectangular distribution field form. The traveling of the resultant field and the secondary magnetomotive forces is again the same as in previous cases.

The tertiary voltage is in this case evidently of the same value and curve shape as the impressed voltage, the zig-zag and belt leakage being obviously zero.

### CASE NO. 5

### *Primary Sinusoidal Distribution, Infinite Number of Secondary Phases With Negligible Resistance and Leakage*

One of the reasons why the previous cases with a concentrated primary winding were taken up so much in detail, is because it can be definitely proved in these cases, that the resultant rotating field approaches a sinusoidal distribution very closely in spite of the very unfavorable rectangular field form set up by the primary winding. With this fact established, it is obvious that with a distributed primary winding giving in itself a sinusoidal field distribution, a similar resultant field will exist. If we assume further an infinite number of secondary coils, together with the previous assumptions of an impressed primary sinusoidal voltage, a true sinusoidal resultant field will undoubtedly obtain and can simply be used as the basis for all calculations. Similar considerations, together with results obtained in Case No. 4, justify, under these conditions, the assumption of a sinusoidal primary current wave, shifted 90 deg. against the impressed voltage wave. Such an ideal case may now be considered to good advantage because the knowledge of this and the previous opposite extremes is helpful in the understanding of the more practical cases.

With the self-evident assumptions just made, the resultant ampere conductors, that is, the sum of the primary and secondary at each point of the circumference, can again be found from our fifth fundamental condition, that the ampere conductors of each point must be the difference of the magnetic densities at either side of the point under consideration.

Having thus a means for determining the resultant ampere conductors, and knowing laws for the time curve of the primary current, as well as for the distribution of the primary and secondary conductors, we can easily write down a number of equations expressing these laws, and which are necessary for determining the numerical values of the currents.

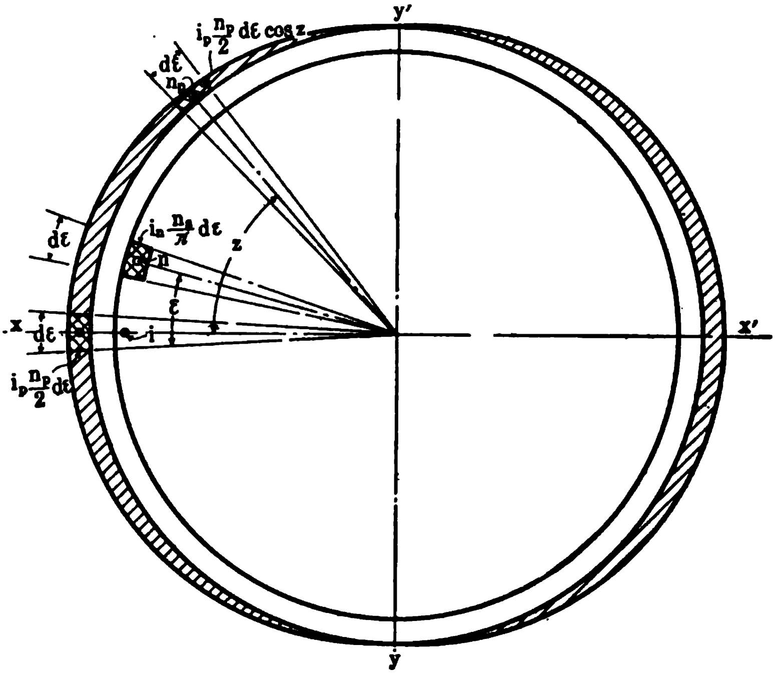

FIG. 22

It is well known that the flux needed to induce the proper counter e. m. f. with a sinusoidal winding distribution, is as a matter of course, larger than with a concentrated coil, which can be taken care of by the usual voltage winding coefficient. The latter is in the present case $c = \frac{\pi}{4}$ so that

$$\varphi = \frac{4}{\pi} \varphi_c, \qquad (63)$$

if $\varphi_c$ is the flux as found from formula (**2**) for a concentrated primary winding.

If the total number of primary turns is again $n_p$ and if they are first assumed to be uniformly distributed over the pole arc $\pi$ we would have $i_p \frac{n_p \, d \, \epsilon}{\pi}$ ampere turns for an infinitely small angle $d \, \epsilon$. With sinusoidal distribution, as shown in Fig. 22, we find a maximum value of ampere turns, by multiplying this average with $\frac{\pi}{2}$ which gives

$$i_p \frac{n_p}{\pi} \, d \, \epsilon \frac{\pi}{2} = i_p \frac{n_p}{2} \, d \, \epsilon$$

at the center $x \, x'$ of the primary phase belt. A value for the ampere turns at a point $n_p$ an angle $z$ away from this center, is consequently

$$i_p \frac{n_p}{2} \, d \, \epsilon \cos z$$

The instantaneous primary current values following a cosine law are under the assumption of the crest value $I_p$.

$$i_p = I_p \cos \alpha$$

and the ampere turns of a space angle $d \, \epsilon$ are

$$i_p \, n_z = I_p \cos \alpha \, \frac{n_p}{2} \, d \, \epsilon \cos z \qquad (\mathbf{64})$$

We know further from previous considerations, equation (**53**), that the resultant ampere turns at a rotor point $n$, an angle $\epsilon$ ahead of coil number 1, must be

$$i_{rn} \, n_{rn} = - \frac{\varphi}{4} \, \frac{K}{K_l} \, d \, \epsilon \cos (\gamma - \epsilon) \qquad (\mathbf{65})$$

It was also pointed out that the ampere turns furnished by the secondary must be at any point of the circumference and at any time the difference

$$i_p \, n_z - i_{rn} \, n_{rn} = i_n \, n_n \qquad (\mathbf{66})$$

For the sake of simplicity, we may again number the secondary coil points so that point 1 coincides with the center of the primary phase belt at the time $\alpha = 0$ which simply means that $\gamma = 0$ in the formula for $i_{rn} \, n_{rn}$. This also means that the rotor coil $n$ coincides with a stator coil point, which is an angle $\epsilon$ from the center at the time $\alpha = 0$ and angle $\epsilon + \alpha = Z$ at the time $\alpha$.

Thus we can write by setting $n_n = \frac{n_s\, d\,\epsilon}{\pi}$ and by introducing into (66) the values from (64) and (65)

$$I_p \cos\alpha \frac{n_p}{2}\, d\,\epsilon \cos(\epsilon + \alpha) + \frac{\varphi}{4}\, \frac{K}{K_l}\, d\,\epsilon \cos(-\epsilon)$$

$$= i_n \frac{n_s\, d\,\epsilon}{\pi} \qquad \textbf{(66 A)}$$

Since both the maximum primary current $I_p$ and the secondary current are unknown in the single equation derived in (66 A) we again make use of our fourth fundamental condition that the resultant secondary current area over the time $\pi$ must be zero, in order to obtain another equation.

We know from the above that the secondary coil $n$ coincides with the center of the primary phase belt at the time $\alpha = -\epsilon$ and $\alpha = -\epsilon + \pi$ and that the integration of the secondary currents between these times must be zero.

Therefore, we have, if we let $d\,\alpha = d\,\epsilon$

$$\int_{\alpha=-\epsilon}^{\alpha=-\epsilon+\pi} \Big[ I_p \cos\alpha \frac{n_p}{2}\, d\,\alpha \cos(\epsilon + \alpha)$$

$$+ \frac{\varphi}{4}\, \frac{K}{K_l}\, d\,\alpha \cos(-\epsilon) \Big] = 0$$

from which we find

$$I_p = -\frac{\varphi}{n_p}\, \frac{K}{K_l} \qquad \textbf{(67)}$$

For open-circuited secondary, we can find from a simple calculation

$$I_p = -\frac{\varphi}{2\, n_p}\, \frac{K}{K_l} \qquad \textbf{(68)}$$

With $I_p$ known, we can find $i_n\, n_n$ from (66A) for any point $n$ of the secondary,

$$i_n\, n_n = i_n \frac{n_s}{\pi}\, d\,\epsilon = \frac{K}{K_l}\, d\,\epsilon\, \frac{\varphi}{2} \left( \frac{\cos(-\epsilon)}{2} \right.$$

$$\left. - \cos\alpha \cos(\alpha + \epsilon) \right)$$

or

$$\dot{i}_n n_s = \frac{K}{K_l} \frac{\pi}{4} \varphi \cos (2 \alpha + \epsilon) \qquad \textbf{(69)}$$

It will be seen from (**67**) and (**68**) that the primary magnetizing current with closed and synchronously rotating secondary is, therefore, just twice the magnetizing current with open secondary. This is in line with the assumption usually made in commercial designing.

Fig. 23 shows the secondary currents for a number of secondary points corresponding to various values of $\epsilon$, as well as the primary current.

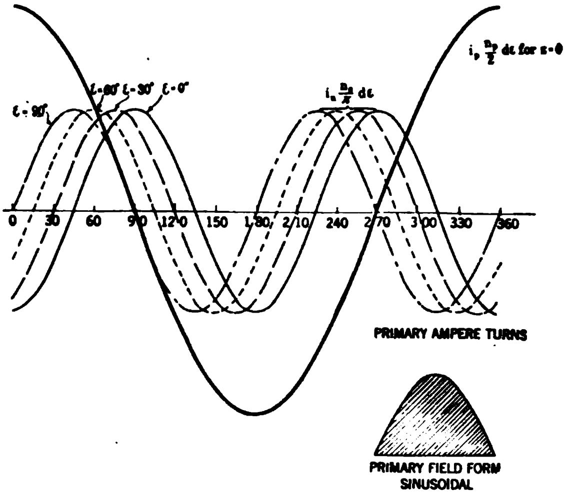

Fig. 23

It will be seen that all the secondary currents are in this case double frequency sinusoidal currents, with the same maximum values of

$$I_n = \frac{\pi}{4} \frac{\varphi}{n_s} \frac{K}{K_l} \qquad \textbf{(69A)}$$

Fig. 23A shows the fields, or rather the magnetomotive forces as furnished by the primary and the secondary, as well as the resultant field, all of which have a sinusoidal local distribution in this particular case. This figure shows better, than those previously given, the fundamental characteristics of the fields and how they travel. The primary magnetomotive force with the axis $z\,z$ is, of course, not rotating, but stationary and alternating; that is, varying in size and polarity. The resultant field with the axis *y y* of practically constant size travels, relative to the stator, at uniform speed and synchronously with the secondary member to the right. The magnetomotive forces furnished by the secondary currents with the axis *x x* are also practically constant in size and travel with equal speed, relative to the stator, to the left. Relative to the synchronously rotating secondary, the resultant field is, therefore, fixed, while the secondary magnetomotive force rotates with double synchronous speed in a backward direction. Such double speed field or rather magnetomotive force in the secondary is, of course, to be expected in so far as the secondary currents are known to be double-frequency polyphase currents. While the primary field may be considered as the sum of the field set up by the secondary and of the resultant field from a purely mathematical point of view, as has been done at times, it should be kept in mind that actually the primary field is primarily induced from the line; its existence, in turn, causes double-frequency currents in a synchronously rotating rotor which, if they are imagined to exist without primary currents, would induce a field rotating opposite in direction

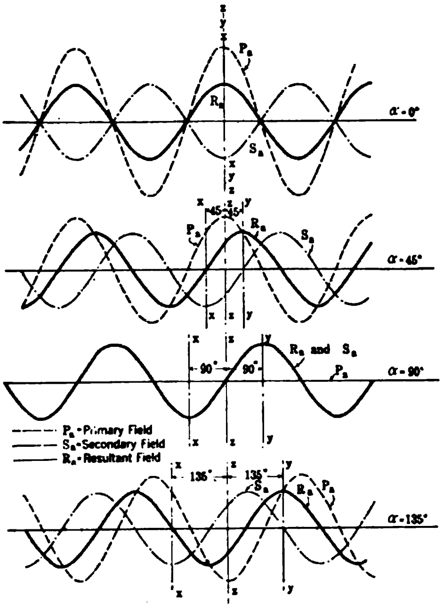

FIG. 23A

to the mechanical direction of rotation. As previously mentioned, the speed of this imaginary field is synchronous relative to the primary and double synchronous relative to the rotor. Stating the facts physically correct, we may say that the stationary and alternating primary magnetomotive forces of line frequency, together with the double-line-frequency secondary magnetomotive forces, combine to induce a resultant field which

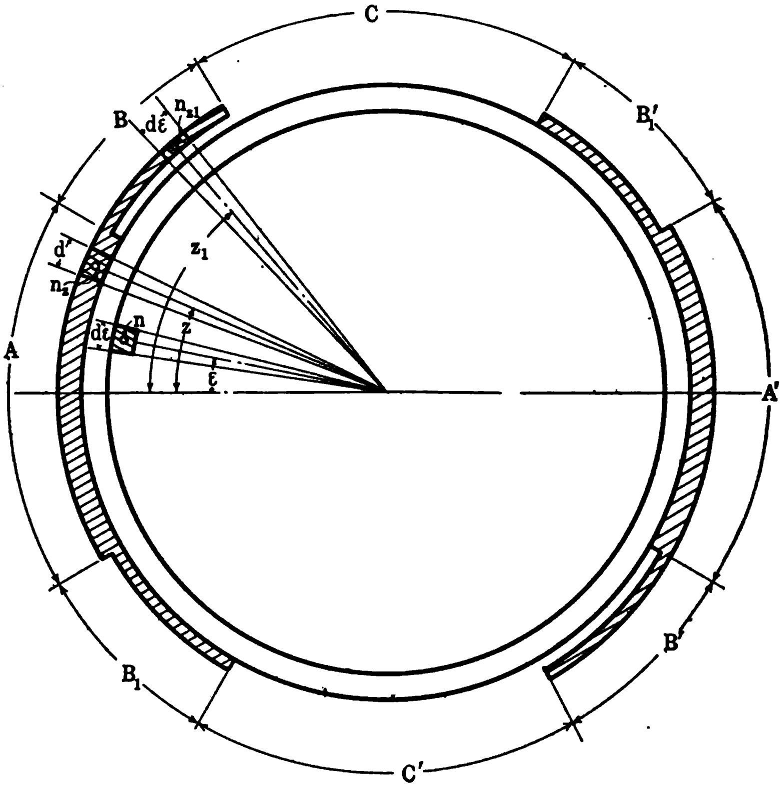

FIG. 24

rotates in the same direction as the mechanical rotation of the secondary and with the same speed relative to the primary.

### CASE NO. 6

### *Primary Field Form Consisting of Straight Line Functions, Infinite Number of Secondary Phases*

In practise, it is difficult to obtain sinusoidal distribution of the primary winding, and, therefore, winding distributions as shown in Fig. 24 are usually resorted to; such distribution can be easily obtained by either concentric windings or chorded diamond coils.

While it is possible with such windings to induce sinusoidal counter e.m.fs. with resultant fields having other than sinusoidal distributions, it may at present be assumed that a sinusoidal distribution of the resultant field exists as in all previous cases of an infinite number of secondary phases. Such a field will always induce sinusoidal counter e.m.fs., thereby satisfying our first fundamental condition.

In view of the strong tendency for the establishment of a resultant sinusoidal field previously demonstrated, this assumption is quite justified, provided, of course, that our other fundamental conditions can be met satisfactorily under this assumption.* The following calculations will show that this is the case.

For similar reasons, a sinusoidal time curve for the primary current may at present also be assumed.

The general method for determining the desired quantities is in the present case the same as in the previous case. The only difference is that the distribution of the primary conductors does not any longer follow a single law all around the circumference, making it necessary to consider the different portions $A$, $B$, and $C$, in Fig. 24 individually in the calculation, this leads naturally to more complicated equations.

If the total primary turns again $n_p$, we find that an angle $d\,\epsilon$ within the angle $A$ has $\frac{n_p\,d\,\epsilon}{(A + B)}$ turns, while an equal angle $d\,\epsilon$ within the angles $B$ and $B_1$ usually has $\frac{n_p\,d\,\epsilon}{2\,(A + B)}$ turns.

A secondary coil point $n$, an angle $\epsilon$ shifted against point 1 must again have the resultant ampere turns—see equation (53)

$$i_{rn}\,n_{rn} = -\frac{\varphi}{4}\,\frac{K}{K_l}\,d\,\epsilon\cos(-\,\epsilon) \text{ if } \gamma = 0 \qquad (70)$$

---

*NOTE: It should be remembered that our considerations so far still assume uniform magnetic reluctance, negligible resistances and leakage reactances, etc. In practical machines, both single-phase and polyphase operated, it is quite possible and in practise often actually experienced, that the resultant field form has higher harmonics, caused by magnetic densities in the iron, bad distribution of resistance between bars and rings of squirrel-cage windings, etc. These are important problems in themselves and will be made the subject of a separate study, apart from the present paper.

While passing the angles $C$ and $C'$, which occurs during the times

$$\alpha = \frac{A}{2} + B - \epsilon \text{ to } \alpha = \frac{A}{2} + B + C - \epsilon$$

and

$$\alpha = \frac{3\,A}{2} + 3\,B + C - \epsilon \text{ to } \alpha = \frac{3\,A}{2} + 3\,B + 2\,C - \epsilon$$

no other ampere turns act upon this point and these ampere turns must, therefore, be furnished by the rotor, so that

$$i_n\, n_{nc} = i_{rn}\, n_{rn}$$

While the same secondary point passes any of the angles $A$ or $B$, we must find the secondary ampere turns as the difference of the primary and the resultant ampere turns.

Point $n$ passes the angles $B$ during the times

$$\alpha = \frac{A}{2} - \epsilon \text{ to } \alpha = \frac{A}{2} + B - \epsilon$$

$$\alpha = \frac{A}{2} + B + C - \epsilon \text{ to } \alpha = \frac{A}{2} + 2\,B + C - \epsilon$$

etc.

During this time, we must have

$$i_n\, n_{nB} = -\, I_p \cos \alpha \frac{n_p\, d\,\epsilon}{2\,(A + B)}$$

$$-\, \frac{\varphi}{4}\; \frac{K}{K_l}\; d\,\epsilon \cos\,(-\,\epsilon) \qquad \textbf{(71)}$$

Point $n$ passes the angles $A$ during the times

$$\alpha = -\, \frac{A}{2} - \epsilon \text{ to } \alpha = \frac{A}{2} - \epsilon$$

and

$$\alpha = \frac{A}{2} + 2\,B + C - \epsilon \text{ to } \alpha = \frac{3\,A}{2} + 2\,B + C - \epsilon$$

During this time, we must have

$$i_n\, n_{nA} = -\, I_p \cos \alpha \frac{n_p\, d\,\epsilon}{A + B}$$

$$-\, \frac{\varphi}{4}\; \frac{K}{K_l}\; d\,\epsilon \cos\,(-\,\epsilon) \qquad \textbf{(72)}$$

We know again that the integral of $i_n\, n_n$ between

$$\alpha = \frac{\pi}{2} - \epsilon \text{ to } \alpha = \frac{3\,\pi}{2} - \epsilon \text{ must be zero,}$$

that is,

$$\int_{\frac{\pi}{2} - \epsilon + \frac{C}{2} + B}^{\frac{\pi}{2} - \epsilon + \frac{C}{2} + B + A} i_n\, n_{nA} + \int_{\frac{\pi}{2} - \epsilon + \frac{C}{2}}^{\frac{\pi}{2} - \epsilon + \frac{C}{2} + B} i_n\, n_{nB}\ .$$

$$+ \int_{\frac{\pi}{2} - \epsilon + \frac{C}{2} + B + A}^{\frac{\pi}{2} - \epsilon + \frac{C}{2} + 2B + A} i_n\, n_{nB} + \int_{\frac{\pi}{2} - \epsilon}^{\frac{\pi}{2} - \epsilon + \frac{C}{2}} i_n\, n_{nC}$$

$$+ \int_{\frac{\pi}{2} - \epsilon + 2\,B + A + \frac{C}{2}}^{\frac{3\,\pi}{2} - \pi} i_n\, n_{nC} = 0 \qquad (73)$$

This equation applies only as long as $\epsilon < \frac{\pi}{2}$ but similar equations follows in other cases. By substitution, we find from this

$$I_p = -\frac{\pi\,(A + B)}{4\left(\sin\frac{A}{2} + \cos\frac{C}{2}\right)}\ \frac{\varphi}{n_p}\ \frac{K}{K_l} \qquad (74)$$

The flux is in this case again

$$\varphi = \frac{1}{c}\,\varphi_c \qquad (75)$$

if $\varphi_c$ the flux required with a concentrated primary coil. The coefficient $c$ is in this case

$$c = \frac{2\sin\frac{A + B}{2}}{A + B}\cos\frac{B}{2} \qquad (75\ A)$$

With diamond shaped chorded coils

$$\frac{2\sin\frac{A+B}{2}}{A + B}$$

corresponds to the usual distribution factor and

$$\cos \frac{B}{2} = \sin \frac{\pi - B}{2}$$

corresponds to the usual chord factor.

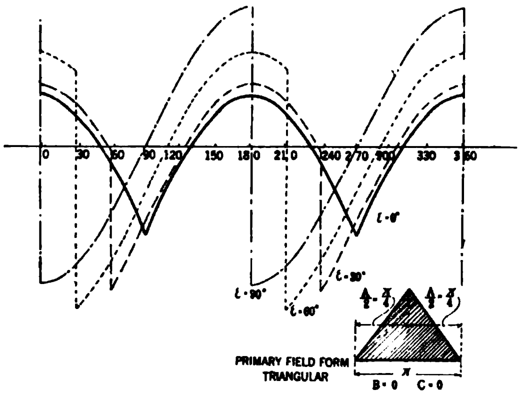

FIG. 25

It will be seen in (**74**) that $\epsilon$ has cancelled out which means that a primary current following a cosine law, together with a sinusoidal resultant field fulfills in each secondary coil located

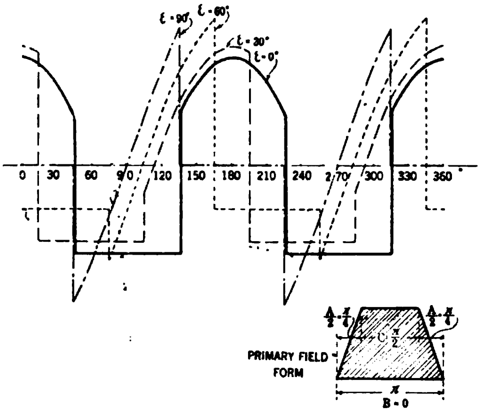

FIG. 26

at any angle $\epsilon$ away from coil 1 the condition that the average current in the coil is zero. This being the case, we have shown that the assumptions of a sinusoidal space distribution of the resultant field and a sinusoidal time curve for the primary current made in the beginning of this case were correct under our present conditions.

Figs. 25 to 29, inclusive, show the curves for the secondary currents for a number of different field forms. Fig. 25 covers the case of a triangular field form for which the currents corresponding to $\epsilon = 0$ are rather small, while those corresponding to $\epsilon = 90$ deg. are rather large. Fig. 28 gives the case of a machine in which the winding is uniformly distributed over 120 deg. as would be the case in a full-pitch three-phase machine run single-phase. As compared with the previous case, the currents corresponding to $\epsilon = 0$ have increased, and those corresponding to $\epsilon = 90$ deg. have decreased. Fig. 27 gives the case of a machine in which the primary winding is uniformly distributed over 90 deg., as would be the case in a full-pitch two-phase machine, run single phase. As compared with the previous case, the currents corresponding to $\epsilon = 0$ have again increased, and those corresponding to $\epsilon = 90$ deg. have decreased. Fig. 28 shows the case of a primary winding which is practically concentrated. It will be realized that even with a primary winding wound into one coil, the field form will not be truly rectangular, but somewhat as shown in Fig. 28. Accordingly, the secondary currents will not assume infinite values as theoretically found in connection with Case No. 4, but it will be somewhat similar to those shown in Fig. 28. Fig. 29 corresponds to a winding in which $A$ is 90 deg., $B$ 35 deg., and $C$ 20 deg. As will be seen from the figure, the primary field form approaches in this case very closely a sinusoidal curve. While the primary current from (**74**) is, in

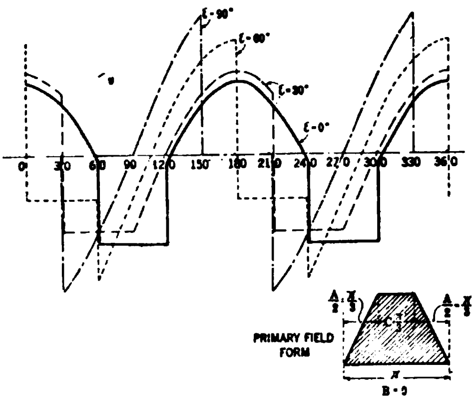

Fig. 27

this case, only one per cent different from that obtained with a sinusoidal distribution, it will be seen that the secondary currents are still very much different from a sinusoidal double-frequency curve. This shows that even the slightest departure from the sinusoidal curve necessary for practical reasons leads to very irregular secondary current curves, although the departure from a sinusoidal curve is the smaller the closer the primary distribution approaches such a curve.

Fig. 27A shows the magnetomotive forces furnished by the primary and the secondary, as well as the resultant field for the

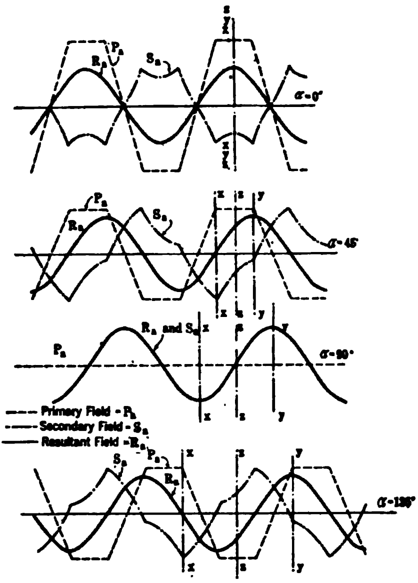

FIG. 27A

case covered in Fig. 27. It can be noted again that the secondary magnetomotive force travels in a direction opposite to the resultant field; its distribution assumes, however, in contradistinction to Fig. 23A, rather irregular shapes, being always the difference between the primary trapezoid and the resultant sine curve.

### CASE NO. 7

### *Primary Distribution According to a Function $f(z)$, Infinite Number of Secondary Phases*

It is evident that the methods used in the previous cases can be applied to any distribution of primary ampere turns. If the primary field form is made up from portions of straight lines, each of them being a function *f* (*z*), the method of Case No. 6 can be applied, no matter what the number of different straight lines is. Naturally, the equations will become rather long and complicated, if the field form is composed of many different lines. In certain cases, it may, therefore, be simpler to resolve the primary field form into a single equation of a fundamental sine wave and its higher harmonics, and then proceed as in Case No. 5. In this case, a single function *f* (*z*) applies for the whole cycle. In such cases, the function may, however, be such as to lead to complications in connection with the inte-

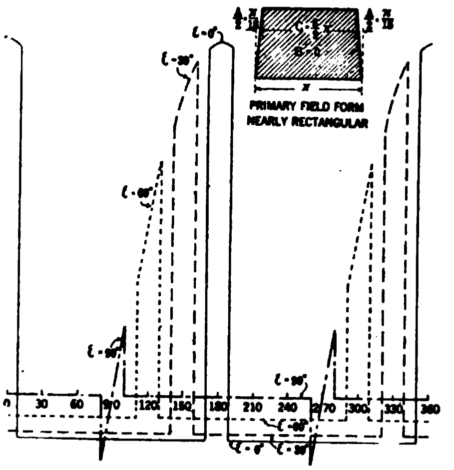

FIG. 28

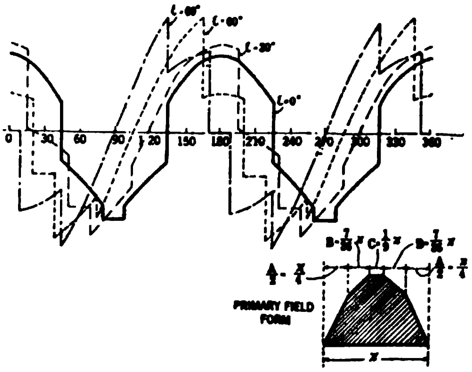

FIG. 29

gration necessary for finding $I_p$. Thus the one or the other method may be found more convenient, depending upon the case.

### CASE NO. 8

### *Primary Winding Distributed in Closed Slots, Infinite Number of Secondary Phases*

In actual practise, field forms and distributions of primary ampere turns can be approximately obtained as shown under Case No. 6 by skewing either the stator or the rotor one primary slot pitch. The practise of skewing slots meets, however, with certain practical difficulties, especially with large machines and, therefore, it is of interest to determine the influence of the straight standard slot. The use of slots for locating the primary ampere turns, which brings about a concentration of the ampere turns in a number of points instead of uniform distribution as assumed in Case No. 6, leads to field forms as shown in a full line Fig. 30a instead of that shown by a dotted line in the same figure and corresponding to the case of Fig. 27. As pointed out under Case No. 7, it is possible to treat such a case along the same lines as described under Case No. 6, which requires, however, a considerable amount of calculation, especially with a large number of slots.

For these reasons, the description of a graphical method for determining the secondary currents may be of interest. This method, which is based on the facts previously determined, introduces a slight theoretical error, which, however, is of no practical importance. On the other hand, the graphical method gives a much clearer picture of what happens in the machine than can be obtained from the previous formulas.

We have previously found that the resultant flux rotating with the rotor of an infinite number of phases is nearly constant and has a sinusoidal local distribution around the surface. From this we further concluded that with uniform air-gap reluctance, we must have at all times at any synchronously rotating rotor point a resultant constant number of ampere turns, the size of which follows a cosine law around the rotor. Line *A B* in Fig. 30c may represent this constant resultant ampere-turn value for a certain point. Now we know that such resultant value is at any time the sum of the rotor ampere turns at this point and of the stator ampere turns which happen to be located opposite to this rotor point during the time element under consideration.

Let us assume, for instance, a case as assumed in Fig. 27 and a rotor point corresponding to $\epsilon = 0$; this means the rotor point passing the center of the primary phase belt at $\alpha = 0$, that is, at the time when the primary current reaches its maximum value.

Fig. 30b shows the dotted areas $a\ a'\ b'\ b$ representing the primary winding belts. While our rotor point travels from $b_1$ to $a$ and from $b$ to $a_2$, it is not acted upon by any primary ampere turns, which means that the secondary ampere turns are identical with the resultant. Therefore, the portions $b_1$ to $a$ and $b$ to $a_2$ of the line $A\ B$ of Fig. 30c represent portions of the secondary ampere-turn time curve.

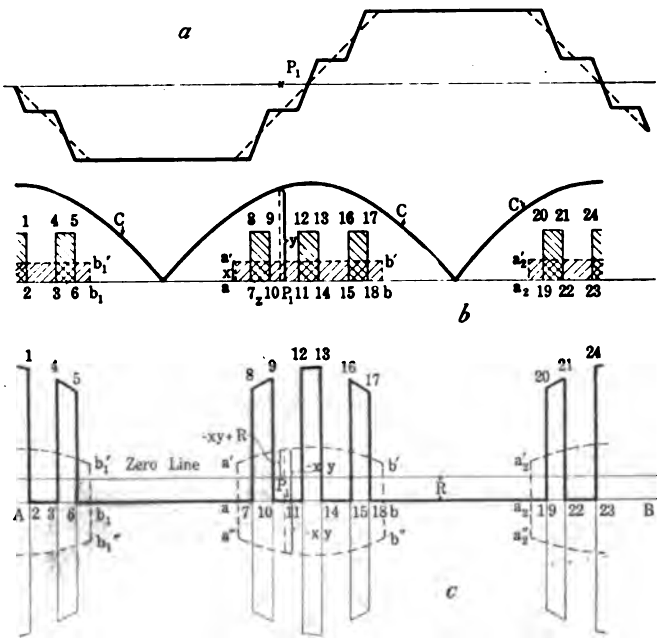

FIG. 30

The sine curve $C$ represents the time curves of the primary current as found from formula (74) for this case plotted so against the circumference, that $y$ represents the time value of the primary current, when our point has reached a point $P_1$ on the stator circumference. If we multiply, therefore, $y$ with the turns located at this point, we get the number of stator ampere turns acting upon our point, when it has reached point $P_1$ on the stator.

These ampere turns $x\ y$ act in the same direction as the resultant ampere turns $R$ represented by line $A\ B$ in Fig. 30c. If we add $x\ y$ to $A\ B$ as shown, we obtain the sum of the primary and resultant ampere turns, and it is also evident that the rotor ampere turns must be the difference between $-x\ y$ and $R$ as shown. By proceeding similarly for all other points, we obtain the heavy dotted line $b_1'\ b_1\ a\ a'\ b'\ b\ a_2\ a_2'$, representing the time curve of the secondary current, which corresponds exactly to the curve for $\epsilon = 0$ in Fig. 27. The distance between $A\ B$ and the light dotted curves $a\ a''\ b''\ b$ etc., represent the primary ampere turns.

Let us assume now, for instance, that the same number of primary turns are concentrated in three equally spaced slots as represented by the areas 7, 8, 9, 10, 11, 12, 13, 14 and 15, 16, 17, 18 in Fig. 30b. We know then from a number of previous considerations that the primary current still follows a sine law, if the number of secondary phases is infinite. The maximum value of the primary current is slightly changed by the change made, but it can be shown that the error in assuming it to be the same as before is less than one or two per cent in the majority of cases. Therefore, we can proceed just as described before and obtain very close values for the secondary ampere turns as represented by the full line curve 1, 2, 3, 4, 5, 6, 7, etc. in Fig. 30c. It is at once evident that the effective or heating value of this curve is considerably larger than that of the dotted curve.

## CASE NO. 9

### *Primary Winding Distributed in Open Slots, Infinite Number of Secondary Phases*

In all previous cases, the resultant ampere turns of a given point were assumed to be proportional to the difference in density of the adjacent surface portions. This assumption is practically correct in most cases of closed and partially closed slots. With wide open slots, it will, however, be necessary to take the variations of the air gap reluctance into account. This can be done very easily with the method shown under Case No. 8. It is merely necessary to replace the straight line $A\ B$ of Fig. 30c by a curve $A\ C\ B$, as shown in Fig. 31, such curve representing the varying reluctance of the air gap caused by the slot openings and with it, the varying resultant ampere turns required to bring about the practically constant flux condition necessary for equilibrium. The secondary ampere turns can then be found just the same as in Fig. 30, except that the distances $x\ y$ are entered, starting from the curve $A\ C\ B$ in Fig. 31 instead of from the straight line $A\ B$ in Fig. 30c.

In this manner, we obtain the heavy line curve 1, 2, 3, 4, 5, etc., giving the secondary current and ampere turns. It will be seen that higher harmonics are introduced all over the secondary current cycle, at the same time, the maximum peaks have been reduced as compared with the previous case.

## CASE No. 10

### *Primary Winding Distributed in Slots, Polyphase Secondary Winding Distributed in Slots*

The case with both primary and secondary winding distributed over a limited number of non-skewed slots found most commonly in practise requires an altogether impractical amount of calculation for its correct treatment. It will be shown, however, that for a well balanced design, very close results can be obtained by the aid of the facts established with the previous cases.

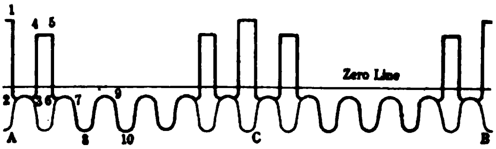

FIG. 31

We have found in connection with cases Nos. 1, 2 and 3 (see in particular Figs. 13 and 16) that each secondary slot reflects into the concentrated primary winding, while passing the same, a very high current peak. Such peak is infinite if the secondary coil is concentrated in a mathematical point and under the assumption of no leakage reactance. In practise, neither of the latter assumptions apply. The fact that the secondary slots, even if practically closed, distribute the effect of the secondary ampere turns over a certain distance on account of the flux dispersion, changes the infinite values of infinitely short duration to definite values still rather large and of relatively short duration. The difference between the theoretical primary current curve and the actual caused by the fact that the effect of the secondary coil is not concentrated in a point is the same as the difference between the secondary currents of Fig. 16 and those of Fig. 28. The leakage reactance present in every machine serves further to reduce the height of the higher harmonic current peaks, especially since they are of rather high frequency. Since the secondary flux dispersion around the slots, as well as the leakage reactance, vary over wide ranges in different designs, it is difficult to give a general rule regarding the exact height of the higher harmonic current peaks actually obtained in case of Fig. 16. It is evident, however, that in this case, peaks of such magnitude may be obtained to cause an appreciable difference between the actual primary current curve and a sinusoidal curve. Since the nature of this actual curve cannot easily be determined, the method of Cases No. 8 and 9 for the determination of the secondary currents, or any of the other methods cannot be readily employed.

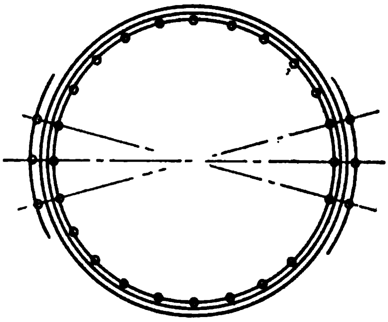

FIG. 32

If the primary winding is distributed over a number of slots of the same pitch as the secondary slot, as shown in Fig. 32, conditions are practically the same as before. Each of the secondary currents has, in the case assumed, three reversed current peaks, as shown in Fig. 30, instead of one, as shown in Fig. 28, each of the three peaks being about one-third of the single peak. Since, however, three secondary coils coincide simultaneously with the three primary coils, three secondary current peaks of one-third size are reflected at the same time into the primary, giving again rather bad peaks in the primary current wave. Experience with noise, dead points, etc. has long ago taught designers to avoid equal tooth pitch in both members, and relative tooth pitches, as shown in Fig. 33 are commonly used. In this case, only one secondary slot coincides at the time with a primary coil so that the secondary reflects for each high peak in Fig. 16, several smaller peaks into the primary. Both the higher frequency and the smaller amplitude of these peaks, together with the effect of the leakage in most practical machines, iron the higher harmonics out to such an extent that the departure of the actual primary current wave from the sinusoidal can be neglected in most well designed machines. Therefore, it is permissible to assume primary currents as calculated from (**57**) for the investigation of most practical cases, although small errors may be introduced even in well designed machines and appreciable errors in some extreme cases.

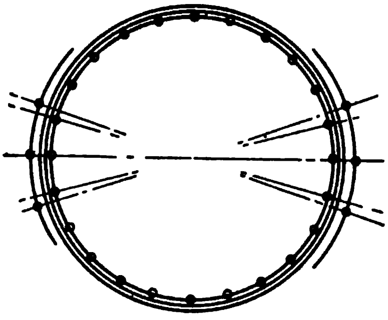

Fig. 33

With a primary current wave established, the method of Case No. 8 may be applied to a limited number of secondary coils with certain modifications.

We must now consider the fact that no matter what the distribution of the primary ampere turns is, the fluxes will always distribute fairly uniform over a secondary tooth. Assume, for instance, again the same primary winding and field as in case of Fig. 27 and Fig. 30. If we have then a secondary slot $n$ located at a point $x$ as shown in Fig. 34a, it is evident that the flux between the points $n - 1$ and $n$ will distribute uniformly over the secondary tooth $N$, as indicated by a dotted line 1, 2. Similarly, the flux between $n$ and $n + 1$ will distribute uniformly over the tooth $N + 1$, as indicated by the dotted line 3, 4. The result is, therefore, the same as if a number of primary turns proportional to the vertical line 2, 3 were concentrated in the point $x$. By determining the length of 2,3 in this manner for a number of locations of the point $x$, we obtain a curve $a$, $b$, $c$, $d$, shown in Fig. 34b which represents the equivalent primary distribution of turns. The corresponding current values are again represented by the sine curve which, as previously pointed out, is only an approximation. By multiplying the values of the two curves, we obtain,

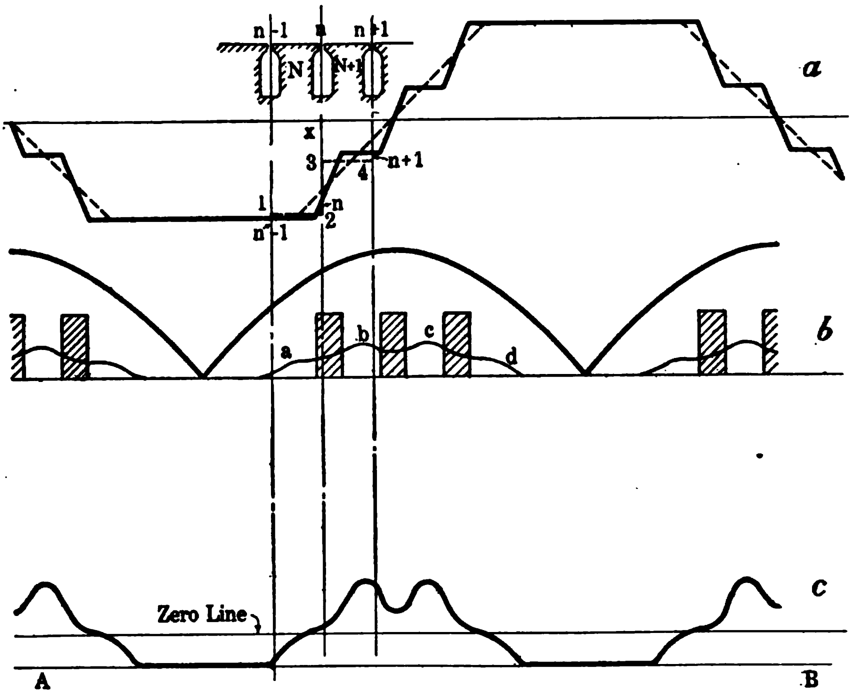

FIG. 34

as before, in Fig. 34c the time curve for the secondary ampere turns for $\epsilon = 0$. By comparison with Fig. 30c, it will be seen that the high peaks, and with them the losses, have been materially reduced by going from an infinite or a very large number of secondary slots to a number of slots only slightly different from the number of primary slots. Similar improvements will be obtained in case of open slots, if the secondary slot pitch approaches the primary slot pitch.

### CASE NO. 11

### *Squirrel-Cage Rotors*

In all previous cases, a polyphase rotor with coils was assumed, and accordingly, the fact that the flux within the secondary coils is approximately constant was used as a basis for the calculation. In practise, short-circuit or damping windings of the squirrel-cage type are more commonly employed. With such windings, each tooth is surrounded by a short-circuited turn, which means that the tendency is to keep the flux within each tooth constant. It has been shown, however, under Case No. 3 that the tooth fluxes must also be constant with a coil winding and it appears, therefore, that the fundamental conditions are the same in both cases and that the results obtained for coils are equally applicable to the bars of squirrel-cage windings.

### CASE NO. 12

### *Synchronous Machines*

The previous considerations apply principally to induction machines, but it is evident that the methods given can be readily used in connection with synchronous machines. The only thing changed is that certain or all current points in the secondary carry a constant direct current. In considering these points, the constant d-c. ampere turns furnished from an outside source must be taken into consideration in the various equations for the ampere turns.

### CASE NO. 13

### *Primary Sinusoidal Distribution, Infinite Number of Secondary Phases, Resistance and Leakage Reactance Taken Into Account*

So far, the resistances were assumed to be negligible in all cases; this assumption holds approximately true at no-load with large machines, as for instance, phase converters for locomotives. Single-phase induction motors are, however, principally built for very small sizes with some of which the influence of the resistances may be noticable even at no-load.

The influence of the secondary resistance in case of sinusoidal primary field distribution with an infinite number of secondary slots may now be investigated.

For zero resistances, we found the instantaneous current values of the primary and secondary, respectively, to be

$$i_p = I_p \cos \alpha$$

$$i_n = I_n \cos (2 \alpha + \epsilon) \text{ (see (69))}$$

Resistance in the secondary may change both the phase relations and the crest values $I_p$ and $I_n$ of the currents. If we designate by $\theta$ the change in phase angle of the primary current and by $\rho$ the corresponding change in the secondary current, we have, therefore, the new relations

$$i_p = I_p \cos(\alpha + \theta) \qquad (76)$$

and

$$i_n = I_n \cos(2\,\alpha + \epsilon + \rho) \qquad (77)$$

wherein at present $I_p$, $I_n$, $\theta$ and $\rho$ are unknown.

In order to send the currents $i_n$ through the secondary coils with a resistance $r_n$, we must induce voltages $e_n = i_n\, r_n$ in the coils. Knowing this, as well as the law of change for $i_n$ from (77), we can easily find the law for the flux changes necessary in each coil to induce the voltages $e_n$. In other words, we find the conditions which the flux has to fulfill to obtain equilibrium in the secondary circuits. By determining further, as in all previous cases, the conditions which the flux has to fulfill for giving equilibrium in the primary circuit, we can definitely determine the actual fluxes satisfying both primary and secondary. The following calculation carried out along these lines gives all desirable information about the fluxes.

Assume again as in the previous cases that each secondary coil $n$ an angle $\epsilon$ ahead of coil 1 passes the center of the primary belt at the time $\alpha = -\epsilon$.

The voltage to be induced in a coil $n$ is according to the above considerations

$$e_n = i_n\, r_n = I_n\, r_n \cos(2\,\alpha + \epsilon + \rho) \qquad (78)$$

Therefore, the rate of change of the flux within the coil must be

$$K_e \frac{d\,\varphi_{ni}}{d\,\alpha} = e_n = I_n\, r_n \cos(2\,\alpha + \epsilon + \rho) \qquad (79)$$

where

$$K_e = 2\,\pi f\, 10^{-8}$$

From which we find

$$K_e\, \varphi_{ni} = \frac{1}{2}\, I_n\, r_n \sin(2\,\alpha + \epsilon + \rho) + C \qquad (80)$$

wherein $C$ is the integration constant.

The latter is found by considering the required flux relations with regard to the primary. Let us assume at present that the flux within each primary coil follows a sine law and that the flux interlinking with the center of the primary coil belt has its maximum value $\varphi$ at the time $\alpha = 0$, as in all previous cases. Then the instantaneous flux value interlinking with this center is

$$\varphi_i = \varphi \cos \alpha$$

Since each secondary coil $n$ coincides with the primary belt center for $\alpha = -\epsilon$, the flux within the secondary coil $n$ at this time must be

$$\varphi_{ni} = \varphi_i = \varphi \cos \alpha = \varphi \cos (-\epsilon) \qquad (81)$$

or if we combine (80) and (81) for $\alpha = -\epsilon$

$$K_e \varphi \cos (-\epsilon) = \frac{1}{2} I_n r_n \sin (-\epsilon + \rho) + C$$

or

$$C = K_e \varphi \cos (-\epsilon) - \frac{1}{2} I_n r_n \sin (-\epsilon + \rho) \qquad (82)$$

and introducing this into (80), we get

$$K_e \varphi_{ni} = K_e \varphi \cos (-\epsilon) + \frac{1}{2} I_n r_n [\sin (2\alpha + \epsilon + \rho) - \sin (-\epsilon + \rho)] \qquad (83)$$

According to previous considerations, we know that the flux $\varphi_N$ of an infinitely small secondary tooth $N$ is one-half the difference between the fluxes of the two adjacent coils $n$ and $n + 1$, an angle $d\,\epsilon$ apart; therefore, we have

$$\varphi_N = \frac{\varphi_{(n+1)i} - \varphi_{ni}}{2}$$

$$= \frac{1}{2K_e} \Big[ K_e \varphi \cos (-(\epsilon + d\,\epsilon)) - K_e \varphi \cos (-\epsilon) + \frac{1}{2} I_n r_n \Big( \sin (2\alpha + \epsilon + d\,\epsilon + \rho) - \sin (2\alpha + \epsilon + \rho) - \sin (-(\epsilon + d\,\epsilon) + \rho) + \sin (-\epsilon + \rho) \Big) \Big] \qquad (84)$$

From this we find the density

$$B_N = \frac{\varphi_N}{d\,\epsilon} K = \frac{K}{2K_e} \Big[ -K_e \varphi \sin (-\epsilon) + \frac{1}{2} I_n r_n \Big( \cos (2\alpha + \epsilon + \rho) - \cos (-\epsilon + \rho) \Big) \Big] \qquad (85)$$

In order to check our previous assumptions regarding the flux interlinking with the center of the primary phase belt, we determine now the voltage induced in the primary. Equation (**85**) represents the flux distribution around the secondary in terms of $\epsilon$. The distribution around the primary in terms of $Z$ follows directly from the fact that we always have

$$z = \epsilon + \alpha \text{ or } \epsilon = z - \alpha$$

Introducing this into (**85**) we get,

$$B_Z = \frac{\varphi_Z}{d_Z} = \frac{K}{2K_e}\Big[- K_e \varphi \sin(\alpha - z)$$

$$+ \frac{1}{2} I_n r_n \Big(\cos(\alpha + z + \rho) - \cos(\alpha - z + \rho)\Big)\Big] \quad \textbf{(86)}$$

The total flux interlinking with a coil $z$ extending from $z$ to $z + \pi$ is

$$\varphi_z = \int_{z=z}^{z=z+\pi} \varphi_Z = \int_{z=z}^{z=z+\pi} \frac{B_Z\, d z}{K}$$

$$= \frac{1}{2K_e} \int_{z=z}^{z=z+\pi} \Big[- K_e \varphi \sin(\alpha - z)\, d z$$

$$+ \frac{1}{2} I_n r_n \cos(\alpha + z + \rho)\, d z$$

$$- \cos(\alpha - z + \rho)\, d z)\Big] = - 2 \varphi \cos(\alpha - z)$$

$$+ \frac{1}{2K_e} I_n r_n [2 \sin(\alpha + \rho + z) - 2 \sin(\alpha + \rho - z)] \quad \textbf{(87)}$$

The voltage $e_z$ induced in coil $z$ with turns

$$n_z = \frac{n_p}{2} \cos z\, d z \quad \text{(See Case No. 5)}$$

is found by determining $K_e \dfrac{d \varphi_z}{d \alpha} n_z$ from (**87**).

We obtain

$$e_z = \frac{n_p}{4} \cos z\, d z\, [2 \varphi K_e \sin(\alpha - z)$$

$$+ \frac{1}{2} I_n r_n [2 \cos(\alpha + \rho + z) - 2 \cos(\alpha + \rho - z)] \quad \textbf{(88)}$$

The total primary voltage follows from this by integrating between the limits $z = -\frac{\pi}{2}$ to $z = \frac{\pi}{2}$, which gives

$$e_p = K_e \frac{\pi}{4} \varphi\, n_p \sin \alpha \qquad (89)$$

which is in line with (63) under Case No. 5 and, therefore, shows that equations (85) and (86) for the flux densities, satisfy the conditions of equilibrium in the primary as well as in the secondary windings.

While the expressions for the densities found in (85) and (86) are correct, they do not offer the complete solution in so far as they contain the secondary current value $I_n$, which is still unknown. It is, therefore, necessary to determine this current before a study of the fields is possible. In doing this, we utilize again our condition, that the resultant ampere turns at any point of the circumference must be proportional to the difference in density at either side of the point. This leads to the following formulas resulting in the determination of the currents $I_p$, $I_n$ and their phase angles.

The resultant ampere turns at a coil $n$ follow from (85)

$$i_r\, n_r = \frac{K}{K_l} \frac{d\,\epsilon}{4 K_e}\Big[- K_e \varphi \cos(-\epsilon)$$
$$- \frac{1}{2} I_n\, r_n (\sin(2\alpha + \epsilon - \rho) - \sin(-\epsilon + \rho))\Big] \qquad (90)$$

From (76) and (77) we know

$$i_p\, n_z = i_p \frac{n_p}{2} \cos z\, d\,\epsilon = I_p \frac{n_p}{2} \cos z \cos(\alpha + \theta)\, d\,\epsilon \qquad (91)$$

and

$$i_n\, n_n = i_n \frac{n_e}{\pi}\, d\,\epsilon = I_n \frac{n_e}{\pi} \cos(2\alpha + \epsilon + \rho)\, d\,\epsilon \qquad (92)$$

and since

$$i_n\, n_n + i_p\, n_z = i_r\, n_r$$

we get the general equation

$$I_n \frac{n_e}{\pi} \cos(2\alpha + \epsilon + \rho) + I_p \frac{n_p}{2} \cos z \cos(\alpha + \theta)$$
$$= - \frac{K}{4 K_l\, K_e}\Big[K_e \varphi \cos(-\epsilon) + \frac{1}{2} I_n\, r_n (\sin(2\alpha + \epsilon - \rho)$$
$$- \sin(-\epsilon + \rho))\Big] \qquad (93)$$

While this equation has four unknown quantities, it applies to any value of $\epsilon$, and we can therefore, obtain from it any number of equations by assuming a corresponding number of definite values for $\epsilon$. As a matter of course, we assume these values so as to get simple expressions.

Some rather simple conditions are obtained by considering coils in line with the line $Y\,Y'$ in Fig. 22, because the primary winding has zero conductors in $Y$ and $Y'$, which eliminates all expressions with $I_p$ in (**93**).

Let us consider for instance a secondary coil, which is an angle $\epsilon = \frac{\pi}{2}$ ahead of coil 1 and passes the points $Y\,Y'$ at the time $\alpha = 0$. Introducing these values into (**93**) we get

$$I_n \frac{n_s}{\pi} \cos\left(\frac{\pi}{2} + \rho\right) =$$

$$- \frac{K}{8\,K_l K_s} I_n\, r_n \left(\sin\left(\frac{\pi}{2} + \rho\right) - \sin\left(- \frac{\pi}{2} + \rho\right)\right)$$

or

$$\frac{n_s}{\pi} \sin \rho = \frac{K}{4\,K_l K_s} r_n \cos \rho$$

or

$$\frac{\sin \rho}{\cos \rho} = \tan \rho = \frac{K}{K_l K_s} \frac{r_n}{n_s} \frac{\pi}{4} \qquad (\mathbf{94})$$

This permits the determination of $\rho$ and any of its functions.

Let us now consider secondary coil 1 with $\epsilon = 0$ passing points $Y$ and $Y'$ at the time $\alpha = \frac{\pi}{2}$. Introducing these values into (**93**) we get

$$I_n \frac{n_s}{\pi} \cos(\pi + \rho) =$$

$$- \frac{K}{4\,K_l K_s} \left[K_s\, \varphi + \frac{1}{2} I_n\, r_n \left(\sin(\pi - \rho) - \sin \rho\right)\right]$$

or

$$I_n \left(\frac{n_s}{\pi} \cos \rho + \frac{K}{4\,K_l K_s} r_n \sin \rho\right) = \frac{K}{4\,K_l} \varphi$$

or

$$I_n = \frac{K}{K_l}\ \frac{\varphi}{4}\ \frac{\pi}{n_s}\ \frac{I}{\cos\rho + \frac{K}{K_l K_s}\ \frac{\pi}{4}\ \frac{r_n}{n_s}\sin\rho}$$

$$= \frac{K}{K_l}\ \frac{\varphi}{4}\ \frac{\pi}{n_s}\ \frac{\cos\rho}{\cos^2\rho + \tan\rho\sin\rho\cos\rho}$$

(See (**94**) for tan $\rho$)

$$I_n = \frac{K}{K_l}\ \frac{\varphi}{4}\ \frac{\pi}{n_s}\cos\rho = \frac{\varphi}{r_n}\tan\rho\cos\rho \qquad (\mathbf{95})$$

$$I_n = \frac{\varphi}{r_n}\sin\rho = \frac{K}{K_l}\ \frac{\varphi}{4}\ \frac{\pi}{n_s}\ \frac{1}{\sqrt{1+\left(\frac{K}{K_l}\ \frac{r_n}{n_s}\ \frac{\pi}{4}\right)}} \qquad (\mathbf{95a})$$

This permits the determination of $I_n$.

Let us now consider the center of the primary phase belt, *i.e.*, point $x$ and $x'$ with $z = 0$, and assume further $\alpha = 0$, at which time the secondary coil 1 with $\epsilon = 0$ coincides with $x$ and $x'$.

Introducing these values into (**93**), we get

$$I_n \frac{n_s}{\pi}\cos\rho + I_p \frac{n_p}{2}\cos\theta =$$

$$-\frac{K}{4K_l}\left[\varphi + \frac{1}{2} I_n r_n\ (\sin(-\rho) - \sin\rho))\right]$$

or

$$\cos\theta = -\frac{2}{I_p n_p}\left(\frac{K}{K_l}\ \frac{\varphi}{4} + I_n \frac{n_s}{\pi}\cos\rho\right)$$

$$\cos\theta = -\frac{\varphi}{2 I_p n_p}\ \frac{K}{K_l}(1+\cos^2\rho) \qquad (\mathbf{96})$$

Let us further consider the center of the primary phase belt for $\alpha = \frac{\pi}{2}$ at which time a secondary coil with $\epsilon = -\frac{\pi}{2}$ coincides with $x$ and $x'$.

Introducing these values into (**93**), we get

$$I_n \frac{n_s}{\pi}\cos\left(\frac{\pi}{2} + \rho\right) + I_p \frac{n_p}{2}\cos\left(\frac{\pi}{2} + \theta\right) = 0$$

or

$$I_p = - I_n \frac{n_s}{n_p} \frac{2}{\pi} \frac{\cos\left(\frac{\pi}{2} + \rho\right)}{\cos\left(\frac{\pi}{2} + \theta\right)}$$

or by introducing $I_n$ as found in (95) we get

$$I_p = - \frac{K}{K_t} \frac{\varphi}{2} \frac{1}{n_p} \frac{\sin \rho \cos \rho}{\sin \theta} \qquad (97)$$

or

$$I_p = - \frac{K}{K_t} \frac{\varphi}{2} \frac{1}{n_p} \frac{\sin 2 \rho}{\sin \theta} \qquad (97\text{-A})$$

By introducing this into (96) we get

$$\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{\sin \rho \cos \rho}{1 + \cos^2 \rho} = \frac{\sin 2 \rho}{1 + \cos^2 \rho} \qquad (98)$$

We are now in a position to calculate all currents and their phase relations correctly for any rotor resistance. We are further in a position now to calculate with $I_n$ known from (85) and (86) the flux values for both the primary and secondary member. This has been done in Figs. 35 and 36 for a case with

$$\rho = 30 \text{ deg.}$$

It is evident from Fig. 35, which is plotted for a case of relatively high rotor resistance that the rotor resistance not only causes a variation in the size of the rotating field but also a variation in its speed relative to the stator. Fig. 36 showing the same case with the field plotted against the secondary surface shows how the field runs at times ahead of the secondary and subsequently falls back, completing such a cycle in the time $\alpha = \pi$. The combination of the variation in size and the running ahead and falling back of the field with regard to the secondary induces the voltage in the rotor, which is required to drive the rotor currents over the rotor resistance. The dotted line for $\alpha = 75$ deg. in Fig. 35 shows the maximum field interlinking with the center coil of the tertiary winding, and indicates that the tertiary voltage is shifted less than 90 deg. against the primary counter e. m. f. and different in size therefrom. The irregular field speed with regard to the primary can be seen by comparing the values of $v$ giving the angle of space travel with the corresponding time angle $\alpha$ of the same curve; it will be seen, for instance, that the field travels 39 space degrees during the first 30 time degrees, etc. The variation in the field size is indicated by the line connecting the crest values of the field, and it will be seen that this curve has double line frequency. In normal machines, the influence of the resistance is very much smaller than indicated in these figures and especially, in case of the sizes used for phase converters it is negligible at no-load, with regard to its influence upon the field sizes and currents, although the phase shifting effect upon the tertiary voltage is appreciable in most cases. In small single-phase induction motors with high rotor resistance it may, however, be necessary to consider the resistance if exact results for the currents are desirable.

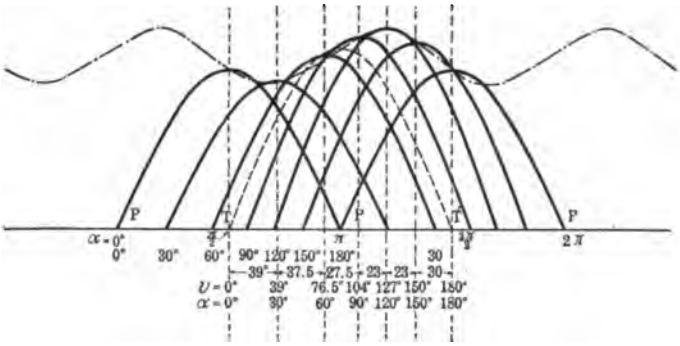

FIG. 35

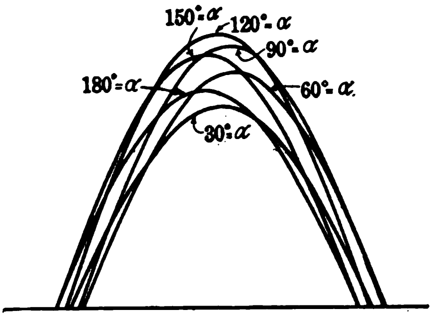

FIG. 36

The tertiary voltage can now also be easily determined. Since we know the laws for the stator flux distributions, we merely need to find the total flux for each tertiary coil by adding all fluxes within the coil. By finding from this $\frac{d\,\varphi_t}{d\,\alpha}\ n_z$ we get the voltage for each coil and subsequently the total voltage by a simple integration of the individual coil voltages.

Equation (87) previously derived gave the flux for any stator coil an angle $z$ away from the center of the primary phase belt. The tertiary winding has its maximum turns at the line $Y\,Y'$ Fig. 22, that is, for $z = \frac{\pi}{2}$ and the turns for a coil an angle $z$ away from $x\,x'$ are, therefore,

$$n_z' = \frac{n_p}{2} \sin z\,d\,z$$

By combining this with $\frac{d\,\varphi_{z'}}{d\,\alpha}$ as found from (87) we obtain therefore,

$$\frac{d\,\varphi_{z'}}{d\,\alpha} = e_z' = \frac{n_p}{4} \sin z\,d\,z \Big[ 2\,\varphi \sin(\alpha - Z) + \frac{1}{2} I_n\,r_n\,\big(2\cos(\alpha + \rho + z) - 2\cos(\alpha + \rho - z)\big)\Big] \qquad (99)$$

The total tertiary voltage follows from this by integrating between the limits $z = 0$ to $z = \pi$ which gives

$$e_t = \frac{n_p}{4}\Big[ -\varphi\left(\frac{1}{2}\sin(\alpha - 2\,z) + z\cos\alpha\right) + \frac{1}{2} I_n\,r_n \left(\frac{1}{2}\cos(2\,z - \alpha - \rho) - z\sin(\alpha + \rho) - \frac{1}{2}\cos(2\,z + \alpha + \rho) - z\sin(\alpha + \rho)\right)\Big]$$

$$e_t = -\frac{\pi}{4}\,n_p\,[\varphi\cos\alpha + I_n\,r_n\sin(\alpha + \rho)] \qquad (100)$$

It will be seen that the tertiary voltage consists of a vector equal and at a right angle to the primary voltage plus an out-of-phase vector proportional to the secondary ohmic drop. In the case assumed in Figs. 35 and 36, the tertiary voltage is larger than the primary and shifted against it.

The influence of resistance in the primary winding is taken into account, the same as in any other a-c. apparatus, and, therefore, does not need to be discussed here.

The influence of the leakage reactances can be taken care of in the same way as the resistances with the only difference that the out-of-phase relation of the leakage reactance drop is taken care of in equation (**78**) which gives an equation

$$\frac{d\,\varphi_{ni}}{d\,\alpha} = I_n\,x_n \cos\left(2\,\alpha + \epsilon + \rho - \frac{\pi}{2}\right)$$

where $x_n$ is the secondary leakage reactance. Otherwise, the procedure will be the same as outlined in connection with the secondary resistance.

The influence of leakage reactance alone can, however, be taken care of in a much simpler way, by the following consideration.

With zero resistances and zero losses, there will be no phase displacements in either the primary or secondary currents, as found under Case No. 5, and, therefore, only the size of these currents will be changed by the presence of leakage reactance. Let us assume a primary leakage coefficient of $\sigma_p$ and a secondary leakage coefficient of $\sigma_s$. This means that, if a certain magnetomotive force in the secondary, for instance, induces a total flux $\varphi_s$, a part $\sigma_s\,\varphi_s$ of the total flux follow the leakage path, while the remaining part $\varphi_s\,(1 - \sigma_s)$ goes across the gap to the stator; similar conditions apply to the primary.

Assume in Fig. 37 the line $x\,x$ to go through the center of the primary phase belt and the line $y\,y$ through the center of the tertiary phase belt.

We found that at the time $\alpha = \frac{\pi}{2}$, the primary m. m. f. is zero, while the secondary m. m. f. sets up a field with the center along $x\,x$. If vector $I_s$ represents the local position and size of the secondary m. m. f. for $\alpha = \frac{\pi}{2}$, the same vector $\varphi_t$ may be assumed to represent the flux sent across the gap by this m. m. f. This flux cuts the tertiary winding on the stator, and, therefore, represents the tertiary flux $\varphi_t$ in size.

Since according to our previous definition the flux $\varphi_t = \varphi_s\,(1 - \sigma_s)$ we find the total secondary flux vector

$$\varphi_s = \frac{\varphi_t}{1 - \sigma_s} \qquad \textbf{(101)}$$

With zero secondary resistance we know this flux to be constant and rotating with synchronous speed. It may, therefore, be represented by the vector $\varphi_s' = \varphi_s$ at the time $\alpha = 0$. The secondary m. m. f. represented by $I_s' = I_s$ acts at that time in a demagnetizing direction, as shown by vector $I_s'$. The secondary leakage fluxes being induced by the secondary currents only are of course in the direction of these currents and may be represented by the vector

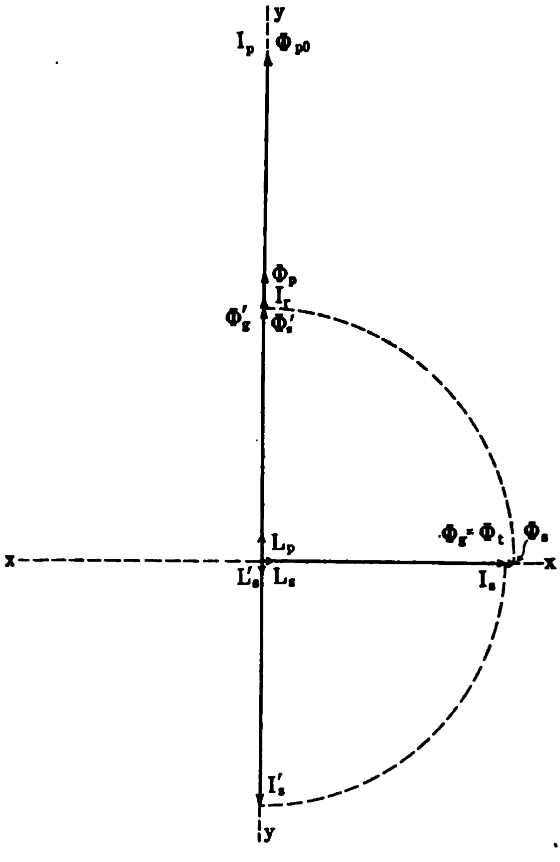

FIG. 37

$$L'_s = \varphi_s' \times \sigma_s \qquad (102)$$

If $L_s$ is the secondary leakage flux acting in negative direction and $\varphi_s'$ the resultant secondary flux, we must have a flux

$$\varphi_g' = \varphi_s' + L_s \qquad (103)$$

cross the air gap into the secondary core. In order to send such flux across the gap, we need a resultant m. m. f. of $I_r$ represented by the same vector. The primary m. m. f. follows therefore as the difference of $I_r$ and $I_s'$ and is represented by $I_p$.

The primary leakage flux produced by this m. m. f. is found from the imaginary primary flux $\varphi_{po}$ to be

$$L_p = \sigma_p \times \varphi_{po} \qquad (104)$$

Adding this to the gap flux $\varphi_g'$ we obtain the total primary flux $\varphi_p$.

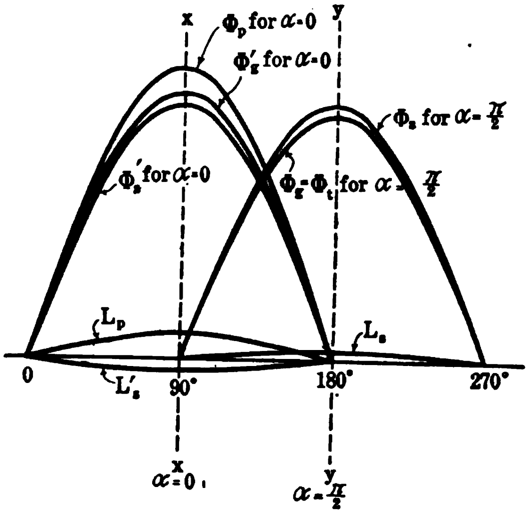

FIG. 38

By making the proper substitutions, we get

$$\varphi_p = \varphi_t \left[\frac{1 + \sigma_s}{1 - \sigma_s} + \sigma_p \left(1 + \frac{1 + \sigma_s}{1 - \sigma_s}\right)\right] = \varphi_t K_2 \qquad (105)$$

Since in all but cases of exceptionally high leakage $\frac{1 + \sigma_s}{1 - \sigma_s}$ can be set approximately equal to $1 + 2\,\sigma_s$, we can write

$$\varphi_p = \varphi_t\,(1 + 2\,\sigma_p + 2\,\sigma_s + 2\,\sigma_s\,\sigma_p)$$

or if

$\sigma_p = \sigma_s = \sigma$, where $\sigma$ = average leakage coefficient for one member.

$$\varphi_t = \frac{\varphi_p}{1 + 4\,\sigma + 2\,\sigma^2} \qquad (106)$$

The value $2\,\sigma^2$ is usually negligible in practise and therefore

$$\varphi_t = \frac{\varphi_p}{1 + 4\,\sigma} = \frac{\varphi_p}{K_2} \tag{107}$$

Fig. 38 shows the fluxes given by vectors in Fig. 37 as they are actually distributed in the machine.

The equations for the current follow from the same considerations and the following simple calculation.

In order to produce an air gap flux $\varphi_g$, we need a primary current

$$I_p = \frac{\varphi_g}{2\,n_p}\ \frac{K}{K_l} \qquad \text{(See equation } \mathbf{68}\text{)}$$

and a secondary current

$$I_n = \frac{\pi}{4}\ \frac{\varphi_g}{n_s}\ \frac{K}{K_l} \qquad \text{(See equation } \mathbf{69A}\text{)}$$

If we find from these the values for $\varphi_g$ and equate them, we obtain the following relation between primary and secondary currents:

$$I_p = I_n\,\frac{2}{\pi}\ \frac{n_s}{n_p} \tag{108}$$

The secondary current is

$$I_n = \frac{\pi}{4}\ \frac{\varphi_g}{n_s}\ \frac{K}{K_l} = \frac{\pi}{4}\ \frac{K}{K_l}\ \frac{\varphi_p}{n_s}\ \frac{1}{K_2} \tag{109}$$

(See equation (**107**) for $\varphi_g$.)

Reflecting this into the primary winding, we get by utilizing (**108**) an equivalent current of

$$I_{ps} = \frac{1}{2}\ \frac{K}{K_l}\ \frac{\varphi_p}{n_p}\ \frac{1}{K_2} \tag{110}$$

In order to send the flux $\varphi_g'$ across the gap, we need a resultant current

$$I_r = \frac{\varphi_g'}{2\,n_p}\ \frac{K}{K_l}$$

where $\varphi_{g'}$ follows from (**103**)
to

$$\varphi_{g'} = \varphi_p - \sigma_p\,I_p\,\frac{K_l}{K}\,2\,n_p$$

Now we know that $I_p$ must be the sum of $I_{ls}$ and $I_r$, therefore,

$$I_p = \frac{1}{2}\ \frac{K}{K_l}\ \frac{\varphi_p}{n_p}\ \frac{1}{K_2} + \frac{\varphi_p}{2\,n_p}\ \frac{K}{K_l} - \sigma_p\,I_p$$

or

$$I_p = \frac{\varphi_p}{2\,n_p}\;\frac{K}{K_l}\;\frac{1+\dfrac{1}{K_2}}{1+\sigma_p} = \frac{\varphi_p}{n_p}\;\frac{K}{K_l}K_3 \quad \textbf{(111)}$$

We may assume approximately

$$1 + \frac{1}{K_2} = 2 - 4\,\sigma = 2\,(1 - 2\,\sigma)$$

if we further set

$$\frac{1 - 2\,\sigma}{1 + \sigma_p} = 1 - 3\,\sigma$$

we get

$$I_p = \frac{\varphi_p}{n_p}\;\frac{K}{K_l}\,(1 - 3\,\sigma) = \frac{\varphi_p}{n_p}\;\frac{K}{K_l}\,K_3 \quad \textbf{(111 A)}$$

It will be seen from comparison of (**110**) and (**69A**), that the secondary currents are the same with and without leakage except for the factor $\frac{1}{K_2}$, which, in normal machines is somewhat smaller than 1.

Similarly, a comparison between (**111**) and (**67**) shows that the primary currents are the same with and without leakage, except for the factor $K_3$, which, in normal machines, is somewhat smaller than 1. The influence of the leakage upon the fields, currents, etc., is usually much larger than that of the resistances and is especially in small machines quite appreciable.

We have so far considered leakage and resistance separately. An exact consideration of the two combined gives rather complicated expressions without giving an appreciably greater degree of correctness, than is obtained by first considering the resistance only and subsequently modifying the currents with the factors $\frac{1}{K_2}$ in case of the secondary, and $K_3$ in case of the primary. A lengthy exact derivation of the combined effect may therefore, be omitted here.

All through the previous considerations, it has been assumed that the rotor runs synchronous. In practise, a very small slip is caused by the torque required to overcome bearing and windage friction. This causes small currents of the slip frequency in the rotor in addition to those found from our calculations. These low-frequency currents are, however, at no-load so small that they are of no practical importance whatsoever.

## CASE NO. 14

### *Primary Winding Distributed in Slots, Short-Circuited Secondary Winding in Slots, (Squirrel-Cage or Individual Coils Short-Circuited) Resistance and Leakage Reactance Taken into Account.*

It is evident from the foregoing that an exact consideration of the influence of resistance and leakage reactance in case of any but the simple sinusoidal distribution of the primary winding leads to altogether impracticable amounts of calculation. It has already been pointed out, however, that with windings distributed into slots the presence of resistance and leakage reactance is beneficial with regard to the elimination of the higher harmonics in the currents and that it is, therefore, permissible in most practical cases to neglect the influence of the slots upon the primary current wave form in all but extreme and undesirable designs. While the presence of resistance and leakage reactance is beneficial with regard to the currents, it must be realized, however, that the smoothing out of the current waves is obtained at the expense of the field form for the resultant field. If we have, therefore, for instance cases with few slots per pole which would call for marked current peaks without leakage reactance and resistance, the reduction of such current peaks caused by the presence of leakage reactance will result in higher harmonics for the resultant field form and may with the loaded motor lead to certain counter torques at certain low speeds resulting in dents in the speed-torque curve frequently observed in practise.*

Whatever higher harmonics in the current waves are not fully eliminated by the leakage reactance will usually result in increased effective current values and ohmic losses which, in themselves, may or may not be sufficient to show up in practical tests of commercial machines. Furthermore, the harmonics may on account of their high frequency cause very appreciable eddy losses in heavy copper conductors often used in larger machines. Large no-load losses otherwise unaccounted for can often be traced to this cause.

Any departures from the primary sinusoidal winding distribution made necessary in practise by the fact that the windings have to be distributed in winding belts, as shown in Fig. 24, with definite steps instead of being graduated according to a sine

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*NOTE: This is not the only cause for dents in speed-torque curve but only one of many causes. A previous footnote has already mentioned that variation in the permeability of iron, bad distribution of resistance in the rotor, etc., may cause undesirable conditions along this line.

law may have effects similar to those just discussed in connection with the influence of the slots. The harmonics caused thereby in the currents are usually, however, of a much lower order than those caused by the teeth, and, therefore, the smoothing out effect of the resistance and leakage reactance will be much smaller and in most practical machines almost negligible. A practical method for determining the no-load currents rather closely consists, therefore, in the determination of the currents by the methods and formulas given under Case No. 6 and by subsequently modifying the currents thus obtained, for the influence of the resistance and leakage reactance through the introduction of the correcting factors as found in connection with the sinusoidal distribution in Case No. 13. A similar procedure will give good results with regard to the tertiary voltage of the phase converter.

The presence of secondary resistance, especially in squirrel-cage windings, as well as the presence of leakage reactance, both of which interfere to a certain extent with the formation of ideal field forms may, on account of this latter fact, introduce a certain amount of differential leakage which is otherwise impossible. It is, therefore, advisable to allow in practical machines for a small amount of differential leakage in the calculation.

## CASE NO. 15

### *Primary Winding Distributed in Slots, Secondary Winding of the Slip-Ring Type Distributed in Slots, (i. e., Secondary Windings with a Small Number of Phases with Distributed Coils of Each Phase Connected in Series)*

Space does not permit to be given here the complete treatment of this case which is rather involved, but a few essential points may be mentioned without giving a full proof for the statements made. With the star-connected three-phase winding frequently used in secondaries, an additional fundamental condition has to be met besides those previously considered; namely, the sum of all currents flowing at any instant towards the star point must be zero. As long as the primary winding has approximately a sinusoidal distribution thereby calling for sinusoidal current waves in the secondary, this condition is naturally fulfilled as in any true three-phase system with sinusoidal waves. In such a case, we may, therefore utilize the results obtained in connection with Cases No. 5 and 13 with regard to the primary currents and the tertiary voltages. The secondary currents follow directly from the following consideration: We have found for zero resistance and leakage reactance in connection with Case No. 5, that the entire resultant field is at times set up by the secondary alone. We have further found that the magnetomotive force of the secondary is constant. We can, therefore, conclude from the combination of these two facts that the magnetomotive force of the secondary alone must be at all times sufficient to set up a field which is equal in size to the resultant field. Since the secondary is a three-phase winding, its currents must, therefore, be the same as the magnetizing currents in the primary of a three-phase motor with the same field and the same winding arrangement. In other words, the secondary current of a three-phase winding of a single-phase motor with zero resistance and leakage can be determined rather closely from the formulas used for the primary magnetizing current of a polyphase induction motor; still assuming, of course, that the primary distribution is sinusoidal. The effect of resistance and leakage reactance can subsequently be again taken into account by introducing the modifying factors found under Case No. 13. For delta connected and other polyphase secondary windings, the fundamental condition is somewhat different, but the conclusions as so far reached apply as well.

With primary distributions not being sinusoidal, a large multitude of different conditions may apply. In a great many cases the tendency for higher harmonic currents in the secondary is materially reduced as compared with the short-circuited (squirrel-cage) winding. With the latter, it was possible for each coil or conductor to individually adjust its current along the lines discussed. With a number of distributed coils connected in series, this is no longer possible and with proper slot relation between primary and secondary, there is a tendency to replace each high current peak of the individual coils by a larger number of smaller peaks in the phase belt. This is the same action as previously described in connection with the primary. For this reason, we find in a great many commercial machines that oscillograms of the secondary currents closely approach a sine curve even though the windings are located in slots and depart a certain amount from a sinusoidal distribution. On the other hand, we may find cases in which the special condition applying to our present case cause, in combination with the other conditions, rather erratic current waves.

The previously mentioned tendency for eliminating harmonics in slip-ring secondary windings is again obtained at the expense of the resultant field form. For this reason, we may again have superimposed rotating fields and corresponding dents in the speed-torque curve; furthermore, the differential leakage may be appreciable in case of slip-ring secondaries.

## CONCLUSIONS

### FIELDS AND MAGNETOMOTIVE FORCES

The actually existing main field in a single-phase induction machine running light is the resultant of magnetomotive forces furnished by the primary and secondary windings. By assuming the magnetomotive forces of only one of these members existing while those of the other members are assumed to be zero, we obtain imaginary fields which may be called primary field if only the primary magnetomotive force is assumed to exist, and the secondary field if only the secondary magnetomotive force is assumed to exist. All machines have, regardless of the primary winding distribution and the primary field form, a strong tendency to form a resultant field of sinusoidal space distribution rotating at no-load with practically the same speed as the rotor, *i. e.*, with synchronous speed relative to the primary. With an infinite number of secondary slots and with zero resistance and leakage reactance, the space distribution of the resultant field is exactly sinusoidal and the speed of the field is zero relative to the rotor and synchronous relative to the stator no matter what the primary field form may be. With a secondary winding having individually short-circuited coils or conductors embedded in a limited number of slots (usually squirrel-cage windings), the values of the average densities of the individual secondary teeth are also constant and follow a sine law, giving thereby a local distribution for the resultant field of a step form with the middle of each step located on a sine wave as shown in Fig. 15; this is still on the assumption of zero resistance and leakage reactance. These ideal field conditions are in practise slightly disturbed by the usual amounts of resistances and leakage reactances of the windings. The true sinusoidal field distribution is preserved only in case of a sinusoidal distribution of the primary winding and with infinite number of secondary slots. The presence of leakage reactance causes, however, even under these assumptions, slight fluctuations of double frequency in the size of the air-gap field. The presence of rotor resistance also causes such fluctuation in size as well as certain variations in the speed of the resultant field so that it rotates at times somewhat slower and at times somewhat faster than the rotor. The influence of leakage reactance upon the air gap field $\varphi_g$ is shown in Fig. 38; the influence of rotor resistance upon the resultant field is shown in Figs. 35 and 36, the former showing the position of the fields relative to the primary, and the latter, showing it relative to the secondary. The presence of primary resistance results, as in all a-c. apparatus, in a slight phase displacement and a small change in the size of the flux.

With non-sinusoidal distribution of the primary winding the influence of resistance and leakage reactance is similar to that described in connection with the sinusoidal primary curve. Moreover, it may, however, cause the departure of the resultant field from the sinusoidal space distribution. Such departures are slight in most commercial machines but may be appreciable in case of exceptionally large secondary resistance or large leakage reactances.

With machines of the slip ring type, which usually have a limited number of secondary phases with a number of series-connected phase coils distributed over phase belts, the resultant field may depart more from the sinusoidal distribution than in all other cases.

The size of the resultant flux in single-phase induction machines is, as in most a-c. apparatus, principally determined by the impressed voltage and frequency and the number of primary conductors and their distribution; the influence of the resistance and leakage reactance upon the size of the flux is usually rather small.

The local distribution of the imaginary primary field is naturally governed by the local distribution of the primary winding and the magnetic reluctances of the machine. Except for minor influences in the variation of the reluctances, the primary field is of constant form and stationary relative to the primary winding, but varying in size and direction.

The imaginary secondary field is, in its form and size, at any time governed by the fact that the primary and secondary densities must, together give the densities called for by the previous rules for the resultant field. This means that the secondary field has often very peculiar shapes and varies in form and size with the primary winding distribution and the reluctance of the machine.

The primary and secondary field forms as obtained when neglecting the influence of resistance and leakage reactance are shown:

In Fig. 2 for a concentrated primary winding with a concentrated single-phase secondary winding. (See Fig. 1.)

In Figs. 11 and 12 for a concentrated primary winding with concentrated two-phase secondary windings. (See Fig. 6.)

In Fig. 20 for a concentrated primary winding with short-circuited secondary having an infinite number of slots.

In Fig. 23a for a primary winding with sinusoidal distribution and a secondary winding with an infinite number of slots. (See Fig. 22.)

In Fig. 27a for a primary winding evenly distributed over 90 deg. and a secondary winding with an infinite number of slots.

The secondary field travels with double synchronous average speed relative to the secondary in a direction opposite to the mechanical rotation of the secondary. It, therefore, travels with synchronous speed relative to the primary winding opposite to the relative mechanical rotation of the secondary.

## PRIMARY MAGNETIZING CURRENTS

The primary magnetizing current has a sinusoidal wave with a sinusoidal voltage impressed upon the machine and an infinite number of secondary slots and phases, regardless of the local distribution of the primary winding. The maximum and effective values of these primary sinusoidal currents are, however, governed to a large extent by the distribution of the primary winding. This is brought out by formulas given under Cases No. 5 and 6. These formulas while applying exactly only under the assumptions made, are correct within one or two per cent for most standard machines if the turns of each primary slot are assumed to be distributed between the centers of the two adjacent teeth.

With a non-skewed slot construction, a slotted primary and a short-circuited secondary with a limited number of secondary slots (squirrel-cage rotor) the curve of the primary magnetizing current may materially depart from a sine curve. Every time a secondary slot coincides with a primary slot a current peak is caused in the primary magnetizing current. These peaks will increase the effective value of the currents above those found in the previous formulas and are furthermore liable to cause appreciable eddy losses in heavy conductors. Some extreme cases of such current peaks are shown in Figs. 3, 7A, 9A and 16A corresponding to windings shown in Figs. 1, 6 and 13. These current peaks will be the more marked, the larger the number of primary and secondary slots which coincide locally at any one time. By keeping such number of coinciding slots small, the departure of the primary current from a sine curve can be made negligible so that the formulas can be applied with a sufficient degree of accuracy.

The leakage reactance of the machine is of material assistance in eliminating current peaks and it may, therefore, be found that, under otherwise equal conditions, the magnetizing current is slightly smaller in machines with large leakage.

The fundamental frequency of the primary currents is of course, the same as the line frequency; whatever higher harmonics exist are usually odd harmonics. If the rotor speed is slightly less than synchronous, the location of the higher harmonics on the fundamental wave changes periodically with the slip.

Outside of the influences previously mentioned, resistances and leakage reactances of the motor tend to decrease the primary magnetizing currents as can be seen by comparing the formulas found under Cases No. 5 and 13.

In motors with phase-wound slip-ring secondaries, most of the previous remarks apply, except that the tendency for higher harmonics in the primary current is usually somewhat smaller than in the case of squirrel-cage rotors.

## SECONDARY MAGNETIZING CURRENTS

With synchronously rotating secondary, the fundamental wave of the secondary currents has double line frequency. With sinusoidal primary winding distribution no higher harmonics exist as indicated in Fig. 23 which shows a number of secondary currents.

Any departure from a sinusoidal distribution of the primary winding will cause the secondary currents to depart materially from a sinusoidal curve and furthermore, cause the currents in different secondary conductors to be different from each other. If the secondary speed is slightly different from synchronous speed, the current wave in each secondary coil changes its shape and size periodically with the slip. The two waves of a secondary current cycle are usually different from each other indicating the presence of even harmonics which are quite large in a great many cases.

Figs. 3, 7B, 7C, 9B, 9C, 16B and 21 show secondary current waves for primary windings concentrated into a mathematical point and having no leakage corresponding to arrangements shown in Figs. 1, 6 and 13. These cases are, of course, extreme, and in practise, impossible. Figs. 25, 26, 27, 28 and 29 show secondary current waves of primary field forms shown in connection with these figures. The heavy line 1, 2, 3, 4, etc., of Fig. 30c shows the influence of the fact that the primary winding is distributed over a number of slots upon the secondary currents, in case of a secondary having a much larger number of slots than the primary. Fig. 34c shows the secondary current for the same case except that the number of secondary slots is assumed to be about the same as the number of primary slots. Fig. 31 shows again a similar case, except that wide open slots are assumed in the primary with the number of secondary slots much larger than the number of primary slots.

It is evident from these figures that the secondary currents are subject to a large number of variations. While, therefore, the simple formulas derived for a sinusoidal primary distribution may give a rough approximation for the secondary currents in single-phase machines, it should be realized that the marked higher harmonics are liable to increase not only the effective current value and the ohmic losses, but may also cause very appreciable eddy losses in the heavy conductors often employed in squirrel-cage windings, as well as in the heavy section end rings.

As in the case of the primary currents, the leakage reactance will again be beneficial in reducing the higher harmonics. Both resistance and leakage reactance also tend to decrease the fundamental wave, as can be seen by comparison of Cases No. 5 and 13.

With slip ring secondaries having phase belts, the tendency of higher harmonics in the secondary is usually much smaller than in short-circuited secondaries, although erratic wave shapes may be obtained in extreme cases.

## TERTIARY VOLTAGES IN PHASE CONVERTERS

The tertiary voltage of a phase converter is sinusoidal if the primary voltage is sinusoidal and the secondary has an infinite number of phases. In case of a short-circuited secondary (squirrel cage) with a limited number of secondary slots the tertiary voltage has a step shape similar to that shown in Figs. 17 and 18 with the center of the steps located on a sine wave. With a distributed tertiary winding of proper choice of tooth pitch and winding distribution, the size of the steps can be reduced so that the tertiary voltage approaches more closely a sine curve. Neglecting the resistance and leakage reactance, the tertiary voltage equals the primary voltage with equal number of turns if the secondary has an infinite number of slots. With a limited number of slots, the tertiary voltage decreases slightly with the number of secondary slots, as indicated in curve No. 19; this influence is the same as usually considered in connection with the zig-zag leakage, in reality it is here caused by the departure of the resultant flux from the sinusoidal wave shape. With the speed slightly different from synchronism, the resultant wave shape of the field varies periodically with the slip which also causes periodic variations in the higher harmonics of the tertiary voltage, without, however, affecting the effective value of the tertiary voltage.

The presence of slot and end connection leakage reactance causes a further difference between the primary and tertiary voltages, in the same direction as that caused by the zig-zag leakage or the difference in wave shape. The presence of resistance in the windings not only affects the amplitude of the tertiary voltage as compared with that of the primary, but also disturbs the 90 deg. phase relation in two-phase converters and the corresponding relation in converters wound for other numbers of phases.

### GENERAL

It is realized that the rather abstract methods used in this paper are not always best adapted to explain the various phenomena in the most simple manner. It is obvious, however, that after the facts have been definitely established by such methods, it will be possible to work out other treatments of the same phenomena; these can be made both simpler to understand, and at the same time correct, by keeping continuously the results derived from this and similar papers in mind. The various methods of assuming fields of oppositely rotating direction appear to be especially advantageous in this connection.

## APPENDIX I

Fig. 6 represents a phase converter with a single concentrated primary coil $P P'$ and a single concentrated tertiary coil $T T'$. The rotor is provided with a two-phase winding consisting of two concentrated coils 1 and 2 displaced 90 deg. against each other. The current and fluxes applying to these coils may be designated by $i_1$, $i_2$, $\varphi_1$ and $\varphi_2$, respectively. The four quadrants of the rotor are marked $A$, $B$, $-A$ and $-B$, and the fluxes carried by these quadrants are $\varphi_A$, $\varphi_B$, $-\varphi_A$, $-\varphi_B$. The densities $B$ are indicated by similar subscripts. Other assumptions are similar to those in case of Fig. 1.

Assume again, that coil 1 coincides with $P$ at the time $\alpha = \gamma$ we know that, the flux of coil 1 has the constant value

$$\varphi_1 = \varphi \cos \gamma \tag{1}$$

The coil 2 coincides with $P$ at a time $\frac{\pi}{2}$ earlier than coil 1, at the time $\alpha = \gamma - \frac{\pi}{2}$, so that it has the constant flux value

$$\varphi_2 = \varphi \cos \left(\gamma - \frac{\pi}{2}\right) \tag{2}$$

For any value $x = 0$ to $x = \frac{\pi}{2}$ corresponding to the time $\alpha = \gamma$ to $\alpha$ to $\gamma + \frac{\pi}{2}$ we have the following equations,

$$\begin{aligned} \varphi_i &= \quad \varphi_a + \varphi_b + \varphi_c && (3) \\ \varphi_1 &= - \varphi_a + \varphi_b + \varphi_c && (4) \\ \varphi_2 &= - \varphi_a - \varphi_b + \varphi_c && (5) \end{aligned}$$

By adding (5) and (3), subtracting (4) from (3), and (5) from (4), and by introducing the values from (1) and (2), we find

$$\varphi_a = \frac{\varphi}{2} (\cos \alpha - \cos \gamma) \tag{6}$$

$$\varphi_b = \frac{\varphi}{2}\left(\cos \gamma - \cos \left(\gamma - \frac{\pi}{2}\right)\right) \tag{7}$$

$$\varphi_c = \frac{\varphi}{2}\left(\cos \alpha + \cos \left(\gamma - \frac{\pi}{2}\right)\right) \tag{8}$$

The densities are, therefore,

$$B_a = \frac{K}{2} \varphi \frac{\cos \alpha - \cos \gamma}{x} \tag{9}$$

$$B_b = \frac{K}{2} \varphi \frac{\cos \gamma - \cos \left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}} \tag{10}$$

$$B_c = \frac{K}{2} \varphi \frac{\cos \alpha + \cos \left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2} - x} \tag{11}$$

Under consideration of the current directions shown in Fig. 6, we have the following relations for the currents, assuming that $n_1$ and $n_2$ are the numbers of turns in coil 1 and 2.

$$B_a = K_1 \left( i_p \, n_p - i_1 \, n_1 + i_2 \, n_2 \right) \tag{12}$$

$$B_b = K_1 \left( i_p \, n_p + i_1 \, n_1 + i_2 \, n_2 \right) \tag{13}$$

$$B_c = K_1 \left( i_p \, n_p + i_1 \, n_1 - i_2 \, n_2 \right) \tag{14}$$

By adding and subtracting these and introducing the values from (9), (10) and (11) with $x = \alpha - \gamma$ we obtain

$$i_p = \frac{K \varphi}{4 K_l n_p} \left[ \frac{\cos \alpha - \cos \gamma}{\alpha - \gamma} + \frac{\cos \alpha + \cos \left( \gamma - \frac{\pi}{2} \right)}{\frac{\pi}{2} - \alpha + \gamma} \right]$$

$$i_1 = \frac{K \varphi}{4 K_l n_1} \left[ \frac{\cos \gamma - \cos \left( \gamma - \frac{\pi}{2} \right)}{\frac{\pi}{2}} - \frac{\cos \alpha - \cos \gamma}{\alpha - \gamma} \right]$$

$$i_2 = \frac{K \varphi}{4 K_l n_2} \left[ \frac{\cos \gamma - \cos \left( \gamma - \frac{\pi}{2} \right)}{\frac{\pi}{2}} - \frac{\cos \alpha + \cos \left( \gamma - \frac{\pi}{2} \right)}{\frac{\pi}{2} - \alpha + \gamma} \right]$$

The tertiary flux is now

$$K \varphi_t = B_b x + B_c \left( \frac{\pi}{2} - x \right) - B_a x - B_b \left( \frac{\pi}{2} - x \right)$$

$$= - B_a x + B_b \left( 2 x - \frac{\pi}{2} \right) + B_c \left( \frac{\pi}{2} - x \right)$$

or by introducing the values for the densities

$$\varphi_t = \frac{\varphi}{2} \left[ 2 \cos \left( \gamma - \frac{\pi}{2} \right) + \frac{4 x}{\pi} \left( \cos \gamma - \cos \left( \gamma - \frac{\pi}{2} \right) \right) \right]$$

By determining $\frac{d\,\varphi_t}{d\,\alpha}$ we get

$$e_t = \frac{2\,\varphi}{\pi}\left(\cos\,\gamma - \cos\left(\gamma - \frac{\pi}{2}\right)\right)$$

Similarly, we find for value $x = \frac{\pi}{2}$ to $x = \pi$ corresponding

to the time $\alpha = \gamma + \frac{\pi}{2}$ to $\alpha = \gamma + \pi$

$$\begin{aligned} \varphi_i &= \varphi_d + \varphi_e + \varphi_f \\ \varphi_1 &= -\varphi_d - \varphi_e + \varphi_f \\ \varphi_2 &= \varphi_d - \varphi_e - \varphi_f \end{aligned}$$

$$\varphi_d = \frac{\varphi}{2}\left(\cos\,\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)\right)$$

$$\varphi_e = -\frac{\varphi}{2}\left(\cos\,\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)\right)$$

$$\varphi_f = \frac{\varphi}{2}\,(\cos\,\alpha + \cos\,\gamma)$$

$$B_d = \frac{K}{2}\,\varphi\,\frac{\cos\,\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)}{x - \frac{\pi}{2}}$$

$$B_e = -\frac{K}{2}\,\varphi\,\frac{\cos\,\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}}$$

$$B_f = \frac{K}{2}\,\varphi\,\frac{\cos\,\alpha + \cos\,\gamma}{\pi - x}$$

$$\begin{aligned} B_d &= K_1\,(i_p\,n_p - i_1\,n_1 - i_2\,n_2) \\ B_e &= K_1\,(i_p\,n_p - i_1\,n_1 + i_2\,n_2) \\ B_f &= K_1\,(i_p\,n_p + i_1\,n_1 + i_2\,n_2) \end{aligned}$$

$$i_p = \frac{K\,\varphi}{4\,K_1\,n_p}\left[\frac{\cos\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)}{\alpha - \gamma - \frac{\pi}{2}} + \frac{\cos\alpha + \cos\gamma}{\pi - \alpha + \gamma}\right]$$

$$i_1 = \frac{K\varphi}{4\,K_l n}\left[\frac{\cos\alpha + \cos\gamma}{\pi - \alpha + \gamma} + \frac{\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}}\right]$$

$$i_2 = \frac{K\varphi}{4\,K_l n_2}\left[\frac{-\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)}{\frac{\pi}{2}} - \frac{\cos\alpha + \cos\left(\gamma - \frac{\pi}{2}\right)}{\alpha - \gamma - \frac{\pi}{2}}\right]$$

$$K\varphi_t = B_e\left(x - \frac{\pi}{2}\right) + B_f\,(\pi - x) - B_d\left(x - \frac{\pi}{2}\right) - B_e(\pi - x)$$

$$\varphi_t = \frac{\varphi}{2}\left[+ 4\cos\gamma + 2\cos\left(\gamma - \frac{\pi}{2}\right) - \frac{4x}{\pi}\left(\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)\right)\right]$$

and

$$\frac{d\varphi_t}{d\alpha} = -\frac{2\varphi}{\pi}\left(\cos\gamma + \cos\left(\gamma - \frac{\pi}{2}\right)\right) = e_t$$

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# A PHYSICAL CONCEPTION OF THE OPERATION OF THE SINGLE-PHASE INDUCTION MOTOR*

BY B. G. LAMME

### ABSTRACT OF PAPER

This paper covers a method of studying the actions of the single-phase induction motor, which the writer has found to be very convenient from the educational standpoint. It is based upon the assumption of two equal and oppositely rotating primary magnetomotive forces combined with a synchronously rotating secondary m. m. f., such as would be produced by direct-current excitation. It follows that there is a resultant rotating primary field just as in the polyphase motor, while in the secondary there are two currents, one of low frequency, corresponding to the polyphase motor, and the other of nearly double the primary frequency. Diagrams and descriptions are given to illustrate the magnetomotive forces and fluxes, showing how, among other conditions, two oppositely rotating fields of unequal value may be possible.

The next step is a consideration of e. m. f. generation by two oppositely rotating fields, showing how both must be taken into account. The effects upon the counter e. m. f. and excitation, of the reduction or suppression of one field is shown. This illustrates, in a simple manner, why the excitation on single-phase must be practically the same as on polyphase at full speed and falls to one-half value at standstill.

The full-load conditions are next considered. A comparison is made between a two-motor unit, consisting of two similar polyphase motors coupled together and connected for opposite rotation, and the straight single-phase induction motor. Various discrepancies are pointed out between the resultant action of the two-motor unit and the single-phase. Modifying conditions are then taken into account which remove the discrepancies. This is followed by a considerable amount of test data which illustrate the principles and actions described in the paper.

THE underlying principles and the operating characteristics of the polyphase induction motor are so well understood that it is found desirable to consider the single-phase induction motor, simply as a special case of the polyphase. On this basis the single-phase motor must be considered primarily as a rotating-flux machine.

Starting with the old assumption that a single-phase alternating magnetic field may be considered as being made up of two

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*It should be understood distinctly that this paper is not to be considered as a presentation of new material, for practically all of the underlying principles are old and relatively well known. It is simply an attempt to describe the operation of the single-phase motor in a way which will be easily understood by those not versed in the mathematics of the subject.

constant fields, each of half the peak value of the single-phase field and rotating at uniform speed in opposite directions, then if the single-phase flux distribution is of sine shape and varies sinusoidally in value, it may be replaced, or represented, by two sine-shaped fields of constant value rotating in opposite directions. This is the simplest case and allows a relatively easy explanation of many single-phase problems. However, when the flux distribution, or field form, due to the single-phase winding, is other than of sine shape, then the oppositely rotating components cannot be considered as of sine shape, but will assume certain varying forms as they rotate, the resultant of each instantaneous pair always giving the single-phase field corresponding to that instant.

As other than sine-shape fields tend toward complications in the physical conception of the single-phase induction motor actions, and lead more or less into the mathematical conception, the following analysis will be limited essentially to sine-shape distributions.

As a starting point and to show reasons for certain later analysis, let us assume a single-phase induction motor operating at no-load, full speed, with its polyphase secondary winding short-circuited. The single-phase primary field, of assumed sine shape, is considered as made up of the two sine-shape equal components of constant value, and of half the peak value of the single-phase field, and rotating synchronously in opposite directions. One of these fields is traveling in the same direction as, and slightly faster than, the rotating secondary. The slip of the secondary with respect to this field is of the same nature as in the ordinary polyphase motor. As the machine is carrying no load the secondary current corresponding to this rotating field is very small, being just large enough to overcome the rotational losses in the motor itself, and its frequency is equal to the slip frequency due to the forward field component.

As there is an assumed backward flux or field component of equal value, the rotating secondary winding cuts this at almost double the frequency of the line. Stated exactly, the sum of the backward and the forward frequencies, in the secondary winding, is equal to exactly double the frequency of the primary supply system. The secondary winding cutting the backward field at this high frequency tends to generate a very considerable e. m. f. and, with the winding closed on itself, short-circuit currents will flow, which tend to damp out or suppress the flux which causes them. This secondary current will rise until its magnetizing effect is practically equal and opposite to the magnetomotive force which produces the backward field, which thus becomes almost zero in value. Consequently there are two distinct sets of secondary currents flowing, one of very small value and of a frequency corresponding to that of the forward rotation, and the other of very much larger value and of almost double the line frequency. Actual tests of the secondary circuit of a single-phase induction motor at small load, taken with an oscillograph, Fig. 1, show both of these currents as above described.

### MAGNETOMOTIVE FORCES AND MAGNETIC FLUXES

It is seen from the preceding that, right at the beginning of our analysis, a new condition is encountered, namely, the introduction of a secondary opposing magnetomotive force which reacts on one of the primary field components and practically neutralizes it. Also, there is a mixture of magnetomotive forces and magnetic fields, which is liable to lead to confusion. Obviously the introduction of the opposing secondary magnetomotive force rotating synchronously with the backward component of the primary introduces some entirely new features. Therefore, before going any further with the above method, it is desirable to set aside for awhile the viewpoint of two equal oppositely rotating fields and begin with a preliminary study of the magnetomotive forces and the magnetic fields resulting from them.

It may be mentioned that while the assumption of two oppositely rotating component fields, in place of a single-phase field, is well known and has been used quite frequently, the corresponding analysis, from the viewpoint of magnetomotive forces, apparently has been but little used. When magnetomotive forces, instead of magnetic fluxes, are considered, then the single-phase primary magnetomotive force, fixed in position, can be replaced by two equal components of constant value, such as would be developed by direct current, each of half the peak value of the single-phase, and rotating at synchronous speeds in opposite directions.

Returning again to our analysis, let us consider two fundamental magnetomotive forces, namely, a primary single-phase one, fixed in position and varying sinusoidally and a secondary one of constant value, of half the peak value of the primary which rotates synchronously in one direction and which is in opposition to the primary in the position where the two coincide.

Let us assume that the primary single-phase magnetomotive force is split into its two equal oppositely rotating components, then the results may be illustrated as in Figs. 2, 3, 4, and 5. In Fig. 2, *C* and *D* represent the two components forming the single-phase magnetomotive force *A*. At the position chosen, *C* and *D* are of equal value and coincide in position and polarity, *B*, which represents the secondary magnetomotive force, is also of half the peak value of *A*, but is of opposite polarity. It, therefore, neutralizes one of the components *C* or *D*, thus leaving a resultant of half the peak value of *A*.

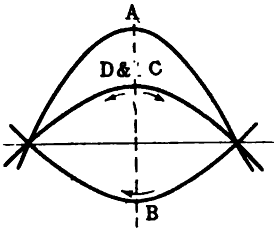

FIG. 2

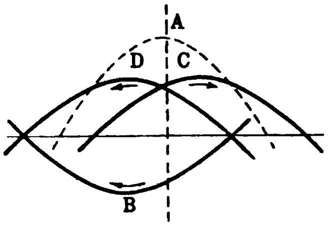

FIG. 3

In Fig. 3, the component *D* has shifted thirty degrees to the left, while *C* has shifted an equal distance to the right. The secondary magnetomotive force *B* is shifted thirty degrees to the left, thus neutralizing *D* and leaving only the component *C*.

In Fig. 4, *D* and *B* are shifted sixty degrees to the left, while *C* is shifted sixty degrees to the right. In the same way, in Fig. 5, *B* and *D* have shifted ninety degrees to the left and *C* has shifted a corresponding amount to the right.

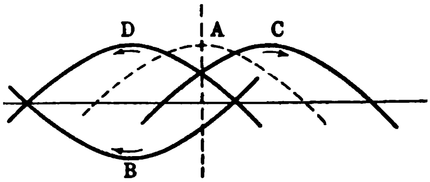

FIG. 4

Thus from the above it is seen that a single-phase magnetomotive force, fixed in position and varying sinusoidally, and a constant magnetomotive force of half the peak value of the single-phase, which is in opposition at the point of coincidence of position, and which rotates synchronously in either direction, will give a resultant constant magnetomotive force, rotating in the opposite direction, but which is of the same polarity as the single-phase magnetomotive force at the position of coincidence.

PLATE XI.
A. I. E. E.
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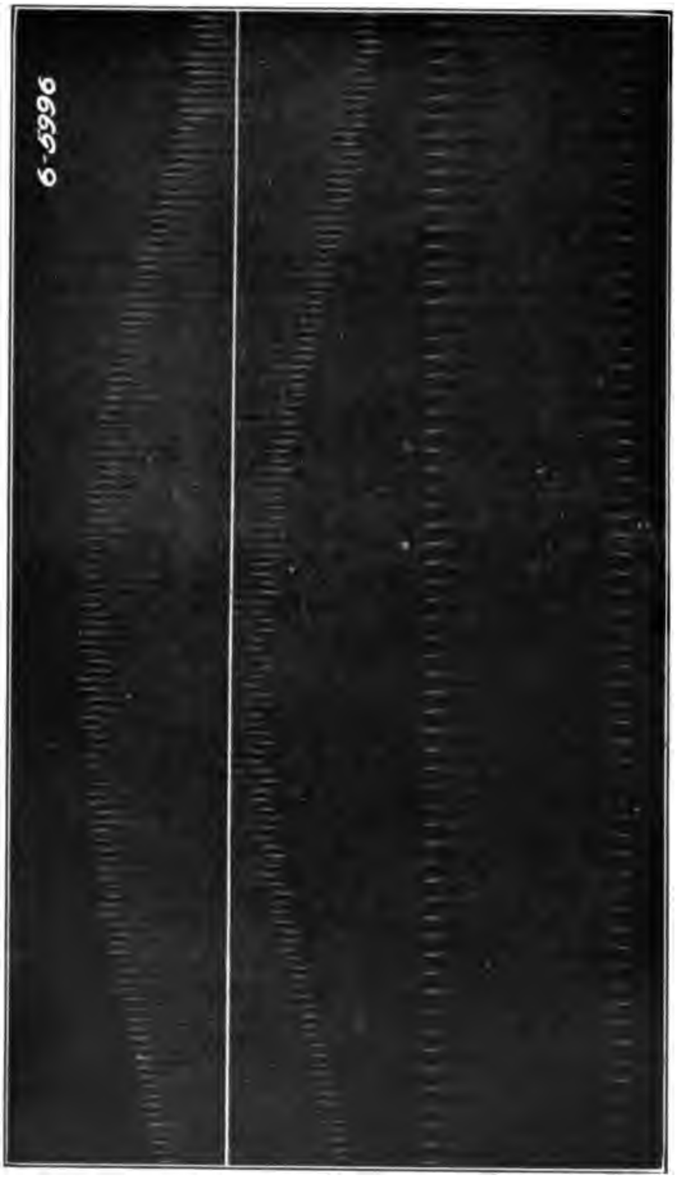


FIG. 1

[LAMME]

In other words, a single-phase magnetomotive force, fixed in position, and an opposing constant one of half the peak value rotating in either direction, will give a resultant rotating magnetomotive force equivalent to that of a polyphase induction motor.

As a continuation of the above, the resultant magnetomotive force $C$ could be replaced by a magnetic field or flux, resulting from such magnetomotive force. If this magnetic field is plotted to the same scale as the magnetomotive force which produces it, then $C$, in Figs. 2 to 5, can represent a magnetic field. This field will be constant in value and of half the peak value of the field which the single-phase magnetomotive force alone would set up.

Thus according to Figs. 2, 3, 4 and 5, by the introduction of an "opposing" magnetomotive force, equal in value to one of the component magnetomotive forces of the single-phase and rotating synchronously with it, one of the two components of the magnetic field can be suppressed and only the other component left, the resultant is thus a rotating magnetic field, just as in the polyphase induction motor.

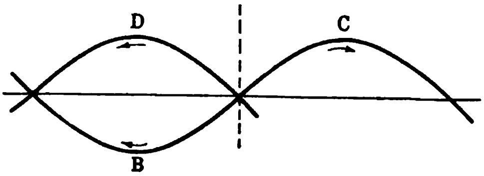


FIG. 5

However, a further modification of this should be considered. Assuming again, that the single-phase primary magnetomotive force is replaced by its two equal rotating components, as in Figs. 2 to 5, then by the addition of an opposing magnetomotive force, similar to $B$ in the same figures, but of *less value than the component $D$*, then the resultant of this opposing magnetomotive force and the component $D$ is a reduced magnetomotive force of the same polarity as $D$. There will then remain two magnetomotive forces, each of constant value, one of half the peak value of $A$ and the other of some smaller value, depending upon the opposing force $B$. These two rotating magnetomotive forces can, therefore, set up two oppositely rotating fields of *unequal* value. These are illustrated in Figs. 6, 7 and 8.

In Fig. 6, $B$ is assumed at some less value than the component $D$. The resultant of $D$ and $B$ is shown as $E$. Therefore, at this position *C* and *E* represent the two resultant magnetomotive forces and the two component fields. In Fig. 7, the conditions are shown for thirty degrees shift and here again *E* and *C* represent the two fields. In Fig. 8 the shift is for sixty degrees.

Thus by the introduction of a constant "opposing" magnetomotive force of less than either of the components of the single phase, two oppositely rotating fields of unequal value may be set up. As extreme cases of this, if the constant opposing magnetomotive force is made zero in value, the magnetic field corresponding to its position and rotation will rise to the full value of the oppositely rotating field; and, on the other hand, if the constant opposing magnetomotive force is made half the peak value of the single phase, the correspondingly rotating field becomes zero. Both of these cases are in accordance with the earlier assumptions.

The above conditions of the single-phase primary magnetomotive force and a constant secondary one, in opposition, which may be of half the peak value, or some less value down to zero, and which rotates synchronously in one direction, resulting in two magnetic fields which may be of equal or unequal value, and which rotate synchronously in opposite directions, all form essential parts in the physical conception or visualization of the actions of the single-phase motor which will be given below.

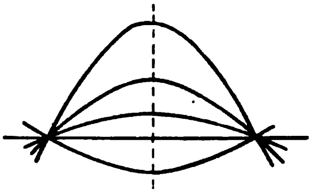


FIG. 6

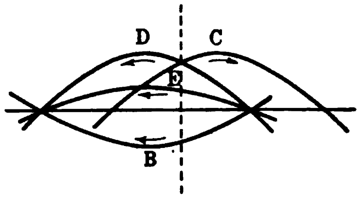


FIG. 7

It should be observed that in the above method of considering the production of a rotating field in the single-phase induction motor, the two primary components of the single-phase magnetomotive force and the secondary damping magnetomotive force *all rotate synchronously*, and such rotation is independent of the speed of the secondary core. In some methods of considering the single-phase induction motor problem, the single-phase primary winding is assumed to generate a magnetomotive force in the secondary which, by rotation of the core, is carried around until it generates a second magnetic field or flux at right angles to the original primary flux, thus giving the equivalent of a polyphase magnetic field. However, the above method does not involve such method of treatment.

It should also be recognized that the foregoing analysis only covers no-load conditions and that with the addition of load new conditions are brought into the problem. These, however, will be brought out later, for the no-load conditions require further consideration, especially as regards the generation of the primary counter e. m. f. by the above described rotating fields. As already shown, there may be a single magnetic field rotating synchronously, or there may be two component fields of equal value rotating in opposite directions, or there may be intermediate conditions of oppositely rotating fields of unequal value, depending upon the value of the damping or opposing secondary magnetomotive force.

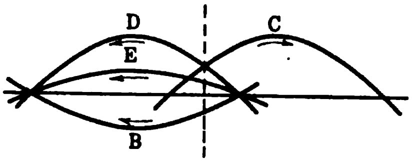


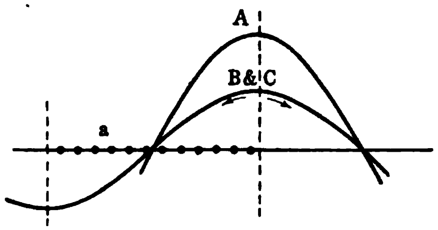
Fig. 8

## Counter E. M. F. Generation and Excitation

Considering next the counter e. m. f. generated in the primary, we should first look into the e. m. f. conditions produced by two oppositely rotating fields of equal values. If the secondary circuits are open, the two component fields are both present and are concerned in the generation of the counter e. m. f. This is true whether the secondary is stationary or is rotated at full speed. If, however, the secondary is closed upon itself, then when running at speed, one of the component fields is practically damped out and the other must generate the entire primary counter e. m. f. Thus, two entirely different conditions are encountered, depending upon whether the secondary is open or closed. To explain this properly, some further analysis is required, as follows:

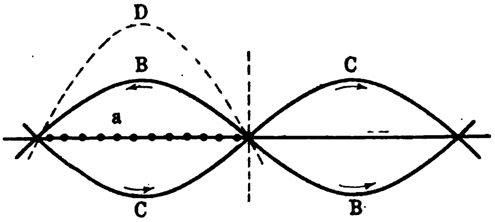
In the first place, it may be stated that the e. m. f., produced in the primary winding by *cutting* its two component fields, is the same as that generated by the single-phase sine-shape field, varying sinusoidally and acting on the primary winding as in a transformer. Herein lies a simple illustration of the equivalence of the transformer and the flux cutting methods for calculating e. m. fs. In Figs. 9, 10 and 11, are shown several positions of the two oppositely rotating fields and their relation to the primary winding.

In Fig. 9 is shown the magnetic flux, or field, *A*, which is set up by a primary winding *a*. This winding, of course, would require a tapered distribution to give such a field. This is mentioned incidentally as it has no direct bearing upon the explanation, except from the mathematical standpoint.



Fɪғ. 9

Assuming the single-phase field at its maximum or peak value, then, at this instant, the two component fields, *B* and *C*, each of half the peak value, will coincide both in position and polarity. From the transformer method of calculation, the e. m. f. generated at this instant, in the winding, will be zero, as the rate of change of the flux is zero. Also from the flux cutting method, the e. m. f. in the primary winding will be zero, for, as is evident from the figure, each belt or group of the primary winding is cutting fields which have equal positive and negative areas or values.



Fɪɢ. 10

Considering next the conditions in Fig. 10, in which the two rotating components have traveled ninety degrees. The fields are shown as *B* and *C*. It is evident that the resultant of these two fields is zero in value, that is, the single-phase field is passing through its zero value, and, accordingly, is generating the maximum e. m. f. by the transformer method. Also, considering component *B* of the rotating fields, obviously, by the cutting method it is generating maximum e. m. f. in the winding *a*. Also, component *C* is generating maximum e. m. f. in winding *a*. However, as one of these fields is positive in this position and is traveling in one direction, while the other field is negative and is rotating in the opposite direction, the two e. m. fs. will be in the same direction, and thus will be added. Thus, from the figure, this position will give the maximum e. m. f. in the winding by the cutting method. It can be shown by calculation that this maximum value is the same with either the cutting or the transformer methods of considering e. m. f. generation.

This shows that both of the component fluxes must be taken into account in generating the total primary e. m. f., and if either component is decreased in value or suppressed, the total e. m. f. generated in the winding will be decreased correspondingly, unless the other component is increased a corresponding amount.

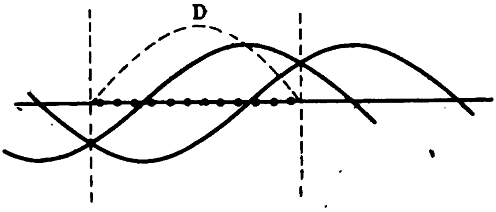


Fig. 11

Fig. 11 is simply a continuation of the conditions of Figs. 9 and 10, in showing an intermediate position of the component field. The result is the same as if the two fields were momentarily replaced by the field *D*.

According to the above analysis, to produce a given counter e. m. f. in the primary, with one of the component fields damped out, the other component must be doubled in value. It was shown before that in the single-phase induction motor, running at full speed with no load, the backward field is practically damped out by the secondary current. Thus with only the forward component field remaining, either the counter e. m. f. will be halved or the forward flux component must be doubled, the latter being the case. This means, in turn, that the primary magnetomotive force must be doubled in value. In other words, suppressing one of the two rotating field components results in doubling the no-load excitation of the motor. Furthermore, doubling the magnetomotive force of the primary and thus doubling the forward component of the field also doubles the backward component, which, in turn, is suppressed by doubled secondary current. The above conditions of doubled excitation is on the basis of sine flux distribution. With other distributions the same result holds approximately, but not exactly, due to conditions involving the shape of the field.

It is evident from the above that, with the secondary circuits open, the excitation required is of constant value regardless of the speed of the rotor core and windings; also when running at speed, the primary excitation is doubled as soon as the secondary circuit is closed. However, it is not obvious, on first consideration, that even with the secondary circuits closed the primary excitation falls to half the full speed value, when the motor is brought to standstill. This involves load conditions which will be treated later, but nevertheless this feature may be brought out at this time. The explanation lies in the fact that at rotor standstill the damping action of the secondary current will be exerted equally on both the forward and backward components of the primary field, so that necessarily these must be maintained at equal value, and, by the above analysis, this requires but half the excitation, compared with the no-load full-speed condition where the backward field is practically completely suppressed.

## LOAD CONDITIONS

When the single-phase induction motor is loaded, the total input current can be considered as made up of two components, namely, the no-load (practically all magnetizing) and the load current. This latter is simply the increased current in the primary due to the load and does not entirely represent energy. This load current, being single-phase, may be represented by two equal oppositely rotating magnetomotive forces in the primary of the motor, just as in the case of the no-load current. The fields which these two magnetomotive forces tend to set up are both practically suppressed by two equivalent *secondary* magnetomotive forces rotating in opposite directions. The forward secondary component corresponds to the secondary load magnetomotive force in the polyphase motor and the interaction between this magnetomotive force and the forward primary field develops torque just as in the polyphase motor. The backward component, at first thought, would appear to develop an opposing torque, corresponding in value to that of the polyphase motor at approximately 200 per cent slip. This, however, is not the case, for at this slip the ordinary polyphase motor takes an excessive primary current tending to develop a large magnetic field, which is suppressed by a correspondingly large secondary magnetomotive force. In the single-phase induction motor, however, the primary backward rotating magnetomotive force component, due to the load current, can be only of the same value as the forward. This fact must be borne in mind as it is a very important factor in the later analysis.

To illustrate the characteristics of the single-phase induction motor, it may be compared with the action of two polyphase induction motors rigidly coupled together, and connected to the line to give opposite rotations. Such a set or unit has certain characteristics which are so similar to those of the usual single-phase induction motor that on first consideration one would assume them to be identical. However, a more careful study of the individual operating conditions shows that the similarity is only a general one, and a number of decided discrepancies are found.

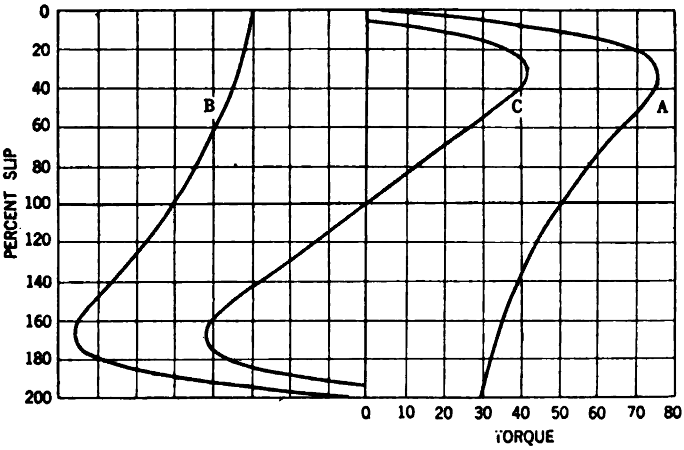


Fig. 12

The characteristics of the above two-motor unit and the single-phase motor may be compared as follows:

(1) The speed torque characteristics of the two motors of the polyphase unit may be represented by *A* and *B* in Fig. 12 and

their resultant by curve *C*. According to this latter curve, the resultant torque is zero at standstill, and a slight change in speed in either direction will give an effective torque tending to speed up the unit in whichever way it is started. This, therefore, corresponds to the well known starting characteristics of the single-phase motor.

(2) It may also be seen that the maximum torque the unit develops is materially less than that of either of the two component motors. This fact is also consistent with single-phase motor operation compared with the same machine on polyphase.

(3) At full speed, according to this resultant curve, the slip for a given torque is very much larger than that of the corresponding polyphase motor. This is not true of the single-phase motor and herein lies one of the discrepancies in this method of illustrating the operation.

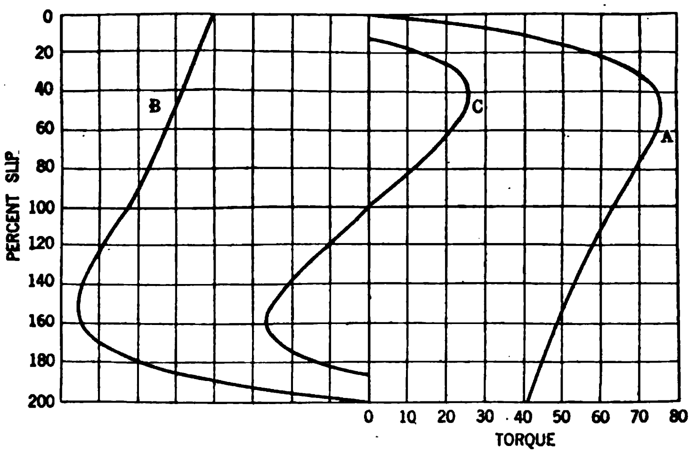


FIG. 13

(4) It is well known that in the polyphase motor the maximum torque it can develop, with constant voltage applied, is independent of the secondary resistance; while, in the single-phase motor, in general, an increase in the secondary resistance will decrease the maximum torque and a decrease will have the opposite effect. This may be illustrated by repeating the curves of Fig. 12 with modified secondary resistance in the two component motors. In Fig. 13 the secondary resistance is increased and in Fig. 14 is decreased relatively to that of Fig. 12. The

resultant speed-torque curves for the three figures show that the maximum torques are materially affected by the secondary resistance. The same holds true for the single-phase induction motor.

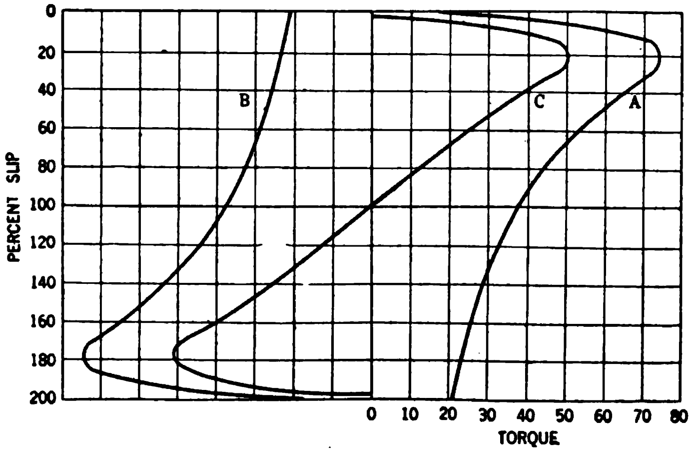


FIG. 14

(5) However, this method of illustrating the characteristics of the single-phase motor torque fails when the condition of secondary resistance is such that the maximum polyphase torque

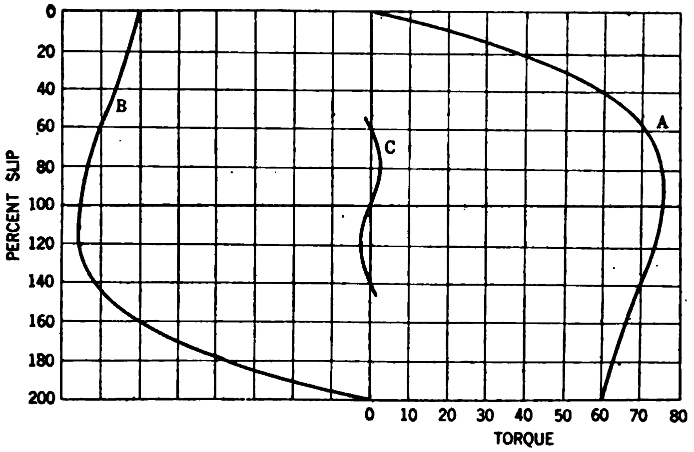


FIG. 15

is developed at about 100 per cent slip. Fig. 15 illustrates this. From this speed-torque curve it appears that the unit has a very low resultant torque, but this is not the case in the single-phase induction motor, for with a polyphase motor developing its maximum torque at 100 per cent slip, the same machine on single-phase will give a very considerable maximum torque. Here again is a discrepancy which the assumed equivalent arrangement does not cover properly.

(6) In Fig. 16, the current-torque curve *D*, for the component motors in the above figures, is shown. This indicates plainly what a wide discrepancy there is between the currents taken by the primaries of the two motors when running at speed. For example, at a given speed *a*, the current taken by the forward rotating motor is *b*, while *c* represents the current taken by the backward motor. Obviously, the current taken from the line, which is the resultant of *b* and *c*, is much greater than that required to produce the resultant torque and the power factor of such a unit must necessarily be very poor. However, such is not the case with the single-phase motor, for the inputs and the power factors are not greatly different from those of polyphase motors of the same capacity. Herein lies a radical difference between the single-phase motor and the above assumed unit.

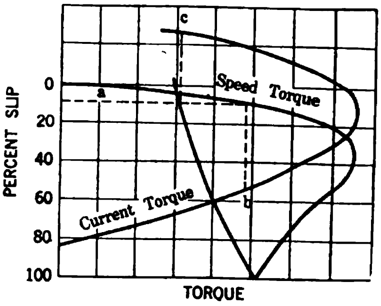


Fig. 16

(7) Another difference between such a unit and the true single-phase motor lies in the no-load or magnetizing input. Obviously, the combined magnetizing components for the two motors will be twice as great as for a single machine, whereas, in the single-phase motor the magnetizing input is practically the same as in the corresponding polyphase machine. Here is another pronounced discrepancy.

It is evident from the above that while this method of illustrating the action of the single-phase motor by means of two polyphase motors, coupled for opposite rotation, is in the right direction, some special modifying conditions must be introduced to account for the discrepancies. The action of this two-motor unit, therefore, will be followed up further, with the introduction of certain modifications derived primarily from consideration of certain characteristics of the single-phase induction motor itself.

In the first place, curves *A*, *B* and *C* of Fig. 12 were based upon equal and constant e. m. fs. applied to the terminals of both motors. That this is not a correct assumption can be determined from the operating conditions in the single-phase motor. From the analysis of the component rotating fields it was shown that at full speed the backward component was practically damped out by a secondary magnetomotive force, thus leaving only the forward component, which then rose to practically double value in order to generate the required e. m. f. However, at standstill, the secondary winding holds the same rotational relation with respect to both component fields and, therefore, neither field can be damped out more than the other. Consequently, at standstill, both component fields are equal in value and the counter e. m. f. of the primary is generated by the two oppositely rotating fields, instead of a single one of double value as is the case at full speed. Therefore, at standstill, the forward field is of only half the value of the full speed field. This corresponds to the operation of the polyphase motor at half field strength, that is, *with half the primary voltage applied*, thus requiring one-quarter the magnetizing input. The same voltage condition applies also for the backward component at zero speed.

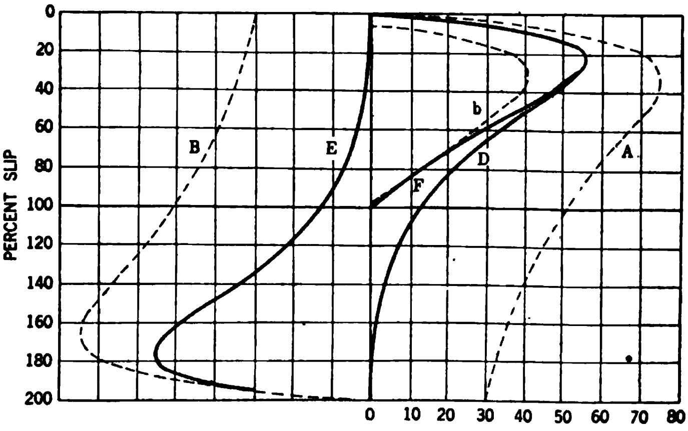


FIG. 17

It would appear, therefore, that in the unit composed of two polyphase motors coupled together, the voltage applied to the terminals of the forward motor should be at practically full value at synchronous speed and should fall to half value at standstill or 100 per cent slip, and should have practically zero value at 200 per cent slip. Then assuming, as a first approximation, that the decrease in voltage from full speed to 200 per cent slip is a straight line law, new speed-torque curves, corresponding to Fig. 13, but with the torques decreasing as the square of the voltage, can be illustrated as in Fig. 17. Here curves *A* and *B* correspond to Fig. 12, while *D* and *E* correspond to the above proportionate reductions in voltage. The resultant *F* of these latter curves is also shown.

This new resultant *F* is similar in general shape to *C* of Fig. 12, but indicates some quite different characteristics. For instance, at the higher speed values it coincides quite closely with the polyphase speed-torque curve, which is actually the case in the single-phase motor. In the second place, with high secondary resistance, as shown in Fig. 15, the speed-torque curves are modified as in Fig. 18, which shows both the former characteristic

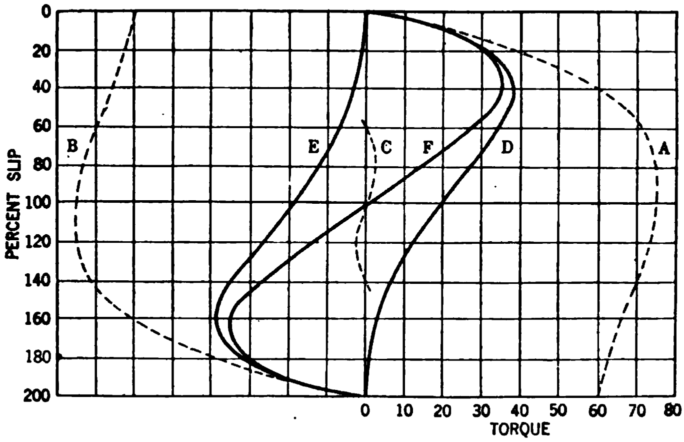


FIG. 18

and the new one. Here the resultant torque, under the new assumption is materially higher and more nearly conforms with the condition in the single-phase motor.

Under the earlier assumption of constant voltage on both motors, it was shown that the magnetizing current would be twice as great as in the single-phase motor. On this new assumption, however, at full speed, with practically full voltage on one motor and zero voltage on the other, the total magnetizing current will be only half as great, and will approximate that of one motor alone, and, therefore, that of the single-phase motor.

Furthermore, under the new assumption, the current taken by the primary of the backwardly rotating motor is quite small at high speed and, therefore, the resultant current taken from the line is not excessive and is more nearly consistent with actual single-phase motor conditions.

Thus, with this new condition of reduced terminal voltage with reduction in speed, practically all the conditions of the single-phase motor are met, except possibly from the quantitative viewpoint. The two-motor combination thus serves as a very good illustration. There is, however, one further condition ***which must be rigidly met*** if the new curves are to be reasonably exact, namely, ***the primary currents taken by the two motors must be equal,*** for, as shown in the early part of this analysis, the forward and backward rotating components of the primary current in the single-phase induction motor are equal at all times. Consequently to duplicate this condition, the primary e. m. fs. impressed upon the terminals of the two polyphase motors should be varied in such a way that the primary currents will always be equal. In addition, it is assumed that the sum of the two impressed voltages is constant. This, however, is only an approximation.

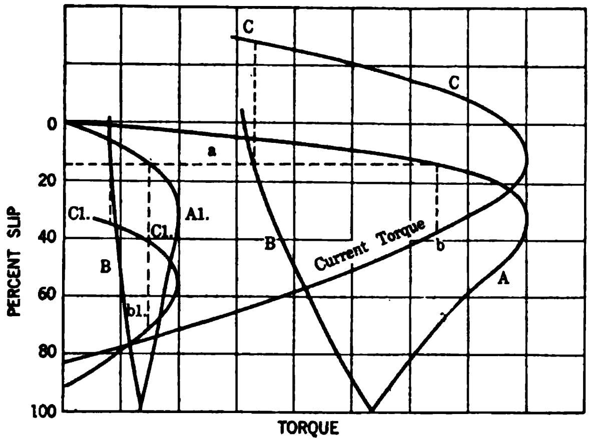


Fig. 19

The next step is to determine what is the actual law of voltage variation which will satisfy the above conditions of current and voltage. A ready means for obtaining this lies in the speed-torque and current-torque curves of the polyphase motor at constant voltage. From the current-torque curve at constant voltage corresponding curves for any other voltage can readily be plotted by varying the abscissas as the square of the voltage and the ordinates directly as the voltage. This is illustrated in Fig. 19. Here *A* is the polyphase motor speed-torque curve at constant voltage. $B$ represents the part below the 100 per cent slip line, but turned above the zero speed line for convenience. $B$ can also be considered as the back torque at full voltage, but thrown to the right of the zero torque line for convenience. Curve $C$ represents the primary current for full voltage conditions. Then at a speed $a$, for example, the primary currents corresponding to the forward and back torque will be $b$ and $c$ respectively.

Assume next that the voltage is halved for both rotations, then the new speed-torque curves will be $A_1$ and $B_1$ in which the torques are reduced as the square of the voltage. The new current curve will be $C_1$. The currents for speed $a$ will now be $b_1$ and $c_1$, or half of $b$ and $c$, as they are varied as the voltage.

The above figure is simply to illustrate the rule for variation of the primary current with the voltage, in the polyphase motor, and does not represent the actual conditions which we are after; for in the above the voltage reductions are the same for both the forward and the back torques. But, according to our former analysis, this condition of equal voltages, for the two rotations, holds only for the 100 per cent slip point. For other speeds the two voltages are reduced unequally, but with the sum of the two approximately constant according to the assumptions.

If, for any speed $a$, we let $x$ represent the percentage of voltage reduction for the forward torque, then $1 - x$ will represent the corresponding reduction for the back torque. Let $I_f$ be the primary current, corresponding to the forward torque for this speed at full voltage, and $I_b$ the current for the back torque at the same speed and also for full voltage. Then $I_f x$ will represent the primary current at the reduced voltage for the forward rotation and $I_b\,(1 - x)$ will be the primary current for the back rotation. One of the conditions of our two-motor unit, to make it correspond with the single-phase motor, is that these two primary currents must be equal. Therefore, $I_f x = I_b\,(1 - x)$, and

$$x = \frac{I_b}{I_f + I_b} \text{ and } (1 - x) = \frac{I_f}{I_f + I_b}.$$

The above allows the determination of the percentage $x$ of full voltage which must apply for each speed between zero and synchronism, when the values of the current $I_f$ and $I_b$ for full voltage are known.

A second method of determining the percentages of voltage for the two rotations is available when the speed-torque curve of the motor on single-phase has been determined, by test or otherwise. By our former assumption this single-phase torque is the difference between the speed-torque curves for the forward and backward rotations with the respective voltages reduced the proper percentages. These torques for any given speed vary as the square of the terminal voltage. For example, calling $T_f$ the forward torque, at full voltage and speed $a$, and $T_b$ the back torque, and $T_1$ the single-phase torque for the same voltage and speed, then $T_f x^2 - T_b (1 - x)^2 = T_1$, from which $x$ may be determined, with $T_f$, $T_b$ and $T_1$ known.

It would appear from the preceding that, if the assumptions made are anyways close to the actual conditions, this method of analysis shows an approximate means for deriving the single-phase speed-torque curve from the polyphase curves of the same machine. Methods of calculating the primary current and speed-torque characteristics of the polyphase motor have been developed quite completely, so that it is not necessary at this place to give any details of such methods. The accuracy of the methods for calculating the polyphase curves depends almost entirely upon the correct determination of the reactance and saturation constants. All methods for the direct determination of the single-phase speed-torque characteristics also involve the use of corresponding reactance and saturation constants. Therefore, the above method brings in no new and more difficult conditions. The primary object of this paper, however, is not to develop a new method of calculation, but simply to give a better conception of the close relation of the single-phase and polyphase characteristics.

After development of the above method, an attempt was made to check it by applying certain existing test data, but without positive results, although the indications were quite satisfactory. It was discovered that in all the existing test data at the writer's command, where the polyphase speed-torque and current-torque curves have been obtained by actual test, constancy of temperature had been more or less disregarded. The effect of change in the secondary resistance on the polyphase speed-torque curve is to change the slips but not the maximum torque. The difficulty, however, in the polyphase tests available was that apparently the resistance had varied very considerably during the tests, especially at the points of high slip, where the secondary losses were very large. As a result the speed-torque curves corresponded to those of motors in which the resistance increased as the load and slip increased. As a consequence, the torques below the zero speed line were considerably too large, which meant that in applying these curves to the above method, the back torques were presumably entirely too great, thus apparently introducing errors in the derivation of the resultant single-phase curve.

The effect of these discrepancies are shown in Fig. 20. Here, *A* shows the speed-torque curve as it should be at constant temperature, whereas, *B* shows the curve with the resistance of the secondary increasing with increased slip. The corresponding current-torque curves are also shown. A consideration of these curves would seem to indicate that the resultant single-phase curves derived from *A* and *B* should differ somewhat.

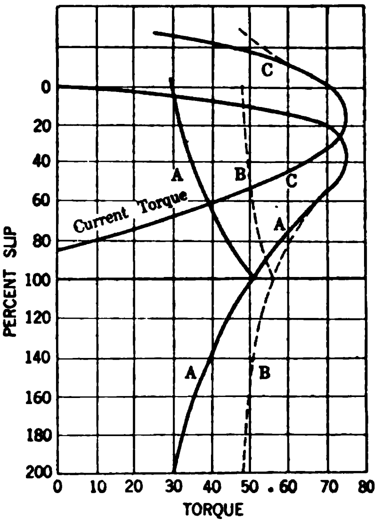


FIG. 20

It was then decided to make a more accurate set of tests on a 10-h.p., 60-cycle four-pole, three-phase motor of the wound-secondary type, so that the secondary resistance could be varied if so desired. It was also decided to obtain a test with two similar motors rigidly coupled together, with their individual primary windings in series, but with their secondaries independent. As already explained, the theory of the foregoing method calls for *equal currents* in the two oppositely rotating fields. This condition is automatically obtained by coupling two primaries in series with each other.* With this arrangement, if the power factors of the two motors were always equal, then it should be equivalent to the method already described. However, these are practically never equal except at the standstill position, although an analysis of the problem shows that the two primary voltages, with this series arrangement, are not greatly out of phase with each other over a very large part of the working range. The

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*In reviewing an early draft of this paper, this suggestion, with a number of other most excellent ones, was made by Mr. R. E. Hellmund. However, it developed later that this same suggestion appeared about twenty years ago in Mr. B. A' Behrend's book, "The Induction Motor."

writer has not yet sufficiently analyzed the series arrangement to be sure that it exactly represents all the conditions of the two rotating fields in the single-phase motor, but is inclined to think that such is the case. However, the approximate method developed in this paper lends itself so readily to calculation, that it was considered worth while to check it up carefully by test to see what degree of accuracy could be obtained.

The following series of tests was planned:

(1) Three-phase speed-torque and primary current curves at 220 volts with one motor alone, with its secondary short-circuited on itself.

(2) Single-phase speed-torque and primary current curves on the same motor as (1) at 220 volts and with the secondary short-circuited on itself.

(3) Three-phase speed-torque and primary current curves on the same motor and at same voltage, but with external resistance in the secondary circuits.

(4) Single-phase speed-torque curves under same conditions as (3).

(5) Speed-torque and primary current curves with two similar motors with their primary windings coupled in series, and with the secondaries independently short-circuited on themselves, one of these motors to be that used in tests (1) and (4).

(6) Similar tests to (5), but with resistance in the secondaries as in (3).

In carrying out these tests, the torque was measured by a special dynamometer brake, the power absorbing element of which consists of a special separately-excited direct-current machine. Below zero speed, power was supplied to the direct-current machine in order to obtain negative rotation.

Difficulties in obtaining consistent tests, especially at negative speeds, soon developed, due to variations in temperature. With the very heavy currents at low and at negative speeds, the motor would heat so rapidly that all kinds of speed-torque readings could be obtained. Test after test was made and while these would agree very well for the higher speed points where the heating was small, they showed all kinds of inconsistencies for the negative speeds, in particular. The currents for these speeds also showed very wide discrepancies. Eventually it was found that those tests taken with extreme rapidity, and which covered only a comparatively small number of points, would plot in quite reasonable curves above zero speed, so that the writer was enabled thus to obtain quite consistent curves for both torque and current between 1800 rev. per min. and standstill. Not only were the curves, consistent in themselves but those taken with different secondary resistances were fairly consistent with each other. It then remained to obtain reasonable readings for the negative speeds. Obviously it was wrong to take a large number of test points and then draw an average curve through them, for it is evident that the errors, due to heating, tend to throw the torques and currents to one side of the proper curves. Consequently the correct curves should really be boundary lines rather than averages. It was noted, in particular, that heating did not appear to affect the speed to the same extent as the torque at very large slips, and, consequently, by plotting the current in terms of speed rather than torque, less erratic curves were obtainable, and it was possible to plot speed-current curves which were quite consistent for the different conditions of secondary resistance. Furthermore, from the speed-torque and speed-current curves above the zero line, which appeared to be reasonably correct, as they were consistent with each other, it was possible to derive the constants for the general equations for speed-torque. It was found that such derived equations fitted these curves quite accurately and, moreover, they held the proper relation of constants for both high- and low-resistance secondaries. The various agreements between the calculations and the tests for the higher speeds were such that one could assume that the derived equations were practically correct and that from them the curves for the negative speeds could be plotted with fair accuracy. In this way the curves for the negative speeds were first obtained and it then remained to check them by actual test. Finally a method of testing was tried which appeared to give quite good results. This consisted in setting the apparatus at about the desired speed and torque conditions; then cooling the motor down to the required temperature preparatory to obtaining the desired test, the power was then thrown on and readings obtained in the shortest possible time, five seconds, for instance. Allowing the motor to run, additional readings were obtained at five-second intervals. A series of consecutive readings, at definite intervals apart, was thus obtained and plotted in a curve. By extending this curve back to the instant of starting, results were obtained which were undoubtedly quite close to those corresponding to the starting temperatures, and were not only quite consistent with each other, but also plotted very close to the negative extensions of the calculated curves. As a result of a series of tests extending over several weeks, data was obtained which plotted in curves which agreed fairly well with each other throughout.

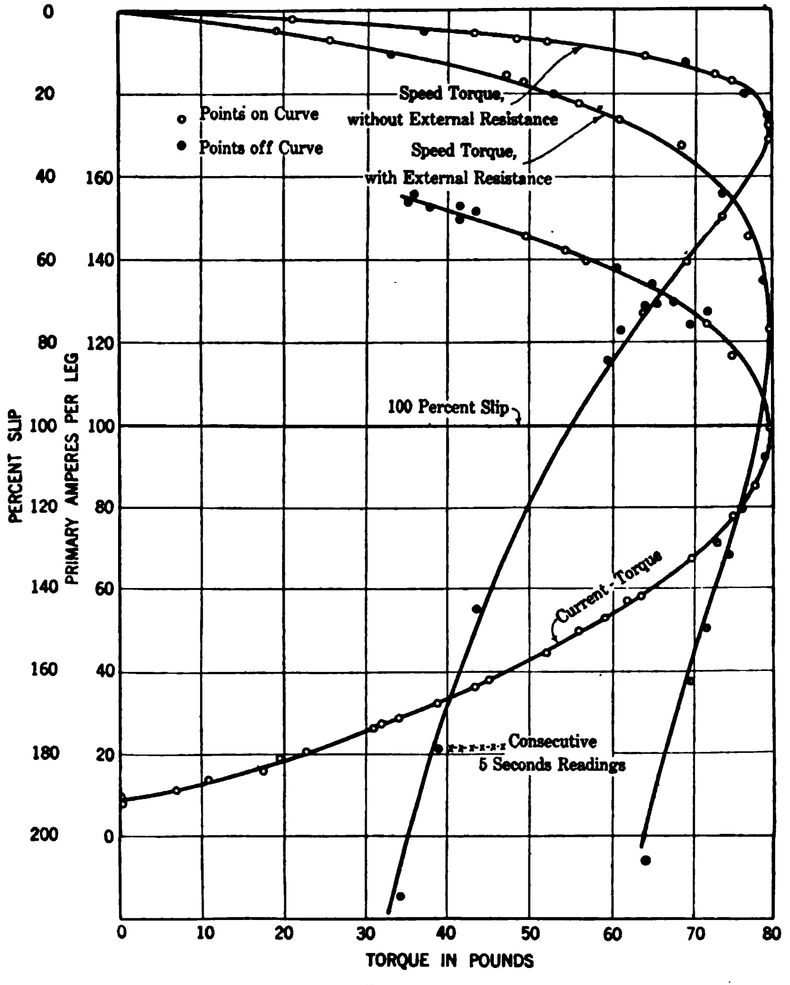

FIG. 21

## RESULTS OF TESTS

### *Polyphase Speed-Torque, Speed-Current and Current-Torque Curves*

In Fig. 21 are shown the polyphase speed-torque and primary current both with the secondary short-circuited, and with resistance added. In the speed-torque curves the circled points represent actual test readings, while the solid line covers the points calculated from the derived equations.

In Table I, covering data on the short-circuited rotor tests, are shown the forward and back torques and the corresponding forward and back currents for the various speeds between zero and 200 per cent slip, as derived from Fig. 21; also the calculated values of the ratio of voltages, $x$ and $(1 - x)$, by which the forward and back torques should be reduced in order to get the equivalent single-phase speed-torque curve. The corresponding reduced values for the forward and back torques are also given as calculated from the values $x$ and $(1 - x)$. The last column shows the difference between the reduced forward and back torques, which should represent the single-phase torque, according to the foregoing analysis.

TABLE I

| Slip | | Torque at full voltage | | Primary amperes per leg at full voltage | | $X =$ | | Reduced torque | | Resultant torque |
|---|---|---|---|---|---|---|---|---|---|---|
| For positive speeds | For negative speeds | $T_f$ | $T_b$ | $I_f$ | $I_b$ | $\frac{I_b}{I_f+I_b}$ | $1-x$ | Forward | Back | |
| 0.02 | 1.98 | 20. | 35.8 | 19.0 | 154.8 | 0.89 | 0.11 | 15.8 | 0.4 | 15.4 |
| 0.05 | 1.95 | 41. | 36.3 | 34.0 | 154.5 | 0.819 | 0.181 | 27.5 | 1.1 | 26.4 |
| 0.10 | 1.90 | 61.7 | 36.9 | 55.5 | 154.0 | 0.735 | 0.265 | 33.3 | 2.6 | 30.7 |
| 0.15 | 1.85 | 71.6 | 37.7 | 71.0 | 153.5 | 0.684 | 0.316 | 33.5 | 3.8 | 29.7 |
| 0.20 | 1.80 | 77.3 | 38.3 | 85.0 | 153.0 | 0.643 | 0.357 | 31.9 | 4.9 | 27.0 |
| 0.25 | 1.75 | 79.4 | 39.2 | 96.0 | 152.5 | 0.614 | 0.386 | 29.9 | 5.9 | 24.0 |
| 0.30 | 1.70 | 79.6 | 39.9 | 104.0 | 152.0 | 0.594 | 0.406 | 28.1 | 6.6 | 21.5 |
| 0.35 | 1.65 | 78.8 | 40.8 | 110.0 | 151.5 | 0.580 | 0.420 | 26.5 | 7.2 | 19.3 |
| 0.40 | 1.60 | 77.6 | 41.6 | 113.0 | 151.0 | 0.572 | 0.428 | 25.4 | 7.6 | 17.8 |
| 0.50 | 1.50 | 74.0 | 43.6 | 121.0 | 150.0 | 0.554 | 0.446 | 22.9 | 8.7 | 14.2 |
| 0.60 | 1.40 | 70.0 | 45.5 | 128.0 | 149.0 | 0.538 | 0.462 | 20.3 | 9.7 | 10.4 |
| 0.70 | 1.30 | 65.9 | 47.8 | 133.0 | 147.3 | 0.526 | 0.474 | 18.2 | 10.8 | 7.4 |
| 0.80 | 1.20 | 62.1 | 50.0 | 136.5 | 145.5 | 0.516 | 0.484 | 16.6 | 11.7 | 4.9 |
| 0.90 | 1.10 | 58.8 | 52.7 | 139.5 | 143.5 | 0.507 | 0.493 | 15.1 | 12.8 | 2.3 |
| 1.00 | 1.00 | 55.5 | 55.5 | 141.5 | 141.5 | 0.50 | 0.500 | 13.9 | 13.9 | 0 |

In Fig. 22 are shown the single-phase speed-torque and current-torque curves with short-circuited secondary, as plotted from Table I, and checked by actual test. The circled dots represent actual test points, while the crosses represent points plotted from the last column in Table I. The agreement of test and calculated values are as close as can really be expected considering the difficulties in obtaining the data, and the possible errors.

Unfortunately, due to the very short time available, it was not possible to make any extended tests on single phase with a view to correcting for temperature. In consequence, the calculated single-phase speed-torque curve, which is on the basis of constant temperature, is compared with tested curves in which no temperature correction has been made. It, therefore, is not known in this case how much of the discrepancy is due to temperature.

In Table II is shown data similar to that of Table I, but for the tests with resistance in the secondary. It will be noted that the resultant of the forward and back torques is considerably lower than in Table I, which is consistent with the fact that increased secondary resistance reduces the maximum torque of the single-phase motor.

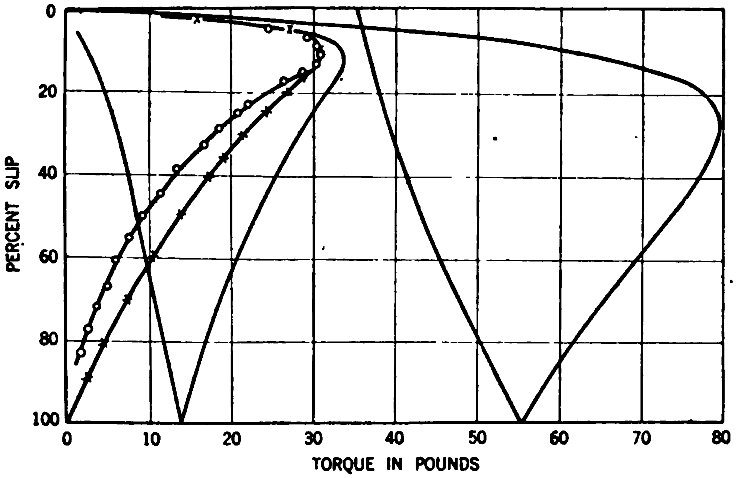

FIG. 22

In Fig. 23 is shown the calculated single-phase speed-torque and the tested torques of the motor with resistance in secondary. Here the circled dots represent the actual test readings and the crosses represent the points obtained from the last column of Table II. The discrepancies are somewhat smaller than in the motor with short-circuited secondary. This should be the case, if heating is responsible for any considerable part of the discrepancy, for the currents are relatively smaller.

In order to get a crude idea as to how much of the difference may be due to this feature of temperature, a polyphase speed-torque test was selected in which no correction had been made for temperature and where the conditions were quite closely comparable with those of the single-phase tests. From the speed-torque and current data of this polyphase test, the resultant single-phase speed-torque curve was calculated, making no attempt at corrections of any sort. This speed-torque curve is represented by the small squares in Fig. 23. This lies much closer to the tested single-phase curve, thus indicating that temperature is possibly an explanation of a considerable part of the discrepancy between the calculations and the tests. This would also indicate that heat effects as referred to in connection with Fig. 20 are not as objectionable as anticipated. However, the writer does not believe that all the discrepancy is due to heating, but considers that this approximate method of dealing with the problem makes the back torque too small. In the arrangement with two motors in series, as mentioned before, the voltages of the two motors will not usually add up directly to give the line voltage, and the motor which represents the back torque, will have a relatively larger percentage of the total voltage than is the case with the above method of considering the problem. This will be considered further under the two-motor tests.

TABLE II.

| Slip | | Torque at full voltage | | Primary amperes per leg at full voltage | | X = | | Reduced torque | | Resultant torque |
|---|---|---|---|---|---|---|---|---|---|---|
| For positive speeds | For negative speeds | $T_f$ | $T_b$ | $I_f$ | $I_b$ | $\frac{I_b}{I_f+I_b}$ | $1-X$ | Forward | Back | |
| 0.02 | 1.98 | 8.2 | 64.3 | 12.5 | 134.2 | 0.915 | 0.085 | 6.9 | 0.5 | 6.4 |
| 0.05 | 1.95 | 18.9 | 64.8 | 18.0 | 133.8 | 0.881 | 0.119 | 14.7 | 0.9 | 13.8 |
| 0.10 | 1.90 | 33.3 | 65.4 | 28.0 | 133.0 | 0.826 | 0.174 | 22.5 | 2.0 | 20.5 |
| 0.15 | 1.85 | 41.5 | 66.1 | 37.0 | 132.3 | 0.781 | 0.219 | 27.5 | 3.2 | 24.3 |
| 0.20 | 1.80 | 53.1 | 66.9 | 46.0 | 131.5 | 0.740 | 0.260 | 29.3 | 4.5 | 24.8 |
| 0.25 | 1.75 | 59.8 | 67.6 | 53.0 | 130.8 | 0.712 | 0.288 | 30.3 | 5.6 | 24.7 |
| 0.30 | 1.70 | 65.1 | 68.4 | 60.0 | 130.0 | 0.684 | 0.316 | 30.5 | 6.8 | 23.7 |
| 0.35 | 1.65 | 69.2 | 69.1 | 66.0 | 129.0 | 0.662 | 0.338 | 30.4 | 7.9 | 22.5 |
| 0.40 | 1.60 | 72.2 | 69.9 | 71.2 | 128.0 | 0.643 | 0.357 | 29.9 | 8.9 | 21.0 |
| 0.50 | 1.50 | 76.5 | 71.4 | 81.0 | 125.5 | 0.608 | 0.392 | 28.2 | 11.0 | 17.2 |
| 0.60 | 1.40 | 78.7 | 72.9 | 88.0 | 122.7 | 0.582 | 0.418 | 26.6 | 12.7 | 15.9 |
| 0.70 | 1.30 | 79.6 | 74.4 | 94.0 | 120.0 | 0.561 | 0.439 | 25.0 | 14.4 | 10.6 |
| 0.80 | 1.20 | 79.6 | 75.9 | 99.5 | 118.1 | 0.542 | 0.458 | 23.4 | 15.9 | 7.5 |
| 0.90 | 1.10 | 79.2 | 77.8 | 104.0 | 112.2 | 0.519 | 0.481 | 21.4 | 18.0 | 3.4 |
| 1.00 | 1.00 | 78.9 | 78.9 | 108.2 | 108.2 | 0.50 | 0 50 | 19.7 | 19.7 | 0 |

### *Two Motors in Series*

In Table III is shown the test data and the calculations derived therefrom, for two motors with their primaries in series and with their secondaries short-circuited independently. In this test no external resistance was used in the secondaries. Considerable difficulty was encountered in making this test, due partly to bad alignment of the machines, as they were rigidly coupled together. Furthermore, in several of the earlier tests, the effects of temperature were disregarded and all indications were that the secondaries were quite hot during the tests. There was so much discrepancy between the various results that the writer cannot feel sure of the data shown in this table, although it was obtained under quite careful conditions of test.

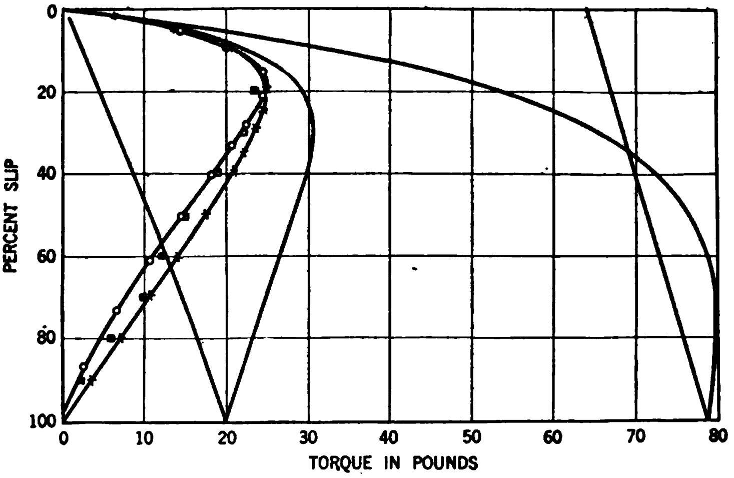

FIG. 23

In the above table the percentage of line voltage applied to each motor is shown. It is of interest to compare these percentages with those shown in Table I. This is illustrated in Fig. 24. This shows that the percentage of voltage on the forward rotating motor is higher at the higher speeds, than in Table I, but is lower at the low speeds. On the other hand, the voltage on the backward-rotating motor is higher at all speeds than in Table I. Thus, the back torque has always a higher value than in Table I. Consequently, with the reduced forward torque at the lower speeds and the higher back torque, the resultant torque

TABLE III.

| Speed | | Torque corrected to 220 volts | Volts | | | Ratio of motor to line volts | | Polyphase torque at 220 volts | | Polyphase torque at reduced voltage | | Resultant torque |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| r. p. m. | Slip | | Line | No. 1 motor | No. 2 motor | No. 1 motor | No. 2 motor | Forward | Back | Forward | Back | |
| 300 | 0.833 | 1.5 | 218.5 | 109.2 | 110.5 | 0.506 | 0.505 | 61.0 | 51.4 | 15.6 | 13.1 | 2.5 |
| 600 | 0.667 | 2.5 | 217.0 | 110.9 | 107.5 | 0.511 | 0.495 | 67.0 | 47.0 | 17.5 | 11.5 | 6.0 |
| 800 | 0.556 | 4.9 | 218.5 | 115.8 | 106.8 | 0.530 | 0.488 | 71.5 | 44.5 | 20.9 | 10.6 | 10.3 |
| 1000 | 0.444 | 8.3 | 219.0 | 118.4 | 105.0 | 0.541 | 0.48 | 76.0 | 42.4 | 22.2 | 9.8 | 12.4 |
| 1100 | 0.389 | 9.7 | 219.0 | 121.2 | 102.5 | 0.553 | 0.468 | 77.9 | 41.4 | 23.9 | 9.1 | 14.8 |
| 1200 | 0.333 | 12.3 | 219.0 | 124.5 | 100.0 | 0.569 | 0.457 | 79.2 | 40.3 | 25.6 | 8.4 | 17.2 |
| 1300 | 0.278 | 15.5 | 219.0 | 126.5 | 100.0 | 0.578 | 0.457 | 79.6 | 39.3 | 26.5 | 8.2 | 18.3 |
| 1400 | 0.222 | 19.6 | 220.0 | 133.4 | 94.0 | 0.605 | 0.427 | 78.8 | 38.5 | 28.9 | 7.1 | 21.8 |
| 1500 | 0.167 | 24.4 | 220.0 | 152.5 | 88.0 | 0.693 | 0.400 | 74.5 | 37.6 | 35.8 | 6.0 | 29.8 |
| 1580 | 0.122 | 27.6 | 220.0 | 157.7 | 78.0 | 0.717 | 0.355 | 67.0 | 37.0 | 34.4 | 4.7 | 29.7 |
| 1620 | 0.100 | 27.9 | 221.0 | 166.4 | 70.0 | 0.753 | 0.317 | 62.0 | 36.7 | 35.1 | 3.7 | 31.4 |
| 1660 | 0.078 | 29.2 | 221.0 | 173.2 | 62.5 | 0.784 | 0.283 | 54.5 | 36.3 | 33.5 | 2.9 | 30.6 |
| 1700 | 0.056 | 26.1 | 221.0 | 187.2 | 52.5 | 0.847 | 0.238 | 44.0 | 36.0 | 31.6 | 2.0 | 29.6 |
| 1740 | 0.033 | 19.9 | 222.5 | 195.5 | 36.0 | 0.879 | 0.162 | 31.0 | 35.7 | 23.9 | 0.9 | 23.0 |
| 1780 | 0.011 | 10. | 223.0 | 202.8 | 25.0 | 0.909 | 0.112 | 14.0 | 35.4 | 10.3 | 0.4 | 9.9 |
| 1800 | 0.0 | .... | 224.0 | 208.0 | 14.0 | 0.961 | 0.062 | .... | 35.3 | .... | .... | .... |

derived from the polyphase curve will naturally be lower than in Table I, which appears to be the case in all the tests made.

The data in Table III indicate that the two motors have their primary voltages very nearly in phase at all times. The sum of the two motor voltages is never much greater than that of the line.

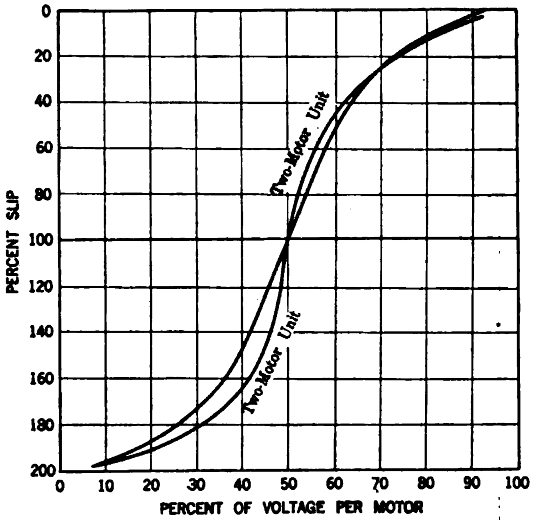

FIG. 24

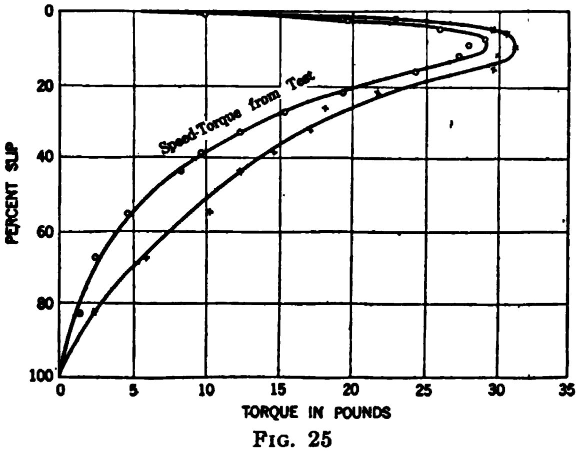

FIG. 25

In Fig. 25 is shown the calculated and test speed-torque results corresponding to Table III. The test result shows lower torques at the low speeds than can be derived from the voltage percent-

TABLE IV.

| Speed | | Torque corrected to 220 volts | Volts | | | Ratio of volts | | Polyphase torque at 220 volts | | Polyphase torque at reduced voltage | | Resultant torque |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| r. p. m. | Slip | | Line | No. 1 motor | No. 2 motor | No. 1 motor | No. 2 motor | Forward | Back | Forward | Back | |
| 340 | 0.811 | 1.71 | 219.0 | 114.3 | 108.0 | 0.521 | 0.493 | 79.6 | 76.1 | 21.6 | 18.5 | 3.1 |
| 480 | 0.734 | 4.93 | 219.0 | 117.8 | 105.0 | 0.538 | 0.480 | 79.7 | 75.0 | 23.0 | 17.3 | 5.7 |
| 640 | 0.645 | 7.85 | 219.0 | 122.1 | 102.0 | 0.552 | 0.470 | 79.1 | 73.5 | 24.1 | 16.2 | 7.9 |
| 810 | 0.555 | 11.2 | 219.5 | 126.4 | 98.0 | 0.576 | 0.446 | 77.8 | 72.0 | 25.8 | 14.3 | 11.5 |
| 1010 | 0.439 | 15.25 | 219.5 | 134.2 | 93.0 | 0.612 | 0.424 | 74.1 | 70.1 | 27.8 | 12.6 | 15.2 |
| 1220 | 0.322 | 20.5 | 220.0 | 146.3 | 83.0 | 0.664 | 0.377 | 67.5 | 68.5 | 29.8 | 9.7 | 20.1 |
| 1300 | 0.278 | 22.2 | 220.0 | 153.3 | 79.0 | 0.697 | 0.359 | 63.1 | 67.8 | 30.8 | 8.7 | 22.1 |
| 1410 | 0.217 | 23.8 | 220.0 | 164.5 | 70.0 | 0.748 | 0.318 | 55.5 | 66.9 | 31.1 | 6.8 | 24.3 |
| 1500 | 0.167 | 23.1 | 220.0 | 174.0 | 61.0 | 0.791 | 0.277 | 48.0 | 66.3 | 30.5 | 5.1 | 25.4 |
| 1600 | 0.111 | 20.1 | 220.5 | 187.0 | 47.0 | 0.848 | 0.213 | 36.5 | 65.5 | 26.3 | 3.0 | 23.3 |
| 1700 | 0.050 | 12.6 | 221.0 | 199.2 | 30.0 | 0.90 | 0.136 | 31.0 | 64.6 | 17.0 | 1.2 | 15.8 |
| 1750 | 0.028 | 6.0 | 221.0 | 204.4 | 24.0 | 0.925 | 0.109 | 12.0 | 64.3 | 10.3 | 0.8 | 9.5 |
| 1780 | 0.011 | 3.2 | 221.0 | 206.1 | 17.0 | 0.933 | 0.076 | 6.0 | 64.0 | 5.2 | 0.4 | 4.8 |

ages applied to the polyphase torques. Part of this difference may be due to temperature conditions.

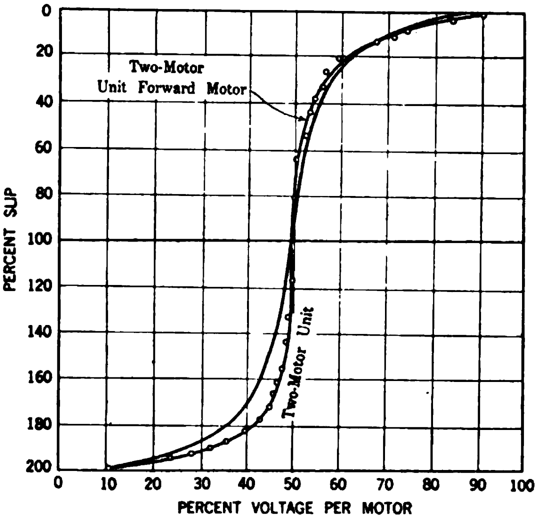

FIG. 26

In Table IV is shown the corresponding data for two motors with resistance in the secondary. Under this condition the various tests made were more consistent with each other and

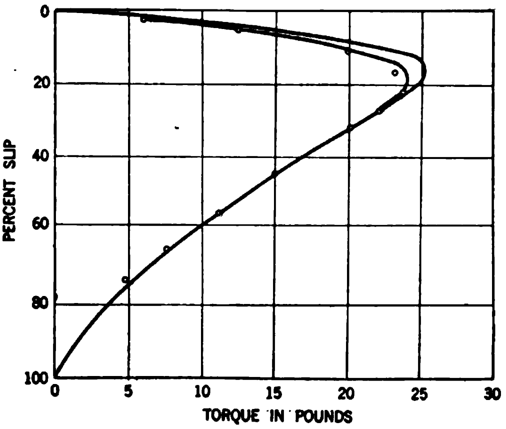

FIG. 27

the writer has more confidence in the data than in the case of Table III.

In Fig. 26 is shown the percentages of line voltage on each of the two motors, compared with those in Table II. These show the same differences as in Fig. 24, where there was no external resistance.

In Fig. 27 is shown the speed-torque curve for both calculation and test, as taken from Table IV. Here the discrepancies are much smaller than in Fig. 25.

## CONCLUSION

While the data are not as exact as the writer would desire, yet he feels that the general results obtained from the various tests have indicated that the method of analysis followed in this paper is along proper lines and that this conception of the action of the single-phase induction motor is of considerable assistance in obtaining a proper understanding of the machine. As stated before, the primary purpose of this paper is not to develop a method of calculation, but is simply to illustrate some of the characteristics of the single-phase motor. It is hoped that this will bring out more clearly the very intimate relation between the polyphase and single-phase induction motors in their operating characteristics.

## DISCUSSION ON "NO-LOAD CONDITIONS OF SINGLE-PHASE INDUCTION MOTORS AND PHASE CONVERTERS" (HELLMUND) AND "A PHYSICAL CONCEPTION OF THE OPERATION OF THE SINGLE-PHASE INDUCTION MOTOR" (LAMME), PITTSBURGH, PA., APRIL 10, 1918 AND NEW YORK, N. Y., APRIL 12, 1918.

### DISCUSSION AT PITTSBURGH

**J. Slepian:** These two papers illustrate what a great difference in the clearness of understanding of a complex system of circuits is made by different choices of the simpler systems out of which it is attempted to build up the more complex systems.

Both papers are concerned with symmetrical induction motors having coils missing. Mr. Lamme takes as his simple system the complete symmetrical machine operating under balanced conditions. He then determines two different balanced conditions such that when super-imposed in one machine their resultant will give zero current in the coils which must be removed to get the unbalanced machine. Mr. Hellmund, however, takes as his simple system the individual circuits of the unbalanced machine. Mr. Hellmund's choice of a simpler system appears perhaps as more fundamental than Mr. Lamme's, inasmuch as the basic laws of electromagnetism were first developed for simple closed circuits, but in the actual study of an unbalanced machine most will agree that Mr. Lamme's choice works better. The papers show quite forcibly that clear conceptions and correct results can be obtained much more easily by trying to resolve the problem of an unbalanced machine into the sum of different balanced conditions, than by considering the individual closed circuits of the machine.

Mr. Lamme's method has been long known, and I believe was used correctly in the treatment of the single-phase induction motor by S. P. Thompson in his "Polyphase Currents," and also by Mr. Behrend in his book on the induction motor. Nevertheless, the method is frequently used incorrectly to this day. I believe one reason for the prevalent error is the failure to distinguish between two different types of single-phase motor, the single-phase *current* motor, and the single-phase *voltage* motor. Take a two-phase squirrel-cage motor and opening one phase, throw the other phase across a single-phase line. The primary current then is certainly confined to the one phase but the voltage when the motor is running is not at all so confined as a voltmeter across the open phase would show. This is the single-phase *current* motor. Now take this same motor and short-circuit the open phase. The primary voltage is now certainly confined to the one phase but the currents are not at all so confined as an ammeter in the short-circuited phase would show. This is the single-phase *voltage* motor.

The error commonly made is to develop a theory of this last type, the single-phase *voltage* motor, and then to apply the results obtained to the generally used motor of the first type, the single-phase *current* motor. Of course, there is lack of agreement between this theory and the observed operation.

The working out of this theory is very simple. The condition of a given voltage on one phase and zero voltage on the other is resolved into the sum of two balanced two-phase voltages, of one-half the magnitude of the given voltage, agreeing in sign in one phase and opposite in sign in the other phase. The resultant currents, torques, etc., of the single-phase *voltage* motor are obtained by adding together the currents, torques, etc. produced by the two two-phase voltages independently. Since the speed-torque curves of an induction motor operated on balanced polyphase constant voltage are well known, the speed-torque curve of the single-phase *voltage* motor may be obtained as described by Mr. Lamme and illustrated in Fig. 12 of his paper. Of course, instead of letting the two two-phase voltages act simultaneously on the same motor, we may let them act on different motors and then add the torques by mechanically connecting the rotors. Furthermore, since the voltages are the same, we may connect the stators in parallel with one phase on one stator reversed. This combination would have the same speed-torque curve as the single-phase *voltage* motor.

The condition obtaining in the single-phase *current* motor, that the current in one phase shall be zero, is satisfied, as is easily seen, by the sum of two balanced two-phase states in which the stator currents of each state are of the same magnitude, agreeing in sign in one phase and opposite in sign in the other. If now, the polyphase and single-phase systems in use were of the constant-current type, the theory of this motor would be exactly similar to that of the single-phase *voltage* motor described above. We would take for the magnitude of the two two-phase currents one-half that of the constant current in the single-phase line. Knowing the torque-speed curves for a balanced motor operated on a balanced polyphase constant-current system, we could obtain the speed-torque curve of the single-phase *current* motor by combining two speed-torque characteristics in the manner of Fig. 12. Similarly, instead of having the two two-phase currents flow in the same machine, we may cause them to flow in two different machines and combine the torques by mechanical coupling of the rotors. Furthermore, since the currents in the two machines are to be the same, the stators may be connected in series with one phase of one stator reversed as Mr. Lamme has done. Of course, in actual practise the voltage of the single-phase line is constant and not the current, but this condition is easily met. Instead of the two two-phase currents being directly given, they must be chosen of such a magnitude that the voltages necessary to produce them must add to the line voltage in the phase receiving current. Thus the currents will have to be changed with each change of slip. This, however, is easily taken care of by anyone familiar with the theory of induction motors under balanced conditions.

The method used by Mr. Lamme is clearly one of superposition, that is, two different possible systems of currents, voltages, torques, etc. are added together and their sum is said to be also a possible system. While no objection can be raised as regards the currents and voltages, there may be some doubt as to the validity of combining the double frequency quantities as torque, Joulian heat, etc. in this way. In fact, such combination in general is not permissible. For example, the ohmic loss produced in a conductor by the sum of two currents of the same frequency is not equal generally to the sum of the ohmic losses which each current would produce in that conductor independently. It is a remarkable fact that for balanced circuits, two systems of currents and voltages, each balanced polyphase of the same frequency but of opposite phase relation may be combined and the mean resultant torque, heating, etc. will be equal to the sum of the mean torques, heatings, etc. which each system would produce separately. This justifies the above procedure.

Of course, the instantaneous torque, heating, etc. cannot be obtained in this way. This points to one difference between the mechanically connected series machines of Mr. Lamme and the single-phase *current* motor. In the former, the torque in each machine is constant in time, so that the same is true of their sum. In the latter the torque is pulsating, vanishing generally four times per cycle. The mean value, however, is the same in the two cases.

Mr. Hellmund, admittedly discusses idealized motors, the idealization consisting in assuming resistance and leakage reactance zero. Discussion of such ideal cases is profitable if this idealization makes the problem easier and if actually used motors with small resistance and small leakage reactance approximate in their operation these resistanceless and reactanceless cases. However, I don't think, that Mr. Hellmund will claim that most of his ideal cases are easy to work out or understand. Also, those cases where the rotor windings are of few phases and concentrated, do not correspond to motors in general use on unbalanced operation as in phase converters or balancers and they bring in complications of infinite currents and higher harmonics. The reason for the detailed consideration of such cases is given at the beginning of Case No. 5. It is because these cases show that as the number of phases in the rotor and the distribution of the rotor windings are increased, the main flux of the motor approaches a synchronously rotating sine distribution, irrespective of the field form which the primary would produce by itself. But could not this result be obtained more directly and more easily. I would like to show here that it can be, following the line of ideas used in Mr. Lamme's paper.

Consider the case of a motor with rotor of an infinite number of phases and having zero resistance and zero leakage reactance. By zero leakage reactance is meant that an arbitrary distribution of m.m.f.'s coming from the stator can be entirely annulled by some properly chosen current distribution on the rotor. Suppose that the stator winding is single-phase and concentrated and gives a square topped m.m.f. wave. Suppose, for simplicity, that the machine is two-pole. The rotor is supposed to be at synchronous speed.

As is well known, the square-topped m.m.f. wave can be resolved by Fourier's theorem into harmonics. The fundamental will be a two-pole, sine-shaped field. The third harmonic will be a six-pole sine field; the fifth harmonic will be a ten-pole sine field, and so on.

Consider first the case of a sine-wave current in the stator. The height of the square topped m.m.f. wave will then change in time according to the sine law. It is clear that each of the harmonics into which the square topped wave is resolved will also have amplitudes varying sine fashion in time. Thus the alternating square topped m.m.f. field is resolved into a two-pole sine-shaped field alternating sine fashion in time; a six-pole sine field alternating sine fashion in time with the same frequency, and so on.

What now will be the reactions of the rotor upon these component fields. Consider first the fundamental m.m.f. field. As Mr. Lamme has so clearly explained, this can be further resolved into a forwardly rotating and a backwardly rotating sine-shaped field. The forwardly rotating component is synchronous with the rotor and therefore the flux it produces induces no currents in the rotor. Therefore, this forwardly rotating component produces its full flux. The backwardly rotating component has, however, a 200 per cent slip relative to the rotor. Rotor currents, therefore, will be induced, which will completely counterbalance this backwardly rotating m.m.f. component. Thus, no backwardly rotating two-pole flux will be produced.

Consider now the third harmonic of the m.m.f. wave, forming a six-pole alternating field. As before, this can be resolved into a forwardly rotating and a backwardly rotating component, but now having six poles the speed of rotation will be only one-third synchronous. Thus the rotor will have a 200 per cent slip relative to the forwardly rotating component and a 400 per cent slip relative to the backwardly rotating component. Hence any fluxes which these components would produce are damped out by currents induced in the rotor. In a similar way, the fifth and higher harmonics of the stator m.m.f. field produce no resultant flux. All that is left is the forwardly and synchronously rotating sine-shaped flux. Such a rotating sine flux would, of course, induce a sine-shaped counter e.m.f. in the stator so that if the impressed voltage is sine shaped, so also will be the stator current. The case where the stator current is not sine shaped offers some interesting features which I shall not go into here.

The real simplicity of the ideal case, just considered is now apparent. As far as concerns the counter e. m. f. generated by the main flux, we see that it remains balanced polyphase under all conditions, if only the rotor is running near synchronism. This is brought about by the selective action of the rotor, which damps out any fluxes other than the synchronously rotating sine field.

Where the leakage reactance and resistance are not zero, but the rotor winding is distributed and has infinitely many phases, it is clear that this selective damping will not be perfect, and rotating components of the field other than the synchronously rotating two-pole field will persist. It is clear that the voltages induced in the stator by each rotating field component will be of fundamental frequency, but now the voltages will not be balanced polyphase. This is the approximate condition in the single-phase motor, phase converter and phase balancer.

When the winding of the rotor is concentrated and of few phases, not only does the damping out of the non-synchronous component of the field flux by the rotor become incomplete, but the rotor currents themselves set up new component rotating fluxes. These give higher harmonics in the stator voltages, as Mr. Hellmund's first cases show. It is easy to show that with the rotor running synchronously, the harmonics of stator voltage are always odd. Thus we see, that the complications in most of the cases considered by Mr. Hellmund arise from his supposition of concentration of the rotor windings. Since this case is quite different from machines actually used under unbalanced conditions, and since the facts concerning the machines of infinitely many phased rotors can be arrived at without considering these cases, its detailed study becomes of less practical importance.

**C. Fortescue** (by letter): The problems of design have occupied the attention of electrical designers so much in the past that there has been comparatively little activity in connection with purely theoretical considerations of machine operation. This state of affairs tends toward a narrow specialization on the part of machine designers which is not favorable to progress. Thus we find a tendency to consider each type of machine as an entity; we have literature on "Theory of Transformers", "Theory of Polyphase Induction Motors", "Theory of Single-phase Induction Motors", "Theory of Synchronous Motors", and so forth, without any inquiry as to the relation between the various types of electromagnetic apparatus. As a result of this there appears to be a distinct loss of generality in modes of thought among electrical designers, so much so that, because a machine has parts that rotate about an axis, it is often considered as being distinct in theory from an ordinary stationary network or even from a network having relative linear motion between its parts as for instance an electromagnet.

Those who have devoted some time to the theoretical investigation of electrical machinery are beginning to see that the same general analytic treatment applies to all the various types of machines. For example the induction motor differs from a stationary network only in this respect, that the coefficients of mutual induction between different branches in the former become time functions and we can no longer express the e.m.f. induced in a given circuit due to a current flowing in another by $M_{12} \frac{d\,i_2}{d\,t}$ as in a simple network but must use instead $\frac{d}{d\,t}(M_{12}\,i_2)$ or $M_{12} \frac{d\,i_2}{d\,t} + i_2 \frac{d\,M_{12}}{d\,t}$. Carrying out the comparison a step further we see that even if $i_2$ is constant an e.m.f. is induced, $i_2 \frac{d\,M_{12}}{d\,t}$, an action familiar to designers of rotating machinery under the name of "rate of cutting of lines of force." Again we find in a-c. generators that if we supply reactive kilovolt amperes to a load a porportionate amount of d-c. ampere-turns must be supplied to the field in order to maintain the e.m.f. Conversely if reactive kilovolt amperes are supplied to the generator by means of condensers the d-c. ampere-turns must be reduced; in some cases even without any d-c. magnetization it is impossible to keep the terminal e.m.f. down to a reasonable value. These actions are strictly analogous to the similar actions in a transformer.

I regard Mr. Hellmund's paper as a bridge between the old methods of considering rotating machinery and the more recent ideas on the subject. His treatment is essentially anlaytic although he uses quantities and terms familiar to designers. Mr. Lamme's paper I regard as a practical demonstration of the truth of the comparatively new physical conception of the operation of a single-phase induction motor as being a special case of that of polyphase machines. This conception might have been deduced from the analytic solution of the single-phase motor which is old, but its full significance was not appreciated until quite recently, when a new analytic method of considering rotating machinery gave the solution in a form which points more directly to this conception.

In regard to Mr. Hellmund's paper, I wish to point out one very common error into which he falls, namely that of associating simplicity of parts with simplicity of theory. In physics it is not generally true, for in considering aggregates or groups it is often possible to simplify the equations without affecting the general result. Thus we are familiar with such expressions as "rigid body" in dynamics, "perfect fluid", etc., which are tacit admissions that the equations used are not absolutely general or fundamental. These expressions and the simple form of the usual electromagnetic equations of electrical circuits are the result of the application of the mathematical device known as the "ignoration of co-ordinates" that is to say the omission of such considerations as have no material weight in the final results. The method of analysis required for practical problems is one which will permit of this and yet permit of further elaboration if desired.

Mr. Hellmund's method of attack lacks this desirable characteristic because it is affected by the number of physical parts that have to be considered simultaneously and, therefore, it is found advantageous in his method to begin with the two simple concentrated circuits in relative motion. The usual analytic treatment of this case with the condition of constraint assumed by Mr. Hellmund is also comparatively easy and leads to similar results. The first few cases which he takes up, though necessary in his train of reasoning, are of academic interest only and are not necessary in the analytic solution of the general problem of unbalanced motors. I think that the method of treatment leads to an over emphasis of non-essential factors, which are largely eliminated in actual designs, and is weak where essential factors such as secondary resistance and magnetic leakage are introduced.

In connection with the wave forms of induction motors; it is not generally appreciated to what a great extent the harmonics are eliminated. I have attached to this discussion a mathematical proof that a uniformly distributed polyphase winding on a cylinder of an infinite number of phases will give a sine wave space distribution of induction. It should be added that a winding of finite number of phases can theoretically be so distributed that it will give a sine wave space distribution of induction. These are interesting facts but others of more importance are that it is not the wave form of the induction which determines the wave form of currents in primary and secondary but the flux linkage distribution or $M_{12}\, i_1$ and $M_{12}\, i_2$. These still more nearly approach sine wave form with properly distributed windings. Thus the secondary flux linkage wave form of one phase of a uniformly distributed three-phase winding due to a balanced primary currents will take the form,

Flux leakage

$$= K\, i_1 \left( \cos\theta + \frac{1}{625} \cos 5\,\theta + \frac{1}{2401} \cos 7\,\theta + \ldots \frac{1}{n^4} \cos n\,\theta \right)$$

where $n$ is always odd. The harmonics are so small that they may be ignored practically. The above does not include tooth harmonics. These may, however, be reduced to a negligible factor in the design.

In a paper presented before the Philadelphia Section of the A. I. E. E. Mr. Gilman and I showed that any unbalanced system of three-phase e.m.f. or current vectors whose vector sum is zero may be resolved into two symmetrical three-phase component systems, one of which is of positive phase sequence and the other of negative phase sequence. The same is true of any two-phase system of e.m.fs. or current vectors. It may be shown analytically that a symmetrically wound induction motor running at a given speed will have a definite impedance to each of these components which will be designated $Z_1$ and $Z_2$. The first impedance is made up of the true resistance and a composite reactance, and a virtual resistance which represents the resistance due to mechanical work. The second impedance is made up of a true resistance, a leakage reactance and a negative virtual resistance which represents the resistance due to mechanical work, required to help circulate the negative phase sequence currents.

The case that Mr. Lamme has considered in his paper is a special case of the symmetrical polyphase induction motor operating under unbalanced terminal e.m.fs. Only one of these terminal e.m.fs. is given directly, namely the single-phase impressed e.m.f., but in this case the currents are perfectly defined in relation to each other, namely (Fig. 1) $\check{I}_B = -\check{I}_c$.

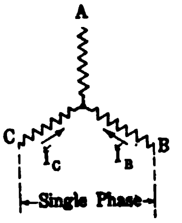

FIG. 1

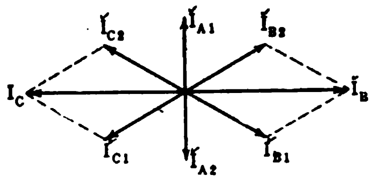

FIG. 2

(Fig. 2) shows graphically how the current vectors $\check{I}_B$ and $\check{I}_c$ may be resolved into two symmetrical current systems $S^1$ $(\check{I}_{a1})$ and $S^2$ $(\check{I}_{a2})$. Fig. 3 shows the same graphical decomposition for a single-phase current into two two-phase currents, $\check{I}_s$ in this case being the single-phase current.

The terminal e.m.f. can likewise be resolved into two symmetrical e.m.fs., the star values being represented by $S^1$ $(\check{E}_{a1})$, and $(S^2\check{E}_{a2})$, and we shall have the relation derived from the regular relations of star and delta e.m.fs.

$$\check{E}_{bc} = j\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \qquad (1)$$

Likewise the currents are subject to the constraint,

$$\check{I}_a = 0,\ \check{I}_b = -\check{I}_c \qquad (2)$$

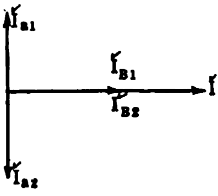

FIG. 3

and therefore by the (Fig. 2)

$$\check{I}_{a1} = -\check{I}_{a2} \qquad (3)$$

or by theory $\dfrac{\check{E}_{a1}}{Z_1} = -\dfrac{\check{E}_{a2}}{Z_2}$

and therefore $\check{E}_{a2} = -\dfrac{Z_2}{Z_1}\check{E}_{a1}$ (4)

Substituting in (1) we obtain

$$\check{E}_{a1} = -\frac{j\,\check{E}_{bc}}{\sqrt{3}} \cdot \frac{Z_1}{Z_1 + Z_2} \qquad (5)$$

$$\check{E}_{a2} = -\frac{j\,\check{E}_{bc}}{\sqrt{3}} \cdot \frac{Z_2}{Z_1 + Z_2} \qquad (6)$$

and therefore

$$\check{I}_{a1} = -j\,\frac{\check{E}_{bc}}{\sqrt{3}} \cdot \frac{1}{Z_1 + Z_2} \qquad (7)$$

$$\check{I}_{a2} = +j\,\frac{\check{E}_{bc}}{\sqrt{3}} \cdot \frac{1}{Z_1 + Z_2} \qquad (8)$$

and

$$\check{I}_b = -\check{I}_c = -\frac{\check{E}_{bc}}{Z_1 + Z_2} \qquad (9)$$

equations (7) and (8) show that the single-phase performance is equivalent to that of two machines coupled together mechanically with windings connected in series, one machine being connected in opposite phase sequence to the other. If $K_1$ and $K_2$ are the ratios of transformation of primary and secondary currents for each symmetrical component, $P_0$ the power output is given by

$$P_0 = I_b^2\left(\frac{\omega_0 - \omega_1}{\omega_1} K_1^2 R_s - \frac{\omega_0 - \omega_1}{2\,\omega_0 - \omega_1} K_2^2 R_s\right) \qquad (10)$$

Where $\omega_0$ is synchronous and $\omega_1$ slip frequency and $R_s$ secondary resistance.

The input in volt-amperes is $P_1 + j\,Q_1$ where

$$P_1 = j\,Q_1 = I_b^2\,(Z_1 + Z_2) + P_F \qquad (11)$$

$P_F$ being the iron losses. The power factor is

$$\cos\alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \qquad (12)$$

If instead of impressing the single-phase e.m.f. across $B\,C$ (Fig. 1) it is impressed between $A$ and $B\,C$ the terminals $B\,C$ being connected together we would have an entirely different story. There the e.m.fs. are determined and therefore, the components are $\check{E}_{a1} = \check{E}_{a2}$ and are given and the two component currents.

Systems $S^1$ $(\check{I}_{a1})$ and $S^2$ $(\check{I}_{a2})$ are inversely proportional to the impedances $Z_1$ and $Z_2$ the operation is, therefore, equivalent to that of two symmetrical motors coupled together, connected to a symmetrical three-phase system one being connected in opposite phase sequence to the other.

Finally if we wish to consider both single-phase primary and single-phase secondary we impose the constraint (3) on both primary and secondary currents with the result that for each harmonic of current in the two windings, there will be two symmetrical components having the same relation as for the simple case except that $Z_1$ and $Z_2$ will have different values for each harmonic.

The above sketch of the principle on which the single-phase motor theory as given by Mr. Lamme depends is presented in order to show how a complex problem may be solved by the superimposition of a train of separate solutions each of which is connected by some known relation to the preceding one. It should be understood that in the case given in the preceding paragraph the effect of slip greatly complicates the problem.

### *Supplement to Discussion*

If $F(\theta)$ be any function of position on a cylindrical surface, which is symmetrical about a diametral plane, it may be expressed by the Fourier series

$$F(\theta) = \Sigma\,(A_n \cos n\,\theta) \qquad (1)$$

where $n$ comprises all odd numbers. The coefficients $A_n$ are obtained from the integral

$$A_n = \frac{2}{\pi}\int_{-\pi/2}^{+\pi/2} F(\theta)\cos n\,\theta\,d\,\theta \qquad (2)$$

If we have a flux distribution $I\,d\,\theta'\ F(\theta)$ round a cylinder due to a filament of current $I\ d\,\theta'$ at $\theta = O$, and if we consider the filaments to be multiplied so as to form a current sheet on the cylinder, of which the current density at any point may be expressed by $I\ d\,\theta' \cos\theta'$. The field density at the point $\theta$ due to the filament at $\theta'$ will be

$$d\,B = I\cos\theta'\ F(\theta - \theta')\,d\,\theta' \qquad (3)$$

The field density at the point $\theta$ due to the current sheet will therefore be

$$B = \int_0^{\pi} I\cos\theta'\ F(\theta-\theta')\,d\,\theta' \qquad (4)$$

Since by (2)

$$F(\theta - \theta') = \Sigma\,A_n \cos n\,(\theta - \theta') \qquad (5)$$

the integral (4) becomes

$$B = \frac{\pi}{2}\,I\,A_1 \cos\theta \qquad (6)$$

$B$ is therefore a simple sine wave field provided that $F(\theta)$ have a finite number of finite discontinuities.

**C. A. M. Weber:** Several years ago, in the course of my regular work, I had occasion to make what I then considered quite complete tests on a standard polyphase motor operating on both three-phase and single-phase over a range of speed from zero to 200 per cent slip.* Upon reviewing these tests with Mr. Lamme, he intimated that he believed it was possible to derive the single-phase speed-torque curve from the polyphase curves on the basis of two oppositely rotating magnetic fields as represented by two motors coupled together. A short study was made of this at that time but on account of lack of knowledge of the voltage relations involved in the two motors no definite results were obtained. However, Mr. Lamme stated that he was going to take the matter up again when he had sufficient time and expected to obtain the real solution along the above lines.

Several months ago he notified me that he had again taken up the problem and believed he could furnish a relatively simple explanation of the actions of the motor. He then explained to me the general method which he used which was practically the same as covered by his paper, but he stated that my former tests were obviously full of errors as he discovered when he applied them as a check to his method. I then proposed to repeat these tests with another set of motors, taking into account the discrepancies and errors involved in the former tests. Mr. Lamme then stated that he expected to carry out similar tests on very much larger motors. It may be of interest to know that my tests were carried out entirely independently of his and that I did not see the results of his tests until after his paper had been sent to press. You may imagine, therefore, my pleasure in noting the very close similarity of my results with his, in view of the fact that my tests were on two-phase motors of quite small capacity while his were on much larger three-phase motors.

For my tests I selected two 1-h. p., 220-volt, 60-cycle 2-phase, 4-pole wound-rotor motors with their shafts rigidly connected together. The torques of the various tests made were registered by an electric dynamometer of just sufficient capacity to handle the motors under test. Considerable pains were taken to obtain practically perfect alignment between the two unit combinations and since special arrangements were provided on the dynamometer for obtaining alignment with apparatus under test I feel that the errors in alignment, while of course not entirely eliminated, have been reduced to a practical minimum. The tests taken were as follows:

(1) Two-phase speed-torque and primary current curves on one of the motors at 220 volts with secondary short-circuited on itself.

(2) Single-phase speed-torque on one winding of one of the two-phase motors at 220 volts with secondary short-circuited on itself.

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*The curves derived from these tests appeared in the *Electric Journal* of September, 1914.

(3) Single-phase speed-torque on the two-unit combination at 220 volts with the secondaries short-circuited on themselves.

The voltages impressed on each motor were also recorded.

Fig. 4 shows the forward and backward torques of one of the two-phase motors at full voltage plotted from test data, the corresponding primary current-torque curve and the single-phase speed-torque curve, shown solid, which was derived from the two-phase speed-torque and primary current curves as shown in Table I.* For comparison, the tested single-phase speed-torque curve on one winding of one of the two-phase motors is also shown. This curve is the one shown dotted.

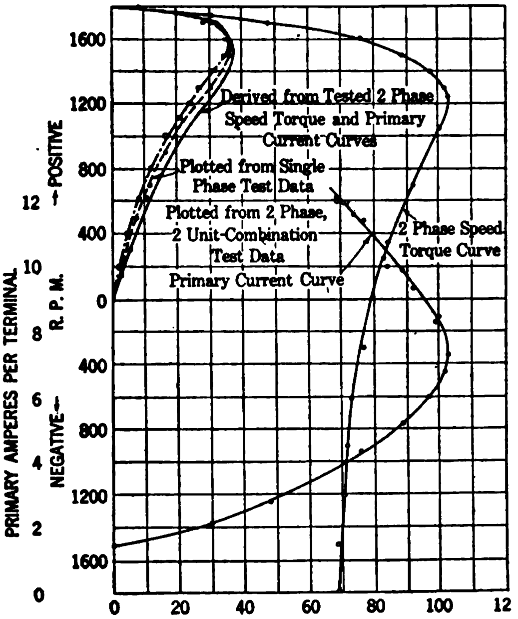

FIG. 4

For further comparison the tested speed-torque curve of the two-unit combination with primary windings in series and shafts rigidly connected together is also shown. This curve was plotted from data in second column of Table II and is the curve in dot and dash.

Considering the nature of the tests, the uncontrollable factors such as heating, saturation, leakage, etc., and the fact that comparisons are being made between several tests which could

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*In order to avoid confusion the tables in this discussion were purposely arranged the same as in Mr. Lamme's paper.

not be made under identical conditions, it is indeed surprising that the various single-phase speed-torque curves check so closely. I wish to call particular attention to one discrepancy and that is the shape of the upper portion of the primary current curve which should bend over, whereas you will note it is practically straight. This result is undoubtedly due to heating as you will note the speed-torque curve at 200 per cent slip is very nearly perpendicular which of course could only occur through heating in this case. I will refer later on to the shape of these two curves in connection with the calculated curves.

Carrying the analysis somewhat further as did Mr. Lamme I measured the two-unit speed-torque recording at each torque reading the voltages impressed on each motor. Table II shows the test data thus obtained together with the calculated resultant torques using the tested voltages on each motor and the two-phase speed-torque curve Fig. 4.

TABLE I.

| Speeds | | Torque at full voltage oz. ft. | | Primary ampere per terminal full voltage | | Per cent voltages | | Reduced Torque oz. ft. | | Resultant. torque. |
|---|---|---|---|---|---|---|---|---|---|---|
| Positive | Negative | *Tf* | *Tb* | *If* | *Ib* | *X* | 1−*X* | Forward | Back | oz. ft. |
| 1750 | 1750 | 30 | 69 | 2.12 | 12.2 | 85.2 | 14.8 | 21.8 | 1.5 | 20.3 |
| 1700 | 1700 | 48 | 69.1 | 2.82 | 12.18 | 81.2 | 18.8 | 31.6 | 2.5 | 29.1 |
| 1600 | 1600 | 76 | 69.3 | 4.27 | 12.15 | 74.0 | 26.0 | 41.6 | 4.7 | 36.9 |
| 1500 | 1500 | 89 | 69.8 | 5.2 | 12.1 | 70.0 | 30.0 | 43.6 | 6.3 | 37.3 |
| 1400 | 1400 | 96.5 | 70.0 | 5.95 | 12.07 | 67.0 | 33.0 | 43.3 | 7.6 | 35.7 |
| 1300 | 1300 | 102 | 70.3 | 6.8 | 12.04 | 63.9 | 36.1 | 41.6 | 9.2 | 32.4 |
| 1200 | 1200 | 103 | 70.5 | 7.3 | 12.03 | 62.2 | 37.8 | 39.9 | 10.1 | 29.8 |
| 1000 | 1000 | 99 | 71.3 | 8.53 | 11.92 | 58.3 | 41.7 | 33.6 | 12.4 | 21.2 |
| 800 | 800 | 94 | 72.4 | 9.21 | 11.81 | 56.2 | 43.8 | 29.6 | 13.9 | 15.7 |
| 600 | 600 | 89.6 | 73.5 | 9.84 | 11.7 | 54.2 | 45.8 | 26.3 | 15.4 | 10.9 |
| 400 | 400 | 85 | 75 | 10.4 | 11.54 | 52.5 | 47.5 | 23.4 | 17.0 | 6.4 |
| 200 | 200 | 81.5 | 76.7 | 10.8 | 11.35 | 51.2 | 48.8 | 21.4 | 18.3 | 3.1 |
| 0 | 0 | 79 | 79 | 11.1 | 11.1 | 50.0 | 50.0 | 19.75 | 19.75 | 0 |

The last column in Table II is to be compared with column No. 2 and it will be noted again that the check is as good as could be expected under the circumstances. The individual resultant torque value in the last column marked with an asterisk is an error probably due to incorrect reading of impressed voltage, etc., and should be thrown out.

In Fig. 5 are plotted the calculated line voltages for forward and backward torques from Table I together with the tested per cent line voltages on each motor of the two-unit combination from Table II.

It will be noted that approximately the same difference between calculated and tested values occur in this case as in the cases described by Mr. Lamme with the one exception, that the tested curve does not cross the calculated curve at the higher voltage values, which may be accounted for by motor size, since the motors used by me were 1-h. p. two-phase whereas those used by Mr. Lamme were 10-h. p. three-phase. The variation of the tested voltage curve from the calculated voltage curve may be accounted for in various ways such as leakage and saturation, and the fact that although both motors of the two-unit combination were manufactured according to the same specifications it could not be expected that they would be identical in every respect as is assumed in the calculations. In Fig. 6 I have plotted the tested torque values of the two-unit combination from the second column Table II, the calculated resultant torque values last column Table II and the tested single-phase torques of one winding of one of the two-phase motors, each against speed.

TABLE II.

| Speed | Torque at 220 volts oz. ft. | Volts per motor | | Per cent line volts | | Polyphase torque at 220 volts oz. ft. | | Polyphase torque at reduced voltage oz. ft. | | Resultant torque oz. ft. |
|---|---|---|---|---|---|---|---|---|---|---|
| r. p. m. | | Motor No. 1 | Motor No. 2 | Motor No. 1 | Motor No. 2 | Forward | Backward | Forward | Backward | |
| 1800 | 0 | 192 | 28 | 87.3 | 12.7 | 0 | 69 | 0 | 1.1 | −1.1 |
| 1700 | 31.5 | 173 | 52 | 78.6 | 23.6 | 48 | 69.1 | 29.6 | 3.9 | 25.7 |
| 1600 | 36.0 | 163.5 | 66 | 74.4 | 30.0 | 76 | 69.3 | 42.0 | 6.2 | 35.8 |
| 1500 | 34.5 | 148.0 | 76 | 67.3 | 34.5 | 89 | 69.8 | 40.2 | 8.3 | 31.9* |
| 1400 | 30.75 | 139 | 79 | 63.2 | 35.9 | 96.5 | 70.0 | 43.6 | 9.0 | 34.6 |
| 1300 | 26.5 | 134 | 86 | 61.0 | 39.1 | 102 | 70.3 | 38.0 | 10.8 | 27.2 |
| 1200 | 24.0 | 129 | 95 | 58.6 | 43.1 | 103 | 70.5 | 35.4 | 12.1 | 22.3 |
| 1000 | 16.5 | 124 | 100 | 56.4 | 45.5 | 99 | 71.3 | 31.5 | 14.8 | 16.7 |
| 800 | 12.0 | 117 | 103.5 | 53.1 | 47 | 94 | 72.4 | 26.5 | 16.0 | 10.5 |
| 600 | 7.5 | 116 | 105 | 52.7 | 47.7 | 89.6 | 73.5 | 24.9 | 16.7 | 8.2 |
| 400 | 4.5 | 112 | 108 | 51.0 | 49 | 85.0 | 75.0 | 21.1 | 18.0 | 3.1 |
| 200 | 2.0 | 111 | 109.5 | 50.5 | 49.7 | 81.5 | 76.7 | 20.7 | 18.9 | 1.8 |
| 0 | 0 | 110 | 110 | 50 | 50 | 79.0 | 79.0 | 19.75 | 19.75 | 0 |

In considering these curves it should be borne in mind that only ordinary precautions were taken to obtain the same temperature conditions as it was discovered at the outset that identical temperature conditions were impossible of attainment.

It will be noted that the derived curve shown solid in Fig. 6 has lower torque values below the break down point than the derived curve shown solid in Fig. 4, which is attributable to the fact that tested voltages were used in deriving the solid single-phase curve, Fig. 6, which takes into account saturation and leakage, whereas when calculated voltages were used as in solid single-phase curve, Fig. 4, saturation and leakage were not taken into account.

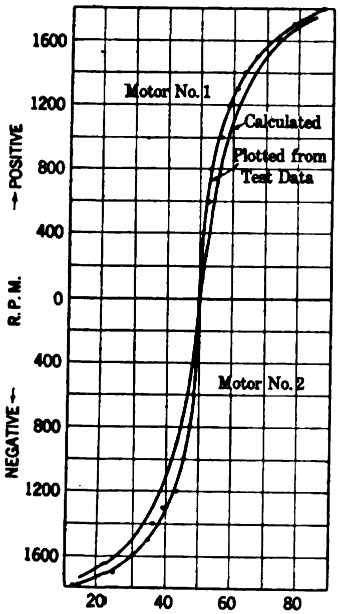

Fig. 5

After making the above tests and obtaining such remarkable checks between the various single-phase curves I decided to calculate the single-phase and polyphase speed-torque and primary current curves from the physical constants of the motor and derive from the polyphase calculated curves the single-phase speed-torque curve to determine how it checks with the calculated single-phase speed-torque curve.

Fig. 7 shows the speed-torque and primary current curves of one of the two-phase motors referred to, calculated from the physical dimensions of the motor. My calculations were based on fixed values of reactance and resistance, whereas under test these values vary. This is quite apparent when a comparison is made between the two-phase tests of Fig. 4 and the calculations, Fig. 7. The torque values from no load to break down calculated higher than the tests due to primary resistance being higher under test than I assumed in the calculations. The break down torque calculated about 10 per cent higher than the tested value due also to primary resistance being assumed lower than

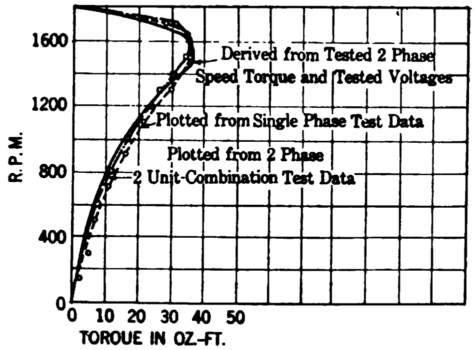

Fig. 6

test. From break down torque to 200 per cent slip the calculated torques are less than tested values for similar reasons. In the calculations I assumed a normal increase in resistance, constant from no load to 200 per cent slip, whereas under actual test the increase in resistance due to temperature was abnormal. This abnormal heating caused the tested primary current curve Fig. 4 to straighten out, whereas normally it should have been as shown in Fig. 7.

Table III shows the calculations of the single-phase speed-torque curve from the calculated two-phase speed-torque and

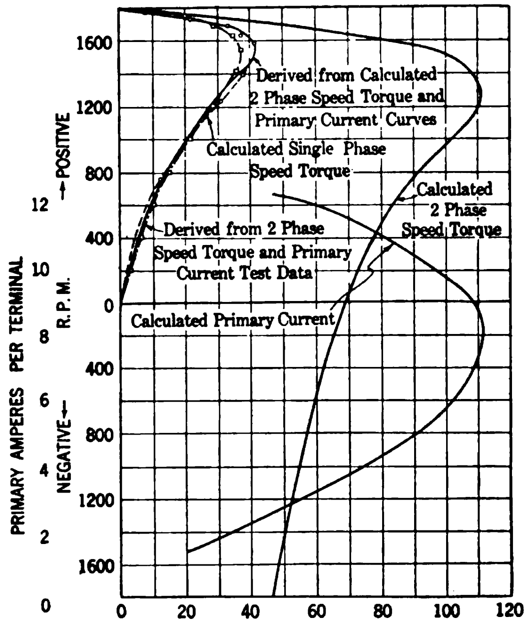

FIG. 7

primary current curves. The values in last column Table III are plotted solid in Fig. 7.

For comparison I have calculated the single-phase speed-torque of one winding of the above two-phase motor and show same dotted in Fig. 7. It will be noted that these two single-phase speed-torque curves, the one derived from calculated two-phase speed-torque and primary current curves and the other calculated from the physical dimensions of the motor, practically coincide. I feel that they would have coincided exactly had I drawn my circle diagrams large enough to reduce the effect of errors in measurement to a minimum.

For further comparison the single-phase speed-torque derived from the two-phase speed-torque and primary current test data are also plotted in Fig. 7. The break down torque of the single-phase curve derived from test data is about 10 per cent less than the calculated value due to temperature. It will be remembered that there was about 10 per cent difference between calculated and tested two-phase break down points.

Reviewing all of the tests, calculations from test data and calculations from the physical dimensions of the motor and the various curves it would appear that the analogy of two oppositely rotating fields by means of two separate polyphase units rigidly connected together with their primary windings connected in series is a correct physical conception of the single-phase induction motor.

TABLE III.

| Speeds | | Torque at full voltage oz. ft. | | Prim. amperes per term. full voltage | | Per cent voltages | | Reduced torque oz. ft. | | Resultant torque oz. ft. |
|---|---|---|---|---|---|---|---|---|---|---|
| Positive | Negative | *Tf* | *Tb* | *If* | *Ib* | *X* | 1−*X* | Forward | Back | |
| 1750 | 1750 | 30 | 47 | 1.8 | 12.33 | 87.3 | 12.7 | 22.8 | .8 | 22 |
| 1700 | 1700 | 52 | 47.2 | 2.86 | 12.32 | 81.2 | 18.8 | 34.2 | 1.7 | 32.5 |
| 1600 | 1600 | 85.5 | 48.3 | 4.62 | 12.31 | 72.7 | 27.3 | 45.4 | 3.6 | 41.8 |
| 1500 | 1500 | 102.7 | 49.4 | 6.0 | 12.29 | 67.2 | 32.8 | 46.4 | 5.3 | 41.3 |
| 1400 | 1400 | 108.8 | 50.4 | 7.0 | 12.27 | 63.6 | 36.4 | 44.1 | 6.7 | 37.4 |
| 1300 | 1300 | 111.4 | 51.5 | 8.02 | 12.23 | 60.4 | 39.6 | 40.6 | 8.1 | 32.5 |
| 1200 | 1200 | 111.0 | 52.6 | 8.75 | 12 21 | 58.3 | 41.7 | 37.7 | 9.2 | 28.5 |
| 1000 | 1000 | 103.0 | 55.0 | 9.57 | 12.14 | 56 | 44. | 32.2 | 10.2 | 22.0 |
| 800 | 800 | 92 | 57.2 | 10.32 | 12.07 | 53.9 | 46.1 | 26.7 | 12.2 | 14.5 |
| 600 | 600 | 84 | 60 | 10.81 | 12 | 52.6 | 47.4 | 23.2 | 13.5 | 9.7 |
| 400 | 400 | 78.3 | 62.6 | 11.13 | 11.89 | 51.6 | 48.4 | 20.9 | 14.6 | 6.3 |
| 200 | 200 | 73.5 | 66 | 11.4 | 11.75 | 50.7 | 49.3 | 18.9 | 17.1 | 1.8 |
| 0 | 0 | 69.5 | 69.5 | 11.58 | 11.58 | 50 | 50 | 17.4 | 17.4 | 0 |

**G. H. Garcelon:** There is one point in Mr. Lamme's paper which, it seems to me, may account for the discrepancy between the tests of a single motor running single phase and a two-motor unit supplied with opposing polyphase current.

The single motor is always excited at full voltage whereas each motor of the two-motor unit is subjected to an excitation varying from one-half normal to practically normal.

This difference would naturally cause variation in the saturation and the reactance so that the coincidence of the two curves seems to me remarkably close considering the conditions.

The two-motor unit has twice as much material, so that some discrepancies are bound to appear.

**R. E. Hellmund:** Believing that those readers of Mr Lamme's paper who are accustomed to work with the circle diagram would be interested to see how the two diagrams of the series connected polyphase motors or the oppositely rotating fields in the single-phase motor join together, I have worked out the diagram for a number of typical load cases. In discussing these diagrams, let us keep the two polyphase motors with oppositely rotating fields and connected in series, in mind. Considering each of these motors by itself, it is evident that the circle diagram applies as usual to the individual motor. If the voltage of the first motor has a direction $O E_1$, (Fig. 8), its resultant field may be represented by a vector $O A_1$ which gives at the same time a measure for the size of the voltage. If the leakage coefficient is represented by the ratio $\frac{O B_1}{O A_1}$, we have at once the circle $B_1$, $D_1$, $A_1$, around the center $C_1$ applying to the first motor. If we assume now the load point $D_1$ corresponding to about full load with a primary current $O\ D_1$ we know that on account of the series connection of the motors this current must apply to both of the motors. If, therefore, $O D_2$ equal and opposite to $O D_1$ represents the current of the second motor, we know that the circle applying to this motor must go through the point $D_2$. If we assume further the voltage of the second motor to be equal to $O A_2$, the direction of $O A_2$ representing the field of the second motor and the basis for the circle is definitely fixed by the knowledge of the point $E_2$ and the ratio $\frac{O B_2}{O A_2}$ which

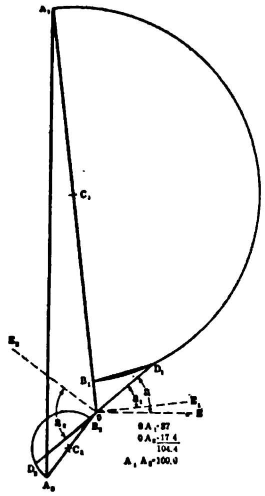

FIG. 8

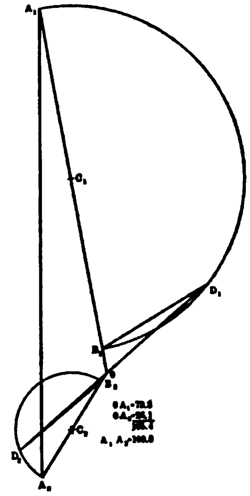

FIG. 9

must be again the leakage coefficient. We find, therefore, the circle $B_2$, $D_2$, $A_2$, around the center $C_2$ corresponding to the second motor. The power factor of the first motor for the load point under consideration corresponds to the angle $a_1$; that of the second motor to the angle $a_2$; and that of the combination to the angle $a$. The secondary current of the first motor is proportional to $B_1 D_1$, and that of the second motor proportional to $D_2 B_2$. Knowing that $O B_1$ and $O B_2$ represent the magnetizing currents of the two motors for their individual voltages, which are also known, we can find all the required scales, and formulas can be derived for the combination under the condition that the sum of the slip of the two motors must be 200 per cent. A complete theory for the series connected polyphase motor and the equivalent single-phase motor could thus be derived. This would be of small practical value, however, since there is small promise that any calculating method along that line would be simpler than those now available for single-phase induction motors. The principal value of the two polyphase motors equivalent being the easy physical conception, the diagram so far derived is sufficient. It will be noticed from the diagram that both the voltages and the fields $O A_1$ and $O A_2$ are somewhat out of phase and that their arithmetical sum is, therefore, slightly larger than the total impressed voltage $A_1 A_2$, which is the geometrical sum. At the same time it will be seen that the difference between the arithmetical and geometrical sums is only 4.4 per cent, so that the small error introduced by figuring with the arithmetical sum as has been done in Mr. Lamme's paper is almost negligible.

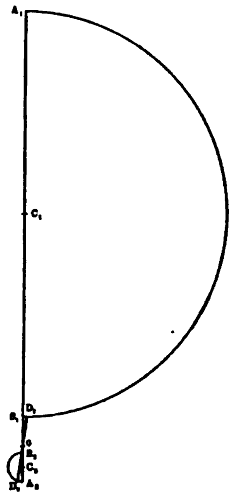

Fig. 10

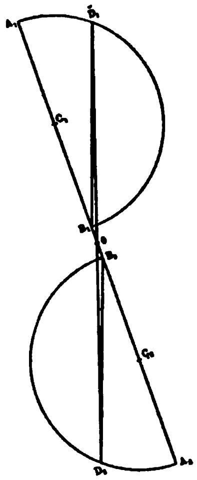

Fig. 11

(Fig. 9) is a similar diagram applying to the same motor for a larger load. It will be noted that the field voltage, and, therefore, the circle for the motor has decreased, while that of the second motor has increased. The difference between the arithmetical and geometrical sums of the two voltages is 5.4 per cent in this case. In practise the difference hardly ever exceeds 10 per cent and is usually much smaller. The two special cases of no-load and standstill are of interest and shown for the same motor in (Figs. 10 and 11;) (Fig. 10) representing the no-load case shows a very small circle $B_2 D_2 A_2$, for the motor which runs at 200 per cent slip. For this condition, the two fields $O A_2$ and $O A_1$ are practically in phase with each other so that the arithmetical and geometrical sums of the two are identical. This is true in nearly all motors, except those with very high resistances, because the no-load power factor of an induction motor is usually not very much different from the power factor obtained at 200 per cent slip. It is interesting in this connection that the no-load test of two coupled motors with oppositely rotating fields gives an excellent means for measuring the leakage coefficient. One of the motors runs at practically no-load while the other motor runs with a current corresponding to 200 per cent slip which is much nearer to the ideal short-circuit current than the current corresponding to 100 per cent slip usually tested. Since with the series connected test the current in both motors is the same, we are testing directly the voltage required to send this current through the motors in one case with practically no-load reactance, and in the other case, with practically ideal short-circuit reactance. The ratio of the voltages represents, therefore, directly the leakage coefficient of the polyphase motors. (Fig. 11) represents the standstill condition with which both motors are on a par, the two voltages being equal and always in phase and the diameters of the two circles being naturally alike.

The fact that the two voltages and fields of the two motors are not always in phase does not indicate by any means that there is a fundamental discrepancy between the single-phase induction motor and the equivalent of two series-connected polyphase motors; this slight out-of-phase condition of the oppositely rotating fields can exist in the single-phase motor as well. As a matter of fact, it seems that the two cases are exactly equivalent in all respects except for slight differences caused by differences in iron saturation, heating, eddy currents, bearing friction, etc., that is, such secondary phenomena which have little to do with the fundamental theory.

The close agreement between the two equivalents may further be emphasized by a number of examples of load cases which are easily understood. At no load, we know that one of two series connected two-phase motors takes practically the full line voltage and, therefore, magnetizing current corresponding to this voltage. This current $i$ flows in both phases of each motor giving a primary copper loss of $4\,r\,i^2$, if $r$ is the resistance per phase in each motor. The single-phase motor takes with the same line voltage a no-load current $2\,i$, which, with the same resistance $r$ in the single-phase, gives the primary losses equal to $(2\,i)^2\,r = 4\,r\,i^2$ that is the same as in the polyphase motor combination. At standstill, each of the polyphase motors has one-half line voltage, giving a certain locked current of $\frac{1}{2}\,I$ and primary copper losses of $\left(\frac{1}{2}\,I\right)^2 \times 4\,r = I^2\,r$. The single-phase motor has full line voltage and, therefore, a locked current of $I$ and primary copper losses of $I^2\,r$, that is again the same as the polyphase motor combination. The same conditions apply to the secondary copper losses. In the single-phase motor secondary, we have two currents of different frequencies, superimposed upon each other. In the two-motor combination, the same currents flow separately in the two secondaries. It can be shown by a simple calculation that currents of different frequencies flowing in the same winding give the same copper losses as if they were flowing in two separate windings, each of the same resistance as is the case in the double motor combination.

The core losses at no load are obviously the same in both cases. With the single-phase motor, we have one rotating field corresponding to full line voltage; with the two polyphase motor combination, one of the motors has a rotating field corresponding practically to line voltage, while the field and core losses of the other motor are practically zero. At standstill, the resultant field of the single-phase motor is alternating in one axis and corresponds in size to full line voltage. In the two motor combination, we have fields of half the strength, which, if they were only alternating in one axis, would give less than one-half the losses per motor, since core losses vary with something like the 1.8th or 2nd power of the field strength. This is, however, practically made up for by the fact that the fields are rotating, giving the maximum density in a larger portion of the iron than would be the case with fields alternating in a single axis. So we have again a very close agreement between the two cases.

The primary ohmic voltage drop at no-load is, in the polyphase motor combination, $i \times 2\,r$ with two motor resistances in series, while in the single-phase motor we have a resistance of only $r$ but a current $2\,i$, therefore, again the same drop. Similarly, we have at standstill an ohmic drop of $\frac{1}{2}\,I \times 2\,r = I\,r$ for the polyphase combination and the same drop $I\,r$ in the single-phase motor.

The same conditions apply to the leakage reactance drop, if the reluctance of the leakage path remains constant as is usually assumed in all induction motor theories. Since, however, the primary currents of the single-phase motor are about double the current in the polyphase motor, they cause larger local leakage fluxes around the slots in which the single-phase winding is located. If there is, therefore, a tendency for saturation in the tooth tips *a*, shown in (Fig. 12) such saturation may cause increased reluctance in the leakage path of a single-phase motor as compared with the polyphase equivalent. The decreasing influence of the increased reluctance upon the leakage field of the single-phase motor tends to give it slightly larger torques than the polyphase equivalent, especially near the pull-out point and speed points below.

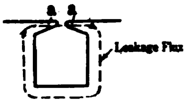

FIG. 12

A comparison between the tested curves of Fig. 22 and Fig. 25 of Mr. Lamme's paper, which correspond to the single-phase motor and the series connected polyphase motors, respectively, shows a difference in this direction. The same fact is also borne out by a comparison of the test curves, Figs. 23 and 27.

There is nothing new, however, in having an otherwise perfectly good theory spoiled by the varying reluctance of the iron. We know, for instance, that the same change in the reluctance of the leakage path introduces marked discrepancies in most theories of the polyphase induction motor and spoils in many cases the true circle of the circle diagram.

Similar discrepancies are caused by the fact that the varying frequency of the induction motor secondary winding bring about varying eddy losses in the copper; this results in a variation of the effective secondary resistance, while most theories are based on constant resistance. The resistance variations caused by heating are even more annoying in any attempts to check theory by tests. The various unavoidable discrepancies in Mr. Lamme's curves can, therefore, not be considered as a reflection upon the correctness of the fundamental ideas advanced, but they are simply discrepancies which confront the designer in nearly all of his work.

**A. M. Dudley and C. A. M. Weber:** After reviewing Mr. Hellmund's paper we decided to wind a phase converter which would meet as nearly as possible some of Mr. Hellmund's fundamental conditions thereby producing by actual tests some of the wave shapes obtained by Mr. Hellmund's analysis. We could not of course obtain zero resistance nor zero leakage and therefore, we would not obtain the infinite current values shown in the paper. However, where infinite values are shown in the paper for zero resistance and leakage our oscillograms show an abrupt rise to a high value during a finite time instead of to an infinite value for an infinitesimally short time.

The converter, which was built and tested after Mr. Hellmund's paper was sent to press, consisted of a 48-slot stator with one primary coil wound pitch in diametrically opposite slots and with one tertiary coil displaced 90 deg. from the primary coil also wound pitch in diametrically opposite slots. This produced

FIG. 13

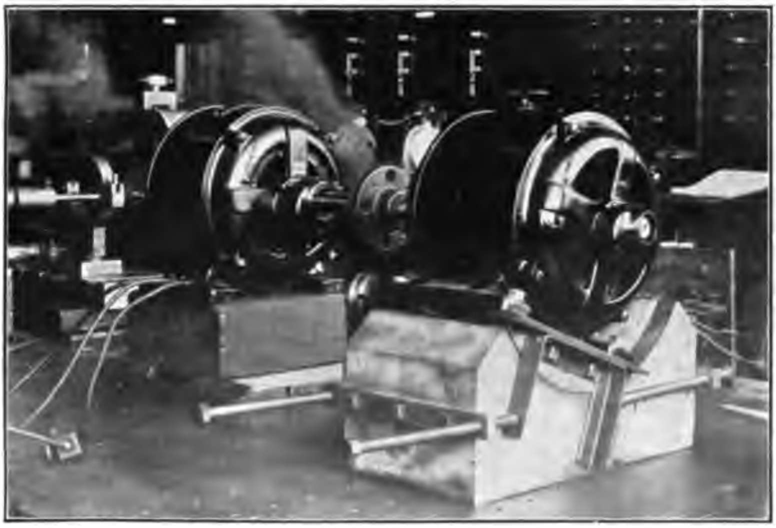

FIG. 14

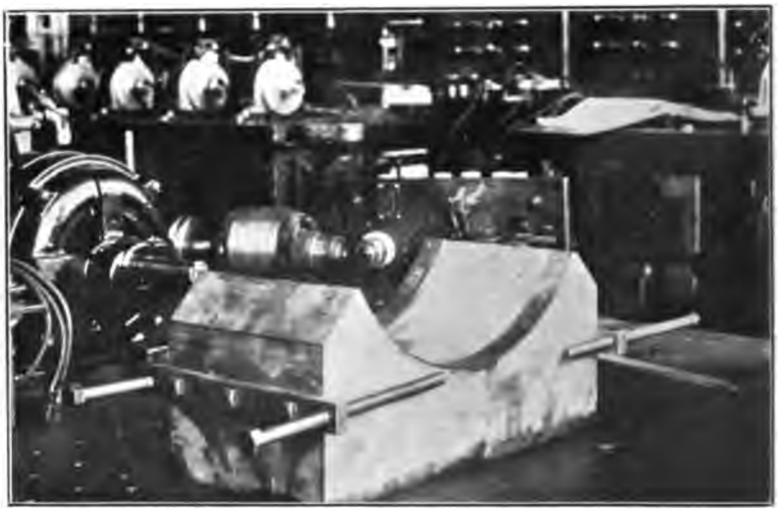

FIG. 15

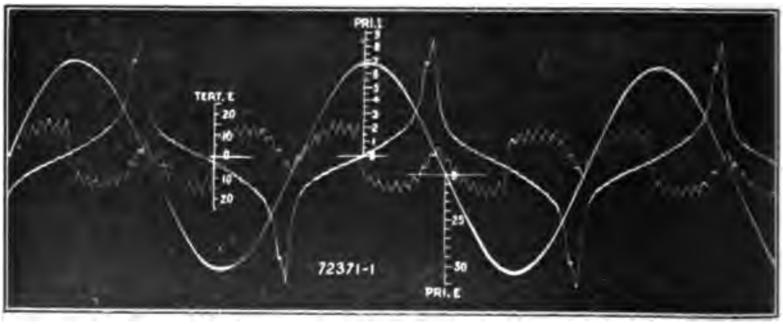

FIG. 16

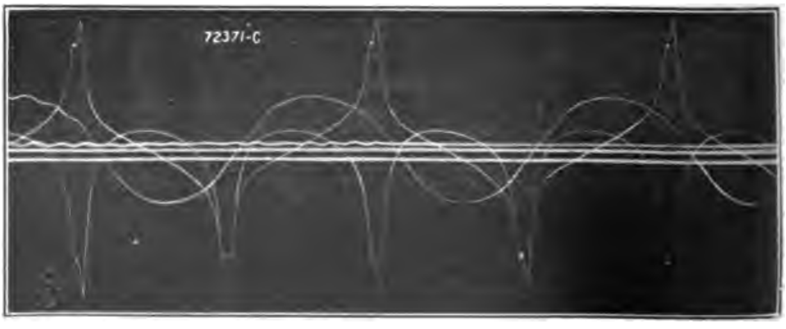

FIG. 17

[DUDLEY & WEBER]

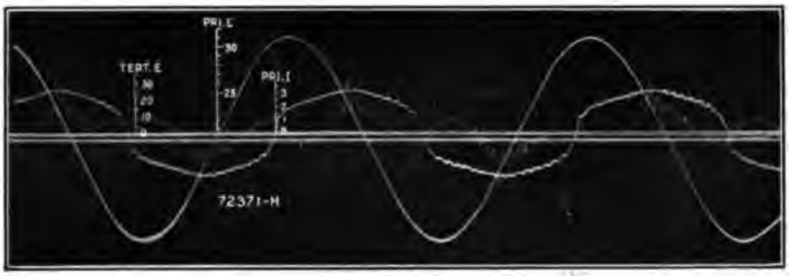

FIG. 18

FIG. 19

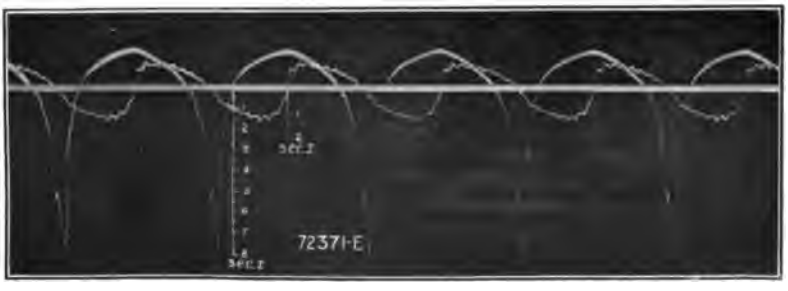

FIG. 20

[DUDLEY & WEBER]

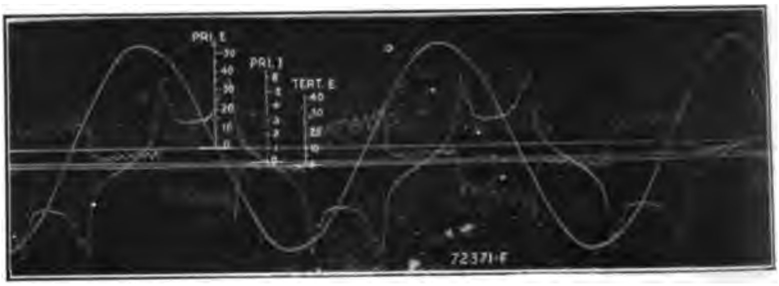

FIG. 21

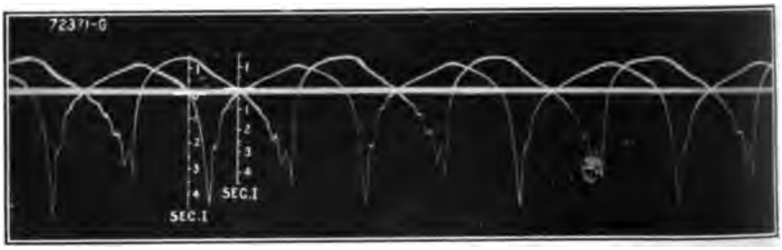

FIG. 22

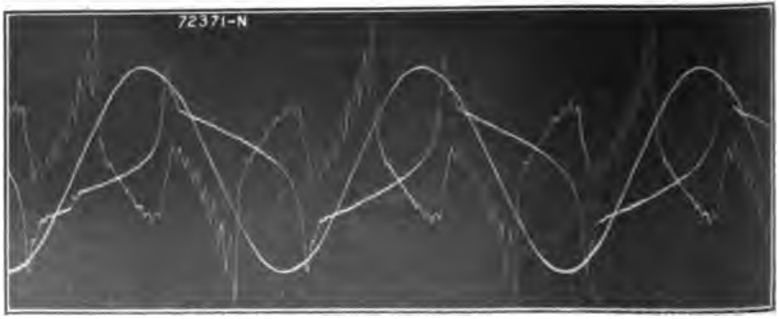

FIG. 23

[DUDLEY & WEBER]

a stator with concentrated primary and tertiary windings and is shown in Fig. 13.

The rotor had 34 slots and was wound similarly to the stator except you will note that the phases are not in exact quadrature which of course does not affect the results obtained with single-phase rotor and very little, as you will see, when the rotor was used two-phase. This rotor is also shown in Fig. 13.

This converter was driven by a two-pole synchronous motor mounted in a cradle. The cradle was necessary in order to obtain and maintain the correct angular position of the primary and secondary coils with respect to the primary flux. Fig. 14 shows the synchronous induction motor mounted in the cradle and coupled to the phase converter. Fig. 15 shows the lower part of the cradle which is simply a block of wood with brass straps for slides.

The synchronous induction motor and primary coil of the phase converter were excited from the same source so as to maintain the proper relationship between primary coil, secondary coil and primary flux. Precautions were also taken to reduce external reactances to a minimum. The only external reactances between the generator and the apparatus under test were two sets of large transformers.

Fig. 16 was taken under the conditions of single-phase secondary coil coinciding with single-phase primary coil at the instant when the primary flux was maximum.

The primary magnetizing current should be compared with Fig. 3A in Mr. Hellmund's paper. The similarity of the primary current wave shape here shown with that in the paper will be noted except that infinite values are modified by leakage, resistance and the fact that the coils are not concentrated in a mathematical point as is assumed in the paper.

The tertiary voltage wave compares with Fig. 4c of the paper. The rotor teeth cause a varying reluctance which in turn has produced a saw tooth effect on the tertiary voltage. It will be noted that there are 34 points in a complete cycle and we used a 34-slot rotor. The effect of the rotor teeth does not show in the primary voltage because this voltage is impressed and the counter e.m.f. adjusts itself exactly. The tertiary voltage crosses the zero line on account of the leakage, resistance, etc.

Fig. 17 was made under the same conditions as Fig. 16. The primary current, the primary voltage and the secondary current are shown.

The primary magnetizing current is the same as in Fig. 16. The primary voltage is also the same as in Fig. 16 except the scale has been reduced.

The secondary current compares with Fig. 3B of the paper except that infinite values do not appear for the same reasons as mentioned in connection with the primary current Fig. 16. Note the strong even harmonics which are indicated by the difference in the positive and negative wave shapes.

Fig. 18 was taken with single-phase primary and single-phase secondary coils coinciding at the instant the primary flux was zero. It will be noted that the presence of the rotor teeth is reflected in the tertiary voltage and primary current.

The tertiary voltage shows the third harmonic with the fundamental almost missing. If the primary and secondary coils had been more nearly in coincidence when the primary flux was zero the fundamental would have practically disappeared.

Fig. 19 was taken with single-phase primary and two-phase secondary. One secondary coil coinciding with primary coil at the instant of maximum primary flux. The primary current and tertiary voltage wave shapes compare with Figs. 7A and 8B in the paper. The effect of the rotor teeth is again reflected in the primary current and tertiary voltage.

The primary current wave shape is similar to Fig. 7A in the paper except infinite values are modified for the same reasons as previously brought out, but otherwise the shape is almost identical with the one in the paper.

The tertiary voltage shape compares with Fig. 8B in the paper. It is rectangular except reflecting the effect of rotor teeth. The jog in the middle of the tertiary wave is probably caused by the element load of the oscillograph. The little jog in the primary current may possibly be due to the same cause or more probably to varying reactances caused by saturation.

Fig. 20 was taken under the same conditions as Fig. 19 and shows the secondary current in each phase. This oscillogram compares with 7B and 7C in the paper except infinite values are modified for reasons previously stated. It will be noted that the secondary currents in the different phases not only vary in value but also in form and in this particular case are quite different from each other.

Fig. 21 was taken with single-phase primary and two-phase secondary. One secondary coil coinciding with the primary coil at about 70 per cent maximum flux, that is, 45 deg. from the position in which Fig. 20 was taken. The primary current compares with Fig. 9A in the paper. The two primary current peaks correspond to the two infinite values shown in the paper. The infinite values are damped out by leakage, resistance, etc.

The tertiary voltage compares with Fig. 10B. It is rectangular in shape and follows the zero line for a certain distance. This wave, compared with the previous tertiary voltage waves, illustrates the fact that the wave form of the tertiary voltage varies with the rotor position, although the effective value (voltmeter value) remains unchanged. The effect of rotor teeth is again reflected in the tertiary voltage.

Fig. 22 was made under the same conditions as Fig. 21 and shows both of the secondary currents. This oscillogram compares with Fig. 9B and 9C in the paper. It will be noted that the shapes of the two current waves are similar and that one is the image of the other, *i. e.* the round part follows the straight part in one wave whereas in the other the straight part follows the round part. It will be noted that the negative areas are equal to the positive areas which is one of the fundamental facts on which calculations in the paper are based.

Fig. 23 was made with single-phase secondary coil coinciding with single-phase primary coil when primary flux was approximately 70 per cent of its maximum value, about half way between Figs. 16 and 17 and shows the primary voltage and current and tertiary voltage. This oscillogram has no equivalent in the paper but is interesting in that it shows the primary current wave shape can be unsymmetrical depending upon the position of the rotor coils. All the previous oscillograms of the primary current showed a symmetrical wave shape. The tertiary voltage shows a strong third harmonic.

The object of these oscillograms was to illustrate that the methods used in the paper were correct and not to illustrate actual conditions in machines that are manufactured today. Having illustrated the correctness of the fundamental principles used in the paper for the simplest form of machine it is reasonable to assume that these same principles when applied to modern machinery will give the correct results. These oscillograms therefore are a practical demonstration of the operations of the fundamental principles used by Mr. Hellmund in his paper and are not to be used in determining the wave shapes in practical machinery.

**R. E. Hellmund:** Various suggestions regarding other methods for investigating the subject have been made. As already emphasized by the discussion, the choice of method is a question of taste and will depend largely upon the mental equipment and upon the mathematical tools which are at the disposal of the engineers studying the problems. There is no doubt whatsoever but that a student who is familiar with the working principles of a polyphase motor and desires to obtain simply a clear physical conception of the single-phase induction motor and its working principles can get what he wants by carefully studying Mr. Lamme's paper. The designer of single-phase machines and phase converters, however, needs something more for his work than a physical conception. It is necessary for him to have formulas by which he can predetermine machine performance with a fair degree of accuracy; further he should have a general idea of what are the best ratios between the numbers of primary and secondary slots; what is the best practical winding distribution for the primary, and so on; in other words, he should know enough to choose the most favorable combinations and keep away from poor combinations which cause unnecessary copper losses, eddy losses, etc. It has been the purpose of my paper to furnish information of this kind. The same or similar results can undoubtedly be obtained by the more mathematical methods intimated by Mr. Fortescue and possibly by methods mentioned by Mr. Slepian. In writing my paper I made it a special point, however, to start in with a few very simple cases. The study of these cases requires nothing but the knowledge of two or three of the simplest and most fundamental laws which everybody who is an electrical engineer must know, as for instance, one of the Kirchhoff laws, the law of magnetic induction, etc. The only other thing necessary for these cases is the ability to solve two or three equations of the simplest kind with two or three unknown quantities. By giving these simple cases first, I had hoped to enable any interested party to understand the fundamental features of the methods used in the paper. After going that far, the reader can either go on or content himself with studying merely the conclusions reached in the paper for the practical cases and using the formulas given, having, however the satisfaction of knowing in a general way how they have been derived. I adopted this procedure realizing that unfortunately the average designing engineer is as a rule rather rusty in mathematics and other subjects.

On the other hand, there is no doubt but that the methods indicated by Mr. Fortescue by which he proposes to go directly after a broad general solution taking everything into account to start in with is a more elegant way of attacking the problem and undoubtedly the one which will appeal to the man who has the required mental aptitude and the mathematical tools at his disposal. I had hardly expected that my method of starting in with a very simple case and building up slowly, considering the various factors, one by one, would appeal to men who are well equipped to handle more difficult problems. I am confident, however, that the average designing engineer will prefer to start the problem slowly with simple means and then progress to the more complicated case as far as required for his purposes. The fact that the problems discussed in my paper do not come any too easily to a great number of engineers is evidenced by the fact that so far as I know, nobody has previously even attempted to study the case much in detail. It is fortunate that in the near future we will have a paper by Mr. Fortescue on a similar subject but using other methods, thereby enabling everybody to take their choice between the two different methods.

**B. G. Lamme:** I think I stated pretty definitely in my paper that it is not intended as a method of calculation, but is merely an attempt to give, in as simple form as possible, a physical conception of what is going on in the motor. Also the general treatment is not claimed as novel or original. There is a very large class of people which is interested in the single-phase induction motor and would like to have some idea of its action, but either cannot or will not follow any mathematical conception or analysis. Twenty-one years ago, I wrote a paper on the characteristics of the polyphase motor. At the time that motor was not very well understood except by a few engineers. I spent a great many months of hard work trying to explain the properties of the polyphase motor and, in doing so, I used considerable mathematics. However, the criticism brought against that paper during its preparation, by a number of people to whom I referred it, was that they did not like the mathematics in it and I was asked to leave out all formulas and equations. This I succeeded in doing after much effort and the general public gave this paper a kindly reception, largely on account of the absence of mathematics in its treatment. I felt, therefore, that a similar treatment of the subject of the single-phase induction motor might be of some value to that portion of the electrical public which does not care to consider papers of a mathematical nature.

## DISCUSSION AT NEW YORK

**M. I. Pupin:** We have to look upon the induction motor from two different points of view; one, from the point of view of the designer, and the other from the point of view of the operator. The operator wants to have a motor which is capable of a certain performance under certain conditions, and the designer wants to know how to design a motor that will give the performance under those conditions.

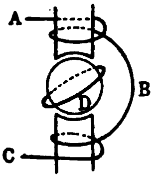

FIG. 24

To design a motor you have to proceed, of course, from certain elements of the motor. Now, what are the elements of an induction motor or any piece of dynamo-electric machinery. Why, its electric and its magnetic circuits. That is all we have to guide us, the constants of the electric and of the magnetic circuits; they are the resistance, inductance, and capacity of each circuit which is employed in the structure. The question arises, how are we to define these constants of the various circuits. The answer to that question is simple. You have to define them as Maxwell defined the constants of a circuit. The attempts of ordinary mortals to do better than Maxwell did, must be discouraged. Let us follow Maxwell as long as we can, then when somebody is born who is more profound than Maxwell, we will bow to him, but for the present, I think Maxwell is the leader.

Maxwell defines the constants of a circuit—the resistance of the circuit, the capacity of the circuit, and the inductance for the circuit—in such way that the action of neighboring circuits is not taken into account. Let Fig. 24 indicate an induction motor, with the rotor circuit *D* and stator circuit *A B C*. What is the inductance of the primary winding *A B C*? It is the inductance which it would have when the secondary winding *D* is open. That is the only satisfactory way of defining it; not the effective, but the value which is found when the secondary is open, and that you can calculate, of course, from the number of turns and from the constants of the magnetic circuit. In the same way, the primary resistance is the resistance which is obtained with that electromotive force which we intend to apply, but the secondary circuit is supposed to be open. The same thing is true for the other circuits.

The elements, then, of the circuit which we must have in order to construct a theory which the designer can use, are the inductance, the resistance, and the capacity of each electrical circuit, when the other circuits are open, Now, when we do that, and introduce the further condition that the primary and secondary currents produce a magnetic field, which has a sinusoidal distribution, then the mathematical theory of the single-phase induction motor becomes very simple; indeed, remarkably simple, so that anybody who knows the elements of the theory of the electrical circuit can sit down and work it out. An undergraduate can work it out—when once he is shown how to do it.

Now, what does that simple theory show? It shows that we have two magnetic fields produced by the *rotor* currents. The presence of these two magnetic fields, rotating in opposite directions, is not a theory; it is a fact. We do not have to make an assumption of that kind at all. It is a fact, from which you cannot escape. You have in each secondary winding two alternating currents. Take an oscillograph and photograph the e. m. f. in the secondary winding, and you will find that it consists of two components of different frequencies; one, which I call the *additive* frequency and another which I call the *differential* frequency. If you impress an e. m. f. of 60-cycles upon the primary or stator circuit and the motor rotates fifty-five times per second, you will have in the secondary windings two frequencies, one of frequency 60 plus the speed of rotation, which is 55, and that makes the additive frequency 115, and the other 60, minus 55, which is 5, that is to say the differential frequency is 5 periods per second. By photographing suitably, you will find that these two frequencies are distributed in such a way that the additive frequency of 115 periods per second produces a rotary magnetic field rotating contrary to the motion of the rotor, and the differential frequency produces a rotation in the direction of the rotor. It is assumed, of course, that the rotor has a symmetrical winding.

The differential frequency is the working frequency—it is the busy bee, the bee that makes the honey, and the additive frequency is the drone which produces the drag and tends to stop the rotor. The additive frequency produces a rotation in opposite direction to the motion of the rotor, trying to stop it; it is a brake. The differential frequency produces a rotation in the direction of the rotor motion; it pulls the rotor and thus produces mechanical work. So that the presence of two rotary magnetic fields is not a theory resting upon a lucky guess; it is an actual physical fact. It is not necessary to assume two rotary magnetic fields produced by the stator current at all; in fact, they have no physical existence; but the presence of two rotary magnetic fields produced by the rotor currents is a fact. That is what the mathematical theory gives you, and that is what experiments show to be actually the case. So much, then, for the mathematical theory.

Now, the mathematical theory, following the methods of Maxwell, which he first pointed out in a very simple and elegant way in 1865, suggests that, just as Maxwell defined the performance of a transformer by means of its effective inductance and effective resistance—I shall define these presently—so you can in the case of an induction motor describe the performance of a motor by finding out the effective inductance and the effective resistance of the primary winding, which is the winding where the impressed force is located.

If that were possible, it would be a very convenient thing to do, namely, to consider first the ideal inductance, the resistance and capacity of the various circuits of the motor, and from these to calculate what is the effective resistance and the effective inductance of the motor under various conditions of load. A comparison between this calculation and experiment must of course follow. The calculation is the same which Maxwell first applied in the case of a transformer.

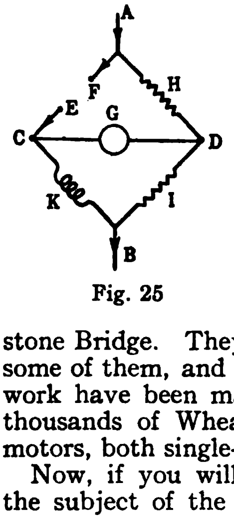

Fig. 25

The question arises then, how are we going to perform the experiment? Very simply,—when you know how to do it. We use the Wheatstone bridge. We actually take the motor and put it in a Wheatstone bridge. Several years ago, about ten years ago, I was giving a short talk to the Radio Engineers, at their request and I told them that one of the simplest ways to determine the performance of an aerial is to determine its effective resistance and its effective inductance under various conditions by means of a Wheatstone Bridge. They are doing it now in this manner, at any rate some of them, and thousands of Wheatstone bridges for aerial work have been made and I hope that some day we will make thousands of Wheatstone bridges for the study of induction motors, both single-phase and polyphase.

Now, if you will allow me I shall amplify a little because the subject of the single-phase induction motor has grown to be much bigger than some of us think. Take the well known arrangement of the Wheatstone bridge, indicated in Fig. 25 *K* is an inductance, *H* and *I* are resistances, and between *E* and *F* we may introduce anything we wish to measure by the indicated Wheatstone bridge arrangement. The two points *E F* may be points through which power is sent anywhere, to a whole district beyond *E F*. If we wish to know how much power is consumed in the district and at what efficiency all you have to do is to connect them to the Wheatstone bridge and measure the effective resistance and effective inductance of that whole district and from the two you can calculate everything else. In the same manner if you put the primary of an induction motor between *E* and *F* you can measure its effective inductance and effective resistance at various speeds, and you get curves something like those in Fig. 26. In this figure the abscissas measure the ratio of rotor speed to frequency speed of the impressed e. m. f., the ordinates give the values of the effective resistance (curve *R*) and of the effective inductance (curve *L*). The effective resistance starts at a certain value, goes up, and is at a maximum a little before synchronous speed is reached; this is also the approximate location of the maximum torque. The effective inductance is represented by the curve *L*. From these two you can calculate everything else. I have done it, and it works like a charm. You can do that for a single-phase induction motor, and you can do it for a polyphase induction motor, with this distinction—that the bridge for a polyphase motor is not quite as simple as that for a single-phase motor, but it can be done, and it can be done in very simple fashion; the method is very accurate.

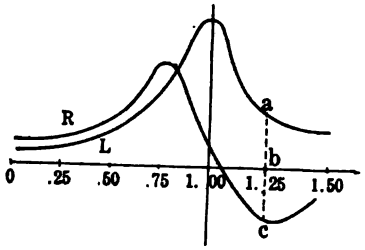

Fig. 26

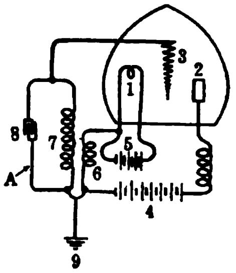

Fig. 27

You will find that the mathematical theory following strictly the simple method of Maxwell gives you results, which agree with the theory very closely, so close that the experimental points obtained on the curve agree with the theory remarkably well, particularly when feeble magnetizations are employed such as I used in my work. Of course, as I have already said, all is based on the fact that we are dealing with a sinusoidal distribution of the magnetic flux, and that is not a serious limitation, because most induction motors have a distribution of flux which approaches the sinusoidal very closely.

That being so, then, it makes the theory very acceptable. And why is the theory so desirable? It is desirable, because, so far as I know, it is the simplest method that can be put in the hands of a designer of single-phase or polyphase induction motors. But that is only one reason. The other reason is that it really presents the whole theory in a much simpler way than anything I have heard of before; simpler even than the simple presentation which Mr. Lamme gave this evening, although I must say, that when I come to read his paper a little more closely, it may perhaps not be quite as simple as he made it.

You know the action of a single-phase induction motor is extremely similar to the action of a vacuum tube amplifier, particularly those vacuum tubes which are used today as oscillators. One of the most beautiful recent inventions, made by a student of mine, Mr. E. Armstrong, is the pliotron oscillator; that is really a wonder. I am digressing, I know, but the subject is connected with the action of the single-phase induction motor, and therefore, the digression is permissible.

Fig. 27 is a diagram of a vacuum tube invented by Mr. De Forest about twelve years ago. It is a vacuum tube, with a third electrode, which is called the grid. 2 is the cold positive and 1 is the hot negative electrode. This electrode is a tungsten filament heated by battery 5, and battery 4 maintains the electron current between 1 and 2. 7 and 8 are the inductance and the capacity of a separate circuit which is coupled by mutual induction to the circuit 2, 4, 6, 1, through which the electron current circulates. Circuit 7, 8 has a period of its own depending upon its capacity 8 and its inductance 7 in the well known way. When the oscillations are started they will not die out if the inductive relation between 7 and 6 are suitably adjusted but will continue until the power of battery 4 is used up. They are so persistent and so steady that the period of oscillation will not change more than one-tenth of one per cent for a whole day. Lately we have been operating a circuit like that, and one of the mechanicians watched it a whole afternoon to see that nothing happened to the apparatus; he did not touch it during the whole afternoon and the period was not changed one-tenth of one per cent; it gave 90,000 periods per second. It is a wonderful piece of apparatus, and the invention consists in introducing this energizing circuit in inductive relation with this oscillating circuit, and that is what Mr. Armstrong did.

It was not accidentally that Armstrong made this invention. He was working in my laboratory where the theory of the induction motor was developed by experimental tests, and he heard my lectures on the subject and applied it, perhaps unconsciously, to the action of pliotrons in which he was interested. I will make now a comparison between the two actions. When the rotor passes beyond synchronous speed, then as you see from Fig. 26 you get negative effective resistance. Negative resistance sounds very strange but it is simple enough when you stop to think about it. Of course, you cannot measure directly in a Wheatstone bridge a negative resistance. It means that we have to introduce a sufficient amount of positive resistance to prevent the occurrence of trouble in the bridge, which I will describe presently.

Consider now the primary effective inductance *a b*, an effective resistance *b c* in Fig. 26. Insert a capacity in the primary, in multiple or in series with the inductance so as to overcome this inductance and introduce a resistance in series so that you will have in the primary winding zero inductance and zero resistance, what will happen? You may say that this is an absurdity. It is not. It is true that you never can have a current equal to infinity; but the current starts toward infinity, but pretty soon the pole pieces are drawn toward the armature, and the belt thrown off and the machine stops. Theory and experiment actually agree in that case.

Another thing which happens is this; when you introduce this capacity to overcome the effective inductance the current grows and the magnetization gets so strong that the inductance changes on account of the change of the permeability; the effective resistance increases also, so that the critical relation which demands an infinite current is upset. Now, the point which I wish to bring out is this: Under the conditions specified above, the primary circuit has a period of its own; it is that due to inductance *a b* in Fig. 26 and the capacity inserted, oscillations being started in this circuit will be maintained if the effective resistance is negative, that is if the power put into the circuit by the rotation of the rotor is equal to or greater than the power consumed by the circuit. The conditions then under which these oscillations are produced by putting power into the circuit are determined by the two curves in Fig. 26.

In the same way, if at *A* you break circuit 7, 8 Fig. 27 and connect the two terminals to *E F* of the Wheatstone bridge, Fig. 25 then apply an alternating electromotive force, varying gradually its frequency and determine the effective resistance and the effective inductance of circuit 7, 8, you will get curves which are almost exactly like the two curves in Fig. 26 and you can tell that, as soon as the effective resistance begins to be negative for a given frequency, the circuit will act as an oscillator in consequence of the power supplied by battery 4.

This battery performs the same function as the mechanical power does which turns the rotor. The period is determined in this case in the same way as in the other, and therefore the two phenomena are the same with this difference, however, that whereas you can produce with the vacuum tube any frequency, you cannot produce it with the single-phase induction motor because you are limited by the speed of rotation.

It is a difficult undertaking to design a dynamo-electric machine that will give you 10,000 periods per second, to say nothing of 50,000 periods per second or even more, whereas here in the case of the vacuum tube we are not limited, and we can get any frequencies depending on the inductance 7 and the capacity 8 and the coupling of circuit 7, 8 to the rest of the system. Pliotrons are made with 1000 volts applied between 1 and 2 which will easily give you 10,000 volts in 7, 8, and a frequency anywhere between 50 and 1,000,000, and power up to 150 watts, from one single pliotron. That is a great deal of power at 1,000,000 cycles, and it will give you a perfectly satisfactory alternating electromotive force so steady that, as I said, the frequency would not change more than one-tenth of one per cent for hours.

The fact that the theory of the induction motor which I described to you as developed along the lines first drawn by Maxwell gives correct results, which not only agree with the actual performance of the induction motor, but enable you to predict what will happen under certain conditions you never thought of, shows that the theory is correct, because after all, the supreme test of a theory is whether it can prophesy or not, and this theory can prophesy results which otherwise would not have been thought of.

The fundamental physical conception in the theory is the presence of the two rotary magnetic fields rotating in opposite directions, as Mr. Lamme pointed out. These contrary magnetic fields, as I said, are not a theory, they are an actual, physical fact, and they exist there, and you must accept them, because the oscillograph shows their presence.

**E. F. W. Alexanderson:** The single-phase motor has been the subject of perhaps more theoretical speculation than any other dynamo-electric machine, and the reason for this is, undoubtedly that it is in its functioning, the most complicated of all dynamo-electric machines, although in its structure it is the simplest of them all. If we are to have a thorough understanding of any dynamo-electric machine I believe it is necessary to form a mental picture of the real physical relations between every volt and ampere and line of force that exists in the machine. that is particularly difficult in the case of the induction motor, and the approximations to a mental picture that we are able to draw depend upon personal preference to a great extent.

Mr. Lamme has given us a theory which is very clearly stated and very easy to follow, and no doubt will prove of great value for students of the technique. However, as I said, for academic reasons it may be worth while pointing out that although the theory Mr. Lamme gives is for all practical purposes accurate enough for induction motors, it is, after all, an approximate theory, and I intend to show to what degree it is approximate and how it differs from an exact theory.

The theory of the single-phase induction motor has been treated on the basis of two rotating magnetic fields, and I was going to say that those fields are a mathematical abstraction. Prof. Pupin told us they should not be regarded so. Now, with the qualifications that he made, we may accept that, because he claims that they are in existence only in the rotor, but not as we ordinarily assume, in the stator. There is, however, one statement in connection with the theory of the single-phase induction motor that represents the actual physical facts, and that is the discovery of Mr. Behrend that the single-phase motor can be replaced by two polyphase motors in series.

Starting with this discovery as a premise, we can draw a few logical conclusions and arrive at practically the same result as Mr. Lamme has, although these conclusions will show just what discrepancy there is.

I have prepared a diagram showing in one composite picture,

Fig. 28, what these relations are. This diagram has exactly the same form as the well-known Behrend-Heyland circle diagram, although it is to be interpreted a little differently. The actual physical substitution that Mr. Behrend has made is that we assume that we put on a motor two windings on one phase and two windings on the other phase. We may assume that we feed the windings on the first phase with one ampere from one single-phase alternator, and feed the windings of the other phase with one ampere from another single-phase alternator in quarter-phase relation. The two second phase windings are in opposition, and therefore the second alternator has absolutely no inductive effect upon the motor. The primary alternator delivers the power to the motor exactly as it does in the single-phase motor. However, the combination Fig. 29 of winding *A* with its current, and winding *D* with its current is a real polyphase winding with the rotation in one direction. The combination of winding *B* with its current, and winding *C* with its current is the real polyphase winding with the rotation in the opposite direction. If we apply the current separately to windings, *A* and *D*, first we get certain voltage reactions; if we apply the current to windings *B* and *C*, afterwards we get certain other voltage reactions upon the total winding.

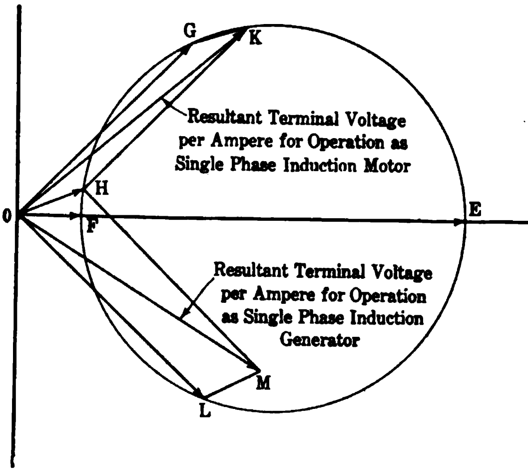

FIG. 28—INVERTED CIRCLE DIAGRAM FOR SINGLE-PHASE INDUCTION MOTOR AND GENERATOR SHOWING VOLTAGES DEVELOPED WITH CONSTANT CURRENT APPLIED

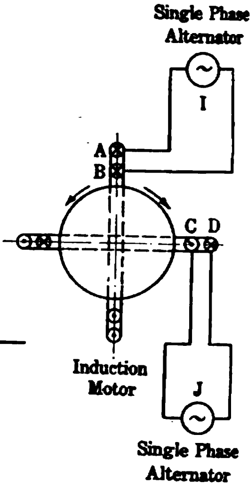

FIG. 29

This circuit follows the same law as the other alternating current circuit, that is, the application of voltage reactions from two sources, will combine vectorially, and therefore, the combined voltage reaction must necessarily be the vectorial sum of the two individual voltage reactions. The voltage reactions of each of the polyphase motors can be calculated by well-known methods, and the calculation is most directly expressed in the Behrend-Heyland circle diagram. That diagram shows what currents flow with a constant voltage applied. I have inverted the diagram showing what voltages are produced with a constant current applied. We must do this in order to get a composite picture, because the windings are in series, and we cannot combine the currents, but must combine the voltages.

The proof why the inverted or voltage diagram has the same shape as the current diagram is a simple proposition of geometry which I will give as a supplement to the written discussion.

On the basis of the known relations in a polyphase motor, referring to the inverted diagram, $O\,E$ is the magnetizing voltage, $O\,F$ is the short-circuit voltage, and $O\,G$ is the normal load voltage for forward rotation. $O\,H$ is the voltage for backward rotation. Now, we let one ampere flow from alternator $I$, and one ampere flow from alternator $J$, in windings $A$, $B$, $C$ and $D$. The voltage in the forward winding is $O\,G$. The voltage in the backward winding is $O\,H$. Therefore, the combined voltage is the vectorial sum of voltage $O\,G$ and voltage $O\,H$ and that means $O\,K$.

Mr. Lamme's theory has assumed that the voltages combine arithmetically, as shown in that diagram, so we are not very far from the truth to assume that it is the arithmetic sum, but after all that is not theoretically correct. If, however, we drive the machine over synchronous speed, then one of the motors consumes energy and the other begins to generate. That means the power factors of the two motors are opposed—Prof. Pupin would say one has positive resistance and the other has negative resistance. Then the diagram becomes entirely different, and it is not correct to assume we can take the arithmetic sum. As seen here, $O\,L$ is the voltage of the motor with the negative resistance, and $O\,H$ is the voltage of the motor with the positive resistance. $O\,M$ is the vectorial combination. The two vectors are nearly at right angles, and the vectorial combination represents the accurate solution, whereas the arithmetical sum is in this case not even an approximation.

I had the opportunity to direct the designing of the first phase balancer installation, the successful completion of which was reported to the Institute in 1915, and in the processes of this undertaking I succeeded in making clear to a number of associates not only the function of the phase converter, but how they can be designed and how automatic control apparatus could be worked out for automatic balancing of the voltages, and I found that in those processes it was not necessary to go into very elaborate theoretical explanations.

**B. A. Behrend:** From Prof. Pupin's remarks one might easily be led to the impression that the theory of alternating-current apparatus did not now follow the conceptions laid down by Maxwell. It need hardly be said that this is not the case and that Maxwell's work underlies all our alternating-current theory. When Prof. Pupin tells us how he uses the Wheatstone bridge to measure the quantities which we measure in practise with ammeters and voltmeters, it is just a little professional mystery injected into a subject which, on account of its inherent complexity, can readily dispense with it.

Mr. Alexanderson shows how to obtain the operating characteristics of the single-phase induction motor for constant current and using the method of inversion or of images, first used by Lord Kelvin, and since frequently applied by Dr. Bedell, Dr. Steinmetz, and the speaker. As it was proved in the case of the polyphase induction motor that the locus of the primary current at constant voltage is a circle, so I proved twenty-two years ago that the same condition prevails in the single-phase motor. I obtained this result by assuming that the single-phase induction motor could be represented by two polyphase motors with their stator windings connected in series acting upon the same rotor winding. From this it follows that, since the slip of the dragging motor is above 100 per cent, it is approximately true to assume the effect of the second motor to be an induced e.m.f. in quadrature with the primary current for the entire range of operation. With this approximation, I worked out a circle diagram for the single-phase induction motor.

Since that time rigorous solutions have been worked out by Messrs. Sumec and Thomaelen demonstrating that the locus of the primary current in the single-phase motor is also a circle. Mr. Alexanderson has now given us another and very interesting method of demonstration of this characteristic of the single-phase induction motor which is to be welcomed by all who see in the vivid and accurate presentation of theory the foundation of all new, great, and enduring engineering work.

**L. W. Chubb:** I agree with Dr. Pupin's contention that there are two opposite rotating fields.

**Prof. Pupin:** In the rotor only.

**Mr. Chubb:** I think that what is in the rotor is in the gap and what is in the gap is in the stator. The best way to get a conception of, and a working system for, a magnetic field is to get it into its components.

In Mr. Alexanderson's discussion, I understood him to say that the two machine analogy is not exactly correct. I wonder if this is not because we have two machines on the same shaft and add together the heat in the two machines. A slight discrepancy may possibly be due to the summation, rather than partial cancellation, of the two resistance losses.

I would like to call attention to another little point. If we have a polyphase winding and apply a single-phase line to two terminals of it, we then have a single-phase current and a polyphase voltage. If now we connect two terminals together, and apply the single-phase voltage, we get polyphase current and single-phase voltage. There are other combinations of connecting the machines in parallel and connecting the machines in series, any of which can readily be treated by the method of oppositely rotating fields.

In going over these papers, it struck me that the differences between the two methods of analysis are very clearly brought out, but not as well brought out as they would have been had the single-phase motor been another special case of the unsymmetrical polyphase treatment. The single-phase motor is one of the special limiting cases of unbalanced polyphase operation and is therefore relatively more difficult, because by Mr. Hellmunds method it is one of the simple but long solutions.

In Mr. Hellmund's analysis he starts by what is, by that method, the simple case, the concentrated winding, and works up to his case Number 5 which is the first practical case. With the double rotation system, if we wish to consider Mr. Hellmund's case Number 1 it would be rather laborious, because with the concentrated winding the distribution of the field is such that we have many harmonics to deal with. Any of the later practical cases with distributed windings, can, however, be more readily treated by Mr. Lamme's type of analysis.

Again the double rotational method is by far the better to prove that the field distribution is sinusoidal with distributed secondary independent of the primary distribution. It can be seen from simple mental conception only and without any long graphical or mathematical analysis. The concentrated primary produces a square topped m.m.f. which can be represented by the equation

$$H = K\left(\sin\theta + 1/3\sin 3\theta + 1/5\sin 5\theta + 1/7\sin 7\theta + \ldots\right)$$

The single-phase alternating primary current will vary the amplitude of the square topped wave with fundamental frequency. Each component will vary proportionally and each component therefore can be represented by two oppositely rotating fields of $n$ pairs of poles, rotating at $1/n$ synchronous speed. All of these component fields, except the forwardly rotating fundamental component, will have a relatively high slip with regard to the synchronously running rotor. All components will therefore be damped out except the fundamental component rotating with the secondary and the field distribution will therefore be sinusoidal. I wish to call your attention to the fact that the distribution will not be sinusoidal except when the rotor is running at synchronous speed. If the rotor runs at one-third speed, there will be a field distribution having three times the number of poles, traveling at the speed of the rotor, if at one-fifth speed, there will be five times the normal number of poles, traveling with the rotor.

This type of treatment applies generally to all special cases of rotating machines, symmetrical or unsymmetrical, balanced or unbalanced, single or polyphase, and such treatment as given in Mr. Lamme's paper should be encouraged as the logical and economical method.

***J. Slepian** (by letter): For text of this discussion, see page 659.

**Alex M. Gray:** Mr. Lamme wrote his paper principally because he had found that college students while they understood the polyphase induction motor had no physical conception of the operation of the single-phase induction motor.

The theory of a-c. machinery is developed in the classroom as follows: After the polyphase alternator and polyphase armature reaction have been discussed, single-phase armature reaction is then considered as the result of two polyphase fields rotating in opposite directions. The inverse field is wiped out by the rotor structure. Unfortunately, the same idea has been applied in discussing the subject of single-phase induction motors, and many teachers do not make it clear that the two phenomena are different, in that the alternator fields are determined only by the current, whereas in the induction motor the applied voltage is fixed, so that any decrease in one of the fields is accompanied by a corresponding increase in the other, in order that the flux threading the stator winding may remain approximately constant.

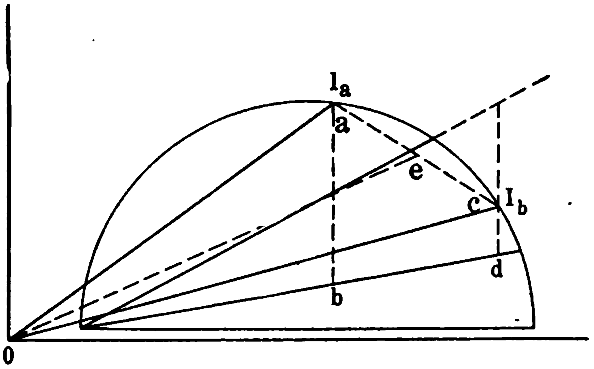

FIG. 30

We can consider the two component fields as produced by two separate polyphase induction motors connected in series, the fields rotating in opposite direction, but the rotors coupled together. The reader can convince himself that this representation is exact by referring to the literature of the subject. We will assume first that the total applied voltage is divided equally

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*Discussion presented both at Pittsburgh April 10 and New York April 12, 1918.

between the two machines, and that the circle diagram representing operation of these polyphase machines is as shown in (Fig. 30). Let us consider the operation at half synchronous speed. In the case of motor *A* the current would be $I_a$ and the torque represented by *a b*, while in machine *B*, which is operating as a generator, the current would be $I_b$ and the torque represented by *c d*. The two motors, however, are in series so that $I_a$ and $I_b$ must be equal and in phase, and therefore the assumption that each motor absorbs half of the applied voltage is wrong. The voltages are in the inverse ratio of the currents $I_a$ and $I_b$ and their resultant is the applied voltage. In (Fig. 30), therefore, the motor torque *a b* has to be multiplied by the ratio $\left(\frac{o\,c}{o\,e}\right)^2$ and the generator torque *c d* by the ratio $\left(\frac{o\,a}{o\,e}\right)^2$.

Another method of treatment is that developed largely in this country by Dr. McAllister and will be found discussed in his book on a-c. motors, and also in recent texts such as Lawrence's "Alternating-Current Machinery". In these texts it is shown that there are two fields 90 deg. out of phase with one another both in time and space, one field being proportional to the speed and the other to the frequency, so that the resultant field is elliptical and is a uniform rotating field at synchronous speed. This latter method is the one now taught to students and it is not improbable that it does not give the student a clear conception of what is happening, since there are four rotor voltages and two rotor currents, and since also phase relations in time and space are liable to be mixed up.

Some teachers, without further introduction, will draw a circuit on the blackboard and state that it is exactly equivalent to an induction motor. The analytical treatment thereafter is easy, but the student does not know the machine when he sees it, far less does he know how the machine works. To prove that the circuit is equivalent to the motor and then to solve the circuit is a roundabout way of going after the matter. The equivalent circuit should never be mentioned until the machine has been thoroughly discussed along the lines indicated above.

There is one other method of treatment which is suitable only for mathematicians who happen at the same time to be electrical engineers, and that is the treatment by complex quantities. Complex quantities are very useful if you know how to humor them and if you are using them frequently, but they do have a habit of slipping out of one's mind. An instructor who revels in this method of treatment can be cured if he is put in charge of the laboratory, because it is very difficult to point out to the student just which is the imaginary part of an actual machine. I am convinced that the use of complex quantities in undergraduate instruction should be restricted to the solution of circuits and networks and should never be used in machinery. After graduation the student may study Steinmetz's texts with advantage.

***R. E. Hellmund** (Read by A. M. Dudley): For text of this discussion, see page 675.

***A. M. Dudley and C. M. Weber:** For text of this discussion, see page 680.

***C. A. Weber** (by letter): For text of this discussion see page 669.

**Chas. F. Scott:** The author uses a type of physical analysis in this paper which is a characteristic of his method of handling engineering and design problems. A notable former instance was in connection with the polyphase motor. The form of motor which first came into general use employed secondary resistance for starting.

Then came one of the notable results of Mr. Lamme's analysis, of this motor, for he so proportioned it that the squirrel-cage rotor could be employed with acceptable starting conditions. It was this kind of physical analysis by Mr. Lamme which led to the proportions in his design and gave us the squirrel-cage or "Type C" motor as it was then called. It was this type which made a reputation for the a-c. motor, and established it as a simple, useful and dependable machine.

Mr. Lamme presented a paper before the National Electric Light Association, at Niagara Falls in 1897, in which he described in a physical way the operation and characteristics of induction motors. It was a sort of practical paper, appealing to those who like to understand fundamental relations, but who are not mathematicians, which has become a classic. Now, the same general method which has done so much to make the operations of the polyphase induction motor known and understood, are extended by Mr. Lamme to a consideration of the single-phase motor.

**Michael I. Pupin:** One of the brightest remarks made this evening was made by a professor, when he said, that when they get an induction motor before the student he understands its true theory when he reaches the laboratory. I agree with him perfectly that the introduction of the complex variable into alternating-current electrical engineering has done a tremendous amount of mischief among the students. Not among men of my age, because I cannot be fooled so easily, but the student has used a complex quantity where the complex quantity ought not to be used. There is no doubt that the introduction of the complex quantity into alternating-current calculations introduces a simplicity which is remarkable in certain types of problems, but certain other types of problems should not be treated by the complex quantity, and this is one of the types. The man who uses complex quantities in the treatment of the theory

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*Discussions presented both at Pittsburgh, April 10 and New York, April 12, 1918.

of the induction motor commits an unpardonable crime, he ought to be interned as a dangerous individual—he should stick to the old fashioned method of using the real quantities in connection with real problems, whenever he can.

It is not easy to teach a student to use a typewriter before he learns how to write. It would be a dangerous thing to introduce it into the public schools, before the boys and girls knew how to write, and the same thing is true in the electrical engineering class room. To introduce complex quantities before the students know how to handle real quantities is dangerous and misleading, and the students themselves do not like it. I never use a complex quantity, until I come to the power transmission lines, and the problem of the distribution of alternating-currents in a network of conductors.

In this problem you cannot use it. When it comes to a problem where there is a single-phase winding in the primary and a single-phase winding in the secondary, God help you, it is a most complex problem, and I warn you not to be misled by your curves, because if you experiment you will find that you do not get periodic curves at all. The theory of the alternator is not the converse of the theory of the single-phase induction motor. In the alternator you have a single-phase winding in the rotor, and you have a single-phase winding in the stator, but in the induction motor you have a tremendous number of circuits in the rotor. In the alternator, the field is saturated so that the magnetic reaction in the primary produced by the armature current does not amount to much. But if you should try an alternator with a laminated field, and have a few turns only in the field winding, you will get electromotive force and current curves which seem to baffle all understanding. When the primary is fed by an alternating current you do not get periodic curves at all, because, although in the primary you impress a simple harmonic alternating electromotive force, you get an infinite series of additive and differential frequencies. I observed them in some of the results shown tonight, when you have a single-phase winding in the secondary of the induction motor.

This is a case which has been handled for us by Goldsmith in his famous alternator which is used in wireless telegraphy, where he impresses 10,000 periods per sec. on the primary and gets 40,000 or 50,000 periods per sec. in the secondary by reflections as he calls it. It is a very interesting case, but it has no practical value except to make high-frequency alternators without resorting to excessive number of poles and excessive speed.

I warn the gentleman who presented the second paper, Mr. Hellmund, to proceed rather slowly. He has given a most complicated case, and I am sure that he will get into a mess, into a labyrinth of all kinds of complications, if he does not stick to the single-phase induction motor with sinusoidal distribution.

**Selby Haar:** When I began to study precision of measurements I was told that there was a great deal of difficult mathemathics in the theory, but that after I had learned the theory I could forget the mathematics, and the actual process of using the theory would be very simple, and would require nothing more than a knowledge of arithmetic. The circle diagram may be very much simpler than the method of complex quantities for purposes of instruction, but when I was a commercial designer of induction motors, we had to spend quite a large amount of time in educating inspectors and others who determined motor performance curves by the circle diagram, and naturally obtained different results than we did. The theory of complex quantities may be difficult, but the calculation of induction motor curves by this method is as simple as the application of the rules of precision of measurements, and, at the same time, is accurate. With a slide rule a whole set of curves covering the usual range from one-half load to one and one-quarter load may be completed well within an hour. In the circle diagram the most important points are near the end of the horizontal diameter, where the errors of measurement are greatest.

**B. G. Lamme:** It has been brought out in the various discussions that the method given in my paper is only an approximate one. That is correct and it is so stated in the paper itself. However, certain of the numerical tables and curves in the paper show how small the error is. If, in order to avoid minor approximations, I had introduced the complications shown in some of the discussions, it would have spoiled the paper for the purpose for which it was written. The intention was to give a simple physical conception of the motor and to do so it was necessary to make certain approximations as stated before. It may be said that none of the discussions this evening have simplified the subject in the sense of making it easier to comprehend.

As to the different methods of looking at the problem of the induction motor, I have tried this experiment,—I have taken three different men and put up to them certain new problems of induction motors and then asked them what was the first thing they thought about. One man said that the first form in which the problem presented itself to him was in the nature of an equation; the second man said that the first thing he thought about was a circle diagram; the third man said that his first thought was about the distribution of flux in the machine and the currents in the windings. If these three men had equal mathematical ability, I believe the one with the physical conception of magnetic flux and the currents could tackle the problem better than the others. In general, I have found that a man with good physical conception and fair mathematical ability can tackle new problems most successfully. Consequently, I have made a practise, with the young men under my charge, to try to teach them the physical conception first of all and then take up the mathematical aspect. I find, as a rule, that they do much better after they get the physical conception.

My purpose in preparing this paper was not to assist that small group of people who have a very good grasp of the fundamentals of the single-phase motor, but to meet the needs of the very much larger class of people who are interested in such apparatus, but who cannot read mathematics very well and do not know what a circle diagram means, or what a vector diagram is. With many such pepole, if an equation is encountered when reading the paper, they will not study it out and, furthermore, if there are one or two equations on a page they will not even read that page. This applies to a very large percentage of the readers of the TRANSACTIONS, and I find that if a subject can be so treated as to leave out all formulas, etc., a great many people will read it who would not otherwise do so.

I wish to add that apparently there has been some misunderstanding as regards the intent of this paper. It was not the intention of the writer to claim anything new or novel in the general treatment. Many of the various methods used in this paper have been brought out long ago, some of them so many years ago that I do not recall where they originated. Mr. Behrend would be a much better man to give this than myself, as he brings out a number of these points in his book on the induction motor, published some eighteen years ago, I believe. The only credit I can claim in regard to this paper, is for the attempt at simplification, to bring it within the range of non-mathematical readers.

**Mr. Hellmund:** It has been brought out quite clearly in my paper that it was not intended as material for the classroom and for giving to beginners a physical conception of the working principles of a single-phase motor. It was rather intended to answer certain questions to the designing engineer and advanced student, who have raised such questions from time to time regarding secondary magnetizing currents, losses, etc. Unfortunately, it was found simply impossible to get all the results I was after without quite a good deal of mathematics.

Mr. Slepian has supplemented my paper by a very interesting discussion, showing how it is possible for an engineer familiar with higher harmonics and some other characteristics of the motor, to arrive without mathematics at one of the conclusions arrived at in the paper, namely, that a sinusoidal field distribution prevails under certain assumptions irrespective of the primary winding distribution. While this fact was known to me for a considerable number of years from certain investigation of the polyphase motor,* I knew little about the influences of the winding distributions upon the currents, and the like, until I figured them out by the use of mathematics. Only after thus definitely establishing certain facts, I was able to devise the

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*See my paper A. I. E. E. 1908, p. 1373, in particular p. 1382. Also Articles in *Electrical World*, 1906, p. 329.

graphical method described in connection with Fig. 34 of my paper for determining the secondary magnetizing currents of commercial machines.

While the treatment of practical cases is given much in detail in my paper, it is evident from the discussion that this has been overlooked by some of the speakers, who have apparently paid too much attention to a few rather theoretical cases, which I have given for reasons previously stated. Referring, for instance, to Prof. Pupin's brief discussion of my paper, it should be pointed out that the case of sinusoidal primary distribution has been treated in full both under consideration of leakages and resistances. This part of the paper alone would, however, not give any answer to all questions arising in connection with commercial machines. The mere fact that the treatment of the latter is more complicated does not make the knowledge of what actually happens in practise less desirable. It was just on account of the peculiar phenomena caused by the non-sinusoidal distribution of windings in slots that designers have been puzzled from time to time and unable to explain, for instance, why certain losses were much higher than expected. I fully agree, however, that the treatment of the more complicated cases requires great care to avoid confusion and wrong conclusions. I am, therefore, very grateful to Messrs. Dudley and Weber for the trouble they went to in checking my methods by actual tests and thus demonstrated their correctness. They deserve much credit for removing thereby any doubts, which are only too natural with some of the peculiar results arrived at in my paper.

*Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.*

# A REVIEW OF ELECTRICAL ENGINEERING PROGRESS

## PRESIDENT'S ADDRESS

BY E. W. RICE, JR.

ELECTRICAL engineering now covers such a wide field of scientific and technical activities that your President is presented with an embarrassment of riches in the attempt to select a subject for his address. He has, therefore, decided not to confine himself to any one feature of our Society's work, but to pick out here and there a few of many items to talk about which seemed of present interest and importance.

It is a pleasure to call your attention to the fact that we have added 1235 members of all classes during the year, our total membership being now 9370. This is a most encouraging result in these times of stress and change.

Our Institute is a national asset of increasing value to its members and to the nation. Every person who has the necessary qualifications should identify himself with the Institute, not only for the great benefits which he personally will receive, but in order that the usefulness and power of the great army of electrical engineers may be increased and rendered more available for the highest and most efficient service to our country and to the world.

The engineer is the hope of the nation, not only now, when we are at war, but even more so in the future in the days of reconstruction following the great peace. The engineer may perform valuable service when working alone, but his usefulness and power is manifestly greatly increased when acting in co-operation with thousands of his brothers.

The Institute is not only a democracy but is a democracy of educated men and such men have a heavy responsibility to society at present and will have in the future. They should be leaders and exemplars for those who have not been so fortunate as to have enjoyed their opportunities. The Institute needs its members and the members need the Institute, that the electrical engineer may fulfill his high destiny.

That the work of the Institute is of the highest quality is evidenced by the character of its meetings, its papers and discussions and the splendid work of its various committees. Its value and usefulness has increased every year of its existence and it should continue to gain in strength and usefulness because its methods are in accord with the spirit of the times.

But in order to accomplish this desirable result, it must continue, as at present, to be representative of the electrical engineering profession of the country, and therefore, must continue to expand its membership. This growth will bring with it problems inherent in all great institutions, democratic or autocratic, but I have confidence that all difficulties will be met successfully for the reason that the members of our profession are trained in the scientific view-point and methods of solving problems.

In the early days the progress of the electric science and arts was so rapid that it was relatively easy to find each year plenty of material for a review. Progress has continued and will continue, but naturally a decided tendency to saturation is shown in many directions. In some instances, this saturation can be demonstrated to be due to the fact that limits of perfection have been so closely approached that little remains of possible accomplishment. In other instances the slowing up is due to lack of knowledge, or, especially at the present time, to lack of workers, such workers having been diverted to the work imperatively needed to secure us against the attack of our enemy on the foundations of our existence.

There has been no material improvement for several years in the matter of efficiency in electrical units, such as dynamos, motors, transformers, etc. The efficiencies stated in Past-President Lincoln's address, in 1915, still remain almost exactly of the same values, and for the reasons which he so clearly pointed out.

The efficiency of conversion of mechanical into electrical energy, or the reverse, of electrical into mechanical energy, is still about 90 per cent in the average case, under practical conditions of operation; the efficiency reaching as high as 97 per cent or 98 per cent in the most favorable cases, with the large units, and falling below 90 per cent in unfavorable cases, or in the small units. The efficiency of conversion of electricity from high to low potential, as in transformers, also remains substantially the same, reaching as high as 98 plus per cent in the largest units. It is obvious, as Lincoln pointed out that no material change can be expected where such practical perfection has been reached.

The conversion of mechanical power of falling water into electrical energy by our water-wheels and electric generators has increased from about 87 per cent to 90 per cent in the largest units of 40,000 h. p. This represents about the limit which may be expected.

In the field of thermodynamic engines, represented largely by the steam turbo-generator unit, some improvement has been obtained. Lincoln stated that 75 per cent of Rankine efficiency had been obtained in some large modern steam turbo units in 1915. This has now been increased to about 80 per cent in the largest units of 35,000 to 40,000 kw. and 75 per cent is quite common practise even in such moderate sized units as 10,000 kw. This improvement, while not large, is doubly important because of the great increase in the cost of fuel. It has been realized mainly by bringing the practical design more nearly in accord with the theoretical, by increasing the number of stages or processes of steam extraction, reducing various losses, and by improving many details which, when properly looked after make in the aggregate, gains of practical importance.

Increase in the initial pressure of steam and lowering of terminal pressure, by better condenser arrangements, have also contributed to improvement, as it enables an increase in the range of temperature to be utilized. This makes possible better thermal efficiencies, even with the same per cent of Rankine efficiencies.

The following information illustrates the improvement in efficiency of turbo-electric units beginning with the first 5000 kw. installed in this country, in 1903, and continuing up to the close of 1917:

| Year | Size, kw. | Steam Conditions | | | Lbs. per kw-hr. | Per Cent of rankine efficiency |
|---|---|---|---|---|---|---|
| | | Steam pressure | Superheat fahrenheit | Back pressure | | |
| 1903 | 5,000 | 175 lb. | 0 | 2 in. | 24.00 | 37.8 |
| 1908 | 14,000 | 200 " | 125° | 1½ " | 13.50 | 66.1 |
| 1911 | 20,000 | 235 " | 100° | 1½ " | 13.20 | 67.0 |
| 1913 | 20,000 | 200 " | 200° | 1 " | 10.74 | 75.9 |
| 1916 | 20,000 | 250 " | 250° | 1 " | 10.00 | 76.5 |
| 1917 | 35,000 | 230 " | 200° | 1 " | 10.14 | 78.7 |

It is gratifying to note that a percentage of Rankine efficiency of approximately 80 has been reached. This progress reflects great credit upon the designers of turbo-electric machines and is a record of achievement found only in electrical development.

Concurrently with this improvement in the turbo-electric machines, great advances have been made in the design and operation of steam producing devices—the boilers, and in auxiliaries and other features of the modern power station. As a result the thermal efficiency has been rapidly improved. The thermal efficiency to which I refer may be stated as the ratio of the total energy produced at the terminals of the generator, to the total energy in the fuel burned — expressed as a percentage. It takes account of all losses from the coal under the boiler to the electricity at the dynamo terminals. It is the ratio of the heat units equivalent to one kw-hr., divided by the similar heat units in the fuel consumed to produce one kw-hr. at the generator terminals.

This thermal efficiency is after all, to the electrical engineer, the most important measure of progress. It measures the advance in station fuel economy, and as stated, many factors in addition to the improvement in turbo-generators have contributed to the result. Thermal efficiency may obviously be used to express the results of a single unit, consisting of turbo-generator, with its bank of boilers and other accessories, or it may be used to designate the combined result of all the units in a given power station.

The progress in the case of a combination unit, *i.e.* turbo-generator, with its boilers, auxiliaries, etc. has been as follows:

| Year | Size of unit kw. | Thermal efficiency per cent |
|---|---|---|
| 1903 | 5,000 | 10.15 |
| 1908 | 14,000 | 15 |
| 1913 | 20,000 | 18 |
| 1917-18 | 35,000 | 21.6 |

For comparison, I may state that large gas engines in steel mill practise, under best test conditions, show 25 per cent thermal efficiency, but in actual operation, an efficiency higher than 18 to 20 per cent is rare.

High compression oil engines of the Diesel type, driving electric generators, realize 25 to 26 per cent thermal efficiency when new, but are difficult to maintain at such efficiency.

The figures given must not be confused with the much higher thermal efficiencies often quoted for gas and oil engines, which refer to indicated horse power and not to electrical output.

The steam turbo-electric unit has not reached its limit of thermal efficiency. Calculations show that, with pressures of the order of 500 lb. gage, a thermal efficiency of 26 per cent should be easily realized. For any further substantial improvement, we must look to new methods, such as the use of two fluids, for example mercury and steam, as planned by Mr. W. L. R. Emmet. This method is still under development but its progress has been hampered by the pressure of war work.

As a matter of interest to electrical engineers, I may say, parenthetically, that the steam turbine in this country owes its existence and development almost entirely to the electrical engineer, and this is not surprising as the electrical engineer was familiar with the advantages of rotary machines, and perhaps it is not too much to say, prejudiced in their favor.

While, as stated, the efficiency of electrical units reached about its limit some years ago, those familiar with electrical engineering development are aware that progress has been made and is still possible in the generation, transmission and utilization of electrical energy. The struggle for improvement in efficiency has been transferred from the unit to the aggregate, called the system. We cannot have a system of maximum efficiency without units of maximum efficiency, but individual units of highest efficiency do not, of themselves, insure that the system upon which they are used will be of the highest efficiency, so progress has been made in the direction of improving the system economy or system efficiency.

To obtain the highest efficiency in practical operation, the element of time enters as a powerful factor. Our conception of efficiency should not be limited to a consideration of the relation between the instantaneous value of available heat units in coal and the electrical units produced at the point or points of consumption, but should consider the relation between the total number of heat units in fuel consumed in a given time, say 24 hours, to the total number of electrical units produced and used in the same time. The attempt to improve the efficiency of the system has shown the necessity for utilizing the generator units and transmission and distributing systems, for the maximum possible time.

This has led to the study of such questions as load factors of generators, of stations, and of the system as a whole, to the study of the diversity factor, to the reduction of idle currents in alternating-current systems by the use of synchronous condensers, and to means for the reduction of the constant and no-load losses in all machinery, in transformers, etc.

The resulting improvement has been effected, not only by changes in designs of the units themselves, but also by their method of use, based upon the recognition of the fact that the elimination or reduction of the losses at light load will greatly improve the total efficiency, especially when the time of use of the apparatus under load is a small part of the total time.

Automatic substations for transformers and synchronous converters have come into existence; different power houses of the same system have been tied together electrically; transmission lines of different systems have been interconnected, so that the units may be usefully employed for the maximum period, or lie idle or unloaded for the minimum time.

This general development has led to marked improvement in total energy efficiency, represented by the amount of fuel burned per electrical unit sold or utilized, and has also reduced cost of operation and charges for investment. There is still room for continued improvement in this direction and the progress will be rapid due to the pressure for maximum efficiency in the use of coal and of existing investment at the present time.

Many interesting examples of the methods and devices adopted to improve station and system economy and efficiency may be found throughout the country. In California, large electrical systems have been arranged to be tied together electrically, for exchange of power. In Washington and Idaho, power systems under different management have made similar arrangements. In the South, all important hydroelectric systems have been tied together for exchange of power. The advantage, as I have stated, of such arrangements is better utilization of variable stream flow, improvement in load factor, increased reliability of service, and the net result is to improve the efficiency of the system, not only financially, but in a purely technical sense. One most important advantage is the obvious reduction of the necessary investment in reserve machinery of every description.

In Montana, eight hydroelectric plants successively use the same stream flow, the total effective head amounting to 600 feet, and not only is the natural flow of the stream thus successively utilized, but all the storage water is effectively used by each plant in series. In this same system, the yearly load factor is stated to reach 75 per cent and the mean monthly load factor to reach 80 per cent.

The interconnection of hydroelectric plants brings about another extremely important saving, based upon the variation of rainfall in amount and time on the different watersheds which are thereby brought to serve a common system. It frequently happens that there will be plenty of precipitation on one watershed, while another watershed may suffer from long continued drought. This condition varies not only in the same year but in different years. Interconnection serves to eliminate these variations by a process of averaging, and where the inter-connected system covers a sufficiently wide area, a remarkable increase in total useful power is made available.

It has frequently happened that thousands of horse power have been wasted over the dams of one system, the watersheds of whose plants happened to have a wet year, and at the same time, a nearby hydroelectric plant, supplied by another watershed, was without water power. The result has been that one system wasted power, while the other was suffering from a power shortage which would frequently be made up by burning a large amount of high grade coal, in the operation of an auxiliary steam plant. This condition has to a large extent been remedied by the interconnections to which I refer.

It has been estimated, and it seems a conservative estimate, that through the saving in reserve equipment, improvement in load factor, and the diversity of different loads, the useful output of groups of large systems may through inter-connection be increased about 25 per cent.

Electric regeneration of power, that is, the utilization of the weight of trains running on a down grade due to the force of gravity to generate electricity which is fed back into the electric system to help other trains up grade, is an illustration of the same important improvement in the system efficiency.

I have thought it desirable to call your attention to the improvements obtained in system economy or efficiency because of the important savings in investment, in coal, in transportation, in labor and material, which in the aggregate, have already been realized. It illustrates the wonderful flexibility, value and economy of a general system transmitting energy by electricity, compared with any other possible method.

These advances have been more rapid during the last year, due to the imperative demands for economy saving and increased efficiency imposed by the war. It is a great satisfaction that the foundation had all been well prepared during the times of peace.

The development of our industry has been so rapid that the need of intelligent and constructive standardization was realized some years ago. The Standards Committee of the Institute, formed in 1898, has been of inestimable value to the profession and to the industry. The standards adopted have been flexible enough to ensure progress and yet to discourage variations which were valueless. The standards promulgated by our committee have so appealed to the profession and to the industry that they have been cheerfully followed, and I am convinced that, as a result, the cost of electrical apparatus to the consumer has been greatly reduced over a number of years and the quality has not, been sacrificed, but has been improved. I consider that the money value of the work so done could be conservatively placed at many millions of dollars.

Sixty-cycle systems have shown, during the past few years, a more rapid growth than 25-cycle, and it is now estimated that 60-cycle systems represent about 70 per cent of the total power supplied in the country. This is undoubtedly due to the lowered cost of transformers, generators, induction motors, and similar apparatus. The relative growth of 60-cycle as compared with 25-cycle systems is reflected in steam turbine installations. In 1910 about 60 per cent of the steam turbine electric energy of the country was supplied from 60-cycle units; in 1917, this had risen to approximately 75 per cent.

This is an instance where standardization is desirable and economical. It will hasten the time so often predicted, when a network of transmission lines, carrying electrical energy, will cover the country. These will be fed by super-power stations, suitably located with respect to cheap reliable supplies of coal for fuel, and water for condensing purposes, and into the same network will also be fed energy from the various hydroelectric installations.

Marked advances have been made during the past year in the application of electricity to the electric furnace. It is estimated that the number of electric furnaces in the United States has been increased about 40 per cent in the past year and that there are now in operation over five times the number that existed five years ago. The world's output of steel from electric furnaces has now grown to approximately four million tons per annum.

Experience has demonstrated that the electric furnaces can utilize the cheapest and most inferior raw material to produce steel of the most uniform and highest quality, with the greatest regularity. The cost of steel so produced, while reasonable, considering its quality, was higher, until recently, than that produced by the open-hearth method. It is now possible to produce electric steel at substantially the cost of that produced by the open-hearth method. This result has been brought about partly by the increased cost of the open-hearth method, due to a variety of well known causes, but largely by a reduction in the cost of electric furnace operation. The marked change which has taken place in the reduction of the cost of operating electric furnaces is based upon greatly increasing the rate at which energy is delivered to the metal, both during the melting and the refining period. This has reduced the time required for an individual heat and also the kilowatt hours required per ton of metal melted, with a net result of increasing the daily output of the furnace.

As a concrete example, I mention the history of a five-ton furnace. It was originally supplied with 800 kv-a. at 80 volts. This was increased to 2000 kv-a. at 150 volts for the melting period and about 1400 kv-a. at 100 volts for the refining period. The time for the heat was reduced from six to three hours, power consumption was reduced from 877 kw-hr. to 588 kw-hr. per ton, and the number of heats per 24 hours was increased from three to five, increasing the net output from 15 to 25 tons.

Electric resistance furnaces of large sizes, for special heat treatment requiring unusual exactness, are being extensively used, producing results greatly superior to oil or gas fire furnaces.

Electric welding, both by the arc and incandescent method, is being rapidly extended and is destined to greater development in ship-building and similar operations.

Electric engineers have been devoting much time to the solution of many war problems. It is not desirable or possible to review such work at present, but when the veil is lifted, we will all be gratified with the result. We must content ourselves with the mere statement that this work has covered means for the detection of the pirate submarine; wireless signalling and telephoning for army and navy, and aircraft devices; searchlights of novel design and great power; improved methods in manufacture of ammunition and ordnance; electro-chemical work of every description; electric welding; X-ray sets of greater simplicity and accuracy; and many other lines too numerous even to mention.

The great industrial research laboratories, the educational and governmental research departments have all co-operated enthusiastically and effectively, and the members of their staffs have labored day and night, without regard to pecuniary reward or public applause, sustained entirely by the high purpose of giving their best to the service of the country. I hope the time may come when the story may be told, so that the world may realize the debt which it owes to scientific men and engineers, without whose arduous, unselfish and almost inspired work, our cause, righteous as it is, would have no chance of a victorious conclusion.

In my address at the opening of the mid-winter convention of the Institute, in February, 1918, I called attention to the advantages which it seemed to me would follow a more general electrification of the steam railroads of the country. I merely repeat at the present time that electric locomotives have been so improved and simplified that they are competent to haul the heaviest train that can be held together with the present train construction; to operate at the highest speed permissible by the alignment of the road and independent of its grades; and that the electric locomotives can meet in the most efficient and adequate manner the transportation problems confronting the country, and offer better results than are now obtained or seem possible with steam locomotives.

There can be no question that railroad electrification is not only economical but imperatively needed to improve the present standards of steam operation. Our mountain districts are congested almost entirely by the limitations of the steam railroad systems, and the addition of more tracks, under such conditions, is not the best solution of the problem. The electrified divisions of the steam roads have been free from troubles during the past severe winter and I repeat that the coal famine which the country suffered last winter could have been largely avoided if the steam railroads had been electrified. Moreover, it should not be forgotten that steam locomotives burn about 25 per cent of the entire coal mined in the United States and that 12 per cent of the entire ton mileage movement of freight and passengers carried over our railroad tracks is represented in cars and tenders required to haul coal to supply steam for the locomotives.

It is a truism, which has been frequently stated, that war requires the mobilization of the nation's industries and their devotion to essential work. This is especially true in this country, as it has been necessary in addition to create substantially new industries on an enormous scale, such as the production of ships, ordnance, ammunition, airplanes, chemicals, etc. To operate these industries, it has been necessary to mobilize to the fullest extent our available material and labor, but material and labor can only be converted into war work by the application of power. This power, in view of its great economy and flexibility, must be electrical.

While this country was fortunate in having available a magnificent system of power stations, so great was the magnitude of the demand for increased power, created by the war industries, that it is estimated that there will be a shortage of at least 500,000 kw. of electric power in the Eastern district.

It takes from one to two years to build and equip the large units which are essential for the production of such power. This illustrates the importance of all of the methods which I have mentioned to conserve, utilize and increase the efficiency of existing equipment and investment, as such methods can produce results in a much shorter time.

It is, however, vitally important that the great electrical power producing companies of this country should be helped in every way to meet the heavy demand which is placed upon them. It has been demonstrated that the quickest, most efficient, and altogether best way to meet the demand for power is through the expansion of such existing organizations and installations.

Fortunately, there is general appreciation of the fact and comprehensive schemes are under consideration which will provide for the erection of large steam electric power stations in the mining regions. Favorable locations exist which are within reach by transmission lines of electric power stations now serving large industrial areas. By interconnection, present investment and machinery will be better utilized and a large amount of additional electric power made available, without making any increased demand upon our congested railroad facilities.

It is evident, therefore, that we need to consider and put into effect, every practical method for conserving our existing developments, and also, we should take a courageous view of the future; we should provide, for the future growth at least as liberally as has been the custom of the managers of the great public service systems in the past. It has been their custom to build from two to three years in advance of existing requirements, in anticipation of the future. I have yet to learn of a single important instance where such foresight has not been amply justified.

I would say in conclusion that the saving in fuel, by such improvements as I have mentioned in various parts of my address, amounts to many millions of tons every year; the saving in material and investment represents millions of dollars, which manifestly represent service of the highest value to the industry and to the country. Such work is just as much the province of the electrical engineer as improvements in the design and efficiency of the electrical units, and requires the same scientific ability, vision and industry.

While I admit to considerable prejudice in favor of things electrical, I think that in no other field of engineering has there been such a remarkable improvement and a condition which so nearly approaches, in the matter of efficiency, to 100 per cent, as has been shown in the field of electricity. This phenomenal record is not the result of accident. It has been due to the enthusiastic devotion of the scientist and engineer and executives to their work. They have not been satisfied with things as they are, or with mediocrity. They have wanted the best; have not been contented with a 75 per cent to 80 per cent efficiency when something better was obtainable. The causes of inefficiency have been scientifically attacked; the losses have been studied and their causes discovered and removed.

Concurrently with the improvements in the efficiency of conversion, the engineer has studied ways and means to reduce the amount of material and the amount of labor required to produce a given effect, and has been equally successful in increasing the effective use of material and labor, and as a result, until interrupted by the war, the cost of electrical machinery and devices of every description has shown a progressive reduction, not only without sacrifice of quality, but with great improvement in quality. This truly marvelous work, we can safely affirm, is the foundation of the phenomenal growth, prosperity and present commanding position of the electrical industry, which is a monument to the broad vision, intellectual honesty, faithful work and the correct economic viewpoint of the electrical engineer and his co-workers.

*Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.*

# ANNUAL REPORT OF TRACTION AND TRANSPORTATION COMMITTEE

*To the Board of Directors,*

The past year has been one of unprecedented difficulties for all railway transportation companies. With labor conditions at their worst, the maximum tonnage ever offered, rolling stock and motive power sadly deteriorated due to a long period of lean income—all added to a winter of unprecedented severity, it is little wonder that the trunk lines practically collapsed under the strain. The collapse of the steam railways, however, did not extend to electrified lines, which once more demonstrated the superiority of electricity as a motive power, particularly in cold weather. This was especially true on the electrified steam railways, such as the Norfolk & Western, and the Chicago, Milwaukee & St. Paul Railways, the latest railways to be electrified; and the Pennsylvania, New York Central, New Haven Railways, and others. These roads, one and all, operated successfully through the hardest weather. The record is one of which all electrical engineers may be proud.

The advantages of electrification are now almost universally recognized. The main features are:

1st, Increased capacity.
2nd, Economy in fuel consumption.
3rd, More reliable service.
4th, Greater safety in operation on mountain grades and through tunnels.
5th, A higher class of service.

The first two advantages were especially emphasized by President Rice in his masterly plea before the Midwinter Convention for the electrification of railways as a war measure, and the Committee urges the most careful consideration of this matter by the Government and the railways.

There are many places where electrification will show an actual economy in operation, especially on lines such as the Norfolk & Western and in the Broad Street Terminal of the Pennsylvania Railroad. In both of these cases, the capacity of the railroads had been reached with steam, and it would have been utterly impossible to handle the traffic which they have had in the past year with steam locomotives. It is impossible to estimate or to appraise all of the advantages of electrification. Electrification will eventually be adopted because the character of service will be so much better that the railways cannot afford to do otherwise. During the war, however, the amount of electrification work done will probably be limited to places where it is absolutely essential, or especially advantageous, in order to facilitate handling the traffic necessary for the proper prosecution of the war.

In this connection, it is interesting to note that the C. M. & St. P. Ry., in continuing its electrification work with 3000 volts d-c., is electrifying the two engine divisions from Seattle east, which will give them a total of approximately 650 miles of electrified main line. Work is progressing on this new line very rapidly, and it is expected that operation will begin early in 1919. It is reported that this line will be the official route from Chicago to Seattle.

The Pennsylvania Railroad has recently put in electric operation the Chestnut Hill division of the line running into Broad Street Station, practically duplicating the single-phase equipments on the Paoli division.

In street railway circles, the past year has been one of great hardships. The long continued cold weather, combined with the heavy falls of snow, pyramided the troubles and left the street railways in many cases almost as bad off as the steam railways.

A number of things combined to produce this result. First and foremost, is the general labor and material situation which makes it difficult, if not impossible, to secure good maintenance. In addition, the street railways have been working for several years under increasing financial difficulties, so that in many cases the equipments were in no condition to stand the extra strain. This led to a very unusual number of break-downs when the severe weather was encountered, and the long continued cold gave no opportunity to recuperate.

Another feature which contributed in a greater or less degree was the light weight campaign which has taken such a strong hold of all of the railways of the country. Ten years ago there was great need for this campaign. The standard street railway equipment consisted of nothing less than four 40-h.p. motors, which, being of the non-ventilated type, were comparatively heavy. Very few people had paid any particular attention to the weight of railway equipments, either of motors, car bodies or trucks. Consequently, the weight per passenger was very high on practically all street railways. The idea became prevalent that it costs a railway 5 cents per lb. per annum to haul their equipments around, so that it has been the end and aim of practically every man having anything to do with the design of cars and equipments, to cut the weight. This has been done in a more or less scientific manner, but like all campaigns of this kind, the pendulum has swung past the limit in some respects. probably due to the fact that it is impossible to recognize the limit until it has been passed and trouble encountered. Fortunately, this has not been sufficiently widespread to do more than point to the limit.

Possibilities for trouble with light weight motors have been approached from four angles: First, open ventilation leading to the introduction of snow into the working parts of the motor, with resulting danger to insulation; second, higher armature speeds, leading to more rapid deterioration of armatures and wear of bearings; third, reduction of weight by increasing the stresses in material; fourth, the danger from overloading due to lack of sufficient thermal capacity to withstand abnormal loads.

The ventilated motor has come to stay. Its advantages are too many to give up because of the small amount of damage resulting from snow. The logical thing to do when there is danger from this, is to put tight covers over the motor openings in the winter time. The additional margin due to the lower ambient temperature in the winter will, in practically all cases, be sufficient to keep the motor temperature within safe limits.

Higher armature speeds should be approached with conservatism, taking care that the armature is sufficiently substantial to withstand the additional strains, and that bearings are of liberal dimensions, with adequate lubrication.

The use of high-grade steel is recommended, but is preferred as an additional factor of safety, rather than to get minimum weight. It must be remembered that it is not always possible to secure special grades of material for making repairs.

Trouble from overloading with the light weight motors has come as a very disagreeable surprise to operators. It is subject to careful analysis and the reasons are easily understood. The chief reason is that the limitations of the ventilated motor are not generally understood.

The trade has been educated to believe that the continuous rating of the railway motor is the one that determines its service capacity; and that if its continuous rating is equal to the integrated loads in service, it will be ample to perform the work. This method of selecting motors was quite satisfactory with the old non-ventilated type of motors, where the motor had sufficient thermal capacity to absorb the losses generated in the short applications of heavy loads, without reaching abnormal temperatures. The modern highly ventilated motor, however, has relatively a very high continuous rating, as compared with the one-hour rating. The latter, as is well known, is really the gage of the thermal capacity of the motor and of its capacity for handling heavy intermittent loads. It will be seen at once that when the motor that is selected for its continuous rating is required to develop four times this load for a few minutes under some abnormal condition, a non-ventilated motor would be loaded to only about 60 per cent above its one-hour rating, while the highly ventilated motor would be loaded to two and one-half or three times its one-hour rating, resulting in a much greater rise in temperature. Where this abnormal load is applied at low speed, as is apt to be the case, the trouble is further accentuated, due to the decreased ventilation secured with fan-cooled motors.

The logic of this situation is simply that motors of the ventilated type for a given service will require a higher continuous rating than one of the non-ventilated type. Due regard must be taken to the capacity for short-time over loads in order to avoid reaching a dangerous temperature under abnormal conditions.

There seems to be a distinct tendency towards more conservative selection of equipments for street car service, since it has been definitely established that the cost of maintenance of motors which are worked beyond their capacity, added to the cost of unreliable service, is so high as to off-set any possible saving resulting from the lighter weight. This is also leading to a return to four-motor equipments, simply because of their greater reliability under abnormal conditions. It is hoped, however, that any return along these lines will be taken with the greatest care and with the maximum utilization of the experience that has been secured up to date. Having been at both extremes, it should now be possible to adopt a mean position which will give the very best results.

Great savings in energy can be effected without cutting the weight of the equipment, by attention to improved methods of operation and, especially, by the further use of field control. It is hoped also that eventually it will be possible to make use of regenerative control for elevated and subway equipments, at least. This has worked out so satisfactorily on mountain grades that its extension to car equipment seems very desirable.

The activity of the Railway Committee in the past year has been at a low ebb, largely due to the fact that everyone in any way connected with transportation business, has been too busy to take up much committee work. It is hoped that another year may see an improvement in this respect, so that a constructive programme may be carried out.

N. W. STORER, *Chairman.*

## ANNUAL REPORT OF THE COMMITTEE ON ELECTRICAL MACHINERY

*To the Board of Directors,*

The Committee on Electrical Machinery submits the following report for the year of 1917-18.

At the first meeting of this new committee called in October to discuss the policy to be adopted, it was found that industrial conditions were such as to make full and regular meetings an impossibility, and that the activities of the Committee would therefore have to be restricted. There was, moreover, a feeling among those present that it would be difficult this year to secure papers from the men in the industry, and it was decided that the Committee limit its activities to the presentation of the subject of "Polyphase Commutator Motors," and to be prepared if necessary to present a critical review of "The Development of Turbo-Alternators". Contrary to our expectations, however, an unusual number of papers were presented for consideration and the Committee was kept busy sifting out the desirable material.

It was clear from the discussion of the three papers on "A-C. Commutator Motors" that little is known about the possibilities of the polyphase commutator machine, and that the available literature in English is so meagre and the methods of attack of the various writers so radically different that it would seem to be the duty of the Institute to find someone who knew the subject and who also knew how to present it to work up the existing material into a comprehensive treatise for publication in the TRANSACTIONS.

We believe that one of the principal functions of the technical committees is to keep the literature of their subject in good shape. The manufacturers will see to it that new developments are brought to the attention of the members. In line with this point of view the Committee has had a critical bibliography prepared on the subject of "Unbalanced Pull in Electrical Machinery" to be attached to a paper to be published in the PROCEEDINGS this summer.

In the April PROCEEDINGS there was published a paper by Mr. Lamme which created considerable discussion. Mr. Lamme had found that the graduates of our universities have not a clear conception of the operation of single-phase induction motors, and he wrote a paper giving a method of treatment which he had found satisfactory for the graduate students of the Westinghouse Co. Such papers, although they do not present new material, are of special interest to the younger members of the Society and might well be presented under the auspices of the Educational Committee. From the discussion of Mr. Lamme's paper it was obvious that there were decided differences of opinion among teachers and practising engineers as to the relative advantages of vector diagrams, equivalent circuits and complex quantities from an educational point of view.

While the chief function of the technical committees is to secure papers in their respective fields, The Committee on Electrical Machinery considers that it might well be used as a clearing house for suggested changes to the Standardization Rules, a place where suggestions dealing with machinery might be thoroughly thrashed out before being submitted to the Standards Committee.

ALEXANDER GRAY, *Chairman.*

## ANNUAL REPORT OF THE LIGHTING AND ILLUMINATION COMMITTEE

*To the Board of Directors,*

I beg to submit on behalf of the Lighting and Illumination Committee the following report for the year 1917-18.

It is with regret that the Committee finds it necessary to report that again this year it did not seem feasible to hold a session of the Institute for the consideration of papers on illumination. The Committee held a meeting early in the year and proposed the following subjects as suitable ones for presentation before the Institute:

(1) Intensive and Ornamental Street Lighting, as projected for a number of cities in the South and Southwest.

(2) A general discussion of Industrial Lighting Codes;

(3) A discussion of Standardized Methods of Lighting Cantonments, Aviation Fields, etc., provided the report of the I.E.S. Committee on this subject will be available for public presentation.

The Chairman was authorized to make an inquiry regarding the possiblitiy of securing papers on one or more of these subjects and report the results subsequently to the Committee, but following a canvass of the situation it was found impossible to arrange such a program. Consequently, any thought of requesting one of the sessions of the Institute to discuss papers on illumination had to be abandoned. When conditions once again become normal, it will be possible to provide interesting programs on this aspect of electrical engineering, but for the time being it would seem that the Committee can do no more than remain intact and wait.

A brief summary of progress in electric illumination during the past year is appended.

### *Progress in Electric Illumination*

The general trend of practise for direct lighting is very decidedly toward units of low brightness. .The extended use of the high-powered incandescent lamps has stimulated the appreciation of good diffusing devices which will give satisfactory light distribution but by their low brightness minimize glare. The enormous increase in commercial activities, particularly in those lines which are connected with supplies for the Government, has made night work the rule and brought a realization of the importance of proper illumination from the standpoint both of the maintenance of quality and quantity in production and of the health and comfort of the worker. Progress toward this end is evidenced in the revision of industrial lighting codes in several states and by the appointment of a National Committee on Lighting to act as a sub-committee of the Advisory Commission-Council for National Defense for the preparation of suggested regulations to govern industrial lighting, which have subsequently been published in the form of a Code of Lighting by the Committee on Labor with a suggestion that the Code be put into effect in every state in the country.

War conditions have also brought about a more careful consideration of protective lighting and the best way to utilize it. Thus it has been found that in many cases inexpensive reflectors of the ordinary type may be used for lighting open spaces in and around a plant leaving the special flood lighting units for those locations requiring particular treatment. In many cases the use of a large number of properly shaded low-intensity units will avoid dangerous shadows better than high powered sources, even though the light flux from the latter is greater.

A sphere formerly considered impregnably held by the arc lamp has been finally invaded by the incandescent lamp. Motion picture projection work required light flux of extremely great intensity and the small area and high intrinsic brilliancy of the source of light in the arc has enabled it to meet the requirements in a way hard to duplicate. By using a mirror back of the filament and for a condensing lens one of the Fresnel type, it has been found possible to make an incandescent lamp which will give satisfactory results within a certain limited field of motion picture work.

The motion picture theatre has in itself become an arena in which unique lighting effects are being experimented with continuously. Thus in several cases, by the use of several circuits in each fixture, lamps of different colors may be lighted and thereby give a color tone to the whole illumination.

The action of the Government in attempting to save fuel by restricting its use for lighting purposes has shown in many localities the important part played by display lighting in maintaining the illumination of streets and sidewalks.

EDWARD P. HYDE, *Chairman.*

## ANNUAL REPORT OF THE COMMITTEE ON TRANSMISSION AND DISTRIBUTION

*To the Board of Directors,*

The Committee on Transmission and Distribution submits the following report for the year 1917-1918:

Experience during the preceding year indicated that the Committee was entirely too large. At the writer's suggestion the membership was reduced during the present year from 24 to 14 members. We now suggest that the Committees be further reduced to not exceeding 10 members. It would be an act of courtesy on the part of a member to decline the appointment when it is offered him if he can take no part in the work of the Committee.

It has been very difficult to make progress in the problems before us on account of every one, almost without exception, being employed on war work or very urgent duties contributing to the war. Some of the members who were most helpful in the past have gone into the government service and have not been able to continue the work which they started last year. Mr. W. D. Peaslee of the Oregon Agricultural College, who has been investigating the insulator problem from a chemical and microscopic standpoint, is now captain in the United States Army. Professor Harris J. Ryan, who is our strong right arm in insulator matters, is giving practically all of his time to government work. However, some progress has been made.

### *High-Tension Insulators*

Last year we had papers by Messrs. Austin, Peaslee and Ryan pointing out clearly that progress in the design of high-tension insulators must provide for (1) reduction of porosity to the lowest possible limit. (2) joints designed to avoid cracking from expansion and contraction and (3) ample mechanical strength. Mr. G. I. Gilchrest has now made some very careful studies of insulators, both from the laboratory point of view where distribution of electric stresses was considered, and from the practical point of view where troubles and failures by operating companies under wide varieties of conditions were examined. We hope that insulator manufacturers will give careful consideration to the design of insulators that Mr. Gilchrest has evolved. This paper shows very clearly where and why many of the old insulator designs failed. While it is quite probable that a perfect insulator will never be obtained, one has but to compare the insulator of today with that of five years ago to see that great progress has been made.

At least one large transmission company is very hopeful of the wood stick insulator for voltages up to 100,000. This insulator has been in successful service in the West for some years on 60,000-volt service. We had a paper listed for this meeting on the wood stick insulator by Mr. H. H. Cochrane, and hoped to get full details of it and record of the service it has given, but at the last moment Mr. Cochrane asked to have the matter go over until a later date, as some difficulties of impregnation had been encountered.

### *Lead Sheath Cables*

Last year at the annual meeting a whole session of the Institute was devoted to the discussion of dielectric losses in cables. It was shown that cables insulated with mineral-base compounds had greatly reduced dielectric losses over those insulated with vegetable-base compounds. It was further shown that cable ratings under some conditions were more than doubled and on the average could be increased 20 or 30 per cent when the mineral-base insulating compounds are used. A start has been made in the matter of preparing specifications covering dielectric losses in cables. Engineers of some of the principal cable manufacturers have agreed to cooperate. Before such specifications can be formulated, at least two fundamental points must be considered and agreed upon:

First, a standard method of making tests must be established. Very few, even of the cable manufacturers, are equipped for measuring dielectric losses and probably no users of cables have facilities for properly making these measurements. Some very much simpler and more easily workable apparatus than is now available must be developed before commercial routine tests of this kind can be applied to the output of the cable factories. If a portable testing set for measuring dielectric loss were devised it could be used to test newly installed cable as well as to secure experimental data on old feeder cables under various conditions of age, temperature, charred insulation, etc.

Mr. S. M. Farmer recently presented a paper describing a method of determining power loss in three-conductor cables, the loss being measured directly under three-phase conditions. It is hoped that the method he followed may be developed for factory tests.

In the second place, data must be collected showing the limits of the losses. To secure the data on losses is a difficult matter at this time when men as well as laboratories are occupied to full capacity on war work.

Additional information is being obtained on the characteristics of insulating compounds. For instance, it has been found that cable insulated with mineral-base compound will not withstand a high insulation test when cold, as the insulation resistance is much reduced under such conditions. All this complicates the problem of preparing specifications. Evidently much research work must yet be done, but as before stated, all work of this sort is much hampered by the war.

Mr. E. B. Meyer in a very practical paper gives the experience of one large distributing company in supplying high-voltage cable service to customers in cities where overhead wires are permissible and where the expense of conduit is not justified. The method while adopted as a war expedient has doubtless a wide field as a permanent method of installation. At least it can be used until the demand for energy in the particular locality is large enough to justify the expense of underground conduit.

Mr. W. H. Cole this year presents a paper as result of the experience of Edison Electric Illuminating Company of Boston with split-conductor cables and balanced-relay protection against interruptions caused by short circuit or grounds. It is very gratifying to note the success which has been attained notwithstanding the complexity of the system and the great care which must be exercised in installation of the equipment. All users of underground cables will appreciate Mr. Cole's work in this field.

### *Suggestions for the Future*

For the future work of this Committee we would like to recommend that investigation of the insulator problem be continued. While at present porcelain seems to be the most available material, yet it is not beyond possibility that some other material such as fused quartz or even glass may be used.

Professor Ryan is now supervising some extensive tests at Leland Stanford, Jr. University of aging effect on insulators carried through a large number of temperature cycles corresponding to the daily and seasonal variations experienced in practise. The result of these tests will be of great value.

Further investigation of the fused quartz insulator along the lines suggested last year by Mr. Peaslee will be well worth while as soon as some one can find time to do it.

As indicated above, the matter of dielectric loss in cables should receive most careful attention, both by cable manufacturers and users. Manufacturers must not be hampered by impractical and half-baked specifications. On the other hand, conservatism of manufacturers must not block the road to progress, and as soon as the laws governing dielectric loss can be determined and a practical method of measuring these losses developed, the most efficient cable will be called for and must be produced.

L. E. IMLAY, *Chairman*

## ANNUAL REPORT OF ELECTROCHEMISTRY AND ELECTROMETALLURGY COMMITTEE

*To the Board of Directors,*

The Committee on Electrochemistry and Electrometallurgy submits the following report for the year 1917-1918:

It did not appear to be possible during the past year to secure any papers of interest on electrochemical and electrometallurgical subjects. One paper was submitted to the chairman of this committee for consideration which seemed more suitable for presentation to the American Electrochemical Society than to the Institute.

One of the most vitally important matters to Electrochemistry and Electrometallurgy is the development of water powers and consequently it appeared to be a subject which would properly come under the consideration of this Committee more particularly as regards the matter of legislation since this has probably done more than anything else to hamper their development. Correspondence with the Chairman of the Public Policy Committee brought out the fact that this subject has been referred to the standing Committee on Public Affairs of the Engineering Council.

Both as regards the question of securing papers for presentation to the Institute and as regards all matters pertaining to electrochemistry and electrometallurgy much more valuable results could probably be reached were some scheme worked out by which there could be cooperation of this committee and the American Electrochemical Society. With this end in view a suggestion was made to the Board of Directors of the American Electrochemical Society that it consider the question of approaching the A.I.E.E. with the object of forming a joint committee which would take care of subjects that are of common interest to the A.E.S. and the A.I.E.E.

Such a Committee would be in a good position to take care of various papers which may be of importance to both societies, to bring to the attention of both societies matters in which they have a common interest and finally to make arrangements when possible for joint meetings of the Societies.

At a meeting of the Board of the American Electrochemical Society held April 28, 1918, the Directors acting on this suggestion appointed a Committee "to co-operate with" this Committee. The Committee appointed by the American Electrochemical Society is as follows:—

Dr. Colin G. Fink, Chile Exploration Co., 202nd Street and Tenth Ave., New York.

Howard C. Parmelee, Metallurgical & Chemical Engineering, McGraw-Hill Co., Inc., 36th St., and Tenth Ave., New York.

C. G. Schluederberg, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

A letter was received from Professor Karapetoff of Cornell University describing the Research Section of the *Electrical World* which he is conducting and asking for suggestions from this committee. The members of the committee were notified of Professor Karapetoff's request.

### *Suggestion for Future Activities*

In view of the action of the Board of the American Electrochemical Society it is suggested that this Committee should communicate with that appointed by the A.E.S. with the object of discussing the feasibility of cooperation for the benefit of the Institute and the Society. This matter, however, should be left to the Committee for 1918-1919 as the term of office of the present committee is so near an end.

As regards hydroelectric power there is no more important matter for consideration by this Committee and if some scheme for cooperation with the American Electrochemical Society is developed it should be specially studied by this Committee.

### *Electrochemistry and Electrometallurgy* 1917-1918

The entrance of the United States into the Great War had the effect of stimulating enormously electrochemical and electrometallurgical processes.

As regards electrolytic work probably one of the most highly stimulated manufactures is that for the production of chlorine; but the most noticeable stimulation is found in electrometallurgical work.

For years there has been much experimental work carried out in electric furnaces for the melting of non-ferrous metals like brass, bronzes, etc., but no really satisfactory furnace was developed. Then, as one of the consequences of the Great War graphite crucibles became scarce and expensive with the result that renewed attempts were made on the electric furnace with the result that during the past year various furnaces have been designed of which some show distinct promise of meeting with ultimate success.

There has also during the past year been an enormous increase in the production of electric furnace ferro alloys of all sorts particularly ferrosilicon and ferromanganese. The latter had been produced in electric furnaces before the war but could not compete with the blast furnace.

There has also been a considerable increase in the use of electric steel furnaces and this probably would have been much greater were it not for the great difficulty of getting a sufficient supply of carbon electrodes.

The most interesting electrochemical development of the year, however, is the construction of the cyanamid nitrogen-fixation plant at Muscle Shoals, Ala. Two unique features in this plant are the use of steam for the generation of the electric energy and the use of 60-cycle current for the calcium carbide furnaces. These features will give interesting subjects for study; on the one hand the energy item in the cost of making ammonium nitrate by the cyanamid process and on the other the power factor of carbide furnaces with high-frequency current.

FRANCIS A. J. FITZGERALD, *Chairman*

## ANNUAL REPORT OF INDUSTRIAL AND DOMESTIC POWER COMMITTEE

*To the Board of Directors,*

In presenting our annual report, your Committee on Industrial and Domestic Power desires first to call attention to two of our members who are now in Military Service in the war of 1917, viz: our chairman, Capt. E. H. Martindale now in active service in France, and Lieut. (j. g.) A. M. MacCutcheon, U. S. Navy, also in active service, U. S. S. Louisiana.

We honor their action, and have missed their advice in our work this year, particularly that of our Chairman.

Others of our Committee from whom we have not heard may be similarly employed, and in that event their names should be joined with the above.

During the two years beginning in August, 1914, under the leadership of Mr. David B. Rushmore, work of the committee began to be centered on a study of our activity by industries. This thought was carried out in several meetings of the Institute.

In the Fall of 1916 under Capt. Martindale, it was decided that the major portion of our activities should be devoted to this study. To show the value thereof, it was decided to make an investigation of the machinery, motors, controllers and accessories involved in each process in three industries or parts thereof, and reach conclusions from this as to next procedure. On this basis, the whole committee was divided into three subcommittees headed by Lieut. A. M. MacCutcheon, to investigate a portion of the Metal Working Industry; Mr. R. B. Williamson, to investigate the Cement Industry; Mr. R. M. Goodwillie, to investigate the Passenger Elevator Industry.

The committees have worked hard, collecting much data, but without having their work completed by the end of the last term.

In the Fall of 1917, by reason of his military service, Capt. Martindale was obliged to relinquish active handling of his work as Chairman, the position of Vice Chairman was created, and the writer appointed to the position. The executive duties have been carried out in the way it was believed Capt. Martindale desired.

On account of the absence of our Chairman, and also because of the many immediate and pressing duties of service of all individually, in connection with the war, it was deemed advisable to maintain our plan just as outlined one year previously, center our thought on its progress and application, and not attempt the preparation or presentation of papers to the Meetings and Papers Committee, and this course has been followed.

The work of the sub-committees in collecting data is well in hand. In the examination of the cement industry for example, Mr. Williamson's committee has first tabulated all the processes involved from the point when the raw product is brought to the cement plant up to the point when the finished product is delivered, barrelled and ready for shipment. Each process may be considered a movement of some kind. Where electric power is applied to this movement, the preferred type of motor, control, and accessories and alternates are indicated. In a similar way, Mr. MacCutcheon's Committee has examined a portion of the metal working industry. Thus mainly the principal data have been collected and compiled. The work has given our whole committee opportunity to thoroughly consider the good features and objections, and in this connection two immediate difficulties; viz: to know when the work is done, and how to present it to the Institute, are not solved. These two points with others brought out by this investigation have been carefully canvassed by the whole committee, and the discussion and recommendations which immediately follow are based upon a majority vote also of the whole committee, with no dissensions.

The work has been carried far enough so that each feels sure that if the data could be gotten together for all processes, kept correct and made available, these would be of immense benefit to the industry as a whole, and particularly at this time. The collecting and preparing of the data presents very serious difficulties as follows:

1. *Tabulation of what industries are within the scope of our committee's activities.* The scope of this committee overlaps that of the committees on mines, steel mills, electrochemical work, marine work, and perhaps others. It is recommended that its scope be plainly defined.

2. *What constitutes an industry.* A tabulation of industries can be prepared by our committee. Such tabulation is bound to be at variance in thought from any tabulation prepared by another committee. We believe that a tabulation should be prepared for the electrical industry, as a whole, and that this can-

not be done short of a careful study of the subject, and the interrelated processes. As Mr. Goodwillie of our committee states, the real essential element is the load characteristics of the machine itself from the starting, accelerating, running, slowdown and stopping conditions which in turn compel considerations of starting friction, inertia, running frictions and load characteristics. It is probably a matter of consideration by some special technical committee, and not by a technical committee devoted to some special work like our own unless the work as a whole for some reason be specially assigned to us. Much study has been put already on this subject by allied associations, and by private enterprises. Much of the benefit of this study can be secured very likely. The list of industries and processes involved will be subject to constant modification as the work proceeds. A plan for caring for these changes is essential.

3. *Magnitude of the work involved in the study.* It is clearly beyond the power of our committee to perform this work alone in any measurable period of time. External assistance is vital and some plan to secure the assistance necessary. Even if it were completed, it is doubtful if the committee alone as it stands could keep it completed.

4. *Preparation and filing for permanent record.* It is apparent that the proper place for recording this data is not in the PROCEEDINGS of the Institute. The expense of doing it in this way would be prohibitive. It would lack availability for use and change as required. A plan for permanently caring for the data should be perfected as will be separately discussed.*

5. *Authority for work.* In the execution of this work, time and expense beyond the limits of the committee would be bound to accrue. Definite authorization from the Board of Directors to proceed is necessary, and with it the statement that the work would be continued from year to year, for it is obvious that to start work and continue for a year or so only to change or abandon would involve needless loss of time, and discouragement.

6. The ethics of the study from the standpoint of the professional engineer, from that of patent rights and from the unconscious publicity given to private *enterprise is comparing correct process attainment.* This is considered one of the most difficult phases to cover correctly. As Mr. Dudley states, it is believed that with the exercise of tact with each case, that difficulty should not be encountered. Thus far in its work the sub-committees have been able to steer clear of this as a diffi-

culty. The work must be undertaken strictly from a technical standpoint.

7. *Conflict with other technical committees in other associations.* The whole reason for this work is to avoid duplication of effort. The American Institute of Electrical Engineers is the recognized association devoted to our art from an unbiased technical and professional standpoint. If this work be undertaken by us with carefully prepared plan and be definitely endorsed, the need for similar work by other bodies such as the N.E.L.A., A.I. & S.E.E., etc., should disappear. The committee is confident that help from them will be fully available and given. There should be no conflict with other committees in the Institute. For this, the management must care.

8. *Miscellaneous Difficulties.* Undoubtedly these exist but it is believed that what has been previously stated will cover the major difficulties, and that the minor ones will settle themselves as they arise.

The study which our committee has thus far put on the work has convinced us that it is broader in scope than that of our committee. If it is right to investigate the large group of industrial applications with which our committee is concerned, why should not the whole field be covered. Consideration of this thought has been inevitable, and is crystallized into a number of suggestions or recommendations which are presented as the opinion of a majority of your committee.

We believe that there should be instituted in the office of the Secretary of the Institute in New York City, and under his complete charge, a file of industries using electrical power, the processes involved in each industry, the movements involved in each process, and the electrical apparatus, which is recommended for each movement. We believe that this recommendation should be strictly from the technical standpoint, the apparatus to be described so that it will be technically understood. We believe that the file should be kept up by the machinery of the Secretary's office, and in such form as will make it most available to all who should have access to it. It is assumed that this file should be in card form, and in this connection, Messrs. Weichsel and Dudley of our committee suggest the compilation of pamphlets or a hand book from these files in the belief that they would have ready sale.

It is further suggested by the committee that as Institute Sections are able to take care of them properly, copies of the files be placed on record at Section Headquarters all under control of the Secretary of the Institute.

Manifestly the data for these files can be secured only through the cooperation of a large number of members, and one perhaps the hardest study would be to enlist this cooperation. Fundamentally, the plan must have the absolute endorsement of the Board of Directors, and then it should be planned to secure this large support. Our committee feels that either the duties of the Industrial and Domestic Power Committee should be broadened to assume the work, or (and this is favored by a majority of the committee) that a new technical committee should be formed to take care of the job. No technical committee alone can get the data unless it have broad powers, and a membership large enough to reach into the activity of the Nation industrially and geographically, in a very comprehensive manner indeed. As a committee, we feel very sure that in some way the individual member must be reached; must be encouraged to furnish data, with proper recognition. We feel that if this action on the part of the individual member in being of service to the Institute and to the industry causes him to recognize his true relation to the Institute of giving to it rather than of receiving from it, it should be of the greatest value.

We further respectfully suggest that in the event of establishing a committee to secure this data that it should work in very close conjunction with the Secretary of the Institute, and with the other technical committees.

Briefly we suggest, that the new committee plan and secure the data, the existing committees censor the data and advise with the new committee, and that the Secretary's office compile and carry the data.

Such a file should be the truest permanent record possible of the application of electrical power and its progress. It should tend to harmonize policies of application of electrical power; not to standardize details. It should prevent an enormous amount of duplication of effort, not only in association work, but in private enterprise as time goes on; should tend to pool, for the good of all, data which are today largely open to all and yet not collected so as to be of mutual benefit.

Your committee sincerely hopes that these views will have your consideration.

We desire to comment on another matter. Our committee's scope today includes domestic as well as industrial power. It has seemed to some that the domestic end of our activity should be taken from us and given to some other committee whose work is more nearly allied thereto. On a poll of opinion seven were in favor of this action, five did not reply and two were in opposition thereto.

The past year has been crowded with work which your committee feels covers very largely application of earlier progress than it does the invention or perfection of new work. Constant effort to keep existing apparatus operating at the maximum rather than to perfect or develop more efficient operation has marked this period when our Nation for the first time in twenty years was plunged in war. This holds particularly for the production of textiles and clothing generally, steel, coal, and in matters affecting transportation.

In the Chicago territory there has been installed during the past year a 7000-h.p. induction motor having a maximum torque equivalent to 30,000 h.p., which so far as normal rating and maximum torque is concerned is the largest industrial motor which we believe has ever been built.

In the textile industry, the matter of individual drive of spinning frames is progressing, many hundred frames having been equipped during the past year.

The first electrically propelled battleship has been launched during the past year. The application is new.

The year marks the first application of small electric generators, air driven, to air-planes. This extends the use of industrial power to the air along with gasoline engines.

In summary, your committee would value quite specific instructions as to its duty, and scope, and shall be much interested to note any action that may be taken on its recommendations.

A. G. Pierce, *Vice Chairman*

## ANNUAL REPORT OF COMMITTEE ON ELECTROPHYSICS

*To the Board of Directors,*

The Committee on Electrophysics submits the following report for the year 1917-18:

During the year two sessions of the Institute have been devoted to electrophysics under the auspices of this committee as follows:

On the evening of Nov. 9, 1917, Mr. Chester W. Rice presented a paper entitled "An Experimental Method of Obtaining the Solution of Electrostatic Problems with Notes on High-Voltage Bushing Design" which forms a very valuable contribution to our knowledge of the electrostatic field. The paper, published in the November PROCEEDINGS, has a direct bearing on the problem of insulator design and the mathematical appendix gives the solution of two electrostatic problems, the usefulness of which extends beyond the immediate problem of design.

On the evening of Feb. 15, 1918, at the Midwinter Convention, the Institute listened to a most interesting lecture by Dr. A. C. Crehore on "Some Applications of the Electromagnetic Theory of Matter"; the lecture is published in the April, 1918, PROCEEDINGS. Great progress is being made in the theory of atomic structure, to which Dr. Crehore's own work has formed no small contribution, and the placing of these most recent advances by Dr. Crehore before the Institute in non-mathematical form was indeed most opportune. It will be recalled that a similar lecture on "Modern Physics" was given by Prof. R. A. Millikan at the Midwinter Convention in 1917.

The committee has felt that the cooperation and mutual understanding between physicists and engineers, as pointed out in the report of the committee for last year, are of the utmost importance. The close relation has been maintained between this committee and the Technical Physics Committee of the Physical Society. It is believed that joint meetings of the Institute and the Physical Society from time to time should be continued as in the past and such a meeting is planned for the Philadelphia session next October.

The maintenance of a supply of trained physicists and engineers, at all times important, the committee has considered to be particularly important in time of war in order to meet the needs of governmental and industrial service. This matter was discussed on April 11th at a meeting of the committee held in New York for the purpose, and the conclusion was reached that to insure the continuous output of technically trained men from the universities and technical schools of the United States it was most important that steps be taken to provide for the maintenance of adequate teaching staffs, which are in danger of being depleted through the application of the draft and through voluntary enlistment. This matter was brought to the attention of your Board on April 12 and it was the unanimous opinion of the members of the board that immediate action was necessary.

FREDERICK BEDELL, *Chairman*

## ANNUAL REPORT OF PROTECTIVE DEVICES COMMITTEE

*To the Board of Directors,*

The Committee on Protective Devices submits the following report for the year 1917-18.

On account of the decision of a number of companies to cancel all committee work during the period of the war the work of this Committee has been carried on largely by correspondence. One of the members of the Committee entered the military service of the country and a number of others have had increased duties on account of their assistants entering the Army or Navy. These changes have served to considerably retard the work of the Committee.

During the year the Committee has issued the questionnaire on relays, referred to in the previous report, to about fifty of the leading operating companies in the country. Not enough replies have been received from the questionnaire up to the present writing to permit of any summary or resume from the inquiries. It is recommended that the work on the questionnaire be pushed to a conclusion during the coming year.

Several members of the Committee have called attention to the proposed interconnection of transmission and distribution systems throughout the country as a measure of economy and fuel saving, and it is recommended that the Committee investigate this subject in particular to determine what protective features are necessary in such tie lines for the purpose of ensuring continuity of service and stability of operation.

Other subjects which might be taken up by the Committee are the following:

Relays on generators, transformers, synchronous converters, etc.

Lightning arresters for transmission lines.

Preparation of definite recommendaions regarding the standardization of relay nomenclature and rating of circuit breakers.

Detrimental effect of power reactors.

D. W. Roper, *Chairman*

## ANNUAL REPORT OF COMMITTEE ON APPLICATION OF ELECTRICITY TO MINES

*To the Board of Directors,*

I give below a brief report of the activities of the Committee on the Application of Electricity to Mines.

Owing to the resignation of Mr. H. H. Clark, the writer was appointed as chairman of the Committee on the Application of Electricity to Mines. Pressure of other work has prevented devoting as much time to Committee work as I would like to have given. No meetings of the Committee as a whole have been held, but the chairman has arranged with the Meetings and Papers Committee to take charge of the October meeting which will be held in Philadelphia. We propose at this meeting to present three papers bearing on the subject of the Application of Electric Power to Coal Mining. We also hope at this time to have an informal talk by a member of the Fuel Administration Bureau at the dinner preceding the evening session.

The American Institute of Mining Engineers has a committee on the Application of Electricity to Mining and I do not believe that the best results can be accomplished by the independent action of these committees in two entirely independent societies; and I would suggest as a future activity for the Committee on the Application of Electricity to Mines in the A.I.E.E. that of making some working arrangements with the corresponding committee of the American Institute of Mining Engineers whereby both national societies may get the benefit of the work of the Committees of each. Possibly this can be accomplished by making the annual meeting of these two committees joint meetings, the A.I.E.E. joining with the mining engineers at one meeting and vice versa for the next, the papers being published by both societies.

K. A. PAULY, *Chairman*

# ANNUAL REPORT OF INSTRUMENTS AND MEASUREMENTS COMMITTEE

*To the Board of Directors,*

This committee was appointed during the year of 1917 with the general purpose of promoting interest through the presentation of papers and discussion along the lines covered by the name of the committee. The field for the consideration of instruments and for measurements had not heretofore been made a matter of separate committee work and discussion heretofore has been coupled with the consideration given papers in which both instruments and measurements were incidental to another subject.

Meetings of the committee were held and arrangements were made for the Meetings and Papers Committee to assign one session of the mid-winter convention in February for the presentation of papers on measurements. One afternoon session was assigned and four papers were presented. Attendance at the session and the discussion indicated sufficient interest to justify the continuance of the Committee's activities along these lines.

Two impressions were obtained from the papers and discussion, which while obvious, seem of sufficient interest to report. Two of the papers presented dealt with the investigation and development of a substitute for the standard cell for certain classes of work. This substitute was a thermocouple, eliminating entirely the use of the chemicals required by the standard cell.

One of the papers presented dealt with the measurement of dielectric losses in cables, and while it was a single paper only, it was in reality the latest of a series of several papers on the same subject presented before the Institute in the last few months. The discussion indicated a desire or a necessity for the standardization of methods of measuring the very small energy losses in the dielectric and the specifications covering the purchase and acceptance tests of cables.

No matters of nomenclature or standardization arose requiring the attention of the committee on standards. No matters of policy or coordination with other committees arose requiring the attention or action on the part of the Board of Directors.

Commenting briefly on the progress of the industry falling within the scope of the committee, it can be stated that as might be expected no new development work along purely commercial lines is being undertaken by the manufacturers of apparatus at this time. Developments have undoubtedly taken place, however, in apparatus along the lines of military and naval activity, such as radio and other signal apparatus which will without doubt furnish valuable and interesting material when available at some time in the future.

S. G. RHODES, *Chairman*

## ANNUAL REPORT OF EDUCATIONAL COMMITTEE

*To the Board of Directors,*

Lack of time on the part of the members of the Educational Committee is responsible for the regrettable fact that only one meeting could be arranged during the year.

At this meeting it was decided that two papers, if possible, be prepared; one dealing with electrical engineering education given in colleges at the present time, and the other giving a summary of the educational facilities offered by the large manufacturing and power companies.

The second paper was not completed, the first is given in the synopsis prepared by the Chairman and presented herewith.

The Committee recommends that the particular scope suggested above be considered as an important part of the duties of future Committees so that the members may be able to keep in touch with educational methods and ideas, and assist the leaders of education in shaping their policies.

### *Synopsis of Electrical Engineering Education given in American Colleges,* 1917

The importance of engineering and engineering education, always recognized in this country, has never been more fully realized than today. The engineer is called upon not only to supervise engineering work and to design machines but is more and more involved in administrative work so that—large as his task has been—it will be greater in the future.

It seemed, therefore, opportune to the Educational Committee that a report be presented to the Institute giving a brief summary of the present status of electrical engineering education in this country, as shown by the latest catalogues. A questionnaire was mailed to a large number of colleges giving four-year courses in electrical engineering and the returns are tabulated below.

The studies were divided in ten groups as shown and the figures given represent the percentage of time out of the entire four-year curriculum devoted to each group.

Group 1, includes strictly electrical engineering subjects.

Group 2, includes English, English literature, rhetoric, etc.

Group 3, Foreign languages.

Group 4, Mathematics, excluding descriptive geometry.

Group 5, Physics, including elementary mechanics.

Group 6, Chemistry.

Group 7, General engineering subjects. This heading includes such phases of the engineering curriculum as are usually given to all branches of engineering students. It includes, for instance, drawing, surveying, descriptive geometry, advanced mechanics, applied mechanics, thermodynamics, etc.

Group 8, General subjects. These include prescribed courses in history, law, economics, etc.

Group 9, Electives, technical and general.

Group 10, Physical training, physiology, hygiene, military work, etc.

Table I gives in alphabetical order the colleges which responded. At the bottom of the tabulations are given the total averages.* This should, therefore, indicate what weight is given to the various groups in the average college at the present time.

It is interesting to note that general engineering subjects, that is, subjects which are essentially common to all classes of engineering students, cover 31 per cent of the entire time, and the purely electrical engineering subjects are given 21.6 per cent. Thus the engineering topics occupy approximately one-half of the entire time of the students.

English, including literature and rhetoric, is given only 5.5 per cent, foreign languages 3.2 per cent, general subjects such as economics, history, law, etc., are given 3.4 per cent. Science subjects, mathematics, physics, and chemistry are given 27 per cent of the total time.

Tables II, III, IV, V, VI, emphasize particular studies. So for instance, Table II gives approximately one-half of the entire list in accordance with the prominence of purely electrical studies. Norwich University leads, it devoted 33.5 per cent, or roughly, 50 per cent more than the average time to that subject.

Table III emphasizes English studies. The Agricultural and Mechanical College of Texas leads with 12.5 per cent or more than twice as much as the average. Some leading colleges give no instruction in English. These may, however, to some extent, take care of this feature in a more rigid entrance examination.

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*Since deducing the average a couple of colleges have been added which may have very slightly modified the actual value.

Table IV emphasizes foreign languages. It is of interest to note that a large percentage of the colleges give no foreign language course at all.

The average time given to electives is 3.9 per cent which seems rather low. There are, however, a number of institutions which permit of a wide choice. Tufts leads with a percentage of 21.

The preparation of these tables is intended to facilitate a discussion on the very vital subject of electrical engineering education as given in colleges. It is not intended that a "standard" course should be evolved—that would indeed be unfortunate—but it does look as if almost all colleges could to advantage increase some of the non-technical courses, such as English, economics and foreign languages. Some colleges should perhaps adopt a course along scientific lines, neglecting somewhat intruction strictly in engineering and concentrating on mathematics, physics and chemistry. Others may also lay less stress on their engineering subjects and devote more time to English, foreign languages and to problems of economics. The third class, perhaps the largest, should do essentially what the average college is doing today.

ERNEST J. BERG, *Chairman*

## TABLE I

| INSTITUTION | Electrical engineering | English | Foreign languages | Mathematics excluding descriptive | Physics including el. mechanics | Chemistry | General engineering | General subjects | Electives | Physical Training |
|---|---|---|---|---|---|---|---|---|---|---|
| Agricultural and Mechanical College of Texas | 28.8 | 12.5 | 00.0 | 13.3 | 4.5 | 4.5 | 26.7 | 3.3 | 1.7 | 6.7 |
| Armour Institute of Technology | 28.6 | 2.4 | 00.0 | 7.2 | 8.8 | 6.4 | 35.0* | 6.8 | 17.9† | 4.8 |
| Bucknell University | 23.0 | 6.0 | 5.5 | 9.0 | 12.6 | 9.4 | 23.0 | 8.0 | 3.0 | 0.5 |
| Carnegie Institute | 28.3 | 2.9 | 4.6 | 8.0 | 9.6 | 4.6 | 30.0 | 3.3 | 0.0 | 8.7 |
| Case School | 25.2 | 6.4 | 6.4 | 10.4 | 9.1 | 5.5 | 25.9 | 1.3 | 1.3 | 8.5 |
| Clemson Agricultural College | 13.4 | 9.7 | 0.0 | 9.9 | 8.0 | 4.2 | 34.6 | 6.9 | 0.0 | 13.3 |
| Colorado College | 26.5 | 4.7 | 7.8 | 20.3 | 13.3 | 4.7 | 15.7 | 4.7 | 0.0 | 2.3 |
| Cornell University | 20.6 | 0.0 | 0.0 | 7.8 | 9.0 | 7.0 | 46.5 | 3.9 | 2.6 | 2.6 |
| Drexel Institute | 20.5 | 7.0 | 0.0 | 8.0 | 10.0 | 8.0 | 31.0 | 4.5 | 0.0 | 3.0 |
| George Washington University | 25.7 | 4.3 | 4.3 | 8.6 | 7.1 | 8.6 | 33.6 | 2.8 | 5.0 | 0.0 |
| Georgia School of Technology | 21.1 | 8.5 | 6.1 | 11.0 | 10.8 | 6.0 | 25.3 | 2.5 | 0.0 | 8.7 |
| Iowa State College | 21.7 | 7.5 | 0.0 | 15.0 | 11.5 | 6.2 | 23.7 | 2.7 | 7.5 | 3.6 |
| Johns Hopkins University | 20.3 | 5.5 | 3.2 | 9.0 | 13.6 | 7.0 | 26.6 | 4.7 | 2.3 | 7.8 |
| Lafayette College | 27.7 | 3.9 | 3.9 | 11.0 | 6.4 | 6.4 | 24.5 | 11.0 | 3.9 | 1.3 |
| Lehigh University | 28.6 | 6.5 | 7.8 | 11.0 | 13.0 | 6.5 | 16.2 | 1.9 | 1.9 | 6.5 |
| Maryland State College . of Agriculture | 25.7 | 10.8 | 5.6 | 10.5 | 4.5 | 4.5 | 21.6 | 9.3 | 0.0 | 7.5 |
| Massachusetts Institute of Technology | 23.8 | 5.2 | 3.2 | 8.8 | 8.2 | 8.3 | 25.5 | 4.2 | 8.3 | 4.5 |
| Mississippi Agricultural & Mechanical College | 15.3 | 10.9 | 0.0 | 18.8 | 7.0 | 4.5 | 26.1 | 5.9 | 6.3 | 5.2 |
| Montana State College of Agriculture and Mechanic Arts | 33.0 | 3.0 | 0.0 | 13.0 | 16.0 | 5.2 | 15.2 | 7.0 | 5.2 | 2.4 |
| New Hampshire College | 21.0 | 4.1 | 0.0 | 12.2 | 10.1 | 6.1 | 35.7 | 0.0* | 5.4 | 5.4 |
| New Mexico College of Agriculture & Mechanics Arts | 17.8 | 2.8 | 3.8 | 9.1 | 8.2 | 5.8 | 42.7 | 2.8 | 1.0 | 5.8 |
| North Carolina State College | 19.4 | 7.7 | 5.3 | 11.9 | 8.5 | 6.0 | 29.4 | 5.3 | 0.0 | 6.6 |
| Norwich University | 33.5 | 3.9 | 7.9 | 0.9 | 9.9 | 3.9 | 13.2 | 5.9 | 1.3 | 10.5 |
| Ohio State University | 20.1 | 2.6 | 5.2 | 13.0 | 13.0 | 5.2 | 31.2 | 0.0 | 5.8 | 3.9 |
| Oklahoma Agriculture and Mechanical College | 25.0 | 5.9 | 0.0 | 12.5 | 11.0 | 5.7 | 30.9 | 1.5 | 3.0 | 4.5 |
| Oregon Agricultural College | 25.0 | 4.5 | 0.0 | 13.0 | 9.5 | 4.5 | 30.0 | 0.0 | 5.7 | 7.5 |
| Pennsylvania College | 15.0 | 9.2 | 3.9 | 12.4 | 14.4 | 9.2 | 23.5 | 11.8 | 0.0 | 0.7 |
| Pennsylvania State College | 22.0 | 7.5 | 7.5 | 12.5 | 7.5 | 5.5 | 30.0 | 7.5 | 0.0 | 0.0 |
| Polytechnic Institute of Brooklyn | 28.0 | 6.6 | 3.7 | 10.7 | 9.4 | 7.0 | 28.0 | 2.8 | 1.9 | 1.9 |
| Purdue University | 20.4 | 7.9 | 6.6 | 14.4 | 9.6 | 3.5 | 21.5 | 5.9 | 7.6 | 2.6 |
| Rensselaer Polytechnic Institute | 23.3 | 5.9 | 6.2 | 13.0 | 9.2 | 6.2 | 33.8 | 1.1 | 0.0 | 1.3 |
| State Agricultural College of Colorado | 25.5 | 5.0 | 0.0 | 10.0 | 8.5 | 7.0 | 21.3 | 2.5 | 16.2 | 4.0 |
| State College of Washington | 27.2 | 6.2 | 0.0 | 12.4 | 6.2 | 6.8 | 29.7 | 4.7 | 3.1 | 3.7 |
| State University of Iowa | 26.9 | 6.6 | 0.0 | 14.7 | 8.8 | 5.9 | 28.3 | 6.6 | 0.0 | 2.2 |
| State University of Kentucky | 12.7 | 6.2 | 0.0 | 10.6 | 7.8 | 5.3 | 53.2 | 0.0 | 0.0 | 4.2 |

* Armour—Includes shop work

†New Hampshire College—General subjects included in electives

TABLE I—Continued

| INSTITUTION | Electrical engineering | English | Foreign languages | Mathematics excluding descriptive | Physics including el. mechanics | Chemistry | General engineering | General subjects | Electives | Physical Training |
|---|---|---|---|---|---|---|---|---|---|---|
| Stevens Institute of Technology | 8.1 | 3.9 | 4.2 | 5.6 | 6.3 | 7.5 | 53.9 | 2.1 | 0.0 | 8.4 |
| Syracuse | 25.0 | 4.0 | 0.0 | 16.3 | 10.2 | 7.5 | 35.0 | 2.0 | 0.0 | 0.0 |
| Throop College of Technology | 19.5 | 8.4 | 7.0 | 8.2 | 10.0 | 6.0 | 30.0 | 6.7 | 0.0 | 4.2 |
| Tufts College | 24.0 | 4.0 | 4.0 | 9.0 | 6.0 | 6.0 | 22.0 | 2.0 | 21.0 | 2.0 |
| Tulane University of Louisiana | 12.3 | 6.4 | 0.0 | 12.8 | 7.9 | 8.9 | 48.4* | 0.0 | 2.0 | 1.3 |
| Union College | 22.8 | 8.6 | 8.6 | 17.2 | 5.6 | 4.5 | 20.0 | 5.5 | 4.4 | 2.8 |
| University of Alabama | 18.4 | 3.2 | 0.0 | 13.2 | 10.5 | 5.3 | 34.7 | 4.7 | 5.8 | 4.2 |
| University of Arizona | 17.3 | 6.4 | 5.1 | 10.9 | 5.1 | 5.1 | 32.7 | 0.0 | 7.1 | 10.3 |
| University of Arkansas | 28.5 | 4.2 | 4.2 | 14.6 | 5.5 | 5.5 | 22.9 | 0.0 | 10.4 | 4.2 |
| University of California | 15.0 | 0.0 | 0.0 | 11.0 | 12.0 | 7.0 | 40.0 | 0.0 | 8.0 | 7.0 |
| University of Cincinnati | 22.4 | 5.3 | 5.0 | 8.0 | 10.8 | 10.4 | 34.2 | 2.3 | 0.0 | 1.6 |
| University of Colorado | 30.0 | 7.1 | 0.0 | 14.3 | 7.1 | 7.1 | 31.4 | 3.0 | 0.0 | 0.0 |
| University of Detroit | 16.7 | 3.8 | 4.2 | 13.8 | 18.6 | 10.5 | 39.5 | 2.9 | 0.0 | 0.0 |
| University of Florida | 13.4 | 5.2 | 7.9 | 11.8 | 10.5 | 6.6 | 41.9† | 0.0 | 0.0 | 2.7 |
| University of Idaho | 21.5 | 7.8 | 0.0 | 15.0 | 7.8 | 5.2 | 36.2 | 1.3 | 0.0 | 5.2 |
| University of Illinois | 27.7 | 4.3 | 5.7 | 14.0 | 9.2 | 5.7 | 25.6 | 8.5 | 0.0 | 4.3 |
| University of Kansas | 17.0 | 7.0 | 7.0 | 12.0 | 11.0 | 9.0 | 28.0 | 5.0 | 2.0 | 2.0 |
| University of Maine | 20.2 | 6.5 | 6.5 | 13.4 | 8.7 | 6.0 | 26.5 | 4.6 | 4.0 | 3.6 |
| University of Michigan | 20.7 | 4.3 | 11.4‡ | 12.9 | 10.0 | 5.7 | 25.0 | 0.0 | 10.0 | 0.0 |
| University of Minnesota | 20.6 | 4.1 | 0.0 | 13.7 | 8.2 | 4.1 | 33.3 | 4.1 | 8.9 | 2.8 |
| University of Missouri (Rolla) | 18.6 | 6.4 | 3.5 | 10.5 | 7.6 | 8.5 | 26.7 | 1.2 | 14.8 | 2.3 |
| University of Missouri (Columbia) | 16.16 | 4.8 | 0.0 | 16.1 | 8.1 | 4.0 | 29.8 | 4.0 | 13.7 | 3.2 |
| University of Nebraska | 17.8 | 3.2 | 0.0 | 16.0 | 12.0 | 4.8 | 32.8 | 0.0 | 10.4 | 3.2 |
| University of New Mexico | 28.0 | 4.2 | 0.0 | 14.0 | 9.1 | 5.5 | 25.9 | 2.1 | 11.2 | ....§ |
| University of North Dakota | 21.0 | 7.0 | 5.5 | 11.0 | 9.5 | 8.0 | 33.0 | 2.0 | 1.5 | 1.5 |
| University of Notre Dame | 29.6 | 4.7 | 6.0 | 11.8 | 10.2 | 0.0 | 22.0 | 10.7 | 1.2 | 3.6 |
| University of Oklahoma | 25.2 | 6.8 | 0.0 | 13.7 | 14.3 | 5.3 | 29.0 | 3.4 | 0.0 | 1.3 |
| University of Pennsylvania | 26.0 | 3.5 | 4.6 | 8.0 | 7.6 | 6.1 | 33.5 | 1.5 | 0.0 | 9.2 |
| University of Tennessee | 10.4 | 6.9 | 6.9 | 11.6 | 4.6 | 5.2 | 39.4 | 0.0 | 3.4 | 11.6 |
| University of Texas | 20.7 | 9.5 | 0.0 | 14.3 | 15.8 | 8.0 | 27.0 | 4.7 | 0.0 | 0.0 |
| University of Utah | 14.6 | 4.6 | 0.0 | 12.0 | 7.7 | 9.2 | 41.5 | 4.5 | 0.0 | 1.5 |
| University of Vermont | 27.6 | 4.3 | 6.5 | 8.6 | 5.4 | 5.4 | 27.0 | 2.2 | 2.2 | 10.8 |
| University of Virginia | 23.4 | 0.0 | 0.0 | 11.8 | 9.0 | 5.9 | 44.0 | 0.0 | 0.0 | 5.9 |
| University of Washington | 22.4 | 1.5 | 0.0 | 9.4 | 9.4 | 7.9 | 26.0 | 0.0 | 17.2 | 6.2 |
| University of Wisconsin | 21.0 | 4.0 | 0.0 | 11.9 | 7.9 | 4.0 | 31.5 | 4.0 | 11.8 | 4.0 |
| University of Wyoming | 16.7 | 4.8 | 0.0 | 14.3 | 7.9 | 6.3 | 38.1 | 0.0 | 8.7 | 3.2 |
| Washington University | 22.3 | 5.0 | 6.3 | 11.3 | 8.8 | 5.4 | 32.7 | 4.4 | 1.3 | 2.5 |
| University of West Virginia | 21.0 | 6.0 | 0.0 | 11.0 | 7.0 | 7.0 | 44.0 | 0.0 | 0.0 | 4.0 |
| Worcester Polytechnic Institute | 20.9 | 4.8 | 7.7 | 11.4 | 8.8 | 6.8 | 32.1 | 5.3 | 0.0 | 2.2 |
| AVERAGE | 21.6 | 5.5 | 3.2 | 11.8 | 9.1 | 6.3 | 31.1 | 3.4 | 3.9 | 4.1 |

*Tulane—Includes shop work

†Florida—Includes shop work

‡Michigan—Includes cultural electives

§New Mexico—Freshman gymnasium not included in percentages.

## TABLE II.
## ELECTRICAL ENGINEERING

| INSTITUTION | Electrical engineering | English | Foreign languages | Mathematics excluding descriptive | Physics including el. mechanics | Chemistry | General engineering | General subjects | Electives | Physical Training |
|---|---|---|---|---|---|---|---|---|---|---|
| Norwich University...... | 33.5 | 3.9 | 7.9 | 9.9 | 9.9 | 3.9 | 13.2 | 5.9 | 1.3 | 10.5 |
| Montana State College of Agriculture and Mechanic Art......... | 33.0 | 3.0 | 0.0 | 13.0 | 16.0 | 5.2 | 15.2 | 7.0 | 5.2 | 2.4 |
| University of Colorado... | 30.0 | 7.1 | 0.0 | 14.3 | 7.1 | 7.1 | 31.4 | 3.0 | 0.0 | 0.0 |
| University of Notre Dame | 29.6 | 4.7 | 6.0 | 11.8 | 10.2 | 0.0 | 22.0 | 10.7 | 1.2 | 3.6 |
| Agric. & Mech. College of Texas............... | 28.8 | 12.5 | 00.0 | 13.3 | 4.5 | 4.5 | 26.7 | 3.3 | 1.7 | 6.7 |
| Lehigh University....... | 28.6 | 6.5 | 7.8 | 11.0 | 13.0 | 6.5 | 16.2 | 1.9 | 1.9 | 6.5 |
| Armour Institute........ | 28.6 | 2.4 | 00.0 | 7.2 | 8.8 | 6.4 | 35.0 | 6.8 | 17.9 | 4.8 |
| University of Arkansas .. | 28.5 | 4.2 | 4.2 | 14.6 | 5.5 | 5.5 | 22.9 | 0.0 | 10.4 | 4.2 |
| Carnegie Institute....... | 28.3 | 2.9 | 4.6 | 8.0 | 9.6 | 4.6 | 30.0 | 3.3 | 0.0 | 8.7 |
| Polytechnic Institute of Brooklyn............. | 28.0 | 6.6 | 3.7 | 10.7 | 9.4 | 7.0 | 28.0 | 2.8 | 1.9 | 1.9 |
| University of Mexico..... | 28.0 | 4.2 | 0.0 | 14.0 | 9.1 | 5.5 | 25.9 | 2.1 | 11.2 | ....* |
| Lafayette College........ | 27.7 | 3.9 | 3.9 | 11.0 | 6.4 | 6.4 | 24.5 | 11.0 | 3.9 | 1.3 |
| University of Illinois .... | 27.7 | 4.3 | 5.7 | 14.0 | 9.2 | 5.7 | 25.6 | 8.5 | 0.0 | 4.3 |
| University of Vermont... | 27.6 | 4.3 | 6.5 | 8.6 | 5.4 | 5.4 | 27.0 | 2.2 | 2.2 | 10.8 |
| State College of Washington........... | 27.2 | 6.2 | 0.0 | 12.7 | 6.2 | 6.8 | 29.7 | 4.7 | 3.1 | 3.7 |
| State University of Iowa. | 26.9 | 6.6 | 0.0 | 14.7 | 8.8 | 5.9 | 28.3 | 6.6 | 0.0 | 2.2 |
| Colorado College........ | 26.5 | 4.7 | 7.8 | 20.3 | 13.3 | 4.7 | 15.7 | 4.7 | 0.0 | 2.3 |
| University of Oklahoma.. | 26.2 | 6.8 | 0.0 | 13.7 | 14.3 | 5.3 | 29.0 | 3.4 | 0.0 | 1.3 |
| Univ. of Pennsylvania.... | 26.0 | 3.5 | 4.6 | 8.0 | 7.6 | 6.1 | 33.5 | 1.5 | 0.0 | 9.2 |
| George Washington University............ | 25.7 | 4.3 | 4.3 | 8.6 | 7.1 | 8.6 | 33.6 | 2.8 | 5.0 | 0.0 |
| Maryland State College of Agriculture......... | 25.7 | 10.8 | 5.6 | 10.5 | 4.5 | 4.5 | 21.6 | 9.3 | 0.0 | 7.5 |
| State University of Colorado............. | 25.5 | 5.0 | 0.0 | 10.0 | 8.5 | 7.0 | 21.3 | 2.5 | 16.2 | 4.0 |
| Case School............. | 25.2 | 6.4 | 6.4 | 10.4 | 9.1 | 5.5 | 25.9 | 1.3 | 1.3 | 8.5 |
| Oklahoma Agric. and Mechanical College.... | 25.0 | 5.9 | 0.0 | 12.5 | 11.0 | 5.7 | 30.9 | 1.5 | 3.0 | 4.5 |
| Oregon Agric. College.... | 25.0 | 4.5 | 0.0 | 13.0 | 9.5 | 4.5 | 30.0 | 0.0 | 5.7 | 7.5 |
| Tufts College........... | 24.0 | 4.0 | 4.0 | 9.0 | 6.0 | 6.0 | 22.0 | 2.0 | 21.0 | 2.0 |
| Massachusetts Inst. of Technology........... | 23.8 | 5.2 | 3.2 | 8.8 | 8.2 | 8.3 | 25.5 | 4.2 | 8.3 | 4.5 |
| University of Virginia ... | 23.4 | 0.0 | 0.0 | 11.8 | 9.0 | 5.9 | 44.0 | 0.0 | 0.0 | 5.9 |
| Rensselaer Polytechnic Institute............. | 23.3 | 5.9 | 6.2 | 13.0 | 9.2 | 6.2 | 33.8 | 1.1 | 0.0 | 1.3 |
| Bucknell University...... | 23.0 | 6.0 | 5.5 | 9.0 | 12.6 | 9.4 | 23.0 | 8.0 | 3.0 | 0.5 |
| Union College............ | 22.8 | 8.6 | 8.6 | 17.2 | 5.6 | 4.5 | 20.0 | 5.5 | 4.4 | 2.8 |
| Univ. of Cincinnati...... | 22.4 | 5.3 | 5.0 | 8.0 | 10.8 | 10.4 | 34.2 | 2.3 | 0.0 | 1.6 |
| Univ. of Washington..... | 22.4 | 1.5 | 0.0 | 9.4 | 9.4 | 7.9 | 26.0 | 0.0 | 17.2 | 6.2 |
| Washington University... | 22.3 | 5.0 | 6.3 | 11.3 | 8.8 | 5.4 | 32.7 | 4.4 | 1.3 | 2.5 |
| Pennsylvania State College............... | 22.0 | 7.5 | 7.5 | 12.5 | 7.5 | 5.5 | 30.0 | 7.5 | 0.0 | 0.0 |
| Iowa State College....... | 21.7 | 7.5 | 0.0 | 15.0 | 11.5 | 6.8 | 23.7 | 2.7 | 7.5 | 3.6 |
| University of Idaho...... | 21.5 | 7.8 | 0.0 | 15.0 | 7.8 | 5.2 | 36.2 | 1.3 | 0.0 | 5.2 |
| Georgia School of Technology........... | 21.1 | 8.5 | 6.1 | 11.0 | 10.8 | 6.0 | 25.3 | 2.5 | 0.0 | 8.7 |

(33 Colleges in all)

## TABLE III.

## ENGLISH

| INSTITUTION | Electrical engineering | English | Foreign languages | Mathematics excluding descriptive | Physics including el. mechanics | Chemistry | General engineering | General subjects | Electives | Physical Training |
|---|---|---|---|---|---|---|---|---|---|---|
| Agricultural and Mechanical College of Texas | 28.8 | 12.5 | 00.0 | 13.3 | 4.5 | 4.5 | 26.7 | 3.3 | 1.7 | 6.7 |
| Mississippi Agric. and Mechanical College | 15.3 | 10.9 | 0.0 | 18.8 | 7.0 | 4.5 | 26.1 | 5.9 | 6.3 | 5.2 |
| Maryland State College of Agriculture | 25.7 | 10.8 | 5.6 | 10.5 | 4.5 | 4.5 | 21.6 | 9.3 | 0.0 | 7.5 |
| Clemson College | 13.4 | 9.7 | 0.0 | 9.7 | 8.0 | 4.2 | 34.6 | 6.9 | 0.0 | 13.3 |
| University of Texas | 20.7 | 9.5 | 0.0 | 14.3 | 15.8 | 8.0 | 27.0 | 4.7 | 0.0 | 0.0 |
| Pennsylvania College | 15.0 | 9.2 | 3.9 | 12.4 | 14.4 | 9.2 | 23.5 | 11.8 | 0.0 | 0.7 |
| Union College | 22.8 | 8.6 | 8.6 | 17.2 | 5.6 | 4.5 | 20.0 | 5.5 | 4.4 | 2.8 |
| Georgia School of Technology | 21.1 | 8.5 | 6.1 | 11.0 | 10.8 | 6.0 | 25.3 | 2.5 | 0.0 | 4.2 |
| Throop College | 19.5 | 8.4 | 7.0 | 8.2 | 10.0 | 6.0 | 30.0 | 6.7 | 0.0 | 4.2 |
| Purdue University | 20.4 | 7.9 | 6.6 | 14.4 | 9.6 | 3.5 | 21.5 | 5.9 | 7.6 | 2.6 |
| University of Idaho | 21.5 | 7.8 | 0.0 | 15.0 | 7.8 | 5.2 | 36.2 | 1.3 | 0.0 | 5.2 |
| North Carolina State College | 19.4 | 7.7 | 5.3 | 11.9 | 8.5 | 6.0 | 29.4 | 5.3 | 0.0 | 6.6 |
| Iowa State College | 21.7 | 7.5 | 0.0 | 15.0 | 11.5 | 6.8 | 23.7 | 2.7 | 7 5 | 3.6 |
| Penn. State College | 22.0 | 7.5 | 7.5 | 12.5 | 7.5 | 5.5 | 20.0 | 7.5 | 0.0 | 0.0 |
| University of Colorado | 30.0 | 7.1 | 0.0 | 14.3 | 7.1 | 7.1 | 31.4 | 3.0 | 0.0 | 0.0 |
| Drexel Institute | 20.5 | 7.0 | 0.0 | 8.0 | 10.0 | 8.0 | 31.0 | 4.5 | 0.0 | 3.0 |
| University of Kansas | 17.0 | 7.0 | 7.0 | 12.0 | 11.0 | 9.0 | 28.0 | 5.0 | 2.0 | 2.0 |
| Univ. of Tennessee | 10.4 | 6.9 | 6.9 | 11.6 | 4.6 | 5.2 | 39.4 | 0.0 | 3.4 | 11.6 |
| University of Oklahoma | 26.2 | 6.8 | 0.0 | 13.7 | 14.3 | 5.3 | 29.0 | 3.4 | 0.0 | 1.3 |
| Polyt. of Brooklyn | 28.0 | 6.6 | 3.7 | 10.7 | 9.4 | 7.0 | 28.0 | 2.8 | 1.9 | 1.9 |
| State University of Iowa | 26.9 | 6.6 | 0.0 | 14.7 | 8.8 | 5.9 | 28.3 | 6.6 | 0.0 | 2.2 |
| University of Maine | 20.2 | 6.5 | 6.5 | 13.4 | 8.7 | 6.0 | 26.5 | 4.6 | 4.0 | 3.6 |
| Lehigh University | 28.6 | 6.5 | 7.8 | 11.0 | 13.0 | 6.5 | 16.2 | 1.9 | 1.9 | 6.5 |
| Tulane University | 12.3 | 6.4 | 0.0 | 12.8 | 7.9 | 8.9 | 48.4 | 0.0 | 2.0 | 1.3 |
| University of Arizona | 17.3 | 6.4 | 5.1 | 10.9 | 5.1 | 5.1 | 32.7 | 0.0 | 7.1 | 10.3 |
| University of Missouri | 18.6 | 6.4 | 3.5 | 10.5 | 7.6 | 8.5 | 26.7 | 1.2 | 14.8 | 2.3 |
| State University of Kentucky | 12.7 | 6.2 | 0.0 | 10.6 | 7.8 | 5.3 | 53.2 | 0.0 | 0.0 | 4.2 |
| State College of Washington | 27.2 | 6.2 | 0.0 | 12.4 | 6.2 | 6.8 | 29.7 | 4.7 | 3.1 | 3.7 |
| Bucknell University | 23.0 | 6.0 | 5.5 | 9.0 | 12.6 | 9.4 | 23.0 | 8.0 | 3.0 | 0.5 |
| Univ. of West Virginia | 21.0 | 6.0 | 0.0 | 11.0 | 7.0 | 7.0 | 44.0 | 0.0 | 0.0 | 4.0 |
| Oklahoma Agric. and Mech. College | 25.0 | 5.9 | 0.0 | 12.5 | 11.0 | 5.7 | 30.9 | 1.5 | 3.0 | 4.5 |
| Rensselaer Polytechnic Institute | 23.3 | 5.9 | 6.2 | 13.0 | 9.2 | 6.2 | 33.8 | 1.1 | 0.0 | 1.3 |
| Johns Hopkins University | 20.3 | 5.5 | 3.2 | 9.0 | 13.6 | 7.0 | 26.6 | 4.7 | 2.3 | 7.8 |
| University of Cincinnati | 22.4 | 5.3 | 5.0 | 8.0 | 10.8 | 10.4 | 34.2 | 2.3 | 0.0 | 1.6 |
| Massachusetts Institute of Technology | 23.8 | 5.2 | 3.2 | 8.8 | 8.2 | 8.3 | 25.5 | 4.2 | 8.3 | 4.5 |
| University of Florida | 13.4 | 5.2 | 7.9 | 11.8 | 10.5 | 6.6 | 41.9 | 0.0 | 0.0 | 2.7 |

## TABLE IV.

## FOREIGN LANGUAGES

| INSTITUTION | Electrical engineering | English | Foreign languages | Mathematics excluding descriptive | Physics including el. mechanics | Chemistry | General engineering | General subjects | Electives | Physical Training |
|---|---|---|---|---|---|---|---|---|---|---|
| University of Michigan .. | 20.7 | 4.3 | 11.4 | 12.9 | 10.0 | 5.7 | 25.0 | 0.0 | 10.0 | 0.0 |
| Union college........... | 22.8 | 8.6 | 8.6 | 17.2 | 5.6 | 4.5 | 20.0 | 5.5 | 4.4 | 2.8 |
| University of Florida..... | 13.4 | 5.2 | 7.9 | 11.8 | 10.5 | 6.6 | 41.9 | 0.0 | 0.0 | 2.7 |
| Norwich University...... | 33.5 | 3.9 | 7.9 | 9.9 | 9.9 | 3.9 | 13.2 | 5.9 | 1.3 | 10.5 |
| Colorado College........ | 26.5 | 4.7 | 7.8 | 20.3 | 13.3 | 4.7 | 15.7 | 4.7 | 0.0 | 2.3 |
| Lehigh University....... | 28.6 | 6.5 | 7.8 | 11.0 | 13.0 | 6.5 | 16.2 | 1.9 | 1.9 | 6.5 |
| Worcester Polytechnic... | 20.9 | 4.8 | 7.7 | 11.4 | 8.8 | 6.8 | 32.1 | 5.3 | 0.0 | 2.2 |
| Pennsylvania State College.............. | 22.0 | 7.5 | 7.5 | 12.5 | 7.5 | 5.5 | 30.0 | 7.5 | 0.0 | 0.0 |
| Throop College.......... | 19.5 | 8.4 | 7.0 | 8.2 | 10.0 | 6.0 | 30.0 | 6.7 | 0.0 | 4.2 |
| University of Kansas..... | 17.0 | 7.0 | 7.0 | 12.2 | 11.0 | 9.0 | 28.0 | 5.0 | 2.0 | 2.0 |
| University of Tennessee.. | 10.4 | 5.9 | 6.9 | 11.6 | 4.6 | 5.2 | 39.4 | 0.0 | 3.4 | 11.6 |
| Purdue University....... | 20.4 | 7.9 | 6.6 | 14.4 | 9.6 | 3.5 | 21.5 | 5.9 | 7.6 | 2.6 |
| University of Maine..... | 20.2 | 6.5 | 6.5 | 13.4 | 8.7 | 6.0 | 26.5 | 4.6 | 4.0 | 3.6 |
| University of Vermont... | 27.6 | 4.3 | 6.5 | 8.6 | 5.4 | 5.4 | 27.0 | 2.2 | 2.2 | 10.8 |
| Case School............. | 25.2 | 6.4 | 6.4 | 10.4 | 9.1 | 5.5 | 25.9 | 1.3 | 1.3 | 8.5 |
| Washington University... | 22.3 | 5.0 | 6.3 | 11.3 | 8.8 | 5.4 | 32.7 | 4.4 | 1.3 | 2.5 |
| Rensselaer Polyt. Inst. .. | 23.3 | 5.9 | 6.2 | 13.0 | 9.2 | 6.2 | 33.8 | 1.1 | 0.0 | 1.3 |
| Georgia School of Technology........... | 21.1 | 8.5 | 6.1 | 11.0 | 10.8 | 6.0 | 25.3 | 2.5 | 0.0 | 8.7 |
| University of Notre Dame | 29.6 | 4.7 | 6.0 | 11.8 | 10.2 | 0.0 | 22.0 | 10.7 | 1.2 | 3.6 |
| University of Illinois..... | 27.7 | 4.3 | 5.7 | 14.0 | 9.2 | 5.7 | 25.6 | 8.5 | 0.0 | 4.3 |
| Maryland State College of Agriculture . | 25.7 | 10.8 | 5.6 | 10.5 | 4.5 | 4.5 | 21.6 | 9.3 | 0.0 | 7.5 |
| Bucknell University...... | 23.0 | 6.0 | 5.5 | 9.0 | 12.6 | 9.4 | 23.0 | 8.0 | 3.0 | 0.5 |
| University of North Dakota ....... | 21.0 | 7.0 | 5.5 | 11.0 | 9.5 | 8.0 | 33.0 | 2.0 | 1.5 | 1.5 |
| North Carolina State College.............. | 19.4 | 7.7 | 5.3 | 11.9 | 8.5 | 6.0 | 29.4 | 5.3 | 0.0 | 6.6 |
| Ohio State University ... | 20.1 | 2.6 | 5.2 | 13.0 | 13.0 | 5.2 | 31.2 | 0.0 | 5.8 | 3.9 |
| University of Arizona.... | 17.3 | 6.4 | 5.1 | 10.9 | 5.1 | 5.1 | 32.7 | 0.0 | 7.1 | 10.3 |
| University of Cincinnati . | 22.4 | 5.3 | 8.0 | 8.0 | 10.8 | 10.4 | 34.2 | 2.3 | 0.0 | 1.6 |
| Carnegie Institute ...... | 28.3 | 2.9 | 4.6 | 8.0 | 9.6 | 4.6 | 30.0 | 3.3 | 0.0 | 8.7 |
| Univ. of Pennsylvania ... | 26.0 | 3.5 | 4.6 | 8.0 | 7.6 | 6.1 | 33.5 | 1.5 | 0.0 | 9.2 |
| George Washington University ........... | 25.7 | 4.3 | 4.3 | 8.6 | 7.1 | 8.6 | 33.6 | 2.8 | 5.0 | 0.0 |
| Stevens Institute ....... | 8.1 | 3.9 | 4.2 | 5.6 | 6.3 | 7.5 | 53.9 | 2.1 | 0.0 | 8.4 |
| University of Arkansas .. | 28.5 | 4.2 | 4.2 | 14.6 | 5.5 | 5.5 | 22.9 | 0.0 | 10.4 | 4.2 |
| University of Detroit.... | 16.7 | 3.8 | 4.2 | 13.8 | 18.6 | 10.5 | 39.5 | 2.9 | 0.0 | 0.0 |
| Tufts College........... | 24.0 | 4.0 | 4.0 | 9.0 | 6.0 | 6.0 | 22.0 | 2.0 | 21.0 | 2.0 |
| Lafayette College ....... | 27.7 | 3.9 | 3.9 | 11.0 | 6.4 | 6.4 | 24.5 | 11.0 | 3.9 | 1.3 |
| Pennsylvania College.... | 15.0 | 9.2 | 3.9 | 12.4 | 14.4 | 9.2 | 23.5 | 11.8 | 0.0 | 0.7 |
| New Mexico College of Agric. and Mech Art.. | 17.8 | 2.8 | 3.8 | 9.1 | 8.2 | 5.8 | 42.7 | 2.8 | 1.0 | 5.8 |

## TABLE V.

## MATHEMATICS EXC. DESCRIPTIVE

| INSTITUTION | Electrical engineering | English | Foreign languages | Mathematics excluding descriptive | Physics including el. mechanics | Chemistry | General engineering | General subjects | Electives | Physical Training |
|---|---|---|---|---|---|---|---|---|---|---|
| Colorado College........ | 26.5 | 4.7 | 7.8 | 20.3 | 13.3 | 4.7 | 15.7 | 4.7 | 0.0 | 2 3 |
| Mississippi Agric. and Mech. College......... | 15.3 | 10.9 | 0.0 | 18.8 | 7.0 | 4.5 | 26.1 | 5.9 | 6.3 | 5.2 |
| Union College........... | 22.8 | 8.6 | 8.6 | 17.2 | 5.6 | 4.5 | 20.0 | 5.5 | 4.4 | 2 8 |
| Syracuse University...... | 25.0 | 4.0 | 0.0 | 16.3 | 10.2 | 7.5 | 35.0 | 2.0 | 0.0 | 0.0 |
| University of Missouri (Columbia)........... | 16.1 | 4.8 | 0.0 | 16.1 | 8.1 | 4.0 | 29.8 | 4.0 | 13.7 | 3.2 |
| University of Nebraska... | 17.8 | 3.2 | 0.0 | 16.0 | 12.0 | 4.8 | 32.8 | 0.0 | 10.4 | 3.2 |
| Iowa State College....... | 21.7 | 7.5 | 0.0 | 15.0 | 11.5 | 6.8 | 23.7 | 2.7 | 7.5 | 3.6 |
| University of Idaho...... | 21.5 | 7.8 | 0.0 | 15.0 | 7.8 | 5.2 | 36.2 | 1.3 | 0.0 | 5.2 |
| State University of Iowa. | 26.9 | 6.6 | 0.0 | 14.7 | 8.8 | 5.9 | 28.3 | 6.6 | 0.0 | 2 2 |
| University of Arkansas... | 28.5 | 4.2 | 4.2 | 14.6 | 5.5 | 5.5 | 22.9 | 0.0 | 10.4 | 4.2 |
| Purdue University....... | 20.4 | 7.9 | 6.6 | 14.4 | 9.6 | 3.5 | 21.5 | 5.9 | 7.6 | 2.6 |
| University of Texas...... | 20.7 | 9.5 | 0.0 | 14.3 | 15.8 | 8.0 | 27.0 | 4.7 | 0.0 | 0.0 |
| University of Colorado... | 30.0 | 7.1 | 0.0 | 14.3 | 7.1 | 7.1 | 31.4 | 3.0 | 0.0 | 0 u |
| University of Wyoming.. | 16.7 | 4.8 | 0.0 | 14.3 | 7.9 | 6.3 | 38.1 | 0.0 | 8.7 | 3.2 |
| University of Illinois..... | 27.7 | 4.3 | 5.7 | 14.0 | 9.2 | 5.7 | 25.6 | 8.5 | 0.0 | 4.3 |
| Univ. of New Mexico.... | 28.0 | 4.2 | 0.0 | 14.0 | 9.1 | 5.5 | 25.9 | 2.1 | 11.2 | ....* |
| University of Detroit.... | 16.7 | 3.8 | 4.2 | 13.8 | 18.6 | 10.5 | 39.5 | 2.9 | 0.0 | 0 0 |
| Univ. of Minnesota...... | 20.6 | 4.1 | 0.0 | 13.7 | 8.2 | 4 1 | 33.3 | 4.1 | 8.9 | 2 8 |
| University of Oklahoma.. | 26.2 | 6.8 | 0.0 | 13.7 | 14.3 | 5.3 | 29.0 | 3.4 | 0.0 | 1.3 |
| University of Maine..... | 20.2 | 6.5 | 6.5 | 13.4 | 8.7 | 6.0 | 26.5 | 4.6 | 4.0 | 3.6 |
| Agric. and Mech. College of Texas............. | 28.8 | 12.5 | 00.0 | 13.3 | 4.5 | 4 5 | 26.7 | 3.3 | 1.7 | 6.7 |
| University of Alabama .. | 18.4 | 3.2 | 0.0 | 13.2 | 10.5 | 5.3 | 34.7 | 4.7 | 5.8 | 4.2 |
| Montana State College of Agric. and Mech. Arts. | 33.0 | 3.0 | 0.0 | 13.0 | 16.0 | 5 2 | 15.2 | 7.0 | 5.2 | 2.4 |
| Ohio State University.... | 20.1 | 2.6 | 5.2 | 13.0 | 13.0 | 5.2 | 31.2 | 0.0 | 5.8 | 3.9 |
| Oregon Agric. and Mech. College......... | 25.0 | 4.5 | 0.0 | 13.0 | 9.5 | 4.5 | 30.0 | 0.0 | 5.7 | 7.5 |
| Rensselaer Polyt......... | 23.3 | 5.9 | 6.2 | 13.0 | 9.2 | 6.2 | 33.8 | 1.1 | 0.0 | 1.3 |
| University of Michigan.... | 20.7 | 4.3 | 11.4 | 12.9 | 10.0 | 5 7 | 25.0 | 0.0 | 10.0 | 0 0 |
| Tulane University....... | 12.3 | 6.4 | 0.0 | 12.8 | 7.9 | 8 9 | 48.4 | 0.0 | 2.0 | 1.3 |
| Olkahoma Agric. and Mech. College. ....... | 25.0 | 5.9 | 0.0 | 12.5 | 11.0 | 5 7 | 30.9 | 1.5 | 3.0 | 4.5 |
| Penn. State College...... | 22.0 | 7.5 | 7.5 | 12.5 | 7.5 | 5 5 | 30.0 | 7.5 | 0.0 | 0.0 |
| Pensylvania College...... | 15.0 | 9.2 | 3 9 | 12.4 | 14.4 | 9.2 | 23.5 | 11.8 | 0.0 | 0.7 |
| State College of Washington........... | 27.2 | 6.2 | 0 0 | 12.4 | 6.2 | 6.8 | 29.7 | 4.7 | 3.1 | 3.7 |
| New Hampshire College . | 21.0 | 4.1 | 0.0 | 12.2 | 10.1 | 6 1 | 35.7 | 0.0 | 5.4 | 5.4 |
| University of Kansas..... | 17.0 | 7.0 | 7 0 | 12.0 | 11.0 | 9 0 | 28.0 | 5.0 | 2.0 | 2.0 |
| University of Utah...... | 14.6 | 4.6 | 0 0 | 12.0 | 7.7 | 9 2 | 42.5 | 4.5 | 0.0 | 1.5 |
| North Carolina State College.... .......... | 19.4 | 7.7 | 5.3 | 11.9 | 8.5 | 6 0 | 29.4 | 5.3 | 0.0 | 6.6 |
| Univ. of Wisconsin...... | 21.0 | 4.0 | 0 0 | 11.9 | 7.9 | 4 0 | 31.5 | 4.0 | 11.8 | 4.0 |

## TABLE VI.

## ELECTIVES

| INSTITUTION | Electrical engineering | English | Foreign languages | Mathematics excluding descriptive | Physics including el. mechanics | Chemistry | General engineering | General subjects | Electives | Physical Training |
|---|---|---|---|---|---|---|---|---|---|---|
| Tufts College........... | 24.0 | 4.0 | 4.0 | 9.0 | 6.0 | 6.0 | 22.0 | 2.0 | 21.0 | 2.0 |
| Armour Inst. of Technology........... | 28.6 | 2.4 | 00.0 | 7.2 | 8.8 | 6.4 | 35.0 | 6.8 | 17.9 | 4.8 |
| University of Washington........... | 22.4 | 1.5 | 0.0 | 9.4 | 9.4 | 7.9 | 26.0 | 0.0 | 17.2 | 6.2 |
| State Agric. College of Colorado............. | 25.5 | 5.0 | 00.0 | 10.0 | 8.5 | 7.0 | 21.3 | 2.5 | 16.2 | 4.0 |
| University of Missouri (Rolla)............... | 18.6 | 6.4 | 3.5 | 10.5 | 7.6 | 8.5 | 26.7 | 1.2 | 14.8 | 2.3 |
| University of Missouri (Columbia)........... | 16.1 | 4.8 | 0.0 | 16.1 | 8.1 | 4.0 | 29.8 | 4.0 | 13.7 | 3.2 |
| University of Wisconsin . | 21.0 | 4.0 | 0.0 | 11.9 | 7.9 | 4.0 | 31.5 | 4.0 | 11.8 | 4.0 |
| Univ. of New Mexico ... | 28.0 | 4.2 | 0.0 | 14.0 | 9.1 | 5.5 | 25.9 | 2.1 | 11.2 | .... |
| University of Arkansas... | 28.5 | 4.2 | 4.2 | 14.6 | 5.5 | 5.5 | 22.9 | 0.0 | 10.4 | 4.2 |
| University of Nebraska... | 17.8 | 3.2 | 0.0 | 16.0 | 12.0 | 4.8 | 32.8 | 0.0 | 10.4 | 3.2 |
| University of Minnesota . | 20.6 | 4.1 | 0.0 | 13.7 | 8.2 | 4.1 | 33.3 | 4.1 | 8.9 | 2.8 |
| University of Wyoming.. | 16.7 | 4.8 | 0.0 | 14.3 | 7.9 | 6.3 | 38.1 | 0.0 | 8.7 | 3.2 |
| Massachusetts Institute of Technology ........ | 23.8 | 5.2 | 3.2 | 8.8 | 8.2 | 8.3 | 25.5 | 4.2 | 8.3 | 4.5 |
| University of California.. | 15.0 | 0.0 | 0.0 | 11.0 | 12.0 | 7.0 | 40.0 | 0.0 | 8.0 | 7.0 |
| Purdue University....... | 20.4 | 7.9 | 6.6 | 14.4 | 9.6 | 3.5 | 21.5 | 5.9 | 7.6 | 2.6 |
| Iowa State College....... | 21.7 | 7.5 | 0.0 | 15.0 | 11.5 | 6.8 | 23.7 | 2.7 | 7.5 | 3.6 |
| University of Arizona.... | 17.3 | 6.4 | 5.1 | 10.9 | 5.1 | 5.1 | 32.7 | 0.0 | 7.1 | 10.3 |
| Massachusetts Agric. and Mech. College......... | 15.3 | 10.9 | 0.0 | 18.8 | 7.0 | 4.5 | 26.1 | 5.9 | 6.3 | 5.2 |
| Ohio State University.... | 20.1 | 2.6 | 5.2 | 13.0 | 13.0 | 5.2 | 31.2 | 0.0 | 5.8 | 3.9 |
| University of Alabama... | 18.4 | 3.2 | 0.0 | 13.2 | 10.5 | 5.3 | 34.7 | 4.7 | 5.8 | 4.2 |
| Oregon Agric. College.... | 25.0 | 4.5 | 0.0 | 13.0 | 9.5 | 4.5 | 30.0 | 0.0 | 5.7 | 7.5 |
| New Hampshire College | 21.0 | 4.1 | 0.0 | 12.0 | 10.1 | 6.1 | 35.7 | 0.0 | 5.4 | 5.4 |
| Montana State Agric. and Mech. College..... | 33.0 | 3.0 | 0.0 | 13.0 | 16.0 | 5.2 | 15.2 | 7.0 | 5.2 | 2.4 |
| George Washington University............ | 25.7 | 4.3 | 4.3 | 8.6 | 7.1 | 8.6 | 33.6 | 2.8 | 5.0 | 0.0 |
| Union College........... | 22.8 | 8.6 | 8.6 | 17.2 | 5.6 | 4.5 | 20.0 | 5.5 | 4.4 | 2.8 |
| University of Maine..... | 20.3 | 6.5 | 6.5 | 13.4 | 8.7 | 6.0 | 26.5 | 4.6 | 4.0 | 3.6 |
| Lafayette College........ | 27.7 | 3.9 | 3.9 | 11.0 | 6.4 | 6.4 | 24.5 | 11.0 | 3.9 | 1.3 |
| University of Tennessee . | 10.4 | 6.9 | 6.9 | 11.6 | 4.6 | 5.2 | 39.4 | 0.0 | 3.4 | 11.6 |
| State College of Washington........... | 27.2 | 6.2 | 0.0 | 12.4 | 6.2 | 6.8 | 29.7 | 4.7 | 3.1 | 3.7 |
| Bucknell University...... | 23.0 | 6.0 | 5.5 | 9.0 | 12.6 | 9.4 | 23.0 | 8.0 | 3.0 | 0.5 |
| Oklahoma Agric and. Mech. College......... | 25.0 | 5.9 | 0.0 | 12.5 | 11.0 | 5.7 | 30.9 | 1.5 | 3.0 | 4.5 |
| Cornell University....... | 20.6 | 0.0 | 0.0 | 7.8 | 9.0 | 7.0 | 46.5 | 3.9 | 2.6 | 2.6 |
| Johns Hopkins University | 20.3 | 5.5 | 3.2 | 9.0 | 13.6 | 7.0 | 26.6 | 4.7 | 2.3 | 7.8 |
| University of Vermont... | 27.6 | 4.3 | 6.5 | 8.6 | 5.4 | 5.4 | 27.0 | 2.2 | 2.2 | 10. |
| Tulane University....... | 12.3 | 6.4 | 0.0 | 12.8 | 7.9 | 8.9 | 48.4 | 0.0 | 2.0 | 1 · |

## ANNUAL REPORT OF THE MARINE COMMITTEE

*To the Board of Directors,*

The Marine Committee submits the following report for the year 1917-18.

Two merchant vessels equipped with alternating-current lighting and motor service including engine room auxiliaries as well as deck axuiliaries have been completed and put in service. As mentioned in last year's report these equipments followed land practise adopting 250-volt, 60-cycle, three-phase alternating current. The total power provided was 200 kw. divided in two equal units.

These equipments were thoroughly tested and found satisfactory by extensive trial trips of the vessels but the vessels themselves have not been in service a sufficient time to warrant conclusions to be drawn as to service conditions. The vessels are both oil carriers and the heaviest auxiliary loads were those connected with the cargo oil pumps. It is to be expected that the owners will maintain records to show whether the vessels show marked improvement in the loading and discharging of cargo. It was upon this basis and the danger coincident to the use of d-c. motors that the application was made. The two electrically propelled merchant vessels have not yet been completed. Their equipments, however, are now under construction.

Much work has recently been projected on the basis of using oil engine-driven generators and electric motors for ship propulsion. For reasons connected with the low speed of the oil engine and increased efficiency, these plants have been designed for 25 cycles. It is understood that two such vessels may be so equipped.

### *Present Activities*

Your committee has not been able to make further progress this year regarding the full revision of the electrical rules of Lloyd's Register of British and Foreign Shipping. The war conditions have prevented the necessary conferences, but minor matters of installation have been referred to the Lloyd's Register from time to time and approval given in accord with American practise. It was the consensus of opinion of this committee at its last meeting that the present time would not permit of the preparation of technical papers either for presentation at meetings or for publication in the TRANSACTIONS. There has been, however, a favorable tendency towards the writing of popular articles on marine subjects in order to aid the general public in its conception of the extent of the uses of electricity in the marine field.

### *Suggestions for the Future*

It is the purpose of your committee to continue to make suggested changes in the rules of the various classification Societies, and as experimental equipments emerge into established practise this committee will make the proper recommendations. As was inferred above, the intensive work of the individual members of the committee now prevents the writing of technical papers. It is believed that the time is approaching when it will not only be expedient but necessary to have such papers prepared and published in the TRANSACTIONS. The tendency in the field of ship propulsion seems to be approaching nearer to the use of electric drive due probably to increased interest on the part of ship owners and marine engineers in the efficiency of such systems, and the possibility of obtaining electrical apparatus with less difficulty than other types of propulsive machinery.

The committee desires to call your attention to the desirability of closer coordination of its work with the other technical committees of the Institute. The growth of the shipbuilding industry in this country and allied problems makes this most desirable.

H. A. HORNOR, *Chairman*

## ANNUAL REPORT OF THE POWER STATIONS COMMITTEE

*To the Board of Directors,*

The Committee held two meetings during the year, which, however, were poorly attended. It was early recognized undesirable to request from any engineer any labor for committee work except that which would be absolutely necessary or would become of vital importance to the operation of plants during the war.

At the last meeting, held December 14, 1917, two members in addition to the Chairman undertook to investigate and collect all available information on the broad questions of savings in production and utilization of power. Mr. Gorsuch has collected a good deal of information on what has been accomplished in utilizing waste gases in industries and tying together the electric power distributing lines with such by-product power plants; also studies of fundamental factors affecting economies in operation of power plants and favorable conditions under which they may be secured.

Mr. Putnam and the Chairman undertook to review the present-day relative economic values of new water power developments vs. steam power developments and their dependency and co-ordination, having in view the advisability or not of investing new capital in new water power developments during the time of the war, in contra-distinction of what the economic factors would have been previous to the war and what may be after the war.

The Chairman in collecting these subjects had in view the possibility of eventually securing proper papers for presentation at one of the meetings of the Institute if it were found desirable to cover at such meeting the subjects from the standpoint of the broad national policy during the war period.

In pursuing the study, the Chairman soon found out that the subject of power is a complex one and has many ramifications, so that a comprehensive solution could not be attained without securing the co-operation of representatives of different organizations interested in the application of water powers, steam powers, best methods of securing highest fuel economy by concentration of power generation, inter-connection of systems, possibilities of economies in use of wastes and gases from by-product coke ovens, powdered fuel, etc.

The Chairman, on the occasion of the Mid-Winter Convention, took the opportunity of suggesting to the President that it might be advisable to initiate the organization of a National Engineering Commission for considering and discussing plans and ways of advancing the recommendations made in his address. Such a Commission would naturally broaden out to study and report on policies affecting economics of power generation for general power application, steam electrification and special industries requiring continuous use of power. It was believed that such a study and recommendation would be of immense value to the industries, Government and State in shaping their policies in the generation and utilization of power.

A conference to discuss the subject thoroughly and outline plans could not, on account of the pressure of other matters, be arranged to include all who, in the opinion of the Chairman, should be present.

Some individual work was, however, done and considerable material is now available for use if the new administration should decide to carry out the plan.

Respectfully submitted,
PHILIP TORCHIO, *Chairman*

*Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.*

# SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION

BY W. H. COLE

## Abstract of Paper

Primarily, this paper is intended to be a brief history of the principal experiences of the Edison Electric Illuminating Company of Boston in the design and application of selective balanced-protection schemes to parallel connected transmission conductors.

Split-conductor cables are discussed at considerable length, both as to design and operation. Paired ordinary conductors also are discussed, and their relation to so-called split-conductors pointed out.

Special apparatus and devices required in connection with current-balancing schemes are illustrated and discussed.

A partial nomenclature is proposed, to assist in clearing the way for intelligent discussion and a uniform understanding of the general subject of current-balance protection for paired conductors.

A schedule of installations in the Boston system is given in order that the extent of the work described may be visualized, supplemented by a description of the results obtained in actual operation.

No general conclusions are drawn since the paper is of the nature of a report on progress; specific conclusions are drawn, however, in a number of cases where the evidence or experience appears to be reasonably conclusive.

A mathematical discussion of a number of reactive end-impedance devices, by Professor C. A. Adams, is appended.

IT IS not the intention to offer this paper as a complete treatise on the general subject of balanced protection for lines and apparatus. It seems advisable to deal as briefly as possible with some of the experiences of the Edison Electric Illuminating Company of Boston in its pioneer work. Only such detailed description and discussion as appears necessary to explain the scope of the work, will be attempted. Such a recital may be of general interest, and of some value to transmission engineers. It is submitted with the hope that a general discussion will follow, disclosing experiences of other engineers along related lines.

Some years ago our engineers became convinced that line protection devices, which function with respect to time, to value of current, or to direction of power flow, were, in an extensive and rapidly growing transmission network, exceedingly difficult, and frequently impossible to adjust and maintain in such a relation one to another, as to provide for the automatic disconnection of any faulty element, without simultaneously permitting the disconnection or shutdown of elements not themselves involved in the fault.

Following this recognition of the inadequacy of such protective gear, careful investigation was made of fault discriminating systems then in use abroad. As a result our company determined to make use of one or more methods based on the current balance principle. Up to that time the input-output method commonly known abroad as the Merz-Price system, seemed to be the most popular, although the older idea of balancing conductors in pairs also had seen some application.

Our company at first gave serious consideration to the input-output method, as it appeared to have enjoyed a considerable degree of success. Before we were prepared to install this scheme in connection with transmission cables, however, attention was directed to a reassembly of the older proposition of balanced pairs, which consisted of an arrangement of paired conductors in the form of a special cable. Since the pairs were to operate parallel connected at each end, and therefore with substantially no potential between the members of a pair, the amount of insulation between them could be comparatively light, and, consequently, one belt of primary insulation only, was required for each two paired conductors. This resulted in a cheaper and more compact transmission unit, for a given capacity, than two separate or independent primary insulated conductors, of equivalent capacity. It also appeared to be a less costly proposition than the "input-output" scheme, especially when the cost of duct space for the necessary pilot cables was considered.

Weighing the advantages of the two methods, led to a decision to make one or more installations of the special form of paired-conductor cable. This type of cable has now become quite well known as the "split-conductor" type. This name may not be the most expressive, since the arrangement is obviously not so much the splitting of conductors, as it is the assembly in an economical manner, of two separate conductors to be operated as paired conductors in a balanced protection scheme. It may be, however, desirable to perpetuate it when referring to any arrangement of paired conductors assembled and operated with a common primary dielectric. The nomenclature used herein is, therefore, based on an assumed division or splitting of conductors into an equivalent arrangement.

While paired conductors have been arranged or proposed in forms other than the "concentric twin," Fig. 1 and Fig. 2, such as the so-called "D twin," Fig. 3, and the "sector twin" Fig. 4, no form other than the "concentric twin" will be specifically considered herein, since this form, at least up to the present moment, seems to embody more desirable features than any other.

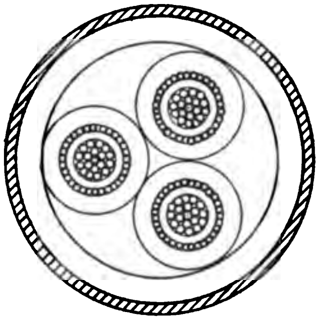

FIG. 1

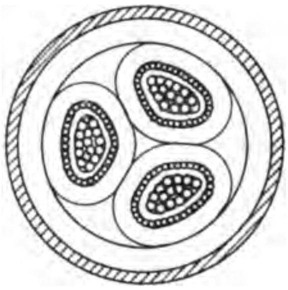

FIG. 2

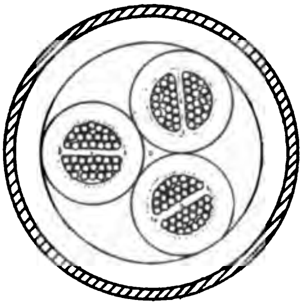

FIG. 3

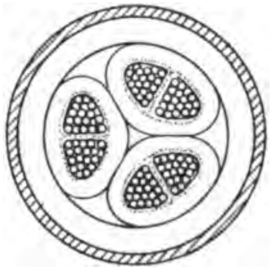

FIG. 4

As a matter of historical interest, it may with propriety be stated here, that the generic type of balanced protection hereinafter discussed, had its birth about 17 years ago, when it was suggested by Mr. E. M. Hewlett of Schenectady, N. Y. A form of balanced protection for paired conductors also was proposed by Mr. Leonard Andrews of Hastings, England, in 1902, embodying the first use of the differential reactor hereinafter described. Balanced protection schemes are, therefore, relatively old suggestions.

In order that the nomenclature used in this paper may be clear in meaning, it seems advisable to define some of the terms particularly applicable to "split-conductor" cable, and to balanced protection, as follows:

1. *Split-Conductor* refers to a conductor divided into two strands separated from one another by comparatively thin insulation, the strands assembled in various shapes and surrounded by insulation commensurate with the operating voltage of the system in which it is to operate.

2. *Conductor Member* refers to one of the conducting strands of a split-conductor.

3. *Primary Dielectric* refers to the dielectric surrounding an assembled group of conductor members, also to the dielectric surrounding an assembly of split-conductors.

4. *Secondary Dielectric* refers to the dielectric between the conductor members. It is also sometimes called "split-insulation."

5. *Impedance Differential* refers to the degree of unbalance between the impedances of the current paths of a split-conductor line. It is usually expressed as the percentage, which the difference between the impedances, bears to the impedance of one of the paths.

6. *Resistance Differential* refers to the degree of unbalance between the resistances of the members of an individual cable section. It is usually expressed as the percentage, which the difference between the resistances, bears to the resistance of one of the members.

7. *Normal Differential Current* refers to the vector differential current in an unfaulted split-conductor line. It is a direct measure of the "Impedance Differential." Its value is directly proportional to the total current and, therefore, reaches its maximum when a through short-circuit occurs.

8. *Tripping Differential Current* refers to the minimum value of differential current to which the relays are responsive.

9. *Through Short Circuit* refers to a short circuit, current to which is fed through any line under consideration. Such current flow is usually termed "through short-circuit current."

10. *Balancing or Differential Transformer* refers to the transformer devices used for comparing the currents in the conductor members, whereby a current proportional to the vector differential current is derived and circulated through the relay circuit.

11. *End-Impedance or End-Reactor* refers to an impedance device used to insure the creation of tripping differential current at either end of a faulted line, when the fault is at or near the opposite end of the line.

12. *Split-Contact Switch* refers to a special form of switch or breaker having three fixed contacts per pole. These contacts are bridged by a three-point yoke or blade. Two of the contacts are the termini of the two members of a split-conductor. The normal closing movement of

the yoke or blade is such that the two conductor-member contacts are connected together, prior to their connection through to the bus. The switching sequence is exactly the reverse when disconnecting the line from the bus. The special function of the two-conductor member contacts with the associated portion of the yoke or blade, is to introduce impedance into one path to an end fault, in order to insure a tripping current differential at the remote end of the line.

13. *End Fault* refers to a primary fault which occurs at or so near the end of a line as to require external devices, such as end-impedances or split-contact switches, to assist in creating tripping differential currents at the opposite end of the line, to insure disconnection at that end.

14. *Middle Fault* refers to a fault which occurs toward or at the middle of the length of the line, and which does not require end-impedance devices to insure tripping differential current at both ends of the line.

15. *Secondary Fault* refers to a fault in the secondary dielectric permitting current flow from one conductor member to the other.

16. *Arithmetical Balancing* refers to any method of balanced protection, wherein the effect of arithmetical difference only, between the compared currents, is operative to cause operation of the disconnecting gear.

17. *Vectorial Balancing* refers to any method of balanced protection, wherein the effect of the vectorial difference between the compared currents, is operative to cause operation of the disconnecting gear.

18. *Primary Capacitance* refers to the ordinary capacitance of a cable, such as is usually measured between one conductor and the other two bunched with the sheath.

19. *Secondary Capacitance* refers to the capacitance of the secondary dielectric, as measured between one conductor member and its mate.

20. *Transposition Joint* refers to any joint in which the members of one or more of the conductors are transposed.

21. *Straight Joint* refers to any joint in which the conductor members are held in the same relative position through the joint.

22. *Self Healing* refers to automatic healing of a punctured dielectric of the saturated type. With balanced protection, it often happens that a line is so rapidly disconnected, the fault current does not attain high values, nor does it carbonize much material. The result is a fairly clean puncture which immediately becomes more or less completely sealed with hot oil or compound. If the potential is restored, it may take considerable time for the carbon particles to align sufficiently to cause repetition of the breakdown.

## SPLIT-CONDUCTOR CABLE DESIGN

In split-conductor cable design, the engineering problems do not differ materially from those arising in ordinary cable design, except with respect to the division of the conductors. The chief additional considerations are, first, the dimensions and type of the secondary dielectric and, second, the permissible resistance and impedance differentials.

In order intelligently to design the secondary dielectric, it is necessary to predict the probable value and duration of voltage stresses in the secondary dielectric, and when they may be expected to occur.

Cables are subjected to two classes of potential stresses, *i.e.*, those occurring during preliminary high potential tests, and those occurring in service. In general, only those occurring in service need be regarded as factors affecting cable design.

If, while in service, one member of a split-conductor becomes involved in a primary fault, say to earth, it is clear that some portion of the other member will for a finite period of time be at star potential above the faulty one. The rapidity of the change from this difference of pressure to a lesser one will depend upon the position of the fault with respect to the end of the cable; the frequency at which the greater one occurs, upon the type of system, *i.e.*, whether grounded or isolated neutral. The current flowing to the fault causes a differential *IR* drop from the terminals to the fault, and under some conditions this drop may result in establishing the star potential across the secondary dielectric. The linear amount of secondary dielectric affected by the higher stresses is dependent upon the position of the fault with reference to the length of the cable, and also upon the type of protective gear. The cumulative duration of stresses in the secondary dielectric for a given fault, is a function of the time required by the switch gear to disconnect the line from all sources of supply.

Since balanced protection schemes provide for rapid disconnection of faulted lines, it is a reasonable assumption that the higher secondary stresses will be more or less transient, for which a conservative allowance may be made when determining the thickness and grade of the secondary dielectric.

The possible effect of high potential testing of the primary dielectric, upon the secondary dielectric, requires further consideration. As stated above, the probable maximum secondary pressure due to operation, will not exceed the star potential of the system. Primary testing at double operating pressure may, if a primary failure occurs, result in stressing a part of the secondary dielectric with nearly four times the star potential. Occasional secondary failures, consequent upon primary failures due to such high-tension testing may, therefore, be expected.

Since high potential testing after installation, if accomplished without failure, causes no abnormal stress in the secondary dielectric, it is good commercial judgment to ignore the possibility of secondary failures resulting from primary high-tension tests, and design for operating star pressure maxima only.

The local heat set up by the arc, upon the occurrence of a primary fault, and its possible effect upon the secondary dielectric, must be recognized. As the outer member usually is very thin, we may expect it to be destroyed rapidly at the point of fault, accompanied by extremely local and intense heating or burning of the adjacent secondary dielectric. On the same basis of reasoning, the secondary dielectric often may be completely destroyed at this point, even though insulating material substantially thicker than required to withstand star pressure is provided. This risk coupled with the fact that the maximum pressure across this dielectric is coincident with the point of fault, makes it extremely probable that if any breakdown of the secondary dielectric does occur, it will occur at this point. It is assumed, of course, that this dielectric is uniform in value throughout its length.

At the beginning, we were forced to consider a choice beween a secondary dielectric of sufficient thickness to withstand successfully at least the star voltage of the system impressed across a dielectric being rapidly weakened by the arc at a fault, and a dielectric of minimum safe thickness from a mechanical standpoint, but of more than ample value for the normal operating voltage between conductor members. A choice of the first meant more expensive cable, while the second involved the danger of failure of the protective apparatus then available, to operate, if a secondary fault should occur. Our reasoning was, that the major portion of the secondary dielectric being substantially an idle investment under all normal conditions, any reduction in its thickness with consequent cheapening of the cable was justified, provided a form of protective gear could be devised whereby a failure of the secondary dielectric under any circumstance would not prevent prompt disconnection of the line so affected.

The required form of gear subsequently became available, resulting in our standardizing, tentatively at least, a secondary dielectric thickness of 3/64 in. (1.19 mm.) paper for all round type, paper-insulated, concentric split-conductors forl 5,000-volt working pressure; 1/16 in. (1.58 mm.) paper for the same class of conductors for 25,000 working pressure; and 5/64 in. (3.96 mm.) paper for both 15,000 and 25,000-volt sector type, paper-insulated, concentric split-conductors. The additional thickness in the sector split type is due to what appeared to be mechanical necessity, *i.e.*, relatively sharp corners of the inner member, but it will undoubtedly be reduced as the art of manufacture improves. So far as our experience indicates, these thicknesses, if properly applied and thoroughly impregnated, with the conductor members free from mechanical defects, are ample to meet the conditions imposed by manufacture, installation, and service, with our standard form of protective gear.

For economic reasons it seemed better to assume some risk of a possible secondary failure, if the cable was also simultaneously involved in a primary fault and, therefore, bound to be disconnected, than to specify a secondary dielectric of a thickness calculated to withstand any possible stress to which it might be subjected, knowing that this maximum stress cannot occur except under primary fault conditions in the same line.

The impedance differential in a completed line must be considered jointly with the characteristics of the protective gear at the ends of the line, and its magnitude limited in accordance therewith, since if the maximum normal differential current is permitted to be high, thereby requiring a high setting of the relays, the means for creating the required value of tripping differential current under and fault conditions must be of corresponding magnitude. This limitation is of particular importance, if the end-impedance devices are the reactive type.

In order that the impedance differential of a completed cable line shall be as small as desired, it is first necessary to care properly for the resistance differential in the individual cable sections, by limiting its value, or preferably its variation from a predetermined percentage of the resistance of one of the conductor members. In other words, it is not so important that the conductors agree with each other to a small percentage, as it is that the ratio of their resistances, one to the other, shall not differ by more than a small percentage. Second, it is necessary that the conductor members shall be so transposed in the jointing of the cable sections as to insure that the resistance differential of the completed line is within the desired limits, which also provides for relative changes in temperature and resistance of the conductor members. Third, the reactance differential of the completed line must be kept down to the proper value, and it is ordinarily accomplished by the transposing required for resistance balancing. As it is a much larger differential in the individual cable sections than the resistance differential, running as high as 40 per cent, it may be the larger component of the impedance differential. It is very necessary that this differential be carefully equalized in a completed line. Its value is fixed by the design of the conductor and, therefore, is out of the control of the manufacturer.

A reasonable specification for resistance differential of a cable section is that it shall not exceed two per cent when the nominal resistance ratio of the conductor members is required to be 100: 100. If it is desired to specify a nominal resistance ratio of, say 100: 102, then the requirement should be that the ratio of resistance of the conductor members shall be not greater than 100:101, nor less than 100:103.

Our experience, without feeder reactors, has shown that no material difficulty need be expected in getting completed lines to balance closely enough to permit the maximum through short-circuit current to flow, without setting up normal differential currents of tripping value. Short lines originating at a power plant are, of course, subject to the heaviest through short circuits and, therefore, require the most care in balancing. While long lines do not require as close balancing, they are easier to balance than short lines, since the law of averages has a more effective application.

During the war, the copper wire market has been such, that considerable difficulty has been experienced by manufacturers, in securing copper drawn with the accuracy desirable for the production of well balanced split-conductors. This has forced the acceptance of cable not quite up to the pre-war standard, and has required more attention in the matter of transposing conductors, in order to secure the desired balance in the completed lines. It is the aim to secure impedance balance in completed lines to within one tenth of one per cent. With uniformly well balanced cable sections, this result is ordinarily secured with from one to five transpositions. The present practise is to use no less than three, dividing the line into four sections.

### RELAYS FOR CURRENT-BALANCE PROTECTION

Relays should be of the instantaneous overload type, operative on small amounts of energy, hand resetting, and the moving parts should have small inertia. Secondary devices are necessary to close the contacts in the tripping circuit. An auxiliary break in the tripping circuit should be provided, to be actuated by the movement of the main switch mechanism. At least two manufacturers in this country have produced a satisfactory form.

The hand-reset feature is very important, since it constitutes reliable means of diagnosing the class of fault causing the operation of the protective gear.

## Split-Conductor Protective Gear

Single-line diagrams, Figures 5, 6, 7, and 8, show diagrammatically some of the forms of protective gear we have used or considered, the form shown in Fig. 5 now being fairly well standardized.

At the risk of some repetition it may be advisable to explain

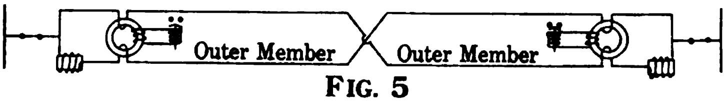

Fig. 5

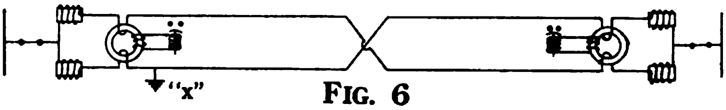

Fig. 6

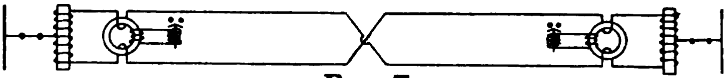

Fig. 7

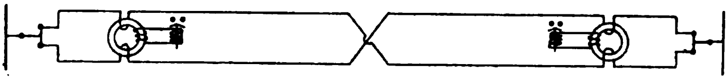

Fig. 8

the functions of the several devices shown, and discuss their different characteristics.

Common to all current-balancing schemes, an apparatus for weighing or comparing the currents in the paired conductors is required. The simple differential transformer is undoubtedly the best device for the purpose so far suggested. It consists of a standard type of current transformer core upon which are wound two primary coils, and one secondary coil. The primary coils are each respectively connected into one of the paired conductors in such a manner that under normal conditions of current flow, the coils will have equal and opposite effects upon the transformer core. If the primary coils are symmetrically disposed with reference to each other and to the core, no core flux will be set up, and consequently no current will flow in the secondary winding or relay circuit connected thereto. This is the condition that should obtain in a perfectly balanced split-conductor under normal conditions. Should a fault occur, however, resulting in a disturbance of the normal current flow, then the balanced-current condition no longer obtains, and the difference of current values in the balancing transformer primaries results in establishing a core flux, and consequently a current flow in the secondary coil and relay circuit. As soon as this secondary current reaches the tripping value, the line is automatically disconnected at one or both ends, depending upon the position of the fault.

If all faults in split-conductor lines were confined to the middle of the length of the line, and to one conductor member only, tripping differential current would be assured at both ends of the line, even though the balancing transformer windings have very little impedance. Balanced-current protection for paired conductors would, with such premises, be an ideally simple proposition.

Since it is absolutely necessary that the scope of the protection shall include the entire line, and extremely desirable that line disconnection be effected while the fault current is moderate in value, it is necessary to provide for what are known as end faults.

End fault protection is secured by the use of so-called end-impedance devices. Up to the present, two types have been developed, the reactive type and the non-reactive type. The reactive type has been proposed in several forms, three of which are shown in Figures, 5, 6 and 7. The non-reactive type as embodied in the three point per pole, or split-contact switch, Fig. 8, is the best known form of that type.

The theory of design and application of end-impedances is discussed quantitatively in the appendix to this paper, kindly prepared by Prof. C. A. Adams, but possibly a non-mathematical statement of the action of end-impedances may be helpful.

Assume that a failure occurs on a split-conductor, say at $X$, Fig. 6, affecting one member only, and for the moment that no impedance devices are in circuit, current will flow to or from the fault in two directions. At the fault end of the conductor, the flow through one primary coil of the balancing transformer will be reversed in direction with respect to the flow through the other primary coil. The current flowing in the first mentioned primary coil will be supplemented by current from the bus at the fault end of the conductor, if this bus is supplied by conductors other than the faulty one, or possibly by current from synchronous apparatus connected to this bus. As the effect of the current flow in the transformer primaries becomes cumulative under conditions of relative reversal of current, it will be seen that ample secondary or tripping current is assured, with consequent disconnection of the conductor at this end.

From the above, it is clear that end-impedance devices ***are not*** required in order to insure operation of the disconnecting gear at the end of a line adjacent to an end fault. Since, however, an end fault is so located with respect to the distant end of the line, that the impedance in one current path from the bus at the distant end is substantially equal to the impedance in the other current path, the two components of the total current flow from the distant end will also be substantially equal, or at least they will not differ by more than the normal differential current, due to the inherent impedance differential of the conductor. As the relays must not operate on normal differential currents, it is necessary, in order to produce disconnection of the distant end, that the impedance of one of the current paths be altered with respect to the other, to the extent required to produce a tripping differential current in the balancing transformer at the distant end of the line.

Referring again to single-line diagrams Figs. 5, 6, 7 and 8, and assuming an end fault on each, in a position equivalent to that shown in Fig. 6, and that all lines have been disconnected from the bus at the fault end in the manner previously described, the tripping differential current to insure disconnection at the distant end is assured as follows: In Fig. 5, the impedance in one current path to the fault becomes increased with respect to the other, due to the inclusion in the first path of both reactors. In Fig. 6 the impedance of one path is increased, due to the inclusion therein of three of the reactors as against one reactor only in the other path. In Fig. 7 the impedance of one path is increased, as the coils of the reactor at the fault end become cumulative in effect, due to relative reversal of current in one of them. In Fig. 8 the impedance of one path is increased (to an infinite value) by the actual opening of one path to the fault at the so called split-contacts.

## DISCUSSION OF END-IMPEDANCE DEVICES

While any of the types or forms shown, may be designed to provide adequate end-fault protection, they differ enough in their characteristics to warrant considerable study before a choice is made. Choice should be governed by such considerations as, strength of design, space required for their installation, effect on regulation, effect on switchgear and switchboard design, rapidity of action, cost, etc.

The form shown in Fig. 5 is a special arrangement of simple reactors, rather than a distinct form. It was developed as a result of a desire to reduce, for a given duty, the reactor capacity necessary in the forms shown in Figures 6 and 7, and as well to insure that tripping current differentials would be set up, should both the conductor members be simultaneously involved in a fault.

The form shown in Fig. 6 has had but little application and is shown and described merely for purposes of comparison. It is obviously the least desirable of all forms shown, since the capacity required for a given duty, the space required for installation, and the effect upon regulation, are all of an order twice that inherent to the scheme shown in Fig. 5. The arrangement compares favorably, however, with that shown in Fig. 7, except as to iron losses and effect upon regulation.

The form shown in Fig. 7 is best described as the differential type. It was suggested by Mr. Leonard Andrews of Hastings, England, about 16 years ago, and has had some application abroad, not only in the form illustrated, but also when provided with a super-imposed secondary winding of proper impedance, thereby combining in one device, the function of an end-impedance with that of a balancing transformer. Its chief claim for consideration, is its non-inductive characteristic, and absence of core loss, under conditions of normal current flow. At first glance it might be thought that this form of reactor could be designed with a minimum amount of iron in its core, since under normal conditions there is no flux therein, and thus it could be smaller and cheaper than other reactive forms of end-impedance. This type of reactor is, however, not only differential itself, but cooperates differentially with its mate at the opposite end of the line, so that it is the vectorial difference between their respective effects that produces the required differential current for tripping purposes, at the end remote from an end fault. Any attempt, therefore, to reduce the size and cost of this form of reactor, by designing it for high saturation in the core when functioning for end-faults, is ineffective, since the volt-ampere capacity must be so increased, as to offset any hoped for gains due to reduction of iron in the core.

The form shown in Fig. 8, is merely the combination of a main and auxiliary switch, the auxiliary contacts when opened separating the conductor members from each other. This isolation of the conductor members from each other is, in effect, equivalent to the introduction of infinite impedance into one path to an end-fault, when the opening occurs as a result of such a fault. If the opening is due to regular switching, the members are merely simultaneously disconnected from the bus at that end.

As before stated, the selection of end-impedance devices should be made with several considerations in mind.

From the standpoint of strength of design, the balance appears to be in favor of the reactive type, since it has no moving or wearing parts requiring adjustment or renewal.

The space requirement usually is a local consideration. American practise in line cell construction, however, seems to favor the reactive type, as the reactors in most cases may be installed in what would otherwise be unused space. The additional space required for split-contact switches usually is more difficult to obtain.

The reactive type, in some forms, admittedly has some effect upon voltage regulation. In any form the effect is not serious, particularly when we consider the growing use of current limiting reactors. The form shown in Fig. 5, in the capacities used by the Boston company, has about the same effect upon regulation as would a one per cent feeder reactor. The reactive form shown in Fig. 7, as well as the split-contact type shown in Fig. 8, has no effect upon regulation.

The relative effect of the two types of end impedances upon switch gear design, is of great importance. It is obvious that standard types of switches are adequate with reactive schemes of end protection; no departure from standard practise or design is necessary. The non-reactive type however, as embodied in the split-contact switch, requires a special design of switch. For small capacities this may not be a serious matter. For the large capacities, such as are common in American practise, a serious factor is introduced, *i. e.*, the necessity of designing the three-point or split-contact switches, so that all three breaks per pole have the same breaking capacity. This means that if a switch is of a type requiring two pots per pole for standard work, it will require an additional pot of equal capacity in order to convert it into a split-contact switch.

It may be thought that since the so-called split-contacts each carry only half the full line current, each might be designed for half duty, when compared with the third or so-called non-split contact. This assumption may be correct if the breaking of through current only is considered. If we assume a case when this type of switch is called upon to perform a dual duty, such as clearing an end fault, it can be shown that one of the split-contact breaks may have a duty exceeding that of the main or non-split-contact break.

Three-point or split-contact switches must be so designed that when being closed both conductor members shall be connected together before either one of them is connected to the bus. This must be assured, for if by chance one of them should be closed on a bus before the other, the instantaneous current flow through one conductor member only, will result in the simultaneous opening of the switches at both ends. In order to prevent such an occurrence, the relative position of the contacts and yoke or blades is made such, that the split-contacts will be connected together just prior to their joint connection to the bus or non-split-contact. The time interval between the two closures must be extremely short, in order to make the switch as effective as possible when rupturing current, since in opening, it is of great importance that all breaks per pole occur as nearly simultaneously as possible, in order to divide the breaking duty equally.

When a three-point switch operates on an end fault, the first break takes place at the main or non-split contact, closely followed by a break at the split-contacts. One of the split-contact breaks handles the component of the fault current flowing over the corresponding conductor member. The other split-contact break must handle the same component, plus any current flowing to the fault from the bus at the fault end of the line via the main break. The third or non-split contact break handles only the current flowing to the fault from the last mentioned bus. Thus it will be seen that one of the split-contact breaks may have a far greater duty to perform than any other.

Close analysis of the relative effect of the two types of end-impedances upon switch gear and switchboard design seems to indicate the superiority of the reactive form of end-impedances.

It has been claimed that the split-contact switch provides for more sensitive control, by virtue of its function of inserting infinite impedance into one path to an end-fault, thereby concentrating the fault current in one path only, at the end remote from the fault, thus creating at that end a greater tripping differential current than would be obtained if reactive impedance devices of permissible size were used. This advantage is real only in a limited degree, and only if the fault current is so small, that when it is divided between the two paths to the fault, as must be the case if reactive end-impedances are used, the differential current is less than that required to operate the relay. Such small fault currents are encountered only when the fault is one to earth, in a system having small capacitance to earth. In such systems the use of the split-contact switch may be justified, particularly as such a system will ordinarily be of small magnitude. In systems of large magnitude, the cable systems also are usually extensive, and fault currents to earth are ample to insure tripping differential current with reactive end-impedance devices of reasonable size, even if the system is operated with an isolated neutral. Whenever the system is of such size or type as to justify the use of reactive end-impedance devices, disconnection of lines under end-fault conditions is effected more rapidly with such devices than would be the case with split-contact switches, since tripping differential currents are set up simultaneously at each end. With split-contact switches, a tripping current differential exists at the fault end, only, until the operation of the switch at the fault end, effects the introduction of impedance and the setting up of a tripping current differential at the opposite end of the line.

On the whole, it would therefore appear, that considering the rapidity of action, at least on large systems, the reactive form of end-impedance device has a distinct advantage over the non-reactive type.

The matter of relative cost of the two types is at present unsettled, as the development in this country has proceeded along reactive lines. It is believed, however, that taking all features into consideration, the cost differential will be in favor of the reactive type.

In order to check the soundness of our conclusions as to end-impedance devices, it has been thought advisable to secure some actual operating experience with the more promising forms. As a result, we have placed in operation during the past two years two installations of the split-contact type of end-impedance, Fig. 8, also we have on order for installation during the current year, two installations of the differential type of end-impedance, Fig. 7.

### BALANCED PROTECTION FOR ORDINARY OR INDEPENDENT CONDUCTORS

While the experience of the Boston company in the balanced protection of paired ordinary conductors has not been extensive, the work to date has been of great importance in establishing a basis for future practise. It is believed, therefore, that a few references to this phase of the subject are permissible at this time.

As it is obvious that there is no material difference in operating principle between the older and well known arrangement of paired independent conductors, and the so-called split-conductor scheme, any choice between the two arrangements tends to be an economic one. It is incorrect, as is sometimes done, to refer to the earlier method as a combination of ordinary conductors on the "split-conductor plan", since such arrangements were proposed many years previous to the split-conductor suggestion.

In general the protective apparatus already described is applicable to any combination of paired conductors, but when the paired conductors are situated each in different cables or lines, they are each insulated for the primary voltage of the system, and are, therefore, independently suitable for operation if for any reason the companion conductor is out of service. It follows, therefore, with any arrangement of paired conductors, other than actual split-conductors, that each of the paired conductors ought to and usually does have its own switches independent of those for its parallel associate. The use of individual switches for each conductor, moreover, results in a combination equivalent in most respects to the split-conductor arrangement, employing split-contact switches in place of end-impedances or reactances, so that end-faults are adequately provided for.

With a paired independent conductor line, it is not possible to secure and maintain the degree of balance inherent to a split-conductor line, for even were it possible to start out with a fair balance, conditions local to one conductor or the other would soon affect the balance to a considerable degree; for example, should one line of a pair be out for repair and the other remain in service, it is obvious that when put in parallel again, the difference in conductor temperature, which might then exist, would be the cause of a material impedance differential. If a through short circuit should occur under such circumstances, the normal differential current would attain a very high value.

It is frequently desirable to consider paralleling, with balanced protection, conductors which differ materially in length, cross section, or reactance. In such cases it is impossible to operate without correspondingly large normal differential currents. Since the end fault condition in such lines will be cared for by the independent switches for each conductor, the remaining necessity is that of compensating for the large normal differential currents set up by through short circuits, and with the same devices provide for selective disconnection, should faults occur that result in differential currents materially smaller than those due to through currents of maximum values.

If the ordinary type of overload relay is used in this class of balanced protection, the settings might need to be so high as to preclude operation on section faults, or in any event to prevent obtaining the maximum benefit of balanced protection.

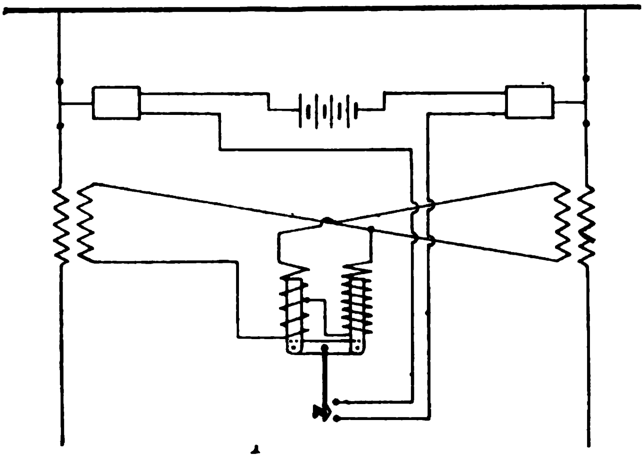

FIG. 9

In order to obviate the necessity of the high relay settings referred to, and at the same time to insure non-operation on through short circuits, relays have been developed both in this country and abroad, variously called, "biased relays", "percentage-balance relays", and "ratio-balance relays".

We have experimented for nearly four years with ratio-balance relays in the forms shown in Figs. 9 and 10. One pair of cables has been equipped with the form shown in Fig. 9, and has been in operation for over three years. In all cases of actual trouble, as well as under artificial fault conditions, the relays have functioned as predicted. While in the case of a single pair of lines, both are disconnected upon the occurrence of a fault in one, the other may be immediately put to work with straight instantaneous overload protection, since the relays may be arranged to so function for either line, when the companion line is out of service.

The form shown in Fig. 10 is particularly well adapted for use with groups of three or more parallel lines, in which case the relays may be electrically interlocked, so that the faulty line only is disconnected. When so applied, all the discriminative features of balanced current protection may be secured without the use of pilot wires, special cables, reactors, or split-contact switches.

The relays shown in Figs. 9 and 10 are so constructed, that the restraining force is proportional to the vector sum of two compared currents, and the operating force is proportional to the vector difference between the compared currents. With proper winding ratios, the restraining effect will predominate until the normal relation between the compared currents is altered, due to a fault involving one of the conductors, to such an extent as to cause the ratio between the vector sum and vector difference of the compared currents, to exceed a predetermined limit. After this limit is reached, the operating force becomes predominative and the relay operates to disconnect the faulty conductor.

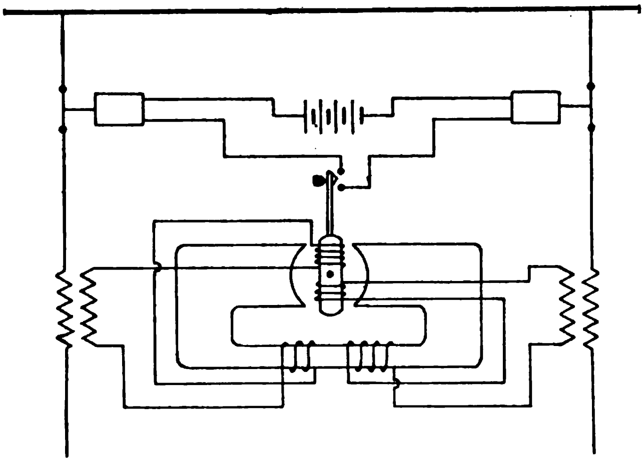

FIG. 10

In order to choose the proper winding ratio for ratio-balance relays, it is necessary to know what the normal differential current will be under extreme conditions, and the percentage it bears to the simultaneous vector sum of the compared currents. Once adjusted for this normal relation, they cannot operate on any value of through current, since the restraining force will then always be in excess of the operating force.

It is expected that upon the return of more normal conditions, further progress will be made in the application of ratio-balance protection to many of our standard transmission lines, by means of the devices described, or that may hereafter become available.

## SCHEDULE OF SPLIT-CONDUCTOR CABLE INSTALLATIONS

The following table shows the installations of split-conductor cable now included in the Boston Edison Company system:

| Line No. | Length in feet | Size | Voltage | Date placed in service |
|---|---|---|---|---|
| 34-82 | 32,032 | 3x4/0 | 15,000 | 9- 4-13 |
| 31-76 | 24,054 | 3x1/0 | 25,000 | 8-31-14 |
| 26-76 | 29,273 | 3x.10 sq. in. | 25,000 | 1-10-15 |
| 26-77 | 29,305 | 3x.10 sq. in. | 25,000 | 5-16-15 |
| 47-76 | 41,478 | 3x4/0 | 15,000 | 11-14-15 |
| 47-75 | 41,478 | 3x4/0 | 15,000 | 11-30-15 |
| 31-77 | 24,069 | 3x.10 sq. in. | 25,000 | 5-2 -16 |
| 16-99 | 7,599 | 3x4/0 | 15,000 | 8-25-16 |
| * 49-98 | 4,742 | 3x4/0 | 15,000 | 9-10-16 |
| * 49-95 | 4,742 | 3x4/0 | 15,000 | 11- 4-16 |
| † 23-75 | 23,652 | 3x3/0 | 25,000 | 11-26-16 |
| † 23-76 | 23,652 | 3x3/0 | 25,000 | 11-26-16 |
| 54-99 | 33,488 | 3x4/0 | 15,000 | 4- 7-17 |
| * 43-59 | 32,205 | 3x3/0 | 25,000 | 4- 1-17 |
| ‡ 8-53 | 6,510 | 3x4/0 | 15,000 | 7- 8-17 |
| ‡ 8-54 | 6,510 | 3x4/0 | 15,000 | 7 - 8-17 |
| ‡ 30-87 | 9,439 | 3x4/0 | 15,000 | 7-15-17 |
| ‡ 30-96 | 9,436 | 3x4/0 | 15,000 | 7-15-17 |
| 49-51 | 24,889 | 3x4/0 | 15,000 | 8-12 17 |
| 39-51 | 23,480 | 3x4/0 | 15,000 | 8-12-17 |
| 51-69 | 8,789 | 3x4/0 | 15,000 | 10-21-17 |
| * 57-69 | 9,189 | 3x4/0 | 15,000 | 10-21-17 |
| * 9-69 | 6,926 | 3x4/0 | 15,000 | 10-21-17 |
| 52-77 | 3,758 | 3x4/0 | 15,000 | 10-21-17 |
| 52-78 | 3,772 | 3x4/0 | 15,000 | 10 21-17 |
| ‡ 8-87 | 7,384 | 3x4/0 | 15,000 | 11-18-17 |
| * 58-80 | 6,742 | 3x4/0 | 15,000 | 11-18-17 |
| § 53-53 | 7,754 | 3x4/0 | 15,000 | |
| § 53-54 | 7,754 | 3x4/0 | 15,000 | |
| § 53-70 | 3,516 | 3x4/0 | 15,000 | |
| § 21-72 | 10,202 | 3x4/0 | 15,000 | |
| § 21-73 | 10,202 | 3x4/0 | 15,000 | |
| 101-102 | 50,044 | 3x4/0 | 15,000 | 5-1-18 |
| 101-103 | 49,272 | 3x4/0 | 15,000 | 5-1-18 |
| §101-104 | 48,733 | 3x4/0 | 15,000 | |
| §101-105 | 48,691 | 3x4/0 | 15,000 | |
| §103-59 | 11,552 | 3x3/0 | 25,000 | |
| § 20-59 | 21,841 | 3x3/0 | 25,000 | |

*Line not yet operating as split-conductor line.

†Lines at present operating in series with standard overhead lines.

‡Lines at present operating in series with standard conductor cable.

§Lines under construction to go in service 1918.

The total length of the above cables is 747,590 ft., or approximately 142 miles.

Lines 26-77 and 31-77 are connected in series at present and operate as one line, which is protected with the English form of split-contact switch gear, Fig. 8. Line 26-76 is equipped with a modification of the form of gear shown in Fig. 6. Lines 26-77 and 31-77 are to be separated during the current year and the new terminals are to be protected by the form of gear shown in Fig. 7, the opposite terminals remaining as before, protected with split-contact switch gear. The remaining lines are protected, or will be upon their completion as split-conductor lines, with the form of gear shown in Fig. 5.

## RESULTS OF OPERATION

Not including the preliminary testing of lines under artificial fault conditions, to determine their characteristics, and to check the theory of operation, a sufficient number and variety of actual faults have occurred to demonstrate that full reliance may be placed on the system of protection, provided the fundamental requirements are met in its design, installation, and operation.

Line 34-82 has now been in service nearly five years and with the possible exception of one case, to be referred to later, the balanced protection apparatus in connection therewith has functioned in accordance with theory, each time the line has been involved in a fault, or when subjected to the stress of feeding through short-circuit current to faults external to the line. This line up to the present has been involved in one secondary fault (due to defective stranding of the outer member of one conductor), one end fault, and at least six through faults, three of which were bus short circuits at one terminal of this line.

Line 47-75 has been in service about two and one half years and has been involved in one fault, a primary fault in a joint, one conductor to earth. This fault created so little disturbance that the station operator was inclined to feel doubtful as to the need of the switches opening. Tests were made to locate the fault, but after a while the operators concluded that none existed, and the line was placed back in service. Outside of some static discharges of small volume at the line terminal, which soon ceased, the line appeared as good as ever. In order to clear up the action of the relays, it was deemed advisable to inspect the line. Men were sent over the line, opening manholes and examining the cable therein, with the result that a joint was found with a hole blown through the lead sleeve, and fresh compound expelled in considerable quantity. The line was then promptly taken out of service, the faulty joint opened, whereupon it was found that one conductor had broken down to earth (corresponding to the relays that operated), but the fault had become healed due to hot compound running back into the wound after the line was disconnected. From all outward appearance, this line probably would have operated for years in its then condition. This case is described at some length, as such an occurrence may have been the cause of relays operating on our first line, No. 34-82, on one occasion without the cause ever having been determined. Self healing, therefore, may be expected. This experience, confirmed by results obtained in high potential testing, teaches that any case of apparently unnecessary operation should be carefully investigated before conclusions are drawn.

Line No. 31-77 has been in service about two years and has been involved in two faults. The first fault was due to a connector in a split-conductor pothead (English Type) becoming loosened, thereby increasing the impedance of one of the current paths and setting up a tripping differential. This caused several operations of the protective gear at each end before a diagnosis of the fault was made by our operators. The second fault was due to mechanical injury; a laborer driving a pick into the cable, grounding one conductor. The protective gear, (split-contact switch type) operating in the expected manner, isolating the line at both terminals. This line was again made alive, but was automatically disconnected again, since the fault was well established. The second operation resulted in breaking down both inner and outer members at the point of fault. The cable at the point of injury was laid on the solid system, *i.e.* armored, laid directly in the ground and covered with a two-in. (5.08-cm.) spruce plank. Repairs were effected by making a new joint in this location.

Line No. 31-76 has been in operation for nearly four years, but on account of defective joints and some dry spots in the secondary dielectric which were eliminated by high-tension tests, it was not finally accepted until about three years after installation. In the course of making the many primary high potential (50,000-volt) tests, the secondary dielectric failed, due to the stresses set up therein as a result of primary failures, at four or five of the dry spots, also in two cases where the outer member was defective, once as to stranding (crossed strands) and once where a strand had been brazed. This line could not be kept continuously out of service for testing, however, and on one occasion it was put back in service with the positive knowledge that an incipient secondary fault due to the testing above referred to was present in one conductor. No cable was available at the time to replace the faulted section, and as the fault was of very high resistance, it was decided to assume the risk of operating the line with balanced protection. This proved to be poor judgment on our part, since this fault developed in service to such an extent as to cause operation of the protective gear, the operation, of course, being perfectly normal for this type of fault. Unfortunately it occurred simultaneously with one of the outages of line No. 31-77 described above with which it was ring connected, and thereby contributed to an interruption of service. It was also unfortunate in this case that the first mentioned fault in line No. 31-77 had not been diagnosed and remedied at an earlier date. Such experiences, however, are of great importance as affecting future operating practise.

Line No. 54-99 was placed in service April 7, 1917 and within a few weeks afterward broke down between main conductors, resulting in its disconnection by the balancing gear. This fault occurred about one and one half miles from the power house. No disturbance was noticed when it was disconnected. Relays operated on two of the conductors, normally indicating a short between them. The operators put the line back again, however, and it ran about three minutes when it came out again, the same relays operating as before. The line was again made alive from a separate bus and generator at somewhat higher pressure than the system pressure, and remained in for four minutes, coming out for the third time upon the operation of the same two relays. This appeared to satisfy the operator that the line was faulty. The fault was located and removed, and was found to be due to a breakdown between the two phase wires corresponding to the two relays that operated in each of the three cases of disconnection. Since a short circuit of similar type and location, on a feeder protected with overload relays with as short a setting as one half second, usually makes itself known in a more or less violent manner, the performance of the protective devices on this line appears to be a substantial tribute to the efficacy of the balanced method of protection. In this case, it appears that each disconnection of the line was so rapid that the fault current built up to a value not materially greater than sufficient to create a tripping differential, and as soon as disconnection occurred the puncture healed up, due to the hot cable compound flowing into the wound. Undoubtedly enough carbon was suspended in the puncture path to gradually line up and bridge between phases, thus causing the second and third disconnection. It is a question how long this process could have been continued, but it would seem that two disconnections with the same relay operation in each case would have been conclusive evidence that a fault did in fact exist. Operators will undoubtedly require considerable experience in order to become convinced that quiet disconnection of transmission cables equipped with balanced protection is sufficient evidence of line failure.

As before stated, some trouble has been experienced by the manufacturers in getting perfect saturation of the secondary dielectric. In the case of the above mentioned primary fault in line No. 54-99, the condition of the secondary dielectric was not checked prior to putting the line back in service after repair, as is now our practise. It was soon found that one of the conductors involved in the primary fault had a secondary fault which resulted in the protective gear again operating. This secondary fault was located and removed, and was found to be due to defective stranding of the outer conductor member. The continued attempts to operate under the main fault condition stressed this defective point severely, and to an amount in excess of the stresses caused by the factory or installed test. It is significant that this secondary fault, as well as the two others (three in all) occurring in service to date, were located in defective construction, which permits of the reasonable assumption that none would have occurred if the construction had been perfect, and also that the values of secondary dielectric chosen for our service, are ample if properly incorporated in the construction.

It is perhaps unfortunate, but nevertheless unavoidable, that so much time is required to obtain extensive operating experience. It is obvious, however, that new cables are not expected to fail soon after having been carefully installed and tested.

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# APPENDIX

BY COMFORT A. ADAMS

The purpose of this appendix is to show the relative amounts of reactance required to produce a given current differential in the three arrangements shown in Figs. 5, 6 and 7, under end-fault conditions, since it is only under these conditions that reactance is required.

In order to simplify the final general analysis assume that the added reactance has in each case a constant value. The effect of saturation will be considered later.

Let $x$ equal the value of a single reactance in the arrangements of Figs. 5 and 6, and of the series or round-the-end reactance of one of the differential reactors of Fig. 7.

Assume that the switch at the fault end has opened—

Let $\dot{I}_1$ be the current from the far end to the fault, by the longer path, $\dot{I}_2$ that by the shorter path, both counted positive in the same direction from supply end to fault end, and $\dot{I} = \dot{I}_1 + \dot{I}_2$ the total fault current.

Let $r_1$ = the resistance of a single conductor member.

Let $x_1$ = the combined reactance of the two conductor members considered as a single conductor, in which the current is the vector sum of the two currents in the two members. This is sufficiently accurate for all practical purposes.

*Arrangement of Fig.* 5. Equating the potential drops from supply to faults by the two paths gives

$$\dot{I}_1 r_1 + 2jx\dot{I}_1 + jx_1(\dot{I}_1 + \dot{I}_2) = \dot{I}_2 r_1 + jx_1(\dot{I}_1 + \dot{I}_2)$$

$$\dot{I}_1(r_1 + 2jx) = \dot{I}_2 r_1$$

$$\frac{\dot{I}_2}{\dot{I}_1} = \frac{r_1 + 2jx}{r_1}$$

$$\frac{\dot{I}_2 - \dot{I}_1}{\dot{I}_2 + \dot{I}_1} = \frac{\dot{I}_d}{\dot{I}} = \frac{jx}{r_1 + jx}$$ where $\dot{I}_d$ is the differential current.

Or in numerical values —

$$\frac{I_d}{I} = q_d = \frac{x}{\sqrt{r_1^2 + x^2}}$$ where $q_d$ is the per cent differential current.

Let $\frac{x}{r_1} = q_x$ then

$q_d = \frac{q_x}{\sqrt{1 + q_x^2}}$ or for a given $q_d$ the necessary $q_x$ is

$$q_x = \frac{q_d}{\sqrt{1 - q_d^2}} \tag{1}$$

*Arrangement of Fig.* 6. Equating potential drops to fault as before —

$$\dot{I}_1 r_1 + 3jx\dot{I}_1 + jx_1\dot{I} = \dot{I}_2 r_1 + jx\dot{I}_2 + jx_1\dot{I}$$

$$\dot{I}_1 (r_1 + 3jx) = \dot{I}_2 (r_1 \times jx)$$

$$\frac{\dot{I}_2}{\dot{I}_1} = \frac{r_1 + 3jx}{r_1 + jx}$$

$$\frac{\dot{I}_2 - \dot{I}_1}{\dot{I}_2 + \dot{I}_1} = \frac{\dot{I}_d}{\dot{I}} = \frac{jx}{r_1 + 2jx}$$

Or in numerical values —

$$\frac{I_d}{I} = q_d = \frac{x}{\sqrt{r_1^2 + 4x^2}} = \frac{q_x}{\sqrt{1 + 4q_x^2}}$$

$$q_x = \frac{q_d}{\sqrt{1 - 4q_d^2}}$$

For purposes of comparison with Fig. 5 this result must be multiplied by two since there are twice as many reactors in this case. The relative magnitude of the reactance required will then be —

$$2q_x = \frac{2q_d}{\sqrt{1 - 4q_d^2}} \tag{2}$$

*Arrangement of Fig.* 7. Equating the potential drops to fault as before

$$\dot{I}_1 r_1 + j\frac{x}{4}(\dot{I}_1 - \dot{I}_2) + jx_1\dot{I} + jx\dot{I}_1$$

$$= \dot{I}_2 r_1 + j\frac{x}{4}(\dot{I}_2 - \dot{I}_1) + jx_1\dot{I}$$

$$\dot{I}_1\left(r_1 + \frac{3}{2}jx\right) = \dot{I}_2\left(r_1 + \frac{1}{2}jx\right)$$

$$\frac{\dot{I}_2 - \dot{I}_1}{\dot{I}_2 + \dot{I}_1} = \frac{\dot{I}_d}{\dot{I}} = \frac{j\,x}{2\,(r_1 + j\,x)}$$

Numerically

$$\frac{I_d}{I} = q_d = \frac{x}{2\sqrt{r_1^2 + x^2}} = \frac{q_x}{2\sqrt{1 + q_x^2}}$$

or, for a given per cent differential current.

$$q_x = \frac{2\,q_d}{\sqrt{1 - 4\,q_d^2}} \qquad (3)$$

*Conclusion.* Referring to equations (1), (2) and (3) it appears that, for the same per cent differential current, the arrangements shown in Figs. 6 and 7 require the same total reactance, and in each case slightly more than double that required by the arrangement of Fig. 5.

As between Figs. 6 and 7, the latter has the advantage, since although the total reactance is the same, it is in two units for Fig. 7 against four units for Fig. 6, which will cost less and take up less space.

Since $q_x$ and $q_d$ are so nearly equal within normal range of values, these formulas supply a simple and convenient method of computing the approximate per cent reactance necessary for any particular differential current. For example, if $q_d = 0.20 = q_x$ (approx.) (Fig. 5), and the normal ohmic drop on the line inquestion is 0.05, the full-load reactive drop due to the reactors will be $0.20 \times 0.05 = 0.01$ or 1 per cent of the line voltage. This is an outside figure.

For the case of Fig. 6, this will be twice as great or two per cent.

In the case of Fig. 7, the reactors offer no appreciable reactance under through conditions.

*Saturation of Reactor Cores.* The effect of the core saturation is to reduce the reactance of each reactor and also to distort the voltage drop across each reactor.

The reduction of reactance works differently in the three cases. In Fig. 5 it reduces the differential current in almost the same proportion; but in Figs. 6 and 7 the differential current is still further reduced by the fact that the reactance in the short-path to ground, which is undesirable as it reduces the current differential, is reduced less than that in the long path to ground, since its core is less saturated. This difference is however not considerable although, as far as it goes, it increases the advantages of the arrangement shown in Fig. 5.

Since the reactance of the reactors absorbs such a small part of the voltage of the system, the saturation of the reactor cores does not influence the current wave shape appreciably but does seriously influence the wave shape of the voltage drop across each reactor. The flux wave is flat topped and the voltage wave very peaked under conditions of core-saturation.

Under fault conditions this distortion results in very high-frequency harmonic voltages between splits or conductor members, which in turn produce high-frequency charging currents between conductor members, which in part reduces the voltage distortion by supplying harmonic exciting currents for the reactors.

This voltage distortion and consequent danger of breakdown of the secondary insulation will be approximately twice as great for Figs. 6 and 7 as for Fig. 5.

Under normal conditions this phenomenon appears to some extent in Fig. 5, but the currents are not large enough to approach the danger point.

These distortion effects can be considerably reduced by the employment of a gap in the magnetic circuit of the reactors.

## DISCUSSION ON "SPLIT-CONDUCTOR CABLE—BALANCED PROTECTION" (COLE), ATLANTIC CITY, N. J., JUNE 26, 1918.

**John B. Taylor:** What is the practise of this company as regards grounding the neutral, which would determine the extent of the unbalancing on the usual cable fault? That is, how much earth resistance, how much limited current, may there be on the system?

**W. H. Cole:** Replying to Mr. Taylor's question as to the grounding of the neutral, our 15,000-volt system neutral is grounded through an 8-ohm resistance, which limits the short-circuit current in the case of fault to earth to a maximum value approximating 1000 amperes.

The setting of the relays used with split-conductor cables depends upon what value of current we are willing to allow to develop before the relays can operate. Our split-conductor relays are standardized on the basis of operation at values of fault current not to exceed 100 per cent full line current when the fault is located near one end of a line, or in the most unfavorable position to create a tripping differential current. Under such conditions, the tripping differential current at the opposite end of the line will be equal to about 10 per cent of the fault current, or 20 amperes with a fault current of 200 amperes. The extent of current unbalance in a split-conductor line, due to a fault therein, is affected more by the position of the fault with reference to the length of the line, than by the value of the fault current, the unbalancing being greater the nearer the fault is to the centre of the line. It is also true that a certain minimum value of fault current must flow in any case to produce the required tripping differential current at both ends of a split-conductor line.

**W. A. Del Mar:** I would like to know whether the extra layer of insulation which is between the two conductors has any appreciable effect on the carrying capacity of the cables?

**W. H. Cole:** We really know little about it. We believe, however, that we need not rate a split-conductor cable any lower than a standard cable of equivalent conductor cross-section, particularly as such a split-conductor cable will ordinarily have a larger outside diameter. I believe Mr. Meyer of the Public Service Electric Company has had experience which has enabled him to draw some conclusions with respect to the relative carrying capacity of the two types of cable under discussion.

**Paul M. Lincoln:** We have all been faced of late years with the importance of operating systems in parallel in order to take advantage of diversity factor. As soon as we begin to connect systems in parallel, it becomes all the more essential that faults of cables and other parts of the system shall be eliminated as soon as they occur, and this method outlined here of eliminating these faults as soon as they occur is, I think, one of the most promising that has been suggested. It is a method which is bound to come into use more and more, as its advantages are understood.

I would like to ask how much more it costs to install cables on the split-conductor plan than if the cables were installed on the straight single-conductor plan? I do not believe that the cost is a great deal more, but if Mr. Cole can give us any specific figures on the difference in cost, I think it would be of interest to all of us.

**W. H. Cole:** In reply to Mr. Lincoln's question as to costs, I dislike to discuss the matter of relative costs of standard conductor and split-conductor cables, for the very obvious reason that until the demand for a new article of manufacture is well established, it is more or less special and is subject to comparatively heavy development charges.

A prominent manufacturer's cable engineer has informed me that in his opinion a 3-conductor 4/0, 15,000-volt, paper-insulated cable such as is extensively used by our company ought not to cost much above 7 per cent in excess of the cost of a standard conductor cable of equivalent capacity. This figure was quoted when cable of the capacity referred to was selling for approximately $1.00 per foot.

Against the excess cost of a split-conductor cable, we compared the cost of pilot cables required with the input-output method of balanced protection, such pilot cables costing approximately 10 cents per foot exclusive of the cost of duct space required by them. Including the cost of duct space for the pilot cables which will run from ten to fifty or more cents per foot, depending upon the number of such cables installed in one duct, we felt that the split-conductor cable had a better basis for consideration in any scheme of balanced protection for new lines.

In the manufacture of split-conductor cable there is one more stranding and one more insulating operation. Some difficulty has been experienced in saturating the secondary insulation in paper-insulated cables. I am quite sure that manufacturers will soon be able to supply the demand for split-conductor cables and at such a price that no engineer will feel that he is paying an unreasonable price for the sake of having balanced protection.

The excess cost of installation over that of ordinary conductor cables is principally due to the splicing or jointing, as jointing of cables is of such a nature that it ordinarily must be a continuous process to prevent the entrance of moisture. It has not been found expedient for a splicer to make more than one of our standard split-conductor joints in a working day of eight hours as compared with two joints on an equivalent ordinary conductor cable.

Including the extra cost of installation, our experience indicates that split-conductor cable costs at the present date from 10 per cent to 15 per cent more than the ordinary conductor type.

**E. B. Meyer:** In reference to the question of carrying capacity, I think that Mr. Cole is slightly mistaken in the statement he made. We had quite some difficulty last summer in keeping a number of our tie feeders in operation. Some of these were of the split-conductor type, but were not operating at the time as split-conductor cables, and the failures due to overheating were caused primarily by the peculiar position in which the cables were placed in the duct, and the way it was necessary to operate them. We were carrying the load at that time at a high load factor, and so I am not prepared to say whether the carrying capacity of the split-conductor cable will be reduced. My opinion is it will be about the same as the solid-conductor cable.

**R. W. Atkinson:** I think I can throw some light on the question of relative carrying capacity of split-conductor and ordinary cables. The difference is so very slight that a theoretical analysis is more direct and conclusive than would be the result of tests unless these were of extended nature; therefore, I will consider the question from that standpoint. The split-conductor cable is larger than the ordinary, both in conductor surface and outside surface; therefore the same number of watts per foot dissipated will produce less temperature rise of sheath above surroundings and less temperature rise of copper above sheath, than in the case of the ordinary cable.

This takes account only of the main insulation. The insulation between members, and the inner member itself will be somewhat warmer than the outer member. The difference is small in actual amount and is unimportant because the inner insulation may safely be subjected to somewhat higher temperature than the outer. When these different considerations are carefully weighed, it is found that the difference in carrying capacity of the two types of cables is entirely negligible, fully justifying the practise of considering them equal.

**J. R. Craighead:** Mr. Cole says it is practicable to arrange two split-conductor cables, after final adjustment, so that their impedance differential is 0.1 of one per cent. He made the statement a moment ago that the fault current was usually not over 200 amperes, and that this, corresponding to a tripping current of 20 amperes, means that the tripping current is 10 per cent of the total fault current. If that is the case, what is the point of attempting to reduce the differential to as low a point as 0.1 of one per cent. Applied to the 200 amperes, a differential of 0.1 of one per cent would mean 0.2 of an ampere or one per cent of the tripping current of the relay.

This also affects the design of the balancing transformer. It is possible to make current transformers for balancing purposes to practically any degree of perfection of balance that is desirable, but when the requirement is beyond reasonable figures, or such as can be obtained by the use of separate coils insulated from one another by simple means, the expense and the difficulty of design of the transformer are considerably increased. I recently experimented with some transformers, in which a part of the requested design was that the secondary current due to passing normal primary current through both primary windings in opposite directions should be limited, to about 0.04 of one per cent of the normal secondary current. That is closer than 0.1 of one per cent.

If these figures are necessary we should know why, and how far they are necessary, and if not, it would be well to know about what limit can be practically used in order to keep design down to the cheapest and smallest practicable sizes.

**Oliver C. Traver:** We have talked about relays with brains and intelligence, etc., and I think that this is the time to feel that we have a relay which has something more than brains, this is a device which has entered into the superhuman class.

I was told a short while ago by one of the engineers of a very large operating company, that if they could operate their system in parallel they would be able to save a million dollars' worth of cable, rather, they could remove from the system a million dollars' worth of cable. This is on the order of being able to pay one thousand dollars a pole for relays in order to accomplish this. That particular company is actually working toward the accomplishment of this vast saving. As operating companies begin to realize the importance of the matter, they stop expecting a twenty-dollar device to hold a thousand-dollar position.

The shorter the run of cable, and the fewer cables in parallel, the greater will the split-conductor method appear to advantage. This is true from the standpoints of both first cost and operation. The longer the split-conductor cable, the greater will be the extra price paid for the protection of the line. This extra price must then be balanced against the cost of the relays of high accuracy and reliability, which are now available, for conditions today are different from "some time ago" when as Mr. Cole suggests, "line protection devices, which function with respect to time, to value of current, or to direction of power flow, were,—exceedingly difficult, and frequently impossible to adjust and maintain" in proper relation one to another.

Longer lines result in higher impedances. Higher impedance puts a limit on fault currents such that a relay can more easily distinguish the fault on which it should function from the one which it should allow to pass.

For multiple lines, balanced groups of relays can be used with greater flexibility, which will give rapid relief from a fault. The more lines in multiple, the easier the problem becomes.

Although two standard cables are more expensive than one split-conductor cable for the same total ampere capacity, still it is also legitimate to take into consideration the advantage of having one workable line in case of trouble in its companion. On the other hand, two standard cables are less costly than two split-conductor cables of the same capacity. So where two or more parallel lines are involved, cost favors the use of balanced protection for these lines, rather than split-conductor protection.

When two standard cables are used, standard types of current transformers which also carry some instrument load can be used in place of the differentially wound current transformers illustrated in Fig. 8 of the paper.

A similar scheme is shown in Fig. 9 and I would like to mention that a "biased" relay is available to select the switch in the line, which carries the greater current. The use of this discriminating device to me, seems to be one of the most promising of the various balanced-relay schemes for parallel line protection.

I have herein suggested some preferable alternatives to the use of split-conductor cables for certain cases. This is not intended to, nor should it, detract from the importance of the work done by Mr. Cole, for I believe the split-conductor cable to be one of the most important developments in selective protection in some time.

If one after, careful investigation, concludes that a system of protection is needed for any line, which will be independent in its selective action of any other part of the system, he should consider split-conductor protection, for it apparently comes nearest to giving complete protection of any scheme so far proposed.

**W. H. Cole:** In closing the discussion, I will first reply to Mr. Craighead's inquiry regarding impedance differential. In the paper I made the statement that we aim to secure impedance differentials in completed lines not to exceed 0.1 per cent. The purpose of limiting the impedance differential to low values is to insure correspondingly low values of normal differential current under through short-circuit conditions, thereby permitting low settings for the relays. If the relays are set to operate on a differential current equal to 10 per cent full line current, it is obvious that the impedance differential must be small enough to insure that this value of differential current shall not be reached upon the occurrence of any possible value of through short-circuit current, due to faults or conditions beyond the cable under consideration. The reference in my reply to Mr. Taylor, of a minimum tripping differential of 20 amperes with a fault current of say 200 amperes, referred of course to conditions involving a fault in the cable under consideration.

Regarding balancing transformers, their design is a matter to which attention must be given by manufacturers in connection with the production of suitable relays for split-conductor protection.

In general it may be said that the performance of balancing transformers should be such that their "normal differential current" shall not be greater than that permissible in the line itself under through short-circuit conditions. We have made use of standard double-wound current transformers with the primaries connected differentially, which under test with high current values indicate that no extreme or expensive designs are required for the duty under discussion.

Mr. Traver has called attention to the desirability of pairing or grouping ordinary conductors under some conditions, as being more desirable than the indiscriminate use of split-conductors. With this suggestion, I am in full agreement with Mr. Traver, particularly when the system is large and consequently involves a relatively large number of lines between distribution and generating centers. Split-conductor cables, however, are self-contained units and have many advantages not inherent to cables previously considered as standard. There is room for both of the general schemes discussed in the paper. Any choice between them "tends to be an economic one" in practically every case where they are under consideration.

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# AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION

BY E. B. MEYER

ABSTRACT OF PAPER

This paper deals with the problem of supplying high-tension electric service where conditions do not permit of the use of open wire or underground circuits.

The methods of overcoming difficulties incidental to providing for high-tension service are discussed in detail, together with a description of the types of cable used and methods of installation.

The experience of a large central station company operating several hundred miles of overhead and underground cable is given and the paper brings out the fact that the type of construction described may be used advantageously for both 13,200- and 26,400-volt service.

CENTRAL station companies have had to meet a number of difficult problems during the past three years but the most important has been that of supplying enormous power demands imposed upon them, particularly since the United States has become one of the allies in the European war.

On account of the rapidity with which most of the materials covered by war contracts must be delivered, industrial companies found that the building of isolated plants was out of the question, not only because of the time necessary for erection, but because of the low rates and excellent service furnished by utility companies.

At the present time the central station engineer in dealing with the customer has to provide for thousands of kilowatts rather than hundreds, which were the usual demands previous to the war.

These large demands have made it necessary to solve numerous operating problems in connection with the transmission system and to devise special methods of construction in order to serve the industries upon which the Government is depending to help win the war.

The Public Service Electric Company, which operates in 200 municipalities throughout the State of New Jersey, supplies light and power to approximately 170 manufacturing plants engaged directly or indirectly on Government contracts. The material furnished under such contracts consists of high explosives, chemicals, shells, textiles, rubber goods, motor cars, castings, wire and cable, wireless apparatus, ships of various types, and many other accessories. In addition to this, the Public Service Electric Company has contracts with the United States Government for furnishing light and power to Camp Dix, Camp Merritt, Raritan Ordnance Depot, Colonia Base Hospital, and the Quartermaster's Department located at the Port Newark Terminal. The supplying of these industries has made enormous demands on the generating system of the company so that at the present time about 80 per cent of the company's output in commercial power is for war work.

One of the special methods adopted by the Public Service Electric Company in meeting war time demands, was that of furnishing the customer with primary service by the use of aerial cable run on poles and supported by messenger wire, a type of construction similar to that used in telephone work.

This type of construction was first used by the company about seven years ago when it was found necessary to connect two large generating stations through tie feeders.

The matter of running overhead wire was considered but found impracticable because the line in several places would have to cross freight yards, trestles, and bridges, and the owners of these structures objected to open-wire high-tension lines.

Most of the section between these two stations was soil of a marshy character, through which it would have been impossible to run a duct line without the use of foundation piling.

It was therefore concluded that the use of aerial cable furnished the most satisfactory solution of the problem. In this installation ordinary lead-covered cable of the same type as that used for underground work was run on a pole line with 50-ft. (15.2-m.) pole spacing. To protect the sheath from mechanical injury there was applied a covering consisting of several layers of jute and marlin with an outer armor of soft steel tape.

The use of lead-covered cable for aerial work was found undesirable, however, because of the excessive weight of the cable and the fact that it could not be installed on standard pole line construction, and a special form of cable was developed to overcome these objections.

In Fig. 1 is shown the modified form of cable for 13,200-volt operation, which is made up with 7/32-in. (5.5-mm.) paper conductor insulation, a 3/32-in. (2.3-mm.) paper jacket and a

4/32-in. (3.17-mm.) reinforced rubber covering over the paper jacket. The reinforced rubber covering is similar in construction to that of the ordinary garden hose, being made up of several plies of fabric and rubber. The entire cable is saturated with rubber compound and covered with tape and a weatherproof braid, thoroughly impregnated with a waterproofing compound. For mechanical protection, the whole core is encased in an armor made up of galvanized steel tape. The use of this form of construction reduces the weight of the cable approximately 50 per cent and permits the use of lighter pole line construction.

Reinforced rubber cable was developed by Mr. Philip Torchio, Chief Electrical Engineer of the New York Edison Co. who has furnished the following information regarding the process of manufacture.

"The process of manufacture of the reinforced rubber covering consists in calendering both sides of the cotton fabric, previously dried and waterproofed, with a 30 per cent Para rubber compound, so as to obtain a thorough filling of rubber, which, under the process of calendering, becomes partially vulcanized. The prepared fabric is then cut into tapes. These are applied to the electrical conductor in the usual manner, all contact surfaces and interstices being filled with a rubber cement. The insulated conductor is then dried under moderate heat. According to whether the reinforced rubber covering is applied over an insulating layer of rubber compound, or a layer of cambric or paper, the finished cable may or may not be subjected to vulcanization. In the latter case, the partial vulcanization of the rubber in the reinforced rubber is further advanced during the drying process and during leading in the case of leaded cables; otherwise further vulcanization takes place with aging and under service.

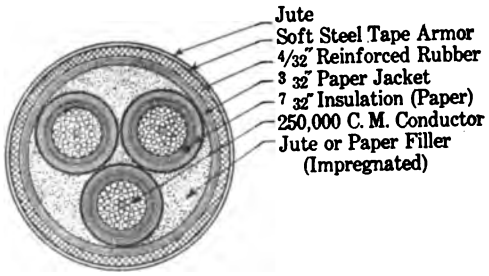


FIG. 1—REINFORCED RUBBER COVERED AERIAL CABLE

The finished material is perfectly homogeneous. Its specific insulating and dielectric constants are lower than those of rubber, paper and varnished cambric insulation, and for that reason, among others, it is preferable to combine a thickness of reinforced rubber with one of the other materials. By placing the reinforced rubber outside a thickness of a higher dielectric compound near the copper wire, the potential gradient is reduced so that the lower dielectric strength of the reinforced rubber does not materially decrease the total dielectric strength of the cable."

Many engineers have been of the opinion that paper insulated cable with the reinforced rubber jacket would not give satisfactory service when subjected to the heat of the summer sun, but in spite of the fact that the cable is exposed to the elements throughout the year the Public Service Electric Company has never experienced a service interruption through the failure of any of the aerial cable in use in the transmission system.

In Fig. 2 are shown several types of reinforced rubber multiconductor cable with and without lead covering.

The following table gives approximate weights and outside diameters of three-conductor cables, insulated for 13,200-volt operation:

APPROXIMATE WEIGHT AND DIAMETER
OF THREE-CONDUCTOR, 13,200-VOLT-AERIAL CABLE

| Size | Weight per foot-lbs. | Dia. inches |
|---|---|---|
| No. 4 | 3.50 | 2.25 |
| No. 2 | 4.05 | 2.41 |
| No. 1 | 4.45 | 2.50 |
| 1/0 | 4.80 | 2.57 |
| 2/0 | 5.70 | 2.66 |
| 4/0 | 6.70 | 2.91 |
| 250,000 cm. | 7.05 | 3 00 |
| 350,000 cm. | 8.50 | 3.22 |

The principal advantage of aerial cable for tie feeder installations is that it makes little difference how many working lines are carried on a single pole line. Additional cable may be run, existing construction changed, transferred or repaired without taking out of service any line except the one on which the actual work is being done. Lightning discharges seem to have little effect because the messenger wire which carries the aerial cable is permanently grounded.

FIG. 2—REINFORCED RUBBER MULTI-CONDUCTOR CABLE

FIG. 3—AERIAL CABLE LINE WITH FIVE 13.200-VOLT CIRCUITS

[MEYER]

FIG. 7—CONNECTION BETWEEN AERIAL AND UNDERGROUND CABLE.

Fig. 3 shows a pole line carrying five 13,200-volt feeders. This line has been in operation for a period of over seven years without the occurrence of a single failure.

The usual aerial cable installation requires the use of Class B chestnut poles, with a normal spacing of from 90 to 100 ft. (27 to 30 m.). Where conditions make it necessary, sections as long as 150 ft. (45 m.) are permissible, but in such cases the adjacent sections should not exceed 130 ft. (39 m.). Sections longer than 150 ft. (45 m.) should receive special attention, and Class A poles should be used on long sections and at points of special strain. The location and frequency of guys is largely dependent on local conditions and can, in most cases, be decided upon by a competent line superintendent.

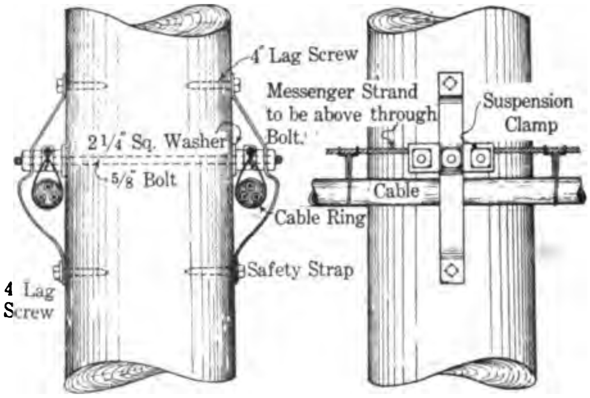


Fig. 4—Method of Suspending Messenger Cable

Attention is called, however, to the fact that the stress at dead ends and corners is very great, frequently being as much as 25,000 lb. (11,339 kg.). These points of special stress need to be well guyed. Both the anchors and the guys should be designed with a factor of safety so high that the messenger will fail before the pole will pull over. In all cases it will be necessary for guy stubs to be reinforced by an anchor guy.

For the suspension of the messenger a double ended 5/8-in. (15.8-mm.) through bolt is recommended, as illustrated in Fig. 4. The use of a safety clamp is also desirable. This clamp serves the double purpose of reinforcing the through bolt and preventing the cable from falling to the ground in case the rings fail. Careful tests made on the method of suspension show that it will withstand the maximum loads to which it will be subjected.

The type of clamp used is similar to that used by the American Telephone and Telegraph Company, the size depending on the diameter of the messenger strand adopted. The clamp is designed expressly for construction of this character and is not built like a guy clamp which is designed to grip two strands instead of one. It affords a greater lever arm for the bolts to work upon in grasping the messenger and supports the messenger strand closer to the bolt, decreasing the bending moment on the bolt due to the weight of the cable.

The messenger strand should always be placed above the bolt in order that the weight of the cable will not be supported by the clamp. Various forms of cable rings may be used in supporting the cable on the messenger wire.

Where two or more cables are to be installed on a pole line, they are usually suspended in pairs, two from each through bolt. The messenger wire is extra strength 5/8-in. (15.8-mm.), seven-strand, galvanized, steel wire. The wire composing the strand should be free from scale, inequalities, splints or other imperfections, not consistent with the best workmanship. It is usual in purchasing galvanized steel wire of this character to have it conform to a specification covering the galvanizing. This is necessary as otherwise inferior grade wire might be obtained.

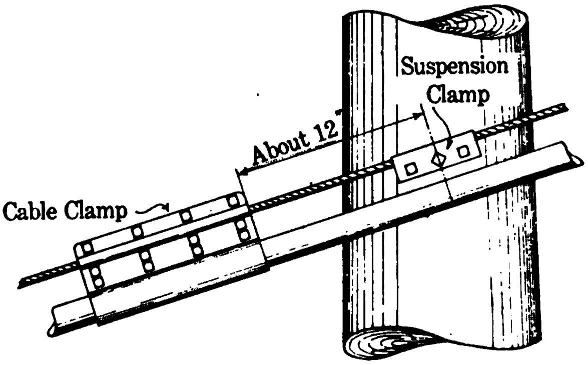


FIG. 5—CLAMPING CABLE TO MESSENGER ON STEEP GRADES

It is very important that the messenger wire be drawn as tight as possible, in order to prevent sagging when subjected to the weight of the transmission cable. If this is not done, an unsightly installation will result. After the messenger wire has been given its final pull and properly dead-ended, the placing of the aerial cable is the next step. In pulling the cable up to the messenger wire it is very important that precautions be exercised to prevent mechanical damage or excessive strains which would tend to weaken or damage the insulation.

It is customary in aerial installations to ground the messenger strand. Where the soil is dry or soil conditions unfavorable for grounding, a ground connection should be installed at every second pole. Where the earth is damp and soil conditions are favorable, a ground should be installed at every fourth pole. In marshy ground and in places where conditions are particularly favorable, a ground at every eighth pole will be sufficient. Where possible, this ground connection should be well bonded to some metallic subsurface structure. If this is not possible, the standard artificial pipe ground should be installed.

It is also desirable that the steel tape on the cable be bonded to the messenger strand with bonding wire at every cable joint, as proper bonding is necessary in order to furnish the required protection against lightning. Where cable is run through trees and likely to be damaged by abrasion it should be protected by several layers of galvanized tape similar to that later described for use in protecting the joint.

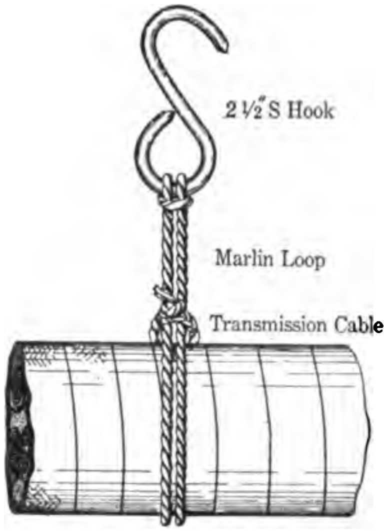


FIG. 6—METHOD OF FASTENING *S* HOOKS TO AERIAL CABLE

In Fig. 5 is illustrated a method of clamping the cable to the messenger wire. On steep grades where the angle between the cable and the horizontal is greater than 30 deg. the use of such a cable clamp is recommended. This clamp can be made up as required and should be used on every fourth pole. It is designed to take the greater portion of the down hill pull on the cable, which otherwise would be carried by the cable rings.

In erecting the cable, the first reel is set up in the usual manner and the cable run off to the first pole, at which is placed a sheave of approximately 12 in. (30 cm.) diameter, the top of the sheave being located about 5 in. (12.7 cm.) below the messenger wire. On the four or five succeeding poles similar sheaves or cable rollers are placed, and in feeding out the cable 2.5-in. (6.35 cm.) "*S*" hooks, spaced 18 in. (45.6 cm.) apart, are fastened to it. These hooks are fastened to the cable with a small piece of marlin, made up in a loop knot, as illustrated in Fig. 6.

A lineman is stationed at each pole to change the "*S*" hooks from one side of the pole to the other, which process is repeated until the entire length of cable has been installed in place.

The "*S*" hooks, which were used as a temporary support, are now removed and permanent rings put in place. This is done by a lineman supported on a boatswain's chair, which is moved along the section supported by the messenger wire.

In running the transmission cable, either a motor truck, horses, hand or power winch may be used.

In the splicing of aerial cable, no special means are employed, but the usual precautions observed in the installation of underground cable must be followed. The jointing of any cable is more or less a matter of individual experience and great care must be exercised in all cases to exclude moisture. The work should be carefully done by a reliable and experienced workman and no splicing should be undertaken when weather conditions are unfavorable.

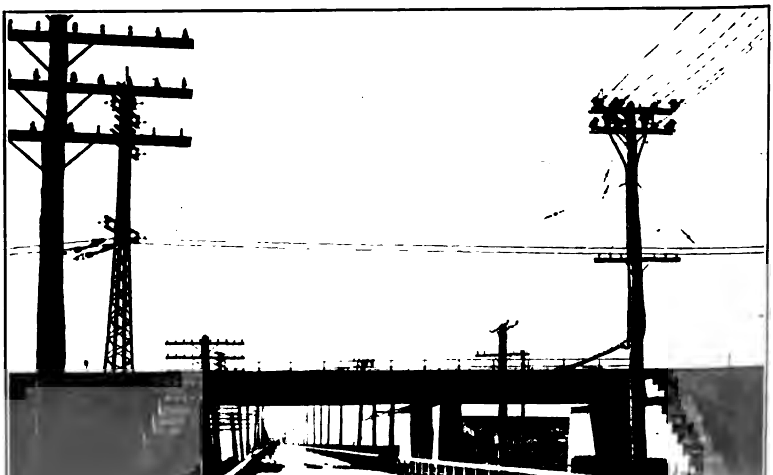
Each conductor of the cable is insulated with black bias-cut varnished cambric tape of a thickness of about 30 per cent greater than the machine applied insulation. Between each layer of tape, varnished cambric insulating compound is applied. After the individual conductors have been insulated a jacket of bias-cut black cambric tape, well painted between layers with an insulating compound, is applied to a thickness of 4/32-in. (3.17 mm.). Over the jacket of cambric tape several layers of the best grade rubber tape, 5/32-in. (3.9 mm.) in thickness, are applied and painted between layers with a high grade rubber compound. The completed joint is then covered with three or four layers of friction tape well painted with rubber compound. The joint is then ready for the application of a soft steel galvanized tape over which is finally applied an outer covering consisting of three or four layers of friction tape painted between the layers with a good grade of waterproof compound.

Where it is necessary to make connection from an aerial cable to an underground system a standard form of lead-covered cable is used and installed in a lateral pipe as shown in Fig. 7. The joint between the underground and aerial cable is made up in the manner just described, and there is slipped over the joint a

FIG. 8—AERIAL CABLE ENTRANCE TO SUBSTATION

FIG. 9—26,000-VOLT AERIAL CABLE INSTALLATION [MEYER]

FIG. 10—CATENARY METHOD OF SUSPENDING AERIAL CABLE



[MEYER]

FIG. 11—26,000-VOLT CABLE CROSSING UNDER RAILROAD BRIDGE AND HIGH TENSION POWER CIRCUITS lead sleeve, one end of which is wiped to the lead-covered cable. The other end is well taped to prevent moisture from penetrating the cable.

Two aerial cables entering a power substation are shown in Fig. 8. In this installation iron pipe laterals are run up the pole and underground lead-covered cable is spliced on to the aerial cable as just described.

While most of the existing circuits are operated at voltages under 15,000, the excellent results obtained with aerial cable has led the company to use this type of construction on all special work for operation at 26,000 volts.

In Fig. 9 is illustrated a completed 26,000-volt aerial cable installation. It should be noted that this method of construction permitted the erection of a high-voltage line without reconstructing the existing low-voltage open-wire distribution circuits.

To keep the cable in good condition it is necessary to paint it every four or five years with some form of insulating paint. This serves to keep the outside jacket from disintegrating and protects it from the action of the elements.

There is in service on the various transmission lines of the Company approximately 65,000 ft. (19,812 m.) of aerial cable operating at 13,200 volts and about 16,000 ft. (4,876 m.) either operating or in course of construction for 26,400-volt service.

Fig. 10 illustrates a catenary method of installing aerial cable. The line so erected was built to furnish 3,000 kw. to a customer who required service within a few weeks after the signing of the contract.

It was impossible within this short length of time to obtain the standard aerial cable with reinforced rubber insulation, and it was found necessary to take ordinary lead-covered cable out of stock.

The erection of lead-covered cable by the methods commonly used in installing aerial cable, on account of the weight and long pole spacing, would have resulted in throwing too great a stress on the messenger wire and lead cable. It was, therefore, decided to use the catenary form of construction so as to reduce the strain with the result that the transmission cable hangs perfectly level and without sag.

In Fig. 11 is shown an illustration of aerial cable crossing under a railroad bridge and electric railway power circuits. Open air potheads or line terminals are installed between the cable and the open wire transmission conductors. No protection in the form of lightning arresters is provided as after a number of years of operation without failure from any source, the cost of the installation of arresters seemed to be unwarranted.

Aerial cable construction is somewhat more expensive than ordinary open wire construction but its cost is less than that of an underground conduit system. As the costs of the various types are so largely dependent on local conditions, no comparative estimates will be given here. In general, the cost of an aerial cable line is about midway between underground and open wire construction.

While this paper deals primarily with the use of reinforced rubber cables, there are numerous installations throughout the country where other forms of insulation have been used with satisfactory results.

It is not the intention of this paper to recommend exclusively the use of reinforced rubber insulation, but primarily to bring out the fact that aerial cable construction may be used advantageously for power transmission, where open-wire construction would be undesirable and the time and cost of underground construction would be prohibitive.

Where line extensions have to be made over marshy ground which would require the use of foundation piling for a conduit line, over private property such as railroad freight yards, trestles or bridges where very special overhead construction would be necessary, or on important streets on which open wires are not permitted, and subway construction would be expensive or impossible, aerial cable furnishes an ideal form of construction.

## DISCUSSION ON "AERIAL CABLE CONSTRUCTION FOR ELECTRIC POWER TRANSMISSION" (MEYER), ATLANTIC CITY, N. J., JUNE 26, 1918.

**W. F. Dawson:** The author pointed to the possibility of having trouble with the insulation of this aerial cable from blistering in the sun. I want to ask if they have any particular protection for the cable, because it makes great difference in the amount of heat attracted, whether it is coated with a light colored paint or an ordinary tar product. Glass thermometers, exposed in the sun will show 15 to 20 deg. fahr. lower temperature than if they are exposed inside of black cases and I would expect a corresponding difference if black paint were used instead of white or gray.

**E. B. Meyer:** No particular attention has been paid to that. I realized that there would be considerable difference where the cable had been painted. Some of the cable has recently been covered with a light coating, but we have not had much experience with it so far.

**H. L. Wallau:** I want to ask the carrying capacity of the aerial installation. Judging from the pictures submitted, in practically all cases these cables have been in series with cables installed in subways, in which case the limiting current will be determined by the underground insulation. I ask if Mr. Meyer has had any of this cable installed exclusively overhead, and can he operate those cables at a higher rating than the subway installation because of that reason?

Another point I notice is that the weight per foot for the 0000 is given at 6.70. Our practise is to use 0000 3-conductor cable either 6 by 6 or 8 by 2 installation, with 1/8 in. lead, if I remember aright, and our cable weighs about 9 lb. per foot, and the weight of the cable listed in the paper is almost 7 lb. per foot, and so I was wondering whether the figure 50 per cent was correct or not.

In reference to lightning protection, has any trouble ever been experienced by the connection of a stretch of cable to two stretches of overhead line?

**E. B. Meyer:** In reference to current carrying capacity, I do not recall a single installation entirely made up of aerial cable. All of the circuits have more or less underground cable in series with the overhead cable, and, in addition, quite a little of the open wire construction. Our principal difficulty has been the fact that the aerial cable, which, in the past, we have made the same size as the open wire, would limit us to the capacity of the underground and overhead cable sections.

It is to be our practise in future work to increase the size of the aerial cable, and also the underground cable, where in series with the open wire, so as to get the full capacity of the circuit.

There are a number of installations where aerial cable is used in the center of connection to open wire lines. To my knowledge, no difficulty has been experienced, even when subjected to very severe lightning storms.

As to the weight, it is my impression from figures that were prepared, that the aerial cable was about 50 per cent less in weight than the underground cable. There may be some slight discrepancy I have not checked up.

**W. A. Del Mar:** I am somewhat surprised to hear the last two speakers, especially Mr. Meyer, express an opinion that the limiting feature in carrying capacity is the underground cable. I should have thought it would have been the overhead cable in sunlight. Cable can easily attain a temperature in direct sunlight of 120 or 130 deg. fahr. There is not very much margin between that and the maximum permissible operating temperature of the insulation. It, therefore, seems to me, that in obtaining a result showing the contrary, the underground cables may have been operated at less than their normal carrying capacity.

At pressures higher than 20,000 or 30,000 volts, there would be practically no carrying capacity left after the cables attained the maximum temperature which they could attain in sunlight.

**E. B. Meyer:** Most of our aerial cable installations are out in the open where they are subjected to the rays of the summer sun, but apparently there is a sufficient circulation of air, and in that case the opportunity of dissipating heat seems to be somewhat better than a cable enclosed in a conduit line, so we have experienced no difficulty in the way of a diminished carrying capacity.

On the 350,000-cir. mil 3-conductor cable, particularly on the tie feeders, we have limited the current to 300 amperes at 13,000 volts, and a number of the 00 cables have carried in excess of 200 amperes without failure. The principal difficulty has been in carrying the current for a long period of time or at a high load factor, particularly where there are a number of cables in the duct.

**John B. Taylor:** The possibility of working cables to the utmost is very important, when we hear about saving a million dollars by a device that will release some of our cables .

I note that the New York Central Railroad use in the summer time a sort of cheese cloth tent over their cables, so I assume their experience is that cables out in the open have a harder time keeping cool than those under ground. Cables should be painted black or white, it depends on whether the greater amount of heat to be disposed of comes from the sun on the cable or from the copper outwardly. In one case the cable should be painted white for the best results, and in the other case it should be painted black for the best results. I do not know why the New York Central use the contrivance I have referred to, but if there is some New York Central man here he would be able, possibly, to give us some of the reasons which cause them to go to the trouble of stretching several miles of saturated fabric over the lines of cables out doors.

**Philip Torchio:** I think Mr. Taylor is making a comparison for two different conditions. Mr. Meyer has an open overhead cable, while the New York Central has a cable installed in a pipe. In the former case there is free circulation of air and, as Mr. Meyer explained, it is pretty hard even with load to raise the temperature of the body as a whole considerably above the temperature of, say, 125 deg. fahr. due to the direct sun's rays, but if the body is enclosed in a pipe and the circulation of air limited, the temperature may go up higher. I think that may explain the difference in results.

**J. Allen Johnson:** In a certain installation with which I am somewhat familiar, of a 2200-volt lead cable, strung on a messenger wire, considerable difficulty was experienced with the joints. The unequal expansion on the lead cable and messenger cable resulted in the breaking of the sheath at the joint away from the sheath of the cable.

I would like to ask Mr. Meyer if they have experienced any difficulty of that character, and if so, what course has been taken to overcome it?

**E. B. Meyer:** Answering Mr. Johnson's inquiry, I have heard of a number of cases where lead cable has given some difficulty, particularly at the joints, due to difference in expansion, perhaps, of the wire and the cable too. I will say that in the Public Service Electric Company's installation, particularly the earlier installation, made with the standard form of underground cable, and the splice made up in exactly the same manner as it is made in the manhole, to my knowledge we have never lost a joint, and have had no difficulty in keeping the cable in alignment. The cable is supported with the messenger ring, which clamps under the messenger wire, and I am unable to account for the trouble experienced by other companies. I can only speak of the satisfactory results we have obtained.

**H. R. Summerhayes:** I was very much interested in this cable installation, Mr. Meyer mentions, as they use all lead-covered cables, and also the cable covered with rubber fabric, and he speaks of other cables, but is it not true that there is nothing else that will stand the out of doors conditions, nothing else but rubber or lead which will keep the insulation dry and will stand the electrical and weather conditions to which the cable is subjected?

**E. B. Meyer:** Answering the last question, I will say that we are experimenting with other types of cable. There are some insulated with a varnished-cambric jacket with the ordinary rubber jackets on the outside. That particular type has not been in service long enough for us to obtain any definite results with it. I do understand that in Pittsburgh they have used the varnished-cambric cable, and that it has operated in a satisfactory manner, but they also have had some trouble in making up the joints.

*Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 26, 1918.*

# APPLICATION OF THEORY AND PRACTISE TO THE DESIGN OF TRANSMISSION LINE INSULATORS

BY G. I. GILCHREST

## Abstract of Paper

The paper first gives a summation of the items that are apparently the main causes of pin-type insulator failures in service. Each item is thereafter briefly discussed and the opinions of operating men are cited.

A brief description is given of the method used to determine the form of the dielectric field about porcelain insulators under normal line voltage. Diagrams of the dielectric field and photographs of flash over tests of theoretical designs are shown. Thereafter, the necessary modifications of the theoretical designs, in order to meet operating and manufacturing conditions, are discussed.

In the latter part of the paper, diagrams and illustrations are shown of a proposed type of commercial insulator design which has been evolved by linking together the theoretical and practical phases of the problem. A comparison is then made between the older types of design and the proposed type, as regards the resistance of each to the conditions that cause failure of the insulator in service.

In conclusion, a summary is made of the advantages it is believed that the new type of design has over the present commercial insulator designs.

## Introduction

USUALLY any design problem of engineering may be quite easily separated into two rather distinct phases. The one phase is termed "theoretical" and infers that the service experience, processes of manufacture, cost of materials, cost of manufacture, etc., are placed secondary in importance in the search of an ideal design. The other phase is termed "practical" and infers that the design has been evolved mostly from a consideration of service experience. It is generally conceded that a design evolved by either method may have certain advantages. The object of the following investigations has been to link together these two phases in a specific application, namely; the design of pin-type-transmission-line insulators.

In order to save repetition, the present article will deal, for the most part, with laboratory tests of various new designs and the comparison of these designs with those now in commercial use. The theoretical elements of the problem which laid the foundation for the following developments are clearly given in two papers published in the 1913 A. I. E. E. TRANSACTIONS. One paper, by C. Fortescue, is subjected *The Application of Theorem of Electrostatics to Insulation Problems*. The other, by C. Fortescue and S. Farnsworth, is subjected *Air as an Insulator*.

. Since an attempt will be made to deal with the practical and theoretical phases of the problem, two logical questions at once arise:

First, are the insulator designs installed in service at the present time satisfactory?

Second, can one type of design be developed that will be satisfactory in all localities?

The first question is answered by a resume of current engineering literature which offers convincing evidence that there is a field for improvement. A comparison of the flashover voltage versus overall dimensions and weight of the present insulator designs would seem to warrant an attempt toward uniformity. Furthermore, the divergence of certain characteristics of some designs from the average curves indicates that some of the designs must be far from efficient.

The causes of such chaos in the present day insulator designs are quite obvious and have been frequently stated. First of all, the progress in transmission engineering has been rapid. The expanding transmission companies demanded insulator designs which would offer a good factor of safety. There was no previous operating experience to use as a basis in new developments and consequently it was often necessary for the transmission engineer to propose his own design. Moreover, the majority of our present insulator types were designed when the electrical and mechanical characteristics of porcelain were less understood than at the present day. As a result, the type of design has fluctuated and various features were, at one time or another, accentuated as most important. That is, at one period a long leakage path was required, regardless of voltage distribution per shell from capacity current or leakage current; then again a high puncture voltage, then a high mechanical strength, and so on. Naturally, many mistakes were made and a large proportion of the older insulator designs have failed in service application.

## CAUSES OF INSULATOR FAILURES

The knowledge that certain insulator types have failed in service is of little value in the redesign of insulators unless actual conditions of service and cause of failure are known. Also, the cause of failure of a particular type in one locality should be compared to the cause of failure in other localities. Hence, before attempting to develop a new type of design, a study was made of available data on insulator deterioration and the opinions of operating engineers in various parts of the country were obtained.

From discussions with these engineers, from published data on insulator deterioration, and from observations of insulators that had failed in operation, it would seem that the following items are the main causes of pin-type insulator failures:

1. Improper distribution of dielectric field.
2. Improper distribution of surface leakage.
3. Porosity.
4. Mechanical breakage.
    a. From handling.
    b. Mischievous shooting and stone throwing.
    c. Insufficient strength as a support.
    d. Brittle material.
5. Lightning.
6. Birds and animals short-circuiting line.
7. Unequal expansion of metal, cement and porcelain.
8. Internal stresses in material.
9. Defective batches.

Items 3, 4d and 9 are the problems in which the ceramic engineer is vitally concerned and will not be considered further in this paper. These items have doubtless been of great importance in the past but more scientific and painstaking factory control must minimize them in the future. ("Electrical Porcelain", *Electric Journal*, February and March, 1918, by T. A. Klinefelter and G. I. Gilchrest).

The manner in which these items have caused ultimate failure of certain designs are very briefly enumerated below:

1. *Improper Distribution of the Dielectric Field.* Failure to consider the electrostatic field distribution as regards every part of the unit has resulted in designs which have an unequal voltage distribution per shell even when the unit is dry and clean. When the rain sheds are closely spaced the air between them is ionized with the result that preliminary discharges take place before flashover. Parts of the unit in the dielectric field are,

thereby partially short-circuited and other parts overstressed. Flashover voltage will, therefore, be low in relation to overall dimensions. These conditions are usually augmented as the insulator becomes dirty and wet.

2. *Improper Distribution of Surface Leakage.* The most serious trouble from surface leakage is probably experienced on sections of line along the California coast, sections near Great Salt Lake, Utah, etc. Moreover, the climatic conditions of certain localities, especially where a "dry season" is followed by a "rainy season", augments the difficulty. For example, along the California coast the insulator surface gradually becomes coated with dirt during the "dry season" of the year. At night a strong breeze drives a fog containing more or less salt spray over the transmission lines. The dirt on the insulator surface then becomes moist and conducting and a rather high leakage current is often the result. Where wooden construction is used, charring of the wood at points of highest ohmic resistance may take place, resulting in the burning of pins and cross arms. The leakage resistance per shell of many of the older designs are such as to give a very uneven voltage distribution per shell under service conditions. Moreover, the voltage drop over the surface between closely spaced sheds often becomes sufficient to cause static discharges between them. The effective leakage surface of the insulator is thereby decreased and the arcing imposes an electrical impact on the insulator at the same time. Of course, this same trouble occurs on sections of lines near factories, steam railroads, cement mills, smelter plants, etc., to a more or less degree.

3. *Porosity.* The deterioration of porcelain insulators in service was given little consideration during the early days of transmission engineering. The majority of transmission engineers preferred an insulator having a porcelain body which offered a high resistance to mechanical breakage. As a consequence, the porosity of the material, which varies inversely to the mechanical strength as regards resistance to mechanical impact, was considered of secondary importance. The results that the condition has caused in service have been clearly presented before the Institute by Professor H. J. Ryan.[1]

4. *Mechanical Breakage.* Mechanical breakage has been a frequent source of annoyance, and has worked havoc in a number of ways.

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[1] "*Ceramics in Relation to the Durability of Porcelain Suspension Insulators.*" A. I. E. E. TRANSACTIONS, Vol. XXXV, 1916.

(a) The deep thin sectioned sheds are easily broken in handling. This results in a loss of insulator units. What is of more importance, the danger of installing defective units is considerable, since many fine cracks may pass the usual construction crews' inspection.

(b) Some of the operating engineers, especially those located in the middle West, or near mining camps, claim that 80 to 90 per cent of the defective insulators removed from the line were first injured by rifle shooting or stone throwing.

(c) Many designs are not sufficiently strong as a support due to the thin sections of porcelain and small area under mechanical stress, or to the fact that the deep center shed necessitates a high pin. Such designs fail in service when unusual stresses occur, such as are caused by sleet storms, by poles giving way during freezing and thawing of the ground, or heavy rains, etc.

5. *Lightning.* It is generally conceded that a direct stroke of lightning will destroy any insulator that comes within its path. However, some of the older designs, especially those having deep inner shells and heads of large diameter, were very vulnerable to any sudden impact voltage. In the first place, the impulse ratio (flashover voltage at high frequency divided by flashover voltage at normal frequency) of such insulators is rather high and in the second place the ratio between flashover voltage in air and puncture voltage under oil is comparatively low.

6. *Birds and Animals Short-Circuiting Line.* Some transmission companies have found it necessary to place shields on the poles in certain sections in order to prevent squirrels climbing the poles and short-circuiting the lines. In other localities it has been necessary to increase the height of insulator pins or the wire spacing in order to prevent cranes, eagles, etc., from short-circuiting or grounding the line.

7. *Unequal Expansion of Metal, Cement and Porcelain.* In many cases solid metal pins or heavy cast thimbles have been cemented into the insulator. Apparently the unequal expansion of the metal, cement and porcelain has caused the cracking of the porcelain and ultimate failure of the insulator.

8. *Internal Stresses in the Material.* Corners of small radii and non-uniform sections of the porcelain shells have possibly produced internal stresses in the material during the manufacturing processes and these have developed cracks later on in service. Also, the relation between the shape of the shells in

the cemented area and the shape of the cemented area itself has been such as to allow the full effect of unequal expansion of the porcelain and cement which is caused by temperature changes or absorption of moisture.

## INVESTIGATION OF DIELECTRIC FIELD

In the papers of Fortescue and Farnsworth, several insulator forms were evolved mathematically, and the dielectric field explored by means of an electrolytic bath. It was believed that the data from which these papers were written in conjunction with the available data of other investigators, of both analytical and experimental nature, afforded sufficient basis from which to formulate preliminary designs.[2]

After a careful summation of the data at hand, it seemed that the logical method of attacking the problem would be to have several theoretical insulator designs produced out of a usual commercial porcelain body. The dielectric field of these should then be investigated under a voltage of approximately the same value that would be impressed in service. Thereafter practical considerations, such as deterioration of the various commercial units in service, manufacturing limitations, etc., should be taken into account with the intent of arriving at a compromise between the theoretical and practical features.

## METHOD OF DETERMINING FORM OF DIELECTRIC FIELD

The dielectric field was determined by the following procedure: The insulator was fastened rigidly in a position such that the plane of the field to be determined extended horizontally. A piece of fullerboard was fitted over a half section of the insulator in this plane. In all cases the apparatus was so arranged that the cross-arm supporting the insulator was grounded as in service where steel construction is used. Finely divided asbestos was then sifted evenly onto the sheet of fullerboard, voltage at 60 cycles of the desired value applied, and the sheet was gently tapped until the particles had adjusted themselves. Permanent records were obtained by placing a sheet of photographic printing paper over the fullerboard, obtaining the field as above, and exposing the paper after the particles had become arranged.

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[2] "Distribution of Potential about High-Voltage Line Insulators," by C. T. Allcutt and W. K. Skolfield. *Journal of Electricity, Power and Gas*, June 17, 1916. *An Experimental Method of Obtaining the Solution of Electrostatic Problems with Notes on High-Voltage Bushing Design*, by C. W. Rice. A. I. E. E. TRANSACTIONS, Vol. XXXVI, 1917, p. 905.

That the stronger portion of the field around an insulator was not disturbed materially by the presence of the fullerboard or the asbestos particles was proven by suspending a piece of finely drawn glass in parts of the field by means of a silk fibre supported by small insulated rods. As nearly as could be checked, the glass indicated the same direction of the field as the asbestos particles.

## THEORETICAL INSULATOR DESIGNS

The dielectric fields of five theoretical designs were determined. Wherever a customary transmission cross-arm and line-wire are used, there are two principal planes of the dielectric field which show the greatest difference, *i.e.*, the plane of the cross-arm and the plane of the line wire. These two planes are 90 degrees apart and in passing from one to the other the transition is gradual. During the investigation, records were taken of the dielectric field of these two principal planes and of a plane midway between the two. In this paper the diagram taken in one plane of the unit is usually shown. Diagrams of three planes of two designs are shown in order to illustrate the variations that occur. The plane in which the particular field was taken is indicated by the reduced top projection at the upper left portion of each figure.

## DIELECTRIC FIELD FORMS AND ILLUSTRATIONS OF THEORETICAL DESIGNS

Fig. 1 shows the field form of a bushing having dimensions of ring and rod chosen such as to give maximum breakdown voltage over the surface for the mean diameter of torus ring.

Fig. 2 shows a 60-cycle flashover on bushing of Fig. 1.

Fig. 3 gives the field form of a design using a confocal system of ellipsoids and hyperboloids of revolution.

Fig. 7A, 60-cycle flashover on shape shown in Fig. 3.

Fig. 4, field form between special metal cap and pin as might be used as terminals of a line insulator.

Fig. 7B, 60-cycle flashover between cap and pin as shown in Fig. 4.

Fig. 5, field form of line insulator without rain sheds.

Fig. 7C, 60-cycle flashover on shape shown in Fig. 5.

Fig. 6, field form of pin-type insulator, the porcelain of which has a curvature similar to that of Fig. 5. However, the porcelain body is separated into three sections and metal rainsheds added to give wet arcing distance.

Fig. 7D, 60-cycle flashover on unit given in Fig. 6.

In Table I are given the length of path over the insulator surface between electrodes and the 60-cycle flashover voltage.

TABLE I

| Shape in figure | Length of surface | | Effective kilovolts flashover voltage | | |
|---|---|---|---|---|---|
| | Inches | Centimeters | Total | Per inch | Per centimeter |
| 1 | 4.25 | 10.8 | 87 | 20.4 | 8.1 |
| 7*a* | 6.5 | 16.5 | 148 | 22.8 | 9.0 |
| 7*c* | 8 | 20.3 | 115 | 14.4 | 5.7 |

From a consideration of Table I it is evident that a flashover value of between 20 and 23 kilovolts per inch (8 and 9 kilovolts per centimeter) of surface may reasonably be expected if the unit is designed with contours of the surfaces approximating the flow lines of the dielectric field. Of course, the flashover on the unit without rain sheds is somewhat lower, being 14.4 kilovolts per inch. The lower flashover on this unit is due to two conditions, *i.e.*, the porcelain surface does not follow the dielectric field in all planes and the small tie wire produces corona and subsequent static discharges at a relatively low voltage. Placing a static shield on the top of this unit increased the flashover voltage 18 per cent.

With the field form between cap and pin as given in Fig. 4, and the voltage values given above in Table I, theoretical insulator designs could be determined for such electrodes. Such designs should follow surfaces indicated on Fig. 4, as (*a*) and (*b*). The highest flashover voltage for a given surface distance between electrodes would thereby be obtained. Moreover, the flashover voltage of such a unit could be closely approximated if the electrodes have sufficient radius of curvature at points of contact with the insulating material and a good seal is made between the metal and the insulating material.

## MODIFICATIONS OF THEORETICAL DESIGN TO MEET OPERATING AND MANUFACTURING CONDITIONS

Insulators based on such theoretical data would be excellent from the electrical and mechanical standpoints if they were to operate in clean, dry air. However, the commercial insulator must maintain the transmission system during the heaviest of

PLATE XIX.
A. I. E. E.
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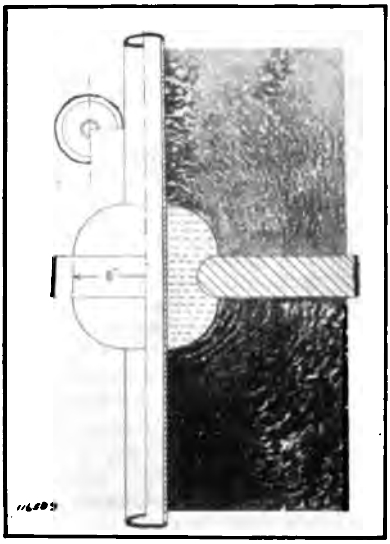


FIG. 1—DIELECTRIC FIELD ABOUT THEORETICAL BUSHING

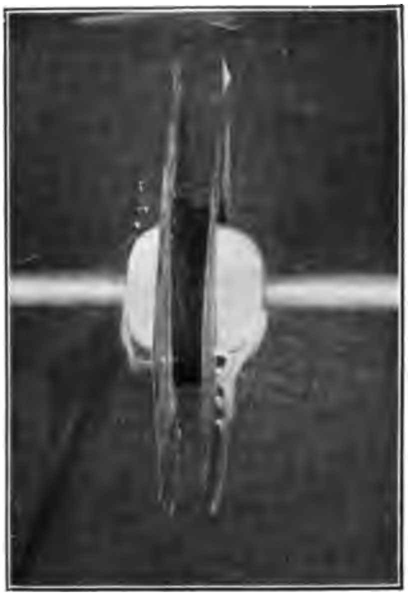


FIG. 2—60-CYCLE FLASHOVER BUSHING OF FIG. 1

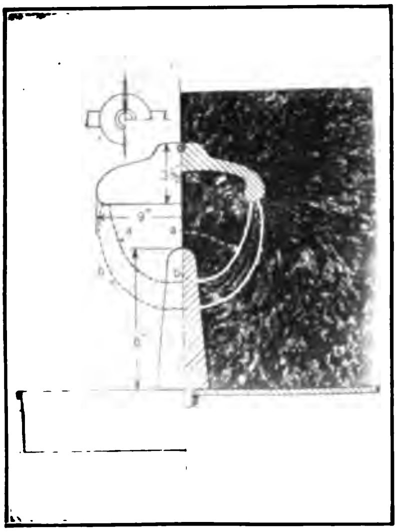
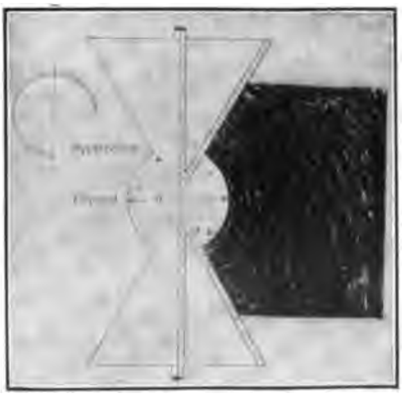


FIG. 4—DIELECTRIC FIELD BETWEEN METAL CAP AND PIN



[GILCHREST]

FIG. 3—DIELECTRIC FIELD ABOUT THEORETICAL DESIGN USING CONFOCAL SURFACES OF REVOLUTION

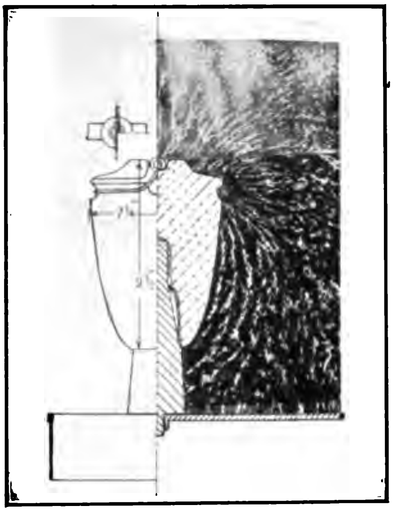


FIG. 5—DIELECTRIC FIELD ABOUT LINE INSULATOR WITHOUT RAIN SHEDS

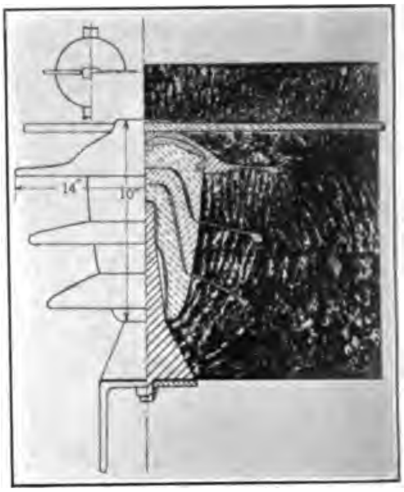


FIG. 6—DIELECTRIC FIELD ABOUT INSULATOR WITH METAL RAIN SHEDS

[GILCHREST]

FIG. 7—60-CYCLE FLASHOVERS ON THEORETICAL PORCELAIN SHAPES snow and rain storms. Moreover, it must have sufficient leakage distance to prevent flashover or even high power loss from surface leakage when the surface becomes dirty and wet.

The production of one-piece insulators for high-voltage service, although possible, would be costly. Also, the puncturing voltage of a one-piece unit would be low for a given thickness, since the stress in an insulating material between metal electrodes of different potential varies as a logarithmic function. The separation of the unit into parts that are cemented together, more uniformly distributes the stress of the dielectric if the unit is properly designed. It also decreases the probability of complete failure of the insulator and facilitates factory production, lessening the cost of the commercial unit.

The use of a special cap would be desirable from a dielectric standpoint. However, the voltage characteristics under rain are the same whether the usual line and tie wire or a special cap are used. Moreover, the cost and ease of replacement, cost of construction, etc., favor the line and tie wire construction.

## PROPOSED COMMERCIAL INSULATOR DESIGN

With the above limitations of the theoretical designs and the causes of insulator failures in mind, the type of unit indicated in Fig. 8 was evolved.

Summed up briefly this type of design embodies the following features:

1. Surfaces *a* conform to the flow lines of the electrostatic field.
2. Surfaces *b* of the rain sheds conform to the equipotential surfaces.
3. Lines of mechanical stress are parallel to the electrostatic flow lines.
4. The leakage resistance per shell is about equal, being increased gradually from the head to the center shell.
5. Approximately equal capacity per shell.

## COMPARISON WITH OLDER DESIGNS

It is not possible to much more than indicate in the following discussion the methods employed to compare the proposed type of design given in Figs. 8A and 8B with the older commercial insulators. Samples of various commercial designs were produced and were subjected to rather thorough laboratory tests at the same time tests were made on insulators of the proposed design.

It should be noted that the insulators of the new type used in the comparative tests do not exactly correspond to the proportions of Fig. 8. In order to lessen the cost of investigation, insulator sheds of several diameters were obtained from one set of molds by trimming the individual shells before burning. This also accounts for the unfinished appearance of the edges of sheds, etc., in some of the experimental designs.

In the following comparison it is not assumed that the evolved design should be final in each detail. The main goal toward which work is being directed is uniformity of all the elements entering into the designs with the idea in view of arriving at a type of design which will be equally successful in resisting failure in service whatever the requirements are in that particular section. In the following comparisons the items causing failure in service are discussed in the order given at the beginning of the paper.

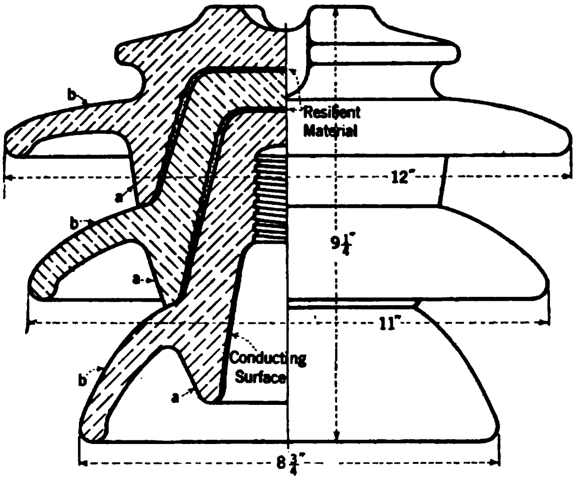


FIG. 8A—THREE-PIECE INSULATOR OF THE PROPOSED TYPE OF DESIGN—INSULATOR *A*

1. *Dielectric Field Distribution.* The shortest air path under electrostatic stress should be at least long enough to prevent overstressing of the air at any point. In the theoretical discussions referred to in the introduction it was proved that wherever porcelain and air are in series in a dielectric field the voltage gradient per unit distance through the porcelain will be ¼ to 1/5 the voltage gradient through the air. It is obvious that

any thin section of air between porcelain sheds of a customary line insulator will be over-stressed even at the normal line voltage of the insulator.

In order to make a comparison of the dielectric fields of various insulators, their field forms were determined as in the investigation of the theoretical designs. It is believed that the following field forms and illustrations sufficiently indicate that many present types have not been designed with a full appreciation of the advantages of shapes that conform to the electrostatic flow lines in obtaining the most efficient distribution of the stresses in the dielectric field.

Fig. 9 (insulator *C*) gives the dielectric field of a unit of the type used in the early developments of high-voltage transmission. The air between sheds just below the cement section is highly stressed. Because of the height of the pin in proportion to other dimensions of the unit the stress toward the base of the pin and the supporting cross arm is very low. Moreover, the third shell of the insulator is spaced so close to the insulator pin that it does not take its proportion of voltage stress when either dry or wet flashover occurs.

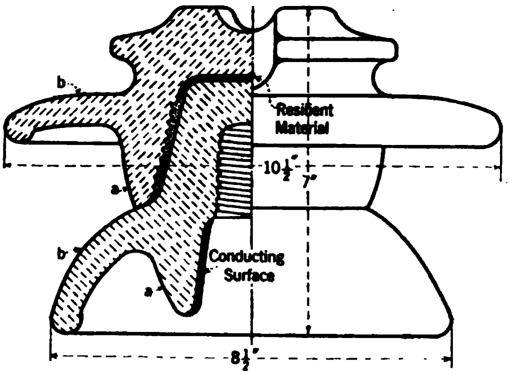


FIG. 8B—TWO-PIECE INSULATOR OF THE PROPOSED TYPE OF DESIGN —INSULATOR *B*

Fig. 10 (insulator *D*) shows the dielectric field of a three-piece insulator of a somewhat more recent design. The center shed is better spaced than in insulator *C*. However, the air just below the cement sections is highly stressed and the short rain shed of the second shell gives an unequal voltage distribution at flashover, dry or wet.

Figs. 11, 12 and 13 (insulator *E*) show the dielectric field of a four-piece unit of comparatively recent design. The sheds of this design are more uniformly spaced, but the air between sheds just below the cement sections is highly stressed. The stress throughout the dielectric field of this unit is an improvement over the types *C* and *D*. However, the short second shed and protected fourth shed give unequal voltage distribution at flashover dry or wet.

Figs. 14, 15 and 16 (insulator *F*) show the dielectric field of a unit of the proposed design. The shortest air path between shells is sufficient so that the air is not overstressed at working voltage of the insulator or until flashover occurs. Moreover, the rain sheds are so spaced that each section of the unit takes its share of the stress at flashover, dry or wet.

Fig. 17 (insulator *G*) shows the dielectric field of a unit similar to insulator *F*, but having rain sheds of greater diameter. The diameter of the head of this unit is probably out of proportion and greater than would be most satisfactory for service. However, the stress in the dielectric is well proportioned and the voltage distribution per shell at flashover, dry or wet, is fairly well proportioned.

Fig. 18 (insulator *F*) shows the dielectric field of the insulator having upper surfaces of the rain sheds covered with a conducting paint. This field form which approximates the rain conditions indicates that the stress per shell on the unit during rain would be approximately equal. Moreover it indicates that the stress in the dielectric field is more uniform during rain.

Fig. 19 (insulator *F*) shows the dielectric field of the insulator when equipped with Nicholson Arcing Rings, and indicates that the most highly stressed portion of the field about the insulator is not changed. However, the most highly stressed portion of the field between the line wire and cross arm is now between arcing rings and flashovers would, therefore, occur between rings.

Fig. 20 (insulator *F*) shows the dielectric field when static shields are placed at the top and base of the insulator. This combination would give a very fine distribution of stresses in the dielectric but would be rather expensive commercially.

### 60-CYCLE FLASHOVER TESTS

Flashover on most of the older insulator types is caused by the corona formation at the line and tie wires and the edges of the

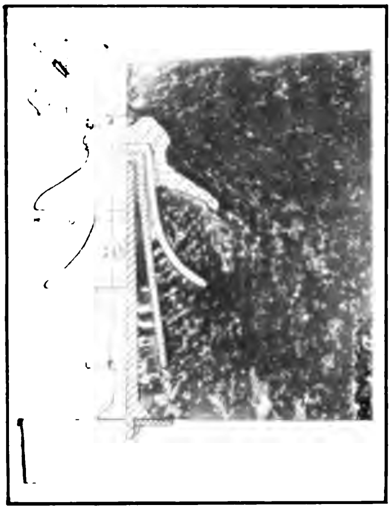


FIG. 9—INSULATOR *C*—DIELECTRIC FIELD ABOUT LINE INSULATOR OF EARLY DESIGN

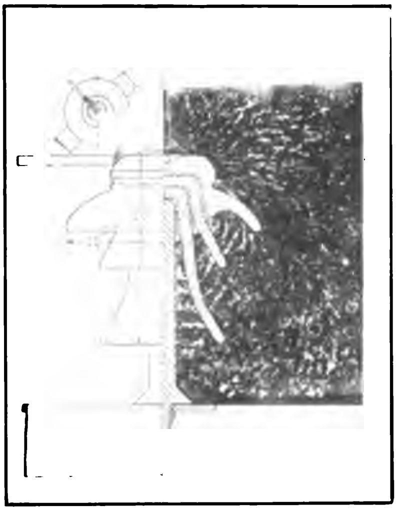


FIG. 10—INSULATOR *D*—DIELECTRIC FIELD OF INSULATOR OF FAIRLY RECENT DESIGN

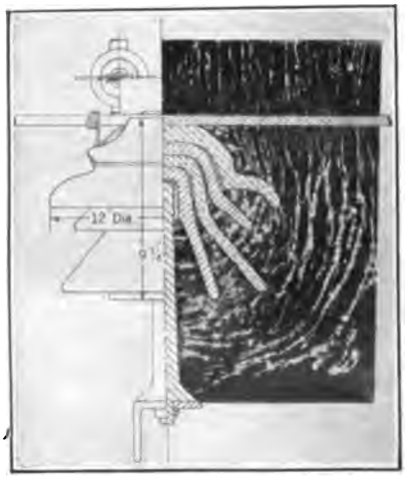


FIG. 11—INSULATOR *E*—DIELECTRIC FIELD ABOUT INSULATOR OF RECENT DESIGN

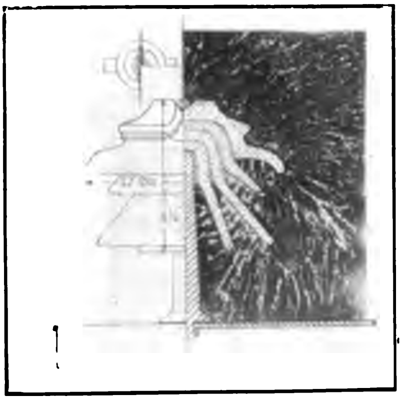


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FIG. 12—(SEE FIG. 11)

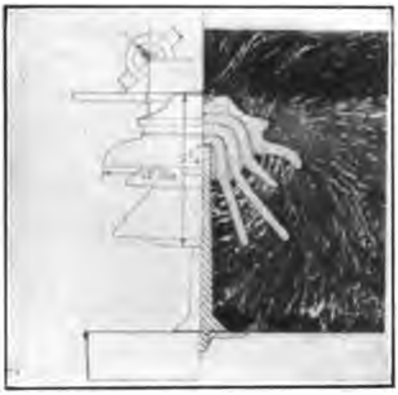

FIG. 13—(SEE FIG. 11)

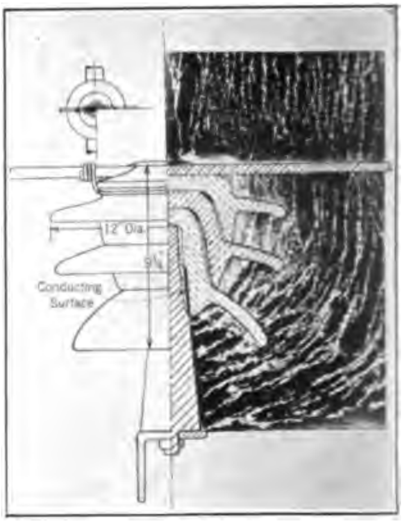

FIG. 14—INSULATOR *F*—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN

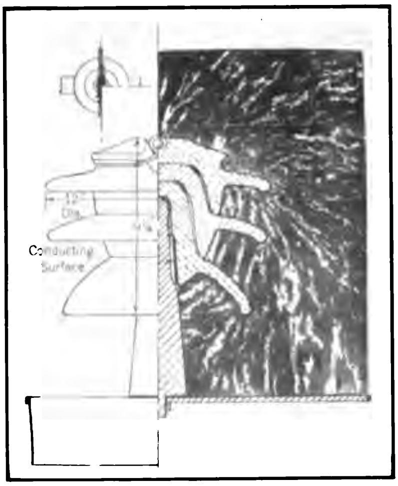

FIG. 15—(SEE FIG. 14)

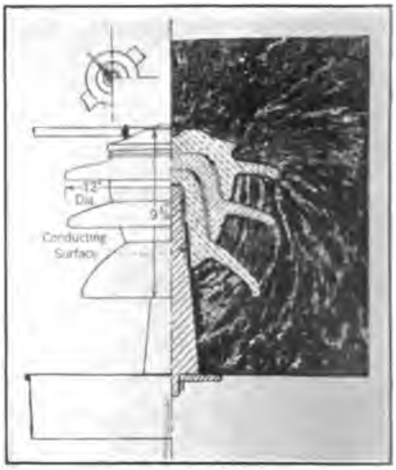

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FIG. 16—(SEE FIG. 14)

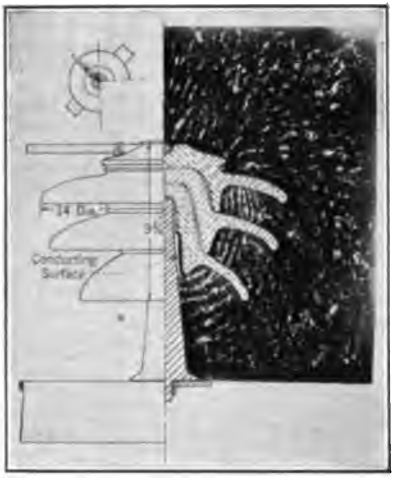

FIG. 17—INSULATOR *G*—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN

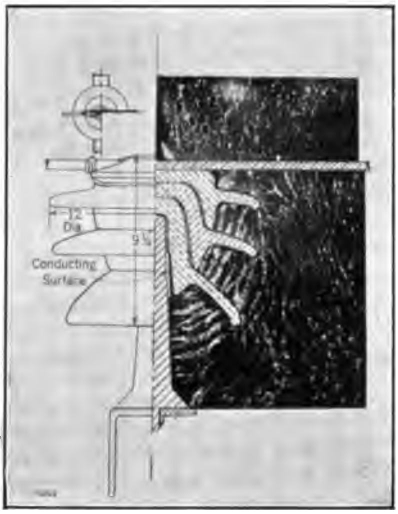

FIG. 18—INSULATOR *F*—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN UNDER CONDITIONS APPROXIMATING RAIN

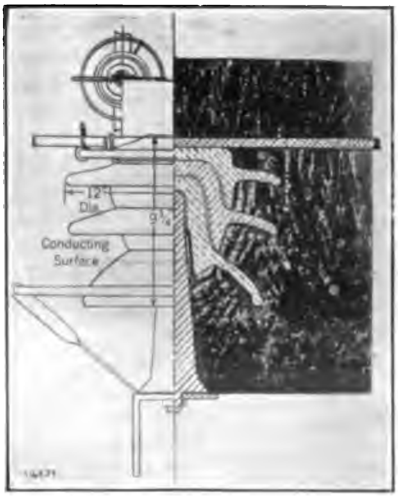

FIG. 19—INSULATOR *F*—DIELECTRIC FIELD ABOUT INSULATOR OF PROPOSED DESIGN INSTALLED WITH NICHOLSON ARCING RINGS

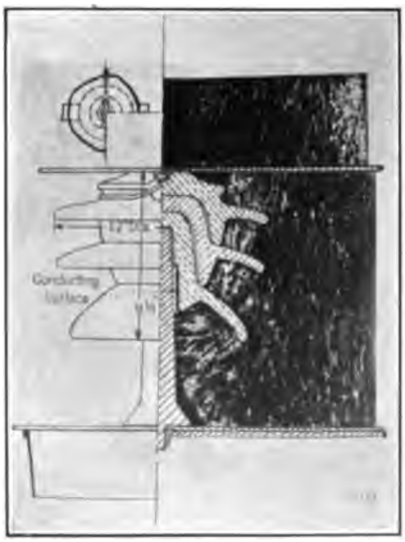

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FIG. 20—INSULATOR *F*—DIELECTRIC FIELD ABOUT INSULATOR HAVING STATIC SHIELDS AT TOP AND BASE

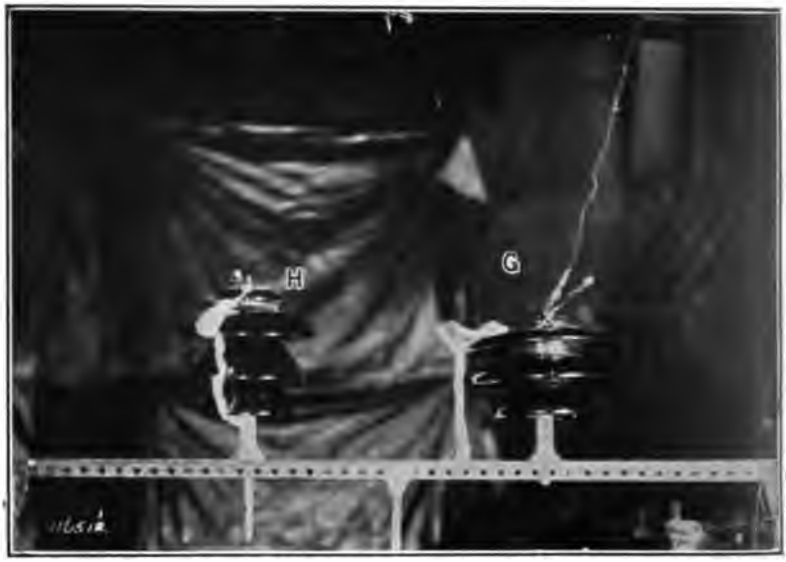

Fig. 22—60-Cycle Dry Flashover

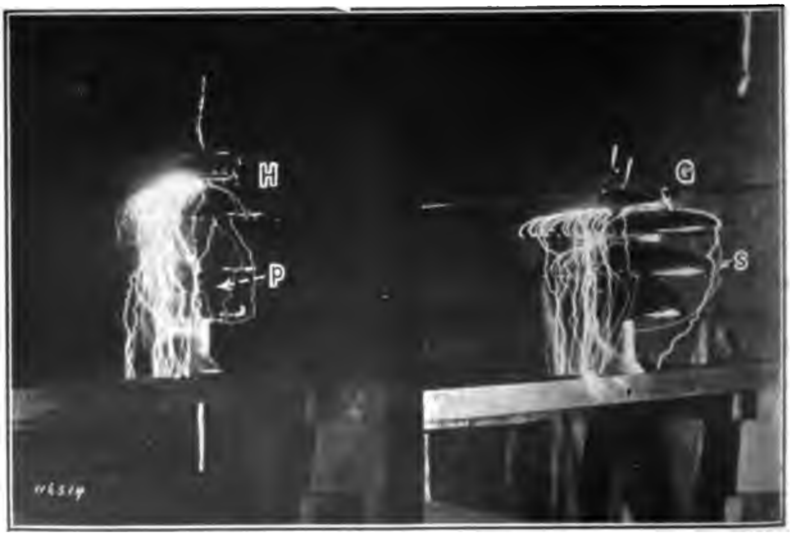

Fig. 23—60-Cycle Wet Flashover [Gilchrest]

cement joints between shells. As the voltage applied to the insulator is increased, the area of the corona formation increases and static streamers gradually spread over the surface of the insulator sheds. The static streamers increase in length until the air insulation between them finally fails and flashover follows. Obviously, the path of the flashover will start along the path of these streamers and thus trouble may be caused by the intense heat of the power arc and rain sheds may be stripped from the insulator.

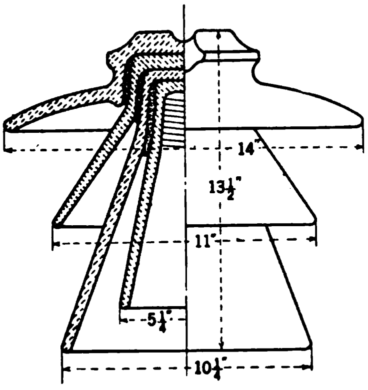

FIG. 21A—INSULATOR *I*

In the proposed type of design there are no static streamers from the edges of the cement section between shells up to flash-over voltage. The corona formation at the tie and line wires therefore, builds up until flashover occurs by breaking down an air path between the line and pin or cross arm. The proof of these statements may be seen in the following illustrations. The axes of the two units in each of the following figures giving comparative flashovers were at the same distance from the camera lens and hence the dimensions are directly comparable.

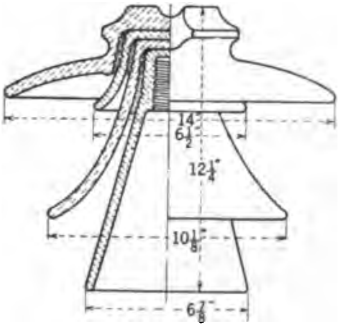

FIG. 21B—INSULATOR *K*

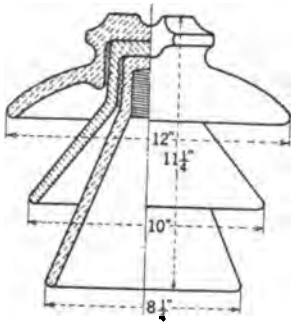

FIG. 21C—INSULATOR *H*

Figs. 22 and 23 show illustrations of dry and wet flashovers respectively, on insulators *G* and *H*. The design of insulator *H* is given in Fig. 21c.

Fig. 24 shows an illustration of wet flashover on insulators *J* and *I*. The design of insulator *I* is given in Fig. 21A. Insulator *J* is of the proposed type similar to the unit given in Fig. 8.

Fig. 25 illustrates of one of the early types of high-voltage insulators and insulator *J* in parallel. The finer lines over the surface of the old type unit are preliminary static discharges. The final power arc passes from the left of the insulator head around in front of and finally to the pin at the back of the insulator.

Fig. 26 shows insulator *F* in parallel with the unit of early design shown in Fig. 25, and shows the corona formation and static discharge over the head and between the head and second shell of the old type unit. The camera exposure was $\frac{3}{4}$ of a minute at *F*-8.

The difference in the stress in the air around the insulators just below flashover voltage dry was very marked. Insulators *F*, *G* and *J* of the proposed type of design showed no appreciable corona except at line and tie wires until flashover occurred. Flashover occurred from tie wire or line wire to pin or cross arm, there being no tendency for the arc to start between the rain sheds. Considerable corona formation and static streamers could be detected on insulators *E*, *H* and *I*. Static streamers began to spread out over the surfaces between shells of insulators *H* and *I* at 80 per cent of flashover voltage, and unless these units are mounted on rather low pins the power arc holds close to the insulator surfaces. Of course, the old type design of Figs. 25 and 26 has been entirely superseded but these two figures clearly indicate the entire neglect of a consideration of the dielectric field.

The difference of distribution of stress before wet flashover is even more noticable. In insulator *E*, *H* and *I* the unequal spacing of rain sheds and consequent unequal wet arcing distances, combined with a highly stressed air between the sheds below the cement sections produces preliminary discharges (marked *p*) between rain sheds. These preliminary discharges throw electrical impacts onto parts of the insulator and short-circuit portions of the porcelain between the line and pin. Consequently, when a line surge occurs during a rain storm or when the unit is wet and dirty the factor of safety of these insulators in resisting puncture or flashover is actually no more, and sometimes is less, than it would be minus one of the shells.

Insulators *F*, *G* and *J* of the proposed design show no preliminary discharges except static from tie or line wires to pin or cross arm. Static discharges (marked $s$) are shown on each of these units. All of the leakage surface and thickness of porcelain between line and pin are, therefore, effective up to failure by flashover.

2—*Surface Leakage.* As previously stated, the leakage surface of many of the older designs gives a very uneven voltage distribution per shell. Table II gives the resistance per shell of various insulators tested during this investigation. The values were obtained by an integration of the surface, *i.e.*, surface resistance equals $S \frac{d\,s}{2\,\pi\,y}$ where $d\,s$ is an element of surface and $y$ the radius of that element from the axis of the insulator.

It is obvious from this table that certain of the older designs, especially those having a short second shell, long inner shells, etc., have a very unequal surface resistance per shell. If the insulator surface becomes dirty and wet so as to pass a leakage current of even a thousandth of an ampere the voltage distribution would depend upon this current and the capacity current could be neglected. The voltage gradient over the insulator surface thus often becomes sufficient to cause discharge between sheds and pin or cross arm or over the short sheds. An electrical impact is thereby applied to parts of the insulator and portions of the porcelain body between line and pin are short-circuited. It is believed that the continued overstressing of parts has been the cause of many insulator failures in the past.

TABLE II

SURFACE RESISTANCE PER SHED IN PER CENT OF TOTAL RESISTANCE

| Insulator | Number of shed | | | |
|---|---|---|---|---|
| | First | Second | Third | Fourth |
| *A* | 28 | 30 | 42 | .. |
| *B* | 45 | 55 | .. | .. |
| *E* | 14 | 13 | 32 | 41 |
| *F* | 26 | 29 | 45 | .. |
| *G* | 26 | 31 | 43 | .. |
| *H* | 18 | 29 | 48 | .. |
| *I* | 12 | 16 | 32 | 40 |
| *K* | 15 | 11 | 30 | 44 |

The surface resistance of the proposed designs as typified by insulators *A* and *B* in Table II is gradually increased from the top to center shells, the increase being considered as an advantage since the center sheds will usually become dirtiest.

A novel feature of the proposed design is illustrated in Fig. 27 showing insulators *D*, *E*, *F* and *H*. These units were set on a cross-arm line, and tie wire attached as in service, voltage applied and plaster of paris dust blown around them. The surfaces along the lines *a* of the proposed design (Fig. 8) are practically free of dust.

The reason for this is quite apparent. All the force acting in the dielectric field along this surface *a* is tangential and would tend to force the particles to the sheds above or below. The same action was noted when the units were subjected to atomized salt water. This feature would doubtless have some value in dust laden sections since the dust would tend to settle mostly on the lower shed and rain and wind would clean this to some extent.

It is necessary to clean the insulators in long portions of line in certain sections of country as the coast districts of California. It is very apparent that the proposed type of design may be cleaned much more readily and thoroughly than any of the older types.

3. *Porosity.* As denoted previously, the porosity of porcelain is a specific problem of the ceramic engineer rather than the designer. As clearly pointed out in a recent paper[*] by Prof. Ryan we apparently have no method of detecting the very slightly porous material which may cause trouble. Since porosity is a function of the body composition, manufacturing process and burning, even with the most careful production and testing, a small amount of this slightly porous material is not detected. The thicker portions of the porcelain in the cemented area of the proposed type of design should minimize the number of the pieces that will give trouble later in service.

4. *Mechanical Breakage.*

(a) From Handling: The increase of thickness of the rain sheds and addition of a drip edge will materially decrease the percentage loss from this cause.

(b) Mischievous Stone Throwing and Rifle Shooting: (1) The following photographs give comparative flashover voltages dry and wet, on units having various rain sheds broken by throwing a small weight at the insulator. Figs. 22 and 23 show the dry and wet flashovers on insulators *G* and *H* respectively. Table III gives the flashover voltages of broken units in per

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[*] loc. cit.

FIG. 24—60-CYCLE WET FLASHOVER

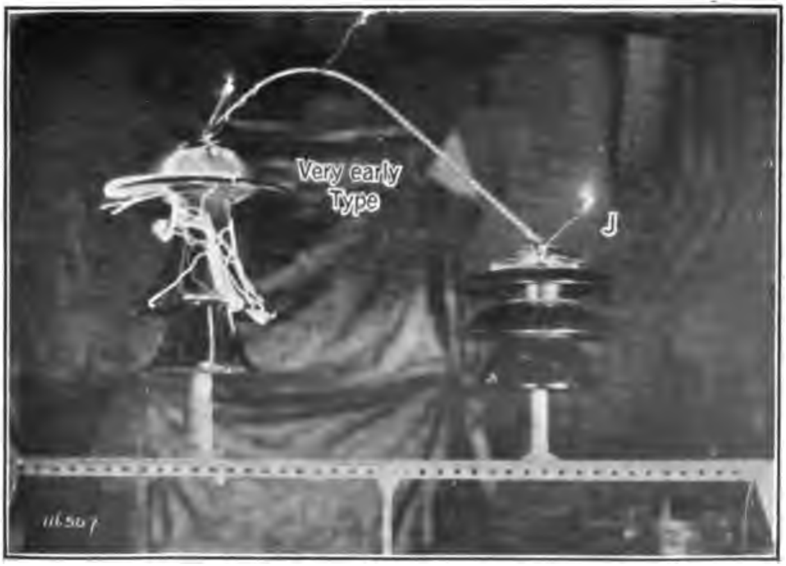

FIG. 25—60-CYCLE DRY FLASHOVER [GILCHREST]

FIG. 26—60-CYCLE TEST—COMPARATIVE CORONA FORMATION

[GILCHREST]

FIG. 27—PLASTER PARIS DUST DEPOSITED WITH INSULATORS UNDER 60-CYCLE VOLTAGE

FIG. 28—TOP SHED BROKEN—60-CYCLE WET FLASHOVER

[GILCHREST]

FIG. 29—SECOND SHED BROKEN—60-CYCLE DRY FLASHOVER

Fig. 30—Second Shed Broken—60-Cycle Wet Flashover

[Gilchrest]

Fig. 31—Second and Third Sheds Broken—60-Cycle Dry Flashover

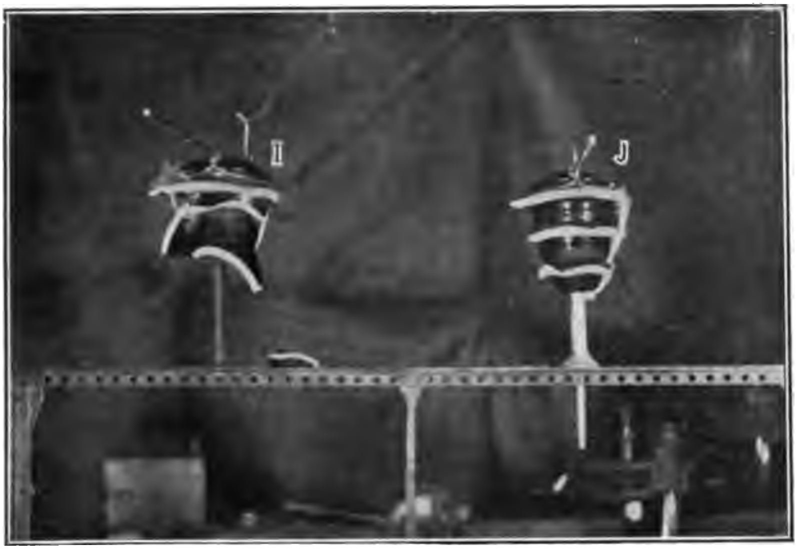

Fig. 32—All Sheds Broken—60-Cycle Dry Flashover

[GILCHREST]

Fig. 33—After 15 Shots Were Fired at Each

Fig. 34—After Number of Shots as Indicated

PLATE XXX.
A. I. E. E.
VOL. XXXVII, 1918

FIG. 35—60-CYCLE DRY FLASHOVERS ON INSULATORS *H*, *I* AND *J* AFTER BREAKAGE FROM RIFLE SHOTS

FIG. 36—RESISTANCE TO SIDE-PULL [GILCHREST]

cent of flashover of the unit when unbroken. Reference to illustrations is made in the Table. Fig. 24 shows the wet flashover on units *J* and *I*. Table IV gives comparative flashovers of broken units, as in Table III.

TABLE III

| Sheds broken.... | Top | | Second | | Second and third | |
|---|---|---|---|---|---|---|
| Illustrated in Fig. | | 28 | 29 | 30 | 31 | |
| Dry or wet...... | dry | wet | dry | wet | dry | wet |
| Insulator *G*...... | 85 | 80 | 100 | 100 | 68 | 74 |
| Insulator *H*...... | 79 | 77 | 82 | 97 | 54 | 70 |

TABLE IV

| Sheds Broken......... | Top | Top and third | All sheds | |
|---|---|---|---|---|
| Photographs in Fig........................................32 | | | | |
| Dry or wet........... | dry | dry | dry | wet |
| Insulator *J*........... | 87 | 78 | 59 | 30 |
| Insulator *I*........... | 85 | 70 | 34 | 15 |

As would be expected from a study of the dielectric field diagrams, the breaking of the second shed of the proposed type of design has practically no effect upon the flashover values of the insulator. In fact, as shown in the illustrations, the paths of the dry and wet flashovers did not follow over the broken shed. When sheds are broken, the corona formation and static streamers build out over the surface of the older type of design at a lower voltage than when the units are intact. The paths of flashover over these older types, therefore, follow the surface of the insulator. In the proposed type of design the absence of streamers from the porcelain surface causes the arc to keep clear of the insulator. A power arc will, therefore, be less liable to cause complete failure of a broken unit of the proposed design.

One of the most important features of the proposed design is that when the units are hit by stones, etc., the rain sheds will not crack or break beyond line *a* Fig. 8, due to the shape of the individual parts. The rain sheds of the older types of designs when hit are very likely to crack or break up into the cemented section. The first voltage surge or even normal line voltage will, therefore, often puncture the remaining shells. In fact, in the two series of tests photographed, both the older type of units punctured during the dry arcover after sheds were broken. Static streamers shot over surface of insulator *H* in Fig. 31 and then puncture occurred. Insulator *I*, Fig. 32, flashed over and then punctured before the circuit breaker of the testing transformer operated.

(2) One each of units *H*, *I* and *K* and two of *J* were subjected to rifle shots. Twenty-two caliber long bullets were shot at the insulators from about 30 yards distance and in a line at 45 deg. to their axes. The following photographs show the comparative breakage and the ability of the broken units to thereafter withstand electrical test. The shooting was done by men disinterested in the design of the insulators and they were requested to do as much damage as possible.

Fig. 33 shows insulators *I* and *J* after 15 shots were fired at each. The top, second and third shells of *I* were broken, the second shell being cracked into the cemented section. The second shell of *J* was chipped in two places, the rest of the insulator being intact. Fig. 34 shows insulators *H*, *K* and *I* after 14 shots were fired at *H*, 12 at *K* and 28 at *I*. The second and center shells of *H* were cracked and the center of *K*. The sheds of *J* were chipped off in a few places but the shells were not cracked. These five units were then set with their axes at right angles to the line of fire. Not more than 5 or 10 shots were necessary to strip the main part of the remaining sheds from Insulators *H*, *K* and *I* while one unit of type *J* still retained a considerable portion of its sheds after approximately 100 shots had been fired at it. The sheds remaining on the two units *J* were then knocked off by a hammer, to illustrate to those present that the surface of the insulator that follows flow line *a* would not be cracked thereby.

Fig. 35 shows the first dry flashover test made on these units after the shooting. Units *H*, *K* and *I* punctured at voltage of 33, 43 and 56 kilovolts, respectively. Unit *J* flashed over at 105 kilovolts, the remaining porcelain body bounded by line *a* still being intact.

(*C*) *Insufficient Strength as a Support*: Two samples as per Fig. 36, were tested to determine the resistance to side pull. In each case load was applied at the wire groove which was one foot from the base of pin. The parts from which insulator *L* was formed were obtained by trimming off the rain sheds of individual shells of unit *J* before burning and the mechanical test should, therefore, be about the same as of unit *J*. The pin of unit *L* was cemented directly into the insulator. A separable pressed steel thimble was cemented into insulator *F*. The one-inch bolt of the pin cemented into insulator *L* failed at 4400 ft-lb., and 3100 ft-lb. bent the pin of insulator *F* as shown, the position of insulator being such that additional load could not be applied. Both units were electrically intact after these tests.

(d) *Brittle Material.* All units used in these comparative tests were made of the same porcelain body and hence the question of brittleness, which is a ceramic problem, does not enter.

5. *Lightning.* The impulse ratio of the proposed design is lower than that of most of the older types of design. The actual value has not been determined. This statement is based on tests of the proposed design in parallel with older types. The old types of design were set on a pin of such height above the cross arm as to cause the old type of unit to flash over when set up alone at a voltage slightly greater than that of the proposed type of design alone. When tested in parallel the static discharges over the porcelain surface of the older type would often cause the proposed type to flashover first.

Furthermore, the body of the porcelain bounded by the flow lines *a* should have an impulse ratio close to one. A very high impulse voltage might, therefore, puncture through the rain sheds of the insulator leaving this body of the unit intact. The thicker section of porcelain between line and pin will also materially increase the factor of safety of the unit.

(6) *Unequal Expanion of Metal, Cement and Porcelain.* The introduction of a resilient material between tops of shells should eliminate the tendency of certain older designs to split off. Greater radii of curvature at the tops of the insulator shells and a cement section sloped from the axis should tend to eliminate the trouble from any difference of coefficient of expansion of the porcelain and cement.

(7) *Internal Stresses in the Material.* Internal stresses set up in the insulator parts during manufacture should be very much decreased by the elimination of small radii in corners and sudden changes of cross section of the material.

## CONCLUSIONS

Briefly stated, it is believed that the advantages of the proposed type over the older commercial types in resisting failure in service would be as follows:

1. When the insulator is dry, the corona and static forma-

tions are practically limited to the tie wire and line wire up to flashover voltage.

2. When the insulator is wet, no corona or static formation occurs up to flashover voltage. The flashover voltages for given overall dimensions are thereby increased.

3. The leakage resistance per shell is increased gradually from the head to the center shell. This takes into account the probability of the lower sheds becoming dirtier than the tops. The voltage distribution per shell is, therefore, equal when the insulator becomes dirty and wet and a heavy leakage current passes over the insulator.

4. Since the capacity per shell is about equal, the voltage distribution per shell will be equal when the insulator is clean and in dry air.

5. Since the distribution of voltage per shell depends upon the capacity current and leakage current, the distribution of voltage per shell in these designs should be approximately equal under all operating conditions.

6. The resistance of the insulator to side pull for a given weight and given electrical strength is relatively high. This is due to the feature of the design whereby the flow line *a* of the electrostatic field and the mechanical stress lines coincide.

7. The design of the individual shells is such that when they are tested before assembly the surface conforms to the electrostatic flow lines *a*. This allows testing of the individual parts to a higher percentage of service voltage than was possible in case of the individual shells of older designs.

8. Due to the shape of individual parts and of the assembled unit, the insulator sheds when hit by stones, rifle, balls, etc., do not break beyond surface *a*. The unit, therefore, offers a considerable percentage of its original resistance to flashover after the sheds are broken. The same feature tends to protect the insulator from complete failure during flashover in service.

9. Each characteristic of the insulator which would vitally affect durability in service has been treated uniformly throughout the line.

## DISCUSSION ON "APPLICATION OF THEORY AND PRACTISE TO THE DESIGN OF TRANSMISSION LINE INSULATORS" (GILCHREST), ATLANTIC CITY, N. J., JUNE 26, 1918.

**Charles Fortescue**: The primary facts to be dealt with in the application of electricity, the two main things, are conduction and insulation. These two facts involve other elements which should be termed the electric circuit, the dielectric circuit and the magnetic circuit. The former and the latter have been given a great deal of attention, but not much attention has been given to the dielectric circuit.

There are two reasons for this. One reason is that in the early days the methods of insulation for low voltages were so obvious that there was not much necessity for giving the dielectric circuit much consideration. Latterly, however, as the conduction problem became more difficult it was necessary to increase the voltages used, and, therefore, for that reason the dielectric circuit became of increasingly greater importance. The problems involved in the dielectric circuit are in three dimensions. They are not easy of solution. They involve some of the most difficult mathematics that are known. Therefore, it is essential to find some practical method of dealing with these problems, and some of these methods have been outlined in various reports from time to time. Mr. Farnsworth and myself brought out a method of plotting out the equipotential surfaces by using an electrolyte. Mr. Rice in a paper a short while ago also mentioned some methods, in fact, he took our method and improved upon it very materially.

Mr. Gilchrest has made use of some of these methods and other methods evolved by other people to produce a practical insulator.

In a paper presented to the Institute some years ago I brought forward an apparent paradox, namely, that by the use of conductors the effectiveness of an insulator can be increased. There is real truth in this. To insulate a body or system of bodies, it is necessary not only to consider the insulating body, but we must also consider the shapes of the bodies to be insulated. We must so form these bodies that they may be insulated with the maximum effectiveness.

In the large porcelain insulator problem, we have to insulate a line wire, we will say, from a cross arm, and we have to evolve an efficient form of insulator, and consider what form of conducting surface will be best suited under the various conditions; that is to say, when it is raining, or when the surfaces are dirty, etc., and this brings up another point, and that is the question of what is meant by theory and practise.

The term "theory" is being used rather loosely to cover an approximate solution of a problem, taking into account what are considered the most important factors. Strictly speaking, theory should account for all the facts, as we know them, and as we observe other facts they are to be introduced into the theory. There are many ways of doing this. One way is to make as close an approximation as possible with known theory and then find out how much it is necessary to deviate from this approximation in order to take account of, or care for, other considerations that come from actual practical work.

Now, in this insulator of Mr. Gilchrest's, we have to consider, as I said before, the best form for the insulating body and conductors and we must take into account the fact that certain surfaces attract dirt, or dirt under certain conditions tends to accumulate upon them. The problem is to produce something in which the distribution of stress will be affected as little as possible by changes in the external conditions. The working surfaces, properly speaking, should conform to the flow lines, or the Faraday tubes. There is a very good reason for this. One is, it cuts down the actual intensity at the surfaces of the insulator. You do not have as much ionization. Another reason is, since the intensity is parallel to the surface, it is preventive of the accumulation of dirt on the actual working surfaces. On the other hand, if, instead of conforming to the flow lines, the boundary surfaces are at right angles to the flow lines, even if they get dirty, they will have the effect of equipotential surfaces, just the same as if we extended the internal equipotential surfaces out into the air. As we know, if a conducting surface coinciding with an equipotential surface is not very thick, it has almost no influence on the shape of the Faraday tubes.

There are other considerations, such as the proper proportioning of the capacity of each part of the insulator, etc., the theory of which has been presented in papers before the Institute. These have all been taken into account in the design of this insulator and have been dealt with in this paper.

There is a common belief that there are two ways of looking at these matters, the theoretical way and the practical way. I wish to emphasize the fact that there should not be—, that there should be only one way to consider them, and that without casting any reflections upon practical experience, might be termed the theoretical way. In other words, the theoretical way should account for all that practical experience shows.

There is a school which takes the stand that in designing insulators practical experience alone counts; in other words, we are expected to arrive at an efficient design of an insulator by a process of evolution or the survival of the fittest. I think that such a process of arriving at a design is rather hard on the people who have to bear the brunt of being the innocent victims of the experimentors. In a few words, I think a proper consideration of all the facts involved ought to enable one to arrive at a good design, which shall be theoretically correct, and this means, also, that the insulators should stand up in practise, and I believe that the work which Mr. Gilchrest has done is a big step in this direction.

**Charles F. Scott:** When a new device is described, we do not get the full significance of it unless we get it in historial perspective. An interesting chapter in insulator history was furnished by the International Electrical Congress, held in 1904 in connection with the Exposition at St. Louis. The Transmission Section brought together the transmission engineers from the Pacific Coast and from the Rocky Mountains, as well as from the East. They were the actual, practical operating men, who came together to interchange experiences and to learn what they could.

The foremost problem in several days of papers and discussions was the insulator. Voltages had risen to 50,000 volts, 60,000 volts, and I think 66,000 volts. They had been of that order for a number of years and no advances had been made. Transformers of double ratio had been installed for 88,000 volts and a lower voltage, but the high voltage was not used because no suitable insulator had appeared.

These papers—some of them specifically on insulators and insulator design, and many of them with regard to operating conditions—tell of insulator troubles, and in discussing the future use of higher voltages, the insulator was the one thing which seemed to set the limit. The pin form of insulator was the only one considered. These apprehensions have been justified by subsequent history, as no plants are operating in this country at any substantially higher voltages with pin insulators than there were at that time. There was no suggestion offered, as I recall now, except some modification of a pin-type insulator, which it was proposed to be make larger and larger, until one engineer suggested that it would resemble a Chinese pagoda in size. There was a limit. The solution came, and the method of operating at higher voltages was worked out by leaving that type of insulator and going to something else, the suspension insulator.

Mr. Gerry, of the Missouri River Power Company, read a paper at this Congress, in which he discussed the insulator, and described a number of elementary experiments to show that the electrostatic field has a good deal to do with the character of the discharge on insulator surfaces, and said that these elements must be considered in insulator design. Dr. Perrine, one of the leading engineers of that time, both in practical experience in one of the largest transmission plants in the West, and also because of his well known theoretical knowledge, said that Mr. Gerry had put his finger on the point to which attention must be given in the future, as we had neglected the electrostatic element in insulator design. The Institute TRANSACTIONS of the past fourteen years since the Electrical Congress contain, almost annually, papers descibing the difficulties and troubles with insulators in one way or another, which have been epitomized very well at the beginning of the paper presented today. One must have all these difficulties in mind to appreciate this paper, which runs along so beautifully, showing how theory has been applied to practise and the two together have produced something that looks so simple and obvious, that it almost makes us feel that there is nothing striking about it. A new step has been taken in the evolution of the insulator. There have been scores of different types, many of which follow some one idea, but have carried it to an extreme and violated some other principle.

The paper shows very nicely, as Mr. Fortescue pointed out, that as long as we deal with the electric circuit alone we get along very well until we get up into the region of high voltage, and then the electrostatic field comes in; at first its importance was not recognized, but now it has been recognized, and this new solution results.

This paper indicates how admirably the subject has been handled from the standpoint of research and experimental development.

The great value of the paper, then, comes in this notable step in design which leads to a more simple and practicable and desirable form, in a field in which insulators by the thousand, yes, by the million, are in use, and it becomes, therefore, of the highest importance in adding to the safety and reliability of high-tension transmission.

Another point that has not been brought up, which is one of interest and consequence, is this,—if this insulator is so much better for the field which has been occupied by pin-type insulators on the order of 66,000 volts, may not this new design enable transmission lines of still higher voltage to use the pin type of insulator advantageously?

**V. Karapetoff:** One point that is not brought out in Mr. Gilchrest's paper is the question of refraction of lines of force when they pass from the dielectric of one permittivity $\kappa_1$ into a dielectric having a different value of permittivity, $\kappa_2$. The lines of electric force experience a refraction similar to the refraction of the magnetic lines of force when they pass at an angle from a ferrous material into the air. This refraction is different from that of rays of light, in that the law which holds here is that of tangents and not of sines.

A college girl in her examination paper, which had the question, "State briefly the law of refraction of light." wrote "The sins of the angels are constant." Well, here it is the ratio of the tangents that is constant.

If the angle of incidence is $\theta_1$ and the angle of refraction $\theta_2$, then the law of refraction is

$$\tan \theta_1 / \tan \theta_2 = \kappa_1 / \kappa_2$$

For a proof of this law see for example, V. Karapetoff's "Electric Circuit," p. 163 (second edition.) I mention this fact, because it is not mentioned in the paper, yet the field is being studied there outside the insulator, and not within the insulator, while we are primarily concerned with the dielectric stresses within the insulator.

It should not be assumed that by mentally extending the lines of force as shown by asbestos dust on the outside we can judge about the stresses in the porcelain. This law of refraction should be taken into account. For instance, assume the ratio $\kappa_1/\kappa_2$ to be equal to 5. Then if $\theta_2$ in the air is equal to 45 deg., the angle $\theta_1$ in porcelain comes out equal to 78 deg., so that refraction is of very considerable magnitude. In determining what takes place in the insulator from the field outside the insulator, this law of refraction must by all means be considered, and, therefore, it would be very desirable to evolve a method similar to that outlined in the paper, but in which the medium in which we are experimenting is the body of the insulator itself and not the surrounding air.

I wish to cite in this connection an old experiment in which an electrostatic field is investigated in liquid paraffine, which later is solidified. A very light dust, lycopodium powder which is obtainable in drug stores, is mixed with the paraffine while it is in a molten state. A vessel is filled with liquid paraffine, electrodes of a desired shape are dipped into the paraffine, and an electrostatic field is produced between the electrodes. The individual particles of powder assume the directions of the lines of force. Then the paraffine is allowed to solidify, and it can be cut in any desired plane, and the actual field investigated. This method had the advantage over the method described in the paper in that it is a 3-dimensional method, whereas the method described by Mr. Gilchrest gives results in only one plane at a time.

**L. W. Chubb:** In Mr. Gilchrest's work on the new insulator, some of the practical men made light of the idea of trying to develop an insulator by scientific and theoretical consideratons. Such things as rain, dust, dirt, salt spray, and other things, they said, made the practical considerations paramount.

Mr. Gilchrest has taken the theoretical considerations into account in connection with the practical, and so placed the cemented points and formed the surfaces that the usual disadvantages of the theoretical design are not present.

Prof. Karapetoff's discussion regarding the refraction of the lines of force is very interesting. The refraction was, of course, considered in the design of the insulator. If he takes the limiting values of 90 deg. to zero for $\phi$, it can be shown that the refraction does not play any part.

It was also mentioned that we are interested in the field within the porcelain. The shape of the field does not necessarily change appreciably if the insulator is properly designed.

**S. Barfoed** (communicated after adjournment): In 1914 I attempted to design an insulator making use of the same sources of theory as Mr. Gilchrest mentions, but devoted my attention to the suspension type. It also occurred to me that if the electrical stress in the insulating material was placed at right angles to the stress as usually found in commercial insulators somewhat less attention need be paid to the porosity of the insulating materials. Further, that if cracks were developed running more or less parallel with the lines of electrostatic force, such cracks would be of approximately the same length as the leakage surface path over the insulator and thus not constitute a puncture in the sense it now is taken. Should this reasoning result in the use of more cheaply manufactured insulating materials, the various processes involved might be conducted with greater certainty as to the results. This thought led to some designs of which some samples are herewith given.

Fig. 1 shows a type of insulator where use is made of fibrous impregnated material of such strength that large mechanical forces can be transmitted. It may be hollow or solid, and its surface may be covered in any suitable manner. At either end of this structure are placed metallic shields for uniformly distributing an electrostatic field over the surface of the insulating body. If hollow, the interior space may be filled in with an insulating compound or oil to prevent the accumulation of moisture. It is seen that the electrostatic stress is lengthwise of the insulating body instead of transversely thereto, giving the highest possible puncture value.

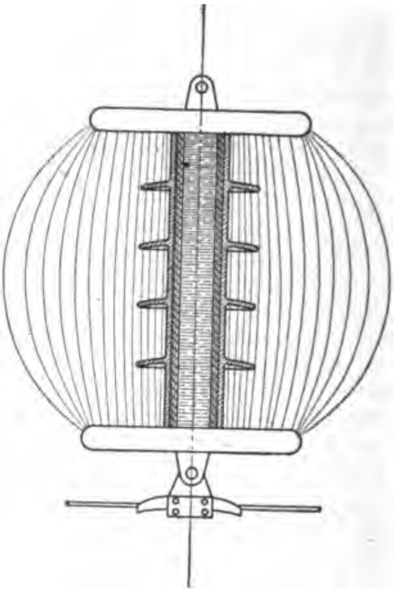

FIG. 1

If this insulator was made sufficiently long no rain sheds need be employed whatever. The outer water-proof covering would merely be a glass or porcelain tube which need not be of the same high class material as now demanded from the insulator manufacturers.

In Fig. 2 the insulating material has been put in compression instead of in tension as in the former case, but here likewise a control of the electrostatic lines of force over the surface of the insulator is had by means of a circular band surrounding the disk shaped insulator body. The electrostatic stress is lengthwise, instead of transverse to the body.

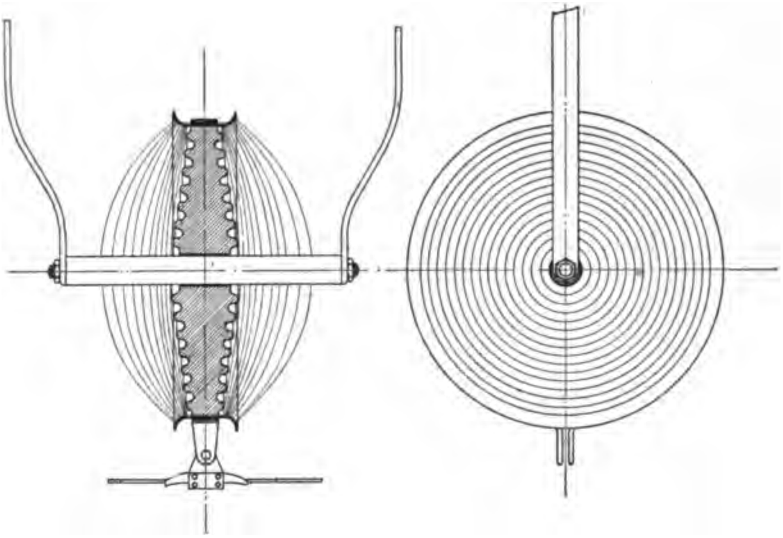

Fig. 2

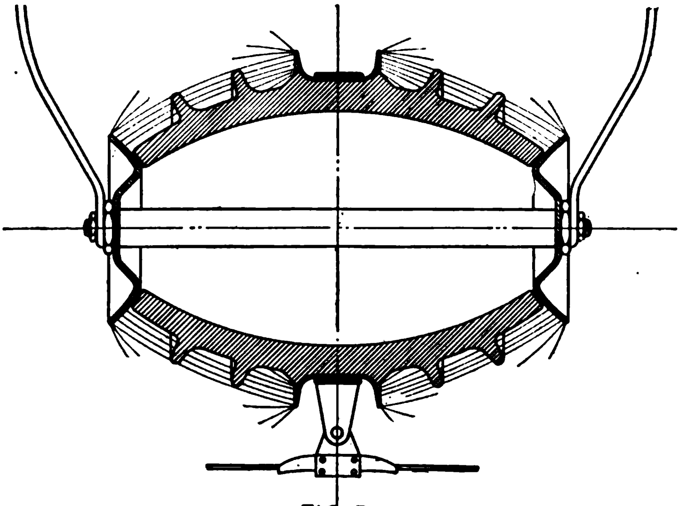

FIG. 3.

In Fig. 3 the same idea is merely carried out in a different manner.

The paper by Mr. Gilchrest is of very great value in showing the lines which must be followed in the future if we are to remove one of the most serious causes preventing commercial progress in high-voltage engineering as applied to the transmission of power.

**G. I. Gilchrest:** In reply to Mr. Scott's inquiry as to the range of voltages, I might say that we have had units made up which we believe are entirely satisfactory for 66,000-volt operation—that is not a catalogue rating, but actual operating voltage, and we have contemplated going to higher voltages possibly 110,000.

I am inclined to take issue with Prof. Karapetoff in regard to the field inside of the insulator being of vital importance. In the operating field the flashover of the unit is the limiting feature of the design, and that is determined by the distribution of the stress over the surface of contact between the air and the insulator. In this design, if the insulating material is made of sufficiently good dielectric, there is no trouble in the inside of the unit.

# INDEX

NOTE: For complete topical and synoptical index see end of Part II

## PAPERS

# INDEX TO AUTHORS