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THE COMMERCIAL LOADING OF TELEPHONE CIRCUITS IN THE BELL SYSTEM

BY BANCROFT GHERARDI

The year 1900 may be considered to have marked the beginning of a very important period in the development of telephony. For some time prior to that date it had been known to those who gave study to the matter that the transmission efficiency of long telephone circuits would be improved by increasing the uniformly distributed inductance of the circuit. This knowledge, however, did not lead to any direct commercial results for the reason that no one was able to point out any practical method of increasing the distributed inductance of a telephone circuit without bringing in difficulties of one kind or another which were fatal.

This apparently insuperable difficulty in obtaining improved results by the use of distributed inductance directed the attention of the mathematicians and physicists to the question of whether the results sought for might not be attained by the use of inductance coils placed at intervals in the circuit instead of by undertaking to distribute the inductance uniformly along the circuit. Vaschy, Heaviside and others either suggested or unsuccessfully tested the insertion of self-induction in coils on actual lines as a means of increasing their transmission efficiency. No practical results followed from their work and no actual progress was made in the matter of using lumped inductance to improve the transmission efficiency of telephone circuits until the year 1900, when the patents of Professor M. I. Pupin, dealing with the reducing of attenuation of electrical waves, were issued. Professor Pupin,[1] and Dr. George A. Campbell,[2] who had also worked on this problem, have published several papers dealing with it. These authors discuss fully the mathematical theory of improving the transmission efficiency of telephone circuits by means of placing thereon inductance coils at intervals. The papers referred to establish the fact that inductance in coils spaced suitably along a telephone circuit will produce the same effects as uniformly distributed inductance and show us how to suitably locate these inductance coils. The matter of suitably spacing the inductance coils is of vital importance and must be kept in mind because the spacing of the inductance coils is the key to the use of lumped inductance and it is from the failure to establish this fact and give due weight to it that the earlier workers failed.

I shall not undertake here to go over the theoretical ground which has already been so well covered in the papers referred to, but before describing some of the practical applications of loading to open wire lines and cables, and discussing the results which have been obtained and mentioning some of the difficulties which have been overcome, I will state the results of the fundamental work on this question for those who do not care to follow through the mathematical papers on this subject.

In the study of any telephone line, with reference to its efficiency as a means of transmitting speech, two important factors must be considered. These we ordinarily characterize as " volume " and " quality."

Volume refers to the loudness of the sound which may be obtained from the telephone receiver at the distant end of the line, and the efficiency of the line as regards this factor is determined by the ratio of the amount of energy received at the distant end of the line to the amount of energy put into the line at the transmitting end.

Quality refers to the clearness of the speech which may be obtained from the current at the receiving end of the line. The efficiency of the line as regards quality is determined by the degree that the shape of the current wave at the receiving end of the line approximates to that of the current wave impressed upon

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[1] *Transactions of American Mathematical Society*, page 259, July, 1900; TRANSACTIONS A. I. E. E., 1900, XVII, p. 445; TRANSACTIONS A. I. E. E., 1899, XVI, p. 93; *Electrical World and Engineer*, October 12, 1901, page 587.

[2] *Philosophical Magazine*, March, 1903.

the line at the sending end. A line which would transmit all the frequencies concerned in the telephonic current with equal attenuations of all these frequencies and with the same velocity would transmit without distortion, and its quality might be considered to be perfect.

For the present I will confine my attention to the matter of the volume of speech transmitted. The formula for the attenuation of a telephone circuit, excluding the effects of terminal conditions which react on the portions of the line near the ends, is

$$\alpha=\sqrt{\tfrac{1}{2}\sqrt{(R^2+p^2L^2)\,(S^2+p^2C^2)}+\tfrac{1}{2}\,(S\,R-p^2\,L\,C)} \qquad (1)$$

in which the symbols have the following significance:

α = attenuation constant per unit length.
R = resistance " " "
L = inductance " " "
C = capacity " " "
S = conductance " " "
$p = 2\,\pi$ (frequency).

This attenuation constant is a measure of the efficiency of the line, for on any line if the amount of current impressed on it at the sending end is known, and the attenuation constant is known, the amount of current at any point along the line may be determined by the formula

$$i=I\,e^{-m\alpha} \qquad (2)$$

in which i = the current at any point;
I = the initial current, and
m = the distance from the initial point of the line to the point at which the current (i) is to be ascertained.

Equation (1) may be simplified somewhat by making

$$S=0,$$

as this item S, representing leakage and dielectric dissipation, is always made as small as is commercially practicable and is, in most cases, sufficiently small to be negligible in its effect upon the general result obtained. With this simplification, equation (1) becomes

$$\alpha=\sqrt{\frac{p\,C}{2}\,(\sqrt{R^2+p^2\,L^2}-p\,L)} \qquad (3)$$

An examination of this equation shows that the attenuation constant will be reduced as the inductance of the circuit, L, is increased. I have already mentioned the fact that no satisfactory commercial method has been suggested or devised for uniformly increasing the inductance. The result of the work published about 1900 shows us, however, that we can obtain the same favorable effects by placing suitably spaced lumped inductance in the circuit as would be obtained from uniformly distributed inductance without the fatal difficulties encountered in endeavoring to place any considerable amount of distributed inductance. This work shows us that if more than π inductance coils per wave length are placed upon a telephone circuit the effect of these coils will be approximately the same as the effect of an equal amount of inductance uniformly distributed along the circuit. If less than π coils per wave length are placed the results are unfavorable. As the number of coils per wave length is increased above π the effect very rapidly approaches the effect produced by uniformly distributed inductance so that if as many as six coils per wave length are placed on the circuit the effect of the inductance thus placed is within 4 per cent of that of uniformly distributed inductance. The wave length in a telephone circuit is easily computed by the following formula:

$$\frac{2\pi}{\sqrt{\frac{1}{2}\sqrt{(R^2+p^2L^2)(S^2+p^2C^2)}-\frac{1}{2}(SR-p^2LC)}} \qquad (4)$$

in which the symbols have the same meaning as in (1) above. To apply this formula to a circuit which it is proposed to load, it is necessary of course to take account of the reaction of the inductance which is to be added to the circuit in the form of loading coils, because this inductance affects the wave length. Therefore, the inductance value in the formula given above must include the proposed loading. In order that all frequencies which are of importance in connection with the transmission of articulate speech shall be cared for by the proposed loading, the frequency employed in this formula must be the highest frequency which takes a necessary part in the clear transmission of speech. It is not found that it is necessary that this spacing of the loading coils should be absolutely uniform. It is important, however, that the spacing should not depart substantially from uniformity.

Without undertaking here to give the equations dealing with the effects of loading on the quality or clearness of speech, it may be stated that it is found both mathematically and experimentally that loading makes the attenuation more nearly equal as between the different frequencies which are important in connection with speech transmission and that loading, therefore, increases the clearness of such speech and improves the quality of the line, particularly on underground cable circuits where, on account of the relatively large amount of capacity in relation to the other characteristics of the circuit, the quality without loading is noticeably inferior on long circuits.

Broadly speaking, it may be stated that what we have done in applying loading to long telephone lines is to adopt the necessary measures to enable us to obtain the advantages of a high potential transmission system, that is, of transmitting the electrical energy from the sending end to the receiving end of the line with a high voltage and a small current as compared with the smaller voltage and higher current which is found on unloaded lines. That this is the result which we are actually accomplishing by loading our lines is shown not only from the study of the equations dealing mathematically with the propagation of telephone currents over loaded and unloaded lines, but it is also further established by actual studies and tests on working lines. We have, therefore, broadly speaking, taken the same step which electric light and power people have taken to improve the efficiency of their long lines. On account, however, of the very different electrical characteristics of telephone lines and currents and high tension electric light and power lines and currents the means which it has been necessary for us to adopt to accomplish this result have been very different from those adopted by the electric light and power engineers.

In the case of power which is to be transmitted at relatively low frequencies, usually 66 cycles a second or less, over lines of relatively short length, say, cables 10 miles long and open wire circuits 100 miles long, it is sufficient in order to transmit the energy with a high voltage and a small current to arrange the receiving end of the line by means of a transformer or otherwise so as to give a great reactance, and to place high potential apparatus at the sending end of the line. Electrically the line is so short that the reactance at the receiving end may be considered to be working directly against the high potential generating apparatus at the sending end and hence the desired result is obtained. With the telephone current, however, which is of high frequencies, and telephone lines which are often very long, the situation is entirely different. Increased reactance placed at the receiving end of the line does not make itself substantially felt at all at the sending end of the line. To obtain the proper reactance for high potential sending apparatus to work against, it is necessary, therefore, in the case of telephone lines to distribute the reactance along the line.

The electrical characteristics of open wire lines and of telephone cables are substantially different from each other. The following table gives the constants of two typical telephone cables and of a typical open wire line:

Type of circuit	Cable	Cable	Open wire
Loop resistance, ohms per mile	88	28	10.5
Mutual capacity, microfarads per mile	0.07	0.07	0.008
Inductance, henrys per mile	0.001	0.001	0.004
Insulation resistance, megohms per mile	500+	500+	1 to 100

The essential differences between the characteristics of these circuits, from the point of view of loading, will be seen by examining the figures in the above table. In the case of underground cable, the capacity is relatively high and the inductance, on the other hand, is relatively small. On overhead lines, however, the capacity is relatively small and the inductance of the circuit, even before it is loaded, is not negligible. The insulation of cables is constant while that of aerial lines varies with the weather conditions. On account of these differences in the electrical characteristics of these two types of circuit the design of the loading coils used on them, the proper spacing of the loading coils, and the results obtained from loading are somewhat different in the two cases. The principles employed are the same in both cases, but the different characteristics of the lines bring about in the loading different dimensions both electrical and mechanical.

The problem of loading underground cables is as a whole easier than that of loading open wire lines and will, therefore, first be discussed.

Loading Telephone Cables

At the present time we are making very general use of two styles of loading for telephone cables, one of which is characterized as " medium " loading and the other " heavy " loading.

In a few cases a third style of loading, known as " light " loading, is employed. These three styles of loading differ in the amount of inductance placed upon the lines and hence in the improvement in transmission obtained from the loading. The heavy loading, as the term would suggest, places the greatest amount of inductance on the line and requires coils placed at closer intervals. This heavy loading employs about 0.2 henry per mile and the spacing between coils is about 1¼ miles; medium loading provides 0.1 henry per mile with the coil spacing of about 1¾ miles; light loading employs 0.05 henry per mile with a spacing of about 2½ miles. The coils for these various loadings are made up on similar cores so that the coils differ only in reference to the windings placed thereon. A description of the coil employed for medium loading and the method of applying it to the lines is as follows:

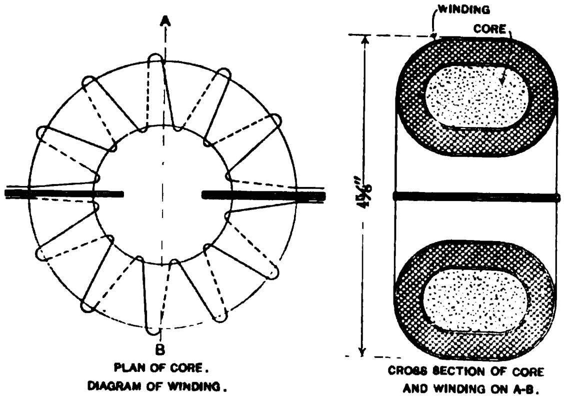


FIG. 1.—Cable loading coil

The coil is of the toroidal type, that is, it has an iron core like a ring. Fig. 1 shows a cable coil. The core is made up of very fine iron wires. Two copper windings are placed on the core, one to go on each side of the line. These windings are also indicated on Fig. 2 which shows the way the coils are connected in to the line. The windings magnetize the core in the same direction, that is, the mutual induction is added to the self-induction. Over these windings there is placed a layer of tape for mechanical protection and the whole coil is then thoroughly dried and immersed in a compound to keep out moisture.

Before leaving the design of the coil it is interesting to consider the precautions which are taken to reduce losses in it. This is a very important matter as, while the inductance added to the circuit by the coil produces an improvement in the transmission efficiency of the circuit, any loss of energy in the coil means, of course, a reduction in the efficiency of the circuit, and with poorly designed coils it would be easily possible to bring in losses so large that they would over-balance all the gain from the loading and leave the circuit poorer than before. These losses must be particularly guarded against because they are cumulative.

The first loss to be considered is a dissipation loss resulting from the resistance of the copper winding of the loading coil itself. This loss cannot be entirely eliminated, but it may be reduced as much as may be desired by increasing the cross-section of the winding. There are, however, practical limits to this, as the increase in the size of the winding increases all the mechanical dimensions of the coil and also tends to increase certain other losses.

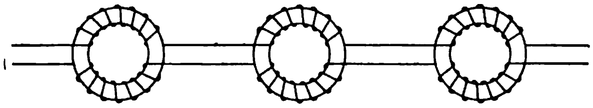


FIG. 2.—Diagram of loading coil windings and method of connecting into a line

A second possible loss in loading coils may also develop in the copper winding. If the cross-section of the conductor used for winding the coil is large, eddy currents may form within the conductor itself and these would produce substantial losses.

Practically this factor is small in cable loading coils, but in open wire coils where a conductor of greater cross-section must be employed it has been found necessary to strand this conductor and to insulate the various strands from each other.

In the iron core of the loading coil there are also several possible losses. In the first place, there is the hysteresis loss. After a great deal of work it has been found practicable to obtain special grades of iron which when worked within certain limits of magnetization are substantially free from hysteresis losses. This at once imposes a limitation on the design of the loading coil and affects the proportioning of all of its parts.

Another loss which may take place in the core of a loading coil is that due to eddy or Foucault currents which may develop in the material of the core itself. In the case of loading coils it has been necessary to construct the cores of extremely fine wire and to coat this wire with a film of insulating material.

Certain other losses must also be guarded against. If a loading coil when magnetized gave an external magnetic field and there were any conducting materials in this field, eddy currents would be produced in this conducting field and would represent a loss of energy through the circuit. Similarly, if there were any iron or other magnetic substances in this field there would be likely to be hysteresis losses in this iron. These losses have been guarded against by adopting the toroidal form of coil already described and placing the windings on it in such a manner that practically no external magnetic field is in any case produced. The designing of loading coils in this manner has not only eliminated external losses but makes it possible to place loading coils in close proximity to each other without introducing cross talk between the circuits involved. It is interesting to consider that in so far as the losses in loading coils are concerned they very closely resemble the losses which must be guarded against in the construction of, for example, dynamo armatures in electric light and

FIG. 3.—Underground man-hole containing loading coils

power practice. The resemblance, however, stops at this point and the experiences of electric light and power people in connection with these losses have not been of great assistance in the design of loading coils because the magnitude of current, its frequency and other characteristics have been so different from those involved in electric light and power practice. The great importance of reducing losses, which from the nature of loading are cumulative in character, has made the problem very different.

The direct current resistance of the medium loading coil is about four ohms. The alternating current resistance of this coil, at a frequency of about 800 periods a second, is about nine ohms. This relatively small increase of resistance at such a high frequency is a good indication of the skillful design and careful manufacture of these coils.

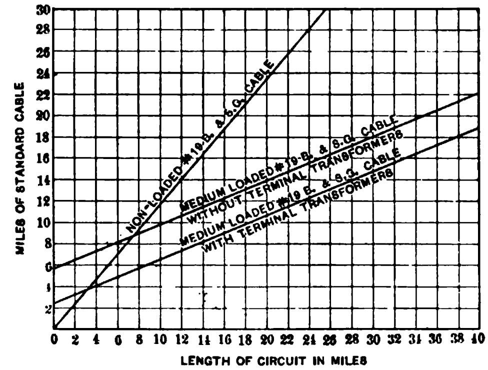


FIG. 4.—Efficiency of No. 19 B. & S. gauge cable unloaded and medium loaded, with substation apparatus located directly at the ends of the circuits

The method of placing loading coils in cable circuits is illustrated by Fig. 3, which shows the photograph of a manhole containing loading coil cases. The cases which you see in this photograph each contain 49 loading coils. These are arranged in the case on seven spindles, there being one spindle in the middle of the case and six spindles arranged around it. The case is filled with insulating compound and special precautions are taken between the flanges to exclude water. Extending from the case to the main cable is a stub cable containing the wires leading into and out of the loading coils. This is spliced into the main cable so as to loop a loading coil winding into each side of each circuit which it is desired to load. In some cases where it is desired to load aerial cables, cases similar to those shown in Fig. 3 are buried at the base of the pole and the stub containing the wires run up the pole to connect with the aerial cable.

The results obtained by applying medium loading to No. 19 B. & S. gauge underground cable, having a capacity of about 0.070 microfarads per mile, are shown in Fig. 4. In this figure is also shown the transmission efficiency of the unloaded cable. All of these transmission efficiencies are measured in terms of what we call standard No. 19 gauge cable. This standard 19 gauge cable is cable having a resistance of 88 ohms per loop mile and a mutual capacity of 0.054 microfarads per mile. Cables as now manufactured have in general higher capacities than this, but when the earlier transmission tests were made cable having a mutual capacity of 0.054 was in common use and was therefore adopted as the standard and it has been convenient to retain it as such. Wherever in this paper, unless otherwise stated, the efficiency of a circuit is spoken of as being equivalent to so many miles of standard cable or of so many miles of cable, the cable referred to as the unit of measurement is 19 gauge standard cable as defined above. It is for this reason that the efficiency of the unloaded cable in Fig. 4 is not quite equal to that of the standard cable used as the unit of measurement.

The efficiencies of the loaded circuits, shown in Fig. 4, are those obtained with substation instruments located directly at the ends of the loaded circuits. One curve is given showing the results obtained by the use of terminal transformers and the other without the use of such devices. These curves bring out clearly a peculiarity of loaded circuits which should be considered at this point. The loading of a circuit, either underground or aerial, brings in at the ends of the loaded circuit whether the circuit is connected directly to substation instruments or to unloaded circuits an effect which we have called reflection losses. That is to say, at the ends of the loaded circuit there are certain losses which take place which are substantially independent of the length of the loaded circuit involved. These losses as between loaded and unloaded circuits are dependent upon the relative electrical characteristics of the loaded and unloaded circuits. In the case of substation instruments, connected directly to the ends of a loaded circuit, the terminal losses are dependent upon the relative electrical characteristics of the substation instrument and of the loaded line. Terminal or reflection losses, of course, also exist between unloaded lines of different characteristics, for instance, as between unloaded underground cable and unloaded aerial lines. They would also be found between substation instruments and unloaded lines of various kinds. In all these cases, however, they are small in magnitude and it is only in the case of loaded lines that they assume such proportions that they must be given consideration in connection with general engineering.

I have been speaking of the terminal losses observed in connection with loading as though they pertained particularly to the loaded line. It has been convenient to so speak of them but, of course, they actually pertain to the combination of loaded lines with unloaded lines or standard substation equipment, and properly are no more associated with one than with the other but rather result from a combination of the two. As, however, the substation instrument and the unloaded lines were in service for a considerable time before the use of loaded lines, it has been convenient to associate these reflection losses with the loaded lines and consider the electrical efficiencies of the unloaded lines and the substation instruments to remain unchanged.

Various devices have been successfully employed to reduce terminal losses, although no plan has been worked out which would completely eliminate them. The plan used most successfully to-day for eliminating terminal losses is a device known as a terminal transformer. This is a transformer or repeating coil which differs from the standard telephone repeating coil in that there are more turns on one winding than on the other. It is used in the same way as an ordinary transformer in electric light practice, the high potential side being connected with the loaded line and the low potential side with the unloaded line. For ordinary conditions of loading the ratio of the two windings is usually either 1 to 1.4 or 1 to 2. The effect of the terminal transformer in reducing terminal losses is well shown in Fig. 4. The terminal loss for the two ends of a medium loaded line without terminal transformers is about 5.75 miles of standard cable. With terminal transformers this is reduced to 2.5 miles. These figures are for cases where the telephone substation apparatus is located directly at the ends of the loaded line. Where there is a length of unloaded line between the loaded line and the substation apparatus it is found that terminal losses are not so great as if the terminal apparatus were directly at the ends of the loaded line. The extent to which the varying amounts of unloaded line reduce terminal losses is shown by the following figures:

Amount of 19 gauge unloaded cable at each end of loaded circuit	Amount of terminal losses at both ends of a loaded circuit—medium loaded	
	Without terminal transformers	With terminal transformers
None	5.75	2.5
1 mile	4.0	2.0
2 miles	3.0	1.50
3 miles	2.2	1.15
4 miles	1.6	1.0
5 miles	1.25	0.9

While terminal losses are substantially independent of the length of the loaded line, they are affected by the character of the loading of the line. The heavier the loading the greater the reflection losses. While with medium loading the reflection losses, as shown by the above table, with no unloaded cable at the ends of the circuit, are 5.75 miles without terminal transformers and 2.5 miles with terminal transformers, the corresponding figures for heavy loading are 7.8 miles and 3.5 miles. These figures are, of course, reduced as in the case of medium loading, by having unloaded cable at the ends of the loaded circuit.

If it were not for these terminal losses, and neglecting leakage, it would be practicable, theoretically at any rate, to indefinitely increase the efficiency of a 19 gauge circuit by adding more and more inductance to it. Even without terminal losses, however, this could not practically be done, because more and more loading coils would have to be added as more inductance was added, and these would in turn bring in larger and larger losses until finally the further gains from adding inductance would be offset or more than offset by coil losses. The increasing effect of leakage as the weight of loading is increased also has a limiting effect. Aside from these effects, terminal losses, which increase as the amount of inductance added to the circuit is increased, place a limitation beyond which it is not advantageous to increase the amount of loading on the circuits. This is illustrated by Fig. 5, which gives the efficiency of medium and heavily loaded 19 gauge conductors and on which, for the purpose of comparison, the efficiencies of various gauges of unloaded circuits are also given. It will be seen from this that by taking into account the reflection losses which are here given at about an average for various conditions, the medium loaded circuit is more efficient than the heavily loaded circuit in lengths up to four miles. Beyond that point the heavily loaded circuit is more efficient, although even at 12 miles the difference is not largely in favor of the heavy loading.

It is interesting to consider from this diagram what results have been obtained from loading 19 gauge conductors. For a length of circuit equal to 4.5 miles the 19 gauge conductor with medium loading is as good as a 16 gauge conductor unloaded; that is to say, loading has made the circuit containing half the copper as good as the larger circuit without loading. For a length of 11 miles a loaded 19 is as good as unloaded 13 weighing four times as much as the 19 gauge conductor.

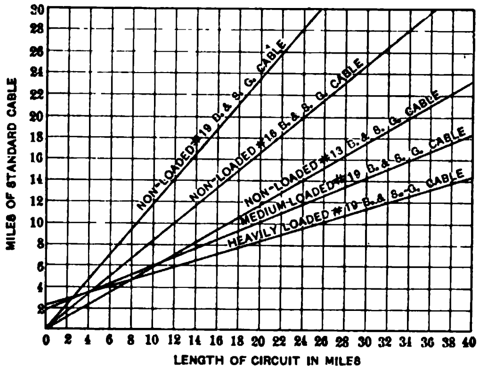


FIG. 5.—Efficiency of No. 19 B. & S. gauge cable loaded, (medium and heavy), with terminal transformers and one mile unloaded loop at each end; also efficiency of unloaded cables of various gauges

For underground circuits, requiring a greater transmission efficiency than can be obtained by the use of heavily loaded 19 gauge cable circuits, it has been found best, all things considered, to employ a larger gauge of wire rather than to load more heavily. Many cables containing 16, 14 and 13 B. & S. gauge conductors have been laid and loaded.

The most interesting existing cable containing large gauge conductors is the New York-Philadelphia cable which was laid several years ago between these two points in order to provide additional circuits and to insure the service against storm damage. Most of the circuits in this cable are of 14 B. & S. gauge and they are heavily loaded. That is, there is 0.2 henry per mile added to the cable circuits by means of loading coils. The results obtained on these circuits are given in Fig. 6, assuming average terminal conditions. The results obtained by this loading are most satisfactory. The distance between New York and Philadelphia, following the route of the cable, is 90 miles and this distance is indicated on the diagram. It will be seen that for this distance the circuits have an efficiency of 13 miles of standard cable. If unloaded, these circuits would have an efficiency of about 60 miles of standard cable. Over such circuits no commercial service could be given between New York and Philadelphia. Without loading, even 10 gauge conductors would have given unsatisfactory results, so that by means of loading we have been able to obtain results between New York and Philadelphia over 14 gauge conductors that could not have been obtained with conductors of much greater weight without loading.

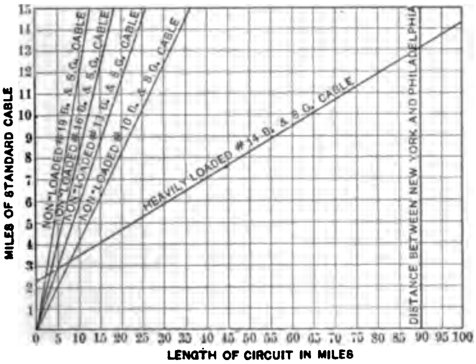


Fig. 6.—Efficiency of No. 14 B. & S. gauge cable with heavy loading, (New York-Philadelphia cable); terminal losses with terminal transformers and one mile loop; also unloaded cables of various gauges

While this New York-Philadelphia cable represents the longest loaded underground cable to-day in use anywhere in the world, a project has been approved and is now under way which will far surpass it. This project is for an underground cable extending between New York and Washington in one direction and New York and Boston in the other. The largest conductors in this cable will be No. 10 B. & S. gauge and by means of loading we confidently expect that we will be able to obtain thoroughly satisfactory results between New York and either Boston or Washington, and that it will even be possible to obtain service through this cable between Washington and Boston should storm damage to the open wire lines render this necessary. The loading of this cable will be of particular interest as it will represent the first large application of the phantoming of loaded cable circuits and the loading of phantom cable circuits. In order to accomplish these results it is necessary to adopt special arrangements of the windings of the loading coils. The windings on the loading coil used for loading the side circuits are arranged as shown in Fig. 7. With this arrangement of winding, the effect of the telephone current in the side circuit, which current flows in opposite directions in the two sides of the circuit, is to magnetize the core of the loading coil in the same direction throughout and to produce no consequent poles. The loading coil thus acts, in as far as the side circuit is concerned, exactly like the loading coil with the simple winding shown in Figs. 1 and 2. In building up a phantom circuit this side circuit, however, becomes one side of the phantom, and the telephone current in the phantom circuit traverses the two wires of the side circuit in parallel. Such a current flowing through the windings of such loading coils as are shown in Figs. 1 and 2 would magnetize the two sides of the core in opposite directions and produce consequent poles at the two sides of the core. This would not only bring in

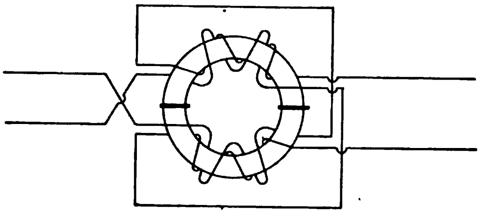
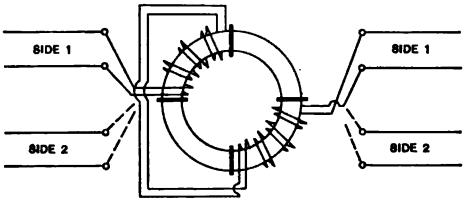


FIG. 7.—Diagram of winding of loading coil for circuits forming the sides of phantom



NOTE:-TO AVOID CONFUSION THE WINDINGS FOR BUT ONE SIDE OF THE PHANTOM ARE SHOWN.

FIG. 8.—Diagram of winding of loading coil for use on phantom circuits

serious losses, but also crosstalk which would be very difficult to deal with. The current in the phantom circuit flowing through a coil with windings on it as shown in Fig. 7 does not magnetize the loading coil at all, as each winding is neutralized by a corresponding winding on the other side of the line. Such coils may be used to effectively load the side circuits of a phantom combination and in as far as the phantom itself is concerned they act only as resistance which is very small in amount. This, however, does not load the phantom circuit, which of course is a desirable result to attain. To load the phantom circuit a coil with windings as shown in Fig. 8 is employed. It will be seen by tracing out the winding of this coil that to the current flowing in the phantom circuit it acts like an ordinary loading coil, that is, all windings magnetize the core in the same direction. To currents flowing in either of the side circuits, however, it acts only as dead resistance. The arrangements described above in connection with the application of loading coils to phantom circuits and to the side circuits of phantom circuits have been successfully applied to open wire lines as well as to underground cables.

Loading Open Wire Lines

Gratifying as have been the results obtained from the use of loading on underground cables, its application to open wire circuits has also resulted in most noteworthy improvements in their efficiency. The general plan of loading open wire circuits is similar to that used on underground cable wires. On account of the differences in the electrical characteristics of open wire circuits, as compared with cable circuits, however, the design and spacing of the loading coils is materially different from the coils and the spacings used on underground cables. In general, only heavy loading is used on open wire lines on account of the greater length of these lines. The coils themselves are substantially larger than those employed on cable circuits for two reasons—in the first place the coil is subject to damage by lightning. In order to minimize this difficulty the coils must be built to have high breakdown strengths against lightning discharges. All aerial coils are built so that they will stand a breakdown test of 8,000 volts. This, of course, considerably increases the size of the coil. The coil is further increased in size by the fact that, as an aerial circuit is initially a good deal more efficient than an underground circuit, it is necessary that the coils used in loading it should have smaller losses than are permissible in underground cable loading coils. The outside diameter of the aerial loading coil is about 10 inches and the resistance of the two windings of the coil to direct current is 2.5 ohms while to alternating currents of a frequency of 800 periods a second it is 6.5 ohms. In addition to building the coil so that it has a breakdown strength of 8,000 volts, each coil is protected by a pair of lightning arresters adjusted to operate at 3,500 volts. These precautions have proved sufficient to reduce trouble from lightning on aerial loading coils to a negligible amount.

Low insulation which is sometimes encountered on open wire lines, particularly in periods of bad weather, proved at the start a serious difficulty in connection with the loading of open wire lines. This, however, has been overcome by removing the worst causes of low insulation and in some cases by the use of improved insulators, and at the present time low insulation is not proving a serious factor in connection with the application of loading to overhead lines. In wet weather the transmission efficiency of overhead lines is not quite as good as in dry weather, but it may be stated that it is very seldom that the loaded circuits fall in efficiency sufficiently to seriously affect the transmission, except in cases where the lines are in actual trouble from branches of trees falling into the line or other similar causes.

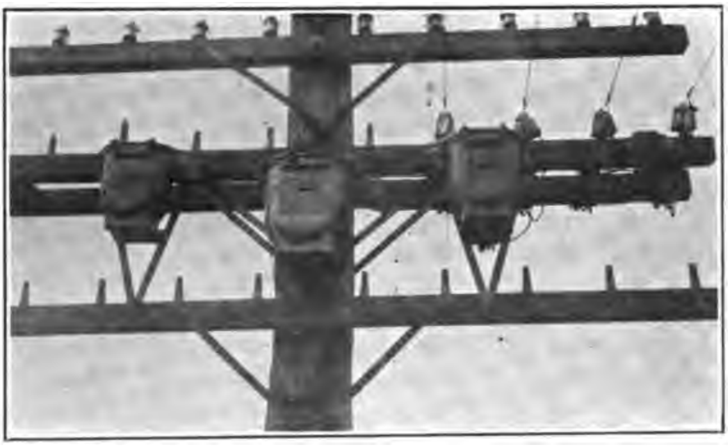


FIG. 9.—Loading coils placed on an open wire line

In connection with the loading of open wire lines the problem of arranging them so that they will be free from substantial crosstalk has required attention. Due to the fact that much of the crosstalk disturbance in telephony is from electrostatic induction it is to be expected and it is found in practice that the loading of a circuit increases its tendency to crosstalk. In the early applications of loading to open wire lines some trouble was experienced from crosstalk and special transposition systems were devised to overcome these difficulties and it was further found necessary to locate the loading coils with special reference to the transpositions. On underground cables it has been found that loading also increases the crosstalk somewhat, and in these we arrange to reduce the capacity unbalances in order to reduce crosstalk to a sufficiently low point.

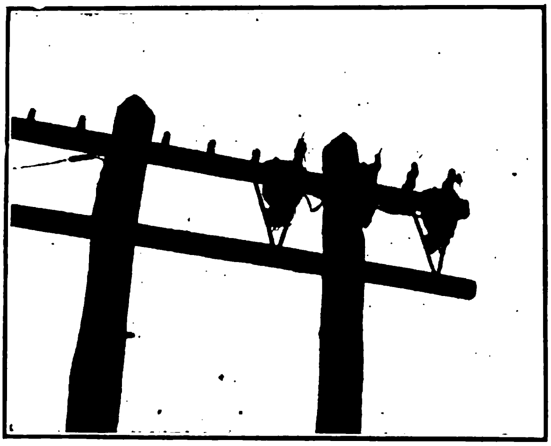


FIG. 10.—Loading coils placed on an open wire line on "*H*" Fixtures

The loading coils for use on open wire circuits are placed in individual cast iron cases and these are placed on the poles or crossarms as in Figs. 9 and 10 which show illustrations of two common arrangements. The line is carried to the loading coil by means of bridle wire. The lightning arrester is placed in a separate case and bridle wire is also extended to it.

The results obtained by the use of loading coils on the No. 8 B. W. and No. 12 N. B. S. gauge wires, which are the standard sizes used in the long distance plant of the Bell system, are shown in Fig. 11, in which for the purpose of comparison the efficiencies of the unloaded circuits are also given. This diagram shows terminal losses with the conditions which may be considered as average. The general reflection loss problem with overhead lines is the same as with underground cables, and having been discussed in detail in connection with such circuits it need not be repeated here. As will be seen from Fig. 11 the results obtained from loading open wire lines have been to more than double their transmission efficiency when used in considerable lengths.

One of the very valuable results which has followed from the loading of open wire lines has been that it has made a very substantial extension to the distance over which long distance service can be given. Up to the time that loading was successfully applied to No. 8 B. W. G. wires, about 1,000 miles represented the greatest distances over which commercial service was maintained. By the loading of No. 8 circuits this has just been extended to 2,000 miles, so that to-day it is practicable to obtain a successful telephone conversation from New York to Denver. The actual distance between these two points by the route which the line follows is 2,010 miles.

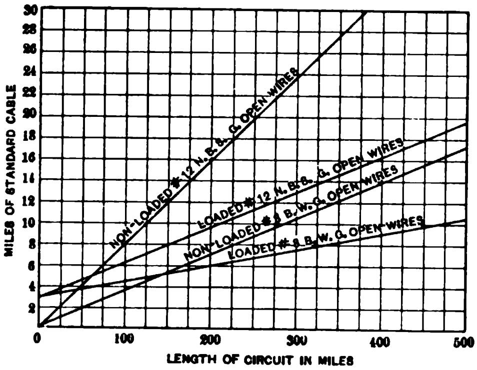


FIG. 11.—Efficiency of No. 8 B. W. G. and No. 12 N. B. S. G. open wire lines loaded and unloaded with average terminal conditions

A very important use has been made of loaded cables in bringing loaded open wire lines into large cities. Loaded cables have also been used to bring unloaded open wire lines into thickly settled communities, but in this case the gains have not been so great because it is not practicable to place terminal transformers for reducing the reflection losses between the outer end of the underground cable circuits and the aerial lines, and hence reflection losses are experienced.

This practical impossibility of placing reflection reducing devices between open wire and cable has also made it impossible to obtain the full benefits from loading in those cases where short stretches of cable are placed in the middle of toll lines—*e.g.*, in communities where the appearance of the open wires is objected to and requests are made to place them underground. If the open wires are loaded the trouble is not so great as a comparatively heavy loading can be used without excessive reflection losses. For non-loaded open wires, the loading of the cable if attempted at all must be very light if the reflection losses are not to offset any gain from the reduced attenuation.

For these reasons and also because cable circuits loaded or non-loaded are inherently less efficient than open wire circuits it is important that every effort be made to keep toll lines as free as possible from cable.

LOADING OF SUBMARINE CABLES

The problem of loading submarine cables is a mechanical one rather than one concerning the principles of loading. In this country a 16 B. & S. G. dry core paper submarine cable about 5 miles long and containing loading coils spaced about 2 miles apart has been successfully laid across the Chesapeake Bay and is now in operation. There has also been talk of laying a dry core loaded submarine cable across Puget Sound for a distance of 16 miles. The situation in the United States, however, is not such that large numbers of loaded submarine cables of any great length are required. Abroad, however, particularly between England and the continent and between England and Ireland submarine cables of considerable length are needed for telephone purposes and in these situations loaded gutta-percha submarine cables have been employed with successful results.

EXTENT OF APPLICATION OF LOADING

It is evident that a device making such great improvements in the transmission efficiency of telephone circuits as does loading would have a large place in the plant of any company providing a comprehensive system of telephone service over a large area, a condition necessarily requiring long cables and long open wire lines. Although the loading art was born in 1900, and its successful commercial application on an extensive scale to our plant dates only from about 1903, there are already in the United States over 85,000 miles of loaded open wire circuits and over

170,000 miles of loaded underground cable circuits. To load these circuits there have been employed about 125,000 loading coils. The day-by-day experience with this enormous loaded line plant has fully demonstrated that we are in practice obtaining the successful results which the engineering data that I have set forth would lead us to expect.

The facts which have been herewith developed have, I think, been more than sufficient to justify the statement at the beginning of this paper, that the year 1900, in which the loading patents were issued to Professor Pupin, marked the beginning of an important era in telephone work. I do not know of any problem in electrical science and electrical engineering which has required greater skill, technical ability and diligence on the part of those engaged in its solution—inventors, engineers, manufacturers and construction people—than has the working out and the application of loading, and it is a pleasure to be able to present to you a statement of the results which they have accomplished.

DISCUSSION ON "COMMERCIAL LOADING OF TELEPHONE CIRCUITS IN THE BELL SYSTEM." CHICAGO, JUNE 28, 1911

E. H. Bangs: One of the most valuable features of Mr. Gherardi's paper is its frank admissions in regard to the practical limitations of circuit loading. In the past there have been some extravagant claims made for the possibilities of this practice, and this paper, in its description of the actual results obtained in the best practice is most welcome.

Mr. Gherardi answers, in a way, a question that has been put to me on several occasions by engineers experienced in power transmission as to why it was that the telephone engineers did not long ago take advantage of the common knowledge of the power transmission engineers, on the subject of added inductance. In asking this question the power engineers say, in effect, " We gave you the idea. All that you had to do was to work out the details." This statement contains its own answer, an answer which is illustrated by Mr. Gherardi's paper. As a matter of fact, it has taken from 10 to 15 years of very earnest effort on the part of a number of people to simply " work out the details." Telephone transmission is only power transmission in its most minute form, and of course the same laws apply to both methods of conveying energy. The difficulties encountered have not been found so much in applying known laws, as in perfecting practical details, and the ingenuity displayed in simply that portion of the problem relating to the loading of phantom circuits and the phantoming of loaded circuits is of a high order. I think that the development of this whole problem has proceeded with dispatch, along logical lines, and abreast of, or well in advance of the economical demand for its solution.

F. B. Jewett: While there is not much that I can add to what Mr. Gherardi has said on the subject of commercial loading of telephone circuits in the plant of the Bell system, there are one or two points which I think may be of interest to those who are not familiar with loading. One of these is with regard to the mathematical development which forms the foundation upon which our present loading practice has been built. All of our experience has tended to confirm this mathematical theory and so far as I am aware it is entirely rigorous. Although the first results on actual loaded circuits seemed not to agree with theoretical expectation, we soon learned that discrepancies which existed resulted from a lack of reliable data as to line constants at high frequencies and not to any imperfection in the mathematical theory of loading. With increasing knowledge as to the high frequency constants of different classes of circuits, the discrepancies between the computed results and the results of actual transmission tests are each day becoming less, and the theoretical forecasts more amply verified.

Mr. Gherardi shows that the general equation for the attenuation constant, which involves the four factors, series resistance, series inductance, shunt capacity and shunt conductance or leakage can be greatly simplified by the elimination of the last of these constants. In the early stages of our work it was assumed that on both cable circuits and open wire lines having high insulation, this could be done. Later work showed us, however, that the direct current insulation was not necessarily a criterion for the high frequency shunt conductance even in the case of open wire circuits where dissipation of energy in the dielectric would, at first sight, seem to be almost negligible.

High frequency measurements during the last few years on both open wires and cable circuits have given us values of the shunt conductance which, when substituted in the general formula, tend to bring the computed and measured attenuation constants into close agreement. I have every reason to believe, therefore, that when we are able to determine the four line constants at high frequencies with the accuracy with which we can now determine direct current resistance, capacity and insulation, we will find that the formulæ used in our computation work are rigorously in accord with the results of actual observations on loaded lines.

The other point which I wish to mention briefly has to do with the enormous change which the introduction of loaded circuits has brought about in the terminal apparatus and outside plant, both in the original installation and in the matter of maintenance. The deleterious effect of low line insulation is so much more pronounced on loaded circuits than on non-loaded circuits that the introduction of loading on a large scale has necessitated a very much higher grade of line maintenance than was ever found necessary in the case of non-loaded circuits even of large gauge. Also in the case of any extensive loading, especially on cable circuits, the necessity of placing a large number of loading coils in the plant has made it absolutely essential that the loading applied to the various circuits be such as to give a resultant plant of maximum simplicity. It was early seen that any general attempt to engineer the loading for each particular line so as to give maximum efficiency to this line would result in a plant that was unmaintainable on account of its complexity. The result has been that for both open wires and cables a system of standard loadings has been developed and the loading of any particular circuit is that standard which most nearly conforms to the theoretical best loading.

In the case of cable loading, especially on congested runs where a great number of circuits are to be loaded, it is very desirable that the loading coils for all circuits having a given type of loading be placed at the same points. This factor has reacted on the art of cable manufacture to the extent that all cable for loading is now manufactured to meet a standard capacity requirement irrespective of gauge or the number of conductors in the sheath —the standard capacity being that which will result in the lowest annual charge on the loaded circuits of given efficiency.

Also the fact that the sending end impedance of loaded lines is very much larger than for non-loaded lines and varies greatly with the type of loading, with resultant high and variable reflection losses, has meant that we cannot connect loaded lines together and to non-loaded lines in the indiscriminate fashion permissible with non-loaded circuits. We have, therefore, been forced to design our loading so that the circuits of various character, whether loaded or non-loaded, may be connected together or to the terminal apparatus with the minimum amount of reflection loss. Further the necessity for having maximum circuit flexibility in a large and growing plant has made it imperative that all loading be planned with the view to future growth and the necessity for avoiding special operating practices as much as possible.

Although a great step in advance has been made since the first introduction of loading on commercial circuits, there is still a large amount of development work to be done before we have succeeded in deriving all of the benefits possible from the use of inductance coils on telephone circuits.

E. H. Colpitts: I wish to call attention to one or two features connected with loading; that is, the necessary engineering studies and scientific investigation in connection with the application of loading are making telephony, from the standpoint of line transmission, an exact art.

Ten years ago when the rule for loading was first given us by Professor Pupin, the only method of determining whether a certain piece of apparatus introduced into the telephone line would cause a transmission loss or a transmission gain, was to actually make a transmission talking test over the circuit. When the art of loading was introduced it was recognized that much more refined methods of measuring the electrical constants of telephone apparatus and telephone lines under the electrical and magnetic conditions of telephone service must be developed. It was seen that actual telephone currents could not be employed, but that it would be necessary to employ alternating currents of telephonic frequencies and amplitudes. At that time no high frequency generators had been designed which would satisfactorily deliver the necessary high frequency currents. Therefore, one of the first problems the telephone engineer had to undertake was to design high frequency generators capable of delivering alternating currents at frequencies of from 200 to 3,000 periods per second. In order to employ this current in the methods which seemed most practical and desirable it was necessary that the open circuit electromotive force of these generators should be as nearly as possible free from harmonics. Such generators were designed.

Among the problems of a physical nature which confronted the telephone engineer it was necessary to devise methods of measuring accurately the losses in iron and its various alloys when subjected to magnetizing forces as small as 0.005 c.g.s.

units and when fluctuating corresponding to alternating currents at frequencies of from 200 to 3,000 periods per second. The telephone engineer today is able to measure such losses with a high degree of accuracy. The engineer had to devise methods of measuring losses in dielectrics at the telephonic frequencies which I have mentioned and at the low potentials which are involved in actual telephone transmission.

In addition to this, methods of measuring very accurately the inductance and effective resistance of loading coils were developed. The measurement of inductance accurately in terms of inductance standards is not at all difficult, but the measurement of effective resistance with a small current and at a high frequency requires most carefully designed apparatus and a complete knowledge of all the factors involved, many of which factors it would seem at first sight to the ordinary engineer could be neglected.

In connection with loaded lines themselves, as Dr. Jewett has already indicated, we soon found that we had to take account of facts which had hitherto been neglected. As an instance, it became evident that we had to measure accurately and consider the effect of bridged impedances on the efficiency of loaded circuits; whereas, in case of unloaded circuits the effects of such bridges could very generally be neglected. Also, in the design of cables which are to be loaded it is very important that electrical symmetry or balance be preserved, and the engineer is interested in capacities and differences of capacities as small as one-millionth of a microfarad in lengths of 400 or 500 ft. of cable. He can today measure and take account of these small differences in capacities between the wires of one pair and the adjacent pairs.

So far I have referred merely to measurements upon the elements used in loading. A large amount of work has been done in connection with the development of methods of making measurements upon telephone circuits, in order to determine the various terms involved in their transmission efficiency. I mention these merely as illustrations of the general nature of the development work necessary to the present condition of the loading art.

E. B. Craft: It may be of interest to note some features in the construction and design of the loading coils themselves. The art of constructing inductance coils is an old one. The use of inductance coils in loading, where we must deal with currents of very high frequency, however, involves a consideration of details and methods of reducing losses to a point which I doubt is reached in ordinary practice.

As an example we may consider a coil such as is being used in loading the new New York-Washington underground cable circuits. The core of this coil consists of a toroid built up of approximately 90,000 turns of No. 38 gauge iron wire. To reduce losses each of these turns must be insulated from the other. This in itself was a problem of considerable magnitude and it eventually led to the development and use of an insulating enamel coating having a thickness of less than 0.0005 in.

In working out the design of the coil windings we also had to consider means for reducing losses, that led to departures from ordinarily used methods. In order that the copper losses might be reduced to a permissible figure, the conductor was subdivided, using a number of strands of enamel insulated wire, which, in turn, were insulated in the usual manner to provide a stranded copper conductor. The winding itself was then subdivided into sections to properly control capacity balances between adjacent sections of the windings.

In the construction of the coil such as we are using in the New York-Washington cable there is required a total length of 2½ miles of copper wire wound around a core composed of 30 miles of iron wire. The mechanical problems involved in the application of these windings were quite difficult. Toroidal coils have been used in the past and were usually wound up by hand, involving a great deal of time and labor. In order that coils of this type might be turned out in large quantities on a commercial basis, it was necessary to develop special machinery by which these windings could be applied automatically. This has resulted in a considerable reduction in cost and increase in efficiency.

After the cores are prepared and the windings applied, it is necessary also to give consideration to the proper control of the losses in the dielectrics. This has led to the application of refined methods of manufacture in the way of vacuum drying and impregnation.

Consideration of the amount of material used in their construction will indicate that, when completed, these loading coils are of no mean proportions. As they are applied to the line or cable they are usually grouped in pots or cases similar to transformer cases, and the larger cases containing 21 coils weigh in the neighborhood of 2,000 lb. and because of size and weight are usually located in the manhole of the subway.

Various speakers have dwelt upon the necessity for adhering to accurate mathematical methods in calculating the electrical constants of the completed circuits, and the development of the design of the coils themselves has reached the stage where we can, by the application of proper methods of calculation, predict with certainty the various electrical characteristics of the completed coil. When we consider that these electrical characteristics involve not only the ohmic resistance and inductance of the coil, but such factors as capacity unbalance and alternating current conductance as well, it can be seen that it has been necessary to employ methods that have not heretofore been commonly used in the design and construction of inductance coils for various commercial uses.

Allard Smith: It might be of interest to get a brief idea of how we have applied loading in conjunction with the A. T. & T.

Co. engineers to a large telephone system like that in Chicago. We have developed a comprehensive loading plan for the entire city, and have advanced the work under that plan as rapidly as conditions would permit. Briefly, only 16 and 19 gauge cables in Chicago are loaded, although there is one cable which carries 14 gauge loaded conductors between Chicago and Milwaukee.

Radiating out from the Chicago Telephone Co's. toll building, in the center of Chicago, there are six loading paths, and these paths bring into the center of Chicago the trunk and toll lines from the outside and outlying exchanges. All of the work is underground. We have no aerial loading because the lines going out into the country are not long enough.

There are distributed around Chicago about 10,000 loading coils, which load about 19,000 pair miles of wire. These coils are incased in sealed iron pots and are placed in manholes, spaced one mile and three-quarters apart, except the coils on the Chicago Milwaukee cable, which are placed one mile and a quarter apart. These cables run out north as far as Waukegan, with 16 gauge conductors, and with 19 gauge conductors to Evanston—about 12 miles. Waukegan is about 25 miles.

Another cable runs south to Hammond, about 25 miles, with 16 gauge conductors and 19-gauge conductors as far as South Chicago, about 16 miles. Also one directly west, going to Maywood, with 16 guage conductors; another southwest, along the C. B. & Q. R.R. to La Grange; another directly south to West Pullman. All of these cables bring the 19 gauge pairs from the outlying Chicago exchanges within a belt line, say, four to five miles out from the city, into the toll board. The 16 gauge conductors bring in the exchanges still further out called the "neighborhood exchanges" and the immediate suburban exchanges. Now we are just starting another loading path, which will be a belt line, continuing in a radius between exchanges about five miles from the center of the city.

J. G. Wray: I would supplement what Mr. Smith has said by stating that in Chicago if our trunk lines were not equipped with loading coils, it would be impossible to carry on commercial conversations between offices on opposite sides of the city. If loading coils were not available and there were no satisfactory substitute, it would be necessary to provide over-head open wire construction for trunk lines between such offices. Such construction would involve high pole lines with many cross arms, would congest the streets and alleys and make them unsightly and would add greatly to the cost of construction and maintenance and would provide a less satisfactory service than is provided at present. As a matter of fact, it would be physically practically impossible to provide pole lines enough to carry the open wire construction which would be necessary.

The problem of local telephone transmission in a big city like Chicago or New York would be exceedingly difficult if it were not for the loading coil. I believe that this fact is not generally realized by engineers who have considered the advantages of the loading coil usually from the standpoint of long line transmission such as the New York-Denver circuit which Mr. Gherardi has just described.

The problem of telephone transmission in a big city, I think, however, has hardly received the attention that it deserves.

Frank F. Fowle: One of the very earliest practical tests of Dr. Pupin's system of inductance loading was made on a 670-mile circuit of No. 12 N. B. S. G. (173-lb.) open wire (hard drawn copper), in 1900. The coils then employed had an ohmic resistance of 9 ohms, an inductance of 0.225 henry and a shunt capacity of 0.0042 microfarad. These coils had no iron and were enclosed in wooden boxes, filled or impregnated with an insulating compound. They were spaced 2½ miles apart. Comparative transmission tests conducted during clear dry weather showed that this loaded circuit was substantially equal to a non-loaded circuit of No. 8 B. W. G. (435 lb.) open copper wire.

The successful result of this trial gave immediate impetus to the development of better loading coils, with the result that the original type was abandoned in favor of the coil described by Mr. Gherardi. In coils of the type now used extreme precautions are necessary to keep the effective resistance at a minimum and not in excess of the resistance stated in the paper.

It is interesting to note that the toroidal coil finally developed is identical in its form with Faraday's original experimental transformer, which consisted of a simple iron ring with two windings, one on either half of the ring.

Quite extensive loading was soon undertaken on commercial circuits, but two prominent difficulties arose. The first was the effect of lightning, which necessitated the development of suitable protection. The second arose from low insulation. It was quite generally assumed at first that the leakage conductance, S, could be neglected, as Mr. Gherardi points out. But it developed that this was not the case. This fact was not clearly demonstrated until several thousand miles of line had been loaded and the commercial operation had shown clearly that during extremes of wet weather the insulation of open lines was inadequate. The reasons for this are clear both mathematically and physically.

This trouble affected the No. 8 circuits most, because they had lower resistance and impedance, non-loaded, than No. 12 circuits and were therefore more efficient initially. Being the more efficient circuit to start with, the gain due to loading was relatively less and the injurious effect of leakage was felt sooner.

Substantial improvements in line insulators did not follow for several years and then glass was abandoned in favor of glazed porcelain.

Experiments were also carried on in relation to the effect of different spacings, varying from two miles to eight miles. It was found that there was little or no gain by short spacing and heavy loading, because the gain secured by increased inductance was practically nullified by the increased apparent resistance of the coils. This will be seen by comparing the apparent resistance of the coils with the resistance per mile of No. 8 conductors.

It was first pointed out by the writer that the proper location of coils with respect to cross-talk disturbances was every eight miles, at the junction of 8-mile transposition sections, or every *S* pole in Barrett's 1898 standard transposition system. This fact, taken in conjunction with the advantages of light loading in respect both to the effects of leakage, and the cost of installation, led to the adoption of the 8-mile spacing.

The increase in cross-talk as a result of loading is the direct consequence, in a large measure, of the increased transmission efficiency, which increases the volume of cross-talk as well as direct talk. At the same time the relative proportions of electrostatic and electromagnetic induction are disturbed and the former is increased. This necessitates increased attention to the electrical balance of loaded metallic circuits, both in open wires and cables. Barrett's earlier system of transposition, known as the A B C system, has certain untransposed exposures and in some cases will permit considerable cross-talk between non-loaded circuits, so that its use is not to be recommended. The standard system of 1898 contains no untransposed exposures, and in general is much superior to the early system.

The use of loaded circuits undoubtedly calls for a high standard of maintenance, and increases the maintenance expense in comparison with non-loaded circuits. While this slightly diminishes the net commercial gain secured by loading, it presents no particular difficulties and its consequences need not be feared.

There is a lower limit of line length which it pays to load, just as in the case of cables. Therefore loaded circuits cannot economically be cut up into short-haul lines, for use part of the time. Equally true, loaded circuits should not be connected to intermediate or way stations, both for the reason just given and further because bridged impedances are of harmful effect on loaded circuits in general.

Some difficulty was experienced with the coil shown in Fig. 1, of the paper, from magnetization caused apparently by telegraph currents when a loaded circuit was composited. Morse currents in either side of the coil magnetized it at once, of course, and in some cases set up a permanent flux which impaired the properties of the coil telephonically. The dividing current of the simplex system was much less harmful, tending only to pole the core at opposite ends of the diameter dividing the windings. The coil shown in Fig. 7 prevents any magnetization by the simplex, through its balanced arrangement of windings. There is probably some advantage in using a polar duplex on either side of the composite, because of the very frequent reversals in the direction of current.

Bancroft Gherardi: Professor Shepardson has asked a question concerning the connecting of loaded cable circuits with open wire circuits. As I understand the question it refers to what special arrangements, if any, are needed when loaded cable circuits are to be connected to open wire lines. Is that the question, Professor Shepardson?

G. D. Shepardson: The question in my mind is whether you use the transformer or use the tapered loading to minimize the reflection.

Bancroft Gherardi: In general we do not do either. There are practical difficulties in the placing of terminal transformers at the outer end of underground cable circuits; there are also a large number of practical difficulties brought in by undertaking to taper off the loading of a circuit. One of the greatest of these difficulties in connection with tapering off the loading would be the extensive changes and re-arrangements which would become necessary from time to time as it became advisable to increase the length of the loaded cable involved, or otherwise change the layout of circuits. Suppose you had a cable that ran out five miles and later was going to be extended two miles more. In case the loading on this cable was tapered there would be a variation in the inductance of the loading, the spacing, or both. When, however, the cable was to be extended, substantial changes would have to be made in the loading coils and their arrangement. In general, we aim as far as possible, to load open wire circuits and cables so that the sending end impedance on the different classes of circuits will be the same, and so that the spacing of the loading coils on the two types of circuits are relatively the same electrically, that is to say, the relative spacings bear a definite relation to the relative electrical characteristics of the two circuits. If these conditions are attained there are no reflection losses where loaded underground cable circuits are connected with loaded open wire circuits. In practice these conditions are so nearly attained that the reflection losses in the case mentioned above are negligibly small.

The discussion of this paper has taken so much the form of a statement of additional facts in regard to loading and so little question has been raised in regard to any of the points brought out in the paper that I do not see that I can add anything in closing except to thank those who have participated in the discussion for bringing out many points of interest in connection with the subject which for the sake of keeping the paper down to a reasonable length were either omitted from the paper or only touched upon briefly.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 28, 1911.

PROBLEMS IN TELEPHONE TRAFFIC ENGINEERING

BY F. P. VALENTINE

It is the purpose of this paper to deal with a few of the many operating problems that confront a telephone company, serving a territory made up of several hundred communities of varying types ranging from a large Metropolitan area to sparsely settled rural districts.

A telephone company has but one line of goods to sell, *i.e.*, service. The entire capital investment in telephone plant and all running expenses are for the purpose of handling the telephone traffic of its subscribers or customers, and from this service is derived the revenue of the company.

Under the prevailing form of telephone operating organization, the responsibility for handling the telephone traffic devolves upon the Traffic Department, which usually represents about one-half the employees and payroll expense of a telephone company.

In carrying out its duties, the traffic department has a twofold responsibility; first, to the subscribers for the quality of the service that it renders to them; second, to the stockholders and subscribers for the economy with which it renders such service.

The quality of the service is directly dependent upon the effective application of suitable methods of operation. The effective application of suitable methods also bears directly upon the economy, for only by the use of such methods is it possible to obtain the maximum efficiency of force and plant. Further, both the quality and economy of service bear directly upon the satisfaction of the telephone user or customer. The quality should be such as to meet every reasonable requirement; the economy such as to enable the company to supply service to its customers at lowest possible rates compatible with a reasonable return on the investment, in order to encourage increase in subscriber development, thus constantly enhancing the value of the service to the individual subscriber.

To meet these requirements it is essential that a traffic department should have, in addition to a proper administrative force, an efficient traffic engineering staff. As the work of this staff is what might be termed the production engineering of the telephone business, it is obvious that it requires a qualitative and quantitative analysis of all factors in any way related to the production of telephone service.

The principal factors entering into the production of telephone service, which must be the subject of engineering study, are as follows:

1. Quality of service.
2. Efficiency of labor.
3. Efficiency of operating methods.
4. Production efficiency of central office equipment.
5. Production efficiency of trunk and toll circuits.

The following discussion of certain phases of traffic engineering problems, in the order above indicated, is based upon some of the results of studies made in the territory of the New England Telephone & Telegraph Company, as it is with the work in this territory that the writer is most familiar. This engineering work, however, is a part of and is carried on in conjunction with that of the engineering staff of the American Telephone & Telegraph Company, for the purpose of making comprehensive fundamental studies.

QUALITY OF SERVICE

From the standpoint of the telephone user, the essential qualities of telephone service are accuracy, speed of connection, and uniformity in both speed and methods of handling calls. These requirements are equally essential from the standpoint of the telephone company. Accuracy and speed of connection mean a minimum of waste effort, motion and time. Uniformity in speed and in the method of handling calls trains the telephone user to coöperate with the operator to the same end, with the resultant effect of satisfaction to the telephone user in smooth, speedy service, and for the company a maximum production efficiency by the operating force and an economical use of switchboard equipment.

As the result of constant study there has been developed a systematic method of service inspection which is now practically standardized among the larger operating companies. By means of suitable apparatus, observations are made on every step in the handling of a telephone call. By the use of stop watches the time intervals of each step in the operation are accurately observed and a code record made, sufficiently comprehensive to permit the analysis of the tabulated results of these observations. A sufficient number of calls is observed during the hours of regular traffic to insure the results being representative of the general grade of service being given in the office under observation. This service observation is practically continuous in all large exchange districts.

Equally careful analysis is made of the phraseology used by the operator in dealing with the subscriber. This latter is no small factor in accurate and speedy service. A telephone operator has to handle calls from subscribers of all types and characteristics and under all conditions. Every word and phrase used by the operator in dealing with a subscriber is the result of long and careful study and is designed to convey essential and significant information or direction in briefest possible form.

Another record essential to the analysis of service quality is what is termed the "record of service criticisms." This is especially valuable as it furnishes a record of the subscribers' opinions of the quality of service. This is a record not only of all formal complaints which are received, but also of data collected by all supervising employees in the central offices who have occasion to deal with any subscriber in connection with an abnormal telephone call. The record is made by noting in code, on a proper form, any comments received indicating the fact and nature of any unsatisfactory service that the subscriber has had, either then or at some previous time. Every one of these cases is followed up and before the record is filed for summarizing, the reasons for the unsatisfactory condition are entered thereon. Besides affording an opportunity to adjust matters to the satisfaction of the subscriber, the analysis of these criticisms furnishes valuable data as to the most frequent causes of unfavorable opinion of service quality, and affords the opportunity of eliminating or minimizing these causes. It is of the utmost importance, not only as good business policy, but also to assist in the analysis of service quality, that every effort be made to obtain fair criticisms from the viewpoint of the customer.

Continuous study and analysis of the results of service observations together with the data obtained from the record of criticisms, has made possible the adoption of standard operating methods designed to give satisfactory service. All calls of abnormal type involving special handling, are transferred to special operators whose sole duty is to care for this class of traffic. This results in removing the most disturbing elements from the work of the " A " or subscribers' operators, and in providing special attention for such calls on the part of employees especially trained for that work.

The adoption of standard methods of operation and the setting of standards of speed and accuracy, has made it possible to reduce all operating procedure to standard sets of instructions for the guidance of all employees concerned. The tabulated results of service observations furnish the administrative force with the information as to how nearly the standards are attained, while the instructions furnish information as to how to attain them.

EFFICIENCY OF LABOR

In determining the efficiency of labor, or the production efficiency of an operating force, it is necessary that standards of labor production be established, and that provision be made for significant reports or summaries which will indicate how nearly these standards are approached. It is not sufficient merely to compare results in various exchanges or in the same exchange at different times on the basis of the traffic handled regardless of the conditions attendant upon its handling. Such a comparison assumes to set as a standard the operation in some particular case. It assumes that at some other place or time an operation was performed with greater efficiency, and, therefore, it should be performed here or now at the same efficiency regardless of whether it is exactly the same proposition in both places or at both times.

The operation of a telephone switchboard comprehends the handling of many different kinds of traffic, and a separate standard is required for the handling of each. To express the degree of efficiency attained as a per cent of these various standards would not only be an interminable task but would give so many individual results that their study and co-relation would be impracticable.

It is necessary, therefore, that a method be employed which in a single percentage figure will show the degree of efficiency attained. This result is accomplished in the following way:

All calls handled are expressed as though they had been of a single class. This is done by multiplying the number of calls of each class by a factor representing the ratio of the labor of handling the calls of a particular class to the labor of handling calls of the class selected as the unit class.

The standard number of these unit calls which should be handled being determined, the per cent of efficiency is readily obtained by comparing the results obtained with those specified as a standard. From the foregoing, it will be seen that this method involves not only the selection of some unit, but the determination of the number of unit calls and the number of calls of other types that may be handled by average skilled operators within a given period, and the establishment of coefficients representing the ratio of the loads of each type of call to that of the unit class.

A flat rate non-trunked call handled on a No. 1 relay board was selected as the unit call for two reasons. It is a very simple type of call and one which predominates in number in most offices. Furthermore, it is of such a class of call that the number an operator can handle depends primarily upon her ability, and not upon circuit conditions.

The determination of the number of unit calls and the number of calls of other classes that operators could handle, involved a most exhaustive study of operating loads. This study consumed a period of over two years during which many thousands of observations were made covering all types of calls, normal and abnormal, from all classes of service. As a result of this study, it was found that an average skilled operator, working in a team of five or more operators, could carry 230 unit calls per hour and maintain that rate during the entire working day, without undue physical or nervous strain, at the same time giving a standard quality of service. Furthermore it was found that, for a single hour in each half day period an operator could, without material impairment of the service, handle 25 per cent more than this number. Even this load could be exceeded at times of unusually sharp peaks.

The ability of the force to carry the peaks at loads in excess of the average busy hour loads, is an important factor in determining operating force requirements. Fig. 1 shows in percentages the actual distribution of traffic in two offices for the day hours. The curve marked A is that of an office of the best type of residential subscribers, while the curve marked B is that of an exclusively business office. The operators at *A* can carry the busy hour traffic at loads exceeding those of the average busy hours, while the operators at *B* will carry the standard load. With an equal amount of daily traffic in both offices, the same number of operators will handle the busy hour traffic of *A* as are required to handle the busy hour traffic of *B*.

In the evolution of this system of equating calls covering two years of exhaustive study and some four years of practical application, its value and accuracy have been thoroughly tested. Its value lies not only in the fact that it furnishes a means of measuring the efficiency of labor, and is essential in measuring the efficiency of some of the other four factors mentioned in opening this discussion, but in the fact that it has opened the way to a most exhaustive analytical study of the human element upon which telephone operation is so dependent.

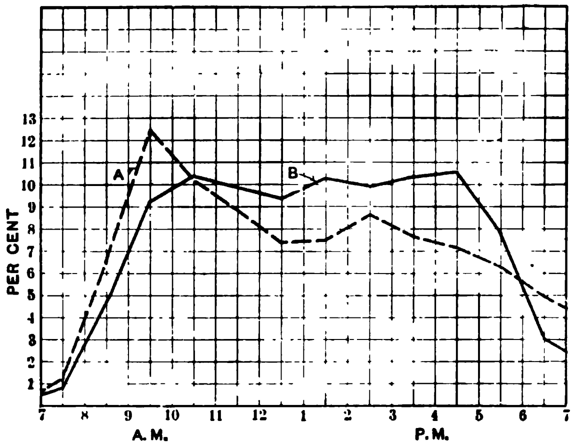


Fig. 1

As a result of the establishment of standards of service quality and labor efficiency, it has been possible to institute methods of selecting and training an operating force with a degree of success otherwise impossible. The question of fitness is of primary importance and the matter of intelligent selection of operators is the result of many years of careful study of the qualities essential to success. All accepted candidates for operating positions are required to spend one month in the operators' training school, listening to lectures in regard to the handling of all types of calls, with the necessary study and examination hours, and in practical operating work under capable instructors on a regular working switchboard representing a typical central office, the calls being originated by instructors. No operator is allowed to work at a central office and handle the calls of telephone users until she has graduated from the school course and met a certain standard of requirement. With this training she has a thorough knowledge of how all classes of calls should be handled and only needs experience to settle that knowledge and acquire speed. This same course of training in modified form, is carried on in the smaller central offices where it is not economical to maintain a regular operators' school.

On an average it takes about a year for an operator to attain maximum efficiency. In view of the fact that, as in any large body of employees, there are constantly resignations and occasionally dismissals for cause, together with the steady growth in subscribers and traffic, it is obvious that the problem of maintaining at all times an adequate and suitably trained operating force is a very important one. Even with the most constant care in selection incompetency will develop in a certain percentage of the cases either during the school training course, or later, in the work in the central offices. On account of the reasons enumerated, it has been found that in order to produce a force of one year's average experience it has been necessary to start and partially train, or train and keep in service for a short period only, at least three for every one who stayed a year. As it costs over $200,000 per year to train central office operators in the territory of the New England Telephone and Telegraph Company, it is obvious that this is a problem well worthy of careful study.

In order to meet this problem, all conditions of employment and work have received most critical analysis. The operating loads have been most carefully determined, having in mind the physical well-being of the operating force. Working conditions have been made as pleasant and comfortable as possible. In addition to a liberal lunch hour, a fifteen minute relief period is insisted upon in the midst of each half day period. Commodious and tastefully furnished rest rooms are provided, magazines and periodicals supplied, and everything possible done to make the work attractive and free from excessive nervous strain. Vacations, with pay, are allowed once during each calendar year. To further the careful study of working conditions and of the human element as it enters into the problem, for over two years the

New England Company has employed a physician of highest standing to make a special study of all phases of these conditions.

A careful record is kept of each employee from the time she enters the service of the company to the time she leaves. This record shows the character of the employee's work, comments of her superiors, results of service observations, and every detail that has a bearing on her efficiency. Such a record is essential in the determination of proper wage increases, and the record of reasons for resignation or dismissal is of especial value in the study of ways and means of maintaining a stable operating force.

This carefully studied system of selection, training and caring for employees, with adequate compensation for efficient work, based solely on merit, has had good results, as is indicated by the fact that during the past four years, it has been possible to increase the average length of employment of the operating forces by 20 per cent, while there is always a waiting list from which to select candidates. It would seem obvious, therefore, that as a large factor in the efficiency of labor the study of the development and maintenance of a proper operating force is essential.

EFFICIENCY OF OPERATING METHODS

The efficiency of operating methods is in their adaptability to produce the required quality of service with the minimum of effort, labor and operating time per call, together with an economical use of switchboards, trunk and toll circuits.

The methods of handling local calls have been fairly uniform for a number of years, the greatest improvements having been made in specializing the operation for irregular calls, or those which cannot be completed at once by the *A* or subscriber's operator. Another step toward the application of efficient operating methods, brought about by the careful study of recent years, is the gradual elimination or diminution of abnormal types of service, such as heavily loaded party lines in congested centers, etc., which tend to degrade the quality and economy of service.

In the field of toll service, except in a few notable instances, progress was not as great until comparatively recently. Toll service must be considered in two classes—"short-haul," or suburban toll, and "long-haul," or "long distance" toll. Within a comparatively short radius of most large telephonic centers are usually found a number of smaller communities closely related to the large center and to each other in a business and social way. The telephone traffic between such offices is of the class first mentioned and usually constitutes a very substantial portion of the toll business, the traffic to more distant points, covered by the second class mentioned, being in the minority.

Toll operating methods have been the subject of most critical analysis for the past few years, resulting in the standardization of several different types of toll operation, each designed to be applied under given conditions.

The subject of operating methods is so fundamental to efficient operation in all its phases that it has received and is receiving very comprehensive study. Every suggestion must receive most careful and critical consideration in order to detect any inherent weakness and to guard against the introduction of a method which, while apparently harmless and even efficient under existing conditions, might fail to meet the requirements of a constantly increasing traffic.

So far as concerns the production efficiency of various operating methods, the use of the unit system, hereinbefore outlined, provided a means of determining the relative efficiency of the different methods. The effect of any change from one method to another can be predetermined by the application of the proper coefficients to the traffic involved.

PRODUCTION EFFICIENCY OF CENTRAL OFFICE EQUIPMENT

The amount of central office equipment necessary is directly dependent upon the number of equated or unit calls to be handled during the average busy hours, or, in other words, upon the number of employees for whom it is necessary to provide switchboard positions during the average busy hours. With service of standard quality, labor production at standard efficiency, and efficient operating methods, by use of the unit system applied to the amount of traffic to be handled the switchboard requirements can be accurately determined. The discussion of operators' loads and Fig. 1 in the preceding section apply also to the determination of amount of switchboard.

In engineering central office equipment it is customary to engineer on a basis of a fifteen to twenty year ultimate capacity and to install initial equipment sufficient to cover from two and one-half to three years' growth. As it is necessary to know the ultimate requirements for buildings, etc., these are developed through special studies or "fundamental plans" which include the consideration of all factors that have a bearing on the future development. In order to determine the probable growth in subscribers, it is necessary to maintain curves of growth for past years for each class of service sold. These curves are projected to the ultimate period, and corrected periodically from actual growth. With the curves of station growth is maintained a curve of the station calling rate in each class of service. From these records, supplemented by the yearly estimates of growth, can be determined the number of stations, and volume and type of traffic to be expected at any future period. By application of the equating process to the various classes of traffic and obtaining the number of unit calls to be handled, it is possible to forecast accurately the amount of switchboard required.

The production efficiency of central office equipment is determined by the output per position, or per dollar of investment. The unit system furnishes means of determining the relative efficiency of different types of switchboard for handling particular classes of service under given conditions. Given the desired standard quality of service, the production capacity of a given switchboard unit of any type can be predetermined, and actual results attained can be checked, by the proper use of the unit system. The relative economy of different types of boards must be balanced against other costs incidental to the case under consideration.

PRODUCTION EFFICIENCY OF TRUNK AND TOLL CIRCUITS

The production efficiency of trunk and toll circuits is affected by the quality of service which it is desired to produce, by the efficiency of operating labor, and by the efficiency of operating methods, and to a certain degree, by the efficiency of central office equipment. It is also affected by the volume of traffic to be handled and the contract conditions under which the service is sold.

Analysis of this subject has demonstrated that while operating and equipment costs vary directly with the number of unit calls, circuit costs vary indirectly with the number of calls, due to the varying efficiency of circuits in groups of different sizes, and more or less directly with the length of haul under a given amount of traffic, except so far as qualified by variation in type of plant necessary to meet transmission requirements.

The production capacity of a circuit under any method of operation is determined by the average circuit holding time per call and the number of calls to be handled over the particular group of circuits in a given time.

For proper consideration of the subject of circuits, distinction will be made between trunk and toll circuits. Trunk circuits may be considered as local circuits connecting offices covering an area of too great telephonic density to be served economically from one central office, and in which case local service, so called, is sold covering intercommunication between the several central offices. Toll circuits will be defined as circuits between offices more distant from each other and between which the service is sold on a message basis, the charge being graded according to duration of connection and distance involved.

Toll circuits, however, must be considered in two classes, one embracing the " short-haul " or suburban toll traffic, and the other the " long-haul " or " long distance " traffic. The former class may be considered on the same basis as trunk circuits, so far as concerns production efficiency and engineering.

The trunk circuits and " short-haul " toll circuits are provided for the purpose of handling calls on a local basis; that is, the telephone subscriber passes the desired number to his operator and remains at the telephone until the connection is completed or a report returned to him of inability to complete. This feature of the trunk and " short-haul " toll operation necessitates providing a sufficient number of circuits between two offices to handle the traffic offered during the busiest period so that there is always a circuit available when a call is to be handled.

The " long-haul " circuits are for economical reasons provided on a less liberal basis which contemplates that the calls will not be handled while the subscriber remains at his telephone.

A comprehensive series of studies on the subject of circuit use has resulted in the development of methods by the use of which all the elements entering into circuit use can be given proper consideration, and the most efficient operating methods applied to existing conditions.

In figuring circuit loads, as a matter of convenience, it is usual to consider the load for a given group of circuits, rather than for a single circuit. As the circuit holding time is made up of the time of conversation plus the use of the line for operating purposes, the total holding time for a call will differ according to the operating method used; for example, the local or " short-haul " toll requires less holding time than that handled on the " long-haul " basis. Also the efficiency of the individual circuits increases as the size of the group increases. These various factors have been studied and reduced to known quantities.

Having ascertained the production capacity of circuits under various conditions of volume of traffic and methods of operation, it is possible to set the proper standards to apply, and with which to check actual results, so far as concerns individual groups of circuits. The study of this matter, however, has developed the fact that in order to properly engineer the circuit plant for handling the traffic between several hundred offices, the subject is far more complex than would appear at first sight. In considering the circuit efficiency between two offices, as these same circuits may carry in addition the traffic to other points, the plant must be considered as a whole. The comprehensive study which has been referred to, has developed methods by which the circuit plant can be intelligently laid out with predetermined results. The problem in laying out a circuit plant is to furnish the proper standard of service as to accuracy, speed and transmission, with a minimum investment in line plant. It is obvious when one considers the variety of types and grades of trunks and toll circuits that this is no small matter.

The production efficiency of a trunk and toll circuit plant has usually been figured either by computing the number of calls handled per circuit or the earnings per circuit mile. Neither of these methods, however, gives the true measure of circuit production efficiency, inasmuch as for economical reasons it is often necessary to switch long-haul calls through one or more intermediate offices, and on account of the varying size of the circuit groups and the consequent variation in the capacity of individual circuits, the real production efficiency of the circuits cannot thus readily be determined. On the other hand, comparison of the earnings per circuit mile would be very misleading if any change in rates had been made between the periods under consideration.

A far more logical means of measuring the production efficiency of a trunk and toll circuit plant is afforded by computing the number of message miles produced per circuit mile. By computing the former on a basis of air line measurements between points of connection and the latter on the basis of actual circuit route mileage, a means is afforded of checking the efficiency of circuit plant layout, not only from the standpoint of the efficiency of labor and methods, but also from the standpoint of economical plant layout.

In the foregoing sections there have been discussed in a somewhat incomplete way some of the principal factors entering into the consideration of traffic engineering problems. Each of these factors has involved most comprehensive study, a discussion of which would involve a paper of considerable length. There is a close relation and inter-dependence between them all. The quality of service is affected by all the other factors, and itself has to be considered in connection with each one. To illustrate the complexity of the problems, and to indicate the possibilities in constructive application of the principles deduced from their comprehensive analysis, it is proposed to discuss the concrete application of some of the results of the engineering studies of the last few years in the development of a toll operating system and plant.

From the standpoint of the telephone user, the ideal toll service would afford connection with the most distant point with substantially the same celerity as obtains in case of a local connection. The development of a toll operating system should be toward this ideal as far as economically consistent.

The handling of toll business at special toll boards, or at separate positions on the local board, developed for two reasons. While the operating and equipment charges, under a given toll operating method, are fairly constant, the circuit charges increase in weight with the length of haul. In view of the large investment involved in long toll circuits, it was deemed necessary for economical reasons, to make each circuit handle all the messages possible, and it was generally customary to handle such calls on the basis of arranging to have both parties ready to talk before the circuits were put up for the actual conversation. From the nature of operating procedure involved in handling traffic on such a basis, it was obviously necessary to remove the handling of this type of call, involving extra labor, from the *A* or subscriber's operator, as it would, if handled by the latter, prevent the smooth, quick answering and handling of local connections. Further, on account of the extra labor involved, the productive efficiency of the operator and switchboard were very substantially reduced. It was therefore necessary to specialize and segregate this business. Where the local board was of fair size or an expensive multiple board, it was economical to set up an entirely separate board carrying only the toll circuits, the connection with local subscribers being made through the medium of switching circuits between the boards.

For a clear understanding of the explanation to follow, it will be well to briefly describe some of the most generally used methods of toll operation.

The operating methods employed in handling the toll traffic may be divided into two classes, " short-haul " and " long-haul ". In the case of the " short haul " method the subscriber, who must pass his call by number, waits at his telephone until the called station answers, as he would on a local call. In the " long-haul " method, however, the subscriber, having given his call to a recording operator—specifying a particular party if he so desires—hangs up his receiver and waits for the toll operator to call him.

Calls of the latter class are further subdivided according to the operating method employed, namely, the two-ticket method, single ticket ring down method, and single ticket call circuit method.

Under the two-ticket method of toll operating the subscriber signals his local operator, asks for and is switched to the toll recording operator, to whom he passes information sufficient to insure the completion of his call, hangs up his receiver and waits to be called. The recording operator having written the necessary information on the toll ticket, forwards it to the line operator, who secures a toll line to the called office, rings, and when the inward operator answers, passes the details of the call, which are again recorded by the inward operator. The latter operator assumes the responsibility of obtaining the particular party; having done this she notifies the outward operator, who thereupon rings the calling subscriber. Both operators supervise and time the call, entering on the tickets the length of conversation and both are responsible for the prompt release of the toll circuits after conversation ends.

The single ticket ring down method of handling a toll call is the same as the two-ticket as far as the recording of the call is concerned and up to the time of passing the call by the outward operator. The latter does not pass the details of the call, but simply the called number to the inward operator, who, without making a second ticket, rings the desired number. The outward operator announces the call to the called subscriber, and assumes sole responsibility of timing and supervising the call, and of releasing the circuits after conversation ends.

The operation of the single ticket call circuit method is similar to the single ticket ring down. It is faster in that the outward operator secures the called station over call circuit trunks to a *B* switching operator in the distant office, thereby eliminating the services of an inward toll line operator.

The " short-haul " methods are subdivided into the " toll board " and the " *A-B* " method.

Under the former, the *A* or local operator receiving from the subscriber the number desired, passes the calling and called numbers directly to a toll line operator, thus eliminating the recording operator as a separate operator. The toll line operator records the call and after ordering the calling subscriber's line to be held on a trunk to the local switching operator, secures the called number as under the single ticket methods of operating.

Under the *A-B* method the toll operator is eliminated, the *A* operator recording the toll ticket and completing the call over a trunk to the distant office. She alone is responsible for supervising and timing the call. This *A-B* method may be on either a ring down or call circuit basis as may be most economical.

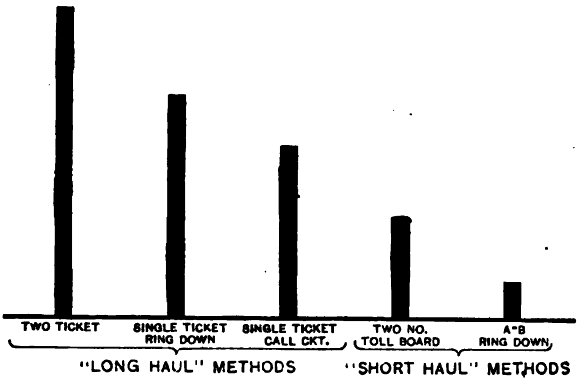


FIG. 2—Relative amount of operating labor required to handle a toll call by the various operating methods

There are further refinements of these methods, in the way of " tandem " toll operation, etc., which are purposely omitted from this discussion, as it is impossible to go too much into detil, and the typical methods shown will serve to illustrate the processes described.

Considering first the efficiency of labor involved in handling a toll call, Fig. 2 shows the relative requirements of operating labor under each of the above described methods.

Considering the production efficiency of central office equipment under the same operating methods, Fig. 3 shows the comparison of relative requirements in the way of switchboards and accessory equipment for the operation under each method.

It will be noted that in case of both the central office labor and central office equipment, the "short-haul" methods are relatively more economical than the "long-haul" methods. Bearing in mind that the toll circuit efficiency is affected by many variables, as has been explained, chiefly volume of traffic, length of haul and circuit holding time, it is necessary to consider this factor in connection with the efficiency of labor and equipment. Obviously, to handle traffic on a "short haul" basis, requires more circuits than to handle the same traffic on the "long-haul" basis. It is necessary, therefore, to ascertain under what conditions it will be economical to apply the more efficient operating methods, considering all factors involved.

The process by which the principles deduced from engineering studies are applied to a territory involving a large number of offices of varying sizes and volumes of traffic, may be best illustrated by taking a hypothetical case, as shown in Fig. 4.

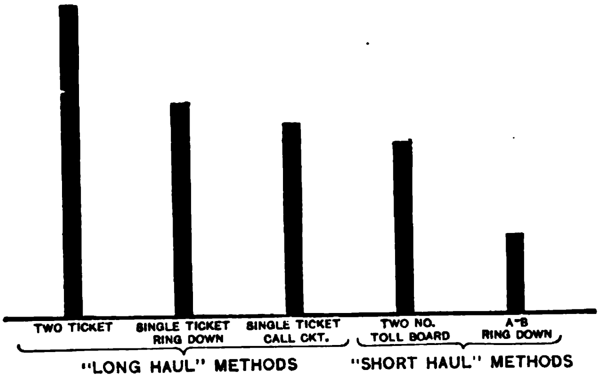


FIG. 3.—Relative amount of switchboard required to handle a toll call by the various operating methods

A and *B* are fair sized offices 30 miles apart. *C*, *D* and *E* are offices separated from *B* by 5, 8 and 20 miles respectively. *F* is between *A* and *B* and slightly off the direct line, being 24 miles from *A* and 8 miles from *B*. The traffic between these offices is indicated in the table adjoining Plan 1. This traffic is shown in number of calls per day rather than per busy hour, for the sake of clearness. In the actual engineering, the busy hour loads are used.

It will be noted in Plan 1, Fig. 4, that the circuits are laid out in a way generally typical in past practice, the tendency being to furnish direct circuits between points having a fair volume of daily traffic. Under this Plan 1, there are toll positions or toll boards at each of the offices. It will be noted that there are 14 circuits between *A* and *B* divided into several groups, also that there is a fair amount of traffic between *B* and *C* and *B* and *D*, with slightly less between *B* and *D*.

Under Plan 2, the traffic between *B* and *C*, *B* and *D* and *B* and *F*, has been taken from the toll position or toll board and is handled on the local boards on the " short-haul " *A*-*B* basis. *C*, *D*, *E* and *F* continue to handle their toll traffic with *A* on toll positions and over direct circuits. At the right of Fig. 4 are shown the comparative requirements under the two plans. It will be noted that, in order to handle the traffic between *B* and *C*, *B* and *D* and *B* and *F* on a local basis through the local board, additional circuits had to be provided to place this traffic on a " short-haul " basis. This increased the circuit requirements, but made a material reduction in the operating requirements as shown, while the equipment requirements increased slightly owing to the transfer of the toll business to the more expensive local board. The composite result, however, is a substantial gain in economy.

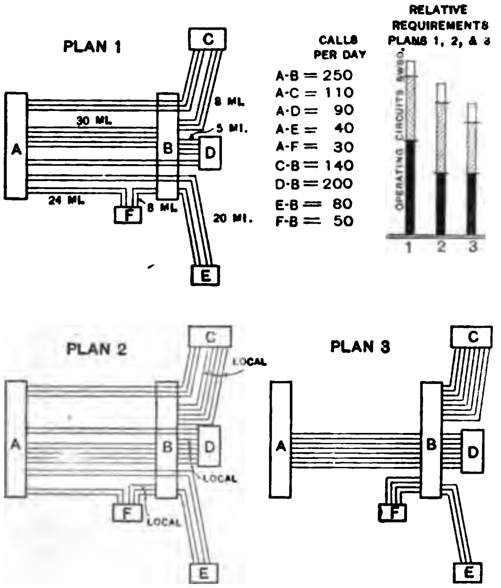


FIG. 4

Having placed the traffic between *B* and *C*, *B* and *E*, *B* and *D* and *B* and *F* on a " short-haul " operating basis, the way is open for still further modification. Plan 3, Fig. 4, shows the next step, which is the centering of all toll operation for *B*, *C*, *D*, *E* and *F* on the toll board at *B*. Placing the traffic between these tributary offices and *B* on a " short-haul " basis will permit of recording at *B* all toll calls from these points to *A*, practically without loss in time. The operators on the toll board at *B* can order up connections through toll switching operators at the tributary offices with the same efficiency and speed as with subscribers at *B*.

This plan, which has been termed " toll center operation ", has several advantages. Applying the principles outlined in the section on the production efficiency of trunk and toll circuits, by combining into one group all toll circuits between *A* and *B*, each circuit can handle more calls than was the case where they were split into smaller groups as in Plan 1. The offices tributary to *B* receive practically local service with *B* and among themselves. They receive the benefits of the large team of toll operators at *B*, and the net result is a great improvement in the quality of service and greater economy in operating and in the use of equipment and circuits, as indicated in comparison No. 3 of " Relative Requirements ". It will be noted that the circuits between *A* and *B* have been reduced from 14 to 8, while the total circuit mileages involved under Plans 1, 2 and 3 are 551, 620 and 436 miles respectively.

This hypothetical case illustrates the application of the engineering principles which have been outlined in the discussion of the various factors entering into efficient production. As indicating how substantial may be the results of the application of these principles to a large territory, it will be of interest to state that in the territory of the New England Telephone and Telegraph Company are over 730 central offices, nearly every one of which formerly had its own separate toll position or toll board, although in the smallest offices the toll operation was handled in combination with the local. The application of the above principles has been carried to the extent that to-day the toll traffic between these 730 offices is handled at 79 toll centers. The general effect on service has been to place all short-haul traffic on an approximately forty second or better basis; to concentrate the longer haul high grade circuits in large groups between centers; to concentrate the long-haul toll operation at these centres, with the increased economy and efficiency of a larger and more highly developed toll force; to remove from the small offices the burden of labor entailed in handling toll calls, this resulting in a smoother local service; and, through making available large groups of trunks on main routes, affording a more speedy, dependable long-haul toll service to all offices. Thus by the use of the comparative values established through development of the unit system of equating traffic and the establishment of standards of production efficiency for force, equipment and circuits, it has been possible to improve the quality of service, conserve the investment and reduce operating expenses.

Undoubtedly the most striking development in the analysis of the toll problem is that which has pointed out the way for the evolution and systematic development of the " short-haul " system of handling toll calls.

Reference has been made to the fact that from the viewpoint of the telephone user the ideal toll service would be that which afforded the same speed of connection to a distant point as that given in making a local connection. Having this ideal in mind, studies were made to determine to what extent this type of service could be developed economically. It was demonstrated that laws could be derived from these analyses whereby it could be predetermined, with a given volume of traffic and given average holding time per call, up to what point a suitable number and grade of circuits could be provided and furnish this type of service economically.

It was found essential, in order to furnish this " short-haul " toll service, that the calling subscriber should pass his call for the distant point by number and not for a designated party. This brought out two important factors as requisite to furnishing the means and the incentive to the subscriber for so placing his calls: first, the matter of the telephone directory, that the subscriber might ascertain the number desired in the distant place; second, whether the subscriber could fairly be asked to place his call by number and pay for the connection if the distant station were reached, whether his party were there or not.

Investigation of the first factor showed that, as the toll calls originated from a comparatively small proportion of the subscribers, where the amount of business warranted, directories for such other places as would meet the needs could be furnished without great expense, or special lists could be made up for subscribers doing considerable business with the distant points.

In case of the second factor, analysis of several thousand toll calls to points at varying distances from the originating office showed that in 89 per cent of the calls where the station was reached, the desired person either answered the telephone or was on the premises and available to talk, and that in 6 per cent more there was some one at the station with whom the business could be transacted. It was thus demonstrated that in 95 per cent of the calls, business could be successfully transacted. It further developed that there was less demand and necessity for locating a particular person on the short-haul calls than on the long haul calls, which is logical in that the more expensive long haul toll calls are not usually made unless the matter is of more importance than is the case in the ordinary short haul toll calls.

Based on these results, a plan was developed by which the " short-haul " traffic would be handled on an exclusively " two number-no delay " basis; while for greater distances, the particular person " long-haul " methods would be employed.

This plan has been gradually developed with the result that while in the year 1906 but 17 per cent of the toll traffic of the New England Company was handled on the " short-haul " basis, in 1910 over 50 per cent was handled on that basis, while the plans under development and already partially in effect for 1911, contemplate that by the end of the year at least 85 per cent of the toll traffic of the Company will be so handled. The results so far indicate that the telephone users greatly prefer this type of fast service to the slower " particular person " service and do not find it a serious disadvantage to forego the " particular person " privilege.

It is probable that the New England territory, with its great density of towns and cities, offers ideal conditions for the development of this type of service, but it seems reasonable to expect that the telephone using public generally will eventually express preference for toll service of this character.

As the volume of traffic increases with the growth in the business, it will be possible to offer such service on an economical basis to a far greater extent than would seem possible without a full knowledge of the facts.

It is possible to obtain a view of some of the future possibilities in the development of " short-haul " toll service by examing the chart in Fig. 5.

The curve shown in Fig. 5 is one of a set of curves made up to cover various conditions. The basic assumptions for this curve are, the use of No. 12 copper line circuit, common battery type of central office equipment, with standard operating, equipment and line production efficiency, and an eight hour telephone day: *i.e.*, one-eighth of the day's traffic is handled during the busiest hour.

The comparisons are between the generally used single ticket particular party "long-haul" method, and the "short-haul" *A-B* ring-down method. From this chart may be determined, assuming a given number of messages per day, up to what distance it will be possible to furnish enough circuits to permit of operating on the "short-haul" *A-B* ring-down basis more economically than on the "long-haul" particular party single ticket basis. This takes into account all the factors involved that are necessary to a fair comparison.

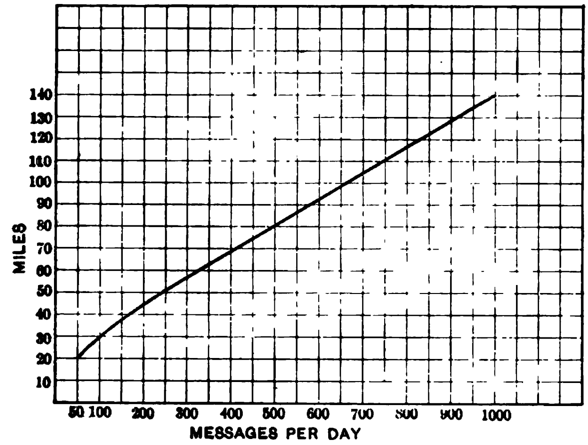


FIG. 5

It would seem obvious that once there is an appreciation of the latent possibilities developed by this analysis, there will be steady progress toward the development of this highly desirable type of service, which can be supplied economically in case of a substantial majority of the toll calls in a territory where there are a number of closely related communities within reasonable distance of each other.

The foregoing discussion has simply covered in a broad way a few of the important problems which have been the subject of traffic engineering study for the past few years. The general problem of telephone service is so broad, and its full consideration involves so many and such a diversity of factors, that it is difficult, within the limits of a paper of this nature, to discuss comprehensively any one of its phases.

It should be apparent that the problems in connection with the production of telephone service are of sufficient importance to call for most careful analysis. The fact that the gross annual revenue of a telephone company is but a fraction of its capital investment, wherein it differs from most lines of commercial endeavor, makes such analysis a necessity. Solution of the problems of handling the traffic is fundamental to the construction engineering and equally so to intelligent consideration of matters affecting the economic and public policies of a telephone company. The determination of values in a problem affected by so many variables, and the development of simple methods of making use of them, have offered and still offer endless opportunities for a high grade of engineering.

The results of the studies so far made have demonstrated that it is not necessary to rely solely on opinion or judgment from a certain point of view; that by analytical process many factors of apparently intangible values may be reduced to known quantities; that relations between them can be definitely and accurately established; and that from these relations certain laws can be derived which are capable of general application in the solution of telephone traffic problems.

DISCUSSION ON "PROBLEMS IN TELEPHONE TRAFFIC ENGINEERING." CHICAGO, JUNE 28, 1911.

A. P. Allen: The " problems of telephone traffic engineering " used to be to *guess what was going to happen*—and we are only just a little bit better off today. As I listened to some of the other papers, I wanted to congratulate the light, power, and construction engineers because they could fix at least something and depend on it. This paper is, however, comforting to a man in my position, because it gives the idea that even in telephone traffic engineering things are coming around so that you can depend on them; so that you can say that an average operator's load ought to be so much, and expect to get it; you can say that the public will give you more calls between nine and ten in the morning than they will between three and four in the afternoon, and have it come true.

But in practice almost everything that the traffic engineer touches goes back on him sooner or later. He gets figures that show that the normal load in a city residence district is heaviest in the morning, and some day he gets the larger load in the middle of the afternoon—larger than he ever had in the morning. That is something that we cannot control. It is the same way with the handling of the toll traffic. The business between two towns will be, perhaps, only 15 per cent taking place in the busy hour, while the business between other towns that have apparently the same conditions will be 25 per cent in the busy hour. It is very hard to establish standards, or formulae, that you can depend upon on all occasions, when the conditions simply *look* similar.

A few papers out of my files illustrate that point. For instance, in Chicago, the long distance business—making a series of tests every day for one week: on Tuesday the busy hour, in the afternoon, was 103 per cent of the busy hour in the morning. (Of course we have to plan our operators for the busy hours.) The very next day the afternoon busy hour was only 64 per cent as heavy as it was in the morning. In the same way with long distance traffic of New York City, comparing different months. Taking April as 100 per cent; one year, the August business is 91 per cent while the next year it is only 85 per cent; but in both years December is 110 per cent of the April business.

All such figures demonstrate that the use of the service by the public is very uncertain. We get up formulas and say that if the average holding time is two minutes and fourteen seconds, a group of ten trunks will carry so many calls. As a matter of fact, our average is not a reliable average, because the variation between the shortest and the longest connection is very great. The formula holds true *if* they are all nearly alike, as to the average time; but as they are plotted out, they run from five-second calls up to about ten minutes. Can we, under such conditions, accurately foretell how many trunks we will need to give instantaneous service to a given number of patrons? That is what we have to contend with in city traffic: we have to give instantaneous connection, we cannot wait for circuits. Between two offices a certain number of calls will require a certain number of trunks; between two other offices the same apparent load may take nearly twice as many.

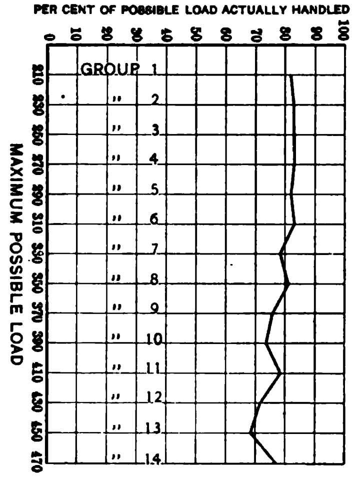


FIG. 1

The charts shown in Figs. 1 and 2 are original, perhaps—at least I have never seen anything just like them. They show the "guessing" we have to do on the ability of the average operator. Each line represents roughly, a group of operators, graded as to what they can do in fifteen minutes, and also showing what

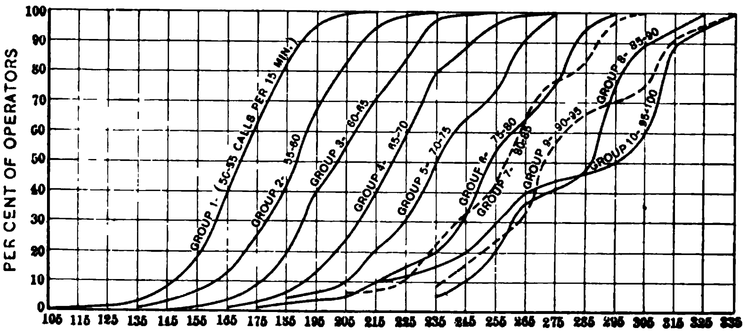


FIG. 2

they did do in the total hour. A total of 1250 operators were thus analyzed, their ability being represented by what they did in fifteen minutes. It is reasonable to suppose that they could do four times as much in an hour if they had it to do; and that the fluctuation in their hourly load is due to the fact that the total loads vary so much during an hour. Due to that fact, the per cent of efficiency that all grades give is almost exactly the same. Girls that handle 55 calls per quarter hour reach 82 per cent efficiency for the total hour. Girls that can handle 50 per cent more in 15 minutes have only the same relative efficiency for the total hour.

I certainly hope that Mr. Valentine's paper gives the right idea, that in the end we will be able, by analysis and careful tabulation and watching of results, to get some real, accurate data on which to base fundamental plans; but it does not appear to me that we have yet reached that condition. And a great many of the problems that we have to contend with are not within our control at all—that is, they depend entirely upon the use of the service by the public.

W. Lee Campbell: In manufacturing, mining, bricklaying and similar occupations great advances have been made in the last few years by engineers studying efficiency methods and making motion studies with the idea of increasing the efficiency of the individual operator. To one who has been familiar with this, the question naturally occurs what can be done along this line, or what has been done in manual telephone work toward increasing the efficiency of the operators? I suppose the principal difficulty is in supplying each operator with as much work as she can do. As Mr. Allen has just brought out, in one moment the operator may have all she can do; the next moment she may not have as much as she can do. The result is that if standards were set, the operators could not be compelled to come up to them by force, nor could they be coaxed to come up to them by increased remuneration. Consequently, it would appear to me that if some device should be introduced between the subscriber's lines and the operator, which would furnish her with work at the greatest speed she could take it, as determined scientifically, that the operators could be brought to a very much higher standard of efficiency. The work could be distributed, starting at one end of the board, to the operators as far as it would go and at the other end of the board a certain per cent of operators could be kept for emergencies. I am not a manual telephone engineer, but I should like to ask of some of these gentlemen who are, if there are not possibilities in such a scheme.

Bancroft Gherardi: I feel that we are all very much indebted to Mr. Campbell for raising the question which he has and for pointing out to us such an interesting problem concerning traffic engineering. It is a most valuable illustration of the type of problem which traffic engineers are sometimes confronted with because it requires not solely the work of traffic engineers, but also the work of plant engineers and requires the coöperation of these two sets of engineers to obtain satisfactory results. The traffic engineer can point out what kind of call distributing system he would like to have from an operating standpoint, and the results which he would expect such a system to attain. Based upon this, the plant engineer would undertake the design of suitable apparatus, and would, in general, sooner or later be confronted by some traffic requirements which could not practically be met or which could only be attained at such great cost or such excessive complications as would not be warranted. So the working out of this problem and similar problems requires coöperation between the traffic engineer and the plant engineer in order that from the general idea at first proposed there may be finally evolved the best arrangement, taking into account all of the circumstances of the case—operating savings, maintenance, character of service, first cost of apparatus, etc. While a discussion of the specific problem which Mr. Campbell has mentioned would carry us somewhat beyond the scope of this paper, I might say in reference to the matter that while a good deal of work has been done on this question by a large number of people and extending over many years, so far as I know no large application of the principle of call distribution has ever been made anywhere. I do not feel that we are now warranted in saying that we can ignore this problem and say there is nothing in it. It is my opinion at the present time that it is a problem which has not as yet been worked out and that there are certainly no data available at the present time to permit us to give an affirmative answer to the question " Is call distribution a satisfactory and economical feature of a telephone switchboard?"

Mr. Valentine has, I think, placed us all under obligations to him in presenting to us his paper dealing with traffic engineering problems. While traffic engineering does not deal with volts and amperes, and in that sense is not electrical engineering, it is nevertheless a most important and interesting class of engineering work, the nature of which is but little appreciated at the present time except by those who come in direct contact with it in their work.

A. P. Allen: What I wanted to make clear was that, regardless of any system of distributing, no one can tell in advance what the load is going to be—that is, how many calls the public is going to give to us—at any particular period of the day; and even if we had a perfect distributing system for those calls, we would not know whether to put on one set of operators, or more, in order to give a certain grade of service.

W. Lee Campbell: It is found in manufacturing practice as shown by the studies of Mr. Harrington Emerson, Mr. F. W. Taylor and others, that the efficiency of the average operator paid by the day, is only about 67 per cent of what is attained if an operator is paid in proportion to his or her efficiency. It is impossible to bring an operator up to the highest efficiency unless she is supplied with all the work she can do. The work must be, in some way, gauged and the operator must have before her definite standards to which she will be expected to rise. As Mr. Allen says, in the use of switchboard operators, the trouble is that the public does not furnish a regular amount of work but if some of the automatic or mechanical devices should be used between the public and the operators to distribute the calls to each at her best rate of speed and the efficiency of the operators could thus be brought from 67 per cent up to 100 per cent, a great saving would be secured. A certain percentage of operators could be held to take care of the irregularities. With the present plan, each operator is given a fixed number of lines to handle, and consequently all of them are subject to irregularities of load with the result that each has an excuse for not doing her best.

F. P. Valentine: While this discussion has brought out one or two typical problems which the traffic engineer has to face, it also indicates that there is not a general understanding of how far these studies have been carried. It is perhaps not generally known that for some years telephone traffic problems have been receiving constant study as the result of which many of the apparently intangible factors have been reduced to known quantities and their inter-relation established, so that due weight may be given to variations. The analytical work of the past few years has been merely outlined in brief form in this paper to point out that the work can be carried on intelligently having accurate knowledge of values and their relations.

We cannot today consider all the problems solved, but by the methods pointed out and with the facts and laws already established as a foundation the way seems fairly clear to the ultimate solution of the various elusive problems which constantly confront the traffic engineer. Traffic engineering in itself is a comparatively young branch of engineering work, in which developments have come very fast within the last few years, and in regard to which there is comparatively little knowledge or appreciation outside of the comparatively few who have been fortunate enough to participate in the comprehensive studies which have been carried on. It is safe to say that the developments of the next few years will bring about a better realization of the fundamental importance of the study of these problems as well as a better appreciation of the progress already made.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 29, 1911.

ELECTRIC LINE OSCILLATIONS

BY G. FACCIOLI

In the summer of 1910 some tests were performed on the lines of the Great Western Power Company, which are operated at 100,000 volts, to obtain information on oscillations and rises of potential, due to switching operations. In these experiments the oscillograph was extensively used and this paper deals with some of the most important and representative records.

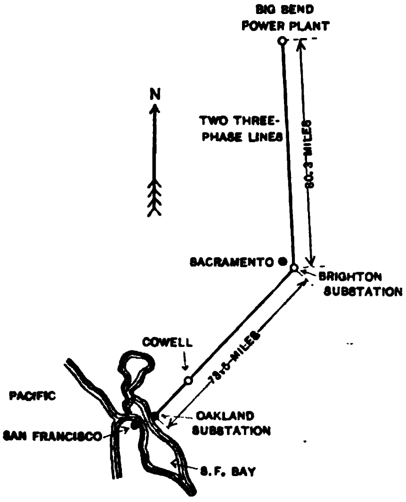


FIG. 1

Since it was necessary to conduct the investigation without interrupting the operation of the system, the study of switching phenomena is rather incomplete and limited to no-load conditions.

Mr. W. W. Lewis, who assisted in these experiments, gives a description of the methods and apparatus employed, in an appendix to this paper.

Fig. 1 is a map of the system of the Great Western Power Company. Two three-phase lines run on the same towers (Fig. 2) the conductors of each line being situated in a vertical plane. Each conductor consists of a copper cable No. 000, B. & S. gauge (seven strands, outside diameter ½ in. approximately). The vertical distance between conductors is ten feet, and the horizontal distance between lines is 14 ft.

The following tests were performed:

1. Switching in and out an open three-phase line at the generating station by high-tension switches.

2. Switching in and out an open three-phase line and step-up three-phase transformer at the generating station by low-tension switches.

Fig. 2.—Transmission line of Great Western Pr. Co. at Brighton

3. Switching in and out a three-phase line, connected at the end to an unloaded three-phase step-down transformer, by high-tension switches, at the generating station.

4. Switching in and out a three-phase step-down transformer at the end of an unloaded three-phase line by high-tension switches.

5. Switching, by high-tension switches, one of the three-phase lines unloaded, on and off the end of the other three-phase line, carrying normal load.

Before entering into the discussion of the oscillographic records, it is advisable to study the constants of the line which, as we said before, consist of three No. 000 B. & S. copper cables situated in a vertical plane. The distance between conductors is 10 ft. and the length of the line is 154 miles.

Line Inductance. The total inductance of each conductor is 0.323 henry.

Line Capacity. The total capacity of each conductor to the neutral is figured as 2.2 microfarads. It is interesting to compare this figure, obtained by calculation, with the figure derived from actual test.

From a series of experiments, which it is superfluous to enumerate, we find that when the voltage at the generator end of the line is 51,700 volts from conductor to neutral, the voltage at the far end of the open line is 64,000 volts from conductor to neutral. In this case, the charging current per phase is 48 amperes.

Assuming the capacity of the line concentrated in two identical condensers, one at the beginning of the line, and the other at the end of the line, we have

$$48 = 2\,\pi\,f\,c\,(51{,}700 + 64{,}000)$$

where f is the frequency, 60 cycles.

c is the capacity of each of the two condensers into which the line capacity was divided.

We find $c = 1.1$ microfarads.

This gives for the total line capacity from each conductor to neutral the value of 2.2 microfarads, which is identical with the value obtained by calculation.

Line Resistance. The ohmic resistance of each conductor is 50 ohms.

Generator Reactance. In all the experiments, the source of power is a 10,000 kv-a. three-phase generator. The windings of the generator are *Y*-connected, and the neutral is not grounded. The speed of the alternator is 400 rev. per min., its normal voltage is 11,000 volts. The machine has 18 poles and three slots per pole per phase.

In later calculations, we will frequently have occasion to use the value of the synchronous reactance, and of the leakage reactance of the generator.

The value of the synchronous reactance may be deduced by test, as follows:

Referring to Fig. 3, when the line is disconnected (high-tension switches open) the voltage of the generator is 4,430 volts between phase conductors.

When the line is connected (high-tension switches closed) the voltage of the generator for the same exciting current is 8,730 volts, and the current supplied by the generator is 400 amperes.

Neglecting the ohmic resistance and the exciting current of the step-up transformer, we have

$$2\,\pi\,f\,L_g\,400 = \frac{8730-4430}{1.73}$$

where L_g is the synchronous inductance per leg and f is the frequency, 60 cycles.

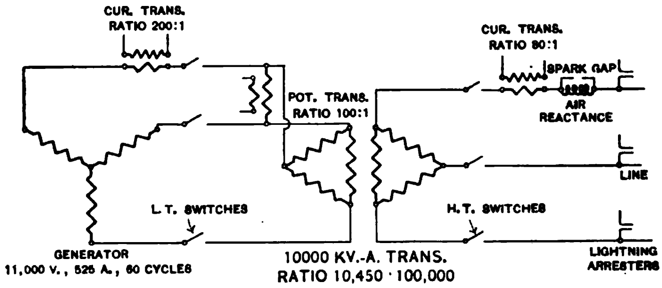


FIG. 3

$$L_g = 0.01645 \text{ henry}$$

If we refer the synchronous inductance to the high-tension side of the step-up transformer, whose ratio of transformation is 9.56, we obtain a value of $L_g = 1.52$ henry.

The leakage inductance of the generator deduced by calculation is 0.00276 henry, and referred to the high tension side of the step-up transformer is 0.253 henry.

Transformer Reactance. The transformers used in all the tests—step-up or step-down transformers—are 10,000 kv-a. three-phase transformers, delta connected on both primary and secondary sides.

The leakage inductance of the transformer referred to its high tension side (L_t) is 0.088 henry, from each conductor to neutral.

Electrostatic Capacity of the Apparatus. The capacity of the generator windings and of the transformer windings plays an important role in some of the oscillations, as we will see later. However, no attempt is made to determine the value of these capacities.

The theory of the phenomena involved in this investigation is well understood, and will, therefore, be omitted with the exception of a few instances. All the calculations and theoretical considerations in this paper had their origin or inspiration from Dr. Steinmetz's treatise on " Transient Electric Phenomena and Oscillations."

TEST No. 1

Fig. 3 shows the arrangement of the apparatus. The potential was recorded across one phase of the low-tension side of the step-up transformer.

The current was taken either across the secondary of the current transformer connected in the low-tension circuit, or across the secondary of the current transformer connected in the high-tension circuit. All the switches in this and the following tests were of the oil type.

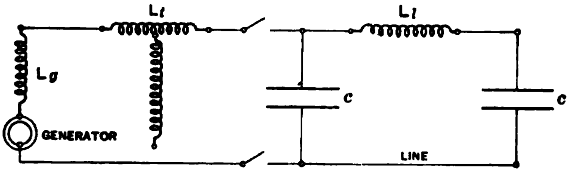


FIG. 4

The low-tension switches were closed, and the generator and step-up transformer excited at a certain voltage. Then the 154 miles of three-phase line were switched in or out by means of the high-tension switches and the waves of e.m.f. and current were recorded at the moment of closing or opening the high tension switches.

The line under test was open at the far end.

Fig. 4 represents a simplified diagram of the connections.

L_g is the reactance of the generator.

L_t is the leakage reactance of the step-up transformer.

L_l is the reactance of the line.

c is one-half of the line capacity.

We neglect ohmic resistances and assume the susceptance of the step-up transformer equal to infinity. We neglect also the electrostatic capacity of the apparatus.

If, for a first approximation, we consider L_l as negligible, the closing and opening of the high-tension switches connects and disconnects the condenser given by the line capacity on and from the inductance of the generating system.

It is known that in a circuit consisting of a condenser in series with an inductance, the general expression of the e.m.f. across the condenser and of the current contains three terms.

The e.m.f. across the condenser is

$$e = e_1 + e_2 + e_3$$

And the current is

$$i = i_1 + i_2 + i_3$$

i_1 and e_1 are the values corresponding to stationary conditions of charging current and condenser potential at fundamental impressed frequency. The current leads the e.m.f. by 90 deg. and the effective value of the current is equal to the effective value of the e.m.f. divided by the line condensive reactance.

i_2 and e_2 represent a damped oscillation at a frequency independent of the impressed frequency but depending on the constants of the circuit, as follows:

$$\text{Frequency} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \left(\frac{r}{2L}\right)^2}$$

Where L is the inductance of the circuit, C the capacity, and r the resistance.

This term depends mainly upon the point of the impressed e.m.f. at which the oscillation starts. There is no point at which this term is absent, and it reaches its maximum value when the oscillation starts at the maximum of the impressed e.m.f.

i_3 and e_3 depend on the constants of the circuit and upon the instantaneous values of current and of difference of potential in the circuit at the moment the oscillation starts.

A. SWITCHING IN OPEN LINE BY HIGH-TENSION SWITCHES

In the special case of switching in a line, if the operation is instantaneous and no secondary phenomena occur, the third term disappears since the e.m.f. and current in the line are zero at the moment the switches are closed.

The complex oscillation produced by closing the high-tension switches, thereby connecting suddenly the line condenser to the inductance of the generating system, will then consist in our case of a fundamental wave at 60 cycles on which is superimposed a damped oscillation whose frequency is—neglecting ohmic resistance—$f = \frac{1}{2\pi\sqrt{LC}}$.

Referring to Figs. 3 and 4, $L = L_g + L_t$ (line inductance neglected).

C = total line capacity.

For L_g we must take the value of the true self-inductance of the armature of the generator, and not the value of the synchronous inductance, as the latter includes the effect of armature reaction which must act on the field circuit where any change in flux is retarded by a change in field current.

If, however, the oscillation lasts a comparatively long time, then the armature reaction may enter in the phenomenon.

$L = 0.253 + 0.088 = 0.341$ henry.

$C = 2.2$ microfarads.

Frequency of oscillation = 139 cycles.

If L_t is not neglected and the circuit is treated as represented in Fig. 4, the frequency of oscillation is 167 cycles. Finally, by taking the total synchronous inductance of the generator in the circuit represented in Fig. 4, the frequency of oscillation is 83 cycles.

In these calculations we have neglected the ohmic resistances, which are actually very low, so that they not only allow the occurrence of a free oscillation (r^2 less than $4\,L/C$) but do not practically affect the value of the frequency of oscillation.

This frequency is then comparatively low, and is between 83 and 167 cycles.

In Fig. 5, a 60-cycle wave is superimposed on a 120-cycle damped oscillation, and the resultant wave is almost identical to those obtained in the oscillograms, showing that the frequency of free oscillation found by test is of the same order of magnitude as calculated and approximately equal to 120 cycles.

However, the oscillograms show that closing the high-tension switches (Fig. 3) produces a more complicated phenomenon than theoretically assumed, as the switches do not close the circuit at once and permanently. Several secondary phenomena occur, as follows:

First: The three switches may not close the circuit at the same instant.

If one switch only closes its circuit, then one wire only of the three phase line is energized, and the potential of the system to ground is unbalanced (the system has no point permanently grounded). A charging current from this wire returns to the generator through the line capacity in series with the capacity to ground of the generating apparatus (transformer included), giving rise to a high frequency oscillation.

If two switches close and the third remains open, single-phase unbalanced load is thrown on the generator, giving rise to odd harmonics of the generated e.m.f. (third, fifth, etc.,) as shown clearly in some of the records.

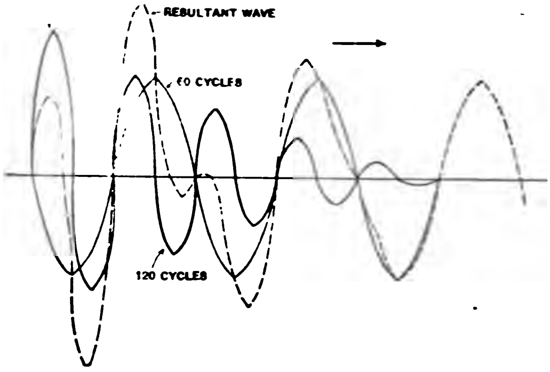


Fig. 5

Since the first closing of a high-tension switch is generally due to an arc before the metallic contact is established, it is natural to expect that the three switches of a three-phase system will not begin to close the circuit at the same instant.

Second: Arcs strike between contacts and die out before the metallic circuit is permanently established. It follows that instead of one starting oscillation, we obtain a series of oscillations, due to the successive closing and opening of the circuit.

We should then expect to find in the records a number of successive oscillations, which consist of a fundamental 60-cycle wave superimposed on a damped oscillation of 120 cycles, when all the three phases of the line are being connected to the generating system. Other frequencies will appear when the three phases are not connected simultaneously, and at the instant when the line and the generating system become disconnected, the line on one side and the generating system on the other side will oscillate at their own natural frequency. Since the oscillograms of the electromotive force are taken on the low-tension side of the generating system, our records do not show the free oscillations of the line, and, likewise, do not show some of the oscillations which occur at high-frequency between the high-tension side of the transformer and the line and which do not go through the iron of the apparatus.

Record No. 1. Referring to Fig. 3, this record is taken at the moment of switching 154 miles of unloaded three-phase line onto the high tension winding of the step-up transformer by closing the high-tension switches.

Before the switches are closed the wave of e.m.f. gives the generator e.m.f. (4,500 volts, 60 cycles), which shows the harmonic due to the teeth of the machine, while the current—taken in the high-tension circuit—is obviously zero.

After a high frequency impulse in the e.m.f. and current, due probably to an arc in one of the switches, the circuit is closed and the oscillation has the appearance of the resultant wave of Fig. 5 (120 cycles superimposed on 60 cycles).

The current has two long zeros before the metallic contact is permanently established, due to arcs in the switch, and finally the last oscillation occurs, which is damped in less than three fundamental cycles.

The new conditions of equilibrium are now reached, and the record shows an e.m.f. of 9,300 volts and a charging current of 48 amperes.

It must be remembered that the e.m.f. is taken across two conductors of the low-tension circuit, while the current is taken in one conductor of the high-tension circuit. Therefore, if the current leads the corresponding e.m.f. by 90 deg., the record should show the high-tension current leading the e.m.f. by 60 deg. or lagging 120 deg. behind the e.m.f. In many of the records either the current or the e.m.f. should be reversed to obtain the proper phase relation.

After the complex oscillation has occurred, the e.m.f. is smooth and does not contain the harmonic due to the teeth of the alternator, because of the very high impedance offered by the generator reactance to the flow of this harmonic (17th harmonic), which *lags*, the inductive reactance of the generator being several times the condensive reactance of the line at that frequency.

Record No. 1

It is interesting to note that the first oscillation differs from the last in the wave of e.m.f. The first oscillation of e.m.f. shows a higher degree of distortion and contains odd harmonics (3rd and 5th) in addition to the natural frequency at which energy is exchanged between inductance and capacity. As pointed out above, this is due to the fact that all the three switches were closed at the last oscillation, while, very probably, all the three switches were not closed when the first oscillation started. Furthermore, while the first oscillation began with zero volts across the condenser, the successive oscillations started under different conditions. In fact, when the current is zero and the circuit is opened by the dying out of an arc in the switch, the e.m.f. across the condenser is near its maximum value and the energy thus stored in the line is dissipated at the natural frequency of the line, but this oscillation may not have died out entirely before the circuit is reëstablished. This introduces a third term in the expression of the e.m.f. and current, as mentioned above.

The record shows also high frequency impulses in the e.m.f., due mainly to a change of energy from leading to lagging or vice versa. For instance, high frequency appears in the e.m.f. whenever the current goes to zero and the arc in the switch dies out. This occurs especially when the current maintains a low value for a comparatively long time due to the interference between the 60-cycle and the 120-cycle waves.

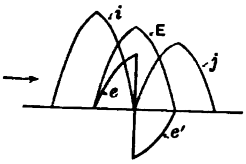


FIG. 6

Referring to Fig. 6, let us assume that the line is disconnected when the charging current, which is leading, is passing through zero.

At this instant the impressed e.m.f. E is maximum. On E is superimposed the e.m.f. e, which is generated by the flow of the charging current i through the reactance of the generating system, so that $E+e$ is active across the condenser. At the next instant the high tension current remains zero, because the arc in the high tension switch has died out, but the generator must excite the transformer, which is still connected. Hence, lagging energy is supplied, and a current j flows. The current j produces an e.m.f. e' across the reactance of the generating system.

It follows that a sudden change of difference of potential has occurred with the corresponding change of energy stored in the capacity of the generating system. The new equilibrium is reached through an oscillation at the natural frequency of the apparatus.

This high-frequency appears in the e.m.f. of the low-tension circuit and obviously not in the high-tension current, but we must expect to find it in the low-tension current, as we will see later.

The *switching-in* is then complicated by arcs and by the irregular closing of the three switches.

The record shows only the phenomena occurring in one particular conductor and phase, and does not show—as stated before—the free oscillations of the line. This line—154 miles long—oscillates at a frequency of 300 cycles when the length of the line corresponds to a quarter wave length.

The records also do not show, or show incompletely, what happens at the very first establishing of the circuit, as only the oscillations passing through the iron of the step-up transformer and of the measuring transformer are recorded. At the very first establishing of the circuit, the energy stored in the high-tension winding of the step-up transformer is thrown into the line, and a direct record of the high-tension e.m.f. would show this phenomenon and the process by which the potential is established on the line.

Finally, the records do not show what happens at the open end of the line, where the wave of e.m.f. undergoes a total deflection.

Record 1 gives as maximum over-tension 60 per cent above final maximum voltage. The current reaches a maximum of 160 per cent above final maximum value. The total complex oscillation lasts for a length of 10 fundamental cycles, or one-sixth of a second.

Record No. 2. This record is taken under the same conditions as the preceding record, but it gives the current in the low-tension circuit instead of the current in the high-tension circuit.

We start again with an e.m.f. of 4,500 volts and the exciting current of the transformer, which is so small that it does not show in the record. Then an oscillation similar to the one of Record 1 occurs, and finally we have 9,300 volts and 450 amperes.

All the remarks concerning the preceding record apply to this record, and the low-tension current shows, as expected, high frequency impulses when the arcs in the high-tension switches die out.

The maximum over-tension is 55 per cent above the final

Record No. 2

Record No. 3

maximum value, while the current reaches a maximum of 175 per cent above the final maximum value.

The complex oscillation lasts nine fundamental cycles, or three-twentieths of a second.

B. SWITCHING OUT OPEN LINE BY HIGH-TENSION SWITCHES

Fig. 3 shows the connections used, and the records are taken at the moment of opening the high-tension switches, thereby disconnecting the line from the high-tension winding of the step-up transformer.

If the switching is instantaneous, then the sudden falling of the current I to zero gives an increase of potential E, such that $\frac{C E^2}{2} = \frac{L I^2}{2}$ where C is the capacity and L the inductance of the line. In other words, the electromagnetic energy stored in the system is transformed into electrostatic energy, and will be gradually dissipated.

Let us consider now the other limit case where the switch opens the circuit very gradually through a continuous arc, which increases slowly in length and resistance, so that the successive adjustments in value and phase of current and e.m.f. follow the progressive change in the conditions of the circuit. In this case, the disturbance is reduced to a minimum, but it is evident that the gradual modification of the circuit and of its constants occurs in the switch, where the most active consumption of the energy stored in the system takes place. It follows that, generally speaking, the opening of the circuit imposes severe conditions on the switch.

Between these two limit cases, approaching one or the other, according to the constants of the circuit and the characteristics of the switch, lies the actual opening of the circuit in practice.

The phenomenon is complicated, however, by the fact that the arc in the switch is apt to rupture and reëstablish itself several times before the circuit is permanently opened.

From this point of view we see that the phenomena and oscillations which accompany the opening of the circuit are very similar to those met with in closing the circuit. Two main differences exist, however. First, in opening the circuit the movable contact of the switch travels away from the other contact, while in closing the circuit, the movable contact travels toward the other contact.

In the latter case, then, the first arcs which strike are long, and there may be long intervals between arcs, while there is little or no probability of rupturing small arcs near the end of the stroke. In the former case short arcs may die out at the zero points of the wave of current, but will be at once reëstablished owing to the short distance between contacts, while the striking of long arcs at the end of the stroke appears improbable when the movable contact is rapidly moving away from the fixed contact.

Second, in opening the circuit, if the interval between successive arcs is short, then the oscillation produced when the circuit is reëstablished finds the potential across the condenser very nearly equal to the impressed e.m.f., while in closing the circuit, at the first instant the e.m.f. across the condenser is zero, and in the successive starting oscillations the difference between the impressed e.m.f. and the condenser e.m.f. may be considerable owing to the comparatively long time during which the circuit was left open.

Record No. 3. This record starts with an e.m.f. of 9,300 volts and 49 high-tension amperes.

The high-tension line is disconnected from the generating system through an oscillation which lasts four fundamental cycles, or one-fifteenth of a second, and then the e.m.f. falls to 4,500 volts and the high-tension current to zero amperes.

Before the oscillation of current begins the peaks of the waves of e.m.f. for five half-cycles show an almost instantaneous break apparently due to arcs which occur near the maximum value of the e.m.f. on the rising side.

This shows that the switch started to open the circuit $\frac{5}{2 \times 60}$ $= 1/24$th of a second before the current in this particular conductor starts its oscillation.

Here again we note the absence of long zeros between arcs and the absence of high frequency impulses, except at the moment of final opening when the energy supplied by the generator changes from leading to lagging.

It is also remarkable that the wave ot e.m.f. is hardly affected and that only a slight over-tension is produced, while the current reaches very high values. This oscillation of the current is similar to that obtained in closing the circuit and has the usual frequency of 120 cycles superimposed on the fundamental.

An inspection of the current wave shows, however, that the 120 cycles oscillating current does not start opposite in phase to the fundamental current as seen in the previous records, but its starting zero is shifted 60 deg. from the zero of the fundamental, showing that this oscillation of current is started by an arc opening the circuit and re-closing it in one of the two switches other than that in series with the current transformer from which the record is taken.

This may explain also why the e.m.f., which is the e.m.f. of one phase of the delta, is not distorted.

We must remember, however, that the record gives the e.m.f. in the low-tension circuit and therefore does not show the phenomena which occur in the line and in the high-tension winding of the transformer, where the sudden change of conditions may produce high frequency impulses affecting the capacity of the line and the air reactance only of the high-tension winding of the transformer.

Record No. 4 is similar to record No. 3. It starts with 9400 volts and 450 low-tension amperes. After the line is disconnected the volts fall to 4400 and the current becomes the exciting current of the step-up transformer. In Record No. 4 the oscillation is limited again to the current and shows two successive arcs, the first, quickly reëstablished, in the switch in series with the current transformer, originating high frequency in the e.m.f.; the other probably in one of the other switches.

Here again the over-tension is small (30 per cent above normal maximum value), while the current reaches a very high maximum value. The total time of the oscillation is 3½ fundamental cycles, or a little more than one-twentieth of a second.

TEST No. 2

A. SWITCHING IN OPEN LINE AND STEP-UP TRANSFORMER BY LOW-TENSION SWITCHES

Fig. 3 gives the diagram of connections.

The low-tension switches are left open and the generator excited to a certain voltage. The high-tension winding of the transformer is connected to the line (high-tension switches closed). The records are taken at the moment of closing the low-tension switches, thereby energizing step-up transformer and line.

The switching by low-tension switches differs from the switching by high-tension switches mainly in the following points: First, the switches are of the oil type but of different design. Second, the energy must go through the transformer before reaching the line and therefore there will be no steep wave fronts

Record No. 4

Record No. 5

entering the line. Third, the magnetization of the transformer calls for an oscillation of magnetizing current depending on the point of the wave of e.m.f. at which the switching occurs. Generally speaking, if the switches are closed at the maximum of the wave of e.m.f. no rush of current will occur provided the residual magnetism is zero. If the switching occurs at the zero point of the e.m.f. then in the first half-cycle the induction in the iron must grow to twice the normal maximum value and a rush of current follows, etc.

These oscillations of magnetizing current will appear, of course, in the waves of low-tension current only.

Fourth, owing to the small voltage and the small distance between contacts the arcs in the low-tension switch are less severe and less frequent.

Substantially, however, the oscillations will present the same character as in the high-tension switching.

Record No. 5 gives the low-tension volts and high-tension amperes when the low-tension switches are closed. (Fig. 3).

For this record the potential transformer through which the wave of e.m.f. is taken is connected on the generator side of the low-tension switches (instead of on the line side as shown in Fig. 3). The record, therefore, starts with the no-load e.m.f. of the generator (4400 volts) and zero high-tension current.

After the oscillation is completed, the e.m.f. is 9300 volts, and the high-tension current is 48 amperes. The conditions are therefore identical to those of Record 1. The maximum overtension is 50 per cent above the final maximum value, and the current reaches a maximum value of 108 per cent higher than the final maximum value.

The oscillation lasts three fundamental cycles, or one-twentieth of a second.

The oscillation consists of two parts: First, a very short high frequency impulse in the e.m.f. due very likely to only one switch closing the circuit, then the usual oscillation of e.m.f. and current resulting from the 120 cycles damped wave superimposed on the fundamental.

It must be noted that in this case there is practically *one* fundamental oscillation, that is to say, the switches did not produce arcs which opened and closed the circuit several times before establishing permanent contact. As mentioned above, the oscillation of current due to the magnetization of the transformer does not show in this record, which gives high-tension current.

Record No. 6

Record No. 7

Another interesting point shown by the record is the rapidity with which the oscillation is damped.

Record No. 6. This is taken under conditions identical to those of Record 5. Since the potential transformer through which the wave of e.m.f. is taken is connected in this case on the line side of the switches this record starts with zero e.m.f. However, the idle line experimented upon runs parallel to the other three-phase line, which was carrying normal load during the test, and, therefore, the small e.m.f. due to the mutual induction between the two lines appears before the switches are closed.

The oscillation is very similar to that shown in the preceding record, and it is apparent that the switches did not close simultaneously, as the wave of e.m.f. starts ahead of the wave of current.

The maximum over-tension is 55 per cent above the final maximum value of the e.m.f. and the maximum value reached by the current is 165 per cent higher than the final maximum value of the current. The oscillation lasts 4½ cycles, or a little less than one-fourteenth of a second.

Record No. 7 gives the low-tension e.m.f. and low-tension current under conditions similar to those of the two preceding records.

The potential transformer through which the wave of e.m.f. is taken is on the generator side of the switches, and the record starts with the no-load e.m.f. of the generator (4450 volts) and zero low-tension current.

After the oscillation is completed, the e.m.f. is 9400 volts and the current is 450 amperes.

The maximum over-tension is 40 per cent above the final maximum value of the e.m.f. and the maximum value reached by the current is 125 per cent higher than the final maximum value of the current. The total oscillation lasts 8½ fundamental cycles, or seven-fiftieths of a second.

This record shows clearly how the switches may not close the circuit at the same instant. In fact, before the current has started the e.m.f. is distorted for 4½ cycles. This distortion of the e.m.f. shows the existence of a single-phase load, and the dissymmetry between the positive and negative waves of the e.m.f. reveal the presence of a considerable rush of magnetizing current. Then the third switch closes and the oscillation is similar to those obtained in the two previous records.

Record No. 8

Record No. 9

B. SWITCHING OUT OPEN LINE AND STEP-UP TRANSFORMER BY LOW-TENSION SWITCHES

Fig. 3 gives the diagram of connections. The generator excites the line through the step-up transformer, and the oscillation is recorded at the moment of opening the low-tension switches.

Record No. 8 gives the low-tension e.m.f. and low-tension current. It starts with 9300 volts and 450 amperes. After the oscillation is completed, the voltage and current fall to zero, since the potential transformer through which the wave of e.m.f. is taken is connected on the line side of the switch.

The maximum over-tension is 43 per cent above the normal maximum value of the e.m.f. and the maximum value reached by the current is 56 per cent higher than the normal maximum value.

The oscillation lasts a comparatively long time and is not completed in the record.

After the low-tension current has entirely died out and the low-tension switches have disconnected permanently the transformer and line from the generating system, the e.m.f. across the low-tension winding of the transformer still continues and reaches the zero value through a long, slow oscillation.

When the low-tension switches are entirely open, the high-tension winding of the transformer is still connected to the line and the presence of the e.m.f. on the low-tension side of the transformer is explained by an exchange of energy between the capacity of the line and the open-circuited inductance of the transformer.

Record No. 9 shows this very clearly. This record gives the low-tension e.m.f. and high-tension current under conditions similar to those of Record 8.

The record starts with 8650 volts and 44 amperes. In this case both current and e.m.f. do not rise above their normal maximum value. The oscillation lasts for 15 cycles, or one-quarter of a second.

After the low-tension switches are opened the line and the transformer slowly synchronize. The high-tension current (which obviously has no image in the open low-tension) consists of impulses having the characteristic shape of currents exciting iron circuits, and shows a strong decrement due to losses. The frequency of oscillation is very low, as the oscillating circuit is composed of the line capacity and of the total open-circuited inductance of the transformer. The frequency also decreases gradually because the e.m.f. and flux density decrease and therefore the inductance of the transformer increases, lowering the frequency.

TEST No. 3

A. SWITCHING IN BY HIGH-TENSION SWITCHES UNLOADED LINE CONNECTED AT THE FAR END TO STEP-DOWN TRANSFORMERS

Fig. 7 is the diagram of connections used in this test. The end of the line is connected to an unloaded 10,000 kv-a. step-down transformer, delta-connected on both primary and secondary sides.

The generator excites the step-up transformer and the oscillation is recorded at the moment of closing the high-tension switches at the power house.

This test is similar to test 1-A, with the difference that the three-phase line is not open at the far end, but closed by an unloaded three-phase step-down transformer.

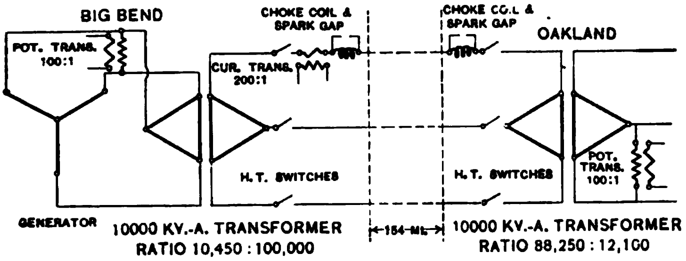


FIG. 7

Fig. 8 gives the equivalent circuits corresponding to the connections of Fig. 7. If we compare Fig. 8 with Fig. 4, we find that in the case of Fig. 4 the oscillations occur between the line capacity (neglecting line inductance) and the inductance of the generating system, while in the case of Fig. 8 the oscillations occur between the same line capacity and the inductance of the generating system connected in parallel to the open-circuit inductance of the step-down transformer. Since this open-circuit inductance of the transformer is exceedingly high in comparison with the inductance of the generating system, we conclude that the frequency of oscillation and in general all the phenomena met with in Test 1-A will be reproduced in Test 3-A. The switches used in both tests are the same.

Record No. 10 gives the low-tension e.m.f. and high-tension current when the high-tension switches connect the unloaded line and step-down transformer to the generating system.

The record starts with 4400 volts and zero current. After the oscillation is completed, the volts are 8700 and the high-tension current 43 amperes.

This record shows all the features pointed out in Test 1-A, namely, irregular closing of the switches, long zero of current, e.m.f. distorted by single-phase load, and oscillations due to a damped 120 cycles wave superimposed on the fundamental.

A new feature, however, which was not given by Record 1, is the distortion of the line current due to the magnetization of the step-down transformer. The line current assumes regular form after approximately 20 cycles following the completion of the main oscillation.

The maximum over-tension is 50 per cent above the final maximum value of the e.m.f. and the maximum value of the current is 200 per cent higher than the final maximum value of the current.

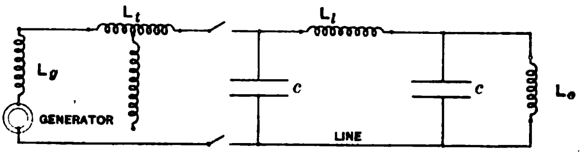
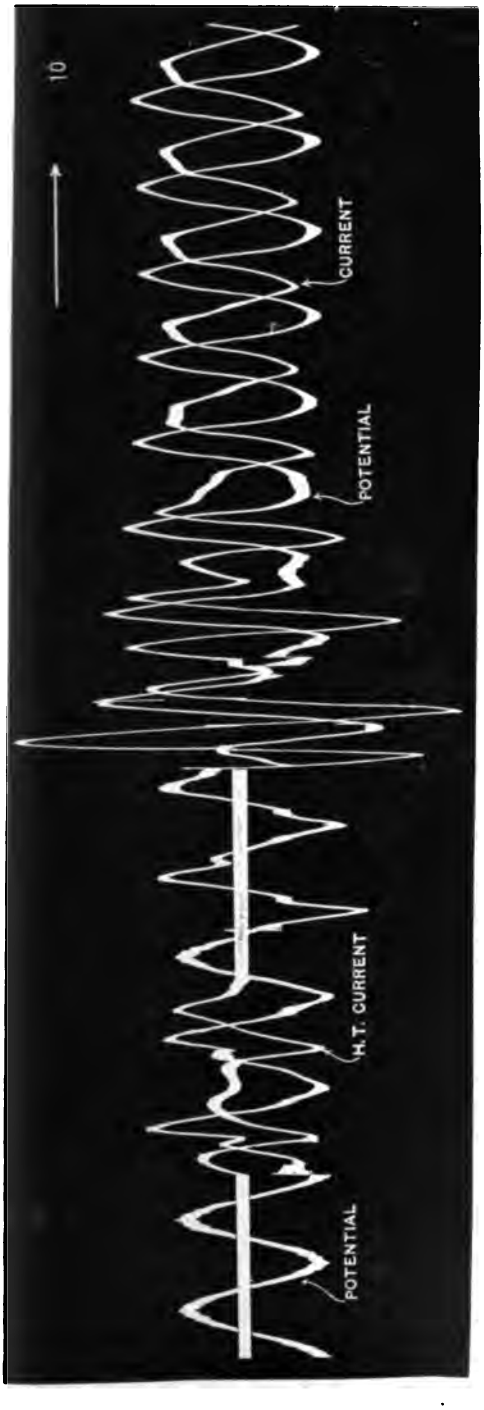


FIG. 8

The total oscillation (disregarding the distortion due to the magnetization of the step-down transformer) lasts about 8 cycles, or a little less than fourteen-hundredths of a second.

In conclusion, the phenomena are substantially the same whether the line is open or closed by an unloaded transformer.

Record No. 11 is taken across one of the low-tension windings of the step-down transformer and shows the establishing of the electromotive force on the low-tension side when the high-tension switches are closed at the power house 154 miles away. The record starts showing a small e.m.f., due to the mutual inductance on the line under experiment from the parallel loaded line, and the final voltage is 12,000 volts. This corresponds to a voltage of 87,500 volts on the high-tension side of the step-down transformer, while the voltage on the high-



Record No. 10

Record No. 11

tension side of the step-up transformer at the power house is 81,400 volts. Of course, the record does not show what occurs on the high tension side of the step-down transformer, where the incoming wave of electromotive force is partially reflected.

B. SWITCHING OUT BY HIGH-TENSION SWITCHES UNLOADED LINE CONNECTED AT THE FAR END TO STEP-DOWN TRANSFORMER

Fig. 7 gives the diagram of connections used in this test. The oscillation is recorded at the moment of opening the high-tension switches at the power house, thereby disconnecting the line and step-down transformer from the generating system.

This test is similar to Test 1-B, with the difference that the three-phase line is not open at the far end, but connected to a step-down transformer.

Record No. 12 gives the low-tension electromotive force and the high-tension current at the moment of opening the high-tension switches (see Fig. 7).

The record starts with 8700 volts and 43 amperes. After the oscillation is completed, the current falls to zero and the e.m.f. becomes the no-load voltage of the generating system, 4400 volts.

All the considerations mentioned in connection with Tests 1-A, 1-B and 3-A apply to this case, since the high-tension switches are the same in the four experiments.

The wave of e.m.f. shows that an arc dies out and is immediately reëstablished in one of the switches, then—after less than half a cycle—a similar phenomenon is repeated, and the oscillation proper begins. At this moment the arcs in the switches are of such nature as to modify the conditions of the circuit, and the current shows the characteristic resultant wave due to a 120 cycle damped oscillation superimposed on the fundamental wave.

Here, again, the oscillation is damped very quickly, a considerable part of the energy being dissipated in the long arcs of the switches.

Record No. 13 is the complement of the preceding record, and gives the potential across one phase of the low-tension winding of the step-down transformer at the end of the line when the switches are opened at the power house under conditions similar to those of Record 12.

Record 13 starts with 12,000 volts, corresponding to 87,500 volts between line conductors at the receiving end of the line, the voltage at the power house being 81,400 volts.

Record No. 12

Record No. 13

Record 13 shows that the dying out of the electromotive force across the secondary of the step-down transformer consists of two distinct periods.

First, the high-tension switches at the power house are opening the circuit and modify the conditions of the circuit for the length of about one fundamental cycle. Second, the high-tension switches at the power house are permanently opened, and the typical low-frequency oscillation takes place between the capacity of the line and the open-circuited inductance of the step-down transformer, as in the case of Test 2-B.

In Record No. 12 the maximum over-tension is 20 per cent above the maximum value with line connected, and the current reaches a value 116 per cent higher than the maximum value of the charging current. The oscillation lasts three cycles, or one-twentieth of a second.

In Record No. 13 the maximum over-tension is 26 per cent above the maximum normal value.

TEST No. 4

A. SWITCHING IN BY HIGH-TENSION SWITCHES THE STEP-DOWN TRANSFORMER (10,000 KV-A.) AT THE RECEIVING END OF THE LINE

Referring to Fig. 7, the line is energized by the generator through the step-up transformer, and the records are taken at the moment of closing the high-tension switches at the far end of the line, thereby energizing the step-down transformer.

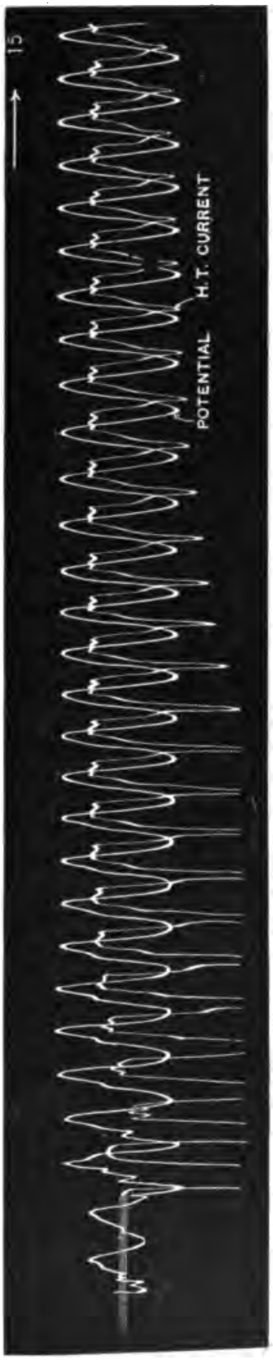
The high-tension oil type switches used in this test are different in design from the high-tension switches used in Tests 1 and 3.

However, the remarks made before on the irregular closing of the switches hold in this case also.

From a theoretical point of view, the most convenient way to picture the phenomenon consists, perhaps, in considering the winding of the transformer as a high-tension line, closed at the far end and under peculiar conditions of electrostatic capacity and inductance. Since the high-tension winding of this step-down transformer has a total length of approximately ten miles, this view of the phenomenon is plausible.

The records are taken on the low-tension side of the step-down transformer, therefore they reveal the establishing of the difference of potential across the low-tension windings, but fail to show the phenomena connected with the first instant of closing the circuit, when the line is, so to speak, drained by the high-tension winding of the transformer.

Record No. 14



Record No. 15

Record No. 14 gives the low-tension e.m.f. across one phase of the low-tension delta of the step-down transformer (which is delta-connected on both primary and secondary sides.)

The record starts, obviously, with zero electromotive force, and after the oscillation is completed the e.m.f. is 12,000 volts, corresponding to 87,500 volts across line conductors.

Record 14 shows an oscillation consisting of three distinct periods: First, a difference of potential is established across the phase of the secondary winding under investigation, and this difference of potential falls to zero through an oscillation, due very probably to the striking of an arc in one of the switches only. Second, another switch closes the circuit, and a normal wave of e.m.f. of small value is established across the secondary winding. In this connection it must be noted that if two switches close the circuit, the full potential is applied at the terminals of the winding connected across the two switches, while each of the other two windings of the delta receive one-half of the full difference of potential. This explains the low value of the e.m.f. in the second period of the oscillation. Third, all the three switches have closed the circuit, and the oscillation proper begins, showing an asymmetrical wave of e.m.f. due to the rush of current exciting the step-down transformer.

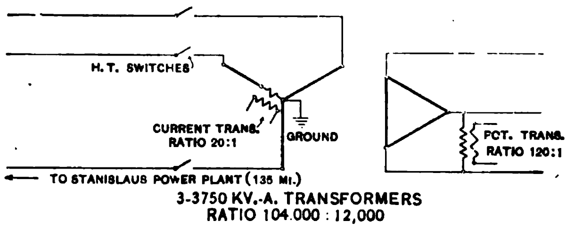


FIG. 9

Record No. 15 shows these phenomena more clearly and gives the high-tension current as well as the low-tension e.m.f. at the moment of switching in the step-down transformers at the end of the line.

Record 15 was taken on the system of the Sierra & San Francisco Power Company, when three 3,750 kv-a. transformers, unloaded, were switched in at the end of 135 miles of three-phase line.

Fig. 9 gives the connections used in this case, the step-down transformers being Y-connected, with grounded neutral, on the high-tension side, and delta-connected on the low-tension side.

The e.m.f. was taken across one phase of the low-tension delta and the current is the current flowing to ground through one of the high-tension windings.

Record 15 shows again the three periods in the oscillation of the e.m.f. and the rush of exciting current. The final voltage is 12,000 volts, corresponding to 104,000 volts across phase conductors.

The oscillation of e.m.f. in Record 14 lasts 10 cycles, or one-sixth of a second, and apparently no over-tension occurs.

A record of the generator e.m.f. and current taken at the power house at the moment of switching in the step-down transformer at the end of the line, under conditions described in connection with Record 14, shows no distortion whatever of the e.m.f. of the generator, but a distortion of the wave of current, due to the magnetization of the transformer.

B. SWITCHING OUT BY HIGH-TENSION SWITCHES THE STEP-DOWN TRANSFORMER (10,000 KV-A.) AT THE RECEIVING END OF THE LINE

Fig. 7 gives the connections used in this test.

The records are taken at the moment of opening the high-tension switches at the end of the line, thereby disconnecting the 10,000 kv-a. step-down transformer from the line.

All the apparatus and connections are the same as those used in Test No. 4-A.

Record No. 16 gives the electromotive force across one phase of the secondary delta at the moment of opening the high-tension switches, which disconnect the step-down transformer from the line.

The record starts with an e.m.f. of 12,000 volts, corresponding to 87,500 volts across line conductors, and after the oscillation is completed the voltage is obviously zero. The main oscillation lasts 4 cycles, or one-fifteenth of a second, and the maximum over-tension is 20 per cent above the normal maximum value.

The most important frequency in this oscillation is the natural frequency of the transformer, which appears more and more prominent the more elastic and unstable is the connection between the transformer and the line, *i.e.*, when the switches begin to open the circuit their arcs are short and steady and the fundamental frequency applied to the transformer is but little

Record No. 16

Record No. 17

disturbed by the oscillation of the transformer. As the arcs become longer and the supply of energy to the transformer becomes more difficult, the natural frequency of the apparatus asserts itself more and more until, at the very end, the transformer is free to oscillate at its natural frequency. This natural frequency of the transformer is approximately 1700 cycles.

After the main oscillation is completed another short oscillation occurs, due probably to an arc in one of the switches. This last oscillation recalls in its character the beginning of the oscillation of Record 14, when the transformer was switched onto the line.

Record No. 17 is taken under identical conditions to Record No. 15, with the difference that the three step-down transformers (see Fig. 9) are switched off the end of the line of the Sierra & San Francisco Power Co. The apparatus, connections and conditions of Record 17 are identical with those of Record 15.

Record 17 confirms entirely the results obtained by Record 16.

A wave of generator e.m.f. and current taken at the power house at the moment of switching out the step-down transformer under the conditions of Record 16, does not show any distortion in the waves of current and e.m.f. of the generator

TEST No. 5

A. SWITCHING BY HIGH-TENSION SWITCHES ONE OF THE THREE-PHASE LINES, UNLOADED AND CONNECTED AT THE FAR END TO A STEP-DOWN TRANSFORMER, ONTO THE END OF THE OTHER THREE-PHASE LINE CARRYING NORMAL LOAD

Fig. 10 gives the diagram of connections used in this test.

Line No. 2, excited by three 10,000-kv-a. three-phase alternators and three 10,000-kv-a. three-phase step-up transformers, carries a load of approximately 20,000 kilowatts, the majority of which is delivered at the end of the line—Oakland substation—154 miles away from the power house.

Line No. 1 is unloaded and connected at the power house to an idle 10,000 kv-a. step-down transformer.

Records are taken when the beginning of Line No. 1 is thrown at the Oakland substation by high-tension switches onto the high-tension bus bars, energized by Line No. 2. The high-tension switches are of the same type as those used in Test No. 4.

As stated before, the two lines are identical and run on the same towers for the whole length—154 miles. Therefore, when the high-tension switches are closed at the Oakland substation the circuit consists of a loop 308 miles long, whose beginning and end are both at the power house.

Record No. 18 gives the electromotive force on the low-tension side of the generator end of the loop, and the electromotive force on the low-tension side of the step-down transformer at the receiving end of the loop. One of the electromotive forces is reversed in phase to avoid confusion.

It being necessary to maintain the load of Line No. 2 in operation and in consequence to avoid great variation in voltage, the voltage at the generator end of the loop was regulated by hand at the moment the switching occurred at the Oakland substation. The records, therefore, do not give the true oscillation in voltage and over-tension, which would have occurred if the fields of the generators had been kept constant.

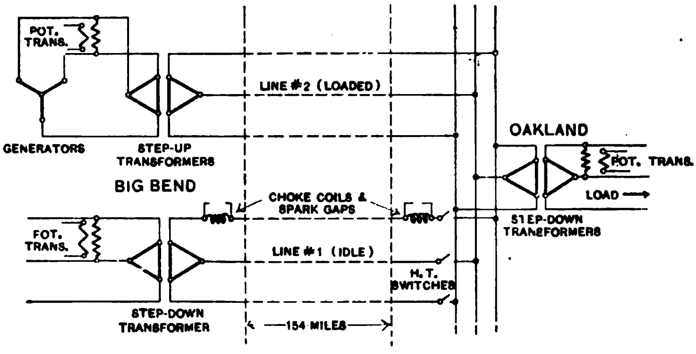


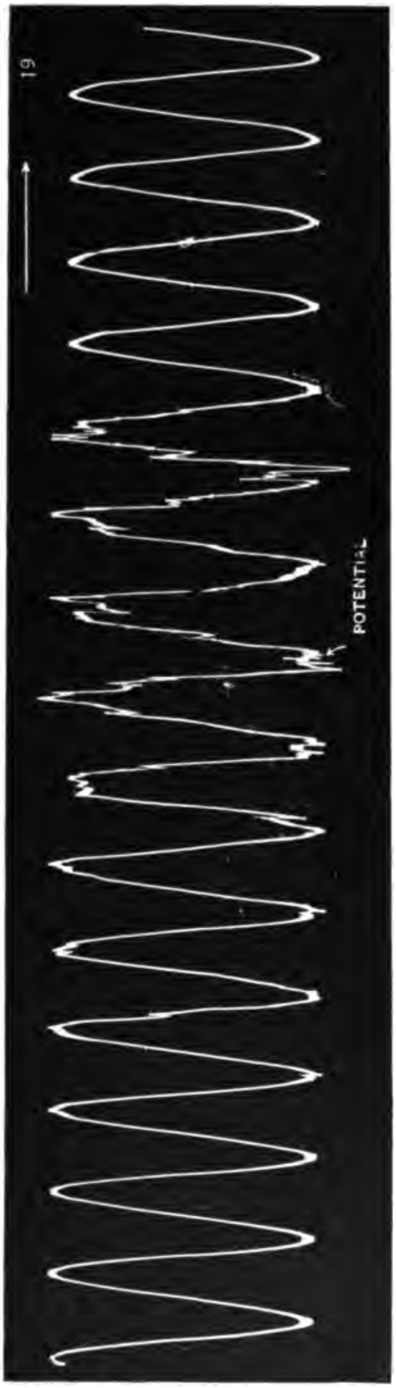
FIG. 10

When one line is connected to the other, the most prominent frequencies which appear are the full-wave oscillations of the total loop and of each line. Since the total loop is 308 miles long, its full-wave oscillation has a frequency of 600 cycles, while each line—154 miles long—has a full-wave oscillation of 1200 cycles, neglecting ohmic resistance.

Record No. 18 starts with 11,100 volts at the generator end of Line No. 2 and zero volts at the receiving end of Line No. 1. After the oscillation is completed, the regulated voltage at the generator end of Line No. 2 is 11,225 volts, while the voltage at the receiving end of Line No. 1 is 9,850 volts.

These last voltages correspond to 107,500 volts between line conductors at the generator end of the loop and 94,300 volts between conductors at the receiving end of the loop.

Record No. 18



Record No. 19

All the remarks made before on the performance of high-tension switches apply, obviously, to this case and are not repeated.

The oscillation lasts 3 cycles, or one-twentieth of a second. The main frequency superimposed on the fundamental is approximately 1200 cycles.

Record No. 19 covers the same experiment as Record No. 18, but is taken at the switching point—the Oakland substation—on the low-tension side of the transformers carrying the load.

The record starts with a voltage of 11,300 volts and after the oscillation is completed, the voltage is 11,900 volts. This last voltage corresponds to 86,800 volts between line conductors at Oakland, while the voltage at the power house, at the generator end of the loop, is 105,200 volts.

The oscillation lasts nine cycles, or less than one-sixth of a second. A frequency of about 1200 cycles is superimposed on the fundamental.

No conclusion can be drawn as to the over-tension because the voltage was regulated at the power house, as explained above.

B. SWITCHING BY HIGH-TENSION SWITCHES ONE OF THE THREE-PHASE LINES, UNLOADED AND CONNECTED AT THE FAR END TO A STEP-DOWN TRANSFORMER OFF THE END OF THE OTHER THREE-PHASE LINE CARRYING NORMAL LOAD

Fig. 10 gives again the diagram of connections.

Record No. 20 is taken under conditions similar to those of Record 18, but it gives the low-tension voltages at the generator end and at the receiving end of the loop when the high-tension switches are opened at the Oakland substation and the idle Line No. 1 is disconnected from the loaded Line No. 2.

The record starts with 11,225 volts (107,500 volts between line conductors) at the generator end of the loop, and 9,850 volts (94,300 volts between line conductors) at the receiving end of the loop. After the oscillation is completed, the volts at the generator end of the loop are 11,100 (106,000 volts between line conductors) and the voltage at the receiving end of the loop is zero.

The oscillation of voltage at the beginning of the loop lasts six cycles, or one-tenth of a second. At the receiving end the oscillation is composed of two parts: The first part covers the period during which the high-tension switches are opening the circuit. The second part occurs after the switches are entirely opened and shows the characteristic exchange of energy be-

Record No. 20

Record No. 21

tween the disconnected line and the step-down transformer at low frequency.

The two parts of the oscillation over-lap, and a frequency of 600 cycles (half-wave length oscillation of 154 miles of line) is superimposed on the low frequency surge between line and transformer.

The first part of the oscillation shows a frequency of about 1200 cycles, increasing in magnitude and superimposed on the fundamental.

Record No. 21 covers the same experiment as Record No. 20, but is taken at the switching point—at the Oakland substation—and gives the voltage on the low-tension side of the transformers carrying the load.

The record starts with a potential of 11,900 (86,800 volts between line conductors) and ends with 11,300 volts (82,500 volts between line conductors).

Before the switches are opened, the voltage at the power house, at the generator end of the loop, is 102,500 volts.

The oscillation lasts eight cycles, or a little less than one-seventh of a second, and shows the usual frequency of about 1200 cycles superimposed on the fundamental.

In the last two records, at the moment of opening the high-tension switches the field of the energizing generators was strengthened in order to compensate for the drop in the line due to the disappearance of the charging current required by the idle Line No. 1. Under these conditions Record No. 20 shows a maximum over-tension of 22 per cent above the final maximum voltage of the loaded Line No. 2, and a maximum over-tension of 21 per cent above the final maximum voltage of the idle Line No. 1. In Record No. 21 the maximum over-tension is 27 per cent above the final maximum voltage.

The records, as repeatedly mentioned above, show only the potentials on the low-tension side of the step-up or step-down transformer, and in order to form an idea of the phenomena occurring on the high-tension line these tests were supplemented by measuring—at the instant of performing the different switching operations—the maximum sparking distance across a choke-coil inserted in series with a line conductor. Figs. 3, 7 and 10 show the exact location of the choke-coils shunted by spark gap.

Each choke-coil consists of 34 turns of half-inch copper rod. The inside diameter is 5½ in., the outside diameter 6½ in., the total height 32 in.

The inductance of each coil is 0.0326 millihenry, so that 100 amperes at 60 cycles would give across the choke-coil a drop of 1.26 volts.

Each of the different switching operations was repeated several times, and the following table gives the maximum sparking distances obtained across the coil.

Test No.	Diagram	Record No.	Sparking distances
1-A	3	1 and 2	¾ in. at switching point.
1-B	3	3 and 4	⅜ " " "
2-A	3	5, 6 and 7	Less than ⅛ in.
2-B	3	8 and 9	Less than ⅛ in.
3-A	7	10	¾ in. at generating station—switching point.
		11	Less than ¼ in. at receiving end.
3-B	7	12	⅝ in. at generating station—switching point.
		13	Less than ⅛ in. at receiving end.
4-A	7	—	Less than ⅛ in. at generating station.
		14	1¾ in. at receiving end—switching point.
4-B	7	—	Less than ⅛ in. at generating station.
		16	⅝ in. at receiving end—switching point.
5-A	10	18	Less than ⅛ in. at generating station.
		19	¾ in. at switching point.
5-B	10	20	Less than ⅛ in. at generating station.
		21	⅜ in. at switching point.

These sparking distances represent the product of the intensity of the current times the steepness of its wave front. In Test 1-A, for instance, the maximum instantaneous value reached by the current is 130 amperes. A sine-wave current whose maximum value is 130 amperes would give across the choke-coil at 60 cycles a drop whose maximum value is 1.64 volts. In our test, the maximum difference of potential across the choke-coil rose to 5,500 volts, from which value we can form an idea of the steepness of the wave front. In the case of Test 4-A the difference of potential reached the value of 24,000 volts.

These results, supplemented by a general inspection of the records, allow us to draw the following conclusions:

In the class of phenomena which we have investigated, abnormally high potentials to ground or between line conductors, or across windings of apparatus, are not to be feared. The maximum over-tension which the records show is 60 per cent above normal operating value. This corroborates the theory of these phenomena, whereby the maximum possible over-tension is equal to the normal tension when the damping effects are neglected.

On the other hand, a glance at the records impresses one with the fact that, if high potential is absent, high frequency is a common occurrence. The spark gap tests tabulated above confirm this impression and point out the danger resulting to the end turns of apparatus from these switching operations.

It seems, therefore, that our common method of protecting transmission systems by lightning arresters connected in series with a spark gap does not answer the purpose effectively in all cases.

Of course, atmospheric disturbances, arcing grounds, switching heavy loads, etc., constitute a class of disturbances which may be taken care of by our present protecting system, since they may give origin to high differences of potential across conductors or to ground; but the switching phenomena with which this paper deals call mainly for protection against high frequency impulses, whose existence has been repeatedly shown by theoretical considerations, but perhaps has not been properly realized in practical operation.

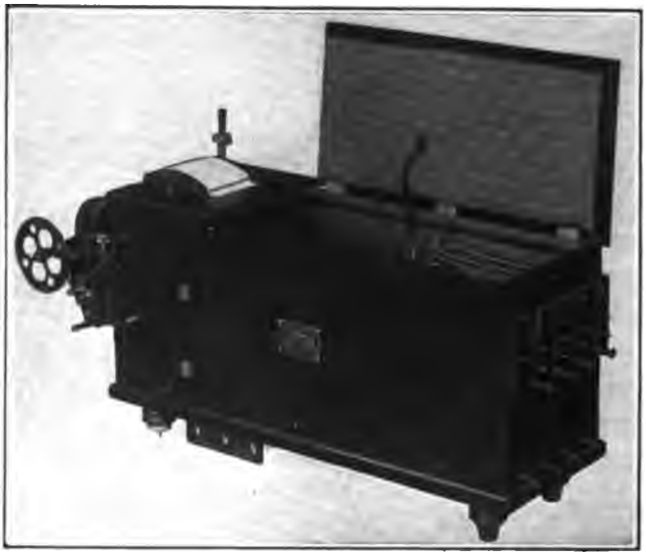


FIG. 11.—Oscillograph

The fact remains, however, that these tests were performed for a long period of time—several months—on systems operated at the highest voltages in use to-day. The switching operations performed, for instance, on the system of the Great Western Power Company during this investigation were more numerous and more severe than the system will experience in a long time of normal operation, and yet no damage whatever resulted to the line or to the apparatus, showing that the confidence of the engineers in high-voltage installations is justified.

The only danger, perhaps, consists in the possibility that some of the frequencies, at which inductances and capacities exchange energy, will coincide with the frequency of the generating system or its harmonics. A simple calculation of the constants of the circuits is, however, sufficient to obtain the value of these frequencies and to ascertain the absence of resonance.

The high frequency impulses, which originate at the point where the switching is performed, extend over a short length of the circuit only. The steepness of such impulses is quickly reduced and smoothed over and their effect is localized.

The oil high-tension switches open the circuit, as generally admitted, at the zero point of the wave of current, and since the establishing of the circuit is generally due to an arc, they close the circuit at the maximum point of the wave of e.m.f. or in its neighborhood. No attempt was made in these experiments to close the switches at different points of the wave of e.m.f., but it was preferred to obtain information as to the point of the wave at which the switches would naturally close the circuit.

Low-tension switching is preferable—when possible—to high-tension switching. In this connection, it is interesting to note that high-tension switching may be undesirable when transformers are near the switching point, in which case low-tension switching may be generally arranged for.

In energizing a line two methods of procedure may be followed (*a*) the open line is connected to the generating system, and the step-down transformer is thrown onto the end of the live line; (*b*) the step-down transformer is connected to the dead line and then line and transformer are connected to the generating system.

Our records show that the second method is the best, as it produces one oscillation only and this oscillation is of the same character as the less severe of the two oscillations which take place in the first procedure.

This paper discusses, naturally, only a small number of the numerous oscillograms taken. For each switching operation several records were obtained, and the ones published are representative.

APPENDIX

BY W. W. LEWIS

The method of recording the waves shown in the foregoing paper is described in this appendix.

Apparatus Used. The oscillograph used was of the three-element electromagnetic type illustrated in exterior view in Fig. 11 and previously described in papers before the Institute.

This is regularly equipped with the film holder, shown in Fig. 12*a* and 12*b*, which consists of three parts; a cylindrical drum to which the film is fastened, by means of spring-clips, a light-tight hollow semi-cylindrical holder in which the drum revolves, and a cover which fits over the end of the holder after the drum is in place. The holder is provided with a rectangular aperture and shutter, and a suitable arrangement for holding in place on the oscillograph.

FIG. 12A.—Standard film holder

The standard film for which this holder is adapted is $3\frac{7}{8}$ in. wide by $12\frac{3}{4}$ in. long.

An auxiliary arrangement was devised for films longer than the standard. This is illustrated in Figs. 13*a* and 13*b*. The following modification in the regular holder was necessary. The clips for holding the regulation film to the drum were removed and a slot $\frac{1}{32}$ in. wide by 2 in. long cut in the drum parallel to its axis. An aperture was cut in the lower part of the holder

FIG. 12B.—Standard film holder

¾ in. wide by 3⅞ in. long. A small wooden box with open top and inside dimensions 1½ in. by 3¾ in. was made. This fitted over dowel pins and was fastened to the holder by small hooks. The box was made for a No. 2 Bull's Eye Kodak spool, as this was a convenient size and permitted of ready means of holding in place. These spools have a hole bored throughout their length, and a brass rod with a knurled head served to hold them in place and form an axis on which they could revolve. A piece of spring brass pressed a small brass cylinder, through a hole in the side of the box, against the end of the spool and served to act as a brake. This was later found to be unnecessary.

FIG. 13A.—Film holder with attachment for long films

Method of Operation. The method of operation with the regulation film will first be described, assuming that all the preliminary steps have been taken and there remains only the exposure of the film. The procedure in making the exposure is ordinarily as follows: First, start motor which revolves the film; second, open aperture in the film holder; third, operate electromagnetic shutter.

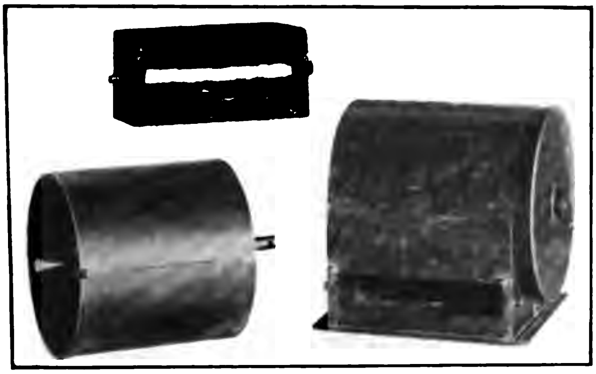


FIG. 13B.—Film holder with attachment for long films

The oscillograph shutter may be operated in two different ways, first by a worm contact, which opens the shutter and commences the exposure at the beginning of the film, stopping the exposure at the end of one revolution; after the lever is pulled the film holder makes about one revolution before the shutter opens. Second, by the instantaneous or disk contact, which opens the shutter instantaneously when the releasing lever is pulled and keeps it open during one revolution of the film.

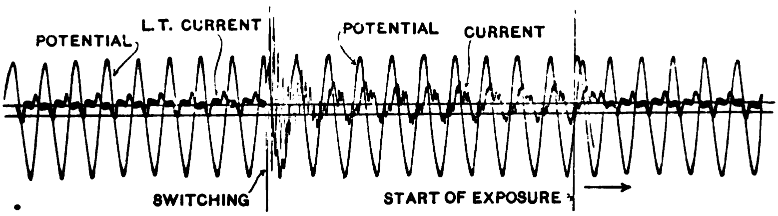


Fig. 14

For ordinary normal waves of 60 cycles frequency the film is revolved at the rate of 600 rev. per min. In recording the oscillations, however, this speed was reduced to 200 rev. per min. That is, the 12⅞-in. film passed the light aperture in three-tenths of a second. At this rate of speed 18 cycles of a 60-cycle wave could be recorded. This, in most cases of switching, would be ample to record the complete oscillation, providing the exposure of the film could be so timed that the oscillation commenced sufficiently near the beginning of the film.

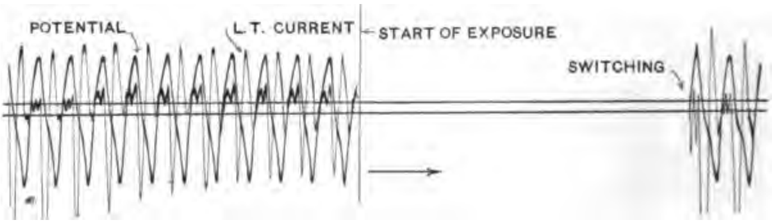


Fig. 15

The first method of operating the shutter, that is, by the worm contact, requires that the shutter lever be operated a trifle more than three-tenths of a second before the oscillation commences; the second method requires that the shutter lever be operated only in time to make sure that it is open before the oscillation takes place, probably one one-hundredth of a second before the oscillation begins, and it has the disadvantage that the exposure may begin at any point on the film.

The second method was found to be the most suitable and some excellent results were obtained. For example see Fig. 14, which shows the complete record. Here the exposure commenced about one-fourth the film length from the end, ran off the end, commenced again at the beginning of the film and ran along to the starting point. The oscillation took place about one-third of the length from the beginning of the film or about one-sixth of a second after the shutter opened. Fig. 15 shows a record that is not satisfactory. The exposure began about the middle of the film. The oscillation commenced nearly at the end of the film and had not been completed when the starting point was again reached. Other records were obtained in which the exposure commenced at the beginning of the film and the oscillation was wholly completed within the length of the film, Fig. 16 shows one of these.

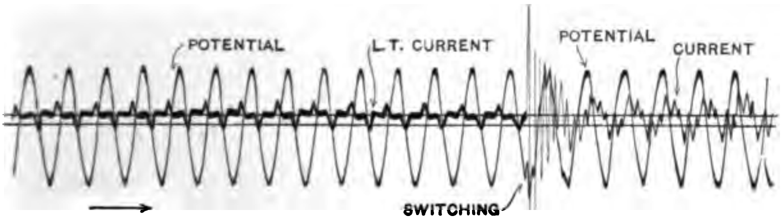


FIG. 16

It will be readily appreciated that this required very accurate timing. Automatic timing devices were tried, but were found to be unduly complicated, in view of the nature of the power plant wiring and switching; and especially so when the switching was done in one station and the oscillograph located in another.

To overcome the shortcomings of the regular film and insure more certain results, the arrangement for long films was devised. This arrangement immediately extended the usefulness and accuracy of the apparatus. The films used were 3½ in. wide by 48 in. long, so that instead of three-tenths of a second in which to make the exposure, about 1.1 seconds were now available, and approximately 66 cycles could be recorded; the exposure always commencing at the beginning of the film.

The procedure in loading these films is as follows: They are loaded in daylight as with an ordinary roll film camera, the film being placed in the box and the end of the black paper covering drawn through the aperture in the holder and inserted in the slot in the drum (see Fig. 17). About one-half turn is made around the drum with the black paper and the box drawn up and fastened to the holder.

The routine in exposing the standard or short films was to first start the motor that revolved the film, next open the aperture in the film holders and lastly to operate the electromagnetic shutter at the proper instant. With the long films the routine was in the reverse order, that is, to first open the light shutter and keep it open, next open the aperture in the film holder, the black paper protecting the film from exposure, and finally at the proper instant to start the motor and wind the film from the spool onto the drum.

FIG. 17.—Showing method of loading a long film

The motor was operated at high speed and the reduced speed required by the film secured by means of counter-shafting, which is clearly shown in Fig. 18. The belts, which consisted of heavy twine, were made very tight. By this means acceleration was secured in probably one-fiftieth of a second, or practically instantaneously. The film was preceded by 20 in. of black paper of which about one-third had already been wrapped around the drum, leaving two-thirds, or one turn, to be wound around the drum before the film could be exposed. This allowed about three-tenths of a second to elapse between the starting of the motor and the exposure of the film, giving a wide factor of safety for acceleration.

As familiarity was gained with this device, the efficiency of operation was increased from probably 50 per cent with the short films to approximately 95 per cent with the long ones. At one particular station where fifteen exposures were made, all were successful—an efficiency of 100 per cent.

The significance of this can be appreciated when it is stated that all the signals were manual and vocal. When the switching was done in the station where the oscillograph was located, the signal passed from the oscillograph operator to the switchboard attendant either direct or through an intermediate person. When the switching took place at one station with the oscillograph located at another, a telephone and two or three additional men were needed in the signalling chain. As described in the paper, Records 18 and 20 were taken at Big Bend Power Plant, while the switching was done at Oakland, 154 miles distant (see map, Fig. 1). The oscillograph set-up and the station layouts necessitated a signaling chain consisting of five men; the oscillograph operator, an intermediary and a telephone attendant at Big Bend; a telephone attendant and a switchboard attendant at Oakland. In addition to allowing time for

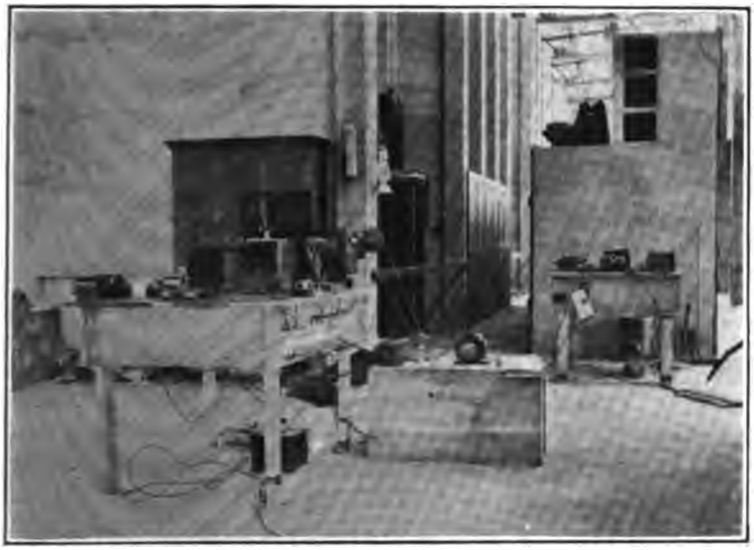


FIG. 18.—Oscillograph and testing equipment at Big Bend

the transmission of the signals, it was necessary to make an allowance for the lag in the switch mechanism, which was sometimes appreciable.

Several exposures were made in which a line was switched on and off the generating system and both operations recorded on the same film.

To assist in judging the proper time to be allowed for the transmission of the signals, etc., one trial switching operation was usually performed, the movement of the beam of light being noted on a ground glass inserted in place of the film.

None of the records given in the foregoing paper show more than 10 in. of the 48 in. of the film exposed, with the exception of Record 15, which shows 20 in.

In most of the exposures the zero line was not recorded, as it was not necessary and only complicated the records. To record the zero line the procedure with the short films is to make a second exposure with the vibrator switches open. With the long films a second exposure is not possible, but the zero line is recorded at the time of the original exposure by using the beam from the third or idle vibrator.

When two vibrators were in use it was found most satisfactory to cause their zero positions to coincide, in some cases reversing one of the waves, to better distinguish them when the two waves were nearly in phase. A few records were taken in which the zero positions of two waves were widely separated, thus separating the waves and preventing confusion. This, however, necessitated waves of small amplitude, and owing to the uncertainty as to the probable amplitude during an oscillation the method of using coinciding zeros in the center of the film was found most suitable.

The unloading is done in a dark room. To unload the standard film, the cover is removed, the drum taken out and the film unfastened—the reverse of the loading operation. With the long films it is only necessary to loosen the hooks and remove the wooden box. The film is then pulled out through the aperture in the holder. The cover and the driving pin need never be removed.

During all the switching operations one terminal of the vibrator was connected to one of the poles of the electromagnet between which it vibrated, in order to eliminate the effects due to electrostatic differences of potential, which were sometimes present.

Discussion on "Electric Line Oscillations." Chicago, June 29, 1911.

C. P. Steinmetz: Mr. Faccioli's paper gives an oscillographic investigation of the transients, that is the oscillations, traveling waves, etc., produced by switching operations in transmission systems at the utmost limits of voltage and distance reached today: a 100,000-volt system, comprising a three-phase circuit of over one hundred and fifty miles, and in some of these oscillographs, records are presented in which two of these lines have been connected in series, so giving a circuit of over 300 miles operating at 100,000 volts. The taking of these oscillograms naturally involved considerable difficulties, quite a number of problems, which are discussed by Mr. Lewis. Specially favorable conditions for the investigation of the phenomena existed in this circuit, as oscillograms could be taken at the generator end, at the receiving end and in the middle of the circuit, and thereby it was possible to investigate the propagation and the attenuation of line disturbances, created at one point of the line, and the effect produced by them at other points, at the middle and ends of the line and at the transition points between line and other apparatus. Tests were made on opening and closing the circuit at the generating end of the high potential lines, by high potential switching and by low potential switching, with the line open at the other end, and with the line connected to step-down transformers. Tests are also given of the phenomena occurring when 150 miles of line is switched to another 150 mile line, or switched off, at 100,000 volts.

An interesting and important conclusion from these tests is that the oscillations and disturbances of such a system are not those of a line or circuit of uniformly distributed capacity and inductance, are not the phenomena usually described in the text-books, but that in the oscillation the transformers and the generating system participate, and the oscillation thus is a compound oscillation of the system, comprising different sections of different characteristics and constants: transmission line, step-up transformer, generator. Two conditions exist, one, where the circuit consists of the transmission line, the leakage reactance of the step-up transformer and the generating system, with transition points between these circuits. The other condition results by disconnecting the generating system from the line, and gives a circuit comprising the transmission line and the mutual inductive reactance of the transformer, which is very many times greater than the leakage reactance, and is an ironclad reactance, while the leakage reactance is essentially an air reactance. The latter case results in a frequency of oscillation of the compound circuit which is very low, below machine frequency, reaching as low as from 20 to 30 cycles. As seen in Figs. 8, 9, 13, 20, etc., voltage and current waves are distorted greatly by the periodic variation of the ironclad inductance, and successive half waves increase in length, due to the increase of inductance with the decrease of voltage and thus of magnetic density. When the generating system is implicated and surges with the line, the transformer inductance is much lower, as it is the leakage inductance, and the frequency of oscillation of the compound circuit is higher, is between the second and third harmonics, approximating 120 cycles. A number of interesting phenomena result by a superposition of this frequency of oscillation on the fundamental generator frequency, since the oscillation frequency is approximately twice the generator frequency. This means in the resulting current alternate half waves are subtracted from each other and added, so that there are successive periods where the current is nearly extinguished by superposition of two opposing currents of the two different frequencies, and periods where it is exaggerated. Such a current passing through a high potential switch gives the effect that the arc is extinguished at the low current values and rekindles again at the high current values. This gives not a single transient oscillation, but a series of successive transients, as shown in the oscillographic Records 1, 2, 10, etc. Record 2 especially gives a very long extinction of current. We must consider that in switching at these high voltages the contact is not made and broken as metallic contact, but before the switch terminals meet, the spark jumps ahead and closes the circuit by an arc. When the switch contacts separate in opening the circuit, the arc follows for quite a number of cycles.

It is obvious that the arcs of the different phases do not necessarily extinguish at the same moment, nor start at the same moment, and that, therefore, even if the switch mechanism of all phases is absolutely simultaneous in its action, the actual closing and opening of the circuit of the different phases is not simultaneous, but some of these open or close ahead of the others. This gives a momentary unbalancing of the circuits, and leads to some interesting phenomena as shown in Records 7, 14 and 15, etc. Specially noteworthy is the frequent appearance of high frequency waves, as shown in most of the oscillograms, Records 1, 2, 3, 10, 12, 16, 17, 18, 19, 20, 21, etc., which indicates that high frequency is an ever present phenomenon in high voltage transmission lines. The observations made with spark gaps across small inductances give further evidence of extremely high frequencies. Especially interesting is the appearance of a high frequency oscillation in a transformer when disconnecting it from the line, as in Records 16, 17, etc., as this shows the danger to which the transformer is exposed. Record 20 is interesting by the superposition of a high frequency oscillation of rapid decay, on the low frequency oscillation of the compound circuit consisting of 150 miles of line with step-down transformer, produced by disconnecting this circuit from another 150 miles of line. Here the origin of the high frequency seems to be the readjustment of the stored energy between the sections of the compound circuit, which precedes its oscillation as a whole.

Especially important is the absence of high frequency in the oscillograms taken at a considerable distance from the starting point of the disturbance, as the switch. This illustrates that high frequency does not travel very far, but is local, and that the more, the higher the frequency: the extremely high frequencies observed by spark gap across a small reactance, had vanished already at a short distance from their origin.

The starting transient of a transformer is shown in Record 15, which illustrates the excessive starting current, lasting for many cycles, and its unsymmetrical shape, resulting from even harmonics.

It is obvious that I cannot discuss in detail the oscillograms, but they have to be carefully studied from the records. However, the conclusions from these investigations are essentially that switching in a high potential circuit of considerable capacity is a dangerous operation, even with the oil circuit breaker, and therefore it should be avoided as far as possible. It is dangerous, however, only if the switching occurs at or near a transition point between circuit sections of different constants, near a transformer, but no danger results where switching is done in the middle, or inside of a homogeneous circuit, as from one transmission line on to another transmission line with no transition point near it. Therefore, where a change in the circuit constants by a transition point is near, as a transformer, you should arrange the switching on the low tension side, as this is safe, but where there is no transformer nearby, you have to switch in the high tension side, but there all the circuits which are involved are line circuits of the same or similar constants, and there switching is relatively harmless. Furthermore, these records show that whatever takes place, any change of the circuit conditions results in the appearance of high frequency, and high frequency is in those high voltage systems an ever present phenomenon, as much as the waves of the ocean are ever present. High frequency impulses, as oscillations of current, and corresponding oscillations of voltage, are of moderate amplitude, usually not high enough to raise the voltage of the system sufficiently to discharge over the lightning protective apparatus, and lightning arresters therefore usually can not take care of these high frequency oscillations. Their danger consists in the possibility of locally piling up the voltage, in inductive parts of the circuit, as in the end turns of the transformers, generators, etc., etc. The danger of these high frequency surges thus results not from an increase of the voltage of the system, but from the local excessive potential differences which they may create in apparatus.

Max H. Collbohm: The main points brought out in Mr. Faccioli's paper are, in the speakers' opinion, the following two:

1. The fact that the higher harmonics in the generator wave at no load practically disappear with a certain increase in current.
2. The appearance of high frequency oscillations during switching operations in long distance transmission systems.

I have made the same observations in a 66,000 volt 25-cycle 63 mile transmission system where the generator wave contained at no load the 17th harmonic of about five per cent amplitude. The influence of this 17th harmonic could, however, not be detected by any appreciable increase in the charging current over that due to the fundamental frequency.

This seems to indicate that in a high voltage transmission system the requirements for a smooth generator voltage wave without harmonics, which heretofore have been strongly advocated, lose a great deal of their importance.

Regarding the second point, *viz.*, the high frequency oscillations of moderate amplitude and the consequent building up of high voltages when meeting inductive reactance, it must be said that our present American types of arresters, including the electrolytic arrester, do not afford any protection whatever under such conditions.

As pointed out in my discussion on Dr. Steinmetz's paper on "Central Station Development," there has however recently been developed in Europe an arrester to meet just that particular condition. It consists of one or more banks of dry condenser elements shunted by a choke coil of very high reactance, the condensers acting on high frequency while the choke coil takes care of static accumulations. This type of arrester has been installed in various prominent European transmission plants with great success.

In the same discussion there has also been mentioned a scheme devised by the speaker for protecting against high frequency disturbances, namely, the use of solid iron wire of high permeability for station wiring, extending from the high tension transformer taps to the point where the arrester taps on to the transmission line, the choke coils being also wound with soft Swedish iron.

The protection afforded by this scheme is obtained by the very high resistance of the iron wire under high frequency, due to the skin effect.

The same discussion also mentions a scheme to protect the high tension series transformer from damage due to the high frequency, which consists in the use of a small electrolytic cell shunted across the high tension terminals of the series transformer. The before mentioned schemes have been tried out with entire success in a prominent hydroelectric plant which has passed through a great number of very severe lightning storms which caused the arresters to act very frequently without the least damage to the station apparatus.

D. B. Rushmore: The real motive of Mr. Faccioli's paper is that high tension transmission systems have been causing disturbances, and as the voltages have been increased there have been certain breakdowns in insulations and in apparatus, which were not understood, and a great many theoretical explanations have been offered for them, but so far as I know this is about the first actual investigation of this particular class of phenomena on an actual transmission line of high voltage to find out what really existed there. Our transformers may break down, and it is very difficult to tell why they do break down. As both manufacturers and operators are desirous of avoiding failures, it is necessary to actually investigate the phenomena taking place on transmission lines, so as to have facts at hand, instead of more or less empirical hypotheses. We have certain elements of mass, velocity, inertia, elasticity in mechanical or electrical systems, and it helps very much to have us understand what is taking place in electrical systems, if we form some conception of the mechanical analyses. Interrupting a high tension circuit often makes me think of a fact in our daily experience, in earlier times, in getting the running light losses on synchronous motors, by throwing the belt, which used to be an exceedingly interesting thing with large motors—we used to throw it across the shop, anywhere in the neighborhood to get it out of the way, and you have just about the same element when the transmission of energy is interrupted very suddenly; we have about the same class of phenomena, with very different appearance. After having investigated and discovered what the real facts are in high tension switching, we can understand the matter more clearly; and I want to offer here a word of sincere appreciation for this investigation and for the elucidation which this matter had from Dr. Steinmetz both before and at the meeting, in clearing up a great many vague ideas which we had of what was taking place on transmission systems.

As a matter of fact, on high tension transmission systems the insulation is getting to be so strong, that if every thing is in proper shape, the number of disturbances is going to be very much smaller, because while these oscillations may exist the voltages induced by them are below the factor of safety of the insulating material on the line and on the apparatus. But, as Dr. Steinmetz very clearly brought out, the line is not the only thing to be considered, because when we have an oscillation of electricity, as it is playing back and forth into the capacity of the line, or through some leak of the line into the capacity of the ground, the place where the harm is done is where the voltage piles up, and that is always across the reactances.

Now, in the future design and operation of transmission systems we are going to find out through that wonderful instrument the oscillograph—and it is rather interesting to note in these scientific investigations the important part the oscillograph has played—more and more nearly exactly what is taking place, and we shall have an increased ability to interpret the results, something that is not always done without a considerable expenditure of time and money. But as a general result, somewhat repeating what has been said, the result of this very important and interesting investigation is a reassurance concerning our ability to deliver uninterrupted power to our future high tension transmission systems.

C. P. Steinmetz: I would like to make a few remarks regarding the spark gap test that showed a voltage of some 20,000. From the characteristics of the line, you get the relation between voltage and current of the transient: $\sqrt{\frac{L}{C}}$, and from the starting voltage of the transient, we get the transient current. This, with the inductance of the reactive coil, gives the frequency required to produce the observed spark voltage of this traveling wave. This calculation gives an approximate frequency of 2,000,000 cycles. We thus find that there are frequencies of oscillation moving around the system, varying from 20 cycles, which is the oscillation of the transformer, up to the magnitude of 2,000,000 cycles.

Percy H. Thomas: As Mr. Rushmore has said, this work of Mr. Faccioli's is one of the first efforts that has been made on a full sized plant to determine just what happens by actual measurement in generators and step-up transformers at the time of switching. It would be well to consider in connection with this paper a paper presented at the Asheville Convention of the Institute in 1905, which was a somewhat similar study of the static effect produced by switching. These results, you will remember, were obtained in 1902 on some plants in the middle western part of this country. The tests were limited strictly to the experimental study of static phenomena, which, as Dr. Steinmetz said, is not the subject of the present paper—these two papers thus become complementary.

For fear it should be considered a misprint, I will call your attention to a point in the table (p. 1841) under 4-*a* opposite 14 in the last column, you will note that 1½ in. spark-gap was jumped at the receiving end across a small choke coil in series with the line, representing a momentary voltage of 24,000 volts. This is a purely static phenomenon of course, this measurement of voltage across the choke coils having been an incidental test in connection with the main experiments. The point about that extra high voltage, this being higher than in any other test, is brought out in the other paper of which I speak, namely, that the most serious results of a static nature from switching come when you connect a piece of apparatus of a different potential to a line which has stored in electrostatic capacity a sufficient quantity of energy so that it can *instantly* charge the idle apparatus with which you are connecting it. This particular case was a transmission line, on the end of which was a transformer, the transmission line having a large capacity of energy stored, ready for immediate delivery, if you will accept that expression, to charge up the transformer initially or nearly. The result was that a choke coil connected to the lead of the transformer received a very excessive strain.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 29, 1911.

THE ELECTRIC STRENGTH OF AIR.—II

BY JOHN B. WHITEHEAD

In a former paper[1] with the title of the present one the author described a series of investigations of the conditions under which the air breaks down in the neighborhood of clean, round wires subjected to high voltage. A principal feature of that paper was the description of a method for observing with a close degree of accuracy the critical or corona voltage for various sizes of wire when centred in cylinders forming the opposite side of the source of voltage. There has been a great diversity in the values of critical voltage as given by other observers, who for the most part have used the appearance of the visible corona and the readings of instruments in the primary circuits of transformers as indications of the voltage at which the air breaks down. The method referred to was developed as the result of a conviction that the laws governing the loss between high-tension lines could not be satisfactorily determined without a study and knowledge of the fundamental phenomena. So far therefore these investigations have been concerned only with the conditions under which the air actually breaks down, causing a large increase in conductivity and power loss. The results of the former paper show among other things that when corrected for wave form, temperature and pressure the electric intensity at the surface of a clean, round conductor, corresponding to the voltage at which corona starts and loss sets in, is a constant for each size of wire. This value of surface intensity varies with the temperature and pressure and is that corresponding to the maximum value of the voltage wave. It is different for different sizes of wire but is independent of the material

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1. J. B. Whitehead, Transactions A.I.E.E., 1910, XXIX, II, p. 1159.

of the wire, of the moisture content, and of the amount of free ionization in the air. In the present paper some further facts bearing on the fundamental relation between diameter and critical surface intensity are given, and a series of investigations of the influence of stranding a conductor, of variations of atmospheric pressure, and of frequency on the critical electric intensity are also described.

Critical Surface Intensity. As used in these papers the term " critical surface intensity " refers to the voltage gradient at the surface of a conductor at which the visible corona appears and ionization of the neighboring air with accompanying conductivity begins. These two phenomena are exactly contemporaneous as has been shown in the foregoing paper and by numerous later observations. No investigations have been undertaken showing the variation of the loss above the critical voltage. For the details of the method by which the critical intensity is observed to a close degree of accuracy the original paper must be consulted. The principle is simple, however, and may be described briefly. The wire is stretched along the axis of a metal cylinder and the voltage is applied between them. Air may be passed through the cylinder by means of two lateral tubes near the ends, the walls of the cylinder at these points being drilled with a number of small holes. Close to one set of these holes a wire mesh electrode, connected through an insulating bushing to a sensitive electroscope, is placed. As soon as the air around the wire breaks down under increasing voltage a rapid and sharply marked leak of the charged electroscope begins. Observations may be repeated at will, and after any interval; when corrected for temperature and pressure a most satisfactory constancy of results is obtained. For convenience of reference a sketch of the apparatus is shown in

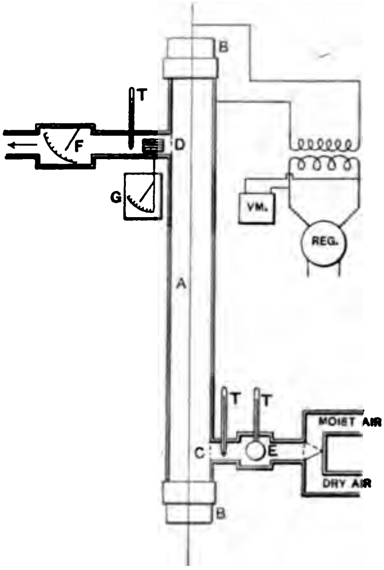

FIG. 1

Fig. 1. The constancy of the relation between critical surface intensity and diameter of wire was shown by various combinations of material and sizes of wire and cylinder. The results on this portion of the work are plotted in Fig. 2 in which the letters *A* and *S* indicate points observed with aluminum and steel wires respectively; the remaining points are for observations with copper wire. A formula connecting the value of the critical surface intensity with the diameter of the wire has been added

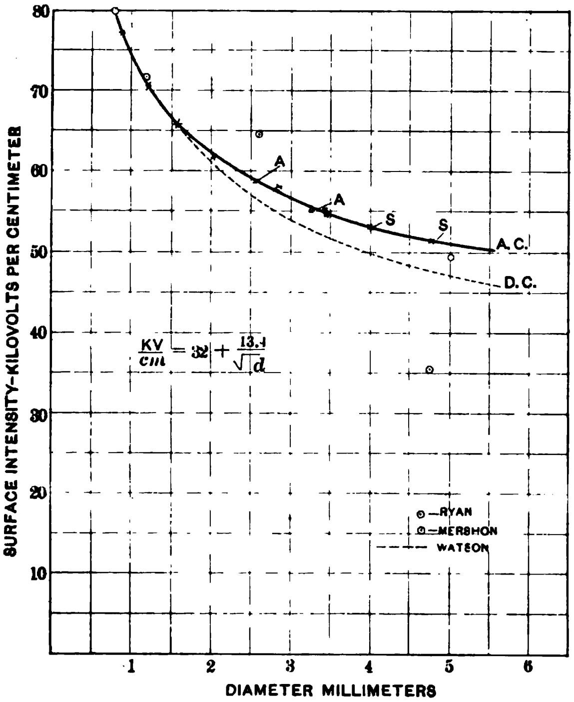

FIG. 2

to Fig. 2. This formula which refers to the conditions of 760 mm. pressure and 21 deg. cent. temperature is as follows:

$$E = 32 + \frac{13.4}{\sqrt{d}} \qquad (1)$$

in which *E* is the critical surface intensity in kilovolts per centimeter and *d* is the diameter of the round conductor in centimeters. For the discovery of this simple relation I am indebted to Dr. Alexander Russell. Over the range of diameters covered by the observations this law is obeyed with remarkable closeness. In Table I values calculated from formula (1) are compared with those observed and plotted in the curve of Fig. 2. A column showing the percentage error referred to the calculated values is also given. It is seen that this error is less than 1 per cent throughout the whole curve with the exception of one point which represents a reading on aluminum wire. This point actually falls outside the curve of Fig. 2 a fact which is due to the impossibility of obtaining a smooth polished surface on an aluminum wire. A roughened surface invariably causes a lowering of the critical intensity. The closeness with which this law is obeyed makes it reasonably certain that it obtains over a wider range of diameter. If this be so the value of critical surface intensity for a wire of 0.25 in. (0.635 cm.) diameter is 48.8 kilovolts per cm., and for 1 cm. diameter, 45.4 kilovolts per cm. Hence the value of critical intensity is still varying considerably for wires in the neighborhood of No. 4/0 B. & S. Such a uniform and regular law should prevent all future use of such artificial suggestions as that the air in the neighborhood of a wire has a greater electric strength than at a distance, and that the thickness of this so-called layer becomes constant above a certain diameter.

TABLE I

OBSERVED AND CALCULATED RELATION BETWEEN DIAMETER OF WIRE AND CRITICAL SURFACE INTENSITY

Diameter cm.	Surface intensity		Difference per cent
	Calculated	Observed	
0.089	76,950	77,100	+0.19
0.122	70,400	70,875	+0.67
0.156	65,950	65,880	—0.1
0.205	61,600	61,680	+0.13
0.254	58,600	58,750	+0.25
0.276	57,500	58,000	+0.87
0.235	55,600	55,000	—1.08
0.340	54,980	55,100	+0.21
0.399	53,230	53,050	—0.33
0.347	54,780	54,500	—0.51
0.475	51,460	51,400	—0.11

The relation which has been described above will only obtain for fixed conditions of pressure and temperature. The laws covering the variation of the critical intensity with temperature and pressure are apparently within easy reach. It is also to be noted however that in actual transmission lines a loss begins at values considerably below those corresponding to formula (1). There are other disturbing factors which are not so readily located but which apparently all take their rise in conditions which affect the value of the surface potential gradient of the conductor. Thus dirt, or any other irregularities, and as shown later in this paper, the stranding of a conductor will lower the critical surface intensity. Moisture content of the air has no influence. The state of the air as regards the amount of free ionization present has been suggested as an important factor. Ionization means conductivity and there is a certain amount present at all times in the atmosphere. This amount however is extremely small. It has been estimated that the number of ions present is about 1000 per cubic centimeter of air. The charge on each one of these ions is about 4.6×10^{-10} c. g. s. electrostatic units, or 1.5×10^{-20} electromagnetic units. Under the influence of ionizations of this amount the most sensitive electroscopes require an extremely long time, say of the order of several days, to lose their charge. The variation from time to time and place to place, in the amount of this free ionization is very small, say from 1 to 4 or 6 times. The amount of ionization caused by the corona is incomparably greater in amount. The air becomes extremely conducting and the electroscope loses its charge within a second or two. It follows therefore that if the presence of a greater or less quantity of ionization in the air has any influence on the point at which corona sets in, a foregoing presence of corona should materially affect the voltage at which corona begins again. For example, let us suppose that the voltage on a clean round wire is raised gradually and carried above the corona voltage and then gradually lowered. If the presence of a large amount of ionization lowers the critical voltage then as the voltage on the wire is lowered the corona should continue down to a value lower than that at which it started. This is not the case however; the corona ceases at exactly the same voltage at which it begins. Further it has been shown by the author that for a voltage well above that at which corona appears the corona is periodic and begins and ends on the voltage wave at approximately the same value. It is readily possible to obtain extraneous sources of ionization and in order to secure direct evidence as to the influence of the amount of ionization in the atmosphere the following simple experiment was performed: A clean copper wire 0.156 cm. in diameter was stretched at the axis of a cylinder 17.5 cm. in diameter, the cylinder being constructed from coarse wire screen with a mesh about one centimeter square. A large X-ray tube enclosed in a light-tight box was set up immediately adjoining this apparatus. When in operation the X-rays ionized the entire region in the neighborhood. A rough laboratory electroscope placed on the far side of the cylinder lost its charge quite rapidly. The voltage at which the visible corona appears on the wire is absolutely independent of the state of the X-ray tube. The visible corona under these circumstances can be read to an accuracy of ½ per cent or even closer, and the above experiment was carried out by several observers. The only claim based on experiment that such an influence of ionization exists is that of Ryan[2] described in his recent A. I. E. E. paper. In this experiment the central one of three parallel wires in one plane was connected to a static induction machine. The alternating voltage corona appeared on the outer wires at a lower voltage when the central wire was discharging than when it was not excited. Ryan's conclusion is that the central discharging wire furnishes a supply of ions which enable the outer wires to discharge at a lower value of voltage. It is obvious as pointed out by the writer in discussing the paper that the presence of the central wire raises (or lowers) the value of the absolute potential of the outer wires above that indicated by the voltage between them. This higher potential causes the normal critical surface gradient corresponding to the size of the outer wire to be reached at a lower voltage.

All the facts and phenomena so far observed indicate that the state of the air as regards ionization has no influence on the value of critical surface intensity. So far as the writer is aware there is no experimental evidence in support of the contrary contention. It is quite possible that the presence of considerable amounts of ionization may cause a small loss before the principal and far greater loss due to the presence of corona sets in. Such a loss would be due to the actual conductivity due to the presence of the ions. This conductivity as already pointed out is extremely small. In the present state of uncertainty as to the conditions controlling the critical voltage and

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[2] H. J. Ryan, TRANSACTIONS A. I. E. E., 1911, XXX, I, p. 1.

the variation of the loss it appears unwise to confuse the problem by the introduction of the ionization theory. This theory has been pushed to extreme lengths and used in the vaguest manner to explain discrepancies and unrecognized phenomena. Its basic principles are undoubtedly correct and it has been a most valuable instrument in the hands of physicists. Corona formation is undoubtedly due to secondary ionization, or ionization by collision, but it is of doubtful wisdom to discuss the ultimate nature of these phenomena when the simple laws they follow have not yet been definitely fixed. To the engineer these laws are more important than their explanation in terms of deeper-lying and often invisible phenomena. The language of the ionization theory is therefore in this paper confined to a discussion at the end.

EFFECT OF STRANDING THE CONDUCTOR

It is quite obvious that if the surface intensity is the determining factor in the voltage at which the corona appears on a given conductor, a stranded conductor should have a critical voltage lower than that of a solid conductor of a diameter equal to that of a circle tangent to the strand. The influence of stranding has been studied by Mershon[3] and his results are contrary to the above conclusion. In fact, he states that under certain circumstances the stranding of a conductor may actually have the effect of raising the critical voltage above that of a solid wire of diameter equal to the overall diameter of the cable. Jona[4] has given an expression due to Levi-Civita from which the value of the maximum surface electric intensity for cables of various numbers of strands may be computed. This expression involves a hypergeometrical series whose evaluation requires some labor. Jona gives a solution for the particular case of six strands in the outer layer. This solution states that the maximum surface intensity occurring in a cable having six strands uniformly spaced in the outer layer is 1.23 times that corresponding to a solid wire having the same cross section. The expression of Levi-Civita makes no allowance for the spiral of a cable. The spiral undoubtedly has the effect of lowering the intensity on the outer portions of the strand. Further it is much more important to refer the behavior of a stranded conductor to its outside diameter since in many cases the interior

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[3] Mershon, TRANSACTIONS A. I. E. E., 1908, XXVII, II, p. 886.

[4] Jona, *Trans. Int. Elect. Congress*, St. Louis 1904, Vol. II, p. 550.

of such conductors is made up of a material different from that of the strand.

An investigation was therefore made of the critical voltage of a number of cables of stranding ranging from three to nine conductors uniformly filling the outer layer. The interior space of these conductors was filled with a single wire, or several wires of suitable size, but in each conductor the wires of the outer layer were all of the same size, 0.162 cm. diameter. The cables were subjected to no special treatment for cleaning or making their surfaces smooth other than to run over them with a piece of crocus cloth in order to remove any points or other imperfections on the round surface of the strands. For the sizes three, four and five strands the experiments were performed by means of the electroscope method of Fig. 1. Beyond these sizes the critical voltage was too high for the insulation of that apparatus and resort was had to the visible corona as indication of critical voltage. In the smaller sizes mentioned the visible corona and electroscope leak were contemporaneous. For the five and six strand conductors the outer cylinder was of woven wire made carefully circular by wooden forms and of diameter 17.13 cm. For the largest sizes the outer cylinder was of 12 in. tin pipe about five feet long. Owing to the want of rigidity of this pipe its diameter as affecting the value of critical surface intensity could not be determined accurately. Comparisons of the values in the different cylinders were readily obtained however with solid wires and the critical voltage was extremely sharply marked by means of the visible corona and also by the sound of the discharge. The corona was observed through a narrow slit cut in the side of the pipe, the latter being maintained at ground potential. By either of these methods it was possible to repeat the readings of the critical voltage to an accuracy well within one per cent. A summary of the observations is given in Table II. This table gives the diameter over all and for comparison the behavior of a single wire having the same over-all diameter. The values of critical kilovolts are the product of the observed primary voltage and the ratio of transformation. Up to 30 kilovolts the transformer described in the earlier paper was used, for higher values a 10-kw. 100,000-volt transformer and a separate generator had to be employed. A comparison between the two as regards wave form was obtained by oscillograms and by observations on the same conductor excited from each source. In the last column of Table II the diameters of the round conductor which would discharge at the same voltage as the stranded conductor are given. At the foot of the table there are given the results of observations on conductors of three and

TABLE II
INFLUENCE OF STRANDING ON CRITICAL VOLTAGE

Number of strands in outer layer	Diameter each strand cm.	Diameter over all cm.	Diameter outer cylinder cm.	Critical volts	Diameter of equivalent round conductor
3	0.162	0.349	9.52	18,300	
3	0.162	0.349	9.52	18,350	0.247
1	0.349	0.349	9.52	21,540	
4	0.162	0.404	9.52	20,750	0.32
1	0.404	0.404	9.52	24,250	
5	0.162	0.45	9.52	22,425	0.373
1	0.45	0.45	9.52	24,375	
5	0.162	0.45	17.13	26,250	0.37
1	0.45	0.45	17.13	29,050	
6	0.162	0.49	17.13	28,100	0.42
1	0.49	0.49	17.13	30,550	
7	0.162	0.541	29.2	32,880	0.465
1	0.541	0.541	29.2	38,600	
8	0.162	0.589	29.2	35,000	0.516
1	0.589	0.589	29.2	40,500	
9	0.162	0.64	29.2	36,900	0.567
1	0.64	0.64	29.2	42,500	
3	0.157	0.336	9.52	16,675	0.205
4	0.157	0.378	9.52	18,500	0.25

four strands in which there was no spiral and having approximately the same size of strand as that on the others. These were made up from carefully cleaned and polished wires with the aid of a fine blow flame for soldering.

The results are presented in somewhat different form in Table III. The columns A, B and C give the diameter of the cable over all, the diameter of a solid conductor of equal section, and the diameter of a solid round conductor having a critical voltage equal to that of the stranded conductor. The ratios of C to B and C to A and the pitch of the spiral, both actual and in terms of the corresponding diameter, are given in the remaining columns. The results of Table III are plotted in the curves of Fig. 3. The upper curve showing the variation of the ratio C to B with the number of strands shows that the critical voltage of a stranded conductor has an approximately constant relation to that of a solid wire having the same cross section when the number of strands on the outer layer is above five. The relation is that the diameter of a solid wire with the same critical voltage is about 97 per cent that of the wire having the same cross section as the cable. For fewer strands than five there is a sharp decrease in this percentage value showing that cables with fewer strands form corona at very much lower voltages.

TABLE III

COMPARISON OF STRANDED AND SOLID CONDUCTORS WITH REFERENCE TO CRITICAL VOLTAGE

Number of strands in outer layer	Diameter over all cm.	Diameter of conductor of equal section	Diameter of conductor with equal critical volts	Ratio $\frac{C}{B}$	Ratio $\frac{C}{A}$	Pitch of spiral	
	(A)	(B)	(C)			cm.	Diameters
3	0.349	0.272	0.247	0.907	0.708	3.81	10.9
4	0.404	0.332	0.32	0.965	0.792	3.49	8.6
5	0.45	0.381	0.37	0.971	0.822	4.44	9.9
6	0.49	0.430	0.42	0.975	0.857	6.02	12.3
7	0.541	0.48	0.465	0.969	0.868	6.66	12.3
8	0.589	0.53	0.516	0.975	0.877	6.35	10.8
9	0.64	0.581	0.567	0.977	0.886	6.98	10.9
3	0.336	0.27	0.207	0.767	0.616	None	
4	0.378	0.312	0.25	0.802	0.665	None	

The curve showing the relation between the ratio C to A and the number of strands is a very much more convenient indication, however, of the behavior of the stranded conductor. This refers the behavior of the cable to its own outside diameter, and the curve shows, for different numbers of strands, the fraction of this outside diameter which as a solid conductor would discharge at the same voltage as the cable. It is seen that with a seven-strand cable, *i.e.*, with six strands on the outer layer, the critical voltage is that corresponding to a solid wire of diameter 0.85 that of the outside diameter of the cable. With nine strands on the outer layer the relation is still less than 0.90.

In a stranded conductor the strands are always spiralled.

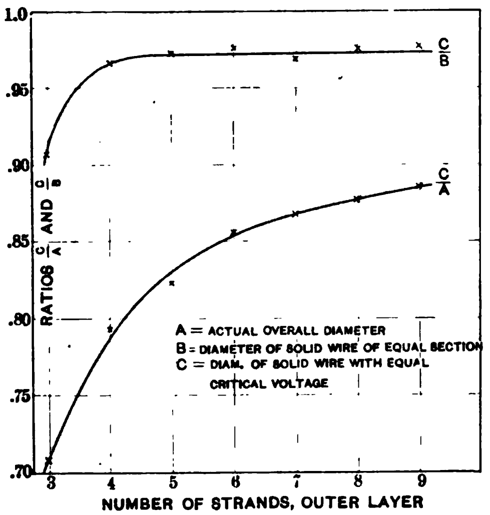

FIG. 3.—Critical voltage of stranded conductors referred to diameters over all and of equal section

The pitch of the spirals of the cables described above is given in Table III. The spiral arrangement of the strands tends to lessen the value of the electric intensity on the outer surfaces since the equipotential surfaces are rendered more nearly cylindrical about the axis of the cable. At the bottom of Table III are given the results of observations on three- and four-strand cables in which there is no spiral. The results indicate the further lowering of the critical voltage when spiralling is absent. The ratio *C* to *A* falls from 0.71 to 0.61 for the three-strand cable, and the difference for the four-strand is somewhat greater. The pitch of the spirals investigated does not appear to follow any regular rule. This irregularity however does not appear to have any corresponding effect on the points of curves of Fig. 3. From this it may be concluded that for a pitch of spiral less than 12 diameters there is no gain on the ground of lessened surface intensity due to a more uniform distribution of the electric field.

The presence of the spiral prevents an exact comparison between the values of surface intensity as calculated from Levi-Civita's expression and those observed here. The critical surface intensity for a 0.162-cm. wire is 66,500 volts per centimeter; the critical intensity for a 0.4285-cm. wire which is equal in cross section to a cable made up of seven wires 0.162 cm. in diameter, is 52,240. According to Jona and Levi-Civita the maximum intensity on this seven-strand cable is 1.23×52,240 =66,050 which corresponds to a primary voltage of 113.9, but this cable actually discharges at 112.7, at which voltage the maximum intensity using the same relation is 65,300; but it is impossible to say whether this lowering is due to the presence of the neighboring strands or to the fact that the 0.162-cm. wire may discharge at a different intensity when made up into a cable than when it stands alone. We can, however, say that this 0.162-cm. wire does form the corona when made up into a cable at a lower intensity than that at which it will form around the wire alone.

A comparison between the actual intensity as calculated by Levi-Civita and that at which corona starts can only be had by the use of cables without spirals. As already indicated the solution of Levi-Civita's expression has only been given for the six strand conductor which is impossible to make up without a spiral. At this writing the author has been unable to obtain the solutions of Levi-Civita's expression as applied to cables having three and four strands. When these are obtained, however, they will permit from the foregoing results a comparison between the maximum corona intensity for a single round wire and that obtaining at the surface of the same wire when made up into a three or four-strand cable without spiral.

INFLUENCE OF FREQUENCY AND WAVE FORM

By use of a cathode ray oscillograph in the high-voltage circuit Ryan in 1904 showed that the appearance of corona was accompanied by a hump or peak on the charging current wave in the neighborhood of the maximum of the voltage wave. The writer by stroboscopic methods has shown that the corona is periodic, appearing every half cycle, and that its first appearance with increasing voltage coincides accurately with the maximum of the voltage wave. Also the duration of the corona may be reduced with lessening voltage to a very small fraction of the period of the alternating voltage. Thus a corona which was found to exist for only 1/20 of a period at the crest of the voltage wave of a 60-cycle circuit was plainly visible in a darkened room. It is evident therefore that the interval of time involved in corona formation and cessation is extremely short. For these reasons it has been supposed that the appearance of corona depends only on the maximum value of voltage occurring in the cycle and is therefore independent of the frequency. Experience with existing lines indicates that if there is an influence of frequency it is small for the range between 25 and 60 cycles. The closeness with which the critical voltage may be read by the methods used in this work gave promise of discovery of any small differences due to a variation of frequency. Several series of observations were therefore made with different sizes of solid round conductor. The method of observation was that of the visible corona and the sound accompanying its start. The range from 15 to 90 cycles was obtained from two generators and the voltage from a 10-kw., 25-cycle, 100,000-volt transformer. The transformer had also a low voltage secondary coil. The method of procedure was to raise the voltage gradually for each value of the frequency and with room darkened to observe the wire through a narrow slit in the wall of the outer cylinder. As soon as the corona appeared the voltage was read from a voltmeter connected to the terminals of the low-voltage secondary coil of the transformer. It was found that the sound accompanying the corona was quite as reliable as the visible corona as an indication of the critical voltage. The results of three sets of observations taken on different days are given in Table IV. An inspection of the readings will show that the voltage could be determined to a close degree of accuracy. Indeed the limiting condition of accuracy when the wires are carefully straightened and polished is found to lie in the constancy of the voltage of the circuit rather than in the sharpness with which the corona begins. These excellent conditions for observations are somewhat impaired at the lowest frequencies where the flicker of the corona is perceptible and where the low pitch of the sound of discharge renders it difficult to distinguish it from other sounds. The generators were excited from storage batteries and no load other than the transformer was on the generator, so that the circuit conditions were very constant.

The results of Table IV are plotted directly in Fig. 4 which shows, therefore, the variation of the voltage on the low tension secondary winding when the frequency is varied from 17 to 92 cycles per second for wires 0.343, 0.635 and 0.716 cm. in diameter when placed in the center of a cylinder 120 cm. long and about 29 cm. in diameter. The dotted breaks in the three curves represent the passage from one generator to the other. The observations were taken as continuous sets, interruption being necessary for only a few seconds to change generators. There were consequently no appreciable variations in temperature or pressure. Ascending and descending frequency is indicated by crosses and circles respectively. The irregular shape of the

TABLE IV
INFLUENCE OF FREQUENCY ON CRITICAL VOLTAGE

0.635 cm. diam. pressure 755 mm. temp. 14.4°–16° C.				0.716 cm. diam				0.343 cm. diam. pressure 752 mm. temp. 14° C.			
Frequency	Test coil volts	Frequency	Test coil volts	Frequency	Test coil volts	Frequency	Test coil volts	Frequency	Test coil volts	Frequency	Test coil volts
19.6	49	54	48.7	17.5	50.7	63	50.7	20	33.3	59	33.8
21.6	48.7	55	48.8	18.7	51.2	69	49.8	22.5	33.3	55	34.1
21.7	48	58.5	48.7	20.5	51.2	74	49	23.5	33.5	55	34.1
25.5	48.7	60	48.5	20.5	51.2	80	48.3	32	33	50.5	33.6
27.2	48.5	63.7	47.2	22	51.5	87	47.2	37	32.7	50.5	33.4
29.7	48	69	46.2	25.5	51.6	92	46.6	40.5	32.5	45	32.5
31	48.5	73.7	45.7	27.5	51.4	77	49.2	43.5	32.5	45	32.5
32.5	47.1	78.7	44.9	29.5	52.5	67	50.1	48.5	33.2	42	32.4
34	47.1	85	44	37	50.7	61.5	51.6	53	34.1	38.5	32.4
37.5	46.7	57.5	49	43.5	50.4	56	52.4	53	34.1	38.5	32.4
38.5	46.7	45	46.7	48.5	51.1	51	52	58	34.1	35	32.6
41	46.7	49.5	48.1	53	52.4	46	50.3	58	34.1	35	32.6
44	46.5	45	46.7	57	52.4	42.5	50.1	63	32.9	32.5	32.7
45	46.7	49.5	48.1	62.5	50.8	39	50.3	63	32.9		
49.5	48.2			59	52.4			59	33.6		

curves of Fig. 4 repeated itself accurately in other series of experiments over the same range of frequency. Since the generators were rated at 5 kw., and since the transformer was operating over a wide range of frequency at approximately the same value of voltage, and its magnetizing current was therefore variable, a variation of wave form due to the armature reaction of the generator appeared probable. The transformer had a capacity of 10 kw. and was designed for 25 cycles. In these experiments the maximum voltage on the primary winding was only one half of the rated value. The magnetizing current of the transformer at full voltage is 15 per cent of full load current.

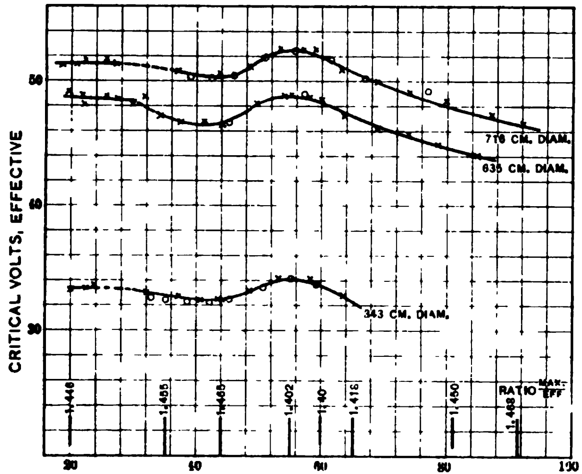

FIG. 4.—Influence of frequency on critical voltage

It will be seen from these figures that the lagging component of the generator current was not unduly large. Both generators were designed for operating the transformer and they have smooth body armatures with flat surface coils. Nevertheless it was realized that at the low frequencies the magnetic density in the field circuit of the generators varied widely from that obtaining at the high values of frequency, and a set of oscillograms was carefully taken at seven different frequencies covering the range shown on the curve. These oscillograms were all taken from the low voltage secondary coil of the transformer and its potential was maintained constant at 50 volts. This voltage is about the value of the mean obtaining over the curve for the

0.716-cm. wire. The ratios of maximum to effective values of these waves were then taken from micrometer measurements of ordinates spaced 7.5 deg. apart over two half waves. The figures of these measurements may be omitted here, but in order to give. an idea of the conditions of accuracy obtaining the following figures are given for two half waves measured from the oscillogram corresponding to 44 cycles. These two half waves were taken on different portions of the oscillogram. On the first half 20 ordinates 2 mm. apart were ruled on a dividing engine; the height of these successive ordinates could be measured to 0.1 of a mm. on the same machine. The maximum ordinate was 31.27 mm. high and the square root of the mean square of all the ordinates was 21.26, giving a ratio of maximum to effective of 1.474. Similar measurements on another half wave gave 1.456 as the ratio of maximum to effective value. A similar treatment of other waves at different frequencies gave the results shown in Table V. It is seen that the ratio of maximum to effective is a minimum somewhere between 50 and 60 cycles, *i.e.*, the region corresponding to the peculiar hump on the curves of Fig. 4.

TABLE V
INFLUENCE OF FREQUENCY ON WAVE FORM

Frequency	Ratio of maximum to effective value		
	1st half wave	2nd half wave	Mean
35	1.462	1.449	1.445
44	1.474	1.456	1.465
55	1.404	1.398	1.402
60		1.40	1.40
65	1.426	1.41	1.418
81	1.45		
91.5	1.466	1.469	1.468

In the curves of Fig. 5 the points indicated are obtained by multiplying the values in Fig. 4 by the corresponding ratio of maximum to effective for the voltage wave. These curves show a lowering of the critical voltage with increasing frequency. They leave something to be desired in the accurate location of the points upon the curves. It should be noted, however, that owing to the magnification of the scale the error of the points falling off the curve and of those lying on the low frequency portion of the curves of Fig. 4 is only about 1 per cent. The measurement of the ratio of maximum to effective value from an oscillogram is subject to considerable error. The maximum at 55 cycles on the curves of Fig. 4 is brought below the values for lower frequencies in Fig. 5 when the correcting factor is introduced; and particularly the lowering at 91 cycles is far too great to be questioned on the score of a possible error of this nature. The curves therefore show with a fair accuracy the nature of the variation of the critical voltage with the frequency. This variation within the range of commercial frequencies is only about 2 per cent. The lowering of the critical voltage at 90 cycles is about 6 per cent. The frequency was measured with a Hartmann & Braun vibration frequency indicator which had been calibrated carefully by means of a tachometer. A possible influence of the frequency variation on the readings of the Weston electrodynamometer type voltmeter was investigated by careful comparison between this instrument and a Kelvin multicellular electrostatic voltmeter at the various frequencies, and then a check by comparing these two instruments with a standard direct-current Weston voltmeter. No variation was found among the readings of any of the instruments in these several conditions.

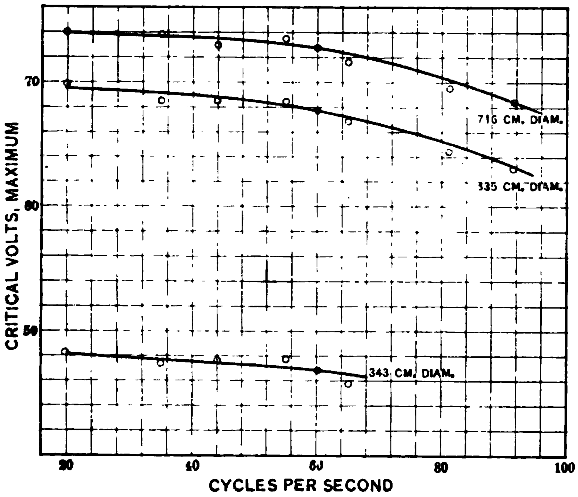

FIG. 5—Influence of frequency on critical voltage

The variation of the critical voltage with the frequency has not been noticed before. Mershon records results for 40, 73 and 93 cycles which indicate a rise of the critical voltage with the frequency. It should be noted, however, that Mershon's definition of critical voltage is taken arbitrarily from his loss curves and therefore has no direct or necessary relation to the critical voltage as defined in these papers. Ryan worked at 130 cycles but the description of the apparatus and conditions under which his experiments were made are not sufficient to permit a comparison between his values and those described here. In addition Ryan used the visible corona and did not make an accurate investigation of the variation of the wave form of alternating voltage. As a result his values, a few of which are indicated in Fig. 2, do not show a regularity which would permit differences of 5 or 6 per cent from the values of this paper to be detected. In a discussion given later in this paper some further comment on the variation of the critical intensity with the frequency is given.

INFLUENCE OF PRESSURE

The influence of pressure on the various forms of spark discharge has been closely studied. Paschen's law[5] states that the sparking potential for a given spark length is directly proportional to the pressure; his investigations cover the range of pressure between 10 and 75 cm. of mercury. Carr[6] has shown that this linear relation extends down to pressures of a few millimeters if the spark lengths are not greater than one centimeter, but does not obtain for lower pressures. Townsend[7] has shown that the potential gradient at which secondary ionization sets in when electricity is passing through a gas is directly proportional to the pressure. Watson[8] investigated the spark length between spheres up to 15 atmospheres and found that the sparking potential increases with the pressure in an approximately linear relation. From the general similarity between the corona and the brush form of the spark discharge a linear relation therefore between pressure and critical surface intensity, or the potential gradient at which corona begins, is to be expected. Apparently the only study of the influence of pressure on the formation of the alternating corona is a single set of

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[5] Paschen, *Wied. Ann.*, 1889, XXXVII. p. 79.

[6] Carr, *Proc. Roy. Soc.*, 1903, LXXI, p. 374.

[7] Townsend, *Phil. Mag.*, 1901, VI, I, p. 198.

[8] Watson, *Electrician*, 1909, pp. 62, 851.

observations by Ryan[9] on a wire 0.32 cm. in diameter placed at the centre of a cylinder 22.2 cm. in diameter. He observed the alternating voltage at which the visible corona appeared for the range of pressure between 45 and 90 cm. of mercury. The alternating frequency was 130. The resulting linear relation is given as between the kilovolts K actually applied and inches of mercury B. $K = 2.93 + 0.902\ B$.

A series of observations on several sizes of wire with varying values of pressure was therefore undertaken. Five sizes of wire were investigated with diameters 0.122, 0.156, 0.276, 0.340 and 0.475 cm. The wires were carefully straightened and cleaned and centered accurately on the axis of the apparatus which has been briefly described above. The outer cylinder has a diameter of 9.52 cm. In the pressure experiments the ends were closed with ebonite caps fastened by insulating rods to the metal cylinder; this arrangement was necessary to withstand the pressures above that of the atmosphere. The side tubes were also closed with ebonite caps and the leading-in wire to the discharge electrode passed through a column of sulphur supported in hard rubber. The electroscope was thus outside the apparatus proper. No troubles with either insulation or air leak were encountered with this arrangement within the range of pressure 30 to 108 cm. All joints were sealed with a mixture of beeswax and resin. The discharge electrode was placed inside the upper side tube and within one or two millimeters of the grating formed by the holes drilled in the outer cylinder. In the earlier work it was found that a flow of air from the cylinder over the electrode contributed little to the sharpness with which the starting of the corona was indicated, the initial discharge of the electroscope occurring at the same value for both moving and stationary air. The results of a typical series of observations are given in Table VI. The values are those for a wire 0.156 cm. in diameter. It will be noted that at each pressure several readings of voltage were taken. This was done by raising the voltage gradually until the initial discharge of the electroscope set in, then lowering, then repeating the process. The table also indicates that observations were taken over the same range of pressure for ascending and descending values. The column showing the critical primary volts indicates a very satisfactory constancy in the values. The results on the other sizes of wire show a corresponding degree of

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[9] Ryan, TRANSACTIONS A. I. E. E., 1904, XXIII, p. 101.

accuracy and the readings are omitted. The degree of constancy, however, is indicated by the curves of Fig. 6 on which are plotted the results for the five sizes of wire which were studied. It is seen that the observations as plotted show beyond any question that the relation between critical voltage and air pressure is a linear one for each size of wire. The values of critical voltage are those taken at the primary terminals of the transformer. This voltage is directly proportional to the corresponding value of potential gradient at the surface of the wire. It was shown in the earlier paper that with the transformer used in these experiments the primary voltage was amply sufficient indication by means of the ratio of turns of the actual

TABLE VI

INFLUENCE OF PRESSURE ON CRITICAL VOLTAGE. CLEAN COPPER WIRE 156 CM. DIAMETER IN 9.52 CM. OUTER CYLINDER. ATMOSPHERE PRESSURE 759.5 MM. TEMPERATURE IN TUBE 24 DEG. C.

Crit. prim. volts Ratio 1:125			Manometer			Pressure mm. mercury
			Right	Left	Difference	
102.2,	102.2,	102.2	487.5	587.5	—100	659.5
97.5,	97.5,	97.2	459	605.5	146	613.5
91.3,	91.3,	91.2	427	628.5	201.5	558
87.2,	87.5,	87.8	407.5	642	234.5	525
83 ,	83.2,	83.4	386.5	656	269.5	490
79.9,	80 ,	80	367.5	669.5	302	457.5
80.5,	80.7,	80.6	371	666.5	295.5	464
74 ,	74 ,	74	340	688.5	348.5	411
68.1,	68.1,	68.1	313.5	707	393.5	366
94.2,	94.2,	94.2	439	617	178	581.5
106.5,	106 ,	106.2	499	576.5	77.5	682
114.5,	114.9,	114.8	545.5	545.5	0	759.5
	Ratio 1:250					
57.5,	57.4,	57.5	545.5	545.5	0	759.5
59.8,	59.9,	59.8	570.5	530.5	+40	799.5
61.8,	61.6,	61.7	592	516.5	75.5	835
64 ,	64 ,	63.8	618.5	499.5	119	878.5
66 ,			641	486	155	914.5
67.7,	67.7,	67.7	661	473.5	197.5	957
69.8,	70 ,	69.9	687.5	457	230.5	989.5
71.6,	71.6,	71.7	710	444	266	1025.5

voltage applied between the wire and the cylinder. It is of great interest to note that the slope of the line showing the relation between critical voltage and pressure varies with the size of wire, and that this slope is greater the greater the diameter of the wire. It should be noted, however, that the values of critical voltage as given in Fig. 6 have no particular significance in that they apply to a particular combination of wire and opposite conductor. A corresponding type of variation is to be expected, however, for any particular arrangement of wire and opposite conductor. Referring to the readings of Table VI, the ratios of transformation were 1 to 125 and 1 to 250, the frequency 60 cycles, and the ratio of the maximum to the effective value of alternating wave of electromotive force of the generator at 100 volts was 1.46. The temperature was 24 deg. cent. The equations of the lines of Fig. 6 are readily deduced, but since, as already stated, they apply to a particular combination of wire and outer cylinder they need not be given here in any further

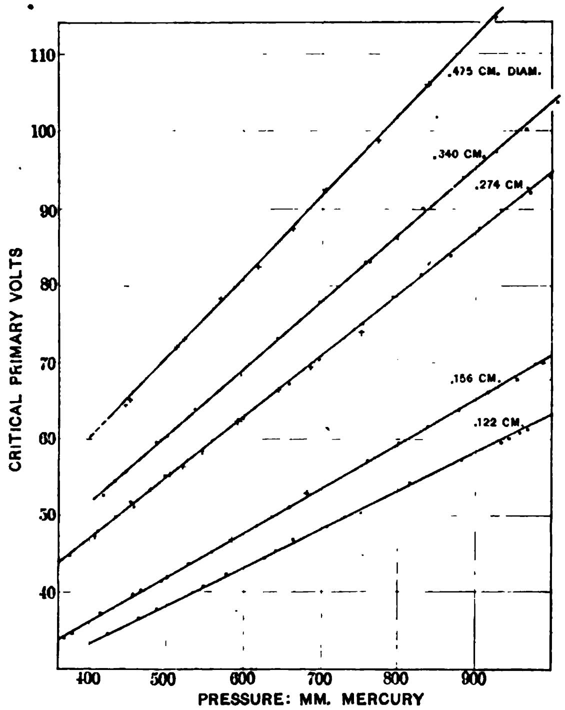


Fig. 6.—Influence of pressure on critical voltage

elaboration than to state that the equation for the 0.340-cm. wire is

$$V = 17 + 0.087\,p \qquad (2)$$

where V is the actual observed critical primary voltage and p is measured in millimeters of mercury. Similar expressions apply for the other sizes of wire. It is necessary for the purposes of universal application to express the relation between the behavior of a wire and the pressure in terms of the critical surface intensity, or potential gradient at the surface corresponding to the appearance of corona. The values of this critical surface intensity have therefore been calculated from the expression

$$\frac{d\,V}{d\,r} = \frac{E}{r \log \frac{R}{r}} \qquad (3)$$

in which E is the maximum value of the potential difference between wire of radius r and outer cylinder of radius R. Expressed in terms of electric intensity at which corona begins in kilovolts per cm. and pressure in cm. of mercury the equations for the five wires of diameters as given above are as follows: In these equations correction has been made for the slight alteration in wave-form, and by reduction to a common basis of values at 760 mm. pressure.

Diameter of wire

0.122 cm. $\quad \frac{d\,(\text{kv.})}{d\,r} = 16.7 + 0.691\,p \qquad (4)$

0.156 cm. $\quad \frac{d\,(\text{kv.})}{d\,r} = 14.95 + 0.66\,p \qquad (5)$

0.276 cm. $\quad \frac{d\,(\text{kv.})}{d\,r} = 11.6 + 0.595\,p \qquad (6)$

0.340 cm. $\quad \frac{d\,(\text{kv.})}{d\,r} = 10.94 + 0.56\,p \qquad (7)$

0.475 cm. $\quad \frac{d\,(\text{kv.})}{d\,r} = 9.56 + 0.534\,p \qquad (8)$

These equations have been plotted in the lines of Fig. 7. The variation of the slope of the linear relation between pressure and critical surface intensity is still evident although it is not so pronounced. It is also to be noted that the variation of the critical intensity with pressure is greater the smaller the size of wire. The slope of the linear relation apparently approaches a minimum with increasing size of wire.

If Ryan's results quoted above for 0.317-cm. wire be expressed in the same terms used in the formulas above, the resulting equation is

$$\frac{d\,(\text{kv.})}{d\,r}=6.15+0.744\,p \tag{9}$$

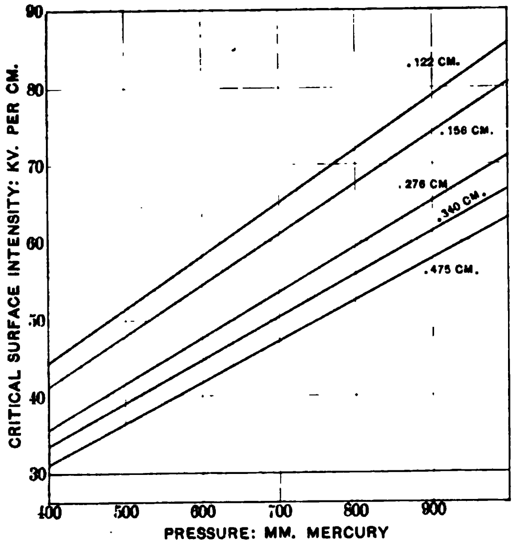


FIG. 7.—Influence of pressure on critical surface intensity

The slope of this line is greater than that of any of the wires as expressed in equations (4) to (8), although the larger size of wire should cause the slope to be less than those of the three smaller sizes. It is to be noticed further that the initial constant term of formula (9) is considerably less than any of those in formulas (4) to (8). Further, the value of critical surface intensity at 76 cm. pressure indicated by formula (9) is 62.6, while that calculated from formula (1), and therefore frequently observed by the writer, is 55.7. Ryan used invariably the visible corona for indication of initial break down; some of his results on wires of different sizes are given as circles in Fig. 2 where they are seen to be very irregularly located. Aside from the uncertainty of the method of observation, the wave form and frequency may have introduced considerable error in the results as reported, although that due to frequency would have tended to a lower rather than a higher value than those obtained here at 60 cycles.

The variation of the linear relation between pressure and critical surface intensity with the size wire adds a further difficulty in the attempt to express in simple terms the behavior of any circuit as regards the safe limiting voltage. The fact that the critical surface intensity at any one pressure varies with the diameter of the wire as shown in Fig. 2 can be taken care of quite readily since the relation is a simple one. It is of interest therefore to study by inspection the constants of formulas (**4**) to (**8**), and see if a simple relation between them and the diameter can be found. These formulas are of the general type

$$\frac{dV}{dr}=K_1+K_2\,p \qquad \textbf{(10)}$$

In Table VII the values of the diameter of the wire expressed in millimeters and the corresponding values of K_1 and K_2, together with the logarithms of these several quantities, are given. The relation of d and K_1 and d and K_2 was studied by plotting the curves between d and K_1, d and $\log K_1$, etc. The only obviously simple relation that appeared was that between $\log d$ and $\log K_1$ which resulted in a straight line. The equation of this straight line is

$$\log K_1=\log 18.07-0.41\log d \qquad \textbf{(11)}$$

from which it may readily be deduced that the relation between K_1 and d in centimeters is as follows:

$$K_1=7.03\,d^{-.41} \qquad \textbf{(12)}$$

In Table VII a column is also given showing the values of K_1 as calculated from formula (**12**).

A similar study was made of a possible relation between d and K_2, the slope of the line showing the relation between pressure and critical surface intensity. Curves were plotted as before between the several pairs of quantities d, K_2, $\log d$, K_2, etc. In this case also a straight line resulted when log d was plotted in combination with log K_2. The equation of the resulting line is as follows:

$$\log K_2 = \bar{9}.8554 - 0.188 \log d \qquad (13)$$

from which it may also be deduced that the relation between K_2 and d with d now expressed in centimeters is as follows:

$$K_2 = 0.464\, d^{-0.188} \qquad (14)$$

Values of K_2 calculated from formula (**14**) are given in Table VII and show a fairly satisfactory relation to those observed.

The relations expressed by formulas (**12**) and (**14**) do not offer promise of any simple factor to take account of pressure variation. The expression connecting the critical surface intensity in kilovolts per centimeter with the diameter of the wire in centimeters and the pressure in centimeters of mercury, at 21 deg. cent. is

TABLE VII
INFLUENCE OF PRESSURE ON CRITICAL SURFACE INTENSITY AS AFFECTED BY SIZE OF WIRE

d mm.	log d	K_1	log K_1	K_2	log K_2	Calculated values	
						K_1	K_2
1.218	0.085	16.7	1.223	0.691	$\bar{9}.839$	16.67	0.69
1.56	0.193	14.95	1.175	0.66	$\bar{9}.82$	15.05	0.658
2.76	0.441	11.6	1.064	0.595	$\bar{9}.775$	11.9	0.591
3.408	0.532	10.94	1.039	0.56	$\bar{9}$ 748	11.1	0.568
4.75	0.677	9.56	0.980	0.534	$\bar{9}.728$	9.55	0.53

$$\frac{d\,(\text{kv.})}{d\,x} = 7.03\, d^{-0.41} + 0.464\, p\, d^{-0.188} \qquad (15)$$

This expression includes the relation given by formula (**1**). For example the critical surface intensity for a wire 0.276 cm. in diameter calculated from formula (**1**) is 57.2; from formula (**15**) with p taken as 76 the value is 56.8. It would be very desirable to extend the investigation of pressure over a wider range of sizes of wire. It is reasonably certain, however, in view of the consistency with which observations like those above may be carried out that the linear relation between pressure and critical intensity varies with the size of wire. The exponents of *d* in the two formulas mentioned may suffer some modification with further experiment, but it seems quite conclusive from the above relations that the relation between pressure variation and diameter of wire while simple in form will nevertheless introduce an unfortunate complication in any final expression which aims to give the critical corona voltage for any particular separation and size of parallel conductors for the whole range of pressure variation encountered in practice.

DISCUSSION

So far as the question of the value of voltage at which corona will start on a given transmission line is concerned, it is probable that a sufficiently accurate solution will be reached sooner or later by means of experiments of the general character of those described above, supplemented by observations on existing lines. There is also a fair reason to suppose that a comparatively simple law will be found. For the surface intensity for any arrangement and size of cylindrical conductors corresponding to a given voltage may be expressed in terms of these constants, for standard conditions of temperature and pressure. While as above shown the relation between pressure and critical voltage varies with the size wire, the law of this variation is simple and it may be possible to adopt a mean value of the slope of this linear relation which will apply with sufficient accuracy to the sizes of wire encountered in practice. Thus, from Fig. 7, it is seen that for wires in the neighborhood of ½ cm. in diameter the slopes of the lines are approaching each other rapidly. In view of the results from the investigation of the influence of pressure it seems probable that the variation with the temperature will also be of different form for different sizes of wire. investigations in this direction are very desirable. It may be recalled that in the writer's earlier paper, and in the experiments of Ryan, it is shown that a linear relation exists for a definite size of wire between temperature and critical voltage. The effect of stranding the conductor has been studied for only one size of strand as yet, but it appears a simple matter with some further investigation to express the effect of each of these influences in terms of the diameter of the conductor.

The influence of frequency does not offer promise of expression as a simple relation. This influence is so small, however, within the limits of frequency met in practice that it may be neglected. The state of the atmosphere appears to be of small importance, for moisture does not influence the critical voltage nor does its state as regards ionization, as is indicated by several considerations given in an earlier paragraph. Dirt and impurities which on settling on the wire cause irregularities of surface may lead to brush discharges, and if these are sufficient in number they may cause a noticeable loss below the normal critical voltage. It is this type of loss which causes the slow initial rise and gradual bending in the loss curve below the critical voltage. These facts will probably be taken care of by a factor of safety multiplying the calculated corona voltage and taking into account all of the influences which are sufficiently great to play a part.

It is of great interest to consider the results so far obtained in their relation to present theories of the nature of the electric conductivity and breakdown of a gas. Under this theory the neutral atoms and molecules of matter may under some circumstances be separated into smaller charged particles. The motion of these particles under electric force constitutes an electric current. In a gas there are always a small number of these free ions present; this number may be greatly augmented by Röntgen rays, ultra-violet light, and other well-known ionizing agents. When so ionized currents of magnitudes within easy measuring range are obtained between terminals subjected to a difference of potential. If this difference of potential is increased continuously a point is reached where the current is constant over a wide range of voltage, which shows that the ions are swept out as rapidly as they are formed. On further increase of potential the current increases sharply showing the presence of some new source of ionization. The theory states that these new ions are formed by the impact of those already existing, and moving with higher velocity in the increased electric field, with the neutral molecules of the gas. This phenomenon has been called ionization by collision, or secondary ionization.

The results of the experiments which have been described above are for the most part consistent with the ionization theory. The various circumstances surrounding the appearance of corona all indicate that it is an instance of secondary ionization. Formula (1) indicates that near a conductor of large radius, or near a plane conductor, the corona intensity approaches a value 32 kilovolts per cm. Secondary ionization between plane electrodes in closed vessels at atmospheric pressure has been noticed by several physicists to begin in the neighborhood of 30,000 volts per cm. The mass of the elementary negative ion or electron is approximately 5.9×10^{-28} grams and the charge it carries is 4.6×10^{-10} c.g.s. electrostatic units. In an electric field the mechanical force acting on the electron is the product of the charge and the strength of field. Hence by the laws of simple mechanics it is possible to calculate the acceleration, the velocity and the kinetic energy attained by an electron in moving a given distance under a given electric intensity. If the mean free path of the electron, about 6×10^{-5} cm. at atmospheric pressure, be the distance between collisions it is thus easy to calculate the kinetic energy of the electron due to the electric field when it collides with a molecule. This energy is readily seen to be equal to $p\ V\ e$, where p is the mean free path, V the electric intensity in electrostatic units, and e the charge of the electron. If, now, the voltage between plane parallel electrodes be raised until secondary ionization begins the value of the voltage makes it possible to calculate the energy required to ionize a molecule of a gas. In fact the values of the energy required to ionize a molecule which are now generally accepted are largely based on the determinations of the value of electric intensity at which secondary ionization begins. It has been pointed out above that the values of this intensity as determined by Townsend and others are in close agreement with the value 32,000 volts per cm. indicated by equation (**1**) as the lowest value at which corona appears. To one skeptical as to the correctness of the electron theory, therefore, (and there are many such) all that may be said so far is that the phenomena of sudden increase of current through a gas above a certain value of electric intensity as observed by Townsend, and that of corona formation, are probably due to the same causes. But there are several other independent methods of determining the energy required to ionize a molecule of a gas. The values are commonly expressed in terms of the potential difference in volts through which the electron must pass in order to acquire energy sufficient to produce an ion by collision. The value pertaining to the method described above is from 10 to 12 volts. Rutherford from the relation between the heating effect of radium and the number of ions it produces gives the value of 24 volts. Stark and Langevin by independent methods conclude that the values are 45 and 60 volts respectively. While the extreme values differ by the factor 5 or 6, it must be remembered that the actual amount of energy required to produce an ion is about 5×10^{-11} ergs, so that all of these values indicate the same order of magnitude. Therefore when taken together they constitute a very strong reason for supposing the value 5×10^{-11} ergs is close to the correct one. If this be true it is good evidence that the formation of the corona is actually due to the liberation of ions from the neutral molecules of the gas when the latter suffer collision from a free electron moving under the force of the electric field. That the electron and not a gaseous ion or aggregate is the active agent is shown by the shorter free paths of these latter which by the relation already given results in a lower value of kinetic energy at the time of collision than the values given above. It is well known that since secondary ionization depends only on the velocity of the ions, and thus on the electric intensity, it should within wide limits be independent of the number of ions already existing in the gas. That the electric intensity corresponding to the appearance of corona is independent of the state of ionization of the air has been shown conclusively in an earlier paragraph.

The general influence of a decrease in pressure or an increase in temperature in lowering the critical voltage is quite consistent with the ionization theory, for under the kinetic theory of gases the free paths of the vibrating molecules and ions are lengthened in these two conditions. During the free path or interval between collisions the ions are accelerated by the electric force, and the longer the interval the greater the velocity acquired and the more kinetic energy and ionizing power. Hence a given amount of energy will be acquired at a lower voltage if the free path is lengthened.

The lowering of the critical voltage by an increase in frequency is not to be explained so simply. However, if within the molecule or atom there are a number of electrons in motion or free to move, and there is some indirect evidence to this effect, it is evident that the forced vibrations set up in such a system of electrons by an external alternating field will with the increase in frequency of these vibrations cause the mutual attractions within the structure of the atom to become less and less strong and therefore more liable to be broken when in collision with an extraneous ion. It is surprising, however, that this effect should be noticeable at frequencies so low as 60 to 90 cycles, for these frequencies are incomparably slower than those suggested by theory for the vibrations within the atom. The close relation between the first appearance of corona and the peak or maximum of the voltage wave is natural in the light of theory, for at atmospheric pressure the mean free path of an electron is about 6×10^{-5} cms. long and under a field sufficiently strong to ionize the gas this path is traversed in about 2×10^{-12} secs.

Perhaps the most interesting problem in connection with the phenomenon of corona formation is the explanation of the greater values of electric intensity at which corona starts around smaller wires, *i.e.*, the upward trend of the curve of Fig. 2. Why should the properties of the air change with a slight alteration in the size of a conductor whose diameter is 50,000 times as great as the mean free path of the molecule? No tenable explanation of the curve of Fig. 2 has yet been offered. The attraction to the surface of the conductor of oppositely charged ions which pile up as it were and reduce the actual gradient below that calculated, and at the same time increase the gas pressure, has been suggested in explanation. Both suppositions immediately include an influence on the value of corona voltage of the amount of ionization present in the gas, and this as already noticed is contrary to observation. Simple calculation also will show that the charge sufficient to materially reduce the gradient at the surface of a conductor at corona potential would require a number of ions far in excess of the numbers commonly present in the atmosphere. The writer, using a sensitive optical method, could find no indication of an increase of pressure at the surface of the conductor. It appears probable that the explanation of the higher values for smaller wires will be found in the lesser surface of these smaller conductors. Secondary ionization probably begins with the collisions of a few electrons which have free paths longer than the average. With decreasing area of the conductor the number of neighboring electrons whose free paths exceed a certain length and at the same time are subject to the maximum electric intensity will be decreased, and consequently the corona forming the electric intensity must be higher.

RESULTS AND CONCLUSIONS

1. The relation between critical surface intensity, *i.e.*, the intensity at which corona starts, and the diameter of a clean round conductor may be expressed by the simple law $E = 32 + 13.4\, d^{-0.5}$, E being the surface intensity in kilovolts per cm. and d the diameter of conductor in cm.

2. Stranding a conductor lowers the critical voltage. The lowering is greater the fewer the number of strands in the outer layer. Expressing the lowering in terms of the diameter of wire giving the same critical voltage, the fraction of the overall diameter of the stranded conductor for three strands is 0.7 and for nine-strand, 0.88. The values for intermediate numbers of strands are also given.

3. With increasing frequency the corona starts on a given conductor at lower values of voltage; the lowering between 25 and 60 cycles is about 2 per cent, at 90 cycles about 6 per cent.

4. A linear relation exists between the atmospheric pressure and the corona-forming voltage for the range between 30 and 109 cm. of mercury. The slope of this relation varies however with the size wire and the rate of change of critical voltage with the pressure is greater the smaller the diameter of the wire.

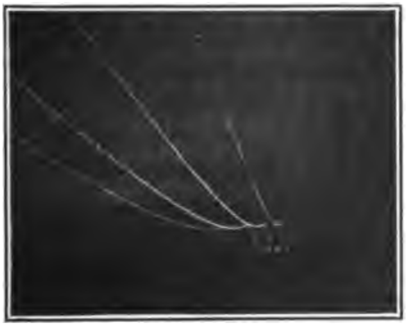
A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 29, 1911.

THE LAW OF CORONA AND THE DIELECTRIC STRENGTH OF AIR

BY F. W. PEEK, JR.

I. SUMMARY

For several years an extensive investigation of the *dielectric strength of air*, and more particularly, of the law of corona, has been carried on by the Consulting Department of the General Electric Company, under the general supervision of Dr. C. P. Steinmetz. The facilities were practically unlimited in regard to power, apparatus, instruments and their standardization, and available engineering skill. Thanks for their active assistance in this investigation are due to Dr. E. J. Berg, Messrs. C. M. Davis, J. L. R. Hayden, A. B. Hendricks, W. K. Page, L. T. Robinson, L. A. Schloss, W. I. Slichter, C. W. Stone and J. B. Taylor.



Corona, standard line A, No. 3/0 cable, 310 cm. spacing, 230,000 volts

The investigation gives the following results:

1. In alternating-current transmission lines at very high voltages a loss occurs by dissipation of power into the air. This is accompanied by luminosity of the air surrounding the line conductor—the so-called corona.

Loss begins at some critical voltage, which depends on the size and spacing of line conductors, etc., and increases very rapidly above this voltage. Fig. 1 shows a typical curve of corona loss.

The extended investigation on an experimental transmission line shows that the corona loss in *fair weather* can be expressed by the equation

$$p = a f (e - e_0)^2 \tag{1}$$

where p = loss, in kilowatts per kilometer length of single line conductor,

e = effective value of the voltage between the line conductors and neutral in kilovolts,*

f = frequency,

and a is given by the equation

$$a = \frac{k}{\delta} \sqrt{\frac{r}{s}} \tag{2}$$

where r = the radius of conductor in cm.

s = the distance between conductor and return conductor in cm.

δ = the density of the air, referred to the density at 25 deg. cent. and 76 cm. barometer as unity.

k = a constant.

e_0 = the effective *disruptive critical* voltage to neutral, and is given by the equation

$$e_0 = m_0\, g_0\, \delta\, r \log_\epsilon \frac{s}{r} \text{ kv. to neutral} \tag{3}$$

where g_0 is the disruptive gradient of air in kilovolts per cm. at 25 deg. cent. and 76 cm. barometer, and is constant for all sizes of wires, frequencies, etc. If the effective value of g_0 is taken, e_0 is given in effective kilovolts.

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*Hence, in single-phase circuits, e is one-half the voltage between conductors. In three-phase circuits e is $\frac{1}{\sqrt{3}}$ times the voltage between conductors.

From our investigations it follows that

$$g_0 = 29.8 \text{ kv. per cm. (maximum)}$$

$$g_0 = 21.1 \text{ kv. per cm. (effective)}$$

m_0 is a constant depending on the surface condition of the conductors, and

$m_0 = 1$ for perfectly smooth polished wires.

$m_0 = 0.98$ to 0.93 for roughened or weathered wires, and decreases to

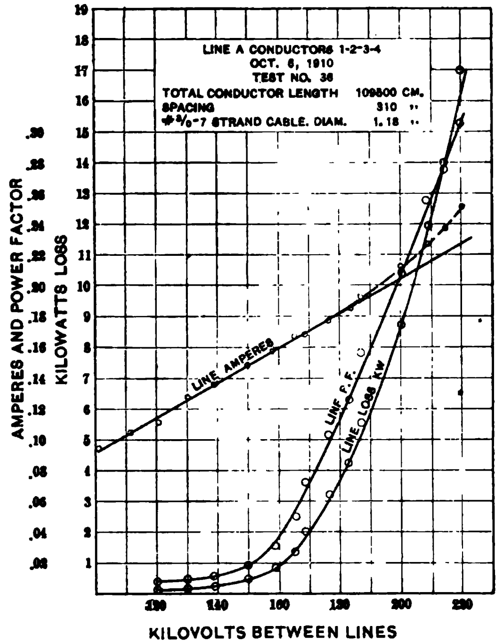

FIG. 1.—Corona loss characteristic curves.

$m_0 = 0.87$ to 0.83 for 7-strand cables (where the radius is taken as the outer radius of the cable).

2. Luminosity of the air surrounding the line conductors does not begin at the *disruptive critical voltage* e_0, but at a higher voltage e_v, the *visual critical voltage.*

The *visual critical voltage* e_v is much higher for small wires than the *disruptive critical voltage*, e_0; it is also higher for large wire, but to a lesser extent.

While theoretically no loss of power should occur below the *visual voltage*, e_v, some loss does occur, due to irregularities of the wire surface, dirt, etc. Thus, for the line voltage, e, near the critical voltage, e_0, the corona loss is greater than given by equation (**1**) for large wires, and smaller for small wires, as shown by curves, Figs. 2 and 3.

The curves as drawn are calculated from equation (**1**); the points shown by circles are the measured experimental values.

At the low values of corona loss two effects occur, which cause this deviation of the loss from the quadratic law, equation (**1**), and which affect the loss in opposite directions:

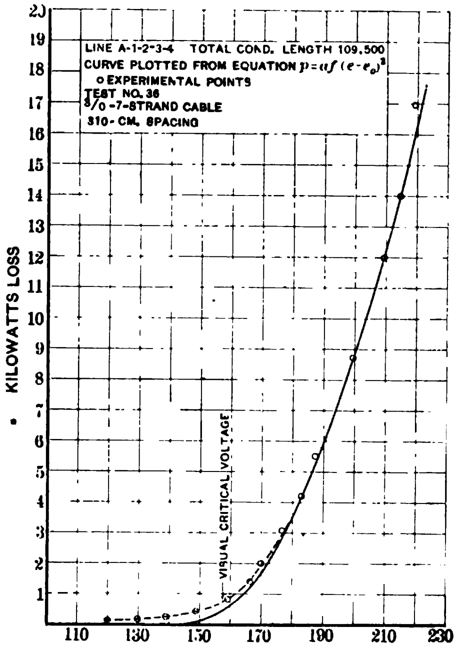

FIG. 2.—Corona loss characteristic curves

a. The loss of power does not begin at the voltage e_0, at which the disruptive gradient is reached at the conductor surface, but only after the disruptive strength of air has been exceeded over a finite and appreciable distance from the conductor, that is, at a higher voltage e_v. As the convergency of the lines of dielectric force is great at the surface of small conductors, a considerable increase of the voltage is required to extend the disruptive gradient to some distance from the conductor, and e_v is considerably higher than e_0. Therefore the decrease of loss below that given by equation (**1**) is appreciable with small conductors within the range between e_0 and e_v, as seen in Fig. 3. With large conductors, however, the lesser convergency of the lines of dielectric force at the conductor surface requires a lesser voltage increase beyond e_0, to extend the disruptive gradient for some distance from the conductor, and e_0 and e_v are therefore closer together, and this decrease of the loss below the theoretical value given by equation (**1**) is not appreciable.

b. As the conductor surface can never be perfect, some loss of power occurs at and below the disruptive critical voltage at isolated points of the conductor, where irregularities of the surface, scratches, spots of mud or dirt, etc., give a higher potential gradient than that corresponding to the curvature of the conductor surface. With small conductors, this loss is rarely appreciable, since the curvature of the conductor surface is of the same magnitude as that of its irregularities. It becomes appreciable however for larger conductors, as seen in Fig. 2. This excess of the loss beyond that given by the quadratic law,

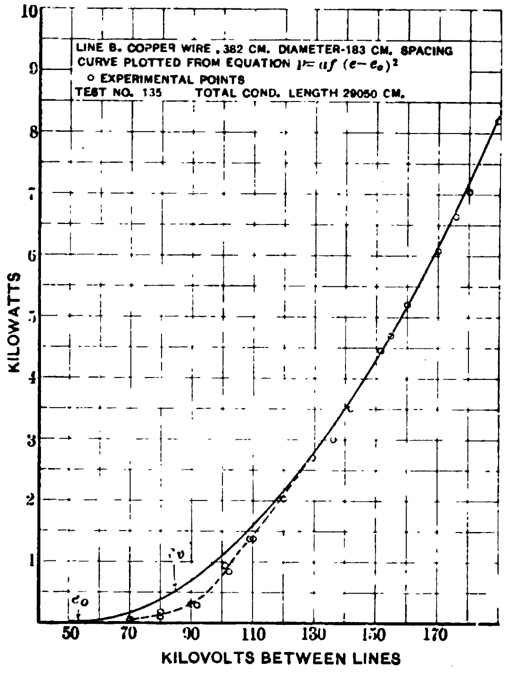

FIG. 3.—Corona loss

equation (**1**), essentially depends on the conductor surface, and is larger, the rougher or dirtier the surface is. It is a maximum at the disruptive critical voltage e_0, and decreases above and below e_0. It is with fair accuracy represented by the probability curve:

$$p_1 = q\,\epsilon^{-h\,(e_0 - e)^2} \qquad (\mathbf{4})$$

where q is a coefficient depending on the number of spots, and h is a coefficient depending on the size of spots.

Snow, sleet and rain losses seem to be of the same nature, but frequently of far greater magnitude.

The visual critical voltage, e_v, is derived from the disruptive gradient, g_0, by the equation

$$e_v = m_v\, g_0\, \delta\, r \left(1 + \frac{0.301}{\sqrt{r}}\right) \log_\epsilon \frac{s}{r} \text{ kv. to neutral} \qquad (\mathbf{5})^*$$

where $m_v = m_0 = 1$ to 0.93 for wires.

$$m_v = \begin{cases} 0.72 \text{ local corona all along conductor;} \\ 0.82 \text{ decided corona all along conductor,} \end{cases} \text{ for seven-strand cables}$$

If g_0 (maximum) is used, e_v is obtained in maximum kilovolts.

From (**1**), (**2**) and (**3**) then follows the complete expression of the corona loss in fair weather, at voltages above the visual critical voltage, e_v,

$$p = \frac{k}{\delta} f \sqrt{\frac{r}{s}} \left\{ e - g_0\, m_0\, r\, \delta \log_\epsilon \frac{s}{r} \right\}^2 10^{-5} \text{ kw. per km. of single conductor} \qquad (\mathbf{6})$$

where e = effective kilovolts to neutral.

$k = 344$.

$g_0 = 21.1$ kv. per cm. (effective).

δ = air density factor $= \frac{3.92\,b}{273 + t}$

$\delta = 1$ at 25 deg. cent. and 76 cm. barometric pressure.

b = barometric pressure, cm.

t = temperature, deg. cent.

r = radius of conductor, cm.

s = distance between centers of conductors, cm.

f = frequency, cycles per second.

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*NOTE:—The visual critical point for wires is very sharp and definite, while for cables it is not so well defined.

m_0 = irregularity factor.
$m_0 = 1$ for polished wires.
$= 0.98 - 0.93$ for roughened or weathered wires.
$= 0.87 - 0.83$ for seven-strand cable.

The corona loss is

a. Proportional to the frequency f.*

b. Proportional to the square of the excess voltage above the disruptive critical voltage, e_0.

c. Proportional to the square root of the conductor radius r, and inversely proportional to the square root of the conductor distance.

The disruptive critical voltage, e_0, is that voltage at which the disruptive voltage gradient of the air is reached at the conductor surface. Hence, it is:

a. Proportional to the conductor radius, r, and the $\log_\epsilon \frac{s}{r}$.

b. Proportional to the air density.

c. Depending somewhat on the condition of the conductor surface as represented by m.

3. The effects of various atmospheric conditions and storms on the critical voltage and loss are as follows:

Humidity or " vapor products " have no effect on either the critical voltage or the loss.

Smoke lowers the critical voltage and increases the loss.

Heavy wind has no effect on the loss or critical voltage at ordinary commercial frequencies.

The weather conditions that really count practically and which must be seriously considered in the design of transmission lines are as follows:

Fog lowers the critical voltage and increases the loss.

Sleet on the wires, or falling sleet, lowers the critical voltage and increases the loss. High voltages do not entirely eliminate sleet formation.

Rain storms lower the critical voltage and increase the loss.

Snow storms lower the critical voltage and increase the loss. The effect of snow is greater than that of any other weather condition.

Fig. 4 shows the loss during fair weather, and a corresponding loss during a snow storm.

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*It is probable that at extremely low frequencies the loss decreases less than proportionally to the frequency, and approaches a small finite limiting value at continuous impressed voltage.

The excess loss during sleet, rain or snow storms, over the fair weather loss, seems with increasing voltage to approach a maximum and then to decrease again (the latter at values very far above the disruptive critical voltage), and the curves of excess loss seem to have the general shape of probability curves, as is to be expected theoretically.

4. The following problem is given in order to show the method of applying the results to an actual transmission line.

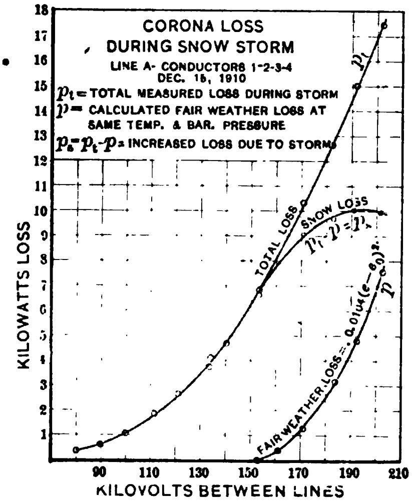

FIG. 4

EXAMPLE

Given a line—three-phase—60 cycles.

Length = 100 miles.
Spacing = 10 feet.
Conductor = No. 0 cable, diameter = 0.375 in.
Maximum temperature = 100 deg. fahr.
Elevation = 1,000 ft.

Required

(*a*) Disruptive critical voltage.
(*b*) Visual critical voltage.
(*c*) Loss (fair weather).
(*d*) Loss (during snow storm) assuming $e_0 = 80$ per cent of fair weather e_0.

Solution

Reducing to metric units
Length = 161 km.
Spacing = 305 cm.
Conductor radius = 0.478 cm.
Maximum temperature = 38 deg. cent.
Elevation being 317 m.
Barometer corresponding = 73.4 cm.

(*a*) Disruptive critical voltage.

From formula (3)

$$e_0 = m_0\, g_0\, \delta\, r \log_\epsilon \frac{s}{r}$$

$m_0 = 0.87$ value for seven-strand cables

$g_0 = 21.1$ kilovolts per cm. (effective).

$$\delta = \frac{3.92\, b}{273+t} = \frac{3.92 \times 73.4}{273+38} = 0.929$$

$$r = 0.478 \text{ cm.}$$

$$\log_\epsilon \frac{s}{r} = 6.46$$

Evaluating

$e_0 = 52.7$ kilovolts to neutral

Therefore the disruptive critical voltage between conductors $= 52.7 \times \sqrt{3} = 91.3$.

(*b*) Visual critical voltage.

From formula (5)

$$e_v = m_v\, g_0\, \delta\, r \left(1 + \frac{0.301}{\sqrt{r}}\right) \log_\epsilon \frac{s}{r}$$

Evaluating
$e_v = 71.1$ kv. for decided corona.
$e_v = 62.5$ kv. for local corona.
Therefore, visual critical voltage between wires
$= e_v \times \sqrt{3}$.
$= 125$ kv. for decided corona.
108 kv. for local corona.

(*c*) Power loss (fair weather).

From formula (6).

$$p=\frac{k}{\delta}f\sqrt{\frac{r}{s}}\left\{e-g_0 m_0 r\delta\log_\epsilon\frac{s}{r}\right\}^2 10^{-5}\text{ kw. per km. of single conductor.}$$

e = effective kilovolts to neutral.
$k=344.$
$\delta=0.929.$
$f=60.$
$\sqrt{\frac{r}{s}}=0.0396.$
$g_0=21.1.$
$\log_\epsilon\frac{s}{r}=6.46.$

Evaluating and multiplying by three, since we have three conductors, we obtain

$$p=2.64\ (e-52.7)^2\ 10^{-2}\text{ kw. per km.}$$

Substituting values for line volts

Fair weather				Storm	
e between line	e to neutral	p	$p$total $=p\times 161$	p	$p$total $=p\times 161$
90	51.9	——	——	2.48	400
100	57.7	0.66	106	6.33	1020
110	63.4	3.02	485	11.90	1920
120	69.2	7.2	1160	19.20	3090
130	75.1	13.3	2140	28.70	4610
140	80.9	21.0	3380	39.50	6360
150	86.6	30.4	4890	52.10	8390

(*d*) For approximate loss during storm consider $e_0=80$ per cent of its value in fair weather.

$$e_0=0.8\times 52.7=42.2\text{ kv.}$$

Therefore:

$$p=2.64\ (e-42.2)^2\ 10^{-2}\text{ kw. per km.}$$

See curves, Fig. 5

The formulas are given below in English units for the benefit of those who may desire them in this form.

1. e_0, the *effective disruptive critical voltage to neutral* is given by the equation:

$$e_0 = 2.302\, m_0\, g_0\, r\, \delta \log_{10} \frac{s}{r} \qquad (3a)$$

2. e_v, the *visual critical voltage* in effective kv. to neutral is obtained from the equation:

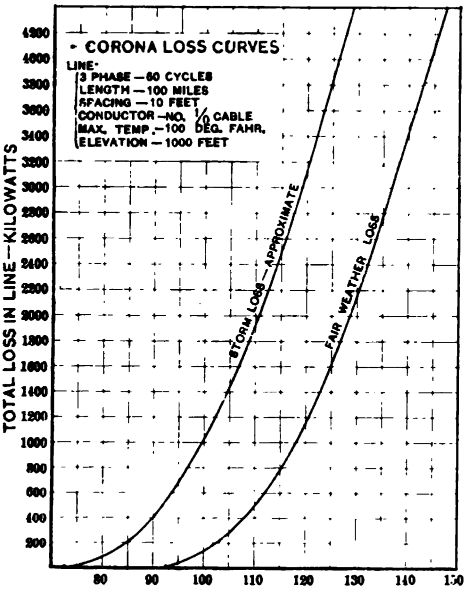

FIG. 5.

$$e_v = 2.302\, m_v\, g_0\, \delta\, r \left(1 + \frac{0.189}{\sqrt{r}}\right) \log_{10} \frac{s}{r} \qquad (5a)$$

3. p = the total power loss due to corona in fair weather, in kw. per mile of single conductor:

$$p = \frac{k'}{\delta} f \sqrt{\frac{r}{s}} \left\{ e - 2.302\, g_0\, m_0\, r\, \delta \log_{10} \frac{s}{r} \right\}^2 10^{-5} \qquad (6a)$$

In the above formulas:

e = effective kilovolts to neutral.

$k' = 552$.

$g_0 = 53.6$ kv. per in. (effective).

δ = air density factor $= \dfrac{17.91\, b}{459 + t}$

$\delta = 1$ at 77 deg. fahr. and 29.92 in. barometric pressure.

b = barometric pressure in inches.

t = temperature, deg. fahr.

r = radius of conductor, inches.

s = distance between centers of conductors, inches.

f = frequency, cycles per second.

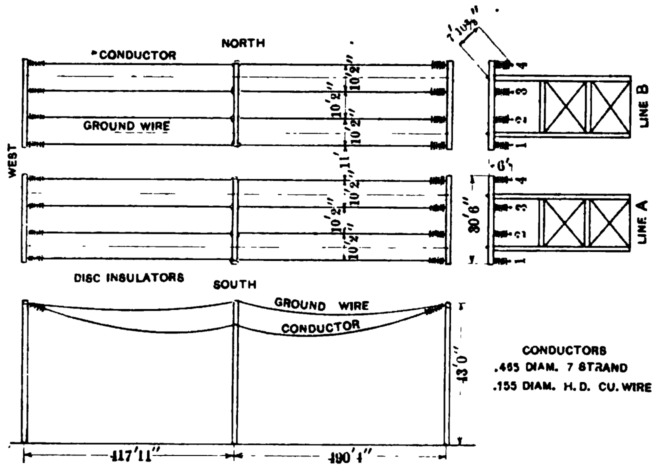

FIG. 6.—Corona loss—Original line arrangement used for preliminary tests.

m_0 = irregularity factor.

$m_0 = 1$ for polished wires.

$= 0.98$ to 0.93 for roughened or weathered wires.

$= 0.87$ to 0.83 for seven-strand cables.

$m_v = m_0$ for wires.

$$m_v = \left\{\begin{array}{l} 0.72 \text{ local corona all along cable;} \\ 0.82 \text{ decided corona all along cable,} \end{array}\right\} \text{for seven-strand cables.}$$

If g_0 maximum is used, e_v is obtained in maximum kilovolts.

II. LINES, APPARATUS AND METHOD OF TEST

The Lines. The conductors used in the investigations were supported by metal towers arranged in two parallel lines of two spans each. The length of each span was approximately 150 m. These tower lines will be designated by *A* and *B* respectively. The conductors were strung in a horizontal plane with seven disk suspension insulators at each point of support. For preliminary tests four No. 3/0 B. & S. (1.18 cm. diameter) seven-strand hard-drawn copper cables were put in place on each line. A seven-strand steel ground cable was also strung. (See Fig. 6). After a number of tests had been made the ground cables were removed from line *A*. The conductors on line *A*, however, were kept in place as a standard throughout all the investigations. The conductors and ground cables were removed from *B*, and the first span of this line was used to support various sizes of conductors at various spaces (see Fig. 7).

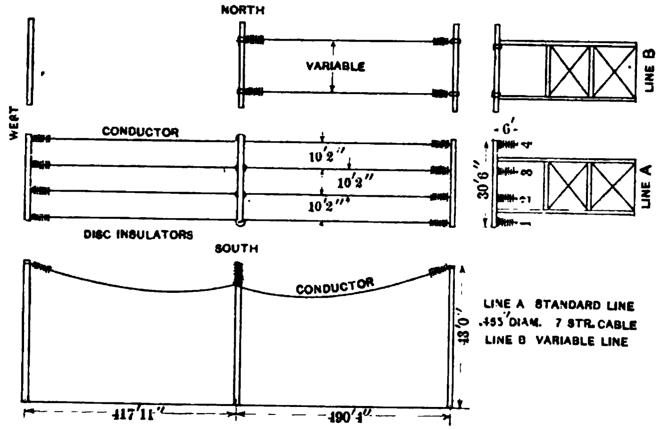

FIG. 7.—Corona loss—Final line arrangement

The lines were erected in a large field. The prevailing winds were from the west over open country, that is, free from smoke from the city and the factory on the east. Figs. 8 and 9 show the towers, and Fig. 10 shows one link of the insulators used.

Test Apparatus. A railroad track was run directly under the line, and the testing apparatus was housed in three box cars. This proved a very convenient arrangement, as the cars could be quickly run back to the factory when changes or repairs were necessary.

Power was received in substation car No. 1 at 550 volts, 40 cycles, and changed to direct current by a converter. This in turn was used to supply a direct-current motor, belted to a single-phase alternator. This last was an old Thomson-Houston machine with a smooth core pan-cake winding on the armature; it gave a very good wave, and was used in all tests. See oscillogram and analyzed wave, Fig. 11. It was rated at 35 kw., but this rating was quite conservative.

The high-voltage transformer and the testing apparatus were placed in car No. 2. The portion of the car roof over the transformer was made of heavy canvas. This could be rolled back, and the leads from the line dropped directly to the transformer terminals. By means of a framework and canvas cover the transformer could be protected from the weather, and investigations carried on during rain and snow storms. The power supply, speed and voltage, were all controlled at the test table in car No. 2. In fact all of the adjustments could be made from this car (see Figs. 12 and 13). The transformer was rated at 100 kw., 200,000 volts and 60 cycles. On the low side were four 500-volt

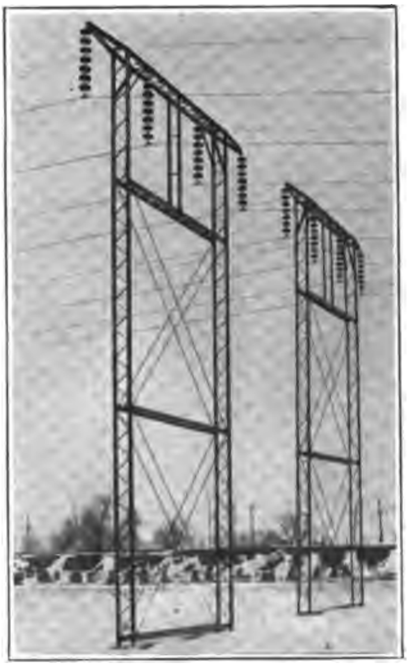

FIG. 8.—Intermediate transmission towers

coils. These coils could be connected in multiple or series for change of ratio. The high-tension winding was opened at the neutral and taps were brought out for the ammeter, and current coil of the wattmeter. Three taps were also brought out here from the main winding for voltage measurement. See Fig. 13. The following tap ratios were thus obtained—100/200,000; 200/200,000 and 400/200,000.

FIG. 9.—Experimental transmission line, looking east

The transformer was specially designed for this work by Mr. A. B. Hendricks, Jr., from experience gained in previous tests.*

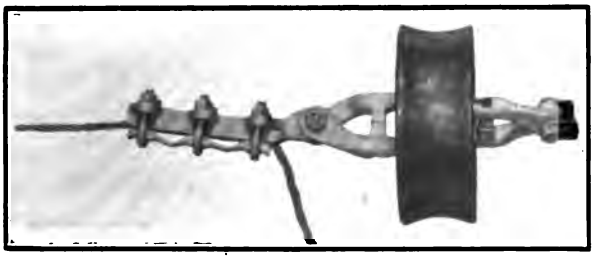

FIG. 10.—25,000-volt link strain insulator

Car No. 3 served as a dark room for making photographs, and visual tests on short wires and cables. Fig. 13*a* shows the framework used for supporting conductors in the visual tests.

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*See TRANSACTIONS A. I. E. E., 1911, XXX, I, p. 167.

Methods of Test. Accurate power measurements of corona are very difficult to make, because of the nature of the load—low power factor and high voltage. It is not desirable to make the measurements on the low side because of the difficulty in separating the transformer iron and load losses, and these may be sometimes as large as the corona losses. In these tests the current coil of the wattmeter, and the ammeter, were put in the high tension winding of the transformer at the neutral point, and the neutral was grounded. See Fig. 13. The voltage coil of the wattmeter was connected to a few turns of the high tension winding at the neutral.* Voltage was also read in this manner. Another method of voltage measurement was also tried as a check, namely, by using the above turns as a separate winding. The results obtained by the two methods checked very well.

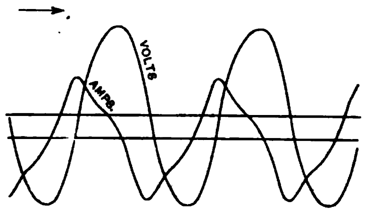

Fig. 11 (*a*).—Line *A* conductors 1, 2, 3, 4—191 kv.

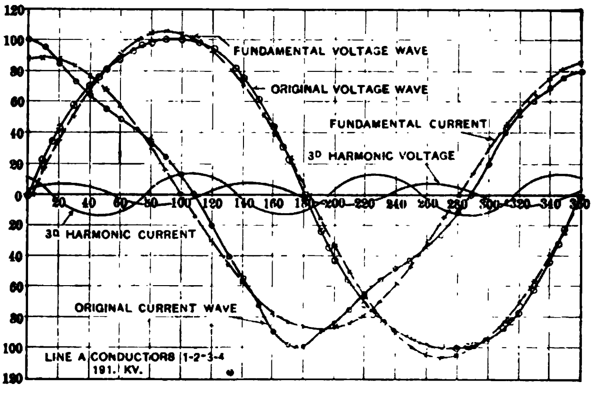

Fig. 11 (*b*).—Corona loss wave form analysis oscillogram *T* 40

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*See paper by A. B. Hendricks. Jr., Transactions A. I. E. E., 1911. XXX, I, p. 167.

All the loss measurements were also duplicated on the low side as a check. Frequency was held at the test table by means of the motor field and a vibrating reed type of frequency meter.

Voltage was controlled in two ways—by the potentiometer method and by rheostats in the alternator field. By the potentiometer method is meant a series resistance on the low side of the transformer for voltage control and a multiple resistance across the transformer, taking about three to five times the exciting current, to prevent wave distortion. When the leading current was very high a reactance was arranged to shunt the generator and approximately unity power factor could be held. This prevented overloading the generator and reduced wave shape distortion. For a set of tests at a given frequency the ratio of the main transformer was kept the same. Where losses at several frequencies were to be compared the main transformer ratio also was changed to keep the flux on the generator as nearly

Fig. 12.—Cars Nos. 1, 2 and 3

Fig. 12 (a).—Interior of car No. 2

constant as possible—for instance at 45 cycles a ratio of 500/200,000 would be used, while at 90 cycles a ratio of 1,000 to 200,000 would be used. Wattmeters with scales especially adapted to the tests were constructed. These were of the dynamometer type; they were each provided with a 75-volt and 150-volt tap. The voltmeter coil ratio on the transformer, and the wattmeter tap were always changed to give the best reading. Four wattmeters were used in these tests. The meters were all carefully calibrated in the laboratory at unity power factor and at 0.10 leading power factor, at both 25 and 60 cycles. See Fig. 14 for the meter scales.

Fig. 12 (*b*).—Transformer end of car No. 2

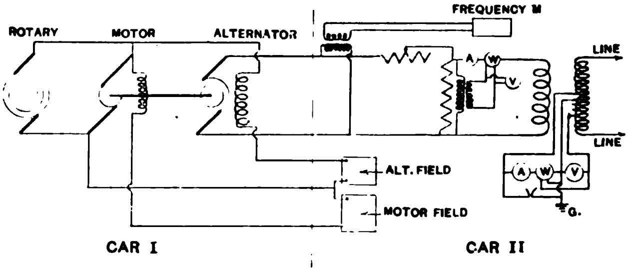

Fig. 13.—Arrangement and connections of apparatus used in corona tests

Humidity, temperature, and barometric pressures, as well as general weather observations, were taken during each test.

III. The Quadratic Law

Table I is a typical data sheet for Line *A*.

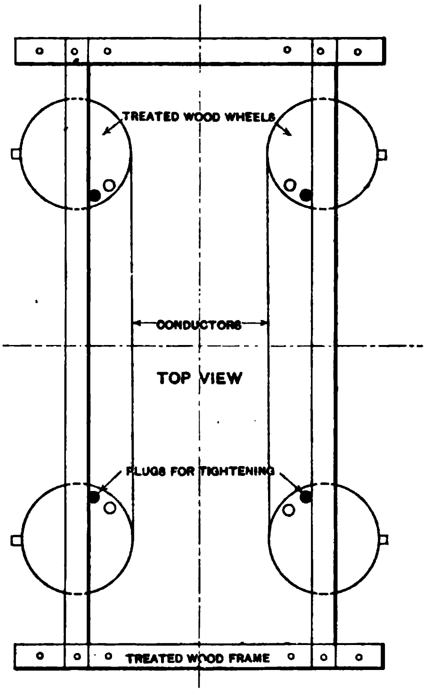

FIG. 13 (*a*)

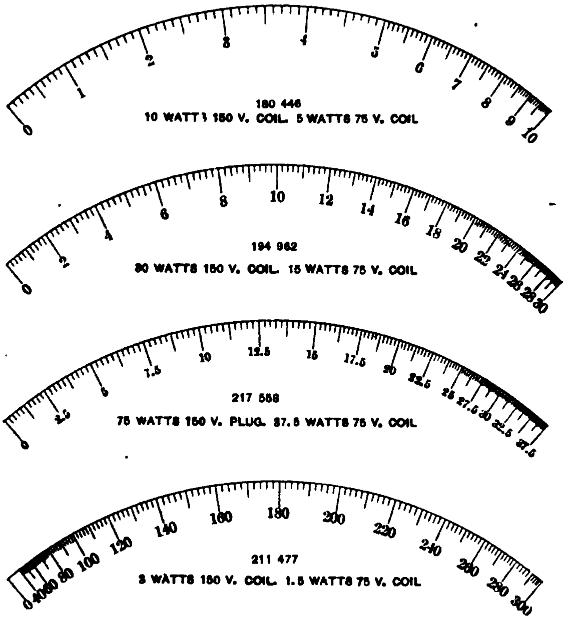

FIG. 14.—Wattmeter scales

TABLE I
EXPERIMENTAL LINE. 10-6-10 4 p.m.

Line on

Low side Total readings			High side Total readings		
Volts	Amperes	Kilowatts	Kilovolts	Amperes	Kilowatts
395	16.5	0.40	80.5	0.077	0.10
435	17.9	0.60	90.5	0.087	0.13
490	20.5	0.70	101.6	0.101	0.17
535	22.6	0.80	111.1	0.112	0.22
590	24.7	1.00	120.4	0.119	0.28
635	27.1	1.10	130.2	0.135	0.35
680	29.2	1.40	139.2	0.145	0.45
735	31.6	1.80	150.0	0.158	0.68
780	33.4	2.40	159.0	0.169	1.10
812	35.3	3.30	165.8	0.178	1.80
830	36.3	3.60	169.0	0.181	2.40
862	37.5	5.12	176.6	0.190	3.60
893	39.2	6.35	183.2	0.199	4.70
914	40.5	7.50	187.0	0.207	6.00
975	43.9	11.40	200.0	0.227	9.30
1020	47.8	14.50	209.0	0.243	12.60
1050	49.3	17.00	214.2	0.253	14.60
1080	52.7	19.50	220.2	0.267	17.60
1125	55.5	22.80	223.4	0.283	20.30
Line off					
400	1.14	0.40	80.0	0.005	0.055
500	1.37	0.62	100.5	0.007	0.10
600	1.63	0.82	121.5	0.008	0.15
718	1.87	1.18	143.5	0.010	0.21
812	2.05	1.45	161.0	0.011	0.28
905	2.29	1.80	181.0	0.013	0.35
1015	2.69	2.20	202.2	0.015	0.44
1095	3.25	3.64	217.0	0.017	0.54

Weather

Cloudy—rain in morning. Barometer 75 cm.—Temperature, wet 10 deg. cent., dry 12 deg. cent.

Line and connections

(1 and 3)—(2 and 4) Ground wires in place.
Total conductor length..109,500 cm.
Spacing.. 310 cm.
No. 3/0 seven-strand cable—diameter............................ 1.18 cm.
Transformer ratio..1000/200,000
Frequency...60 cycles.

Fig. 1 shows the characteristic corona curves. The corrected values for Table I are recorded in Table II.

TABLE II
CORONA LOSS
OBSERVED VALUES CORRECTED FROM TABLE 1

Kilovolts between lines e'	Line amperes	Kilovolts to neutral e	Kilowatts line loss p	Kilovolt-amperes	Power factor
80.5	0.072	40.2	—	5.80	—
90.5	0.081	45.2	—	7.33	—
101.6	0.094	50.8	—	9.55	—
111.1	0.104	55.5	—	11.55	—
120.4	0.111	60.2	0.11	13.40	0.008
130.2	0.126	65.1	0.15	16.40	0.009
139.2	0.135	69.6	0.22	18.80	0.012
150.0	0.147	75.0	0.40	22.10	0.018
159.0	0.157	79.5	0.79	25.00	0.032
165.8	0.166	82.9	1.42	27.60	0.051
169.0	0.168	84.5	2.04	28.40	0.072
176.6	0.177	88.3	3.21	31.20	0.103
183.2	0.185	91.6	4.28	33.90	0.126
187.0	0.193	93.5	5.55	36.10	0.154
200.0	0.212	100.0	8.78	42.40	0.207
209.0	0.227	104.5	12.02	47.50	0.253
214.2	0.237	107.1	13.99	50.70	0.276
220.2	0.250	110.1	16.94	55.10	0.307

The shape of the curve between kilovolts and kilowatts suggests a parabola. After trial it was found that the losses above the knee of the curve follow a quadratic law. Below the knee it was found that the curve deviates from the quadratic law. This variation near the critical voltage is due to dirt spots, irregularities and other causes that will be discussed in section VII. We will confine ourselves in this section to the main part of the curve expressed by

$$p = c^2 (e - e_0)^2$$

where

p = the line loss.

e = kilovolts to neutral.

e_0 is called the *disruptive critical voltage*, and is measured in kilovolts to neutral.

The meaning of e_0 and c^2 will be considered later.

Let us now see the best mechanism of evaluation of constants for a given set of tests. We may write (1)

$$\sqrt{p} = c\,(e - e_0)$$

then, if the quadratic law holds, the curve between the square root of p, and e, will be a straight line. e_0 will be the point where the line cuts the e axis, and c will be the slope of the line (see Fig. 15). e_0 and c might be evaluated graphically in this way, but it is difficult to know how to draw the line accurately and give each point the proper weight. To do this we will use the $\Sigma\Delta$ method,* and proceed as follows:

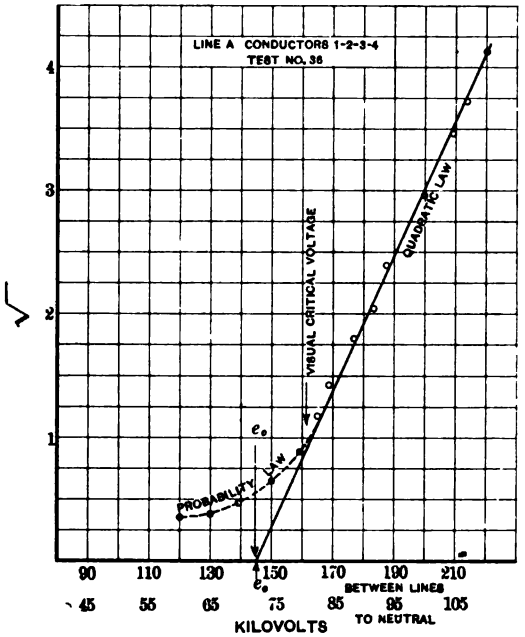

FIG. 15.—Corona loss—method of reducing

The values of e and the $\sqrt{p}$ for the set of readings to be investigated are first tabulated and a curve plotted (Fig. 15). All points that differ greatly from the straight line are eliminated as probably in error or, as at the lower part of the curve, following a different law. Then taking the remaining

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*See "Engineering Mathematics", Steinmetz, page 232.

readings and forming two groups, each of an equal number of readings, we have:

$$\text{Group 1} \quad \Sigma_1 e \qquad \Sigma_1 \sqrt{p}$$

$$\text{Group 2} \quad \Sigma_2 e \qquad \Sigma_2 \sqrt{p}$$

Then

$$\Delta \Sigma e = \Sigma_1 e - \Sigma_2 e \qquad \Delta \Sigma \sqrt{p} = \Sigma_1 \sqrt{p} - \Sigma_2 \sqrt{p}$$

$$\Sigma \Sigma e = \Sigma_1 e + \Sigma_2 e \qquad \Sigma \Sigma \sqrt{p} = \Sigma_1 \sqrt{p} + \Sigma_2 \sqrt{p}$$

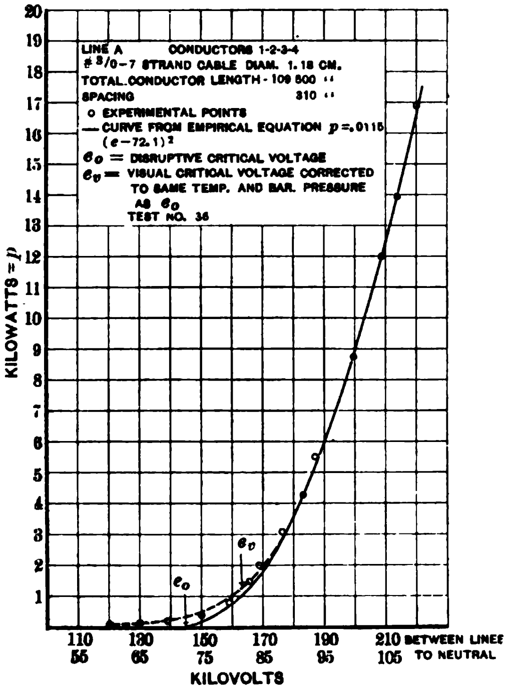

Fig. 16.—Corona loss

$$c = \frac{\Delta \Sigma \sqrt{p}}{\Delta \Sigma e}$$

$$e_0 = \frac{\Sigma \Sigma e - \dfrac{\Sigma \Sigma \sqrt{p}}{c}}{n}$$

where n is the number of points used.

Thus e_0 and c are determined,

TABLE III

CORONA LOSS

METHOD OF REDUCING (10-6-10)

Kilovolts between lines e'	Kilovolts to neutral e	Kilowatts loss p	$\sqrt{p}$
120.4	60.2	0.11	0.332
130.2	65.1	0.15	0.388
139.2	69.6	0.22	0.470
150.0	75.0	0.40	0.632
159.0	79.5	0.79	0.889
165.8	82.9	1.42	1.192
169.0	84.5	2.04	1.428
176.6	88.3	3.21	1.792
183.2	91.6	4.28	2.069
187.0	93.5	5.55	2.356
200.0	100.0	8.78	2.963
209.0	104.5	12.02	3.467
214.2	107.1	13.99	3.740
220.2	110.1	16.94	4.116

Total conductor length..109,500 cm.
Spacing..310 cm.
No. 3/0 seven-strand cable diameter.........................1.18 cm.

e	$\sqrt{p}$
91.6	2.069
93.5	2.356
100.0	2.963
104.2	3.467
107.1	3.740
110.1	4.116

$\Delta \Sigma e = 36.6$ $\Delta \Sigma \sqrt{p} = 3.935$

$\Sigma \Sigma e = 606.8$ $\Sigma \Sigma \sqrt{p} = 18.711$

$\Sigma_2 e = 285.1$ $\Sigma_2 \sqrt{p} = 7.388$

$$c = \frac{\Delta \Sigma \sqrt{p}}{\Delta \Sigma e} = 0.107$$

$$c^2 = 0.0115$$

$$e_0 = \frac{\Sigma \Sigma e - \frac{\Sigma \Sigma \sqrt{p}}{c}}{n}$$

$\Sigma_1 e = 321.7$ $\Sigma_1 \sqrt{p} = 11.323$

$e_0 = 72.1$

$e_0' = 144.2$

Table III shows the method of reducing. The curve, Fig. 16, is drawn from the equation $p = 0.0115\ (e - 72.1)^2$. The circles show the experimental values, and where the losses deviate from the quadratic law.

TABLE IV

CALCULATED VALUES FOR FIG. 16

$$p = c^2 (e - e_0)^2$$

$$p = 0.0115 (e - 72.1)^2$$

Kilovolts between lines e'	Kilovolts to neutral e	Kilowatts $p = c^2 (e - e_0)^2$
144.2	71.2	0.0
150.0	75.0	0.10
159.0	79.5	0.63
165.8	82.9	1.34
169.0	84.5	1.77
176.6	88.3	3.02
183.2	91.6	4.17
187.0	93.5	5.08
200.0	100.0	9.03
209.0	104.5	12.10
214.2	107.1	14.10
220.2	110.1	16.70

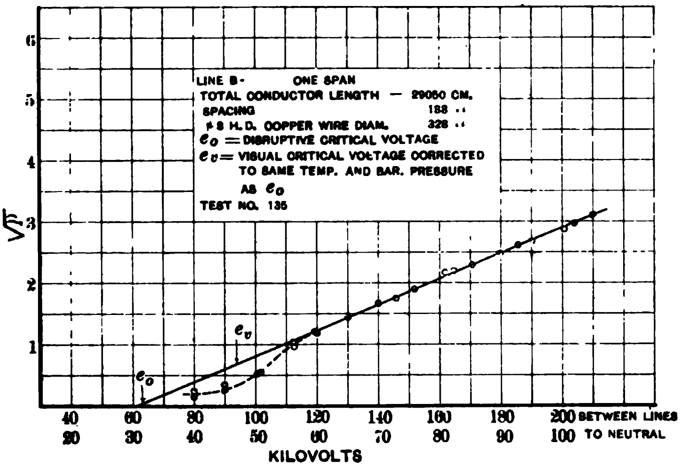

FIG. 17.—Corona loss—Method of reducing

Table V gives a similar set of data for a small wire. The results are plotted in Figs. 17 and 18.

All data taken under various conditions were first reduced in this way and tabulated as in Table VI (*a*), (*b*), (*c*).

TABLE V
CORONA LOSS
$\Sigma \Delta$ METHOD OF REDUCING

Kilovolts between lines e'	Kilovolts to neutral e	Kilowatt line loss p	$\sqrt{p}$
70.0	35.0	0.02	0.14
80.0	40.0	0.07	0.26
91.2	45.6	0.26	0.51
101.3	50.6	0.85	0.92
110.0	55.0	1.42	1.19
120.0	60.0	2.02	1.42
130.0	65.0	2.71	1.65
141.5	70.7	3.51	1.87
70.0	35.0	0.06	0.24
80.0	40.0	0.10	0.32
90.5	45.2	0.26	0.51
101.3	50.6	0.96	0.98
109.9	54.9	1.43	1.20
152.0	76.0	4.45	2.11
160.4	80.2	5.17	2.27
170.0	85.0	6.06	2.46
180.6	90.3	7.04	2.65
190.6	95.3	8.26	2.87
200.0	100.0	9.52	3.08
193.6	96.8	8.60	2.93
176.0	88.0	6.65	2.58
155.0	77.5	4.66	2.16
136.0	68.0	3.01	1.73

Total conductor length...29,050 cm.
Spacing...183 cm.
No. 8 H. D. copper wire—diameter...328 cm.

e	$\sqrt{p}$	e	$\sqrt{p}$
100.0	3.08	80.2	2.27
96.8	2.93	77.5	2.15
95.3	2.87	68.0	1.73
90.3	2.65	60.0	1.42
85.0	2.46	88.0	2.58
467.4	13.99	373.7	10.16

$$\Sigma \Sigma e = 841.1$$

$$\Sigma \Sigma \sqrt{p} = 24.15$$

$$e_0 = \frac{84.11 - \frac{24.15}{0.0406}}{10}$$

$$= 24.6$$

$$\Delta \Sigma e = 93.7$$

$$\Delta \Sigma \sqrt{p} = 3.80$$

$$c = \frac{3.80}{93.7} = 0.0406 \qquad c^2 = 0.00164 \qquad p = 0.00164\,(e - 24.6)^2$$

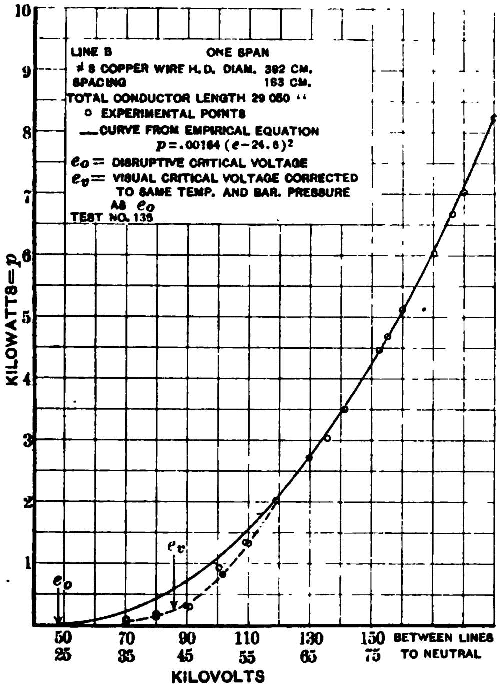

FIG. 18—Corona loss

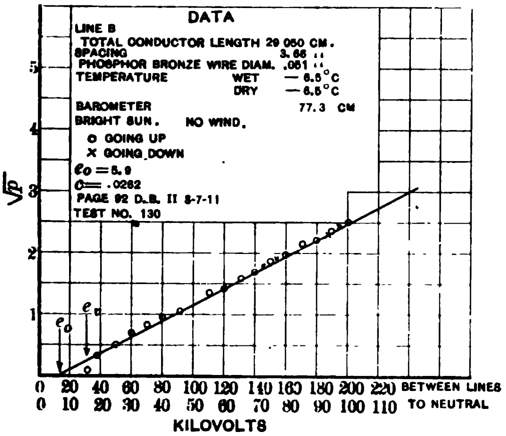

FIG. 19.—Corona loss

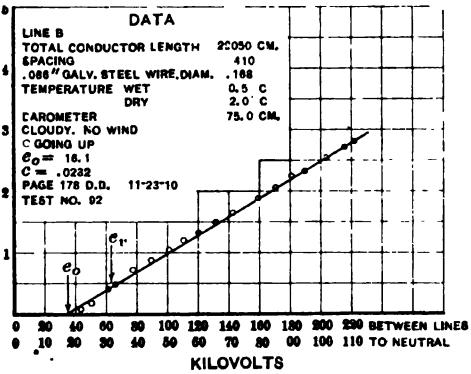

FIG. 20.—Corona loss

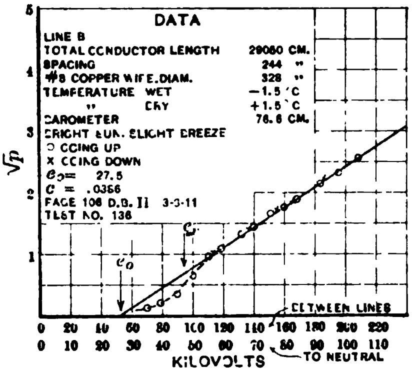

FIG. 21.—Corona loss

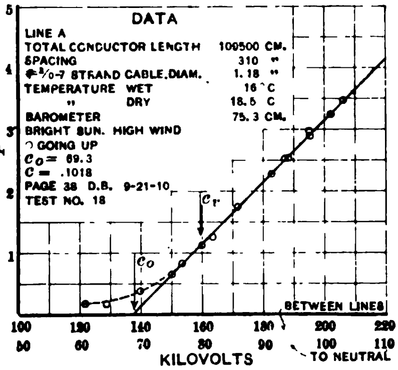

FIG. 22.—Corona loss

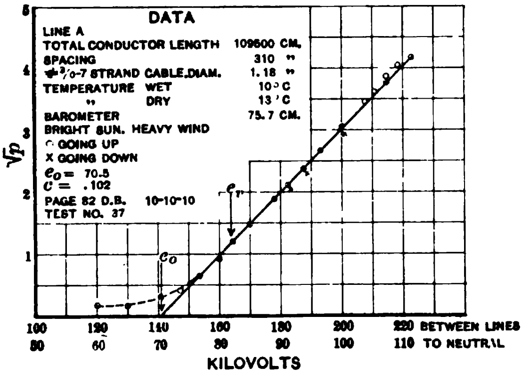

FIG. 23.—Corona loss

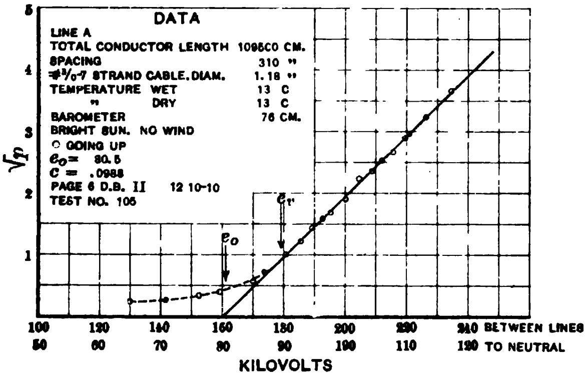

FIG. 24.—Corona loss

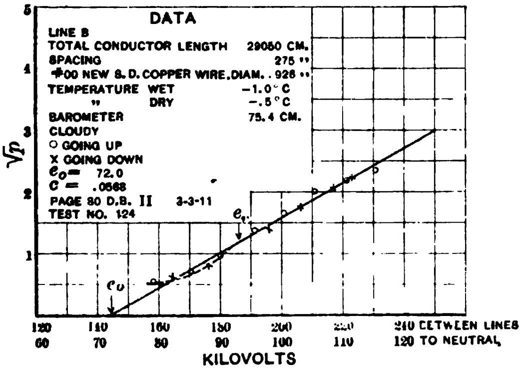

FIG. 25.—Corona loss

TABLE VIa

Σ Δ REDUCTION OF CORONA TESTS CABLES

Test No.	Temperature		Bar cm.	Freq. cycles	c^2/f per cm. of total cond.	e_0	g_0 Eff	g_0 max.	c^2/f at 25° C. 76 cm. bar	g_0 max. at 25° C. 76 cm. bar	Spacing cm.	Weight			Weather
	Wet	Dry										Limits		No. points used	
STANDARD—Line A—Conductors 1-2-3-4.						Total length 109,500 cm.			No. 3/0 7-strand copper cable diameter 1.18 cm.						
18	16	18.5	75.5	60	15.80×10-10	69.3	18.9	26.1	16.10×10-10	25.5	310	160	206	8	Bright sun—wind.
36	10	12	75.0	60	17.57 "	72.1	19.5	26.9	18.20 "	26.0	310	180	220	6	Cloudy.
37	10	13	75.7	60	15.80 "	70.5	19.05	26.3	16.48 "	25.3	310	164	222	16	Bright sun. Heavy wind.
84	1	3	75.2	60	15.80 "	74.5	20.1	27.8	16.93 "	25.8	310	166	213	4	Cloudy.
102	— 1	— 1	74.7	60	15.88 "	76.9	20.7	28.6	17.17 "	26.5	310	170	211	6	Cloudy.
103	— 4.9	— 4.5	75.7	60	16.24 "	77.7	21.0	29.0	18.03 "	26.1	310	174	221	8	Sun—heavy wind.
104	— 9.5	— 9.5	76.2	60	14.93 "	78.5	21.2	29.3	17.00 "	25.7	310	177	233	14	Sun—cold wind.
105	—13	—13	76.2	60	14.84 "	80.5	21.7	30.0	17.15 "	26.0	310	181	234	12	Bright sun. No wind Snow on ground.
109	— 3	— 2	73.9	60	14.48 "	74.4	20.1	27.8	15.61 "	25.8	310	160	214	12	Hazy sun. Slight wind Snow on ground.
119	— 6.5	— 6	76.5	60	12.00 "	75.1	20.3	28.0	13.51 "	24.8	310	171.2	217.6	8	Bright sun. Heavy wind Going up—frost, on lines.
119	— 6.5	— 6	76.5	60	16.56 "	78.3	21.2	29.3	18.65 "	25.9	310	174.4	215.4	4	Bright sun. Heavy wind. Going down. Lines dried by current.
Line B—Total conductor length 29,050 cm.									0.375" galv. steel cable diameter 0.953 cm.						
100	— 1	— 1	74.7	60	26.40×10-10	50.0	20.0	27.6	28.55×10-10	25.5	91.4	161	212	8	Cloudy.
115	— 7.8	— 7	76.5	60	30.40 "	54.7	21.4	29.5	34.40 "	26.2	91.4	160	207.6	10	Bright sun—heavy wind.
Line B—Total conductor length 29,050 cm.									0.230" galv. steel cable diameter 0.585 cm.						
73	1	3	75.2	60	16.96×10-10	34.7	20.6	28.7	18.20×10-10	26.5	91.4	140	156	4	Cloudy.
80	1	3	75.2	60	13.08 "	34.3	18.7	25.8	14.04 "	24.0	152	181	213	4	"
76	1	3	75.2	60	9.08 "	37.9	18.6	25.7	9.76 "	23.9	310	160	221	6	"
83	1	3	75.2	60	8.56 "	41.5	19.4	26.8	9.20 "	25.0	432	180	210	4	"

TABLE VI*b*

Σ Δ REDUCTION OF CORONA TESTS
LINE *B* WIRES

Test No.	Temperature		Bar cm.	Freq. cycles	c^2/f per cm. of total cond.	e_0	g_0 Eff	g_0 max.	c^2/f at 25° C. 76 cm. bar	g_0 max. at 25° C. 76 cm. bar	Spacing cm.	Weight			Weather
	Wet	Dry										Limits		No. points used	
	0.02″ phosphor bronze wire—diameter 0.051 cm.									Total conductor length cm.					
129	—	— 8	77.3	60	——	5.3	24.6	33.9	——	29.8	122	90.5	173.8	14	Bright sun—no wind.
133	—	— 1.5	77.3	60	——	5.5	24.1	33.2	——	29.8	183	91.0	203	12	" "
131	—	— 8	77.3	60	——	5.8	24.6	33.9	——	29.8	244	109.7	203	18	" "
130	—	— 6.5	77.3	60	——	5.9	24.5	33.8	——	29.8	366	100	201	14	" "
132	—	— 1.5	77.3	60	——	6.0	24.1	33.2	——	29.8	488	102	210	16	" "
	0.066-in. galvanized steel wire—diameter 0.168 cm.									Total conductor length 29,050 cm.					
91	0.5	2.0	75.0	60	8.44×10⁻¹⁰	15.5	24.4	33.7	9.07×10⁻¹⁰	31.3	152	101	218	8	Cloudy—no wind.
94	0.5	2.0	75.0	60	6.84 "	16.4	24.6	33.9	7.34 "	31.6	229	118	222	6	" "
92	0.5	2.0	75.0	60	4.28 "	16.1	22.7	31.3	4.60 "	29.1	410	131	222	8	" "
95	1.0	3.0	75.0	60	4.68 "	18.0	24.4	33.7	5.00 "	31.4	550	120	222	6	" "
	No. 8 copper wire H. D. Diameter 0.328 cm.									Total conductor length 29,050 cm.					
134	— 1.5	+ 1.5	76.6	60	12.89×10⁻¹⁰	25.2	22.9	31.6	14.11×10⁻¹⁰	28.8	122	112	201	16	Bright sun. Slight breeze
135	— 1.5	+ 1.5	76.6	60	9.52 "	24.6	21.5	29.7	10.42 "	27.1	183	120	200	10	" "
136	— 1.5	+ 1.5	76.6	60	8.56 "	27.5	23.0	31.7	9.38 "	29.0	244	110.5	203.8	10	" "
137	— 1.5	+ 1.5	76.6	60	6.52 "	26.4	20.4	28.1	7.14 "	25.7	366	124	186	10	" "
138	— 1.5	+ 1.5	76.6	60	5.52 "	26.0	20.1	27.7	6.05 "	25.2	488	131.5	202.2	10	" "
	No. 4 copper wire H. D. diameter 0.518 cm.									Total conductor length 29,050 cm.					
125	— 5	— 4.5	75.9	60	17.88×10⁻¹⁰	34.7	22.7	31.3	19.85×10⁻¹⁰	28.2	91.4	128	213	16	Cloudy Slight breeze.
126	— 5	— 4.5	75.9	60	11.44 "	36.3	21.3	29.4	12.70 "	26.5	183	151	231	12	" "
127	— 3.5	— 2.5	75.9	60	8.68 "	37.5	20.7	28.6	9.57 "	26.0	275	159.2	216	10	" "
128	— 2	— 1.5	75.9	60	6.88 "	40.0	20.0	27.6	7.64 "	26.2	397	170	221	8	" "
	No. 2/0 copper wire S. D. Diameter 0.928 cm.									Total conductor length 29,050 cm.					
122	— 1	— 0.5	75.4	60	23.80×10⁻¹⁰	55.3	22.6	31.2	25.85×10⁻¹⁰	28.7	91.4	185.6	233	10	Cloudy.
123	— 1	— 0.5	75.4	60	19.80 "	66.3	23.9	33.0	21.51 "	30.4	183	191	225	8	"

TABLE VI*c*

Σ Δ REDUCTION—DIFFERENT FREQUENCIES. CORONA TESTS

Test No.	Temperature		Bar cm.	Freq. cycles	c^2/f per cm. of total cond.	e_0	g_0 Eff	g_0 max.	c^2/f at 25° C. 76 cm. bar	g_0 max. at 25° C. 76 cm. bar	Spacing cm.	Weight			Weather
	Wet	Dry										Limits		No. points used	
Line *A*—conductors 1-2-3-4. Total length 109,500 cm. No. 3/0 7-strand copper cable diameter 1.18 cm.															
14	20	22	75.2	50	17.20×10^{-10}	69.3	18.7	25.8	17.26×10^{-10}	25.7	310	164	195	6	Fair—sun at intervals.
15	20	22	75.2	60	17.48 "	70.0	18.9	26.1	17.54 "	25.9	310	161	202	6	" "
17	20	22	75.2	70	18.77 "	70.5	19.0	26.2	18.83 "	26.1	310	163	200	6	" "
16	20	22	75.2	80	17.20 "	69.5	18.75	25.9	17.26 "	25.8	310	158	190	4	" "
33	20	22	76.2	90	17.91 "	68.2	18.4	25.4	18.21 "	25.0	310	153	188	4	Cloudy.
Line *A* conductors 2-3 total length 54,750 cm. No. 3/0 seven-strand copper cable diameter 1.18 cm.															
68	3	5	75.2	47	11.40×10^{-10}	72	19.5	26.9	12.16×10^{-10}	25.3	310	170	197	4	Damp and Cloudy.
69	3	5	75.2	60	12.42 "	70.2	19.0	26.2	13.30 "	24.6	310	170	212	6	" "
62	3	5	75.2	80	13.78 "	71.5	19.2	26.5	14.68 "	24.9	310	183	213	4	" "
65	3	5	75.2	90	13.88 "	73.2	19.8	27.3	14.80 "	25.6	310	180	211	4	" "
66	3	5	75.2	100	13.14 "	72.2	19.5	26.9	14.02 "	25.2	310	179	211	6	" "

To further investigate the law it is now necessary to determine the various factors affecting e_0 and c^2. These will be taken up under separate headings. The loss near the critical point will then be discussed.

In Fig. 19 the $\sqrt{p}$ and e are plotted. This is an especially interesting curve on account of its range. The measurements are taken up to approximately 20 times the disruptive critical voltage, and show how well the quadratic law holds. Figs. 20 to 25 are plotted in the same way to illustrate the quadratic law.

IV. Frequency

To determine the way that frequency enters into the power equation

$$p = c^2 \, (e - e_0)^2$$

a series of loss curves were taken at various frequencies between 47 and 115 cycles. Points taken from the volt-watt curves are tabulated below in Table VII and VIII and plotted in Figs. 26 and 27.

TABLE VII

LINE A. CONDUCTORS 2-3. TOTAL LENGTH 54,750 cm.

	210	200	190	180 kilovolts between lines
		Kilowatts	loss	
f				
47	4.9	3.2	2.0	1.30
60	5.1	3.7	2.6	1.6
70	6.1	4.3	2.8	1.70
80	6.9	5.0	3.4	1.9
			3.6	2.1
90	7.5	5.6	3.8	2.6
100	8.0	5.6	3.70	2.4
115	9.3	7.1	5.0	3.4

TABLE VIII

LINE A. CONDUCTORS 1-2-3-4. TOTAL LENGTH 109,500 cm.

	210	200	190	180 kilovolts between lines
		Kilowatts	Loss	
50	9.24	6.30	3.80	2.20
60	10.50	7.30	4.65	2.65
70	12.1	8.60	5.50	3.20
80	14.0	10.20	6.80	3.95

For a given voltage the points lie on a straight line through the zero point. That is, the loss seems to vary directly with the frequency. The data are further investigated and reduced by the Σ Δ method. The results are tabulated below in Tables IX and X.

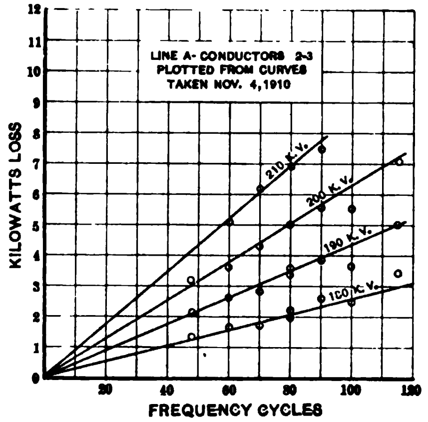

FIG. 26.—Corona loss—Different frequencies

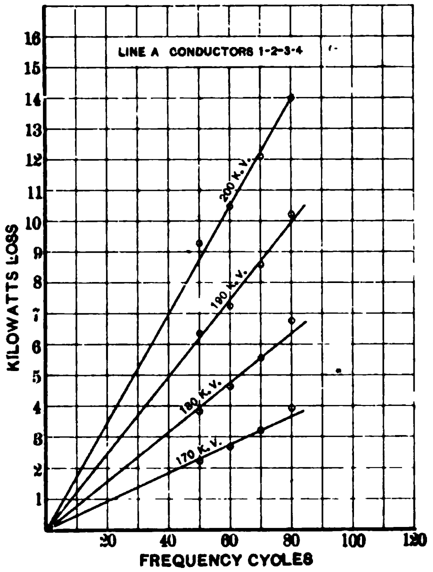

FIG. 27.—Corona loss—Different frequencies

TABLE IX
LINE A. CONDUCTORS 2-3

f	e_0	c^2	Total $c^2/f=a$
47	72	0.00292	6.00×10^{-5}
60	70.2	0.00408	6.80
80	71.5	0.00600	7.50
90	73.2	0.00684	7.60
100	72.2	0.0072	7.20
		Average,	7.10

TOTAL LENGTH 54,750 cm.

TABLE X
LINE A. CONDUCTORS 1-2-3-4. TOTAL LENGTH 109,500 cm.

f	e_0	c^2	Total $c^2/f=a$
50	69.3	0.0094	$18.80\times10^{-}$
60	70.0	0.0115	19.10
70	70.5	0.0140	20.60
80	69.5	0.0151	18.80
90	68.2	0.0177	19.60
		Average,	19.40

The values of c^2/f should be constant if the loss varies directly as the frequency. The tables show c^2/f constant within the limits of experimental error.

Care was taken to keep the wave shape as nearly constant as possible in these tests. Variations in e_0 and c^2 are accounted for by wave shape changes. For instance a slight progressive change in the wave shape would change the slope c of the line between $\sqrt{p}$ and e. This would cause a variation in both e_0 and c^2.

The reduced values of c^2 are plotted with frequency in curves 28 and 29. The points lie on a straight line passing through the zero point. The tabulated values show that e_0 is independent of the frequency. The power equation may now be written:

$$p=a f\,(e-e_0)^2$$

that is, at a given sine wave voltage the loss per cycle is constant for a given conductor at a given spacing.

The fact that the curves pass through the zero point does not necessarily mean that there is no loss at zero frequency or continuous impressed voltage. What takes place is probably this: when excessive continuous voltages are applied to a conductor, the air is broken down and a transfer of energy which appears as corona takes place. Now if the conditions were constant, such as still air, constant temperature, and no electrostatic repulsion, there would be no further loss than the first energy rush. However, as this overstrained air is probably driven away and replaced by fresh air, which is in turn broken down, there is actually a power loss with continuous voltage. Hence the above frequency relations do not contradict the observed fact of corona loss with direct current. If observations could

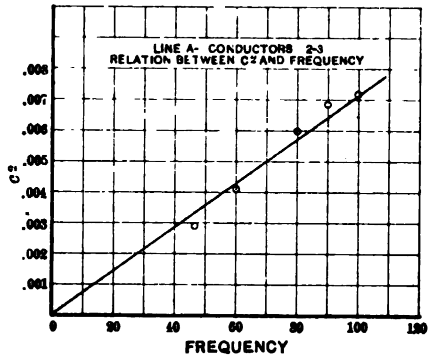

FIG. 28.—Corona loss

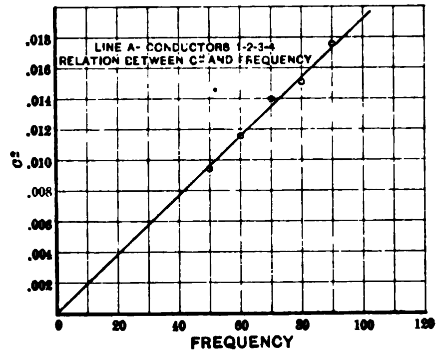

FIG. 29.—Corona loss

be made at very low frequencies, or continuous impressed voltage, the curve would probably be obtained as shown in Fig. 30.

V. RELATION BETWEEN $c^2/f = a$ AND s/r

The power equation may now be written:

$$p = a\,f\,(e - e_0)^2$$

where $a = c^2/f$ is a factor that has not to this point been investigated.

In Table XI are values of c^2/f for various sizes of wire and cable and various spacings. Upon investigation it is found that a varies greatly with the radius of the conductor and the spacing. Plotting s/r and a a curve is obtained that suggests a hyperbola.

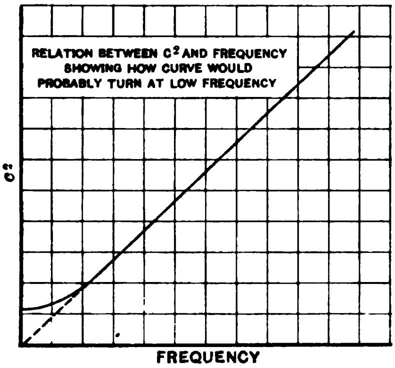

FIG. 30.—Corona loss

Where r = the radius of the conductor in cm., s = distance between conductor centers. The curve between $\log_\epsilon a$ and $\log_\epsilon s/r$ is a straight line. Therefore the following relation between a and s/r is established:

$$a = c^2/f = k\,(s/r)^d$$

The constants k and d are calculated by the $\Sigma\,\Delta$ method in Table XII, and

$$a = c^2/f = 3.70\,(r/s)^{.523} \times 10^{-8} \qquad (2)$$

These calculations are merely preliminary.

The fair weather value of a for standard line A 1-2-3-4 may now be examined in Table XIII.

It is seen that these values are not exactly constant, but apparently vary with the temperature and barometric pressure. $1/\delta$ in Table XIII is explained in section XI. It is sufficient here to call it the air density correction factor; it is unity at 25 deg. cent. and 76 cm. barometer. a and δ in curve Fig. 31 suggest that a varies as $1/\delta$. Multiplying by $1/\delta$ then reduces a to the standard temperature of 25 deg. cent. and 76 cm. barometric pressure. The constants for equation (2) were determined from values of a corresponding to various temperatures and barometric pressures. In order to properly evaluate the constants it is now necessary to reduce all a values

TABLE XI
(EXPERIMENTAL VALUES FROM TABLE VI)

Relation between $c^2/f = a$ and s/r

Tests No.	Diam. cm.	$\frac{s}{r}$	$c^2/f \times 10^{10}$ $= a \times 10^{10}$	Style of conductor	Material
95	0.168	6550	4.68	Wire	Gal. iron
92	0.168	4880	4.28	"	"
86	0.168	3700	5.44	"	"
138	0.328	2980	5.52	"	Copper
94	0.168	2730	6.84	"	Gal. iron
137	0.328	2230	6.52	"	Copper
91	0.168	1820	8.44	"	Gal. iron
128	0.518	1530	6.87	"	Copper
136	0.328	1490	8.56	"	"
82	0.585	1480	8.56	Cable	Gal. iron
135	0.328	1120	9.52	Wire	Copper
94*a*	0.168	1090	10.56	"	Gal. iron
77	0.585	1060	9.08	Cable	"
79	0.585	770	9.52	"	"
134	0.328	740	12.88	Wire	Copper
126	0.518	700	11.44	"	"
18	1.181	525	15.80	Cable	"
80	0.585	520	13.08	"	Gal. iron
125	0.518	350	17.88	Wire	Copper
73	0.585	310	16.96	Cable	Gal. iron
100	0.953	193	26.40	"	"

TABLE XII

RELATION OF $\frac{s}{r}$ to a. $\Sigma\,\Delta$ REDUCTION

Test No.	Diam. cm.	$\frac{s}{r}$	$\left(\frac{c^2}{f}\times 10^{10}\right)$ per cm. $= a \times 10^{10}$	$\log_\epsilon \frac{s}{r}$	$\log_\epsilon\left(\frac{c^2}{f}\times 10^{10}\right)$	Kind of cond.	Material
92	0.168	4880	4.28	8.492	1.454	Wire	Gal. iron
86	0.168	3700	5.44	8.215	1.694	"	"
138	0.328	2980	5.52	8.000	1.708	"	Copper
137	0.328	2230	6.52	7.709	1.875	"	"
136	0.328	1490	8.56	7.306	2.147	"	"
82	0.585	1480	8.56	7.299	2.147	Cable	Gal. iron
135	0.328	1120	9.52	7.020	2.253	Wire	Copper
94a	0.168	1090	10.56	6.993	2.357	"	Gal. iron
134	0.328	740	12.88	6.607	2.556	"	Copper
126	0.518	700	11.44	6.551	2.437	"	"
18	1.181	525	15.80	6.263	2.760	Cable	"
80	0.585	520	13.08	6.254	2.575	"	Gal. iron
125	0.518	350	17.88	5.858	2.884	Wire	Copper
73	0.585	310	16.96	5.736	2.831	Cable	Gal. iron

$$\Sigma \log_\epsilon \frac{s}{r} = 54.04 \qquad \Sigma \log_\epsilon (c^2/f \times 10^{10}) = 13.284$$

$$\Sigma \log_\epsilon \frac{s}{r} = 44.26 \qquad \Sigma \log_\epsilon (c^2/f \times 10^{10}) = 18.400$$

$$\Delta\,\Sigma \log_\epsilon \frac{s}{r} = 9.78 \qquad \Delta\,\Sigma \log_\epsilon (c^2/f \times 10^{10}) = -5.116$$

$$\Sigma\,\Sigma \log_\epsilon \frac{s}{r} = 98.30 \qquad \Sigma\,\Sigma \log_\epsilon (c^2/f \times 10^{10}) = 31.67$$

$$d = \frac{\Delta\,\Sigma \log_\epsilon (c^2/f \times 10^{10})}{\Delta\,\Sigma \log_\epsilon \frac{s}{r}} = -0.523$$

$$c' = \frac{\Sigma\,\Sigma \log_\epsilon (c^2/f \times 10^{10}) - d\,\Sigma\,\Sigma \log_\epsilon \frac{s}{r}}{n} = 5.91$$

$$-0.523 \log_\epsilon \frac{s}{r} = \log_\epsilon (c^2/f \times 10^{10}) - 5.91$$

$$(c^2/f) = 3.70 \left(\frac{r}{s}\right)^{0.523} \times 10^{-8}$$

to a standard value of temperature and barometric pressure. In Table XIV all values of a are corrected to 25 deg. cent. and 76 cm. barometer, and the constants recalculated by the $\Sigma\,\Delta$ method in Table XV.

TABLE XIII

RELATION OF a TO $\frac{1}{\delta}$ FOR MAIN EXPERIMENTAL LINES

(STANDARD LINE A)
Line A 1-2-3-4

Test No.	$(c^2/f)\times 10^{10}$ per cm.	δ	$\frac{1}{\delta}$
18	15.80	0.982	1.020
36	17.56	0.966	1.037
37	15.80	0.959	1.043
84	15.80	0.933	1.074
101	15.88	0.925	1.081
103	16.24	0.901	1.112
104	14.92	0.878	1.138
105	14.84	0.866	1.158
109	14.48	0.928	1.078
119	16.56	0.888	1.127

TABLE XIV

$\frac{a}{\delta}$ and $\frac{s}{r}$

Test No.	Diam. cm.	s/r	$\frac{c^2}{f}\times 10^{10}$ $=(a\times 10^{10})$	$\frac{c^2}{f}\times\frac{10^{10}}{\delta}$	Corr. factor	$\log_e \frac{c^2}{f}\times\frac{10^{10}}{\delta}$	$\log_e \frac{s}{r}$	Style of conductor
95	0.168	6550	4.68	5.00	0.936	1.609	8.787	Wire
92	0.168	4880	4.28	4.60	0.932	1.526	8.492	"
86	0.168	3700	5.44	5.80	0.930	1.758	8.215	"
138	0.328	2980	5.52	6.04	0.913	1.798	8.000	"
137	0.328	2230	6.52	7.16	0.913	1.968	7.709	"
77	0.585	1060	9.08	9.76	0.932	2.278	6.966	Cable
126	0.518	700	11.44	12.72	0.898	2.543	6.551	Wire
18	1.181	525	15.80	16.08	0.982	2.777	6.263	Cable
80	0.585	520	13.08	14.00	0.932	2.639	6.254	"
125	0.518	350	17.88	19.88	0.898	2.989	5.858	Wire
73	0.585	310	16.96	18.20	0.932	2.901	5.736	Cable
100	0.953	193	26.40	28.52	0.925	3.341	5.263	

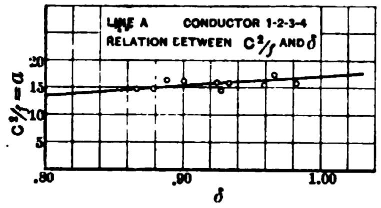

FIG. 31.—Corona loss

TABLE XV

$\Sigma \Delta$ REDUCTION OF RELATION OF c^2/f TO $\frac{s}{r}$

c^2/f CORRECTED TO 25 DEG. CENT.—76 CM. BAROMETRIC PRESSURE

Test No.	$\log_e \left((c^2/f) \times \frac{10^{10}}{\delta}\right)$	$\log_e \frac{s}{r}$	Test No.	$\log_e \left((c^2/f) \times \frac{10^{10}}{\delta}\right)$	$\log_e \frac{s}{r}$
95	1.609	8.787	126	2.543	6.551
92	1.526	8.492	18	2.777	6.263
86	1.758	8.215	80	2.639	6.254
138	1.798	8.000	125	2.989	5.858
137	1.968	7.709	73	2.901	5.736
77	2.278	6.966	100	3.341	5.263
	10.94	48.169		17.20	35.925

$$\Delta\, \Sigma \log_e \left((c^2/f) \times \frac{10^{10}}{\delta}\right) = -6.260$$

$$\Sigma\, \Sigma \log_e \left((c^2/f) \times \frac{10^{10}}{\delta}\right) = 28.14$$

$$\Delta\, \Sigma \log_e \frac{s}{r} = 12.244$$

$$\Sigma \log_e \frac{s}{r} = 84.094$$

$$\tan \theta = \frac{-6.26}{12.244}$$

$$= -0.511 \cong -0.5$$

$$\log_e \left((c^2/f) \times \frac{10^{10}}{\delta}\right) = -0.5 \log_e \frac{s}{r} + c'$$

$$c^2/f \times \frac{10^{10}}{\delta} = c'' \left(\frac{s}{r}\right)^{-\frac{1}{2}}$$

$$c^2/\delta f = c'' \sqrt{\frac{r}{s}}\, 10^{-10}$$

$$a = c^2/f.. = 344 \sqrt{\frac{r}{s}} \times 10^{-10}$$

This gives:

$$a = 344\sqrt{\frac{r}{s}} \times 10^{-10} \text{ per cm. of total cond. at 25 deg. cent.,}$$

76 cm. barometer,

and this is the final value of the constant.

TABLE XVI

CALCULATION OF CURVE 32 FROM $(c^2/f) \times 10^{10} = 344\sqrt{\frac{r}{s}}$

$\frac{s}{r}$	$\sqrt{\frac{r}{s}}$	$\frac{c^2}{f}\,10^{10}$
250	0.0633	21.84
500	0.0447	15.44
1000	0.0316	10.92
1500	0.0258	8.92
2000	0.0224	7.72
3000	0.0183	6.32
4000	0.0158	5.44
6000	0.0128	4.44

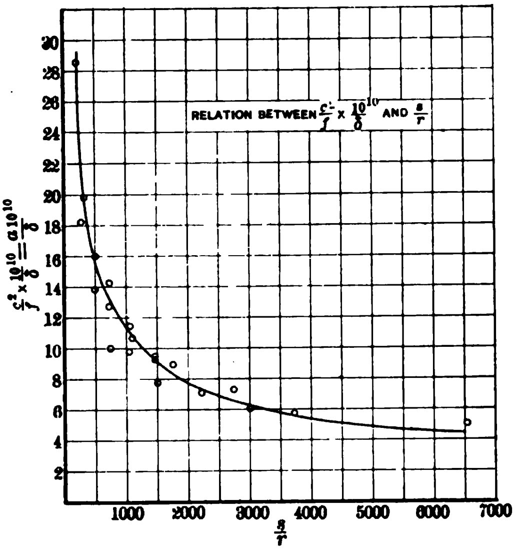

FIG. 32.—Corona loss

Curve 32 is plotted from the points calculated in Table XVI, while the circles show the actual experimental points.

Curve 33 shows a straight line relation between $\log_\epsilon s/r$ and $\log a$. We may now write the equation for the power loss at 25 deg. cent. and 76 cm. barometric pressure:

$$p = 344\, f \sqrt{\frac{r}{s}} (e - e_0)^2\, 10^{-10}$$

where p = the energy loss per centimeter of total conductor in kilowatts.

e = kilovolts to neutral.

e_0 = disruptive critical kilovolts to neutral at 25 deg. cent. and 76 cm. barometric pressure.

f = the frequency in cycles per second.

r = the radius of the conductor in cm.

s = is the distance between conductor centers in cm.

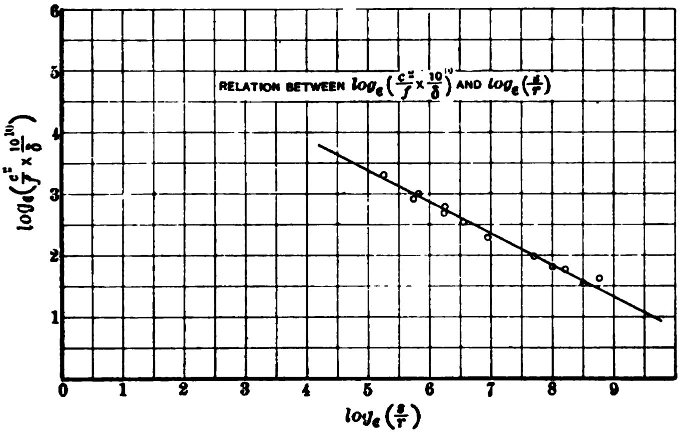

FIG. 33.—Corona loss

The value of e_0 varies with the radius of the conductor, r, and the spacing s, and will be discussed in section VI.

VI. THE DISRUPTIVE CRITICAL VOLTAGE

The point of greatest stress for a cylindrical conductor is at its surface. Where s/r is large the gradient at the surface of the conductor may be expressed:

$$g = \frac{d\,e}{d\,x} = \frac{e}{r \log_\epsilon \frac{s}{r}}$$

where e = the voltage to neutral.

s = distance between conductor centers.
r = the radius.

Where r and s are in cm. and e is in kilovolts, g is expressed in kilovolts per cm. Now if e_0 the disruptive critical voltage is taken for e we have:

$$g_0 = \frac{e_0}{r \log_\epsilon \frac{s}{r}}$$

TABLE XVII
DISRUPTIVE CRITICAL VOLTAGE GRADIENT FOR WIRES. VALUES CORRECTED TO 76 CM. BAROMETER AND 25 DEG. CENT.

Test No.	Spacing cm.	Diam. cm.	g. Kv/cm. max.	Per cent variation from mean	Per cent variation max. to min.
129	122	0.051	29.8		
133	183		29.8		
131	244		29.8		
130	366		29.8		
132	488		29.8		
			Avg = 29.8	—	—
91	152	0.168	31.3		
94	229		31.6		
92	410		29.1		
95	550		36.5		
			Avg = 30.9	5.8	7.9
134	122	0.328	28.8		
135	183		27.1		
136	244		29.0		
137	366		25.7		
138	488		25.3		
			Avg = 27.2	7.0	12.7
125	91.4	0.518	28.7		
126	183		26.5		
127	275		26.0		
128	397		26.2		
			Avg = 26.9	6.7	9.4
122	91.4	0.927	28.7		
123	183		30.4		
120	214		30.5		
124	275		31.0		
			Avg = 30.1	4.8	7.6
			Total Avg. 29.0		

TABLE XVIII

DISRUPTIVE CRITICAL VOLTAGE GRADIENT FOR CABLES. VALUES CORRECTED TO 76 CM. BAR. AND 25 DEG. CENT.

Test No.	Spacing cm.	Diam. cm.	g. Kv/cm. max.	Per cent variation from mean	Per cent variation max. to min.
73	91.4	0.585	26.5		
80	152		24.0		
79	244		23.9		
77	310		23.9		
82	432		25.0		
			Avg =24.7	7.3	9.8
100	91.4	0.953	25.5		
115	91.4		26.2		
116	183		26.0		
117	275		26.4		
118	366		28.1		
			Avg =26.4	6.4	9.3
			Line A 1-2-3-4		
18	310	1.181	25.5		
36	"		26.0		
37	"		25.3		
84	"		25.8		
101	"		26.5		
103	"		26.1		
104	"		25.7		
105	"		26.0		
109	"		25.8		
119	"		25.1		
119	"		26.0		
			Avg =25.8	2.7	5.3
			Total Avg. 25.7	5.5	8.1

g_0 then is the stress at the conductor surface corresponding to e_0, and will be called the *disruptive gradient*, to distinguish it from the visual gradient g_v. Values of g_0 for wires and cables taken under a great variety of conditions are given in Tables XVII and XVIII. These values are corrected to standard temperature and pressure as explained in section XI.

If we examine the values of e_0 for standard line *A* (1.8 cm. 7-strand cable) we find that the average value of g_0 is 25.8 kv. per cm. maximum. For cables between 0.583 cm. and 1.18 cm.

in diameter and various spacings, the average value of g_0 is 25.6 kv. per cm. maximum, or, in other words, *g_0 is constant for all sizes of cables, at all spacings, and is* 25.6 *kv.* per cm. maximum. In determining the value g_0 for seven-strand cables r was taken for convenience as the outside radius. Hence the above g_0 is not the *actual* g_0 as obtained for wires, but is an *apparent* g_0. The actual g_0 would be obtained by taking some mean radius r_2 between the outside radius r, and the radius to the point of contact of the outside strands r_1. r_2 approaches r in value as the number of strands are increased.

The values of g_0 for wires varying in diameter from 0.0508 cm. to 0.928 cm., and for spacings from 90 to 600 cm. are constant within the limits of experimental error. The mean value is:

$g_0 = 29$ kv. per cm. at 25 deg. cent and 76 cm. barometric pressure

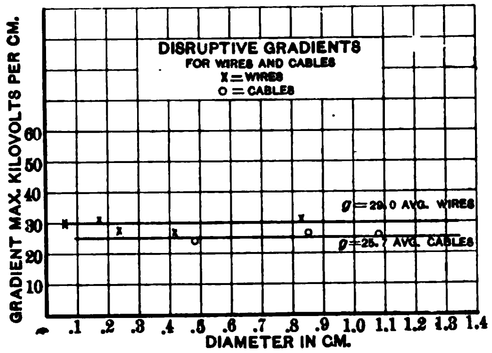

FIG. 34

Considerable variation should be expected in g_0 values due to:

1. Necessarily imperfect conductors, kinks, etc., in an outdoor line of this length.
2. Progressive change in the value of successive points on a given curve due to slight changes of wave shape, etc., as the voltage is increased, and the apparent shift of e_0.

The close agreement of g_0 for wires is for the above reasons remarkable.

Discrepancies due to progressive change were not to be expected for standard line A to any great extent as the conductor spacing was always the same, and test conditions were kept as nearly constant as possible.

From the above, then, *g_0 is constant for all diameters of con-*

ductors and all spacings. This immediately suggests that g_0 is the actual rupturing gradient of air. More will be said of this in Section IX. Also

$$\frac{g_0 \text{ cables}}{g_0 \text{ wires}} = m_0$$

where m_0 is a fraction which approaches unity as the irregularity of the surface is reduced, or number of strands increased. This factor is further discussed in Section X.

Then

$$e_0 = \delta\, m_0\, g_0\, r \log_\epsilon \frac{s}{r} \qquad (5)$$

(See curve Fig. 34).

The loss equation may now be written

$$p = \frac{k}{\delta} f \sqrt{\frac{r}{s}} \left\{ e - \delta\, m\, g_0\, r \log_\epsilon \frac{s}{r} \right\}^2 \qquad (6)$$

where p expresses the loss above the visual critical voltage e_v. The loss at the lower part of the curve will now be considered.

VII. LOSSES NEAR THE DISRUPTIVE CRITICAL VOLTAGE — e_0

If the conductors could be made perfect no loss would occur below the visual critical voltage. However, at low values of corona, two effects occur, which cause a deviation of the loss from the quadratic law, equation (1), and which affect the loss in opposite directions:

a. The loss of power does not begin at the voltage e_0, at which the disruptive gradient is reached at the conductor surface, but only after the disruptive strength of air has been exceeded over a finite and appreciable distance X from the conductor, that is, at a higher voltage e_v, as fully explained in Section VIII. See Fig. 39. Since the convergency of the lines of dielectric force is great at the surface of small conductors, with such conductors a considerable increase of the voltage is required to extend the disruptive gradient to some distance from the conductor, and e_v is considerably higher than e_0, and thereby the decrease of loss below that given by equation (1) is appreciable with small conductors within the range between e_0 and e_v, as seen in Fig. 37.

With large conductors, however, the lesser convergency of the lines of dielectric force at the conductor surface requires a lesser voltage increase beyond e_0 to extend the disruptive gradient to some distance from the conductor; e_0 and e_v are therefore closer together, and this decrease of the loss below the theoretical value given by equation (1) is not appreciable.

b. As the conductor surface can never be perfect, some loss of power occurs at and below the disruptive critical voltage at isolated points of the conductor, where irregularities of the surface, scratches, spots of mud or dirt, etc., give a higher potential gradient than that corresponding to the curvature of the conductor surface. With small conductors, this loss is rarely appreciable, since the curvature of the conductor surface is of the same magnitude as that of its irregularities. It becomes appreciable, however, for larger conductors, as seen in Fig. 35. This excess of the loss beyond that given by the quadratic law equation (1) essentially depends on the conductor surface, and is the larger, the rougher or dirtier the surface is. It is a maximum at the disruptive critical voltage e_0, and decreases above and below e_0, and is with fair accuracy represented by the probability curve,

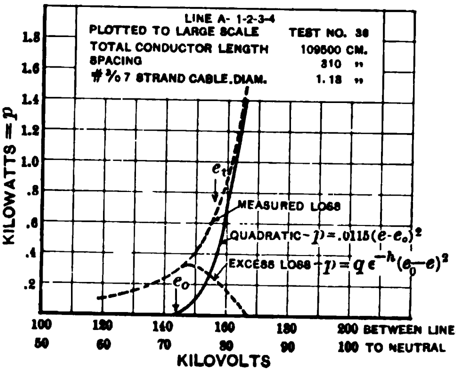

FIG. 35.—Corona loss near critical point

$$p = q\,\epsilon^{-h(e_0-e)^2} \qquad (4)$$

where q is a coefficient depending on the number of spots, and h is a coefficient depending upon the size of spots.

Snow, sleet and rain losses seem to be of the same nature, but frequently of far greater magnitude.

Equation (4) is probably of no practical importance, as the loss is small, and q and h naturally cover a wide range of values depending upon the condition of the conductor surface. Experimental values near e_0 are taken from Table I and tabulated in Table XIX, together with values calculated by the quadratic law. Corresponding experimental and calculated values are subtracted and also tabulated.

TABLE XIX

LINE A 1-2-3-4. FROM TABLE I. TEST 36. $c^2 = 0.0115$, $e_0 = 72.1$

Kilovolts between conductors e'	Kw. exp. p_e	Kw. $p = 0.0115\,(e - e_0)^2$	Excess loss $p_1 = (p_e - p)$	$\log_\epsilon\,(p_1 \times 10^2)$	$(e_0 - e)^2$
120.4	0.11	——	0.11	2.40	141.6
130.2	0.15	——	0.15	2.71	49.0
139.2	0.22	——	0.22	3.09	6.2
150	0.40	0.10	0.30	3.40	8.5
159	0.79	0.63	0.16	2.77	54.7
165.8	1.42	1.34	0.08	2.10	117.0

Fig. 35 is plotted to a large scale to show the excess loss near e_0. This is for large conductors, e_0 and e_v are near together, and the effect of (b) predominates. In order to see if equation (4) holds, write

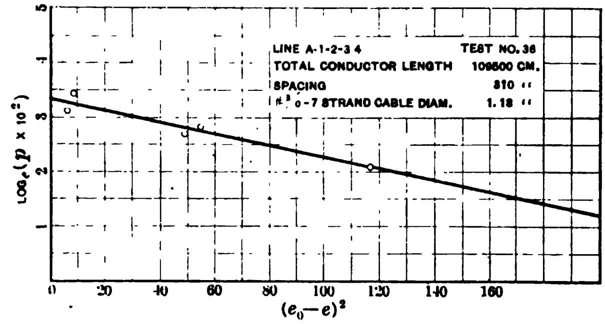

FIG. 36.—Corona loss. Method of reducing loss near critical point

$$\log_\epsilon p_1 = \log_\epsilon q - h\,(e_0 - e)^2$$

Then the curve between $\log_\epsilon p_1$ and $(e_0 - e)^2$ should be a straight line. This is shown in Fig. 36.

Values of q and h are of the following order for line A:

Test No.	q per cm. total conductor	h
30	3.19×10^{-6}	-0.0220
105	2.47 "	-0.0208
103	2.74 "	-0.0304

Fig. 37 is plotted from values for a small smooth conductor. Here e_0 and e_v are far apart, and as the curvature of the conductor surface is of the same magnitude as its irregularities they do not greatly influence the loss. The (a) effect here predominates—that is, the loss near e_0 is lower than that shown by the quadratic law.

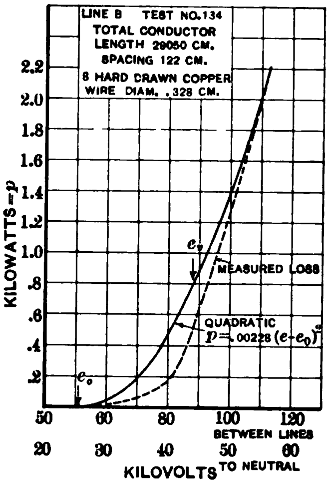

Fig. 37.—Corona loss near critical point

A phenomenon of the same nature as the above seems to occur in the striking distance between needle points. " Theoretically " the relation between volts and striking distance should be a straight line. This is practically so at high voltages, while at low voltages the curve deviates from the straight line. The voltage for small striking distances is higher than " theory " would warrant. This seems to be because a certain finite amount of energy must be stored in the dielectric about the point of discharge or, in other words, the voltage must be raised sufficiently above the " theoretical " voltage in order to extend the rupturing gradient over a finite distance. This is further discussed for visual corona on wires in Section VIII.

VIII. VISUAL TESTS

Visual tests were made on two parallel conductors, supported indoors on a frame of treated wood. Wooden pins were used throughout the construction of this frame. The end supports were wooden wheels. This circular curvature prevented a flux concentration at the ends of the conductors. Two separate frames were used. One gave an active length per conductor of 308 cm., the other an active length of 610 cm. The distance above the floor was 150 cm. The maximum spacing between conductors was 107 cm.

The *visual critical point*, or point where visual corona starts, is very definite for polished conductors, and can be repeatedly checked within a small per cent. The tests were made in a dark room and the method of procedure was as follows: Conductors of a given size were placed on the framework. Critical points were then taken at various spacings up to 107 cm.

The maximum intensity at the surface of one of two parallel conductors may be written

$$\frac{d\,e}{d\,r} = g_v = \frac{e_v}{r \log_\epsilon \frac{s}{r}}$$

where e_v = the (maximum) voltage to neutral.

r = the radius of the conductor in centimeters.

s = the distance between the centers of conductors in centimeters.

This holds where s/r is large, or when the flux is uniformly distributed at the surface of the conductors.

When s/r is small, that is, when the conductors are large and the spacing small, the flux is not uniform and g_v may be more closely approximated thus:

$$g_v = \frac{e\left(1+\dfrac{r}{\frac{s}{2}-r}\right)}{r \log_\epsilon \frac{s}{r}}$$

The following Table XX shows a typical set of readings. Note that g_v is *constant* for *all spacings*, for a *given size conductor.*

TABLE XX
VISUAL CRITICAL VOLTAGES
(g_v WITH VARYING SPACING AND CONSTANT DIAMETER)
POLISHED COPPER CONDUCTOR, DIAMETER 0.0343 cm.

s cm.	e_v' kilovolts between conductors (effective)	e_v' kilovolts between conductors (maximum)	g_v *kv/cm* (maximum)
2.54	12.1	17	99.5
2.93	12.4	17.4	99
3.18	12.5	17.7	98.5
3.81	13.0	18.4	99
4.45	13.5	19.0	99.5
5.08	13.8	19.4	100
5.73	14.0	19.8	99
7.62	14.5	20.5	99
15.2	16.0	22.6	97.2
30.5	17.7	25	97.2
45.6	18.7	26.3	96.1
61.0	19.4	27.4	98
106.8	20.6	29	97.2
			Average 99

Sets of readings similar to the above were taken on wires varying in diameter from 0.02 to 0.93 cm. The average values are tabulated in Table XXI.

Note that these values are taken for a number of different metals. The points all fall on the curve plotted between g_v and diameter. That is, the critical gradient is independent of the conductor material.

Now it seems reasonable to assume that air under constant conditions should break down at a constant potential gradient g. An examination of Table XXI shows that g_v increases as the diameter of the conductor decreases. This is also shown on the curve Fig. 38. The apparent increase in the dielectric strength of air surrounding small conductors was explained by Steinmetz some years ago by the assumption of a condensed air film at the surface of the conductor. If this were so, a greater critical gradient would be expected for tungsten than for aluminum. That is, the air film should be denser around the denser metals. As already noted, Table XXI and curve, Fig. 38, show that the gradient is not affected by the material or density of the conductor. In a paper read at the Annual Convention of the

A. I. E. E., June, 1910,* by Hayden and Steinmetz, it was shown that energy as well as voltage was necessary to rupture insulation. Also see Ryan's† explanation by the ionization theory. Applying this definite energy theory to corona formation, it means that the potential gradient at the surface of the conductor must be raised above the actual breakdown gradient in order to store sufficient energy in the air immediately surrounding the conductor to cause breakdown at a distance x from the conductor surface. This distance x from the surface must be finite and the gradient at the point x a constant and equal to the dielectric strength of air g. See Fig. 39.

TABLE XXI

VARIATION OF g_v WITH DIAMETER OF CONDUCTORS (AVERAGE VALUES FOR POLISHED WIRE). CORRECTED TO 25 DEG. CENT. 76 CM. BAROMETRIC PRESSURE

Diameter cm.	$\frac{de}{dr}=g_v$ kv/cm. max.	Material
0.0196	116	Tungsten
0.0343	99	Copper
0.0351	94	"
0.0508	84	Aluminum
0.0577	82	"
0.0635	81	Tungsten
0.0780	76	Copper
0.0813	74	"
0.1637	64	"
0.1660	64	Iron
0.2043	59	Copper
0 2560	57	Aluminum
0.3200	54	Copper
0.3230	50.5	"
0.5130	49	"
0.5180	46	"
0.6550	44	"
0.8260	42.5	"
0.9280	41	"

***Disruptive Strength with Transient Voltages,* by J. L. R. Hayden and C. P. Steinmetz, TRANSACTIONS A.I.E.E., 1910, XXIX, II, p. 1125.

†*Open Atmosphere and Dry Transformer Oil as High-Voltage Insulators,* by Harris J. Ryan, TRANSACTIONS A.I.E.E., 1911, XXX, I, p. 1.

On the above assumptions we may write

$$g_v = \frac{e_v}{r\left(\log_\epsilon \frac{s}{r}\right)} = \text{visual gradient}$$

$$g = \frac{e_v}{(r+x)\log_\epsilon \frac{s}{r}} = \text{constant}$$

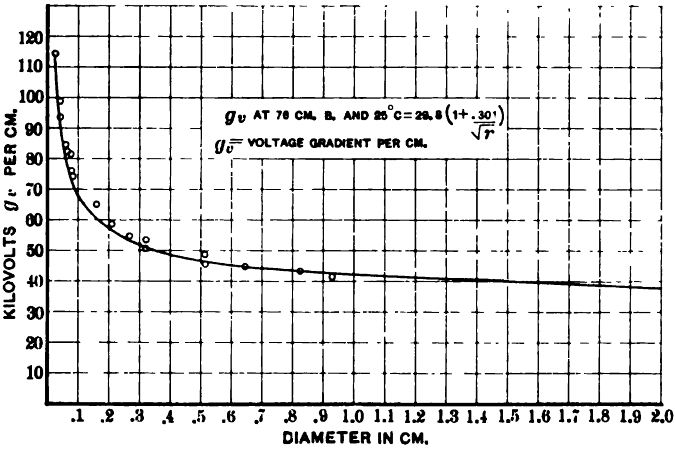

FIG. 38.—Visual critical voltage gradient—Two parallel wires

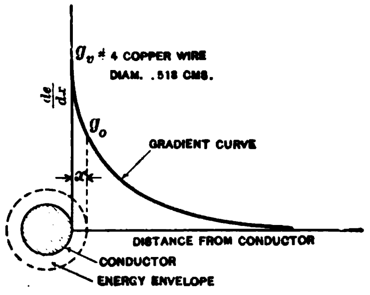

FIG. 39.—Potential gradient in air surrounding one of two parallel conductors

Now theoretically one is led to expect that x is not constant for all values of r, but

$$x = \phi(r)$$

Then we may write

$$g = \frac{e_v}{(r+\phi(r))\log_\epsilon \frac{s}{r}}$$

TABLE XXII

RELATION OF VISUAL CRITICAL VOLTAGE GRADIENT TO RADIUS(WIRES)
(EXPERIMENTAL VALUES)

CORRECTED TO 76 CM. BAROMETER AND 25 DEG. CENT.

Diam. cm.	g_v kv/cm.	Radius r = cm.	$\frac{1}{\sqrt{r}}$ cm.
0.0196	116.0	0.0098	10.10
0.0343	99.0	0.0172	7.65
0.0350	94.0	0.0175	7.58
0.0508	84.0	0.0254	6.27
0.0577	81.5	0.0288	5.90
0.0635	81.0	0.0317	5.64
0.078	76.0	0.0390	5.08
0.0813	73.5	0.0406	4.96
0.1635	63.8	0.0818	3.51
0.1660	63.4	0.0830	3.45
0.202	59.1	0.1010	3.13
0.257	56.7	0.128	2.76
0.320	54.3	0.160	2.51
0.322	49.6	0.161	2.51
0.513	48.8	0.256	2.01
0.518	44.5	0.259	1.94
0.655	43.7	0.327	1.82
0.826	42.2	0.413	1.57
0.928	40.6	0.464	1.44

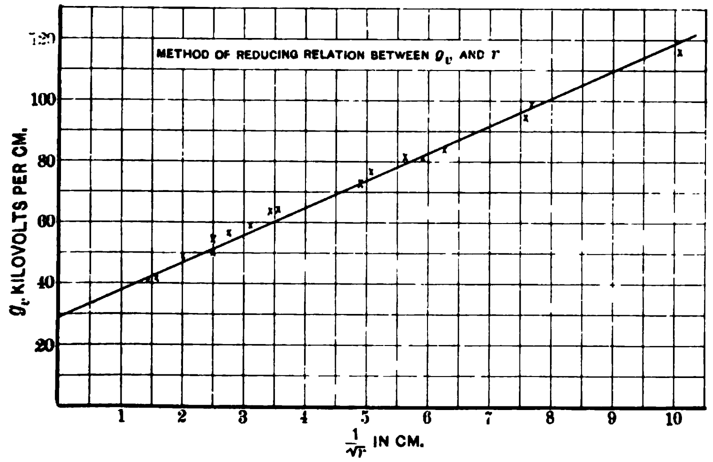

FIG. 40.—Visual critical voltage gradient

A study of the experimental values shows that if g_v is plotted with $1/\sqrt{r}$, the curve is a straight line. See Table XXII and Fig. 40. That is,

$$g_v = g\left(1+\frac{k}{\sqrt{r}}\right)$$

In order to give proper weight to all of the experimental points the $\Sigma\,\Delta$ method is used in the evaluation of the constants. See Table XXIII.

TABLE XXIII

RELATION OF g_v AND $\frac{1}{\sqrt{r}}$

($\Sigma\,\Delta$ reduction)

g_v	$\frac{1}{\sqrt{r}}$	g_v	$\frac{1}{\sqrt{r}}$
99	7.65	59	3.13
82	5.90	54	2.51
81	5.64	50.5	2.51
76	5.08	49	2.01
74	4.96	41	1.44
Σ 412	Σ 29.23	Σ 253.5	Σ 11.60

$$\Delta\,\Sigma\,g_v = 158.5 \qquad \Delta\,\Sigma\,\frac{1}{\sqrt{r}} = 17.63$$

$$\Sigma\,\Sigma\,g_v = 665.5 \qquad \Sigma\,\Sigma\,\frac{1}{\sqrt{r}} = 40.83.$$

$$c_1 = \frac{158.5}{17.63} = 9.00$$

$$g = \frac{665.5 - 9 \times 40.8}{10} = 29.8$$

herefore

$$g_v = 29.8 + \frac{9}{\sqrt{r}}$$

$$= 29.8\left(1+\frac{0.301}{\sqrt{r}}\right)$$

These values give 29.8 kilovolts per cm. maximum as a rupturing gradient of air at 25 deg. cent. and 76 cm. barometer. We may find x thus:

$$\frac{g_v}{g} = \left(1+\frac{0.301}{\sqrt{r}}\right)$$

$$\frac{g_r}{g} = \frac{\dfrac{e_v}{r\,(\log_\epsilon s/r)}}{\dfrac{e_v}{(r+x)\log_\epsilon s/r}} = \frac{r+x}{r}$$

therefore

$$\frac{r+x}{r} = \left(1+\frac{0.301}{\sqrt{r}}\right)$$

$$x = 0.301\ \sqrt{r}$$

Fig. 38 is plotted from equation (6). The experimental points are shown as circles. Taking the correction factor given in Section XI for barometric pressure and temperature we may now write:

$$g_v = 29.8\ m_v\ \delta\ \left(1+\frac{0.301}{\sqrt{r}}\right) \text{ kv. per cm. maximum}$$

$$= 21.1\ m_v\ \delta\ \left(1+\frac{0.301}{\sqrt{r}}\right) \text{ kv. per cm. effective}$$

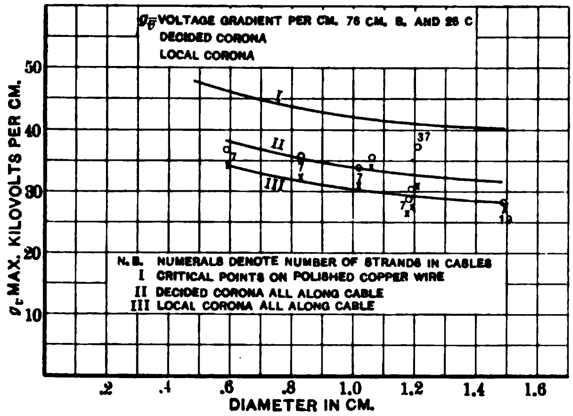

FIG. 41.—Visual critical voltage gradient—Two parallel cables

Also

$$e_v = 21.1\ m_v\ \delta\ r \left(1+\frac{0.301}{\sqrt{r}}\right) \log_\epsilon \frac{s}{r} \text{ effective kv. to neutral} \quad (5)$$

The *numerical value of g determined above is practically the same as the numerical value of g_0 determined from the disruptive critical voltage.* The above values are for polished wires.

While the visual critical point is quite sharp and definite for wires it is not so for cables. This point for cables seems to come on gradually and cover a considerable range.

Table XXIV and Fig. 41 give average values for cables.

TABLE XXIV

VISUAL TESTS ON CABLES

Diam. cm.	strand	Decided corona			Local corona		
		g_v max. per cm.	g_v max. corr. per cm. 25° C. 76 cm.	Avg.	g_v max. per cm.	corr. 25° C. 76 cm.	Avg.
0.585	7	40.4	38.9	37.1	39.2	37.8	35.3
0.585	7	39.3	36.2		36.5	33.7	
0.585	7	38.4	36.3		36.1	34.3	
0.831	7	37.0	35.7	35.7	33.4	32.4	32.4
1.020	7	34.9	33.7	33.7	32.0	31.9	31.9
1.063	19	36.7	35.4	35.4	34.8	33.7	33.7
1.181	7	33.4	32.7	28.9	31.4	30.9	26.9
1.181	7	24.3	23.0		21.5	20.4	
1.181	7	28.2	27.0		25.4	24.4	
1.181	7	34.2	33.0		32.0	31.9	
1.193	19	30.6	30.1	30.1	28.2	27.7	27.7
1.209	37	37.2	36.4	36.4	35.8	35.1	35.1
1.485	19	28.7	28.3	28.3	28.0	27.6	27.6

The values for wires are plotted on the same sheet. There is nearly a constant difference between the two curves, that is, the visual critical gradient for cables, g_v', equals the visual critical gradient for polished wires, g_v, times the constant m_v. Increasing the number of strands in a cable appears to increase the value of m_v.

Photographic Study. A photographic study of corona on wires and cables was made as follows: Two parallel conductors were spaced 122 cm. between centers. The camera was focused on one conductor only. The distance to the lens was such as to show the conductors at approximately actual size. An exposure was made for a given time at a given voltage. The plate was then shifted slightly, the voltage was raised, and an exposure made for the same time. That is, a given series shows the same part of the same single wire at different voltages. This operation was repeated until the series for a given wire was complete. See Fig. 42. These photographs are shown in Figs. 43–50.

Fig. 43 shows corona on a bright tinned phosphor bronze wire—0.051 cm. in diameter.

Fig. 44 shows corona on a bright copper wire (0.186 cm.) diameter—polished after each exposure.

Fig. 45 shows the same wire used in Fig. 44, but allowed to remain idle a few hours after running at 200 kv.

Fig. 46 shows a weathered (0.168-cm. diameter) galvanized iron wire.

Fig. 47 shows a bright polished rod, and a new unpolished 3/0 copper cable.

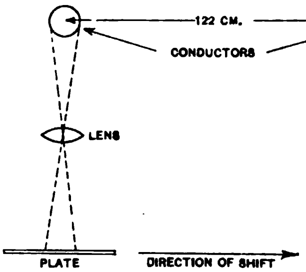

Fig. 42.—Method of making photographs of corona on short length conductors

These photographs explain much in the study of corona, for instance, variations in k, and g_0, the effect of moisture, loss at low voltage, etc. A complete discussion must be left to another paper. The conditions shown cover more than the practical range.

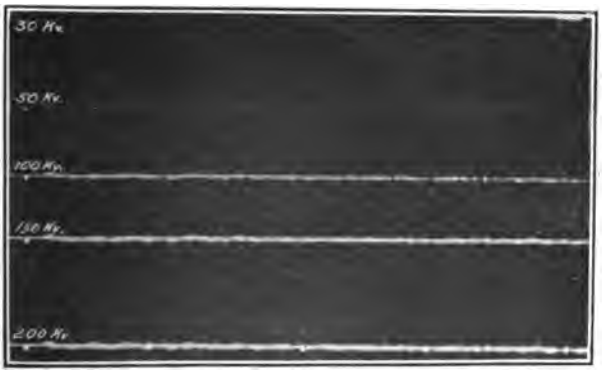

Fig. 43.—Phosphor bronze bright tinned wire. Diameter 0.051 cm.

IX. Relation between Visual Critical Voltage and Disruptive Critical Voltage

In Section VIII it was shown by loss measurements that for wires:

$$g_0 = \text{constant} = 29 \text{ kv. per cm. maximum}$$

By visual measurements, in Section VIII, it was shown that

$$g_v = g\left(1+\frac{0.301}{\sqrt{r}}\right)$$

where $g = 29.8$.

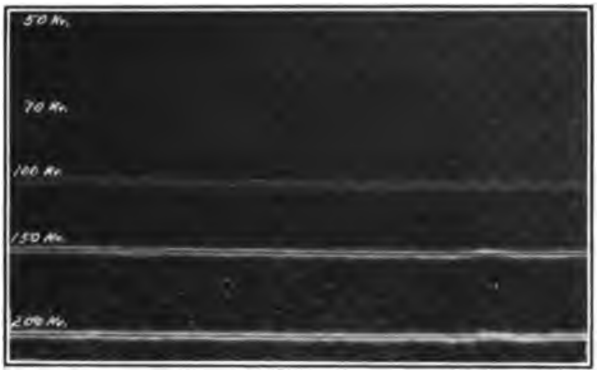

Fig. 44.—Polished copper wire. Diameter 0.186 cm. Polished after each exposure

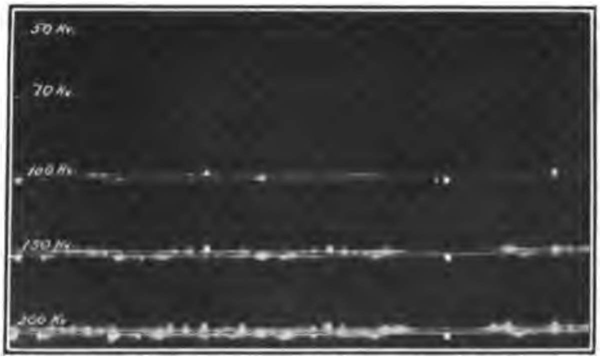

Fig. 45.—Polished copper wire. 0.186 cm. Run at 200 kv., then allowed to stand idle. (This shows effect of oxidation)

Then where $r = \infty$, or for flat surfaces, $g_v = g = 29.8$.

The agreement between g as determined visually and g_0 as determined by loss measurement is remarkable, considering the different methods followed. It means that $g_0 = g = 29.8$ *kv. per cm. = dielectric strength of air at* 25 *deg. cent. and* 76 *cm.*

More weight is given to 29.8 as the value for the dielectric strength of air, as it was made on polished wire, and by a direct method, while g_0 was necessarily determined on long lengths of unpolished and more or less kinky wires by an indirect method. See Fig. 51.

FIG. 46.—Weathered galvanized iron wire. Diameter 0.168 cm.

FIG. 47.—1.25-cm. polished brass rod and unpolished copper cable

X. IRREGULARITY FACTOR

Two factors affecting e_0 and the *apparent* g_0 have been used in equations and discussed. The numerical values covering the range of transmission practise are:

For wires:

$m_v = m_0 = 1$, polished wire.

$= 0.98$ to 0.93, roughened or weathered wire.

FIG. 48.—No. 3/0 weathered cable

FIG. 49.—No 3/0 line cable—Dry

FIG. 50.—No. 3/0 line cable—Wet

For cables:

$$m_0 = 0.87 \text{ to } 0.83$$

$$m_v = 0.82 \text{ to } 0.72$$

k in equation (6) will vary somewhat with wave shape, etc., But this variation can generally be neglected, as it enters the equation directly, while the e_0 variation enters as the square. The value given should cover practical conditions.

XI. TEMPERATURE AND BAROMETRIC PRESSURE

Values of the *disruptive critical voltage* e_0 covering a considerable temperature range are tabulated below. In Table XXV correction is made to a barometric pressure of 76 cm. on the assumption that e_0' varies directly with the pressure. In Fig. 52 $1/e_0'$ is plotted with temperature. The straight line through *these points cuts the temperature axis* at -273 deg. cent. or *absolute zero.* Temperature was always measured in the shade. The points that do not fall well on the curve are the summer sunny day points. This is what would be expected as the conductors were at a higher temperature than the temperature read.

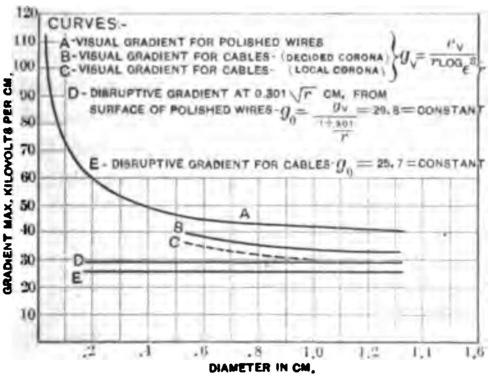

FIG. 51.—Visual and disruptive gradients for wires and cables

Fig. 52 shows that the critical voltage or the rupturing gradient varies inversely as the air density.

The density of air at 25 deg. cent. and 76 cm. barometric pressure is used as the standard in this paper.

TABLE XXV
TEMPERATURE AND DISRUPTIVE CRITICAL VOLTAGE e_0
(Standard line *A* 1-2-3-4)

Test No.	Temperature		Bar cm.	e_0' kv. between lines	e_0' corr. to 76 cm.	$1/e_0'$	Weather
	Wet	Dry					
18	16	18.5	75.5	138.5	140	0.00715	Bright sun
15	20	22	75.2	140	142	0.00705	Cloudy sun
37	10	13	75.7	141	14	0.00705	Bright sun
36	10	12	75.0	144.2	146.5	0.00683	Cloudy
109	- 3	- 2	73.9	148.8	150.8	0.00663	Hazy sun
84	1	3	75.2	149	151	0.00662	Cloudy
101	- 1	- 1	74.7	153.8	156.8	0.00638	Cloudy
103	- 4.9	- 4.5	75.7	155.3	156.7	0.00638	Sun
104	- 9.5	- 9.5	76.2	157	157.0	0.00637	Sun
119	- 6.5	- 6	76.5	156.6	156.1	0.00642	Sun
105	-13	-13	76.2	161	161	0.00622	Sun

The factor for reducing the density of air, taken at a given standard temperature and pressure, to any desired temperature and pressure, may be deduced as follows, and the factor which has been called δ in the previous discussion obtained.*

$$w=\frac{0.00465\, b}{273+t}$$

where w = the weight of air in grams per cubic cm.
b = barometric pressure in cm.
t = the temperature in degrees centigrade.

At 25 deg. cent., 76 cm.,

$$w_{25°-76\text{ cm}}=\frac{0.00465\times 76}{273+25}=0.001185 \text{ grams}$$

$$w \text{ at desired temperature and pressure } w_{tb}=\frac{b}{273+t}$$

$$\frac{w_{tb}}{w_{25°\text{ C}-76\text{ cm}}}=\frac{0.00465\, b}{(273+t)\, 0.001185}=\frac{3.92\, b}{(273+t)}=\delta$$

---

*This is the method used by Professor Ryan as the result of his laboratory experiments. The above table is a check on his work on a large scale.

where $(273+t)=T=$ absolute temperature, deg. cent.

$$\delta=\frac{3.92\,b}{273+t}=\text{correction factor.}$$

Then if e_0 increases directly with the air density,

$$e_0''=\frac{3.92\,b}{273+t}\,e_0$$

where e_0 is the critical voltage at 25 deg. cent. and 76 cm. barometric pressure, and e_0'' is the critical voltage at the desired temperature and pressure—of course the corresponding values of g_0 may be found the same way.

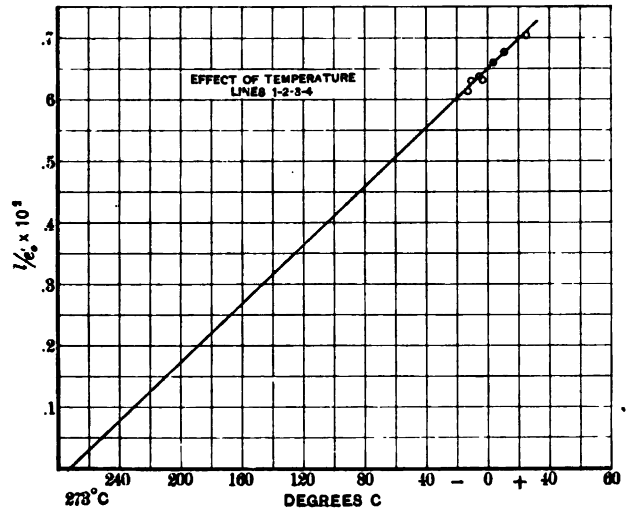

FIG. 52.—Corona loss

XII. HUMIDITY, SMOKE, WIND

Humidity. Line *A* was kept as a standard throughout the tests. A careful study of the disruptive critical voltage and c^2/f shows no effect of either humidity or " vapor products ".* Visual tests made on two short parallel wires indoors and over a great humidity range also bear this out.

***High Voltage Measurements at Niagara,* by R. D. Mershon, TRANSACTIONS A. I. E. E., 1908, XXVII, II, p. 845.

TABLE XXVI

†STANDARD LINE *A*. CONDUCTORS 1-2-3-4

Test No.	Temperatures		Relative humidity	Vapor product	g_0 reduced to 25° C. 76 cm. bar.
	Wet	Dry			
15	20	22	0.84	0.55	18.8
18	16	18.5	0.75	0.35	18.5
36	10	12	0.78	0.25	18.8
37	10	13	0.67	0.21	18.3
84	1	3	0.69	0.11	18.7
101	- 1	- 1	1.00	0.17	19.2
103	- 4.9	- 4.9	1.00	0.13	18.9
104	- 9.5	- 9.5	1.00	0.08	18.6
105	-13	-13	1.00	0.06	18.8
109	- 2	- 2	1.00	0.12	18.7

Theoretically there should be no appreciable effect due to humidity, since even if the water vapor, which may be considered as a gas dissolved in air, has a different disruptive gradient from air, the percentage of the gas in the mixture should be too small to cause any appreciable change. It has been suggested that " vapor products " is a measure of ionization and, in that way, the critical voltage varies with vapor products. This does not seem likely, because with all ordinary atmospheric air the percentage of ionization is so small that it would not be expected to produce any effect. To test this, the visual critical point was determined on two parallel wires. The room was then closed and the wires were run at a point very much above the critical point for about an hour, or until a very intense odor of ozone filled the room. The voltage was then taken off and the surface of the wires cleaned in order to remove oxidization. The critical point was then redetermined and found to be the same, although the amount of ionized air was many times that which could be expected in free atmospheric air. Of course if the percentage of ionization is enough to change the constitution of the air, as for instance in an ozone machine, a change in the disruptive strength would then be expected.

It has been claimed that ultra-violet light reduces the sparking point. This is not borne out in tests, where any quantity of power is involved. Though ultra-violet light, ionized air,

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†Line *A* was kept at constant test conditions for use as a standard in the study of varying atmospheric conditions, etc.

and various radiations, cause the small energy in condensers to discharge when applied over comparatively great time, in case of large energy discharge in a very short time, as spark discharge and corona, no appreciable effect should be expected, or can be observed, since the discharge which takes place by ionized air is not of the same order of magnitude as the spark discharge.

The conditions are not the same with fog; here there is not a mixture of two gases, air and water vapor, as is the case where humidity is concerned, but actual water particles are in the air. It has been observed by Steinmetz that fog actually raises the striking distance between needle points. This is because the conducting water particles have the effect of increasing the size of the discharge points. With balls as electrodes, or where the electrodes are already large, fog would probably decrease the striking distance. Greater loss should be expected in corona measurements during fog due to charge and discharge of the water particles. This causes loss at lower voltages and has the effect of decreasing the critical point; it is more fully discussed in Section XIII.

Smoke. It was difficult to get measurements to show the effect of smoke, as the prevailing winds were from over the fields and towards the city. At one time, however, during a change in the wind thick smoke was blown over the line from rubbish dump and smokestacks of a factory. The loss was increased. This, however, will probably not be a serious consideration in practise.

Wind. Losses measured during very heavy winds show no variation from losses measured during calm weather.

XIII. MOISTURE, FROST, FOG, SLEET, RAIN AND SNOW

During some of the first tests it was noted that the losses were sometimes greater on the "going up curve" than on the "coming down curve", especially in the early mornings after heavy dew. The losses became less after the line had been run for a while at high voltage; Fig. 53 shows this well for a conductor with a coating of frost. This excess loss was thought at first to be due to leakage through moisture on the insulators. Insulators were put up without line wires, but measurements showed a very small insulation loss even during storms. It was then concluded to be due to moisture on the conductors themselves. Visual tests made on short lengths of wet and dry cables showed this in a very striking manner. Two parallel dry cables were brought up to the critical point. Water was then thrown on the cables. What was a glow on the surface of the dry cables now became, at the wet spots, a discharge extending as much as 5 to 8 cm. from the cable surface. This discharge reminded one of an illuminated atomizer. Illustrations, Figs. 49 and 50, show this, but a great part of the effect is lost in reproduction. The wires became quite dry and down to normal discharge after running at high voltage for a very few minutes.

The curves, Fig. 54, taken during fog also show the combined effect of condensed moisture on the cables, and free water particles in the air. The moisture particles on the conductor become charged and are repelled. The particles in the air also become charged and discharged, thus increasing the loss very greatly above that for dry conductors.

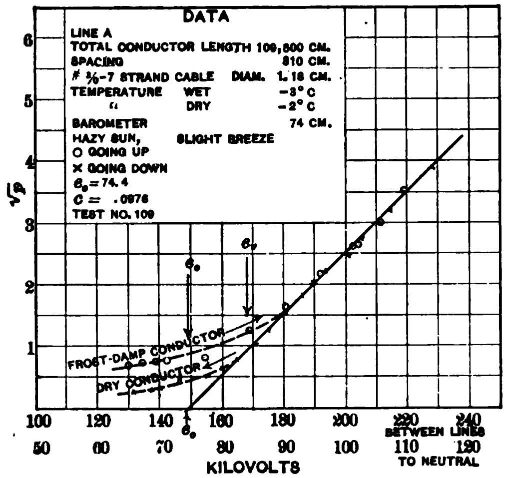

FIG. 53.—Corona loss

The losses during snow and rain storms are much greater than fair weather losses at the same temperature and barometric pressure. In Fig. 56 the actual measured loss is plotted, and also a corresponding calculated fair weather loss. The difference between the two curves shows the excess loss due to snow. The effect of snow is greater than that of any other storm condition. This is because the particles are larger and a greater number strike the line, or come near the line.

The sleet curves are of special interest. Sleet had already started to form on the conductors, and was still falling when the

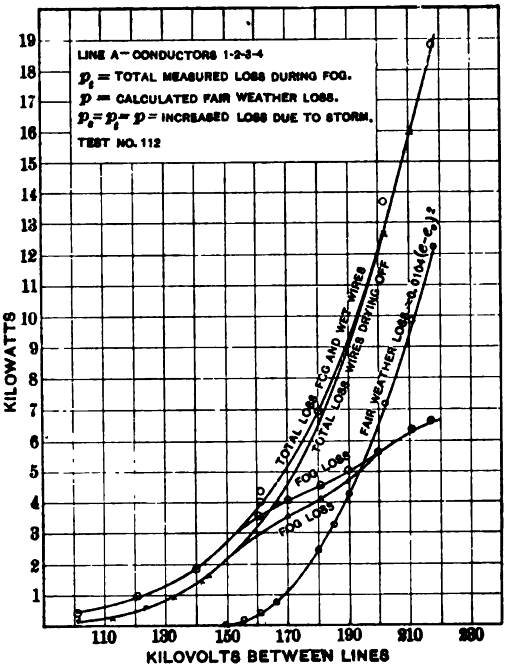

FIG. 54.—Corona loss during fog. Wires wet

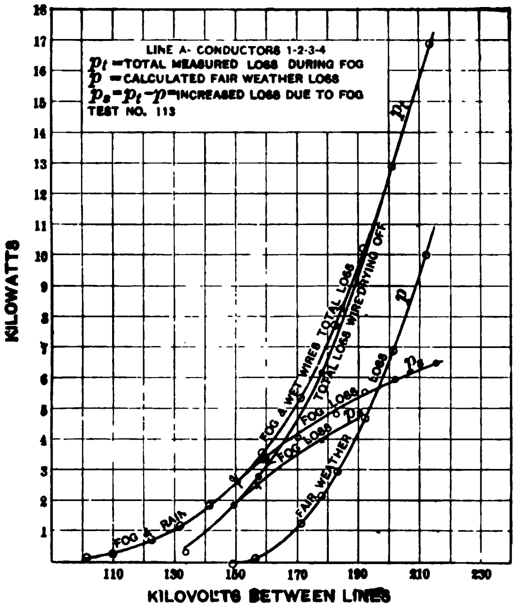

FIG. 55.—Corona loss during fog. Wires wet

tests were started. Fig. 57 shows the loss curves. After the curves were taken the line was kept at 200,000 volts for over an hour with no apparent diminution of sleet. This seems to show that sleet will form on high voltage transmission lines.

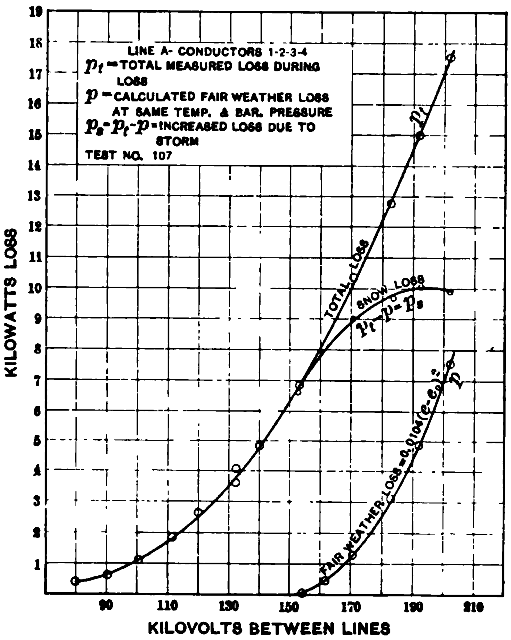

FIG. 56.—Corona loss during snow storm

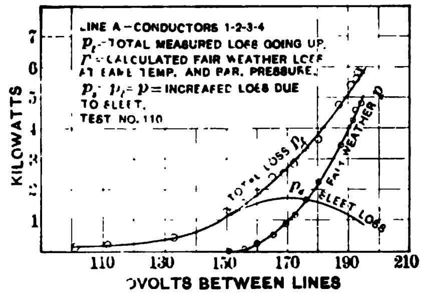

FIG. 57.—Corona loss during sleet storm

The day after these tests were made was bright and clear and the conductors were still coated with sleet. A set of readings was taken, and it is interesting to note that the excess loss here was as great as when sleet was falling. See Figs. 58 and 59.

The excess loss for sleet, rain or snow storms (over the fair weather loss) seems with increasing voltage to approach a maximum and then to decrease again (the latter at values very far above the disruptive critical voltage), and the curves of loss seem

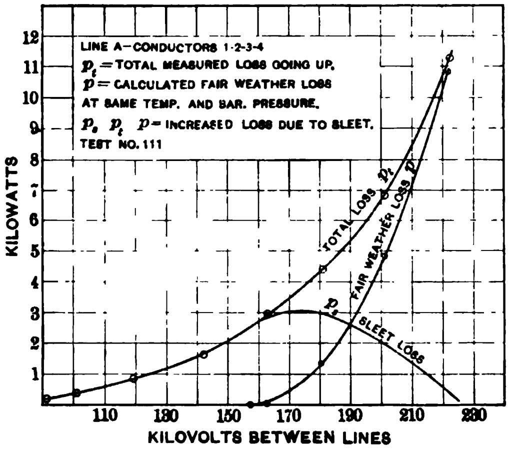

Fig. 58.—Corona loss—Wires coated with sleet

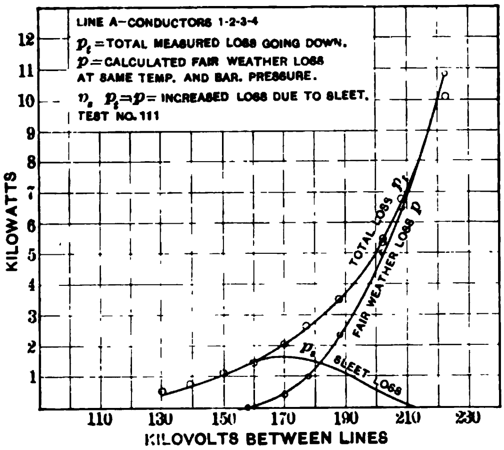

Fig. 59.—Corona loss—Wires coated with sleet

to have the general shape of the probability curve, as is to be expected theoretically.

The above readings show the importance of taking weather conditions into account in the design of high voltage transmission lines.

TABLE XXVII
CORONA LOSS. DURING FOG

(Data for curves Fig. 54)

Kv. between lines e'	Amperes total	Kw. read total p_t	Fair weather loss calculated $p = a f (e - e_0)^2$	Kw. excess loss or snow loss $p_t - p = p_s$
101.0	0.109	0.41	——	0.41
121.0	0.130	0.93	——	0.93
140.0	0.156	1.81	——	1.81
161.2	0.185	4.25	0.36	3.89
180.0	0.214	6.94	2.42	4.52
202.0	0.257	13.66	7.15	6.51
202.0	0.252	13.07	7.15	5.92
202.0	0.252	12.77	7.15	5.62
218.0	0.288	18.79	12.20	6.59
211.0	0.272	16.02	9.85	6.17
202.0	0.250	12.77	7.15	5.62
190.0	0.229	9.21	4.27	4.94
185.0	0.217	7.63	3.28	4.35
180.0	0.215	6.60	2.42	4.18
180.0	0.200	4.95	2.42	2.53
166.0	0.189	3.99	0.71	3.66
156.0	0.177	2.82	0.11	2.71
144.0	0.160	1.65	——	1.65
142.0	0.159	1.45	——	1.45
132.5	0.146	0.97	——	0.97
123.7	0.131	0.62	——	0.62
112.5	0.119	0.26	——	0.26
102.5	0.110	0.15	——	0.15

Line *A*. 1-2-3-4. Test No. 112

Total conductor length	109,500 cm.
No. 3/0 7-strand cable diameter	1.18 cm.
Spacing	310 cm.
Temperature	Wet 2 deg. cent. Dry 2 deg. cent.
Barometer	75.5 cm
e_0 fair weather	74.7

TABLE XXVIII
CORONA LOSS. DURING FOG
(Data for curves Fig. 55)

Kv. between lines e'	Amperes total	Kw. read total p_t	Fair weather loss calculated $p = a f (e - e_0)^2$	Kw. excess loss or snow loss $p_t - p = p_s$
101.5	0.103	0.16	——	0.16
110.0	0.117	0.27	——	0.27
121.5	0.129	0.64	——	0.64
131.5	0.145	1.12	——	1.12
141.5	0.157	1.85	——	1.85
150.0	0.170	2.73	——	2.73
159.6	0.182	3.61	0.26	3.35
171.0	0.196	5.31	1.21	4.10
183.0	0.219	7.74	2.92	4.82
192.0	0.232	10.21	4.68	5.53
201.0	0.247	12.87	6.90	5.97
213.0	0.272	16.92	10.50	6.42
201.0	0.247	12.87	6.92	5.97
178.0	0.207	6.16	2.11	4.05
157.0	0.177	2.81	0.15	2.65
134.0	0.150	0.38	——	0.38

Line *A*. 1-2-3-4. Test No. 113

Total conductor length.................................109,500 cm.
No. 3/0 7-strand cable diameter............................ 1.18 cm.
Spacing.. 310 cm.
Temperature.. Wet 2 deg. cent. Dry 2 deg. cent.
Barometer... 75.5 cm.
e_0 fair weather... 74.7

TABLE XXIX
CORONA LOSS. DURING SNOW STORM
POINTS TAKEN GOING UP ON LINES 1-2-3-4

(Data for curves Fig. 56)

Kv. between lines e'	Amperes	Kw. read total p_t	Fair weather loss calculated $p = a f (e - e_0)^2$	Kw. excess loss or snow loss $p_t - p = p_s$
80		0.38	——	0.38
90.5		0.62	——	0.62
100.5		1.09	——	1.09
111.5		1.88	——	1.88
120.5		2.63	——	2.63
132.5		4.08	——	4.08
132.5		3.68	——	3.68
140.5		4.86	——	4.86
153.0		6.83	0.06	6.77
161.4		7.46	0.46	7.00
170.4		10.28	1.28	9.00
182.8		12.75	3.11	9.64
192.0		15.02	4.98	10.06
202.0		17.46	7.53	9.93

Line *A*. 1-2-3-4. Test No. 107

Total conductor length.................................... 109,500 cm.
No. 3/0 7-strand cable diameter.......................... 1.18 cm.
Spacing.. 310 cm.
Temperature.. Wet –0 deg. cent. Dry –0 deg. cent.
Barometer.. 74.2 cm.
e_0 fair weather.. 74.1

TABLE XXX
CORONA LOSS. DURING SLEET STORM
POINTS TAKEN GOING UP ON LINES 1-2-3-4
(Data for curves Fig. 57)

Kv. between lines e'	Amperes	Kw. read total p_t	Fair weather loss calculated $p = a\,f\,(e - e_0)^2$	Kw. excess loss or snow loss $p_t - p = p_s$
111.3		0.21	——	0.21
150.0		1.25	——	1.25
160.0		1.81	0.21	1.60
169.0		2.51	0.83	1.68
180.6		3.58	2.26	1.32
191.6		5.35	4.26	1.09
102.0		0.13	——	0.13
133.0		0.48	——	0.48
151.0		1.25	——	1.25
172.4		2.80	1.18	1.62
.192.9		5.75	4.52	1.23
194.0		5.74	4.78	0.96
192.8		5.54	4.52	1.02
187.6		4.76	3.46	1.30
176.6		3.34	1.69	1.65
165.0		2.40	0.50	1.90
156.0		1.59	0.06	1.53
194.0		5.74	4.78	0.96
194.0		5.74	4.78	0.96

Line *A*. 1-2-3-4. Test No. 110

Total conductor length	109,500 cm.
No. 3/0 7-strand cable diameter	1.18 cm.
Spacing	310 cm.
Temperature	Wet −1 deg. cent. Dry −1 deg. cent.
Barometer	75.4 cm.
e_0 fair weather	75.6

TABLE XXXI

CORONA LOSS

WITH SLEET ON WIRES. BRIGHT SUNNY DAY. POINTS TAKEN ON LINES 1-2-3-4

(Data for curves Figs. 58 and 59)

Kv. between lines e'	Amperes	Kw. read total p_t	Fair weather loss calculated $p = af(e - e_0)^2$	Kw. excess loss or snow loss $p_t - p = p_s$
		Going down		
202.0		5.46	5.10	0.36
222.4		10.10	10.87	-0.77
208.0		6.74	6.58	0.66
2 .0		5.38	5.10	0.28
188.0		3.55	2.39	0.16
177.6		2.67	1.03	1.64
170.0		2.07	0.41	1.66
160.0		1.47	0.02	1.45
150.0		1.10	——	1.10
139.0		0.71	——	0.71
130.0		0.52	——	0.52
		Going up		
60.0		0.03	——	0.03
70.0		0.07	——	0.07
80.5		0.13	——	0.13
91.4		0.23	——	0.23
101.0		0.40	——	0.40
119.3		0.83	——	0.83
142.5		1.66	——	1.66
162.6		2.94	0.06	2.88
180.8		4.41	1.38	3.03
201.0		6.81	4.87	1.94
222.4		11.29	10.87	0.42

Line *A*. 1-2-3-4. Test No. 111

Total conductor length.................................... 109,500 cm.
No. 3/0 7-strand cable diameter.......................... 1.18 cm.
Spacing.. 310 cm.
Temperature.. Wet -10 deg. cent.
Dry -10 deg. cent.
Barometer.. 76 cm.
e_0 fair weather.. 78,8

XIV. REMARKS

There is still much left for further discussion, and many speculations to put forth, which, for lack of time, must be left to a future and more theoretical paper. Under this heading may be mentioned:

Rationalization of equations.

*Energy storage at the conductor surface through the distance x before the visual point is reached.

*Further relations between capacity and a in equation (2) and value of a for very small conductors.

The extension of corona from the conductor surface and the resulting increase in capacity.

A further discussion of the quadratic law, and the losses at low voltages.

Storm losses.

Photographic study.

Unexplained corona phenomena.

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*Work on rationalization of x and a seems to show that the values given are the first terms of a series.

DISCUSSION ON "ELECTRIC STRENGTH OF AIR, II," "THE LAW OF CORONA AND DIELECTRIC STRENGTH OF AIR." CHICAGO, JUNE 29, 1911.

C. P. Steinmetz: The papers of Dr. Whitehead and Mr. Peek, Jr., are of momentous importance by solving the problem which threatened to stop the further advance of high voltage power transmission, the corona or loss of power by discharge into the air. From an unknown danger, corona thus becomes a phenomenon amenable to exact calculation and predetermination.

Mr. Peek's paper, while very voluminous, is only a short abstract of some of the more important results of an investigation, which is probably one of the most extensive experimental electrical engineering investigations ever undertaken.

Some of its results are:

The loss of power by corona is proportional to the frequency and to the square of the excess voltage above a certain critical voltage, the disruptive critical voltage.

This disruptive critical voltage is that voltage at which at the conductor surface a certain definite and constant potential gradient is reached, which is 30 kv. per centimeter, independent of size of wires and distance, but is proportional to the air density, that is, the barometric pressure and the absolute temperature, and may be considered as the dielectric strength of air.

The corona, involving luminosity and conductivity of the air and loss of power, does not yet begin when the dielectric strength of air, that is, the disruptive voltage gradient, has been reached at the conductor surface, but only after the disruptive gradient has extended a finite distance from the conductor, that is, after an envelope of dielectrically over-strained air of a finite thickness has been formed around the conductor, and a definite amount of energy has become available thereby. In this respect, corona formation is analogous to other phenomena of gas conduction, as the electric arc, which latter also requires a definite amount of energy for its starting.

There are thus two definite critical points: the disruptive critical voltage, e_0, at which the disruptive critical gradient, that is, the dielectric strength of air, of 30 kv. per cm., is reached at the conductor surface, and the visual critical voltage e_v, at which the disruptive critical gradient has spread from the conductor to a sufficient distance to give the energy of corona formation, and luminosity and power loss begins. At the latter voltage, the gradient at the conductor surface is higher than the disruptive critical gradient, the more so, the smaller the conductor diameter, and the ratio between the visual and the disruptive voltage gradients, respectively, is given by equation (1) of Dr. Whitehead's paper, which is, not only in its form, but even in its numerical constants, the same as the corresponding equation of Mr. Peek's paper.

The theoretical loss curve, that is, that of a conductor of perfect surface, would then have the shape as shown in Fig. 1, that is, start abruptly with a finite value at e_v, while the dotted part from e_0 to e_v is unstable, just as in the arc characteristic a similar discontinuity occurs at the intersection with the "stability curve."

Another interesting result of Mr. Peek's paper is, that the lower part of the numerous previously published corona loss curves, which has been variously expressed as "sub-corona," or "part corona," etc., is nothing but the effect of the irregularity of the conductor surface, and as such follows the probability law.

Humidity of the air is by Peek's paper finally eliminated as one of the factors which might have an effect on corona. So also is the free ionization of the air. A pronounced effect, however, is produced by any solid or liquid contained in the air, as smoke, rain, snow, sleet, etc., and the effect is a great increase of loss and decrease of the voltage where the loss begins.

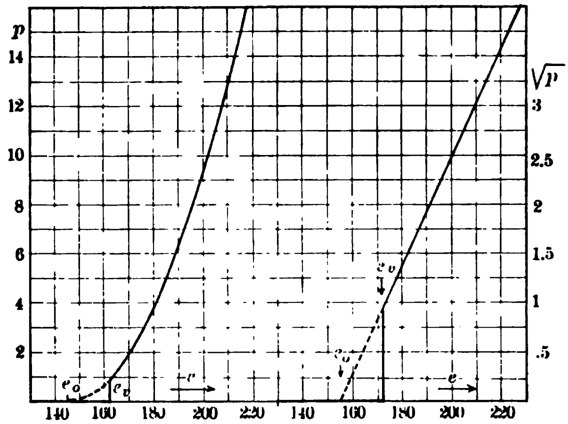


Fig. 1

Dr. Whitehead's paper specifically deals with the quantity, which in Peek's paper is called the "visual critical voltage e_v" and the "visual critical gradient g_v," the values of voltage at which corona begins, that is, at which, according to Peek, the disruptive strength of air has been exceeded for a sufficient distance from the conductor to supply the energy required to start the luminous discharge or corona.

There are some apparent discrepancies between the results of Dr. Whitehead and those of Peek: Peek shows that the disruptive critical voltage, the dielectric strength of air, and the constants of the corona equation are independent of the frequency, while Dr. Whitehead shows that the visual critical voltage slightly decreases with the frequency. Peek shows that the dielectric strength of air is proportional to the density, while Whitehead, extending his observations over a wide range of pressure, shows the visual critical voltage to vary less than proportionally to the air density. However, these discrepancies are only apparent, as Peek refers to the disruptive critical voltage e_0, Whitehead to the visual critical voltage e_v, and the effect observed by Whitehead should rationally be expected from Peek's results: Peek shows the disruptive critical gradient g_0 to be proportional to the air density, but corona to begin when the disruptive gradient has extended a finite distance from the conductor, that is, after the formation of an envelope which supplies the energy of disruption. At this point, at the conductor surface a higher gradient g_v has been reached. At lower air densities, the energy of the dielectric field is less, and it should therefore be expected that a greater thickness of envelope would be required to supply the energy of disruption, that is, the visual gradient g_v should vary less than proportionally to the air density. Similarly, at higher frequency and thus greater flow of power into the dielectric field of the conductor, less thickness of the envelope of overstrained air would be required to give the energy of corona formation, and increase of frequency thus should have the effect of decreasing the visual critical voltage e_v, at constant disruptive critical voltage e_0. That is, in Peek's equation relating the visual gradient g_v and the disruptive gradient g_0

$$\frac{g_v}{g_0}=1+\frac{0.301}{\sqrt{r}}$$

(which is equation (**1**) of Whitehead's paper), the constant 0.301 should contain the air density δ and the frequency f as terms.

This is a field for further investigation.

In conclusion, I wish to congratulate Dr. Whitehead, not only on the valuable results of his paper, but more still on the form in which the results are given, as empirical facts without coloration by the terminology of any metaphysical speculation, as unfortunately is done so often today, but the speculative interpretation is separated from the statements of the facts. However, from the point of view of the ionic theory—in which I do not believe—the facts of Peek's and Whitehead's papers appear easy of interpretation.

The dielectric strength of air or disruptive critical gradient is that field intensity, which during the main free path of an ion gives it a velocity sufficient to produce ions by collision. The dielectric strength of air thus must be a constant, independent of the size and distance of the terminals. The corona then is the result of ionization by collision, and the density of the ions in the corona is enormously high compared with the highest natural ionization of air. Starting then from the few free ions existing near the conductor, in the formation of corona these ions accelerate by the dielectric field to form new ions by collision, these again accelerate to collision velocity, and in this manner, by successive collisions of the free ions formed by collision, gradually the ionic density increases to the value required to give conductivity and luminosity, that is, corona. As a very large number of successive collisions are required to increase the ionic density to that in the corona, it follows that the ionization density of the dielectric field must extend over a distance from the conductor which is very large compared with the free path of the ion, that is, an envelope of finite thickness of over-strained air is required for corona formation, as brought out by the facts of Peek's paper. Or, from the energy point of view, a finite amount of energy is required for corona formation: the $\sum \frac{m\,v^2}{2}$ of all the ions which have been accelerated to collision velocity, until corona density of ions is reached.

At lower air densities, the free path of the ions is longer and the field intensity required to produce collision velocity, that is, the disruptive critical gradient, or dielectric strength of air, proportionally lower. However, at the longer free path, a greater thickness of the envelope of over-strained air is required to increase the ionic density by successive collisions to that required by the corona, and thus the visual critical gradient must vary less than proportionally to the air density, in agreement with the results of Whitehead's paper, while the disruptive critical gradient should be directly proportional to the air density, as given by Peek's paper.

The free ionization of the air can have no effect on the critical alternating voltage, where corona begins, nor on the power loss, since at every half wave of alternating voltage the corona formation starts not with the free ionization of the air, but with the residual ionization of the preceding half wave, which, while extremely small compared with the ionic density in the corona, is large compared with the free ionization of the air. This residual ionization, however, should to some extent vary with the frequency, and at higher frequency be higher, thus requiring a lesser thickness of the ionizing envelope, that is, a lower visual critical voltage e_v, in accordance with Whitehead's paper. Following an exponential function, the ionic density at the formation of the corona, and at its disappearance, should pass through the critical corona value very abruptly, that is, the point where corona forms, and where it disappears, should be the same and very sharply defined, as in agreement with empirical evidence.

While the free ionization of the air can have no effect on alternating corona, it should have an effect on corona formation by a single voltage impulse of very short duration, and such a very short transient voltage should either give no corona at all—if its duration is less than the time required to reach corona density by successive collisions—or only at a very much higher, and more or less indefinite voltage. That is, a much greater thickness of ionizing envelope should be required to reach corona ionization from the low natural ionization of the air, than from the higher residual ionization of the alternating corona. With alternating corona, this should give a corona transient probably extending over several half waves. Experience seems to point to the absence, or the much lesser extent of corona with transient voltages, than with alternating voltages. Thus for instance insulators, designed to flash around much below their puncturing voltage, frequently puncture with transient voltages of short duration, as lightning, without flashing over. That is, with an alternating impressed voltage, corona spreads over the insulator until followed by a disruptive discharge over the surface, long before the disruptive strength of the insulator material is reached. At a short transient voltage, however, none or very little corona seems to form, and the flash-over voltage thus becomes much higher, and above the puncture voltage. However, this entire field of corona formation with transient voltages, though of high industrial importance, is still practically unknown and work in its exploration is contemplated.

Cassius M. Davis: During the investigations made to determine the relation between the visual corona voltage, the diameter of the conductors and the distances between them, the observations were carried down to very small distances of separation, even below the point where corona ceases to form and disruptive discharge takes place, and some rather interesting results were obtained.

Where voltage is applied to two parallel wires or conductors the distribution of potential between the conductors is not uniform, but the potential gradient is greater at the conductor surface than at any other point. If the applied voltage is high enough the air immediately surrounding the wires becomes conducting and luminous as far as the potential gradient exceeds the dielectric strength of air. Since the medium surrounding the wires is now conducting, the real surface is no longer the metallic surface of the wire but some surface distant from it by the depth of the conducting envelope about it. The boundary of the corona is determined not by the equipotential surfaces surrounding the conductor, but by the equigradient surfaces. These curves are not concentric with the metallic conductors, but are nearer together.

The general mechanism of corona formation may be described as follows: Upon the gradual increase of the applied voltage a value is reached which gives, at the conductor surface, a potential gradient sufficient to break down the air. In this way the air becomes conducting as far from the conductor as the gradient exceeds the breakdown value. This decreases the distance between the conductors, but increases their diameter. The former tends to increase, the latter to decrease the gradient at the surface of the effective conductor. If then the distance between the wires is large compared with their diameter, the decrease of gradient due to the increasing conductor diameter is greater than the increase due to their decreasing distance, since the decrease of distance is a relatively small part of the total distance between wires. At the outside of the effective conductor, that is, outside of the space filled by the conducting air or corona, the gradient is less than it would be at the surface of the wires, and the discharge between wires thus limits itself to a corona extending as a more or less uniform glow, up to the distance at which the potential gradient is the breakdown gradient. If, however, the distance between the conductors is so small that the increase of potential gradient due to the decrease of distance between the effective conductors, resulting from the corona formation, would be greater than the decrease

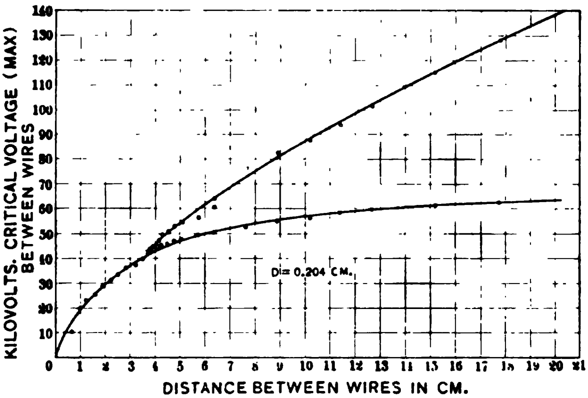


Fig. 1

due to the increasing conductor diameter, corona cannot form, but as soon as the discharge into the air begins at the conductor surface, it increases the potential gradient and thereby passes across between conductors as a disruptive discharge. Thus if the conducting envelope produces a decrease of the potential gradient between the wires, corona appears; if it produces an increase, disruptive discharge takes place.

The transition from disruptive discharge to corona is rather interesting to observe. Starting with the conductors very close together and gradually separating them, maintaining the potential at the critical value all the while, a point is reached where the disruptive discharges become less sharply defined and less snappy; and intermittently between the more noisy discharges are very quiet and what may be called "puff" discharges. These are intermediate in intensity between the disruptive and corona discharges. If the distance is further increased a point will be found where corona begins, and by carefully adjusting the distance a position can be determined where, at the same applied potential difference, all three discharges take place one after another in various orders of succession. This shows very nicely that corona, like the Geissler tube discharge, is a form of disruptive discharge.

If the wires are separated by a greater amount than this critical distance and corona is produced at the critical voltage, a disruptive discharge may again be brought about by raising the voltage above the critical value of corona formation. Thus if a curve is plotted between voltage or potential gradient and separation between conductors it consists of a single line up to the critical distance, then at that point it divides, forming two branches, one for disruptive discharge and one for corona. Two such curves, one showing the voltage, the other the potential gradient at the conductor surface, are given in Figs. 1 and 2.

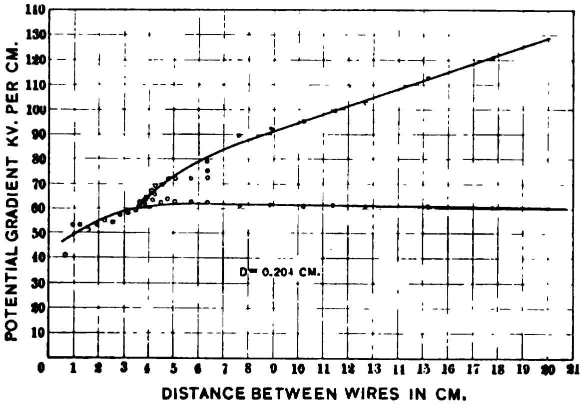


FIG. 2

The critical distance where corona begins increases with increasing diameter and should probably be of such a value as to give a constant ratio between the separation distance and the diameter. It is rather difficult to calculate this ratio because of the complex shape of the equigradient surfaces. Based on cylinders whose sections are circles concentric with the metallic conductor, the ratio is about 3.3; and based on cylinders whose sections are eccentric circles with constant distance between the outside surfaces, the ratio is about 4.1. Experimentally this ratio seems to be about 25, indicating that the precise shape of the equigradient surfaces should be determined and used in the calculation. The accompanying table gives the experimental values of the diameter, critical distance, critical potential difference to neutral in kilovolts, gradient at the surface and ratio of critical distance to diameter.

Diameter cm.	Critical distance cm.	Visual critical potential	Potential gradient at surface	Ratio $\frac{\text{critical distance}}{\text{diameter}}$
0.0344	1.8	7.6	96.4	52.3
0.0780	2.7	13.0	81.1	34.6
0.1662	3.6	19.7	66.4	21.6
0.204	3.6	20.7	60.6	17.6
0.320	7.8	35.0	58.6	24.4
0.513	13.3	49.1	50.2	25.1

No great accuracy is claimed for these results, since the apparatus used in the experiments was designed to be employed for comparatively great distances, and small distances were difficult to measure accurately; however, they serve to show the general magnitude of the values.

The disruptive branches beyond the critical distance (giving the relation between voltage and size of conductors) approach each other, and beyond separation distances of 10 cm. have practically the same values for wire diameters within the range used in the tests. The lower end of the disruptive branch, that is, the part below the critical distance, seems to point toward approximately the value 30 kv., at zero distance, determining the dielectric strength of air in another way. This is reasonable because the surfaces when very close together become planes and the distance between them becomes less than the thickness of any envelope of conducting air around them, and thus the gradient at the surface of the conductor becomes the breakdown gradient of air.

The accuracy with which the critical voltage can be determined visually is quite remarkable. Observations can be made with very little practise which check within one per cent of each other. This applies as well for laboratory measurements as for determinations made upon an actual transmission line. The visual method has the advantage over other methods in that the effect of point discharges can be eliminated since a portion of the conductor can be selected for observations which is free from irregularities. That the visual point is the point where ionization begins is very nicely illustrated in the papers by Messrs. Whitehead and Peek, in which, working independently, each derives almost the identical equation for critical voltage: one using the ionization point, the other the visual point; one using a conductor concentrically located in a cylinder, the other two parallel wires.

The appearance of corona is greatly dependent upon the surface condition of the conductor. A perfectly smooth and polished copper wire exhibits corona as a very soft uniform glow; however, if the discharge is allowed to continue a short time the glow becomes less homogeneous and soon appears in tufts with the space between almost non-luminous, giving the wire the appearance of a string of bright beads. An examination of the conductor after this " beading " has continued a while shows alternate stripes of the bright wire and dull oxide which latter rubs off easily as a black substance.

The mass of information now available upon the subject of corona formation on transmission lines is quite voluminous, and high-potential lines can be designed with considerable accuracy with regard to the amount of energy dissipated into the air; also the critical voltages can be calculated very accurately and avoided. With other apparatus conditions are different, and while the critical voltage can be estimated very closely, and in a measure avoided, yet with insulation other than air, which is hardly ever homogeneous, and very limited distances of separation, the means of eliminating corona and its accompanying deleterious effects are less understood and a very large field is offered for investigation.

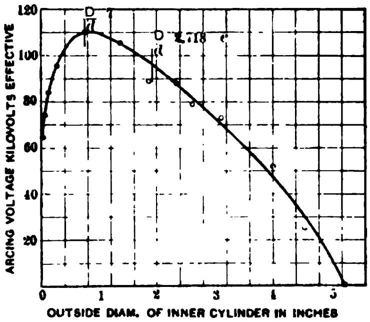
A. B. Hendricks, Jr.: One of the most striking features of the papers of Whitehead and Peek is the great accuracy of the results reported, especially in the determination of the voltage required to produce visual and audible corona. This at once suggests the possibility of constructing a corona voltmeter as proposed by Professor Ryan and others—and its use for high-tension measurements in place of the needle point spark gap. Such an instrument could be made to give both visible and audible indications with a probable accuracy of one-half per cent.

The form and arrangement of electrodes should of course be such as to produce corona at a voltage far below the arcing voltage, thus avoiding short circuits and high-frequency oscillations in the high-potential system. This feature, together with the reliability of the indications, would render the device very valuable for high-tension investigations and much superior to the needle point spark gap.

The accuracy of Whitehead and Peek's measurements also demonstrates the usefulness of the voltmeter coil on the high-tension transformer core, as nearly all of the determinations were made by this means, though checked by independent methods. This method has met with some criticism on the score of alleged inaccuracy, therefore the results are very gratifying and furnish sufficient proof of reliability.

Theory seems to indicate that the dielectric strength of the air between concentric cylinders is a maximum for a ratio of cylinder diameters equal to 2.718 or the base of the natural logarithms. Experiment shows that this is not the case but that the actual ratio is very much greater. For example—with a diameter of 5¼ in. for the outer cylinder, the maximum was reached with an inner cylinder one-eighth inch in diameter, whereas theory indicates that this should be one and five-eighths inches. The most definite results I have obtained illustrating this point, are on transformer oil and are best shown by the accompanying curve Fig. 1. In this case, the ratio of diameters for maximum dielectric strength was found equal to seven. Possibly from their own investigations, the authors may be able to give an explanation of the cause of this discrepancy. At any rate, the fact seems to be established and should be taken into account in high-tension design.

Another interesting phenomenon, though not directly connected with the subject of these papers, is the effect of pressure on the dielectric strength of oil. I have investigated this point but slightly and find that the dielectric strength is increased about 50 per cent by a rise in gage pressure from zero to 200 lb. per square inch. Presumably others have more complete information regarding this effect, and, if so, it is suggested that their results would be of great value and interest if presented to this Institute. This property of oil may render possible the construction of apparatus for extreme high tensions, normally operating under high oil pressure.



D = inside diameter of outer cylinder = 5.25 in. constant

d = outside diameter of inner cylinder = 0 005 - 4.56 in.

FIG. 1.—Arcing voltage between concentric cylinders under oil

Charles F. Scott: I will not attempt to contribute to the technical discussion of the subject. The former speakers have dwelt upon these technical points and given a most excellent discussion. I would like, for a moment, however, to take a general survey, to show the progress we are making in transmission.

It is not so very many years since 40,000 volts marked the high range of potential of operating pressure. A paper before the Institute a dozen or thirteen years ago, describing Mr. Mershon's work at Telluride, gives tests which were limited to two and a half or three miles in length, in which the voltages used were in the main from 50,000 to 70,000, although a few tests ran up to 110,000 volts, which I believe was the limit of insulation of the transformer, as demonstrated by the test itself, with inadequate instruments and methods devised by Mr. Mershon on the ground under the greatest difficulties—those were the conditions only a few years ago.

A few years later, in the static investigations to which the Chairman referred, which were made by Mr. Thomas himself, the potential was as high as seventy thousand volts, and the tests were made on a number of commercial lines in the West.

Now, we have tests covering three hundred miles of line, for normal operation, at 100,000 volts, and we have a fundamental knowledge of the phenomena on which to base the tests, which are made with instruments which were not in existence until a few years ago. The oscillograph has opened up and given us photographs of just what is going on which made possible a new order of investigation and enlightenment.

A word should be said as to the amount of time, labor and money which must be expended on tests of this kind; also commendation should be made of the excellent policy of presenting so freely and fully these measurements before the Institute, not only with the results obtained on lines in the west, but those on the experimental lines at Schenectady. The general result of all these investigations has been most valuable. All papers of this morning have contributed to our knowledge of the conditions in the high-voltage field, and have not busied themselves in bringing up impediments and difficulties, but they have defined the conditions of operation, and have been extending the possibilities and the certainties of high-tension transmission by showing definitely what those conditions are. Sometimes papers deal with the discovery of some phenomena which are impediments, sometimes papers deal with special devices for overcoming special difficulties, but here we have a larger scientific investigation, showing what the conditions are, and fortunately, if I am not mistaken, not bringing up new hindrances, but blazing the way for the extended commercial development of high-tension transmission and making its operation more sure.

Harris J. Ryan: Dr. Whitehead has evolved out of the original crude concentric cylinders outfit for the laboratory study of corona an instrument of precision that offers a wide range of application in the exact study of the essentials of corona phenomena. In the concentric cylinders method the atmosphere between the cylinders is stressed in a balanced electric field. When the surface of the central cylinder is true and free from all foreign material, ideal conditions exist and the turbulent elements that enter into the practical case, giving rise to the "irregularity factor," are absent. Dr. Whitehead's studies are therefore particularly valuable because his results constitute a foundation or base upon which to rest judgment of the conditions and relations that exist in the practical cases as they arise. He has found that the electric surface-intensities which start to cover a clean cylindrical conductor with visible corona in a balanced electric field are

$$E_1 = E_0 = 32 + \frac{13.4}{\sqrt{d}},$$

while Mr. Peek has found that these same corresponding electric intensities which start to cover a clean cylindrical conductor in the unbalanced field set up between two equal parallel cylinders are

$$E_1 = 29.8\left(1+\frac{0.301}{\sqrt{r}}\right) = 29.8 + \frac{12.7}{\sqrt{d}}$$

The ratios of these corresponding values are nearly constant for all diameters likely to be employed in engineering, as may be seen in the table below:

d cms.	E_0 kv/cm.	E_1 kv/cm.	E_1/E_0
0.2	61.95	58.17	0.939
2.0	41.47	38.77	0.935
6.0	37.46	34.98	0.934

When these formulas are applied to the case of parallel plate electrodes, $\sqrt{d} = \infty$, and the values of E_0 and E_1 are the corresponding ultimate dielectric strengths of the normal atmosphere stressed by a uniform electric field:

$E_0 = 32$ kilovolts per centimeter,

$E_1 = 29.8$ kilovolts per centimeter.

There should be no question about the integrity of each of these values. Their ratio is

$$\frac{E_1}{E_0} = \frac{29.8}{32} = 0.93$$

For some time physicists have concluded through methods peculiar to their studies of the conduction of electricity through gases that the dielectric strength against rupture of the normal atmosphere using unbalanced electric fields in the open is quite close to 30 kv. per cm. Mr. Peek's value of 29.8 found by totally different methods is quite in agreement therewith. Dr. Whitehead's value of 32 applies to the normal atmosphere enclosed and stressed in a balanced electric field. It applies, therefore, to conditions in which all irregularity effects are quite completely eliminated. Therefore, one may safely use 32 kv. per cm. for the ultimate dielectric strength of the normal atmosphere in constructing a rational formula for the critical visual corona voltage provided the factor 0.93 is included, due to the change in the electric fields from one that is enclosed and balanced to one that is in the open and unbalanced, *i.e.*, as set up between two equal parallel cylinders.

In the present papers we have placed before us conclusive evidence that corona starts to cover a round conductor in the open when the normal air is stressed to 30 kv. per cm. at a distance of $a = 0.301\sqrt{r}$ cm. radially from the conductor surface.

The theory of ionization by collision which accounts for this phenomenon insists that the normal rupturing strength, 30 kv. per cm., must be directly proportional to the atmospheric density factor

$$\delta = \frac{3.92\,b}{273+t}$$

and to $0.301\sqrt{r}.f(\delta)$. There is substantial evidence* that $f(\delta)$ in $0.301\sqrt{r}.f(\delta)$ is so nearly constant and so near unity over the whole range of natural atmospheric variations observed by barometer and thermometer that it is entirely safe to use $f(\delta)=1$ in $a=0.301\sqrt{r}.f(\delta)$. Thus beyond all doubt, so far as theoretical reasoning can settle such matters, the barometer-thermometer correction factor in the rational visual corona critical voltage formula must be

$$\delta = \frac{3.92\,b}{273+t}$$

Dr. Whitehead has again undertaken to show that I am wrong in regard to the effect of electrified atmosphere upon the visual corona critical voltage; he does this by proving what I have all along stoutly maintained, *viz.*, that no amount of electrification will alter the critical voltage as observed in his apparatus. The theory of ionization by collision that applies here necessarily makes this so. In the open atmosphere about parallel cylinders which never remain " clean " and with their unbalanced electric field it is a different matter. The " coring up " phenomenon comes in and electrification or " antecedent ionization " introduces part-corona or local corona which starts at lower surface gradients and therefore at lower critical voltages. He also renews his objection to the experiment referred to in Fig. 28† of my January 13, 1911, paper. My reply to that objection appears‡ in the corona discussion published in the TRANSACTIONS, 1911, XXX, I. He offers also the conjecture that some of the surface intensities reported in my paper are erratic as to position because of lack of care as to high-voltage wave forms employed and in making the visual settings for the initial corona. I do not think this is so. I refer to this matter later on so as to reply to the question here raised.

Regarding Mershon's work at Niagara Falls referred to several times in the present corona papers, anyone who repeats Mershon's observations at the same point as to location and under the same circumstances as to wind and weather should find just about the same values that he found. In doing this the be-

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*TRANSACTIONS A. I. E. E., 1911, XXX, Part I, curves I and III Fig. 4, p. 6; also, compare the Townsend values Fig. 15*b*, p. 21 and the *B-P-S* values of Fig. 6, p. 8 of the same paper.

†See TRANSACTIONS, 1911, XXX, I, p. 52. ‡ p. 127.

havior of the high-voltage circuit with ungrounded neutral must be kept in mind, as well as the importance of the electrification of the atmosphere produced by the falls in connection with conductors having surfaces in various physical states.

All who are interested in the corona problem will find Mr. Peek's paper a mine of valuable results, well produced and applied. His work has decided a number of things about which uncertainties have heretofore existed. He shows conclusively that:

1. The dielectric strength of the normal atmosphere at rupture is very close to 30 kv. per cm. This is in agreement with the corresponding value found by the physicists who have employed entirely different methods.
2. Corona will form and cover the surface of a clean conductor in the normal air whenever the potential of the conductor is raised to the value whereat the air is stressed to the breaking point, 30 kv. per cm., at a *critical distance* normal to the conductor surface, thus vindicating the conclusions drawn from the results of Steinmetz and Hayden by the energy-storage method, of Townsend through his established theory of ionization by collision and of Baille-Paschen-Schuster by the critical sparking distances between metal spheres.
3. The initial corona-forming voltage is proportional to the density of the atmosphere.
4. An *irregularity factor* must be applied in the rational formula for the initial corona voltage in the practical case because of turbulent elements that are not subject to systematic coördination. Judgment with a knowledge of the conditions must be employed to determine the value of this irregularity factor.

In addition thereto the paper gives the law governing the loss of power in corona about a high-tension transmission line as the voltage is elevated from the corona-starting value. This is likewise of much importance. It will help the engineer to know in advance how much power will be lost by corona in a high-tension transmission that must in emergencies or otherwise be operated at a voltage above that which inaugurates corona. It is altogether possible, too, that high-tension transmission lines may be operated at critical corona voltage over strategic sections or over branches, loaded or unloaded, as an aid to or as a substitute for the static arrester. Should such practise come about this corona power law will become invaluable in connection therewith.

Mr. Peek has found that the critical radial distance from the cylindrical conductor surface through the initial corona envelope is such as to make the radius of the outer surface of the envelope

$$r_v = r\left(1 + \frac{0.301}{\sqrt{r}}\right) = r + 0.301\sqrt{r}$$

wherein r_v is the radius of such corona envelope;
r is the radius of the conductor.

It is a great satisfaction to find that the thickness of the initial corona envelope is a simple function of the radius of the conductor, because the rational derivation of the initial corona-forming voltage formula must include this thickness. There can be no question about the integrity of this result, applicable to engineering sizes. There are at hand three independent checks, as follows:

Dr. Whitehead's corona-starting surface-intensities are given by the expression

$$E = g_v = 32 + \frac{13.4}{\sqrt{d}}$$

which may be written in the form

$$g_v = 32 \left(\frac{r + 0.296 \sqrt{r}}{r} \right)$$

$$a = 0.296 \sqrt{r}$$

wherein a is the corona striking distance or thickness of the initial corona envelope, corresponding to Mr. Peek's value:

$$a = 0.301 \sqrt{r}$$

The critical striking distances between metal spheres in the normal atmosphere observed by continuous voltage many years ago* give an average of

$$a = 0.300 \sqrt{r}$$

Mr. Peek's method for evaluating a has been applied to the surface-intensity values observed some years ago by the concentric cylinders visual method.† (See accompanying diagram Fig. 1.) The resulting value of the initial corona thickness is

$$a = 0.316 \sqrt{r}$$

These are all in remarkable agreement, considering the circumstances. Less confidence is placed in the last value than in the others. It is higher by five per cent, due more likely to the character of the surfaces of the conductors used than to errors in observation. All settings were repeated several times; wave-forms were watched by means of a special oscillograph connected in series with the high-tension test circuit, also in series with an air condenser across such circuit, besides, the maximums of all high-voltage waves were carefully checked with needle

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*TRANSACTIONS A. I. E. E., 1911, Vol. XXX, I, p. 8.

†TRANSACTIONS A. I. E. E., 1904, Vol. XXIII, p. 101.

point spark gaps. It is not thought that these 1904 values are erratic so much through errors of observation as suggested by Dr. Whitehead in his present paper. The cause is probably the condition of the surfaces of the brass rods used for all sizes above 0.2 inch. While clean in the ordinary sense they were not " clean " as the present day corona term implies. It is a happy event to have this corona feature cleared up so well by Mr. Peek. It establishes the integrity of the rational formula for the visual critical corona voltage, which is written by him thus:

$$e_r = 2.302\ m_v\, g_0\, \delta\, r\ 1 + \frac{0.189}{\sqrt{r}}\ \log_{10} \frac{s}{r} \ldots\ldots\ldots [(5a)\ \text{p. } 1899]$$

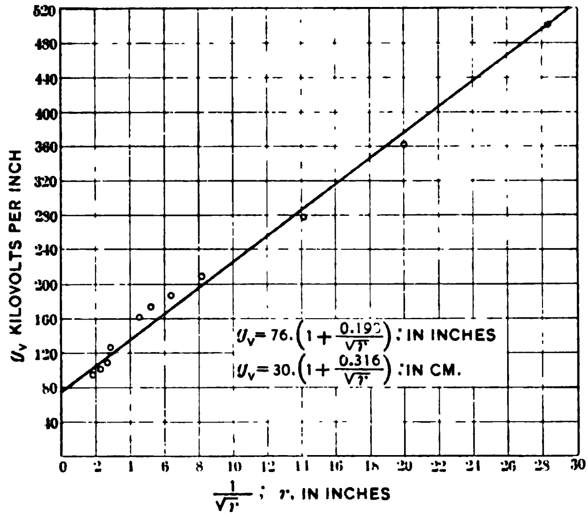

FIG. 1

and as given in the January 1911 corona paper thus:

$$E_{crit} = 0.455\ k\ \frac{17.9\, b}{459 + t} \cdot \frac{d + 2\, a}{C} \ldots\ldots\ldots\ldots\ldots [(3)\ \text{p. } 69,\ \text{Part I}]$$

wherein E_{crit} = critical loss in kilovolts, effective sine-wave. (This is the same as visual critical voltage.)

k = factor selected by judgment guided by the results of practical tests and experience.

d = outer diameter of wires or cables in inches.

a = corona striking distance in inches obtained from curve in Fig. 2, p. 4, Part I.

b = barometer, inches of mercury.

t = temperature, degs. fahr.

C = capacity of the line in microfarads per 1000 feet.

This applies to the single-phase line. For the three-phase line, E_{crit} and C are taken to neutral.

Substituting Mr. Peek's function for the conductor radius, $0.301 \sqrt{r}$, for the corona striking distance or thickness of the initial corona envelope (a) the above formula [(**3**) p. 69] becomes in English measure

$$E_{crit} = 0.455 \; k \; \frac{17.9 \, b}{459+t} \; \frac{d+0.267 \sqrt{d}}{C} \; \ldots\ldots\ldots \quad (\mathbf{3}a)$$

or, in Mr. Peek's notation:

$$e_g = 0.455 \; m_v \; \delta \; \frac{d+0.267 \sqrt{d}}{C}$$

This is a somewhat more convenient form of the rational formula for use. One can take the values of C from the usual capacity tables in lieu of having to work out $\log s/r$. This formula has now been checked for all conductor sizes varying from diameters of 0.005 to 0.5 of an inch, while there is good laboratory evidence through the critical surface intensities and striking distances found with spherical electrodes that the formula will be correct for all conductor diameters below 2.5 in.

It is a matter, for the present at least, more of theoretical than of practical interest, that $a = 0.189 \sqrt{r}$ does not hold for diameters much above 2.5 in. Where $r = \infty$, *i.e.*, with plate electrodes, this value for a becomes ∞, while under these circumstances the critical surface-intensity occurs at a striking distance already known to be

$$a = 0.268 \text{ inch.}$$

Available evidence shows that the value of a as a function of the conductor radius or rather of the diameter d between the limits of $d = 0.25$ in. and $d = \infty$ will be found to be quite close to

$$2\,a = \frac{d}{1.4+1.86\,d}$$

$$2\,d = \frac{1}{\frac{1.4}{d}+1.86}$$

When $d = \infty$,

$$a = 0.268 \text{ inch.}$$

The corresponding rational formula for the critical visual corona voltage would be

$$e_v = 0.455\ m_v\ \delta\left(1 + \frac{1}{1.4 + 1.86\,d}\right)\frac{d}{C}$$

This formula is in close agreement with Mr. Peek's corresponding formula for all diameters from 0.25 to 2.5 in. while it is probably quite correct for all larger diameters. As just stated, it is perhaps more of theoretical than of practical interest. However, it is in further support of the integrity of Mr. Peek's values, $a = 0.189\ \sqrt{r}$, because the relation

$$2a = \frac{d}{1.4 + 1.86\,d}$$

is known to be true for the largest diameter, $d = \infty$, and at the same time covers his values between the limits $d = 0.25$ and $d = 2.5$ in.

The critical striking distance a is now known, therefore, quite exactly between the limits of $d = 0.005$ and $d = 0.5$ in. and its function of the conductor diameter for all larger sizes is probably known within five per cent of the true values.

Mr. Peek's conclusions in regard to the values of the irregularity factor are important, based as they are on such extensive outdoor line tests. They are further supported by a considerable number of less comprehensive tests reported in the present TRANSACTIONS.* His formulas (3) and (5) for "disruptive critical voltage" are rational in their derivation and their constants are now all known with fair exactness; exception is made for the irregularity factor (m_v), which in the nature of things must, in part at least, be settled by judgment. The facts brought out in this paper are a great help to decide upon the value of this factor in most actual cases as they arise. In due course of time, doubtless, more data will be brought forward to help one to decide upon this factor under the most trying or extraordinary conditions. These formulas (3) and (5) will hereafter be a reliable guide to the high-tension engineer in designing transmission lines so that waste of power in corona will be avoided.

The high-voltage measurements here reported upon were made on circuits operated with the neutral grounded. This should always be the case when definite results are to be obtained. Initial corona values on a completely insulated high-voltage circuit in the open will vary somewhat because the absolute potential of the line, under these circumstances, will vary, due to the electrification of the atmosphere. Unless the purpose is

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*TRANSACTIONS A. I. E. E., 1911, Vol. XXX, I, p. 70.

to watch the varying effects upon initial corona values due to the electrification of the atmosphere surrounding a completely insulated high-tension circuit such circuit should be grounded at the neutral.

John B. Whitehead: Before replying to the discussion of my own paper I shall comment briefly on that of Mr. Peek. The Institute is to be congratulated on the mass of data which it contains and on the liberal spirit of the company he represents not only in conducting the experiments, but in offering the results to the profession. Such experiments on a scale approaching the conditions of practise, paralleled by accurate laboratory investigations, will quickly settle the uncertainties of this great corona problem or indeed of any other of the many constantly before us in the applications of electricity. It is a great pleasure to find that where the work of Mr. Peek has coincided in sphere with my own, the results are in such good agreement.

The title of my paper is the "Electric Strength of Air." The word *electric* indicates that the air is considered as regards its behavior under electric intensity or strain. The word "*dielectric*" by its origin and long usage refers to the inductive property of a substance, which property has no definite relation to its disruptive strength. The term "dielectric strength" therefore suggests the specific inductive capacity or perhaps some limit of specific inductive capacity. The term as adopted by Mr. Peek to describe the disruptive strength has been used rather commonly in this country, but the origin and present sense of the word *dielectric* would seem to indicate that its use in this connection is a mistaken one.

A further question of terminology is suggested by the expression "disruptive critical voltage." As defined by Mr. Peek the "disruptive gradient" for air is constant; it is the same for all sizes of wire and cable and has the value 30 kv. per cm. The obvious objection to this definition lies in the fact that the air will break down under this gradient only in the theoretical case of infinite parallel planes, and that it may be subjected to far higher gradients in the neighborhood of wires and cables without giving any evidence whatever of breakdown or loss. Mr. Peek and Dr. Steinmetz, to explain this inconsistency, state that in the case of wires or cables the "disruptive gradient" must be reached at a distance from the surface. Now this means either that breakdown occurs first at a distance from the surface, a condition of which there is no experimental evidence, or that the first breakdown occurs at a higher gradient than 30 kv. per cm. Since it is certain that a higher gradient than 30 kv. per cm. can be applied to air without breaking it down, we are hardly justified in saying that its electric strength is constant, or in calling the constant term of my formula (1) [Mr. Peek's corresponding formula has no reference number], a "critical" gradient.

By producing the quadratic loss curves backward to the voltage of zero loss Mr. Peek deduces a series of values of voltage gradient whose mean is very close to the value 30 kv. per cm. above referred to and this is his principle reason for defining this constant gradient as " disruptive gradient " as already discussed. This approximate constancy is extremely interesting as there is no obvious reason why it should occur. It must be remembered, however, that the quadratic law is obeyed with great accuracy by all the observations on the upper portions of the curve, whereas there is a rather wide percentage variation among the gradients calculated as above described, almost too wide to be attributed to experimental error. Further, since the quadratic law is deduced from the portion of the curve above the visual corona voltage, and since the latter is always greater than that corresponding to the constant term of my equation (**1**), it may be that the approximate agreement of the zero-loss-gradient, as obtained by extending the quadratic curve backward, and the constant term is fortuitous.

A third reason for defining the " disruptive critical voltage " is that suggested by Mr. Davis in his experiments on the sparking voltage of two wires brought close together. He states that sparking sometimes takes place at lower voltages than those corresponding to corona voltage. I should be very much interested to see a more detailed account of his experiments and observations. There are several evident reasons which might explain a lowering of the sparking voltage below that corresponding to visual corona intensity. First, with parallel wires close together the equipotential gradient surfaces, if I may use the term, are not concentric with the wires. This means that the total circumference of such a surface is greater in the case of the parallel wires than in the case of concentric cylinders. This means that the volume of air involved in the strain of a given value of gradient is greater in the former case. We have already seen that in the case of larger volumes, as for example with larger wires in the case of concentric cylinders, the breakdown takes place at a lower value of gradient. Another possible cause for lowering of the sparking gradient is the drawing together of the wires by electrostatic attraction or a lack of true straightness. It is possible that errors of this nature were provided against, but the apparatus described by Mr. Peek shows a span rather long as compared with the small distances suggested by Mr. Davis.

Mr. Peek has accepted Ryan's conclusion that the disruptive critical voltage or rupturing gradient varies directly as the atmospheric pressure. He also finds from his own experiments that these quantities vary inversely as the absolute temperature. My experiments are at considerable variance with these conclusions. As regards temperature neither Mr. Peek's nor my own experiments are sufficiently extended to justify a discussion of their differences. In the matter of pressure, however, my paper describes experiments showing that while the relation between critical gradient and pressure is linear, it is not one of direct proportionality, and further, the slope of the linear relation varies with the size of wire. Dr. Steinmetz states that there is no real discrepancy, and explains in terms of his idea of the storage of energy in an envelope of air around the wire. Among other things he states: " At lower air densities the energy of the dielectric field is less." This idea is also adopted by Mr. Peek in a reference to the Hayden-Steinmetz paper in which Dr. Steinmetz first introduced it. The suggestion is a novel one, but in the absence of confirmatory experimental evidence or further explanation of the form of the stored energy and of its relation to present physical theory, it is hardly possible that it will be accepted in explanation of the phenomena we are now discussing. Dr. Steinmetz makes some further interesting suggestions in his explanations in terms of the ionization theory. It is regrettable that he has no interest in this theory. If there is one thing it does do it is to permit quantitative calculations; in the hands of Dr. Steinmetz, with his splendid power of analysis, I believe that many of our present uncertainties in this most fascinating field would be cleared up, and new problems set before us. Professor Ryan remarks that my conclusions as to the absence of the influence of foregoing ionization on the corona voltage are in accord with his own, and that therefore my reference to his three-wire experiment is pointless. Replying I would state that even if a distinction between " part corona " and " full corona " be granted, no conclusions are permissible from his experiment since the voltage gradients are necessarily different in the two conditions which he compares.

F. W. Peek, Jr.: When the investigations given in the present paper were first planned it was decided to erect a short experimental line to represent practical conditions, and to supplement this study by extensive laboratory work. A short line of this sort has the advantage over a long operating line in that there are no impossible corrections to make due to voltage rise and variable elevations, weather, temperature, etc., along the line. Thousands of readings were taken for various conditions. The present paper though quite long is necessarily very much condensed on account of the vast amount of material to discuss. While the formulas given cover any conditions to be met with in practise, as frequency range, natural range of barometric pressure, etc., it is hoped at some future date to go further into the relations determined, more from the speculative and theoretical standpoint. For instance in the expression:

$$g_v = g_0 \quad 1+\frac{0.301}{\sqrt{r}}$$

$\sqrt{r}$ is probably the first term of a convergent series, etc.

Dr. Whitehead's paper deals with what comes under visual test, Section VIII, of my paper. Dr. Whitehead is to be congratulated on the ingenious and accurate method which he has devised to determine the critical surface intensity and also upon the general arrangement of his data. In our laboratory tests we have preferred the visual method in determining the critical intensity as it has been found to give consistent results, is easy of manipulation, and the effects of slight imperfections on the conductor surface can be seen at once and eliminated. Dr. Steinmetz has pointed out that the discrepancies between Dr. Whitehead's paper and mine are only apparent. Thus g_0 should be independent of the frequency and proportional to the air density, but the visual critical gradient g_v, covered by Dr. Whitehead's experiments, should be dependent on, and increase with the frequency, to some extent, as greater energy is stored in the " disruptive envelope " for a given thickness, as the frequency is raised. For a similar reason g_v should probably vary more slowly than the air density. Thus theoretically both δ and f should enter the constant 0.301 in the relation between g_v and g_0. However, this correction is very small in the practical ranges of frequency and natural atmospheric conditions. The ionic theory can be nicely applied here, as Dr. Steinmetz and Professor Ryan have shown.

Dr. Whitehead objects to the term " disruptive critical voltage ". As there is every evidence that two critical voltages exist, they must be distinguished. The term was introduced to separate this constant gradient critical voltage from the already known visual critical voltage. The various evidences of the disruptive critical voltage or a constant breakdown gradient for air are:

1. The e_0 of the quadratic law gives a constant gradient.
2. The e_0 in the equation $e_v = e_0 \ 1 + \frac{0.301}{\sqrt{r}}$.
3. Observations of Professor Ryan and others, who have determined a constant breakdown gradient of air of about 30 kilovolts per cm.
4. The reason suggested in the discussion of Mr. Davis.

All the values agree in magnitude and point to this constant breakdown gradient.

Dr. Whitehead states that the g_0 values vary to a considerable extent. It must be remembered that these values were not determined on short lengths of carefully polished conductors in a laboratory as were the g_v values, but on weathered wires out of doors, on an actual line, and having a considerable range of surface conditions or irregularity factors. It also must be remembered that e_0 is not determined directly as e_v but by an indirect method which would also account for variations, as discussed in the paper.

Mr. Hendricks' experimental observations that the strength of air between two concentric cylinders is not a maximum when the ratio of diameters is 2.718, as theory would at first seem to indicate, but at a greater ratio, I think is what would be expected. Thus before the spark-over potential is reached the inner cylinder has a much greater diameter, due to the conducting corona envelope, and the actual metallic ratio is not the true ratio.

Professor Ryan has added a very interesting and instructive discussion. Of special interest is the value of a that he has deduced. Thus he has taken values from his paper of 1904 and deduced values of a, the thickness of the " energy envelope ", by the method described in my paper, for comparison. The close agreement between these values and those of the present papers is quite remarkable. Professor Ryan has also shown that this is in agreement with measurements made on spheres some years ago.

A discussion at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 29, 1911.

HIGH-TENSION TRANSMISSION

INTRODUCTION

BY PERCY H. THOMAS

The High-Tension Transmission Committee this year wished to give opportunity for the consideration of matters pertaining to the design and operation of extra high tension transmission systems, that is, systems of 80,000 volts or higher. As we have a number of these systems in actual operation, the time appears ripe for a first discussion or interchange of ideas on this subject. The committee during the early part of the year sent out a list of questions to some seven plants, which are operating at such voltages, with a request for brief categorical answers to the questions which had been prepared. The committee has received from Messrs. Jollyman, Lee, Hebgen and Hanscom the four following communications.

The original questions were as follows:

1. Do you operate with a grounded neutral? If so, with or without resistance, with how much and of what type?
2. Do you use overhead ground wires, if so how many and how spaced and how frequently grounded?
3. Standard span, length, material of conductor, spacing of conductors, clearance from the earth, etc. Also, data on special spans of interest.
4. Occurrences of lightning, how frequent, its effects on the system.
5. What is the charging current of your line? Give if possible accurate data as to length of line, size and spacing of conductors, frequency, etc., at the time of the making of the measurement. Note any peculiarity in charging current.
6. Experience with insulators, electrical and mechanical, swinging of conductors with suspension insulators.
7. Experience with high-tension switches, opening charging current and load current, type of switch used.
8. Supply and control of charging current; is there synchronous receiving apparatus on line? Minimum size of generator and transformer units. Actual number and capacity of generators connected to the system.
9. Experience with outlets and terminals.
10. Experience with electrolytic and other high-tension arresters.
11. Experience with wind and sleet.

12. Experience with corona effects.
13. Experience with telephone communication and disturbance thereof.
14. Regulation.
15. Notes on towers, type, foundations, corrosion, etc.
16. How long has your system been operating at its present voltage?

I will take a few minutes to bring out some of the matters which appear to be of greatest interest in these reports, and make some comments based on other data. The general conclusion from these communications seems to be that the operation of 80,000 and 100,000 volt systems is satisfactory, not that operation is perfect in all cases, but that it is as satisfactory as operation at lower voltages.

It appears from the actual experiences of these plants that some of them are very close to the practical corona limit. Although no plant is actually handicapped in its operation by corona loss, there is good evidence that the voltage could not be greatly raised in these cases without a serious loss. We can now figure quite closely, however, in future plants on the results we may expect to get.

The feature of greatest interest is the operation and design of the constructions utilizing conductors hung on suspended insulators, conductors which are not mechanically rigidly fixed to the tower at the insulator, as with the pin type. You will all recognize that wires suspended in this manner become what may be called an elastic medium, that is, if there is a displacement in the wire at any tower, lengthwise or sideways, there is a returning force, which gives the effect of elasticity to the wires. The wires also have inertia. We have, then, elasticity and inertia, which are two quantities necessary to produce wave motion in any other mechanical device. Elasticity and inertia correspond to capacity and inductance, respectively, in transmission circuits.

We must then consider whether we are to experience in suspended conductors the sort of waves that we have found so troublesome in alternating circuits. Contrary to the first expectations of the engineers laying out suspended inductors, it turns out that trouble from these waves is to be apprehended in some cases. Some of our plants it is true have operated without a suspicion of trouble from mechanical waves, or mechanical motions of suspended conductors, but others have had difficulties, and it becomes of importance to outline and set forth and study, especially study, what are the conditions which will magnify the effects of mechanical waves and which will restrain them; also under what conditions they may be expected to be serious.

The laws of wave motion are well worked out in other directions. They may be briefly summarized by saying that a wave in any system, passing along on a uniform conductor, will tend to maintain its velocity and its amplitude, except for friction losses, until it strikes a reflecting point or dividing point. If it strikes a point of total reflection, that is, a rigid support, the wave will be returned in the reverse direction without losing strength by the reflection, but at the reflecting point there is a strain of double amplitude. With a recurrent source of waves and a reflecting point, there will be outcoming and returning waves, continually passing and re-passing, and where they pass they make stationary waves of double magnitude. Any particular suspended transmission conductor will have a definite period of swing. It swings like a pendulum. The velocity of these waves passing along will be relatively small.

When any point of the transmission line is thrown to one side and released, it produces a wave, or if it is thrown and suspended in the thrown position, the first act of throwing sends a wave along the line.

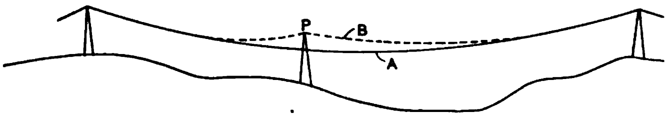
We must consider the matter of resonance. If for any cause we have a portion of a line swinging in a definite period, waves will be sent along the line, and if they are reflected and return to the original point, there may be a building up of the amplitude of the swing, as with a coil and condenser, or any other form of resonance. This condition is not likely to occur very often in suspended conductors. The independent, constant, source of energy which will maintain an exciting oscillation is likely to be absent. There might be a case where a line of heavy mass has been set swinging, and some line of lesser mass is connected to it, and in that case the stored energy in the swinging of the heavier mass may serve to send out a number of impulses into the branch circuit.

The thing that is likely to give difficulty in a suspended conductor in practical operation is the amplitude of the swing. If the swing is great enough, the conductor may come near enough to its support to permit a spark to pass. This has occurred in many instances in the last two or three years.

What is it that determines the amplitude of the swing? In the first place, the amplitude is restrained by the fact that as the wire swings to the side it is lifted and gravity tends to return it—if the suspending connection, the chain of insulators, is short, the length of the pendulum is short, and the conductor rapidly rises against gravity as it swings to the side, and the motion becomes a short motion. If the length of the supporting chain is long, however, the amplitude of the swing in feet will be greater. The amplitude in angular degrees may be the same, but the amplitude in feet will be greater. Thus, if instead of using the necessary four suspension insulators, in a certain case, a big margin of safety is demanded and seven or eight insulators are used, there would result a much wider amplitude of swing from the added disks with possibly disastrous results. In avoiding one trouble there would be danger of getting into another.

The length of the span will, of course, have a good deal to do with the amplitude, because the sag will be greater on long spans. The greater the sag, from the same reasoning, the greater will be the amplitude of the side swing in the center of the span; that is, if you take a small conductor, and to keep within its strength you use a very great sag, that sag exposes the line to a much wider amplitude of swing for a given disturbing impulse than a line with a lesser sag, and that is a fact to be borne carefully in mind in design work. This condition may in some cases be the limiting factor in determining the length of span with small wires.

There is another difficulty, which is shown in the accompanying sketch. Suppose the custom of the country would naturally give us a very long span as shown at *A* and suppose this is a little too long and a tower is placed at the point *P*, making the contour like the line *B*. If it were not for the central support, the tendency would be for the line to have a long swing at the point *P*, corresponding to the long span, because it has a large sag from the point of support. The effect of the tower at the point *P* is to relieve the long span of only a small portion of its weight. That is, if the line starts to swing at the center the weight supported by the tower at *P* has very little restraining force and the energy of the main portion of the span may carry the conductor up against the cross arm. This is a real danger and a matter which is very easily overlooked in laying out a line. Be very sure that there is a sufficient weight hanging on the intermediate insulator so that as the wire rises at this insulator due to the restraint of the short link, a sufficient weight is affected so that there is a big natural returning force.



There is another situation with suspended conductors which tends to give trouble. Assume a two-circuit line, with two overhead ground wires, the ground wires being steel and the line conductors aluminum. If, now, swinging is set up the steel and the aluminum will not swing in the same time period and clearance must be provided for the most unfavorable positions of various conductors.

Ordinarily in systems of suspended conductors there is very little swinging, and what occurs is apt to be very slow and steady, but there do come conditions in which the oscillations are not uniform and in which they are very severe. Wind is, of course, the chief exciting cause. It is not, however, necessarily the only exciting cause. There is the possible effect of current on the swinging of suspended conductors. With a short-circuit on two parallel wires of a circuit, the wires would tend to spread. If you have two loops, one on each side of a system, situated parallel, a short-circuit may tend to spread both wires and bring together the two inside wires or throw them against some other wire. The spreading force depends on the square of the current and is thus not very large with very high tension systems, except when of enormous capacity. To show what may occur in some instances, however, I will tell of one instance, which occurred on a 10,000-volt plant. There were two wires something like 20 or 30 in. apart with spans of about 150 feet. A short-circuit occurred somewhere on this line, a short way from the power house, and the force of repulsion, due to the short-circuit current, was sufficiently great to separate the wires under great tension, storing considerable energy in the elasticity of the wire. The action of a circuit breaker cutting off the current allowed these wires to come together until they wrapped around each other, and stayed permanently there, and a good deal of force was required to unwind them. A force of that magnitude, in most of our present suspended conductors, would make trouble, but, of course, on 50,000 or 100,000-volt systems, no such currents are obtainable as are met with near a large power house on a 10,000-volt circuit.

There is still another point which must be borne in mind in connection with suspended conductors. If we have a short span and a long span adjacent, and these two spans are set with the suspension link vertical on a warm day, in cold weather these spans are much tighter, and the stresses will no longer be equal on account of the difference in lengths of the spans. The wire will thus pull toward the shorter span as the temperature gets lower. If the spans are long this may pull the insulator far out of plumb, reducing somewhat the clearance, and throwing twisting strains on the cross arm. That is one of the objections to having a long span and a short span closely adjacent.

Mr. Hebgen, I believe, suggested providing for slipping of a clamp on the conductor, changing its location as between hot weather and cold weather. That might be a practicable thing to do.

There are many other interesting points brought out in these communications, which I will not take the time to mention. The maximum span is about 3,000 feet. Telephone lines are successfully operated through split inductance. The matter of tower foundations is called to our attention—do not sit in the office and prescribe a certain type of foundation for the whole line, to be used both in rock bottom and in the marshes. It is necessary to consider each foundation in the light of the character of the ground, and not only the character of the ground when inspected, but the condition it is likely to be in at the worst time of year.

The experience with lightning appears to be the same as with the ordinary high-tension lines. Some plants had trouble, and others did not. There are some cases where the hooks in cement types of insulators have pulled out. Sleet has been reported weighing 1.9 lb. per running foot on No. 0 wire. This means over two inches in diameter, outside of the wire, if symmetrically spaced. One report was brought in of six inches of sleet, but the manager of the plant suggested that "possibly that was an exaggeration."

A communication presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 29, 1911.

TRANSMISSION SYSTEM OF THE GREAT WESTERN POWER COMPANY

BY J. P. JOLLYMAN

The Great Western Power Company operates a long-distance, extra high-tension transmission system which extends from its hydroelectric plant at Big Bend, 16 miles north of Oroville, Cal., to Oakland, Cal. The nominal line voltage is 100,000, at 60 cycles. The length of the main line is 153.6 miles. A branch line 1.1 miles long taps the main line 136.5 miles from Big Bend. Both the main and branch lines are double circuit. Both are supported on steel towers.

1. The high-tension sides of all the transformers are delta connected. There is no connection to ground, except through the electrolytic lightning arresters.

2. One ground wire, supported on the apex of the towers, is used. This is grounded at every tower.

3. The standard span is 750 feet. 3/0, seven-strand copper cable is used on the main line, No. 6 copper on the branch. Each circuit is completely transposed about every 10 miles. The arrangement of the conductors is shown in Fig. 1.

The longest span with regular towers and cable is 1998 feet. The longest span is 2740 feet. This is across the San Joaquin River near Antioch. Special towers are used. The conductors clear high water 125 feet. The six conductors are in a horizontal plane, 15 feet apart.

4. We have never yet had any lightning whatever.

5. The measured value of the charging current is very nearly the same as the value determined by using the ordinary formula for charging current based on the capacity of two parallel con-

ductors. The distance between the conductors is taken as the average distance of 13.33 feet.

VALUES OF CHARGING CURRENT

Volts at Big Bend	Computed current	Observed	
		Current	Kv-a.
70,000	35.7	36.0	4360
80,000	40.8	41.5	5750
90,000	46	47	7330

6. Line insulators used. G. E. strain type, Locke strain type, Thomas strain and suspension types. Only three or four insulators have broken down during the past year. In no case have we any reason to believe that the insulator may not have been damaged mechanically before it failed electrically. We consider that the suspension insulator is a success.

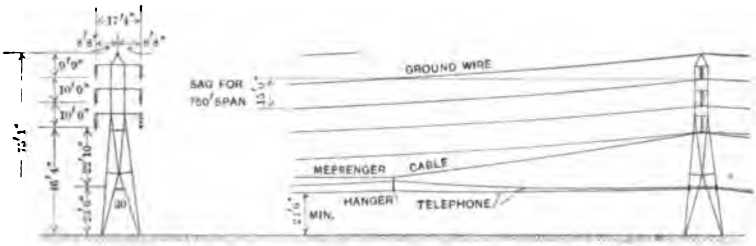


FIG. 1

We have had no trouble with swinging of conductors with suspension insulators.

7. G. E. type T and type K-10 oil switches are both used. We have had no difficulty in opening load, short-circuit or charging current. The indications are that these switches could be safely used on a considerably larger system than we now operate.

8. We have all kinds of load connected to our system, including synchronous motors. No attempt is made to neutralize the charging current by reducing the field on any synchronous load. Under ordinary conditions the power factor at the generating station is nearly unity.

For the convenience, speed and safety of operation, each generating unit should have a capacity of at least equal to the charging current of one line. In our case this means units of about 10,000 kw. We have at present four 10,000-kw. generators at Big Bend, each with a 10,000-kw., three-phase transformer. The station is planned to ultimately contain eight 10,000-kw. units.

9. *Transformer Terminals.* Oil-filled terminals were supplied. It has been found difficult to maintain them free from the possibility of leakage. They have been refilled with compounds designed to reduce the chance of leakage.

Oil Switch Terminals. We have used compound-filled terminals. Experience has shown that compounds for use in high-tension terminals must be very carefully tested in practical operation before their suitability can be definitely decided upon.

Outlets. The outlets from our buildings are primarily composed of plate glass windows five feet square, through the center of which the conductor passes. Various arrangements of extra insulation around the conductor have been tried.. Our experience seems to show that the best arrangement of plate glass windows is to use a bare conductor of large diameter and leave as large an opening around the conductor as climatic conditions permit. The window should be so protected that rain may not run over the surface of the glass.

10. We are using electrolytic lightning arresters at each end of the main line. Having had no lightning, we have had no experience with them as lightning arresters. We have operated one circuit without lightning arresters at either end for some time. Our experience has shown that they can not absorb much energy without being damaged and therefore their critical voltage must be higher than any dynamic voltage that may ever occur upon the system.

11. We have had no sleet, nor any trouble from wind since the line was finished.

12. There is no appreciable corona effect on the main line. Probably no very great increase in voltage would produce corona on the branch line of No. 6 copper wire. No part of the line is over 2,000 feet elevation above sea level.

13. The telephone line is strung on the towers. In regular spans it is supported by three hangers which are carried by a grounded messenger cable. Drainage coils consisting of the 2200-volt winding of a two-kw. transformer with the center point grounded are used at both ends and in the center of the line. These reduce the voltage to ground of the telephone line to a very low value, probably less than 100 volts. This obviates the necessity for especially high insulation and reduces the noise due

to unequal leakage. Highly insulated repeating coils are used where connections are made to telephones at which it is impossible to insure adequate insulation to ground of the person using the instrument. The telephone line is transposed at every tower where the towers are evenly spaced. Where the tower spacing is uneven the distance between transpositions is made as even as possible. The service obtained from this line is very satisfactory and is not more than momentarily interrupted by disturbances on the power line.

14. The only difficulty in maintaining satisfactory voltage regulation is at times when changes in load amounting to a large percentage of the total occur suddenly.

15. Steel towers whose general dimensions are given in Fig. 1 are used. Foundations suitable to the character of the ground must be used. Piles are used in very soft ground, cut off below the ground water level. The tower footing is attached to the top of the pile. All the tower footings are set in concrete. There have been no signs of corrosion in two years.

16. The system has been operated at 100,000 volts since November 1, 1909.

GENERAL COMMENTS

Our experience indicates that the following points should receive special attention when planning an extra high tension, long distance transmission system:

Generator capacity to handle the charging current.

Provision for the proper control of the system under all possible emergency conditions.

If delta operation is contemplated, insulation should be provided that will safely withstand the full delta voltage. The insulation should be of a character especially adapted to withstand transient voltage strains considerably in excess of the delta voltage.

A communication presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 29, 1911.

TRANSMISSION SYSTEM OF THE SOUTHERN POWER COMPANY

BY W. S. LEE

The following data are submitted in regard to the operation of the high-tension transmission lines of the Southern Power Company, of Charlotte, N. C.

1. 100,000 volt system. Transformers for stepping up delta-connected on 44,000-volt side and star-connected on 100,000-volt side, with neutral grounded. No resistance. 44,000-volt system is delta-delta non-grounded.

2. Overhead ground wires are used. One from peak to peak of the twin circuit steel towers upon which the two three-phase circuits are arranged in vertical planes, one on each side of each tower. Towers are not set in concrete, thus the ground wire is grounded at each tower, as each of the four legs is set about eight feet in the earth. Ground wire is ⅜-in. S. M. strand.

3. Conductors are 2/0 seven-strand copper and 2/0 seven-strand aluminum. Our standard span on 100,000-volt circuit is 600 ft., and we always have 20 ft. clearance at the middle point of sag. We use both copper and aluminum and make very little difference in sag. On 100,000-volt towers there are two circuits; three wires held vertically on each side of towers. Length of cross-arm 15 ft. 10 in. on old towers, 18 ft. 4 in. on new towers. Vertical distance between cross-arms 8 ft. 4 in. on old towers, 10 ft. 6 in. on new towers. We have many spans over 1000 ft. long and several 1500-ft. and 1600-ft. spans in both the copper and aluminum. We use strain towers at each end of these spans and keep the strain on the conductor the same as on the short spans.

4. Frequent lightning storms. The 100,000-volt lines stood

the lightning better the one season since they have been in service than the lower voltage lines.

From January 1, 1910 to date the Great Falls-Greenville line, 96½ miles long, 858 towers, had eight shut-downs due to lightning damaging either the 100,000-volt apparatus or lines. Time lost, 39 minutes.

From January 1, 1910 to date the Great Falls-Greensboro line, 161 miles long, of which 18 miles is a branch 100,000-volt pole line with 338 poles—main line with 1300 towers—had 14 shut-downs due to lightning damaging 100,000-volt apparatus or lines. Time lost, 59 minutes.

5. The only time the charging current was measured was when we started up the first 143 miles. We had 88,000 volts at the step-up transformers and we measured on the 2440-volt side of the transformers 1436 amperes, which is 6185 kv-a. The calculated current is 1450. This was on 2/0 copper at 60 cycles.

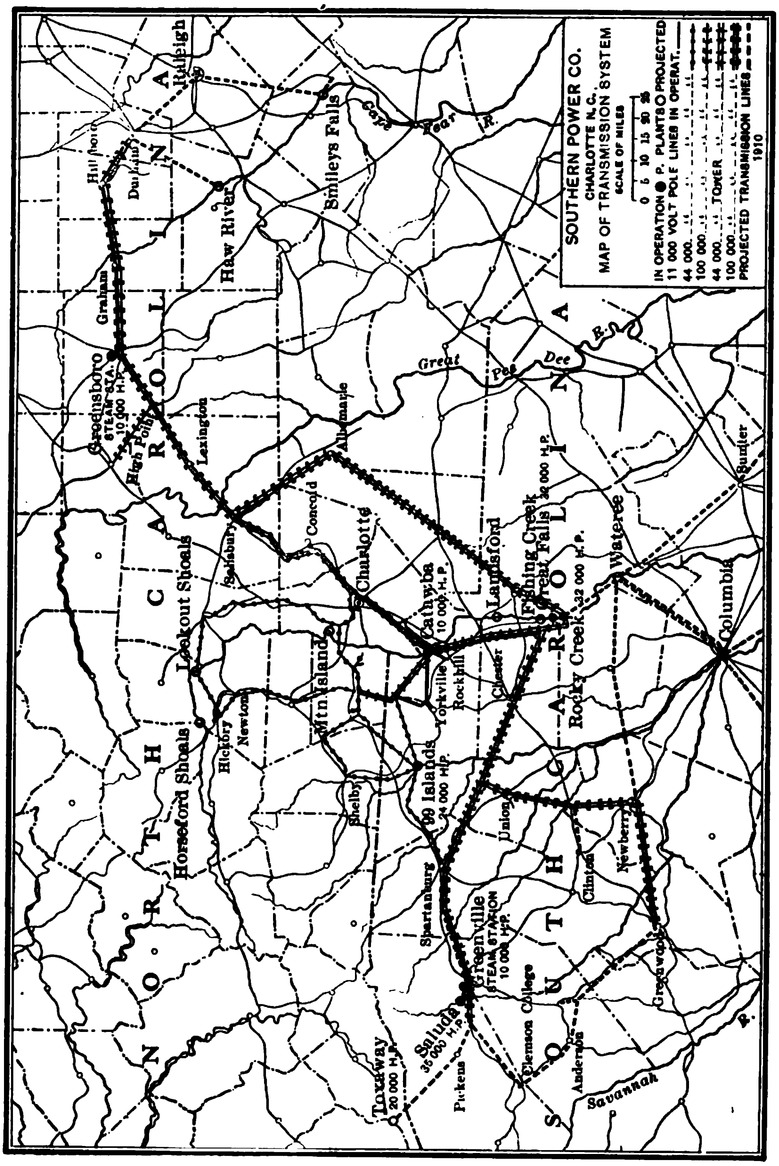
6. Some hooks have pulled out of insulators; these were the only defects we have found. We have never had a line put out of service due to failure of insulators, except during a sleet storm when three hooks pulled out. Insulators swing with the line in the wind. We have had no trouble from circuits on opposite sides of the tower swinging together and have never observed any dangerous or noticeable tendency to do so.

7. G. A. type switches with condenser terminals are used. There has been no more trouble switching and handling the 100,000-volt line than we had with our 44,000-volt and 10,000-volt lines. The same methods of operation, repair, control and distribution used on the 100,000-volt line as on other lines. There are 13 100,000-volt substations, in most of which transformation is made direct from 100,000 volts to 2300 volts by a bank of three 1000-kw. transformers, connected delta-delta.

8. There is practically no synchronous receiving apparatus on the lines. The minimum size of transformers is 1,000 kv-a. The minimum size of generators is 750 kw. in the Catawba plant, which is often in parallel with other stations, all of which supply the 100,000-volt transformers from the 44,000-volt system. The smallest generators ever used exclusively on 100,000-volt lines are 3,000 kw. and never less than three for the two lines.

22............3,000-kw. generators
4............ 900 " "
4............ 750 " "

are connected to the 44,000-volt system from which the 100,000-volt system is derived through six 4000-kv-a. transformers.


SOUTHERN POWER CO.
CHARLOTTE N.C.
MAP OF TRANSMISSION SYSTEM
SCALE OF MILES
0 5 10 15 20 25
IN OPERATION ● P. PLANTS ○ PROJECTED
11 000 VOLT POLE LINES IN OPERAT.
44 000 " " " "
100 000 " " " "
44 000 " TOWER " "
100 000 " " " "
PROJECTED TRANSMISSION LINES
1910
NORTH CAROLINA
SOUTH CAROLINA
Raleigh
Hillsboro
Durham
Haw River
Smilleys Falls
Cape Fear R.
Graham
Greensboro
STEAM STA.
10 000 H.P.
High Point
Lexington
Albemarle
Great Pee Dee R.
Salisbury
Concord
Charlotte
Lookout Shoals
Horseford Shoals
Hickory
Newton
Mtn Island
Catawba
10 000 H.P.
Yorkville
Rockhill
Chester
Lansford
Fishing Creek
Great Falls 32 000 H.P.
Rocky Creek 32 000 H.P.
Wateree
Sumter
Columbia
Shelby
99 Islands
4 000 H.P.
Union
Spartanburg
Greenville
STEAM STATION
10 000 H.P.
Clinton
Newberry
Greenwood
Saluda
36 000 H.P.
Pickens
Toxaway
20 000 H.P.
Clemson College
Anderson
Savannah R.

9. Porcelain bushings set into 5-ft. square slate slabs are used as outlets. Only one has broken down, and this was caused by bushing being cracked. There has been very little trouble with terminals except in a few damp places.

10. Two sets of 100,000-volt electrolytic arresters, each consisting of four units, are in successful and satisfactory operation.

11. Wind has caused no trouble whatever, and outside of three hooks pulling out soon after the line was put up, sleet has caused no trouble.

12. Have no corona effects except that in one case a very high resistance ground through a line entrance bushing caused the small No. 2 copper equivalent aluminum to glow.

13. We have had more telephone disturbance than with any other line.

14. In regard to the matter of voltage regulation, there is no distinction between the 100,000-volt lines and the 44,000-volt lines.

15. The towers which are set in earth have been erected about 18 months and have given no trouble because of corrosion or through unstable anchorages.

16. We have been operating at 100,000 volts since October 24, 1909.

A communication presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 29, 1911.

TRANSMISSION SYSTEM OF THE GREAT FALLS POWER COMPANY

BY M. HEBGEN

The Great Falls Power Company is at present operating a total of 282 miles of transmission line at 100,000 volts. The principal transmission is from Great Falls to Butte, Montana, a distance of 130 miles. Over this distance two separate, single-circuit, tower lines are used, a single line being extended from Butte to Anaconda, a distance of 22 miles.

1. All transformers connected to the system are delta-connected, this form of connection being very satisfactory in every way.

2. Above the transmission wires are located two ground wires consisting of ⅜-in. seven-wire, Siemens Martin steel strand, galvanized. These ground wires are clamped to the steel towers and are thus grounded at every tower through the tower legs. The tower legs extend six feet into the earth, and terminate in flat steel feet, which act as ground plates. No additional ground is provided.

3. The standard span for level country is 600 ft. In hilly country, however, there is no regularity in the length of the spans, and spans of from 1000 to 2000 ft. are common, no special construction being employed except to side-guy the towers. The conductors are spaced 10 ft. 4 in., and all three conductors lie in a horizontal plane with no transpositions whatever. The conductor is No. 0 B. & S. gauge, six-wire, hard drawn copper strand with hemp center. The normal clearance of wires from the earth is 29 feet.

The longest span in the line is 3034 ft., and occurs at the crossing of the Missouri River. In this span ⅜-in., Siemens Martin steel strand is substituted for the copper conductors, and a spacing of 20 ft. between wires is employed, two conductors only being supported by each tower. Standard towers and insulators were used, the insulators being doubled.

4. During the summer severe lightning storms are frequent in the vicinity of the line, and several of these have occurred since the line was put in commission at 100,000 volts, but as yet no shut-downs have been occasioned by lightning and only mild discharges have taken place over the lightning arresters. It is believed that the overhead ground wires are largely responsible for this immunity from lightning.

5. The charging current of 130 miles of single line at 60 cycles and 100,000 volts at the generating end is 39 amperes per wire or 0.3 ampere per mile. This current was measured directly with a high-voltage ammeter, and checks closely with the calculated charging current.

6. The insulators used are of the suspension type and consist of six units, 10 in. in diameter, with the under side of each unit corrugated. The caps and pins are cemented to the porcelain with Portland cement. The insulators have an ultimate strength of approximately 10,000 lb. and will flash over, wet, at somewhat over 300,000 volts. Up to the present time there has not been a single insulator failure, either mechanical or electrical.

It is true that shortly after the line was erected an insulator was shot in two by a high powered rifle, the bullet cutting a deep groove in the forged steel pin, cutting in two the clevis ears and destroying the porcelain of one unit. This, however, could hardly be classed as a failure.

No difficulty has been experienced with the insulators swinging in the wind. With a 60-mile wind blowing at right angles to the line it is believed that the insulators will swing side-ways and stand at an angle of approximately 40 deg. with the vertical. The maximum deflection observed so far, however, has not been over 30 deg. A considerable deflection has been noticed in insulators which are located between a long span and a short span, due to changes in temperature. The tension in a short span varies more with changes in temperature than does the tension in a long span, consequently during cold weather the insulators deflect along the line toward the short span while in hot weather the deflection is in the opposite direction. This action deserves considerable attention, and to accommodate the different positions of the insulator at different times of the year a wire clamp should be employed with a long bearing surface on the wire and a relatively short distance between the wire and the hinged point which supports the clamp. This form of clamp is desirable to prevent a sharp bend in the wire when the insulator stands at either extreme of its swing.

7. The line is controlled at each end by 100,000-volt oil switches, solenoid operated, having a double vertical break. The switches are top-connected through oil filled bushings. These switches have been entirely satisfactory and have readily broken the short-circuit current supplied by the 21,000-kilowatt plant feeding the lines. To sectionalize the line out-doors, double air break switches are used of the three-pole, revolving arm type. The switch blades and jaws are mounted upon pedestal insulators made up of six insulator units in series very similar to the regular suspension insulators except that the cap of one unit is extended upward to form the pin of the next, thus making the insulators as a whole rigid. These switches have not as yet been used to break any charging current but from the standpoint of good insulation and mechanical strength and ease of operation they have been very satisfactory.

8. The charging current of the line is at present entirely supplied by the main generating plant, which has a total capacity of 21,000 kw., divided into six generating units of 3,500 kw. each.

The output of two generating units is utilized near the generating plant, consequently there are at present installed only four banks of 100,000-volt step-up transformers, each bank having a capacity of 3,600 kw. The charging current of one line is well above the normal rated capacity of one generating unit. It has been found, however, that one generating unit and bank of transformers can readily supply the charging current of one line for a short time.

The plant is now fully loaded with an induction motor load and the lagging current taken by the induction motors so nearly balances the charging current of the line that the power factor at the generating plant averages 99 per cent.

There are installed in Butte three 1200-h.p. synchronous motors direct-connected to air compressors. These are not yet connected to the line but will be in a short time, and it is expected that they will aid slightly in regulating the voltage at Butte.

9. The 100,000-volt outlets at the Rainbow Plant and at the

Butte substation, as well as the switching station at the middle of the line, are through the roofs of the buildings. Oil-filled porcelain bushings are used, and these have proved entirely satisfactory. All transformer bushings and switch bushings are oil-filled and as yet have caused no trouble.

10. Electrolytic lightning arresters are used at both ends of the line and in the middle, the latter being installed in the switching station. These arresters have discharged during lightning storms, and usually discharge when any high-tension switching is done.

The original pedestal insulators upon which the horn gaps for these arresters were mounted proved weak mechanically. Two insulators broke off at the bottom connection without apparent cause. These have since been replaced with the type of insulator used on the line-sectionalizing switches and no further trouble has been experienced.

11. The line has successfully withstood winds estimated at 60 miles per hour, and has been through one sleet storm where sleet formed on the wires " to a diameter of six inches ", as reported by one patrolman. This, however, was doubtless an exaggeration.

12. When the line was first put in commission corona was plainly visible on every live part of the system. The corona gradually became less until after about three weeks no corona at all was visible except in a few places in the stations, such as the points of switch blades and other sharp projections. It is believed that this corona formed on small points or other roughnesses caused by the rough handling of the wire and that these have gradually worn off or burned off.

13. A private telephone line parallels the transmission line from Great Falls to Butte. This line is erected on wooden poles and runs midway between the two tower lines at a distance of 30 ft. from each. There was considerable static and much noise on this line when first put into operation. Reactance coils bridged across the line with their middle points grounded were installed at each end of the line and in the middle, and these reduced the induced voltage to about 50 volts between wires and ground, and did away almost altogether with the noise. The line now gives excellent service, and is easier to talk over than the lines of the Bell Company.

14. At no load there is a rise in voltage on the line of 3.5 per cent. With a load of 15,000 kw. at 85 per cent power factor on

the two lines the drop in voltage is 7 per cent. With all this load on one line the drop in voltage is 17.5 per cent. Taps are provided on the high-tension winding of the step-up and step-down transformers, and the step-down transformers are connected on a 10 per cent lower tap than are the step-up transformers, thus compensating for a drop of 10 per cent. When it is necessary to carry the full load on one line the generating plant raises its voltage 10.5 per cent to take care of the additional drop.

The switching station, which has been referred to, will provide a cross-over in the middle of the line so that it will be necessary to cut out only half of one line in case of accident. This will considerably improve the regulation of the line as a whole.

15. The towers have four legs and a single horizontal cross arm. The wires are supported at the two ends and in the middle of the cross arm and hang 40 ft. from the ground at the tower. The cross arm is supported at two points, each midway between the middle wire and the outside wire. The two ground wires are supported above the cross arm near its points of support.

The tower is composed entirely of angles and flats, all connections being by means of bolts. The cross arm is made of two 4-in. channels placed back to back and separated in the middle. The tower is erected on four angle iron stubs set six feet in the ground, the bottoms of these stubs terminating in flat feet made of short pieces of channel section having an area of 144 sq. in. each. All parts of the tower are galvanized.

16. The system has been operating at 100,000 volts for six months and nothing has developed as yet to indicate that the operation of a 100,000-volt system is any more difficult than the operation of a 50,000-volt system. On the other hand, it is believed that the extra high insulation provided for this voltage prevents many breakdowns, due to lightning and surges, which would occur on a system operating at 50,000 or 60,000 volts with insulators designed for a factor of safety of two or three, as is common with such systems.

After all, the normal voltage of a long transmission system has little to do with the voltage which must be insulated against. It is the abnormal voltages, caused by lightning, switching and accidental grounds, which really test the insulation of a system such as this.

A communication presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 29, 1911.

TRANSMISSION SYSTEM OF THE CENTRAL COLORADO POWER CO.

BY P. T. HANSCOM

1. The transformers at the Glenwood power house and at all substations are delta-connected. The transformers at the Boulder power house are Y-connected on account of better voltage ratio, and are operated with ungrounded neutral.

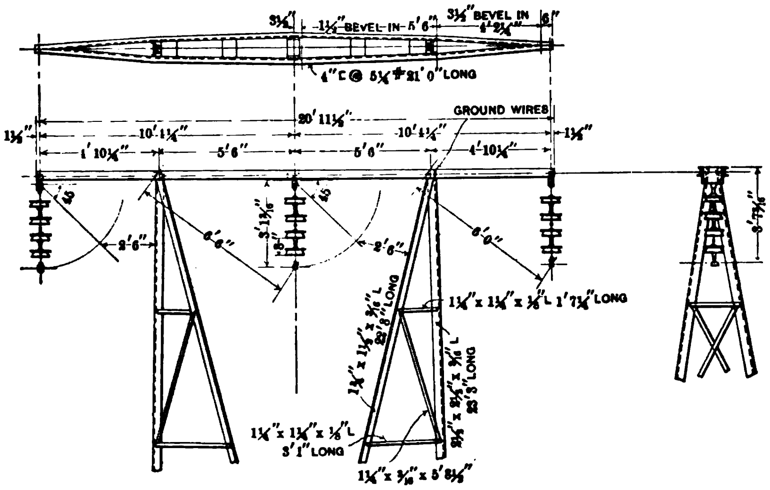


FIG. 1.

2. Two ground wires are used throughout on the Boulder line and for a distance of five miles from any station on the Glenwood line, also between Leadville and Dillon and across the Argentine and Haggerman Passes. The wires are grounded at each tower and spaced as shown in Fig. 1.

3. The standard span is 660 ft. and the average throughout the lines 750 ft.

The conductors on the Glenwood line are No. 0 B. & S. copper, composed of six strands with a hemp center, except between Leadville and Dillon where the conductors are No. 1 copper. The Boulder line is also No. 1 copper.

The conductors are drawn to an elastic limit of 35,000 lb. per sq. in. and ultimate strength of 60,000 lb. per sq. in.

There are a number of spans between 1,500 ft. and 2,000 ft. and one span of about 2,900 ft. with standard copper conductors.

At some of the railroad crossings and in a few spans, exposed to severe winds, ½-in. high-strength stranded steel cable is used for conductors.

4. The records of the operating department show 10 interruptions from lightning during the season of 1910 from 63 recorded lightning storms on the high-tension lines.

The records of the operating department are as follows:

Central Colorado Power Company

Record of lightning storms, June 1 to November 1, 1910

June 13. In the afternoon lightning storms were general over all Central Colorado Power Company's lines; no disturbance on Company's system during the day.

June 14. Lightning and rain east of Argentine Pass; high winds between Leadville and Dillon. Lightning lasted about six hours. A disturbance interrupted Dillon and Denver service once. On patrol next day broken insulator discs were found at the following towers near Leadville: Nos. 372, 378, 381 and 435. These were repaired later during a prearranged shutdown, service being maintained over them in the meantime. The transmission line was not dead during the day.

June 15. From 8:00 to 9:00 p.m. heavy lightning storm on Argentine Pass. One disturbance caused by a discharge to tower 752 from the clips of the disconnecting switch. This arc caused a shut-down on the transmission line of 3 min., being the only disturbance experienced during the storm.

June 16. Lightning and snow on Argentine Pass. No disturbance on line.

June 17. Lightning and rain on Argentine Pass. No disturbance on line.

June 18. Lightning and rain on Argentine Pass, at Idaho Springs and around Boulder. Generally stormy along Eastern Slope. No disturbance on system.

June 20. Lightning and rain on west side of Argentine Pass, between Dillon and Leadville and west of Hagerman Pass. No disturbance on transmission line. One slight disturbance on 13,000-volt feeder out of Dillon substation. One discharge on 13,000-volt circuit in the Boulder district about 5 p.m. interrupting one customer.

June 21.	Severe lightning around Barker and other parts of Boulder district. No disturbance on line.
June 22.	Lightning around Golden and Idaho Springs. No disturbance on system.
June 23.	Cloudy, with lightning around Boulder. No disturbance on system.
June 24.	Severe lightning between Denver and Boulder. No disturbance on system.
June 27.	Lightning around Denver, Golden and Boulder, also Idaho Springs. One disturbance which caused Denver Gas & Electric Company's switch in its West Side station to open, but did not affect service to Northern Colorado Power Co., or any other customer.
June 28.	Severe lightning around Boulder and Dillon all the afternoon up to 9 p.m. No disturbance on transmission system. One 13,000-volt feeder circuit out of Boulder and one 13,000-volt circuit out of Dillon substation were interrupted.
June 29.	Lightning in Boulder district from 4.00 to 8.00 p.m. One disturbance on transmission line, caused by trouble originating on a 13,000-volt feeder out of Boulder. This did not affect service over the transmission line, but the Denver Gas & Electric Company's switch opened, separating the system.
June 30.	Generally cloudy and stormy. Severe lightning around Boulder in the afternoon. No disturbance on transmission system. One disturbance on a 13,000-volt feeder from Boulder.
July 1.	Lightning severe around Idaho Springs; some lightning around Denver in the afternoon. No disturbance on system.
July 3.	Storm around Dillon from 6.30 to 9 p.m. One disturbance on line but not affecting delivery to substations. Denver Gas & Electric Company tripped off of their end of the feeder. One disturbance on a 13,000-volt circuit out of Dillon substation.
July 11.	Lightning around Idaho Springs about 7.00 p.m. No disturbance on system.
July 12.	Severe lightning around Denver. No disturbance on Central Company's system.
July 16.	Severe lightning storm on Argentine Pass and at Idaho Springs. No disturbance on system.
July 17.	Generally cloudy with some lightning. No disturbance on system.
July 18.	Generally cloudy with lightning over almost the entire line system. One disturbance due to the discharge which was observed to strike between towers 938 and 939. Transmission line was not dead, but regulation was poor for one-half min. The Denver Gas & Electric Company's switch tripped out on this occurrence and they were separated from Central Company's service for 15 min.

July 19. Severe lightning around Georgetown; cloudy over transmission line. Lightning broke insulators at towers 431, 437 and 604, all between Dillon and Leadville, causing two interruptions. After the first one service was resumed, but after the second one the broken insulators prevented recharge of the line until they were replaced.

July 20. Lightning storm general during the afternoon. One discharge observed to hit ground wires on towers 885 and 886 but no customers were interrupted except Denver Gas & Electric Company, whose switch opened automatically, separating the systems for 7 min., Leadville and Dillon substation instruments showing continuity of service and no disturbance.

July 21. General lightning storm in the afternoon. One disturbance on the 13,000-volt feeder out of Boulder which caused slight fluctuation on transmission line, but not affecting delivery to substations. During this disturbance the Denver Gas switch opened automatically, separating the systems for 6½ min.

July 27. Lightning around Dillon, Argentine and Leadville all day, around Shoshone and Boulder in the afternoon. One disturbance on transmission line caused momentary variation and separated Denver Gas from Central Company's system for about 6 min. Voltage low on transmission line for about 20 sec. During this storm other parts of the State were seriously affected.

July 28. Lightning around Dillon and Boulder in the morning. During the afternoon very heavy rain with severe lightning was general over the whole state. One roof bushing at Boulder was damaged but was automatically disconnected without interrupting service in Leadville, Dillon or Denver substations. The automatic switch in the Denver Gas Company's station opened, separating the system for 6 min. Four other slight disturbances, the first of which was caused by lightning breaking insulators at tower 389 near Leadville, the other three being caused by the remaining insulators furnishing insufficient insulation; which broke down at intervals until the line was disconnected by prearrangement to replace the damaged insulators. The Colorado Telephone Company's service was seriously interferred with, both locally and on the toll lines. A great deal of cable on their Denver circuits was damaged, and it was reported that about 600 telephones in Denver were out of service.

July 29. Heavy rains all day over the entire system. Lightning general. One disturbance; insulators on middle conductor at tower 1008½ damaged.

July 30. After 11.00 a.m. lightning was general around Barber, Dillon, Leadville and on the western slope. One disturbance at 2.00 p.m. Transmission line not dead.

July 31. Rain and lightning between Argentine Pass and Hagerman Pass. No disturbance.

Aug. 1. About noon small local storm just west of Denver caused no disturbances on Central Company's system.

Aug. 2. During the afternoon from 2.00 p.m. to 6.00 p.m. there was some lightning and slight showers along the transmission line. No disturbance.

Aug. 3. Partly cloudy during the day. No rain. Small lightning discharges in the afternoon. No disturbances.

Aug. 4. Weather cloudy. Showers in the afternoon. Lightning general over whole line. One severe discharge at 1.28 p.m. destroyed a roof bushing at the Denver substation and interrupted service to Denver and Boulder 17 min. No interruption at Dillon or Leadville. Heavy rains at Boulder in the afternoon, accompanied by severe lightning. Caused two disturbances on the 13,000-volt distribution line without disturbing the transmission system.

Aug. 5. Wind, rain and lightning in the afternoon generally. No disturbance on transmission system.

Aug. 6. Lightning in afternoon east of Leadville, north of Dillon and around Idaho Springs. No disturbance on transmission system.

Aug. 7. Rain near Idaho Springs. Lightning in the afternoon west of Leadville and on Argentine Pass. No disturbances on transmission system.

Aug. 8. Wind and rain storm from Idaho Springs to Denver in the afternoon, with some lightning. Very heavy lightning storm at Denver, in the evening, lasting about three hours. No disturbance.

Aug. 10. Partly cloudy. Storm around Idaho Springs and Boulder in the afternoon with rain at Boulder and some lightning. No disturbance.

Aug. 12. Weather cloudy. Rain and lightning between Leadville and Shoshone all day. Rain and lightning in Denver in the evening. No lightning disturbances on transmission system. Broken conductor between towers 126–127. Break caused by bullet from rifle.

Aug. 13. General storm with rain from Argentine east. No disturbance on transmission system.

Aug. 14. Severe lightning and rain at Idaho Springs from 5 to 6 p.m. One disturbance caused a discharge across a transformer bushing at the Dillon substation.

Aug. 15. Rain west of Dillon with some lightning. No disturbance on transmission line.

Aug. 16. Lightning around Idaho Springs beginning at 1.30 p.m. Lightning around Boulder at about 3 p.m. Insulators punctured on tower 472 caused shutdown of transmission from Leadville east.

Aug. 20. Weather cloudy over whole system. Rain and lightning at Waldorf in the afternoon.

Aug. 22.	Weather cloudy and stormy. Lightning around Idaho Springs east of Denver. No disturbance.
Aug. 23.	Stormy and rain from Shoshone to Leadville in the afternoon. Lightning from Shoshone to Argentine. One discharge near Shoshone caused the horn gap outside of Shoshone station to break down, shutting down Dillon and Leadville and causing the Denver Gas Company's switch to trip. The attendant at the Denver substation disconnected the Shoshone line quickly enough to prevent an interruption to the Boulder customers, and load continued to be carried by the Boulder Plant. Service to Denver substation from Boulder was thus momentarily interrupted.
Aug. 24.	Lightning around Denver in the afternoon. No disturbance on transmission line.
Sept. 1.	Lightning storm in the evening from Basalt to Dillon, from Idaho Springs to Denver and Boulder. No disturbance on transmission system. One disturbance on a 13,000-volt line from Boulder.
Sept. 2.	Lightning in the evening along line from Dillon to Denver. No disturbance.
Sept. 11.	Lightning around Denver in the afternoon. No disturbance on system.
Sept. 13.	Heavy lightning storm all along transmission line in afternoon. No disturbance on Central Company's system.
Sept. 14.	Some lightning along transmission line in the evening. No disturbance.
Sept. 15.	Lightning around Idaho Springs in the afternoon. No disturbance.
Sept. 18.	Severe lightning in afternoon and evening. Four disturbances on transmission system. One of these disturbances interrupted service.
Sept. 19.	Severe lightning storm during early evening. No disturbances.
Sept. 20.	Severe lightning storm with rain east of Argentine Pass. No disturbance.
Sept. 21.	Severe lightning storm in afternoon around Denver over Boulder and Shoshone. No disturbance.
Sept. 22.	Severe lightning storm in the afternoon broke insulator on tower 919. Interrupted no customers except Denver and Idaho Springs.
Sept. 25.	Very severe lightning storm over Denver in the afternoon. No disturbance.
Oct. 14.	Severe lightning storm in the afternoon and evening from Dillon west to Georgetown and north to Boulder. No disturbance on high-tension transmission system. One interruption of all distribution lines from Boulder; three additional interruptions on one distribution circuit from Boulder with additional interruption on each of two other distribution circuits from Boulder.

Oct. 15. Some lightning on Argentine Pass in the afternoon. No disturbance.

Oct. 16. Some lightning on Argentine Pass in the afternoon. No disturbance.

NOTE.— 10 interruptions due to lightning on high-tension lines.
63 recorded storms.
135 days.

5. Information on charging current is given in Part I of the 1911 TRANSACTIONS. See paper by Faccioli and communication by West, pages 337 and 77.

6. No electrical difficulties have occurred chargeable to insulators and mechanical difficulties of insulators have been limited to two or three defective connecting links between units. The normal strength of links is equal to the No. 0 conductor.

Some difficulties have been experienced from lack of stability of conductors in the more exposed sections of the line due to low horizontal stress and low vertical component at point of support. Such difficulties have occurred in local sections exposed to the more violent winds.

In some of the spans on the eastern slope where the line crosses the openings of canons, the wind at times comes whirling out of the canon at excessive velocity, intersecting other currents of air which produce wind eddies with the effect of lifting the entire span, shaking the conductors violently and mixing them up generally. To meet such conditions it has been necessary to dead-end the spans, increasing the horizontal stress, and in one span it has been found desirable to increase the spacing of conductors.

7. No difficulties have been experienced from high-tension switches or operation of same with and without load. High-tension switches of the oil-break type are installed at all substations, but none are installed at either of the two power stations.

8. Under present operating conditions the power factor at all power stations is about unity, hence no attempt has been made to control the charging current of the line.

The system is operated in multiple with the following power stations:

Leadville Light & Power Co. (steam).
United Hydro Electric Co. (hydraulic).
Denver Gas & Electric Co. (steam).
Northern Colorado Power Co. (steam).

There are two 5000-kw. generators installed in the Glenwood power house and two generators of the same capacity are installed at the Boulder power house. The combined waterwheel capacity at both plants is 39,000 h.p. The available generator capacity for continuous service is considerably in excess of the generator ratings given above.

9. Some difficulty was experienced with high-tension bushings and terminals during the first few months of operation, but these difficulties have been practically eliminated by changes in design and method of supports.

10. Aluminum cell lightning arresters are installed at Dillon, Denver and Boulder and their performance has been satisfactory. No difficulty has been experienced with the arresters.

11. Experience with wind conditions is given above. During spring and fall months sleet and snow has collected on wires at the lower elevations, and in one instance before the line was placed in commission the accumulation of sleet and snow on a No. 0 conductor was found to be 1.9 lb. per linear foot.

12. Data on corona are given in Part I of the 1911 TRANSACTIONS. See paper by Ryan, pp. 70-73; West, p. 77; Faccioli, p. 337; discussion by Faccioli, pp. 89-90; discussion by Ryan, p. 125.

13. The telephone communication may be considered good, as there is no disturbance or interference under normal operating conditions.

14. The voltage regulation at the Denver substation under normal operating conditions is within 1 per cent of a given value and the departure from this value for different periods of the day is about 3 per cent.

15. The towers are angle construction for horizontal circuit.

DISCUSSION ON " TRANSMISSION SYSTEMS OF THE GREAT WESTERN POWER CO.," "THE SOUTHERN POWER CO.," "THE GREAT FALLS POWER CO.," and "THE CENTRAL COLORADO POWER CO." CHICAGO, JUNE 29, 1911.

M. H. Collbohm: I note that some of the operating companies are using the ungrounded delta system. I would like to know what the considerations were which guided these companies in deciding for such a system. It would seem to me that the disturbances set up by an arcing ground in a system with exceptionally long lines operating under very high voltage would be very marked. It has been stated that the aluminum arrester would be able to take care of such disturbances, but I have had experience in an ungrounded delta system, where such a type of arrester, properly installed and pronounced in good order by the manufacturer, had been blown up three or four times, without lightning being the cause, simply by surging, and it seems to me that perhaps the grounded Y system might be preferable.

Paul M. Lincoln: My experience with the grounded neutral on high-tension lines has been of a secondary nature for the last five or six years—it has come second-hand, so that anything I may say on the matter of grounded neutral is necessarily information that came from some one else to me. However, making a general analysis of such information as I have been able to gather, I have formed the general opinion that the grounding of a neutral of high-tension transmission lines, preferably through a resistance, is a desirable practise. Some transmission lines, particularly those in the western states, have made it a practise to dead-ground their neutrals, and in some cases have gone so far as to use the ground as a conductor. Some single-phase customers of these transmission lines have been served by a single conductor, using the ground as a return. That is carrying the grounded neutral idea to its further point. However, in general, from such observation as I have been able to carry out on the transmission lines, I find that the grounding of the neutrals, although it tends to emphasize some difficulties, tends to reduce others, and in my opinion the reduction of the difficulties is greater than the increase, due to that method of operation.

L. C. Nicholson: Concerning the question of grounding the neutral of a high-voltage transmission system, I agree with Mr. Lincoln that grounding through a limiting resistance gives best results. A neutral completely grounded is open to several grave objections. A ground on one wire constitutes a short-circuit through the earth as a return. This usually interrupts the delivery of power, and causes destructive inductance in parallel telephone or telegraph circuits.

With the neutral insulated, arcing grounds composed of the charging current of the system set up dangerous surges which often cause failure at some other point and thus develop into a phase-to-phase short-circuit.

The advantages of inserting say 1,000 ohms in the neutral of a 60,000-volt line are several. First, the earth current is limited to a value which does not disturb the voltage of the system and therefore does not disturb the load. Furthermore, such an arc is not persistent and will usually cease after a few seconds without the necessity of cutting out the line, which is not true of an arc carrying large current as in the case of a neutral grounded without resistance. The chief advantage of resistance in the neutral is that it largely overcomes the fundamental objection to an ungrounded neutral, that is, surges. The fact that the arc carries a small power current, and not condenser current only, tends to make the arc a better and more continuous conductor, which prevents the rapid make and break effect of a true capacity arc, and in this way an arcing ground loses its power to produce destructive voltage surges.

An operating advantage of a resistance is that a line or section of line equipped with oil switches may be cut out automatically in case of a ground on one conductor. This is not true of an ungrounded neutral.

N. J. Neall: The question as to whether a line should be operated without grounded neutral or with grounded neutral is, of course, an old one that we have all battled with, and I suppose, judging from what has been said this morning, that neither method is objectionable, provided it does not interfere with continuity of service.

There is one point, however, which the inquiries sent out by the Committee did not cover, and I think it of great interest, and that is the question—How do these large operating companies take care of customers on branch lines? Do they permit the branch customers to be tied into the main line directly, or do they operate them through oil switches or fuse connections? There seems to be no information on that score, and it is, I think, one of some importance.

As far as lightning protection is concerned, I observe that Mr. Jollyman speaks of the electrolytic lightning arrester as having a definite limit of operation and recommends that the gaps shall presumably be set above the critical voltage of the transmission line. This is rather an extraordinary point of view, and does not coincide with the opinions expressed in the report made to the Committee of the National Electric Light Association, on Lightning Protection, of which I happen to be a member this year, and in which we had some very full replies. The general impression in reading the returns would be that the electrolytic lightning arrester had established itself positively in operation without very much trouble. Its operation, however, has created difficulties which are being recognized and corrected, and I can simply say to you as a matter of record, because it is no secret, that you must be very careful not to take your electrolytic lightning arrester for granted. The electrolytic lightning arrester and other forms of lightning protective arresters are not widely different from the class which we call fuses, and while they represent a certain outlay, and loss is undesirable, yet I see no reason to object, if the loss of the electrolytic or any other type of lightning arrester is simply, presumably, to save your main generating apparatus. It represents, however, a standard which the users of apparatus are already creating, namely, that as generating apparatus, transformers and other types, are becoming stronger against static disturbances, and they are having less failures, they are demanding that the protective apparatus which they were formerly willing to see fail, provided it saved the rest of the station, shall likewise increase in strength, all of which is piling up the insulation at all points, and they are thus expecting to have no interruptions whatsoever. I think that is a very difficult ideal to attain at the present time. The general improvement of arresters, plus the value of overhead line protection, has materially changed conditions, as they were reported to the Committee, and in this connection, I have a communication from Mr. Vaughan, which follows:

J. F. Vaughan (communicated): The chairman of your High-Tension Transmission Committee has asked for information on the results of the operation of the lightning protection devices adopted for the Taylor's Falls transmission line of The Minneapolis General Electric Company, following the experiments which were published by the writer in the TRANSACTIONS of the A. I. E. E., 1908, XXVII, I, p. 397.

The original line, about 40 miles long, was built in 1905, operating at from 50,000 to 55,000 volts. It consists of wooden pole construction, carrying a three-phase line of No. 4/0 B. & S. gage copper cable, arranged on a 72-in. upright equilateral triangle. The original protection consisted in "low equivalent" multigap arresters at each end of the line, a double gap selective resistance horn arrester at the power station, a single horn gap water resistance arrester at the substation, a triple gap selective resistance horn arrester at the middle of the line, and an electrolytic arrester without oil insulation at the substation. The line protection originally consisted of four experimental half-mile sections of different forms of overhead ground wire, with occasional lightning rods on the transmission poles and also a few rods on separate poles alongside the line. The most pronounced effect was produced by the type of overhead ground wire consisting of two No. 6 ground wires mounted 6 ft. apart 18 in. above the top transmission wire, which appeared to shield the line effectively, preventing spillover and breakdown of insulators. These troubles were frequent and serious on the unprotected portions of the line. The station protection was satisfactory.

The success with the overhead ground wire led the company to extend it year by year until the whole line was protected. Line failures and disturbances decreased in proportion as the ground wire was extended.

This year a duplicate line was built alongside the first line, using semi-flexible " A " frame steel towers, with the wires arranged in an inverted 72-in. equilateral triangle with a ⅜-in. steel cable ground wire at the apex of the tower. Electrolytic arresters were installed in connection with the new line at both stations and the power station horn gap arrester was replaced by an electrolytic arrester as well. The horn arrester at the middle of the line was found unnecessary and removed.

This season, up to the present writing, there have been 22 thunder storms without causing any interruption to service on either the pole line or the tower line. In this and other points the operation of the system has confirmed the conclusions drawn from the original experiments as reported in the above-mentioned Institute paper.

Hugh Pastoriza: Mr. Hebgen mentions the swinging of suspension insulators in the direction of the line due to temperature effects on adjacent spans of unequal length. This is an interesting point and one which we have investigated in some detail in connection with line design for the Telluride System.

As would be expected, the amount of deflection of insulators is greater the longer the spans concerned. With a given maximum span the greatest deflection occurs when the adjacent span is approximately half the size. The worst condition appears to be five or six equal long spans adjacent to a series of five or six spans half as long.

This swinging may be minimized by a proper proportioning of sags in the various spans. Ordinary practise is to stress all spans to the same amount (usually to the elastic limit) under maximum load and at fairly low temperature. This results in a very large unbalancing of tension between a long and a short span at low temperature when unloaded. If the tensions are equalized at average temperature and loading instead, the unbalancing of tension under extreme conditions is much diminished and the swinging of insulators considerably reduced. This method of stringing results in greater sags on the shorter spans than are given by the usual method, but as difficulties with ground clearance usually occur on the longest rather than the shorter spans, this is not objectionable.

In some cases computed where tensions were not thus equalized, insulators were found to deflect as much as 30 degrees. This emphasizes Mr. Hebgen's point regarding the desirable construction for wire clamps.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago. Ill., June 30, 1911

THE ECONOMICAL DESIGN OF DIRECT-CURRENT ELECTROMAGNETS

BY R. WIKANDER

The design of an electromagnet for a given duty can as a rule be varied considerably, and while it is comparatively easy to design a magnet that will serve a certain purpose, it requires careful consideration to find the most economical design in any given case.

In some cases the most economical design of a magnet will be the one for which the annual cost of the energy which it consumes added to the depreciation and the interest on the price will be a minimum. In other cases it is of importance that the magnet should be of compact design and of light weight in order to be transported conveniently, or it forms a part of some apparatus and should occupy a minimum of space.

In this paper we will limit our investigations to the design of the cheapest, the most compact or the lightest magnet which can perform a given duty.

The following fundamental formulas apply to all types of electromagnets.

The pull $\mathcal{P}$ of a magnet of the pole surface S sq. cm. and the magnetic induction $\mathcal{B}$ lines per sq. cm. is

$$\mathcal{P} = S \frac{\mathcal{B}^2}{8\pi} \text{ dynes,} \tag{1}$$

provided that the air gap between the plunger and the stop is very small in proportion to the diameter of the core.

The magnetic induction $\mathcal{B}$ is equal to the flux Φ in lines divided by the pole surface of the magnet or the section of the flux S in sq. cm.

$$\mathcal{B} = \frac{\Phi}{S} \text{ lines per sq. cm.} \qquad (2)$$

The flux Φ can be figured from the magnetomotive force $\mathcal{F}$ and the reluctance $\mathcal{R}$ of the magnetic circuit.

The magnetomotive force depends upon the number of ampere-turns of the electric circuit and can be expressed as follows:

$$\mathcal{F} = \frac{4\,\pi}{10} \times A \text{ gilbert} \qquad (3)$$

where A is the number of ampere turns of the electric circuit.

The reluctance $\mathcal{R}$ of the magnetic circuit is figured from the reluctances of its various series or parallel connected parts in the same way as we figure the resistance of an electric circuit from the resistances of its constituent parts.

The reluctance of an air gap is equal to its length, l, in cm., divided by its section in sq. cm.

$$\mathcal{R} = \frac{l}{S} \text{ oersteds} \qquad (4)$$

The reluctance of a magnetic metal is

$$\mathcal{R} = \frac{l}{\mu\,.\,S} \text{ oersteds} \qquad (5)$$

where μ is the permeability of the metal, generally iron or steel.

The magnetic flux Φ in maxwells is:

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} \qquad (6)$$

It will be seen from (1), (2), (3) and (6) that we can express the pull $\mathcal{P}$ in various ways, as follows:

$$\mathcal{P} = \frac{\mathcal{B}^2}{8\,\pi} S = \frac{\Phi^2}{8\,\pi\,S} \qquad (7)$$

These formulas seem very simple but it is often difficult or impossible to apply them.

Figs. 1 and 2 represent typical cases where the above formulas cannot be applied directly. *A* and *B* are iron cores and *C* is the magnetizing coil. We see that the section of the flux in the air is infinite and that the length of the different parts of same varies between a limited value and ∞ *. Magnets of such types are mostly used for instruments or relays, while our discussions will be limited to magnets for comparatively heavy duty. Magnets of this kind have as a rule a magnetic circuit of definite shape (see Fig. 3).

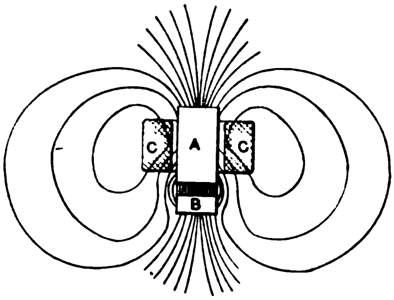


FIG. 1

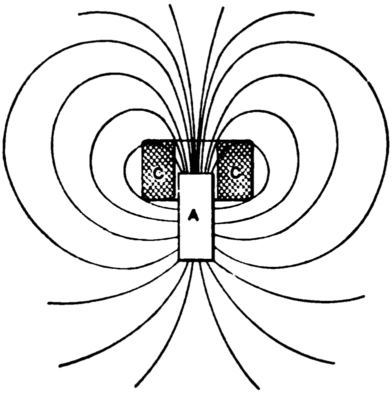


FIG. 2

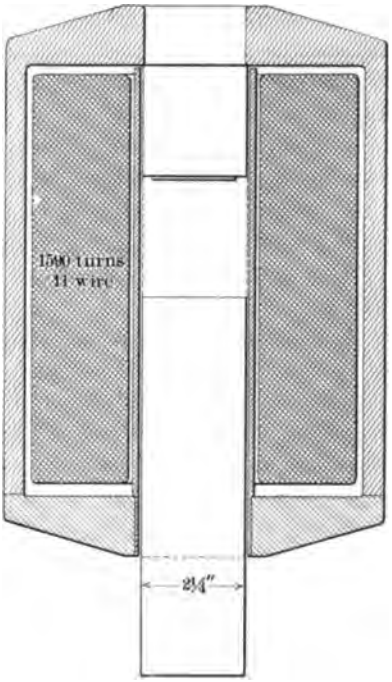


FIG. 3

The principles governing the design of electromagnets are entirely different for direct and alternating current. At present

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*In his book on "Solenoid Electromagnets and Electromagnetic Windings" Mr. C. R. Underhill has published several tests made on magnets of such types, and from a comparison of the conditions and results of those tests it will in many cases be possible to calculate the approximate pull that will be obtained with such magnets.

we will only treat the design of the former class, reserving the alternating-current magnets for a future presentation.

We distinguish between magnets which have to carry the exciting current continuously and those in which the current is admitted to the coil for only a few seconds or minutes in order to perform a certain mechanical operation.

We may further distinguish between magnets whose main function is to exert a certain *final* pull or pressure, and those which perform a certain amount of work, as when the product of the *initial* pull and the stroke is given, though it is often difficult to draw the line between these various classes.

According to the above distinctions we will discuss the following cases separately.

1. Continuously excited magnets for exertion of a certain pull or pressure or for a given *final* pull.
2. Intermittently excited magnets for the same purpose.
3. Continuously excited magnets for the performance of a certain amount of work or for a given product of *initial* pull and stroke.
4. Intermittently excited magnets for the same purpose.

In our analytical investigations regarding the most suitable dimensions of plunger type magnets we will make use of the following relations between the ordinate of a curve and the distance from the origin at which the tangent to the same point intersects the axis of the ordinates.

If b is a function of the variable g, the equation

$$b^x = K \,.\, g$$

expresses a curve which we can make tangent to any curve in the plane at any point whatsoever by assigning the proper values to x and K.

We will in the following call this curve "a parabola of the degree x".

Any tangent to the curve expressed by the above equation intersects the axis of the ordinates at a distance b_1 from origin.

$$b_1 = b - g\left(\frac{d\,b}{d\,g}\right)$$

or, in the present case, where

$$b = K^{\frac{1}{x}} \,.\, g^{\frac{1}{x}}$$

and

$$\frac{d\,b}{d\,g}=\frac{1}{x}\,.\,K^{\frac{1}{x}}\,.\,g^{\frac{1}{x}-1}$$

$$b_1=b-\frac{1}{x}\,.\,K^{\frac{1}{x}}\,.\,g^{\frac{1}{x}}$$

or

$$b_1=b\left(1-\frac{1}{x}\right)$$

If we know the value of x this latter equation enables us to find by trial the parts of any curve whatsoever which are tangent to a parabola of the degree x.

If we further have a function y which is proportional to the value of the expression $D^{f\,(x)}$ or

$$y=k'\,.\,D^{f\,(x)}$$

we know that y will *decrease* with *decreasing* values of D if the value of $f\,(x)$ is positive and with *increasing* values of D if the value of $f\,(x)$ is negative.

If, further, the values of D corresponding to positive values of $f\,(x)$ are all larger than any value of D corresponding to negative values of the same function we can conclude that y will become a minimum for the values of D and x which correspond to the equation

$$f\,(x)=0$$

In the following examples we will use these formulas in order to find the most suitable parts of the magnetization curves, (or in some cases the pull curves) for various types of magnets.

NOMENCLATURE

A = the number of ampere-turns of the winding.
a = the number of ampere-turns per inch of air gap.
B = the flux density in the air gap in maxwells per sq. in.
b = a function of g.
b_1 = the distance from origin at which the tangent to a certain curve intersects with the axis of the ordinate.

C = the cost of the electromagnet in cents.

C_1 = the cost of one cu. in. of the winding in cents.

C_2 = the cost of one cu. in. of the magnetic circuit.

D = the outside diameter of the winding in inches.

d = the diameter of the core (= inside diameter of winding) in inches.

$\delta = \frac{d}{D}$ = the ratio of the core diameter to the outside diameter of the winding.

E = the voltage of the current supply system.

e = the ratio of the volume of the total magnetic circuit to the core of the magnet.

Φ = magnetic flux in lines (maxwells).

G = the weight of the electromagnet in lb.

G_1 = the weight of one cubic inch of winding in lb.

G_2 = the weight of one cu. in. of the magnetic circuit.

G_3 = the weight of π cu. in. of winding.

g = a variable quantity.

I = the exciting current in amperes.

K, K_1, K_2, K_3, etc., constants.

$K_2 = e - 1$, if the function y expresses the volume of the magnet.

$= \frac{e \,.\, G_2 - G_1}{G_1}$ if the function y expresses the weight of the magnet.

$= \frac{e \,.\, C_2 - C_1}{C_1}$ if the function y expresses the cost of the magnet.

L = the length of the winding in inches.

l = the air-gap in inches.

N = the number of turns per sq. in. of the section of the winding.

n = the total number of turns of the winding.

P = the pull required in lb.

R = the resistance of π cu. in. of the winding.

r = the resistance of the winding in ohms.

S = section of magnet core in sq. cm.

S_c = the cooling surface of the winding in sq. in.

V = the volume of the electromagnet in cu. in.

W = the energy required for the excitation of the magnet in watts.

W_1 = the ratio of this energy to the cooling surface of the coil in watts per sq. in.

W_2 = the ratio of this energy to the volume of the coil in watts per cu. in. of the winding.

x, z = exponents which express the degree of a parabola.

y = a function which may express V, G or C depending upon the values of certain constants (K_2 and K_3).

1. CONTINUOUSLY EXCITED MAGNETS FOR A GIVEN FINAL PULL OR PRESSURE

The limiting condition for the compact and cheap design of this kind of electromagnet is that the coil must be able to carry the exciting current continuously without overheating.

The magnet must further be able to produce the required pressure or pull with a certain air gap which is required in order to prevent "freezing", to allow for the wear of contacts, or to meet other conditions depending upon the application of the magnet.

The pull P in pounds can be expressed by the formula

$$P = 1.09 \;.\; B^2 d^2 \;.\; 10^{-8} \ldots\ldots \text{in lb.} \qquad (8)$$

Transposing, we can write

$$B = \sqrt{\frac{P \;.\; 10^8}{1.09 \;.\; d^2}} \qquad (9)$$

or

$$B = \frac{9600}{d} \;.\; \sqrt{P} \ldots \text{in maxwells per sq. in.} \qquad (10)$$

or

$$d = 9600 \, \frac{\sqrt{P}}{B} \ldots \text{in inches} \qquad (11)$$

The air-gap is supposed to be so short that the influence of the "fringing" or the pull produced by the flux around the edges outside of the air gap can be neglected.

The induction B for a given magnet is a function of the ampere-turns of the coil. At low densities of the iron B increases approximately in proportion to the ampere-turns, while at higher densities the induction increases more slowly than the magnetizing current. We will first investigate which part of the magnetizing curve is the most economical one to work on. For any magnet of given dimensions we can assume that the relation of the flux density to the magnetizing ampere-turns is expressed by the equation

$$B^x = K \cdot A \qquad \text{or} \qquad A = \frac{1}{K} \cdot B^x \tag{12}$$

where x is a positive quantity increasing with the magnetization.

We have

$$A = I \cdot n = I \cdot N \cdot L \cdot \frac{D-d}{2} \tag{13}$$

or

$$I^2 = \frac{A^2 \cdot 4}{N^2 \cdot L^2 \cdot (D-d)^2} = \frac{B^{2x} \cdot 4}{K^2 \cdot N^2 \cdot L^2 (D-d)^2} \tag{14}$$

or if we substitute the value of B from equation (**10**) we can write

$$I^2 = \frac{9600^{2x} \cdot P^x \cdot 4}{K^2 \cdot d^{2x} \cdot N^2 \cdot L^2 (D-d)^2} \tag{15}$$

The resistance of the winding can be expressed as follows:

$$r = R \frac{D^2 - d^2}{4} \cdot L \tag{16}$$

The surface of the coil which is available for heat dissipation or cooling can be expressed as follows:

$$S_c = L (D+d) \pi \tag{17}$$

We neglect the end surfaces of the winding because they do not help to conduct any heat from the center or hottest part of the coil. We count, however, the inside surface of the winding as cooling because no heat is produced in the adjacent core and therefore the high thermal conductivity of the same helps to cool the winding.

We can express the energy of excitation as follows:

$$W = L (D+d) \pi \cdot W_1 \tag{18}$$

But it can also be expressed as a function of the current or the voltage and the resistance if the winding

$$W = I^2 \cdot r = \frac{E^2}{r} \tag{19}$$

Substituting the values of I^2 and r from (**16**) and (**17**) we can write

$$L\,(D+d)\,\pi\,W_1=\frac{9600^{2x}\,.\,P^x\,.\,R\,.\,(D^2-d^2)\,L}{K^2\,.\,d^{2x}\,.\,N^2\,.\,L^2\,(D-d)^2}\qquad(\mathbf{20})$$

or

$$L^2\,(D-d)\,d^{2x}=\frac{9600^{2x}\,.\,P^x}{K^2\,\pi\,W_1}\,.\,\frac{R}{N^2}\qquad(\mathbf{21})$$

For any given value of X corresponding to a certain part of the magnetizing curve, the quantities at the right-hand side of the equation (**21**) do not vary with L, D or d.

The ratio $\frac{R}{N^2}$ varies with the material of the winding and the space factor of same, but is comparatively constant for different sizes of wire.

We can therefore write, approximately,

$$L^2\,(D-d)\,d^{2x}=K_1\qquad(\mathbf{22})$$

or

$$L^2\,D^{1+2x}\,(1-\delta)\,\delta^{2x}=K_1\qquad(\mathbf{23})$$

The volume of the electromagnet is equal to the sum of the volume of winding and magnetic circuit and can be expressed as follows:

$$V=L\,(D^2-d^2)\,\frac{\pi}{4}+e\,.\,L\,\frac{d^2\,\pi}{4}\qquad(\mathbf{24})$$

or

$$V=L\,(D^2+(e-1)\,d^2)\,\frac{\pi}{4}\qquad(\mathbf{25})$$

or

$$V=L\,D^2\,(1+(e-1)\,\delta^2)\,\frac{\pi}{4}\qquad(\mathbf{26})$$

The weight of the electromagnet can be expressed

$$G=L\,(D^2-d^2)\,\frac{\pi}{4}\,.\,G_1+e\,L\,\frac{d^2\,\pi}{4}\,.\,G_2\qquad(\mathbf{27})$$

or

$$G=L\left(D^2+\frac{e\,G_2-G_1}{G_1}\,.\,d^2\right)\,G_1\,.\,\frac{\pi}{4}\qquad(\mathbf{28})$$

or

$$G=L\,D^2\left(1+\frac{e\,G_2-G_1}{G_1}\ .\ \delta^2\right)G_1\ .\ \frac{\pi}{4} \tag{29}$$

and the cost can be expressed as follows:

$$C=L\ \left(D^2+\frac{e\,C_2-C_1}{C_1}\,d^2\right)\ C^1\ .\ \frac{\pi}{4} \tag{30}$$

or

$$C=L\,D^2\left(1+\frac{e\,C_2-C_1}{C_1}\,\delta^2\right)C_1\ .\ \frac{\pi}{4} \tag{31}$$

The equations (**25**), (**26**), (**28**), (**29**), (**30**) and (**31**) are of the general form

$$y=L\ (D^2+K_2\,d^2)\ K_3 \tag{32}$$

or

$$y=L\,D^2\ (1+K_2\,\delta^2)\ K_3 \tag{33}$$

where y expresses any of the quantities V, G or C depending upon the values we attribute to the constants K-2 and K-3.

In order to keep the volume, weight or cost of the electromagnet as low as possible we should reduce the value of y to a minimum. We may vary the quantities L, D and d, but they must always satisfy the equations (**22**) and (**23**).

From the equation (**23**) in which δ is a positive quantity <1 we can deduce the following equation:

$$L\,D^2=K_1^{\frac{1}{2}}\ .\ (1-\delta)^{-\frac{1}{2}}\ .\ \delta^{-x}\ .\ D^{2-\frac{1+2x}{2}} \tag{34}$$

and this expression substituted in equation (**33**) gives the following expression for y:

$$y=K_1^{\frac{1}{2}}\ .\ (1-\delta)^{-\frac{1}{2}}\ .\ \delta^{-x}\ .\ D^{2-\frac{1+2x}{2}}\ (1+K_2\,\delta^2)\ K_3 \tag{35}$$

If the exponent of D is positive, which is the case for low values of x and low densities, the function y will decrease with a decrease in the diameter of the magnet core and winding. If this exponent is negative, which corresponds to high values of x and high densities, y will decrease with an increase in the diameter of the magnet core and winding. By varying the diameter, or, which amounts to the same, by varying the flux density, we can thus in either case obtain a decrease in the value of y and consequently of the volume, weight or cost of the magnet.

If the exponent of D is equal to 0 the value of y will not change with variations of the diameter. It follows that y is a minimum if we work on the part of the curve for which the value of x satisfies the equation

$$2-\frac{1+2x}{2}=0 \tag{36}$$

which gives

$$x=\frac{3}{2} \tag{37}$$

and consequently

$$B^{\frac{3}{2}}=K\ .\ A \tag{38}$$

The most economical value of the flux density is thus to be found on the part of the magnetization curve which is a parabola of the degree $\frac{3}{2}$.

The tangent at any point intersects the axis of the ordinate at a distance from the origin

$$b_1=\left(1-\frac{1}{x}\right)g$$

and in the present case where $x=\frac{3}{2}$ this distance will be

$$b_1=(1-\tfrac{2}{3})\ g=\tfrac{1}{3}g \tag{39}$$

The magnetization curve of the magnet is not identical with that of the iron alone, because it includes the air gaps as well, and its shape varies with the ratio of the air gap to the total length of the magnetic circuit.

In practical cases this ratio may vary from about 0.25 per cent (for a magnet of $\frac{1}{2}$-in. core diameter, $\frac{1}{64}$-in. total air gap and 6-in. length of the magnetic circuit) to about 1.5 per cent (for a magnet of 5-in. core diameter, $\frac{1}{4}$-in. total air gap and 17-in. length of the magnetic circuit.)

In Fig. 4, $A\ B$ is the magnetization curve for one inch of a high grade steel, the line $O\ C$ representing the magnetization of an air gap of 0.0025 in. By adding the ampere-turns for each value of B we find the magnetizing curve $O\ D$ of one inch of iron *and* 0.0025 in. of air, which has the same shape as the magnetizing curve of the whole magnetic circuit.

By trial we find the point E on this curve for which the ordinate $E\ F$ is three times larger than the distance $O\ G$. G is the point where the tangent at E intersects the axis $O\ Y$. The corresponding flux density is 81,000 lines per sq. in. or 12,600 lines per sq. cm. If the air gap is 1.5 per cent of the length of the magnetic circuit the line $O\ C$ represents the magnetization of the air gap corresponding to $\frac{1}{6}$ in. of iron. If we add to this line the ampere-turns required for the magnetization of $\frac{1}{6}$ in. of iron, we find the curve $O\ H$, which represents the magnetization of $\frac{1}{6}$ in. of iron and 0.0025 in. of air and is of the same shape (only the scale of the ampere-turns is different) as the magnetization curve of any magnet with 1.5 per cent air gap. We find by trial the point K on this curve, for which the tangent to the curve intersects the axis $O\ Y$ at the point L and $K\ M = 3 \times O\ L$. The corresponding flux density is consequently 96,000 lines per sq. in. or 14,900 lines per sq. cm.

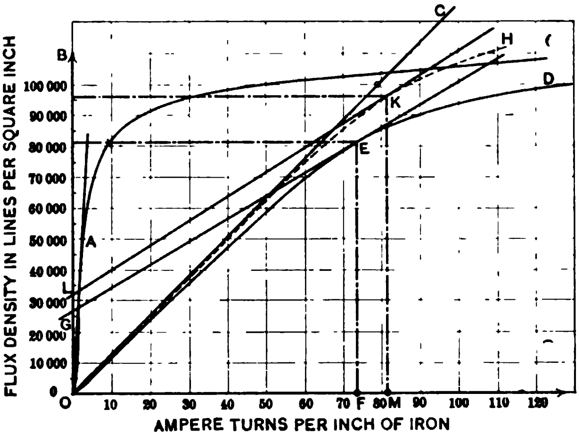

FIG. 4

The most economical flux density varies, consequently, between 81,000 lines per sq. in. for magnets with small air gaps and 96,000 lines for magnets with large air gaps. This density increases with the air gap. For average conditions the most economical flux density will be approximately 90,000 lines per sq. in. or 14,000 lines per sq. cm. for steel or wrought iron of high magnetic permeability. For cast iron a corresponding point on the magnetizing curve of that material should be chosen.

As soon as the flux density to be used has been decided, the core diameter can be determined from equation (**11**). There still remain to be determined the values of L and D and these can be found as follows:

Substitute the value $\frac{3}{2}$ for x in the equation (**22**), which then takes the form

$$L^2 (D-d)\, d^3 = K_1 \tag{40}$$

or

$$L = \sqrt{\frac{K_1}{(D-d)\, d^3}} \tag{41}$$

Substituting this value of L in equation (**32**) we have

$$y = K_3 (D^2 + K_2 d^2) \sqrt{\frac{K_1}{(D-d)\, d^3}} \tag{42}$$

Differentiating this expression with regard to D we find that y becomes a minimum for

$$D = d\, \frac{2 + \sqrt{4 + 3\, K_2}}{3} \tag{43}$$

or

$$\delta = \frac{d}{D} = \frac{3}{2 + \sqrt{4 + 3\, K_2}} \tag{44}$$

From equations (**23**) to (**28**) we can see that K-2 always is > -1 and δ is therefore always positive and < 1.

The curve Fig. 5 shows the variation of δ with regard to K-2.

From the values of d and δ we find the corresponding value of the outside diameter of the winding:

$$D = \frac{d}{\delta}$$

From the known value of the air gap we can calculate the corresponding number of ampere-turns required to carry the flux density B through the gap and the iron.

$$A = 0.314\ .\ B\ .\ l \tag{45}$$

This equation gives us the number of ampere-turns required to carry the flux through the air gaps. For the magnetization of the iron this value has to be increased from 7 per cent for magnets with comparatively large air gaps to 14 per cent for magnets with comparatively small gaps (see Fig. 4).

From equations (**14**), (**16**), (**18**) and (**19**) can be deduced

$$R \frac{D^2-d^2}{4} \cdot L \cdot \frac{A^2 \cdot 4}{N^2 L^2 \cdot (D-d)^2} = L\,(D+d)\,\pi \cdot W_1 \quad \textbf{(46)}$$

or

$$L = A \cdot \sqrt{\frac{R}{N^2} \cdot \frac{1}{(D-d)\,\pi\, W_1}} \quad \textbf{(47)}$$

wherein the value of $\frac{R}{N^2}$ depends upon the character of the wire and its insulation. It also varies slightly with the size of the wire. The wire table given on another page shows the variation of this quantity for cotton covered wire of the usual sizes.

For a first approximation we may choose

$$\frac{R}{N^2} = 3.8 \cdot 10^{-6}$$

The value of W_1 or the watts dissipated per square inch of the winding depends of course upon the conditions of the case. Under average conditions it may be assumed that 1 watt per square inch of the cooling surface will produce a rise of temperature of 100 deg. cent. If we allow a maximum rise of 40 deg. cent. we should choose $W_1 = 0.40$.

Substituting these constants in equation (**47**) we find the value of *L*.

Equation (**18**) gives the value of *W*.

Transposing equation (**19**) gives

$$r = \frac{E^2}{W} \quad \textbf{(48)}$$

and transposing equation (**16**) gives

$$R = \frac{4\,r}{(D^2-d^2)\,L} \quad \textbf{(49)}$$

From the wire table accompanying this paper we find the corresponding size of wire. From the same table we find the corresponding number of turns N per square inch of the winding section.*

The total number of turns is of course

$$n = N \cdot \frac{L\,(D-d)}{2} \qquad (50)$$

The dimensions of the most economical magnet for a given duty are thereby determined.

From the above deductions we can draw some interesting conclusions concerning the design of electromagnets of this class:

1. The most economical density is always the one for which the corresponding part of the magnetization curve is a parabola of the form:

$$B^{\frac{3}{2}} = K \cdot A$$

2. The absolute value of B depends upon the permeability of the iron part of the circuit and upon the length of the air gap, but is the same whether we design a magnet of minimum volume, minimum weight, or minimum cost.

3. The ratio of core diameter to outside diameter of the winding is independent of air gap, flux density and ampere-turns, and depends only upon the value of the constant K_2, which in its turn is dependent upon whether we wish to design a magnet of minimum volume, weight, or cost and upon the relation of the volume of the total magnetic circuit to the core inside of the coil.

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*This table gives the values of R and N which correspond to the theoretical winding space, obtained by assuming that the space occupied by each wire is equal to the square of its outside diameter. For machine wound coils it is always possible to meet this figure, but for hand wound coils we must allow from 7 to 18 per cent more winding space. The values of R and N decrease while the value of $\frac{R}{N^2}$ increases in the same proportion.

For coils of small diameter as used for telephones and instruments the increase in winding space amounts to about 7 to 9 per cent while for larger coils as used for electromagnetically operated switches, etc., the space actually required is about 16 to 18 per cent greater than the one theoretically necessary. In either case the excess space required is somewhat larger for finer wires.

2. INTERMITTENTLY EXCITED MAGNETS FOR EXERTION OF CERTAIN PULL OR PRESSURE

Magnets of this class are as a rule excited for a few moments only and at considerable intervals, allowing the winding to cool off after each operation.

The limiting condition for compact and cheap design of such magnets is that the thermal capacity of the winding must be sufficient for the absorption of the energy dissipated during the operation without overheating.

If we assume W_2 watts to be absorbed per cubic inch of the winding, the expression for the total energy, corresponding to equation (**18**), is

$$W = L\,\frac{D^2-d^2}{4}\;.\;\pi\;.\;W_2 \qquad \textbf{(51)}$$

This expression must be equal to the right hand side of equation (**20**) and we can write

$$L\,\frac{D^2-d^2}{4}\,\pi\,W_2 = \frac{9600^{2x}\;.\;P^x\;.\;R\;.\;(D^2-d^2)\;.\;L}{K^2\;.\;d^{2x}\;.\;N^2\;.\;L^2\,(D-d)^2} \qquad \textbf{(52)}$$

or, corresponding to equation (**22**),

$$L^2\,(D-d)^2\,d^{2x} = K_4 \qquad \textbf{(53)}$$

Equations (**24**) to (**33**) apply of course to this class of magnets, also.

If we substitute $D\delta$ for d in equation (**53**) we have

$$L^2\;D^{2x+2}\;(1-\delta)^2\;.\;\delta^{2x} = K_4$$

hence

$$L\,D^2 = K_4^{\frac{1}{2}}\;.\;(1-\delta)^{-1}\;.\;\delta^{-x}\;.\;D^{2-\frac{2+2x}{2}}$$

Substituting this value in equation (**33**) we have

$$y = K_4^{\frac{1}{2}}\;.\;(1-\delta)^{-1}\;.\;\delta^{-x}\;.\;D^{2-\frac{2+2x}{2}}\;.\;(1+K_2\,\delta^2)\;.\;K_3 \qquad \textbf{(54)}$$

The most economical saturation will be the one for which

$$2-\frac{2+2\;x}{2} = 0$$

or

$$x = 1 \text{ and } B = K\,.\,A$$

For this class of magnets we should thus work on the part of the magnetizing curve for which the induction increases proportionately to the ampere-turns, or at the point of the magnetization curve for which the tangent goes through the origin. It is interesting to note that this point is independent of the air gap and can consequently be found from the magnetization curve of the iron alone.

For the iron of which the magnetization curve is shown in Fig. 5 the most economical flux density for this kind of magnets will be found by tracing the tangent $O\,H$ which gives us the point A, corresponding to a flux density of 50,000 lines per sq. in. or about 7,750 gauss (=lines per sq. cm.).

Knowing the value of B, equation (**11**) gives us the corresponding core diameter for any pull required. If we insert the value of $x=1$ in equation (**53**) it takes the form

$$\left.\begin{aligned} &L^2\,(D-d)^2\,d^2=K_4 \\ \text{or}\qquad &L=\frac{\sqrt{K_4}}{(D-d)\,d} \end{aligned}\right\} \tag{55}$$

If we substitute this value of L in equation (**32**) it takes the form

$$y=K_3\ .\ \sqrt{K_4}\ .\ \frac{D^2+K_2\,d^2}{(D-d)\,d}$$

Differentiating this expression with regard to D we find that y becomes a minimum for

$$D=d\,(1+\sqrt{1+K_2})$$

or

$$\delta=\frac{d}{D}=\frac{1}{1+\sqrt{1+K_2}} \tag{56}$$

The dotted line in Fig. 5 shows the variation of δ with regard to K_2.

From the values of d and δ we find the corresponding value of the outside diameter of the winding:

$$D=\frac{d}{\delta} \tag{57}$$

The required number of ampere-turns is found from equation (**45**) in the same way as for magnets of class No. 1.

From equations (**14**), (**16**), (**51**) and (**19**) we can deduce the following expression for the length of the winding:

$$L = \frac{2A}{D-d} \cdot \sqrt{\frac{R}{N^2} \cdot \frac{1}{\pi W_2}} \qquad (58)$$

The value of $\frac{R}{N^2}$ can be found from the wire table.

The value of W_2 or the watts to be dissipated per cubic inch of the winding depends of course upon the quantity of metal contained in one cubic inch of the winding, also upon the material of the wire and the space occupied by the insulation.

The wire table gives the weight G_3 of copper contained in π cubic inches of winding for different sizes of cotton covered copper wires.

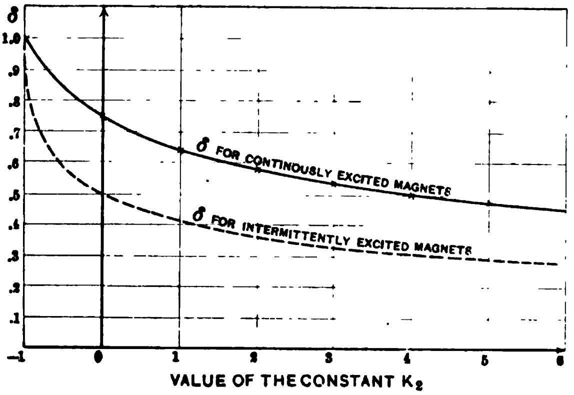

Fig. 5.—Economical ratio of core diameter to outside diameter of winding for direct-current plunger magnets

If we allow a rise of temperature of 100 deg. cent. after 100 seconds and considering that 1 watt-sec. = 0.000527 pound-calories and the specific heat of copper = 0.0951 we find the value of W_2 as follows:

$$\pi\ W_2 = G_3 \cdot \frac{100\ .\ .0951}{100\ .\ .000527} = 180\, G_3 \qquad (59)$$

or

$$W_2 = 57.5\ .\ G_3 \qquad (60)$$

G_3 varies from 0.352 for No. 35 wire to 0.70 for No. 10 wire. For average conditions we can assume $G_3 = 0.60$ as a first approximation.

Equation (51) gives us the value of W.

Equation (48) gives us the value of r.

Equation (49) gives us the value of R.

Equation (50) gives us the value of n.

The dimensions of the magnet are thereby determined.

Summing up the above investigations we can draw the following conclusion concerning the design of magnets of this class.

1. The most economical density is on the part of the magnetizing curve just below the " knee ".
2. The value of B depends only upon the magnetic properties of the iron.
3. The ratio of core diameter to outside diameter of winding depends only upon the value of the constant K_2 and is independent of air gap, flux density and ampere-turns.

3. Continuously Excited Magnets for the Performance of a Certain Amount of Work

For magnets of this class it is generally required that the product of the *initial* pull and the stroke of the magnet shall correspond to a certain amount of work.

In order to calculate the *initial* pull of a magnet we must consider the influence of the " fringing " which takes place as soon as the air gap reaches an appreciable value.

One effect of the fringing is an additional flux outside of the cylindrical space between the end surfaces of the magnet core, and this flux adds considerably to the pull of the magnet.

Another effect is that this flux increases the induction in the iron core to a value considerably in excess of the flux density in the air gap, and this effect produces indirectly a decrease of the pull.

Both these effects vary in different ways with the ampere-turns of the magnet, the material of the magnet core, the length of the air gap, the diameter of the core and the shape of the magnet. To determine analytically the pull of a magnet with considerable air gap from the magnetizing curve of the iron and the shape of the magnet, with any degree of accuracy, is rather complicated.

Any general formulas which might be derived to express the pull as a function of these data would be too complicated for the purpose of our present investigation and we have therefore resorted to another method.

For magnets of approximately the same type, the same quality of the iron, the same proportion between air gap and plunger diameter, and the same number of ampere-turns per inch of the air gap, it will be found that the flux density for corresponding points is the same for all sizes of magnets.

If, therefore, we take a magnet of any size and determine by test the relation between the pull per square inch of the whole surface and the excitation in ampere-turns per inch of air gap for different values of the ratio between air gap and pole diameter, we will obtain a set of curves which apply to all sizes of magnets of the type under consideration.

Fig. 6 represents a set of curves which has been obtained from tests of the magnet shown in Fig. 3.

In order to find the most economical value of the pull per square inch of pole surface we apply the following method, which is based upon the same principle as the one applied to the magnets of the classes 1 and 2.

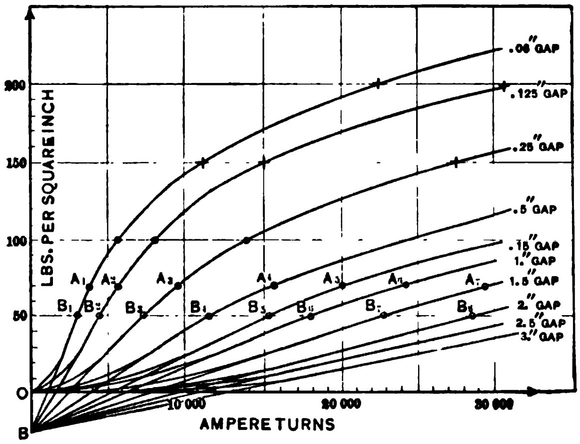

FIG. 6.—Pull curves for constant air-gap and variable current for plunger type magnets

We express the pull P as a function of the diameter of the core d, the ampere-turns per inch of air gap, a, and a quantity K_5 which may be different for different parts of the curve, but is a constant for small variations of P, d and a on any part of the curve.

$$P=K_5 \ . \ a^{\frac{1}{x}} \ . \ d^2 \tag{61}$$

The amount of work which the magnet should perform may be expressed as

$$Q=P \ . \ l \text{ or } P=\frac{Q}{l} \tag{62}$$

From equations (**61**) and (**62**) we derive

$$a = K_5^{-x} \ . \ Q^x \ . \ l^{-x} \ . \ d^{-2x} \tag{63}$$

And the number of ampere-turns

$$A = a \ . \ l = K_5^{-x} \ . \ Q^x \ . \ l^{1-x} \ . \ d^{-2x} \tag{64}$$

Considering that the ratio $\frac{l}{d}$ is constant, we may write

$$A = K_6 \ . \ d^{1-3x} \tag{65}$$

From equations (**13**) and (**65**) we can deduce

$$I = \frac{2\,A}{N \ . \ L\,(D-d)} = \frac{2\,K_6 \ . \ d^{1-3x}}{N \ . \ L\,(D-d)} \tag{66}$$

Inserting the value of W from equation (**18**), the value of I from (**66**) and the value of r from equation (**16**) in the equation (**19**), we have

$$L\,(D+d)\,\pi\,W_1 = \frac{4}{N^2\,L^2\,(D-d)^2} \cdot K_6^2 \,.\, d^{2-6x} \,.\, \frac{R\,(D^2-d^2)\,.\,L}{4} \tag{67}$$

or

$$L^2\,(D-d)\,d^{6x-2} = K_7 \tag{68}$$

or, introducing the value δ,

$$L^2\,D^{6x-1}\,(1-\delta)\,\delta^{6x-2} = K_7 \tag{69}$$

or

$$L\,D^2 = K_8 \ . \ D^{2-\frac{6x-}{2}} = K_8 \ . \ D^{\frac{5}{2}-3x} \tag{70}$$

This value of $L\,D^2$ substituted in equation (**33**) gives

$$y = K_9 \ . \ D^{\frac{5}{2}-3x} \tag{71}$$

The quantity y which may express the volume, weight or cost of the magnet becomes a minimum when

$$\frac{5}{2} - 3\,x = 0 \tag{72}$$

which gives

$$x = \frac{5}{6} \tag{73}$$

and consequently if we substitute this value of x in equation (61)

$$P=K_8\ .\ a^{\frac{6}{5}}\ .\ d^2 \tag{74}$$

The most economical values of the flux density are thus to be found on the parts of the curves, Fig. 6, which are parabolas of the degree $\frac{5}{6}$.

The tangent of any parabola of this degree intersects with the axis of the ordinate at a distance from the origin

$$b_1=\left(1-\frac{1}{\frac{5}{6}}\right)P=-\frac{1}{5}P \tag{75}$$

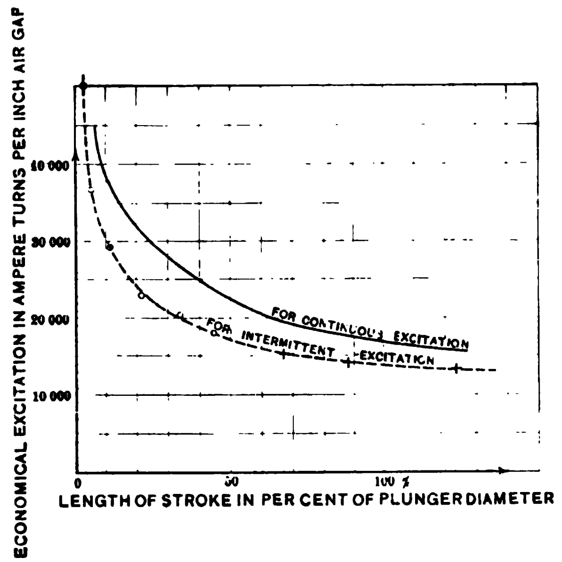

Fig. 7.—Economical excitation per inch of air-gap for plunger type magnets

From the curves, Fig. 6, we find by trying the most economical points on the P curve, the points for which the tangents intersect the negative part of the axis of the ordinates at a distance equal to $\frac{1}{5}P$.

We find that this condition is approximately fulfilled by the part of the P curves around the points $A1$, $A2$, $A3$, etc., and conclude that the function y will be a minimum for the values of pull and excitation corresponding to the points $A1$, $A2$, $A3$, etc.

It is interesting to note that within the range of our test the most economical pull per square inch is approximately the same for all values of $\frac{l}{d}$ and in this case about 70 lb. per sq. in.

The number of ampere-turns per inch of gap, varies, however, with the ratio $\frac{l}{d}$ as shown in Fig. 7, which represents the value of a plotted against $\frac{l}{d}$ for the most economical points of the various P curves.

In most practical cases the values of the initial pull P and the stroke l are given, and we can find the corresponding value of d from the equation

$$P = K_{10} \; . \; d^2 \tag{76}$$

or, in our case,

$$P = 70 \; . \; \frac{d^2 \; \pi}{4} = 55 \; d^2 \tag{77}$$

or

$$d = 0.135 \; \sqrt{P} \tag{78}$$

From d and l we find the ratio $\frac{l}{d}$ and from the curve Fig. 7 we find the corresponding number of ampere-turns per inch of air gap.

Substituting the value of $x = \frac{5}{6}$ in equation (68) it takes the form

$$L^2 \, (D - d) \, d^3 = K_{11} \tag{79}$$

or

$$L = \sqrt{K_{11}} \; . \; \frac{1}{\sqrt{(D-d) \, d^3}} \tag{80}$$

Inserting this value of L in equation (32) it takes the form

$$y = \sqrt{K_{11}} \; . \; \frac{D^2 + K_2 \, d^2}{\sqrt{(D-d) \, d^3}} \, . \, K_3 \tag{81}$$

Differentiating this expression with regard to D we find that y becomes a minimum for

$$D = d \cdot \frac{2+\sqrt{4+3\,K_2}}{3} \tag{82}$$

or

$$\delta = \frac{d}{D} = \frac{3}{2+\sqrt{4+3\,K_2}} \tag{83}$$

This formula is identical with equation (**44**) and consequently the solid line curve, Fig. 5, represents the value of δ corresponding to different values of K_2 for this class of magnets as well.

From the value of D we find

$$D = \frac{d}{\delta} \tag{84}$$

The ampere-turns required are

$$A = a \;.\; l \tag{64}$$

Equation (**47**) gives us the corresponding value of L.

Equation (**18**) gives us the corresponding value of W.

Equation (**48**) gives us the corresponding value of r.

Equation (**49**) gives us the corresponding value of R.

Equation (**50**) gives us the corresponding value of n.

The dimensions of the most economical magnet for the given pull and stroke are thereby determined.

In case the value of L obtained from equation (**47**) should be less than about twice the value of l it is recommended to increase L to the said amount and figure the corresponding value of D from equation (**47**), which can be transposed to read

$$D = \frac{R}{N^2} \cdot \frac{A^2}{L^2\,\pi\,W_1} + d \tag{85}$$

We then find the other dimensions as indicated above.

If only the product $Q = P\,l$ were given it may be of interest to see how the values of P and l should be chosen in order to obtain the most economical magnet for a given duty.

Let us assume that for the most favorable ratio of $\frac{l}{d}$ the ampere-turns per inch of air gap can be expressed as follows:

$$a = K_{12} \ . \ \left(\frac{l}{d}\right)^{z} \tag{86}$$

The number of ampere-turns is

$$A = a \ . \ l = K_{12} \ . \ l^{z+1} \ . \ d^{-z} \tag{87}$$

and the corresponding value of I^2 is found by substituting the value of A from equation (87) in equation (14)

$$I^2 = \frac{4 \, K_{12}^2 \ . \ l^{2z+2} \, d^{-2z}}{N^2 \, L^2 \, (D-d)^2} \tag{88}$$

Substituting the value of I^2 from (88), the value of r from (16), and the values of W from (18) in equation (19) we derive

$$L^2 \, (D-d) = l^{2z+2} \, d^{-2z} \ . \ K_{13} \tag{89}$$

or

$$L \, D^2 = K_{14} \ . \ l^{z+1} \ . \ d^{\frac{3}{2}-z} \tag{90}$$

Substituting this value in equation (33) it can be written

$$y = K_{15} \ . \ l^{z+1} \ . \ d^{\frac{3}{2}-z} \tag{91}$$

$$= K_{15} \, (l \, d^2)^{z+1} \ . \ d^{-\frac{1}{2}-3z} \tag{92}$$

From equations (62) and (76) we deduce

$$Q = K_{10} \ . \ l \, d^2 \tag{93}$$

or

$$l \, d^2 = Q \ . \ K_{10}^{-1} \tag{94}$$

Substituting the value of $l \, d^2$ from equation (94) in equation (92), and considering that Q is constant, we may write

$$y = K_{16} \ . \ d^{-\frac{1}{2}-3z} \tag{95}$$

y becomes a minimum for

$$\tfrac{1}{2} + 3 \, z = 0 \tag{96}$$

or

$$z = -\tfrac{1}{6} \tag{97}$$

This value of z corresponds to a point on the curve a for which the tangent intersects the positive side of the axis of the ordinate at a distance

$$b_1=\left(1-\left(-\frac{1}{6}\right)\right)\ a=\frac{7}{6}\,a \tag{99}$$

This part of the curve a lies, however, outside of the range of our test.

Analyzing the part of the curve a represented in Fig. 7 we find that the distance of said intersection from origin, in proportion to the value of a, is approximately

$$b_1=\frac{3}{2}\,a$$

and consequently

$$z=-\tfrac{1}{2} \tag{100}$$

which value substituted in equation (95) gives

$$Y=K_{16}\ .\ d \tag{101}$$

This means that within the range of our test the volume, weight or cost of any magnet of this kind increases approximately in proportion to the core diameter, or, which amounts to the same, in inverse proportion to the square root of the length of the stroke.

From the above investigations we can draw the following conclusions regarding the economical design of this class of magnets.

1. In order to obtain an economical magnet the stroke should be chosen as long as the conditions of the case permit.
2. The section of the core should be chosen so as to give a certain pull per square inch, in this case about 70 lb.
3. From the ratio of diameter of core to length of stroke we find the necessary ampere-turns per inch of stroke.
4. The ratio of outside diameter of coil to diameter of core depends only upon the value of the constant K_2 and is the same as for magnets of class No. 1.

4. INTERMITTENTLY EXCITED MAGNETS FOR THE PERFORMANCE OF A CERTAIN AMOUNT OF WORK

For magnets of this class the same method as described in the previous chapter can be used in order to find the most economical design.

Equations (**62**) to (**66**) apply to this class as well.

The equation corresponding to (**67**) takes the form

$$L\ \frac{D^2-d^2}{4}\ \pi W_2 = \frac{K_6^2\ .\ d^{2-6x}\ .\ (D^2-d^2)\ L}{L^2\ (D-d)^2}\ .\ \frac{R}{N^2} \tag{102}$$

or

$$L^2\ (D-d)^2\ d^{6x-2} = K_{16} \tag{103}$$

or, introducing the value of $\delta = \frac{d}{D}$,

$$L^2\ D^{6x}\ (1-\delta)^2\ .\ \delta^{6x-2} = K_{16} \tag{104}$$

or

$$L\ D^2 = K_{17}\ .\ D^{2-3x} \tag{105}$$

This value of $L\ D^2$ inserted in equation (**33**) gives us

$$y = K_{18}\ .\ D^{2-3x} \tag{106}$$

and y becomes a minimum for

$$2-3\,x=0 \tag{107}$$

or

$$x = \tfrac{2}{3}$$

Substituting this valuc of x in equation (**61**) we obtain

$$P = K_5\ .\ a^{\frac{3}{2}}\ .\ d^2 \tag{108}$$

The most economical values of P are to be found on the parts of the curves, Fig. 6, which are parabolas of the degree $\tfrac{2}{3}$.

The tangents of same intersect with the axis of the ordinate at a distance from origin:

$$b_1 = \left(1-\frac{3}{2}\right) P = -\frac{1}{2} P \tag{109}$$

From the curves, Fig. 6, we find by trial the most economical points on the corresponding P curves.

These points B, $B2$, $B3$, $B4$, etc., all correspond approximately to the value of P equal to 50 lb. per square inch.

The number of ampere-turns per inch of air gap for the most economical points of the various P curves vary as shown by the dotted line in Fig. 7.

If P and l are given we find the corresponding values of d from the equation

$$P=K_{19}\ .\ d^2 \tag{110}$$

or, in our case,

$$P=50\,\frac{d^2\,\pi}{4}=39.3\,d^2 \tag{111}$$

or

$$d=0.16\,\sqrt{P} \tag{112}$$

From d and l we find the ratio of $\frac{l}{d}$ and from the dotted line in Fig. 7 we find the corresponding number of ampere-turns per inch of stroke.

Substituting the value $x=\frac{2}{3}$ in equation (**103**) it takes the form

$$L^2\,(D-d)^2\,d^2=K_{16} \tag{113}$$

or

$$L=\frac{\sqrt{K_{16}}}{d\,(D-d)} \tag{114}$$

If we substitute this value of L in equation (**32**) it takes the form

$$y=\frac{\sqrt{K_{16}}}{d\,(D-d)}\cdot(D^2+K_2\,d^2)\,K_3 \tag{115}$$

Differentiating this expression with regard to D we find that y becomes a minimum for

$$D=d\,(1+\sqrt{1+K_2}) \tag{116}$$

or

$$\delta=\frac{d}{D}=\frac{1}{1+\sqrt{1+K_2}} \tag{117}$$

This formula is identical with equation (**56**) and consequently the dotted line in Fig. 5 will show the relation of δ to K_2 for this class of magnets as well.

Equation (**84**) gives us the value of D.

Equation (**64**) gives us the value of A.

Equation (**59**) gives us the value of $\pi\ W_2$.

Equation (**58**) gives us the value of L.

Equation (**51**) gives us the value of W.

Equation (**48**) gives us the value of r.

Equation (**49**) gives us the value of R.

Equation (**50**) gives us the value of n.

The dimensions of the magnet are thereby determined.

In case the value of L which we obtain from equation (**58**) should be less than about twice the value of l it is recommended to increase L to the said amounts and figure the corresponding value of D from the following equation, which can be derived from equation (**58**):

$$D = \frac{2 . A}{L} \sqrt{\frac{R}{N^2} \cdot \frac{1}{\pi W_2}} + d \qquad \textbf{(118)}$$

We then find the other dimensions as indicated above.

If only the product $Q = P . l$ were given it may be of interest to see how the values of P and l should be chosen so as to obtain the most economical magnet for a given duty.

We may express the values of a, A and I^2 as in equations (**86**), (**87**) and (**88**).

Substituting the values of I^2 from (**88**), r from (**16**) and W from (**51**) in equation (**19**) we derive

$$L^2 (D - d)^2 = l^{2z+2} d^{-2z} . K_{17} \qquad \textbf{(119)}$$

or

$$L D^2 = K_{18} . l^{z+1} d^{1-z} \qquad \textbf{(120)}$$

Inserting this value of $L D^2$ in equation (**33**) it can be written:

$$y = K_{19} . (l d^2)^{z+1} . d^{1-z-2z-2} \qquad \textbf{(121)}$$

or

$$y = K_{19} (l d^2)^{z+1} . d^{-1-3z} \qquad \textbf{(122)}$$

Substituting the value of $l d^2$ from equation (**94**) in (**122**) gives

$$y = K_{20} . d^{-1-3z} \qquad \textbf{(123)}$$

Substituting the value of z from equation (**100**), which approximately applies to the dotted curve as well (see Fig. 7), gives

$$y = K_{20} . d^{\frac{1}{2}} \qquad \textbf{(124)}$$

This equation shows that for magnets of this class, and within the range of our tests, the weight, cost or volume increases as the square root of the diameter.

Size of wire B. & S. No.	Diameter of bare wire in inches	Diameter of insulated wire in inches	Square of diameter of insulated wire in square inches	Resistance of π cubic inches of winding at 68 deg. fahr. R in ohms	Number of turns per sq. in. winding section N	Ratio $\frac{R}{N^2}$ to 10^{-6}	Weight of copper in π cubic inches of winding. G_s in lb.	Resistance per lb. of winding 68 deg. fahr. in ohms	Note
10	0.1019	0.1099	0.0121	0.02155	82.7	3.17	0.7	0.0308	Double cotton covered wire.
11	0.09074	0.09874	0.00975	0.03375	102.7	3.2	0.689	0.049	
12	0.08081	0.08881	0.00788	0.0527	127.0	3.26	0.675	0.0777	
13	0.07196	0.07996	0.0064	0.0818	156.0	3.36	0.665	0.123	
14	0.06408	0.07208	0.0052	0.127	192.3	3.45	0.651	0.195	
15	0.05707	0.06507	0.00423	0.1965	236.0	3.53	0.638	0.3085	
16	0.05082	0.05882	0.00346	0.303	289.0	3.63	0.621	0.488	
17	0.04526	0.05326	0.002835	0.467	354.0	3.73	0.608	0.768	
18	0.04030	0.04830	0.002335	0.713	428.0	3.88	0.588	1.213	
19	0.03589	0.04389	0.001925	1.0925	520.0	4.03	0.570	1.915	
20	0.03196	0.03596	0.001293	2.05	773.0	3.41	0.65	3.15	Single cotton covered wire
21	0.02846	0.03246	0.001053	3.18	953.0	3.50	0.64	4.97	
22	0.02535	0.02935	0.000862	4.895	1160.0	3.62	0.622	7.87	
23	0.02257	0.02657	0.000705	7.55	1420.0	3.74	0.606	12.45	
24	0.0201	0.0241	0.000580	11.56	1724.0	3.87	0.59	19.65	
25	0.0179	0.0219	0.000479	17.66	2090.0	4.06	0.572	30.9	
26	0.01594	0.01994	0.000397	26.86	2520.0	4.22	0.554	48.5	
27	0.0142	0.0182	0.000332	40.5	3010	4.45	0.530	76.5	
28	0.01264	0.01664	0.000277	61.2	3620.0	4.70	0.510	120.0	
29	0.01126	0.01526	0.000233	91.8	5400.0	4.97	0.482	190.5	
30	0.01003	0.01403	.0.000197	136.8	5080.0	5.29	0.464	294.5	
31	0.008928	0.012928	0.000167	203.5	6000.0	5.67	0.441	461.0	
32	0.00795	0.01195	0.000143	299.8	7000.0	6.12	0.418	717.0	
33	0.00708	0.01108	0.000123	439.5	8130.0	6.66	0.394	1115.0	
34	0.006305	0.010305	0.000106	643.0	9530.0	7.2	0.375	1715.0	
35	0.005615	0.009615	0.0000925	930	10800.0	7.95	0.352	2640.0	

We can sum up the results of these investigations of this class of magnets as follows:

1. If the stroke is not given it should be chosen as long as the conditions permit.

2. The section of the core should be chosen so as to give a certain pull, in the present case about 50 lb. per sq. in. of plunger section.

3. The required number of ampere-turns per square inch depends upon the ratio of diameter of core to length of stroke.

4. The ratio of outside diameter of the winding to the core diameter depends only upon the value of the constant K_2 and is the same as for magnets of class No. 2.

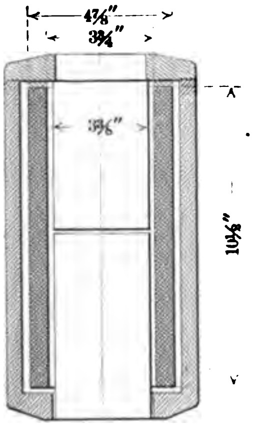

Fig. 8

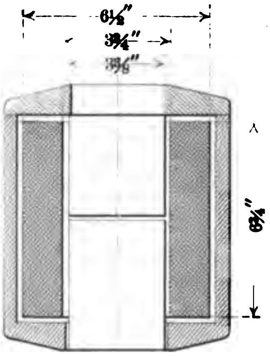

Fig. 9

Remarks Concerning the Application of the Preceding Theory

When we apply the preceding theory to practical cases we should bear in mind that it is based upon several approximations, which must be considered by the designer.

1. The core diameter is supposed to be equal to the inside diameter of the winding. The latter must, however, be somewhat bigger in order to allow for the necessary space for insulation and the outside diameter should be increased in the same proportion.

2. The length of the core is supposed to be equal to the length of the winding, but the latter must be somewhat shorter for the same reason.

3. The ratio of the volume of the total magnetic circuit to the volume of the core inside of the winding is supposed to be constant, but varies always somewhat with the ratio of diameter to length of winding, and considerably so for freakish designs as represented by Figs. 10 and 11.

4. For hand-wound coils the extra space required by the winding should be taken into consideration. Figs. 8, 9, 10 and 11 represent magnets with such coils figured on the basis of 16 per cent extra winding space.

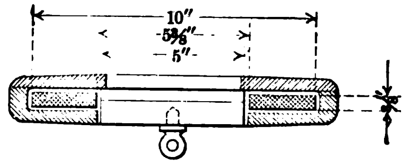

FIG. 10

5. The accompanying table gives the resistance of copper wires at 20 deg. cent., but the magnets should of course be figured so as to give the required pull or do the work at the maximum temperature for which they are designed.

6. The theory of the magnets for performance of a certain amount of work is based upon tests on a plunger type magnet of the most usual form and with comparatively limited stroke.

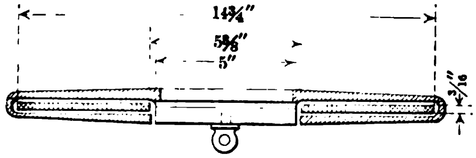

FIG. 11

For very different types this theory should be applied with discretion and for comparatively long strokes it will be preferable to reduce the pull per square inch of core section.

Figs. 8 and 9 represent the cheapest and the most compact magnets of the first class for a final pull of 1000 lb. and a maximum temperature rise of 50 deg. cent. for continuous excitation.

Figs. 10 and 11 represent the corresponding magnets of the second class for a final pull of 1000 lb. and a temperature rise of 100 deg. cent. in 100 seconds. The figures show the freak designs which we would obtain if we did not take the point 3 into consideration.

The preceding magnets were designed for hand wound coils while the following are supposed to have machine wound coils.

Figs. 12 and 13 represent the cheapest and the most compact magnets for the performance of 1000 lb.-inches at 1½″ stroke, continuous excitation and 50 deg. cent. maximum temperature rise. The designer would probably prefer shorter and less economical magnets or would choose a different type.

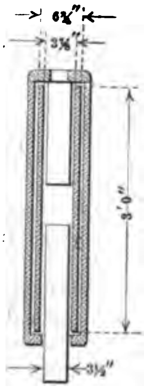

Fig. 12.—The most compact magnet

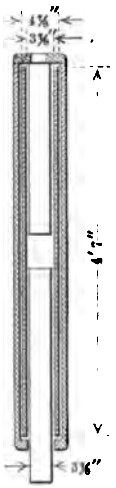

Fig. 13.—The cheapest magnet

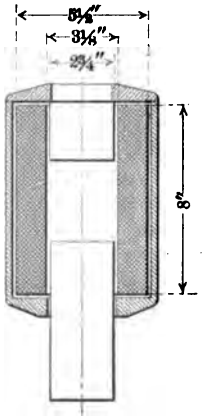

Fig. 14

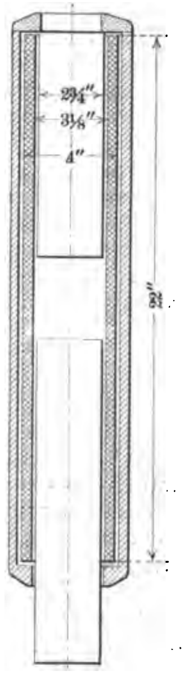

Fig. 15

Figs. 14 and 15 represent the corresponding magnets for the performance of the same amount of work at 3½″ stroke, with intermittent excitation and 100 deg. cent. temperature rise in 100 seconds. The designer would probably always choose the more compact of these two magnets, which weighs about 40 per cent less than the cheapest and costs only 9 per cent more.

DISCUSSION ON "THE ECONOMICAL DESIGN OF DIRECT-CURRENT ELECTROMAGNETS." CHICAGO, JUNE 30, 1911.

Frank F. Fowle: Mr. Wikander's paper is a very comprehensive discussion of the design of direct-current electromagnets, limited to the types for comparatively heavy duty. Electromagnets for light duty, mainly in closing one or more local contacts, are used in tremendous numbers in telephony, telegraphy and signaling. Their design is probably as important commercially as that of the heavy duty type.

The problem broadly stated is to design magnets for minimum annual cost—or minimum interest, depreciation, taxes, maintenance and cost of operating energy. In the case of constant-current circuits, or in circuits with an intermittent current of constant maximum value, the least energy is consumed when the resistance is a minimum. Again in the case of direct-current telegraphy the question of resistance becomes doubly important, because it affects the efficiency and cost of transmission, as shown in the writer's paper* on "Telegraph Transmission" presented at this Convention.

Mr. Wikander's expression (**7**) is identical with the writer's expression (**134**) in the paper just referred to above, or

$$P = \frac{\phi^2}{8 \pi S} \qquad \textbf{(1)}$$

In the case of ordinary electromagnets such as those employed in telegraph relays and generally similar apparatus (numerous examples in telephone and signaling practise), the magnetic circuit is nearly closed and the air gap is normally short in comparison with its breadth. The poles are usually extensions of the cores and the armature is a flat piece of equal breadth, spanning from pole to pole in the familiar manner.

In such cases the reluctance of the whole magnetic circuit is ordinarily composed in great part—over 90 per cent—of the air-gap reluctance. By neglecting the relatively small iron reluctance the writer shows that expression (**1**) can be put in the form

$$P = \left(\frac{16 \pi n^2 I^2}{800 l^2} \right) S \qquad \textbf{(2)}$$

when $n\,I$ represents the ampere-turns and l the length of air gap. This shows that for a constant value of ampere-turns the pull increases proportionately with the cross-section of the air gap or with the area of the poles.

This is easily seen from expression (**1**): doubling the pole area halves the reluctance and doubles the total flux, while the flux density remains constant; thus B^2 is the same as before, but S has been doubled and therefore the pull P has been doubled also.

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*Part II, p. 1683.

This result is not capable of being extended indefinitely, for when the pole area has been increased roughly ten times, the air-gap reluctance is probably no longer the principal part of the total reluctance and then further increase of area has a rapidly diminishing value.

It is essential to observe that the enlargement of one pole, but not both, is of little or no value in case the one not enlarged is of controlling importance in its relation to the total reluctance. Thus the introduction of a third air gap may destroy the whole principle here described.

In order to determine whether polar enlargement will be generally useful, the criterion is as follows. Calculate the reluctances of those gaps across which the working attractive force is made use of, and then calculate the reluctance of the remainder of the magnetic circuit; if the former is several or many times the latter, then polar enlargements will increase the pull materially.

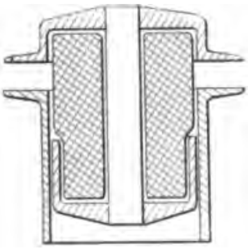

FIG. 1

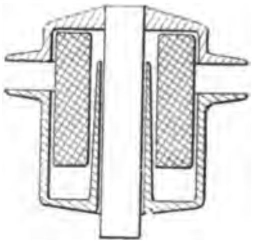

FIG. 2

Expression (**2**) can be written as follows:

$$P = \left(\frac{\pi}{50\, l^2}\right) (n\, I)^2\, S \qquad (\mathbf{3})$$

which shows that for a constant value of P, with increasing S, the ampere-turns may be diminished in the ratio of the square root of the increase in S. That is, if S is quadrupled, the ampere-turns may be halved.

Figs. 10 and 11 of the writer's paper illustrate two methods of enlarging the pole areas, with telegraph relays. The enlargement, it may be noted, is relative and refers to the area of the pole with respect to the cross-section of the core. In the type of plunger magnets considered by Mr. Wikander there are several ways of securing polar enlargements. Two of these methods are shown in Figs. 1 and 2 herewith.

Charles R. Moore: This paper certainly marks a distinct step in advance along the lines of electrical design. Many electromagnets have been designed and their performance accurately predetermined, but the designer seldom knew without carrying out several designs whether the one in hand was the best one for the purpose. The work oftentimes stopped without being carried to the point where there was little doubt about a given design being the most economical for the work intended.

Increased attention is now being paid to the design of traction electromagnets on account of their usefulness in control apparatus. In order to make this kind of apparatus satisfactory, such considerations as efficiency, first cost, and space required must receive the careful attention of the designer.

Mr. Wikander, by carefully analyzing the form of the equations resulting from the mathematical statement of the above considerations, has been able to derive formulas giving at once the best and most economical dimensions, for a given design to fulfil a certain set of requirements. As one would naturally expect, the cheapest magnet is not the most economical in the use of energy.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

ELECTROLYTIC CORROSION IN REINFORCED CONCRETE

BY C. EDWARD MAGNUSSON AND G. H. SMITH

While reinforced concrete was coming into general use as a structural material much space in the technical press was given to discussions and reports on the durability of the encased iron. The results from a large number of experiments gave fairly conclusive evidence that under ordinary conditions the iron is protected, and that even if it was rusty when placed in the concrete, it will be free from the oxide after remaining in the concrete for some time. The time test on the durability of practical structures is, of course, the final arbiter, and for each year the increasing data bear out the assumption that properly constructed concrete-steel structures will stand indefinitely.

With so much evidence tending to prove that iron encased in concrete will remain in good condition for any length of time, it was quite natural to infer that when failures did occur, the crack in the concrete preceded the corrosion of the iron, and that the presence of the iron in no way entered as a factor in causing the failure of the concrete. The complex chemical changes taking place for a considerable time in the hardening process of cement makes it difficult to secure any chemical basis on which this inference might be refuted. According to Le Chatelier the hardening process consists in a slow hydration and hydrolysis of the compounds formed by the fusion of the cement materials. The process being a change from the tri-calcium silicate ($3CaO.SiO_2$), by adding water, to a lower hydrated silicate, ($CaO.SiO_2 + 2.5\ H_2O$) and calcium hydroxide ($2Ca(OH)_2$), or $2(3CaO.SiO_2) + 3H_2O = 2(2CaO.SiO_2)\ H_2O + 2Ca\ (OH)_2$, and the hydration of the tri-calcium-aluminate, ($CaO.Al_2O_3 + Aq.$

= ($3CaO.Al_2O_3.12H_2O$). On the solid solution theory Richardson gives the formation as a tri-calcium-aluminate, ($3CaO.Al_2O_3$) dissolved in tri-calcium-silicate, ($3CaO.SiO_2$) in solution with an accessory compound consisting of di-calcium-aluminate, ($2CaO.Al_2O_3$) dissolved in di-calcium-silicate, ($2CaO.SiO_2$). In a recent paper[1] Dr. O. Schott reports on an investigation of the several compounds comprising Portland cement and in his conclusions he holds that "tri-calcium-silicate cannot be present in Portland cement", and further, that it "represents merely a fused mixture in the molecular proportions $3CaO$ and $1SiO_2$ which contains free lime in addition to a chemical compound". From the above it is evident that chemical analysis does not offer a simple basis for determining changes in the properties of cement.

Besides the complexity of the chemical changes, many variables of poorly defined range enter into the making of concrete, both in quality of material and manner of construction, so it becomes very difficult to prove that any portion of a structure where failure develops was constructed in a proper manner and from good material. Under these conditions it has become quite customary to assume that a crack in the concrete is in itself evidence of poor cement, careless construction, faulty design or some other similar factor.

Outside causes like vibrations from periodic impulses have been investigated in special cases, and for the past four years or more some attention has been given to the possible effects of electric currents. In the discussion[2] of Knudsen's paper a marked difference of opinion was manifest and it appeared that more experimental data would be necessary to determine the true nature of the phenomena. Accordingly, a series of experiments was begun in September, 1907, in the Electrical Engineering Laboratory of the University of Washington, for the purpose of studying the electrolytic effects on iron in reinforced concrete. The work may be grouped under three heads.

I. To determine if failure in reinforced concrete can be produced by the electric current.
II. To analyze the process involved.
III. To find means of protecting the concrete against the action of the electric currents.

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1. *Cement and Engineering News*, Vol. XX, Nov. and Dec., 1910, page 511.
2. TRANSACTIONS A. I. E. E., XXVI, 265.
3. TRANSACTIONS A. I. E. E., XXVI, 231.

It may be noted that these experiments take a long time; the mere preparation of a block before any observations are taken requires from thirty to eighty days, while the readings with a single block in circuit may continue for a few days or months, or even a year. The work is still incomplete but it seemed wise to make a progress report at this time in the hope that the data may prove helpful to others. Unless exception is noted, the following points will apply in these experiments:

(*a*) The cement used was of the " Washington " brand made at Concrete, Washington. This brand is of good commercial quality and is used extensively in the Puget Sound region. For lack of time only a few tensile strength tests were made, but these gave results well within the specifications for a standard Portland cement as given by the American Society for Testing Materials.

Table I gives data from one test of tensile strength.

TABLE I

Time	Samples			Average
	1	2	3	
24 hours	105	103	115	108 lb. per sq. in.
7 days	447	495	555	499 " " " "
28 "	730	726	695	717 " " " "
Specific gravity	3.09	3.19	3.11	3.13

(*b*) The iron was cut into eight-inch (20.3-cm.) lengths from commercial stock of ¾ in. (19 mm.) Johnson steel bar. A copper wire was soldered to one end for making electrical contact.

(*c*) The ratio of cement to sand was one to three for the concrete. The sand was of good commercial quality.

(*d*) Cylindrical blocks nine inches high and six inches in diameter were formed in moulds. The iron bar was placed along the axis and extended to within two and one-half inches of the lower end, thus leaving a layer of about two and one-half inches of concrete between the water in the tank and the iron bar. Each block was kept in a moist condition for at least thirty days before the electromotive force was applied. Three blocks were made at a time, two were used in the circuit and the third kept as a check.

I. WILL THE ELECTRIC CURRENT CAUSE FAILURE IN REINFORCED CONCRETE?

This has been investigated and papers published[4] by Knudsen, Toch, Crim, Langsdorf, Nicholas and others and all come to an affirmative conclusion. A number of experiments, similar to those referred to above, were made and with like results. Modifications in applied voltage, current density, and kinds of aqueous solutions did not alter the general result.

TABLE II

BLOCKS NO. 27 AND 31 IN SERIES ON 90 VOLTS, DIRECT CURRENT. UNPAINTED IRON BARS IN CONCRETE BLOCKS, IMMERSED IN FRESH WATER

Time from Start	Amperes	Volts	Remarks
Start	0.105	86.5	
30 sec.	0.105	86.5	
1 min.	0.105	86.5	
3 "	0.106	86.0	
5 "	0.107	86.0	
10 "	0.110	85.6	
15 "	0.114	85.6	
30 "	0.116	85.6	
1 hr.	0.116	85.7	
2½ "	0.112	86.5	
3½ "	0.118	99.0	
5½ "	0.118	95.0	
1 day	0.091	101.0	Noticed three small cracks from iron to circumference of block No. 31, 19 hours. Cracks widened in No. 31.
3 "	0.058	92.5	
4 "	0.043	89.0	
5 "	0.045	89.0	
5½ "	0.078	91.5	
6 "	0.079	93.0	
7 "	0.070	96.0	
8 "	0.072	91.2	
9 "	0.069	90.0	
10 "	0.081	92.3	
11 "	0.093	92.0	
12 "	0.097	87.6	
13 "	0.088	81.6	Wide crack in No. 31.
14 "	0.064	89.5	
15 "	0.066	92.1	
16 "	0.070	91.8	
17 "	0.061	92.6	
19 "	0.053	90.8	
21 "	0.042	90.6	
23 "	0.030	104.5	
26 "	0.026	84.0	
29 "			Experiment stopped.

No. 27 was not affected while No. 31 was readily pried apart with a screwdriver.

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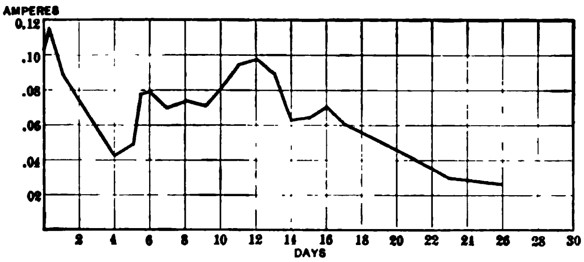
[4] See appended bibliography.

In Curve I and Table II is given a typical set of data, and Fig. 1 and Fig. 2 show the arrangement of the material under test, while Fig. 3 and Fig. 4 show the appearance of blocks at the end of the experiments. In most cases the e.m.f. was applied between the tank and the iron bar in the block. When two blocks were in series, with the positive to one of the iron bars and the negative to the other, wooden tanks were used.

To add more data on this point would be useless repetition, for in the papers already referred to it has been shown that an electric current passing from the iron through the concrete will cause corrosion where it leaves the iron, and, if the process be continued for a sufficient length of time, the surrounding concrete will crack.



Curve I

II. Analysis of the Processes Involved

This fact being established, it becomes important to determine the processes involved by which the current causes a failure in the concrete.

The following suggestions will be discussed:

(*a*) Temperature rise due to $R I^2$ losses.

(*b*) Hydrostatic pressure at the anode due to the current and to changes in solution density.

(*c*) Pressure caused by the generated gases.

(*d*) A chemical change in the cement, due to the current directly, destroying the cohesive strength of the concrete.

(*e*) During corrosion, the iron changes to an oxide or a salt; this involves an increase in volume, the compound taking more space than the iron from which it is formed. As the process continues the point may be reached when the stress becomes sufficient to rupture the surrounding concrete.

(*a*) In cases where the current is comparatively large and more heat is generated than can be dissipated without a great rise in temperature, this factor may be the cause of the failure. With sufficient heat generated to cause the water to change into steam, failure of the concrete will quickly follow. For example, in Table XII, No. 9, steam was given off and the block cracked in five hours. Such cases are, however, rare and readily noticed, and the flow of the current stopped. The large majority of cases deal with small currents, a few hundredths of an ampere, and the rise in temperature is slight. Hence, except in a few very special cases, the heat will readily be dissipated and the concrete will not be affected by this factor.

(*b*) While the hydrostatic pressure at the anode is sufficient to force a few drops of discolored water up around the edges of the iron on top of the block, the porous nature of the blocks makes it improbable that any considerable stress should come from this source. Whether the pressure is caused by the flow of the current radially from the iron, or by the increase in solution density through the formation of salts near the iron, it seems obvious that the liquid should readily escape through the pores of the concrete upwards along the iron bar. The drops of discolored water that appear on top of the block when action is in progress. show that this path is open. The water will rise to the top before, as well as long after, the crack appears. With the form of block used, the water was raised only a few inches and the pressure under these conditions should be negligible.

FIG. 1

(*c*) From the porous nature of the surrounding material it seems quite impossible that any large force should come from this source. Moreover, when two blocks are connected in series (Table II) the volume of hydrogen gas will be twice that of the oxygen, still no action was observed where the hydrogen was liberated, while the block receiving the oxygen cracked in nineteen hours.

(*d*) In order to determine what action, if any, was produced directly on the concrete by the electric current the resistance of cement and concrete was first measured under specified conditions· Only limited data were available on the resistance of cement and concrete. A conclusion from very limited experimental data, that[5] "in no sense can concrete be considered an insulator, and as shown, it is from all appearances just as good an electrolyte as any of the soils of the earth ", can hardly be considered final.

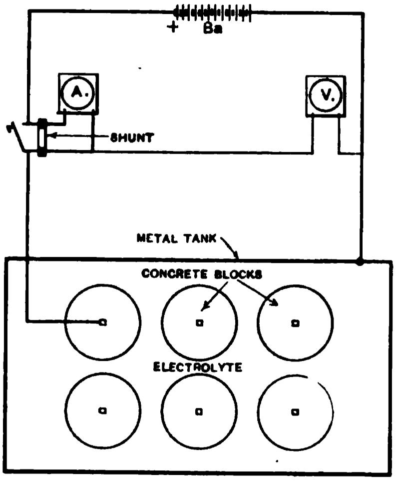


FIG. 2

In connection with Creighton's[6] work in lightning arresters the resistance of concrete and cement was investigated by Marvin. The observations deal particularly with the effect of high temperatures.

The tests[7] " tend to show the following conclusions: 'At moderate temperatures, the resistance depends simply on the

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[5] TRANSACTIONS A. I. E. E., 1907, XXVI, I, 245.

[6] TRANSACTIONS A. I. E. E., 1908, XXVII, I, 669.

[7] TRANSACTIONS A. I. E. E., 1908, XXVII, I, 732.

amount of moisture in the cement and becomes extremely high if the moisture is removed, either by long drying or by artificial heating. The addition of sand increases the resistance, acting apparently as an insulator distributed through it. When cement is heated, it at first increases enormously in resistance as the moisture is driven off, but at a red heat it again becomes as good a conductor as when cool and damp. With the same voltage per unit of length, a moderate voltage such as 600 volts will not heat the material above 100 deg. cent. so as to pass the interval of high resistance; but a higher voltage such as 8,000 volts can pass the interval and heat the resistance to incandescence.' "

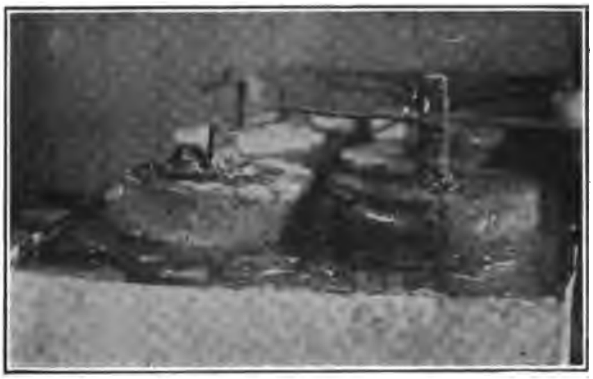

FIG. 3

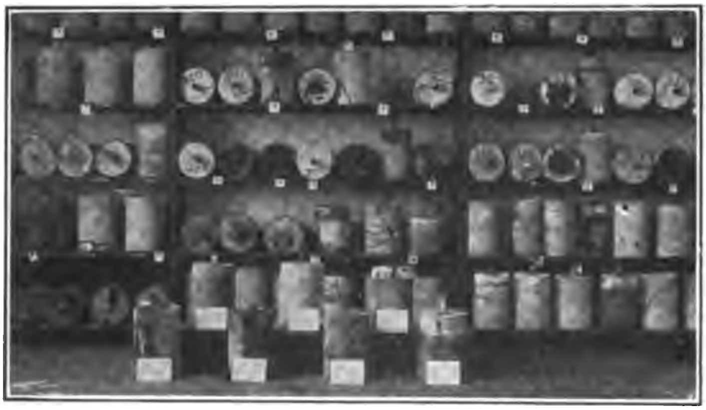

FIG. 4

Since conductivity measurements could not readily be made with the cylindrical blocks containing the iron bars, as used in the other experiments, eight sets of cubical blocks were made, using standard brass moulds from the cement laboratory. Each set consists of 24 cubes. Of these, 12 were of neat cement and the other 12 of concrete. Twelve moulds were available and 12 cubes were made at a time; three three-inch (7.6-cm.) cubes, six two-inch (5-cm.) cubes and three one-inch (2.5 cm.) cubes. In making the cubes, care was taken to secure uniform conditions. The consistency was adjusted to give, when tested by the Vicat needle, a reading of 10 on the scale. Numerous air bubbles appeared in the cement cubes, and probably expert moulders would have secured better results. After removing the cubes from the moulds they were kept in a moist condition for over twenty days, and then under water for ten days longer. After that, sets No. 1, 2, 3, 4, 7 and 8 were placed on a shelf in a steam heated room and sets No. 5 and 6 were placed on top of the steam radiators in the same room. This drying period extended over 40 days. The cubes were weighed before and after drying; and the two-inch (5-cm.) cubes from sets No. 5 and 6 showed a loss in weight of about 9 per cent for cement and 13.6 per cent for the concrete. The other cubes gave a slightly smaller loss, indicating that in the shelf-dried sets some moisture still remained.

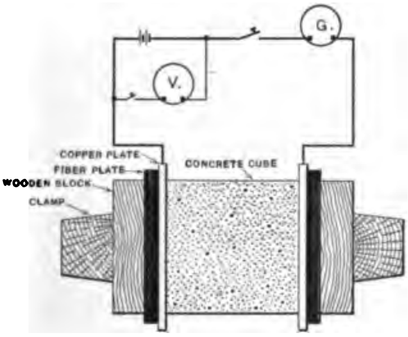

FIG. 5

The twenty-four two-inch (5-cm.) cubes from sets No. 5 and 6 were then tested for their electrical conductivity by means of a deflection galvanometer, with the circuits arranged as shown in Fig. 5.

The galvanometer constant for the part of the scale used was

$19.5 \, . \, 10^{-9}$ amperes for one millimeter deflection on the scale. The cube was placed between two copper plates, these in turn between two fibre boards for insulation. Outside of the fibre, blocks of wood were placed, and the whole put under pressure by means of a clamp, thus securing fairly good contact between the copper plates and the cube. In series with the cube by means of the copper plates the galvanometer was connected to three storage cells giving 7.51 volts. The deflection on the galvanometer when the cubes of set No. 4 were tested showed considerable variations; for the cement the specific resistance for set No. 4 varied from 0.8×10^8 ohms to 6×10^8 ohms, and for the concrete from 1.8×10^8 to 3.5×10^8 ohms per cu. cm. As the loss in weight had indicated that the drying process was not complete, this variation was expected. For set No. 5 (dried over the steam radiator) the readings were fairly uniform and gave the specific resistances for dry cement and concrete given in Table III. The galvanometer showed some "creeping," coming up to a maximum deflection and then decreasing during a few seconds. The readings used in the calculation were taken when this movement had practically ceased, about ten or fifteen seconds after the contact was made.

TABLE III

	Specific resistance (1 cu. cm.)
For cement	12.7×10^8 ohms
For concrete	10.6×10^8 "

TABLE IV

		Specific resistance (cu. cm.)
Wet cement	(Cedar River water)	8700 ohms
" "	(Lake Washington water)	7000 "
" "	(Fifth normal NaCl solution)	5500 "
" "	(Half " " ")	3850 "
Wet concrete	(Cedar River water)	7400 "
" "	(Lake Washington water)	6700 "
" "	(Fifth normal NaCl solution)	4500 "
" "	(Half normal NaCl solution)	2900 "

The cubes were then placed in the following solutions:

Set No. 5 in Cedar River water.

Sets No. 4 and No. 7 in Lake Washington water.

Sets No. 1 and No. 3 in 1/5 normal NaCl solution.

Sets No. 2 and No. 8 in 1/2 normal NaCl solution.

They were left in these solutions over two weeks so as to allow the water to penetrate to all parts, and then tested for the specific resistance by the volt-ammeter method. Readings were taken at three pressures, 46.8, 66.8 and 86.7 volts, with a milliammeter. While some difficulty was found in keeping the cubes at the same dampness, the results were fairly uniform. In Table IV the average values in round numbers are given for the 2-in. (5-cm.) cubes and at about 20 deg. cent. temperature.

It may be of interest to note the specific resistances of the several solutions, although these would be modified when used with the cubes, as the sulphate and other parts from the cement would go into solution and change the conductivity.

TABLE V

	20 deg. cent.	30 deg. cent. (1 cu. cm.)
Cedar River water	20,100 ohms	15,600 ohms
Lake Washington water	15,400 "	12,100 "
Fifth normal NaCl solution	54.4 "	45.5 "
Half normal " "	24.2 "	19.6 "

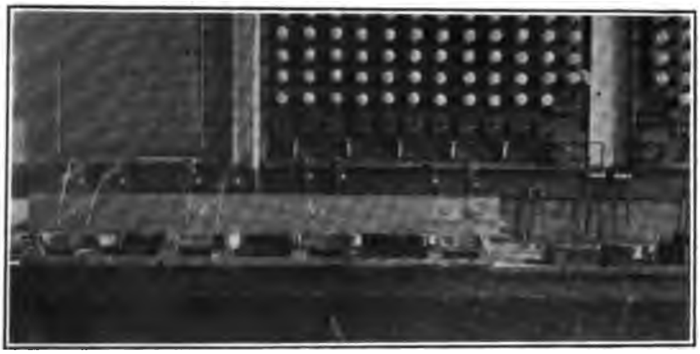

FIG. 6

In order to secure direct evidence of what action, if any, the electric current will produce on the cohesive property of the concrete, experiments were made using the arrangement shown in Fig. 6 and with the circuits as shown in Fig. 7.

Four 2-in. (5-cm.) cubes were placed in a row on a glass plate. At each end was placed a plate of iron extending a little beyond the surface and having a copper wire soldered to one edge for electrical connection. Glass plates were placed outside of the iron for insulation and all put in a wooden clamp, by which pressure could be applied so as to give a fairly good contact between the iron plates and the cubes as well as to bring the four cubes into close contact in a series. Glass strips were cut

2 by 8 in. (5 by 20.3 cm.) so as to fit the sides of the four cubes placed in a row and then held in place by a steel clip. Another glass plate was placed on top and in this two small holes had been drilled about five inches (12.7 cm.) apart. By means of bottles and tubes as shown in Fig. 6, a salt solution was continually supplied through the holes in the cover glass and the cubes kept wet. The flow was adjusted by small clamps on the tubes and satisfactory results were obtained in this manner. The cubes were kept in a fairly uniform state of dampness with a minimum by-path for the current outside of the cubes between the iron plates. A storage battery was connected in series with the four cubes, letting the current pass between the iron plates and then through an adjustable outside resistance. The electromotive force was kept on continuously for 25 days for one set, and 30 days for the second set. At the anode, the iron corroded and the two cubes nearest this end became covered with the oxide. At the cathode no rust appeared. In Table VI are given the data of the e.m.f. applied, the current passing between the iron plates, and the time in days for one set of cement cubes.

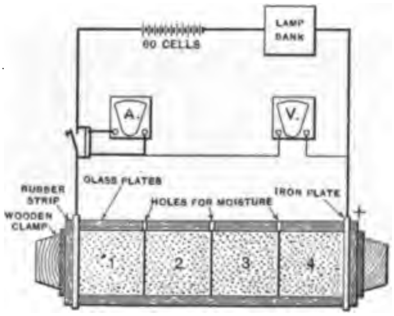

FIG. 7

These data are typical for all the cubes and show variations of resistance similar to those recorded in Table II, relating to the cylindrical blocks. In all 32 cubes were treated in this manner.

These cubes were next tested for their compressive strength, using an Olsen 30,000-lb. (13,607-kg.) testing machine for the concrete and a Riehle 100,000-lb. (45,359-kg.) machine for the cement. Check cubes from the same set were also tested and the results are tabulated in Table VII. In the same table are recorded the average e.m.f. applied, the average current passing between the iron plates, and the total number of days the cubes were kept in circuit. The cubes are numbered in the following order; No. 1 was nearest the cathode, then No. 2 and 3 and No. 4 nearest the anode. Nos. 5 and 6 are check cubes not acted upon by the current. The iron plate next to No. 4 showed much corrosion; cube No. 4 was covered with the oxide and the others were somewhat discolored. No. 4 gave off a strong odor of chlorine when taken from the circuit.

From data given in Table VII it is readily seen that an electric current of low density passing through the cement and concrete does not reduce the compressive strength of the cubes. The current was sufficient to produce chlorine gas at the cathode and was of about the density used in many of the experiments causing failure in a few days. While experiments of a wider range will be required to determine the limits within which the current produces no effect, the results are deemed sufficient to show that the deterioration of the cement was not the chief factor causing the failures in the other experiments recorded in this paper.

TABLE VI

CUBES EXPOSED TO ACTION OF CURRENT 30 DAYS. (SEE FIG. 6 AND FIG. 7)

Time days from start	Cement cubes, set No. 2		Concrete cubes, set No. 2	
	Current amperes	Voltage	Current amperes	Voltage
0	0.040	114.0	0.061	99.2
1	0.050	99.0	0.043	102.0
4	0.020	110.3	0.017	110.0
6	0.037	103.8	0.023	108.3
7	0.041	100.0	0.029	103.0
8	0.033	102.3	0.030	102.3
9	0.061	89.0	0.036	100.0
10	0.070	86.5	0.055	90.1
13	0.031	105.0	0.031	102.0
14	0.019	108.0	0.035	100.0
15	0.027	104.0	0.026	104.1
17	0.028	106.7	0.030	104.3
19	0.036	102.5	0.041	102.5
21	0.030	104.0	0.030	103.9
22	0.031	104.0	0.018	109.0
23	0.044	98.6	0.035	103.1
26	0.045	99.1	0.032	104.0
28	0.060	92.0	0.024	104.0
30	0.054	98.0	0.025	104.0

TABLE VII

	Cube No.	Maximum crushing strength — pounds per sq. in.	Maximum crushing strength — kg. per sq. cm.	
Concrete Set No. I	1	2325	163	Average e.m.f. 113.3 volts.
	2	2237	157	Average current 0.061 amperes.
	3	2075	146	Time in circuit 25 days.
	4	1950	137	
	5	——	——	
	6	1975	139	
Concrete Set No. II	1	1825	128	Average e.m.f. 101.8 volts.
	2	1712	120	Average current 0.031 amperes.
	3	1725	121	Time in circuit 30 days.
	4	1912	134	
	5	1800	126	
	6	1775	125	
Concrete Set No. III	1	2776	195	Average e.m.f. 116.6 volts.
	2	2825	199	Average current 0.059 amperes
	3	2975	209	Time in circuit 25 days.
	4	3125	220	
	5	3025	213	
	6	2875	202	
Concrete Set No. VIII	1	2700	189	Average e.m.f. 103.0 volts.
	2	2725	192	Average current 0.027 amperes.
	3	2225	156	Time in circuit 30 days.
	4	2787	196	
	5	2425	170	
	6	2825	199	
Cement Set No. I	1	7625	537	Average e.m.f. 116.5 volts.
	2	6385	449	Average current 0.057 amperes.
	3	8200	575	Time in circuit 25 days.
	4	8425	593	
	5	7625	537	
	6	7125	502	
Cement Set No. II	1	9675	681	Average e.m.f. 100.7 volts.
	2	8450	955	Average current 0.040 amperes.
	3	6625	466	Time in circuit 30 days.
	4	9325	656	
	5	6600	464	
	6	6000	422	
Cement Set No. III	1	8250	580	Average e.m.f 118.8 volts.
	2	10725	755	Average current 0.042 amperes
	3	7625	537	Time in circuit 25 days.
	4	7650	539	
	5	7375	519	
	6	7775	547	
Cement Set No VIII	1	6750	475	Average e.m.f. 91.3 volts.
	2	9025	635	Average current 0.062 amperes
	3	8750	616	Time in circuit 30 days.
	4	10550	742	
	5	7850	553	
	6	7625	536	

The following summary may be made:

1. Concrete and cement, when dry, are good insulators, their specific resistances being of the order of 1,000 megohms per cubic centimeter.
2. The conductivity of concrete and cement depends upon the porosity of the material and the nature of the solution in the pores. When the pores are full with an aqueous sodium chloride solution both cement and concrete are fairly good conductors of electricity.
3. The current flowing through the concrete probably follows the ordinary laws of electrolysis, with the liquid in the porous spaces of the concrete mass as the conductor.
4. An electric current of low density passing through cement or concrete does not affect the compressive strength of the cement or concrete.

(*e*) When the current leaves the iron through an aqueous solution, oxygen, and under some conditions, chlorine, will be liberated at the iron surface. An iron oxide or a salt will be formed and these chemical changes are accompanied by an increase in volume. When iron is changed to ferric oxide, (Fe_2O_3), the volume is increased in the ratio 1 to 2.2 and a similar increase occurs when the other compounds are formed. No attempt was made to determine the magnitude of the stress that may be developed in this manner but it is known that enormous forces are required to prevent chemical action by mechanical pressure. Without doubt the forces produced by chemical action are ample to break the concrete, and all the observed facts can readily be explained on this basis. The observed data give evidence that the corrosion of the iron, caused by the electric current, precedes the crack in the concrete; that the increase in volume when the iron changes into an oxide or a salt is the direct cause of the failure of the concrete.

III. MEANS AND METHODS FOR PROTECTING THE CONCRETE FROM THE ACTION OF THE ELECTRIC CURRENTS

Granting that the current produces no direct effect on the strength of the cement, that the current through the concrete follows the usual laws of electrolysis, and that the failure of the concrete is caused by the increase in volume which accompanies the changing of the iron into an oxide or a salt, then it becomes a problem of how the corrosion of the iron may be prevented.

Five methods will be discussed in order:

(*a*) By keeping the iron negative to the surrounding concrete so the current will flow to the iron in all cases.

(*b*) By using alternating currents.

(*c*) By filling the pores of the surrounding concrete with some non-conducting material.

(*d*) By coating the iron with a metal so that the conduction will be metallic instead of electrolytic when the current reaches the iron.

(*e*) By coating the iron with some insulating material before it is placed in the concrete.

(*a*) When the current flows toward the iron in an aqueous solution, hydrogen is given off at the iron surface and escapes as a gas, producing no reaction on the iron or the concrete. Observations were made in a number of experiments which showed no action at the cathode, but this point has been amply covered in articles already published. It is also evident that while this condition can readily be obtained in the laboratory, it is seldom possible or practicable on outside structures and hence can find little or no commercial application.

(*b*) That alternating currents will greatly reduce if not entirely eliminate electrolytic action is well known. Theoretically, the electrolytic effect should be nil if the currents flowing in both directions are exactly equal. To observe the action when applied to reinforced concrete the following experiment was made with 60-cycle current. Two blocks, Nos. 416 and 417, were connected in series and placed in a 3 per cent sodium chloride solution. A similar pair, Nos. 419 and 420, were placed in the same tank and the two pairs connected in parallel · In series with both pairs an adjustable resistance was placed, reducing the 110-volt pressure at the mains to about 43 volts across the blocks. The current was kept on continually for 258 days and readings were taken on an average every other day. As the current and e.m.f. stayed almost constant for the whole period, the average values only are given in Table XII, No. 24.

After the blocks had been in circuit for 125 days, a small hole, four inches (10.16 cm.) deep, was drilled from the top of block No. 417 and parallel to the iron bar, in which a thermometer was inserted. The temperature inside the block was found to be 3.8 deg. cent. higher than the water in the tank. At the close of the experiment all the blocks appeared in perfect condition. An account of failure caused by heat due to a larger alternating current has been published.[8] These results are in accord with principles already established for the electrolytic action of alternating currents and give evidence that unless the currents are large enough to produce excessive heating, no deteriorating effect will come to the reinforced concrete from alternating currents of equal positive and negative wave forms.

(*c*) If the current is conducted electrolytically through the pores of the concrete, it seems likely that if these spaces were filled with a non-conducting material, no current would flow. In a laboratory experiment this end is readily gained. That blocks like those used in the experiments can first be dried and then covered by an insulating paint is obvious, and as long as the coating continues in good condition no current can flow and hence no corrosion can take place. It is also evident that such methods are impracticable for any commercial structure.

TABLE VIII

Days from start	Amperes	Volts
0	0.238	33.8
½	0.335	40.0
1	0.202	28.5
2	0.099	28.3
3	0.108	30.4
5	0.092	30.3
6	0.093	30.2
7	0.086	30.5
10	0.082	30.2
13	0.102	30.2
14	0.108	27.5
21	0.184	32.6
22	Taken out of circuit, cracked.	

Much work has been done in connection with water reservoirs to reduce the porosity of concrete and cement. The problem of making cement a retainer of water is, however, much simpler than to exclude the moisture so as to make concrete a non-conductor. In order to note what difference would be indicated, a set of blocks was treated by Sylvester's process. The proportions used were the same as those employed by the United States Army engineers: one part of soap to one hundred of water, and one part of finely powdered alum to one hundred parts of cement. The blocks were then placed in a circuit and observations made as shown in Table VIII.

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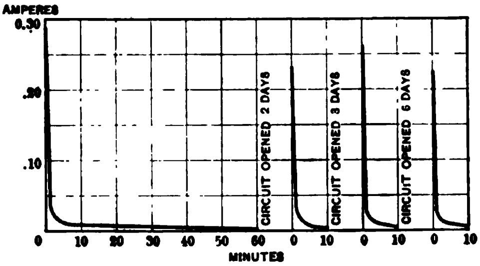
[8] Nicholas, *Engineering News*, Dec. 1, 1910, page 259.

TABLE IX

Time from instant of closing circuit	Amperes	Volts
———	0.292	50.5
10 sec.	0.102	50.5
20 "	0.062	50.5
30 "	0.044	50.5
40 "	0.037	50.5
50 "	0.031	50.5
1 min.	0.026	50.5
2 "	0.018	50.5
3 "	0.012	50.4
4 "	0.011	50.4
5 "	0.010	50.4
10 "	0.006	50.3
15 "	0.004	50.3
20 "	0.003	50.3
25 "	0.003	50.3
30 "	0.002	50.2
45 "	0.002	50.0
60 "	0.002	50.0
Circuit opened, left open two days		
———	0.272	57.0
10 sec.	0.077	57.0
20 "	0.037	57.0
30 "	0.030	57.0
40 "	0.022	57.0
50 "	0.019	57.0
1 min.	0.017	57.0
2 "	0.011	57.0
3 "	0.008	57.0
4 "	0.007	57.0
5 "	0.006	57.0
10 "	0.004	57.2
Circuit opened, left open three days.		
———	0.312	60.5
10 sec.	0.092	60.5
20 "	0.042	60.5
30 "	0.031	60.5
40 "	0.024	60.5
50 "	0.020	60.7
1 min.	0.016	60.7
2 "	0.011	60.7
3 "	0.009	60.7
4 "	0.007	60.7
5 "	0.005	60.7
10 "	0.003	60.8

The blocks cracked after 22 days in circuit and the results indicate that little or nothing was gained by the treatment. Other methods used in waterproofing cement may be more effective. Still, it is not likely that any combination can be found which will so fill the pores that the moisture will be excluded and the concrete become a non-conductor of electricity.

(*d*) If the current should leave the iron through some other metal so that the oxygen ions could not come in contact with the iron, the corrosion would be prevented. Naturally the metallic covering should be so selected that the evil would not be transferred instead of eliminated. Coating the iron with zinc proved useless, as seen in Table XII, No. 9. Bearing in mind the electrolytic properties of aluminum, which have become so generally known through the electrolytic lightning arrester, it was thought that this material would automatically produce an insulating layer which would effectively prevent the flow of the current. To test this directly on the aluminum, blocks were made using aluminum wire instead of the iron bars. In Table IX and Curve II data are recorded indicating the action of the aluminum in a fresh water solution, and showing the recovery of the conductivity when the circuit was broken.



CURVE II

The characteristic drop in current occurring during the first few minutes when in circuit should be noted. After a short time the current has become almost negligible.

Using a 3 per cent salt solution the results were similar. The data are given for two voltages in Tables X and XI.

For Table X readings were continued for 153 days and the current remained at 0.001 ampere with the pressure about 44 volts.

For Table XI readings were continued for 127 days with an impressed voltage of about 110 volts and a current of approximately 0.001 amperes. The concrete remained unchanged.

Since the same increase in resistance, and recovery on open circuit, was obtained with the sodium chloride solution as with the fresh water, it is inferred that the same chemical change took place on the surface of the aluminum. Bare aluminum placed in a 3 per cent sodium chloride solution will not become covered by an insulating coating if the action is fairly rapid, as will be the case with even a few volts pressure. The escaping gas, the convection currents in the water, and other factors prevent the formation of a layer giving high resistance. Inside the concrete the conditions are different and the observations tend to prove that an insulating layer will also be formed with the salt solution as electrolyte. Further tests will be made on this point.

TABLE X

Aluminum wire in concrete, salt water		
Time from start	Amperes	Volts
—	0.180	36.0
10 sec.	0.155	36.0
20 "	0.140	36.0
30 "	0.120	36.0
40 "	0.105	36.0
50 "	0.095	36.0
1 min.	0.085	36.0
2 "	0.060	36.0
3 "	0.044	36.0
4 "	0.035	36.0
5 "	0.030	36.0
10 "	0.015	36.0
45 "	0.007	37.0
3 days	0.001	42.6
5 "	0.001	44.1
153 " (continued)	Ave. 0.001	Ave. 44.0

To transfer these conditions to the iron, the bars were covered with two or three coats of aluminum paint, using banana oil (amyl acetate) as a binder.

With aluminum paint, the characteristic drop in current—that is, the rapid rise in resistance due to the formation of aluminum oxide—is similar to the results shown for the aluminum wire. The current is reduced to a small amount in a short time, and if the circuit is broken, recovery occurs with the paint similarly to that noted under the aluminum bars. With higher pressures, the coating is seemingly broken through quite rapidly and the resistance does not increase as much as for solid aluminum. Although the current for voltages like those in Table XII, No. 13, is reduced to a small quantity, the paint with the amyl acetate as a binder does not protect the iron from corrosion. The time element before failure of the concrete occurred was considerably increased, but the blocks cracked as in the case of bare iron. Two explanations are suggested. First, in salt water solution, the chlorine given off at the cathode attacks the aluminum and thus destroys the protective qualities of the oxide film, and when the aluminum has all been changed to a salt, the current can attack the iron. Second, the oxygen ions pass between the layers of aluminum flakes and thus reach the iron. The latter process seems to be the most probable. By using other binders, it was hoped that better results would be obtained. Bars were coated with aluminum mixed in varnish and several of these were tested. In Table XII, No. 14, the results from one block are given. By using the varnish alone similar results were obtained, as may be noted in Table XII, No. 15. Evidently, the insulating properties are due to the varnish more than to the aluminum, and the permanency of the protection would depend on the stability of the varnish.

TABLE XI

Aluminum wire in concrete, salt water		
Time from start	Amperes	Volts
———	0.80	82.0
10 sec.	0.64	83.0
20 "	0.56	85.0
30 "	0.49	87.0
40 "	0.44	89.0
50 "	0.41	91.0
1 min.	0.38	93.0
2 "	0.26	99.0
3 "	0.20	102.0
4 "	0.16	104.0
5 "	0.13	106.0
10 "	0.065	110.0
20 "	0.032	112.5
30 "	0.022	113.5
1 hr.	0.011	114.5
2 "	0.007	115.8
4 "	0.003	116.0
5 "	0.003	116.0
1 day	0.001	108.0
2 days	0.001	110.0
127 " (continued)	Ave. 0.001	Ave. 110.0

TABLE XII

	Electrolyte	Time in circuit	Volts average	Amperes average	Ampere-hr. approximate	Results
1. Bare iron..	Fresh water	26 da.	92.0	0.071	44.3	Block cracked in 29 hours. Block finally split into four pieces and iron bar deeply corroded.
2. Bare iron..	Salt water	35 da.	8.0	0.04	33.6	Block cracked in 7 days. Block finally split into two pieces. Iron corroded.
3. Bare iron..	Salt water	5 da	29.0	0.25	30.0	Block cracked. Iron corroded.
4. Bare iron Concrete mixed with soap and alum..........	Salt water	21 da.	30.2	0.128	64.5	Block cracked. Iron corroded.
5. Red lead in linseed oil.....	Salt water	47 da.	30.0	0.076	85.7	Block cracked, iron corroded, paint decomposed.
6. Red lead in linseed oil.....	Salt water	7 da.	31.0	0.145	24.4	Block cracked down side and across top. Iron bar slightly corroded, paint film decomposed.
7. Graphite paint..........	Salt water	9 da.	31.0	0.205	44.3	Block crackedacross top and down side. Iron deeply corroded.
9. Zinc coated iron bar........	Salt water	4½ hr.	101.0	1.65	7 4	Block cracked.. Chlorine and steam given off next to iron bar.
10. One coat of aluminum in amyl acetate...	Fresh water	38 da.	42.0	0.123	112.0	Block cracked. Iron deeply corroded.
11. Two coats of aluminum in amyl acetate...	Sea water	14 da.	43.5	0.040	13.4	Block cracked, iron corroded.
12. Three coats of aluminum in amyl acetate...	Fresh water	26 da.	92.0	0.019	11.9	Block cracked, iron corroded.
13. Three coats of aluminum in amyl acetate...	Sea water	37 da.	103.7	0.019	16.9	Block cracked, iron corroded.
14. Three coats of aluminum in Florette varnish	Sea water	257 da.	100.0	0.001	6.1	Rusty water at top of block in 16 days. No further evidence of action.
15. One coat Florette varnish	Salt water	179 da.	42.0	0.007	30.1	Block cracked, iron corroded.

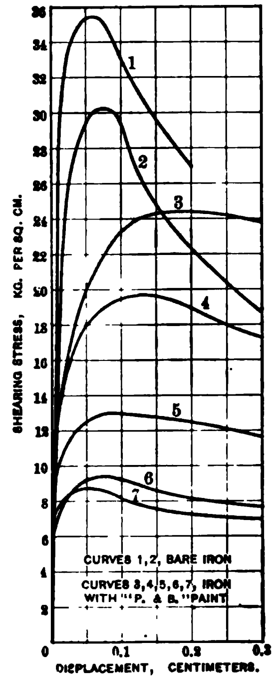
TABLE XII—Continued

	Electrolyte	Time in circuit	Volts average	Amperes average	Ampere-hr. approximate	Results
16. Two coats marineTockolith	Salt water	73 da.	100.0	0.007	12.3	Block cracked, iron corroded.
17. Two coats P.&B. paint....	Salt water	312 da.	40.0	0.003	22.5	No evidence of corrosion.
18. Two coats P.&B. paint....	Salt water	198 da.	39.2	0.004	19.0	No action observed for four months. Block finally cracked, iron corroded.
19. Two coats asphalt paint, T. C. U. Co....	Salt water	258 da.	104.0	0.001	6.2	No evidence of corrosion.
21. Two coats R. I. W. No. 110 paint..........	Salt water	41 da.	31.0	0.005	4.9	Block cracked, iron corroded.
22. Two coats of Tockolith and one coat of R. I. W. No. 110....	Salt water	184 da.	107.0	0.000	0.0	No evidence of corrosion.
24. Bare iron alternating current...........	Salt water	258 da.	40.2 (Two blocks in series)	0.445	2760.	No evidence of corrosion.

With the iron coated by a continuous layer of aluminum so as to eliminate the passageway for the oxygen ions to the iron, the desired results might be achieved. Iron bars plated with aluminum were not available but it seems likely that the automatic increase in resistance would follow in a manner similar to the experiments on aluminum wire. With a practical method of coating the iron with aluminum, the problem of protecting the reinforcing bars in concrete from electrolytic action may be solved.

(*e*) If the iron bar be coated with an insulating compound before being placed in the concrete, thus preventing the flow of electricity, it is evident that there will be no corrosion of the iron. To find a material with which such an insulating coating can readily be placed on the iron so as to produce commercially practical results is no easy task. Work has been done on this problem by Dr. Maximilian Toch, who has prepared a special paint for which he claims protective properties. This and a number of other commercial paints were tested and a summary of the observations is given in Table XII. In all cases the paint was thoroughly dried before the block was made; and a period of at least thirty days was given for the concrete to set before the block was placed in the circuit. The density of the salt water electrolyte was approximately 1.026. In the first column is noted the material with which the iron was coated before being placed in the concrete. Current was supplied by a storage battery and the e.m.f. was subject to changes similar to those recorded in Table II.

The results indicate that the time element is an important factor in determining the protective qualities of a paint. The deterioration of the insulating power of the paint is probably due to a slow absorption of moisture. No. 18 gave no indication of corrosion during the first four months, while complete failure followed by continuing the process two months longer. No. 17, prepared in the same manner as No. 18, showed no signs of corrosion or failure although the e.m.f. was applied for over ten months. Similarly, the time required to produce failure in No. 5 was over six times as long as for No. 6. The data recorded in Table XII, excepting for Nos. 5 and 7, are for the block in each group which failed in the shortest time. The time for producing failure varied considerably for the several blocks in a group. The sets of blocks represented by Nos. 17, 19 and 22 gave no evidence of corrosion, but whether the concrete will be protected indefinitely or failure will follow by continuing the process for a few more months remains to be seen. Since the insulating power of any material is greatly affected by changes in the moisture content, it seems evident that the covering which will give permanent protection must not absorb any water.



Curve III.

If a paint be found that will permanently protect the iron from corrosion it becomes necessary to determine what effect the layer of paint will have on the reinforcing power of the iron. For a preliminary test ten blocks were made using smooth ¾-in. (19-mm.) iron bars that had previously been covered with two coats of " P. & B." paint. For every two blocks with painted bars a third was made with bare iron. The bars were let down to the bottom of the moulds so as to give uniform conditions. For testing, the iron bar was moved relative to the concrete by means of an Olsen 30,000-lb. (13,607-kg.) testing machine. The stress applied was read in pounds directly on the scale of the machine, while the displacement of the iron relative to the concrete was measured by means of a scale, telescope and mirror. The mirror was mounted on a small tripod, which rested with two points on the concrete block and a third point on a small clamp screwed to the iron bar. The contact surface between the iron and the concrete was 25.5 sq. in. (166 sq. cm.). The relations between the stresses applied and the corresponding displacements are given in Curve III. It is readily seen that the presence of the paint greatly reduced the maximum shearing force. The data given in Curve III are for smooth bars. It is quite likely the effect would be less marked when using the Johnson or other commercial form of reinforcing bar, but the reduction factor should be determined.

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DISCUSSION ON "ELECTROLYTIC CORROSION IN REINFORCED CONCRETE." CHICAGO, JUNE 30, 1911.

Burton McCollum: The paper by Professors Magnusson and Smith is an interesting and important one since it deals with a subject that has recently sprung into considerable prominence. A number of investigators have published papers on this subject during the last four or five years and much information has been brought to light. This paper carries our knowledge a step farther in several directions, but much still remains to be done, and it is encouraging to note that the authors are continuing this work.

The conclusion reached by recent investigators and which appears to be concurred in by the authors of this paper is that the failure of the concrete is due not directly to the passage of the current through the concrete, but indirectly to the corrosion of the imbedded metal and has therefore been described as an anode effect, since the cathode does not corrode. It cannot be doubted that the most conspicuous effect occurs at the anode and the cracking of the concrete is no doubt due entirely to this cause. The Bureau of Standards has been carrying out investigations on this subject during the past year and among other things these investigations have brought out the fact that there is a cathode effect quite independent of the effect at the anode. The specimens in which this effect has been observed were of a size and form very similar to that adopted by Professors Magnusson and Smith, being cylinders with metal rods imbedded at the center. When these specimens have been placed in circuit with 15 volts pressure with the imbedded metal as cathode a distinct softening of the concrete near the cathode has been observed. The softening is confined to the region near the cathode, extending in some cases to about half an inch therefrom. No cracks, however, developed at the cathode. The cause of this softening has not yet been definitely determined, but the matter is now the subject of investigation.

The effect of various voltages on reinforced concrete has also been studied and it appears that the time required for cracking of the concrete around the anode decreases much more rapidly than the current increases. The variation between individual specimens under the same treatment is so great that no definite law has been determined, but it is safe in general to say that the number of ampere-hours required to crack a given specimen is much greater in case a low voltage is used than when a high voltage is applied, and this is true even when the voltages do not reach a value sufficient to produce serious heating.

A number of different brands of Portland cement have been tested. The specimens used in this test were 1:2½ :4 of Portland cement, coarse sand, and crushed trap. Specimens were made of each of the following brands of cement: Atlas, Dragon, Alpha, Lehigh, Pennsylvania, Giant. All contained imbedded electrodes of 1-in. pipe cleaned and weighed, except the specimens of Giant cement, which contained $\frac{3}{4}$-in. solid iron rods. Six specimens of each brand were used, but while the results differ considerably in some cases, this difference is but little greater than is to be found in different specimens of the same kind of cement made at different times, so that it is not to be expected that any one cement will prove materially more resistant to the effects of electrolysis than the others.

Several series of experiments were carried out using brass, carbon, copper and other electrodes. In the first series a pair of each were put in parallel on 15 volts direct current, one of the pairs being anode and the other cathode (the other electrodes were sheet iron surrounding the cylinders in tap water).

All specimens are still uncracked after seven months' run. After a month or two the cathode carbon specimen began to discharge a thick tarry substance at the top end of the carbon, apparently forced up through the core by pressure from below.

On the top of all the cathode specimens, the mortar had apparently disintegrated for about 1 inch around the cathode. To see whether this extended downward into the concrete, the carbon specimens were broken open. It was found that there was a softening around the core for about $\frac{1}{16}$ in. only, the softening at the top being confined to the surface layers.

The surface of the anode carbon had disintegrated for about $\frac{1}{32}$ in. No other effect was apparent. There was no certain evidence of loss of strength in either anode or cathode specimens.

An interesting comparison may be made by computing the number of ampere-hours passed per sq. in. of anode surface. Cracking has occurred in the iron anode specimens when this quantity has amounted to a value ranging from 0.8 to 3.0. That for the carbon is 3.0 with no cracking.

In the second series, two pairs of specimens, having brass and copper electrodes, were arranged so that 1 copper and 1 brass specimen were put in series across 115-volt direct current, both being anode, and 1 copper and 1 brass in series as cathodes on 115 volts.

No cracking of any specimen occurred during seven months' run. At the end of this time all four were broken open, and the following points were noted:

The anode specimens were sound and hard, with no evidences of deterioration. Both anodes were considerably pitted, the losses being about 10 grams apiece, rather more for the brass than the copper. Greenish products of copper had penetrated $\frac{1}{4}$ in. to $\frac{1}{2}$ in. into the concrete from the electrodes. Ampere-hr. per sq. in., 10.1.

The cathode specimens showed evidence of deterioration as described in the first series. The surfaces of the cathodes had the appearance of weathered copper, and a distinctly dark layer of disintegrated mortar extended for about $\frac{1}{4}$ in. to $\frac{1}{2}$ in. all around the metal. Ampere-hr. per sq. in., 36.8. A peculiar feature was noticeable in that the final resistance of the copper anode specimen is about 1/10 that of the brass anode specimen, while the reverse was the case in the cathode specimens.

Much work has been done with a view to throwing light on possible methods of protecting concrete from electrolysis. Since the most destructive effect is the cracking of the concrete due to corrosion of the anode it follows that if the anode can be protected from corrosion this trouble will be eliminated. The authors suggest doing this by preventing the current flow. Their observations on the effect of insulating materials on the bond between iron and concrete are confirmed by our experiments, although even greater reductions in the shearing stress have been noted. In one case, in which the rod had a rather heavy coating of pitch, it was forced out of the concrete by pressure which gradually developed from within. This would seem to indicate that the method of insulation should be resorted to with great caution if at all.

The suggestion made by the authors that iron might be coated with aluminum to protect it from corrosion is novel and worthy of serious consideration. It appears to have been established that if iron can be successfully coated with aluminum commercially this method could be used to protect the concrete from cracking. Perhaps the most obvious method of attacking this problem, however, would be to take advantage of the insulating properties of dry concrete. Concrete, when dry, is a fairly good insulator, good enough in fact to prevent any appreciable corrosion under any conditions that might arise. We have therefore done a great deal of work on the subject of water-proofing concrete. A large number of waterproofing agents now on the market have been tried, but with indifferent success. These tests have included those waterproofing agents that are to be mixed with the cement and also those that are to be applied to the surface of the concrete, as a paint. Not one of these agents, however, has shown sufficient tendency to reduce the damage or prolong the life of the cement, to justify its use in practise. Experiments now under way using treated papers, felts, etc., give promise of much better success. The success of these coatings, however, depends mainly on the insulating properties of the coatings themselves, so that it is immaterial whether the concrete is wet or dry. This is obviously important since it is difficult to secure dry concrete under practical conditions.

Another method of reducing damage due to electric currents is to employ some means of reducing the efficiency of corrosion, and a great deal of work has been done along this line at the Bureau of Standards. It has been found that in ordinary concrete the efficiency of corrosion is quite low, the actual amount of corrosion caused by the passage of a given number of ampere-hours being of the order of from eight to 15 per cent of the theoretical maximum. This is due mainly to the presence of a large amount of calcium hydroxid in all Portland cements. Attempts have been made to reduce the efficiency of corrosion still further by increasing the amount of calcium hydroxid and it has been found that this does reduce the corrosion to a considerable extent and its addition does not injure the cement, but on the contrary increases its strength somewhat and renders it somewhat less porous. The difference, however, even when 20 per cent by weight of calcium hydroxid was added to the cement, was not great enough to warrant the use of this method in practise, since at best it could but delay the trouble for a little while. Other and more soluble hydroxids will doubtless give better results, but it is not clear as yet what effect these may have on the properties of the concrete. Barium hydroxid has been shown to reduce the efficiency of corrosion very greatly. This is probably due to its reduction of the calcium sulphate present. All Portland cements contain a certain amount of calcium sulphate, the amount being in most cases of the order of 3 per cent by weight of the cement. This is added just before grinding the clinker and serves the purpose of retarding the setting of the cement. Experiments carried out at the Bureau of Standards have shown that the greater part of the electrolysis is due to the presence of this added calcium sulphate. When barium hydroxid is added to the cement the sulphate is precipitated out as the insoluble barium sulphate and in this way the efficiency of corrosion has been reduced to a fraction of one per cent. Other experiments have been made using specially ground clinker to which no calcium sulphate has been added. This, however, was not entirely free from sulphate, there being a residuum of about 3/10 per cent present in the ordinary course of manufacture, or about 1/10 the amount present in ordinary commercial cements. Specimens made up with this cement showed a corrosion efficiency of but a small fraction of 1 per cent. The great difficulty in using this lies in the difficulty of making sound concrete when the calcium sulphate is absent, owing to the very rapid setting of the cement. This is very troublesome even in the laboratory and in practise it would no doubt be prohibitive unless some means other than the use of calcium sulphate can be used to retard the setting of the cement. In making laboratory specimens this trouble may be largely overcome by using ice water in mixing the cement, and it is possible that this might also be accomplished in practise. Much additional work remains to be done, however, before the practical value of this method of procedure can be demonstrated.

The importance of this and similar papers mainly lies, of course, in the fact that they demonstrate the possibility of damage to reinforced concrete structures such as buildings, bridges, etc., due to stray currents from railways or from other sources. That such possibility exists has been amply proved by laboratory experiments and it becomes a matter of importance to determine whether or not such damage is likely to occur in practise, and if so, to seek the most suitable remedy. Numerous cases of damage to reinforced concrete buildings and bridges have been reported during the last few years, in which the damage has been attributed to electrolysis. We have had occasion to examine a number of such cases and in every case thus far investigated there has been evidence that stray currents had little if anything to do with the deterioration of the concrete. That the damage is in some cases due to corrosion of the reinforcing material by local galvanic currents there can be no question, and in a sense therefore it may be said to be due to electrolysis, but in each case a careful examination has convinced us that stray currents from railways or other power circuits could not be held responsible for the trouble.

It should not be inferred, however, that cases of damage due to stray currents do not exist or may not occur. The laboratory experiments of numerous investigators have shown that such damage may result if conditions are favorable, and thorough investigations should be made to determine whether or not such conditions often arise under actual conditions. But until such cases are found and it can be shown beyond doubt that stray currents are responsible for the damage it is best that we be cautious about drawing conclusions based solely on laboratory experiments, lest we create unnecessary alarm among builders and property owners and work an injustice to electrical interests already somewhat overburdened with troubles arising from electrolysis. The work has reached a stage which calls for extensive investigations in the field to determine precisely the magnitude of the danger. Such investigations are now being carried out by the Bureau of Standards.

E. W. Stevenson: I would like to ask the last speaker, if he saw a recent report in the *Engineering News* which described the effects of electrolysis on a whole building, including the foundations, ceilings and every story, where the stray currents were being dissipated. The building is a new one, the business is that of a cold storage packing house, and it is situated within about 300 ft., I think, from the power house of a traction company. Several investigations have been made by different companies, but this was the first public report as to the actual results of this electrolysis.

Of course it was presumed that it was the traction company's current doing all the damage. An engineer was called in by the traction company, and he proved, apparently to the satisfaction of both parties, that the current used by the traction company had nothing to do with it and the destruction was due to the cold storage company's own electric light plant. The engineer's proofs appear to have been very simple, for he merely connected two voltmeters with the steel structure of the building, connecting the other sides, in one case to the track of the trolley company, and in the other case to the mains of the company's own electric light plant. Then by manipulating switches in each circuit, he proved to the owners of the building that it was their own current, and not the stray current from the trolley system, which was doing the damage.

All who have anything to do with installation of electric wires in buildings are aware of the strict rules governing every part of the work, under the Board of Fire Underwriters. And it seems to me almost impossible for any inspector of this department to have passed such an installation before the current could have been turned on. For, if the engineer's exposure of the situation was correct, the circuits must have been grounded all over the building.

What I would like to know is this: How would it be possible to prevent a similar occurrence, supposing that the conditions were the same? For there are not many people who would look for such disintegration of their buildings, unless they were experienced in it.

Speaking from my own personal experience, the company I am connected with recently erected a building as an addition to its plant. The basement is constructed of reinforced concrete, and just out of curiosity, a few weeks ago, as I happened to have a low-reading voltmeter, I tested to see if there were any currents escaping from the trolley rails, which run past the building, to the columns of the structure. I found when a car was passing, a pressure rising from ½ a volt to 2 volts. The needle was deflected till the car got past and for about a mile further, when it gradually came to rest. I had no method of obtaining the quantity of current, but the local traction promised to look into the matter at once and to remedy it.

G. D. Shepardson: We are likely to make ourselves more trouble than is necessary over this question of electrolysis, because, in so many cases the corrosion which occurs is due to a purely chemical action, or to local action on account of non-homogeneous composition of the metal. I presume that many of us have had cases brought to our attention of trouble, apparently caused by electrolysis, where it could be shown, at least to the satisfaction of interested parties, that the trouble might have been caused by non-electrical causes, or at least by purely local currents. So it seems to me that we should guard ourselves against too much alarm on account of stray currents coming in from outside. In short, other sources of trouble may come in which will relieve the railway companies from responsibility.

George A. Hoadley: Since it seems to be evident, from the proof that we have had, that there is a kind of disintegration taking place in concrete due to the passing of the current from the iron to the concrete or from the concrete to the iron, I want to ask Professor Magnusson whether it would not be possible for the construction companies to so arrange a metallic by-pass with the inside imbedded iron that the current could be shunted around the concrete? It seems to me that a shunt circuit could be provided that would take care of this.

A. S. Langsdorf: About four years ago, I published the results of a series of experiments on this same subject, and in general they confirmed those which Professor Magnusson has reported here. I did not go quite so far in an attempt to prevent the corrosion as he has done, but confined the work to a study of what would happen if the current passed through the concrete specimens. It was found that even with a very small current, 2/100 of an ampere, the damage was just as great, if allowed to continue, as with much heavier currents for a shorter time; but the question that arises is whether, under actual conditions, with structures buried in damp soil or water, there is likely to be any voltage of sufficient magnitude to force through the current that could do the damage that the 2/100 of an ampere did in the case of my specimens. The apparent resistance of the specimen, after the action has gone on for a short while, is very high, due, no doubt, to polarization that takes place; it amounts to about 500 or 600 ohms per specimen, with a thickness of 2½ or 3 in. of concrete; and that means that the voltages that are likely to be found in practise due to stray currents from railways, say, perhaps, from one volt up to a maximum of 10 or 12 volts, are hardly large enough to produce a current of destructive magnitude. With reference to what Professor Hoadley has just said, it may be of interest to remark that after I explained the results of my work to the Engineers' Club of St. Louis, the engineer of the Board of Education took the opportunity, in the construction of the concrete footings of a new school building, to bond the reinforcement in such a way that if current did get in from a neighboring street car track, it could get out again without going through the concrete. That is the only case of the kind to my knowledge where concrete footings have been bonded in that way.

Maxmilian Toch: C. Edward Magnusson and G. H. Smith's paper on the " Electrolytic Corrosion in Reinforced Concrete " is a welcome addition to the literature on the subject. They have mentioned me as one of the earlier investigators on this subject, but since I began these investigations 10 years ago, a dozen or 15 excellent papers have been written on the subject, all of which tend to throw light on it.

What I established was

1. That there is such a thing as electrolytic corrosion.
2. That corrosion always takes place at the anode.
3. If pressure of surrounding concrete is less than the pressure exerted by the formation or the conversion of iron into iron hydroxid, the concrete is bound to split.
4. That no corrosion can occur if a suitable insulating paint be used, which

A. Must be proof against the action of alkali.
B. Proof against the action of water.
C. Of such composition that concrete will adhere to it.

Shaffer has shown that the strength of the currents with which he worked, and which in some instances were so weak that water could not be dissociated by them, produced corrosion just the same.

It must be understood that I am only a quasi-instructor on the subject of the chemistry of paints, but in former years I was frequently consulted on matters pertaining to corrosion, outside of my regular business, and during such consultations I was informed and saw where much damage had been done. I recall one case particularly, where huge pieces had been eaten out of the side of a steel bridge. The corrosion was not chemical in the true sense, but was electrolytic, and the damage was done entirely by both a voltage and amperage of less than 2.

Messrs. Magnusson and Smith make a statement that I have worked on this problem and have prepared a special paint. I refer to test No. 22, *two coats of Tockolith and one coat of R. I. W., No.* 110; electrolyte, salt water; time in circuit, 124 days; average volts, 107; showed no evidence of corrosion. This is the only test in the entire lot made by the investigators which shows that the paints in question insulated perfectly. Every one of the other paints used shows a leakage, which indicates that the insulating properties are not as good as the test just mentioned. I dislike to criticize the tests that have been made, because the work deserves great commendation, but the voltage and amperage are entirely too strong, for whenever electrolytic corrosion takes place, either through induction or through some leaky wire, both the e.m.f. and the current values are usually less than 2, and the surprise to me is that any paint should stand at all a voltage of 107 for 124 days.

I made an examination of some grillage beams which had been coated with these same paints and placed in salty concrete, and after five years, when the grillages were uncovered, the paint was still intact and there was no evidence of corrosion.

What Magnusson and Smith have shown with reference to red lead, graphite, and, in fact, all paints in which a linseed oil varnish is used, is not new, but deserves the careful consideration of every engineer who still uses linseed oil paints on structural steel, for, even if there is no electrolytic corrosion, or any danger of it, alkaline moisture tends to destroy paint completely in which the binder is *oil*. Excellent work on this subject has been done by Mr. G. F. Shaffer.

It is to be hoped that Messrs. Magnusson and Smith will continue their researches, the next time with weaker currents, and also with cements that are waterproof and that contain broken stone, so that a fair comparison can be made as to the action in concrete. It would be well to try out the waterproofing compounds on the market which are added to the cement, and the waterproof coatings which are often applied after concrete has set.

Guy F. Shaffer: This series of tests, on a subject of such vital interest to concrete firms, structural, bridge and electrical engineers, railroad companies using electric motive power and street railway companies, should bring forth the fullest discussion as to the limits of investigation in theoretical work of this character. It is the object of the writer to define, from one point of view, some of the limitations and give reasons for such definition.

A. *Stray Currents.* The first factor in the problem is; at what stray current voltage will corrosion of ordinary unstressed steel occur? Now by " stray currents " an upper limit is usually defined in the neighborhood of 15 volts. Even lower voltage drops should be secured with correct electrical installation, but any thing above that must be considered a decided " leak ". Therefore any test which is carried above that limit is forcing conditions on the concrete which are not likely to remain long undiscovered in any electrical circuit. It is current flowing at lower and less likely to be discovered voltages that needs study. The writer has published a report on a test to destruction at 3.4 volts and unless investigators wish to spend months in useless duplication, no further tests should be made above that limit. To add further data to this point let me say that other tests have just been completed in the laboratories of the Massachusetts Institute of Technology showing that at 1.1 volts corrosion will occur, but these tests were not carried to destruction.

Outside the question of inhibiting or minimizing stray current corrosion by waterproofing or other specially applied methods, this voltage drop is dependent upon several variables:

1. The thickness of the concrete.
2. The per cent proportion of salt solution in the surrounding electrolyte.
3. The density of the mass (involving the proportioning of ingredients).

With a decreasing voltage these factors simply reduce the amount of current flowing, but it is the law of the change in the *rate of corrosion* of imbedded steel which a careful study of these variables would define.

B. *Ingredients in Concrete.* Another factor is the kind of cement, involving as well the kind of stone. This question brings up the discussion of the cause of failure in the concrete. If the action going on is simply an accumulation of rust, and nothing more, the problem of the ingredients vanishes. A test with a non-corrodible anode, such as an arc light carbon, would give some data on that question. But, if there is a chemical reaction going on in the block itself, the chemical nature of the cement and the chemical nature of the stone should be known. Under this head the fact should be pointed out that what should be studied is *concrete* as " concrete," not as cement and sand.

C. *Imbedded Metal.* Another factor is the kind of steel, iron or other imbedded metal used and the stress to which it is subjected. The question of stress involves not only the stress on the metal, both internal and applied, but stress in the concrete as well. Before the bars are imbedded or tested in any way electrolytically their mechanical and chemical properties should be known, as these properties may have a decided effect on the results, which effect would only be brought out by careful comparative experiments.

A complete study of the three factors outlined above, developed in such a way that the results given are comparable with other experimental data, will determine very closely the law of imbedded corrosion.

D. *Prevention.* A fourth factor is the question of prevention and under this head should be studied all the typical waterproofing compounds and iron preservative paints so that no question of commercialism enters into the application of the results. Now to examine critically the work done by Messrs. Magnusson and Smith as it comes under these several heads.

Under A: Every test reported was carried on at such a high voltage that the five factors they name as involved, (*a*), (*b*), (*c*), (*d*) and (*e*) were distorted out of their true relation to the practical side of the problem.

The ohmic resistance across the cement and sand should have been reported both before and after the blocks were put in circuit.

If by "fresh water" (mentioned in Table II) is meant the river or lake water, the tests are not comparative, as the per cent of impurities should be definitely known.

The mass was not examined as to density.

Under B: Report is given as to the brand of cement and its mechanical properties, but no chemical analysis is given.

As no stone was used the question of "concrete" is immediately eliminated.

Under C: Not only was the iron not reported on chemically, but no report is given as to the weight of the imbedded bars in order to determine the variation in the rate of corrosion from the well defined electrochemical law of 1.045 grams per ampere-hour.

Under D: The report on waterproofing and paint tests is interesting and valuable to engineers, but it narrows some of the results down to a basis that is unfair commercially. If time was lacking for general tests on iron preservative paint films no specific report should have been published, as this is unfair to other manufacturers who may have materials equally as good as those described.

The specific resistances of cement and of cement and sand as given are valuable.

One other test not commented on is the test on the crushing strength of blocks *after* being subjected to current. There may be such a question as *recovery* occur, so that blocks should be subjected to a coincident current and crushing test.

A general summary of this criticism then is that according to the report of these investigators a great amount of needless duplication is being done. It is hoped that a broader view of the variables of the problem is being studied in the work shown in Fig. 1 and Fig. 4.

The writer feels that the subject needs all the study that men of an investigative turn of mind can give, but the wide range of variables must not be lost sight of, as otherwise the data developed will be useless.

Harry Barker and W. L. Upson: The paper by Messrs. Magnusson and Smith has special interest at this time, as there have recently been published[1] the details of a case where the heretofore vague fears of engineers have become quite definitely realized.

The writers, in this discussion, first desire to call attention to the conditions of the structural failures in this building and to point out certain ways in which prevention of trouble in similar cases may be secured. Secondly, there is one point in the studies of Messrs. Magnusson and Smith which may be critically examined. Thirdly, the writers desire to place on record the results of certain studies which have been made during the past year at the University of Vermont, which seem to throw additional light on the mechanism and seat of the action which leads to the disruption of concrete surrounding iron and subject to electric current.

The destruction described, in the article referred to, was in a reinforced-concrete packing house where there were various rendering processes going on which liberated great amounts of vapor laden with organic acids. These condensed on the concrete walls and ceilings and on any cold metal exposed. The 110-volt, direct-current lighting circuits were run in ordinary metal ducts unconcealed and held in contact with the steel reinforcement of the beams and girders by special sockets and bolts. Under the severe conditions which prevailed the concrete became water-soaked and the moisture even penetrated the conduits. Evidently the insulation of a negative wire at one point and of the positive wire at some other point broke down so that current could flow to the reinforcement in one place through the concrete mass to the reinforcement and positive lead in another place, with corrosion and disintegration wherever it left the reinforcement. Beams and girders were cracked and the grip of the reinforcement was loosened for some 60 per cent of the total span of each. The original design of the structure appears not to have been very economical, which perhaps was the salvation from complete structural failure. There was a considerable length of beam (40 per cent) near the supports, which was apparently unaffected and this was where the greatest bond stress would be imposed by normal loading. Tests showed no abnormal deflection up to 125 per cent normal load and it was decided to patch up the old beams for service.

Certain preventives of such troubles suggest themselves to electrical engineers and among them may be noted (1) the use of strictly marine installations of lighting circuits where bad

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[1] *Engineering News*, June 8, 1911. "Serious Injury to a Reinforced-Concrete Building by Electrolysis."

moisture conditions prevail; (2) practical insulation of the conduits from the reinforcement of the building, and (3) prevention of water-soaked walls and ceiling by covering with a waterproof membrane like water-repellant paint, etc.

Messrs. Magnusson and Smith seem to conclude from their studies that there is no diminution of the strength of concrete, or mortar, due to the mere passage of current as contrasted with some mechanical action at the oxidized electrode.

In the experiments where the blocks were enclosed in a glass box and fed with brine, certain masking conditions are at once recognized. The strips must necessarily have been loose, so that there was a layer of electrolyte all over the outside of the blocks, forming a path for the current in parallel with that presented by the blocks themselves. Indeed, the authors remark that they hoped to secure " a minimum by-path for the current outside the cubes." It is worth while comparing this by-path with the resistance of the cubes themselves.

From the uneven surface of the cubes and the easily broken edges which usually become slightly rounded, it is probable that the liquid layer between each glass plate and the cubes would not be any less than the equivalent of 1/64 in. thick. The four sides in parallel would form a path of salt solution 1/16 in. thick, 2 in. wide and 8 in. long. The resistance of this, according to the figures given by Messrs. Magnusson and Smith, would be 1,390 ohms, using the value for fifth-normal sodium-chloride solution. The resistance of the concrete-block path works out, following the same data, to be 3,540 ohms. The combination of these parallel resistances should result in an apparent resistance between electrodes of 1,000 ohms. Compared with this we find that the resistance between plates was as low as 1,250 ohms, which in a general way confirms the assumption. Polarization voltages have been neglected. We may reasonably suspect that only about a third of the current went through the concrete blocks. This reduces the actual density in the concrete to such a low figure that, in the short space of time covered by the experiments, no diminution of strength was apparent. The iron oxide formed seems to have traveled with the current along the outside of the cubes and not to have penetrated the concrete. This is, in itself, an evidence of the masking phenomena. It also prevents a study of the mechanical effect of the molecules of iron oxide inside the concrete.

Those experiments by Magnusson and Smith, relating to the protection of iron against corrosion, are very useful though they show no practical remedy. It is seen that most of the protective paints depend for their effectiveness on their imperviousness. When, in the course of time, this quality disappears, the protection ceases. It is surprising perhaps that the experimenters did not use also paraffine, which comes the nearest to being truly non-porous, though, of course, it is not a practical paint or coating for such work.

The necessity for the greatest possible natural bond between the concrete and reinforcing steel needs to be borne in mind in considering the practical use of these insulating paints. Their use will not be entertained for reinforced-concrete structures by designing engineers. The other remedies of (1) keeping the concrete fairly dry; (2) isolating it from ground waters, and (3) preventing random electric potentials upon steel or concrete, must be relied upon, and so far as present rather academic experiments and practical experience go there is little or no danger to be expected if these conditions are satisfactory.

The University of Vermont Experiments. Two series of experiments were planned by the writers a year ago and now have been carried out (under their general supervision and with the assistance of Professor J. W. Elliot), by Messrs. O. J. Olgiati and H. Roberts, senior students in the departments of electrical and civil engineering, as their bachelor thesis. The series were outlined in the manner hereinafter shown, after having followed the progress of such experiments for several years and with the belief that added light could be thrown on certain obscure points.

For instance, Mr. U. J. Nicholas, in describing[2] experiments carried on at the Massachusetts Institute of Technology, found that specimen blocks could be cracked irrespective of the direction of current—whether the central imbedded electrode was anode or cathode. These tests added a little to our knowledge in that they indicated that the cracking of the concrete was due perhaps more to the indirect localized heating or to the mechanical action of disseminated rust. This left us to guess whether the softening and very general disintegration was due to the electric current or to some secondary chemical reactions of the iron, or to mere mechanical stress.

Professor A. S. Langsdorf, in 1909, added[3] to the corroborative evidence then accumulated. Progressive corrosion of imbedded steel was shown; all specimens developed cracks and some showed soft disintegration; a coating of rust developed on the walls of cracks.

Eltinge and Beers have described[4] certain experiments carried on at the Rensselaer Polytechnic Institute in which they duplicated the cracking and soft disintegration of electrolysis by externally heating the projecting end of the imbedded bar.

The first series of tests planned by the writers was intended to be wholly qualitative and using the studies already mentioned as points of departure. Several points should be specifically noted.

1. Collapsible iron electrodes, shown in Fig. 2, were used in the hope of showing whether or not local thermal expansion, or the demand of the oxidized iron for more room at the electrode

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[2] *Engineering News*, December 24, 1908.

[3] *Journal of the Association of Engineering Societies*, February, 1909; Abstracted in *Engineering News*, April 29, 1909.

[4] *Engineering News*, March 31, 1910.

surface would crush the electrode instead of cracking and softening the concrete.

2. The use of imbedded carbon electrodes, in one set of specimens, was tried to study the effect of eliminating iron and iron oxide.

3. By using loose carbon electrodes the effect on cracking, of eliminating the mechanical forces at the electrode surface and the iron oxides in the concrete, was studied.

4. Duplicate specimens with these several types of electrodes were to be subjected to alternating current, keeping all conditions, except the kind of current, the same as in the direct-current tests.

5. Special specimens were subjected to thermal stresses according to the temperatures actually noted in the other trials.

All of the specimens, except two, were of the same general shape and make-up except as noted. Fig. 1 shows the arrangement with iron pipe, and Fig. 2 shows the collapsible electrode used in its place.

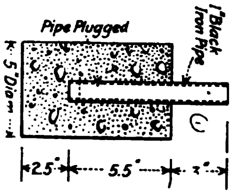

FIG. 1

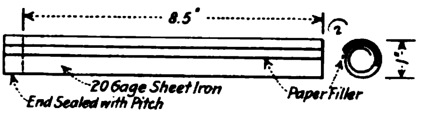

FIG. 2

" Ironclad " brand Portland cement was used and the quality-test results compared well with the specifications of the American Society for Testing Materials, as shown in Table I.

TABLE I.—QUALITY TESTS OF " IRONCLAD " CEMENT USED IN UNIVERSITY OF VERMONT ELECTROLYSIS EXPERIMENTS

Fineness:

Left on 200 sieve 23.80 per cent

Left on 100 sieve 5.85 per cent

Specific Gravity:

$$\frac{\text{Weight cement in grams}}{\text{Displaced volume in cu. cm.}} = \frac{65}{20.6} = 3.11$$

Breaking Strengths,

Neat cement briquettes:				
Days age	No. 1	No. 2	No. 3	Average
7	480	550	588	539
28	625	645	667	646
1:3 sand-cement mortar:*				
7	90	95	107	97
28	185	190	103	159

*The low breaking strength of the mortar was probably due to the excessive dryness of the sand.

The sand used in all tests and experiments was of uniform size, passing through a 10-mesh and remaining on a 30-mesh screen, and was thoroughly dry. The aggregate was ½-in. broken gray sandstone, very dry. The specimens subjected to current were of a 1:2:4 mixture of cement, sand and stone. In all, 28 specimens were made for these qualitative studies, six with each of the four types of electrodes and two of each special type shown in Figs. 3 and 4.

The specimens were connected in series-parallel circuits, as shown in Figs. 5 and 6, to facilitate regulation of the current. Each group contained one block with each type of electrode.

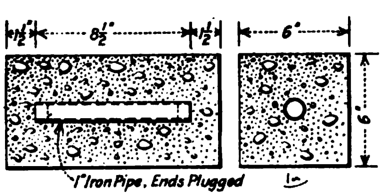

FIG. 3

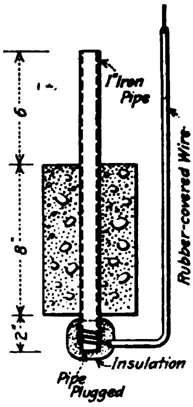

FIG. 4

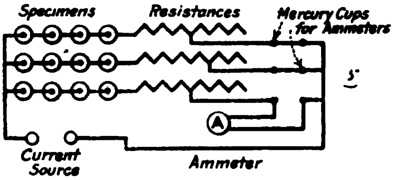

FIG. 5

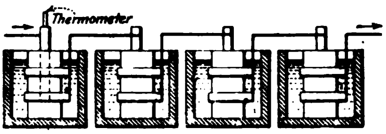

FIG. 6

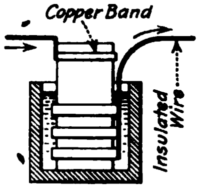

FIG. 7

Two duplicate tests were run simultaneously, one with alternating and one with direct current. Current was regulated several times a day to the value of 0.1 ampere. A small generator set was used for the direct current and the city supply system for the alternating current.

To determine temperature effects, the special specimens, shown in Fig. 4, were tested with the others. They were placed in water as in the electrolysis experiments, but the current was confined to the metallic circuit and regulated to secure the same temperature of a thermometer in oil in the pipe as in the electrolytic runs. The temperatures found in the latter runs were so low that a current of 0.1 ampere through the pipe of the temperature specimen was enough.

For simplicity, the results will be presented separately for each type of electrode.

Solid Pipe. The behavior on direct current was in accordance with results of other experimenters. When current was first turned on, there was an evident washing out of cement or apparently similar materials, with the evolution of much gas at the imbedded electrodes. At the end of 24 hours a notable amount of iron oxide had been forced out around these electrodes. After a few days the entire top of the specimens, which were out of water as shown in Fig. 6, was covered with oxide. During the experiments gases were given off more or less from all parts of the concrete. The average centigrade temperatures were as shown below:

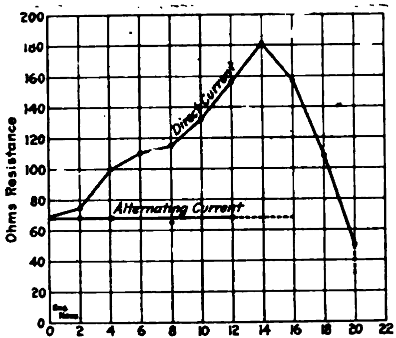

FIG. 8

Room	Thermal test specimen	Direct-current set	Alternating-current set
22 deg.	22.1 deg.	22 deg.	22.15 deg.

The " apparent " resistances of all blocks were measured by the voltmeter-ammeter method. Table II shows typical results on one set and the data are plotted in Fig. 8.

TABLE II.

No. days	Volts drop	Amperes	Resistance
RESISTANCE OF DIRECT-CURRENT SPECIMENS			
2	7.5	0.1	75.0 ohms
4	10.0	0.1	100.0 "
6	11.0	0.1	110.0 "
8	11.5	0.1	115.0 "
10	13.2	0.1	132.0 "
12	15.7	0.1	157.0 "
14	18.1	0.1	181.0 "
16	15.8	0.1	158.0 "
18	10.8	0.1	108.0 "
20	5.0	0.1	50.0 "
RESISTANCE OF ALTERNATING-CURRENT SPECIMENS			
2	6.8	0.1	68 "
4	6.8	0.1	68 "
8	6.6	0.1	68 "
12	6.8	0.1	68 "

The gradual increase in resistance may perhaps be due to the accumulation of iron oxide, decreasing porosity, accumulated gas, or to some chemical change in the cement. It seems probable that the iron goes into solution as a hydrate and is later converted to oxide by evolved oxygen and perhaps by the cement compounds. If the last action was general throughout the mass of concrete, it might be expected to diminish the strength. If, however, the action was confined to the neighborhood of the electrode there might be no appreciable change in the general strength of the concrete mass. The sharp peak of the direct-current resistance curve, Fig. 8, suggests the beginning of mechanical failure.

A specimen on direct current showed distress at the end of 20 days and ruptured, on the following day, so that it could be pried apart with a screw driver. The first indication of failure was the appearance of small drops of colored water on the upper surface of the concrete. These places developed into the cracks along which the blocks broke. As expected, the iron was considerably corroded and the rust disseminated throughout, though more marked along the line of failure. The concrete itself could easily be broken with the fingers.

FIG. 9

FIG. 10

The specimens subjected to alternating current did not break at low current density, nor was there any corrosion of the electrode. The appearance of the concrete and general strength were not changed. No gas was liberated by the current, and it was not manifest by other signs. At the completion of the tests alternating current of up to 3 amperes was sent through the blocks. At 2 amperes the internal temperature increased to 45 deg. cent. and the blocks split after being in circuit 8 to 10 hours. They were broken apart with a hammer and, as seen in Fig. 9, the iron was not corroded. The disruption seems to have been the result of temperature stresses. The specimens with various solid imbedded electrodes behaved similarly, all splitting apparently from the same cause.

Collapsible Electrodes. The specimens with collapsible electrodes, on direct current, were the second in order of breaking. This tardiness might have been due in a small measure to slightly larger electrode diameter and probably, to a greater degree, to the yielding of the collapsible electrode to the demand of the oxide for more room. The disruption then might have come finally from the stresses set up by the formation of iron oxide in the pores of the concrete a little distance in from the electrode. On alternating current the collapsible-electrode specimens showed no signs of distress. The concrete was not softened and the blocks did not split on increasing the current to 3 amperes.

Imbedded Carbon Electrodes. It was surprising to find that the behavior of these specimens under direct current was similar to that of those with iron electrodes. In place of iron oxide, a thick black material, apparently a combination of carbon and some ingredient of the cement, was forced up and out around the electrode. The composition of this material was not studied.

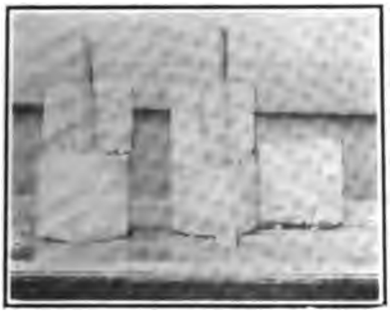

FIG. 11

FIG. 12

The carbon electrode was eaten away and considerable dissemination of the corrosion product was found on prying the specimens apart, as seen in Fig. 12. The increase in resistance of these specimens was not so decided as in the case of iron electrodes. Cracks developed in 25 days. The softening of the concrete was noticeable, but not as marked as with iron electrodes. On alternating current no signs of damage were found. The electrode was not corroded and the concrete was not softened.

Loose Carbon Electrodes. With direct current, gas was freely evolved at the carbon electrode and a decided smell of chlorine noted, though no chlorine salts or other chemicals were added to the water. A black compound was formed, apparently identical with that found with the imbedded carbon electrodes and the electrode was deeply pitted. After being in circuit for some four weeks, no disintegration of the concrete was found. A current of one ampere was then forced through for 12 hr., but without apparent effect. The specimens were broken open with a hammer, and appeared as shown in Fig. 14.

On alternating current no ill effects were noted, as may be inferred from Fig. 15.

Completely Imbedded Iron. These specimens were made up to approximate actual construction conditions, where the random potentials might not be actually applied to the reinforcement. The current was led into the concrete as shown in Fig. 7. After 18 days, with 0.1 ampere, direct current, the block split, showing typical corrosion of the iron and dissemination of oxide. The specimen on alternating current was not affected, and is shown, at the left in Fig. 16, after being broken open with a hammer.

Thermal-test Specimen. The steel in this was kept at the observed temperature of the iron-pipe specimen on direct current during the entire run. The temperatures varied but slightly from those of the room and the specimen remained sound. The pipe was not found rusted, on breaking open at the end of the tests.

Quantitative Studies. It was hoped by subjecting a large number of specimens, with the various types of electrodes previously mentioned, to similar treatment, and by breaking samples at regular intervals, to determine whether the destructive actions were gradually developed or suddenly. The specimens were made in standard 4-in. molds and were set under water for three weeks before use. A 1:2 cement-sand mortar was used, the cement and sand being the same as already noted. The current was limited to 0.03 ampere, to eliminate chance of temperature stresses. Fig. 17 shows the general make-up of the blocks, the only difference in the several specimens being in the use of iron-pipe, collapsible iron, imbedded and loose carbon electrodes.

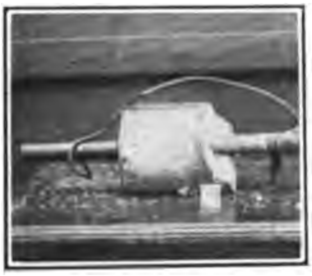

FIG. 13

FIG. 14

In all, 22 blocks with each of the four types of electrode were made; 20 blocks of each type were placed under current. In addition, 20 dummy cubes, without electrodes, were made and used under same conditions, except as to current. At the start, two blocks of each type were crushed and at regular intervals two more of each type were crushed. The iron-pipe electrodes were weighed before being placed in the blocks and as broken to determine the loss of iron during the test. Even loading in the testing machine was secured by using blotting paper between the block and the crushing heads of the machine. The load was applied at right angles to the direction of electrode axis. The results of the tests are shown in Table III. The electrical circuits were the same as for the qualitative series, in general.

FIG. 15

Conclusions. All these experiments, studied in connection with previous ones, variously noted, allow several tentative conclusions, concerning the seat and mechanism of electrolytic disruption of reinforced concrete. It is to be emphasized that these are only tentative and do not affect any recognized practical measures which may be taken to prevent conditions leading to structural failure in the reinforced concrete, as already noted.

FIG. 16

1. The disruption of reinforced concrete may result from stresses produced by local thermal expansion where the current density is high, with either alternating or direct current and irrespective of the direction of current in the latter case.

TABLE III

RESULTS OF QUANTITATIVE TESTS ON ELECTROLYTIC DISINTEGRATION OF REINFORCED CONCRETE.

Iron-pipe electrodes					Collapsible iron electrodes			Imbedded carbon electrodes			Loose carbon electrodes		
No. speci-men	No. hr. in cir-cuit	Average loss per day, grams	Breaking load of specimen	Breaking load of dummies	No. of speci-men	No. hr. in cir-cuit	Breaking strength	No of speci-men	No. hr. in cir-cuit	Breaking strength	No. of speci-men	No. hr. in cir-cuit	Breaking strength
0*a*	0	0.0	20,100		0*b*	0	14,400	0*c*	0	30,000	0*d*	0	14,810
1*a*	66	0.215	19,600	18,070	1*b*	66	14,230	1*c*	66	22,700	1*d*	66	15,000
2*a*	66	0.175	18,680		2*b*	66	12,910	2*c*	66	19,630	2*d*	66	18,000
3*a*	138	0.202	19,550	17,900	3*b*	138	13,670	3*c*	138	25,120	3*d*	138	18,560
4*a*	138	0.178	18,750		4*b*	138	12,800	4*c*	138	25,740	4*d*	138	16,900
5*a*	186	0.194	19,600	20,900	5*b*	186	14,250	5*c*	186	24,250	5*d*	186	22,010
6*a*	186	0.195	24,200		6*b*	186	10,000	6*c*	186	23,800	6*d*	186	19,020
7*a*	282	0.202	20,800	20,320	7*b*	282	14,720	7*c*	282	24,340	7*d*	282	18,450
8*a*	282	0.182	19,950		8*b*	282	13,870	8*c*	282	21,700	8*d*	282	17,180
9*a*	354	0.197	22,630	24,325	9*b*	354	14,800	9*c*	354	23,200	9*d*	354	18,110
10*a*	354	0.203	22,740		10*b*	354	10,460	10*c*	354	17,650	10*d*	354	15,500
11*a*	426	0.211	19,860	23,500	11*b*	426	12,530	11*c*	426	18,350	11*d*	426	16,880
12*a*	426	0.214	19,200		12*b*	426	12,570	12*c*	426	24,000	12*d*	426	15,440
											13*d*	498	19,200
Test stopped for three days					Test suspended for three days			The test was stopped for three days			14*d*	498	15,300
											15*d*	570	12,000
13*a*	498	0.210	28,710	25,980	13*b*	478	15,590	13*c*	498	33,500	16*d*	570	12,460
14*a*	498	0.198	26,110		14*b*	478	14,900	14*c*	498	32,100	17*d*	642	19,400
15*a*	570	0.197	30,450	27,010	15*b*	570	17,370	15*c*	570	37,250	18*d*	642	16,700
16*a*	570	0.176	28,350		16*b*	570	19,100	16*c*	570	29,250	19*d*	744	19,700
17*a*	642	0.223	20,500	31,300	17*b*	642	18,300	17*c*	642	30,300	20*d*	744	17,650
18*a*	642	0.190	29,600		18*b*	642	18,200	18*c*	642	26,100			
19*a*	744	0.194	36,000	31,245	19*b*	744	12,500	19*c*	744	24,500			
20*a*	744	0.188	27,740		20*b*	744	15,600	20*c*	744	31,700			

2. With small current densities and without temperature stresses being developed, disruption finally results from the accumulation of iron oxide at and near the imbedded iron where direct current leaves the iron to enter the concrete.

3. The bursting stresses developed with low current densities are largely caused by the formation of iron oxide, immediately at the anodic surfaces to a slight extent, but largely in the concrete near the anodes, the depth of penetration depending on the porosity of the concrete, the gas pressure, current strength, etc., among other things.

4. No gradual reduction of the strength of the concrete has been shown to accompany progressive corrosion of the iron. The oxygen for the formation of oxide probably is furnished mostly by the electrolysis of water and only to a slight degree is taken from the cement compounds.

5. Disruption may result with low current densities, where iron is not the anode material, if other compounds are formed from the anode or from the concrete and expand during the secondary reactions. In all cases, pressure of gases developed may slightly assist or hasten disruption.

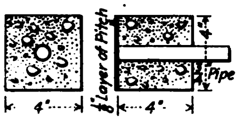

Fig. 17

6. The general softening of the concrete after rupture is similar to that in specimens crushed mechanically and may be here regarded as largely the result of the bursting stresses.

7. A single current of small density may cause corrosion and disruption at several points by passing to and leaving a number of pieces of reinforcing steel, each completely imbedded.

8. With large leakages of either direct or alternating current to reinforced concrete, structural failures may be expected within a few hours of the start of current, and as the result of thermal effects.

9. With small leakages of direct-current an insidious undermining of structural strength must be expected.

10. With alternating currents, when the current density is low enough to eliminate local temperature stresses, repeated tests have shown no danger of structural troubles. This is in accord with the findings reported by Mr. J. L. R. Hayden, before the A. I. E. E., March, 1907, but does not mean that there is absolutely no effect from alternating currents. It would indicate rather that the secondary reactions were so slow that the primary reactions were practically reversed each cycle.

11. Structural trouble is not to be expected with dry concrete unless random high direct-current voltages are noted on the structural members.

12. Present methods of electrical survey seem sufficient to detect dangerous (to the structure) voltages about reinforced concrete structures and remedies are available for application as each individual case seems to demand.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

WAVE SHAPE OF CURRENTS IN AN INDIVIDUAL ROTOR CONDUCTOR OF A SINGLE-PHASE INDUCTION MOTOR

BY H. WEICHSEL

The phenomena which take place in the rotor of a single-phase motor, can be explained in two different ways. The first is that proposed by Galileo Ferraris, and consists in the resolution of a single-phase field into two fields rotating in opposite directions. The second method, which we might call the "cross field method," has been proposed by several authors.* Mr. V. Fynn lately developed this method to a very large extent, treating alternating-current motor problems on a basis similar to that of the direct-current motor. The cross field method has given a big impetus to the general understanding of the working conditions of the single-phase motor. Both methods lead naturally to the same result. Frequently it is of interest and very instructive to solve certain problems by the use of both methods, since a comparison between the two frequently throws more light on the subject. In this paper the wave shape of the currents in an individual conductor of a single-phase motor will be determined, first by the rotating field method; and secondly by the cross field method.

Ferraris has shown that a single-phase field can be resolved into two fields of equal magnitude rotating in opposite directions

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*A. Poitier, *Bulletin de la Societé Internationale des Electriciens*, May 27, 1894; H. Goerges, *Elektrotechnische Zeitschrift*, Jan. 17 and Nov. 28, 1895; Atkinson, Feb. 22, 1898; McAllister, *American Electrician*, June, 1902; Franklin, TRANSACTIONS A. I. E. E., 1904, XXIII, p. 429; V. Fynn, British Institution of Electrical Engineers, Feb., 1906; McAllister, *Electrical World*, Aug. 1906; V. Fynn, British Institution of Electrical Engineers. Dec., 1907.

with equal angular velocity. Let the amplitude of the single-phase field be $\bar{N}$ and it follows that the amplitude of each of the rotating fields must be $\frac{\bar{N}}{2}$. One of these fields rotates clockwise and the other rotates counter-clockwise. Let us assume that the armature runs clockwise also, and with synchronous speed with respect to the line frequency, or what means the same thing, with the speed of the field which rotates clockwise. The rotor bars, therefore, do not cut this field; no e.m.f. can be induced in the bars due to this field. But the rotor bars rotate against the second component or the counter-clockwise rotating field, N_2, with a speed equal to double the synchronous speed. The e.m.fs. set up in the bars must therefore be of double frequency, and have a time phase displacement with respect to each other equal to their space displacement expressed in electrical degrees. If the rotor bars are all short-circuited, as is the case in the squirrel cage construction, it is evident that currents of double frequency will be set up in these bars having the same time phase displacement against each other as the e.m.fs. producing them. That is, the time phase displacement of the rotor currents with respect to each other is equal to the space displacement of their conductors. From this it is evident that these are polyphase currents, and the field produced by them is rotating against the individual conductors with a speed equal to twice the synchronous speed of the line. This field rotating with double frequency in a direction opposite to that of the rotor bars, has a speed with respect to a fixed point in space equal to synchronous speed, referred to line frequency, and in a direction opposite to that of the rotor rotation.

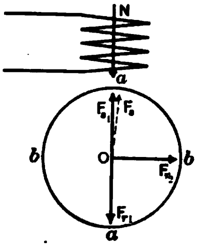

Fig. 1

The rotor in a single-phase motor is completely surrounded by iron. Therefore if we neglect the effect of ohmic resistance the currents produced by the e.m.fs. in the rotor bars will have a time displacement of 90 deg. behind their e.m.fs. The field set up by these currents, as previously explained, is a rotating field with direction of rotation opposite to that of armature. We can gain a better insight into the phenomena occurring in the rotor if we resolve this field, due to the polyphase rotor currents, into two fields at right angles. One of these components must be in line with, but directly opposite the main stator field. Referring to Fig. 1, the line $O\ F_{r1}$ represents the component of the field set up by the rotor currents which is in line with the main stator field $O\,F_{s_1}$. These two fields $O\,F_{r_1}$ and $O\,F_{s_1}$ annul each other, but since the main stator field cannot be eliminated, the m.m.f. of the stator must be increased until the original number of lines are set up. The additional field of the stator is represented in Fig. 1 by $O\,F_s$, and this is the final remaining field in that axis counterbalancing the line voltage. The other component of the rotating field produced by the rotor currents is shown by the vector $O\,F_{r_2}$. The resultant of these two fields which are at right angles produces the true rotating field of the induction motor. This resultant field rotates in the same direction as the rotor and with line frequency. This will be seen if we remember that the resultant of the fields $O\,F_{r_1}$ and $O\,F_{r_2}$ is the field produced by the rotor currents and rotating in direction opposite to that of the rotor bars. But one of these components has been annulled by the stator field $O\,F_{s_1}$ and the main field $O\,F_s$ is the only one remaining in this axis. This, however, is displaced 180 deg. from $O\,F_{r_1}$ and therefore the resultant of $O\,F_s$ and $O\,F_{r_2}$ must produce a rotating field which is rotating in direction opposite to that of the field produced by $O\,F_{r_1}$ and $O\,F_{r_2}$. It is clear also from the above discussion that the m.m.f. of the stator is just twice what it would be if the rotor had no conductors.

The results we have obtained so far may be seen more clearly, and their importance more fully realized, if we solve the same problem on a mathematical basis. Assume that the voltage impressed across the stator winding follows the sine law. For convenience we will express this as the cosine function

$$E_t = \overline{E} \cos (m\,t) \text{ where } m = 2\,\pi\,v$$

The induced voltage that counterbalances this is

$$E_t' = \overline{E} \cos (m\,t + 180) = -\frac{d\,Nt}{d\,t}$$

$$N_t = -\int \overline{E} \cos (m\,t + 180)\,d\,t$$

$$N_t = +\int \overline{E} \cos (m\,t)\,d\,t$$

$$N_t = \frac{\overline{E} \sin (m\,t)}{m}$$

This is the main transformer flux. Since the flux is proportional to the impressed e.m.f. we may write

$$N_t = \overline{N} \sin (m\ t)$$

where N_t is the value of the flux at any time t and $\overline{N}$ is the maximum value of the flux.

Now let us assume that the field induction under the pole also follows the cosine law. The induction at any point can therefore be expressed as

$$B_\alpha = +B_c \cos \alpha$$

where B_c represents the instantaneous value of the field induction in the center of the pole (see Fig. 2). The induction may be expressed in terms of the total number of lines as follows:

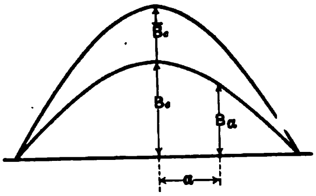

FIG. 2

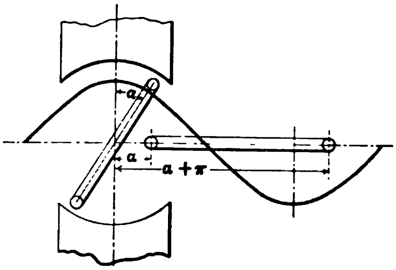

FIG. 3

$$\overline{N} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \overline{B} \cos \alpha = 2 \int_{0}^{\frac{\pi}{2}} \overline{B} \cos \alpha = 2\,\overline{B} \Big[\sin\ \alpha \Big]_0^{\frac{\pi}{2}} = +2\,\overline{B}$$

$$\overline{B} = +\frac{\overline{N}}{2}$$

This is the true maximum of the field induction. The value of the induction in the center of the pole at any other instant is

$$B_{c_t} = +\frac{N_t}{2}$$

The induction at any point other than the center may therefore be expressed as follows:

$$B_{\alpha_t} = +\frac{N_t}{2} \sin (m\ t)$$

$$= +\frac{\overline{N}}{2} \sin (m\ t) \cos \alpha$$

Any armature loop which spans a complete pole pitch and whose position is such that one side of the loop is the angle α from the center line of pole, as shown in Fig. 3, includes N_α lines, where N_α is given by the expression

$$N_\alpha = \int_{\alpha}^{\alpha+\pi} B_{\alpha t}\, d\,\alpha = \int_{\alpha}^{\alpha+\pi} \frac{\overline{N}}{2} \sin{(m\,t)} \cos \alpha$$

$$= \frac{\overline{N}}{2} \sin{(m\,t)} \Big[\sin \alpha\Big]_{\alpha}^{\alpha+\pi} = -\overline{N} \sin{(m\,t)} \sin \alpha$$

If the armature rotates with synchronous speed the angle α is equal to $(m\,t+\varphi)$ where φ is the angle which the armature loop forms with the center line of the stator pole at the time $t=0$. Substituting this value for α we get

$$N_\alpha = -\overline{N} \sin{(m\,t)} \sin{(m\,t+\varphi)}$$

The voltage induced in this armature loop by the N_α lines interlinked with it is given by the equation

$$E_t'' = -\frac{d\,N_\alpha\,s}{d\,t} \times 10^{-8}$$

where s is number of turns in the loop;

$$E_t'' = \frac{d\,\overline{N}\,s \sin{(m\,t)} \sin{(m\,t+\varphi)}}{d\,t}$$

In order to solve this equation for the induced voltage we will simplify the second member.

$$\sin{(m\,t+\varphi)} = \sin{(m\,t)} \cos \varphi + \cos{(m\,t)} \sin \varphi$$

Substituting this we get

$$E_t'' = \frac{d\,\overline{N}\,s\,\{\sin^2{(m\,t)} \cos \varphi + \cos{(m\,t)} \sin{(m\,t)} \sin \varphi\}}{d\,t}$$

We may also write

$$\cos{(m\,t)} \sin{(m\,t)} \sin \varphi = \tfrac{1}{2} \times \sin{(2\,m\,t)} \sin \varphi$$

$$E_t'' = \overline{N}\,s \left\{ \frac{d}{d\,t} \sin^2{(m\,t)} \cos \varphi + \tfrac{1}{2} \frac{d}{d\,t} \sin{(2\,m\,t)} \sin \varphi \right\}$$

$$= \overline{N}\,s \times m \{\sin{(2\,m\,t)} \cos \varphi + \cos{(2\,m\,t)} \sin \varphi\}$$

$$= \overline{N}\,s \times m \sin{(2\,m\,t+\varphi)} = +\overline{E''} \sin{(2\,m\,t+\varphi)}$$

This is the e.m.f. produced in the rotor loop of s turns by rotating the armature conductors in the alternating stator field. This voltage sets up currents in the rotor conductors which in turn produce magnetic lines. These lines set up a counterbalancing e.m.f., E_t''', so that E_t''' is displaced 180 deg. from E_t''.

$$\overline{E}_t''' = -\frac{d\,N_2}{d\,t}$$

where N_2 is the number of lines produced by the rotor currents.

$$\overline{E}'' \sin(2\,m\,t+\varphi+180) = -\frac{d\,N_2}{d\,t}$$

$$N_2 = \int \overline{E}'' \sin(2\,m\,t+\varphi)\,d\,t$$

$$N_2 = \int \overline{E}'' \sin(2\,m\,t)\cos\varphi\,d\,t + \int \overline{E}'' \cos(2\,m\,t)\sin\varphi\,d\,t$$

$$N_2 = -\frac{\overline{E}''}{2\,m}\left\{\cos(2\,m\,t+\varphi)\right\}$$

This is the equation for the lines produced by the secondary current. Since the lines are directly proportional to the secondary current we may write

$$i_2 = -k\cos(2\,m\,t+\varphi)$$

The negative sign refers to its direction compared with the primary; k is a constant. The equation gives the current set up at any time t in an armature loop which formed the angle φ with the center of the pole when t was equal to zero. For the conductor that lies directly under the pole center at this instant $\varphi = 0$ and the equation for the current is

$$i_2 = -k\cos(2\,m\,t)$$

From this it will be seen that the currents in two successive armature loops have a time phase displacement equal to the space displacement of their conductors, and a frequency just twice that of the impressed e.m.f.

In the above discussion we have neglected the effects of rotor resistance as compared with rotor inductance; therefore the currents produced in the secondary conductors will be displaced 90 deg. from the e.m.fs. producing them, as shown in the equations just derived. In order to see the effects of these rotor currents, we must consider the limiting cases of the equation for i_2. In the conductor under the center of the pole at the time $t=0$, the current is a maximum, in direction opposite to that of the impressed voltage. In the conductor 90 deg. from the center of the pole at this same instant the current is zero. This means that no field is set up in the direction of axis $a-a$, Fig. 1, but there is a field set up in the direction of axis $b-b$ having its maximum at this instant. In the same way we will find that at a time $t, =\frac{T}{4}$ the field in direction of $b-b$ is zero, and the field in direction $a-a$ has its maximum. We assumed at the outset an alternating field N in axis $a-a$ of constant magnitude due to the impressed voltage. We now see that the rotor currents produce a field in this same axis. In order, then, for the main field N to remain, an additional field (or currents) must be set up in the stator to annul the effects of rotor currents. This explains the fact that a single-phase motor running at synchronism and no load draws more current with the rotor short-circuited, than when the rotor is open-circuited.

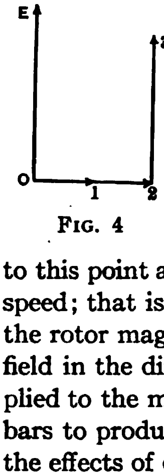

FIG. 4

The currents that we have been discussing up to this point are those which flow in the rotor bars at synchronous speed; that is, at no load. We might therefore call these currents the rotor magnetizing currents because it is due to them that the field in the direction of axis $b-b$ exists. As soon as a load is applied to the motor additional currents will be set up in the rotor bars to produce the necessary torque. If we neglect at present the effects of ohmic resistance and leakage it will be evident that the vector diagram for the stator conditions can be represented by Fig. 4. There are flowing in the stator three different current components. The vector 0–1 represents the magnetizing current of the stator when the rotor is open-circuited. The vector 1–2 represents the additional magnetizing current drawn from the line when the rotor bars become short-circuited. This component counterbalances the effect in axis $a-a$ produced by the no-load currents in the rotor. In the case of the ideal motor without leakage or resistance in the rotor, the component 0–1 equals the component 1–2. The third current component 2–3 is at 90 deg. phase displacement from these first two components, or what means the same thing, it is in phase with the impressed voltage. It is therefore the watt component of the total current. Since only the magnetic field produced by the wattless component 0–1 can exist in the axis $a-a$, as otherwise the voltage $O-E$ could not be counterbalanced, it follows that the magnetic action of the current component 2–3 must be equalized by currents flowing in the rotor. For the time being let us assume that the rotor runs at synchronous speed even if loaded. The currents flowing in the individual rotor bars must be of such a nature as to produce a field stationary in space in direction of the axis $a-a$ but alternating in time with the frequency of the line current 2–3. Such a field will be produced by a double-frequency current superposed upon a direct current, as will be seen from the following:

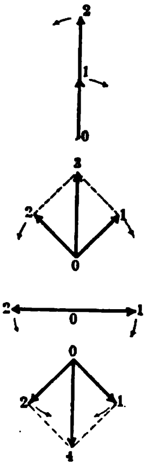

Fig. 5

Double-frequency currents must flow so as to produce a field rotating in space opposite to the direction of the armature rotation, and at double synchronous speed with respect to the individual rotor bars. This field therefore is rotating against a fixed point in space at synchronous speed with respect to line frequency. If at the same time a direct current flows in the rotor bars, it is evident that the field set up by the direct current will be constant and stationary with respect to the rotor bars. But the rotor bars rotate with synchronous speed, so this constant field must rotate against a fixed point in space with the same speed and in the same direction as the armature. The two fields we have just considered are equal in magnitude, and rotate with respect to a fixed point in space with equal speeds but in opposite directions. The resultant field produced is alternating with line frequency, but is stationary in space as represented in Fig. 5. This represents four different instants in the period of the fields. When the components are in phase the resultant is the sum of the two. At a later instant 0–3 is the resultant as shown by the vector addition. Still later the components are opposite in phase, so the resultant is zero. The last view shows the resultant opposite in direction to the first two. These diagrams show clearly that the resultant of the fields produced by the components of the armature working current is an alternating field stationary in space. The strength of this resultant alternating field must be such as to counterbalance the magnetic effect of the stator current component 2–3 as shown in Fig. 4. But as the true maximum of the resultant field equals twice the field strength of one of the rotating field components, it follows that the strength of each rotating field component must be one-half of the true maximum value of the stator field which corresponds to the stator current 2–3.

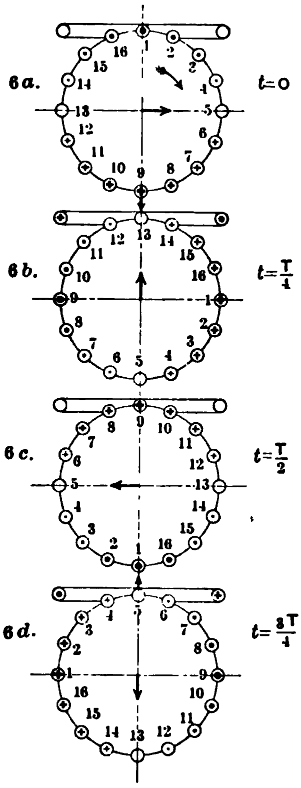

FIG. 6

Let N_a represent one rotating field component due to the working component of the armature current, and let N_s be the field due to the stator current vector 2–3. Then $N_a = \frac{N_s}{2}$.

From a consideration of the above we may determine the value of the direct-current component of the rotor. It must be of such magnitude as to produce a field equal to that produced by the double-frequency current.

It was assumed at the outset that all fields should have sine-shaped distribution. The field produced by the direct current must therefore be sine-shaped, and the only way this is possible is for the currents in each bar to be different at any one given instant, and for this variation to follow the sine law.

A graphical representation of the different currents flowing in one conductor of a single-phase rotor at no load, will show more clearly the physical meaning of the phenomena discussed above. Let us consider that the rotor bars 1, 2, 3, etc., Fig. 6*a*, have, at the time $t = 0$, the position indicated. Assume further that the line voltage has its maximum value at the time $t = 0$. The line voltage may therefore be graphically represented by curve E in Fig. 7. In order to counterbalance this voltage, a current, represented in the figure by the curve i_0, must flow through the primary winding, and this lags 90 deg. behind the impressed e.m.f. The lines produced by this current are in phase with it. At the time $t=0$, therefore, the stator current has its zero value. The rotating field produced by the rotor currents has its maximum in the axis $b-b$ and no component in the axis $a-a$. The maximum rotor current flows therefore at this instant ($t=0$) in conductors No. 1 and No. 9. Conductors No. 2, 3, 4 and 5 carry less current than conductors No. 1 and 9, and the magnitude of the current in each is proportional to the cosine of the angle the rotor bar makes with the center of the stator pole. Fig. 7*b* shows graphically the magnetizing current flowing in rotor bar No. 1. The currents in rotor bars No. 3, 5, 7 and 9 are shown in Figs. 7*c* to 7*f*, each having a phase displacement corresponding to the angular position of the rotor bar. Figs. 6*b* to 6*d* represent the positions of the rotor bars and the directions of currents at the instants $t=\frac{T}{4}$, $t=\frac{T}{2}$ and $t=\frac{3T}{4}$

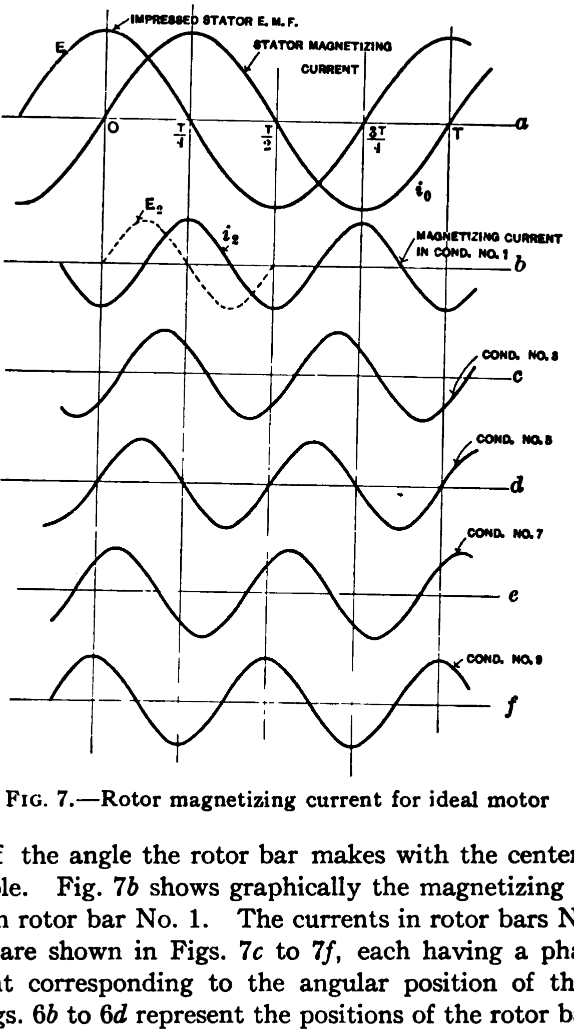

FIG. 7.—Rotor magnetizing current for ideal motor

where T equals the time of one period. In these diagrammatic views the stator has been indicated by a single loop, and the direction of currents by crosses and points. These latter are shown light or heavy to indicate as far as possible the relative magnitudes of the currents. It is plainly evident that the stator winding carries its maximum current at the instant that the rotating field produced by the rotor currents lies in the direction of the center line of stator pole, at the same time the field due to stator currents tends to annul the other field.

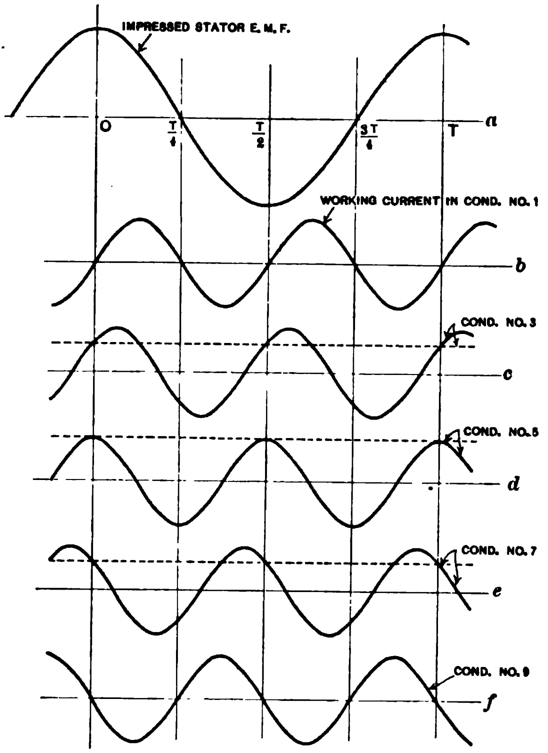

Fig. 8.—Rotor working current components for ideal motor

It must be remembered that the diagrams above referred to show the no-load condition of the ideal motor without ohmic resistance. In a similar manner we may show the currents that flow when the motor is loaded. We will still hold to the assumption that the motor has no ohmic resistance. To simplify the diagrams we will omit the magnetizing currents which have been represented in Figs. 6 and 7. Fig. 8*a* again represents the impressed line voltage. The working current drawn from the line will be in phase with the impressed voltage and can therefore be represented by the same curve as the impressed e.m.f. This curve has a maximum at the time $t=0$, and its magnetic effect must be cancelled by the rotor currents. From a consideration of Fig. 9*a*, it will be seen that the rotor currents must be a maximum in conductors No. 5 and 13 and zero in No. 1 and 9 in order that the resulting field may be in such a direction as to oppose the field due to the stator working currents. This is true for both components of the working currents. After the time $t=\frac{T}{4}$, or what means the same thing, after the time taken by the armature to rotate 90 deg. in a clockwise direction, the currents in the rotor conductors must be such as to produce no field at all. This condition will be fulfilled if that component of the working current which has double frequency is counterbalanced by the other component of the working current which is constant. As represented in Fig. 9*b*, conductor No. 1 has moved 90 deg. in space. In order to show the distinction between the alternating current and direct current the rotor has been represented as consisting of two squirrel cage windings, one carrying the constant working current, and the other the double-frequency working current. In a practical motor these two windings are one, and the double-frequency current is superposed upon the direct current. As explained above, the currents must be such as to produce no resultant field at all. Hence the two fields due to the windings are directly opposing. The currents in each separate rotor conductor have been shown in Figs. 8*b* to 8*f*. The conditions at the time $t=\frac{T}{2}$ are shown in

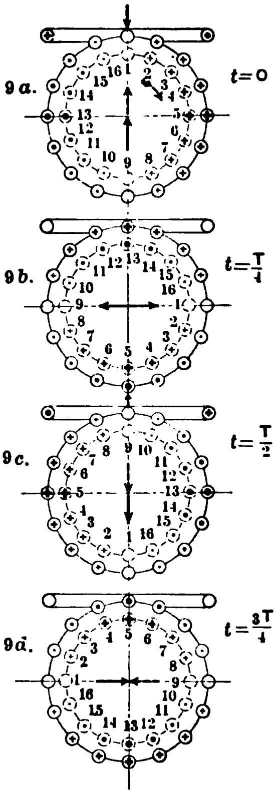

FIG. 9

Fig. 9*c*. The fields due to the two rotor currents are again assisting, but now in opposite direction to that shown in Fig. 9*a*. Ninety degrees later the currents are opposing each other with a resultant of zero, as shown in Fig. 9*d*. These four views indicate a complete cycle in the stator current and the resultant of the two rotor currents.

All of the derivations and graphical representations dealt with in the previous pages have been based on the assumption that the rotor had no ohmic resistance, and was running at synchronous speed even though the motor carried load. In order to make the problem more general, we must consider the conditions that exist when the ohmic resistance of the rotor is taken into account. This means that the speed of the rotor will be no longer equal to the synchronous speed of the line. In other words, the rotor has a certain slip. Let us consider first what is taking place in the rotor so far as the rotor magnetizing currents are concerned. We know that these currents must produce a field in the axis $a-a$ and another field in axis $b-b$ which is displaced 90 deg. both in time and space. This means that the rotor currents produce a rotating field which rotates against the rotation of the armature and which has a synchronous speed s in relation to a fixed point in space. The rotor has a speed s_2 in relation to a fixed point in space. The speed of the rotor field in regard to any rotor conductor therefore must be (s_1+s_2). This, however, necessitates currents with a frequency (v_1+v_2) where v_1 equals line frequency and v_2 equals speed frequency. That is, v_2 is the frequency of the currents which would be set up in the rotor conductors at the speed s_2, if the stator were excited with direct current. A graphical representation of this condition has been given in Fig. 10. The impressed voltage and the stator magnetizing current are again shown in the first figure and here it is also assumed that the voltage is a maximum at the time zero. The rotor magnetizing current in conductor No. 1 must be a negative maximum at this instant, (if we neglect the ohmic resistance as compared with the inductance), but the remainder of the current curve is a junction of (v_1+v_2). In order to show the current graphically we must assume a certain speed, and for convenience in drawing the curves we will say that $s_2=\frac{2}{3}s_1$. The frequency of the currents in the bars, therefore, must be $1\frac{2}{3}$ times the line frequency. The time phase displacement of the currents in adjacent rotor bars equals the space displacement of the bars themselves.

This has been shown in Fig. 10 in representing the currents in conductors No. 1, 3, 5, 7 and 9. In Fig. 11 are shown diagrammatic views of the rotor at the times $t=0, \frac{T}{4}, \frac{T}{2}$ and $\frac{3T}{4}$.

It will be noted that at the time $t=\frac{T}{4}$ conductor No. 1 has moved only 60 deg. from the center line of stator pole, neverthe-less the strength of the currents has changed in such a way that the rotor field is in the direction of the axis $a-a$. This may be made more clear by a study of Fig. 10, where it is seen that the current in conductor No. 2 would be about at its positive maximum at this instant $\frac{T}{4}$. After the time $\frac{T}{2}$ (Fig. 11) the conductor No. 1 is 120 deg. from the center line of stator pole,

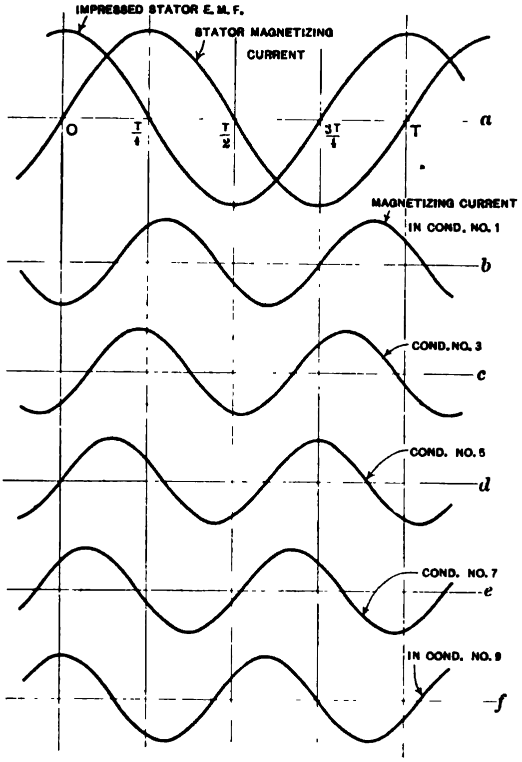

FIG. 10.—Rotor magnetizing current for two-thirds synchronous speed

but now the currents have such a distribution that the field is in the direction of the axis $b-b$. The rotor currents, instead of being exactly 90 deg. behind the e.m.f. that produces them, are slightly less than 90 deg. behind, on account of the effect of rotor resistance which we have here considered.

In a similar manner we may derive the working component of the currents in the rotor when the motor is carrying load. These currents must produce an alternating field which is stationary in space, and whose axis coincides with the axis of the stator poles. As we have seen before, such a field may be considered as the resultant of two fields of equal magnitude rotating in opposite directions against a fixed point in space with a speed equal to the synchronous speed of the line. The component that rotates in the same direction as the rotor, has a speed in relation to the conductor equal to s_1-s_2; and the other component rotating opposite to direction of rotor, has a speed equal to s_1+s_2 in relation to the conductor. Each individual conductor, therefore, carries two different working currents; one with slip frequency and the other with speed plus line frequency. Under the assumption that the rotor runs at two-thirds of synchronous speed, these two currents must have the frequency $1-\frac{2}{3}$ and $1+\frac{2}{3}$ times line frequency. These currents in the different rotor conductors, for this value of slip, are shown in the curves of Fig. 12. These curves were drawn up in exactly the same manner as the curves in Fig. 8, except that instead of a double-frequency current we now have a current with a frequency $1\frac{2}{3}$ times the line frequency; and in place of the direct current we have a current with a frequency $\frac{1}{3}$ that of the line.

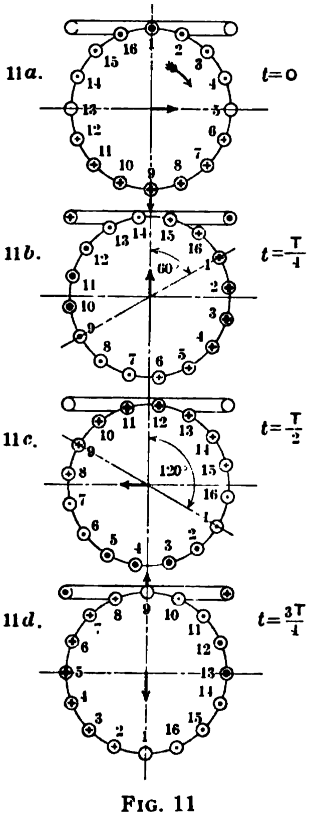

Fig. 11

It is now not a very difficult matter to express in mathematical formulas, the facts we have just stated. The currents producing the rotor magnetization have a frequency equal to the line plus the speed frequency, and therefore may be expressed as follows for conductor No. 1:

$$i_2 = -K \cos (m_1 t + m_2 t)$$

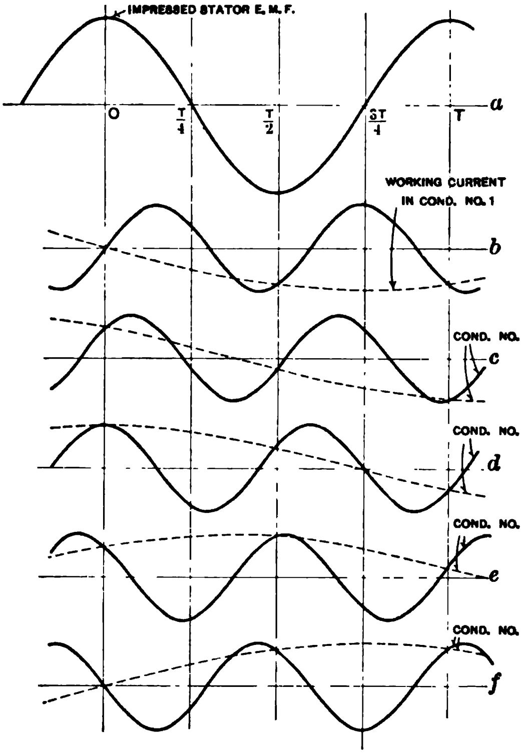

FIG. 12.—Rotor working current for two-thirds synchronous speed

For any other conductor forming the angle φ with the center line of the pole the equation would become

$$i_2 = -K \cos (m_1 t + m_2 t + \varphi)$$

The component of the working current which produces a rotating field in direction of the armature rotation may be expressed by the equation

$$I_{r_1} = -K \sin (m_1 t - m_2 t - \varphi)$$

and the other component, producing a field rotating opposite to the direction of armature rotation, is given by the equation

$$I_{r_2} = +K \sin (m_1 t + m_2 t + \varphi)$$

Since these two components must be equal we may write the equation for the total working current as

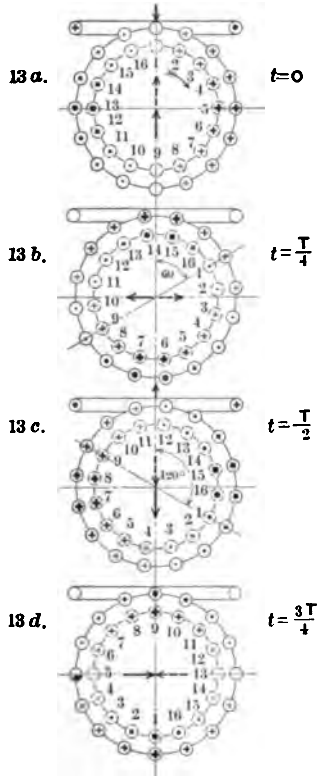

FIG. 13

$$I_r = +K \{\sin (m_1 t + m_2 t + \varphi) \\ - \sin (m_1 t - m_2 t - \varphi)\}$$

where $m_1 = 2\pi V_1$

$m_2 = 2\pi V_2$

V_1 = line frequency

V_2 = speed frequency

φ = angle between any particular conductor and stator pole center line at time zero.

The equations for I_{r_1} and I_{r_2} have been plotted in Fig. 12 for five different rotor conductors.

The low-frequency current is the one producing the field rotating with the armature. As the speed of the armature approaches synchronism, the frequency of this current approaches zero, until at synchronous speed we have a direct current flowing with the resulting conditions as shown previously in Figs. 8 and 9. In Fig. 12 we have omitted the curve of magnetizing current so as to avoid confusion. It is evident however that there are three distinct currents flowing in each rotor bar. The first is the rotor magnetizing current, and the others are the two components of the working current. The magnetizing current happens to be of the same frequency as one component of the working current. *The total resultant current therefore consists of two different waves; one of which has " line plus speed " frequency, and the other has slip frequency.* The total resultant current can easily be obtained from Figs. 10 and 12 by adding together the ordinates of the three curves. This has been done in Fig. 14. Fig. 15 shows these same currents for three complete primary cycles. This figure shows clearly that it will be impossible, with the standard design of oscillograph, to produce a stationary picture of this curve by the visual method, since the shape of the curve repeats only after a comparatively long period. The photographic method, however, will give the exact shape.

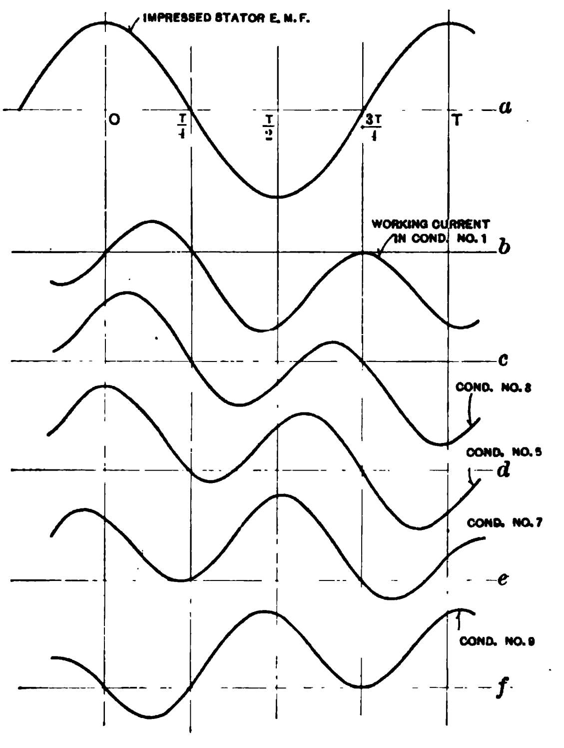

FIG. 14.—Resultant rotor working current for two-thirds synchronous speed

In the beginning of this paper, it was stated that the phenomena taking place in the rotor of a single-phase motor could be explained in two different ways. We have made use in the above discussion of the rotating field theory. We will now explain briefly the same phenomena by the cross field method. We will select first for this purpose the working current, because this is the current of greatest importance. Referring to Fig. 16, the working current flows in axis $a-a$. The current flowing in the connection between the brushes $a-a$ must be a current of line frequency, and the same is true for the current flowing through the connection between the brushes $b-b$. Let us assume for a moment that the rotor is stationary and we find that the working currents produce a field in the axis $a-a$ whose distribution in space we will assume follows the sine law. The maximum induction in the axis $a-a$ occurs in the moment that the working current changes with the time, so the corresponding change in the induction may be expressed as

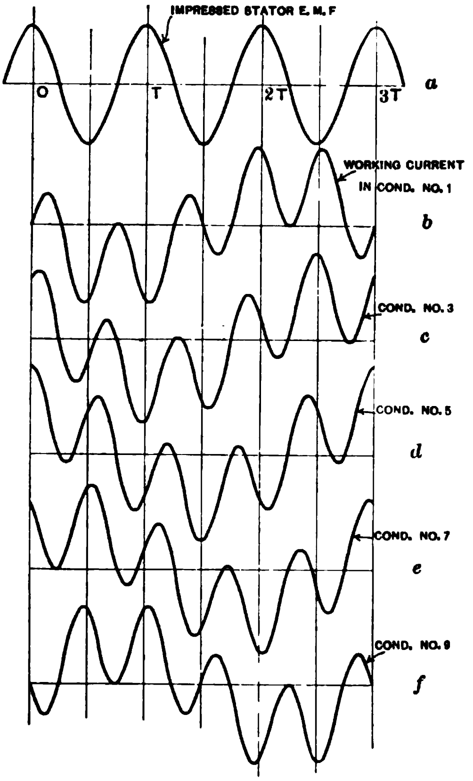

FIG. 15.—Resultant rotor working current for two-thirds synchronous speed

$$B_c = \bar{B} \cos (m\,t)$$

The induction at any point which forms the angle α with the center of the pole may be expressed by the equation

$$B_{\alpha} = B_c \cos \alpha = +\bar{B} \cos (m\,t) \cos \alpha$$

In order to obtain such a field distribution, the current distribution in space must be given by

$$I_\alpha = \bar{I} \cos (m\, t) \sin \alpha$$

where $\bar{I}$ is the maximum current which flows in any conductor at the time t equals zero. This current, however, flows in that conductor which lies in the axis $b-b$. The field distribution will not change if the motor rotates, so the currents in the individual conductors must be such as to produce this same field. Hence the current distribution in space must remain constant. If we assume that the rotor rotates with k per cent of synchronous speed we can express the angle α through which any conductor has moved in t seconds as $\alpha = (k\, m\, t)$. If the conductor under consideration formed the angle φ with the axis $a-a$ at the time $t=0$ the total angle which it makes with the axis $a-a$ after t seconds is

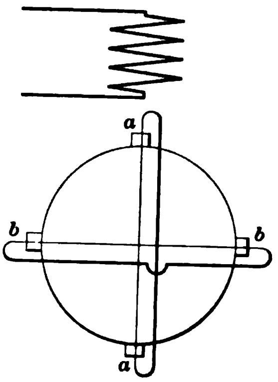

FIG. 16

$$\alpha = (k\, m\, t + \varphi)$$

Substituting this value in the equation for I_α we obtain

$$I_\alpha = \bar{I} \cos (m\, t) \sin (k\, m\, t + \varphi)$$

This equation may easily be changed to a form that will more readily show the shape of the curve.

$$\sin (m\, t + \alpha) = \sin (m\, t) \cos \alpha + \cos (m\, t) \sin \alpha$$

$$\sin (m\, t - \alpha) = \sin (m\, t) \cos \alpha - \cos (m\, t) \sin \alpha$$

Subtracting the second equation from the first, we obtain:

$$\sin(m\,t+\alpha)-\sin(m\,t-\alpha)=2\cos(m\,t)\sin\alpha$$

also let $(k\,m\,t)=m_2\,t$.

By substitution, then, we may obtain:

$$I_a=\frac{\bar{I}}{2}\{\sin(m_1\,t+m_2\,t+\varphi)-\sin(m_1\,t-m_2\,t-\varphi)\}$$

The form of this equation checks exactly with that derived by the rotating field theory. No attention has been paid to the absolute value of the constant K in the equation for I_r but it is evident that the current I_r is identical with the current I_a just now derived. The question of the absolute values of the currents is beyond the scope of this paper, as it was the aim to show only the wave shape and frequency of the currents for the case where primary field is excited by sine-shaped e.m.f. and rotor and stator field have also sine-shaped distribution in space.

It still remains to show that the rotor magnetizing current as derived by the rotating field theory of Ferraris agrees with the results obtained by the cross field method. The following discussion outlines the proof for this, but for a more extended explanation of the theory, the reader is referred to an article by Mr. Fynn in the *Electrical World* of November, 1909, p. 1235, where a complete diagram of the no-load conditions of a single-phase induction motor is given. Here it may be stated simply that in the axis $a-a$ flows a magnetizing current which in the ideal motor is 90 deg. phase displaced against the impressed stator voltage; and in the axis $b-b$ flows a magnetizing current which is in phase with the impressed e.m.f. Further, we know that the magnetizing current in the axis $a-a$ is equal to the magnetizing current in the axis $b-b$. Also, in the same way as for the working current, it follows that the magnetizing current flowing in an individual conductor in the axis $b-b$ may be expressed as

$$i_b=\bar{i}_b\cos(m\,t)\cos\alpha$$

and the magnetizing current in the axis $a-a$ is

$$i_a=-\bar{i}_a\sin(m\,t)\sin\alpha$$

This expression must have the negative sign because it opposes the stator magnetizing current, which follows the law

$$i=\bar{i}\sin(m\,t)$$

The total magnetizing current flowing in an individual conductor is the sum of i_a and i_b

$$i_m = i_a + i_b = \bar{i}_b \cos (m\,t) \cos \alpha - \bar{i}_a \sin (m\,t) \sin \alpha$$

$$= \bar{i}_a \cos (m\,t + \alpha)$$

The value of α for any conductor at any time t is given by the expression

$$\alpha = (k\,m\,t + \varphi) = m_2\,t + \varphi$$

This substituted in the equation for i_m gives

$$i_m = \bar{i}_a \cos (m_1\,t + m_2\,t + \varphi)$$

This equation leads us to the same result which we obtained by the rotating field theory.

A summary of the results obtained above shows that the wave shape of the current in an individual rotor conductor of a single-phase motor consists of three current components. One of these is the rotor magnetizing current, which has a frequency equal to line plus speed frequency. The remaining two are the two components of the rotor working current; one of which has line plus speed frequency, and the other has slip frequency. Two of these three current components have the same frequency, so the total resultant rotor current is made up of two distinct waves, one with line plus speed frequency and the other with slip frequency.

DISCUSSION ON " WAVE SHAPE OF CURRENTS IN AN INDIVIDUAL ROTOR CONDUCTOR OF A SINGLE-PHASE INDUCTION MOTOR." CHICAGO, JUNE 30, 1911.

Theodore Hoock: Mr. Weichsel's interesting theoretical investigation of the wave shape of currents in a single-phase rotor has been made under ideal conditions, *i.e.*, there has been assumed a machine without leakage and rotor resistance so that the rotor exciting current is equal to the stator exciting current, or in other words that the no-load current with short-circuited rotor is twice the no-load current with rotor winding open. The actual measured values, however, are smaller than those in the ideal motor, varying between 1.60 to 1.95. The reactance, and especially the rotor resistance, has a vital influence upon the secondary exciting current, reducing it as low as 60 per cent of its theoretical value. The derived equations in Mr. Weichsel's paper should, therefore, be corrected in figuring the actual amplitude of the current. Bearing this in mind it may be found valuable to apply the equations in investigating local fields, which are set by the rotor currents under certain conditions, and which cause sometimes very inconvenient phenomena in the machine.

H. Weichsel: Referring to Mr. Hoock's discussion, I agree that on actual single-phase motors, we find the no-load current less than twice the magnetizing current. The factor 2 is only a theoretical conclusion, and in the actual machines the total no-load current is usually about 1.6 to 1.9 times the magnetizing current, and this deviation from the factor 2 is caused by two factors, namely, the ohmic resistance and the inertia of the rotor.

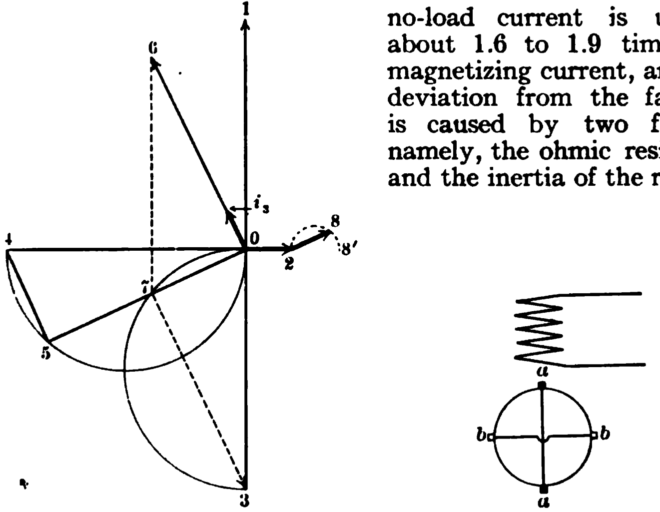

FIG. 1

The relation between no-load current and magnetizing current is shown graphically in diagram Fig. 1, which is based on the assumption that the rotor runs exactly at synchronism and with uniform speed.

0-1 = e.m.f. impressed on stator winding, whose ohmic resistance has been neglected.

0-2 = magnetizing current corresponding to e.m.f. 0-1.

0-3 = e.m.f. of transformation in *a-a* axis of rotor.

0-4 = e.m.f. of rotation generated in *b-b* axis of rotor due to rotation in main transformer field.

0-i_3 = rotor magnetizing current in *b-b* axis.

4-5 is therefore the ohmic drop due to i_3 in *b-b* axis. It is self-evident that 5 lies on a circle over 0-4.

0-6 = e.m.f. of rotation generated in *a-a* axis by rotation in field produced by current i_3.

0-7 = resultant e.m.f. in *a-a* axis = resultant of 0-6 and 0-3. This resultant e.m.f. must be equal to the ohmic drop in axis *a-a* and is therefore proportional to the rotor current in axis *a-a*.

2-8 = primary current counterbalancing the rotor current 0-7. It will be seen that vector 0-6 equals 7-3 and as 0-6 is proportional to 0 i_3 and 0 i_3 proportional to 4-5 it follows that also point 7 lies on a circle over 0-3. But 0-7 is proportional to the resultant rotor current in axis *a-a*, which current must be counterbalanced by the stator current 2-8. The end of this vector must lie, therefore, on a circle. For the ideal case, that rotor has no resistance, the panel 8 moves to 8′ and the total stator current is 0-8″ or 2 times the magnetizing current. For a finite rotor resistance the stator current is 0-8 which, according to diagram, must be smaller than 0-8′ or less than 2 times the magnetizing current.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

CHOICE OF ROTOR DIAMETER AND PERFORMANCE OF POLYPHASE INDUCTION MOTORS

BY THEODORE HOOCK

The theoretical part of polyphase induction motor design has been treated thoroughly, analytically and graphically,* and there is very little left for further investigation. In consideration of the great importance of this type of motor very little has been said about the leading points in the practical design.

In laying out a new line of induction motors it is desirable to have a rational method for determining the influence of the rotor diameter upon the performance rather than using the longer procedure of designing a number of motors under different assumptions and comparing the final results.

The performance is so rigidly interlinked with the mechanical dimensions and the windings that a theoretical design can easily be made which will show clearly the influence of the chosen constants upon the motor characteristic.

The derived formulas in this paper are not intended to supersede the detailed design but they should be used for the first layout.

On the other hand the designer will find the results of the calculations very convenient for comparison and assistance in choosing the proper frame for certain guarantees to be met.

It is undoubtedly of great use in the further development to analyze the design on a practical basis in order to find the limitations and the influence of the variables.

It is fully demonstrated by tests that the $D^2 l_i$ of standard speed motors is not limited by the temperature rise but by the

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*By Adams, Arnold, Steinmetz, Behrend, McAllister, Hellmund, de la Tour and others.

performance and flux density. It is also a sad fact that until the present time these limits could be found only by test and that we have to confine the investigations to the power factor, overload capacity, copper and iron losses.

Instead of dealing with the rotor diameter we express all results in terms of the pole-face proportions, *i.e.*, the effective core length divided by the pole pitch.

We will see later from (Figs. 6 to 10) that the power factor and the copper losses depend largely upon properly proportioning the pole face. There exists always one "best" proportion for the power factor and another "best" proportion for the minimum copper losses. Both conditions do not occur at the same ratio of core length to pole pitch. Also the deviations from the minimum or maximum values obtainable vary considerably with the design; for instance with the slot dimensions, number of poles, type of winding and so on. It is, therefore, advisable to put all these deviations on a percentage basis, because it gives us a convenient method of comparison.

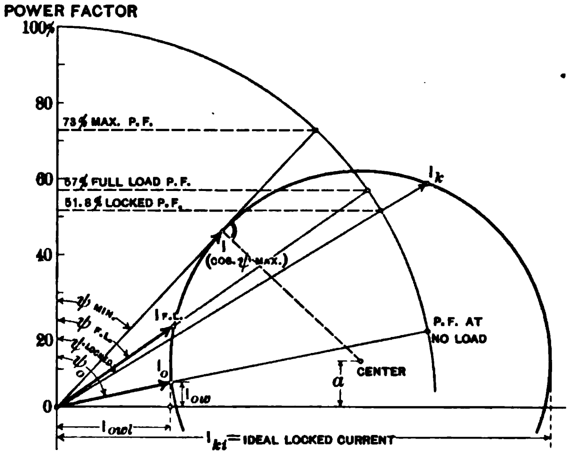

Fig. 1

These investigations can be divided under the following headings:

1. Leakage coefficient.
2. Copper losses.
3. Overload capacity.
4. Iron losses.

THE LEAKAGE COEFFICIENT

The circle diagram, as drawn in Fig. 1, illustrates in the simplest way the relation between the current taken from the line and the power factor for any given load.

Two tests, the no-load test and the locked test, are required in order to draw the diagram. The no-load test determines the no-load current and the no-load power factor while running light at normal voltage. The current and power factor at standstill will be found from the locked test. These four quantities I_0, $\cos\varphi_0$, I_k, $\cos\varphi_k$, determine two points of the primary current circle. The center of the circle lies at a distance a above the base line, which can be found by test, calculation or with the aid of a simple geometrical construction.

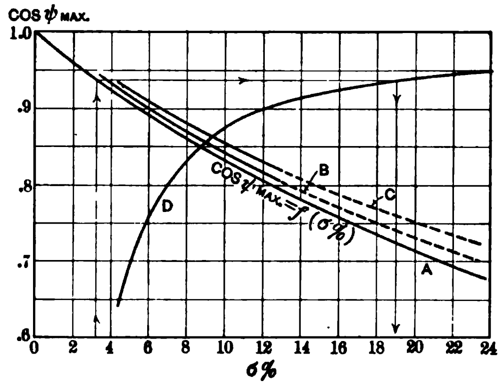

FIG. 2

The tangent to the circle gives the smallest phase displacement φ_{min} and the largest power factor $\cos\varphi_{max}$ ($\cos\varphi_{max} = 73$ per cent in Fig. 1).

The maximum power factor is determined by the circle diameter $(I_{ki} - I_{0wl})$ and the distance from the origin to the circle, *i.e.*, the wattless magnetizing current I_{0wl}. We can also see from Fig. 1 that a shifting of the center up or down from the base line will influence the maximum power factor somewhat (3 to 5 per cent). See curves *A*, *B* and *C* in Fig. 2.

The smallest current which will be taken from the line is the no-load current I_0. In loading the motor the current will increase to the full load current I_{FL}. When we load the motor still more we reach $I_{(\cos\varphi\ max)}$ and finally, it will pull out and come to standstill. In case the normal voltage is still applied, the locked current I_k will flow in the winding.

In case we could decrease the ohmic resistances of the motor the point I_k would move along the circle, *i.e.*, the locked current would increase. For the extreme case having resistances equal to zero the ideal locked current I_{ki} would be taken from the line. This reactive current I_{ki} is determined by the leakage coefficient and the wattless magnetizing current. It is

$$I_{ki}=\frac{I_{0wl}}{\sigma} \tag{1}$$

Where σ is the leakage coefficient or the ratio of the wattless component of the no-load current to the ideal locked current; or rewritten,

$$\sigma=\frac{I_{0wl}}{I_{ki}} \tag{2}$$

The main advantage of using this coefficient is the independence from voltage and magnetizing current. As soon as the punchings, the core length and the number of poles are settled the leakage coefficient is almost fixed (disregarding the iron saturation and fractional pitch windings). Furthermore all the characteristics of the motor are improved by decreasing the leakage coefficient.

It is not necessary to draw the circle diagram in each case in order to find the leakage coefficient, as it can be figured easily from the no-load and locked test data.

The total reactance of a motor at short-circuit is

$$x_k=\sqrt{\left(\frac{P_1}{I_k}\right)^2-\left(\frac{W_k}{m_1 I_k^2}\right)^2}=\frac{P_1}{I_k}\sqrt{1-(\cos\varphi_k)^2}=\frac{P_1}{I_{ki}} \tag{3}$$

and the leakage coefficient

$$\sigma=\frac{I_{0wl}}{P_1}x_k=\frac{I_0\sin\varphi_0}{P_1}x_k=\frac{I_{0wl}}{I_{ki}} \tag{4}$$

The Maximum Power Factor. It has been brought out above that the magnetizing current I_{0wl} and the leakage coefficient determine the maximum power factor. Under the assumption that the center of the circle lies on the base line the maximum power factor is

$$\cos \varphi_{max} = \frac{1}{1+2\,\sigma} = \frac{I_{ki}-I_{0wl}}{I_{ki}+I_{0wl}}\,* \qquad (5)$$

These equations were figured for different values of σ and are plotted in Fig. 2, curve A.

Curves *B* and *C* in Fig. 2 are added to show the effect of the center displacement upon $\cos \varphi_{max}$. Both curves are taken from actual tests and will be found very useful for approximations.

The following details may be kept in mind:

Use curve *A* plus ½ to ¾ per cent for large motors.

Use curve *B* for small and medium size motors and $\cos \varphi_k = 0.3$ to 0.6.

Use curve *C* for small and medium size motors and $\cos \varphi_k \geqq 0.75$.

If the tested power factor is plotted in a curve we can find readily from its maximum value the size of the leakage coefficient, using these curves (Fig. 2). If the no-load current is known, we are in a position to compute also with equation (4) the reactance of the motor without knowing the data of the locked tests.

Another quantity which is convenient for comparison of the overload capacity is the current at which the maximum power factor occurs. It is

$$I_{(\cos \varphi\ max)} = I_{0wl} \sqrt{\frac{1}{\sigma}} \qquad (6)$$

We can now investigate the relations between the leakage coefficient, the rotor diameter and the core length. All practical considerations, as peripheral speed, temperature rise, flywheel effect and so on, will be eliminated in our investigations entirely. It is perfectly feasible to build two motors for the same purpose, one with a large diameter and narrow core and the other with a small diameter and long core. The same $D^2\,l_i$ is assumed in both cases. The rotor diameter is proportional to the pole pitch for a given number of poles and therefore we have all results in relation to the ratio of core length l_i to pole pitch τ.

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* $\frac{I_{ki}}{I_{0wl}} = \frac{1}{\sigma}$ is *the ideal short-circuit ratio.* For $\sigma = 10$ per cent. $= 0.10$ is the ideal locked current $\frac{1}{0.10} = 10$ times the magnetizing current.

The pole face of a motor is the product $l_i \tau$. The square pole face will have then the ratio $\frac{l_i}{\tau}$ equal to 1.

The leakage coefficient σ can be figured with great accuracy from the dimensions of the motor.[*]

It is

$$\sigma = \frac{\text{ampere-turns circuit}}{\text{ampere-turns air gap}} \Sigma\, \sigma = \text{saturation factor } \Sigma\, \sigma \qquad (7)$$

The sum of the leakage coefficients $\Sigma\, \sigma$ consists of the following coefficients:

σ_{n_1} = Stator slot leakage coefficient.
σ_{n_2} = Rotor slot leakage coefficient.
σ_{s_1} = Stator end-connection leakage coefficient.
σ_{s_2} = Rotor end-connection leakage coefficient.
σ_z = Zigzag leakage coefficient.
σ_{b_1} = Stator belt leakage coefficient.
σ_{b_2} = Rotor belt leakage coefficient.

$$\text{Or } \Sigma\, \sigma = \sigma_{n_1} + \sigma_{n_2} + \sigma_{s_1} + \sigma_{s_2} + \sigma_z + \sigma_{b_1} + \sigma_{b_2} \qquad (8)$$

We combine the corresponding leakage of the stator and rotor, and placing

$$\left.\begin{aligned} a &= \delta\, k_1\, c_n\, (\lambda_{n_1} t_1 + \lambda_{n_2} t_2) \\ b &= \delta\, k_1 \left(c_{s_1} \frac{l_{s_1}}{f_{p_1}{}^2 \tau} + c_{s_2} \frac{l_{s_2}}{f_{p_2}{}^2 \tau} \right) \\ c &= c_0\, c_K\, t_1\, t_2 \end{aligned}\right\} \qquad (9)$$

we obtain for the leakage coefficient

$$\begin{aligned} \sigma &= \text{saturation factor} \left(\frac{a+c}{\tau^2} + \frac{b}{l_i} + \sigma_b \right) \\ &= \text{saturation factor} \left(\frac{a+c}{\tau^2} + \frac{b}{\beta \,.\, \tau} + \sigma_b \right) \end{aligned} \qquad (10)$$

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[*] See R. E. Hellmund. *Elektrotechnische Zeitschrift*, 1911, p. 1111.

In order to determine the minimum value of the leakage coefficient $\Sigma\,\sigma$ we neglect the belt leakage as being a small amount and almost independent of the pole face proportion. We take the first derivative of $\Sigma\,\sigma$ with respect to β and place it equal to zero:

$$\frac{d\,\Sigma\,\sigma}{d\,\beta}=0$$

We find that the minimum value for $\Sigma\,\sigma$ occurs when

$$\frac{a+c}{\tau^2}=\frac{b}{\beta\,.\,\tau}=\frac{b}{l_i} \tag{11}$$

This equation indicates that *the leakage coefficient will become a minimum when the total slot plus zigzag leakage is equal to the total end connection leakage.*

The quantities *a*, *b* and *c* are determined by the air gap, slot dimensions and the number of poles. We can also from the quotient of $\frac{a+c}{b}$ find the pole pitch for which the leakage coefficient $\Sigma\,\sigma$ will become a minimum, *i.e.*, when

$$\frac{\tau}{\beta}=\frac{a+c}{b} \tag{12}$$

This relation can be used directly for the layout of the diameter and core length for a certain $D^2\,l_i$ or for a comparison of machines in order to determine the most advantageous frame. We will carry out these calculations on a large motor. We assume a $D^2\,l_i$ equal to 300,000, and split the product in different values D and l_i, varying from 140 to 70 in. (3.58 to 1.79 m.) diameter and 15.3 to 61 in. (0.38 to 1.55 m.) length of core, which gives a variation of the pole face proportion $\beta=\frac{l_i}{\tau}=0.417$ to 3.33 (see Table I).

The fifth column shows the ratio τ/β (88 to 5.5). The values of *a*, *b* and *c* are 5.25, 0.556, 1.135, which are written on the curves in Fig. 3. The ratios $\frac{a+c}{\tau^2}$ and $\frac{b}{\beta\,.\,\tau}$ and the sum $\Sigma\,\sigma$

are plotted also in Fig. 3. The minimum value occurs at $\frac{\tau}{\beta}=\frac{a+c}{b}=\frac{5.25+1.135}{0.556}=11.5$ (upper curve in Fig. 3), giving a ratio $\beta=\frac{l_i}{\tau}=1.95$.

When we add to curve $\frac{a+c}{\tau^2}+\frac{b}{\beta\,.\,\tau}$ the belt leakage, the total $\Sigma\,\sigma$ will be found. These figures are worked out for 12, 16 and 32 poles on six different diameters (see Table I).

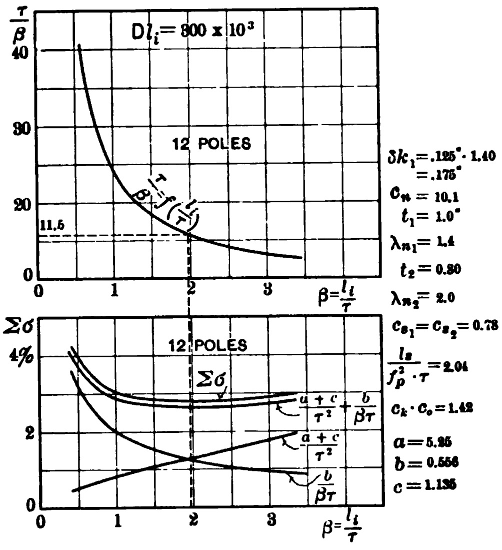

FIG. 3

The results are plotted in Fig. 4, *a*, *b*, *c*, against the ratio of core length to pole pitch. We find that the minimum value for the different number of poles does not occur at the same ratio l_i/τ. The minimum values for the 12, 16 and 32 poles lie at $\frac{l_i}{\tau}=1.7$, 1.55, 1.3 respectively.

We can see from these curves that a design with a large diameter and small core length, as well as with a small diameter and a corresponding extreme core length, has a tendency to increase the leakage coefficient.

The curves of $\Sigma\ \sigma$ have no sharp knee and the deviations from the minimum values are small over a large range of l_i/τ.

In order to bring all curves upon a uniform basis the percentage of increase above the minimum values is plotted in Fig. 4, *d*, *e* and *f*. The smallest value of $\Sigma\ \sigma$ for 12 poles is $= 2.825$ per cent at $\frac{l_i}{\tau} = 1.7$, curve *a*. At $\frac{l_i}{\tau} = 0.417$ we find $\Sigma\ \sigma = 4.26$ per cent. This means an increase of 51 per cent, curve *d*, which is due to the large influence of the end-connection leakage, its coefficient being eight times as large as that of the slot leakage. The curve *e* of the 16-pole machine shows the same characteristic, but not quite so distinctly. The end-connections in the 32-pole design are only of moderate influence while the slot and zigzag leakage in the long motor with a small pole pitch are the largest items. It is certainly of interest to follow these figures to the final result which may be considered to be the maximum power factor (cos φ_{max}). The full load power factor is then only determined by the overload capacity or the ratio of the wattless magnetizing current I_{0wl} to the full load current I_{FL}. Therefore, we multiply the figured leakage coefficient $\Sigma\ \sigma$ by the saturation factor $= 1.15$, assuming 15 per cent of the air ampere-turns for magnetizing the iron path in all three designs. From curve *A* (Fig. 2) or from equation (5) can then be found the maximum power factor. The results are plotted in Fig. 21.

TABLE I

$D^2\, l_i = 300{,}000$—12 poles

D (inches)	l_i (inches)	$\mathfrak{J}$ (inches)	Ratio $\beta = \frac{l_i}{\tau}$	Ratio $\frac{\tau}{\beta}$
140	15.3	36.7	0.417	88
120	20.9	31.4	0.665	47.1
110	24.8	28.7	0.865	33.2
100	30	26.2	1.15	22.8
90	37	23.5	1.58	14.9
80	47	20.9	2.25	9.3
70	61	18.3	3.33	5.5

The maximum values of the cos φ_{max} correspond with the minimum values of $\Sigma\ \sigma$ and occur at the same ratios l_i/τ as the leakage coefficients $\Sigma\,\sigma$ in Fig. 4, *a*, *b* and *c*. It is surprising that for 12 poles ($\Sigma\,\sigma = 2.825 \times 1.15 = 3.25$ per cent), the $\cos\varphi_{max} = 93.7$ per cent, only 3 per cent larger than $\cos\varphi_{max} = 91$ per cent for the 51 per cent larger leakage coefficient. ($\sigma = 4.26 \times 1.15 = 4.9$ per cent at $l_i/\tau = 0.415$.)

We can figure from curve *B*, Fig. 2, the deviation $\Delta\,\sigma$ per cent for 1 per cent change in $\cos\varphi_{max}$ and find curve *D*. This result shows that with a leakage coefficient of 3.25 per cent or $\cos\varphi_{max} = 94$ per cent, 19 per cent deviation is permissible, for which amount the $\cos\varphi_{max}$ will go down to 93 per cent. In our case with 51 per cent deviation the $\cos\varphi_{max} = 93.7$ per cent is decreased $\frac{51}{19}$ per cent $= 2.7$ per cent and we actually find 91 per cent against 93.7 per cent (Fig. 21).

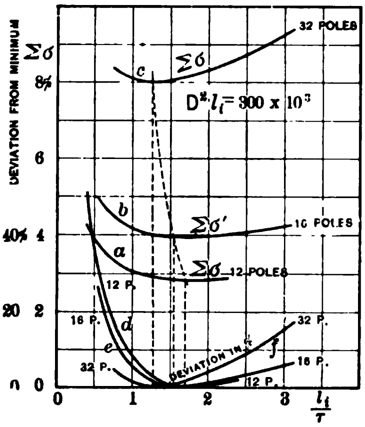

FIG. 4

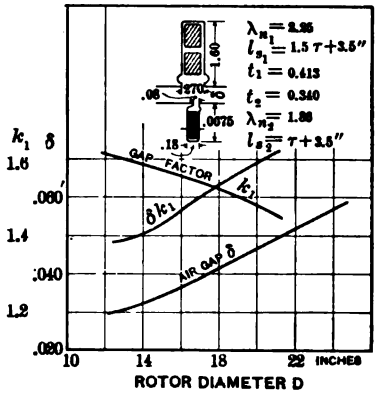

FIG. 5

The " best " pole face of the 32-pole machine gives a $\cos\varphi_{max} = 84.3$ per cent ($\sigma = 9.25$ per cent) for which the permissible allowance is only 8½ per cent for each per cent power factor. It was found for the longest motor $\frac{l_i}{\tau} = 3.06$ ($\sigma = 9.3$ per cent), a deviation of 16 per cent. The power factor $\cos\varphi_{max}$ will be decreased in this design $\frac{16}{8.5}$ per cent $= 1.9$ per cent or it will be $84.3 - 1.9 = 82.4$ per cent.

This influence is greater the larger the leakage coefficient is, or in other words, machines with a low power factor or with a large leakage coefficient will be more sensitive than those with small leakage and high power factor. It is, therefore important to choose the pole face ($l_i \,.\, \tau$) of motors with large number of poles as close to the best values as possible, since every variation of 5 to 7 per cent in the leakage coefficient *decreases* the maximum power factor one per cent and *increases* the full load stator current in the same percentage as the power factor is decreased.

The determination of the pole face by the formula (**12**) is very simple as long as the air-gap δ is kept constant. This condition will not exist, however, in most practical cases. It is general practise to vary the air-gap proportionally with the rotor diameter, Fig. 5, except that very long cores for high speed will have also a bearing upon the air-gap. We will consider here only the standard speed machines for which these investigations are made especially. Not only the air-gap brings a complication in the analytical solution of the problem but also the member $\frac{l_s}{\tau \,.\, f_p^2}$ in quantity b, because the length of the end connections are (const. × pole pitch *plus* 2 *to* 10 *in.*) according to size, voltage and type of winding. This constant addition introduces an error which may lead to incorrect results.

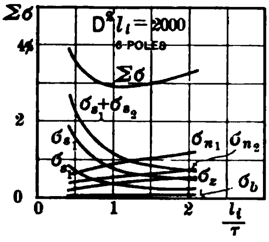

FIG. 6

The smallest leakage coefficient does not occur always between the ratio $\frac{l_i}{\tau} = 1.2$ to 1.7, as it was obtained for $D^2 l_i = 300 \,.\, 10^3$, but its location depends upon the size of the pole face, the type of windings, length of the air-gap and slot pitches. The writer has worked out a few interesting cases which will give an idea of the range of variation.

In Figs. 6 to 10 the leakage coefficients $\Sigma\,\sigma$ for a $D^2 l_i = 2000$ are drawn for various number of poles. The diamond coils of the stator are placed in open slots and slightly chorded. The rotor has partially closed slots and a special squirrel-cage winding with a slot pitch of only 0.34 in. (8.6 mm.). The $D^2 l_i$ is split up again as shown before in Table I. The air gap δ is varied according to Fig. 5. The slot dimensions of stator and rotor are given in Fig. 5. Since the slot openings are kept constant and the air gap varies, the gap factor k_1 varies inversely as the rotor diameter, but the effective air gap $\delta \cdot k_1$ *increases* with the rotor diameter.

The leakage coefficients $\Sigma\ \sigma$ are figured for all diameters and numbers of poles, using formula (8) in order to show the relative magnitude of each kind of leakage.

The upper curves in Figs. 6 to 9 give always the sum $\Sigma\ \sigma$ per cent and all sums are combined in Fig. 10, *a* to *e*.

The minimum values of $\Sigma\ \sigma$ occur for the 6- 8- 10- 12- and 14-pole designs at a ratio $\frac{l_i}{\tau} = 1.15$, 1.40, 1.38, 1.40, 1.30 and 1.20 respectively.

In Fig. 11 the deviations of $\Sigma\ \sigma$ from the minimum are drawn, which have the same character as those of Fig. 4, *d*, *e*, *f*.

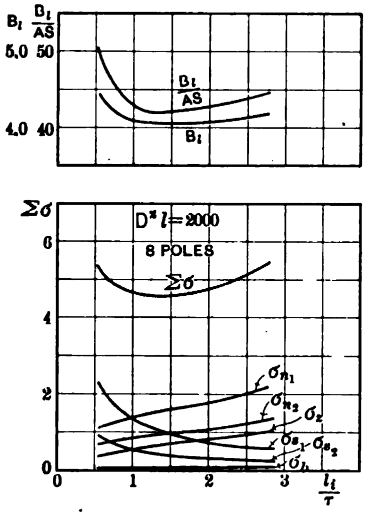

FIG. 7

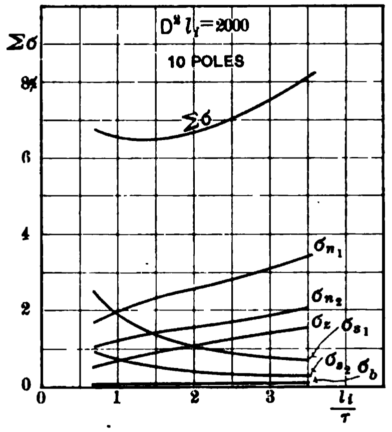

FIG. 8

The curves are somewhat further extended than we would find in actual machines. It is possible, however, that certain conditions, 2- or 4-pole, or 14- to 20-pole designs on standard frames would give $\frac{l_i}{\tau} = 0.45$ to 0.5 or 2.5 to 3.5.

From these curves, Fig. 11, we can judge again, in combination with the actual leakage coefficient $\sigma =$ saturation factor $\times \Sigma\ \sigma$, how much the power factor will be decreased by making the pole face proportions different from those which will give the minimum values.

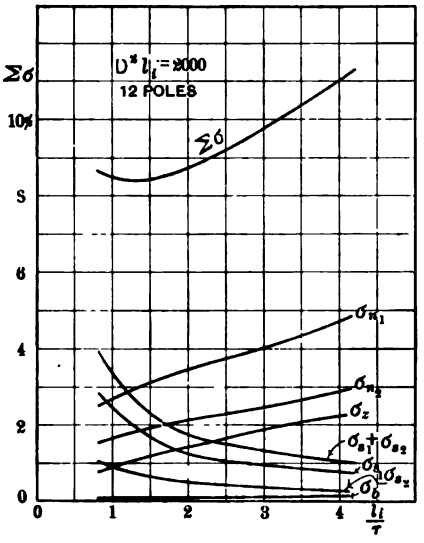
$D^2 l_i = 2000$
12 POLES
$\Sigma\sigma$
10%
8
6
4
2
0
$\Sigma\sigma$
σ_{n_1}
σ_{n_2}
σ_z
$\sigma_{s_1} + \sigma_{s_2}$
σ_{s_1}
σ_{s_2}
σ_b
0
1
2
3
4
$\frac{l_i}{\tau}$

FIG. 9

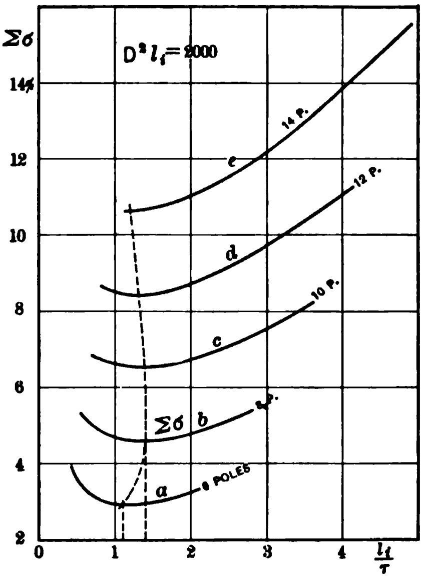
$\Sigma\sigma$
$D^2 l_i = 2000$
14%
12
10
8
6
4
2
e
14 P.
d
12 P.
c
10 P.
$\Sigma\sigma$ b
8 P.
a
8 POLES
0
1
2
3
4
$\frac{l_i}{\tau}$

FIG. 10

It was previously mentioned that the size of the motor, the slot pitches and the type of windings influence the design as to its most economical proportions.

Table II gives a layout for a $D^2 l_i = 289 - 6$ poles.

The stator is in all three designs the same, chorded diamond coils in open slots. The rotor *A* represents a construction with bolted bars and rings with a secondary slot pitch of 0.642 in. (16.2 mm.) for a bar $\frac{3}{8}$ by $\frac{3}{8}$ in. (9.5 by 9.5 mm.). In *B* a rotor is used with very narrow *open* slots with a slot pitch of 0.156 in. (3.97 mm.) for a special rotor winding with $\frac{1}{16}$ by $\frac{1}{2}$ in. (1.5 by 12.7 mm.) copper section. Design *C* has *partially* closed slots with a slot pitch of 0.241 in. (6.3 mm.) and a copper section of $\frac{1}{8}$ by $\frac{1}{2}$ in. (3.17 by 12.7 mm.).

TABLE II

$D^2 l_i = 289—6$ poles

Design	*A*	*B*	*C*
Stator slot dimension	Fig. 12	Fig. 13	Fig. 14
" " constant λ_n	2.20	2.20	2.20
" " pitch τ	0.526 in.	0.526 in.	0.526 in.
Length of end connections	$l_s = 1.3\,\tau + 3.5$ in.		
air gap δ	0.0190 to 0.0276 in.		
gap factor k_1	1.50 to 1.58	1.76 to 1.87	1.50 to 1.58
Rotor slot dimension	Fig. 12	Fig. 13	Fig. 14
" " constant λ_n	1.0	2.4	2.13
" " pitch τ	0.642 in.	0.156 in.	0.241 in.
Length of end connections	$l_s = \tau + 3$ in.		
Copper section	$\frac{3}{8} \times \frac{3}{8}$	$\frac{1}{16} \times \frac{1}{2}$	$\frac{1}{8} \times \frac{1}{2}$

Rotor diameter D	Core length l_i	Pole pitch τ	Ratio $\beta = \frac{l_i}{\tau}$
11 in.	2.38 in.	5.76 in.	0.412
10	2.89	5.23	0.55
9	3.56	4.71	0.755
8	4.52	4.18	1.08
7	5.9	3.66	1.61
6	8.05	3.14	2.56
5½	9.6	2.88	3.33

We bear in mind that the mechanical air gap δ is the same in all three cases *A*, *B* and *C* for the same ratio l_i/τ and compare at first the leakage coefficients $\Sigma\ \sigma$. The minimum values of $\Sigma\ \sigma$ are as follows:

	A	*B*	*C*
$\Sigma\sigma$..................................	6.95%	5.85%	5.7%
At $\frac{l_i}{\tau}$..................................	0.550	0.75	0.80

We see that a smaller motor calls for a considerably smaller ratio l_i/τ than the larger sizes figured.

Designs *B* and *C* show very clearly the good influence of the large number of rotor slots, which decreases $\Sigma\ \sigma$ 20 per cent or increases the maximum power factor 2 per cent.

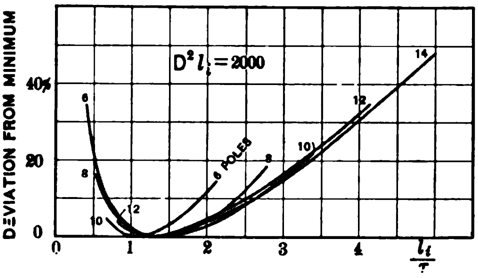

FIG. 11

Design *B* with the open rotor slots is almost as good as *C* with partially closed slots, as far as the maximum power factor is concerned. The air-gap is rather small in this small motor, being 0.02 to 0.0275 in. (0.5 to 0.69 mm.) on one side, so that the gap factor k_1 increases the magnetizing current and affects the full load power factor. This leads to the conclusion that a motor with open rotor slots of this small size could not compete with one having partially closed slots.

The percentage increase of the leakage coefficient $\Sigma\ \sigma$ per cent above the minimum values is plotted in Fig. 15. The slot and zigzag leakage are the largest items in design *A* which overbalance the end-connection leakage considerably and reach a very high percentage in Fig. 15. All three curves have a sharp turn.

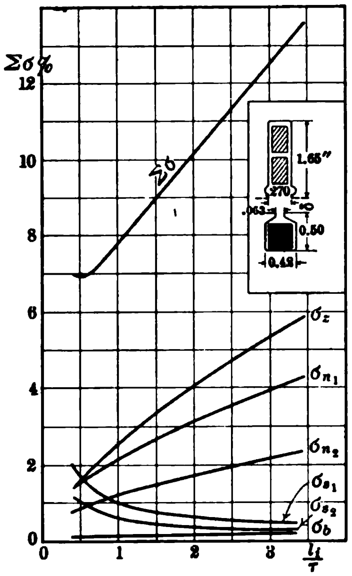

FIG. 12

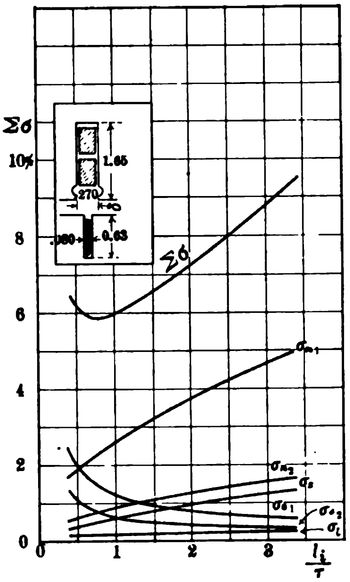

FIG. 13

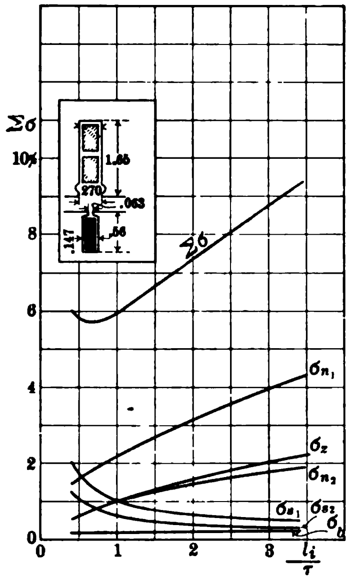

FIG. 14

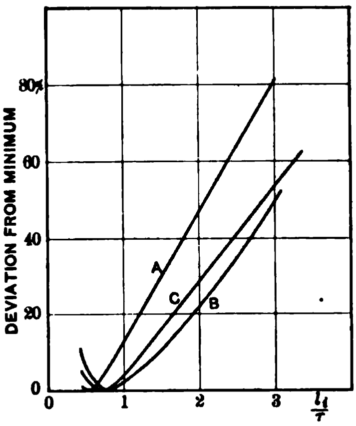

FIG. 15

We see further that a motor can be made longer (large l_i/τ) the smaller the slot and zigzag leakage can be kept in comparison to the end connection leakage.

TABLE III

$D^2 l_i = 24.5$—2 poles— two-phase

Rotor diameter D	Core length l_i	Pole pitch τ	Ratio $\beta = \frac{l_i}{\tau}$	Ratio $\frac{\tau}{\beta}$
4.52 in.	1.20 in.	7.1 in.	0.169	42.0
4.22	1.48	6.62	0.224	29.6
3.92	1.59	6.15	0.259	23.7
3.625	1.87	5.68	0.333	17.05
3.32	2.22	5.20	0.427	12.16
3.02	2.69	4.74	0.569	8.32
2.72	3.30	4.27	0.773	5.52

In Table III and Fig. 16 the dimensions, constants and the complete leakage data of a very small motor are given. When using formula (12), with $a = 0.231$, $b = 0.0227$ and $c = 0.348$, we find the ratio $\frac{\tau}{\beta} = 25.7$ to give the highest maximum power factor at a pole-face ratio $\beta = \frac{l_i}{\tau} = 0.25$.

The belt leakage is the largest item in the group and it changes the ratio l_i/τ to 0.22. It can be seen again that the equation furnishes good results when the air-gap is kept constant.

We have figured previously with the sum $\Sigma\,\sigma$ of the single leakage coefficients. It may happen, however, that the maximum power factor and the derivations for the smallest copper losses are influenced by the saturation of the iron path. Equation (10) expresses the influence of the saturation factor.

$$\text{saturation factor} = \frac{\text{total ampere-turns}}{\text{air ampere-turns}} = 1 + \frac{\text{iron ampere-turns}}{\text{air ampere-turns}} \tag{13}$$

It has been assumed that the iron ampere-turns are constant for all number of poles and ratios l_i/τ. It would lead to considerable complications if we tried to introduce the iron ampere-turns in all derivations. The density in the air-gap and the ampere-turns for the teeth and yoke depend largely upon the motor type, frequency and number of poles. In case we decrease or increase the air-gap with the rotor diameter and keep the density in all parts of the magnetic path constant, the following results may be obtained: Motors with a large number of poles will require a small amount of ampere-turns for the yoke, due to the short length of path τ. The ampere-turns for the teeth will be constant with all poles for constant density and slot depth. The ampere-turns of the air gap will vary with $\delta\, k_1$, which is almost proportional to the air gap δ. In turn we find for a given number of poles an increase of the saturation factor, when decreasing the air gap. From these conclusions we can state that the minimum leakage coefficient and the maximum power factor are always shifted a small amount toward a lower ratio l_i/τ, than found by figuring with the sum $\Sigma\,\sigma$ only. The influence of the iron ampere-turns comes into consideration only in highly saturated machines or in those with very small air gap.* As long as the iron ampere-turns are not more than 25 per cent of the air ampere-turns, the results will not be influenced.

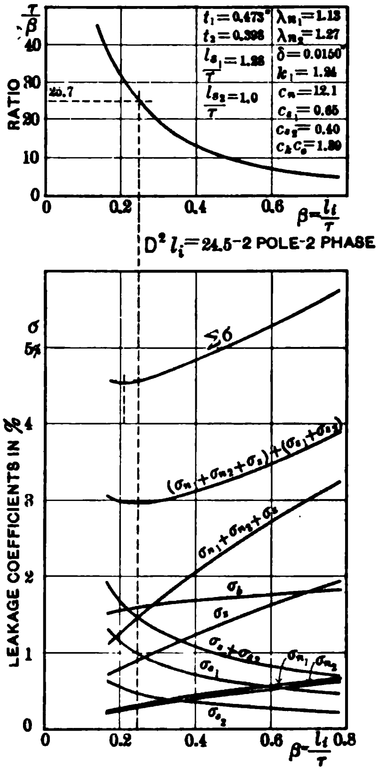

FIG. 16

COPPER LOSSES

The stator copper losses are

$$W_{stator} = \frac{\pi\, D\, A\, S\, s_1\, k_r\, l_a\, (1 + 0.004\, t^\circ)}{14500} \text{ watts} \qquad \textbf{(14)}$$

where $l_a \cong l_i + 1.5\,\tau + 3.5$ in.

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*Th. Hoock and R. E. Hellmund. "Elektrotechnik & Maschinenbau," Wien, 1910, p. 741.

$$k_r = \frac{\text{alternating-current resistance}}{\text{direct-current resistance}}\ *$$

or, for both windings together, double the amount when we assume $A\ S$ and the copper density s for the rotor the same as in the stator. The ampere-turns per inch $A\ S$ and the density s may be assumed constant, so that in varying D and l_i of a certain $D^2\ l_i$ as carried out previously, only D and l_i change in equation (**14**).

We can write then for the total copper weight, in its simplest form,

$$\text{copper weight} = \text{const. } D\ l_a = \text{const. } D\ (l_i + 1.25 \text{ to } 2.0\ \tau)$$

A simple differentiation furnishes the smallest copper weight for

$$l_a = 1.5\ \tau \text{ at a ratio } \frac{l_i}{\tau} = 3.0$$

$$l_a = 2.0\ \tau \text{ at a ratio } \quad = 4.0.$$

The length of conductor is correctly

$$l_a = l_i + \text{const. } \tau + \text{const. allowance} \qquad (\mathbf{15})$$

The neglecting of the additional constant length which is given by the type of the winding involves an error. In order to eliminate it the quantity $D\ l_a$ has been figured for several $D^2\ l_i$ and is then plotted against the ratio l_i/τ. The presence of ventilating ducts in the core increases the constant allowance in equation (**15**) and their influence can be estimated from curves *b* in Figs. 18 to 20.

All results are reduced to a percentage basis, calling the smallest value zero. (See curves in Figs. 17 to 20.) We see from all curves that the copper weights become a minimum at a ratio $\frac{l_i}{\tau} = 3.0$ to 5.0. These values are so high that they are beyond practical applications.

Figs. 17 and 18 refer to a $D^2\ l_i = 1220$–4 poles. Assuming a motor with a pole-face ratio $\frac{l_i}{\tau} = 0.5$, we find from curve *b*,

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*A. B. Field, TRANSACTIONS A. I. E. E., 1905, XXIV, page 761.

Fig. 18, that the copper weight will be 50 per cent higher than the amount required for the " lightest copper " machine. For the square pole face $\frac{l_i}{\tau} = 1.0$, only 18 per cent difference is found.

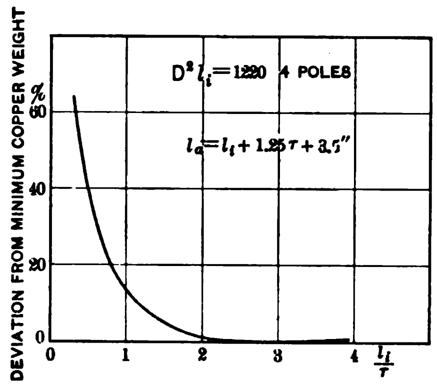


Fig. 17.—Deviation of copper weight from the minimum

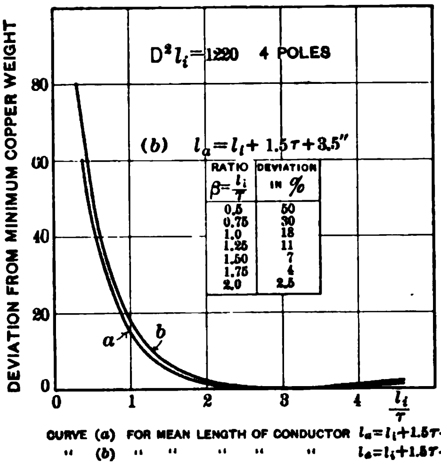
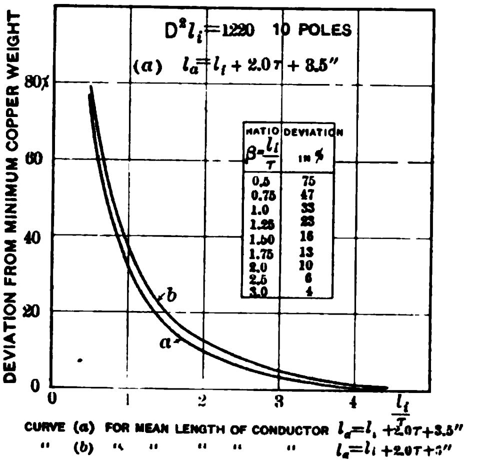


Fig. 18.—Deviation of the copper weight from the minimum

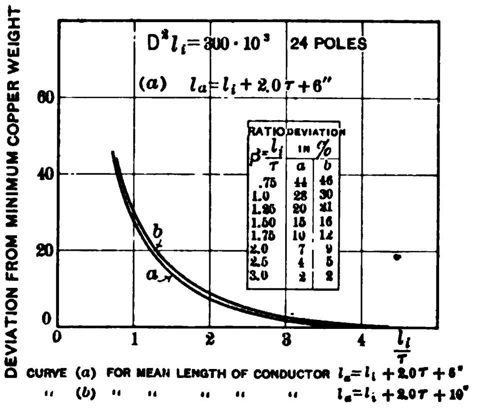
In case we wind the same frame $\left(\frac{l_i}{\tau} = 0.5 \text{ for 4 poles}\right)$ for 10 poles or $\frac{l_i}{\tau} = 1.25$, approximately 20 per cent difference will be found from curve *b*, Fig. 19.

The minimum of the leakage coefficient or the maximum power factor on one side, and the minimum copper weight or smallest resistance (constant copper density assumed) on the other side, occur always at a different ratio $\frac{l_i}{\tau}$. The full-load current is determined in the circle diagram by the no-load current, the leakage coefficient, the no-load and the locked power factor. Assuming the magnetizing current, the no-load and locked power factor to be constant, we find the full-load current



RATIO $\beta = \frac{l_i}{\tau}$	DEVIATION IN %
0.5	75
0.75	47
1.0	33
1.25	23
1.50	16
1.75	13
2.0	10
2.5	6
3.0	4

FIG. 19.—Deviation of the copper weight from the minimum



RATIO $\beta = \frac{l_i}{\tau}$	DEVIATION IN % a	DEVIATION IN % b
.75	44	46
1.0	28	30
1.25	20	21
1.50	15	16
1.75	10	12
2.0	7	9
2.5	4	5
3.0	2	2

FIG. 20.—Deviation of the copper weight from the minimum

varying in the same percentage as the maximum power factor decreases or increases by changing the pole-face proportion.

The following procedure can be used under these considerations for finding the ratio of core length to pole pitch at which the copper losses approach the minimum value. The copper losses vary with the square of the current. Therefore we square the deviations of the full-load current as derived from the leakage coefficient and the maximum power factor, and add to these values the percentage deviations of the resistances from curves in Figs. 17 to 20. An example will better illustrate the application of this method.

We have found the leakage coefficients $\Sigma\,\sigma$ for the motor $D^2\,l_i = 300.10^3$ which are drawn in Fig. 4. The leakage coefficient σ is then calculated by multiplying by the saturation factor 1.15. Curve *A*, Fig. 2, then gives the maximum power factor which is shown in the upper curve of Fig. 21. The highest value $\cos\varphi_{max}$ of the 32-pole design is $84\frac{1}{2}$ per cent, at a ratio $\frac{l_i}{\tau} = 1.28$. For a core dimension three times as long as the pitch, $\cos\varphi_{max} = 82\frac{1}{4}$ per cent is found. This corresponds to a current deviation $\Delta\,I$ of $2\frac{1}{4}$ per cent (see curves in Fig. 21) or $(\Delta\,I)^2$ equal to 5 per cent.

We assume the mean length of conductor $l_a = l_i + 2.0\,\tau + 6$ in., and use the curve *b* from Fig. 20 again, which is copied in Fig. 21 and marked *C u*. The sum of the two curves $(\Delta\,I)^2$ per cent $+ Cu$ per cent shows a distinct minimum at the ratio $\frac{l_i}{\tau} = 3.36$. By reducing the ordinate values to the zero line we find the deviation of the copper losses from the minimum. These results indicate a rather large ratio $\left(\frac{l_i}{\tau} = 3.36\right)$ as far as copper losses are concerned (disregarding the iron losses), while the highest power factor will be obtained at a ratio $\frac{l_i}{\tau} = 1.28$. In case the ratio is made $= 2.25$, only one per cent deviation of each item will result and the apparent efficiency will be near its maximum. If the power factor is a prevailing quantity in the guarantees, the core will be made narrower, approaching the ratio $\frac{l_i}{\tau} = 1.28$. This change, however, will involve an increase of copper losses and higher cost. It can be seen from the flat shape of the curve that the performance of a large motor is less sensitive than a smaller one when the best proportions are not used.

The same method is applied on the small motor $D^2 l_i = 289$ —6 poles, designated as design *A*.

The leakage coefficient $\Sigma \sigma$ increased very rapidly with l_i/τ as shown in Fig. 12. With the aid of curve *A*, Fig. 2, the cos φ_{max} was plotted in Fig. 22. Then using the maximum value 87.5 per cent as a base, the percentage of current increase ΔI per cent and $(\Delta I)^2$ per cent were the figures. For the mean

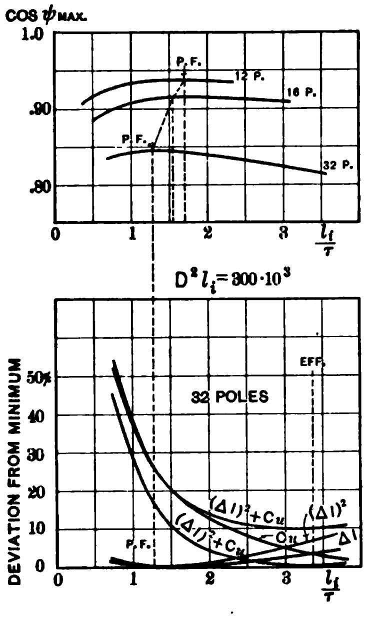


FIG. 21

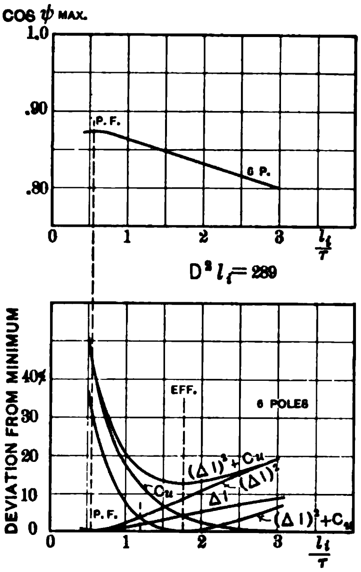


FIG. 22

length of conductor curve *b* in Fig. 18 has been chosen. After adding the curve $(\Delta I)^2$ to *Cu* per cent, their sum, minimum locus $\left(\frac{l_i}{\tau} = 1.75\right)$ and their per cent deviation from the minimum were found. These results show again that the difference between power factor and copper loss loci is large, $\frac{l_i}{\tau} = 0.55$ against 1.75. The deviations at the cutting point, $\frac{l_i}{\tau} = 1.26$, however, are only 2 per cent, with a maximum power factor of 85.5 per cent. If a higher value is desired the core should be made narrower. At a ratio $\frac{l_i}{\tau}=0.55$, a $\cos\varphi_{max}=87.5$ per cent can be attained, but the copper weight will be increased approximately 30 per cent.

These investigations show clearly the opposing influence of the pole-face proportions upon power factor and copper losses. The iron losses introduce another component which makes the problem practically impossible to solve, because of its variable proportions in the sum of the losses. A final decision can only be made on the basis of a complete design.

THE OVERLOAD CAPACITY

The change of the leakage coefficient σ with the pole-face proportion and its influence upon the whole performance of the motor introduce only variables in the problem. A base to start from, however, is given in the maximum torque or the overload capacity. Bearing this in mind we will change all quantities in such a way that the pull-out torque remains constant in our further investigations.

A simple method will lend itself to this purpose. The wattless magnetizing current per phase I_{0wl} can be computed as follows:*

$$I_{0wl}=\frac{1.11\times(\text{total ampere-turns})\times p}{m_1\,w_1\,f_1\,f_{p_1}} \tag{16}$$

where

$$\text{total ampere-turns}=406\,\delta\,k_1\,B_l\times\text{saturation factor} \tag{17}$$

It is customary to express the magnetizing current in percentage of the full load current I_{FL}.

We combine equation (**16**)and (**17**) and divide by the full load current

$$\frac{I_{0wl}}{I_{FL}}=\frac{1.11\,p\,406\,\delta\,k_1\,B_2\times\text{saturation factor}}{I_{FL}\,m_1\,w_1\,f_1\,f_{p_1}} \tag{18}$$

$I_{FL}\,m_1\,w_1$ represents the total ampere-turns in the stator if we place

$$2\,m_1\,I_{FL}\,w_1=\pi\,D_1\,A\,S=2\,p\,\tau\,A\,S, \tag{19}$$

where $A\,S$ = ampere conductors per inch circumference.

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*Arnold-Wechselstromtechnik, V.

Introducing $A\ S$ in equation (**18**) we find

$$\frac{I_{0wl}}{I_{FL}} = \frac{1.11.406\ \delta\ k_1\ B_l \times \text{saturation factor}}{f_1 f_{p_1}\ \tau\ A\ S} \qquad \textbf{(20)}$$

or, for B_l in kilolines per sq. cm. and $A\ S$ in inches,

$$\frac{I_{0wl}}{I_{FL}} = \frac{0.45\ \delta\ k_1\ B_l \times \text{saturation factor}}{\tau\ f_1 f_{p_1} A\ S} \qquad \textbf{(21)}$$

The maximum or pull-out torque in terms of the full load torque can be figured to

$$\frac{\text{maximum torque}}{\text{full load torque}} = \frac{I_{0wl}}{I_{FL}\left(2\,\sigma + \dfrac{2\,r_1\,I_{0wl}}{P_1}\right)\cos\varphi.\eta} \qquad \textbf{(22)}$$

We take only the three prevailing quantities into consideration and plot the pull-out torque against the ratio

$$\frac{I_{0wl}}{\sigma\ I_{FL}} \text{ (see curve } a\text{, Fig. 23)} \qquad \textbf{(23)}$$

The upper curve may be used for highly saturated motors with a small number of slots per pole and a " bent " locked saturation curve.

Now we combine this pull-out relation with the winding and motor dimension of equation (**21**) and write

$$\frac{I_{0wl}}{\sigma\ I_{FL}} = \frac{0.45\ \delta\ k_1 \text{ saturation factor } B_l}{\sigma\ \tau\ f_1 f_{p_1} A\ S} = \frac{0.45\ \delta\ k_1\ B_l}{\Sigma\ \sigma\ \tau\ f_1 f_{p_1} A\ S} \qquad \textbf{(24)}$$

We see that the pull-out torque can be determined without knowing the ampere-turns for the iron when the $\Sigma\ \sigma$ is used instead of the total leakage coefficient σ.

We find therefore the ratio of the specific working quantities B_l and $A\ S$

$$\frac{B_l}{A\ S} = 0.4 \text{ to } 0.8\ \frac{f_1 f_{p_1}\ \tau\ \Sigma\ \sigma}{0.45\ \delta\ k_1} \qquad \textbf{(25)}$$

This ratio will be the smallest for a certain pull-out when $\dfrac{\tau\ \Sigma\ \sigma}{\delta\ k_1}$ becomes a minimum.

The required field strength in the air-gap will then be obtained with the smallest number of ampere-turns per unit of length.

Suppose it is asked that the pull-out torque be not less than 2¾ times full load torque; the ratio $\frac{I_{0wl}}{\sigma\ .\ I_{FL}}$ will then be larger than 0.5. Or from equation

$$\frac{B_l}{A\ S} \geqq \frac{0.5 f_1 f_{p_1} \tau\ \sigma}{0.45\ \delta\ k_1 \times \text{saturation factor}} \geqq 0.5\ \frac{f_1 f_{p_1} \tau\ \Sigma\ \sigma}{0.45\ \delta\ k_1}$$

This ratio gives a relation between the specific working densities and the constants of the motor for a certain overload capacity.

Equation (**25**) may also be used directly to calculate the turns per phase or the correct air density for a given frame and a certain pull-out torque. We figure for this purpose from equation (**25**) the ratio $B_l/A\ S$ and introduce the result into the output equation or the machine constant*

$$\frac{D^2 l_i n}{\text{kv-a.}} = \frac{133\ .\ 10^{-11}}{f_1 f_{p_1} B_l A\ S} \qquad \textbf{(26)}$$

and set

$$B_l = A\ S \left(\frac{B_l}{A\ S}\right)$$

Hence

$$A\ S = \sqrt{\frac{\text{kv-a.}}{D^2 l_i n}\ \frac{133\ .\ 10^{11}}{f_1 f_{p_1} \left(\frac{B_l}{A\ S}\right)}} \qquad \textbf{(27)}$$

And finally the turns per phase

$$w_1 = \frac{A\ S\ \pi\ D}{2\ m_1\ I_{FL}} \qquad \textbf{(28)}$$

or the air density

$$B_l = \sqrt{\frac{\text{kv-a.}}{D^2 l_i\ n}\ \frac{133\ .\ 10^{11} \left(\frac{B_l}{A\ S}\right)}{f_1 f_{p_1}}} \qquad \textbf{(29)}$$

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*Arnold loc. cit.

These equations enable us to determine quickly the lowest air density at which the required pull-out torque will just be met in case the leakage coefficient is known. This refers especially to designs which are limited by the maximum torque.

IRON LOSSES

An incorporation of the iron losses complicates the theoretical design and these losses should preferably be calculated after the main dimensions are settled.

A complete line of iron loss figures was made for the same $D^2 l_i = 2000$ as above, running the motor on 60 cycles with a synchronous speed of 900 rev. per min.

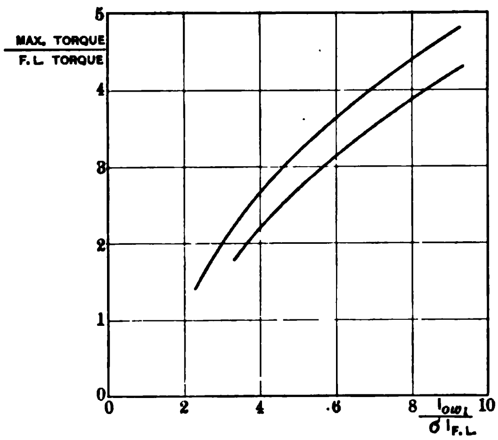

FIG. 23

It was pointed out that the pull-out torque should be kept constant. Assuming in this case a pull-out torque of 2¾ times full load torque, we find from equation (25)

$$\frac{B_l}{A\ S} = 5\ \frac{0.957\ .\ 0.966\ .\ \tau\ .\ \Sigma\ \sigma}{0.45\ .\ \delta\ .\ K_1} = 10\ \ \frac{\tau\ \Sigma\ \sigma}{\delta\ K_1}$$

if we take $f_1 = 0.957$ and $f_{p_1} = 0.966$.

The ratio $B_l/A\ S$ has been figured and plotted in Fig. 7 using the data of the lower curve in Fig. 23.

Finally we obtain with these results in combination with formula (24) the required air-gap density B_l, Fig. 7.

The flux per pole is

$$\phi = \frac{E \,.\, 10^8}{4.44\ c\, w_1\, f_1\, f_{p_1}} \tag{30}$$

and the maximum air density in kilolines per sq. cm.

$$B_l = \frac{\phi}{\frac{2}{\pi}\, 6450\, \tau\, l_i} = \frac{\phi}{4110\, \tau\, l_i} \tag{31}$$

The density in the (90 per cent solid) iron behind the slots

$$B_a = \frac{\phi}{2 \,.\, 6450 \,.\, 0.90 \,.\, l \,.\, h_a} = \frac{\phi}{11600 \,.\, l \,.\, h_a} \tag{32}$$

Combining, we find

$$B_a = 0.354\ \frac{l_i}{l}\ \ \frac{B_l \tau}{h_a} \tag{33}$$

or approximately

$$= 0.36\ B_l\, \frac{\tau}{h_a}$$

The volume of the stator core is

$$\text{vol.}_{St.\,c.} = (\text{D} + 2\, h_{n_1} + h_{a_1})\ \pi\, l\, h_{a_1}\ 0.90 \text{ cu. in.;}$$

of the rotor core

$$\text{vol.}_{Rot.\,c.} = (\text{D} - 2\, h_{n2} - h_{a_2})\ \pi\, l\, h_a\ 0.90 \text{ cu. in.;}$$

of the stator teeth

$$\text{vol.}_{St.\,t.} = Z_1\, h_{n_1}\, z_{m_1}\, l\ 0.90 \text{ cu. in.;}$$

of the rotor teeth

$$\text{vol.}_{Rot.\,t.} = Z_2\, h_{n_2}\, z_{m_2}\, l\ 0.90 \text{ cu. in.}$$

The iron loss is figured under the assumption of a constant density behind the slots in the stator $B_{a_1} = 9{,}000$ and in the rotor $B_{a_2} = 10{,}000$. The variation of the air-gap density, Fig. 7, is evident in the core volume and later also in the losses. The volume of the stator and rotor core reaches a minimum while the stator and rotor teeth increase with the core length. The stator teeth are in this example a big item and their influence is expressed in the volume of the total active iron very distinctly, curve *c*, Fig. 25, which is a measurement of the cost of the punchings. Curve *b* represents the sum of stator core plus stator teeth, while curve *a*, the sum of the stator plus rotor teeth, is a value for estimating the additional losses.

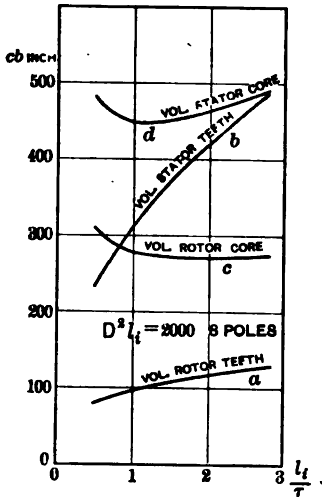

FIG. 24

The hysteresis and eddy losses of the stator core and teeth are figured separately and plotted in terms of the ratio l_i/τ in Fig. 26. We notice the same character in the loss curves as in the curves of the volume in Fig. 24. The total iron loss in core and teeth caused by the rotating field increases slightly with an increase of core length (750 watts at $\frac{l_i}{\tau}=0.5$; 800 watts at $\frac{l_i}{\tau}=1.7$; 900 watts at $\frac{l_i}{\tau}=2.5$).

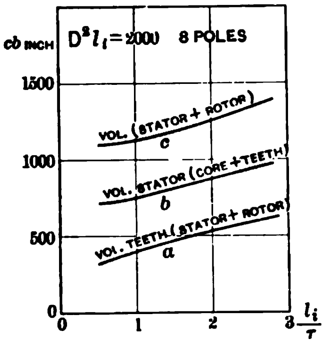

FIG. 25

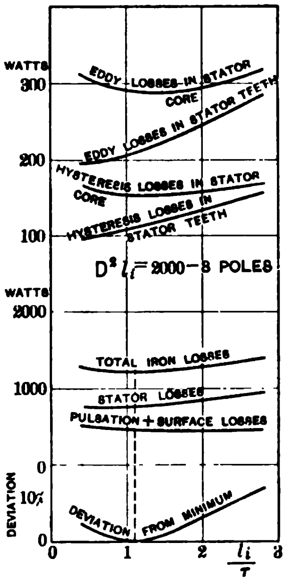

FIG. 26

The total additional iron losses decrease very slightly with a longer core. The pulsation and surface losses in the stator and rotor teeth decrease even despite the fact that the volume (curve *a*, Fig. 25) increases. The explanation is given by the reduction of frequency of these pulsations and the speed in the longer core designs because the number of slots are proportional to the rotor diameter. It is very interesting to note that the increase of one class and a decrease of the second class of losses result in their sum a minimum total iron loss (Fig. 26). The percentage variation of the total iron losses from the minimum is plotted in Fig. 26. The best pole-face proportion of this example lies at $\frac{l_i}{\tau} = 1.2$. The deviation is 10 per cent at $\frac{l_i}{\tau} = 2.4$, which is smaller than the test variations.

These figures are carried out more completely in order to justify our previous assumption of constant no-load losses when investigating the copper losses. In case the iron losses have a great bearing upon the efficiency at full load, the choice of the pole-face proportion will be influenced by the ratio of the constant losses to the copper losses.

SUMMARY

The ratio of the rotor diameter to the core length influences the performance of the motor considerably. The investigations show that there exists for every rating one ratio of rotor diameter to core length for which the performance becomes a maximum. The power factor, the copper losses, iron losses and overload capacity have an opposing influence upon this ratio. In order to work the material in the most advantageous manner for each item we would obtain as many different diameters as there are items. It is not feasible to express all influences in one equation. It is, therefore, the scope of this paper to determine the proper ratio of core length to pole pitch for which each item of the performance will become a maximum or minimum. The introduction of the leakage coefficient, that is, the ratio of the wattless magnetizing current to the ideal locked current, furnishes very simple formulas for practical application.

The highest power factor will always be obtained at a ratio of core length to pole pitch which can easily be computed from formula (**12**). Since the obtained " best " result usually differs from the actual machine dimensions, all calculations in the paper are reduced to a percentage basis in order to judge the magnitude of the deviation from the theoretical values.

The copper losses are based on the full load current and the resistance. A set of curves are calculated and drawn in Figs. 17 to 20 to indicate the relation between resistance or copper weight and the main dimensions of the rotor.

The percentage deviation from the maximum power factor has then been used in order to find in a simple manner the minimum copper losses.

The following table shows the ratio of core length to pole pitch at which the power factor, apparent efficiency and copper weight approach the minimum or maximum values of a certain frame ($D^2 l_i$). The limits vary with the type, length of air gap, type of winding, slot dimensions and number of poles.

$D^2 l_i$	Ratio of core length to pole pitch			
	Best power factor	Best app.eff.	Lowest copper losses*	Lowest copper wt.*
20	0.2 to 0.3	0.5 to 0.8	1.0 to 2.0	2 to 3
200	0.4 " 0.8	1.0 " 2.0	1.5 " 2.5	2.5 " 3.5
2000	0.8 " 1.4	1.5 " 2.5	2 " 3	2.5 " 4
10,000	0.9 " 1.5	2 " 3	2.5 " 3.5	3.5 " 5
$300 . 10^3$	1.3 " 1.8	2 " 3	3 " 4	3.5 " 6

*For stator winding or wound rotor motors only.

The field of application or the characteristic of the type usually settles or limits the main dimensions. The peripheral speed, temperature rise, flywheel effect, method of manufacturing, ventilation, available floor space, shipping weight, load factor, power consumption and factory cost are some of the factors determining the choice of the diameter within small limits and sacrificing certain parts of the performance. In the analysis of a concrete case the above points should therefore be considered carefully.

NOTATION

a = See equation (9).
$A\ S$ = Ampere conductors per inch circumference.
b = See equation (9).
B_a = Density in the iron behind the slot.
B_l = Maximum density in the air gap in kilogausses per sq. cm.
c = Frequency in cycles per second.
c = See equation (9).

c_o = Constant for zigzag leakage.
c_k " " " "
c_n " " slot "
c_s = " " end-connection leakage.
D = Bore of stator punchings in inches.
E = E. M. F. induced in stator winding.
f_1 = Winding factor.
f_{p1} = Pitch or chord factor.
h_a = Depth below slot.
h_n = Slot depth.
I_{FL} = Full load current.
I_{owl} = Wattless component of no-load current.
I_k = Short-circuit or locked current.
$I_{k_i} = \frac{P_1}{x_k} = \frac{I_{owl}}{\sigma}$ = Ideal locked current.
ΔI = Percentage of current valuation.
k_1 = Air gap factor.
k_r = Eddy current factor.
l = Core length in inches.
l_a = Mean length of conductor in inches.
l_i = Effective core length in inches.
l_s = Length of end connections.
m_1 = Number of phases in stator.
n = Rev. per min.
P_1 = Terminal voltage per phase.
p = Number of pairs of poles.
s = Copper density in amperes per square inch.
t = Slot pitch.
W_k = Watts input at short-circuit.
w_1 = Turns per phase of stator winding.
x_k = Reactance per phase.
z_m = Average tooth width.
$\beta = \frac{l_i}{\tau}$ = Ratio of core length to pole pitch.
η = Efficiency.
λ_n = Slot constant.
σ = Leakage coefficient, see equation (**7**).
$\Sigma\sigma$ = Sum of leakage coefficients—see equation (**8**).
τ = Pole pitch in inches.
ϕ = Flux per pole.

DISCUSSION ON "CHOICE OF ROTOR DIAMETER AND PERFORMANCE OF POLYPHASE INDUCTION MOTORS. CHICAGO, JUNE 30, 1911.

E. F. W. Alexanderson: Mr. Hoock's paper is a valuable contribution to the literature of induction motors, not in the sense that it will enable the designers to produce better motors or even enable them to save time in doing so, but from the educational point of view. The value of Mr. Hoock's paper, to my mind, is his method of dealing with the proportions. The problem for the designer is very much circumscribed by practical conditions, and his work consists in compromising between those conditions. Therefore, in order to do so intelligently, he must have a clear conception of the effect of any change in proportions and if he has such physical conception, it is not necessary to use any formula or curves; in fact he would not have time to do so. If Mr. Hoock had written the paper for parties who have an outside interest in induction motors, rather than designers, he probably would have formulated his theories somewhat differently. However, there are a number of points that ought to be of general interest. I particularly refer to what he calls the leakage coefficient. The leakage coefficient, according to his definition, is the ratio of magnetizing current to the ideal short-circuit current; and this factor is most important of all proportions of induction motors. I think the name of " leakage coefficient ", although it may have some precedence in literature, is rather inadequate, because it does not convey to the mind of the general engineer the features of practical importance which it is intended to signify. If it is stated that, in a certain induction motor, the short-circuit current is twenty times the magnetizing current, this statement contains complete information regarding power factor and over-loading capacity. More often the motor is described by saying that the short-circuit current is seven times the full-load current, and this information does not mean anything unless it is coupled with other information on magnetizing current or power factor. The ratio of short-circuit and magnetizing current are inherent characteristics of the motor, whereas the ratio of short-circuit current and full-load current are characteristics of the designer's method of giving the motor a commercial rating. I believe it would be of advantage if the practical significance of the so-called "leakage coefficient," that is, the ratio of short-circuit to magnetizing current, were more generally known and appreciated, because it makes it possible to furnish almost the complete information as to a motor's excellence in power factor and overload characteristics by stating only one figure.

C. J. Fechheimer: The author makes a statement near the beginning of the paper as follows:

" It is fully demonstrated by tests that the $D^2 l_i$ of standard speed motors is not limited by the temperature rise, but by the performance and flux density."

He then follows this by subdividing the investigations under the following headings:

1. Leakage coefficient.
2. Copper losses.
3. Overload capacity.
4. Iron losses.

This would convey the impression that these are the only important limitations to be considered in choosing the diameter. I do not agree with him in the statement that the temperature rise does not affect the $D^2 l_i$, and believe that in a great many motors this should influence the choice of dimensions at least as much as the performance and flux density. He has not considered the effect of the diameter and length upon what I believe to be one of the most important considerations in proportioning motors; that is, the cost. For example, in most 25-cycle and many 60-cycle motors we have no difficulty whatever in obtaining reasonably high power factors, efficiencies and torques, but we find that we would have to use a very great amount of copper to keep the temperature rise within the specified limits, if at the same time we were to obtain just enough torque. In extreme cases, moreover, such design may involve the use of very deep slots in the stator; to such a degree that, for a given internal diameter, we can increase the number and size of conductors, (thereby increasing the flux), and use shallower slots, and thus obtain the same, or possibly a smaller external diameter than with the larger number of conductors. As the heating in the majority of standard speed motors is nearly entirely dependent upon copper loss, it will be seen that for a given temperature rise, the section of conductor is proportional to the number of conductors. Hence, for the same heating, the weight of copper will be practically proportional to the square of the number of conductors. Therefore, the small flux motor has a large amount of copper and a prohibitively great cost. In other words, in many standard speed motors, *temperature rise*, *cost* and *flux density* are the limitations placed upon the dimensions of the motor and not performance; but in some motors, *performance*, *flux density* and *cost* (not temperature rise) are the limitations.

Now, when we come to consider the effect of the diameter and length upon the cost, we shall find that in general the large diameter motor is more expensive than one of small diameter, even though the active material in the two motors be the same. This will be apparent when one considers the great increase in weight in mechanical parts, such as the yoke and shields, and the increase in labor with the increase of external diameter of the stator punchings. We cannot go too far, however, in decreasing the stator diameter (and increasing the length), for after a certain point is passed, we obtain a machine of such length that a great increase in active material would be required to obtain the desired performance and temperature rise.

From the above it follows that the performance, temperature rise and cost are related to the diameter and length to such an extent that we cannot neglect any one of these factors in deciding upon the proportions of the motor. While it may be possible to reduce the relations of the many quantities involved to the form of mathematical equations, the final expressions are liable to be so involved and complicated that they would scarcely admit of solution.

In proportioning the parts of squirrel cage induction motors the importance of the effect of diameter upon the losses which it is possible to obtain at starting for a given temperature rise, should not be lost sight of. It can quite easily be shown that the temperature rise of the conducting material in the rotor circuit is dependent upon the watts per pound of conducting material and the specific heat of this material. This is based on the assumption that all the heat which is generated at starting is absorbed by the material in which it is generated.

It is also well known that the starting torque, expressed in synchronous kilowatts, is equal to the kilowatts lost in the rotor circuit. Hence to obtain high starting torques and at the same time keep the temperature rise within reasonable limits, it is necessary to have a large amount of conducting material in the rotor circuit, as we are limited by the specific heat of this material.

We cannot put too much loss in the rotor bars, as this would result in high temperature rise in the rotor circuit under operating conditions, thus causing undue temperature rise in the stator. In a great many motors we must therefore place most of the loss in the end rings. Assuming a definite specific heat and resistivity of the material to be used in the end rings, the loss in the rings can be increased in proportion to the square of the diameter for the same loss per pound of material.

We would of course increase the loss in the end rings without prohibitive heating at starting by increasing the specific resistance of the material used, which would increase the section and thus enable use of a smaller diameter. This, however, becomes a very serious objection in many cases, as the end ring assumes such huge proportions that the ventilation is seriously impaired. In this, as in nearly everything that the designing engineer has to decide, he must rely upon his judgment for the choice of the diameter as affected by the loss in the rotor.

S. Haar: Mr. Hoock prefixes to the curves of his paper a description of the circle diagram for determining characteristic curves and of the leakage factor as a design constant, in the use of which he follows the general practise in Europe. Here, in the United States, the analytical methods of design and calculation are still in use, which in my opinion are the only ones suitable for the designer in commercial practise. Had Mr. Hoock spoken from the point of view of a teacher or a lecturer before a mixed audience, it would be idle to take exception to his methods because in such a case there would be no question of the results being used in a contract; however, the introductory paragraphs of the paper express the hope that the curves deduced will be useful for the layout of a line of motors, and for this reason I wish to dissent from his methods.

I have investigated every circle diagram which has come to my notice and have yet to find one which always gives reliable results. While the circle diagram is useful for approximating extended characteristic curves or for demonstration purposes, it must always remain an approximation because it is fundamentally wrong. The magnetizing current does not remain constant, and the locus of the end of the current vector is not a circle, as I proved by determining analytically the centers of circles drawn through successive groups of points. Short arcs of the locus may strongly resemble a circle, but neither the centers nor the radii of the successive circles remain constant. Since the determination of part load power factors by the circle diagram is easier than by the analytical approximations, the lack of a thoroughly reliable circle diagram is regrettable. To my mind, one reason for the confidence in the circle diagram is its trial on motors with high power factor and efficiency; for such motors, almost all methods of approximating give good results. Unfortunately, however, some of the motors which are built have low power factor; here the rapid approximations fail so that just when reliability is most needed, the accuracy of a circle diagram is most uncertain.

Furthermore there is no saving of time in most cases. Four points or at most five will determine satisfactorily the characteristics of a motor between half load and 50 per cent overload. A five point curve can be calculated by a person of average skill with a slide rule within an hour, and an unusually rapid calculator can reduce this time to about 40 minutes. In order to secure an accuracy comparable with the analytical method, a circle about 20 in. in diameter is necessary and by the time the diagram is laid out to scale just as much time will be consumed. In the occasional case when the characteristics throughout the whole range of motor action are desired and exact values are not necessary, the circle diagram saves time.

The system of calculations of either Dr. Steinmetz or Professor Pender seems to give the same results; after a trial of both I find practically no difference in the time required. One great advantage of the Steinmetz method is that the whole calculation can be carried out with only a slide rule, while it is necessary to have tables of some of the factors employed in the Pender system. It may be objected that the theory of these methods is difficult of comprehension. This argument does not apply as calculators unacquainted with the theory have no difficulty in obtaining correct results. All that is necessary is to tabulate the successive steps and follow the order systematically.

The leakage coefficient by definition involves both the magnetizing current and the leakage reactance; these vary in quite different manners and the difficulty of their predetermination is not at all the same (it is comparatively simple to calculate the magnetizing current); why then use a factor containing two independent variables when one will accomplish the same purpose? The leakage coefficient is no more constant than the leakage reactance; if it is necessary to calculate either one, the same assumptions are used, and if data from tests are at hand, the same readings in either case will give the necessary information. A designer using the leakage coefficient would probably draw the diagram for the purpose of obtaining the maximum torque and maximum output of a motor; whereas by the use of very simple formulas involving the resistance and reactance, these characteristics can be calculated in a few minutes.

Experience indicates that short cuts such as the curves presented in this paper will be of only moderate value to any one except the author because their aim (selection of motor dimensions to meet certain guarantees) is not the chief criterion by which the dimensions usually are decided, and furthermore because motor design is an art and not a science. By this I mean that the arbitrary assumptions, such as the spread of flux in the gap, the length of coil ends, etc., differ enough among designers so that it would be necessary for every one using the author's methods to refigure the curves to suit his own particular practise. It usually happens that a motor must be developed from patterns and dies already existing; therefore the first approximation will be from the frame of something already built which would have the rating desired at the speed given. The next point, in my opinion, is to settle the maximum output. After this comes the choice of copper densities and finally the efficiency and power factor. It is my experience that a satisfactory design can be arrived at more quickly by this method than by starting from the $D^2 l_i$. This constant must be used with considerable discretion, and I should prefer to consider it only as a check on other methods. For determining the gap diameter, a good beginning can be made by choosing the number of slots per pole per phase and tooth pitch by judgment to conform to the guarantees expected, and selecting the nearest practicable diameter.

It is the exception rather than the rule, however, that motor builders are called upon to meet definite guarantees, and it is probable that as time goes on, the custom will die out. In the early days of motor drive, when power plants were smaller and the cure for troubles from low power factor was not so generally known, high power factor was demanded, and the motors were relatively expensive. Nowadays, it is frequently more economical to use smaller motors of lower constants and adopt additional means for obtaining a satisfactorily high power factor; in the future this tendency will probably increase.

J. D. Nies: I believe that the following is a simpler method of determining the wattless magnetizing current than that used by Mr. Hoock.

Let B_{ave} be the average density in lines per sq. cm. in the gap, or

$$B_{ave} = \frac{\text{flux of one pole}}{\text{area of one pole}}$$

Let h be the ampere-turns required to project this flux density across the air gap of Δ cm. depth, or

$$h=\frac{B_{ave}\,\Delta}{0.4\,\pi}$$

this being the average value of the sinusoidally distributed m.m.f.

Then the wattless magnetizing current per circuit is

$$I=1.16\,\frac{h\sin\frac{\pi}{2\,N_{sp}}}{n_c\sin\frac{\lambda\,\pi}{2}}$$

in which

N_{sp} = slots per pole in the stator.

n_c = half the conductors per slot, or turns per coil in windings having as many coils as slots.

λ = pitch expressed as decimal fraction of full pitch.

$1.16=\frac{\pi^2}{6\sqrt{2}}$ = constant used for three-phase motors.

($1.23=\frac{\pi^2}{8}$ = constant used for two-phase motors).

I = effective amperes per circuit, *i.e.*, in a ring-connected motor with two parallel circuits per phase, I would be the effective current in one circuit only, and the star or line current would be $I\times2\times1.73$.

An example may be taken from Hobart's "Electric Motors," page 421, 1904 edition, where dimensions and tests of the following motor are given:

Horse power	100
Volts	500
Phases	3
Cycles	50
Poles	12
Stator slots	180
Stator slots per pole	15 (N_{sp})
Air gap	0.15 cm. (Δ)
Turns per coil	1.5 (n_c)
Pitch is full	1 (λ)
Connection	star
Circuits per phase	1
Area of 1 pole	625 sq. cm.

Calculation:

Flux per pole 1,480,000 lines.

$$B_{ave}=\frac{1{,}480{,}000}{625}=2{,}370 \text{ lines per sq. cm.}$$

$$h=\frac{2{,}370\times 0.15}{0.4\,\pi}=283 \text{ ampere-turns.}$$

$$I=1.16\ \frac{283 \sin 6 \text{ deg.}}{1.5\times 1}=23 \text{ amperes effective star current.}$$

This result agrees exactly with the tested value as quoted by Hobart.

The above method follows the logical order for such calculations, since it begins with finding the flux density, next the ampere-turns necessary to produce the density, and finally the amperes to give the ampere-turns. Besides being extremely simple the method is exact under all conditions.

Theodore Hoock: Mr. Fechheimer emphasized the importance of cost analysis in induction motor design. When I tried several years ago to combine the cost data with the machine dimensions, I obtained such complicated formulas that they were evidently not fit for practical use. This was chiefly due to the inactive material, which is usually a large percentage of the total factory cost. A quite accurate estimate of the cost can easily be made by means of a set of curves showing the cost of the different parts in ratio of the outside and inside diameter of the punchings, core length, voltage, number of poles, type of winding, number of slots, and so on.

As to Mr. Alexanderson's remarks on the adaptability of the derived formulas, I refer to the statement in my paper. " The derived formulas are not intended to supersede the detailed design but they should be used for the first layout."

The main object in presenting these investigations was to study the relation of the different items of the performance. There are only two publications to my knowledge on the subject of minimum leakage. Hobart* published curves showing a constant decrease of the leakage coefficient when the core length was made smaller (the influence of the end connection caused this misleading conclusion), while Professor Arnold† derived the opposite result, that is, an increased ratio of core length to pole pitch should decrease the leakage. We see however that there are distinct minimum values which may be obtained from equations (**11**) and (**12**).

I made the statement that the $D^2\,l_i$ of standard machines is not limited by the temperature rise. There are certainly exceptions. A definite limit cannot be given on account of the influence of construction and ventilation. In case blowers are used to cool windings and cores, the temperature limits will seldom be reached (at reasonable speeds), but the air gap density will cause disturbance. Either the iron losses will effect the efficiency (open slots) or prohibitive magnetic noise may be the result. I mean the objectionable hum, howling or whistling of

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*H. M. Hobart, St. Louis, 1904. A. I. E. E. TRANSACTIONS.

†E. Arnold. *Wechselstromtechnik.* Vol. V.1., p. 343.

machines which eliminates their application in hospitals, apartments, hotels or offices.

We have at the present time no systematic investigations on this subject, neither on alternating-current or direct-current machines, and only cut and try methods in connection with experience are available. The prediction of noise with certain slot combinations is very uncertain. There are many other sources for magnetic running or starting noise, as the amplitude of variation of the air density, field form, rotor currents, unsymmetrical windings, loose punchings, weak yoke iron, mechanical and electrical resonance, etc. Fundamental research work on magnetic noise should be well worth while undertaking by advanced students. Any results on this subject will be appreciated highly by the designing engineer.

Mr. Haar discussed at length a part of my paper, which is in no way connected with the main scope of it. The superiority of analytical over graphical methods for commercial work is open to discussion. The preference for either one is chiefly a question of personal taste. The circle diagram has proved a success. It might be interesting to note that approximately 30 to 40 per cent of all induction motors built in the United States are designed or guaranteed on the basis of the circle diagram. I personally use either one or both methods whenever the problem demands it. The advantage in using the leakage coefficient or its reciprocal value is brought out in my paper and by Mr. Alexanderson.

The accuracy of the results obtained by the slide rule or circle diagram is practically the same. It is, furthermore, not justifiable to condemn a method because of its inaccuracy in a problem in which the first assumption cannot be predetermined or vary within 5 to 10 per cent. The argument that a calculator who is unacquainted with the theory can obtain correct results when using the slide rule only is in my mind no proof in favor of the analytical, but for the graphical method. A number of lines can easily be drawn and its construction memorized, but the large number of equations have always to be looked up in order to avoid mistakes. On the other hand, I consider it a dangerous procedure to take calculated or graphically obtained results from a person who is not acquainted with the theory.

The results of my paper are based on correct scientific investigations and since art (motor design) is an applied science I do not see why a designer could not use the results in the table given in the Summary to great advantage.

Regarding the remarks of Professor Nies, I can only say that I go through the same steps that he does with the exception that I avoid the complication of two sine functions. Furthermore, he neglects in the cited example the increase of the magnetizing current due to the slot opening and saturation of the iron. Taking these points into consideration his method of calculating the magnetizing current is quite correct.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

THE APPLICATION OF CURRENT TRANSFORMERS TO THREE-PHASE CIRCUITS

BY J. R. CRAIGHEAD

The performance of current transformers when their secondaries supply simple series loads, and the methods of test for determining the errors introduced by the transformer when used with known secondary connected loads, have been considered in previous papers. The secondaries of two or three current transformers whose primaries are supplied from the lines of a three-phase circuit, are, however, frequently interconnected to save room, simplify wiring, and diminish cost. In this case the equivalent load carried by the current transformer secondary can not be determined in the simple manner that applies to an ordinary series connection, since the devices used in a series circuit may constitute a very different equivalent load when used in an interconnected circuit. An understanding of the equivalent load carried by each transformer is necessary in order to determine suitable limits of load from results of tests made in the ordinary manner. The term "equivalent load" is here used to indicate the load carried by the secondary of a current transformer where this may differ from that obtained by combining in series the resistances and reactances of the devices used.

In interconnecting secondary loads for current transformers, the load is placed in the form of a Y, the differences between the various interconnections arising from the various methods of connecting the transformer secondaries to the three load terminals. The difference between this load and the ordinary load connected in Y to power transformers is that the power circuit operates with practically constant voltage, while the current and the impedance of the devices connected change together: while on the Y supplied by current transformers, the impedances of the devices remain constant, the current and voltage changing together.

The following formulas serve to determine the delta voltages (voltages between external terminals) of a Y-connected circuit, when the resistance, reactance and current flowing in each line are known.

Referring to Fig. 1, let A, B and C be any three Y-connected loads for current transformers. Using the ordinary nomenclature,

$$z_A = r_A - j\,x_A$$

$$z_B = r_B - j\,x_B$$

$$z_C = r_C - j\,x_C$$

The currents flowing are I_A, I_B, I_C. If ϕ represents the angle by which I_B lags behind I_A, and S is the ratio of I_B to I_A (r.m.s. values of equivalent sine waves),

$$I_B = S\,I_A\,(\cos\phi + j\sin\phi)$$

Then, since the circuit is a Y-connection,

$$I_C = -I_A - I_B = -I_A - S\,I_A\,(\cos\phi + j\sin\phi)$$

The voltages from the three terminals to the common point, across each of the three loads, are

$$e_A = I_A\,(r_A - j\,x_A)$$

$$e_B = I_B\,(r_B - j\,x_B) = S\,I_A\,(\cos\phi + j\sin\phi)\,(r_B - j\,x_B)$$

$$\begin{aligned} e_C = I_C\,(r_C - j\,x_C) &= (-I_A - S\,I_A\,(\cos\phi + j\sin\phi))\,(r_C - j\,x_C) \\ &= -I_A\,(1 + S\,(\cos\phi + j\sin\phi))\,(r_C - j\,x_C) \end{aligned}$$

The delta voltages across the supply terminals, entering A, B and C respectively, are

$$\begin{aligned} E_1 = e_A - e_B &= I_A\,(r_A - j\,x_A) - S\,I_A\,(\cos\phi + j\sin\phi)\,(r_B - j\,x_B) \\ &= I_A\,\{r_A - j\,x_A - S\,(\cos\phi + j\sin\phi)\,(r_B - j\,x_B)\} \end{aligned} \tag{1}$$

$$\begin{aligned} E_2 = e_B - e_C &= S\,I_A\,(\cos\phi + j\sin\phi)\,(r_B - j\,x_B) + I_A\,\{1 + S\,(\cos\phi \\ &\quad + j\sin\phi)\}\,(r_C - j\,x_C) = I_A\,[S\,(\cos\phi + j\sin\phi)\,(r_B - j\,x_B) \\ &\quad + \{1 + S\,(\cos\phi + j\sin\phi)\}\cdot(r_C - j\,x_C)] \end{aligned} \tag{2}$$

$$E_3 = e_C - e_A = -I_A \,\{1+S\,(\cos\phi + j\sin\phi)\}\,(r_C - j\,x_C) - I_A\,(r_A - j\,x_A) = -I_A\,[\{1+S\,(\cos\phi + j\sin\phi)\}\,(r_C - j\,x_C) + (r_A - j x_A)] \quad (3)$$

Secondary loads for current transformers are expressed in terms of volt-amperes and power factor at a standard current and frequency. The formulas given above are stated in terms of I_A, and the angle

$$\tan^{-1}\frac{\text{imaginary component}}{\text{real component}}$$

gives the angle between E_1, E_2 or E_3 and I_A. The angle between E_2 and I_B may then be obtained by subtracting ϕ, and that between E_3 and I_C by subtracting the angle between I_C and I_A, obtainable in the same manner from the preceding equation for I_C. The cosines of these angles are the power factors of the equivalent secondary loads.

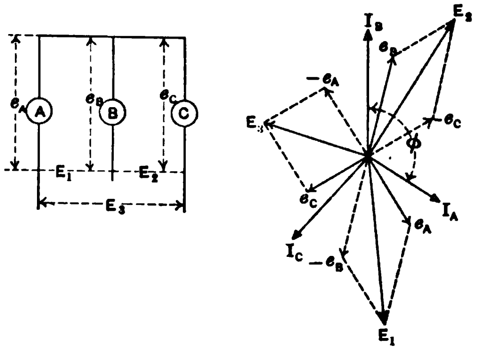

FIG. 1.—Y-connected loads. Connections and theoretical diagram

The volt-amperes supplied by a transformer are the product of the voltage across the secondary by the current flowing in it, but this value must be reduced to standard conditions. If E is the voltage and I is the current, the volt-amperes at 5 amperes (used as a standard in stating current transformer loads) $= E\,I \times \frac{25}{I^2}$ or $\frac{25\,E}{I}$. In all balanced current conditions, the voltages supplied to different parts of the circuit may be compared instead of the volt-amperes, the current being merely a common multiplier which may be neglected for convenience in comparison. In any Y-connected circuit where the conditions are known, the voltages may be obtained by substitution in formulas (**1**), (**2**) and (**3**). These formulas are based on the phase position of I_A, and the angles obtained from (**2**) or (**3**) must be corrected by addition or subtraction of the angle between I_A and I_B or I_C in order to represent the phase angles of the voltages with respect to I_B or I_C.

For any given case these formulas will give the three delta voltages on the loads. If two transformers only are used, two of these voltages are the secondary voltages at which the transformers operate. If three transformers are used, they are connected in Y, and divide the delta voltage into Y components, whose magnitude and phase position for each transformer is a function of the exact characteristics of the transformer as well as of those of the loads. In considering each particular case, the formula may be applied, or an approximate result as to possible maxima may readily be reached by an inspection of the load diagram.

In using two current transformers on a three-phase circuit, they may be connected symmetrically, in two lines, as though one transformer were omitted from a three-transformer Y-connection. This is called "straight". connection. Or the secondary of one transformer may be reversed; this is called "cross" connection, and is equivalent to an open delta connection of the secondaries.

The following causes ordinarily produce negligible effects on the amounts and phase position of the equivalent loads on the current transformers, and will be omitted from the discussion:

1. Variation of wave shape in the primary current.

2. Differences between primary and secondary currents due to the phase angle and inaccuracy of ratio of the transformer.

The following causes may change the equivalent secondary loads carried by the transformer without any alteration of connections:

1. Change in the relative amounts of current in the primary lines.

2. Change in the phase angle between currents in the primary lines.

Variation of load due to these causes (changes of S and ϕ in the above formulas) must be accepted as unavoidable, and a reasonable margin should be allowed for their effect in planning an installation.

The following causes control the amount of equivalent secondary load carried by the current transformers, when conditions stated above do not vary, and are the real basis for selecting combinations which will operate properly on three-phase circuits:

1. The amounts (volt-amperes or impedances) of the secondary loads *A*, *B* and *C*.
2. The power factor of each of these loads and the relation of these power factors to one another.
3. The number and method of connection of the current transformer secondaries.

A short consideration will be given to each of the chief conditions arising from the above mentioned variations, referring more frequently to figures than to the formulas stated above.

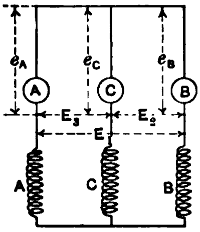

FIG. 2.—Three transformers. Secondary connections

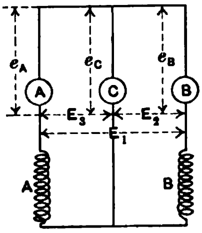

FIG. 3.—Two transformers straight-connected. Secondary connections

A. TWO TRANSFORMERS "STRAIGHT"-CONNECTED, FIG. 3

1. Balanced conditions throughout; equal primary currents 120 deg. apart, equal secondary loads *A*, *B* and *C* of the same power factor. (Fig. 5.)

The voltage of transformer *A* will be $e_A - e_C = -E_3 = (r_A - j\,x_A)\ I_A\ (1-0.5+j\,0.866+1) = e_A\ (1.5+j\ 0.866) = 1.73\ e_A$ $\tan^{-1}\frac{0.866}{1.5}$ or 1.73 e_A lagging behind e_A 30 deg.

The voltage on transformer $B = e_B - e_C = E_2 = I_A\ (r_A - j\,x_A)$ $(-0.5+j\ 0.866+1-0.5+j\,0.866) = e_A\ (i\ 1.73) = 1.73\ e_A$ lagging 90 deg. behind e_A, or 1.73 e_B leading e_B by 30 deg. That is, the volt-amperes on each transformer are equal to 1.73 times the volt-amperes of load *A*, *B* or *C*; but the phase angle between voltage and current is changed 30 deg. in the lagging direction on transformer *A*, and in the leading direction on transformer *B*. Evidently for power factor of secondary loads *A*, *B* and *C* varying from unity to zero, the power factor of the equivalent load on transformer *A* will vary from 0.866 leading to 0.5 lagging; while on transformer *B* it will vary from 0.866 lagging to a negative 0.5, which must be considered as meaning that the input to the transformer is really on the secondary side.

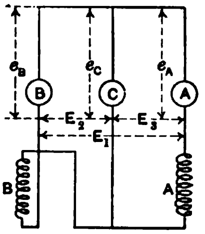

Fig. 4.—Two transformers, cross-connected. Secondary connections

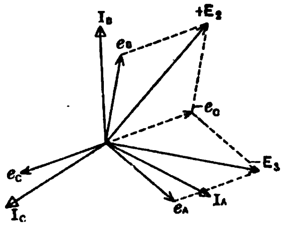

Fig. 5.—Two transformers, straight-connected. Primary currents equal and 120 deg. apart. Loads *A*, *B* and *C* equal and of the same power factor.

2. Equal primary currents, 120 deg. apart, secondary loads varying in amount and power factor. (Figs. 6 and 7, 8, 9, 10.) Fig. 6 shows the effect of varying load *C* from a very low value to a very high value, while the power factors of *A*, *B* and *C* remain

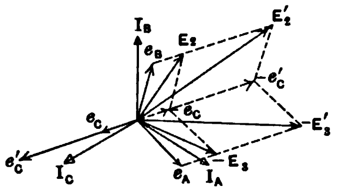

Fig. 6. — Two transformers, straight-connected. Primary currents equal and 120 deg. apart. Loads *A* and *B* equal load $C(e_C)$ less than *A* or *B*, and also (e_C') greater than *A* or *B*.

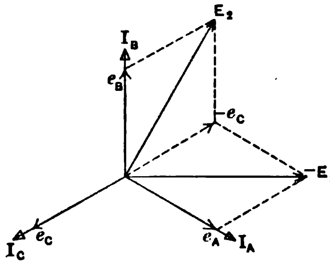

Fig. 7. — Two transformers, straight-connected. Primary currents equal and 120 deg. apart. Secondary loads equal, and non-inductive.

constant. From the formula, when $r_C - j\,x_C = 0$, $E_2 = e_B$ and $-E_3 = e_A$, which means that the circuit is really two separate circuits, electrically in contact at one point only. When $r_C - j\,x_C$ becomes (proportionally) so large that e_B and e_A may be neglected, $E_2 = -e_C$ and $-E_3 = -e_C$.

That is, if load C is diminished or A and B are increased the loads on the two transformers approach the amount and power factor of load A and load B respectively; if load C is increased (or A and B are diminished) the loads on the two transformers approach the value of C, and the angle corresponding to the power factor of the equivalent secondary load approaches the value 60 deg. $+\theta_C$ for transformer B and 60 deg. $-\theta_C$ for transformer A, where θ_C is the angle by which I_C lags behind e_C. Figs. 7, 8 and 9 show the effect of variation of power factor of load C, loads A and B remaining non-inductive. Fig. 7 shows power factor of $C=1$, Fig. 8, 0.5, Fig. 9, 0.1. Fig. 8 evidently represents the maximum equivalent load which can be caused by change of power factor, which is the arithmetical sum of the volt-amperes of A and C occurring where $\theta_C-\theta_A=60$ deg. In Fig. 9, passing lower than 0.5 power factor, when $\theta_C-\theta_A$ is greater than 60 deg., the voltage developed on both transformers decreases. The tendency with lagging power factors in load C is to increase the equivalent load on the transformer A which is connected in the leading phase, and to diminish the equivalent load on transformer B which is connected in the lagging phase.

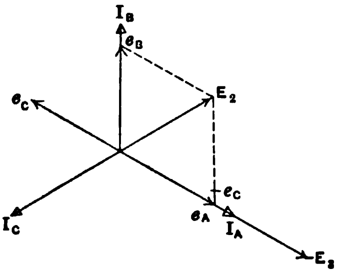

FIG. 8. — Two transformers, straight-connected. Primary currents equal and 120 deg. apart. Secondary loads equal, loads A and B non-inductive, load C 0.5 power factor

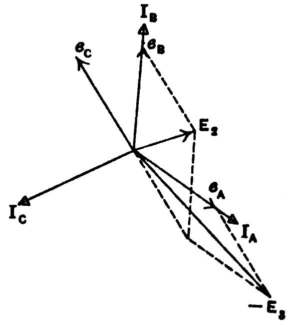

FIG. 9. — Two transformers, straight-connected. Primary currents equal and 120 deg. apart. Secondary loads equal; loads A and B non-inductive, load C 0.1 power factor

Low power factor in loads A and B combined with high power factor in C produces similar conditions, but here the maximum voltage is on transformer B (in the lagging phase) instead of A, (in the leading phase). See Fig. 10, which shows a combination where E_2 is near the maximum limit.

3. Primary currents varying in amount and power factor. To avoid complication in the diagram, the secondary impedances are shown equal and of the same power factor. Fig. 11 shows effect of diminishing I_C and Fig. 12 the effect of increasing I_C. The limit in one direction is reached when I_C becomes zero, in which case the two transformers are working on a single-phase circuit, carrying load *A* and *B* respectively; that is, by formula, $E_2 = e_B$ and $-E_3 = e_A$: and in the other direction when the angle between I_A and I_B diminishes toward zero, when (if the power factors of *A*, *B* and *C* are alike), transformer *A* carries the arithmetical sum of e_A and e_C, while transformer *B* carries the arithmetical sum of e_B and e_C.

When the currents in the two transformers are unequal, (see Fig. 13). The transformer *B* having the larger current carries a load approaching the arithmetical sum of e_C and e_B: while the transformer *A* having the smaller current carries a voltage approaching e_B. As this transformer does not carry full current, its volt-ampere load is not fairly represented unless it be reduced to standard terms for comparison. That is, for transformer *B* (assuming $I_B = 5$ amperes)

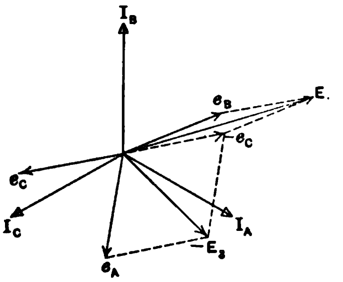

FIG. 10.—Two transformers, straight-connected. Primary currents equal and 120 deg. apart. Secondary loads equal; loads *A* and *B* have low power factors, load *C* a high power factor

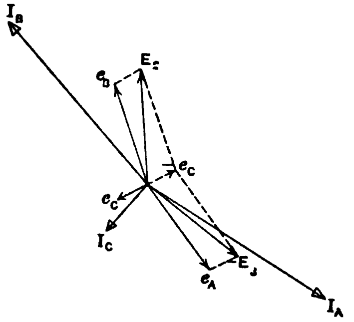

FIG. 11.—Two transformers, straight-connected. Equal primary currents in the two transformers. Smaller current in the line without transformer. Secondary loads equal and of the same power factor

$$\text{volt-amperes} = 5\,E_2$$

For transformer *A*,

$$\text{volt-amperes} = I_A \times E_3 \times \frac{25}{I_A{}^2} \text{ or } = \frac{25\,E_3}{I_A}$$

substituting actual values in (2) and (3), if $I_A = 0.5$ amperes and $I_B = 5$ amperes ($S = 10$), $r_A = r_B = r_C = 1$, $x_A = x_B = x_C = 0$, and $\varphi = 120$ deg., $E_2 = 0.5[10(-0.5 + j0.866) + 1 + 10(-0.5 + j0.866)]$ $= 9.76$ volts. Volt-amperes on transformer $B = 9.76 \times 5 = 48.80$ volt-amperes. $-E_3 = 0.5(1 + 10(-0.5 + j0.866) + 1) = 4.58$ volts.

$$\text{Volt-amperes on transformer } A = \frac{4.58 \times 25}{0.5} = 229 \text{ volt-amperes.}$$

This calculation represents a very extreme case of unbalancing. The loads C and B are almost entirely carried by transformer B with full current, while the volt-ampere load on transformer A is more than four times as great as that on transformer B. It should be noted, however, that the current on this transformer is a very small part of the total amount flowing, and therefore the actual error caused by the overload is small.

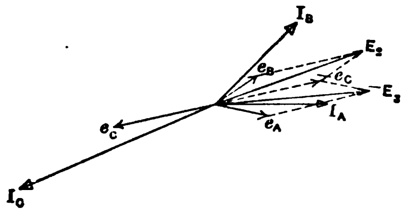

FIG. 12.—Two transformers, straight-connected. Equal primary currents in the two transformers. Larger current in the line without transformer. Secondary loads equal and of the same power factor

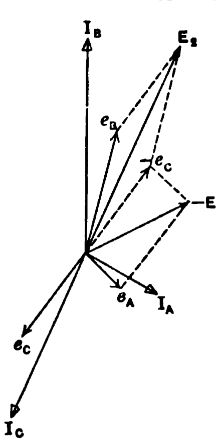

FIG. 13.—Two transformers, straight-connected. Unequal primary currents in the two transformers. Secondary loads equal and of the same power factor

To summarize equivalent loading on the straight connection:

1. Under completely balanced conditions, the load on each transformer is 1.73 times one of the three equal loads, and the power factors of effective secondary loads are altered by a shift of 30 deg. in the corresponding angle, lagging for one, leading for the other.

2. With balanced primary conditions, variations of amount and power factor in the secondary connected loads produce different distributions of load between the two transformers, the maximum load on either transformer not exceeding the arithmetical sum of its load and the load in the secondary line without transformer, and the increase of load on one transformer due

to variation of power factor being in general accompanied by a decrease in the load on the other.

3. Where primary conditions become unbalanced, the tendency is to increase the volt-ampere load on both transformers, especially that carrying the smaller current.

B. Cross Connection, Two Transformers

1. Balanced primary currents, 120 deg. apart, equal secondary loads of the same power factor. (Fig. 4 and 14).

Since the transformer *B* has its secondary reversed, the current I_B is 180 deg. from its previous position. The current I_C is the resultant of two currents 60 deg. apart instead of 120 deg. and is not proportional to the current in any single primary line.

From formula (2),

$$E_2 = I_A\,(r_A - j\,x_A)\,\{1 + 2\,(\cos - 60\text{ deg.} + j\sin - 60\text{ deg.})\}$$

$$= e_A\,(2 - j\sqrt{3}) = e_A\sqrt{7}\text{ leading } e_A \text{ about 41 deg.}$$

or lagging I_B about 19 deg.

From formula (3),

$-E_3 = I_A\ (r_A - j\,x_A)\ (2.5 - j\,0.866) = e_A\ \sqrt{7}$, leading e_A about 19 deg.

The voltages $-E_3$ and E_2 carried by the transformers are each removed only about 19 deg. from e_A and e_B instead of 30 deg., as in the straight connection, and their values are considerably greater than in the straight connection with the same loads, because of the greater I_C and the smaller angle between $-e_C$ and e_A or e_B.

2. Balanced primary currents, 120 deg. apart. Secondary loads varying in amount and power factor.

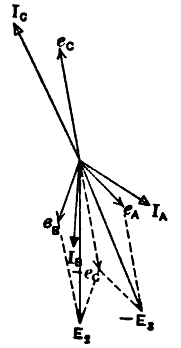

Fig. 14.—Two transformers cross-connected. Equal primary currents, 120 deg. apart. Secondary loads equal and of the same power factor

It is evident from an examination of Fig. 14, that the maximum voltage on either transformer due to changes in the relative size of the load cannot exceed the arithmetical sum of e_C and e_A or e_B. Also, that because of the smaller angle between I_A and I_B, the phase displacements of E_2 and E_3 due to the differing power factors in *A*, *B* and *C* will be in general less than on a straight connection. If load *C* is reduced to zero we have the same condition as on the straight connection; that is, two separate circuits which are in electrical contact at only one point.

3. Primary currents varying in amount and power factor.

Variations in equivalent load and power factor of load caused by this will be of the general nature of those with the " straight " connection, but will be somewhat less because of the smaller angle between I_A and I_B. There is the same tendency for the transformer carrying the smaller current to operate against a comparatively heavy volt-ampere load.

Summary of Cross-Connection. This method gives a true secondary representation of only two of the three primary currents; the total load carried by the transformer is greater than where the same apparatus is used with the straight connection. The effective loads, however, are somewhat less influenced by changes in primary current or differences of power factor of the secondary connected loads than the " straight " connection.

C. THREE TRANSFORMERS WITH SECONDARIES Y-CONNECTED

The voltages carried are those shown in the " straight " connection, with the third voltage which completes the voltage triangle. The transformers, however, are Y-connected, and the division of voltage among them is dependent on the characteristics of the individual transformers and the conditions in the primary lines. For this reason the exact voltage for each transformer in an actual case is difficult to calculate even with full knowledge of the characteristics of the transformers. In practically every case, the mean equivalent loads on a two-transformer straight connection are diminished by the insertion of a third transformer to complete the Y. If the common point of the three loads is connected to the common point of the three transformer secondaries by a lead of negligible impedance, the connection becomes simply three independent circuits which are electrically in contact at one point only.

The following methods have been in use for some time for approximation of the volt-ampere loads on interconnected circuits. They are based on the formulas for balanced conditions of primary current and for secondary loads of the same power factor. They are sufficiently accurate to use as a check to prevent the overloading of transformers.

A. TWO TRANSFORMERS WITH SECONDARIES " STRAIGHT "-CONNECTED

This is best divided under three headings, according to the ratio of total volt-amperes on the secondary line having no current transformer to the total volt-amperes in the line directly connected to the secondary of the transformer considered.

a. Where the ratio is greater than 3.2. Total volt-amperes on the transformer under consideration equals the sum of volt-amperes directly connected to its secondary and volt-amperes in secondary line without transformer.

b. Where the ratio is less than 3.2 and greater than 0.4. Total volt-amperes on the transformer under consideration equals the sum of volt-amperes in the line directly connected to its secondary and 0.75 times the volt-amperes in the secondary line without transformer.

c. Where this ratio is less than 0.4. Total volt-amperes on transformer under consideration equals the sum of volt-amperes in the line directly connected to its secondary and 0.5 times the volt-amperes in the secondary line without transformer.

B. TWO TRANSFORMERS, WITH SECONDARIES "CROSS"-CONNECTED

The total volt-amperes on each transformer equals the sum of the volt-amperes in the two lines directly connected to the two secondaries and three times the volt-amperes in the secondary line without transformer, the whole divided by two.

C. THREE TRANSFORMERS, WITH SECONDARIES Y-CONNECTED

Total volt-amperes on each transformer equals the sum of the volt-amperes of the three secondary loads, divided by three.

GENERAL CONCLUSIONS

Certain methods of interconnection of secondary circuits of current transformers are used because of advantage in cost, space occupied, simplicity and convenience.

The use of these interconnections results in the transformers carrying equivalent secondary loads which differ decidedly from those resulting from the use of the same devices with a plain series secondary connection. The power factor of the effective secondary load may be leading or even negative in extreme cases.

The variations in equivalent secondary load due to the power factors of the separate loads have a general tendency to offset one another; that is, when the power factor of one equivalent load is changed in the leading direction, the other is usually changed in the lagging direction, when one equivalent load is increased, the other is usually diminished. Therefore, these variations may be neglected in making approximate estimates of volt-ampere loads. A method of making estimates based on the assumption that the power factors of the three secondary loads are alike will give results accurate enough to prevent overloading.

Unbalancing of primary currents has a general tendency to increase loads on interconnected current transformers, and where the circuit is known to be unbalanced to an unusual degree, interconnections should be avoided or the loads connected to the secondaries should be kept considerably below the amounts allowable under balanced conditions.

All load estimates made in the approximate way given are of value chiefly as mean results for the combination, and not as definite limits for the equivalent load on each transformer.

The exact volt-amperes and power factor of the equivalent loads of a two-transformer combination may be obtained if re quired from formulas (**1**), (**2**) and (**3**). The results for a three-transformer combination cannot be exactly calculated from the volt-amperes and power factor of the separate loads, because the characteristics of the transformers themselves affect the division of the load among them. This circuit may be changed by the addition of a common return lead to three simple series circuits, whose volt-amperes and power factor are easily obtainable. This is the better connection except where the load in one line is an overload for one transformer, when the interconnected combination divides the load in such a way as to relieve the overloaded transformer.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

COST OF TRANSFORMER LOSSES

BY E. C. STONE AND R. W. ATKINSON

The following paper is an investigation of the losses produced in a system by the distributing transformers, with a view to determining the cost to the central station of supplying these losses. The cost of the losses occurring in a transformer is of the same order of magnitude as the cost of the transformer itself and should, therefore, receive the same amount of consideration. If a transformer had a perfect magnetic (iron) circuit and a perfect electric (copper) circuit, no losses would be produced if it were placed on a system. Hence the losses which do actually occur come into two general divisions, *viz.*: (1) losses due to imperfect iron, and (2) losses due to imperfect copper.

The expense involved may be divided as follows:

1. Elements due to imperfect iron.
    - (*a*) Iron loss, involving:
        - (*A*) Consumption of energy in transformer.
        - (*B*) Station and line capacity to take care of such energy.
    - (*b*) Magnetizing current, involving:
        - (*A*) Copper loss in generator and line.
        - (*B*) Generator and line capacity to take care of this magnetizing current.
2. Elements due to imperfect copper.
    - (*a*) Copper loss, involving:
        - (*A*) Consumption of energy in transformer.
        - (*B*) Station and line capacity to take care of such energy.
    - (*b*) Fluctuating secondary voltage, causing shortening of life of lamps, and perhaps dissatisfaction to consumers.

A great many factors enter into the problem, the value of which can be determined only roughly, hence the absolute result can be only approximate. However, the approximations are of such a nature as to permit of sufficiently accurate selection of the most economical transformer for a given service, from a group of transformers having different performances.

Furthermore, the effect of the losses upon the active material in a transformer will be taken up in a general way, merely to give an idea as to the limitation of the designer with respect to the variation of these losses. Transformers are designed and will be designed and sold to meet the demand, regardless of the cause of the demand. If the operating man specifies a given performance, he will get bids on that performance, though perhaps at prohibitive prices. If he is prepared to accept different performances, knowing exactly what is their relative worth to him, he is in a position to obtain that which will ensure him the greatest ultimate economy, and is able to compare the values of transformers from different manufacturers, different types from one manufacturer and, of not the least importance, different sizes of one type.

It will be assumed in this discussion that all the elements of cost of power supplied to a customer are known, the commercial problems of rate-making being, of course, entirely eliminated. Only the special problems of the cost of the losses will be considered.

The cost of power is ordinarily divided into three parts:

1. Output charge, consisting of all elements of cost proportional to kilowatt-hour output.

2. Capacity or capital charge, consisting of all elements proportional to the kilowatt capacity of the station.

3. Fixed charge consisting of all elements independent of both output and capacity.

The last of these, part of which is often called the " customer charge ", is not affected by transformer losses and requires no further consideration. The cost of the losses is, with certain exceptions, which will be explained later, the same as the wholesale cost of power. The " diversity factor " will be mentioned, in its effect on the copper loss of the transformer. The special problems introduced are the effect of the magnetizing current associated with the iron loss and the regulation due to the copper loss.

The cost of the iron loss of the transformer will include the cost of generating such loss and of supplying the station capacity with which to generate it. It will also include the cost of transmitting the energy consumed by the loss, from the station to the transformer. Iron loss introduces certain peculiar conditions due to the fact that it requires a continuous supply of energy at a power factor very much lower than that of any other part of the load. The power factor of transformer exciting current varies from 10 to 40 per cent. This means that engine and generator capacity much larger than the total iron loss must be kept in service all the time. Energy at 30 per cent power factor requires twice as much generator capacity as energy at 70 per cent power factor and three times as much as at 100 per cent power factor. With full kilovolt-ampere load at such low power factor, the losses in the generator become, of course, a much larger part of the energy output. Suppose that during the day, one engine and generator carrying full kilovolt-ampere load and having 15 per cent engine friction and generator losses, takes care, at times of light load, of transformer exciting current having a power factor of 20 per cent. At full kilovolt-ampere load, the energy generated is 20 per cent, and the loss 15 per cent. Hence the loss which the exciting current produces in the generator and engine is in this case 15/20 or 75 per cent of the iron loss itself. If the power factor of the exciting current had been 100 per cent, the generator would have taken care of four times the energy and the loss would have been only 15 per cent instead of 75 per cent. Likewise, the loss in the line due to the large wattless current becomes a larger percentage of the power.

Furthermore, the greater the exciting current, that is, the lower the power factor for a given iron loss, the greater is the total current at the peak of the load, and hence the greater must be the station and line capacity to take care of the peak. It should be distinctly understood that low power factor of exciting current is not in itself objectionable. For example, suppose that with a constant value of exciting current, the iron loss is reduced. It is evident that the improvement of the transformer has resulted in a lower power factor. It is important, when considering magnetizing current, if the power factor be considered, that it be considered *only* in connection with the core loss. It is ordinarily much simpler and more precise to make no mention of power factor, but only of magnetizing volt-amperes.

The copper losses in station and line produced by the exciting current are, of course, continuous but they are not constant, for they depend on the magnitude, power factor and wave form of the exciting current, and upon the magnitude and power factor of the load. The increase of copper loss caused by the fundamental component of the exciting current depends upon the magnitude of the line current. That part of the exciting current made up of harmonics produces a constant copper loss regardless of the amount of line current. See Fig. 1.

Since magnetizing current increases the line current and continues all day, and therefore at the peak, it necessitates an increase in line and station capacity to take care of it. At the station, the increase of capacity will be only in the electrical end, since obviously, wattless current does not affect engines and boilers.

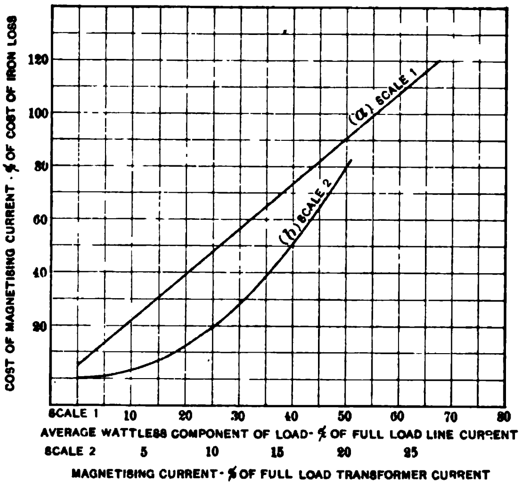

FIG. 1.—Cost of magnetizing current

(*a*) Cost of 5 per cent magnetizing current for various wattless components of load current
(*b*) Cost of various magnetizing currents—wattless component of load = 0

We must next consider copper loss. Several new elements enter here, such as duration of load, relation between station capacity and connected transformer capacity, and effect of regulation.

A transformer is loaded only part of the time, hence, the copper loss is not continuous, so that a kilowatt-year of copper loss costs less than a kilowatt-year of iron loss, which is continuous. The load will vary from 0 to 1⅓ or 1½ times the transformer rating—or may be even greater for short periods. For calculations, the load per day should be reduced to an equivalent number of hours of full load, on the basis of square root of mean square current.

Since not all transformers are on full load at the same time, it will require somewhat less than a kilowatt of station capacity to take care of a kilowatt of peak load transformer copper loss. The ratio varies with the nature of the load and also with the number of separate installations on each transformer. If there were only one installation per transformer, it would be the same as the so-called " diversity factor." As the number of customers on a single unit is increased, the peaks tend more and more to occur at different times and the ratio of station capacity to connected transformer capacity increases. The opposite extreme to one installation per transformer would be where there would be only one transformer on the system, in which case station and transformer capacity would be equal. It may be assumed that

$$\frac{\text{rated station capacity required}}{\text{rated transformer copper loss}} = \frac{\text{peak station capacity}}{\text{connected transformer capacity}}$$

both transformer copper loss and capacity being based on full load rating. The annual charge against copper loss will, therefore, be the capital charge, as previously explained, multiplied by this ratio.

Transformer regulation cannot be considered entirely apart from the regulation of the secondary network. The following general discussion applies whether the regulation considered is entirely or in part due to the transformer.

The variation of voltage due to regulation causes irregularity in the amount of light and may be a cause of " flicker ", hence must be kept within reasonable limits in order to satisfy the customer. If the regulation should become great enough to cause noticeable fluctuations in the light, it might outweigh all other considerations in importance.

The effect of the introduction of tungsten lamps may be noted. A given change of voltage, with tungsten lamps, causes less than two-thirds the change in candle power that would be produced in carbon lamps, so that the tungsten lamp will stand more than 50 per cent greater regulation without causing greater change in candle power or more unsatisfactory service. It is a peculiarity of tungsten lamps, moreover, that they do not change in quality of light as does the carbon lamp when voltage is lowered, hence it is quite probable that a considerably greater regulation than above indicated would be permissible on tungsten lamps without causing dissatisfaction. A common standard on direct-current circuits is a maximum of 2 per cent regulation for carbon lamps. The same standard of service could be met with tungsten lamps on 3 per cent or possibly higher regulation. Ordinarily the standard is lower on alternating-current circuits, and the allowable regulation would be increased as before for tungsten lamps. This question of customer's satisfaction is an exceedingly important one, and in those cases where regulation causes or may cause a noticeable flicker, it cannot be ignored.

If an incandescent lamp is operated on a fluctuating voltage, the cost of a given amount of light will be greater than if it is operated on a constant voltage having the same value as the mean of the fluctuating voltage. That is, the life of the lamp is shortened so that the cost of renewals becomes greater, and this increase in cost varies as the square of the regulation. Two per cent fluctuation in voltage (2 per cent regulation) will produce about 0.6 per cent increase in cost of renewals of carbon lamps or about 0.3 per cent for tungsten lamps. Except when the regulation is quite low, this is of considerable importance in comparison with the copper loss of the transformer. A more complete discussion of this subject appears in the appendix. (See Fig. 2.)

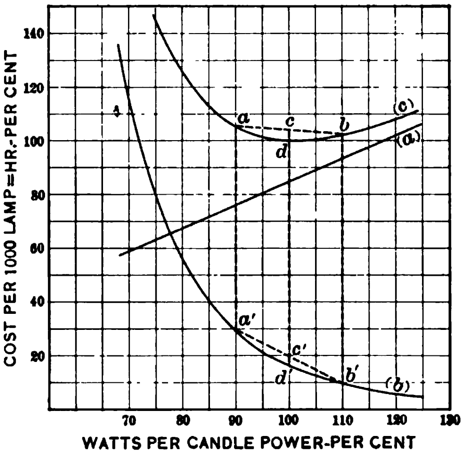

FIG. 2.—Cost curves for carbon lamps

(*a*) Cost of energy
(*b*) Cost of renewals
(*c*) Total cost

It has been said that regulation reduces the voltage upon the load, and therefore, causes a direct loss of revenue by reducing the power sold. Under ordinary commercial operation, regulation causes not a lower voltage, but a fluctuating voltage. There is some voltage which the central station operator desires to maintain at the customer's outlet. If it is possible to maintain this as a constant voltage, provided there is no transformer regulation, then it is obviously just as possible to maintain it as a mean voltage when there is transformer regulation. To make the most desirable voltage the mean rather than the maximum —which is attained only at no-load—is evidently the more economical arrangement. Is it the custom of progressive central stations to regulate their systems so that the really desirable voltage is reached only when there is no load connected? If the mean voltage with transformer regulation is maintained at the same value as the constant voltage without regulation, the power delivered to customers must be the same in both cases, hence there can be no loss of revenue (power sold) due to regulation.

The losses and their effect on the amount of active material required are interesting. Suppose, as an illustration, the voltage of a transformer to be changed, while the kilovolt-ampere output is kept constant. The copper loss will then vary inversely as the $(\text{iron loss})^{\frac{2}{a}}$, where a is the exponent of the iron loss curve, and is about 1.9 in modern distributing transformers. This means that by increasing the iron loss one per cent, the copper loss is reduced in the ratio $\frac{2}{1.9}$ or 1.05 per cent. For example, if the copper loss is twice as great as the iron loss, to reduce the iron loss one watt will increase the copper loss approximately two watts.

In modern transformers, however, the limits of the active materials are such that to change materially the ratio of the losses would require distortion of the design, so that with a given amount of material, to reduce greatly either loss at normal rating would require a much larger increase in the other loss than indicated above. The most efficient shape does not depend upon the ratio of losses except when it must be distorted for the above reasons. In transformers of different voltage ratings and sizes, the relative dimensions, unless distorted as explained above, vary only with space occupied by insulation and relative price of copper and iron.

With the quality of iron at present used in distributing transformers, to decrease the copper loss 1 per cent requires about 1.6 per cent increase in the active material; to decrease the iron loss one per cent requires 1.7 per cent increase; and thus to decrease both losses one per cent requires about 3.3 per cent increase in the cost of the active material.

The above relations are true only when the relative space available for copper is constant, that is, the space-factor is constant. Since the space-factor is improved by increasing the size, neither the additional material required to decrease the losses nor the reduction of material attained by increasing the losses, is always as great as would appear from the above statements, more especially in the small sizes. At all outputs, the question of density comes in, if the size is to be reduced, and makes it possible to go only a very short distance in that direction. There is no limit except cost to the reduction of losses.

It will be of interest to show why the loss and amount of material per kilowatt are less for large capacity. Suppose each dimension of a given transformer be multiplied by two. The amount of material will then be multiplied by $2 \times 2 \times 2$ or 8. If the turns, flux density and current density are kept constant, the losses are also multiplied by 8. The cross-section of copper and iron will each be multiplied by 2×2 or 4; hence the current and voltage will each be multiplied by 4, and the output by 4×4 or 16. From these relations, it will be seen that both the loss per kilowatt (per cent loss) and active material per kilowatt have been *divided* by 2. In many cases, the gain is much greater than this, due to the improved space-factor in the larger sizes.

The problem to be considered in selecting a transformer for a given service is two-fold:

1. The transformer should be operated with the ratio of copper to iron loss such that the total cost of the loss is a minimum.

2. The total cost, taking into consideration the first cost of transformer and the cost of the losses, should be a minimum.

The operator has a certain degree of freedom, inasmuch as there are standard lines of transformers on the market, having different rated primary voltages, for example, 2200 and 2400 volts. He also has a certain degree of freedom as regards the size of the transformer. First the transformer must be large enough to carry its load without overheating. Next it must come as near as possible to satisfying the minimum cost conditions given above. The effect of the ratio of losses can best be seen, as mentioned before, by maintaining the kilowatt load on a given transformer constant, and varying the voltage and current. In this way, any desired ratio of losses may be obtained, according to the relations given above. The curve thus obtained is shown in Fig. 3. As the ratio of losses increases, the copper loss increases much faster than the iron loss decreases, hence the ultimate temperature rise must increase also, but since the transformer with larger ratio is required only for a load of short duration, the maximum rise in that case may not be excessive. Fig. 4 shows the cost of losses for a typical case under the conditions when ratio of copper loss to iron loss is varied from 1 to 4.6. The lower curve is cost of iron loss and copper loss alone.

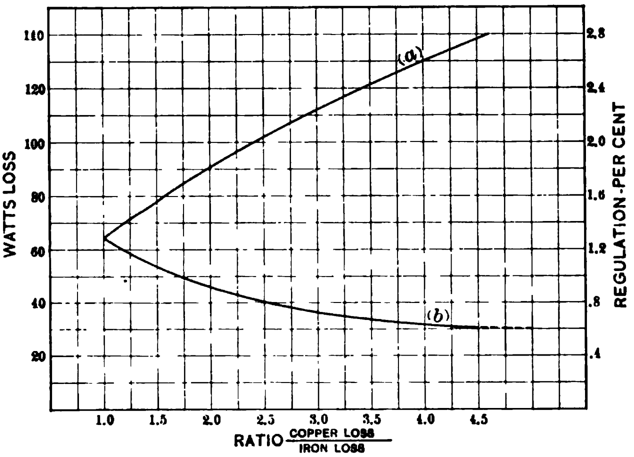

FIG. 3.—2200/220 volts 60 cycle 5 kv-a. transformer. Output constant. Voltage varies

(*a*) Copper loss and regulation
(*b*) Iron loss

The upper curve, Fig. 4, includes the effect of magnetizing current and loss due to regulation, thus giving the total cost of all losses caused by the transformer. It will be seen that here the minimum total loss occurs at a larger ratio of copper to iron loss and that the increase in cost is greater as we depart from the best ratio. This is caused by the high exponent of the saturation curve of the iron which causes the magnetizing current to increase much faster than the iron loss.

The total cost of a transformer performing a given service depends upon the amount which must be paid for the losses during the life of the transformer and upon the price paid for the transformer itself. The smaller the transformer, the greater the cost of the losses and the less the price paid for the transformer and vice versa. In considering losses and price paid for transformer together, the losses may be most conveniently represented as a capital cost by dividing their annual cost by the interest and depreciation factor. The result will be the amount which would be required at the time of installation of the transformer to pay for the losses during the whole life of the transformer, and may be added directly to the first cost in order to determine the total cost of transformer and losses.

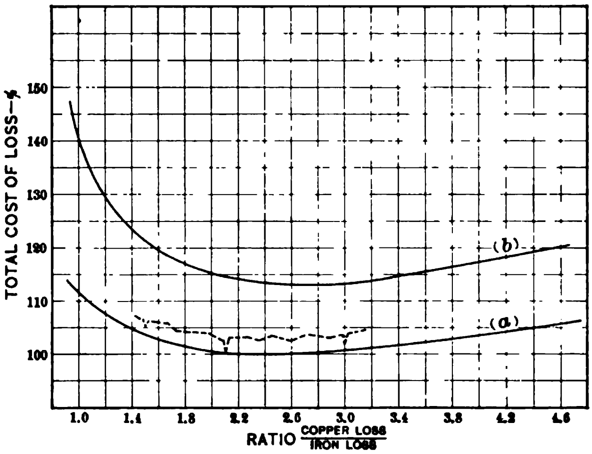

FIG. 4.—Cost of losses

(a) Cost of iron, and copper loss

(b) Cost of all losses including magnetizing current and regulation

For determining the proper capacity of transformer to be used, the use of recording ammeters in the secondary circuits of transformers is well worth while, because of the large saving effected by adaptation of the transformer to its load, which is thus made possible. In general, it will be found that transformers will operate most successfully—losses and first cost considered—when run at their limiting temperature rise. This will mean a small transformer at overload for short-hour load, hence a high ratio of copper to iron loss and small first cost—these being conditions demanded by loads of short duration. The importance of using a transformer of proper capacity for a given load is emphasized. The reduction in cost of losses gained by using a transformer which is too large is not nearly so great as is the increase in first cost.

It may seem from some of the foregoing that the designer is free to vary at will the losses of the transformer, decreasing one at the expense of the other or at the expense of the active material, according to the curves given. This would be approximately true if one were considering the initial design of a standard line of transformers. In this case, the theoretical efficiency of design may be reached, since it is possible to gain full benefit of all such items as special sizes of copper strap, punchings, etc. Otherwise, there would be waste space which is, of course, expensive. In the case of special transformers made in smaller quantities, these items and also the increased proportion of such items as time of engineers, foremen, and inspectors, cause such an increase in cost that it is not possible actually to reach the theoretical curve shown. The true practical condition would be represented by a curve of the form of the dotted one in Fig. 4. The theoretical efficiency will, in general, be reached only on the " standard lines ". In the case of " specials" the departure will be more or less great, depending upon particular conditions. Hence, while general laws may readily be stated as regards the cost of losses from the standpoint of design, yet it cannot be said that any particular desired performance can be obtained in accordance with these curves.

The method of selecting transformers used by the Government seems the most logical. A certain performance is specified, and it is also stated that in comparing transformers of different performances, the iron loss will be evaluated at 88 cents per watt and copper loss at 11 cents per watt. (See Bureau of Standards Bulletin " Specifications for Distributing Transformers.") It would seem quite practical for any purchaser to evaluate the losses similarly, using figures *corresponding to his own conditions*. Were there a general definite idea of the cost of the losses, the standard design would soon be such as to give the minimum total cost of losses and of transformers.

SUMMARY

We have shown (1) a general method of determining the cost of the transformer losses. The cost of energy, the capacity charge, the line loss and the line cost of any part of the system are known very closely. The losses, magnetizing current and heating of the transformer are determined by test as soon as the transformer is received, or perhaps from maker's guarantees. The copper loss has a less cost than the iron loss, due to the reduction in output charge because of its short duration, and also has a slightly less capital cost due to its " diversity factor." The cost of magnetizing current is shown to be of considerable importance in many cases. This is the most variable factor entering into the cost of the losses. The cost of the regulation associated with the copper loss is due, not to any reduction of power sold, but to customer's satisfaction or dissatisfaction and to the effect of the varying voltage upon the performance of the lamps. If the total cost of all losses due to the transformer is 100 per cent, the various elements will be approximately as follows:

Iron loss	40 per cent to 70 per cent
Magnetizing current	1 per cent to 30 per cent
Copper loss	30 per cent to 50 per cent
Regulation loss	1 per cent to 10 per cent

(2) A discussion has been given showing the general relation of the cost of the losses to the amount of material in the transformer. It has been found that for a given amount of material, as the ratio of the losses is varied, the copper loss of the transformer increases faster than the iron loss decreases, thus making the total loss larger, the larger the ratio of the losses. The amount of material in the transformer increases faster than the losses are decreased, when the losses are varied by varying the size of the transformer. For lighting transformers, the amount of active material varies inversely as more than the third power of the loss.

(3) In general, in order to obtain minimum operating costs, transformers of the present standard performances should be used on a load which will bring them to their maximum safe temperature rise.

(4) In the appendix will be found a development of formulas for the numerical determination of the cost of transformer losses; also a special case, worked out in detail with values so chosen as to give, as near as possible, typical results.

We wish to extend our thanks to the Allegheny County Light Co., Rochester Railway and Light Co., the Westinghouse companies, and the National Electric Lamp Association for valuable assistance in the preparation of this paper.

APPENDIX

I. Cost of Iron Loss

Let c_1 = cost of energy per kw-hr., in dollars.
C_1 = capacity charge per kilowatt of station and lines (not including secondary network) in dollars per year.
w = watts iron loss in transformer.

Then, since the iron loss occurs 24×365 = 8760 hours per year, the total cost of the iron loss for a year is:

$$\frac{w}{1000}\,(8760\,c_1+C_1)\text{ dollars} \qquad (1)$$

II. Cost of Magnetizing Current

E = line voltage.
I = line current.
a = power component of line current, expressed as a fraction of the line current, that is, power factor.
b = wattless component of line current expressed as a fraction of the line current.
t = magnetizing component of transformer exciting current, expressed as a fraction of the line current,
$f\,t$ being that part at fundamental frequency,
$h\,t$ being that part made up of harmonics.
k = line loss expressed as a fraction of full load volt-amperes. (If the line loss is given in terms of power output, this value must be divided by the power factor to get k).

The line loss before the transformer is put on is

$$k\,(1)\,E\,I=k\,(a^2+b^2)\,E\,I \qquad (2)$$

After the transformer is on, the line loss becomes

$$k\,(a^2+\overline{b+f\,t}^{\,2}+\overline{h\,t}^{\,2})\,E\,I \qquad (3)$$

or

$$k\,(a^2+b^2+2\,b\,f\,t+\overline{f\,t}^{\,2}+\overline{h\,t}^{\,2})\,E\,I \qquad (4)$$

The increase in line loss is then

$$k\,(2\,b\,f\,t+\overline{f\,t}^{\,2}+\overline{h\,t}^{\,2})\,E\,I=k\,(2\,b\,f\,t+t^2)\,E\,I \qquad (5)\text{ and }(6)$$

If M = magnetizing component of exciting current in volt-amperes, $t = \frac{\text{transformer magnetizing current}}{\text{line current}} = \frac{M}{E\,I}$

and $$k\,M\,(2\,b\,f + t) = \text{increase in line loss.} \qquad (7)$$

the increase in current is proportional to $k\,M\left(b\,f + \frac{t}{2}\right)^{*}$

If b is the average wattless component of the line current per day, this expression will give the average loss caused by the exciting current during the day.

Because of the transformer magnetizing current, the total amount of the peak load is increased, and the line should, therefore, be increased. As a sufficiently close approximation, the line capacity may be assumed to be increased in proportion to the ratio of average increase in current to the rated capacity of the line. Hence, the line loss is increased only in direct proportion to the increase in current. We may then say:

$$\text{Line loss due to magnetizing current} = k\,M\left(b\,f + \frac{t}{2}\right) \text{ watts.} \qquad (9)$$

Cost of line loss due to magnetizing current

$$= \frac{k\,M}{1000}\left(b\,f + \frac{t}{2}\right) \times c_1 \text{ dollars.} \qquad (10)$$

$$\text{The increased capacity of line will be } \frac{M\left(b\,f + \frac{t}{2}\right)}{1000} \text{ kv-a.} \qquad (11)$$

The increased cost of line will be

$$\frac{M\left(b\,f + \frac{t}{2}\right)}{1000} \times \text{cost per kv-a. of line capacity.} \qquad (12)$$

NOTE:—This is only that part of line between transformer and station and only the portion, the cost of which increases with the kilovolt-ampere load.

If k_1 = engine friction and generator loss, in terms of full load kv-a., and b_1 = the average all-day wattless component of total station load, the increase of station loss $= k_1\,M\left(b_1\,f + \frac{t}{2}\right)$ watts.

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*When x is small $\sqrt{1+x^2} = 1 + \frac{x^2}{2}$.

The cost of increased station capacity will be:

$$\frac{M}{1000}\left(b_1 f+\frac{t}{2}\right)\times \text{annual cost per kv-a. of generator capacity.}$$

The total cost of magnetizing current will therefore be: in the line

$$M\left(b f+\frac{t}{2}\right)\frac{(k\times 8760\times c_1+\text{annual cost of line per kv-a.})}{1000} \qquad (13)$$

in the station

$$M\left(b_1 f+\frac{t}{2}\right)\frac{(k_1\times 8760\, c_1+\text{annual cost of generator per kv-a.})}{1000} \qquad (14)$$

COST OF COPPER LOSS

c_1, C_1 = cost of energy and station capacity as in cost of iron loss.

$$D=\frac{\text{peak station capacity}}{\text{connected transformer capacity}}$$

h = hours of day at full load (in case of load not being uniform during use of transformer, h is the number of hours which would be required at full load to give the same copper loss as the load actually existing.)

W_c = copper loss of transformer.

$$\text{Total cost of copper loss per year} = \frac{W_c}{1000}(365 h c_1 + D C_1)\text{ dollars}$$

EFFECT OF VARYING VOLTAGE ON COST OF LAMPS

The characteristics of carbon and tungsten lamps are approximately as follows:

		Carbon lamps	Tungsten* lamps
$\frac{1}{\text{life}}$ (renewals) varies as	....	$E^{21\cdot 5}$	$E^{14\cdot 0}$
Candle power "	....	$E^{5\cdot 7}$	$E^{3\cdot 68}$
Watts "	....	E^{2}	$E^{1\cdot 59}$
$\frac{1}{\text{watts per c.p.}}$ "	....	$E^{3\cdot 7}$	$E^{2\cdot 1}$
Life "	.....	$\left(\frac{\text{watts}}{\text{candle power}}\right)^{5\cdot 8}$	$\left(\frac{\text{watts}}{\text{candle power}}\right)^{6\cdot 2}$

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*For drawn wire lamps, $\frac{1}{\text{life}}$ varies as $E^{15\cdot 8}$

The above figures for tungsten lamps were given by the National Electric Lamp Association.

Using these relations, curves of cost of energy and cost of renewals may be plotted against voltage, or what amounts to the same thing, watts per candle power. This has been done for carbon lamps in Fig. 2. The total cost of the light will then be the sum of the two curves. It will be seen at once that the lowest point of this resultant curve gives the watts per c.p. at which the total cost will be a minimum. If instead of operating constantly at this efficiency, suppose that a lamp is operated half the time at 10 per cent lower watts per c.p. and half the time at 10 per cent higher watts per c.p. The cost of energy is not increased, since the energy cost curve is a straight line, but the cost of renewals is increased by the amount $c'\ d'$, and the total cost by the amount $c\,d$, or about 4 per cent. If instead of operating one-half the time at each extreme voltage, the voltage had varied uniformly, the increase in cost of renewals would have been but ⅓ as great.

Cost of Regulation

The increase in cost of lamp renewals due to variation in voltage can be expressed as a function of the variation, and of the copper loss of the transformer.

Suppose life varies as (watts per c.p.)^a and E^b varies as $\frac{1}{\text{watts per c.p.}}$

Then 1 per cent variation of voltage will cause an increase in the cost of renewals equal to

$$b^2 \times \frac{a\,(a-1)}{8} \times 0.01 \text{ per cent} \qquad \textbf{(15)}$$

of the cost at normal steady voltage, or if the variation is uniform from ½ per cent above to ½ per cent below the average, the increased cost is

$$b^2 \times \frac{a\,(a-1)}{24} \times 0.01 \text{ per cent} \qquad \textbf{(16)}$$

The increased cost depends on the square of the voltage variation, and hence is four times the above value for 2 per cent change in voltage. In figuring a specific case, it should be remembered that maximum regulation must be used, that is, the regulation at the heaviest overload that the transformer frequently carries.

Let r = approximate cost, in cents, of renewals per 1000 lamp hours.

p = energy consumed per 1000 lamp hours, in kilowatts.

x = increase in cost of renewals for 1 per cent regulation expressed in terms of r.

L = the increase in cost, in cents, of renewals per kw-hr. of transformer copper loss, for 1 per cent regulation.

$$L = \frac{x \times r}{p} \tag{17}$$

for 1 per cent regulation, and varies as the square of the regulation.

For 50-watt, 16-c.p. carbon lamps,

$$L = \frac{0.0016 \times 0.35}{0.5} = 0.11 \text{ cent.}$$

For 25-watt tungsten lamps at present prices, $L = 0.20$, and for 60-watt tungsten lamps, $L = 0.12$ cent.

Let R = regulation at maximum overload at which transformer is operated.

W_c = copper loss.

h = hours per day at full load, as in copper loss.

$$C_R = \text{cost of regulation in cents} = L\ R \times 365\ h \times \frac{W_c}{1000} \tag{18}$$

TOTAL COST OF TRANSFORMER LOSSES FOR A SPECIAL CASE

Assume cost of energy $c_1 = \$0.0075$ per kw-hr.

Capacity charge $C_1 = \$30.00$ per kw-hr. per year.

($20.00 for station)

($10.00 for line)

Assume a 5-kw. transformer with an iron loss of 45 watts, a copper loss of 93 watts at full load, and magnetizing current of 2 per cent.

Then $W_1 = 45$ watts, $W_c = 93$ watts, $M = 100$ volt-amperes.

From formula (**1**), cost of iron loss $= \frac{45}{1000}(8760 \times 0.0075 + 30)$ $= \$4.31$.

COST OF MAGNETIZING CURRENT

$M = 100$ volt-amperes

Suppose that cost of line = \$10.00 per kw.

Cost of station (electrical equipment) = \$10.00 per kw.

$$k = 0.10,\ b = b_1 = 0.20,\ k_1 = 0.12,\ f = 0.85,\ \frac{t}{2} = \frac{0.02}{2} = 0.01$$

Substituting in formula (2), cost of the magnetizing current in the line =

$$100\ (0.20 \times 0.85 + 0.01)\ \frac{(0.10 \times 8760 \times 0.0075 + 10.00)}{1000} = \$0.30$$

Cost of magnetizing current at the station =

$$100\ (0.20 \times 0.85 + 0.01)\ \frac{(0.12 \times 8760 \times 0.0075 + 10.00)}{1000} = \$0.32$$

Total cost of magnetizing current = \$0.62.

It will be seen, then, that in this case the magnetizing current costs 14.5 per cent as much as the iron loss itself.

COPPER LOSS

$W_c = 93$

Suppose that $D = 0.8$, $h = 4$, maximum frequent overload = 33 per cent, c_1, C_1, as before.

$$\text{Total cost} = \frac{93}{1000}\ (365 \times 4 \times 0.0075 + 0.8 \times 30.00) = \$3.26.$$

COST OF REGULATION

The regulation at full load is very close to the per cent copper loss or 1.86 per cent. At overload, it is 1.33 × 1.86 or 2.48 per cent.

Assume $L = 0.11$.

Then the cost of regulation, by formula (18), $= 0.11 \times 0.0248 \times 93\ \frac{365 \times 4}{1000} = \$0.37$.

The total cost of losses is, therefore,

Iron loss	\$4.31
Magnetizing current	.62
Copper loss	3.26
Regulation	.37
	\$8.56

Now if a 10-year life be assumed, with interest at 6 per cent, the interest and depreciation factor will be 13.8 per cent. Therefore, the " capital cost " of the losses will be $\frac{8.56}{0.138}=\$62.00$.

This is more than the first cost of the transformer, hence it will be seen that a saving of 10 per cent in the losses in this case would be of greater value than to obtain a reduction of 10 per cent in the price of the transformer.

Similarly, the operating cost of a transformer of the low efficiency type may be calculated: Assume

Iron loss.......................................=62 watts
Copper loss.......................................=93 "
Magnetizing current.......................................=10 per cent
All other factors as before.

Then cost of iron loss.............................. =	$5.94
Magnetizing current.............................. =	3.78
Copper loss.............................. =	3.26
Regulation.............................. =	.37
	$13.35

The " capital cost " $=\frac{13.35}{0.138}=\$97.00$.

If, now, the former transformer cost $60.00 and the latter $50.00, the total costs will be $122.00 and $147.00.

The importance of the cost of the losses is quite evident.

The cost per watt of iron loss or copper loss may be readily calculated. In these cases, the iron loss and magnetizing current costs from 80 cents to $1.10 per watt of iron loss; the copper loss and regulation costs 28 cents per watt of copper loss.

DISCUSSION ON "THE APPLICATION OF CURRENT TRANSFORMERS TO THREE-PHASE CIRCUITS," "COST OF TRANSFORMER LOSSES." CHICAGO, JUNE 30, 1911.

A. H. Pikler: It is now more than 26 years since the first large transformer installation was made in Budapest, Hungary. Both the system of operation of distributing-transformers and the transformers themselves were inventions of Messrs. Zipernowsky, Déry and Bláthy. The parallel system, or as it later was called in this country, the multiple arc system, is used today exactly as it was in use then, and the general design and construction of the transformers of that time has undergone very little change in comparison with the transformers of the present day. Most of the changes or improvements which we find in the present transformers as compared with the first types are due to the improvement in the materials, as insulating materials and the steel used; further to the increase in cost of labor.

That the distributing-transformers are still of acute importance is shown by the present paper. The reason for this is the very large amount of capital invested yearly in such transformers, and the large amount of money expended in operating them. The Bureau of Standards of the United States Government, recognizing the great importance of the subject of distributing-transformers, went into a thorough investigation of the value of the various characteristics of such transformers expressed in dollars and cents, and a few years ago issued the Circular No. 22 where a report of the investigations is found in connection with other useful suggestions relating to the same subject.

The paper written by Messrs. Stone and Atkinson is to my knowledge the first one that takes into consideration *every* single item affecting the operating cost of the transformers, and gives a thorough and full presentation of the whole subject. The manner in which the authors deal with this subject I consider classical because not the least little detail is lost sight of, from the power house to the consumer. The important difference between the government circular and this paper lies in the fact that Messrs. Stone and Atkinson point out the method whereby operating cost corresponding to the exciting current and regulation may be taken into account in the same way as the core loss and copper loss.

By this additional information the transformer purchasing public is put into a better position as to the selection of the most economical distributing-transformer. By the use of a high magnetic density in a transformer core, the dimensions of the magnetic circuit will be decreased and a small transformer will be obtained of considerably lower cost than when using a lower magnetic density. If considering the core and copper loss only in comparing the high magnetic density transformer with one having a low magnetic density, on the basis of evaluation used in the government circular, it may frequently be found that the lower losses in the transformer of generous amount of material may not suffice to quite offset the difference in price between the two transformers. If, however, the exciting current is also and rightly taken into consideration, such will not be the case.

Another important point which the authors bring up and which may bear some further illustration is that the low power factor in a transformer, in itself, is not objectionable when it is accompanied by a low core loss.

I have often heard statements like this:

" The introduction of silicon steel does not mark such a great gain in the transformer as may appear from the core loss watt-per-pound figures of the ordinary and silicon steels. The power factor at open circuit in the silicon steel transformer has decreased considerably as compared with what it was in the transformer built with ordinary steel. Therefore what we gain in core loss in the transformer we lose on the line due to lower power factor."

That the logic of this statement is not correct may be seen from the following:

Let us assume for instance two 5-kw. transformers, one of the old type made with ordinary steel, another of a recent type made of silicon steel. The old type transformer has a core loss of 64 watts, the modern transformer has a core loss of 45 watts, the exciting current of both transformers is the same, that is, say 2 per cent of the full load current. Then the old type transformer will have a no-load power factor of $\frac{64}{0.02 \times 5000}$ $= 64$ per cent. The modern transformer will have a power factor of $\frac{45}{0.02 \times 5000} = 45$ per cent. The power factor of the old transformer is higher than the new one. But is the old transformer therefore a better transformer? It is not, because the exciting current, *i.e.*, the no-load current on the line is just the same, moreover in the new transformer we have a transformer which in connection with that same exciting current has a lower core loss.

Another item of importance that may be pointed out in connection with a magnetizing current which is of value to the purchaser of transformers is as follows: If the normal magnetic density in the transformer core is higher in one transformer than in the other, then a slight increase in line voltage may cause a very large increase in the exciting current of the transformer having the higher magnetic density in its core. This is self explanatory from the well known saturation-curve.

I do not agree with this statement of the authors: " Transformers are designed and will be designed and sold to meet the demands, regardless of the cause of the demand." I consider it rather academic and do not believe that it coincides with the real, actual development of the transformer. Not only a transformer but any kind of machinery is made by the manufacturer so as to produce it with the greatest economy to both the manufacturer and the public. The public buys what is offered to it. Later on competition steps in and causes further improvements in the apparatus. It is therefore the competition that creates further development in the art and not the demand of the public. The public generally specifies the best performance offered by the various competitors,

That the decrease in core loss and copper loss requires a greater expenditure of material can be appreciated from the figures given by the authors stating that 1 per cent decrease of both losses requires about 3.3 per cent increase in the cost of the active material.

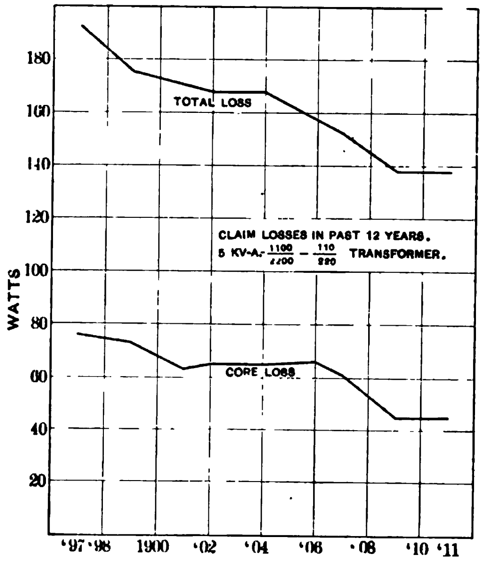
There is a paragraph in the paper where it is explained why the loss of the amount of material per kilowatt is less with increasing capacities. I beg to offer the following explanation, which I think is broader and clearer: Let us consider a transformer or any other electromagnetic apparatus with linear dimensions of unity; if these dimensions are doubled, then the volume of material and with it the weight will be increased as the third power, the cross section of the iron core and with it the total magnetic flux will increase with the second power. The cross section of the copper and with it its current carrying capacity will also increase with the second power. Since, further, the output of any electromagnetic apparatus can be expressed as the product of magnetic flux and ampere-turns, it may be seen at once that by doubling the linear dimensions of the apparatus, its output increases with the fourth power whereas the amount of material increases with the third power. Using the same densities in both cases, the watts lost are directly proportional to the weight of the material. Therefore it will be readily seen that the apparatus of the larger output will have a lower loss per kilowatt output, that is, a higher efficiency. It should have been pointed out, however, by the authors that the radiating surface increases only with the second power and this influences the design in a much greater measure than does a change in the winding space factor, as is the authors' opinion.

In closing I wish to point out the fact that the significance of the various items entering into the real economy of the transformer can be best appreciated by the figures given at the close of the paper, which show that whereas the initial cost of a 5-kw. transformer, for instance, is approximately $60, its operating cost will be of the order of $50; so that the total cost of the transformer to the purchaser will amount to $122 to $147. This is the total cost assuming a 10-year life and capitalization with interest at 6 per cent, taking into account depreciation and interest.

W. C. Smith: Viewing the question of transformer losses from the manufacturers' standpoint, I would like to add a few words to the discussion of the excellent paper by Messrs. Stone and Atkinson.

That the cost of losses is an important consideration in the selection of distributing transformers, even approaching a value equal to that of the transformer itself, has been clearly shown by the authors.

The demand of the central station for lower losses has been insistent, until today, for a given kv-a. rating the distributing transformer is the most efficient piece of electrical apparatus on the market. The improvement in core loss by the introduction of silicon steel has, of course, been one of the greatest contributing factors in this development. The actual watts per lb. at normal working densities, due to hysteresis and eddy current losses, has, within the lifetime of the average transformer, been reduced at least one-half.



Curve of losses

A curve of losses on the 5-kv-a. standard lighting transformer, as given in published claims of a prominent electrical manufacturing company in this country, shows the remarkable decrease that has been accomplished during the past few years.

With such marked reductions in the losses, combined with better mechanical construction in the new types as they were put on the market, a very high depreciation value naturally had to be placed on apparatus already installed. In other words, the " smashing point " was reached early in the life of the old lighting transformer.

Today, conditions are changed. In the best class of this apparatus now on the market, every part has been scientifically proportioned. There is no waste space anywhere. The construction is extremely rugged and the heating uniform. With steel absolutely non-aging, the customer is assured of proper division of losses and constant efficiency. The processes of manufacture are such as to warrant long continued service without disturbance of the various parts.

In other words, the best class of modern distributing transformers is now built primarily for long service. No radical mechanical changes can be anticipated, which will warrant replacing these units. Only a very material reduction in losses can suffice for such replacement.

However, with efficiencies already of 97 to 98.5 per cent, any very material saving in power cannot be anticipated even though the present losses be decreased by a fairly large percentage. No improvement is of course expected in the quality of copper. It is too much to hope for any marked improvement in the permeability of the steel, which means that, even though the core losses per lb. be considerably reduced, it will not be possible to operate at higher flux densities due to magnetizing current considerations. We must, for these reasons, look toward the core losses for any great improvement in efficiency. But even assuming a 25 per cent decrease in core loss (and this is hardly probable) we shall only find 8 per cent decrease in total losses, due to the fact that the present division of losses is two of copper to one of iron.

With the foregoing facts in mind, the value of a small depreciation factor becomes at once apparent. Taking the typical case of a 5-kv-a. transformer already cited, if the life be increased from 10 to 20 years, interest at 6 per cent, the interest and depreciation factor will be 8¾ per cent. Assuming the same original costs, the increased life of 10 years can be shown to be equivalent to a 69 per cent difference in cost of total losses between two transformers giving the same length of service.

This consideration shows such a large saving, due to long life, as to almost eclipse the demand for lower losses which has been pressed so strongly upon the manufacturer during the past few years. Other factors remaining the same, lower losses will still be sought for. But a far greater consideration will be that of long life under continued service.

Rather than any marked decrease in total losses, resulting from improvement in materials, reproportioning of parts, etc., it is possible that the future may see a development toward higher operating temperatures, resulting in a cheaper transformer. Before the advent of silicon steel, aging required an 80 deg. cent. temperature limit. At the present time, the flow point of the impregnating compounds gives, let us say, an ultimate temperature limit of 90 degrees. It is possible that the development of synthetic gums will soon reach a stage to permit of actual operating temperatures of at least 125 deg. to 150 deg. centigrade. Such a development will naturally present a problem to the oil refiner—a situation, however, which can undoubtedly be met.

E. A. Wagner: The paper by Messrs. Stone and Atkinson gives a basis of calculation for the comparative operating costs of different types of transformers which is of considerable value. The exciting currents, or magnetizing currents, are, as a rule, not given consideration when summing up the comparative values of transformers. A study of the formulas will show, however, that the results obtained depend solely on the assumption made at the beginning, so that with different assumptions entirely different values will result.

In the summary of the losses Mr. Atkinson tabulates the value of two transformers of different characteristics, one a so-called high efficiency type, and another one a so-called low efficiency type. In the high efficiency type, according to this formula, the magnetizing current has a value of 62 cents, and in the low efficiency type a value of $3.78, thereby increasing the cost of exciting current more than five times with a transformer showing 50 per cent higher core loss. Such examples, given as typical illustrations, are apt to prove misleading to one not thoroughly acquainted with transformer performances. It is a matter of fact that magnetizing currents do not increase proportionately with the core losses. In transformer design we find that low costs and high efficiencies are arrived at by crowding the magnetic circuit (operating at densities close to the saturation point) and increasing the space factor by eliminating as much of the insulation between layers and coils as possible. Such a transformer, in comparison with one designed to operate at lower densities in the iron, but with a considerably heavier cross section in the magnetic circuit, assuming that modern silicon steel is used in the construction of the core in both cases, will show fully as high, if not higher, exciting currents. With a given quality of silicon steel, and retaining a winding space factor not varying more than 3 per cent, transformers of widely different characteristics in regard to core and copper loss and widely varying weights of copper and iron, will all produce very close to the same full load efficiency. In the design of this type, and assuming the valuations given by Mr. Atkinson, it will show that the so-called cheap transformers, or low efficiency transformers, will be much more desirable than the one which he classified as a high efficiency transformer. In other words, the choice of transformers is dependent almost entirely on the value of the magnetizing current.

H. B. Gear: I wish to emphasize the remarks made by a previous speaker regarding the importance of securing long life in modern transformer design. In the larger systems where the cost of energy has been reduced by large turbo-generators, until it approaches the vanishing point, the item of depreciation assumes greater importance than the cost of the iron and copper losses. The larger companies are therefore becoming more interested in these days in the life of their transformers then formerly.

Although the iron losses in the line transformers in the Chicago system amount to approximately 6,000,000 kw-hr. per year, the cost of producing this energy is only about half the fixed charges—interest and depreciation—on the investment, based on an average life of 15 years.

Improvements in the methods of manufacture and insulation which would add five years to the average life of line transformers, are therefore fully as important as a reduction of 25 per cent in the core losses.

The operating engineer who endeavors to specify to the manufacturer a type of transformer which would be most economical in operation, must choose between a unit which will be most economical with lighting customers having a low load factor and a unit which is most economical for power customers having a high load factor.

The ratio between iron and copper losses should be lower for the consumers with small load factors and higher for the consumers with higher load factors.

It is not practical for operating companies to attempt to carry two kinds of transformers in stock for general distribution work and the operating engineer must therefore strike a happy medium which will be most economical for the average load factor of his customers.

R. W. Atkinson: I have been much interested in the points brought out by the various speakers. I shall refer to an objection made by Mr. Pikler to our statement that the manufacturer will adjust the losses to correspond with the demand. I think, however, that Mr. Pikler's idea is not really different from our original statement, namely: It does not make much difference whether the demand for any performance is for logical or illogical reasons or is because the salesmen of the various companies making transformers have convinced their customers that a certain loss should be reduced or that certain other losses should be left as they are. Two transformers of different performances will have a relative difference in their market value due to a difference in their performance. For instance, suppose that one has 90 watts iron loss and the other 100 watts iron loss. That difference of 10 watts iron loss will make a different sale value of the two transformers. Difference in value may be purely commercial, due to the fact that the salesmen have convinced the average user of the transformer that that difference is worth a certain amount to him, or it may be because the consumer has worked out, from either a logical or illogical standpoint, that that difference means so much to him in dollars and cents. It does not make any difference what the basis of the demand is, the manufacturer will design his transformers to meet the demand of the market.

There has been a constant tendency to increase the temperature rise. As small transformers were built a few years ago, the limits were the efficiency. It was impracticable to build transformers with a high temperature rise, simply because they would have been so inefficient that it would not have paid to use them. At present, due to improvement in materials and design, the dimensions have been decreased more than the losses, thus increasing the densities, with a consequent tendency toward an increased temperature rise. If the insulating people can do what Mr. Smith hopes they may do there is a field now, as there formerly was not, for a material capable of withstanding high temperature.

A word in regard to the question of cost of magnetizing currents in the two transformers used in the problem. These were two particular transformers that were on the market at the time the figures were made. There is a tendency, I believe, for the high iron loss transformer to have high magnetizing current, although this is not necessarily so. I think the reason for this tendency was brought out by Mr. Wagner, that transformers are operated at densities very near the saturation point, and if the iron loss is increased with a given type of transformer there is a tendency for the magnetizing current to be increased in much greater proportion. Of course that can be *modified* by changing the design. The question of magnetizing current is a separate one from the others, and it is true that if the two transformers in the problem had about the same magnetizing current, and with the other figures as we have given them, the ultimate costs would not have been greatly different. The cost of magnetizing current is the most variable factor in the cost of transformer losses and depends largely on local conditions. If the transformer is put on the line with the purely lighting load, where the power factor is high, the magnetizing current will not be as expensive as if it were on a circuit of low power factor.

In regard to the relative ultimate cost of the low efficiency or high efficiency transformer, I would say that it depends very largely upon the purposes for which the transformer is to be used and the conditions of the cost of power. If the transformer is to be used for a long-hour load then it will pay to have a relatively low copper loss and a relatively higher iron loss. I do not want to say, as a general statement, that it will pay to use either a low efficiency, or a high efficiency transformer, that it will pay to use a low ratio of copper loss to iron loss, or to use a high ratio. The point we wish to make is that the cost of the losses is dependent upon specified conditions which may vary with different systems and perhaps with different parts of one system, but in such a way as can easily be determined. Furthermore this cost is of such importance that one cannot afford to determine it by guess.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

SOLUTION TO PROBLEMS IN SAGS AND SPANS

BY WM. LE ROY ROBERTSON

Solutions for sag and span problems occurring in overhead line construction have been developed from time to time by many authors. Frequently during the process of development one author has taken up the work where his predecessor left off and has endeavored to make the formula more complete. Such solutions heretofore developed are approximate, and apply to the case where the sag is very small compared to the span. All solutions up to the present time are based upon the assumption that a span of wire forms the arc of a parabola. The formulas are approximately derived and are mathematically inconsistent. For instance, the formula for length of arc in a span, instead of being derived from the parabola, is derived in an approximate manner from the circle. Again the stress value is taken at the center of span and assumed to be constant at every point along the wire, when in reality it varies at every point, reaching maximum values at the supports. Further, certain small errors occur in formulas dealing with the effect of changes in temperature and stress. Again, in the case where one support is higher than the other, a certain discrepancy occurs which will be explained later under that particular heading. However, when the sag is very small compared to the span, the error introduced by the formulas is of no practical consequence, although when the sag increases sufficiently there may be considerable error introduced. The data produced by the solutions are adaptable to a limited number of span values and usually are adaptable to only one material. Complete recalculation is necessary for a second material.

The question of large sag is important, and must be considered. Relatively larger sags occur in long spans. When abnormal weather conditions cause the wire to stretch, large sags are often the result. Again in the case where a difference in height exists between supports, it will be found, as described later, that the sag is never less than the difference in height between the supports, and may be larger.

In the solution hereafter considered, it is proposed to give mathematically consistent and correct formulas and to supply data sufficient to solve problems with the least additional computation. It is also intended that the solution so far as possible will be universal and not limited to any particular sag and span values, nor to the kind of material.

Selection of Catenary

Plate No. 1 gives a comparison between the equations of various curves and actual measurements made upon a span of wire. The length of the span is exactly 80 ft. and a comparison of the deflection is made as given for a two- and a 10-ft. sag. The span of wire which was measured consisted of No. 12 annealed bare copper. The supports were perfectly level and rigid and the transit was used in taking the measurements.

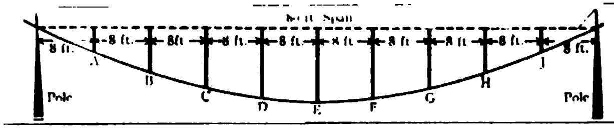
It will be noted that the suspended wire follows the course of the catenary much more closely than it follows the other curves. It is possible that some wires under certain conditions would perhaps more closely follow the curve of the parabola. A stiff, thick wire in a short span would not be expected to follow the catenary.

The results show that when the sag value is small in comparison to the span, it is immaterial which curve is used, except the circle.

Considering that wires and cables tend to greater flexibility in longer lengths; that spans are as long as 2000 ft. and over; that in proportion to the span larger sags are more likely to occur in long spans; that, when the supports are at different levels, larger sags exist; and that the catenary formulas on the whole are probably as simple as any, the catenary seems to be the best curve.

Plate No. 1 also gives a rough idea of the difference existing between various formulas. The greatest difference is in the stress, which in turn would cause an error in the effect of temperature and stress.

Formulas. The basic principles underlying all solutions to sags and spans are set forth in Chapter IV, of J. Weisbach's "Theoretical Mechanics".



	CURVE USED IN COMPUTATIONS			Deflection (length in feet)											
				At pole	At *A*	At *B*	At *C*	At *D*	At *E*	At *F*	At *G*	At *H*	At *J*	At pole
2-foot sag	Circle	$x^2+y^2=2ry$	$r=401$	0	0.721	1.281	1.692	1.940	2.000	1.940	1.692	1.281	0.721	0
	Ellipse	$x=\sqrt{2cy-y^2/3}$	$c=400\frac{1}{3}$	0	0.720	1.280	1.680	1.920	2.000	1.920	1.680	1.280	0.720	0
	Catenary	$y=a/2\,(e^{x/a}+e^{-x/a})$	$a=400.332$	0	0.720	1.280	1.680	1.920	2.000	1.920	1.680	1.280	0.720	0
	Parabola	$x^2=4ay$	$a=200$	0	0.720	1.280	1.680	1.920	2.000	1.920	1.680	1.280	0.720	0
	By actual measurement*			0	0.717	1.276	1.681	1.920	2.000	1.912	1.681	1.280	0.726	0
					0.722	1.278	1.681	1.916		1.916	1.681	1.278	0.722	
10-foot sag	Circle	$x^2+y^2=2ry$	$r=85$	0	3.746	6.541	8.481	9.623	10.000	9.623	8.481	6.541	3.746	0
	Ellipse	$x=\sqrt{2cy-y^2/3}$	$c=81\frac{2}{3}$	0	3.648	6.448	8.428	9.608	10.000	9.608	8.428	6.448	3.648	0
	Catenary	$y=a/2\,(e^{x/a}+e^{-x/a})$	$a=81.615$	0	3.644	6.444	8.425	9.606	10.000	9.606	8.425	6.444	3.644	0
	Parabola	$x^2=4ay$	$a=40$	0	3.600	6.400	8.400	9.600	10.000	9.600	8.400	6.400	3.600	0
	By actual measurement*			0	3.640	6.430	8.415	9.595	10.000	9.599	8.427	6.442	3.643	0
					3.642	6.436	8.421	9.597		9.597	8.421	6.436	3.642	

*Upper row is the actual measurements. Lower row is the average of like values occurring on opposite sides center of span.

Formulas	2 ft. sag in 80 ft. span		10 ft. sag in 80 ft. span	
	Length of arc	Total stress	Length of arc	Total stress
Catenary................	80.1332	*402.331 *W*	83.2414	*91.615 *W*
Parabola (true)...........	80.1331	*402.664 *W*	83.2183	*93.041 *W*
Commonly used (parabolic)	80.1333	†400.000 *W*	83.3333	†80.000 *W*

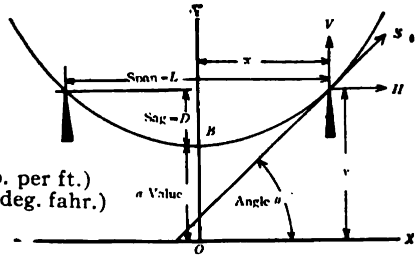
*Total stress at supports. †Total stress at center of span.
W = resultant weight of wire (lb. per foot).

PLATE No. 1.—Sags and spans

Comparison of formulas

Symbols

x = $\frac{1}{2}L$.
D = $y-a$.
O = origin.
B = vertex.
C = length of arc.
$\triangle C_t$ = change in C (temp.).
$\triangle C_s$ = change in C (stress).
A = area of wire (sq. in.).
W = resultant weight of wire (lb. per ft.)
γ = temperature coefficient (per deg. fahr.)
m = modulus of elasticity.
t = degrees temperature (fahr.).
(t_1-t_2) = change in temperature.
e = 2.7182818—base of Naperian logarithms.
K = $L/100$. h = feet difference between height of supports.
H = horizontal stress in wire (total lb.) (constant for every point).
V = vertical stress in wire (total lb.) (varies for every point).
S = stress in direction of wire (total lb.) (varies for every point).
s = stress in direction of wire (lb. per sq. in.).
(s_1-s_2) = change in stress (lb. per sq. in.).



Computations
based on 100-foot span
$K=1$

A values feet	Sag feet	Length of arc feet	Total stress in lbs. along directions of wire		
			At center of span	At supports	Average
25	69.0549	181.3431	25 W	94.055 W	46.274 W
50	27.1540	117.5201	50 W	77.154 W	59.065 W
100	12.7629	104.2185	100 W	112.763 W	104.760 W
200	6.2826	101.0448	200 W	206.283 W	202.704 W
400	3.1290	100.2604	400 W	403.129 W	401.172 W
600	2.0846	100.1158	600 W	602.085 W	601.042 W
1000	1.2503	100.0415	1000 W	1001.250 W	1000.647 W
2000	0.6250	100.0102	2000 W	2000.625 W	2000.312 W
4000	0.3110	100.0032	4000 W	4000.311 W	4000.156 W
8000	0.1564	100.0011	8000 W	8000.156 W	8000.078 W

W = resultant weight of wire per foot

Formulas

(1) $y=\frac{a}{2}\left(e^{x/a}+e^{-x/a}\right).$

(2) $\tan\theta=\frac{dy}{dx}=\frac{1}{2}\left(e^{x/a}-e^{-x/a}\right).$

(3) $C=\int_a^b\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}dx$

(4) $C=a\left(e^{x/a}-e^{-x/a}\right).$

(5) $H=\frac{V}{\tan\theta}=a\,W$

(6) $V=\frac{C\,W}{2}.$

(7) $S=\frac{V}{\sin\theta}=(a+D)\,W=y\,W.$

(8) $\Delta C_t=C\,\gamma\,(t_1-t_2).$

(9) $\Delta C_s=\frac{C\,(s_1-s_2)}{m}=C\times\%$ elongation.

(10) $s=\frac{S}{A}.$

(11) $\frac{D}{K},\ \frac{C}{K},\ \frac{S}{K\,W}$ respectively give $\left\{\begin{array}{l}D \text{ and } C \text{ in terms of ``}K\text{'' and}\\ S \text{ in terms of ``}K\,W\text{''.}\end{array}\right.$

PLATE No. 2.—Sags and spans. Formulas and computations based on catenary. Level supports

Plate No. 2 gives all the necessary formulas based on the catenary and on level supports. Formula (**1**) is the equation of the catenary. Formula (**2**) is derived from formula (**1**). Formula (**3**) is the integral equation for length of arc of any curve in ordinates of x and y. Formula (**2**) is substituted in formula (**3**), giving the formula (**4**) for length of arc. The remaining formulas are self-explanatory. All formulas up to and including (**7**) are derived from the catenary. The others are derived independently of the catenary.

Inspection of the formulas shows that if the a, x, y, C, S, H and V values be multiplied by a constant K the relations of the formulas remain unchanged.

If we take the value x or the value of span and multiply it by two, all the other values in the formulas will be correct only when multiplied by two; in this case K equals 2. In all the following curves and computations, the constant K is used, and span is taken at 100, making the value of K equal to the length of span divided by 100. The use of the K factor renders the following curves adaptable to any value of span.

Computations and Curves. In equation (**1**), by substituting various a values for a given x value, the corresponding value of y and hence the *sag* can be obtained. Likewise, in formula (**4**), for the same given x value, the corresponding *length of arc* can be obtained by substituting the same various a values, and in formula (**7**) the corresponding *maximum stress* can be obtained by substituting the y value corresponding to the same various a values.

Obtaining the stress values for various points along the wire, by substituting in formula (**7**) various y values for a given a value, and averaging them, the *average stress* value may be obtained corresponding to the given a value. This can be repeated for all a values. All of the above substitutions must be made for the one given x value.

Formula (**5**) gives the *stress value at the center of the span.*

On Plate No. 2 a full set of computations for various selected a values is given based on a 100-ft. span where K equals one.

From these computations three sets of curves are plotted in sections—Curve No. 1, between sag and length of arc; Curve No. 2, between sag and stress; and Curve No. 3, plotted for wires and cables from formula (**10**), Brown & Sharpe gage.

Plates No. 3 and 4 give data on wires and cables, which cover tensile strength, modulus of elasticity, ice and wind load on

$W = \sqrt{V^2+H^2}$

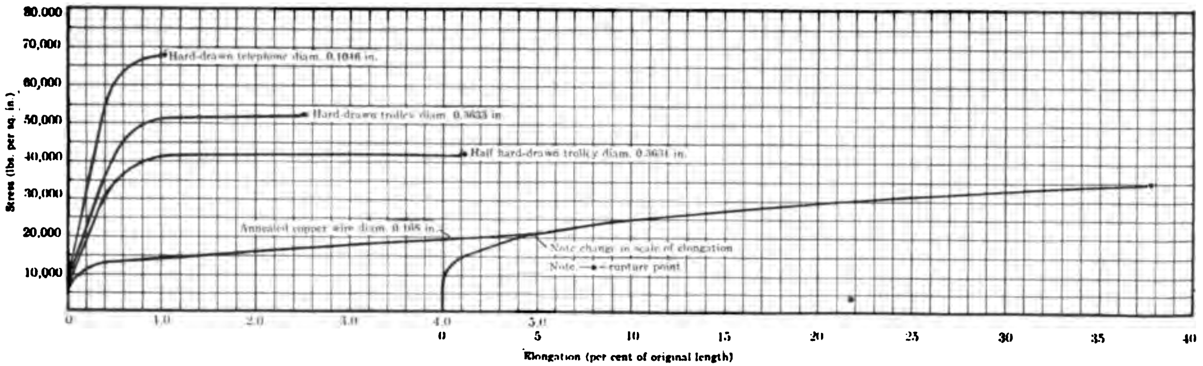
V = vertical component—any value V_1, V_2, V_3, V_4 given in table.

H = horizontal component—any value H_1, H_2, H_3, H_4 . . . . H_8 given in table.

		Bare copper wire lb. per ft.						Three braid insulated copper wire lb. per ft.					
				Wind pressure						Wind pressure			
				50 miles per hr.		60 miles per hr.				50 miles per hr.		60 miles per hr.	
	Gage B. & S.	Wire alone	Wire + ½ in. ice	Wire alone	Wire + ½ in. ice	Wire alone	Wire + ½ in. ice	Wire + insulation	Wire + insulation + ½ in. ice	Wire + insulation	Wire + insulation + ½ in. ice	Wire + insulation	Wire + insulation + ½ in. ice
		V_1	V_2	H_1	H_2	H_3	H_4	V_3	V_4	H_5	H_6	H_7	H_8
Stranded conductor	1,000,000	3.050	4.090	0.606	1.128	0.872	1.630	3.550	4.805	0.805	1.331	1.175	1.920
	750,000	2.288	3.230	0.520	1.043	0.755	1.505	2.713	3.853	0.705	1.230	1.030	1.775
	500,000	1.530	2.358	0.428	0.952	0.616	1.372	1.894	2.914	0.592	1.130	0.852	1.604
	450,000	1.377	2.178	0.406	0.930	0.584	1.340	1.724	2.722	0.575	1.100	0.828	1.584
	400,000	1.223	1.902	0.383	0.905	0.552	1.304	1.553	2.536	0.558	1.080	0.804	1.556
	350,000	1.070	1.803	0.358	0.883	0.516	1.272	1.345	2.287	0.525	1.050	0.756	1.512
	300,000	0.917	1.630	0.330	0.855	0.476	1.232	1.174	2.079	0.492	1.020	0.708	1.464
	250,000	0.764	1.439	0.311	0.828	0.448	1.192	0.985	1.870	0.475	1.000	0.684	1.440
	#0000	0.646	1.294	0.278	0.800	0.400	1.156	0.800	1.649	0.447	0.972	0.644	1.400
	#000	0.512	1.124	0.247	0.772	0.356	1.112	0.653	1.462	0.413	0.938	0.576	1.352
	#00	0.406	0.985	0.219	0.745	0.316	1.072	0.522	1.282	0.372	0.897	0.536	1.292
	#0	0.322	0.872	0.194	0.720	0.280	1.036	0.424	1.145	0.339	0.864	0.488	1.244
Solid conductor	#0000	0.640	1.242	0.242	0.767	0.348	1.104	0.775	1.580	0.408	0.933	0.588	1.344
	#000	0.508	1.081	0.217	0.742	0.312	1.068	0.630	1.384	0.367	0.892	0.528	1.284
	#00	0.403	0.948	0.192	0.717	0.276	1.032	0.490	1.205	0.333	0.858	0.480	1.236
	#0	0.319	0.838	0.169	0.694	0.244	1.000	0.400	1.085	0.311	0.833	0.448	1.200
	#1	0.253	0.748	0.153	0.678	0.220	0.976	0.306	0.968	0.289	0.814	0.416	1.172
	#2	0.201	0.677	0.136	0.661	0.196	0.952	0.268	0.906	0.269	0.795	0.388	1.144
	#4	0.126	0.568	0.108	0.633	0.156	0.912	0.164	0.750	0.225	0.750	0.324	1.080
	#6	0.079	0.495	0.085	0.611	0.124	0.880	0.112	0.652	0.189	0.714	0.272	1.028
	#8	0.050	0.433	0.067	0.593	0.096	0.853	0.078	0.569	0.151	0.684	0.220	0.985
	#10	0.032	0.410	0.054	0.578	0.077	0.771	0.055	0.526	0.134	0.666	0.196	0.961

PLATE No. 3.—Data on wires. Values for W = resultant weight per foot for copper only

Curves between stress and strain for copper wires taken from Mr. P. O. Blackwell's paper. International Congress at St. Louis, 1904

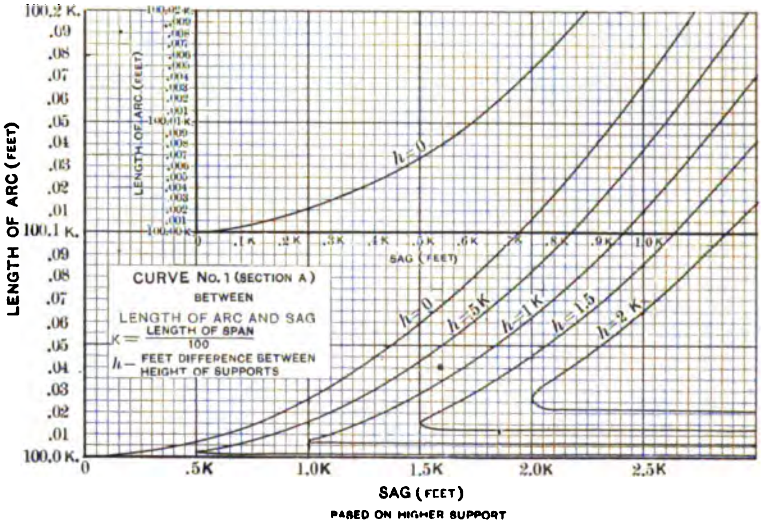


Tensile strength of copper wire (Roebling's)
Total pounds—breaking weight

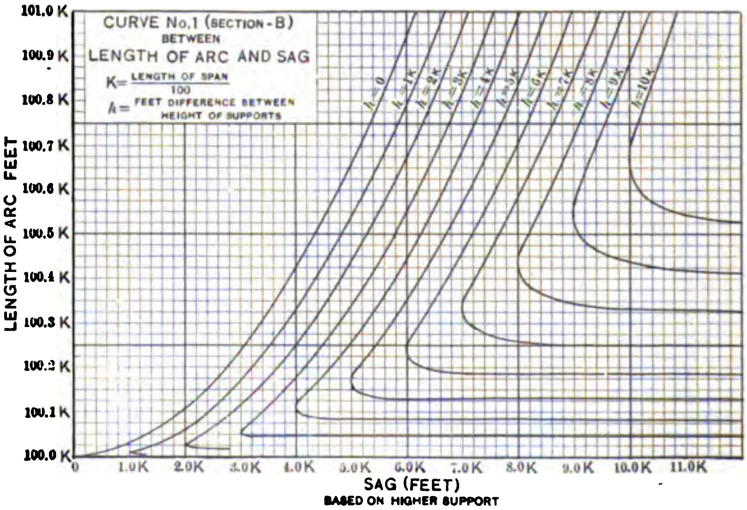
Numbers B. & S. gage	Hard-drawn	Annealed
0000	8310	5650
000	6580	4480
00	5226	3553
0	4558	2818
1	3746	2234
2	3127	1772
4	1967	1414
6	1237	700
8	778	440
10	489	277
12	307	174

	Copper hard-drawn	Copper annealed	Aluminum
Tensile strength.........	50 to 67,000 lb. per sq. inch	34,000 lb. per sq. inch	26,000 lb. per sq. inch
Elastic limit.............	30 to 40,000 lb. per sq. inch	7,000 lb. per sq. inch	12 to 14,000 lb. per sq. inch
Modulus of elasticity....	16,000,000	12,000,000	9,000,000
Temp. coef. of expansion..	0.0000096 per deg. fahr.	0.0000096 per deg. fahr.	0.0000128 per deg. fahr.

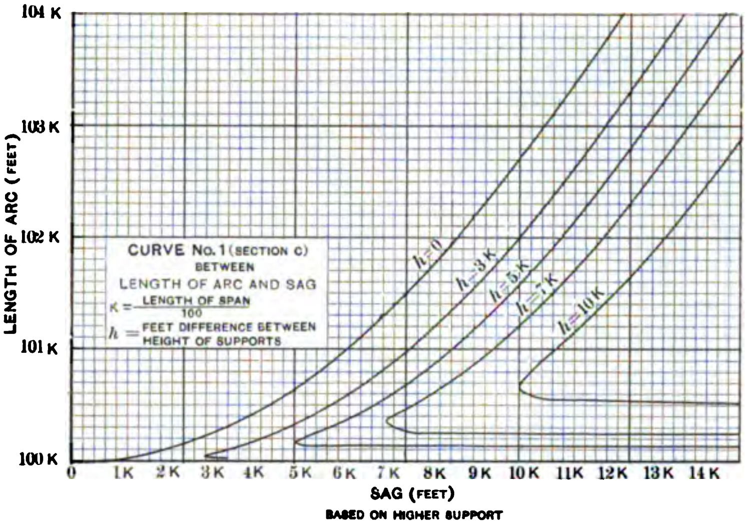
Plate No. 4.—Data on wires
Effect of temperature and stress



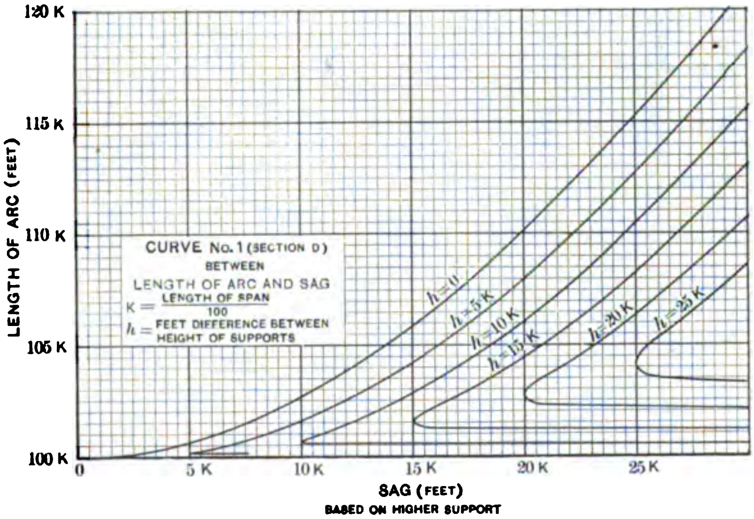
Curve No. 1.—Section A



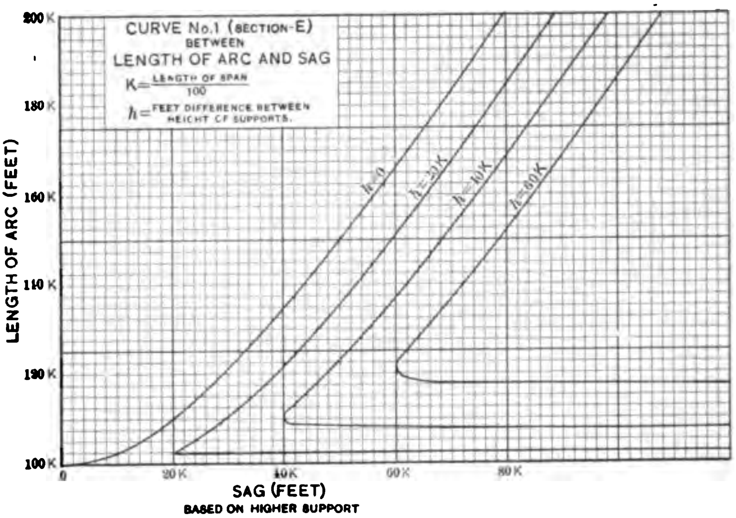
Curve No. 1.—Section B



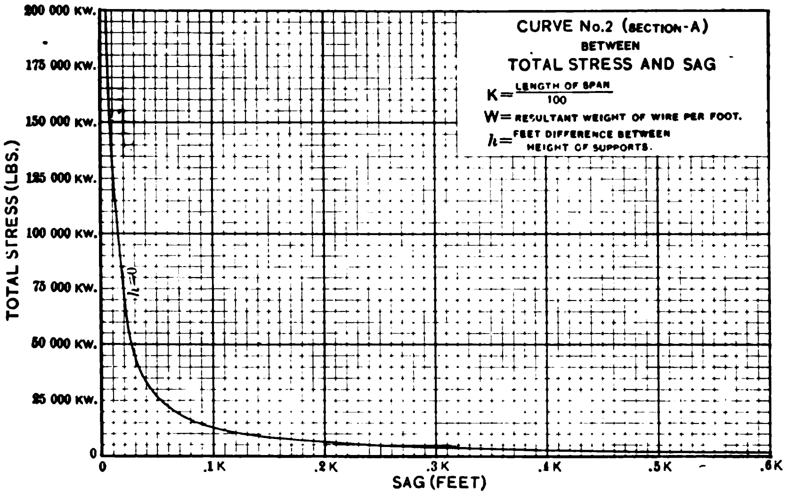
CURVE No. 1.—Section C



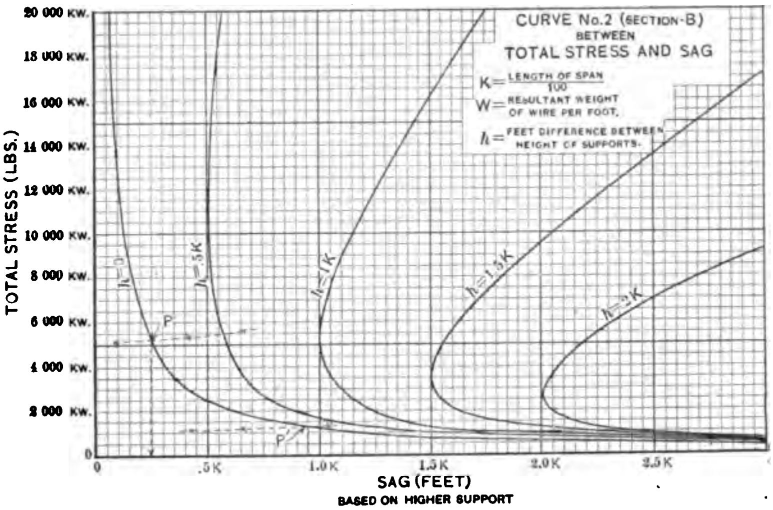
CURVE No. 1.—Section D



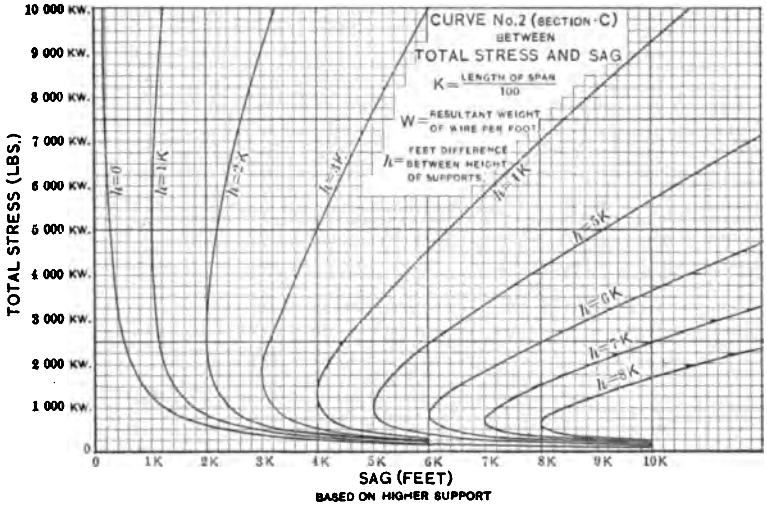
Curve No. 1.—Section E



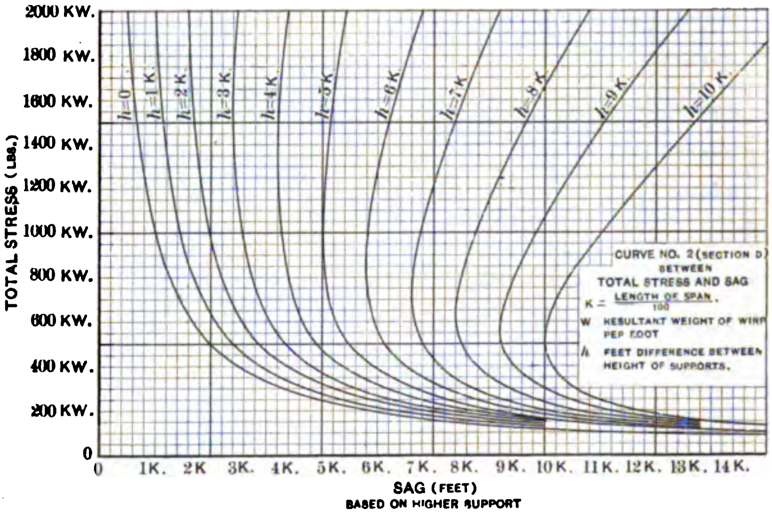
Curve No. 2.—Section A



CURVE No. 2.—Section B



CURVE No. 2.—Section C



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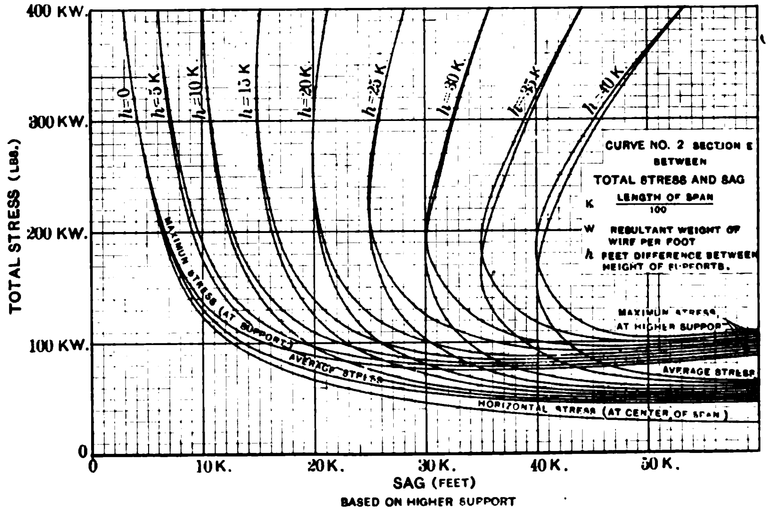
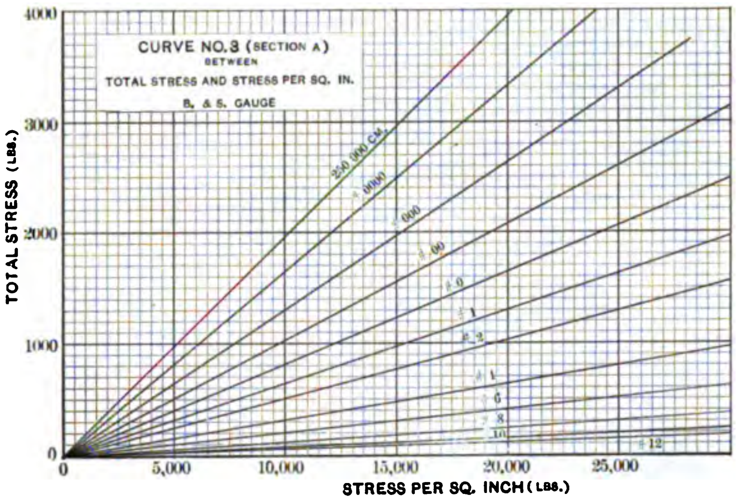
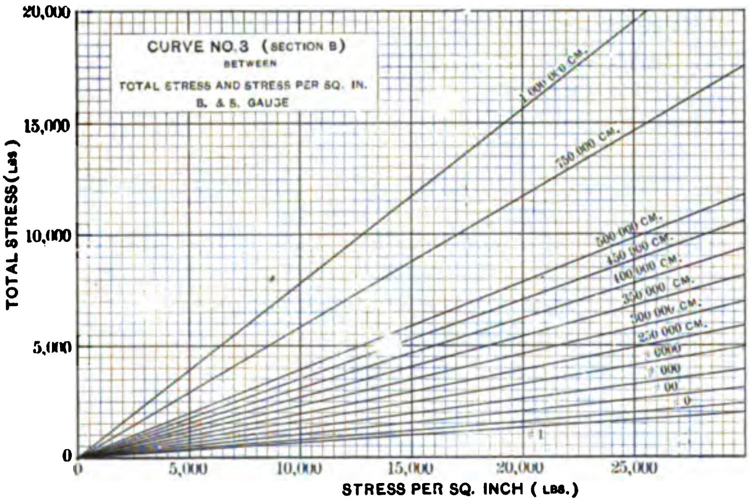


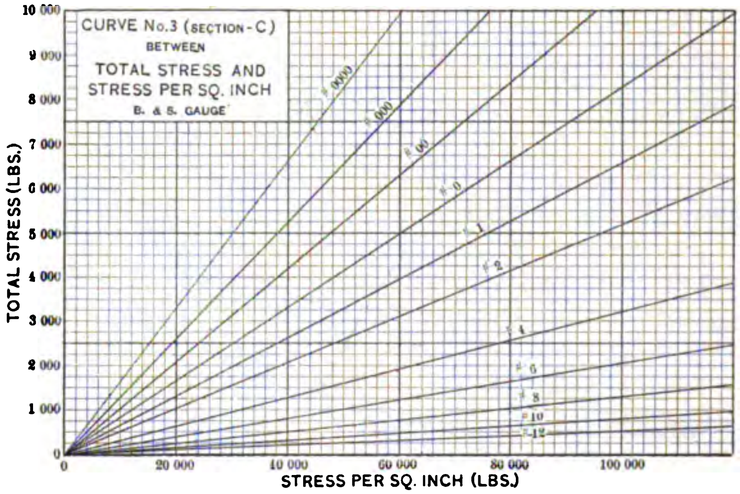
Curve No. 2.—Section E



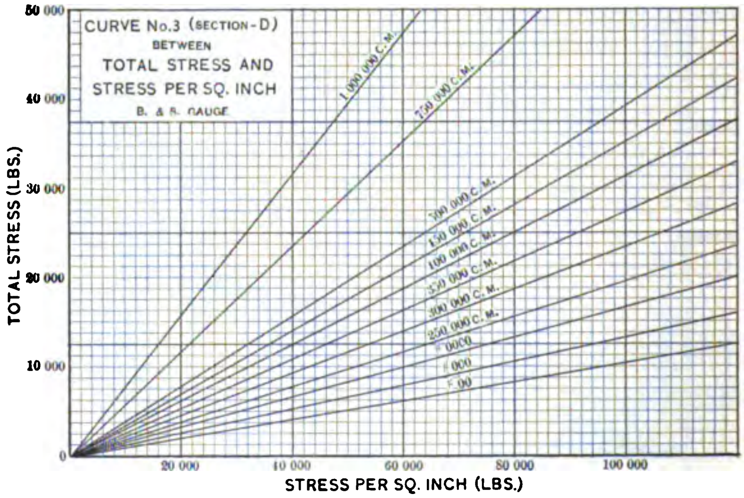
Curve No. 3.—Section A



Curve No. 3.—Section B



Curve No. 3.—Section C



Curve No. 3.—Section D

wires, etc., all of which make up an important factor in the solution of sags and spans. The data which we have at hand to-day are not sufficient. Wires, when stressed, do not return to their original length; there is frequently a permanent set. This fact is overlooked in previous solutions, which are more or less dependent on the wires being perfectly elastic. Mr. O. F. Blackwell, in a paper read before the International Congress at St. Louis, 1904, gives some very good data on the stress and strain in wires, where permanent set has taken place. When an annealed copper wire is stretched, it hardens, becomes somewhat drawn, and increases in strength. There do not seem to be available any detailed data on this property of wire. Full detailed data, of this kind, are desirable.

The data given on Plate No. 3 for wind pressures are based on the formula* $P = 0.0025 V^2$, where P is the wind pressure in lb. per sq. ft. and V is the actual wind velocity in miles per hour. Authorities disagree, however, on wind pressures. The above formula is generally accepted as correct, yet according to an article by Piatt, Lane and Kistler in the *Journal of Electricity, Power and Gas*, July 29, 1911, wind pressures are found to be much higher, and especially so for the smaller wires.

EFFECT OF CHANGES IN TEMPERATURE AND STRESS

The effect of a change in temperature or W value is to increase or decrease the length of wire in a span and thereby alter the sag.

In turn, every change in the sag causes a corresponding change in the stress.

For instance, if an increase in temperature alone takes place, the wire will lengthen and the sag will increase. Because the sag increases, the stress will decrease in turn, causing the wire to shorten, whereby the sag will tend to decrease.

An increase in the W value brings about a similar condition.

The resultant sag is a state of equilibrium between the action of the above forces (forces tending to increase sag and forces tending to decrease sag.)

To compute the resultant sag by means of a formula is a difficult matter, involving complex equations depending on many variable quantities. The following graphical method employing a hypothetical condition is given†

---

*See "Transmission Wire Crossings," F. F. Fowle.

†See paper by H. W. Buck, International Congress, St. Louis, 1904.

Change in Temperature (*Value of W Remaining Constant*). Given a condition of sag and span. From Curve No. 1 find the length of arc and from Curve No. 2 find the (average) stress. Assume hypothetically that the stress in the wire be removed; that the wire contracts to its original length as when unstressed. Find length of arc by applying formula (**9**). For a given change of temperature, find length of arc as it would be changed from the hypothetical length by applying formula (**8**), and find the sag corresponding to this length of arc from Curve No. 1. Now assume hypothetically that the wire is again stressed. Plot a curve showing how the sag increases as the stress is gradually re-applied. This is done by finding the increased length of arc for each value of re-applied stress by formula (**9**), and the corresponding sag from Curve No. 1. Plot the newly found curve *A B*, Fig. 1, and where it cuts the regular curve between sag and stress, Curve No. 2, the point of intersection *P* gives the resultant sag *O C* and stress *O B* due to a change in temperature. In computing the above curve, one must not overlook the fact that the stress value must be expressed in terms of *K W* and the sag in terms of *K*.

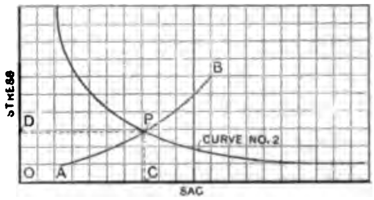


FIG. 1

Change in W Value (*Temperature Remaining Constant*). Given a condition of sag and span. From Curve No. 1 find the length of arc, and from Curve No. 2 the (average) stress, the value of *W* being normal. Assume hypothetically as before that the wire becomes unstressed and find new length of arc by formula (**9**), and the corresponding sag from Curve No. 1. Now assume hypothetically that the stress be resumed, but this time consider the new value of *W*, plot a curve between stress and sag as before, remembering to properly express the ordinates in terms of *K W* and *K*, and find the resultant sag and stress in the same manner.

The curves and formulas are further adaptable to finding resultant sag and stress when the length of span changes, due to poles bending or swaying and crossarms twisting.

PROBLEM

Given a 100-ft. span of No. 4 hard drawn copper wire (three braids weatherproof) strung in summer at 90 deg. fahr. with a sag of six in. Find (1) the resultant sag and stress in winter at 10 deg. below zero, and (2) with ½-in. covering of ice.

Normal $W = 0.164$ lb. per ft. . . . . From Plate No. 3
$K = 1 \ldots K\ W = 0.164$. . . . . From formula (11) on Plate No. 2
Normal sag = 6 in. = 0.5 K. . . . . From formula (11) on Plate No. 2
Normal stress = 2,220 $K\ W = 364$ lb. . . . . From Curve No. 2
Normal stress = 11,200 lb. per sq. in. . . . . From Curve No. 3
Normal length of arc = 100.0068 K = 100.0068 ft. . . . . . . From Curve No. 1

First. Length of arc (hypothetically unstressed)
= 99.9368 ft. = 99.9368 K. . . . . By formula (9)
Length of arc (decreased by change in temperature)
= 99.8409 ft. = 99.8409 K. . . . . By formula (8)

Hypothetical curve as follows (stress resumed):

Assumed stress values		Length of arc by formula (9)	Corresponding sag from Curve No. 1
Lb. per sq. in.	Total lb.		
0	0	99.8409 = 99.8409 K	Hypothetical
11000	—	99.9087 = 99.9087 K	"
22000	—	99.9747 = 99.9747 K	"
26250	860 = 5240 $K\ W$	100.0005 = 100.0005 K	0.090 K
27000	880 = 5360 $K\ W$	100.0051 = 100.0051 K	0.420 K
28000	920 = 5610 $K\ W$	100.0112 = 100.0112 K	0.655 K

Plotting this curve on Curve No. 2 (Section B), using the same ordinates, it is found to intersect at the point P, giving:
Resultant sag = 0.240 K = 0.240 ft. or 2.9 in.
Resultant stress = 5300 $K\ W$ = 870 lb. = 26,600 lb. per sq. in.

Second. New value of $W = 0.750$ lb. per ft. . . . . From Plate No. 3
Length of arc (hypothetically unstressed, normal W and decreased by changes in temperature) = 99.8409 = 99.8409 K as given above.

Hypothetical curve (stress resumed, $W = 0.750$ lb. per ft.) as follows:

Assumed stress values		Length of arc by formula (9)	Corresponding sag from Curve No. 1
Lb. per sq. in.	Total lb.		
0	0	99.8409 = 99.8409 K	Hypothetical
27000	880 = 1170 $K\ W$	100.0051 = 100.0051 K	0.420 K
28000	920 = 1230 $K\ W$	100.0112 = 100.0112 K	0.655 K
31000	1030 = 1380 $K\ W$	100.0294 = 100.0294 K	1.050 K

Plotting curve on Curve No. 2 (Section B), as before, the point of intersection is P, giving:

Resultant sag $=0.950\ K=0.950$ ft. or 11.4 in.
Resultant stress $=0.1350\ K\ W=1000$ lb. $=30{,}000$ lb. per sq. in.

SUPPORTS ON DIFFERENT LEVELS

In dealing with level supports it seemed unnecessary to define sag, but in the case of supports at different levels, it is important to consider this function of spans. There are certain other conditions which must also be understood.

Sag may be defined as the difference in height between the lowest point, or vertex, of the curve, and the point of support. With level supports, the vertex of the curve is at the center of the span, but with supports at different levels, this is not the case. Hereafter, in dealing with supports at different levels, the sag will be reckoned from the higher support. The conditions can best be illustrated in Fig. 2. Here a series of curves, A, B, C, D and E, are shown, representing a single span of wire drawn up at various degrees of tautness between two supports. The curves are produced beyond the lower support in order to complete the catenary curve. Starting with curve E and drawing the wire tighter, the sag represented by S_2 decreases and the vertex moves away from the upper and over toward the lower support. When the vertex reaches the lower support, the sag equals the difference in height between the supports, and if the wire is further tightened, the vertex of the curve moves away from both supports and the sag again increases.

To obtain the relations between sag, stress and length of arc for supports at different levels involves further complex mathematics. However, a semi-graphical solution may be obtained in conjunction with the data and curves already supplied for level supports.

In Fig. 2, take any curve E. It may be considered as made up of two parts, one corresponding to x_1, and the other to x_2. Each part in itself may be considered as half of a separate and independent span having level supports, one half having L_1 for given span value and the other having L_2. Each span will have the same a value. By the proper manipulation of the K factor, all values of sag, stress and length of arc may be obtained for each span from the curves already given for level supports; and then, knowing these values, the relations between difference in height of supports, sag based on higher support, length of arc and stress may be readily computed for the span suspended by the supports on different levels.

To establish a series of curves between the above relations, the following method was employed:

Assume various a values. For each a value select various sets of x_1 and x_2 values (Fig. 2) whose sum is always constant and equal to 100 ft. For each x_1 and x_2 value, determine first the ratio between x_1 and x_2 and then the value of h. From this plot a series of curves, one for each a value, plotted between the ratio of x_1 to x_2 and the value of h.

From these curves the ratio x_1 to x_2 may be obtained for any selected even value of h.

Select a series of h values. For each value of h obtain from the curves the ratio x_1 to x_2 corresponding to each of the already chosen series of a values. In each case, determine the values for x_1 and x_2, sag, length of arc and stress.

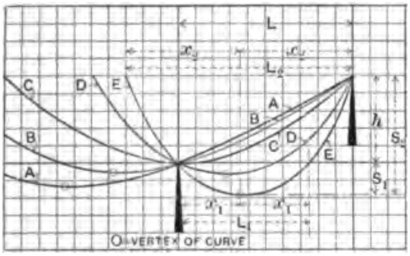


FIG. 2

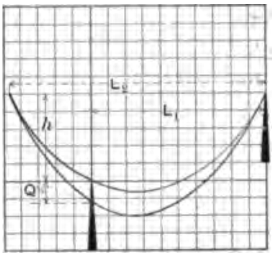


FIG. 3

A series of curves can then be plotted between sag and length of arc, one curve for each value of h. In the same manner, a series of curves can be plotted between sag and stress, one curve for each value of h.

Such sets of curves will be found included with the curves given for level supports. Problems involving the effect of temperature and stress changes may be solved from these curves in the same manner as is done with level supports.

Fig. 3 illustrates a discrepancy which enters into previous solutions offered in sags and spans when considering the effect of temperature and stress changes with non-level supports. They produce the portion of the curve between the supports represented by L_1, until it may be considered as a span on level supports represented by L_2, and solution is made accordingly.

When the effects of temperature and stress changes are considered, the sag may increase or decrease, and the difference in height of supports is altered by an amount represented by Q in Fig. 3. If the supports are fixed, this discrepancy enters.

An interesting feature of spans on supports at different levels is that when the vertex of the curve lies outside of the two supports, there is a small vertical force exerted on the lower support tending to lift the support. This may be demonstrated by inserting a third support between any two level supports which suspend a span of wire. The third support, which should be movable, must be placed lower than the lowest point in the original span. The condition represents two spans of wire where the center support is the lowest and where in each span the vertex of the curve lies outside of its two supports. If the lower support is of material light enough in weight, it will rise.

At first, it might be supposed that when the difference in height of supports increases, the stress values in the span would decrease. Upon inspection of the curves, it is found that very high stress values exist when the vertex of the curve lies outside of both poles. In fact, the stress values approach infinity as the curve of the span approaches a straight line, due to being drawn taut.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

SAG CALCULATIONS FOR SUSPENDED WIRES

BY PERCY H. THOMAS

The method here described for calculating sags and strains in suspended wires was devised to shorten the process of the transmission line computations, especially where the effect of temperature is to be considered. The method is a semi-graphical one and involves no numerical operations other than may be performed by the simplest slide rule manipulation. The method is based on the assumption that the suspended conductor conforms to the catenary, which is generally considered to be the actual fact, although as far as the writer is informed no scientifically accurate verification on a large scale under practical conditions has been attempted. The results obtained by the use of the catenary basis will not differ from those derived from the usual parabola formulas more than 10 per cent in the strain for a sag of 7.5 per cent. For larger sags, however, the difference rapidly increases. A description of the use of the method will be given, followed by a brief statement of the mathematical justification therefor.

METHOD OF MAKING NUMERICAL DETERMINATIONS

The problem is the determination of the various quantities sag, span, strain and angle of wire at support under any definite conditions, and also the effect on these quantities of change in load or change in temperature after the wire has been secured in position.

Imagine the given span to be reduced in size, without changing the shape of the curve, until the span is one foot. The sag will then be reduced in direct proportion to the reduction of span, in other words the percentage of sag will remain the same. The stress in the wire and the length of wire, also, will be reduced in the same ratio. Again, the stress in the wire for a given span for a definite sag is directly proportional to the weight per foot of the wire or the combined effect of weight of wire and ice or of the combined effect of the weight of wire and ice and of wind pressure.

In Plate I the curve marked "*Sag*" shows the relation between the strain in the wire at the point of support, and the sag, in a one-foot span with the total load on the wire of one pound per foot.

From this curve the sag in any span can be found when the length of span, the total load per foot, and the stress to be allowed in the wire are given. Divide this allowable stress in pounds by the span in feet and by the load per foot on the wire. This will give the stress at the support on a span one foot long, of the same shape, having a loading of one pound per foot. From the sag curve on Plate I then read the sag (abscissa) corresponding to this stress (ordinate), which is the actual sag for the one-foot span. This sag multiplied by the span will be the sag in feet of the actual span.

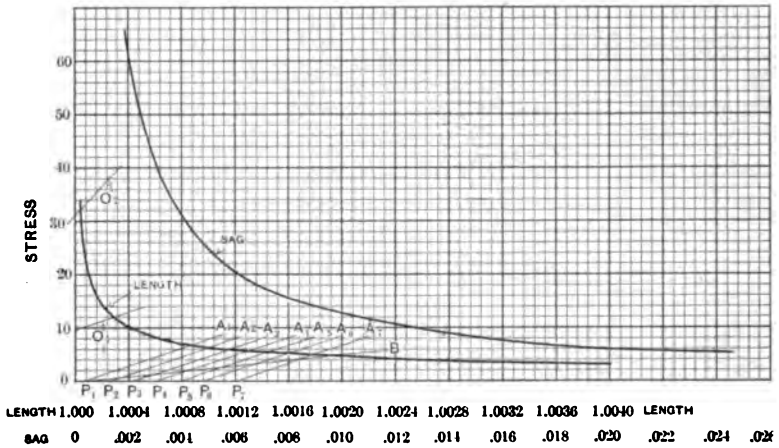


PLATE I.—Sag calculations for suspended wires

In case the sag is given instead of the stress, the corresponding sag for the one-foot span may be obtained by dividing the given sag by the span, both in feet, and the stress for a one-foot span may be obtained from the sag curve. From this the stress in the actual conductor may be obtained by multiplying by the span in feet and the load on the wire in pounds per foot.

To determine the effect of change in temperature, find the length of the wire in the one-foot span (abscissa) shown on the length curve on Plate I as corresponding with the stress in the wire (ordinate). It should be noted that the sag and the length curves in Plate I have the same ordinates, namely, the stress values. Since the wire may be assumed to be fastened at the supports at the original temperature and sag chosen, no subsequent *slipping* will occur, and any subsequent change in temperature will tend to change the length of the wire by *expansion*, and consequently to change the sag. Since the sag is extremely sensitive to the length of the wire, even the very small changes in length resulting from changes in temperature will be important. But with every change in sag there is an important change in stress, which will change the amount of *stretch* in the wire due to the stress. These two effects are simultaneous and are closely interrelated, and must be considered together. Having given the stress and having found the length of wire for the corresponding one-foot span, the length of wire without stress may be calculated from the modulus of elasticity M, *viz.*, by the formula, the elongation or stretch $= \frac{\text{stress}}{M}$. If this unstressed length be marked on the axis of X in Plate I as at P_1, a straight line connecting this point with the point on the length curve corresponding to the actual stress, as already determined, will be the *stretch curve* of the wire with stress. Such a line is marked A.

On the other hand, if it be assumed that the stress remain constant and the temperature change, the wire will change in length proportionally with the temperature change, in accordance with its proper coefficient of expansion. If it be assumed that the sag be desired for every 20 deg. above or below the initial temperature, the length of conductor unstressed may be calculated for these several temperatures, and these lengths marked on the chart on the axis X as P_2, P_3, P_4, etc. These points do not represent actual lengths of wire as hung in the span, since they are unstressed lengths, but the actual lengths taken by the conductor for the different temperatures can be obtained by drawing lines, as A_2, A_3, A_4, etc., through the several points P_2, P_3, P_4, etc., parallel to the line A, the *stretch* curve of the wire at the initial temperature, which lines will be the stretch curves for their respective temperatures. The points of intersection of these ***stretch*** lines with the ***length*** curve will give the stresses which the suspended wire will actually have at the several temperatures; the sag corresponding for the one-foot span can be read on the sag curve from the stress values. The sags of the actual span can then be obtained as before by multiplying by the span.

In some cases where the sag is small the length of wire unstressed will be less than one foot, and so will fall off the plot. In Plate I, for example this would occur on copper wire in a one-foot span stressed to 12. In such a case the stretch line can be found by computing the stresses required to produce any two suitable lengths (as for example, the length under maximum stress and the one-foot length) and drawing the stretch line therethrough.

Such a stretch line is shown at O_1, in Plate I.

The effect of the accumulation of ice and sleet introduces a new condition, since it increases the load per foot on the wire. The former ***stretch*** curve becomes inapplicable, since it is based on a different number of pounds per foot, or load on the wire. The stress in the actual wire represented by a given ordinate, if produced by a new condition of loading, other things being constant, will be changed from that represented by the old loading in the ratio of the change of loading. That is, if the load per foot be doubled, a given ordinate will represent twice as great a stress in the actual wire. Therefore the ***stretch*** under the heavier load can be obtained on Plate I by changing the stretch, that is, the increase of length, at any given ordinate, or by changing the stress for any given length in the same proportion as the change in loading. This will give a second stretch line, making a different angle with the axis of X, and representing the new condition. Such a line for a double loading is shown at B, Plate I. The point where this line intersects the ***length*** curve will, as before, give the stress in the one-foot wire corresponding to the new temperature. Similar stretch lines for the changed loading may be drawn for any temperatures. The new stretch line corresponding to the line O_1, for the same case of new loading is O_2.

Plate II is similar to Plate I, but is intended for actual numerical determinations and is drawn to three scales suited to different classes of work. The curves of sag and stress on Plate II are plotted from accurate equations and contain no approximations. The expansion of metals with temperature and their stretch with

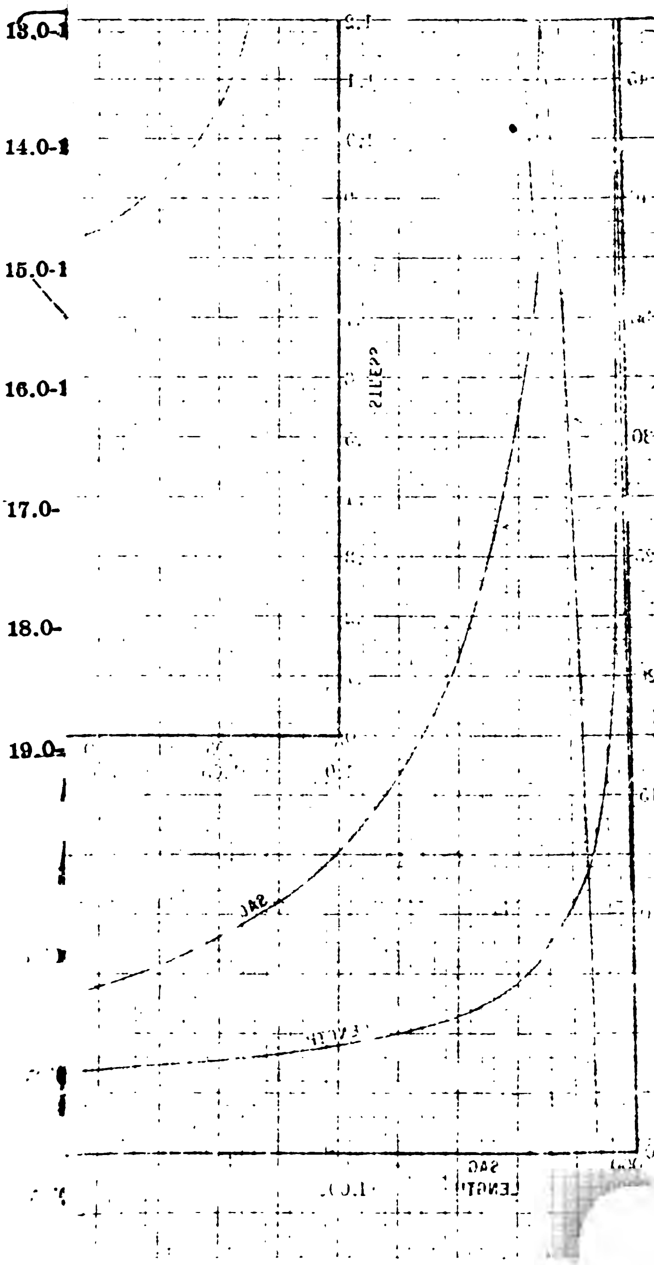


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Plate II is similar to Plate I, but is intended for actual numerical determinations and is drawn to three scales suited to different classes of work. The curves of sag and stress on Plate II are plotted from accurate equations and contain no approximations. The expansion of metals with temperature and their stretch with stress do not follow the straight line law exactly, and introduce a certain error in this as in any method of calculation.

The middle curves are intended for sags up to about 2 per cent, the left-hand curves from about 2 per cent to 15 per cent, and the smaller curves in the upper right-hand corner are useful in special cases to show the effect of very large sags. It is clear from these curves that after the sag has reached 15 to 20 per cent there is little reduction of stress by further increase of sag and an actual increase of stress soon results. These smaller curves will give an indication of the conditions where a wire has to be taken down a precipitous place, giving the equivalent of an abnormally large sag.

Where the height of the two supports is unequal, the length of the wire on the two sides of the lowest point of the span is not equal. However, the form of the curve and the stresses in each part will be the same as if the other part were symmetrical with it. If, then, the horizontal distance from the higher support to the lowest point of the wire is known, the stress and sag in this portion can be determined as though the whole span were equal to twice this distance. If desired, the lesser strain in the other portion can be determined in the same manner. The following formulas give the horizontal distance from the higher support to the lowest point of the wire, x_1:

$$x_1 = \frac{X}{2} + \frac{t\,S}{X} \tag{A}$$

$$x_1 = \frac{d\,X}{t}\left[1 - \sqrt{1 - \frac{t}{d}}\right] = X\,\frac{\sqrt{d}}{\sqrt{d-t} + \sqrt{d}} \tag{B}$$

X is the span in feet.

x_1 is the horizontal distance in feet from the higher support to the lowest point of the wire.

t is the difference in height of the two supports in feet.

S is the stress in pounds in the wire at the highest support, with *one pound per foot load* on the conductor.

d is the sag in feet measured from the higher point of support.

Formula (**A**) is useful when the span and the stress to be allowed in the wire are given, and formula (**B**) when the span and the sag are given.

These formulas are approximate; formula **A** is correct within 2 to 4 per cent when neither sag nor difference in heights of supports exceeds 15 per cent of the span. Formula **B** has an error of less than 1 per cent under these conditions.

The following examples will make the use of the curves clear. Assume the given conditions to be:

Size wire No. 00 B. & S. copper.

Span 500 ft.

Safe strength of wire 3140 lb.

Worst conditions, $\frac{1}{2}$ in. of ice all around the wire, at 0 deg. fahr., and wind pressure of 8 lb. per square foot.

Weight of wire and ice per foot, 0.940 lb.; wind pressure per foot, 0.910 lb.; resultant force per foot 1.308 lb.

Then the stress on a one-foot span, for use on Plate II,

$$=\frac{3140}{1.308\times 500}=4.8.$$

From the curve the sag for the one-foot span is 0.028, and the sag in the actual span $0.026\times 500=13$ ft.

The length of the wire in the one-foot span under these worst conditions is 1.00193. The length unstressed is $1.00193-\frac{30{,}000}{16{,}000{,}000}=1.000055$, where the modulus of elasticity is taken as 16,000,000—a usual value for copper. This is the length that would be taken by the actual wire as it lies tied on the insulators were it unstressed at this temperature, *viz.*, 0 deg. fahr.

To determine the sags for various conditions when the ice is removed and no wind exists, determine the load per foot on the conductor under the new condition. If the weight of the wire only is to be taken, giving 0.403 pounds per foot, the ratio of this to the ice and wind condition above is $\frac{1.308}{0.403}=3\frac{1}{4}$. Therefore, the stretch, when plotted on the curve at the same ordinate as for the ice and wind condition, would be reduced in the same proportion, giving a length 1.000632 instead of 1.00193 as before. The new line showing the stretch curve of the wire for the no-ice or wind condition must now be drawn from the point P_1 on the axis of X to the new length at the stress 4.8. This intersects the length curve at the stress 6.8, the sag for which is the value sought, *viz.*, 0.018 for the one-foot span.

The effect of temperature can be obtained as follows:

Twenty degrees temperature change will mean a change in length of copper wire, other things remaining the same, of

$0.0000096 \times 20 = 0.000192$ feet on Plate II, using the coefficient of expansion 0.0000096. Therefore, the length of wire unstressed for various temperatures will be as follows:

20 deg., 1.000247; 40 deg., 1.000439; 60 deg., 1.000631; 80 deg., 1.000823; 100 deg., 1.001015; 120 deg., 1.001207. By drawing lines from the several points on the axis of X, parallel to the stretch line already determined, (two draftsman's triangles may very conveniently be used for this purpose), a number of points of intersection with the length curve will be obtained, giving the following stress values: for 20 deg., 6.4; for 40 deg., 6.0; 60 deg., 5.7; 80 deg., 5.3; 100 deg., 5.0; 120 deg., 4.7. The sags corresponding are, for 20 deg., 0.0195; 40 deg., 0.0207; 60 deg., 0.022; 80 deg., 0.0235; 100 deg., 0.025; 120 deg., 0.0265 on the one-foot curve. Sags on the actual span are obtained by multiplying by the span, 500 feet, *viz.*, for 20 deg., $0.0195 \times 500 = 9.75$ ft.; for 40 deg., 10.35 ft.; 60 deg., 11.0 ft.; 80 deg., 11.75 ft.; 100 deg., 12.5 ft.; 120 deg., 13.25 ft..

Other conditions of stress, as high wind without ice and high temperature, may be similarly determined.

To illustrate a case in which the height of supports is unequal, the following example is added:

Given, high support 40 ft. above the lower; span 500 ft., conductor No. 00 copper, allowable stress 3140 lb. The stress allowable, divided by the pounds per foot on the wire, assuming that the conditions are the same as the ice conditions of the last example, is $\frac{3140}{1.308} = 2400$. From formula (A) the distance from the high support to the lowest point of the wire is $\frac{500}{2} + \frac{4 \times 2400}{500}$ $= 442$ ft. Then the span would be $2 \times 442 = 884$ ft. if the span were symmetrical and both sides like the higher side. The calculations for the high side can now be made as before, using the span as 884 ft. In determining the effect of temperature changes with unequal heights of supports, a certain inaccuracy is introduced by the assumption that the length of equivalent span remains the same; but this can be neglected, except where the conditions require close working. It is evident that the effect of temperature changes in such a span will be less than they would be in a span twice the value x_1; and more than would be the case with a span twice the length of $X - x_1$; that is, the distance from the lower support to the lowest point on the wire.

Where the span instead of the stress is given, the formula (**B**) may be used and the rest of the computation remains the same.

The sine of the angle made by the wire with the horizontal is $\frac{1}{2}$ the length of wire in the span divided by the strain with one lb. per ft. weight of wire, and may be obtained from the length curve. In Plate I, with the length of wire $=1.002$, the strain $=4.7$ and the sine $=\frac{\frac{1002}{2}}{4.7}=0.107$, and the angle $=6$ deg., 7 min. The sag for this point $=0.0265$ per cent.

Appendix

The curves of the plates in the present paper are obtained in the following manner:

The equation of the catenary corresponding in position to the span wire is

$$y=\frac{h}{2}\left(e^{\frac{x}{h}}+e^{-\frac{x}{h}}\right)$$

where h represents the distance of the lowest point of the catenary above the axis of X (not, however, above the ground).

The sag corresponding to any point $x\,y$ on this curve is $y-h$. The sine of the angle made by the tangent of the curve at this point with the verticle is $\frac{\sqrt{y^2-h^2}}{y}$. The length of the curve from the lowest point to the point $(x\,y)$ is $\sqrt{y^2-h^2}=\frac{1}{2}$ the length shown on the curves. The stress along the wire at the point $x\,y$ is the total weight (one lb. per ft. assumed) divided by the sine of the angle the tangent makes with the vertical, $=$ $\sqrt{y^2-h^2}\div\frac{\sqrt{y^2-h^2}}{y}=y$. This is a very simple and interesting relation. These equations give the basis for all the necessary data for the curve. In the actual calculations a number of suitable values of h and x were assumed and the other quantities calculated from these equations. These values were reduced, of course, to a one-foot span by dividing in each case (except for total length of wire) by $2\,x$. In the case of total length of wire the length $2\sqrt{y^2-h^2}$ must be divided by $2\,x$.

The formulas for supports of uneven height are derived as follows:

Let l_1 and l_2 respectively be the lengths of wire from the lowest point of the conductor to the higher and the lower points of support, and L the total length of the wire. As before $x_1\ y_1$ and $x_2\ y_2$ are the coördinates of the wire respectively at the higher and the lower points of support, and t equals the difference in height of supports.

From the general formula for length on the catenary, $l_1=\sqrt{y_1^2-h^2}$ and $y_1=\sqrt{l_1^2+h^2}$; $l_2=\sqrt{y_2^2-h^2}$ and $y_2=\sqrt{(L-l_1)^2+h^2}$ but $y_1-t=y_2$, then $\sqrt{l_1^2+h^2}-t=\sqrt{(L-l_1)^2+h^2}$

$$t^2-2\,t\sqrt{l_1^2+h^2}=L^2-2\,l_1\,L \qquad \textbf{(C)}$$

But $S=y_1=\sqrt{L_1^2+h^2}$

Therefore $t^2-2\,t\,S=L^2-2\,l_1\,L$ and

$$l_1=\frac{L}{2}+\frac{t}{L}\left(S-\frac{t}{2}\right)$$

Since $\frac{t}{2}$ will be small compared with S it may usually be omitted. Also, since l and L respectively nearly equal x and X for prevalent values of sag, the latter values may be substituted for the former, giving formula (**A**) above, *viz.*,

$$x_1=\frac{X}{t}+\frac{t\,S}{X} \qquad \textbf{(A)}$$

Again: sag $=y_1-h=d$. Combining this with the equation for length $l_1^2=y_1^2-h^2$, $h=\frac{l_1^2-d^2}{2\,d}$; combining this with equation (C) above,

$$t^2-2\,t\sqrt{l_1^2+\left(\frac{l_1^2-d^2}{2\,d}\right)^2}=L^2-2\,l_1\,L$$

and

$$l_1=\frac{d\,L}{t}\left(1\pm\sqrt{1-\frac{t}{d}+\frac{t^2}{L^2}\left(\frac{t}{d}-1\right)}\right)$$

$$=\frac{d\,L}{t}\left(1-\sqrt{\left(1-\frac{t}{d}\right)\left(1-\frac{t^2}{L^2}\right)}\right)$$

In this equation only the negative sign of the radical meets the conditions of the problem. The quantity t^2/L^2 will always be small in practical transmission work, and may be omitted, since t/d will always be less than 1. The same substitution of x for l and X for L may be made as in the case of formula (**A**) and we then have formula (**B**), *viz.*,

$$x_1=\frac{d\,X}{t}\left(1-\sqrt{1-\frac{t}{d}}\right) \tag{B}$$

which, by an algebraic transformation, equals

$$X\frac{\sqrt{d}}{\sqrt{d-t}+\sqrt{d}} \tag{B}$$

It is interesting to note that this latter form is the formula derived from the parabolic curve for determining the same quantity.

By the formula already given, the sine of the angle with the horizontal made by the wire at the point $x\,y$ is $\frac{\sqrt{y^2-h^2}}{y}$ as derived from the catenary equation. But $\sqrt{y^2-h^2}$ is the length of wire from the lowest point to the point $x\,y$, and y is the total strain in the wire with one lb. per ft. load. Therefore, the angle can be readily calculated from the plotted length curve in the one-foot span, taking for $\sqrt{y^2-h^2}$ one-half the length on the length curve.

The values from which the curves are plotted are as follows:

Length	Sag	Strain
1.0000042	0.00125	100.0013
1.0000051	0.00138	90.9105
1.0000061	0.00150	83.3348
1.0000071	0.00162	76.9247
1.0000082	0.00175	71.4303
1.0000094	0.00188	66.6685
1.0000107	0.00200	62.5020
1.0000118	0.00212	58.8257
1.0000136	0.00225	55.5578
1.0000151	0.00238	52.6339
1.0000167	0.00250	50.0025
1.0000261	0.00313	40.0031
1.0000372	0.00375	33.3371
1.0000511	0.00438	28.5758
1.0000667	0.00500	25.0050
1.000104	0.00625	20.0063
1.000150	0.00730	16.6742
1.000266	0.01000	12.5100
1.000417	0.01250	10.0125
1.000598	0.01500	8.3483

Length	Sag	Strain
1.000817	0.01751	7.1604
1.001066	0.02001	6.2700
1.001351	0.02252	5.5781
1.001668	0.02502	5.0250
1.002017	0.02753	4.5730
1.002402	0.03004	4.1967
1.003754	0.03757	3.3709
1.006680	0.05017	2.5502
1.010435	0.06283	2.0628
1.015068	0.07556	1.7422
1.020542	0.08840	1.5170
1.026881	0.10134	1.3513
1.034093	0.11441	1.2255
1.042191	0.12763	1.1276
1.051185	0.14100	1.0501
1.061089	0.15455	0.9879
1.083691	0.18226	0.8965

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

THE MECHANICAL AND ELECTRIC CHARACTERISTICS OF TRANSMISSION LINES

BY HAROLD PENDER AND H. F. THOMSON

The object of this paper is to present in compact form the data and formulas, together with their derivations, required for the determination of the mechanical and electric characteristics of transmission lines, together with a set of charts whereby the various mathematical operations involved may be accomplished rapidly and with the minimum effort. The treatment naturally falls under two headings, A—Mechanical Characteristics, and B—Electric Characteristics. Summaries of the working symbols and equations for Part A and Part B follow the general discussion under these heads.

A. MECHANICAL CHARACTERISTICS

The following discussion will be limited to the consideration of the characteristics of a perfectly flexible wire suspended from two fixed points of support. Actual wire spans closely approximate this condition, particularly when the successive spans are all of the same length, whether the supporting insulators be of the pin or of the suspension type. In the case of successive spans of unequal length the change in tension in the various spans, due to the variation of temperature and the mechanical load on the wire, will not be the same in each span and consequently such changes will produce a resultant force on the insulator, causing thereby a slight deflection of the top of the tower; or a deflection of the insulator only, if the latter is of the suspension type. This motion of the points of support will tend to equalize the tensions in the various spans.

In the design of a wire span two problems must be solved: 1. At what tension (or sag) must the wire be strung in order that the tension in the span may not exceed a definite limit under the worst conditions of temperature and of mechanical loading (due to ice and wind)?; and 2. What will be the maximum vertical sag of the wire for a given variation in loading and temperature? The factors which enter into these two problems are the following:

1. The length of the span.
2. The material of the wire.
3. The size of the wire.
4. The coefficient of linear expansion of the wire. For copper this coefficient is 9.6×10^{-6} and for aluminum 12.8×10^{-6}, the temperature in each case being expressed in deg. fahr.
5. The modulus of elasticity of the wire. The modulus of elasticity for copper or aluminum is not strictly a constant, but may be assumed as such as a first approximation. For copper this modulus varies from 12×10^6 to 16×10^6, depending upon the quality of the wire and upon whether the wire is stranded or solid. The former figure is generally used for stranded copper wires. For aluminum wire, which is always stranded when used for transmission lines, the modulus is equal to 9×10^6. The modulus M is equal to the increase of tension (ΔT) in pounds per square inch required to produce an elongation ($\Delta \lambda$) of a given length of wire (λ) divided by the ratio of the elongation to the original length, *i.e.*,

$$M = \Delta T \div \frac{\Delta \lambda}{\lambda} = \lambda \frac{\Delta T}{\Delta \lambda}$$

6. The maximum tension in pounds per square inch to which the wire should be subjected. This maximum allowable tension is usually taken at one-half the ultimate tensile strength of the wire, about 30,000 lb. per sq. in. for copper and 13,000 lb. per sq. in. for aluminum.
7. The maximum external load to which the wire may be subjected due to the collection of sleet on the wire and the pressure of the wind against it. This, of course, will depend upon climatic conditions, but even for a given section of the country, there is considerable difference of opinion as to what should be assumed as a reasonable external load. The Joint Committee on Overhead Line Construction has recently recom-

mended that the maximum loading for high-tension crossings be taken as that due to an ice coating 0.5-in. thick all around the wire, plus a wind pressure of eight lb. per sq. ft. of the projected area of this ice cylinder. To meet this requirement on a long span, especially with aluminum wire, would require a relatively large sag, from 30 to 100 ft., depending on the length of span and size of wire. This would require a prohibitive height of tower, at least for level country work. For cross country spans it would, therefore, seem advisable to assume a less heavy loading, particularly as an ice coating of 0.5-in. and a wind velocity of 60 miles per hr. (corresponding approximately to eight lb. wind pressure) will seldom, if ever, exist simultaneously.

8. The minimum temperature at which the maximum external load occurs. Here again there is considerable difference of opinion, even for given climatic conditions. The Joint Committee on Overhead Line Construction recommends the assumption of a minimum temperature of −20 deg. fahr. for lines situated in the northern part of this country. For cross country spans the assumption of a higher temperature would seem more reasonable, in view of the fact that sleet seldom exists on a wire when the temperature is much below freezing.

9. The temperature at which the wire is to be strung. As an average stringing temperature 70 deg. fahr. is a reasonable assumption.

10. The maximum temperature to which the wire may be subjected. For lines located in the northern part of this country a maximum temperature of 120 deg. fahr. is reasonable. It should be noted that the temperature of a wire exposed to the sun is considerably higher than the temperature of the surrounding air.

The interrelation of these various quantities in the case of a wire suspended from two fixed points of support will be considered in detail.

DETERMINATION OF THE RATIO (K) OF THE RESULTANT FORCE ON THE WIRE TO THE WEIGHT OF THE WIRE

The total force on a span of wire under any condition of mechanical loading consists in general of three components: 1, its own weight, 2, the weight of the ice coating, and 3, the force acting on it due to wind pressure. It is usual to assume that the wind pressure is horizontal, although it may have a considerable vertical component, particularly if the span is parallel to the base of a steep hill. The other two components of the resultant force on the wire are vertical.

Let w = weight in pounds of one foot of the wire.

d = diameter of wire in inches.

i = thickness of ice coating in inches.

v = wind pressure in pounds per square foot perpendicular to the wire.

K = ratio of total force on the wire to weight of wire under these same conditions.

The vertical force in pounds on one foot length of the wire under a given mechanical load is then

$$w+\frac{57}{144}\cdot\frac{\pi}{4}\left[(d+2\,i)^2-d^2\right]=w+1.24\,i\,(d+i)$$

where the weight of ice is taken as 57 lb. per cu. ft. (Tests show that the sleet formed on a wire weighs less than this, due probably to air bubbles in it—this figure is, however, on the safe side.) The horizontal force acting on one foot of the wire is

$$\frac{v}{12}\,(d+2\,i)$$

The resultant force acting on one foot of the wire is then

$$\sqrt{\left(w+1.24\,i\,(d+i)\right)^2+\left(\frac{v\,(d+2\,i)}{12}\right)^2}$$

and therefore the value of the ratio K is

$$K=\sqrt{\left(1+\frac{1.24\,i\,(d+i)}{w}\right)^2+\left(\frac{v\,(d+2\,i)}{12\,w}\right)^2} \qquad (1)$$

The values of this ratio K for the various sizes of wire between 1,000,000 circular mils and No. 6, B. & S. gage, for both copper and aluminum, for no ice and for ice thicknesses of ¼, ½ and ¾ in., and for wind pressures from 0 to 12 lb. per sq. ft., are plotted on Chart No. 1. On this chart is also given a short table showing the relation between indicated wind velocity and pressure per square foot. The latter is deduced from the formula given by H. W. Buck in the Transactions of the International Electric


K=Ratio of Total
1
1.5
2
3
0
1
40
2

Congress, 1904; this formula is that the wind pressure in pounds per square foot of projected area of a round cylinder of diameter d_1 produced by an actual wind velocity of V miles per hour is $0.00021\ V^2\ d_1$.

THE EQUATIONS OF A WIRE SPAN

In the following discussion the wind pressure will be assumed perpendicular to the vertical plane through the two points of support and the pressure per foot of wire in the direction of the wind will be assumed constant at all points irrespective of the angle between the wire and the direction of the wind. This assumption is not strictly realized, since the wire near the points of support will make an angle of less than 90 deg. with the direction of the wind, and therefore to obtain an exact solution the wind pressure should be resolved into its two components, one acting in the direction of, and the other perpendicular to, the wind. Since the angle between the wire and the direction of the wire is in general very nearly 90 deg., the assumption that the wind pressure actually exerted on the wire is equal to the pressure which would be exerted were the wire in a vertical plane normal to the wind will not introduce an appreciable error, in comparison with the variations in the wind pressure which will occur due to the actual variation in wind velocity at different parts of the span.

The general case of the points of support at different elevations will also be considered, and the wire will be assumed to lie in a plane through the two points of support parallel to the direction of the resultant force. This assumption is not strictly realized when there exists a combination of wind pressure with the two points of support at different elevations, but is strictly true in all cases when the points of support are at the same elevation, and in the former case gives a solution as nearly correct as can be obtained without an elaborate analysis.

Fig. 1 is a diagram of a span of wire in perspective. This diagram shows the general case when the points of support are at different elevations and when there is a horizontal component of the force (wind pressure) acting on the wire. Fig. 2 shows only the plane $A\ E\ F\ B$, which is the plane in which the wire lies. When there is no wind this plane is vertical.

The plane $A\ E\ F\ B$ is determined by the line $A\ B$ and the direction of the resultant external force. The planes $A\ E\ R$ and $B\ F\ U$ are determined by the two components of the resultant force. $A\,G$ is drawn in the plane $A\,E\,F\,B$ through A perpendicular to the resultant force. $E\,O\,F$ is drawn in the plane $A\,E\,F\,B$ perpendicular to the resultant force and through the point O where the wire is also perpendicular to the resultant force.

Let f = tension in pounds at any point P of the wire.

α = angle between the tangent to the wire at this point and the line $A\,G$ drawn perpendicular to the resultant external force.

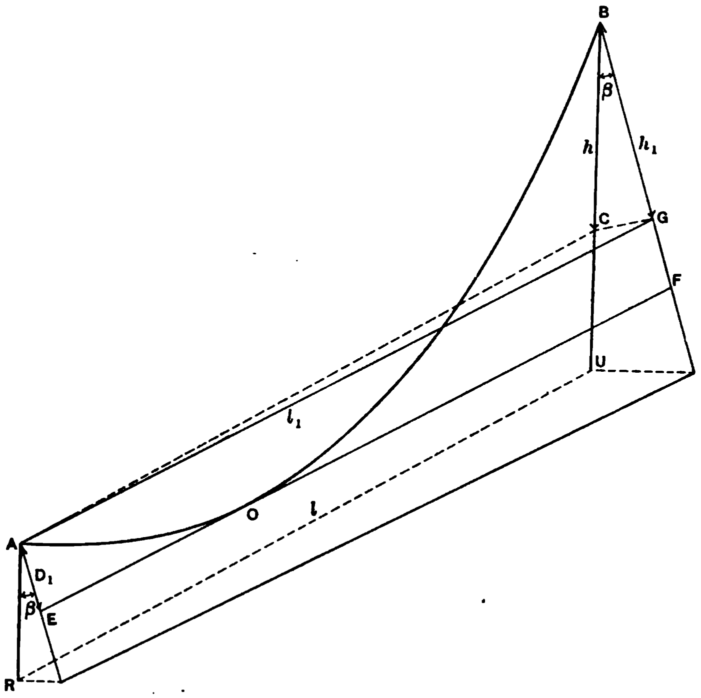

FIG. 1

F = tension in pounds at the point O where the wire is perpendicular to the resultant force.

x = the horizontal distance between the point P and the point O.

y = perpendicular distance from P to the line $O\,X$.

λ = length of wire in feet from O to P.

$d\,\lambda$ = an elementary length of the wire in feet at the point P.

$K\,w$ = resultant force in pounds acting on a one-foot length of the wire due to its own weight, the weight of the ice coating and wind pressure.

m = weight in pounds of a bar of the conductor material one foot long and one square inch in cross section. (For copper $m = 3.85$; for aluminum $m = 1.16$).

T = tension in wire in thousands of pounds per square inch at the point O.

Acting on the elementary length $d\,\lambda$ are three forces in the directions indicated. The resultant of these three forces must be zero. Hence, resolving these forces parallel to $O\,X$ and $O\,Y$ respectively, we have

$$(f+d\,f)\cos(\alpha+d\,\alpha)-f\cos\alpha=0$$

$$(f+d\,f)\sin(\alpha+d\,\alpha)-f\sin\alpha-K\,w\,d\,\lambda=0$$

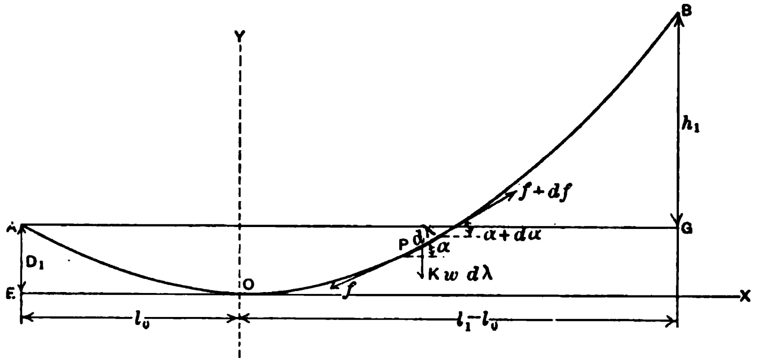

FIG. 2

But since $d\,\alpha$ is an infinitesimal angle its cosine is unity and its sine is equal to $d\,\alpha$. Hence,

$$\cos(\alpha+d\,\alpha)=\cos\alpha-\sin\alpha\,d\,\alpha$$

$$\sin(\alpha+d\,\alpha)=\sin\alpha+\cos\alpha\,d\,\alpha$$

which, substituted in the above equations, give

$$\cos\alpha\,d\,f-f\sin\alpha\,d\,\alpha=0$$

$$\sin\alpha\,d\,f+f\cos\alpha\,d\,\alpha=K\,w\,d\,\lambda$$

which may in turn be written

$$d\,(f\cos\alpha)=0 \qquad (2)$$

$$d\,(f\sin\alpha)=K\,w\,d\,\lambda \qquad (3)$$

Integrating these two equations, we get

$$f \cos \alpha = F \qquad (4)$$

and

$$f \sin \alpha = K w \lambda \qquad (5)$$

Equation (4) shows that the component of the tension perpendicular to the resultant force is the same at all points of the wire. Equation (5) shows that the component of the tension at any point of the wire in the direction of the resultant force on the wire due to its own weight and external load is equal to the value of this force acting on that portion of the wire between this point and point O at which the wire is perpendicular to the resultant force. The ratio of (5) and (4) gives

$$\tan \alpha = \frac{K w \lambda}{F} \qquad (6)$$

This equation involves only the constants K, w and F and the angle α and distance λ; the two latter depend only on the shape of the curve in which the wire hangs. By substituting for α and λ their values in terms of the coördinates x and y, we can obtain the equation of this curve in rectangular coördinates. In (6) substitute $q = \tan \alpha$ and differentiate with respect to x; this gives

$$\frac{d q}{\sqrt{1+q^2}} = \frac{K w}{F} d x$$

The integral of this equation is*

$$q = \sinh \frac{K w x}{F} \qquad (7)$$

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*For any argument u the hyperbolic sine is

$$\sinh u = \frac{e^u - e^{-u}}{2}$$

and the hyperbolic cosine is

$$\cosh u = \frac{e^u + e^{-u}}{2}$$

Chart No. 5 shows the relation between u, sinh u, and cosh u.

Substitute for q its value $\tan \alpha = \frac{d y}{d x}$ and integrate; this gives

$$y = \frac{F}{K w}\left[\cosh \frac{K w x}{F} - 1\right] \tag{8}$$

This is the equation of the curve in which the wire hangs; it is the equation of the catenary.

The substitution in (6) of the value of $q = \tan \alpha$ given by (7) gives

$$\lambda = \frac{F}{K w} \sinh \frac{K w x}{F} \tag{9}$$

which is an expression for the length of wire λ from O to the point P in terms of the corresponding distance x measured along the line $O X$.

Since the weight w of the wire per foot is equal to its cross section multiplied by the weight m of a rod one foot long and one square inch in cross section, and since the tension in pounds F is equal to the cross section of the wire multiplied by 1000 times the tension T in thousands of pounds per square inch,

$$\frac{w}{F} = \frac{m}{1000\, T}$$

Hence equation (8) may be written

$$y = \frac{1000\, T}{K m}\left[\cosh\left(\frac{K m x}{1000\, T}\right) - 1\right] \tag{10}$$

and equation (9) may be written

$$\lambda = \frac{1000\, T}{K m} \sinh\left(\frac{K m x}{1000\, T}\right) \tag{11}$$

The hyperbolic cosine of an argument u may be expanded into the series

$$\cosh u = 1 + \frac{u^2}{2!} + \frac{u^4}{4!} + \frac{u^6}{6!} + \ldots\ldots\ldots$$

and the hyperbolic sine into the series

$$\sinh u = u + \frac{u^3}{3!} + \frac{u^5}{5!} + \frac{u^7}{7!} + \ldots\ldots\ldots$$

Therefore

$$\cosh u - 1 = \frac{u^2}{2}\left(1 + \frac{u^2}{12} + \frac{u^4}{360} + \ldots\ldots\ldots\right)$$

$$\sinh u = u\left(1 + \frac{u^2}{6} + \frac{u^4}{120} + \ldots\ldots\ldots\right)$$

Hence, for $u < 0.48$, we may write, with an error of less than 2 per cent,

$$\cosh u - 1 = \frac{u^2}{2}$$

Also, with an error of less than 0.05 per cent,

$$\sinh u = u\left(1 + \frac{u^2}{6}\right)$$

Hence for

$$K\,m\,x < 480\,T \qquad (12)$$

equation (10) may be written, with an error of less than 2 per cent,

$$y = \frac{K\,m\,x^2}{2000\,T} \qquad (13)$$

which is the equation of a parabola. In all practical cases, except for extreme conditions of loading or for very long spans, the condition (12) is satisfied, and therefore for practical work the equation of a span of wire may be represented with sufficient accuracy by the parabola (13).

Under the same conditions, equation (11) for the length of wire between O and P may be written, with an error of less than 0.05 per cent,

$$\lambda = x\left[1 + \tfrac{1}{6}\left(\frac{K\,m\,x}{1000\,T}\right)^2\right]$$

But from (**13**)

$$\frac{K\,m\,x}{1000\,T}=\frac{2\,y}{x}$$

Therefore, to the same degree of accuracy,

$$\lambda=x+\frac{2}{3}\,\frac{y^2}{x} \tag{14}$$

DEFLECTION AND SAG.

From the above equations the deflection and vertical sag of the wire for a given tension T and loading factor K may be deduced readily. In addition to the above symbols, let (see Figs. 1 and 2)

l = length of span in feet; *i.e.*, the horizontal distance between the points of support.

l_1 = distance $A\,G$ in feet.

l_0 = distance $E\,O$ in feet.

D = deflection in feet of the point O from the horizontal line through the two points of support when *points of support are on the same level.*

$p=\frac{100\,D}{l}$ = percentage deflection when *points of support are on the same level.*

K' = ratio of vertical component of external force to weight of wire (K' is the value of K when there is no wind).

$S=\frac{K'\,D}{K}$ = vertical sag corresponding to the deflection D.

h = difference in elevation in feet between the two points of support.

h_1 = the distance $B\,G$.

D_1 = deflection in feet of the point O from the line $A\,G$ (Figs. 1 and 2.)

S_1 = vertical sag in feet corresponding to the deflection D_1.

From equation (**13**) and Fig. 2,

$$y_a=D_1=\frac{K\,m\,l_0^2}{2000\,T} \tag{15}$$

and

$$y_b=D_1+h_1=\frac{K\,m\,(l_1-l_0)^2}{2000\,T} \tag{15a}$$

Subtracting we get

$$h_1 = \frac{K\,m}{2000\,T}\,(l_1^2 - 2\,l_1\,l_0)$$

or

$$l_0 = \frac{l_1}{2}\left[1 - \frac{2000\,T\,h_1}{K\,m\,l_1^2}\right] \tag{16}$$

The relations between l_1 and l and between h_1 and h may be deduced from Fig. 1. The plane $B\,C\,G$ is perpendicular to $A\,C$ and therefore perpendicular to the plane $A\,C\,G$. Also the angle $B\,G\,A$ is a right angle. Hence the angle $B\,G\,C$ is a right angle. Therefore

$$h_1 = h \cos\beta = \frac{K_1\,h}{K}$$

Also, $A\,C$ is perpendicular to $G\,C$, since $A\,C$ is perpendicular to the plane $B\,G\,U$; therefore

$$l_1 = \sqrt{l^2 + h^2 \sin^2\beta} = l\,\sqrt{1 + \left(\frac{h \sin\beta}{l}\right)^2}$$

The ratio of h to l in any practical case will not exceed 0.25 and the angle β will not exceed 45 deg. Hence l_1 will differ from l by less than 2 per cent. As a first approximation then we may write equation **(16)**

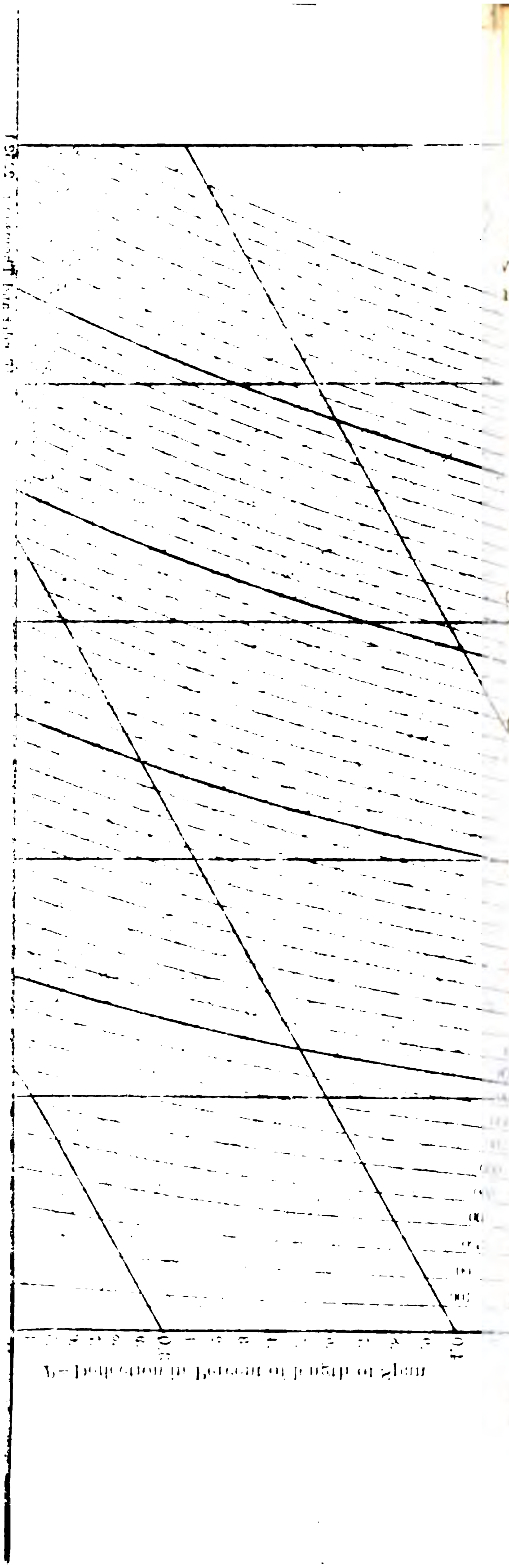
$$l_0 = \frac{l}{2}\left[1 - \frac{2000\,T\,K'\,h}{K^2\,m\,l^2}\right] \tag{16a}$$

For $h = 0$, that is, for the points of support on the same level, $l_0 = \frac{l}{2}$; the point O is then at the center of the span. Hence, for *points of support at the same level*,

$$D = \frac{K\,m\,l^2}{8000\,T} \tag{17}$$

The percentage deflection is then

$$p = \frac{K\,m\,l}{80\,T} \tag{18}$$



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TENSION AND DEFLECTION CHART—NO. 2

Chart No. 2 gives the value of this percentage deflection for both copper and aluminum for the various values of ($K\ l$) and T which are likely to arise in practise. The vertical sag corresponding to D is

$$S=\frac{K'\,D}{K}$$

For a *difference of elevation h* we then have, from (**16***a*) and (**17**),

$$l_0=\frac{l}{2}\left[1-\frac{K'\,h}{4\,K\,D}\right] \qquad \textbf{(19)}$$

D is the deflection corresponding to the same length of span, same tension, and same loading ***but the two points of support at the same elevation***. Substituting (**19**) in (**15**) gives

$$D_1=D\left(1-\frac{K'\,h}{4\,K\,D}\right)^2 \qquad \textbf{(20)}$$

and the vertical sag corresponding to D_1 will not exceed

$$S_1=\frac{K'\,D_1}{K} \qquad \textbf{(21)}$$

(An inspection of Fig. 1 will show that the vertical sag may be slightly less than this; formula (**21**), however, is on the safe side.)

From equation (**19**) it is seen that the distance of the point O to the lower point of support becomes negative when

$$h<\frac{4\,K\,D}{K'}$$

This means that when $h<\frac{4\,K\,D}{K'}$ the point O lies outside of the span; *i.e.*, the wire has a continuous upward slope from the lower to the higher support. This condition is illustrated in Fig. 3. Since the vertical force downward on the insulator due to the wire between A and B is equal to $K'\,w\,l_0$, if l_0 is negative there will be an upward pull at A.

If this force is greater than the downward pull due to the span on the other side of this support, the resultant force on the insulator will be upward. In laying out a line this condition should of course be avoided by relocating the poles.

When the point *O* coincides with the lower point of support, $l_0 = 0$, and the horizontal distance between *O* and the higher point of support is equal to the length of the span *l*. Hence the greatest value of $\frac{K\,m\,x}{1000\,T}$ (see equations **10** and **12**) for any point in the span is equal to $\frac{K\,m\,l}{1000\,T}$ and this in turn is equal to $\frac{8\,p}{100}$ where p is the per cent deflection which would be produced by the same tension and loading were the points of support on the same level (see equation **18**). Hence the condition that the approximate equations (**13**) and (**14**) may be applied is that

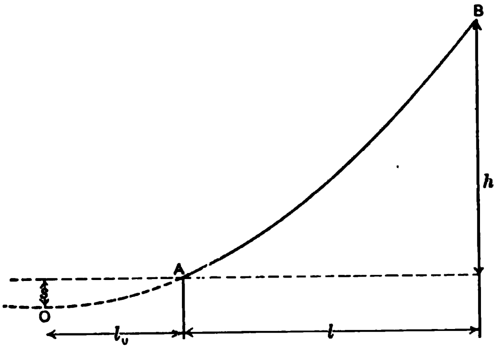

Fig. 3

$$p < 6 \text{ per cent}$$

Hence the maximum error which can arise from the use of Chart No. 2 for the determination of deflection and sag is less than 2 per cent.

EFFECT OF VARIATIONS IN TEMPERATURE AND MECHANICAL LOADING

We wish next to determine how the tension and deflection of a span of wire varies when the temperature of the wire changes and when the mechanical load on the wire changes (due to the formation or melting of the sleet and changes in wind pressure.) We shall assume that the points of support remain rigidly fixed.

Let

t = temperature in deg. fahr. under any given set of conditions.

T = tension at O in thousands of pounds per square inch under these same conditions.

K = ratio of resultant force on wire to weight of wire under these same conditions.

K' = ratio of vertical component of resultant force to weight of wire under these same conditions.

λ_1 = length of the wire in feet under these same conditions.

t_0, T_0, K_0, K_0', λ_0 designate the same quantities for any other set of conditions.

a = coefficient of linear expansion of the wire per deg. fahr.

M = modulus of elasticity of the wire in (pounds per square inch) units.

Other symbols as above.

From equations (**14**) and (**15**) and Figs. 1 and 2 we have that the length of the wire in the span under any conditions of temperature and loading is

$$\lambda_1 = l_0 + \frac{2}{3}\,\frac{D_1^2}{l_0} + l - l_0 + \frac{2}{3}\,\frac{(D_1+h_1)^2}{l-l_0}$$

$$= l + \frac{2}{3}\left(\frac{K\,m}{2000\,T}\right)^2 [l_0^3 + (l-l_0)^3]$$

$$= l + \frac{2}{3}\left(\frac{K\,m}{2000\,T}\right)^2 [l^3 - 3\,l\,l_0\,(l-l_0)]$$

From equation (**16***a*)

$$l_0 = \frac{l}{2}\left[1 - \frac{2000\,T\,h\,K'}{K^2\,m\,l^2}\right]$$

$$l - l_0 = \frac{l}{2}\left[1 + \frac{2000\,T\,h\,K'}{K^2\,m\,l^2}\right]$$

whence

$$\lambda_1 = l + \frac{2}{3}\left(\frac{K\,m}{2000\,T}\right)^2 l^3 \left[\frac{1}{4} + \frac{3}{4}\left(\frac{2000\,T\,h\,K'}{K^2\,m\,l^2}\right)^2\right]$$

or

$$\lambda_1 = l\left[1+\frac{1}{6}\left(\frac{K\,m\,l}{2000\,T}\right)^2+\frac{1}{2}\left(\frac{K'\,h}{K\,l}\right)^2\right]$$

From (**18**)

$$\frac{K\,m\,l}{2000\,T}=\frac{p}{25}$$

where p is the per cent deflection for the same condition of temperature, loading and tension, but for points of support on the same level. Hence the expression for λ_1 may be written

$$\lambda_1 = l\left[1+\frac{p^2}{3750}+\frac{1}{2}\left(\frac{K'\,h}{K\,l}\right)^2\right] \qquad (\mathbf{22})$$

Similarly, for any other conditions of temperature and loading the length of the wire is

$$\lambda_0 = l\left[1+\frac{p_0^2}{3750}+\frac{1}{2}\left(\frac{K_0'\,h}{K_0\,l}\right)^2\right]$$

Hence the change of length due to the change in temperature and loading is

$$\lambda_1-\lambda_0 = l\left[\frac{p^2-p_0^2}{3750}+\frac{1}{2}\left(\frac{K'\,h}{K\,l}\right)^2-\frac{1}{2}\left(\frac{K_0'\,h}{K_0\,l}\right)^2\right] \qquad (\mathbf{23})$$

If the points of support remain fixed this change of length can result only from a stretching (or contraction) of the wire due to change in tension and to an expansion (or contraction) of the wire due to change in temperature. This change in length may, therefore, also be expressed in terms of the temperature coefficient a, the difference of temperature $t-t_0$, the modulus of elasticity M, and the change in the resultant tension in the wire.

The resultant tension in the wire, at any point a distance λ measured along the wire from the point O, is equal to

$$T' = \sqrt{f_0^2+(K\,w\,\lambda)^2} = T\sqrt{1+\left(\frac{K\,m\,\lambda}{1000\,T}\right)^2}$$

See equations (**4**) and (**5**).

The maximum value of T' for a given value of T will occur at the higher point of support, *i.e.*, for

$$\lambda = l - l_0 = \frac{l}{2}\left[1 + \frac{K'h}{4KD}\right]$$

See Fig. 1 and equation (**19**). Also, from equation (**18**)

$$\frac{Kml}{2000\,T} = \frac{p}{25}$$

whence the maximum value of T' is

$$T' = T\sqrt{1 + \frac{p^2}{625}\left(1 + \frac{K'h}{4KD}\right)^2} \qquad \textbf{(24)}$$

The maximum value of p likely to arise in practise is 6 per cent, and the maximum value of h is $\frac{4KD}{K}$ (lowest point of the wire coinciding with lower point of support), and hence the maximum value of T' likely to arise in practise is

$$T' = 1.11\,T$$

The limiting case of a 6 per cent deflection rarely occurs, and even in this case the resultant tension is equal to the higher value, 1.11 T only at the higher point of support; for the greater part of the length of the wire the resultant tension is sensibly equal to the tension T at the point O. Hence in calculating the stretching of the wire by the resultant tension we may take, as a close approximation, the resultant tension at every point equal to the tension at O.

The change in length due to a change of temperature from t_0 to t is

$$a\,(t - t_0)\,\lambda_1$$

and the change in length due to a change in tension from T_0 to T is

$$\frac{1000\,(T - T_0)\,\lambda_1\,[1 + a\,(t - t_0)]}{M}$$

But since the coefficient of expansion a is a very small quantity (of the order of 10^{-5}) the term $a\,(t - t_0)$ may be neglected in comparison with unity for any value of $t-t_0$ likely to arise in practise ($t-t_0$ will never be greater than 150 deg. fahr.). Hence the total change in length ($\lambda_1-\lambda_0$) may be written

$$\lambda_1-\lambda_0=\left[a\ (t-t_0)+\frac{1000\ (T-T_0)}{M}\right]\lambda_1$$

Again, for $p<6$ per cent, λ_1 will differ from l by less than one per cent (see equation **23**); hence, with an error of less than one per cent,

$$\lambda_1-\lambda_0=\left[a\ (t-t_0)+\frac{1000\ (T-T_0)}{M}\right]l \qquad \textbf{(25)}$$

Equating (**23**) and (**25**) we have

$$\left(p^2-\frac{3.75\times10^6\,T}{M}\right)-\left(p_0^2-\frac{3.75\times10^6\,T_0}{M}\right)=$$

$$3750\,a\ \left\{(t-t_0)-\frac{h^2}{2\,a\,l^2}\left[\left(\frac{K'}{K}\right)^2-\left(\frac{K'_0}{K_0}\right)^2\right]\right\} \qquad \textbf{(26)}$$

For the points of support on the same level ($h=0$) or for no change in the position of the plane of the span ($\frac{K_0'}{K_0}=\frac{K'}{K}$; a particular instance of which is no wind pressure, in which case the plane of the span remains vertical), this equation reduces to

$$\left(p^2-\frac{3.75\times10^6\,T}{M}\right)-\left(p_0^2-\frac{3.75\times10^6\,T_0}{M}\right)=3750\,a\ (t-t_0) \quad \textbf{(27)}$$

Hence, when the tension T_0 at temperature t_0 and the loading factor K_0 are known, the tension T at any other temperature t and loading factor K may be determined by solving this equation for T, after substituting for p and p_0 their values

$$p=\frac{K\,m\,l}{80\,T}\ \text{ and }\ p_0=\frac{K_0\,m\,l}{80\,T_0}$$

This equation, however, is a cubic in T, and its solution by algebraic methods is extremely difficult. By making use of Chart

No. 2, however, its solution may be obtained without any arithmetical computations whatever other than a simple multiplication.*

The curved lines on this chart give p^2 as ordinates (actual vertical distances equal p^2; the scale on the left of the chart gives p, which is the square root of the vertical distances) against T as abscissas. The numbers on the curves give the corresponding value of $(K\,l)$. Two scales for the tension T are given, one for copper and one for aluminum, this arrangement making possible the use of the same set of curves for both materials. The three heavy straight lines at the base of the chart are drawn so that the ordinate (actual distance) of any point on any one of these lines is equal to

$$\frac{3.75 \times 10^6\, T}{M}$$

where T is the abscissa of this point on the proper tension scale and M is the modulus of elasticity of the wire.†

The small vertical scales in the lower left hand corner of the chart marked " Temperature Correction " are laid off so that the vertical distance between any two temperature marks t and t_0 is equal to

$$3750\, a\, (t - t_0)$$

SUMMARY

The method of solving equation (**27**) by means of Chart No. 2 is given below. First, however, a summary of the working symbols will be convenient:

l = length of span in feet.
h = difference in elevation of points of support in feet.

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*The physical constants for copper-clad steel wire whose conductivity is 40 per cent of that for the same size of copper wire are: $a = 6.7 \times 10^{-6}$ per deg. fahr.; $M = 22 \times 10^6$ lb. per square in.; $m = 3.58$ lb. per ft. The additions to Chart No. 2 for operating with this wire are as follows:

1. Tension scale equal to 0.93 of copper tension scale per in.
2. Modulus guide line through the origin and the point $p = 2.8$, $T_{\text{copper}} = 50$.
3. Temperature correction scale equal to 1.43 of that per in. for copper.

†A serious objection to another graphical solution for calculations of this sort which was published by one of the authors in the *Electrical World*, Sept. 28, 1907, was the necessity of a separate guide line for every length of span. This difficulty has been obviated in the above method. The angular location of the tension axis in the present form of chart was suggested by Mr. R. S. Brown of Telluride, Colo.

T_0 = the tension at the point O (Figs. 1 and 2) in thousands of pounds per square inch under any given condition of temperature and loading.

t_0 = temperature in deg. fahr. when the tension is T_0.

K_0 = ratio of resultant force due to weight of wire, weight of ice and wind pressure, to weight of wire (Chart No. 1) when the tension is T_0.

K_0' = ratio of the vertical component of this force to weight of wire when the tension is T (K'_0 = value of K from Chart No. 1 corresponding to the given size of wire and the given thickness of ice, but no wind.)

T = tension in thousands of pounds per square inch for any other condition of temperature and loading.

t = temperature in deg. fahr. when the tension is T.

K = ratio of resultant force due to weight of wire, weight of ice and wind pressure, to weight of wire when the tension is T. (For no ice and no wind $K=1$.)

K' = ratio of the vertical component of this force to weight of wire when tension is T. (For no ice and no wind $K'=1$).

D = deflection in feet of the lowest point of the wire from the line through the points of support when the tension is T and the points of support are on the same level. D is measured in the plane of the curve in which the wire hangs.

$p=\frac{100\,D}{l}$ = per cent deflection corresponding to D.

S = vertical sag in feet of the lowest point of wire below the points of support when the tension is T and the points of support are on the same level.

D_1 = deflection in feet of the point O from the line $A\,G$ (see Figs. 1 and 2) when the tension is T and the points of support are not on the same level. (For $h=\frac{4\,K\,D}{K'}$ the lower point of support is the lowest point of the wire.)

S_1 = vertical sag in feet of the lowest point of the wire below the lower point of support.

a = temperature coefficient of wire when temperature is expressed in deg. fahr. (for copper $a=9.6\times10^{-6}$, for aluminum $a=12.8\times10^{-6}$).

To find the tension T and the corresponding deflection and sag when l, h, T_0, t_0, K_0, K_0', t, K and K' are given, Chart No. 2 is used as follows, when $\frac{K'}{K} = \frac{K_0'}{K_0}$:

1. On a straight strip of paper mark a reference point A and vertically above this point mark a second point B such that the distance $A\,B$ is equal to $t-t_0$ measured on the temperature scale.

2. Next place the strip of paper with its edge along the ordinate passing through the tension T_0 and make the point A coincide with the intersection of the edge of the strip and the curve marked with the number equal to $(K_0\,l)$. Mark on the strip the point C where the heavy straight line marked with the proper modulus M intersects the edge of the strip.

3. Keep the strip of paper vertical and slide the point C along the modulus line until the edge of the strip at B intersects the curve marked with the number equal to $(K\,l)$. The abscissa of the point B is then equal to the tension T corresponding to the temperature t and loading factor K.

The ordinate of the point B (read on the vertical scale on the left) is the per cent deflection p corresponding to this tension T and loading factor K when the points of support are on the same level. Then the corresponding deflection in feet is

$$D = \frac{p\,l}{100} \tag{28}$$

and the vertical sag is

$$S = \frac{K'\,D}{K} \tag{29}$$

If the difference in elevation of the two points of support is h, the deflection of the lowest point of the wire is

$$D_1 = D\left(1 - \frac{h\,K'}{4\,K\,D}\right)^2 \tag{30}$$

and the vertical deflection is

$$S_1 = S\left(1 - \frac{h\,K'}{4\,K\,D}\right)^2 \tag{31}$$

The deflection and sag corresponding to T_0 are calculated in exactly the same manner from the per cent deflection p_0 corresponding to the tension T_0 and the loading factor K_0; whether the sag corresponding to T will be greater than that corresponding to T_0 will depend upon the temperatures and loading factors.

In case the points of support are not on the same level and there is a change in the direction of the plane of the wire, (*i.e.*, $\frac{K'}{K} \neq \frac{K_0'}{K_0}$), the new tension T must be found by solving equation (**26**). This can also be done by means of Chart No. **2**, for the term

$$- \frac{h}{2\,a\,l^2} \left[\left(\frac{K'}{K} \right)^2 - \left(\frac{K_0'}{K_0} \right)^2 \right]$$

is equivalent to a decrease of temperature numerically equal to this expression. Hence the only change in the procedure described above is to consider the temperature rise not as $t - t_0$, but as

$$(t - t_0) - \frac{h^2}{2\,a\,l^2} \left[\left(\frac{K'}{K} \right)^2 - \left(\frac{K_0'}{K_0} \right)^2 \right]$$

The rest of the procedure is exactly the same as before. This correction due to the difference between $\frac{K_0'}{K_0}$ and $\frac{K'}{K}$ may be important, particularly in passing from the condition of no ice and no wind to the condition of ice and wind, for in the latter case $\left(\frac{K_0'}{K_0} \right)^2$ may be as great as ½ (45 deg. deflection of the plane of the wire from the vertical) whereas for the former $\frac{K'}{K} = 1$. Similarly, in the reverse problem of passing from the condition of ice and wind to the condition of no ice and no wind there will be a like correction, but the formula without this correction factor will give results on the safe side.

It should also be noted that the maximum resultant tension in the span under any condition of temperature and loading is not equal to the tension T at the point O but in general is equal to

$$T' = T \sqrt{1 + \frac{p^2}{625} \left(1 + \frac{h\,K'}{4\,K\,D} \right)^2} \tag{33}$$

(see equation **24**). Similarly, the maximum resultant tension under the "zero" conditions is

$$T_0' = T_0\sqrt{1 + \frac{p_0^2}{625}\left(1 + \frac{h\,K'_0}{4\,K_0\,D_0}\right)^2} \qquad \textbf{(33}a\textbf{)}$$

EXAMPLE

Given a No. 0 B & S aluminum wire; $l = 200$ ft.; $T_0 = 13$; $(t - t_0) = 70$ deg. fahr. for ½ in. of ice and 8 lb. per sq. ft. of wind $K_0 = 11.6$; $K_0' = 6.7$; for no ice nor wind $K = K' = 1$.

Then	$h = 0.0$ ft.	$h = 10.0$ ft.
$(t-t_0) - \frac{h^2}{2\,a\,l^2}\left[\left(\frac{K'}{K}\right)^2 - \left(\frac{K'_0}{K_0}\right)^2\right]$	70	4.9
K_0l	2320	2320
Kl	200	200
P_0	2.58 %	2.58 %
D_0	5.16 ft.	2.68 ft.
S_0	2.98 ft.	1.55 ft.
T	1.28 thousand lb.	1.92 thousand lb.
p	2.27 %	0.49 %
D	4.54 ft.	0.9 ft.
S	4.54 ft.	0.9 ft.

B. ELECTRIC CHARACTERISTICS

When a given amount of electric power is transmitted over a transmission line to a substation or other receiver, a certain amount of power is lost in the line. There is in general a difference in the voltage between wires at the two ends of the line, and the power factor at the sending and receiving ends will be different. The power loss, voltage loss, and change in power factor in general depend upon the following factors:

1. The amount of power delivered.
2. The voltage at which the power is delivered.
3. The power factor at the receiving end.
4. The frequency of the system.
5. The kind of line—three-phase or single-phase.
6. The length of the line.
7. The size of the wires.
8. The material of the wires.

9. The temperature of the wires.
10. The arrangement of the wires on the poles; particularly their distance apart.

The interrelation of these various factors will be considered in detail.

RESISTANCE, REACTANCE AND CAPACITY, SUSCEPTANCE AND LEAKAGE

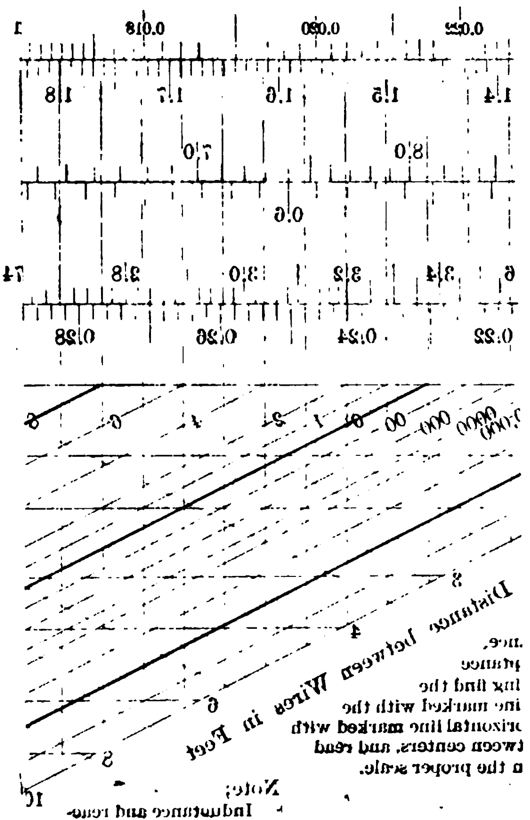
For wires of the size ordinarily used in practise and for any frequency up to 60 cycles, the resistance of a solid wire per unit length depends solely upon the material of the wire and its cross section. A stranded cable has a slightly greater resistance than a solid wire, due to the spiralling of the individual wires which made up the cable. In the table on Chart No. 3 are given the resistance and weight per mile of both copper and aluminum for the various sizes between a No. 16 and 1,000,000 circular mils. This table is calculated for a temperature of 20 deg. cent. (=68 deg. fahr.), assuming a temperature coefficient of 0.42 per cent per degree centigrade; the conductivity of copper is taken as 98 per cent and the conductivity of aluminum as 62 per cent of Matthiessen's standard; and the resistance and the weight of the stranded wires are taken one per cent greater than for solid wire of the same cross section.

The reactance of a wire in ohms is equal to $2\pi f L \times 10^{-3}$, where f is the frequency and L the self-induction in millihenrys. The self-induction of a round solid wire of radius a inches when the return wire is parallel to it and at a distance D inches from it (center to center) is

$$L = 0.7411 \log_{10} \frac{D}{a} + 0.0805 \text{ millihenrys per mile.}$$

This formula applies also when the return circuit consists of two wires each of which is at a distance D from the wire in question; that is, the above formula gives the inductance per mile of *each* wire whether the system is single-phase or three-phase, provided in the latter case the wires are arranged symmetrically (*i.e.*, form the three edges of an equilateral prism). The inductance of a stranded wire is practically equal to that of a solid wire of the same cross section of conducting material; for a given number on the B. & S. gage or for a given area in circular mils the inductance is therefore independent of whether the wire is solid or stranded.

LINE CONSTANTS PER MI



...nce.
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...ing find the
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Note:
Inductance and reactance for solid and stranded wires practically identical.
Capacity and susceptance for stranded wires for solid a distance ¼ in. to the right of for the corresponding strand

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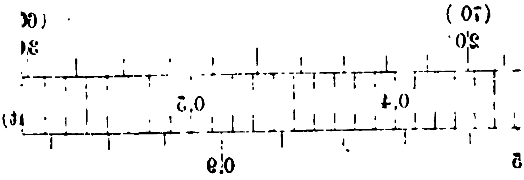
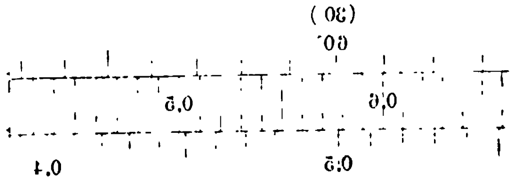
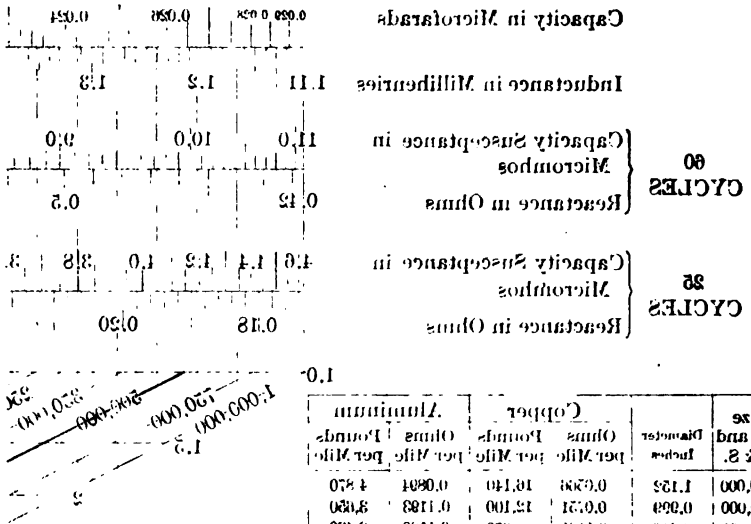




For bracketed functions read bracketed angles. Exa...



Size C.M. and B. & S.	Diameter Inches	Copper Ohms per Mile	Copper Pounds per Mile	Aluminum Ohms per Mile	Aluminum Pounds per Mile
1,000,000	1.152	0.0566	16,140	0.0894	4,870
750,000	0.999	0.0751	12,100	0.1193	3,650
500,000	0.819	0.1118	8,070	0.1788	2,430
350,000	0.679	0.1616	5,650	0.255	1,703
250,000	0.580	0.226	4,040	0.358	1,217
0000	0.530	0.267	3,420	0.423	1,050
000	0.470	0.337	2,710	0.533	817
00	0.420	0.425	2,150	0.672	648
0	0.375	0.536	1,707	0.848	513
1	0.330	0.676	1,351	1.069	407
2	0.291	0.852	1,071	1.347	323
4	0.231	1.355	674	2.14	203
6	0.162	2.13	419.5	3.37	127.9
8	0.1285	3.38	263.8	5.35	79.2
10	0.1019	5.38	166.0	8.51	49.8
12	0.0818	8.55	104.4	13.53	31.3
14	0.0641	13.58	65.6	21.5	19.7
16	0.0508	21.6	41.3	34.2	12.4

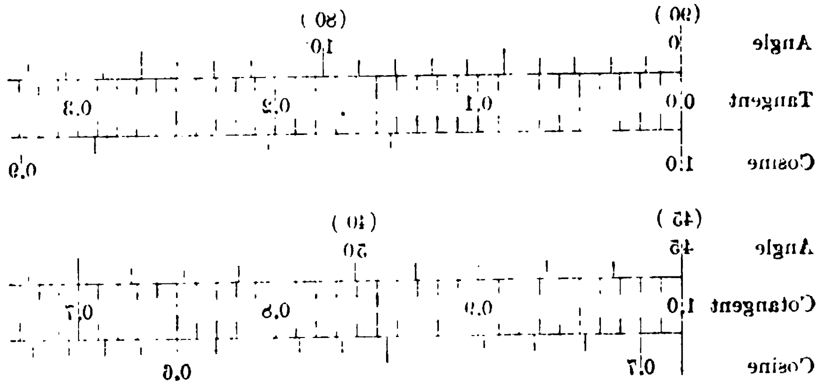
Solid ↕

DIRECTIONS

To determine th inducte
capacity, reactance, or susc
for any size wire on any sp
intersection of the diagonal l
given size of wire with the li
the given number of feet be
vertically above this point o

EXAMPLE

For a Number No. 0000 wi
6 feet from its return wire t
1.830 Millihenries per mile. th
microfarads per mile; the re
0.757 ohms per mile; the susc
6.03 micromhos per mile



The value of the inductance per mile of wire for various sizes of wire and for various spacings is given on Chart No. 3. This particular form of chart arises from the fact that the inductance depends only upon the *ratio* $\frac{D}{a}$; consequently, all combinations of sizes and spacings for which this ratio is constant give the same value of the inductance. The reactance for both 25 and 60 cycles is also given on Chart No. 3.

The capacity susceptance of a wire in ohms is equal to $2\pi f C \times 10^{-6}$, where f is the frequency and C the capacity in microfarads of the wire to neutral; the susceptance of a wire is the ratio of the charging current to the volts to neutral. The capacity to neutral of a round wire of radius a inches when the return wire is parallel to it and at a distance D inches from it (center to center) is

$$C = \frac{0.03883}{\log_{10}\frac{D}{a}} \text{ microfarads per mile,}$$

provided $\frac{D}{a}$ is greater than 12* and the distance from the wire to all other conductors is large compared with D. This formula applies also when the return conductor consists of two wires each of which is at a distance D from the wire in question. That is, the above formula gives the capacity per mile of *each* wire whether the system is single-phase or three-phase, provided the wires are at a great distance, compared with D, from all other conductors (*i.e.*, overhead lines) and provided that in the case of a three-phase system the wires are arranged symmetrically (*i.e.*, from the three edges of an equilateral prism.) The capacity of a stranded wire is greater than that of a solid wire, since its radius is greater. The above formula is not strictly applicable to a stranded wire, since the latter is not round, but as a first approximation it may be used.

In Chart No. 3 is given the capacity of the various sizes of

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*When $\frac{D}{a} < 12$, the exact formula for the capacity, when the two wires are at a great distance from all other conductors, is

$$C = \frac{0.08941}{\cosh^{-1}\frac{D}{2a}}$$

wire on various spacings. In calculating the capacity scale the diameter of the wire in each case was taken 15 per cent greater than the diameter of a solid wire of the same cross section; the ratio of the diameter of a stranded wire to a solid wire is approximately 1.15. The proper correction to obtain the capacity of a solid wire is indicated on the chart. The chart also gives the capacity susceptance at 25 and 60 cycles.

The leakage current between wires under ordinary working conditions is negligible. The leakage need be taken into account only in the case the voltage is sufficiently high to produce the so-called "corona effect". This effect appears only when extra high voltages are used and the wires are comparatively small.

THE EQUATIONS OF A TRANSMISSION LINE.

Fig. 4 is a diagrammatic representation of a section of a transmission line. Each wire of the line, whether single- or three-phase, may be considered separately, and the return for each wire may be represented by a fictitious neutral of zero resistance and zero reactance. Let

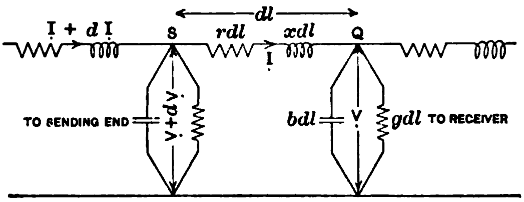

FIG. 4

P = average watts transmitted past the point Q by all of the wires.

E = effective volts between wires at any point Q.

V = effective volts between any one of the line wires and neutral at the point Q. (For a single-phase line $V=\frac{E}{2}$; for a three-phase line $V=\frac{E}{\sqrt{3}}$)

φ = power factor angle at Q (φ taken positive for lagging current).

I = effective amperes at the point Q. (For a single-phase line $I=\frac{P}{E\cos\varphi}$; for a three-phase line $I=\frac{P}{\sqrt{3}\,E\cos\varphi}$)

The same symbols with the subscript " 1 " refer to the receiver.
The same symbols with the subscript " 0 " refer to the sending end.

l = distance in miles of this point Q from receiver.
$d\,l$ = elementary length of line between Q and S.
l_0 = length of line in miles.
r = resistance of wire in ohms per mile.
x = reactance of wire in ohms per mile.
g = leakage conductance, one wire to neutral in mhos per mile.
b = capacity susceptance, one wire to neutral, in mhos per mile.

$$z = \sqrt{r^2+x^2}$$

$$y = \sqrt{g^2+b^2}$$

$$\epsilon = \tan^{-\prime}\frac{x}{r}$$

$$\eta = \tan^{-\prime}\frac{b}{g}$$

For the voltage and current sine functions of the time we may use the symbolic method. Let $\underset{\cdot}{V}$ and $\underset{\cdot}{I}$ be the symbolic expressions for the voltage and current respectively. The symbolic expression for the impedance per unit length of line is

$$\underset{\cdot}{z} = z\,(\cos\epsilon + j\sin\epsilon) = z\,e^{j\epsilon} \qquad (1)$$

and the symbolic expression for the admittance of the leakage circuit per mile of line is

$$\underset{\cdot}{y} = y\,(\cos\eta + j\sin\eta) = y\,e^{j\eta} \qquad (2)$$

Applying Kirchoff's laws to the element $d\,l$ of the line, we get

$$\frac{d\,\underset{\cdot}{V}}{d\,l} = \underset{\cdot}{z}\,\underset{\cdot}{I} \qquad (3)$$

$$\frac{d\,\underset{\cdot}{I}}{d\,l} = \underset{\cdot}{y}\,\underset{\cdot}{V} \qquad (4)$$

Differentiating (3) with respect to l and substituting the value of $\frac{d\,\underset{\cdot}{I}}{d\,l}$ from (4) gives

$$\frac{d^2\,\underset{\cdot}{V}}{d\,l^2} = \underset{\cdot}{y}\,\underset{\cdot}{z}\,\underset{\cdot}{V} \qquad (5)$$

The integral of this equation is

$$\underset{\cdot}{V}=\underset{\cdot}{A}\sinh\left(\sqrt{\underset{\cdot}{y}\,\underset{\cdot}{z}}\,l+\underset{\cdot}{\delta}\right) \tag{6}$$

where $\underset{\cdot}{A}$ and $\underset{\cdot}{\delta}$ are constants of integration.

Substituting (6) in (4), and integrating with respect to l, we get

$$I=\sqrt{\frac{y}{\underset{\cdot}{z}}}\,\underset{\cdot}{A}\cosh\left(\sqrt{\underset{\cdot}{y}\,\underset{\cdot}{z}}\,l+\underset{\cdot}{\delta}\right) \tag{7}$$

Both $\underset{\cdot}{A}$ and $\underset{\cdot}{\delta}$ may be complex; let their symbolic expressions be

$$\underset{\cdot}{A}=A\,(\cos\alpha+j\sin\alpha)=A\,e^{j\alpha} \tag{8}$$

$$\underset{\cdot}{\delta}=\beta+j\,\gamma \tag{9}$$

From (1) and (2)

$$\sqrt{\underset{\cdot}{y}\,\underset{\cdot}{z}}=\sqrt{y\,z}\,e^{j\left(\frac{\eta+\epsilon}{2}\right)}=\sqrt{y\,z}\left[\cos\left(\frac{\eta+\epsilon}{2}\right)+j\sin\left(\frac{\eta+\epsilon}{2}\right)\right] \tag{10}$$

$$\sqrt{\frac{y}{\underset{\cdot}{z}}}=\sqrt{\frac{y}{z}}\,e^{j\left(\frac{\eta-\epsilon}{2}\right)} \tag{11}$$

Put

$$\lambda=\frac{\eta-\epsilon}{2} \tag{12}$$

$$m=\sqrt{y\,z}\cos\left(\frac{\eta+\epsilon}{2}\right) \tag{13}$$

$$n=\sqrt{y\,z}\sin\left(\frac{\eta+\epsilon}{2}\right) \tag{14}$$

Then (6) and (7) become

$$\underset{\cdot}{\gamma}=\underset{\cdot}{A}\sinh\left[(\beta+m\,l)+i\,(\gamma+n\,l)\right] \tag{15}$$

$$\underset{\cdot}{I}=\sqrt{\frac{y}{\underset{\cdot}{z}}}\,\underset{\cdot}{A}\cosh\left[(\beta+m\,l)+i\,(\gamma+n\,l)\right] \tag{16}$$

But

$$\sinh\,[(\beta+m\,l)+j\,(\gamma+n\,l)]$$

$$=\sinh\,(\beta+m\,l)\cos\,(\gamma+n\,l)+j\cosh\,(\beta+m\,l)\sin\,(\gamma+m\,l)$$

$$=M\,e^{j\,\mu} \tag{17}$$

where

$$M=\sqrt{\sinh^2\,(\beta+m\,l)\cos^2\,(\gamma+n\,l)+\cosh^2\,(\beta+m\,l)\sin^2\,(\gamma+n\,l)}$$

and

$$\tan\mu=\frac{\tan\,(\gamma+n\,l)}{\tanh\,(\beta+m\,l)}$$

Making use of the relations that, for any argument u,

$$\sinh^2 u=\frac{\cosh 2\,u-1}{2}$$

$$\cosh^2 u=\frac{\cosh 2\,u+1}{2}$$

$$\sin^2 u=\frac{1-\cos 2\,u}{2}$$

$$\cos^2 u=\frac{1+\cos 2\,u}{2}$$

the above expression for M may be written

$$M=\frac{1}{\sqrt{2}}\sqrt{\cosh 2\,(\beta+m\,l)-\cos 2\,(\gamma+n\,l)} \tag{18}$$

Whence, substituting for $\dot{A}$ its value from (8) and the relation (17), the expression (15) for the voltage at any point along the line becomes

$$\dot{V}=A\,M\,e^{j(\alpha+\mu)} \tag{19}$$

In an exactly similar manner, we may write

$$\cosh\,[(\beta+m\,l)+j\,(\gamma+n\,l)]=N\,e^{j\,\nu}$$

$$N=\frac{1}{\sqrt{2}}\sqrt{\cosh 2\,(\beta+m\,l)+\cos 2\,(\gamma+n\,l)} \tag{20}$$

$$\tan \nu = \tan (\gamma + n\,l) \tanh (\beta + m\,l) \tag{21}$$

Whence the current at any point along the line is

$$I = \sqrt{\frac{y}{z}}\, A\, N\, e^{j(\alpha + \nu + \lambda)} \tag{22}$$

Hence the effective value of the voltage at any point is $A\,M$ or

$$V = A_0 \sqrt{\cosh 2\,(\beta + m\,l) - \cos 2\,(\gamma + n\,l)} \tag{23}$$

and the effective value of the current at this point is

$$I = \sqrt{\frac{y}{z}}\, A_0 \sqrt{\cosh 2\,(\beta + m\,l) + \cos 2\,(\gamma + n\,l)} \tag{24}$$

where A_0, which is a constant yet to be determined, is put for $\frac{A}{\sqrt{2}}$. The angle by which the current lags behind the voltage, or the power factor angle φ, is

$$\varphi = \alpha + \mu - (\alpha + \nu + \lambda)$$

$$= \mu - \nu - \lambda$$

A simple expression for the angle $(\mu - \nu)$ may be derived by making use of the trigonometric relation

$$\tan (\mu - \nu) = \frac{\tan \mu - \tan \nu}{1 + \tan \mu \tan \nu}$$

for, substituting for $\tan \mu$ and $\tan \nu$ their values from (**17**) and (**21**), we get

$$\tan (\mu - \nu) = \frac{\sin 2\,(\gamma + n\,l)}{\sinh 2\,(\beta + m\,l)}$$

Whence the power factor angle at any point along the line is

$$\varphi = \tan^{-1} \left[\frac{\sin 2\,(\gamma + n\,l)}{\sinh 2\,(\beta + m\,l)} \right] - \lambda \tag{25}$$

The power per phase at any point is then

$$P' = V\,I\cos\varphi$$

$$= \sqrt{\frac{y}{z}}\,A_0^2\cos\varphi\,\sqrt{\cosh^2 2\,(\beta+m\,l)-\cos^2 2\,(\gamma\;+nl)}$$

Making use of the relations

$$\cosh^2 u = 1+\sinh^2 u$$

$$\cos^2 u = 1-\sin^2 u$$

we may write this expression

$$P' = \sqrt{\frac{y}{z}}\,A_0^2\cos\varphi\,\sqrt{\sinh^2 2\,(\beta+m\,l)+\sin^2 2\,(\gamma+n\,l)}$$

From (**25**), making use of the trigonometric formula for the cosine of the difference of two angles,

$$\cos\varphi = \frac{\sinh 2\,(\beta+m\,l)\cos\lambda+\sin 2\,(\gamma+n\,l)\sin\lambda}{\sqrt{\sinh^2 2\,(\beta+m\,l)+\sin^2 2\,(\gamma+n\,l)}}$$

Whence

$$P' = \sqrt{\frac{y}{z}}\,A_0^2\,[\sinh 2\,(\beta+m\,l)\cos\lambda+\sin 2\,(\gamma+m\,l)\sin\lambda] \quad (\mathbf{26})$$

At the receiver $l=0$; and at the sending end $l=l_0$. Hence, from (**23**), we have that the ratio of the volts E_0 at the sending end to the volts E_1 at the receiver is

$$\frac{E_0}{E_1} = \frac{V_0}{V_1} = \sqrt{\frac{\cosh 2\,(\beta+m\,l_0)-\cos 2\,(\gamma+n\,l_0)}{\cosh 2\,\beta-\cos 2\,\gamma}} \quad (\mathbf{27})$$

From (**24**) the ratio of the current I_0 at the sending end to current I_1 at the receiver is

$$\frac{I_0}{I_1} = \sqrt{\frac{\cosh 2\,(\beta+m\,l_0)+\cos 2\,(\gamma+n\,l_0)}{\cosh 2\,\beta+\cos 2\,\gamma}} \quad (\mathbf{28})$$

From (**26**) the ratio of the power P_0 at the sending end to the power P_1 at the receiver is

$$\frac{P_0}{P_1} = \frac{P_0'}{P_1'} = \frac{\sinh 2\,(\beta+m\,l_0)\cos\lambda+\sin 2\,(\gamma+n\,l_0)\sin\lambda}{\sinh 2\,\beta\cos\lambda+\sin 2\,\gamma\sin\lambda} \quad (\mathbf{29})$$

We have now to determine the two constants β and γ. From (**25**), putting $l=0$, we have

$$\frac{\sin 2\gamma}{\sinh 2\beta}=\tan(\varphi_1+\lambda) \tag{30}$$

and from (**23**) and (**24**), putting $l=0$, and squaring, we have

$$V_1^2=A_0^2(\cosh 2\beta-\cos 2\gamma)$$

$$\frac{z}{y}I_1^2=A_0^2(\cosh 2\beta+\cos 2\gamma)\ .$$

Adding and subtracting these two expressions and taking the ratio of the results gives

$$\frac{\cosh 2\beta}{\cos 2\gamma}=\frac{z\,I_1^2+y\,V_1^2}{z\,I_1^2-y\,V_1^2}$$

Put

$\rho=\dfrac{z\,l_0\,I_1}{V_1}$ = ratio of total line impedance volts to volts delivered.

$\sigma=\dfrac{y\,l_0\,V_1}{I_1}$ = ratio of total line admittance current to current delivered.

Then

$$\frac{z\,I_1^2+y\,V_1^2}{z\,I_1^2-y\,V_1^2}=\frac{\rho+\sigma}{\rho-\sigma}$$

whence

$$\cosh 2\beta=\frac{\rho+\sigma}{\rho-\sigma}\cos 2\gamma \tag{31}$$

From (**30**)

$$\sinh 2\beta=\frac{\sin 2\gamma}{\tan(\varphi_1+\lambda)} \tag{31a}$$

Squaring and subtracting we get

$$1=\left(\frac{\rho+\sigma}{\rho-\sigma}\right)^2\cos^2 2\gamma-\frac{\sin^2 2\gamma}{\tan^2(\varphi_1+\lambda)}$$

q=Percent Voltage Rise

p=Percent Voltage Drop

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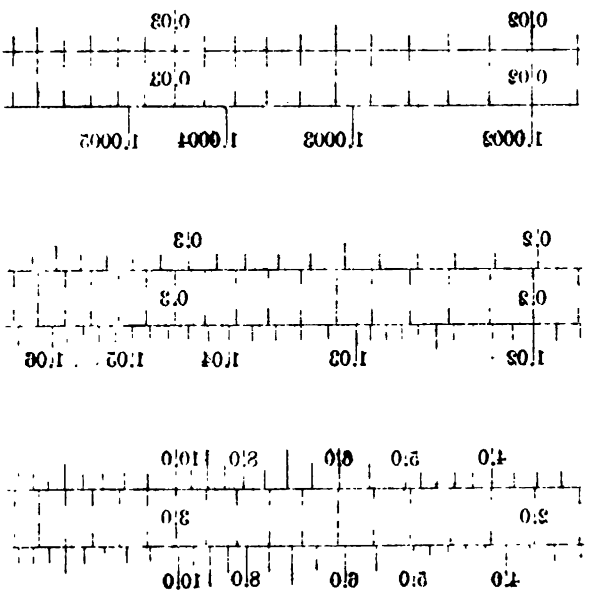
HYPERBOLIC FUNCTIONS



ment less than 0.01, sinh $u = u$ (error less

" " 0.1 , cosh $u = 1+\frac{u^2}{2}$ (" "

" greater " 6.0 , sinh u = cosh $u = \frac{e^u}{2}$ = log

cosh u + sinh u

cosh u − sinh u

Making use of the trigonometric relations

$$\cos^2 2\,\gamma = \frac{1}{1+\tan^2 2\,\gamma}$$

$$\sin^2 2\,\gamma = \frac{\tan^2 2\,\gamma}{1+\tan^2 2\,\gamma}$$

and solving for tan 2 γ, we get

$$\tan 2\,\gamma = \frac{2\,\sqrt{\rho\,\sigma}\,\sin\,(\varphi_1+\lambda)}{\rho-\sigma} \qquad \textbf{(32)}$$

Note that, from (**31**) and (**31***a*), sin 2 γ and cos 2 γ have respectively the same algebraic signs as the numerator and denominator of this fraction. This is important since the algebraic sign of the tangent alone does not fix the quadrant in which an angle lies. The value of 2 γ may be taken directly from the trigonometric scale on Chart No. 3. (If the second member of (**32**) is greater than unity, use the reciprocal which is equal to the cot 2 γ). From (**30**) we then have

$$\sinh 2\,\beta = \frac{\sin 2\,\gamma}{\tan\,(\varphi_1+\lambda)} \qquad \textbf{(33)}$$

from which 2 β may be obtained from Chart No. 5.

Approximate Equations for Ordinary Lines.

In case the length of the line does not exceed 100 miles when the frequency is 60 cycles or 200 miles when the frequency is 25 cycles the above equations may be greatly simplified by making use of an approximation which will not introduce an appreciable error in any case likely to arise in practise.

The simplification results from the following substitutions in equations (**27**) to (**29**)

$$\begin{aligned} \cosh 2\,m\,l &= 1+2\,m^2\,l^2 \\ \sinh 2\,m\,l &= 2\,m\,l \\ \cos 2\,n\,l &= 1-2\,n^2\,l^2 \\ \sin 2\,n\,l &= 2\,n\,l \end{aligned} \qquad \textbf{(34)}$$

where $n\,l$ is expressed in radians. These approximations are accurate to within 2 per cent for $2\,m\,l$ less than 0.35 and for $2\,n\,l$ less than 0.35 radians or 20 deg.

From equations (**31**), (**31***a*) and (**32**) we have

$$\sinh 2\,\beta = \frac{2\,\sqrt{\rho\,\sigma}\cos(\varphi_1+\lambda)\cos 2\,\gamma}{\rho-\sigma} \tag{35}$$

$$\cosh 2\,\beta = \frac{\rho+\sigma}{\rho-\sigma}\cos 2\,\gamma \tag{36}$$

$$\sin 2\,\gamma = \frac{2\,\sqrt{\rho\,\sigma}\sin(\varphi_1+\lambda)\cos 2\,\gamma}{\rho-\sigma} \tag{37}$$

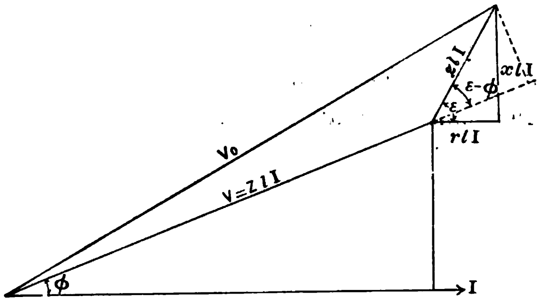


Fig. 5

Substituting equations (**34**) to (**37**) in (**27**) to (**29**) we get, for no leakage,

$$\frac{E_0}{E} = \sqrt{1+2\,\rho\cos(\epsilon-\varphi_1)+\rho^2-b\,x\,l_0^2} \tag{38}$$

$$\frac{I_0}{I} = \sqrt{1+2\,\sigma\cos(90-\varphi_1)+\sigma^2-b\,x\,l_0^2} \tag{39}$$

$$\frac{P_0}{P} = 1+\frac{\rho\cos\epsilon}{\cos\varphi} - b\,r\,l_0^2\tan\varphi_1 \tag{40}$$

It is interesting to note that these last three equations, when b is put equal to zero, are identical with the relations which follow immediately from the vector diagram, Fig. 5, of the line and load when the capacity of the line is neglected.

SUMMARY

In the working formulas given below, the symbols for voltage, current, etc., at the receiver will be written without the subscripts, and l will be used to designate the length of the line from receiver to sending end. The power will also be expressed in kilowatts. The leakage will be assumed zero.

P = kilowatts delivered to receiver.
E = volts between wires at receiver.
K = power factor of receiver.
$\varphi = \cos^{-1} K$ = power factor angle of receiver (see Chart No. 3.)
I = amperes taken by receiver. (For a three-phase line $I = \frac{P \times 1000}{\sqrt{3} E K}$; for a single-phase line $I = \frac{P \times 1000}{E K}$).

The same symbols with the subscript zero refer to the sending end.

l = distance of transmission in miles.
r = resistance of wire in ohms per mile (see Chart No. 3).
x = reactance of wire in ohms per mile (see Chart No. 3).
b = capacity susceptance, one wire to neutral, in mhos per mile. (See Chart No. 3).
$\epsilon = \tan^{-1} \frac{x}{r}$ = power factor angle of the line (see Chart No. 3).
Z = equivalent impedance of receiver terminal to neutral, per mile of line.
$= \frac{E}{\sqrt{3}\, l\, I}$ for a three-phase line;
$= \frac{E}{2\, l\, I}$ for a single-phase line.

Exact Method. Since the leakage is assumed zero, the admittance y is equal to b and the power factor angle of the leakage circuit is 90 deg. Hence to determine the voltage, current, etc., at the sending end proceed as follows:

First calculate the constants

$$\lambda = \frac{90 - \epsilon}{2} \qquad (41)$$

$$u = 2\, l \sqrt{\frac{b\, x}{\sin \epsilon}} \sin \lambda \qquad (42)$$

$$\theta = 114.6\, l \sqrt{\frac{b\,x}{\sin \epsilon}} \cos \lambda \tag{43}$$

$$d = \frac{100\, r}{Z \cos \epsilon} = \text{per cent ratio of impedance volts to volts delivered} \tag{44}$$

$$d' = 100\, b\, Z\, l^2 = \text{per cent ratio of charging current to current delivered.} \tag{45}$$

$$\theta_0 = \tan^{-1}\left[\frac{2\sqrt{d\,d'}\,\sin(\varphi+\lambda)}{d-d'}\right] \tag{46}$$

The algebraic signs of sin θ_0 and cos θ_0 are respectively the same as the numerator and denominator of this fraction; this fixes the quadrant in which θ_0 lies.

$$u_0 = \sinh^{-1}\left[\frac{\sin \theta_0}{\tan(\varphi+\lambda)}\right] \tag{47}$$

Then

$$\frac{E_0}{E} = \sqrt{\frac{\cosh(u_0+u) - \cos(\theta_0+\theta)}{\cosh u_0 - \cos \theta_0}} \tag{48}$$

$$\frac{I_0}{I} = \sqrt{\frac{\cosh(u_0+u) + \cos(\theta_0+\theta)}{\cosh u_0 + \cos \theta_0}} \tag{49}$$

$$\frac{P_0}{P} = \frac{\sinh(u_0+u)\cos\lambda + \sin(\theta_0+\theta)\sin\lambda}{\sinh u_0 \cos\lambda + \sin\theta_0 \sin\lambda} \tag{50}$$

The power factor angle φ at the sending end may also be determined directly from the formula

$$\varphi_0 = \tan^{-1}\left[\frac{\sin(\theta_0+\theta)}{\sinh(u_0+u)}\right] - \lambda \tag{51}$$

An excellent check on the results obtained is to compare the value of cos φ_0 determined from this formula with the value of the power factor calculated from the voltage E_0, current I_0, and power P_0 determined from the three preceding formulas. (For a three-phase line $\cos\varphi_0 = \frac{1000\,P_0}{\sqrt{3}\,E_0\,I_0}$; for a single-phase line $\cos\varphi_0 = \frac{1000\,P_0}{E_0\,I_0}$).

Approximate Method, Neglecting Line Capacity. In the case of a short line (*i.e.*, $10^4\,b\,x\,l^2$ negligible compared with d) the relations between the voltage, current and power at the two ends of the line are as follows. Let

$p = 100 \cdot \frac{E_0 - E}{E}$ = the per cent ratio of the difference in volts at the two ends to the volts delivered, *i.e.*, the per cent " volts drop ".

$q = 100\ \frac{P_0 - P}{P}$ = the per cent ratio of power lost in line to power delivered, *i.e.*, the per cent " power loss ".

Then

$$p = \sqrt{10^4 + 200\,d\cos(\epsilon - \varphi) + d^2} - 100 \qquad (52)$$

$$q = \frac{d\cos\epsilon}{K} = \frac{100\,r}{K\,Z} \qquad (53)$$

$$K_0 = \frac{100+q}{100+p} \cdot K \qquad (54)$$

Chart No. 4 gives a graphical solution of equation (52). The ordinate scale of this chart gives directly the value of the per cent volts drop when the per cent impedance drop d and the difference in the power factor angles of the line and load are known. The method of using the chart is described in detail on the chart itself. Each of the curves on the chart was drawn by plotting the values of

$$200\,d\cos(\epsilon - \varphi) + d^2$$

as actual vertical distance against d as abscissas; the numbers on the curves give the corresponding values of $(\epsilon - \varphi)$. The scale of ordinates marked on the left is a square root scale, the numbers giving the value of

$$\sqrt{10^4 + 200\,d\cos(\epsilon - \varphi) + d^2} - 100$$

Note that since $\cos(\epsilon-\varphi)=\cos(\varphi-\epsilon)$, the algebraic sign of the resultant angle $(\epsilon-\varphi)$ is immaterial. The curve marked with a given number, say 50, is to be used whether $\epsilon-\varphi=+50$ or $\epsilon-\varphi=-50$.

Approximate Method when Line Capacity is Taken into Account. For longer lines (*i.e.*, when $10^4\,b\,x\,l^2$ is not negligible in comparison with d), the following close approximations hold, provided the length of the line is not over 100 miles for 60 cycles or over 200 miles for 25 cycles. Let

p' = true value of the per cent volts drop when corrected for capacity effect.

q' = true value of the per cent power loss when corrected for capacity effect.

$p'_i=100\,\dfrac{I_0-I}{I}$ = the per cent ratio of the difference in the current at the two ends of the line to the current delivered; *i.e.*, the per cent "current loss" (p_i will in general be negative for a lagging current at the receiver).

Then

$$p'=\sqrt{10^4+200\,d\cos(\epsilon-\varphi)+d^2-10^4\,b\,x\,l^2}-100 \tag{55}$$

$$p_i'=\sqrt{10^4+200\,d'\cos(90+\varphi)+(d')^2-10^4\,b\,x\,l^2}-100 \tag{56}$$

$$q'=q-100\,b\,r\,l^2\tan\varphi \tag{57}$$

$$K_0'=\frac{(100+q')\,K}{(100+p')\,(100+p'_i)} \tag{58}$$

Equations (**55**) and (**56**) may be solved graphically by the use of Chart No. 4. Distances on the "capacity correction scale" in the upper left hand corner of this chart are laid off equal to $10^4\,b\,x\,l^2$, the number on this scale giving the corresponding value of $b\,x\,l^2$. The method of making this correction is described on the chart itself.

Chart No. 4 may also be used to calculate the regulation in any case where the voltage drop is due to the impedance of the apparatus. For example, it may be used to calculate the regulation of a transformer when ϵ is taken equal to the power factor angle of the equivalent impedance of the transformer and d is taken equal to the per cent ratio of the equivalent impedance of the transformer to the equivalent impedance (V/I) of the load, or to calculate the regulation of a generator when ϵ is taken equal to the power factor angle of the equivalent impedance of the generator and d is taken equal to the per cent ratio of the synchronous impedance of the generator to the impedance of the load (V/I).

EXAMPLE

Given the same example as noted on Chart No. 4, namely:

$E = 60{,}000$ volts $l = 100$ miles $b = 6.03 \times 10^{-6}$ ohms
$I = 100$ amperes $r = 0.267$ ohms $\epsilon = 69.83$ deg.
$K = 0.95$ lag $x = 0.727$ ohms $\cos \epsilon = 0.3447$

Then, $Z = 10.083$ $d = 22.36$
$\lambda = 10.08$ $d' = 20.89$
$u = 0.07567$ $\theta_0 = 85.89$
$\theta = 24.38$ $u_0 = 1.378$

Whence,

	Exact method		Approximate method (See Chart No. 4)
p''	$= 13.01$ per cent	p'	$= 13.6$ per cent
q''	$= 6.43$ per cent	q'	$= -6.7$ per cent
K''_0	$= 96.68$	K'_0	$= 96.5$

DISCUSSION ON "SOLUTION TO PROBLEMS IN SAGS AND SPANS," "SAG CALCULATIONS FOR SUSPENDED WIRES," AND "MECHANICAL AND ELECTRIC CHARACTERISTICS OF TRANSMISSION LINES." CHICAGO, JUNE 30, 1911.

Paul M. Lincoln: In the paper by Messrs. Pender and Thomson, I notice they lead us through quite a number of equations containing hyperbolic functions, and finally come to our good old friend, the parabola, as representing the curve which a transmission line assumes when it is extended between two supporting structures. The result is one we all recognize, although I must say for myself I do not recognize the process by which it was derived. It is perfectly true that the parabola can be taken as the curve which a transmission line will assume and the error due to this assumption is very small in the ordinary case. The sag must be large in proportion to the span, before the error in making this assumption needs to be taken seriously.

I also notice that Messrs. Pender and Thomson have arrived at results that show that our old and well known method of calculating line drops is perfectly correct for 60 cycle lines up to one hundred miles, and 25-cycle lines up to two hundred miles, and it is not necessary to add the refinements which they have worked out for lines below that length.

In regard to Mr. Thomas's paper, I have been impressed and delighted with the way he has worked the thing out. However, I would like to ask one question. Mr. Thomas has assumed that the metal constituting the line has a certain amount of stretch when it is loaded by wind, ice, or in any other manner. It seems evident, however, that he has assumed in his calculations that the supports of the transmission line are absolutely rigid, that they do not give. Now, I do not believe that this assumption is justified, because the bending of the supports, particularly when there are angles in the line, will, I believe, aggregate nearly as much as the stretch in the metal of the conductor. Therefore any results which are deduced from the stretch of the conductor only, would not necessarily hold in an actual transmission line, particularly when there are angles in the line. We all know that the assumption that the supports of the transmission line are absolutely rigid is far from being a correct one. I ask Mr. Thomas if he has taken any cognizance of this matter of rigidity of supports in working out his theory?

L. C. Nicholson: Those of us who are in the habit of calculating sags will be glad to have such a short-cut and accurate method as Mr. Thomas points out.

There is a point which I wish to mention in connection with the sag and spacing of wires. The ordinary understanding that the distance between conductors should be determined largely by the operating voltage is not entirely rational. For such other factors as span, sag, size, weight and material of conductor should be given consideration, and I believe have more direct bearing upon the problem of properly spacing the conductors than has the operating voltage. In the case of extra high voltage lines electrical considerations of course become important.

N. J. Neall: This subject has made a special appeal to me, for I happen to have had actual experience with the problem which the theoretical presentation this morning relates to. I was particularly impressed in this connection by what Dr. Pender said in Boston when he presented in advance the paper written by Mr. Thomson and himself, in explaining that it was all well enough to have rules and methods, but that most engineers use these rules and methods so infrequently that when they came back to the work again they had to brush up on all the facts connected with that method, and waste considerable time in getting started, so therefore I consider admirable the degree to which Messrs. Pender and Thomson have taken their paper, in finishing the calculations, and giving the one reliable result which may be selected with confidence.

Whereas the papers presented today treat this as a theoretical problem, I think the practical problem necessitates, although it has not been so expressly stated, that every span shall be a self-contained one, irrespective of its shape, that is to say, we do not want to have a span when points of support are at such different levels that the lower support is just at the lower end of the span or perhaps inside of it, even though it might be argued along the lines suggested yesterday by Mr. Thomas that such procedure might be intended to prevent oscillations.

It is highly important to know where the low point of the span will come with respect to the lower point of support. As a practical proposition you are obliged to limit this to a distance of, say, 50 to 75 feet, as a minimum just to save yourself. There may be line constructions of grades of excellence which will give up to all theoretical requirements, but I doubt it, and it is the practical margin you must be guided by in laying out these data for use. Now, the difficulty is also enhanced by the fact that if your line is 500 miles long, or even 100 miles long, you have a variable topography to accommodate the line to, and it is no uncommon thing to get a very irregular profile. A very useful method of laying out transmission lines is by use of a template. One template is used for the minimum sag without wind and another for maximum sag without wind. Both are of great importance in determining respectively the distance of the bottom of span from the lower point of support, and the least clearance above ground.

Another point that has been touched on in these papers is the effect of adjacent spans of unequal length. On the principle that the line is to have self-contained spans with a uniform stress in the conductor throughout, you must carefully allow for the short span adjacent to long ones; and on downhill work the effect is to take away from the top spans and increase the sag of the bottom ones because of the total gravity effect.

There is, however, one point in this matter of spans which is of great importance to our engineers. This relates to railroad and telephone crossings where requirements may call for construction much in excess of what is theoretically necessary for the line as a whole. Lastly, the imperfections introduced by construction may further seriously impair your line—such for example as bad surface cuts due to improper use of line tools.

The cost of spans and of the various details entering into these, is a very serious item. It is almost impossible to combine a result theoretically, by means of a formula, which will tie cost to the other considerations; but there are modifications, even in these methods, which will be necessary, and the exception which will prove the rule. But, broadly speaking, the literature which has been created in these papers is invaluable, and those who have to deal with these problems should study carefully each of the methods presented, and decide which will be the most useful for his particular purpose.

Jean Bart Balcomb: I very much appreciate the practical remarks made by the last speaker, also the idea of elasticity of support, especially of the towers. I believe these enter into the problem of wire tension more largely than most of us have realized in the past. Referring to the papers specifically, the last sentence of Mr. Robertson's paper reads: "In fact, the stress values approach infinity as the curve of the span approaches a straight line, due to being drawn taut." I wish simply to call attention to the fact that this is universal, whether the supports are level, or one is higher than the other.

While it is always disagreeable to adversely criticize a paper, there is one statement contained in the paper by Messrs. Pender and Thomson which I would like to mention. In speaking of the deflection, the paper says: " To meet this requirement on a long span, especially with aluminum wire, would require a relatively large sag, from 30 to 100 ft., depending on the length of span and size of wire." The paper then goes on to say: " This would require a prohibitive height of tower, at least for level country work, and therefore we should not use it." To my mind, since we are trying to be scientifically exact, we should live up to the facts, even though they are not satisfactory. If we can prove the supposed facts to be wrong; they are no longer facts. I wish to congratulate the authors, not only on the excellence of their exact treatment of the subject, but also for the finished thought they give, and an approximation that will answer in most cases. I think both exact and approximate solutions should be included in scientific papers generally.

My work in the mountains and on the Nevada desert has especially impressed me with problems like these, that are not met with so much in the level country and under ordinary conditions. Going into places where the maximum temperature reaches 140 deg.

and where at times the worst conditions of sleet and ice are encountered, these problems are pressed home to one very closely. I might call attention to this—in my experience I have noticed that the maximum sag, wherever there are sleet conditions, is not caused by the maximum temperature, but by the maximum ice conditions at the highest temperature in which sleet will form; so that is the point we need especially to bear in mind when considering how low our wire will hang; also that the maximum stress will be induced by the greatest wind and lowest temperature conditions combined. In approaching these problems I always find it worth while to select a few points that are determinative, then work with these in mind.

A point which has not been touched thus far, I think, is the stress that is produced by the oscillations of the wire while the wind is blowing. In our mathematical determinations, we too often assume that when wind conditions prevail, the wire swings over with the wind and remains at a certain angle; yet the wire instead of doing this, swings back and forth, sometimes with considerable violence. How to arrive at this stress, I am unable to say, but I think the matter deserves very careful study.

As a closing thought—there are some excellent determinations among those offered here this morning, and I would like very much if this Institute another year could have these different formulas compared with the large amount of data, gathered throughout the country, showing actual conditions. It seems to me if some one at the present time could be asked to prepare a paper and present it a year from now, or possibly two years from now, it would be worth while and would be greatly appreciated by engineers.

The above applies to the mechanical side especially. Regarding the electrical end of the work, I think we are in a transitional period and the next few years will show large development—so we are in position to study, but not to determine, these electrical factors for perhaps a number of years to come.

W. L. R. Robertson: I have studied Mr. Thomas' paper carefully and with much interest. He is to be commended for the introduction of the so-called stretch curves. The " curves " are nominally straight and parallel lines, they can be readily computed and plotted, therefore shortening somewhat the process of calculation when temperature and stress are considered.

As a matter of fact, this stretch curve is in principle identical with the so-called hypothetical curve given in my paper; the only difference being in the selection of coördinates. When plotted between stress and length of arc, the curve is the nominal straight line. When plotted between stress and sag it is a curved line. In theory the stretch curves plotted between stress and length of arc are not entirely straight nor parallel. I would like to ask Mr. Thomas whether there are any conditions where the use of perfectly straight parallel stretch curves would introduce undesirable error? When the stress passes the elastic limit, the stretch curve is decidedly a curved line, especially for annealed copper.

I have heard of a method of erecting wires which may, or may not have merit, namely, the use of annealed copper wire erected more or less taut and allowing the weather conditions to stretch the wire until it has sufficient sag to withstand all conditions of load, and never taking up the sag. As the wire stretches it probably increases in tensile strength.

For investigating or determining the extent of the merit of such a method as this, or for investigating any other condition where it is required to calculate beyond the elastic limit, the straight line stretch curve is of no value.

If one-half of the ultimate strength or 1,700 lb. per sq. in. is the allowable stress for annealed copper and this is finally considered to be good engineering, then calculations must be made beyond the elastic limit and it is only the true hypothetical curve that can be used.

Further after one becomes an adept in the use of these solutions he will really find that the application of the true hypothetical curve involves but very little more calculation than the straight line stretch curve, especially if very closely approximate results are all that are desired.

It seems to me that the most vital point to be considered in sag and span is the behavior of the material used in the span. By behavior I mean the relation between strain and stretch, permanent set, tendency to increase in tensile strength upon stretching, temperature changes, etc. That is why more data on wires than we have today are desirable.

Problems involving all of these properties can only be investigated by so-called true stretch or hypothetical curves.

There are a few emphases which I would like to make relative to the claims set forth in my paper, namely, the solutions are intended to be universal, to apply to abnormal as well as normal conditions; to apply to any material; to take care of any such conditions as poles swaying, cross-arms twisting, etc., and to give mathematically correct results with a minimum of additional calculation.

H. F. Thomson: I would like to refer merely to the point made by Mr. Neall in the discussion. We have endeavored to indicate the method of the solution on the charts themselves. The discussion which accompanies is more of a justification than anything else; that is, an engineer could cut out the charts included in the paper, and with the suggestions made on them, could work out a complete example without any additional help from the paper itself, and the discussion merely goes to broaden out the points which are included.

H. V. Carpenter: In adapting tower construction to the use of suspension insulators convenience has probably been the reason for arranging the three conductors of a circuit in a vertical plane as has been done in a number of prominent cases. Experience has brought out the fact that this construction must be used carefully in localities where sleet forms, as it has been found that when a heavy load of sleet suddenly falls from one span while the adjacent spans retain their loads the lightened span may rise into contact with the wire above it.

If we remember that the vertical load on any insulator is only ¼ to ⅛ of the tension on the cable, it is clear that a small percentage of unbalance in the tensions on either side of an insulator may deflect it considerably. Since a small change in the position of the end supports of a span causes a very large change in the sag it can be seen that the trouble described above will be more likely to occur than would be anticipated.

A simple example will show what may be expected. Assume a circuit of 0000 copper strung with 600-ft. spans and 8-ft. vertical spacing, hung on suspension insulators 21 in. long. If the sag is 3 per cent when loaded with ½ in. of sleet (no wind), the tension will be about 3200 lb. total. If all the sleet on one span should fall before any on adjacent spans, the sag of the lightened span will decrease to approximately 1.6 per cent or the middle of the cable will rise about 8.4 ft., giving a permanent short-circuit with the cable above. This involves a deflection of each insulator of about 33 deg. from its normal vertical position.

On account of the elastic character of the entire structure much smaller disturbances than that assumed would be sufficient to cause a momentary throw of a cable that would cause trouble.

Hugh Pastoriza: The chart given by Mr. Thomas offers a very easy method for computing line stringing curves but it may also be applied conveniently to other mechanical problems of transmission lines.

For example, certain types of line crossing embody an auxiliary guy which catches and supports the span wire when it breaks at either insulator. In order to get proper clearances under the broken span wire, its final sag after breaking must be determined. Here Mr. Thomas's chart is of considerable assistance. From the mechanical construction of the crossing span, the increase in length of wire between supports, when the span wire breaks, can be found. Expressing the new length in per cent of the span, the new tension and deflection may be read from the chart. This neglects change in length due to reduction in tension, but an approximate correction may easily be made for this.

Problems connected with flexible transmission structures require the use of a curve between tension in span wire and deflection of support. This curve may be found for any given case from Mr. Thomas' chart. Starting with normal tension and deflection and span length, the new tension and deflection, as supports approach each other by 0.1 per cent of span, for example, may be found approximately by obtaining from the curves the tension and deflection which would obtain on the original span but with a length of span wire 100.0/99.9 of the original. A correction may also be applied here for change in wire length due to change in tension.

R. S. Brown: The calculation of the long transmission line may be simplified considerably by the use of the relation between inductance and capacity:

$$L\,c = \frac{1}{V^2}$$

where V is the velocity of light. This would be rigidly true were it not for the effect of the magnetic flux within the wire. In the actual case the approximation is very close if a reduced value of the velocity of light is used, (183,000 miles per second). Within the range of commercial sizes of nonmagnetic wire, the maximum error thus introduced in the value of capacity is one per cent. By the use of this relation one variable, line capacity, may be eliminated from the problem.

The following symbols are used:

P = the real power at the receiver.
E_0 = the receiver voltage (taken as standard phase).
$\dot{E}$ = the generator voltage.
$\dot{I}_0$ = the load current.
φ = power factor angle of load.
ω = $2\,\pi$ frequency.
l = length of line, miles.
r = resistance per mile of line.
c = capacity per mile of line.
L = inductance per mile of line.
x = reactance per mile of line.
$\dot{z}$ = impedance per mile of line.
$\dot{y}$ = admittance per mile of line.
$R = l\,r$ $X = l\,x$ $\dot{Z} = l\,\dot{z}$ $\dot{Y} = l\,\dot{y}$

Dotted letters represent vectors.

Undotted letters represent reals.

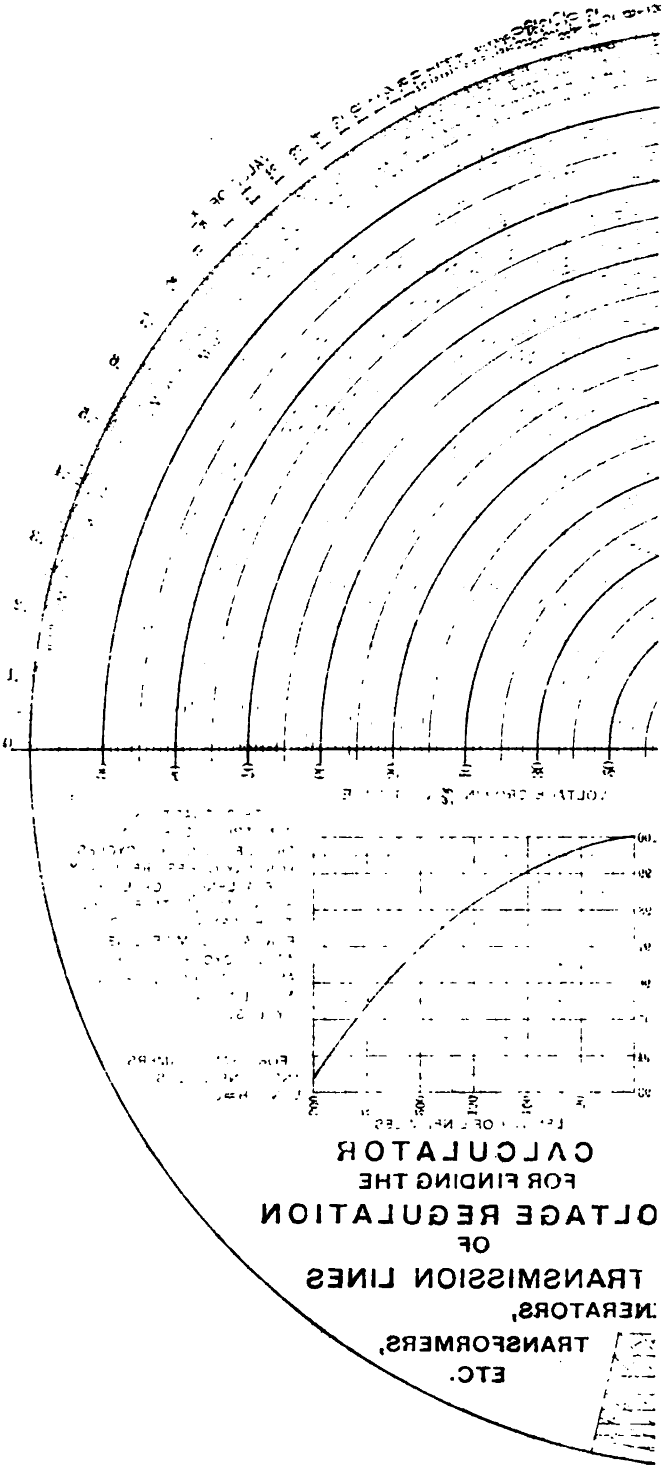
The general equation of the long transmission line as given in Steinmetz's "Engineering Mathematics" is:

$$\dot{E} = \dot{E}_0\,\dot{A} + \dot{I}_0\,\dot{B}\,\dot{Z} \tag{1}$$

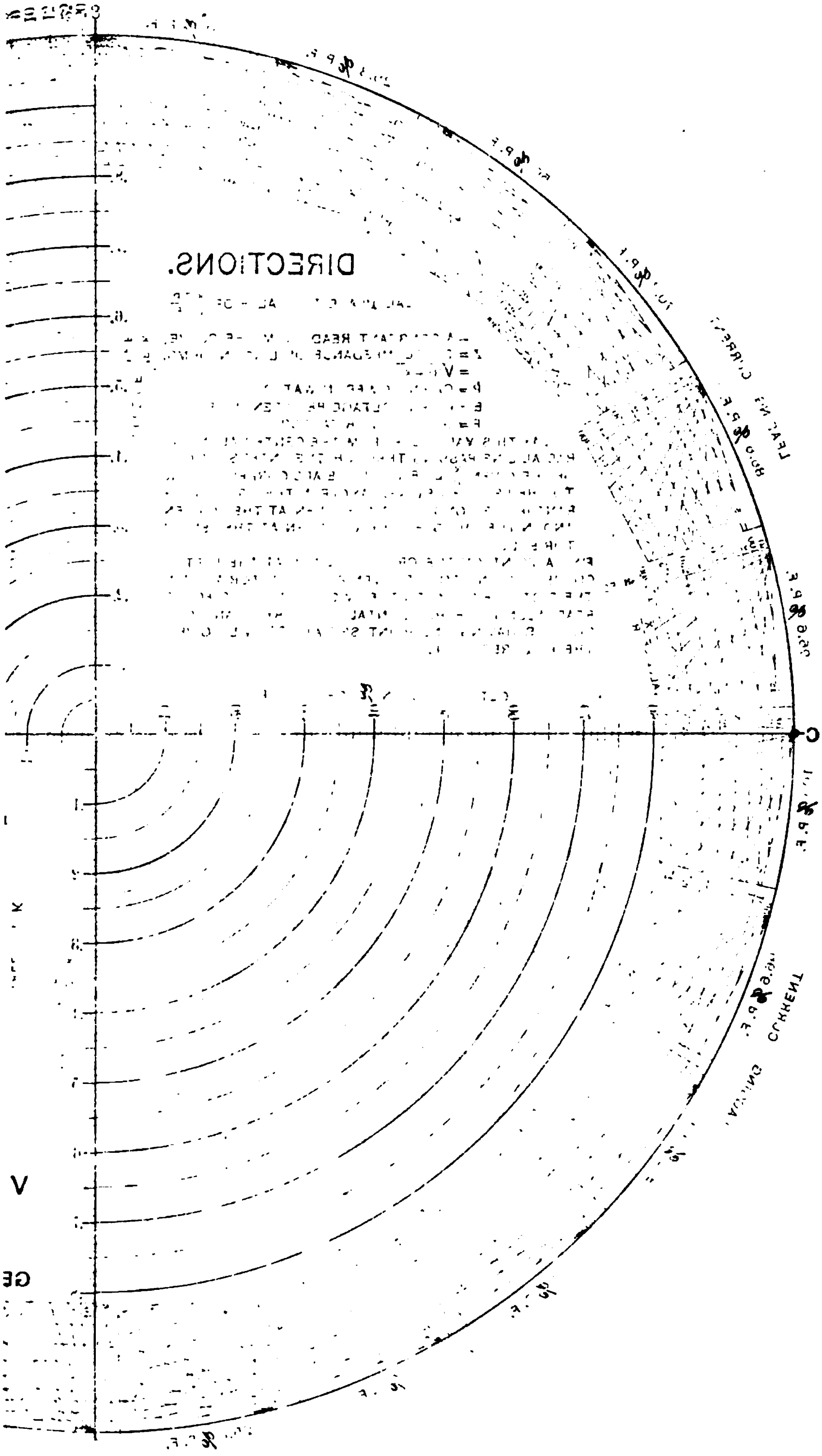
Where $\dot{A}$ and $\dot{B}$ are vector functions of line constants as follows:

$$\dot{A} = 1 + \frac{\dot{Z}\,\dot{Y}}{2} + \frac{(\dot{Z}\,\dot{Y})^2}{24} + \ldots\ldots$$

$$\dot{B} = 1 + \frac{\dot{Z}\,\dot{Y}}{6} + \frac{(\dot{Z}\,\dot{Y})^2}{120} + \ldots\ldots$$


CALCULATOR
FOR FINDING THE
OLTAGE REGULATION
OF
TRANSMISSION LINES
NERATORS,
TRANSFORMERS,
ETC.
VOLTAGE DROP %
LENGTH OF LINE IN MILES

(S. R. Brown p. [illegible])



$$\begin{aligned}\dot{Z}\,\dot{Y} &= l^2\,(r+j\,x)\,j\,\omega\,c\\ &= l^2\,(j\,r-x)\,\omega\,c\end{aligned}$$

This assumes $\dot{y}=j\,\omega\,c$, that is, zero leakage. By means of the relation

$$L\,c=\frac{1}{V^2}$$

$$\dot{Z}\,\dot{Y}=l^2\,(j\,r-x)\,\omega\,\frac{1}{L\,V^2}$$

$$=l^2\,(j\,r-x)\,\frac{\omega^2}{x\,V^2}$$

$$=\left(\frac{\omega\,l}{V}\right)^2\left(j\,\frac{r}{x}\,-1\right)$$

Dividing equation (1) by E_0

$$\frac{\dot{E}}{E_0}=\dot{A}+\frac{\dot{I}_0}{E_0}\,\dot{B}\,\dot{Z}$$

$$\frac{\dot{E}}{E_0}=\dot{A}+\frac{P}{E_0{}^2}\,(1+j\tan\varphi)\,\dot{B}\,\dot{Z}$$

$$1+j\tan\varphi=\frac{\sigma^{j\,\phi}}{\cos\varphi}$$

$$\frac{\dot{E}}{E_0}=\dot{A}+\frac{P\,\dot{B}\,\dot{Z}}{E_0{}^2\cos\varphi}\,\sigma^{j\,\phi}$$

$$\frac{\dot{E}}{E_0}\,\sigma^{-j\,\phi}=\dot{A}\,\sigma^{-j\,\phi}+\frac{P\,\dot{B}\,\dot{Z}}{E_0{}^2\cos\varphi}$$

The regulation is defined as $\frac{E-E_0}{E_0} = \frac{E}{E_0} - 1$ and the magnitude of $\frac{\dot{E}}{E_0} = \frac{E}{E_0}$, is the length of the vector $\frac{\dot{E}}{E_0}\,\sigma^{-j\phi}$. This length is the distance between the termini of the two vectors $-\dot{A}\,\sigma^{-j\phi}$ and $\frac{P\dot{B}\dot{Z}}{E_0^2\cos\varphi}$.

The terminus of $-A\,\sigma^{-j\phi}$ is plotted at the left of the chart, it being a function of φ, x/r, and l.

The second vector is

$$\frac{P\dot{B}\dot{Z}}{E_0^2\cos\varphi} = \frac{P\dot{B}Z}{E_0^2\cos\varphi}\,\sigma^{j\tan^{-1}\frac{x}{r}}$$

The part $\frac{PZ}{E_0^2\cos\varphi}$ is real and it must be multiplied by the vector part $\dot{B}\,\sigma^{j\tan^{-1}\frac{x}{r}}$ whose terminus is plotted in the first quadrant on the chart. The length of vector $\dot{B}\,\sigma^{j\tan^{-1}\frac{x}{r}}$ can be scaled from the chart but is most easily found from the small curve in the fourth quadrant, it being practically independent of $\frac{x}{r}$. It is designated by κ.

The length $\frac{\kappa ZP}{E_0^2\cos\varphi}$ is laid off in the proper direction as determined by a line joining the origin with the intersection of appropriate curves in the first quadrant, and the distance from the terminus of this vector to that of vector $-\dot{A}\,\sigma^{-j\alpha}$ is found by means of dividers.

This length laid off along the horizontal scale will give the required value of voltage regulation.

Frank F. Fowle: The most important problem in the mechanical design of wire spans and structures is the choice of a safe assumption as to loads. In the recent joint report covering specifications for overhead crossings of electric light and power lines, prepared by a joint committee of the National Electric Light Association, American Institute of Electrical Engineers, American Electric Railway Association, Association of Railway Telegraph Superintendents, and American Railway Engineering and Maintenance of Way Association, the subject of loading has been considered with great care. Three classes of loading are there defined, as follows:

Class of loading	Loads		
	Vertical component	Horizontal component	Temperature
A	Dead	15 lb. per sq. ft.	
B	Dead plus ½ inch of ice.	8 lb. per sq. ft.	0 deg. fahr.
C	Dead plus ¾ inch of ice.	11 lb. per sq. ft.	

Class B loading is the stated requirement in the specifications proper and the ordinary temperature range is given as −20 deg. to +120 deg. fahr. The weight of ice is given as 57 lb. per cu. ft. or 0.033 lb. per cu. in. The wind pressure requirement on poles or towers is 13 lb. per sq. ft. on the projected area of closed or solid structures and one and one-half (1½) times the projected area of latticed structures.

Mr. Thomas, who was a member of the joint committee, employs the Class B loading in the example of span calculations given in his paper. Mr. Robertson also employs substantially the same loading. But Professor Pender takes exception to loading as heavy as this and says that the combination of ½ inch of ice and 60 miles per hr. wind velocity will seldom if ever exist simultaneously; he also says that sleet will seldom exist on a wire when the temperature is much below freezing.

Probably no one will dispute that sleet accumulates to thicknesses exceeding ½ inch, or that wind velocities (corrected) sometimes exceed 60 miles per hr., or again that temperatures fall lower than 20 deg. fahr. below zero. How much sleet or how much wind, or how low a temperature will really occur, are matters of probability, only to be determined with some approach to accuracy by studies of weather phenomena extending over long periods. It will be generally admitted that the extremes of wind, sleet and temperature do not occur simultaneously. What we wish to know is how far we must go in assuming simultaneous values in order to design spans and structures so as to be reasonably safe.

Fundamentally we ought to define what constitutes reasonable safety. Should we build expressly to prevent any failure whatever in the light of the most severe combination of loads ever known, or should we take a few chances and proceed to build so that the line will probably fail once in twenty years, or once in ten years—or how often? The question of probability, as applied to the occurrence of simultaneous loads of wind and sleet and the accompanying temperature, enters the problem at many points. Generally speaking, the probability of failure ought to be limited by the risks—that is, it ought to be inversely as the risk, to some extent. How far we *ought* to go in the elimination of risk is one thing; how far we *can* go is another. The engineer who is radical on the side of safety is likely to be criticized by those who pay the bills, and the standard of engineering efficiency is perhaps judged mainly by construction costs, in some cases, when there are other important factors to consider.

But when the risks are considerable, and particularly when human life is involved, it seems proper to reject any assumption that failures at stated periods, even at very long intervals, are permissible. If this is sound, then we can limit the application of probability to the simultaneous values of wind, sleet and temperatures. The gathering of data on this question is a work of great magnitude, if the results are intended to be comprehensive and of substantial value. A preliminary effort in this direction was made by the writer, which consisted of an analysis of the records of the Weather Bureau at Chicago and was published in the *Electrical World* of October 27, 1910. The results of this study do not uphold Professor Pender's comments on Class B loading, but rather indicate that heavier resultant loads would be justified—at least in certain localities.

In regard to the existence of sleet at temperatures lower than freezing, it is a fact, of course, that it will not *form* at lower temperatures, but nevertheless the temperature may fall after the sleet precipitation ceases—and fall considerably. At the same time the wind velocity may increase with the lower temperature, although it is generally stated, and apparently true, that extreme velocities do not occur at the lowest extremes of temperature.

In the matter of formulas expressing wind pressure as a function of velocity there is some disparity. The original form of Mr. Buck's expression, referred to by Professor Pender, is

$$P = 0.0025\ V^2$$

which expresses the pressure on the projected area of a bare stranded cable, in pounds per sq. ft. Mr. Buck's conclusions in relation to wind velocities are also well worth studying in the present connection.

For stranded cables the wind velocities corresponding to pressures of 8, 11 and 15 lb. per sq. ft. of projected cable, with Mr. Buck's constant and with a constant of 0.002, are as follows:

Values of P	Values of V	
	Constant = 0.0025	Constant = 0.002
8.0	56.6	63.3
11.0	66.3	74.2
15.0	77.5	86.6

It is worthy of note that the joint committee report permits an allowable safe stress in hard drawn copper of 50 per cent of the ultimate, or approximately at the elastic limit as determined by the ordinary tests. For sustained loads copper has not as high an elastic limit as this percentage gives; the stretching which will take place, when copper is loaded up to its ordinary elastic limit, will produce considerable increase in the sag when the original span is tightly drawn. On that account it seems conservative to employ a larger factor of safety than two.

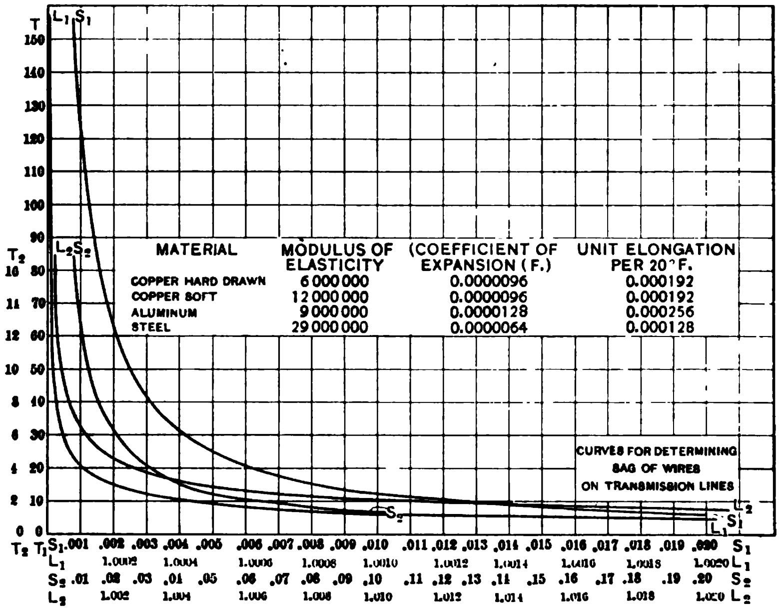
Another reason for increasing the factor of safety lies in the fact that the stresses due to swaying, and the resultant shocks, are not taken account of in the usual span calculations. Failures do not ordinarily occur under steady loads, but during the variable conditions which occur in a storm—when the wind is probably coming in gusts. At the same time any yielding of the poles, towers, or guys may throw additional stresses on the spans.

In the papers on high-tension operation at 100,000 volts, by Messrs. Jollyman, Lee and Hebgen, it is interesting to note that sleet has been experienced on such lines, although it has caused no extensive trouble. Mr. F. W. Peek, Jr., in his paper on "The Law of Corona and the Dielectric Strength of Air," gives some interesting information in relation to sleet. He says, "Sleet had already started to form on the conductors, and was still falling when the tests were started. Fig. 57, (of his paper), shows the loss curve. After the curves were taken the line was kept at 200 kilovolts for over an hour with no apparent diminution of sleet. This seems to show that sleet will form on high-voltage transmission lines. The day after these tests were made was bright and clear and the conductors were still coated with sleet. A set of readings were taken, and it is interesting to note that the excess loss here is as great as when sleet was falling." These results seem to clear up any doubt as to the occurrence of sleet on high-tension lines, with copper conductors.

Mr. Buck, in his 1904 International Congress paper on "The Use of Aluminum as an Electrical Conductor," before alluded to, says: that "aluminum wire gathers much less sleet than copper. This is due perhaps to the grease which is absorbed in the aluminum, due to its porous qualities, in the process of wire drawing or from some other physical condition of its surface." Beyond this statement the writer has seen no definite or comparative data on the behavior of copper and aluminum in sleet storms.

During the winter of 1910-1911 the writer exposed several short spans of copper, iron and aluminum, in different sizes, to observe the effects of sleet precipitation. However, the opportunities for observation proved to be very limited and sleet occurred but once, and then as a trace only. This happened at night, when observations were necessarily hampered, but it was definitely learned that sleet commenced to accumulate on copper, iron and aluminum alike. It did not progress sufficiently, however, to reveal quantitative results for comparison.

R. C. Darrow: In the paper presented by Mr. Thomas a solution is given for the problem of finding the sag of transmission wires at different temperatures when the initial load conditions are given. This method will be found a short and convenient one and all the calculations can be made on a slide rule with sufficient accuracy for practical use. The method is based on the equations of the catenary and it may be interesting to compare the results obtained using this method with those obtained by some method in which the tension is given by the formula $T = \frac{w\,l^2}{8\,d}$, and the length by the formula $L = l + \frac{8}{3}\,\frac{d^2}{l}$



MATERIAL	MODULUS OF ELASTICITY	(COEFFICIENT OF EXPANSION (F.)	UNIT ELONGATION PER 20° F.
COPPER HARD DRAWN	6 000 000	0.0000096	0.000192
COPPER SOFT	12 000 000	0.0000096	0.000192
ALUMINUM	9 000 000	0.0000128	0.000256
STEEL	29 000 000	0.0000064	0.000128

Plate I

where T equals the safe tension in the wire, w the weight per foot along the wire, l the span, and d the sag. Reference is made in Mr. Robertson's paper to an error introduced by considering the tension along the wire as constant and equal to the tension at the point of support. Following out this suggestion a curve has been determined giving a correction factor where greater accuracy is desired.

A set of curves which will be found to cover a wide range of problems and which check those of Mr. Thomas are shown on Plate I. The points (Table I) on this curve have been determined with a ten place logarithm table, except those where the tension for the unit span is greater than one hundred. In getting these points the formula, $T=\frac{1}{8}\;\frac{w\,l^2}{d}$, was used for the unit tension and $l+\frac{8}{3}\;\frac{d^2}{l}$ for the unit length, experience showing that this would be accurate enough for practical problems. This portion of the curve will seldom be used except for very short spans or sizes of wire such as No. 12 or No. 14 N. B. S. copper, or a twisted pair of insulated No. 17 B. & S. steel wire.

TABLE I

Tension for unit span	Sag for unit span	Length for unit span
156.25	0.000800	1.000001707
138.88	0.000900	1.000002160
125.00	0.001000	1.000002667
113.63	0.001100	1.000003227
100.001250	0.001250	1.000004212
90.910466	0.001375	1.000005104
83.334833	0.001500	1.000006058
76.924702	0.001625	1.000007110
71.430321	0.001750	1.000008223
66.668542	0.001875	1.000009441
62.502000	0.002000	1.000010742
58.825654	0.002125	1.000012115
55.557806	0.002250	1.000013571
52.633954	0.002375	1.000015102
50.002500	0.002500	1.000016725
40.003125	0.003125	1.000026072
33.337083	0.003750	1.000037516
28.575804	0.004375	1.00005106
25.005000	0.005000	1.00006667
20.006250	0.006250	1.00010418
16.674167	0.007501	1.00015003
12.510001	0.010001	1.00026669
10.012503	0.012503	1.00041673
8.348337	0.015003	1.00059808
7.160364	0.017507	1.00081687
6.270011	0.020011	1.00106701
5.578071	0.022515	1.00135055
5.025021	0.025021	1.00166751
4.572982	0.027528	1.00201789
4.196703	0.030036	1.00240173
3.370904	0.037570	1.0037542
2.550167	0.050167	1.0066801
2.062833	0.062833	1.0104352
1.742231	0.075564	1.0150676
1.516968	0.088397	1.0205421
1.351340	0.101340	1.0268808
1.225522	0.114411	1.0340934
1.127626	0.127626	1.0421906
1.050092	0.141001	1.0511847
0.987888	0.154554	1.0610893
0.896549	0.182264	1.0836910

Comparison of this Method with One Using Equations Based on the Parabola. In the determination of a temperature-sag curve the initial conditions at worst load are known, together with the tension to be allowed in the conductor, the first step in the solution of the problem being to get the sag at the minimum temperature with the weight of the wire only. Curves given in Fig. 1 show the results for the original conditions obtained from the catenary curves and from the formula, $d=\frac{w\,l^2}{8\,d}$, using a tension of one thousand pounds and a weight per foot of one pound. These curves illustrate the variation in sags as the length of span increases and show that the difference in sags determined by the two methods is quite marked when the per cent sag is increased, quite an error being introduced at the beginning of the problem by the use of the formula $d=\frac{w\,l^2}{8\,T}$.

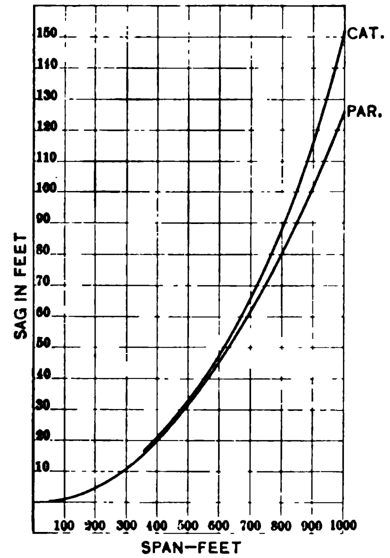


FIG. 1

Curves in Figs. 2, 3 and 4 show some comparisons between temperature-sag curves for various sizes of wire and different spans computed by two methods, those marked 1 being figured from a method based on the formulas

$$T = \frac{w\,l^2}{8\,d} \text{ for tension and}$$

$$L = l + \frac{8}{3}\ \frac{d^2}{l} \text{ for length,}$$

and those marked 2 being figured from the catenary curves. The variations in the two methods is brought out very distinctly by these figures which show that for a 100-ft. span and No. 0000 B. & S. stranded copper wire the curves coincide, while for the larger spans the difference increases as the per cent sag is increased, those figured from the catenary curves giving larger results for the sag at the same temperature. In working out these problems, when the larger spans are used and greater per cent sags result, the points determined will not lie exactly on a smooth curve when the catenary curves shown on Plate I are used, but the discrepancy is so small that a smooth curve can be easily drawn through the points giving satisfactory results, and if more accurate results are desired the curves where the unit tension is small can be drawn on a larger scale. This refinement will not be necessary in practical work because of the inconsistency in the constants used for the material and the inaccuracies arising from the assumptions made in the solution of the problem, one of these being that the tension along the conductor is constant.

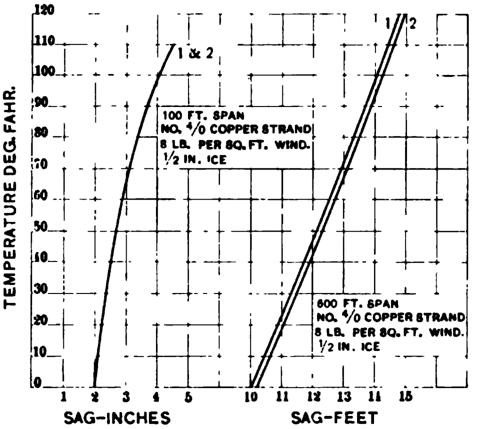


Fig. 2

Variation of the Tension along a Conductor. It may be of interest to develop a curve giving a correction factor for various values of tension at the point of support which may be used where

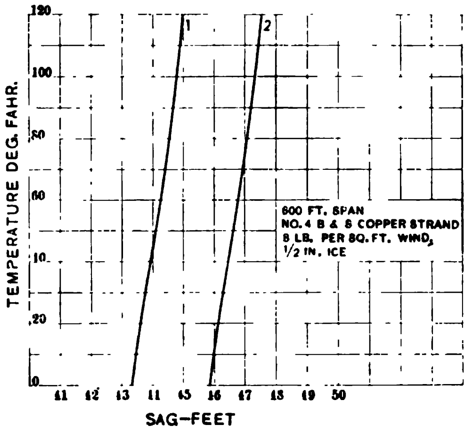


Fig. 3

greater refinement is desired. It can be shown that, if the average tension along the conductor is used, the contraction of the conductor when the stress is removed can be more accurately figured, giving a greater contraction than when the tension at the center of the span is used and a smaller one than that given by using the tension at the point of support. This means that when the tension at the point of support is used the values of the sag at different temperatures will be too small, and when the tension at the center of the span is used the values will be too large. The following method is developed so that the assumptions made may be clearly understood.

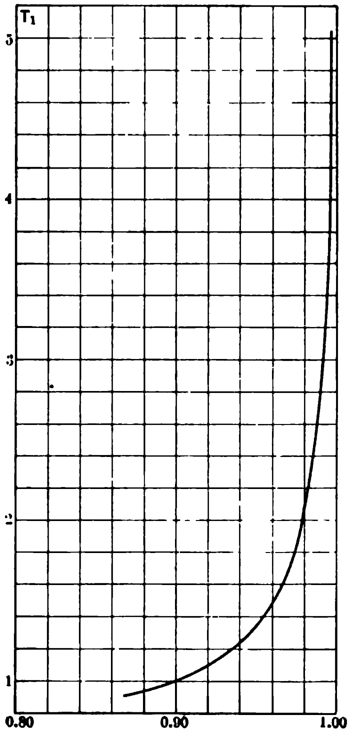


FIG. 4.—Curve for determining average tension along a transmission wire

T_1 = Tension at point of support for unit span

K = Average tension ÷ tension at point of support

Average tension = $T_1 \times K \times w \times l$

Contraction for unit span = $\frac{fK}{E}$

Let λ equal the contraction of any cord of length x, and let the tension along the chord vary as the length, such that $t=f(x)$. Then $d\lambda=\kappa f(x)\,dx$, and the total contraction, $\lambda=\kappa\int_0^x f(x)\,dx$. The term $\int_0^x f(x)\,dx$ will be the area between the chord as the x axis and the curve representing the tension, $t=f(x)$. This area can also be expressed by the length (x) times the average tension ($av.\ t$), so that $\lambda=\kappa x\,(av.\ t)$. If $\kappa=\frac{1}{AE}$, where A equals the cross-sectional area of the chord and E is the modulus of elasticity of the material, the equation becomes $\lambda=\frac{x\,(av.\ t)}{AE}$. If x is unity and the average fibre stress ($av. f$) is $\frac{(av.\ t)}{A}$, the contraction for unit length (λ_1) becomes $\lambda_1=\frac{av. f}{E}$, which, subtracted from the unit length, gives the unstressed unit length. The problem now becomes one of finding the average tension along a catenary when the tension at the point of support or center of the span is given. The equation for the catenary is $y=c/2\,(e^{x/c}+e^{-x/c})$, and the length is $s=c/2\,(e^{x/c}-e^{-x/c})$. The values of y along the curve represent the values of the tension and the average value of y will represent the average tension. The average value of y along the length s is given by the equation

$$av.\ y = \frac{\int_0^s y\,d\,s}{s}$$

Since $y=\frac{c}{2}\,(e^{x/c}+e^{-x/c})$ and $s = \frac{c}{2}\,(e^{x/c}-e^{-x/c})$, the equation becomes,

$$av.\ y = \frac{\int_0^x \frac{C}{4}(e^{+x/c}+e^{-x/c})^2\,d\,x}{\frac{C}{2}(e^{x/c}-e^{-x/c})} = \frac{C\int_0^x \frac{1}{C}\cosh^2\frac{X}{C}\,d\,x}{\sinh\frac{X}{C}}$$

$$=\frac{c}{2}\left(\cosh\frac{x}{c}+\frac{x}{c}\,\frac{1}{\sinh\frac{x}{c}}\right)$$

Since the length of a span equals $2\,x$, the average tension per unit span is given by the formula

$$av.\ y_1=\frac{c}{4\,x}\left(\cosh\frac{x}{c}+\frac{x}{c}\,\frac{1}{\sinh\frac{x}{c}}\right)$$

$$=\frac{c}{4\,x}\left[\frac{(e^{x/c}+e^{-x/c})}{2}+\frac{x}{c}\,\frac{2}{(e^{x/c}-e^{-x/c})}\right]$$

If x/c has the same values as are used in determining the catenary curves, then for every value of the tension at the point of support, the equation will give a corresponding value of the average tension, or for every value of the tension at center of the unit span which equals the tension for unit span at the point of support minus the sag for unit span, we will have a value for the average tension. The value of the average tension divided by the tension at the point of support, will give a factor which may be plotted as absissas with the corresponding tension for unit span as ordinates, Fig. 4. The values used in plotting this curve are given in Table II. From this curve we can get the correction factor for any tension which may be used in determining the contraction from the formula $\lambda_1 = \alpha f/E$, where f is the fibre stress at the point of support. An examination of this curve shows that the factor is negligible except where the tension for unit span is low, and these conditions will be found when the per cent sag is large. It must be remembered, however, that if this refinement is desired a new set of catenary curves must be drawn using average tension as ordinates in place of the tension values given in Table I. These ordinates can be found by multiplying the values of tension in this table by the proper factor. Also average tension should be used throughout the solution of the problem.

TABLE II

Tension for unit span			Sag for unit span	α Factor $= \dfrac{\text{column 3}}{\text{column 1}}$
At point of support	At center of span	Average tension		
5.0250	5.0000	5.0083	0.02502	0.9967
4.5730	4.5455	4.5546	0.02753	0.9960
4.1967	4.1667	4.1767	0.03004	0.9952
3.3709	3.3333	3.3459	0.03757	0.9926
2.5502	2.5000	2.5168	0.05017	0.9865
2.0628	2.0000	2.0211	0.06283	0.9797
1.7422	1.6667	1.6921	0.07556	0.9712
1.5170	1.4286	1.4584	0.08840	0.9614
1.3513	1.2500	1.2843	0.10134	0.9504
1.2255	1.1111	1.1500	0.11441	0.9384
1.1276	1.0000	1.0436	0.12763	0.9255
1.0501	0.9091	0.9573	0.14100	0.9116
0.9879	0.8333	0.8866	0.15455	0.8975
0.8965	0.7143	0.7778	0.18226	0.8676

A value of one pound per foot of conductor was used in determining the above table.

P. H. Thomas: In answer to Mr. Lincoln's question, I will say that it is manifestly impossible to tell in a general statement how much a pole or pin will yield and it is impossible to take mathematical account of a lot of the varying conditions, and if we, by our papers, have enabled you to understand what the result of the known factors is, you can more easily make allowance for the others.

In regard to the effect of flexibility of supports, I think we will have to consider that two cases are likely to arise. There will be times on a long tangent, *i.e.*, a long straight line, where the supports will be only slightly deflected, since the spans support one another. If we have a general increase in tension in the line the cross-arm cannot yield in either direction, because it is pulled in both directions. On the other hand, where we have bends in the line, or where one conductor burns off, or some other unsymmetrical condition arises, we may have a great deflection in the tower, in which case the strains calculated by this formula will be wide of the fact.

I have spoken of broken conductors as a very important case. Assume that all the conductors are burned off on one side of the pole—not only do we have thereafter the strain that results from the tension on one side only, but we have a sudden shock as the wires due to the swing reach the lowest point. With suspension insulators the wire forms a sort of toggle joint and will produce a considerable strain as the maximum of the swing. In practise I have known no trouble on this score.

I wish to add one word to what Mr. Nicholson has said in regard to the proper spacing between wires, that proper spacing depends on other things than the voltage. This matter deserves a great deal of emphasis. I would add to the variables Mr. Nicholson named, the question as to whether the type of insulator is pin or suspension, for the suspension insulator, having a free swinging motion sidewise, requires a much wider spacing of conductors than the pin type of insulator.

Mr. Robertson's statements about the " stretch " curves on my diagram are perfectly just. They are true, I think, within all ordinary ranges; if you go beyond the elastic limit they are not true, but it is true that you can get the exact result by plotting, instead of the straight line, the actual stretch curve, whatever it may be. If you know you are going beyond the elastic limit, and know what the curve of the material is, plot it on the diagram.

I consider the method Mr. Robertson says has been suggested to him, that of stretching soft wire tighter than the ultimate condition will allow, and letting it stretch when the strain exceeds the elastic limit, is a dangerous proceeding. If the wire is perfect, and you know it will stretch uniformly, there is much to be said in favor of the idea. If, however, there are any joints in the wire, or there is any danger of its being kinked, or nicked, the stretching will come at some one point rather than uniformly throughout the wire, and you do not know what condition will result.

W. L. R. Robertson: In Mr. Fowle's discussion, he has very appropriately considered the assumption of proper loading and the proper safety factor. In the 1911 overhead line report of the Pennsylvania Electric Association, the writer makes a few suggestions that are closely in accord with Mr. Fowle's ideas. It seems that in overhead line construction the tendency is to base calculations on very narrow margins of safety. In other engineering problems such as building construction, etc., we assume loads on our structures up to the extreme limits, and in addition to this we employ safety factors up to 5 and larger. We are, indeed, handicapped in our calculations by not having definite conclusives in these matters.

In regard to wind pressures on wires, up to the present time Mr. Buck's expression:

$$P = 0.0025\ V^2$$

is generally conceded to be correct, but in conjunction with the study of Mr. Buck's conclusions, attention should also be given to an article in the *Journal of Electricity, Power and Gas*, July 29, 1911, by Messrs. Piatt, Lane and Kistler, wherein they claim very much higher wind pressures than given by the above expression, especially for the smaller wires.

In answer to Mr. Thomas, when he criticizes the stretching of annealed copper as a dangerous proceeding, the writer has not recommended the stretching of any wire beyond the ultimate conditions. If stretching is carried on too far, it most assuredly is dangerous, but if soft copper is to be used at all in overhead lines and to be erected with reasonably small sag values then a certain amount of stretching cannot be avoided. If 17,000 lb. per sq. in. or ½ the ultimate strength is the proper allowable stress, as recommended in the overhead report of the National Electric Lighting Association, June, 1911, then the stretching of soft copper is considerable. In fact the elastic limit of annealed copper is very low and indeterminate, and it would be impracticable to use annealed copper without stretching. Further, on this basis satisfactory calculations cannot be made for annealed copper unless data on permanent set, increase in tensile strength due to stretching, etc., are available.

P. H. Thomas: Attention should be called to the ingenious method of Mr. R. S. Brown, found in the discussion of these papers, in which he has simplified line calculations. It is well known that in any transmission wire not containing loops there is an inherent relation between inductance and capacity, so that one property bears a definite numerical relation to the other property, this relation being expressed by the equation $L\,c = \frac{1}{V^2}$, where V is a constant approximately equal to the velocity of light. The relationship is as a matter of actual fact, somewhat disturbed by conditions within the conductor, but presumably not sufficiently to interfere with its practical accuracy for general work. While this relation has been well known, as far as I am aware it has not previously been directly applied to the simplification of equations containing both inductance and capacity as apparently independent variables.

Considering Mr. Fowle's discussion of the choice of maximum conditions to be met by any particular structure, I would like to call attention to one saving fact, namely: that, should a strain appear on the wire greater than the elastic limit, the result will not ordinarily be the rupture of the wire but merely a certain stretch which will greatly relieve the tension in the span. This condition serves practically to greatly extend the extreme conditions under which an actual rupture and falling of a line conductor would be expected. This favorable situation exists, of course, only with regard to the wires or cables strung along the poles and does not extend to the line poles, except as against such strains as are produced by the toggle effect of the suspending wire. In the case of a stretched wire it will of course sometimes be necessary to take up slack afterwards.

I should like to call attention to Mr. Darrow's communication on my paper as representing a very careful study of the method and its relation to the present ordinary methods. Mr. Darrow has given the values upon which the curves of Chart No. 2 of my paper are drawn, but has extended the computations to more significant figures in such a way as to permit the application of the curves to much tighter spans, such as are frequently used in telephone and relatively low voltage distribution work.

Harold Pender and H. F. Thomson: The object in presenting the detailed derivation of the formulas in our paper was to put in a readily accessible place a complete discussion of the limitations of the approximate formulas used in practise. We should have been no more surprised than Mr. Lincoln had we found that for short spans the assumption of a parabola introduces a considerable error and that in the case of short transmission lines the simple method of calculation ordinarily used gives erroneous results. In view of the fact that long spans and long lines are becoming more and more frequent it is important that the limitations be recognized. We wish to point out again that the complete rules for the use of the charts are given on the charts and that it is therefore unnecessary in using them to refer to the detailed deductions in the text.

The method of solving the wire span given by Mr. Thomas is exceedingly simple in theory, but requires considerable slide rule calculation. Our chart gives the same results but only a single slide rule computation, a simple multiplication, is necessary. To show the graphical portion of our solution we have reproduced as Fig. 6 the part of Chart No. 2 necessary for the solution of the example of the aluminum wire span given at the foot of the chart. The location of the points A, B and C has been explained previously. It will be noticed that only two of the system of curved lines given on the chart are necessary for any single solution. It should be observed also that for a given span the tensions and deflections for any series of stringing temperatures may be found by locating other points like C by means of lines parallel to $B\ C$ and passing through the corresponding temperature points on $A\ B$. A number of such lines for the above example are indicated in Fig. 6. The advantage of this chart when one has to make a number of calculations for different lengths of span or for various temperatures with a given span is evident.

It is worth noting that the relation $L\,c = \frac{1}{V^2}$ used by Mr.

Brown applies only to overhead transmission lines where the capacity is not influenced by the presence of other conductors or the earth. The relation does not hold for a lead sheathed cable, nor does it hold for a loaded telephone line. This also applies to the capacities given on our Chart No. 3.

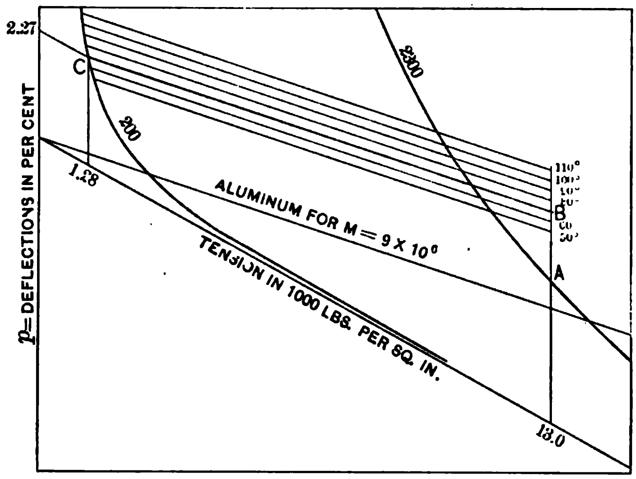


FIG. 6.

Location of points *A*, *B*, and *C* for example of aluminum wire given on Chart No. 2. For tensions and deflections at stringing temperatures other than 70 deg. fahr., draw lines parallel to *B C* through points on *A B* corresponding to the respective temperature corrections, as 50 deg., 60 deg., etc.

In the paper the report of the Joint Committee on Overhead Line Construction is misquoted. Their recommendation for the maximum load is the simultaneous existence of 0.5-in. of ice, a wind pressure of 8 lb. per sq. ft. and a temperature of 0 deg. fahr., not −20 deg. fahr. as stated in the text. Our attention was called to this error by Mr. R. D. Coombs.

The error pointed out by Mr. Robertson in Fig. 3 in his paper, when the points of support are not on the same level, does not enter into our formula (31). The phenomenon referred to is taken account of by the second term in the parenthesis.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

THE HIGH-EFFICIENCY SUSPENSION INSULATOR

BY A. O. AUSTIN

The high-efficiency type of suspension insulator has become an important factor in high-tension transmission within the last few years, and it is hoped that the considerations which led to the design of this type will be of interest.

A very high potential and small current, with a wide range in power factor make quantitative measurements very difficult. For this reason the performance of the insulator is based largely on visual phenomena or comparative test.

In service the insulator is subjected to two classes of stress—mechanical and electrical. Mechanically the insulators must withstand the stresses necessary to support the conductor, and electrically it must prevent failure by the current passing through the insulator, over the surface, or through the air from conductor to support or ground. To satisfy the electrical requirements, dielectric strength, surface resistance and capacity are necessary.

It is not sufficient that these properties be developed for laboratory tests only, but for conditions in service where the effect of depreciation and its causes must be given due consideration.

After making a larger number of tests on the different types of insulators in 1904, it was decided that if an improvement was to be made in the insulator, it would be by improving the efficiency rather than by increasing the size or weight of the insulator. With this idea, a number of experiments were started on different styles of disks to obtain their relative efficiencies as insulating members for the high-tension insulators. As it was desired to design an insulator for the severe conditions around San Francisco Bay, the effect of surface depreciation was of greatest importance, for it was evident that after a few years operation, insulators failed through the surface becoming coated.

In photographing the different types, it was noticed that there was a difference in the nature of the flashover, the arc in some instances following the surface, taking a very long path between conductor and pin, while in others the path of the arc was through the air or partially over surface and through the air. It was noticed, however, that the arc followed the surface in the larger types excepting where a wooden or porcelain pin was used. The reason for this was not at first apparent, but after a study of the characteristic it was decided that this was largely due to the overstressing of a part, the insulator failing by a cascade action. It was well known that certain parts of the insulator were greatly overstressed, causing many insulators to puncture on assembled test, but the remedy for this had not been advanced.

Shortly before this time, considerable improvement had been made in design to obtain higher flashover of the insulator under storm conditions by giving the insulator large striking distances between surfaces. This, however, was carried to an extreme in some of the designs, for it was readily seen that after the insulators were in service, that it would not be possible to utilize the full striking distance owing to there being a weaker path over the surface for the forming of the arc.

As the rating or capacity of the insulator is based largely on the potential necessary to flash over the insulator under storm conditions, it is important that the properties influencing the flashing or arcing be given close attention.

There are two types of insulators shown in Fig. 1; in one, the insulator flashed over, the arc forming over the surface, while in the other one, the arc took the air path. If the arc builds up over the surface on the clean insulator, it will follow that the potential required to cause flashover will be much lower after the insulator surface has depreciated under operating conditions. If, however, the arc forms between surfaces through the air practically the same potential may be necessary to cause flashover even after considerable surface depreciation has set in. This latter will be true as long as the path over the surface shunting the air path will maintain a drop in potential equal to that necessary to rupture the air path.

The amount of depreciation which an insulator will stand and not lower its rating depends upon the excess in surface insulation between the points where the arc forms. In the pin-type insulator it is very difficult to obtain the same ratio between surface resistance and the flashing distance, as a slight change in the distance or conditions varies the relative values to a large extent. In the suspension insulator, however, there are more nearly ideal conditions.

Owing to the limitations of the pin type insulator, engineers were looking for a different type of insulator, and several designs of suspension and post types were proposed in 1904. Tests on some of these showed that they had very good properties, but that the efficiency would have to be very much improved before they would be of importance.

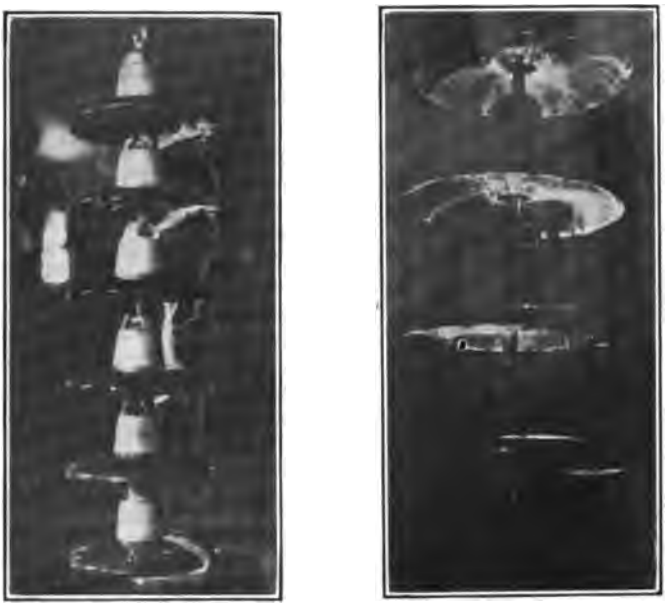


Fig. 1.

In addition to the electrical characteristics, a study of manufacturing methods was made in order to form a basis for practical designing. This work all required much time, and it was necessary to develop not only manufacturing methods for making up some of the pieces, but the porcelain body also. It was later found that a number of the principles had been used in some of the earlier types of insulators with success, but had been practically abandoned and forgotten at the time.

SURFACE RESISTANCE

In service, the insulator must be regarded as a high resistance. The drop in potential over the surface will depend upon the flow of current and the resistance of the surface. As the surface resistance varies greatly with a change in conditions, usually the worst conditions are assumed for the purpose of analysis.

The early telegraph insulators appear to have been very carefully designed so as to give high surface resistance. While the voltage remains low, no trouble is experienced by the overstressing of air-gaps as they are relatively large and high surface resistance may be obtained by providing long leakage paths of small diameter.

When the voltage is increased, larger insulators made along the same lines arc between the ends of petticoats and are noisy at potentials considerably below flashover; for if the difference in potential between any two points in the leakage path becomes equal to the flashing potential for the air distance between the points, overstressing develops and an arc forms.

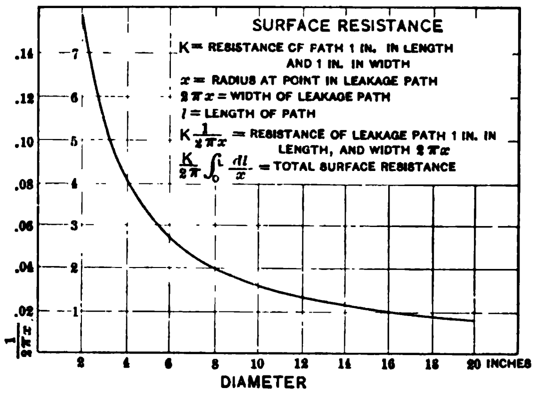


FIG. 2.

The insulator does not necessarily flash over, but is likely to spit and become very noisy. Large air spaces remedy this fault, but if not properly placed low efficiency results.

As surface resistance not only prevents a serious loss of current but is responsible for the potential gradient over the surface of the insulator it is of no little importance.

The resistance of an insulator must be determined by taking the width into account as well as the length of leakage path. Surface resistance will vary directly as to length and inversely as to width. The width of the leakage path at any point will be $2 \pi r$ where r is the radius of the zone at that point. By taking as the unit of resistance, a surface one inch (2.54 cm.) in width and one inch in length, the resistance may be determined in terms of this unit.

Fig. 2 shows the effect of diameter on resistance and shows how very misleading it is to base surface resistance on length of leakage path. The area below the curve gives the total resistance and shows very plainly that a leakage path of large diameter furnishes but little resistance compared to a path of small diameter.

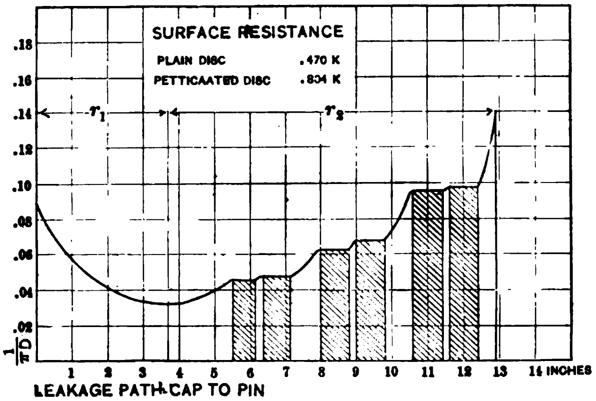


Fig. 3.

Fig. 3 shows the resistance integral curve for a 10-in. (25.4 cm.) high-efficiency disk. The shaded portion represents the resistance furnished by the petticoats. Fig. 3*b* shows a detail of a unit.

The petticoat is a very efficient way in which to increase surface insulation, for 16 per cent of material added in the form of petticoats increases the resistance of the lower surface 100 per cent. In this manner a high surface resistance can be obtained with a small diameter.

The resistance integral curve Fig. 2, plainly shows that where the diameter of an insulator is made large, very little resistance is gained, for in order to produce an effective drop in potential over the lower resistance of a zone of large diameter would require a leakage current so large that the zone of small diameter, in series, would be a mass of fire. Where very large diameters are used for severe conditions it is equivalent to placing a 16-c.p. lamp and a 50 c.p. lamp in series on double voltage and expecting an efficient combination.

The petticoat in addition to providing an increase in surface resistance reduces the electrostatic capacity of the flange. This reduces the charging current and gives the section a 40 per cent higher flashing potential without increasing the distance between cap and pin.

A high charging current on the surface evaporates the water striking the surface, preventing a washing action, and in addition highly conducting compounds are produced greatly depreciating the surface insulation.

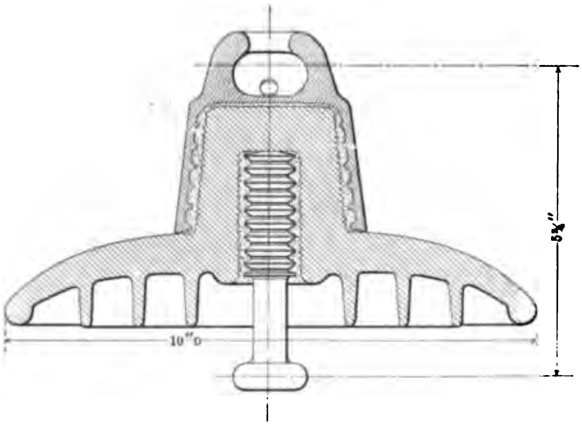


FIG. 3b.

In an endeavor to reduce the charging current and consequent depreciation several all-porcelain types, were developed, shown in Fig. 4.

These insulators had very good properties, but it was found that the high-efficiency disk type with low charging current gave nearly as good results and had decided mechanical advantages.

THE INCLINATION OR SPACING OF THE SKIRT OR FLANGE

After conducting a number of tests with fog shields, and oil zones in an endeavor to protect insulators against depreciation, it was decided that the operation of the insulator could be greatly improved by obtaining a better relation between the surface gradient and spacing of parts.

Where the distance between parts is too small for the stress, an arc is established. The arc reduces the potential between the parts depending on the current in the arc. In some insulators overstressing or arcing starts at a very low potential. The insulator does not necessarily arc over, for the current in the arc may be limited to the leakage or charging current of the insulator or part, there being enough insulation in series to limit the current in the arc.

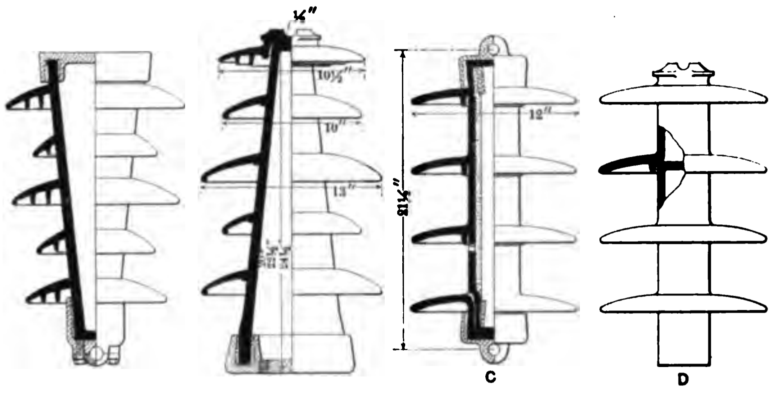
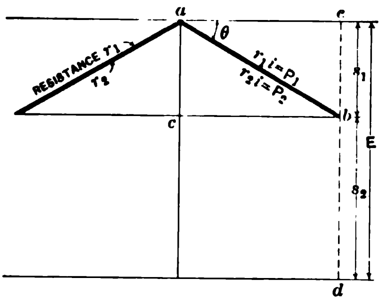


FIG. 4.

Since the arc bridging an overstressed air-gap acts as a conductor shunting the surface resistance, it lowers the efficiency of the insulator and should be prevented by arranging the clearances in proportion to the difference in potential; that this is especially important where the conditions are severe is shown by the following:



r_1 = resistance of upper surface
r_2 = resistance of lower surface

FIG. 5.

Fig. 5 shows a simple insulator, it being desired to find the proper position of the skirt or flange to prevent overstressing and give maximum flashing potential.

E = potential necessary to strike arc between e and d.
s_1 = potential to strike arc between e and b.
s_2 = potential to strike arc between b and d.
i = leakage current.
p_1 = drop in potential over upper surface.
p_2 = drop in potential over lower surface.

The distance between b and ground and b and pin is made equal or $b\,d = b\,c$.

Since the maximum potential which may be applied to the insulator is limited to the flashing potential for E or the shortest air path of the insulator, the flashing efficiency will be the ratio of potential necessary to flash insulator, to this potential. It is evident that for maximum possible efficiency, the following equation must be satisfied:

$$p_1 + p_2 = E \qquad (1)$$

$$\frac{p_1}{p_2} = \frac{s_1}{s_2} \qquad (2)$$

applying Ohm's law gives

$$\frac{p_1}{p_2} = \frac{r_1\,i}{r_2\,i} \qquad (3)$$

For practical purposes

$$\frac{s_1}{s_2} = \frac{b\,e}{b\,d} \qquad (4)$$

Substituting from (3) and (4) in (2) gives

$$\frac{r_1}{r_2} = \frac{b\,e}{b\,d} \qquad (5)$$

Substituting for $\frac{b\,e}{b\,d}$ gives

$$\frac{r_1}{r_2} = \tan\theta \qquad (6)$$

Therefore, to obtain maximum striking efficiency the angle with the conductor must be such that $\tan \theta = \frac{r_1}{r_2}$.

Equation (6) shows that for maximum wet striking distance corresponding to $\theta = 0$ that $\frac{r_1}{r_2} = 0$ which can only be satisfied when $r_1 = 0$ or $r_2 = \infty$. When an insulator is clean and the under surface dry while the upper is wet, $\frac{r_1}{r_2}$ is very small, and a nearly horizontal skirt will give good results. This, however, comes far from representing conditions found in practice where the upper surface may have more resistance than the lower. If the resistance of the upper surface is higher than the lower, then $\tan \theta > 1$ and θ is greater than 45 deg.

That this is no exaggeration in practice is evident when it is considered that where conditions are severe the lower protected surfaces are continually depreciating due to accumulations of conducting material, while the upper surfaces retain a fair state of insulation due to the washing by the rain.

That fogs are likely to give most trouble, is evident when the value of $\frac{r_1}{r_2}$ is considered together with the design of the insulator.

When the insulator has been in service some time, the upper surface is fairly clean compared to the lower, and even in a rain, the wet upper surface may have a resistance comparable to the dry but dirty lower surface. During a fog, however, all surfaces are wet, and the resistance of the dirty lower surface is very much lower than during a rain storm, greatly increasing $\frac{r_1}{r_2}$. For this condition θ should be large, but if θ is small, overstressing may develop and a large part of the surface insulation lost through the shunting or leakage arcs.

From the above considerations it is seen that in practice $\frac{r_1}{r_2}$ may vary from nearly zero to greater than unity and that for a slight change in conditions θ should vary accordingly to give maximum efficiency. It is very desirable to keep $\frac{r_1}{r_2}$

constant for all conditions so that the inclination of the skirt will not have to be changed for a slight variation in conditions, in order to maintain efficiency.

When $\theta = 0$ $\frac{r_1}{r_2} = 0$ for all conditions and efficiency equal 100 per cent, but the resistance of the upper surface r_1 is lost.

By making r_2 large in comparison to r_1 will necessitate only a slight change in the inclination θ to obtain maximum efficiency. Furthermore if r_2 is large θ will be small. The increase in r_2 will reduce the leakage current, and the electrical gradient over the surface will be less. Fig. 3, shows that the petticoats form a very effective means of increasing r_2.

The insulation of the air may be regarded as constant while surface insulation depreciates with time and severe conditions. Then by designing the insulator so that the flashing potential is limited by the breaking down of the air paths, the rating or capacity may not be affected by surface depreciation.

To insure the air characteristic in the insulator, the striking distance between successive sections is made small so that the weakest path for the forming of the arc will be through the air. In providing the air characteristic it is important that the length efficiency remain high; the following example showing the effect of inclination of flange on the length efficiency.

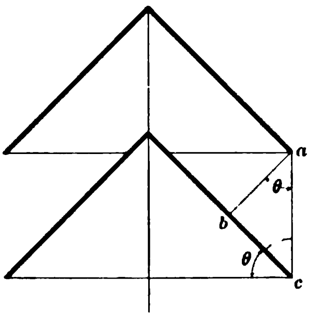


Fig. 6.

Fig. 6 represents two successive sections having section length $a\ c$, it being desired to find the effect of inclination of flange on the length efficiency.

e = potential necessary to flash between a and lower insulator.
E = potential necessary to flash $a\ c$.
p = potential necessary to flash $a\ b$.
d = potential drop from b to c.

$$\text{The efficiency} = \frac{e}{E} = \frac{p+d}{E} \qquad (7)$$

If $\frac{r_1}{r_2}$ is small, *d* becomes very small. If the diameter is large, the surface resistance from *b* to *c* is very small, even compared to r_1, and *d* may be considered as zero without materially affecting results.

When $d=0$

$$\text{Efficiency} = \frac{p}{E} \qquad (8)$$

For practical purposes, the potential is proportional to striking distance, hence

$$\frac{p}{E} = \frac{a\,b}{a\,c} \qquad (9)$$

Substituting for *a b* in (**8**) gives

$$\frac{p}{E} = \frac{a\,c \cos 0}{a\,c} = \cos 0 \qquad (10)$$

Equation (**10**) shows that if $d=0$ the efficiency $=\cos 0$ and for maximum efficiency $\cos 0 = 1$ or $\theta = 0$.

When the diameter of the insulator is large or where the under surface of the insulator has a high resistance compared to the upper, *d* becomes very small and equation (**10**) very nearly approximates conditions in practice.

By consulting the surface resistance curve, it will be seen that *d* will increase as the diameter of the insulator decreases, owing to the higher resistance of the zone of smaller diameter.

From this it follows that in general, the smaller the diameter, the greater the permissible inclination for maximum length efficiency. It also shows that where insulators of large diameter are used, the length efficiency will be lowered greatly with the inclination of the skirt or flange.

Dielectric Strength or the Ability of the Insulator to Carry Electrical Stress

In service, the insulator must withstand two classes of stress; that of the line at normal frequency and voltage, and that of the high frequency surge. The insulator must operate indefinitely under the normal line potential, which affects every insulator on the system. The surge, however, may throw a very high stress on a few of the insulators, but only for an extremely short space of time.

For reliable operation, no insulator should puncture or fail by flashing or spilling. To produce 100 per cent reliability against flashing or puncture, would require a very large investment in the line, but it is possible to obtain a high degree of reliability at a moderate cost for the line by using the suspension insulator. To prevent spillovers would require very large insulators and the greatly increased cost would not be warranted by the small improvement in operation over that afforded by ordinary practice. With a spillover, the line may not be appreciably affected, but when an insulator punctures, the line is usually disabled until the faulty insulator is replaced. Since dielectric strength does not necessarily require an increase in size in the insulator and is of such great importance in affecting reliability, more attention should be given to it in the insulator.

In order to increase the reliability against puncture, it is common practice to test all insulators at a potential several times that of the line. This weeds out a number of the weaker insulators and improves the reliability, but owing to the time element in effecting breakdown, does not insure absolute reliability nor uniform strength.

In the ordinary high-tension insulator, the testing stress compared to thickness is not high enough to puncture perfect material. When insulators of this type are tested, it is found that the breakage becomes less as the time of test increases, but is never entirely eliminated.

In order to draw conclusions as to reliability in the insulator against puncture, it is necessary to study the time puncture curves. These curves are constructed by noting the time that each puncture occurs after the potential has been applied and plotting the per cent of breakage in respect to time.

The greater number of pieces on which the curve is based, the more valuable will it become.

Fig. 7 shows the breakage or time puncture curves for an insulator or part at two different potentials. When the difference in test potential is not very great, a high potential for a short time eliminates practically the same material that a lower potential would, applied for a longer time, the curves being discussed on this basis.

Curve *A* shows a breakage of 2.2 per cent after ½ minute test at 100 kv. In order to have eliminated the same material at 85 kv., curve *B* shows that it would have been necessary to test for 4.7 minutes.

If the insulators which had received the 100-kv. test were tested for another ½ minute at the same potential, there would be a loss of 1.2 per cent. If, however, the insulators which had received the first half minute test had been tested for ½ minute at 85 kv., the loss would have been only 0.2 per cent or about ⅙ what it was at the higher voltage.

The curves show that if the insulators had been tested for 5 minutes at 100 kv., it would take a very long time at 85 kv. to cause a breakage of one per cent. If in place of 85 kv. a potential of 50 kv. was applied, it might be a matter of days before one per cent had punctured.

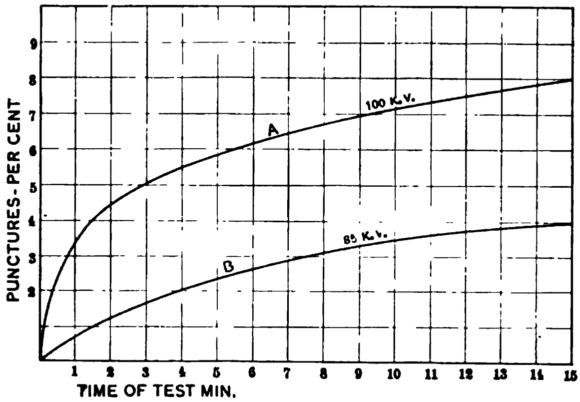


Fig. 7.

By having the breakage time curves, it is then possible to construct the time potential curve which shows the relation between time and potential to produce a certain per cent breakage.

When the time potential curve and the stress carried by the different parts in the insulator are known, it is possible to predict the number of punctures in the complete or assembled insulator for any potential and time.

By limiting the flashing potential, and having the curves and knowing the punctures on the line, some idea as to the relative conditions in operation and test may be obtained.

If a transmission system installed 50,000 insulators shown in curve *A* which had been tested to 100 kv. for one minute and the number of punctures were 10 for the year, the breakage would be 0.02 of one per cent. The stress that would produce this breakage would run from the line potential to that of flashover, most of the breakage being at the higher stresses during lightning storms, as most of the punctures in operation occur at near flashing potential of the insulator the time breakage curve for this potential should furnish a fair basis of comparison for punctures, providing the stress in service can be limited to this value.

Curve *A* shows that after testing the insulators for one minute at 100 kv. the rate of puncture is 1.3 per cent per minute. From this it follows that to produce the same number of punctures at 100 kv. on the 50,000 insulators that occurred on the line, the stress would have to be applied until 0.02 of one per cent were punctured or for $\frac{0.02}{1.3} = 0.0157$ minutes.

Reducing the stress 15 per cent from that in curve *A* gives punctures in accordance with curve *B*. The rate of breakage given by the 85-kv. curve on insulators first tested for one minute at 100 kv. is 0.18 of one per cent per minute as against 1.3 per cent per minute on the 100-kv. curve and the breakage for the same interval of time might be expected to be reduced accordingly.

From this it would follow that by constructing the insulator so that all stresses in practice would be reduced 15 per cent, the punctures would be reduced from 10 to $\frac{0.18}{1.3}$ of 10, or 1.38, estimated from the time breakage curves.

In the above example, the possible 86 per cent reduction in punctures can be made use of in practice if all stresses are reduced 15 per cent. This can be approximated by the addition of a part to the insulator which would take 15 per cent of the stress. Up to 100 kv. on the insulator this would be entirely satisfactory, but if the addition of the part increased the flashover potential, the stress on the original insulators would not be kept down to 85 per cent under heavy surges, and the benefit of the added part would be partially, if not totally, lost under these conditions.

By adding the part such that the flashover is not increased and 15 per cent of the stress is absorbed, the insulator would operate in accordance with curve *B*, with an 86 per cent reduction in punctures over that when operating in accordance with curve *A*.

To make full use of the 86 per cent reduction in loss in the above example, the stress at flashing potential on the insulator would not exceed 85 per cent of that which it received on test, or in other words, the insulators would have a tested factor of safety of $\frac{100}{85}$ or 1.175.

This is accomplished in large pin type insulators by designing the parts so that they will have a high flashing potential compared to the stress which they have to carry at flashover on the complete insulator, permitting of a tested factor of safety. In the suspension insulator, the section length is reduced so that the entire insulator flashes before the tested or flashing potential for a part is reached.

If in addition to providing a tested factor of safety at flashover, the test be continued, the reduction in probable punctures will be made possible in accordance with the increase in reliability shown by the time puncture curves. By testing to five minutes at 100 kv. in the above example, the rate of puncture is reduced from 1.3 per cent per minute at one minute to $\frac{1}{3}$ of 1 per cent per minute, reducing the probability of puncture 75 per cent or to one puncture about every three years.

It may be contended that owing to the time lag in the breakdown of the air, that the impressed potential on surge will be so high that time puncture curves made at normal frequency will be of little value. While the time puncture curves made at different potentials and normal frequency may vary considerably from those made at high frequency and short time, the general characteristics would probably be the same, and a factor of safety based on the curves made at normal frequency would apply in general to operating conditions.

There is such a marked reduction in loss at testing potentials by applying a small factor of safety, that it seems reasonable to assume that by making the tested factor of safety large and the air path over the insulator direct that the insulator may be made to withstand even the direct stroke of lightning.

The chief value derived from the time puncture curves is a basis for determining the relation between cost of insulation and reliability.

To give the same degree of reliability against puncture, the rate of puncture per minute at end of test should be the same, for similar insulators made at different times or by different processes or factories.

Fig. 8 shows the time puncture curves made on the same piece of ware at different factories. Curve *C* shows a loss at the end of the test at the rate 2.4 per cent per minute at 55 kv. Curve *D* gives for the rate of loss one per cent per minute at the end of the test. Some idea as to the effect of the reliability of the two lots of insulators may be gained by the performance of the above on test. If 10,000 of lot *C* were again tested for 10 seconds they would show approximately 60 punctures. While lot *D* when given the same test would show only 17.

If the test potential is low compared to the stress which the part may receive in service, the time breakage curve may not give a proper comparison, for a piece which had a relatively very low breakage rate may have at a higher potential a high rate of puncture and a higher total loss.

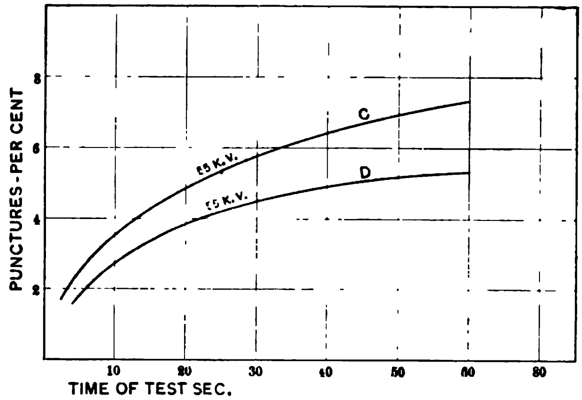


Fig. 8.

It has been shown that insulators of given reliability may have their reliability greatly increased by providing a tested factor of safety for severest conditions. When a tested factor of safety prevails throughout the insulator the flashing characteristics are very interesting.

Fig. 9 shows a four-section insulator flashing from conductor to support, the arc forming through the air. As this insulator has a tested factor of safety, the air path from conductor to pin was broken down before a flashing (or tested potential) was reached on any section. If the mechanical limitations would permit, the tested factor of safety could be further increased by adding another unit without increasing the length.

Fig. 10 shows another four-section insulator of the same length as in Fig. 9, but in this instance the arc is seen forming over the surface of each part, since the arc picked up over the surface in the same way on test, it is reasonable to assume that the stress was approximately the same in each case. From this it would follow that when an insulator has the surface arcing characteristics that it can never have a tested factor of safety greater than one at flashover. If the sections in the insulator could be tested in a denser atmosphere permitting of a higher test potential than that which would cause flashover in the assembled insulator, a tested factor of safety might be gained even though the arc, picked up over the surface in the insulator, at flashover. This method, however, is impracticable.

FIG. 9.

FIG. 10.

Any insulator tested at flashover near sea level acquires a tested factor of safety when installed at a high altitude, owing to the lowered potential necessary to cause flashing at the greater altitude.

Fig. 11 shows the free arcing characteristic in a pin type insulator having a tested factor of safety. Fig. 12 shows a pin type having the surface arcing characteristics. In this insulator

225 kv. was necessary to flash the insulator while the aggregate test potential of the four parts was in excess of 270 kv. That the excess in test potential does not provide a tested factor of safety for some part, is due to the surface arc being formed by flashovers on the parts individually until the arc is formed over the series. The insulator really fails at 200 kv. when a small shell reaches flashing potential and spills, throwing more stress on the remaining parts. By raising the potential slightly, one of the other parts is overstressed, and the arc forms in cascade over the entire insulator, thus producing a flashing potential on every part equivalent to its test potential.

FIG. 11.

Owing to the lack of tested factor of safety and poor reliability in the ware, some designs of the disk type have shown up poorly at flashover and the dielectric strength of the single-piece disk type was unjustly condemned.

THE DISTRIBUTION OF STRESS IN THE INSULATOR

The sections may be regarded as electrostatic condensers in series, and if the same flux was carried by the dielectric in each member, the distribution of stress would be uniform.

E = potential applied to the insulator or series.
t = tested dielectric strength of a member of the series.
e = drop in potential over or stress on a section.
q = charging current for a section.
C = electrostatic capacity of a section.

For the electrostatic condenser $e=\frac{q}{c}$ and for the series

$$E=e_1+e_2+e_3\ldots e_n=\frac{q_1(c_2\,c_3\ldots\ldots c_n)+q_2(c_1\,c_3\ldots\ldots c_n)\ldots\ldots q_n(c_1\,c_2\ldots\ldots c_{n-1})}{c_1\,c_2\,c_3\ldots\ldots c_n} \tag{12}$$

Where there is the same dielectric flux in each insulator $q_1=q_2=q_3=q_4\ldots$ equation (**12**) may be written

$$E = q\;\frac{(c_2\,c_3\ldots c_n)+(c_1\,c_3\ldots c_n)+\ldots(c_1\,c_2\ldots\ldots c_{n-1})}{c_1\,c_2\,c_3\ldots\ldots c_n} \tag{13}$$

FIG. 12.

When $c_1=c_2=c_3\ldots c_n$ corresponding to practice. Equation (**12**) becomes

$$E=\frac{1}{c}\,(q_1+q_2+q_3\ldots q_n) \tag{14}$$

If both c and q are the same for each member of the series,

$$E=\frac{q}{c}\,(1+1+1\ldots n) \tag{15}$$

From equation (**15**) we get the stress on each section

$$\frac{q}{c}=\frac{E}{n}=e \qquad (\mathbf{16})$$

When n is large enough, e becomes less than t and a tested factor of safety $\frac{t}{e}$ is obtained.

If E is the stress necessary to flash insulator, for $\frac{t}{e}>1$, the arc strikes through the air from conductor to support, otherwise a flashing stress t would be placed on each section, and $\frac{t}{e}=1$ or $\frac{t}{e}<1$, when parts have been tested below flashing potential. Tests in the suspension insulator show that e varies for different sections and as c is the same for each section, equation (**14**) represents the series.

The uneven distribution of stress is caused by part of the dielectric flux taking an air path. Owing to the position of the lower section practically all of the flux must pass through from metal to metal, making q larger for this section than any other. The upper section would also be expected to carry more stress than some of the others.

Since q varies on each section, c must vary accordingly in order that uniform distribution of stress may result. To obtain uniform stress distribution in this manner would be very undesirable for each section would be different from every other, and the advantage of interchangeable parts would be lost. Although impracticable for the suspension insulator, this method has been found very valuable in distributing stress in the pin type insulator. This method was used on some of the first high-efficiency disk insulators with good results.

If the series is represented in equation (**14**) a uniformly tested factor of safety for the series may be had by making t proportional to q for each section. This would necessitate sections of different size and would be more impracticable than varying the electrostatic capacity.

Since q depends almost entirely on the electrostatic capacity of the series, a decrease in the electrostatic capacity of the series will produce a decrease in q.

The decrease in electrostatic capacity of a given length is accomplished by decreasing the length of section so as to include more in the series.

With the limits in practice it is possible to make q so low that $\frac{t}{e}$ will provide a tested factor of safety against puncture. Although the tested factor of safety varies for different units in the series. Satisfactory operation with a flashing stress on the insulator depends on providing a sufficiently large factor of safety for the end section.

FIG. 13.

FIG. 14.

Owing to too great a section length in some single-piece disk suspension insulators, no tested factor of safety was provided for any of the units, and when tested to flash over, punctures occurred, giving rise to the opinion that the single-piece disk suspension insulator was inferior to the two-part disk suspension insulator, while the opposite was the case with properly designed insulators.

That stress distribution can be controlled by change in c equation (**13**) is shown by the following cases:

Fig. 13 shows a suspension insulator composed of two sections the upper having a small electrostatic capacity compared to the lower. In the position shown, 57 kv. was required on the small insulator to cause it to arc and a potential of only 62 kv. on the series caused the smaller to be stressed to its flashing potential. The photograph was taken with 62 kv. on the series and shows the charging current of the large insulator forming an arc over the smaller. To flash over the series, required 150 kv.

Fig. 14 shows an insulator of relatively small electrostatic capacity between two sections of larger capacity. When tested alone, flashing potential of the small unit was 57 kv. The photograph was taken with 97 kv. on the series, this being sufficient to overstress the small insulator, while 300 kv. was required to flashover the series. When it is considered that the overstressed member shown in Fig. 14 adds but little to the flashover of the rest of the insulator, it is seen why some designs are very inefficient.

The economic importance of producing reliability by designing the insulators for a tested factor of safety is apparent from the following:

From the time breakage curve it is seen that the rate of breakage decreased very slowly after the knee of the curve is passed and it would take a very long time to produce the reliability that could be gained by testing for a short time and then providing a tested factor of safety. Carrying the test for 15 minutes at 100 kv. gives a rate of breakage of 0.18 of one per cent per minute and a total loss 8 per cent. The same rate of breakage on the 85-kv. curve is obtained after 10 minute test at 85 kv. or a one minute test at 100 kv., the breakage being only 3.5 per cent. Hence by providing a factor of safety of 100/85 the same reliability is obtained with a test only 1/15 as long and a saving of 4.5 per cent material is made.

That it is not necessary to lose any of the valuable characteristics in providing a tested factor of safety is seen by comparing two 100-kv. insulators in the following table:

TABLE SHOWING COMPARISON OF 100 KV. LINE INSULATORS

	Type *A* high-efficiency type	Type *B*
Number of sections	6	4
Number of shells per section	1	2
Diameter	10 in.	14¼ in.
Length of insulator	34¼ in.	41 in.
Mechanical strength	10,000	8,000
Weight of porcelain	30 lb.	62 lb.
Total weight	50 lb.	90 lb.
Number of cemented joints	12	12
Formation of arc—dry	Through air	Over surface
Formation of arc—wet	Through air	Over surface
Total tested dielectric strength	540 kv.	440 kv.
Surface resistance	K 527	K 440
Minimum to maximum width of leakage path in per cent	16	10.6
Wet flashover	265	235
Depreciation due to loss of one section	16⅔%	25%

FIG. 15b. FIG. 15a.

The table shows that type *A* not only has a higher rating but has only half the weight of porcelain and a shorter length.

The efficiency of type *B* is lowered by the overstressing of the small inner shell. The inner shell owing to its smaller electrostatic capacity spills in the same manner as the small insulator in Fig. 13, causing the flashing of the insulator at a comparative low potential.

Fig. 15 shows a comparative test on the two types. The illustration shows a two-section insulator—diameter 14½ in. (36.8 cm.) length 20½ in. (52 cm.) weight 45 lb. (20.4 kg.)—type B, flashing while a three section insulator—diameter 10 in. (25.4 cm.) length 17¼ in. (42.8 cm.) weight 25 lb. (11.3 kg.)—of type *A*, which is in multiple, has not reached flashing potential.

MECHANICAL STRENGTH

The cemented suspension insulator would have come into use at an earlier date for high-tension work if there had not been doubt as to its mechanical reliability. Although some of the insulators made over 40 years ago had the iron cap and pin cemented to the insulator member much in the same way as the modern suspension insulator, the method was practically abandoned. When it was proposed to adopt the type for high-tension work, it was considered that an interlocking feature was highly desirable. There seemed to be no doubt as to the ability to cement porcelain to porcelain successfully, but there was doubt, however, in regard to the successful cementing of porcelain to metal, as there had been some reported failures of large pin-type insulators where a large pin had been cemented into the insulator. A careful consideration of the relative coefficients of temperature and elasticity for porcelain and iron indicated that the two could be used together successfully, for the range in temperature and mechanical stress to which the insulator would be subjected.

The most feasible mechanical arrangements of parts placed the cement in shear and as there was no data at hand, the shearing stress of cement was computed by estimating the shear from cement cubes which failed by shear when tested to compression. Designs made as early as 1904 by using this method proved to be correct within a very small per cent.

Porcelain has a tensile strength of approximately 2,500 lb. (1,133 kg.) per square inch (6.45 sq. cm.), good cement has a shearing ultimate of over 1600 lb. per square inch. By making the gripping surface of the pin and pin hole efficient, the full shearing strength of the cement is developed and a high mechanical ultimate is obtained. Tests on some old insulators of this type gave an ultimate of from 10,000 to 12,000 lb. (4,535 to 5,443 kg.).

Fig. 16 shows an insulator which broke at a little over 12,000 lb. When stress is applied the pin elongates straining the porcelain, and at the ultimate the porcelain fails by combined shear and tension, this causes the break in Fig. 16.

The insulator may be designed so that the pin will pull without breaking the porcelain for the same ultimate, by changing the diameters of pin and gripping surface so that the shearing strength of the cement is reached before a breaking stress is developed in the porcelain.

FIG. 16.

The mechanical reliability of insulators based on the shearing strength of porcelain is well recognized, and insulators used for the highest stress are of this type.

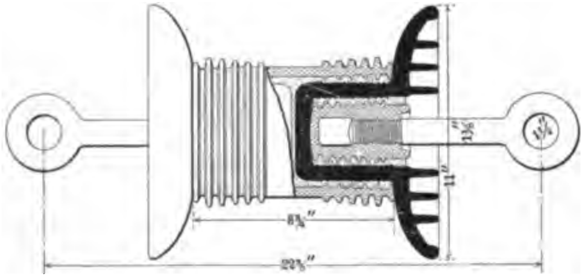


FIG. 17.

Fig. 17 shows the detail of an insulator of this type used on heavy catenary work where a failure would be very serious. The insulator is designed for a combined mechanical and electrical ultimate of 35,000 lb. (15,875 kg.) and 110 kv.

In the economic design for high ultimate mechanical strength careful consideration must be given the stresses, produced by change in temperature and relative coefficients of elasticity for the different materials as well as the shearing stress on the cement, making the problem rather difficult.

For the same cost, the cemented type is much more reliable than any interlocking type, for its connections may be tested, eliminating the personal factor. Practice has shown that unless the interlocking parts are large, the arc at puncture may destroy the connections, as the interlocking connections do not always come in contact.

The cemented type may blow up on short-circuit, but it is a question whether this would not be an advantage in locating a fault.

Reliability in practice depends on testing all insulators and connections, eliminating any weak members. Connections must be simple and positive, otherwise when installed on the line, poor workmanship may lower the mechanical reliability. The connections should be such that the replacement of a broken section may be quickly and easily made.

High mechanical strength is obtained in the high-efficiency type by making the gripping surfaces effective and developing the full shearing strength of the cement, permitting of very small metal parts. This is important as the metal in the insulator is a large part of the cost, and to obtain insulation with large metal parts it is necessary to increase the size of the porcelain for the same amount of insulation.

The remarkable improvement in efficiency has not been confined to the suspension insulator alone, great improvement being made in the pin type as well. With improved manufacturing conditions greater improvements will be possible, reducing the cost of reliability in the transmission system.

The electrical advantages of efficiency in design are greater dielectric strength, high surface insulation and lower depreciation. The economical advantages are lower cost of production, lower weights, resulting in a saving in transportation and erection, greater length efficiency, permitting of a saving, in the suspension type, in the height of towers and length of cross arm.

With the increase in the size of the transmission systems reliability will be more important, and the elements of reliability in the insulator will receive more of the attention that they deserve.

DISCUSSION ON "THE HIGH EFFICIENCY SUSPENSION INSULATOR." CHICAGO, JUNE 30, 1911.

E. E. F. Creighton: The fact has been brought out a number of times during the past year that the great problem at the present time in high tension transmission is really confined to the line, and is a matter mostly of protection against lightning on the line. Personally I have a great admiration for the work the author of this paper has done in perfecting insulators. Although the methods used differ from the methods of observation used in the lightning arrester laboratory, Mr. Austin gets, apparently the same result. I have not had an opportunity to read the entire paper, but have personally talked with Mr. Austin about his methods of observation, and as I see them, they consist in observing the successive flooding of the insulator parts by brush discharge when the potential is gradually increased. In that way the relative stresses can be observed over the different parts of the insulator. In the lightning arrester laboratory the relation of potential to puncture and potential to flash around the skirts of an insulator is determined by a sudden discharge from Leyden jars with a frequency approximating about one million cycles per second.

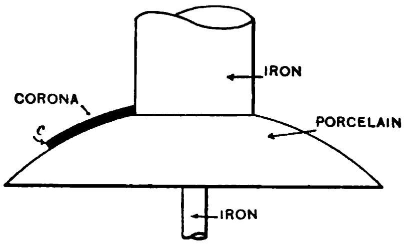


FIG. 1

Since the problem is not dielectric strength for dynamic potential, but the protection against lightning, it is important to have tests that will give an indication of the effect lightning will have. The ordinary method of testing insulators, that is, gradually applying potential, does not give any indication of what the insulator will do under lightning stresses. This point has been brought up before. To illustrate, if this is the porcelain disk (see accompanying illustration Fig. 1) and the potential is applied between the top and the bottom, and is gradually increased, the effect is to produce corona streamers which run out more or less from the terminals, and before the spark takes place the distance between the terminals has been very materially reduced, so that one might say that the spark took place from some point *C* out on the skirt of the insulator around to the other terminal. Now, when a high frequency of anywhere from 500,000 to 3,000,000 cycles is applied to the same insulator, the discharge has to jump the entire distance from metal to metal around through the air or puncture the porcelain. I have made many tests on the pin type of insulator. One example will suffice for illustration. By putting on gradually applied potential at 60 cycles, an insulator flashed around some 500 times. An electrostatic charge was then discharged against the insulator, and it punctured immediately. The methods used by Mr. Austin, watching the gradually growing brush discharges, makes it possible to observe the weak parts individually and to increase the dimensions of the porcelain accordingly, so as to prevent puncture. I tested some of these insulators recently, and a thousand charges were discharged against a single disk, of such a value that they would just not spark around. I wanted to see if there was any possibility of fatiguing the material, and so far as I could see, there was not. By raising the discharge potential slightly, it would spark around the insulator each application. Unfortunately, we have not been able to get enough static potential to spark around a string of these insulators in series.

So far as the surface resistance is concerned, I believe it has comparatively little to do with protection from lightning—I am speaking of such a thing as a slight accumulation of dirt on the surface. Although I have not tested out this particular feature, similar tests with high frequencies show there is very little decrease in the equivalent-needle-gap by using high resistance in parallel. There is one other point, and that is regarding the multi-gap effect—it has been commented on many times, and has been tested by a number of engineers.

There is no doubt from the consideration of Dr. Steinmetz's theory, on the multi-gap arrester which was developed in 1906, that the same conditions exist to a greater or less extent in the suspension type of insulator, and it is quite sure that some types of these insulators will spark very much easier on high frequency than on low frequency. In regard to this, it seems to me that the conditions shown in Fig. 10, would illustrate that it is possible to make a design of these suspension type insulators, in which the multi-gap effect has entirely disappeared. Dr. Steinmetz showed at the time he made his investigations that the multi-gap effect in a multi-gap arrester depended upon the value of the capacity between cylinders relative to the capacity of each cylinder to ground. If the capacity from the cylinder to ground was diminished, then it was possible to do away entirely with the multi-gap effect. The subject was further developed by Messrs. Rushmore and DuBois a year later, and all that material is available at the present time and might be profitably used. In connection with the design of these insulators, referring to Fig. 9 and Fig. 10, I think it is evident that Fig. 9 would give less multi-gap effect than the insulators in Fig. 10.

In closing, I should like to ask Mr. Austin if he has noted any fatigue in insulators, whether there is any possibility that an insulator under test will be damaged. It is an important thing to decide, in order to be able to test insulators installed on a line to eliminate faulty ones at times of service when their failure would do the least amount of harm. I know in one case of apparent fatigue of a 100,000 volt insulator. When it was first put on the line, it was entirely quiet, but after it had operated for several months, it became noisy and finally broke down.

Paul M. Lincoln: I have not read this paper carefully but I notice that Mr. Austin has devoted considerable attention to the question of leakage, leakage current and leakage resistance paths of insulators. It becomes important, in the design of insulators, to determine whether the equipotential surfaces are determined by the question of leakage over the resistance paths of an insulator, or whether it is determined by the question of condenser capacity. For instance, these suspension insulators are in effect conductors which are separated from each other by some insulating material. Line potential is applied at one side and the ground potential is applied at the other side. Now, the total potential is distributed across this whole combination, and the ideal insulator is the one in which the potential is equally distributed across the various units and the individual sections of each unit. When that condition is obtained, we have the ideal condition. It becomes important to determine whether the potential of the equipotential surfaces is determined by the current which leaks across the resistance paths, or whether it is determined by the condenser capacity between the equipotential surfaces.

Now, there are some facts which have been observed which to my mind indicate that the potential of these surfaces is determined almost entirely by the static capacity, and very little by the leakage over the resistance paths. The main fact that so indicates is that some insulators, and in fact many insulators, will have a flash-over voltage wet as nearly as high as when dry. What does that mean? A wet insulator undoubtedly has a much lower resistance over its surface and consequently a much higher leakage current than a dry one. Further, the distribution of potential due to these leakage currents must be different when wet than when dry, since surfaces previously dry are now wet. As a consequence, one would expect to have the distribution of potential and the flash over voltage different in a wet insulator than in a dry. The fact that there is practically no change in the flash over voltage between a wet and a dry insulator would indicate that the equipotential surfaces are determined by static capacity rather than by leakage across resistance paths.

P. H. Thomas: I would like to add one item to Mr. Creighton's explanation of the multigap effect, so called, illustrating how important it is. There was a transmission line put up, I think about 1900 in Colorado, in which lightning arresters were placed out doors in boxes, the arresters being of the multigap type, and they were placed in as small boxes as possible in order to economize space. It was a 20,000-volt line. The gaps connected in the two line wires, as I remember it, were placed in "V" fashion—ground connection being at the point and one line being connected to each leg. For some strange reason, it seemed strange then, the arresters would not hold the line voltage, they would "slop-over", as it was called, extremely easily, much more so than in the laboratory or in other installations. The reason for this is clear. The capacity of cylinders connected near one line, on account of the proximity of the cylinders connected to another line, is much greater than it would have been if isolated—that is the minute charging currents were greater. This condition makes no trouble in the first cylinder, because it is connected directly to the line, but the second cylinder and the others have to receive their small charges by drawing sparks from these cylinders connected to the line so that the current in these gaps is greater, when you have two closely adjacent cylinders connected to opposite lines. This condition caused a great deal of actual trouble. I have forgotten at this time what the solution of the difficulty in this case was, but in 1902, I repeated some experiments to prove the correctness of my theory above stated which is substantially the same as outlined by Professor Creighton. In the experiment cylinders were taken and arranged in a "V" shape upon a board. A very much larger number of units would be jumped by a given voltage in this arrangement than when the cylinders were arranged in a straight line. I think the factor was as great as 2 to 1, the difference between the two arrangements being due to the lesser charging current taken by the cylinders in the straight line arrangement than in the arrangement in the V form. I think this experiment was described in 1905 at the Asheville Convention. It is a matter of importance, not only in the cylinders of lightning arresters, but in all types of electrical apparatus. If you want to be free from sparking trouble, such as jumping over surfaces on bushings, and other such conditions, keep your high potentials as far apart as possible.

For instance, in an ordinary insulator where the ground potential we will say is carried by the pin up pretty close to the conductor, there will be a much greater tendency to jump over the petticoats than if the insulator were made entirely of porcelain and the ground and line wire widely separated.

E. M. Hewlett: We investigated this insulator trouble, and we found right away that the parties who were interested wanted to know a great deal more about the disk insulator than they had found out about the pin insulator, in fact, more than they have found out as yet. We made practical tests. We assumed that if we could get porcelain of the proper dimensions and thickness to withstand puncture—to flash over rather than to puncture—that that was the safest line to follow. Since that time some insulators have been made larger than they should be, that is, they are likely to puncture before they flash over. It seems to me that the best design possible is to make them so that they flash over rather than puncture, and also to space them closely enough together so they will flash over the whole set rather than individually, to bring the arc over and around the disk.

In reference to the fatigue of porcelain, a good many persons consider there is a fatigue in porcelain, but what is often considered to be fatigue, I think is low burning. The porcelain is not thoroughly vitrified, and for a time it stands up satisfactorily. The dampness is absorbed and then the strength is decreased and the insulator will consequently puncture. It would seem therefore that the best general plan would be to make the disk small enough and of good enough material, high enough grade of material, and well enough vitrified, so as to resist puncture. Shape them so they will stand a maximum weather test, and not be reduced too much on the line test, and then use enough disks in series to get the desired result. I think with a small diameter of disk there is much less likelihood of puncture from the lightning as the path of the lightning does not deviate so much from a straight line in going to the ground.

N. J. Neall: Where does the analysis of punctures per minute begin, and, further, is the record of a given batch of insulators after installation of value in this connection? I have seen failures on the test table which seemed to indicate imperfect material. I think there would be a point where the imperfect material would be dismissed and the criterion Mr. Austin selected would begin?

A. O. Austin: There is probably no appreciable fatigue in good porcelain under normal electrical stress. The puncturing being due to mechanical defects or poor material.

When an insulator punctures the effect on operation is the same regardless of the cause of failure, and it matters little whether there is fatigue in the material; the number of failures for a given stress and time, however, is of great importance. With an increase in the dielectric strength of insulators the percentage of breakdowns due to stresses above those which may be shown on the time breakage curves, diminishes rapidly and the time breakage curves become valuable in predicting failure when comparing different insulators.

If a number of insulators are subjected to stress and the rate of puncture becomes less with time, the insulators can hardly be said to show fatigue for the remaining insulators have improved in reliability. If, however, the rate of puncture increased with time, the insulators may be said to show fatigue for the reliability has decreased. Then for practical purposes it may be said that there is no fatigue unless the puncture rate increases with time and applied stress. These characteristics can be shown without going out on the line, and should form the basis for dielectric strength and reliability.

The distribution of stress between the several parts of an insulator may vary widely for different conditions, and it is possible to design an insulator such that the flashing potential is higher under storm conditions than when dry. It was found that too much space would be required to discuss stress distribution, an example, however, will show how the electrostatic capacity and the surface resistance may effect the performance under different conditions.

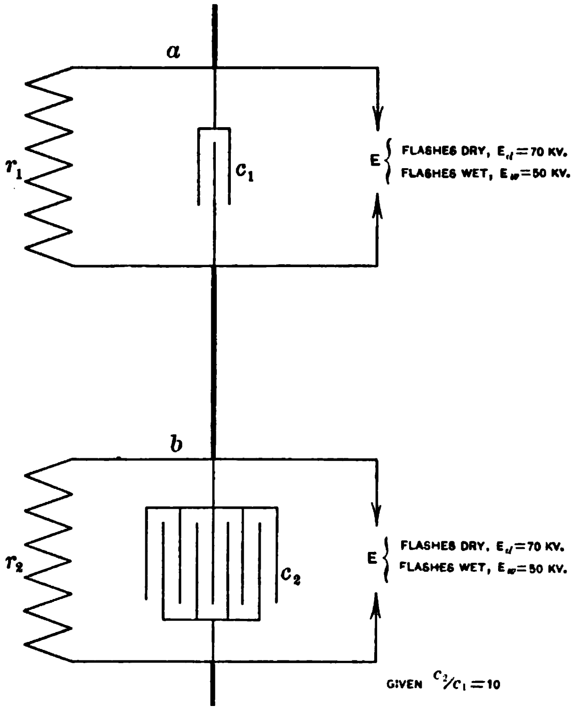


Fig. 1

The accompanying illustration Fig. 1 represents two insulating elements in series, each element having an electrostatic capacity C surface resistance r and flashing potential equivalent to E_d for dry conditions and E_w for wet conditions.

For dry conditions, r_1 and r_2 are very high and the leakage current may be neglected the stress distribution being determined by the members acting as electrostatic condensers.

For this condition a carries 9/10 of the stress and flashes when the voltage applied to the series reaches 78 kv.

When a flashes, the arc forms a shunt reducing the stress on *a* and increasing it on *b* and by increasing the stress on the series by a few kilovolts, the series flashes. The capacities may easily be made large enough so that 85 kv. will cause complete flashing.

When the insulator is wet, the surface resistance may be so low that the surface leakage current will determine the stress distribution. If the elements have practically the same surface resistance the stress would be the same on each part and the flash-over of the series would be 100 kv. which is higher than the dry flashing potential.

The electrostatic capacity, surface resistance and flash-over vary greatly with a change in conditions. It is difficult to predict the performance of an insulator except for limiting cases, which is usually sufficient as the intermediate conditions are less severe.

The effect of design on depreciation was given considerable attention in 1904 as it was seen that where the surface current was large the insulators coated very rapidly. This was largely due to the accumulated material left by the water, upon being evaporated by the current. Where the evaporation is large the washing action of the rain is practically eliminated and the insulator will depreciate rapidly.

In some designs the charging current increases very rapidly with a slight surface depreciation. To eliminate this, the all porcelain designs shown in Fig. 4, were made. Since there was a considerable difference in potential between the upper and lower surface of the flange it formed a condenser with a comparatively large charging current, the performance being considerably below that expected until the electrostatic capacity of the flange was reduced by the addition of petticoats. By making the insulator with discs with low electrostatic capacity the presence of the metal connection was not so noticeable and compared favorably with the best all porcelain types.

The stress distribution under the high frequency surge may be quite different than that at normal frequency, even at flash-over of the insulator. Insulators which have a small air path for the forming of the arc between line and ground, will be much better protected than those having a long air path, although the insulators may have the same flashing potential at normal frequency.

It is a question as to how much insulation is needed for the extremely severe conditions on the line. A consideration of the different characteristics will probably lead to an increase in the dielectric strength over what it is at the present time.

A paper presented at the 28th Annual Convention of the American Institute of Electrical Engineers, Chicago, Ill., June 30, 1911.

TENTATIVE SCHEME OF ORGANIZATION AND ADMINISTRATION, FOR A STATE UNIVERSITY

BY RALPH D. MERSHON

Some time ago the writer had occasion to look into the matter of the organization and conduct of universities, especially state universities, with a view to making some suggestions along these lines in connection with a state university. A search for printed matter bearing upon the subject seemed to indicate a scarcity of available information of a specific nature and most of such matter as was available bore upon certain phases, only, of the subject. It also seemed to indicate that there is considerable room for improvement in the present organization and administration of most, if not all, of the state universities.

After studying such material as an ordinarily careful search revealed, I undertook to draw up a general scheme for the organization and conduct of a state university. The result of my endeavors, in its final form, is given below. It is limited to a state university for the reasons above outlined, since there are certain features in connection with a state institution of this sort which do not exist in endowed universities and which render difficult, if not impossible, some of the methods applicable to the endowed institution.

The object in presenting this scheme is to provoke such discussion, and consequent elucidation of the subject, as will make it possible to draft a thoroughly workable organization scheme generally applicable to state universities. Such scheme, to be generally applicable, must be very general in its nature, dealing mainly, if not entirely, with fundamental matters, since each particular case will involve many conditions peculiar to itself necessitating detailed development of the general scheme to fit local conditions.

The scheme as presented herewith was prepared after discussing the subject with several persons who have had to do with the organization and conduct of large public utility and industrial enterprises, but before any discussion had been had with anyone connected with university work. Since it was drawn up it has been submitted to university workers. Some of their criticisms of it strike me as well grounded (especially such as apply to portions of the scheme which for the sake of discussion I purposely made rather drastic), some as open to question, and some as of little weight. In spite of the fact that some of the criticisms appear to me fully pertinent I have concluded to present the scheme as originally drawn up that the objections to it may be brought out in open discussion.

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SCHEME OF ORGANIZATION

The proposed scheme is shown in the Diagram of Organization and Administration appended hereto. It will be more readily comprehended in connection with the following explanatory matter, which, however, does not attempt to go into minute detail, but merely to outline the more salient and important features.

President. The President is to be the executive head of the university, and is to have the necessary authority to that end. He is to be the representative and general executive of the board of trustees, in all university affairs, except as otherwise directed by the board, and it shall be his duty to enforce all the rules and regulations of the board and of the faculty.

He shall have the right of veto of any action taken by the university faculty and of any action, except such as has to do with the budget, of the university council. But in case of the exercise of such veto power the faculty or council, as the case may be, may pass the matter involved over the president's veto by a two-thirds vote of the full membership of the body in question. In case the matter is passed over the President's veto, he may, whether it be a matter which would regularly come before the board of trustees and the alumni advisory board for their approval or not, submit the matter to the board of trustees and the alumni advisory board, together with a statement of the situation and his recommendations against the action of the council or faculty.

The president's other duties and powers will appear more fully from what follows.

Secretary. The duties of the secretary shall be those usually pertaining to such an office, and more particularly as laid down in the "rules and regulations of the board of trustees". He shall be the financial officer of the university, shall be bursar of the university and shall through the purchasing agent make all purchases for the university.

Treasurer. The duties of the treasurer shall be those usually pertaining to such office, and more particularly as laid down in the "rules and regulations of the board of trustees".

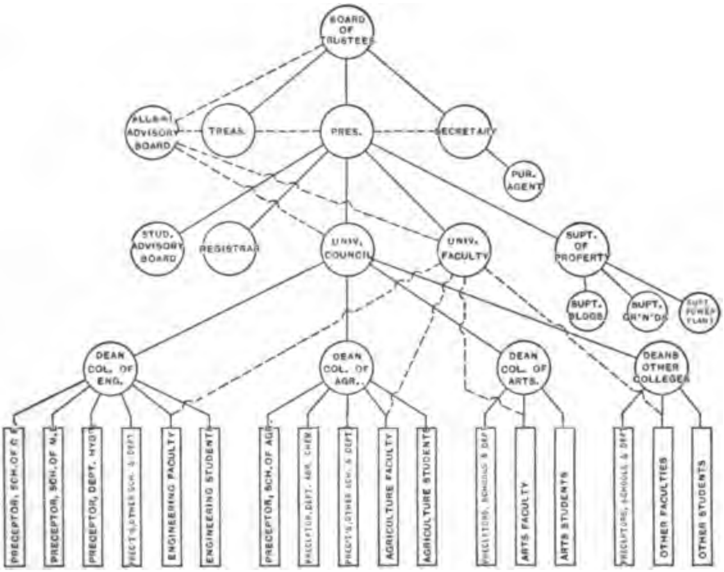

Diagram of organization

Solid lines indicate relations more or less directly subordinate; dotted lines indicate relations co-ordinate, or merely advisory, or both. Only the more important relations are shown.

University Faculty. The university faculty shall be the legislative body of the university in all matters relating to the conduct and control of the student body. The university faculty shall be made up of the deans of the various colleges, preceptors of the various schools, preceptors of the various departments of instruction and all instructors of the university holding the rank of professor or associate professor.

The president of the university shall be, *ex officio*, chairman of the university faculty, and as such shall have no vote except in case of tie.

University Council. The university council shall be made up of the deans of the various colleges of the university.

In consultation with, and through the president, it shall be the executive body of the university.

In consultation with the president, it shall be the coördinating body of the university.

The president shall be, *ex officio*, the chairman of the university council, and as such shall have no vote except in case of a tie.

Alumni Advisory Board. The Alumni advisory board shall consist of twenty alumni elected by the membership at large of the university alumni association. Four members of the board shall be elected each year, so that after the board shall have been established five years the term of office of each member shall be five years.

In order to be eligible to the alumni advisory board, an individual must be an alumnus of at least ten years' standing, who has done at least four years' work at the university in the acquisition of a degree from the university, and who is a member of the university alumni association.

The function of the alumni advisory board shall be to advise and, by its influence, aid the president, the board of trustees, the university council and the university faculty, in any and all matters pertaining to the welfare of theuniversity.

No appointment to any position in connection with the university shall be made without the approval of the alumni advisory board.

The alumni advisory board shall from time to time, through visiting committees appointed by it, investigate the various departments of the university for the purpose of determining their efficiency, and for aiding them, either by suggestion or advice, or by using its influence in the matter of financial or other aid.

These visiting committees may or may not be limited in their membership to the members of the alumni advisory board. That is, the alumni advisory board may avail itself of the assistance of specialists in regard to any matters with which none of the members of the board are particularly conversant.

Visiting committees shall report directly to the alumni advisory board. The reports shall be disposed of in accordance with the judgment of the alumni advisory board, whether they be made public, transmitted to the board of trustees, transmitted to the president or retained by the board without presentation or action. Provided, however, that if any report be transmitted to the board of trustees there shall be transmitted, at the same time, a copy of the report to the president. And provided, that if a report be made public, one week prior to its being so made public there shall be transmitted a copy to the board of trustees and to the president.

No member of the alumni advisory board shall at the same time be a member of the board of trustees, or be in any other way connected with the executive, financial or teaching staff of the university; or hold any office in the alumni association.

Any vacancy occurring in the advisory board by death, resignation, or otherwise, shall be filled by the board until the next regular election of the alumni association; at which meeting, in addition to the members to be regularly elected, there shal be elected a member to fill the remainder of the unexpired terml (See note in regard to alumni advisory board).

Deans. The deans of the various colleges shall be appointed by the president, with the approval of the alumni advisory board, under confirmation by the board of trustees, and presumably with the advice of the faculties of the respective colleges.

Each dean shall be responsible to the university council and the president for the conduct of his college.

A dean shall hold office so long as the administration of his college is such as is productive of satisfactory results.

A dean may do as much or as little teaching as he considers expedient. Such teaching as he does shall be confined to his own college.

The dean shall, at the proper time, make up a budget for his college for presentation to the university council, after the manner hereinafter described under " Budget ". In making up the college budget, the dean shall consult with the preceptors of the various schools of his college and the preceptors of the various departments of his college, endeavoring to arrive at a budget which shall represent the consensus of opinion of the various preceptors and himself.

Where it is not possible to arrive at a unanimous agreement of the preceptors, together with the dean, relative to the budget, the dean shall submit the budget he recommends, together with any exceptions to it which any of the preceptors of his college may desire to make as individuals or as a group. Such exception may take the form of the recommendation of an entirely different budget.

College Faculties. The college faculties shall be made up of all the instructing force of the college above the grade of instructor. In case of an individual giving instruction in more than one college, he shall be considered as a member of the faculty of that college to whose work the greater portion of his time is devoted. If, however, it seems advisable that he should be a member of other college faculties as well, he may, if he so elects, serve upon such other college faculties if elected to them by the respective faculties in question.

The college faculty shall have charge of all minor legislation relative to its own college not covered by the legislation of the university faculty. It shall, through its committees, coördinate the work of its various schools.

Schools and Departments of Instruction. The work of each college shall be grouped into one or more schools, at the head of each of which shall be a preceptor, and into one or more departments, at the head of each of which shall be a preceptor. It may follow in some cases that the preceptor of a school will also be preceptor of a department.

Superintendent of Property. The superintendent of property shall report directly to the president. He shall be responsible for the buildings, the grounds and the power and heating plants, through the superintendents of these various departments.

Student Advisory Board. The student advisory board shall be made up of two seniors, two juniors and one sophomore, to be elected each year by their respective classes. This board may, of its own initiative, bring matters directly to the attention of the president. It may, on request of the president, advise with the university council, the university faculty or the alumni advisory board. It may not, of its own initiative, bring matters to the attention of any of these bodies.

Budget. The president shall, near the end of each year, prepare and present to the board of trustees a budget based upon the estimated income of the university for the next year. This budget shall be made up as follows:

The dean of each college shall prepare a budget, by consultation with the preceptors of his college, as provided under the head of " dean ". The university council, consisting of the deans, shall then meet and in conjunction with the president correlate, coördinate and adjust the budgets of the various deans to accord with the estimated income of the university, and with the university requirements other than those of an instructional nature.

In case a unanimous agreement is arrived at, the result of the budget shall be transmitted to the board of trustees by the president as the budget of the university council and himself.

If a unanimous agreement is not arrived at, the president shall transmit to the board of trustees, for action by the board, a majority report, a minority report and his own recommendation, which may or may not agree with either the majority or minority, together with all the necessary papers, both those having to do more particularly with the deliberations of the university council and those having to do with the deliberations of the various college faculties.

Amendments. Amendments to the scheme of organization and administration will preferably be instituted by the university faculty, with the approval of the alumni advisory board, and will be subject to confirmation by the board of trustees; but, however the amendments be instituted, they shall not become effective unless approved by the alumni advisory board.

NOTE IN REGARD TO ALUMNI ADVISORY BOARD

It is contemplated that the alumni advisory board shall have no direct connection with the alumni association and shall perform no function in connection with the association.

The alumni advisory board will be clothed with a considerable amount of power which should be wisely and carefully exercised as the result of the deliberations of, as nearly as possible, the full board. For this reason, the membership of the board will preferably be drawn from those of the older alumni as have had wide experience, and who, previous to their election, shall have stated their ability and willingness to faithfully attend the meetings of the board.

Failure of the board to act promptly might seriously hamper the conduct of the university. In order to insure prompt and regular attendance at the meetings of the board, it will be advisable to penalize tardiness or absence. Provision might be made that if a member is either tardy or absent from a meeting twice during his term of office, without being excusable, he shall automatically cease to be a member of the board; with the further provision that to be excused from attendance or prompt attendance, a member must have obtained leave from the board meeting immediately preceding the one in question; if the necessity for being excused cannot be foreseen in time to obtain such leave, the member must have been granted leave by the president of the university, which leave shall be valid only if confirmed by the meeting to which it applies.

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As previously intimated the above scheme is presented, not as one which even approaches perfection, but as the basis for discussion which it is hoped will lead to a scheme as nearly as possible perfect. It is very desirable that the discussion include the views of those in responsible charge of large industrial establishments as well as those connected with university work. The attitude of many of the latter, in the past, seems to have been that the problem is one so different from all other administrative problems that it occupied a class entirely by itself. And that no parallels could be drawn and no lessons of value learned from industrial organization. While the underlying basis for such an attitude is undoubtedly sound when properly applied, it does not, it seems to me, apply to the problem in its entirety. While there is a wide difference at some points of the respective processes, between the problem of turning out shoes, for instance, and that of turning out mentally trained men, there are other parts of the processes, or their administration at least, which seem to me closely parallel, if not identical, and, therefore, subject to similar treatment.

DISCUSSION ON "TENTATIVE SCHEME OF ORGANIZATION AND ADMINISTRATION OF A STATE UNIVERSITY." CHICAGO, JUNE 30, 1911.

A. H. Ford: The Institute is to be congratulated upon having a paper of this kind presented to it for consideration. Among the papers dealing with educational subjects, presented in the past, have been those by practicing engineers indicating the kind of training which the young engineers whom they employ should have and those by educators saying that it was impossible to give such a training. Now there is presented to us a problem of organization on which it seems as though the engineers and educators might agree, as the problem of organization is essentially the same no matter whether the product is electrical energy, dynamos or trained men.

As the scheme of organization presented by Mr. Mershon does not differ materially from various schemes now in operation the speaker will point out some of the defects of the present systems.

First: The fixing of authority is indefinite, particularly as to distinctions between management and administration; with the result that frequently the board of trustees decides on matters of administration concerning which it has insufficient knowledge to act intelligently. The natural result is that the administrative officers are unable to maintain a definite policy and consequently find it difficult to obtain proper coöperation of those under them.

Second: The making of the secretary, or other financial officer, responsible to the board of trustees instead of to the president gives rise to a two headed organization which frequently results in the control of the educational policy of the institution by the secretary, through his control of the budget. This is because the board of trustees naturally looks to the secretary rather than the president for advice regarding financial matters and decisions regarding the amount to be paid for equipment and professors' salaries have a material affect on the educational policy of an institution.

Third: The activities of the students outside of the class room ordinarily receive little attention, with the result that many students when freed from restraint in this way acquire habits which make the ordinary routine of business irksome. This is no doubt the cause of the frequently repeated remark that a college graduate is of not much use in the business world until he has been away from college for two years.

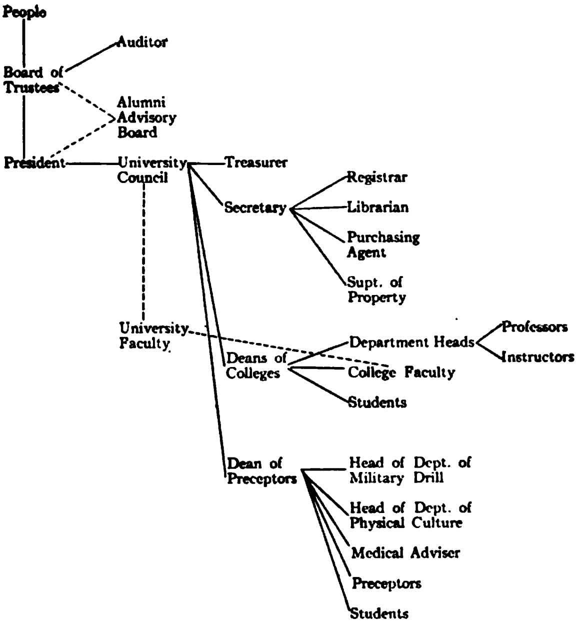
There are two provisions in Mr. Mershon's paper which stand out prominently. The alumni advisory board and the method of fixing a budget to be presented to the board of trustees.

An alumni advisory board of the kind contemplated should prove very useful; but it should not have the duty of confirming appointments to the teaching staff for such appointments frequently require confirmation with great despatch and cannot wait for the assembly, on call or at a regular meeting, of such a large body as the alumni advisory board.

The method of fixing a budget to be acted on by the board of trustees is admirable and should do much to overcome the bad effect of having the secretary report directly to the board of trustees.

The following plan of organization, shown in Fig. 1, is suggested as a means for overcoming the defects noted. The duties of the various officers are given only where they differ from those in the plan proposed by Mr. Mershon.

Board of Trustees. The board of trustees is the direct representative of the people and as such should dictate the general policy of the university and represent the university before the legislature. Being nearer the people than the members of the faculty the members of the board of trustees may see fields which could be profitably covered and which have escaped the observation of the faculty. It should not burden itself with details of administration but should confine its activities to such large matters as determining the general policy of the university, appointing the president, confirming the appointment of deans and other officers of equal grade, passing on the budget and approving plans for new buildings.



Solid lines indicate subordinate relations. Dotted lines indicate advisory relations.

President. In addition to the powers and duties named the president shall, on the advice of the university council and subject to the confirmation of the board of trustees, appoint the deans, secretary and treasurer.

University Council. The university council shall consist of the deans, the secretary and the treasurer. The secretary shall be secretary of the council. The secretary and the treasurer are added to the council in order to have their advice in matters which involve the expenditure of money or which require a knowledge of the past actions of the council or any of the faculties

Secretary. The secretary shall have charge of all records of the university except those relating to finances and shall be secretary of the university council and of all faculties. He shall appoint all his subordinates subject to the confirmation of the president.

Treasurer. The treasurer shall have charge of all moneys and credits and shall keep a record of all financial transactions. He shall be the bursar of the university.

Deans of Colleges. In addition to those duties mentioned, the dean of each college shall appoint, subject to the confirmation of the president, all officers of instruction in the college.

Dean of Preceptors. The dean of preceptors shall be responsible to the university council and the president for the conduct of the students when outside of the class room. He shall have supervision of all student organizations and shall see that the accounts of such organizations are audited by the auditor of the university. The departments of military drill and athletics shall be under the dean of preceptors.

College of .... Each college shall be organized into departments each of which shall be in charge of a professor who shall also be director of research in that department. Under him shall be such associate professors, assistant professors and instructors as may be necessary for the proper conduct of the department.

Preceptors. A preceptor is a person of the rank of professor or assistant professor who has charge of the students at such time as they may be outside of the class room. He shall act as their confidential adviser and shall give them such advice as to methods of study and conduct as may be desirable. There shall be at least one preceptor for each 50 students lodged in the university dormitory. He shall be head of the house and as such shall see to the proper enforcement of the dormitory rules. The different use of the term preceptor from that of Mr. Mershon is to be noted. It is not intended to limit the activities of the preceptors to such as are concerned with those students who live in the dormitories; but each student in the university, is to have a preceptor who shall be in *loco parentis* for that student, the number of students assigned to a preceptor being small enough so that he may become personally acquainted with each.

Medical Adviser. The medical adviser to students shall have the rank of professor and shall be under the dean of preceptors.

His duties shall be the giving of medical advice to students but shall not include treatment when ill. He shall also advise the dean of preceptors in regard to matters affecting the health of the student body.

Registrar. The registrar shall keep a record of the academic work of all students and perform such other duties as may be assigned him by the secretary.

Librarian. The librarian shall perform such duties as usually appertain to that office and in addition shall be superintendent of university publications other than announcements and those of like nature.

Superintendent of Property. The superintendent of property shall have charge of all property of the university except moneys and credits and the erection of all buildings. He shall report directly to the secretary.

Purchasing Agent. All purchases made for the university shall be made through the office of the purchasing agent. He shall report directly to the secretary.

University Faculty. The university faculty shall consist of the members of the university council and all persons on the university staff having the rank of professor or associate professor. Assistant professors shall have seats in the university faculty but shall not have the right to vote.

The university faculty shall be the legislative body of the university in all matters relating to the student body, entrance requirements, graduation requirements and such other requirements as affect more than one college.

College Faculties. Each college faculty shall consist of all the members of the college staff above the rank of instructor. The instructors shall have seats in the faculty but shall not have the right to vote. The college faculty shall be the legislative body of the college in all matters of a pedagogic nature and shall be the final authority in fixing the courses of study.

Auditor. The auditor shall be the direct representative of the board of trustees and as such shall audit the accounts of the treasurer and act in an advisory capacity to that officer. He shall also audit the accounts of all student organizations.

C. F. Harding: For a number of years we have had before the Institute and other conventions considerable discussion of the technical university and its product—by technical men in manufacturing institutions and in the engineering field in general. It seems to me that this is a very healthy condition of affairs. It would be too bad, if the manufacture of an article should be entirely neglected by the consumer of that article, and it is equally serious if the output of the technical university is entirely neglected during the university period by those in charge of the corporations to whom the university graduate is going. I am particularly pleased, therefore, to congratulate the Institute, upon the papers which it has had in past years and upon the very general discussion, which has been entered into by those in charge of university departments and by the engineering profession generally.

The paper which we have this year I regard as confined largely to the organization and administration of the university, and not to those points which, it seems to me, are most vital in the development of the technical graduate, namely, the curriculum and the administration of that curriculum in class room and laboratory. I wish, therefore, the discussion could be broadened out and not limited to the rather narrow question of university organization and administration. If this plan were put in vogue and standardized in the various universities we would not be benefitting, in my opinion, the output of these universities. We would be expediting the business of the university, and possibly indirectly, through the alumni advisory board, improving the output to some extent, but it seems to me that the discussion and the resulting changes in the curriculum, and in the administration of that curriculum in class room and laboratory, would yield very much greater results. Limited as we are, however, to the organization and administration of the university, there are one or two things I wish to say along the lines of this paper.

The university organization could be, in my opinion, quite similar to the organization of an industrial corporation. I see no reason why it should be materially different from such an organization. I do not see, however, how such an organization can be standardized for a number of universities, particularly state universities, where local and legislative conditions are bound to limit such standardization. Nevertheless, if we cannot have a standardized organization, as this paper infers we may have, I think we can gain from this paper a great number of valuable suggestions which can be picked out and applied to the organizations of the various universities.

The most radical departure from the present organization of most universities as has been stated, is probably the formation of the alumni advisory board, and it seems to me that this is directly along the line of coöperation between the university and engineers and manufacturers in the field, which, as I stated at the beginning of my remarks, is of so much value. If the alumni ten years out of the university, who have been through that university, know its problems, who have been through ten years of practical and professional experience, can be brought into vital touch with the organization of the university, it seems to me that this coöperation which I have spoken of so favorably, would be greatly enhanced, to the ultimate benefit of the graduate of that university not only, but to the prestige of that university as well.

The formation of such an alumni advisory board as has been suggested in this paper, while beneficial, as has been stated, would, if carried out in detail as suggested here, greatly cripple, to my mind, the business of the university. I believe that such an advisory board, if organized at all, should be a fairly representative one, should include engineers from all branches of the profession, and from all parts of the country, and, therefore, with a board of twenty of such men, it would be extremely difficult to get them together at a single meeting sufficiently often to enable them to carry out the duties which have been imposed upon them by an organization such as we have in this paper. To reduce these difficulties, without introducing any serious disadvantages, it would be possible to reduce the number of such an alumni advisory board by a large majority, from twenty to four or five, and to limit its jurisdiction to the purely advisory duties as suggested here, with the exception that they should advise, not with regard to the appointment of all the members of the instructional board of the university, but the heads of departments, heads of schools, and deans. Now if the alumni have an active part in suggesting the heads of departments and deans, and if the election or appointment of the subordinate instructors is left to the heads of departments and deans, the up-to-dateness of the faculty, if I may use that word, will be maintained. I assume it is largely that quality which the author of the paper wishes to maintain in the faculty—the prevention of the faculty from getting into a rut, as is often the case, and becoming out of touch with practice, and therefore becoming a detriment to the output of the university. If this alumni advisory board can be limited in this way, it seems to me it would be a great improvement, not only over the present condition, but over that set forth in the organization mentioned in this paper.

I regret that there are comparatively few of the engineers in the field here today, as I had hoped we would have an active discussion from these men. I believe they are the men with whom we should coöperate in turning out the best and most efficient product from our technical universities.

B. B. Brackett: I believe thoroughly that the administration and management of our universities can be improved, and I believe that they can be improved by the advice of those who are outside. I have no patience with those who maintain that the university is a world by itself, and is subject to its own laws and its own rules, methods of management and government.

We are to be congratulated in having this subject brought up. But I do not like the author's suggestion of an alumni advisory board that might be a controlling factor. As I understand it, his alumni board must approve all appointments, and in that way it would possess real veto powers. To my mind there are serious dangers in such a plan. It is possible that it might lead to results quite the opposite of those suggested by the last speaker. You cannot have an alumni board, unless that alumni board is elected by the alumni. Are the representatives of the alumni likely to be any more expert in educational or technical lines than the average member of the alumni association? Is the average alumnus of our colleges and universities an educational expert? Is the average one well enough informed to take part in the management of a university, to the extent of saying who should be on the faculty or who should not?

In comparatively recent years, two of our well known universities have had radical upheavals in management and internal organization. In both cases investigation will show that the alumni were with the old order of things and not with the progressive element. There is a possibility of an institution being away behind the times, in the ability of its faculty, its general organization, and the character of its work. And it is possible for the mass of the alumni to think that this institution is one of the best in the world. In fact, it is quite natural for them to think so, and to stand as strongly as they can for the old order of affairs.

An alumni board with such influence in the appointment of faculty members might foster excessive inbreeding in the personnel of the faculty. In fact there are many possibilities of obstruction to real progress in an alumni board with powers so unusual as those proposed by Mr. Mershon.

The dangers would be especially great in the case of those institutions that have not yet reached true collegiate standing.

A. S. Langsdorf: This paper brings to mind that the study of organization and administration in university work was taken up some time ago by the Carnegie Foundation for the Advancement of Teaching. The investigation, which was made by a mechanical engineer, had for its particular purpose the study of methods in teaching physics. Attention was confined to one subject because of the obvious impossibility of considering the entire field in the time allowed for the work. The results have been published by the Foundation and are available to all who are interested. The report met a rather cool reception and even ridicule, especially from college men, because of certain unique suggestions which imposed factory methods upon the methods of training students.

Mr. Mershon's paper seems to be actuated by the same general motive which prompted the Carnegie Foundation in its undertaking, namely, the conservation of energy in administering purely routine affairs of university work. It would seem that many of the plans outlined by the author should be self-evident to executive officers of reasonable efficiency, always supposing that political conditions (in the case of a state institution) are what they should be. But the question that is likely to present itself to those in the educational field, after reading the paper, is: Where is the Faculty? Its functions, which should be of the greatest importance in a democratic institution like a university, are apparently submerged and made subordinate to those of unduly exalted executives.

I believe with one of the previous speakers that the powers of the alumni council as proposed in the paper are too great.

Alumni in general are not well acquainted with college affairs, especially after an asbence of ten years or more, and there has been a tendency on their part, particularly in the case of state institutions, to meddle in certain features which were better removed from their influence. I refer to their rather pernicious activity in athletics, in which they have frequently exercised a decidedly corrupting influence. But granting that an alumni council of the best type could be secured, its presence would seem to violate a fundamental precept in efficient management by a further division of responsibility instead of a concentration. Instances could be mentioned—such as student "strikes"—in which the interposition of an alumni body between students and faculty would have been a real calamity.

George D. Shepardson: I have tried to orientate this paper with reference to the work of the American Institute of Electrical Engineers, and I confess difficulty in orientating my own ideas with reference to this paper. In attempting to find the relation of the electrical department to the proposed organization, I confess embarrassment. On the other hand, it seems to me that the paper might have begun further back. Considering the problems which a state educational institution has to face, we find that almost without exception the public resources of the state are scattered. In almost every state instead of there being one State University, which should be at the head of the educational system of the State, there are several institutions. Even though the state university is nominally at the head of the state public educational institution, in every state certain parts of the system are outside. Minnesota, for example, has been pointed to as a state in which the entire educational system has been built up symmetrically by having the high schools point up towards the State University, and at first sight it looks like an ideal situation. There are, however, a number of normal schools whose purpose primarily is to train teachers, and which have no thoroughly specified connection with the university, but which every now and then agitate the question of extending their work and granting degrees. There are certain other branches of education which are under state auspices, and yet which do not quite fit in with the ordinary scheme, such as institutions for the blind, or for the deaf, or for the feeble-minded, institutions which too frequently are classed along with the penal institutions. In almost every state we find several governing boards which have to do with educational institutions and when it comes to the legislative sessions, whence come the appropriations, there are a number of different boards generally in sharp competition with one another, or else in some unholy alliance which is not of the best order.

One of the fundamental features which should be included in any general scheme of reorganization, is some provision for harmonizing all public educational institutions in the State. In the State of Ohio, for example, I believe there are four, certainly three, different institutions for higher education that enjoy the receipt of money from the State Treasury, and in almost any other state that situation exists to some extent. It seems to me the greatest need of our state universities is that the entire educational system of the state should be unified. This is much more important than that the university organization itself be overhauled. Another difficulty with the proposed organization is that in most of the states the universities are chartered, which is particularly true of the older institutions. It is a matter of very great difficulty to change that form of organization. As an illustration, in Minnesota there has been no acknowledged arrangement by which the alumni should have representation upon the Board of Regents, corresponding to the Trustees, although such representation seems to be very desirable. Only after several years of agitation was there success in getting one or two alumni members upon the Board of Regents, and that is done, not by right, but by courtesy. The matter was brought up at the psychological moment, before a properly disposed Governor, and he agreed to appoint two alumni upon the Board of Regents when a vacancy occurred. Since that time the succeeding Governors have been prevailed upon to appoint alumni as successors of alumni whose terms expire, and so we have an alumni representation upon the Board which has proven to be very satisfactory.

As to the official relations between the alumni and the university, it is difficult to make rigid statements which will apply everywhere. It seems to me the attitude that has been generally expressed here this afternoon is the correct one; that, generally speaking, the alumni organization should only have advisory power, and not have veto or appointive power. If the alumni are assured of representation upon the general governing Board of Trustees or Regents, then we may be sure that the alumni will be well represented in matters which they desire to bring to the notice of the Board of Trustees or the Board of Regents, and that these matters will be given proper attention.

M. C. Beebe: I am sure that all of the members of university faculties appreciate suggestions made by engineers in the field in fact, we make every effort to get them to coöperate to as great an extent as is possible.

I cannot see any reason why a university, cannot be organized on the same sound principles which obtain in our best corporations. I think there is a chance for decided improvement, especially on the business side. The difficulties in the past have perhaps been due to the very rapid growth of the state universities, so that the development of the organization has tended to lag behind that of the growth of the institution itself but is not the same true of many business organizations?

I think we have got to go slow in applying the same standards of efficiency to the educational side that are applied to the purely manufacturing plant. The matter of economy should be judged in a somewhat different way. True economy in education does not mean a low cost per student for there is no such competition to be met. Employers are not looking to our universities for cheaply educated men.

The only economy worth while and it is very well worth while is to turn out the best possible product with the money available for the purpose. This means among other things a careful selection and retention of good instructors and equipment.

Mr. Ford objected to the two-headed scheme, as he called it. It may be interesting to know that at the University of Wisconsin the two-headed arrangement is being tried out. We have the president of the university, and besides we have a newly appointed business manager, whose duty it is to look after all the purely business matters. He has under him the purchasing department, the Secretary of the Board of Regents, and Bursar and has control of the buildings and grounds.

The argument for this arrangement is that the institution is so large that it is impossible for one man to do justice to both interests, the pedagogical and business, and the work may better be divided. The situation is analagous to the case of a large corporation, where the various vice-presidents are given charge of separate divisions of the organization. Although this departure is somewhat new, it seems to be working very successfully.

W. L. Upson: There are many points in this paper which invite criticism. Some of these are already embodied in university organization and might be discussed from the standpoint of experience. In this respect may be mentioned what the author calls, "Schools and Departments of Instruction." This is proposed as a means of dividing the various college faculties into smaller and more workable bodies, and might be justifiable in an institution of 10,000 students. At present, however, these divisions are apt to be so totally devoid of power as to render them unnecessary and useless complications in the general scheme. In the college faculties, at present, important action is usually taken with departmental agreement and as departments are limited in number, the faculties are not as unwieldly as their actual membership might suggest. The author recognizes the value of alumni coöperation, and this can hardly be overestimated. Alumni representation on Trustee Boards has generally been productive of much good. His method of attacking the problem is novel, but will probably find few supporters. It appears like putting a horse-whip in the hands of an incompetent, but self-confident boy, who proceeds to make general havoc in stirring things up. It is doubtful if most state universities could command the services of 20 alumni who were capable of exercising such functions as are prescribed. Again, incompetency is not a general characteristic of university faculties. On the contrary, failure to obtain better results is rather due to insufficient means—too few and too poorly paid instructors—and to lack of coöperation between faculty and trustees. These two bodies should be brought more closely together, not separated by another all-powerful body of doubtful competency. The interest of alumni should be secured by direct representation on the board of trustees.

A third criticism relates to the student advisory board. As proposed, this board is too small to be properly representative of the student body. It should, however, be recognized as a very important part of the university organization and be given as full and unlimited power over student activities as individual conditions would warrant.

Ralph D. Mershon (by letter): I regret exceedingly the circumstances which rendered it impossible for me to present this paper in person and take part in the oral discussion of it.

It is a matter of regret that no one outside the educational field took part in the discussion. The opinions of men who have had experience in large industrial organizations are most desirable in matters of this kind.

Mr. Ford's contribution is more definite and constructive than any of the rest of the discussion, and gives evidence of careful thought. There are some matters however on which I do not entirely agree with him.

He says that in my scheme " The fixing of authority is indefinite, particularly as to contentions between management and administration, with the result that frequently the Board of Trustees decides on matters of administration concerning which it has insufficient knowledge to act intelligently. The natural result is that the administrative officers are unable to maintain a definite policy, and consequently find it difficult to obtain the proper coöperation of those under them."

The fixing of authority should not be indefinite. My endeavor in drawing up the scheme was to definitely fix the authority; to give the President all the power requisite to make him fully responsible for the success of his administration, and yet not leave him so free from possible restraint as to invite the condition of czarism that has developed, and led to trouble, in some instances. For this reason, on further thought, I agree with Mr. Ford that the Secretary should not report directly to the Board of Trustees. For this reason, also, I prefer to make the Secretary, Registrar and Superintendent of Property report to the President directly, instead indirectly as Mr. Ford does. But there is no good reason that I can see why the Treasurer should report to the President. I would say, also, that if a " Board of Trustees decides on matters of administration concerning which it has insufficient knowledge to act intelligently," then, in my opinion, it is not a good Board of Trustees. A Board that properly does its work will call for such information as is necessary to enable it to act intelligently, and will see to it, in case of conflicting opinions, that information is had from authoritative sources on both sides of the question.

Mr. Ford, and others of those taking part in the discussion, think that there would be difficulty in having the Alumni Board advise the Board of Trustees and confirm appointments to the teaching staff. At Harvard, the Alumni Advisory Board, designated as the " Board of Overseers ", has, for many years done this very thing, and Dr. Eliot, in the Harris lectures on University Administration, sets forth this as one of the valuable and desirable functions of the Board. If such a scheme has worked successfully at Harvard, I see no reason why it should not in other universities.

My scheme contemplates an Alumni Advisory Board with the powers now had by the Harvard Board of Overseers. I recognize the fact that in instituting such an Alumni Advisory Board, in connection with a State University, it would be difficult, and perhaps inadvisable, to procure for them, officially, any such considerable measure of power as this. I believe, however, that an Alumni Advisory Board elected by the Alumni Association of a State University, under the restrictions as to membership provided by my scheme, that goes to work in a conscientious and careful way, will have all the power it needs in any direction in connection with the university, whether it has official recognition or not. I believe that in most state universities such a Board would be welcomed by the Executives and Trustees. That it is unlikely the Board of Trustees and the Alumni Advisory Board would ever come into conflict. But that if they ever should, and it should be deemed advisable by the Advisory Board to compel a course it recommended, it could do this by making a public statement of the matter at issue. I do not believe it would ever be either necessary or advisable to take such drastic action, but the power to take it would lie with the Advisory Board. The fact that the Board had this power would probably obviate the necessity for its ever being exercised; and the fact that such action would be drastic, would be a strong reason why the Board should be extremely cautious about ever resorting to it.

All the other suggestions made by Mr. Ford, insofar as they do not conflict with the exceptions above noted, seem to me eminently desirable and practical. I intend, so soon the leisure for it offers, to prepare a revised scheme embodying them.

Mr. Brackett asks: " Are the representatives of the Alumni likely to be any more expert in educational or technical lines than the average member of the Alumni Association." I believe the value of an Alumni Board will come through the fact that the members of it are not experts in education. Experts in any line are likely to become onesided and unable to take a common sense view of the relation of their work to the outside world. The very object of the Alumni Board is to have the benefit of the views of people not subject to educational myopia.

Some of those in educational work do not realize that an Alumni Board, made up of mature men who have had ten or more years of worldly experience, is not going to interfere in matters unless it has good reason to do so. And that, generally, the individual members of it are in a much better position to acquire reliable information relative to any candidate for the instructional force than is anyone connected with the institution. The mere fact that an inquirer is connected with the institution makes it difficult for him to get information that is reliable.

Mr. Brackett also says that an Alumni Board with such influence may foster excessive inbreeding in the personnel of the faculty. Dr. Eliot's testimony is a sufficient answer to this. But I may say, from my own experience in connection with one State University, that the Alumni, generally, even those on the faculty, hold the view that there is too much inbreeding in most of the State Universities, and that it should be stopped.

Mr. Langsdorf says: " Where is the faculty? Its functions, which should be of the greatest importance in a democratic institution like a university, are apparently submerged and made subordinate to those of unduly exalted executives." I agree with Mr. Langsdorf that the functions of the faculty are of the greatest importance. The very object of the existence of a university is that the faculty may exercise its functions in teaching, but I do not believe that administration is a proper function of a faculty. I do not mean to say that the faculty and its opinions should be ignored in the administration. Quite the contrary. But I do mean to say that the average faculty member has not had that training which fits him to have a part in the administration of an institution and that if he had, the functions of instructing and administering do not properly go together.

Mr. Upson objects to the division of a college into schools and departments of instruction, saying that it might be justifiable in institutions of ten thousand students, but implying that it is not justifiable at the present time. It was an institution of ten thousand students, or more, that I had in mind in drawing up this scheme. The time will come, and is not very far away, when we shall have institutions of such size; and they will have to be handled. It seems to me we might begin now, nominally at least, to handle university work in this way, so that we shall be ready for the larger problem when it comes; to outline a skeleton organization which shall be conformed to now, as nearly as may be without needless duplication, and which can be put more and more nearly into effect as increasing numbers make it necessary. As our State universities become larger and larger the problem of handling them will become increasingly difficult unless there is now laid the foundation for such organization as will enable the proper handling of large numbers.

It has been found in manufacturing work, where one organization makes a great quantity and diversity of product, that the best results are gotten, not by having one great equipment to turn out the various products, but by having an institution consisting of a combination of smaller institutions, each with its own equipment, and each turning out its own line of product. It is by such course, only, that the individual product can receive anything like the individual attention it must receive for best results. And under such an arrangement the product can be turned out of better quality and with less waste of time and money, even though it involves a duplication of equipment. It seems to me this applies to educational institutions, and that eventually we must have in all universities separate and distinct colleges involving more or less duplication of instructive force and equipment, in order that we may turn out the best product; in order that the product shall receive the same individual attention it does in smaller institutions. By such a course we shall retain all the advantages of the large institution while doing away with the disadvantage that the large institution is now considered to have as compared with the smaller one; and at the same time establish lines of growth along which the institution may continually and naturally expand to meet the requirements of continually increasing numbers of students and the demands for additional courses of instruction.

A paper presented at the 264th meeting of the American Institute of Electrical Engineers, New York, October 13, 1911.

A THEORY OF COMMUTATION AND ITS APPLICATION TO INTERPOLE MACHINES

BY B. G. LAMME

In the usual theory of commutation it is considered that, when the current in a coil is commutated or reversed, the local magnetic flux due to the current reverses also, and in so doing sets up an e.m.f. in the coil which opposes the reversal. This is the so-called reactance voltage referred to in commutation problems. The fact that two or more coils may be undergoing commutation at the same time involves consideration of mutual as well as self-induction. The relation of the mutual to the self-induction, the probable value of each, etc., lead to such mathematical complication in the analysis of the problem, that empirical methods have become the usual practice in dealing with commutation. The usual analytical methods do not permit a ready or easy physical conception of what actually takes place. One must think in formulas rather than in the phenomena of the commutation itself.

According to the usual theory, during the commutation of the coil the local magnetic flux due to the coil is assumed to be reversed. However, in the zone in which the commutation occurs, certain of the magnetic fluxes may remain practically constant in value and direction during the entire period of commutation. This is but one instance, of which there are several, to show where there is apparent contradiction of fact in the usual mathematical assumptions made in treating this problem.

The above fact of part of the flux in the zone of commutation remaining practically constant in value and direction, led the author to a method of dealing with the problem of commutation which is based upon consideration of the armature flux as a whole, as set up by the armature ampere turns. The results obtained by the method were very satisfactory, and it was apparent that a much better conception could be obtained of some of the phenomena of commutation than was possible with former methods.

In the following pages the method is indicated in general, and its application to interpole machines is then worked out in greater detail. In non-interpole machines the problem is greatly complicated by the presence of local currents under the brushes which modify the distribution of certain of the armature magnetic fluxes, as will be shown.

This theory of commutation, with the method of calculation, is based upon the broad principle of the *armature conductors cutting across the magnetic field set up by the armature winding and thereby generating an e.m.f. in the short circuited coils which is proportional to the product of the revolutions, the flux which is cut and the number of turns in series.* The usual "reactance" voltage due to reversal of the local flux of an individual coil is not considered, although its equivalent appears under another form.

The method in general is therefore the same as that used for determination of the main armature e.m.f., except that the magnetic fluxes cut by the armature conductors are those due to the armature magnetomotive force instead of those due to the field.

When the armature winding is carrying current its magnetomotive force tends to set up certain magnetic fields or fluxes which have a definite relation to the position of the brushes. Considered broadly, the current after entering the commutator or armature winding, at any brush arm, divides into two paths of opposite direction. As the winding on each of these paths is arranged in exactly the same way, and as the currents flow in opposite directions, the armature windings in these two paths have magnetomotive forces which are in opposite directions. The resultant armature magnetomotive force rises to a maximum at points corresponding to the brush positions. Midway between these points the magnetomotive force is zero. Magnetic fluxes are set up by these magnetomotive forces, which are a function of the force producing them, and the proportions, dimensions and arrangement of the magnetic paths; and these magnetic fluxes will be practically fixed in position corresponding to the brush setting.

The armature conductors cutting across these fluxes set up by the armature magnetomotive forces, will have e.m.fs. generated in them. In those conductors which have their terminals short circuited by the brushes, these e.m.fs. may be called the short circuit e.m.fs.

There are three principal armature fluxes which are cut by the short circuited armature coils. In the order of their usual importance these are,

1. That which crosses from slot to slot. It may be called the slot flux.
2. The interpolar flux which passes from the armature surface to the neighboring poles or yoke surface. It may be called the ***interpolar*** flux, as distinguished from ***interpole*** flux, which term will be used later.
3. That flux set up in the armature end-winding in the zone of the short circuited coil, due to the magnetomotive force of the end windings as a whole. It may be called the end flux.

The short circuited armature coils cutting across these three fluxes generate the short circuit e.m.fs. The whole problem of commutation may be considered as depending upon the predetermination of these three fluxes.

Consider, first, an armature conductor approaching the point of current reversal or commutation. Under this condition the current carried by the coil *always flows in the same direction as the e.m.f. generated by the conductor cutting across the magnetic field or flux set up by the armature winding is induced.* When the terminals of an armature coil pass under the brush and are short circuited, it is obvious that the e.m.f. set up in the coil by the armature flux is unchanged in direction for the coil is still cutting a field of the same polarity. This e.m.f. tends to maintain the current in the short circuited armature coil in the same direction as before but the value the current attains will be dependent upon the short circuit e.m.f. and largely upon the resistance in the circuit, which will usually consist of the resistance of the coil itself and of the brush contact. As the coil passes out of short circuit, that is, as it leaves the brush, the current must flow in the opposite direction, but the e.m.f. set up by the armature flux is still in the same direction as before. Therefore, after commutation, the armature current in the coil is flowing in opposition to the e.m.f. set up in the coil by the armature flux.

The following is a method for calculating approximately the three fluxes before described and the e.m.fs., generated by the armature conductors cutting them. The interpolar fluxes will be considered first, the end fluxes second, and the slot fluxes last, as these latter are greatly complicated by the problem of local currents produced largely by the interpolar and end fluxes.

Interpolar Armature Flux

By this is meant the flux in the interpolar space between the armature core and the field poles and yoke, due to the magnetomotive force of the armature winding, as shown in Fig. 1. This magnetomotive force has its highest value at those parts of the armature winding corresponding to the brush contacts on the commutator and is zero midway between such points. If the brushes are set with zero lead then the maximum magnetomotive force of the armature lies midway between adjacent field poles and will taper off in value from this midpoint toward the adjacent edges of the poles. The flux density between the armature surface and the sides of the poles should therefore tend to taper off as the armature magnetomotive force is reduced but, in most types of field construction, it tends to increase in value due to the relatively shorter magnetic path as the edges of the poles are approached. Usually this increase very considerably overbalances the decrease due to the lower magnetomotive forces and in consequence the interpolar flux density due to the armature generally has a minimum value midway between the poles and rises toward the edges of the poles. This is illustrated by Fig. 2.

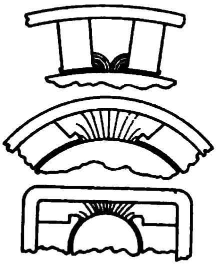


FIG. 1

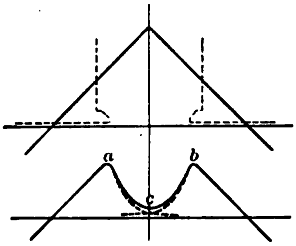


FIG. 2

The density of this flux in the interpolar space is dependent upon many conditions such as the ampere turns of the armature winding per pole, distance between poles, conformation of the poles ,yoke, etc. In Fig. 2 the ordinates of the dotted lines represent the flux densities at the armature interpolar surface due to each of the two adjacent poles. The resultant of these two is the full line *a c b* which represents the distribution of the armature interpolar flux. This interpolar flux might be considered as a true magnetic field fixed in space with respect to the position of the brushes. This field being fixed and the armature conductors rotating it is obvious that any conductor moving across this magnetic field must have e.m.f. generated in it, the value of which depends upon the flux which is cut at any instant. Therefore, the e.m.f. due to this interpolar field can be determined directly, if the intensity of the field itself can be calculated.

During the period of commutation the armature coil is short circuited and has the current reversed in it under certain portions of this field. The problem is to determine the strength of the field corresponding to this point of commutation and then by direct calculation the corresponding e.m.f. can be determined. In the following analysis two cases will be considered, namely, pitch windings, and "chorded" or "fractional pitch" windings.

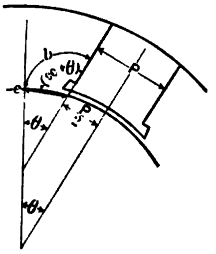


FIG. 3

Pitch Windings. When commutating or reversing a coil with a pitch winding it is evident that if there were no lead at the brushes such a coil would commutate, on the average, at the midpoint between two poles. The e.m.f. generated in the coil by cutting the interpolar field would therefore be proportional to the strength of the interpolar flux at the midpoint. This flux can be determined approximately in a fairly simple manner in the ordinary types of machines in which the poles are relatively long compared with the distance between adjacent pole tips and where the distance from the armature surface to the yoke is relatively great. The following is a method which appears to give reasonably close results:

Let W_t = total number of wires on the armature.
I_c = the current per conductor.
p = number of poles.

Then, the armature ampere turns per pole $= \dfrac{I_c \times W_t}{2p}$, neglecting any change in ampere turns due to the short circuiting action of the brushes.

In Fig. 3 let b represent the length of the mean flux path corresponding to the mid-interpolar position. This is assumed to be a part of a circle which is practically at right angles to the armature surface and the side of the field pole, as indicated in Fig. 3.

Let P = width of body of pole.

Let B_i = the flux density at the midpoint between the poles.

$$\text{Then } B_i = \frac{2 \times 3.19 \; I_c \times W_t}{2\,p\,b}.$$

But $b = 2\,\pi\,a\,\frac{(90+\theta)}{360}$, approximately, as angle $(90+\theta)$ is only approximate.

$$\text{Or } b = 2\,\pi\,a\left(0.25 + \frac{\theta}{360}\right) = 2\,\pi\,a\left(0.25 + \frac{1}{2\,p}\right).$$

Also, $a = \left(\frac{\pi D}{2\,p} - \frac{P}{2}\right)$ approximately.

Therefore

$$b = 2\,\pi\left(0.25 + \frac{1}{2\,p}\right)\left(\frac{\pi\,D - Pp}{2\,p}\right) = \frac{\pi\,(0.25p + 0.5)\,(\pi\,D - Pp}{p^2}$$

Therefore

$$B_i = \frac{2 \times 3.19 \; I_c \; W_t \times p^2}{\pi\,(0.25p + 0.5)\,(\pi\,D - Pp) \times 2\,p}$$

$$\text{Or, } B_i = \frac{I_c\,W_t\,p}{(0.25p + 0.5)\;(\pi\;D - Pp)} \text{ approx.}$$

The above gives the approximate flux density at the midpoint between poles. The flux densities at points at each side of the midpoint can be determined in a similar manner, taking into account the lower armature magnetomotive force as the midpoint is departed from. As the edge of the pole is approached the effect of pole horns may complicate the flux distribution so that the above method of calculating interpolar flux density will not apply for points close to the pole.

E.m.f. Due to Interpolar Flux.

Let E_c = The e.m.f. due to cutting the armature flux.
D = diameter of armature.
L = length of core including ventilating spaces.
T_c = turns per individual armature coil.
R_s = revolutions per second.

Then, the e.m.f. induced in a coil cutting the field at *c* (Fig. 2) can be represented by the formula,

$$E_c = \frac{B_i \times \pi D L \times 2 T_c \times R_s}{10^8}$$

Or,

$$E_c = \frac{I_c W_t p}{(0.25p+0.5)(\pi D - Pp)} \times \frac{2 \pi D L T_c R_s}{10^8}$$

Or

$$E_c = \frac{I_c W_t T_c R_s}{10^8} \times \left(\frac{2p \times \pi D L}{(0.25p+0.5)(\pi D - Pp)} \right)$$

Incidentally, with this method of dealing with the problem the effect of the addition of an interpole can at once be seen. The magnetomotive force of the interpole is superimposed on that of the armature and the resultant flux is then considered. The armature conductors cut this flux and thereby generate e.m.f. If the interpole magnetomotive force is stronger than that of the armature, then the flux established will be in the opposite direction in that part of the armature face which lies under the interpole. Therefore, the flux or field over the commutated coil in the non-commutating pole machine is replaced by flux in the opposite direction. The presence of the interpole does not increase the reactance of the armature coil as sometimes considered, but, on the contrary, the harmful flux is replaced by one which is of direct assistance in commutation.

Effect of Brush Width. In cutting across the interpolar flux it is obvious that the e.m.f. set up in the short circuited coil is not a function of the length of time the coil is short circuited, for this interpolar flux is set up by the armature winding as a whole and not by individual coils. If two or more armature coils in series are short circuited by the brush, then their e.m.fs. will be in series while the total resistance in the path will be very little higher than in the case of a single coil short circuited, for the principal part of the resistance lies in the brush contact. It is evident therefore that considerably higher short circuit currents can be set up by the interpolar field when more commutator bars, and more turns, are short circuited. It can therefore be assumed that, as far as the interpolar field is concerned, the more commutator bars the brush covers the greater will be the short circuit current and the greater will be the difficulty in commutation, assuming there is no external field assisting commutation.

Chord Winding. With a pitch winding, with no lead at the brushes, the commutation of a coil will occur in the lowest part of the armature interpolar flux, as *a a* in Fig. 4. With a chorded winding, as indicated at *b b*, the commutation will occur under somewhat higher flux than with a pitch winding. Therefore in considering the interpolar flux a full pitch winding commutates under better conditions than a chorded winding.

END FLUXES

The armature winding as a whole sets up certain fluxes in the end windings. These fluxes are fixed in position with respect to the brushes, and the armature coils, in cutting across, them, generate e.m.fs. The only part of these end fluxes concerned in the present problem is that which the commutating coils cut during the operation of commutation.

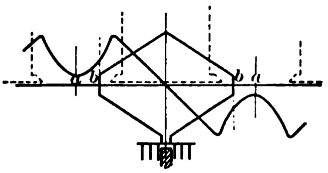


FIG. 4

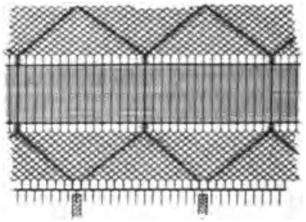


FIG. 5

Fig. 5 illustrates an armature winding in which the heavy lines represent two coils in contact with the brushes and therefore at the position of commutation. It is only the end flux density along the shaded portion or zone of this diagram which need be considered. If the various densities for this zone can be determined, then the e.m.f. in the commutated coil can be calculated. Only the usual cylindrical type of end windings will be considered, as practically all direct current machines at the present time use this type. Such windings are usually arranged in two layers, the coils of which extend straight out from the armature core for a short distance, usually ½ in. to 1½ in., depending upon size and voltage of the machine, and then extend at an angle to the core of 30 deg. to 45 deg. The conductors of the upper and lower layers therefore usually lie almost at right angles to each other.

Pitch Winding. Let Fig. 6 represent a single coil of the end winding located in the commutating zone. Both theory and test show that the maximum flux density in this zone is at *a* and tapers off slightly to *b*, then tapers off more rapidly from *b* until it reaches practically zero value at *c*. It may be assumed with but little error that the decrease from *b* to *c* is at a practically uniform rate. The flux density along the commutating zone of the end winding may therefore be represented by Fig. 7, in which the ordinates represent flux density. On the above assumption the total flux in the commutating zone of the end winding can be determined with sufficient accuracy if the density at *b*, for instance, can be determined and the distances *a b* and *c d* in Figs. 6 and 7 are known. These latter can be determined directly from the winding dimensions.

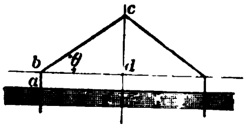


Fig. 6

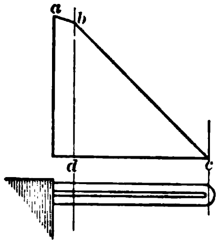


Fig. 7

The following is an approximate formula for the flux density at *b*, including allowance for proximity of iron end plate, core, etc.

$$B_e = \frac{2.15\, I_c\, W_t \times \log 2\, N}{\pi\, D \sin \theta}$$

N = number of slots per pole.
I_c = current per conductor.
W_t = total armature wires.
D = diameter of armature.
Let $a\,b = h$, and $c\,d = m$.

Then the flux cut by one conductor at one end is

$$\pi\, D \left(h + \frac{m}{2}\right) \times \frac{2.15\, I_c\, W_t \times \log\, 2\, N}{\pi\, D \sin \theta}$$

Therefore the e.m.f. per single turn of the armature winding, due to the end flux, considering the end fluxes for both ends of the core, becomes

$$E_c = 2\left(h+\frac{m}{2}\right)\times\frac{2.15\, I_c\, W_t \times \log 2\, N}{\pi\, D \sin\theta}\times\frac{\pi\, D\, R_s \times 2\, T_c}{10^8}$$

Or,

$$E_c = \frac{I_c\, W_t\, T_c\, R_s}{10^8}\times\frac{4.3\,(2\,h+m)}{\sin\theta}\times\log 2\, N$$

This formula is on the basis of non-magnetic paths around the end windings, that is, with no bands of magnetic material and no magnetic supports under the coils. The effect of bands over the end winding is approximately equivalent to cutting the flux path to half length for those parts of the end winding covered by the bands. Therefore, with bands, the diagram representing flux density in the commutating zone of the end winding would be as indicated in Fig. 8. In this case the total flux corresponds to the total area of the curve including the dotted portion. Of course the actual flux distribution would not be exactly as shown in this diagram for there would be some fringing in the neighborhood of the bands. The diagram simply serves to illustrate the general effect of magnetic bands and an approximate method of taking it into account.

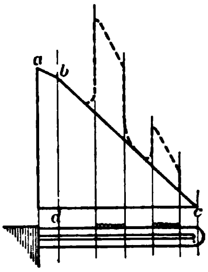


Fig. 8

The effect of a magnetic coil support will be very similar to that of a steel band in reducing the length of path and therefore increasing the flux in the neighborhood of the coil support. However, in case of magnetic bands over the winding and coil supports under it the limit lies in saturation of the bands themselves. This usually represents a comparatively small total flux. The coil support, however, would probably not saturate in any case.

The above formula for end flux can therefore be corrected for magnetic bands and coil supports by multiplying by a suitable constant to cover the increased flux.

It is obvious that the determination of the end flux is, to a certain extent, a question of judgment and experience. No fixed method or formula can be specified for all types of machines, for this flux would be influenced very greatly by the bands, if of magnetic material, and by the material, size and location of the coil supports and their relation to the bands. Also, eddy currents may be set up in the coil supports which will influence the distribution of the end flux in the zone of the commutated coil. However, in each individual case an approximation can be made which will, in general, be much closer than would be obtained from any empirical rule or by neglecting the effect of the end flux altogether.

Chord Winding. The effect of chording the armature winding is to slightly diminish the flux density in the commutating zone which results in a slight reduction in the e.m.f. of the commutating coil. But a relatively much greater gain is obtained by the consequent shortening of the distance $c\ d$ in Fig. 8 and the corresponding reduction of the total end flux. Due to the chording itself the flux density at b is reduced practically in the ratio of $\frac{\log 2\,N_1}{\log 2\,N}$, where N_1 = number of slots spanned by the coil. For example, if the full pitch is 20 slots and the coil spans 18 slots, then the density at b will be reduced in the ratio of $\frac{\log 36}{\log 40} = 0.971$ due to the chording itself; and the flux along $c\ d$, Fig. 7, will be further reduced in the ratio of $\frac{18}{20}$ due to the shorter end extension. The average flux along $c\ d$ therefore will be reduced to $0.9 \times 0.971 = 0.874$, or about 87 per cent of that of a pitch winding.

Effect of Brush Width. As in the case of the interpolar flux the width of the brush, or the number of armature coils short circuited by the brush, has practically no influence on the e.m.f. generated per turn. However, the total effective armature ampere turns will be reduced slightly, if the average current in the short circuited turns is less than the normal current. This will have a very slight effect on the e.m.f.

SLOT FLUX

By this is meant the magnetic flux across and over the armature slots which does not extend to the yoke or field poles.

Two general cases will be considered; first, that in which no local currents are present, which is the case in well designed interpole machines; and second, that in which there are local currents set up in the short circuited coils, which is almost invariably the case in machines without interpoles or some other form of compensation. Also, pitch and chorded windings will be considered.

SLOT FLUX WITH NO LOCAL CURRENTS

Pitch Winding. Let Fig. 9 represent an upper and a lower coil in the same slot, with equal turns and currents. Then if there is no saturation in the adjacent teeth the flux density across the slot will be zero at the bottom of the lower coil and will rise to a maximum value at the top of the upper coil. There will also be a flux across the slot above the upper coil and also from the top of the tooth as indicated in Fig. 9. The total slot flux entering at the bottom of the teeth is therefore equal to the total flux which crosses the two adjacent slots, plus the flux crossing at the top of the slots. The interpolar flux which extends from the armature surface to the poles or yoke is not included in this.

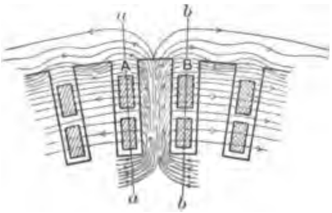


FIG. 9

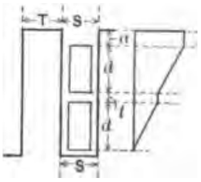


FIG. 10

As this slot flux is practically fixed in position the armature conductor in slot *A*, in passing from *a* to *b* must cut this flux. It is obvious that the flux which crosses above the uppermost conductor in the slot is cut equally by all the conductors in the slot, as the coil passes from position *a* to position *b*; but the flux crossing the slot below the uppermost conductor does not affect all the conductors equally, and therefore, for simplicity of calculation, an equivalent flux of lower value can be used which may be considered as cutting all the conductors equally.

Let d Fig. 10, represent the depth of the conductors of one complete coil.

t represent the distance between the upper and lower coils.

a represent the distance from the upper conductor to the core surface.

s represent the width of the slot, assuming parallel sides.
n represent the ratio of width of armature tooth to the width of the armature slot, *at the surface of the core.*
T_c represent turns per single coil, or per commutator bar.
C_s represent the number of individual coils, or commutator bars, per complete coil.
L represent the width of armature core, including ventilating spaces.
I_c represent the current per armature conductor.

Then, ampere turns per upper or lower coil $= I\ T_c\ C_s$.

$$\text{Total flux across coil space} = \frac{3.19\ I_c\ T_c \times C_s\ L\ (2\ d + t)}{s}$$

$$\text{Flux across slot above coil} = \frac{3.19\ I_c\ T_c\ C_s\ L \times 2 \times a}{s}$$

Flux from tooth top across the slot is approximately,

$$3.19\ I_c\ T_c\ C_s\ L \times 2 \times 0.54\ \sqrt{n}$$

$$\text{Total flux above upper coil} = 3.19\ I_c\ T_c\ C_s\ L\ \frac{(2\ a + 1.08\ s\ \sqrt{n})}{s}$$

The sum of the two fluxes represents the total flux across one slot which enters at the bottom of one tooth. As a similar flux passes across the slot at the other side of the tooth the total flux entering the tooth will be double the above and becomes

$$\text{Total slot flux} = 2 \times 3.19\ I_c\ T_c\ C_s\ L\ \frac{(2\ d + t + 2\ a + 1.08\ s\ \sqrt{n})}{s}$$

This total flux cannot be used directly in the calculations as it does not affect all the conductors equally. It is therefore necessary to determine equivalent fluxes for the upper and lower coils which can be used instead of the above value.

For the lower coil the following value has been calculated:

$$\text{The equivalent flux} = \frac{2 \times 3.19\ I_c\ T_c\ C_s\ L}{s}\ (1.833\ d + t)$$

And for the upper coil,

$$\text{Equivalent flux} = \frac{2 \times 3.19 \times I_c\ T_c\ C_s\ L}{s} \times 0.833\ d$$

To these equivalent fluxes should be added the total flux above the upper coil. This gives the total effective flux for the upper and lower coils. Then, for the lower coil, Total effective flux

$$=2\times3.19\,I_c\,T_c\,C_s\,L\left(\frac{1.833\,d+t+2\,a+1.08\,s\,\sqrt{n}}{s}\right)$$

And for the upper coil,
Total effective flux

$$=2\times3.19\,I_c\,T_c\,Cs\,L\times\frac{(0.833+2\,a+1.08\,s\,\sqrt{n})}{s}$$

The average value of the effective flux for the upper and lower coils then becomes,

$$3.19\,I_c\,T_c\,C_s\,L\,\frac{(2.67\,d+t+4\,a+2.16\,s\,\sqrt{n})}{s}$$

(This average effective value is approximately 80 per cent of the total slot flux.)

On the basis of a pitch winding and the assumption that only one armature coil is short circuited, that is, with the brush covering the width of only one commutator bar, then the above slot flux is cut by all the coils in the slot in passing through one slot pitch. From this the e.m.f. in the commutating coil due to the slot flux can be calculated directly and may be expressed as follows:

$$E_c=\frac{2\,T_c\,R_s}{10^8}\times\text{number of slots}$$

$$\times3.19\,I_c\,T_c\,C_s\,L\,\frac{(2.67\,d+t+4a+2.16\,s\,\sqrt{n})}{s}$$

But $C_s\times$number of slots = No. of commutator bars

$$=\frac{\text{total number of conductors}}{2\,T_c}=\frac{W_t}{2\,T_c}$$

Therefore the above expression for e.m.f. may be changed to the form,

$$E_c=\frac{3.19\,I_c\,W_t\,T_c\,R_s\,L}{10^8}\,\frac{(2.67\,d+4\,a+t+2.16\,s\,\sqrt{n})}{s}$$

If it is desired to compare this expression with a certain well known formula which has been much used heretofore, then let the quantity in the parenthesis in the above expression be represented by c_x. The formula can then be changed to,

$$E = (2 \times 3.19 \times c_x) \times \frac{I_c\, T_c^2 \times \text{number commutator bars} \times R_s \times L}{10^8}$$

It contains the same terms (except in the value of the constant) for the expression of the e.m.f. which has been used heretofore in determining the reactance of the commutated coil.

Effect of Brush Width or Number of Commutator Bars Covered by Brush. The above formulas are on the basis of the brush covering only the width of one commutator bar. In this case all the conductors of one slot cut across the entire slot flux in passing through one tooth pitch. However, if the brush covers more than one commutator bar, then the full slot flux is not cut in passing through one tooth pitch, and a movement greater than one tooth pitch is required for full cutting. For example, if there is one commutator bar per armature slot and the brush covers a width equal to two commutator bars, then the total cutting of the slot flux will take place in two tooth pitches. Again, if there are three commutator bars per armature slot and the brush covers the width of one commutator bar, then the total cutting of the total slot flux would occur in one tooth pitch, while if the brush covered two bars, the total cutting would occur in 1⅓ tooth pitches; and if it covered three bars 1⅔ tooth pitches are required. In other words, the total cutting will occur in a period corresponding to the number of commutator bars per slot plus one less than the number of commutator bars covered by the brush.

On this basis the correction factor for the slot e.m.f. should be expressed by the term $\frac{C_s}{C_s + B_i - 1}$, where C_s = number of commutator bars per slot, and B_i = number of commutator bars spanned by the brush. However, with several coils per slot, and with the brush spanning several bars, the rate of cutting of the tooth flux for the entire period is not quite the same as the rate for one tooth pitch. Taking this into account the correction factor should not be equal to $\frac{C_s}{C_s + B_i - 1}$, but is slightly greater. Up to four commutator bars per slot, and three bars spanned by the brush the correction factor can be expressed by the term $1+\frac{1}{B_i\, C_s} - \frac{1}{C_s}$.

Taking the lengthened period of reversal into account, it would appear that a wide brush covering a large number of commutator bars should be beneficial in reducing the e.m.f. generated by the slot flux. This is true where the local currents are very small, or are absent, as is the case in a properly designed interpole machine. In a non-interpole machine where the local currents in the short circuited coils may be relatively high, this condition does not hold, as will be explained later.

The above formula for e.m.f. due to the slot flux should therefore be modified by multiplying by a factor which takes into account the period of reversal as affected by brush width.

Chord Winding. The armature winding may be chorded one or more slots and, in some instances, where there are several coils side by side there has been chording of part of the conductors in the slot. In Fig. 11 is illustrated the conditions with one-slot chording. The total slot flux now occupies two teeth instead of one. Therefore the e.m.f. set up by cutting across this slot flux will be approximately one-half that which is obtained with a full pitch winding, on the basis of the brush covering the width of one bar only, for the e.m.f. generated by cutting this flux will be reduced in proportion as the period of cutting is increased. There is one slight difference from the flux distribution with a pitch winding, namely, that at the top of the teeth. With a chorded winding this flux will be slightly greater than with a pitch winding, but the total effect of this difference should be relatively so small that ordinarily the value need not be changed. Therefore equivalent fluxes used with chord windings can be taken the same as for pitch windings. In consequence, the e.m.f. ***due to the slot flux***, with one-tooth chording, may be taken as one-half that for a pitch winding, with the brush covering one commutator bar in both cases.

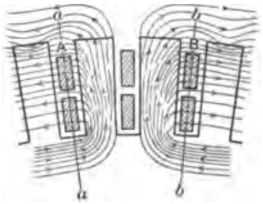


FIG. 11

For two-slot chording the slot flux may be considered as occupying the space of two teeth only, while there will be a magnetically idle tooth at the center. The e.m.f. per coil actually generated by cutting the slot flux will be, for part of the period, the same as for one-slot chording, but there will be an intermediate period where the slot e.m.f. is practically zero, which does not occur with a one-slot chording or with a pitch winding. The average results, however, should be practically the same as if the total slot flux were actually distributed over three teeth instead of two.

Effect of Brush Width with Chord Winding. In the chord winding, when the brush covers two or more commutator bars, the period of cutting the slot flux will be lengthened just as with a pitch winding on the assumption of no local currents. For example, if there are three commutator bars per armature slot and the winding is chorded one slot, then with the brush covering one commutator bar, complete cutting of the slot flux will occur in the space of six commutator bars. If the brush covers three commutator bars instead of one, then complete cutting will occur in the space of eight commutator bars, while in a corresponding full pitch winding it would occur in the space of five bars. Therefore, the wide brush represents an improvement with the chorded winding, but not to the same extent, relatively, as with the pitch winding. This is on the assumption of absence of local currents in the short circuited coils.

Bands on Armature Core. By the preceding method of analysis the effect of bands of magnetic material on the armature core can be readily taken into account. This effect represents simply an addition to the total flux which can pass up the tooth and across the top of the slots. From the ampere turns per slot, the clearance between the bands and the iron core, the total section of the band, etc., the flux due to the band can be calculated. This flux can either be combined directly with the slot flux already described and the resultant e.m.f. can then be calculated; or, the e.m.f. can be calculated independently for the band flux alone. Magnetic bands on the armature introduce a complication into the general e.m.f. formula due to the fact that in many cases the flux into the bands is such as to highly saturate the band material at relatively low armature currents. This flux therefore is usually not proportional to the armature ampere turns. If the e.m.f. due to the band flux is to be calculated separately, the following formula can be used:

ϕ_b represents the total magnetic flux in the band from the armature core considering both directions from the tooth, then

$$E_c = \frac{2\,\phi_b\, N\, p\, T_c\, R_s}{10^8}$$

This formula holds true for the band flux which passes through the one tooth in the pitch winding. Proper allowance must be made for the effect of chord windings and brush width, which can be done by the methods already described.

SLOT FLUX WITH LOCAL CURRENTS

Pitch Winding. In the preceding analysis local currents have not been included, as the method would be greatly complicated by taking such currents into account. In the general method given below the effect of local currents in the short circuited coils can be most easily shown.

As already explained, an armature coil, as it approaches the short-circuit condition, has an e.m.f. generated in it by the interpolar and the end fluxes. After the coil is short circuited this e.m.f. is still generated by the coil and naturally a local or short circuit current tends to flow through the coil, brush contact and brush. In addition, the work, or supply, current is being furnished to the armature winding through the brushes. These two currents are superimposed in the short circuited winding in such a way as to have a very pronounced influence in the distribution of the slot fluxes. This effect can be best seen by first determining the distribution of the work current in the various parts of the short circuited winding on the assumption of *no local current* and second, determining the distribution of the local currents on the assumption of no *work current, but with the same armature magnetomotive force* as in the first assumption. The two distributions can then be combined and the resultant currents in the various parts of the short circuited coils can be obtained.

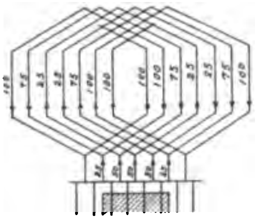


FIG. 12

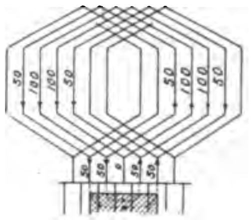


FIG. 13

Let Fig. 12 represent the first assumption in which no local currents are present. In order to illustrate conditions to better advantage, four commutator bars are assumed to be covered by the brush. Uniform distribution of current over the brush contact can be assumed in this case, as there are no local currents.

Tracing out the current in each short circuited coil in Fig. 12, it will be seen that the current decreases at a uniform rate and then rises in the opposite direction at the same rate until the short circuit is removed. The period of commutation is the longest possible with this number of commutator bars short circuited, and the brush conditions are ideal, as the current density at the brush contact is uniform at all parts. The above are the conditions which the designer endeavors to obtain in the construction of good interpole machines, as will be shown later.

In Fig. 13 the same arrangement of winding and brushes is chosen as in Fig. 12 except that only the local currents are shown and the values of these are assumed as proportional to the e.m.fs. in the short circuited coils and the resistance in circuit. In this diagram the current is a minimum in the coils at the moment that short circuit occurs, and rises to a maximum value and then diminishes to zero value again at the end of the short circuit.

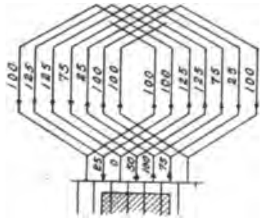


FIG. 14

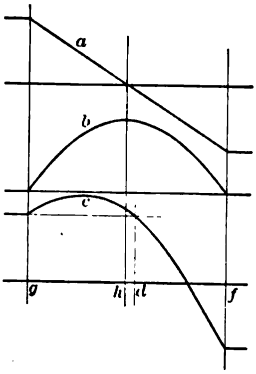


FIG. 15

In Fig. 14 the currents of Fig. 12 and 13 are superimposed. The resultant currents in the various parts of the short circuited winding are seen to rise after short circuit until a maximum value is reached and then decrease rapidly and reverse to normal value in the opposite direction. Therefore, the period from normal value of the current to normal in the opposite direction is very much shorter than when no local currents are present. It may therefore be considered that the period of reversal is much reduced by the presence of the local currents, so that the e.m.f. in the short circuited armature conductors generated by the slot flux is proportionately increased, compared with the value it would have in case the local currents were absent. These conditions can be shown possibly in a somewhat better manner by curves *a*, *b* and *c* in Fig. 15. The curve *a* shows the distribution of current in the short circuited coils without any local currents. Curve *b* shows the distribution of local currents.

while curve *c* shows the resultant of the two. The distance between *d* and *f* on curve *c* gives the period of reversal from normal current in one direction to normal current in the opposite direction. This period is much shorter than the full period represented by *g f* which would be obtained without local currents. The period *d f*, however, may not differ much from the period of commutation with the brush covering the width of only one bar, when the local current is high compared with the work current. In such case the gain in the period of commutation which should be obtained by means of the wider brush may be practically offset by the effect of the local currents which also increase with the wider brush, so that over a considerable range the resultant of the two effects may be practically constant. This is one indication why, in non-interpole machines, the brush width may be varied over quite a range with relatively small noticeable difference in the commutation. This may be illustrated by Fig. 16, in which is shown the current conditions with two to five bars spanned. In this figure *a b*, *b c*, *c d*, etc., each represent the width of one commutator bar. Therefore, curve *A*, extending over the width *a c*, represents two bars spanned. The period of reversal of the current from normal value in one direction to normal in the opposite direction is represented by *g c* for curve *A*, *h d* for curve *B*, *i e* for *C* and *k f* for *D*. A comparison of these values is interesting. Calling *a b* the period of reversal with the brush covering one bar only, then *g c* with two bars covered, is greater than *a b*. *h d* is also greater than *a b*, but less than *g c*, while *i e* is slightly less than *a b*, and *k f* is considerably less. However, the variation between *g c* and *k f* is much less than between *a c* and *a f* which would be the corresponding periods with no local currents.

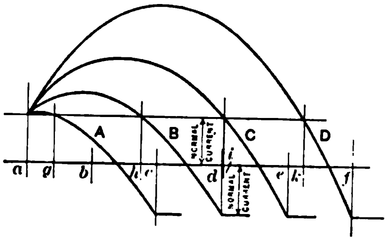


FIG. 16

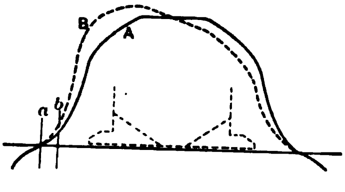


FIG. 17

It should be borne in mind that the above curves are only relative, depending upon the comparative values of the local and work currents and assuming a constant brush resistance which is not correct, but they serve to illustrate the general principle. This method of presentation is simply a skeleton of the problem of commutation when local currents are present in the short circuited coils and it would be beyond the scope of this paper to attempt a full solution.

Effects of Field Distortion. One of the " bugaboos " of the designer of commutating machines has been the question of field distortion. It has usually been considered that when the machine is loaded the magnetic field is more or less distorted or shifted from its normal no-load position and that commutation is affected by this distorted field.

To state the case plainly, the field distortion has practically nothing to do with the problem. The distorted field magnetism is simply a resultant of the no-load main field flux combined with that due to the armature winding. Therefore, the two components of the distorted full-load field are the no-load main field, which is fixed in space and is usually practically constant, and the armature field, which is also fixed in space but varies with the load. If the brushes are set in a certain position with respect to the no-load field, then, as this component of the resultant full load field is practically fixed in space and in value, it has no variable influence on the commutating conditions. The true variable element which does affect the commutation is the armature field, or flux, and it is in this very flux which is the basis of the preceding theory of commutation. Therefore, the distorted resultant field of a loaded machine does not present any new condition in the problem of commutation. One exception, however, can be made to the above, namely, where there is any considerable saturation in the armature teeth or in the main field pole corners. The effect of the armature magnetomotive force is to strengthen one corner or edge of the field pole and to weaken the other edge, but when saturation is pronounced the strengthening action is much less than the weakening action. The resultant of these actions is a decrease in the total value of the main field flux. If, now, this main field flux be brought back to its normal total value, or higher, a very considerable addition to the main field magnetomotive force will be necessary, which will be effective in increasing the field flux at the weaker pole corner to a much greater extent than at the highly saturated pole corner. In consequence, with load, the main field distribution, or field form, may be considered as being changed from its no-load form A, to the form B, as indicated in Fig. 17. It is, in reality, strengthened at a point b, for example. In such case the main field will have a variable influence on the commutation, if the brush is set with a lead, as at *b*, and, to a slight extent, the effect of an interpole is thus obtained.

Effect of Brush Lead. Before taking up the problem of interpoles on direct current machines it might be well to consider the effect of brush lead, as this gives a result intermediate between true interpole and non-interpole commutating conditions.

The preceding formulas apply to non-commutating pole machines without brush lead. However, except in case of reversing machines, such as street railway motors, or hoist motors, etc., it is usual practice to give a forward lead to the brushes of direct current generators or a slight backward lead to direct current motors. The effect of giving a lead at the brushes of a non-interpole direct current machine may be considered as being equivalent to the effect of an interpole with the exception that correct flux conditions and proper commutation, with any given brush setting, are obtained only for one given load.

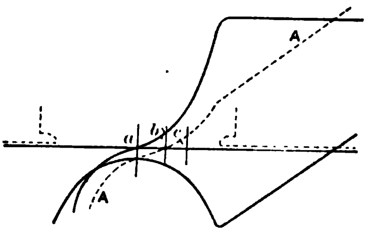


FIG. 18

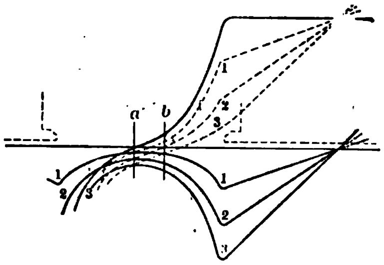


FIG. 19

As described before, with a non-interpole machine the armature winding sets up a flux in the interpolar space. With no-lead at the brushes this flux is usually a minimum midway between the poles and rises toward the polar edges. The flux from the adjacent main poles has a zero value midway between the poles and rises toward the polar edges, but has opposite polarities at the two sides of the midpoint. This is illustrated in Fig. 18. The resultant of the armature and field fluxes is indicated by the dotted line *A*. This resultant falls to zero at one side of the midpoint and then rises in the direction opposite to that of the flux due to the armature ampere turns. At the other side of the midpoint the two fluxes add, giving an increased resultant flux in the same direction as the interpolar flux due to the armature. From this figure it is evident that if the point of commutation is shifted from *a* to the point of zero interpolar flux *b*, then commutation will occur without any interpolar flux to be taken into account, that is, the e.m.f. generated by the short circuited armature conductors may be due to the slot and end winding fluxes only. If the brushes are shifted still further in the same direction to *c*, then, not only will the interpolar armature flux be annulled but a flux in the opposite direction would be cut by the short circuited armature conductor, which will generate an e.m.f. in opposition to that due to the armature fluxes in the slots and end windings. Consequently, the commutation can be materially assisted by such lead at the brushes.

The difficulty in the use of this method of commutation lies in the fact that the commutating or reversing flux at *c* is the resultant of the main field flux and the armature interpolar flux at this point, and the latter flux varies with the load, while the former remains practically constant. Therefore the zero point of the resultant field shifts backwards or forwards with change in load and the density of the commutating field beyond the zero point will therefore change with the armature current. In consequence, if the brushes are shifted into a suitable resultant field *c* at a given current, then with a different load the intensity of this field at *c* will be changed, and unfortunately the change will be *in the opposite direction* from that desired. In other words, the density of this resultant field will decrease with increase in load, whereas just the opposite effect is desired for good commutation over a wide range in load.

In practice, however, an average condition is found which, in many cases, will give reasonably good commutation over a relatively wide range in load. The brushes may be shifted at no-load into an active field in such a way as to generate an e.m.f. in the armature coils of a comparatively high value. This e.m.f. will circulate considerable local current through the brush contacts and the amount of lead which can be given is dependent, to a certain extent, upon the amount of local current which can thus be handled without undue sparking.

As the load is increased the strength of the resultant field, corresponding to this brush position, will be decreased, and with some value of the current this field will be reversed in direction. At this point the e.m.f. due to this field is added to the e.m.f. due to the slot and end winding fluxes. Obviously the limiting condition of commutation will be reached at a much higher current than would be the case if no load at all had been given. This condition is represented in Fig. 19, in which curves 1, 1, 2, 2, 3, 3, etc., represent the armature and resultant flux distributions with various loads. In this figure the brushes are given a lead so that commutation occurs at a point corresponding to *b*.

It is obvious that at heavy load a still greater lead at the brushes might give improved commutating conditions. However, if the load were suddenly removed without moving the brushes toward *a*, then the short circuited coils would be cutting the main field at such density that serious sparking or flashing might occur.

One serious objection to this method of commutation is that the distribution of the resultant field is practically such that equally good commutation cannot be obtained for all the coils in one slot when there are several coils or commutator bars per slot. All the coils of one slot must pass under a given position or value of the interpolar magnetic field at the same instant, while the commutator bars to which these coils are connected must pass under the brush consecutively. If the field intensity is just right for good commutation as the first coil per slot passes under the brush, then it may be entirely too great by the time the last coil is commutated. For good commutation with a number of coils in one slot, the resultant interpolar flux should have practically constant value over the whole range represented by the period of commutation of all the coils in one slot. This condition, however, is extremely difficult, or is frequently impracticable, to obtain with the ordinary non-interpole machine.

The above treatment of the problem of the effect of the brush lead has been based upon the armature interpolar magnetic field being located in the same position with lead as when there is no lead at the brushes. It has been assumed heretofore that the non-interpolar flux due to the armature winding has a minimum value midway between the main poles and rises uniformly toward two adjacent pole corners. This, however, is only true when the point of commutation, or brush setting, is midway between the poles. When the brushes are shifted toward either pole the point of maximum armature magnetic potential is shifted in the same way. This means that the distribution of the armature interpolar flux will be modified directly by the position of the brushes. Instead of rising uniformly toward the two pole corners, with a minimum value midway between, it will have a minimum at one side of the midpoint, this being at the opposite side from the point of brush contact, and will have an increased value on the side toward which the brushes are shifted. This is illustrated in Fig. 20 in which *A* represents the armature interpolar flux distribution with the brush at *a*, while *B* represents it with the brush at *b*.

This increased armature interpolar flux due to the brush shifting means that the resultant interpolar flux due to both the armature and main field fluxes will cross the zero line at a point further removed from the midpoint than in the case of no lead at the brushes. Consequently, in order to obtain a given useful commutating field the brushes must be given a greater amount of lead and this in turn shifts the zero point still further. Thus, the act itself of shifting the brushes makes the commutating conditions more difficult.

The calculation of the commutating conditions with any given lead therefore resolves itself into a determination of the resultant fluxes in which the coil is short circuited or commutated and the e.m.fs. generated by such fluxes. For the slot and end winding fluxes the calculation will be the same as for no-lead at the brushes. The resultant flux in the interpolar space is the only condition which will introduce any variation from the preceding formulæ and methods of calculation. This part of the problem resolves itself simply into the determination of the resultant interpolar flux at the point of commutation for any given load. The corresponding e.m.f. can then be calculated. This, combined with the e.m.fs. due to the slot and end windings, gives the total short-circuit e.m.f. The method is, in principle, exactly the same as given before, except that the determination of the interpolar flux will be modified.

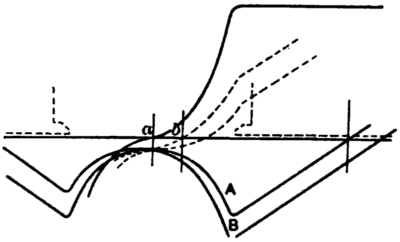


FIG. 20

Summation of Formulæ. In order to obtain the total voltage in the short circuited coil a summation should be made of the four separate voltages which have been derived for the interpolar, end, slot and band fluxes. In reality it is the resultant fluxes which should be combined, but as the voltages to be derived from these fluxes represent somewhat different terms, a better procedure appears to be the summation of the voltages. Also, in practice it is the e.m.fs. generated by the different fluxes, rather than the fluxes themselves, which are desired.

The e.m.f. derived from the interpolar flux is

$$E_c = c_1 \times \frac{I_c\, W_t\, T_c\, R_s}{10^8} \; \frac{2\, p\, \pi\, D\, L}{(0.25\, p + 0.5)\; (\pi\, D - P\, p)}$$

where c_1 is a correcting factor for chord winding, etc.

The formula for the end flux voltage is,

$$E_c = c_2 \times \frac{I_c\, W_t\, T_c\, R_s}{10^8} \times \frac{4.3\, (2\, h + m)}{\sin\, \theta} \times \log 2\, N$$

where c_2 represents the correcting factor for chord windings, etc.

The formula for the slot flux voltage is,

$$E_c = c_3 \times \frac{3.19\, I_c\, W_t\, T_c\, R_s\, L}{10^8} \; \frac{(2.666\, d + 4\, a + t + 2.16\, s\, \sqrt{n})}{s}$$

where c_3 is the correcting factor for the brush width, chord winding, etc., and,

For the bands,

$$E_c = c_4 \frac{2\, \phi_b\, N\, p\, T_c\, R_s}{10^8}$$

where c_4 is the correcting factor for chord winding, brush width, etc.

Therefore,

$$\begin{aligned} E_c \text{ total} = \frac{I_c\, W_t\, T_c\, R_s}{10^8} \Bigg[c_1 \frac{2\, p\, \pi\, D\, L}{(\,(0.25\, p + 0.5)\; (\pi\, D - P\, p)\,)} + \\ c_2\, \frac{4.3\, (2\, h + m)}{\sin\, \theta}\, \log 2\, N + \\ c_3 \left(3.19\, L \frac{(2.666 d + 4 a + t + 2.16 s\, \sqrt{n})}{s} \right) \Bigg] \\ + c_4 \times \frac{2\, \phi\, N\, p\, T_c\, R_s}{10^8} \end{aligned}$$

It is evident from this last equation that when there are no bands over the core the total e.m.f. in the short circuited coil is directly proportional to the current per armature coil or conductor. If the bands saturate, as would usually be the case with any considerable load, then the e.m.f. is no longer directly proportional to the current. Attention is called to this point as it has some bearing in the design of interpole machines.

Condensed Approximate Formula. The above formula can be simplified very considerably by certain approximations which introduce but little error within the range of ordinary design.

First, the expression, $\frac{p}{(0.25\,p+0.5)\;(\pi\,D-P\,p)}$ does not seem to be capable of any general simplification. In fact, as shown from its derivation, it is not a general term, but applies only to certain constructions and may appear in a quite different form for other constructions. Therefore this expression must be used with judgment in any case. Moreover, this term appears only in non-interpole machines or in interpole machines only when the interpoles are narrower than the armature core or the number of interpoles is less than that of the main poles. Therefore this term may be neglected in many cases where interpoles are used.

Second, the expression $4.3\,\frac{(2\,h+m)}{(\sin\theta)}\,\log 2\,N$ can be changed as follows:

$4.3\,\frac{(2\,h+m)}{(\sin\theta)}=\frac{4\,\pi\,D}{p}$, with reasonable accuracy within the ordinary limit of design,

And $\log 2\,N=0.9+0.035\,N$, with an error of about 4 per cent within the range of 6 to 24 slots.

Therefore $4.3\,\frac{(2\,h+m)}{(\sin\theta)}\,\log 2\,N=\frac{4\,\pi\,D}{p}\,(0.9+0.035\,N)$, approximately.

This is simpler to handle, in practice, than the original term.

Third, the expression, $\frac{2.666\,d+4\,a+t+2.16\,s\,\sqrt{n}}{s}$ can be simplified very materially.

Let the total depth of slot be represented by d_s, which is equal to $2\,d+a+1.5\,t$, approximately.

Then, the term, $2.666\,d+4\,a+t$ can be changed to $\frac{4d_s}{3}+\frac{8\,a-3\,t}{3}$

Assuming $a=0.25$ and $t=0.15$, then

$$\frac{8\,a-3\,t}{3}=0.52 \text{ approximately.}$$

$$\text{Therefore, } \frac{2.66\,d+4\,a+t}{s}=\frac{4d_s}{3\,s}+\frac{0.52}{s}$$

This is a very close approximation within the ordinary working range of slot dimensions. Therefore, the above expression becomes, $\frac{4d_s}{3\,s}+\frac{0.52}{s}+2.16\,\sqrt{n}$, which is much simpler to use in practice.

Fourth, in the simplified equation π appears in the first and second terms, and 3.19 appears in the third term. These are so nearly equal that π may be used as a common factor for the three terms.

The combined formulæ for the total voltage per armature coil thus becomes, in approximate form,

$$E_c=\frac{I_c\,W_t\,T_c\,R_s\,\pi}{10^8}\left[c_1\,\frac{2\,p\,D\,L}{(0.25p+0.5)(\pi D-Pp)}\right.$$

$$+c_2\,\frac{4\,D}{p}\,(0.9+0.035\,N)+$$

$$\left.c_3\,L\,\frac{(1.33\,d_s+0.52+2.16\,s\,\sqrt{n})}{s}\right]+c_4\,\frac{2\,\phi_b\,Np\,T_c\,R_s}{10^8}$$

This appears to be about as simple a form as the equation can be put into when all the factors are to be included. It will be shortened for machines without magnetic bands on the core, and in many interpole machines the term derived from the interpolar flux may be omitted. For a given line of machines which are all of similar design, etc., it is probable that the terms can be further combined and simplified.

INTERPOLAR MACHINES

In the interpole machine a small pole is placed between two adjacent main poles for the purpose of setting up a local magnetic flux under which the armature coil is commutated. This local flux, in order to assist commutation, must be opposite in direction to the interpolar flux set up by the armature winding itself. To set up this flux in the opposite direction the magnetomotive force of the interpole winding obviously must be greater than that of the armature winding in the commutating zone.

An armature coil, cutting across this interpole flux, generates an e.m.f. proportional to the flux, the speed and the number of conductors in series. This e.m.f. is in opposition to the e.m.f. in the short circuited coils, generated by the slot and end winding fluxes. For ideal commutation these e.m.fs. are not only in opposition, but they should also be of practically equal value. For perfect commutation the current in a short circuited coil should die down to zero value at about a uniform rate and should then rise to normal value in the opposite direction by the time the coil passes out from under the brush, as was illustrated in Fig. 12. This is the condition when no local currents are developed in the short circuited coils and this can only be obtained when the interpole e.m.f. at all times, balances the armature e.m.fs. in the short circuited coils.

Looking at the problem broadly, the resultant magnetic fluxes and e.m.fs. may be assumed as made up of two components which can be considered singly. One of these components is that which would be obtained *with the armature magnetomotive force alone acting through the various flux paths, including the interpole.* The other would be that which would be obtained with the *full interpole magnetomotive force alone*, the armature magnetomotive force being absent. Saturation is not considered in either case.

Considering the first component, due to the armature magnetomotive force alone, there would be the slot and the end fluxes with their short circuit e.m.fs., as already described, and in addition, there would be a relatively high flux, and short-circuit e.m.f. due to the good magnetic path furnished by the interpole core. In case the interpole does not cover the full width of the armature, or the number of interpoles is less than the main poles, there will also be some interpolar flux and e.m.f., as already described.

Considering the second component, *the entire interpole magnetomotive force* would set up a relatively high flux through the interpole magnetic circuit and a correspondingly high e.m.f. would be generated in a short circuited armature coil cutting this flux.

When these two components are superimposed, it is seen that the interpole flux due to the armature magnetomotive force is in direct opposition to that due to the interpole magnetomotive force and therefore only the e.m.f. due to their difference need be considered. As the interpole winding has the higher magnetomotive force, the resultant interpole e.m.f. is in opposite direction to the armature e.m.fs., and should be sufficient to neutralize them. This way of considering the problem avoids a number of confusing elements which would complicate the explanation, if given in detail.

In practice it is difficult to obtain exact equality between the interpole and armature e.m.fs. That due to the armature fluxes is generated in all parts of the coil including the end winding, while the e.m.f. due to the interpole flux is generated only in that part of the coil which lies in the armature slots. However, it makes no difference in what part of the coil the e.m.f. due to the interpole is generated provided it is of such value that it properly opposes and neutralizes the various e.m.fs., due to the armature fluxes. Therefore, in practice the interpoles need not have the same width as the armature core and, where space and magnetic conditions will permit, the number of interpoles can be made half that of the main poles.

According to the method outlined, the whole problem of the design of the interpole depends, first, upon the determination of the e.m.fs. due to the armature fluxes, and, second, upon the determination of such interpole flux as will generate an e.m.f. in the short circuited armature coils which will equal, or slightly exceed, the armature e.m.fs.

Interpole Calculations. Assuming that all the armature fluxes, except the interpolar, are unaffected by the presence of interpoles, the armature e.m.f. to be balanced by the interpole would be represented by the formula

$$E_c = \frac{I_c\, W_t\, R_s\, T_c\, \pi}{10^8} \left[c_1\, (L - L_1)\, \frac{2\, D\, p}{(0.25\, p + 0.5)\; (\pi\, D - P p)} \right.$$

$$+ c_2 \left(\frac{4\, D}{p} \right) (0.9 + 0.035\, N) +$$

$$\left. c_3\, L\, \frac{1.33\, d_s + 0.52 + 2.16\, s\, \sqrt{n}}{s} \right] + c_4\, \frac{2\, \phi\, N\, p\, T_c\, R_s}{10^8}$$

However, the flux above the slot, from the tooth top, is very considerably modified by the interpolar flux. In fact most of this should be omitted. It may be assumed that the flux across the slot, above the upper coil, simply " bulges " up slightly into the air gap, and the remainder of the usual tooth top flux is absent, except when the interpole does not cover the full armature width. Therefore, in the above formula, the term $L \times 2.16\sqrt{n}$ should be changed to $(L-L_1)\times 2.16\sqrt{n}$ and the term $\frac{1.33\ d_s+0.52}{s}$ replaced by $\frac{1.33\ d_s+0.7}{s}$.

Then, the corrected resultant of all the armature e.m.fs. becomes

$$E_c=\frac{I_c\,W_t\,R_s\,T_c\,\pi}{10^8}\left[c_1\,(L-L_1)\,\frac{2\,D\,p}{(0.25p+0.5)(\pi D-Pp)}\right.$$

$$+c_2\times\frac{4\,D}{p}\ (0.9+0.035\ N)$$

$$\left.+c_3\,L\,\frac{(1.33\,d_s+0.7)}{s}+c_3\,(L-L_1)\times 2.16\ \sqrt{n}\right]+c_4\times\frac{2\phi\ Np\,R_s\,T_c}{10^8}$$

In this formula

L represents the width of the armature core.

L_1 represents the effective width of interpole at the gap on the basis of the full number of interpoles.

$L-L_1$ is the difference between the width of the armature core and the interpole face. This term enters when the interpole is narrower than the armature core. When alternate interpoles are omitted and the remaining interpoles are of the same width as the armature core the conditions are practically the same as when the full number of interpoles are used but with their width equal to half the core width. Other combinations should be treated in the same way so that the above formula can be taken to represent the general conditions.

In practice it is desired that the resultant interpole e.m.f., and therefore the interpole flux, vary in proportion to the armature short-circuit e.m.f. which is to be neutralized. As shown by the last equation, this e.m.f. is proportional to the armature current, except where there is saturation in the armature flux path, as in the case of magnetic bands over the core. Therefore the interpole magnetomotive force should vary in proportion to the armature current, neglecting core bands. In consequence, in practice the interpole winding is always connected in series with the armature winding.

The interpole magnetomotive force can be considered as made up of two components, one of which neutralizes the armature magnetomotive force, and the other component represents the ampere turns which set up the actual interpole flux. The first component will be referred to as the neutralizing ampere turns or neutralizing turns, and the other as the magnetizing ampere turns or magnetizing turns.

Let T represent the total interpole turns for one interpole,

T_i represent the total magnetizing interpole turns for one interpole.

T_a represent the total effective "armature turns per pole" $= \dfrac{\text{total eff. ampere turns of armature}}{\text{number poles} \times \text{total current}}$, and

I represent the amperes per interpole coil.

Then $I\,T_i = I\,T - I\,T_a$, or, $T = T_a + T_i$.

Let g = effective air gap per interpole.

B_i = flux density under the interpole, and

E_i = e.m.f. in an armature coil of turns T_c due to the interpole flux.

Then,

$$B_i = \frac{3.19\, I\, T_i}{g}$$

The e.m.f. due to one interpole is equal to

$$\frac{B_i\, \pi\, D\, L_1\, T_c\, R_s}{10^8}$$

Or, for two interpoles

$$E_i = \frac{3.19\, I\, T_i\, \pi\, D\, L_1 \times 2\, T_c\, R_s}{g \times 10^8}$$

This e.m.f. should be equal to the e.m.f. generated in the same coils by the armature flux, or $E_i = E_c$. Therefore,

$$\frac{3.19\, I\, T_i\, \pi\, D\, L_1 \times 2\, T_c\, R_s}{g \times 10^8} = \frac{I_c\, W_t\, T_c\, R_s\, \pi}{10^8}$$

$$\Big[c_1\, (L - L_1)\, \frac{2\, D\, p}{(0.25\, p + 0.5)\, (\pi\, D - P p)}$$

$$+ c_2 \times \frac{4\, D}{p}\, (0.9 + 0.035\, N) + c_3\, L\, \frac{(1.33\, d + 0.7)}{s}$$

$$+ c_3\, (L - L_1) 2.16\, \sqrt{n}\, \Big] + c_4\, \frac{2\, \phi\, N p\, R_s\, T_c}{10^8}$$

In the second term of this equation $I_c\, W_t = I \times T_a' \times 2p$, where T_a' = total armature turns per pole, as distinguished from effective turns per pole T_a, and $T_a' = \frac{T_a}{1 - b\, p}$, where $b = \frac{B_i\, T_c}{W_t}$ as will be shown later under the subject of " Effective Armature Ampere Turns." Therefore, neglecting magnetic bands on the core, the above expression becomes,

$$T_i = \frac{T_a\, p\, g}{3.19\, D\, L_1\, (1 - b\, p)} \Big[c_1 \times \frac{(L - L_1)\, 2\, D\, p}{(0.25\, p + 0.5)\, (\pi\, D - P p)}$$

$$+ c_2 \times \frac{4\, D}{p}\, (0.9 + 0.035\, N) + c_3\, L\, \frac{(1.33\, d + 0.7)}{s}$$

$$+ c_3\, (L - L_1)\, 2.16\, \sqrt{n} \Big]$$

$$T = T_i + T_a = T_a \Big[1 + \frac{p\, g}{3.19\, D\, L_1\, (1 - b\, p)}$$

$$\Big(c_1\, (L - L_1) \cdot \frac{2\, D\, p}{(0.25\, p + 0.5)\, (\pi\, D - P p)}$$

$$+ c_2 \times \frac{4\, D}{p}\, (0.9 + 0.025\, N) + c_3\, L\, \frac{(1.33\, d_s + 0.7)}{s}$$

$$+ c_3\, (L - L_1)\, 2.16\, \sqrt{n} \Big) \Big]$$

If the full number of interpoles is used, and each covers the full width of the armature, then $L-L_1=0$, and

$$T=T_a\left[1+\frac{p\,g}{3.19\,D\,L_1\,(1-b\,p)}\left(c_2\times\frac{4\,D}{p}\,(0.9+0.035\,N)\right.\right.$$

$$\left.\left.+c_3\,L\,\frac{(1.33\,d_s+0.7)}{s}\right)\right]$$

Therefore the total interpole turns for one pole are equal to the effective armature turns per pole multiplied by a constant which is a function of the proportions of the machine. However, this holds true only for the condition of no saturated path for the armature flux, such as magnetic bands.

The above formula gives the interpole turns for two interpoles acting on each armature coil. With but one interpole per coil the number of conductors per armature coil generating the interpole e.m.f. is halved so that the flux density must be at least doubled, and the effect of the armature flux in the interpolar space over the other half of the armature coil must also be taken into account. This can be done in the preceding formula by using the equivalent value of L_1.

With half the number of interpoles the effective gap length, g, will not be the same as with the full number of interpoles with the same mechanical gap, for the flux from the interpole may be considered as returning across the gap of the two adjacent main poles and the value of g must be increased to represent the total resultant gap.

Let g_e represent the effective resultant gap,
g_m represent the effective gap under the main poles,
A_i represent the area of the interpole gap, and
A_m represent the area of one main pole gap.

These areas can be derived from the field distribution or " field form " of the main and the interpoles.

Then, the effective resultant gap $g_e=g+\frac{A_i}{2\,A_m}\,g_m$, and this should be used instead of g.

With half the number of interpoles and on the basis of the interpole flux returning through the two adjacent main poles, it may be assumed that this flux weakens the total flux in one pole and strengthens that of the other pole a like amount. If there is no saturation in the main poles or armature teeth under them, then no additional ampere turns, other than for the increased gap, will be required on account of the main poles carrying the interpolar fluxes. However, where there is much saturation of the main poles or teeth, then additional interpole ampere turns will be required, as will be described later in connection with effects of saturation.

Chord Windings with Interpoles. Chorded armature windings can be used with interpoles with satisfactory results provided the interpoles are suitably proportioned. There are apparently some advantages with such an arrangement, but there are also disadvantages of such a nature that it is questionable whether it is advisable to use chord windings with such machines, except possibly in special cases. When chorded windings are used with interpoles, the e.m.f. due to the armature flux is usually much smaller than with a pitch winding and thus fewer interpole magnetizing turns are required. Also, the effective armature turns which must be neutralized by the interpole are reduced somewhat, which also means a slight reduction in interpole turns. Against these advantages must be charged the disadvantage of a wider interpole face. This in itself would not be objectionable where there is space for such wider pole face, but if the space between the main poles must be increased it may lead to sacrifice in the proportions of the main poles or changes in the general dimensions, such that the result as a whole is less economical than with a pitch winding.

Effective Armature Ampere Turns. The term T_a representing the effective armature ampere turns should be considered, as the value of this term is influenced by a number of conditions, such as the number of bars covered by the brush, the amount of chording in the armature winding, etc. With a full pitch winding and neglecting the reduction in current in the short circuited coils, the magnetomotive force of the total armature winding is represented by the expression, $\frac{I_c\, W_t}{2}$, and per pole it is, $\frac{I_c\, W_t}{2\, p}$. However, when the brush spans several coils, so that a number of armature coils are short circuited at the same time, the average current in these short circuit coils should be considerably less than the normal value so that the effective ampere turns per pole is correspondingly reduced. Allowance must be made for this reduction as it has considerable influence in determining the correct number of interpole turns.

On the basis of no local currents, the average value of the current in the short circuited coils is just half that of the work currents per conductor.

Let B represent the total number of commutator bars,
B_1 represent the number of bars spanned by the brush,
p_1 represent number of current paths, and
p number of poles.

Then, $\frac{B\,T_c}{p_1\,p}$ = total number of armature turns per pole, and $\frac{B_1\,T_c}{2\,p_1}$ = number of turns by which the total armature turns per pole must be reduced to obtain the effective turns per pole, or,

$$T_a = \frac{B\,T_c}{p\,p_1} - \frac{B_1\,T_c}{2\,p_1}$$

$$B\,T_c = \frac{W_t}{2}, \ldots T_a = \frac{W_t}{2\,p\,p_1} - \frac{B_1\,T_c}{2\,p_1}$$

Let $B_1\,T_c$ be represented as a percentage of W_t, or $B_1\,T_c = b\,W^t$

Then, $T_a = \frac{W_t}{2\,p_1\,p}\,(1 - b\,p)$

$$I\,T_a' = \frac{I_c\,W_t}{2\,p} = \frac{(I_c\,p_1)\,W_t}{2\,p\,p_1} = \frac{I\,W_t}{2\,p\,p_1}, \ldots T_a' = \frac{W_t}{2\,p\,p_1}$$

Therefore, $T_a' = \frac{T_a}{(1 - b\,p)}$.

Chorded windings also have an influence on the effective armature ampere turns per pole. When the winding is chorded one slot, for example, then, in one slot per pole, the upper and lower coils will be carrying current in opposite directions and their magnetizing effects will be neutralized. In consequence, the total effective armature ampere turns are correspondingly reduced and this must be allowed for in determining the interpole turns.

Conditions Affecting Interpole Proportions. The foregoing formulæ have been based upon the use of interpoles of such proportions that the interpole flux varies directly as the magnetizing current and its distribution over the commutating zone is such as will give the proper opposing e.m.f. at all times.

However, the proportionality of flux to current can only be true as long as there is no saturation in the interpole magnetic circuit. Such saturation is liable to be found in practice and not infrequently it is quite a problem of design to avoid it within the working range of the machine.

Also, another difficult problem lies in so designing the interpole face that the flux distribution in the commutating zone is such that its e.m.f. will properly balance the armature e.m.fs. in the short circuited coils, especially as the latter are generated by cutting fluxes which may be distributed in a quite different manner from the interpole flux.

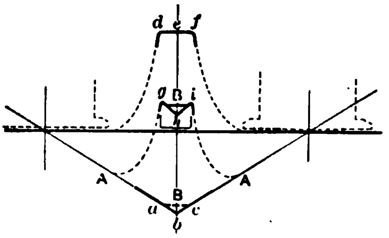


FIG. 21

Shape and Proportion of Interpole Face. As already shown, the effective interpole flux under the pole face is the resultant of the total interpole magnetomotive force and the opposing armature magnetomotive force. As the armature winding is distributed over a surface and the interpole winding is of the polar or concentrated type, the resultant magnetomotive force would normally be such as would not tend to give a uniform flux distribution under the interpole unless the interpole face is properly shaped or proportioned for such distribution. The conditions may be illustrated in Fig. 21. In this figure the lines *A A* represent the armature magnetomotive force, with a full pitch winding, and the brush covering one commutator bar. The heavier part of the lines *a b c* at the peak of the magnetomotive force diagram, represents the armature magnetomotive force which would be obtained under the interpole face, and also the flux distribution which would be obtained, with no interpole magnetomotive force, and with uniform gap under the pole faces. In opposition is shown the interpole magnetomotive force and flux distributions *d e f* for corresponding conditions. The resultant magnetomotive force is represented by *g h i*, and with a uniform gap under the pole, the resultant interpole flux would have a similar distribution. Instead of this, either a flat or, in some cases, the reverse distribution is required, that is, with a slight " hump " in the middle instead of a depression. By properly shaping the pole face so as to give an increased air gap toward the edges, the flux distribution can be made practically anything desired. In some cases a relatively narrow pole tip with a very large air gap will give a close approximation to the desired flux distribution.

However, in practice the above distribution of the armature magnetomotive force is rarely found. The use of brushes which cover more than one commutator bar serves to cut off or flatten out the pointed top of the armature magnetomotive force diagram, as shown by the dotted line *B*, in Fig. 21, and thus lessen the depression at the center of the resultant magnetomotive force distribution.

As intimated before, this problem of proportioning the interpole face turns upon the determination of the armature e.m.fs. in the short circuited coils which have to be balanced by the interpole. If the different armature e.m.fs. are determined for the whole period of commutation and then superimposed, the resultant e.m.f. indicates the flux distribution required under the interpole. Usually the e.m.fs. due to the end winding, and to the interpolar flux, if any, will be practically constant during the whole period of commutation. If no local currents are present the e.m.f. due to the slot flux will also be practically constant, although it may be slightly reduced near the beginning and end of the commutation period. The sum of these e.m.fs. should therefore be practically constant over the whole commutation period and therefore, in a well designed machine, the interpole flux density should be practically constant over the whole commutation zone. As explained before, special shaping of the poles and pole face will be necessary, in most cases, to obtain exactly this proper flux distribution. Large interpole air gaps are obviously advantageous in obtaining such distribution. In fact, a very small interpole gap makes the determination of the proper interpole face dimensions very difficult in many cases. On account of the interpole usually covering less than two armature teeth, the ordinarily accepted methods of determining the effective length of air gap under a pole will not apply, in many cases, which may lead to a slight error in the results. Practically the effective gap under the narrow interpole will usually be longer than determined by the ordinary methods. This partly explains the fact that, in some cases, an increase in mechanical clearance between the interpole face and the armature core does not require anything like a corresponding increase in the interpole magnetizing ampere turns. The effective interpole air gap increases, but at a much less rate than the mechanical gap.

The brush setting in relation to the interpole is of great importance. The point of maximum armature magnetomotive force is definitely fixed by the brush setting. With the interpole fixed in position, any shifting of the brushes backward or forward will obviously change the shape of the resultant magnetomotive force distribution under the interpole face and in consequence the flux distribution will be changed. With but one armature coil per slot and the brush covering but one commutator bar, good commutating conditions might be found over a considerable range of brush adjustment, by suitably varying the interpole ampere turns. However, with two or more coils per slot and with the brush short circuiting several bars, any marked change in the resultant interpole magnetomotive force and flux distribution will mean improper commutation for some of the coils. Proper brush setting is therefore of first importance.

It has been assumed in the foregoing treatment, that an exact balance between the interpole and armature e.m.fs. will give the best conditions. From certain standpoints, this is true, but in practice usually a slight excess in the interpole strength, or " over-compensation " of the interpole, as it is frequently called, is advantageous. Reference to Fig. 14 shows that in a machine without interpoles, and therefore without compensation, the current flowing between the brush contact and the commutator is crowded toward one brush edge, this being the edge at which the commutation of a coil is completed, that is, at the so-called forward brush edge. With over-compensation the opposite effect occurs—that is, the brush current density is below the average at the forward edge. This is, to a certain extent, a desirable condition. Also, if there is any saturation of the interpole circuit at overloads, the over excitation of the interpole winding can take care of the saturation ampere turns, so that normal compensation can be obtained at considerably higher load than in a machine with no over compensation. Furthermore, over compensation is desirable on account of the effect of the resistance of the coils undergoing commutation, which heretofore has been neglected as being of minor importance. Such resistance tends to lower the current density at the middle of the brush contact, and increase it toward the brush edges. Over compensation will oppose this at the forward edge, but increase it at the back edge, which is less objectionable. Also, as shown in Fig. 21, there is liable to be a depression at the center of the interpole flux distribution, if the pole face is not properly shaped. This depression tends to cause higher current densities at the brush edges. Over compensation again tends to reduce this density at the forward brush edge. Thus there are several good reasons for slight over compensation, and practical operation bears this out, especially on high voltage machines, where the short circuit e.m.fs. average higher than in other machines.

Balanced Circuits. It has been assumed that the armature ampere turns per pole have been the same for all poles. This will be true for the usual two-circuit or series type of winding, or its allied combinations, but is not necessarily true of the parallel type of armature winding. In such a winding a number of circuits are connected in parallel at the brushes, and, unless ample provision be made for equalizing the different circuits, they may not carry equal currents at all times. As the resultant interpole flux and e.m.f. is directly dependent upon the opposing armature ampere turns, it is obvious that any inequalities in the armature currents would lead at once to incorrect interpole conditions. A poorly equalized parallel-wound armature might furnish conditions such that the interpoles cannot be adjusted for satisfactory operation. Also paralleling of the interpole windings, unless care be taken to insure equal current division among the circuits, is liable to lead to trouble.

Saturation of the Interpole Circuit. Heretofore the interpole turns T, as determined, have been only those required for forcing the resultant interpole flux across the effective interpole air gap, and nothing has been allowed for any turns required for magnetizing the parts of the interpole circuit other than the gap. Where such additional turns are required they must be added to the turns T, already determined.

Saturation in the interpole magnetic path is the principle cause for such additional turns, but saturation in the various flux paths may occur in such a way as to be either harmful or beneficial, depending upon where it is located. Beneficial saturation may be assumed to be such as will reduce the armature short circuit e.m.fs., while harmful saturation tends to reduce the interpole e.m.f.

While the useful interpole flux passing into the armature may be relatively low—say one-fifth that required for saturation of the interpole material—the leakage flux between the interpole and the two adjacent main poles is often very much greater than the useful flux so that the interpole at the part where it carries the highest total flux may be worked up to possibly half saturation, or higher, with normal load on the machine. The interpole leakage flux is due to the ***total ampere turns on the interpole***, while the useful interpole flux is due only to the magnetizing component of the interpole ampere turns, which may be as low as 15 per cent to 25 per cent of the total interpole ampere turns. The leakage flux is thus liable to be a high percentage of the total interpole flux.

While the ampere turns on the interpole will rise in direct proportion to the current, the effective magnetizing component will rise in direct proportion only below saturation of the interpole circuit. Any ampere turns required for saturating this circuit will be taken from the magnetizing component of the interpole winding. Therefore, when any appreciable saturation occurs, the effective magnetizing component will not vary in proportion to the current, and the interpole e.m.f. will not vary in proportion to the armature e.m.fs. As the magnetizing component of the interpole winding usually represents a relatively small number of ampere turns per pole a comparatively slight saturation in the interpole circuit may have an appreciable effect. It is therefore advisable to work at as low a saturation as possible in the interpole circuit so that practically no saturation occurs within the ordinary working range of the machine.

Where saturation occurs in any of the armature flux paths, as, for instance, with saturated bands over the armature core, the result of such saturation will serve to neutralize the effect of saturation in the interpole magnetic circuit. In other words, the armature e.m.f. will not rise in proportion to the current and therefore the opposing interpole e.m.f. does not need to increase in proportion either.

The principal source of saturation in the interpole circuit lies in the magnetic leakage from the interpole to the adjacent main poles. Serious trouble has often been encountered by not making due allowance for such leakage. However, there may be other causes for saturation. When the full number of interpoles is used the interpole magnetic path or circuit is independent of the main pole magnetic circuit, except in the yoke and in the armature core below the slots, as indicated in Fig. 22. In the yoke it may be seen that the interpole flux is in the same direction as the main flux at one side of the main pole and is in opposition to the main flux at the other side. The same is true in the armature core. Therefore the interpolar flux tends to reduce the flux in one part of the yoke and tends to increase it in the other part. If the saturation in these parts is relatively low, then the magnetomotive force required for forcing the low and the high fluxes through the yoke will be but little greater than if these fluxes were equal. However, if the yoke is highly saturated the increase in ampere turns required for the high part much more than offset the decrease in ampere turns for the low part, so that, as a result, additional ampere turns are required for sending the interpole flux through this path. The interpole ampere turns therefore must be increased on this account, when the saturation is high. The same condition holds for the armature core.

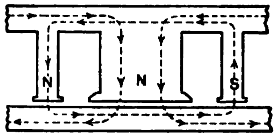


FIG. 22

A similar condition occurs where half the number of interpoles is used and when there is much saturation of the main pole and the armature teeth under it, as already referred to. This condition requires additional interpole ampere turns.

In practice, with the ordinary compact designs of direct current machines, it is usually difficult to keep the total interpole flux as low as one-third that which gives any material saturation and, not infrequently, it is much higher than this. Therefore, by direct proportion it might be assumed that such machines could carry only double to treble load without sparking badly. However, the resistance of the brushes, etc., will be of such assistance that relatively higher loads may be commutated reasonably well. For instance, with the interpole worked at about half saturation at normal load, the machine may be able to commutate considerably more than double load without undue sparking. It is also of material assistance, where heavy overloads are to be carried, to over-excite the interpole winding, that is, to make the magnetizing component somewhat greater than required at normal load, as described before. In this case, at light loads, the interpole e.m.f. exceeds the armature e.m.f. a certain amount which is taken care of by the brush resistance as local currents will be less harmful when the work current is low. As partial saturation is obtained at overload, the two e.m.fs. become equal but at a higher load than would be the case without over-excitation of the interpole.

Commutating Conditions on Short Circuit. When a direct current generator is short circuited across its terminals, either through a low external resistance or without such resistance, a current rush will occur which will rise to a value represented approximately by the generated e.m.f. divided by the resistance in circuit. This current rush is only of short duration as the excessive armature current will react to demagnetize or " kill " the field. If the short circuit is without external resistance the current rush may reach an enormous value as the internal resistance on large machines is usually very low. This means that currents from 25 to 40 times full load may be obtained on " dead " short circuit. Experience shows that under such current rushes, any kind of direct current machine will tend to flash viciously at the brushes.

By the preceding theory and analysis a rough approximation to the commutating conditions on short circuit can readily be obtained. Assuming an interpole machine, the following conditions will be found:

1. The interpole will be highly saturated so that it is of little or no direct benefit.
2. The slot flux will rise to such a value that the armature teeth in the commutating zone are practically saturated.
3. There may be some interpolar flux from the armature, as the high interpole saturation may allow this.
4. The armature end flux, with the exception of that part due to magnetic bands, will rise practically in proportion to the current.

The following short circuit e.m.f. conditions will be obtained:

1. There will be possibly a slight e.m.f. due to the armature interpolar flux.
2. There will be an e.m.f. due to the tooth flux which is almost as high, per conductor, as could be obtained by a conductor cutting the flux *under the main field at no load*, for saturation of the armature teeth may be assumed to be the limit in both cases.
3. There will be an e.m.f. due to the end flux which may be 10 to 20 times larger than at normal full load.

Therefore, the total e.m.f. in the short circuited coil due to cutting the armature flux on dead short circuit may be higher than would be obtained *if the brushes were shifted at no load until the commutated coil lies under the strongest part of the main field.*

As very few machines of large capacity would stand this latter condition without flashing, it may be assumed that they would be no more able to stand a dead short circuit without flashing. In fact, 8 to 10 times full load current will make an interpolar machine of normally good design flash badly, as it is impracticable to make an interpole of the usual type which will not saturate highly at 8 to 10 times normal current.

If, however, the interpole is combined with compensating windings in the main poles, the interpole leakage may be made so small that comparatively low saturation is obtained normally in the interpole circuit. In such case the interpole may be effective with heavier currents and the flashing load may be very much higher than with the usual type of interpole machines.

CONCLUSION

The foregoing is a general presentation of the problem of commutation, which is admittedly crude and incomplete in some points. In particular may be mentioned the part describing the action of local currents. Also, the method of considering the resultant action in interpole machines as the superposition of two components does not tell the whole story, but the actual analysis, in detail, of a number of these phenomena would be so confusing and complicated that a general physical conception of what takes place during commutation would be lost. In the ultimate analysis it will be found that a number of the methods described are, in reality, simply illustrations of the conditions of commutation rather than an analysis of the conditions themselves. However, the method as given throws light on many things which take place during commutation. It also includes a number of conditions which are not covered in the usual methods of dealing with this problem. For example, the number of commutator bars spanned by the brush is an important element in this method of handling the problem, whereas, in many former methods, this point was either omitted, or treated in an empirical manner. In this method the results obtained would be very greatly in error if the brush span were not included.

Any theory or method of calculation is open to question until ti has stood the proof of actual test. In consequence, the above method has been tried on a very large number of direct current machines, including high speed direct current generators, direct current turbo-generators, direct current railway motors of all sizes, moderate- and low-speed generators of all capacities, industrial motors of various designs including adjustable speed motors and machines with half the number of interpoles. In those cases where the actual test data of the machines was very accurately obtained, the agreement between the tests and the calculated results by the above method was found to be close. In fact, the method in some cases indicated errors or inaccuracies in the test results. In a number of cases of early interpole machines there was considerable disagreement between the results of the calculation and the actual test, but, in many of these cases, later experience showed definitely that the proper interpole field strength or proportions had not been obtained in the actual test or that the proper brush setting had not been used. These cases were thus, to a certain extent, a verification of the method, for in general the greatest discrepancies between the calculated and the test results corresponded to the machines which eventually proved to have the poorest proportions or adjustment.

This theory of commutation looks complicated and cumbersome in its practical application, but it should be understood that it is, in reality, an exposition of a general method from which special and simpler methods may be derived for different types and designs of machines. It indicates plainly that the problem is so complicated that no simple formulæ or methods of calculation can be devised which will cover more than individual cases, and that such formulæ, if applied generally, will lead to error sooner or later. If, however, the general derivation of such simplified formulæ is well understood, then they may be used with proper judgment and with much less danger of error in the results. It is evident, from the general analysis, that the whole problem must be handled with judgment, for new or different conditions are encountered in almost every type of machine.

A great many problems, closely allied to that of commutation in interpole machines, have not been considered, because some of them represent special cases of the general theory, while others are somewhat outside the subject of this paper. Of the former class may be mentioned, commutation of synchronous converters, machines with distributed or true compensating windings, the so-called " split-pole " converter, and the commutator type alternating current motors, etc. In the latter class may be included such problems as the effect on commutation of closed circuits around the interpoles, losses due to commutation, current distribution at the brush contact, etc. Some of these subjects were included in this paper as originally prepared, but on account of its undue length they had to be omitted.

DISCUSSION ON "A THEORY OF COMMUTATION AND ITS APPLICATION TO INTERPOLE MACHINES," NEW YORK, OCTOBER 13, 1911.

H. F. T. Erben: When I learned that Mr. Lamme had written a paper on the subject of commutation I was in hopes that at last we would see an exposition on the subject from which would be absent the familiar "K" which is found in all formulae so far developed by various writers. I find however, that the "K" is still freely used and that Mr. Lamme has been unable to develop formula which brings the subject of commutation down to what we might call an exact science. I do not wish to disparage Mr. Lamme's ability to produce such formulae I firmly believe that there are so many variables, both of the mechanical and electrical nature, entering into the subject of commutation that the derivation of a formula without constants is simply an impossibility. I think we will have to remain content with formulae of a skeleton nature, upon which the designer will build the finished product. I think that the merit of Mr. Lamme's paper lies in the method in which the subject matter is presented, rather than any new or fundamental ideas.

Of the three fluxes which Mr. Lamme considers, the first two have always been thoroughly considered in making our calculations. I have not studied the paper sufficiently to get a clear idea of what Mr. Lamme terms the "slot flux". I think, however, that what he terms the "slot flux" is identical with what other authors have termed "flux set up by the coils undergoing commutation". The fact that the formula which Mr. Lamme uses for determining the e.m.f. due to slot flux is very similar to that used by other designers, confirms this opinion.

In the latter part of Mr. Lamme's paper he states that up to the present, little has been done in the way of determining the maximum overload capacity of machines at the time of short circuits. A little over a year ago we carried out a long series of tests on machines of the commutating pole type and compensated type with a view to determining the maximum momentary overload which could be carried without flashing over. The result of these tests show that an overload of 10 to 12 times normal, will cause a complete flash over and that momentary overloads of seven or eight normal, will cause very severe sparking and flashing around the brush holders but the machine is not liable to completely flash over. The compensating winding permits a machine to carry a somewhat heavier momentary overload but the gain is not very great. Oscillograph records taken at the time of short circuits show that in the case of a compensated machine there is less field distortion than if commutating poles alone are used and in consequence a machine fitted with compensated winding will carry a somewhat greater maximum load before the flashing over point is reached.

Photographs taken at the moment of short circuits show that a considerable amount of gas or vapor is generated, due to the brushes and commutator copper being momentarily heated to incandescence. This gas being of high conductivity permits the establishment of an arc between the brush and the commutator segments.

The formation of gas as effecting flashing is clearly shown by the fact that if half the brushes are removed from the commutator the machine will flash over at a lower current value than if the full set of brushes are present.

We are all very much indebted to Mr. Lamme for this paper as he has presented the subject of commutation in rather a new light and has thereby stimulated general interest in a subject which has of late years been neglected by those presenting papers before general technical societies. I hope that this paper will serve the purpose of inducing other members to present their views on this most difficult and complex subject.

Geo. L. Hoxie: The way in which Mr. Lamme has put together his qualitative description of what goes on in a machine seems to me to be open to some criticism. The point I wish to make is that you cannot consider magnetic fluxes separately, each as being a flux due to a particular m.m.f. taken alone, and then add up these component fluxes and get the resultant flux. That is to say, you cannot do this where you have magnetic material in your magnetic circuit, and especially where some parts of the magnetic circuit are saturated, as will usually be the case.

In a direct current generator we have at no load a more or less uniform flux under the pole faces, a very dense flux through the armature teeth, etc., all due to the field currents. With load, armature currents superimpose their m.m.f. upon the m.m.f. of the field and the resultant m.m.f. produces a resultant flux which as a rule is not at all the geometric sum of the fluxes that would be due to each m.m.f. taken separately. This is the very result that electrical machinery designers found it necessary to strive for from the beginning, and they usually expressed the thing by saying that the distortion of field flux due to armature reaction was largely minimized by a strong field, liberal air gap, and by saturation, in pole tips, pole faces, and armature teeth. In the very early machine it was not uncommon to have the magnetism in one set of pole tips reversed by armature reaction. Such a state of affairs meant a tremendous field distortion, and brought with it very bad commutation.

I would take issue with Mr. Lamme on the proposition that field distortion has nothing to do with the problem of commutation. As I understand the problem of commutation it is ideally desirable that during the time a coil is short-circuited by a brush, the coil should have generated within itself an e.m.f. opposed to the current that the particular coil carried a moment earlier. Also that the e.m.f. of the short-circuited coil should be exactly enough to stop the old current in the coil, start a current in the opposite direction, and bring this new current up to the exact number of amperes that are flowing in the coils next beyond. If all of this occurs exactly during the time that the coil is short-circuited, ideal commutation will result. If the brushes be given no lead, zero e.m.f. is generated at no load in the short-circuited coil. The correct lead used to be found by experiment. Then as load increased, the field distorted, and a new commutating position was found, also by experiment. With the introduction of carbon brushes, and saturated magnetic circuits, machines were produced that did not require a constant shifting of brushes to follow up the distorted field. This was largely because only a little distortion occurred.

Now we come to interpole machines, which are rather a new thing. In these machines the effort is made to set up a m.m.f. proportional to the armature m.m.f., and either equal to, or a little greater than, the armature m.m.f. This means practically an effort to balance, or neutralize, armature reaction, at least so far as concerns the field through the coils being commutated. It is found that neutralizing armature reaction in this manner produces very excellent commutating results, and the interpole machines are getting quite popular.

I do not intend to discuss interpole machines but I do want to say that about 21 or 22 years ago Professor H. J. Ryan designed and patented a pole-face winding, so called, which was simply a distributed interpole winding, or was a series winding distributed in the pole faces just outside the armature, having ampere turns equal and opposite to the armature ampere turns. Those machines also commutated excellently. I have always believed, and I still believe, that had the Ryan patents been owned by one of the big companies instead of by a rather small concern we should have seen direct current development proceed upon quite different lines. Probably these patents have expired by this time, and possibly they would cover the interpole machine if still in force.

Perhaps we shall yet see the pole face winding, or some modification of it, such as I take the interpole to be, come into wide use. It will certainly pay students of direct current machine design, and of commutating problems, to look carefully into this early work of Professor Ryan's.

H. M. Hobart: In listening to this very interesting abstract by Mr. Lamme it occurred to me what a capital thing it would be if the International Electro-Technical Commission could standardize some commutation criterion. Each of us has a theory of commutation and each believes that his own theory serves the case admirably; but we have great difficulty in interchanging views. We all speak the same mother tongue, and yet nevertheless language differences exist which make it quite difficult to follow another's exposition of the subject. Now, I have felt this in reading Mr. Lamme's paper. Already at the first reading I could see that a great deal of valuable information was contained in the paper, and yet it took a lot of study on my part before I began to grasp the full significance of Mr. Lamme's point of view, and as my study of the paper progressed, I found that the differences between the way we are often in the habit of looking at it and the way in which I saw Mr. Lamme regarded it, gradually decreased until it seems to me it is largerly a matter of expression. I have usually obtained pretty good results from considering the voltages in the short circuited coils as arising largely from various components and it has only seemed worth while to take the two principal of these components into account. Mr. Lamme speaks of a component voltage associated with slot fluxes and a component voltage associated with end winding fluxes. In addition to these he lays a great deal of stress on a third component, that associated with the interpolar flux.

It is difficult to deal with this third component in any except one of the three cases which naturally present themselves. There is, first, the interpole machine; then secondly there is the non-interpole machine in which you advance the brushes a little to overcome sparking, and thirdly there is the non-interpole machine operated with the brushes in the geometrical neutral position. As to the first of these types, the armature m.m.f. which would (were no interpoles present) set up the interpolar flux, is overcompensated by the m.m.f. of the winding on the interpole. As to the second type, I agree with the previous speaker that Mr. Lamme's method of treatment is faulty, since he combines fluxes instead of adhering to the combination and resolution of m.m.fs. It seems to me hopeless to tackle the problem in this incorrect manner. But in the third type, the difficulty largely vanishes. We there have an arrangement in which the armature m.m.f. is alone of any account in the position midway between pole tips because the main field fluxes are symmetrically disposed with reference to the resultant armature m.m.f. The difficulties met with in determining the interpolar flux in this third type relate mainly to the estimation of the reluctance of the path followed by that flux from the center lines on the armature periphery until the flux finds a landing place in the main magnetic circuit. For my part, I never had the temerity to attempt to calculate that flux. I think Mr. Lamme is entitled to credit for his courage in undertaking the job. In the abstract which he gave us Mr. Lamme states that we need a different formula for calculating the interpolar flux for each different type of magnetic circuit.

I rather anticipate that there will be considerable difficulty in calculating that flux. Nevertheless, it is well worth trying. I wish that Mr. Lamme had given quantitative figures of representative machines, setting forth the amount of each of these three components of the sparking voltage. I am very interested in Mr. Lamme's statement that in a certain design this third component was greater than the sum of the other two components. As far as I can see, that could only have been the case in rather a bad machine. In my designs, I have always sought to minimize the interpolar flux. A chief means to this end is to have as wide a neutral zone as possible. This usually involves having a fairly small pole arc and a considerable distance between pole tips. The interpolar flux may also be maintained low by designing with a weak armature and a strong field, but of course such a design is relatively expensive.

After careful reflection I feel disposed to give very serious consideration to Mr. Lamme's proposal to take into account this third component in addition to the two components usually considered, indeed, we have always done it in one way or another, for of course we all recognize that the reactance voltage formulae we use are empirical, and if we decide on employing three components instead of two, then we shall distribute the resultant voltage amongst the three components in as suitable a proportion as our ability to analyze the data at our disposal will permit us.

Mr. Lamme gives no attention whatever to one point of considerable importance in the study of commutation. We can correctly deal with the reactance voltage nowadays. When the reactance voltage must be high, we neutralize it by the introduction of interpoles. Amongst other advantages this permits of building motors whose speed may be varied over a wide range by controlling the current in the shunt field. But while the reactance voltage now presents no difficulty in such machines the commutation may still be very bad through having too great an average voltage between adjacent commutator segments. At the motor's highest speed the maximum voltage between adjacent segments will often be several times greater than the average voltage between segments. This is because the armature reaction quite overmasters the normal field. It may lead to 40 or 60 volts between the segments when the average voltage between segments is only—say—10 or 15. Now that brings us to the border line of commutation trouble of a kind which has nothing to do with the reactance voltage.

The introduction of interpoles has tempted designers to increase the reactance voltage. Indeed, there has been a temptation to take great liberties in this direction, such as decreasing the number of segments with a view to cheapening the machine. This has resulted in increasing the average voltage between segments and has increased still more the maximum. I have pointed out this omission to deal with the average and maximum voltages between adjacent segments as I consider that in a modern theory of commutation their consideration is of no less importance and may even be of greater importance than the consideration of the reactance voltage.

Returning for a moment to a consideration of Mr. Lamme's third component of the reactance voltage, *i.e.*, the component associated with the interpolar flux, it will obviously be a function of the armature strength in ampere turns per pole. For machines with the same armature strength and the same number of poles, this third component may for armatures with one turn per commutator segment, be expressed as a function of the product of the gross core length (λg) and the speed in revolutions per minute (R). It is interesting to compare this result for Mr. Lamme's additional component with a result in my article published on page 6 of *The Electrician* for April 20, 1906, and which is further developed on page 336 of Hobart and Ellis' "High Speed Dynamo Electric Machinery", from which I abstract the following portions of an extensive table for facilitating estimations of the reactance voltage:

Table of factors to be employed in obtaining the reactance voltage in terms of the gross length of the armature core and the speed in revolutions per minute.

Armature strength in ampere-turns per pole	Values of K in the formula: Reactance voltage $= K \times \lambda g \times R$ for the numbers of poles set forth at the heads of the vertical columns		
	4	**8**	**16**
8000	0.000256	0.000512	0.001024
4000	0.000128	0.000256	0.000512
2000	0.000064	0.000128	0.000256

From the above table it is seen that I have resolved the first two components (slot and end connection components) down to a simple form to which the "interpolar" component can also be reduced. Nevertheless, it would appear to be desirable to keep in mind that the result is a combination of *three* important components instead of comprising only *two* components.

We must always remember that electromagnetic considerations are by no means the only ones affecting commutation. Various mechanical conditions may be and often are of predominating importance. These are often not susceptible of *quantitative* predetermination. Thus who can calculate the extent of the improvement in commutation which will attend grooving out the mica between segments, or the use of a grade of brushes of different composition and construction, or the substitution of a different type of brush holder, or a low peripheral speed of the commutator? We know that we can often very greatly improve commutation by attention to these points and that we may then safely employ higher reactance voltage and higher average and maximum voltage between commutator segments, but we can rarely make quantitative statements of the extent to which we shall obtain advantages by these means.

Malcom MacLaren: The method of treating the theory of commutation proposed by Mr. Lamme throws a great deal of light upon this very complex problem, and should allow a clear analysis to be made in many special cases where the problem had previously appeared hopelessly involved.

The formulae which he derives appear at first sight rather formidable, but, as he explains, these can be materially simplified when applied to a single class of machines. For example for four-pole machines with the poles covering ⅔ of the armature circumference the expression for E_c becomes:

$$E_c = \frac{14.7\, I_c\, W_t\, T_c\, R_s\, L}{10^8}$$

A somewhat closer approximation can be made for four-pole machines by substituting 82 for 90 in the value of b and substituting $\frac{\pi D}{12}$ for $\frac{P}{2}$ in the expression for a. These changes would simply alter the constant in the above expression for E_c and make this 16.8 instead of 14.7. For any greater number of poles the approximations which have been made in the general solution are very close. A further modification might be introduced however, to take care of the effect of armature slots, for the expression given in the general formula shows the e.m.f. which would be produced in the coil under commutation due to the interpolar flux, if this coil were on the surface of the armature. This correction would of course be slight except in the case of comparatively small diameters and deep slots.

It is interesting to note that this method of treatment shows clearly that a full pitch winding commutates under better conditions as regards the interpolar flux than a chorded winding. This effect, however, is not large even with considerable chording when account is taken of the neutralization of the armature conductors in the chorded zone. For example, with the pole covering ⅔ the polar pitch and with chording ½ of the interpolar space the effective ampere turns will be 5/6 of the ampere turns with full pitch winding. The length of path from the coil under commutation to the nearest pole will be ½ the length of path for the pitch winding, and the length of path to the more distant pole will be 1½ times this distance, so that the mean effective path for the chord winding will be ¾ of the path for the pitch and the strength of field will be $5/6 \times 4/3$ or 10 per cent higher than the field with pitch winding.

The effect of chording upon the slot flux as given in the paper on the basis of the brush spanning one commutator bar is apparently greater than experience with actual machines would suggest. Mr. Lamme's analysis clearly shows that a number of factors enter into the determination of the effect of chording upon commutation some of which tend to neutralize each other, but it would seem as though sparking at the commutator would be more a function of the maximum rate of cutting of the slot flux than the average, which Mr. Lamme has used.

Applying the formula, given by Mr. Lamme, to a number of non-interpole machines for which the commutating characteristics were known, it was interesting to note the relative importance of the various elements contributing to the e.m.f.

produced in the short circuited coil. All of the machines investigated had long poles similar to Fig. 3, so that the expression for the interpolar flux in the formula could be used without change. As comparative figures only were desired, the brushes were considered as spanning one commutator bar and were set on the geometric neutral. It was found that for a whole line of slow speed generators the e.m.f. due to the end winding flux remained surprisingly constant at about ½ volt; but for certain high speed machines this value was nearly doubled. The e.m.f. due to the interpolar flux was relatively unimportant in many cases, but for several large generators this became the predominating factor; in the case of a 1500-kw. high speed generator the e.m.f. due to the interpolar flux was 2.8 volts, to the end winding flux 0.9 volt and the slot flux 1.3 volts.

C. A. Adams: Mr. Lamme deserves great credit for the careful and conscientious manner in which he has attacked this ever-new problem of commutation. He has considered several points ordinarily neglected, and has presented the whole subject in such a way as to assist greatly in visualizing a very complicated set of phenomena. The subject is one which in most of its phases does not admit of a very satisfactory oral discussion, since it requires too close an analysis. I should like however to say a few words about the characteristic feature of the present paper, the feature which chiefly distinguishes it from most others on the same subject; it is the assumption that the total e.m.f. induced in a short-circuited coil is induced by the mechanical movement of the coil in question through certain stationary fluxes.

In the more familiar form of analysis a part of this e.m.f. is calculated as if induced by the change in magnitude of the flux linked with and moving with the coil in question, said flux being due and proportional to the current in the coil. When more than one coil are undergoing commutation at the same time, this method demands the consideration of the e.m.f. induced in each coil by the change of current in each of the others, *i.e.*, the consideration of the mutually-induced as well as the self-induced e.m.f. This necessity seriously complicates the problem in the general case, and it was doubtless a desire to avoid this complication that led Mr. Lamme to his present method. The question I wish to raise as to the legitimacy of this method, may be most readily answered by a consideration of the location and magnitude of the various m.m.fs. which go to produce the resultant flux in the commutation zone. *If these m.m.fs. are stationary as a whole, and if the local magnetic circuit is stationary, then will the resulting magnetic flux be stationary.*

Divide the total m.m.f. into three parts: first, the field m.m.f. which is obviously stationary; second, the main part of the armature m.m.f. (exclusive of the current in the commutating zone), also stationary as a whole; and finally the m.m.f. due to the current in the commutation zone, *i.e.*, the current in the coils undergoing short-circuit by the brushes. In order, for the present, to eliminate the question of variable local reluctance, assume a smooth core armature. Assume also a full pitch winding. Then the only possible source of a non-stationary flux is a non-stationary m.m.f. due to the current in the commutation zone.

Consider the current distribution around the periphery of the armature in the vicinity of the commutation zone. In Fig. 1, $\overline{B\,C} = +I$ represents the current in each coil as it approaches the brush, and $\overline{A\,D} = -I$ represents the current in each coil as it leaves the brush; the ordinates of the line $\overline{C\,D}$ representing the time variation of current in each coil during the commutation period, on the assumption of perfect commutation. If there are many segments under the brush and many coils undergoing commutation at the same time, the line $\overline{C'\,C\,D\,D'}$ will represent the approximate peripheral distribution of current in the vicinity

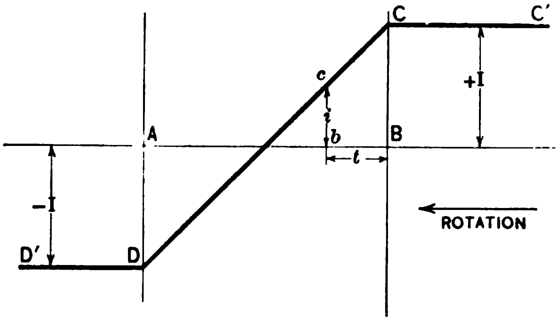


FIG. 1

of the commutation zone at any instant; *i.e.*, each ordinate of this line represents the current in the coil located at that part of the periphery at the instant considered. With any possible number of segments under the brush, each coil will occupy an appreciable part of the zone-width and the peripheral current density will obviously be constant over the coil width. If it were possible to have 16 segments under the brush, the instantaneous current distribution for the case of perfect commutation would be shown by the pair of stairs in Fig. 2, where i_1, i_2, i_3, etc., are the currents in the several coils located at b_1, b_2, b_3, etc. For the case of delayed commutation, shown by the time current curve $\overline{C\,O'\,D}$, the instantaneous space distribution of current would be shown by 16 steps of equal width but of varying height, superimposed on the curve $\overline{C\,O'\,D}$. A little consideration will show that there are slight pulsations in either of these distributions as each coil moves over the width of one step, *e.g.*, from b_1 to b_2, and that these pulsations are less, the larger the number of segments under the brush. In other words, for a large number of segments under the brush, the current and m.m.f. distribution are practically constant. Thus in this case of many segments, the e.m.f. computed on the assumption of a stationary flux (the method being accurately applied) will be exactly the same as that computed on the assumption that there is a self-inductive flux carried along by the current. In the case of a few segments however, the difference is not negligible; *e.g.*, consider the case where the segment width and the brush width are equal. The same assumptions as before are made as to smooth-core armature, full pitch winding, and perfect or straight-line commutation. Then when the coil under the brush has progressed ⅛ of the way across the commutation zone, the current in the coil, and the peripheral current density over that portion of the armature occupied by the coil in question, will have dropped to ¾ of their full precommutation value. The distribution of current density in the vicinity of the commutation zone will then be shown by the lines marked 1 in Fig. 3. When the coil has progressed 2/8 of the way across the zone, the distribution of current density will be shown by the lines marked 2 and so on. If the commutation is not according to the straight line $\overline{CD}$, but according to the curve $\overline{C O' D}$, then will the height of each step be determined by the corresponding ordinate of $C O' D$ rather than of $\overline{CD}$? In any case it is obvious that with only one coil under the brush, and to a lesser degree with two or three, there is a very considerable pulsation of the m.m.f. distribution in the vicinity of the commutation zone. It is thus no longer legitimate from the theoretical point of view to assume a stationary flux distribution, but more accurate to deal with that part of the armature m.m.f.

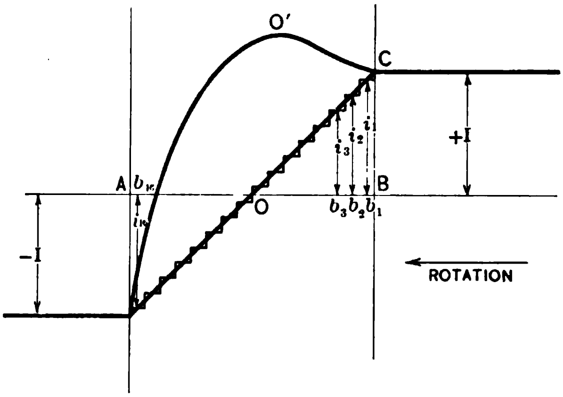


FIG. 2

comprised within the commutation zone as producing a local flux which moves with the armature and varies as the moving current varies. This point of view is still more obviously consistent with facts when it is remembered that in practically all commercial machines the armature core is toothed and that therefore the local magnetic circuit moves with the current. *I.e.*, the reluctance of the local self-inductive magnetic path linked with a group of conductors in a given slot is constant when considered as moving with the conductors, but the reluctance of a local path at any point (stationary with respect to the field poles) of the commutation zone is not constant, but varies as the teeth and slots pass by.

Most of the common simple formulae for commutation reactance e.m.f., give only the approximate *average* value, assuming perfect commutation, and do not take account of the variation

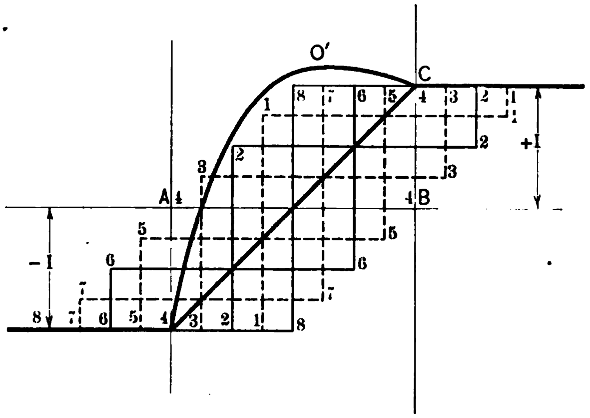


FIG. 3

of the reactance e.m.f. during the commutation cycle. The averaging in this method is equivalent to the assumption of stationary flux in Mr. Lamme's method; *i.e.*, neither takes account of the local pulsations of flux and of the corresponding induced e.m.fs. It is on this account that the principal terms in Mr. Lamme's equations are so familiar. The real essential difference between the common formulae and those of the present paper is that the latter include a number of factors ordinarily neglected.

Mr. Lamme states as evidence of the accuracy and legitimacy of his method, that the results check with the experimental results of tests which give the potential curve under the brush; but it should be remembered that an ordinate of one of these curves is obtained by connecting a direct current voltmeter across the contact surface at the point in question, and is therefore the *average* value of the potential drop, not showing the pulsations.

Thus neither Mr. Lamme's method nor the reactance voltage method as ordinarily applied, takes account of the pulsations above described. To do this requires a much more thorough analysis, such as that which Professor Arnold gives in his classical work, " Die Gleichstrommaschine ".

But there is still another source of flux pulsations and induced e.m.fs. besides that due to the pulsations of m.m.f. in the commutation zone and to the pulsations of local reluctance in the same region; it is the pulsations of the reluctance of the main magnetic circuit linking with the coil undergoing commutation, due to the varying number of teeth under the main field poles. *E.g.*, suppose that the equivalent pole arc is 10½ times the tooth pitch; then in one position there will be eleven teeth under one pole, and in another position only 10, with an approximate change of reluctance of something less than 10 per cent. If the same change takes place under adjacent poles at the same time this will mean pulsations of the main flux, which while much less than 10 per cent owing to the damping effect of the field coils, is nevertheless of sufficient magnitude to induce a considerable e.m.f. in the short-circuited coil. Increasing the number of slots, increasing the length of the gap, chamfering the pole faces, a proper choice of the ratio of the equivalent pole arc to the tooth pitch, or any arrangement which brings the maximum reluctance under one pole at the same time with the minimum reluctance under the adjacent poles, will obviously reduce these pulsations, which though not large on well designed machines, frequently account for otherwise unexplainable eccentricities of commutation.

I was very glad to hear Mr. Lamme agree that with the brushes in the geometrical neutral, the degree of distortion *per se*, has nothing to do with commutation. Since the coil is in the field-neutral, the only e.m.f. is due to the armature flux, which is practically independent of the air gap. With a short air gap there is much greater distortion but no greater flux in the field-neutral zone for a given armature current, and no greater induced e.m.fs. in the short-circuited coils, provided the shortening of the air gap has not increased the pulsations of reluctance of the main magnetic circuit. I have proven this on an actual machine, by reducing the air gap from its normal value of 0.15 in., (1.27 mm.) to 0.02 in. (0.508 mm.), without reducing the current at which sparking commenced. In fact with one armature, of such tooth proportions that the main reluctance pulsations were less with short than with long gap, the commutation actually improved as the gap was reduced. With another armature where the reluctance pulsations were larger with the shorter gap the reverse was true.

It has been claimed or at least intimated in the discussion to-night that because of the numerous non-computable mechanical factors which may seriously affect commutation, it does not pay to go into refinement of calculations in connection with the computable portions of the subject. I cannot agree with this position for two reasons; first, the improvement in the mechanical design and construction of commutators, brushes and brushholders, have vastly reduced the relative magnitudes of the disturbances introduced by these various elements; second, every well developed refinement of predetermination, however slight, is a step in the right direction and well worth while. The time involved in its application is ridiculously small when compared with the importance of the result. Because we can't have the whole loaf is not a good excuse for refusing half or three quarters of it.

I therefore wish to congratulate Mr. Lamme, and to express gratification that a practical designing engineer of his high standing has shown his belief in the importance of careful and refined calculations.

C. F. Scott: I want to comment for a moment on the method which is used by a prominent designer and engineer in doing his electrical work. This paper, aside from the specific results which it gives, is, I think, typical of the man. He uses physical methods of analysis rather than mathematical formulas. He may use mathematical methods; he is a mathematician, but it is rather a method than a kind of machinery that he employs. For example, in simple things he does not happen to use a slide rule, but with a little lead pencil and a good deal of mental arithmetic he can set down the answer quicker than most men can use the slide rule. So in his handling of the larger problem, he has a sort of short cut method which enables him to get his results by a kind of physical analysis which is exhibited to us in the paper without very much mathematical machinery being apparent in it.

In the present paper he has treated the matter of commutation not as a limited and particular problem involving only an isolated coil, but he has somehow made it a part of the general problem of the cutting of a stationary magnetic field by moving conductors; so that commutation comes into the general class of phenomena which are involved in the generation of electromotive force in the armature.

Nine years ago at the opening meeting of the Institute for that year Mr. Lamme presented a paper on single-phase motors. There was a great deal of interest in that paper, and it was thought that he must have some mysterious method which would account for the commutation of alternating current. He simply insisted that there was nothing magical about it, that he simply had a machine of exceptionally excellent commutation. Now, his paper tonight reveals somewhat of the method by which he has presumably handled not merely his direct current commutator problem, but also the problem of the alternating current which led to the motor.

The paper, therefore, can be commended as an excellent example of a method of handling engineering and design problems.

James Burke: I had not intended to make any comments upon this excellent paper of our friend, Mr. Lamme, especially as regards the interpole feature; but Mr. Lamme has explained in this paper the reason why we have been doing many of the things that we have been doing for a great many years, which arouses much interest. I think all manufacturers of commutating machines have for 20 years stated that their machines operated sparklessly, that there was no sparking under any conditions. This paper tells us why they have operated sparklessly.

An interesting point brought out in the paper is the matter of the fractional pitch as compared with the full pitch of winding. Mr. Lamme's paper mentions the influence of fractional pitch where commutating poles are used. Machines have been built by some companies for many years with various degrees of fractional pitch, and many were built as low as 70 per cent of fractional pitch. However, introduction of the interpole necessitated increasing the pitch so as to bring the commutated coil under the interpole. I think that pretty generally it has been found that in interpole machines the pitch of the coil must nearly approach a full pitch in order to get the best results of the introduction of the interpole, except in the exceptional cases where the end winding becomes a large factor.

I think we must not assume that because we have this paper that any one without any additional experience can build a non-sparking machine, because there are a great many other items of very great importance that are not brought into consideration here. For example, this paper does not introduce the question of the width of the mica between commutator bars. The customary machine has a width of mica of about 1/32 (0.794 mm.) of an inch. Under certain conditions of design, very marked improvement can be obtained by increasing the width of the mica. I have had experience with machines in which with the ordinary width of mica the machines would have been impossible of successful operation, but by introducing increased widths of mica into the design, in some extreme cases to about ⅜ of an inch, (9.525 mm.) or about 12 times the usual practice, the machines were entirely successful. The improvement in commutation due to wider mica is a study by itself, but one of the important factors introduced is the increased effect of the resistance of the brush in the path of the short circuit coil, because the path through the brush although variable, has a minimum length equal to the width of the mica, and by increasing the mica, that path is increased so that the effect of the resistance of the brush is very markedly increased. There are difficulties in getting mica that will wear evenly with the copper in the commutator, but this can be overcome by scraping the mica lower than the copper. As an example, I know of one motor that if its criterion of commutation be judged by any of the well known formulae, would be declared impossible, and yet this machine, operating as a reversing motor with its brushes at the geometrical neutral, has been operating for some years carrying intermittent loads in excess of what would be considered normal for this size of machine. The feature of the width of mica is a very important one in the study of commutation.

The question of finding the geometrical neutral is one that is difficult and takes considerable time as mentioned by Mr. Lamme, especially if the coils are fractionally pitched, and more especially if the armature is twisted so that the teeth do not enter evenly under the pole. The proper position as mentioned by Mr. Lamme is very important in interpole machines. A method that we have used for many years might be of interest and is a time saver. It will be better understood by the following diagram, Fig. 1, in which a two-pole machine is used for illustration: *A-A* represent the poles; *B* the armature; *C-C* the brushes; *D* any one of the armature coils, shown in the diagram as being the coil short circuited at that instant by one of the brushes, and in the illustration being at the geometrical neutral, the pitch of the coil being slightly fractional in the illustration. *F-F* represent the field coils excited from an external exciting supply, which field excitation can be thrown on or off by means of switch *H*. Across the brushes is connected a low reading voltmeter *I*. Now if the switch *H* is opened or closed while the armature is stationary, there will be an instantaneous deflection of the indicator of the voltmeter, if the brushes do not happen to be at the geometrical neutral. By moving the brushes, the deflection at opening or closing the switch will decrease if the brushes are moved towards the geometrical neutral, until finally there will be no instantaneous deflection of the voltmeter when the brushes are on the proper neutral position. Instead of making or breaking the field, it is sometimes more convenient to simply use switch *H* for interposing resistance in the field circuit. The foregoing gives the true position both for machines without interpoles and with interpoles for the geometrical neutral. This is the starting point from which the brushes are moved to aid commutation when brush movement is resorted to. Also in the interpole machines in cases where the interpoles are not of the proper strength, some departure from this position will be found necessary to correct for incorrect strength of interpoles,

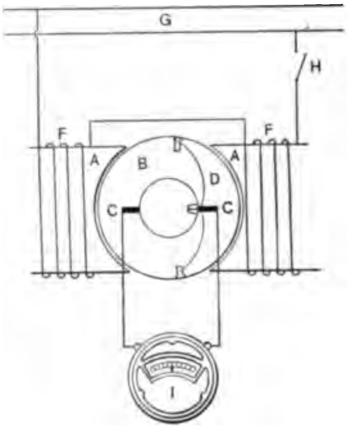


FIG. 1

This paper takes up only one kind of interpole construction, perhaps the most inefficient kind of interpole because it is that which takes the largest number of ampere turns for producing commutating zones of the correct condition for commutation. For instance, another extreme in the comparison with this may be illustrated by the following sketches, Figs. 2 and 3. Fig. 2 shows the ordinary type of interpole which is the form referred to throughout Mr. Lamme's paper. If the armature strength is for example, 10,000 ampere turns per pole, it would be necessary to have somewhere between 12,000 and 14,000 ampere turns on the interpole, all of which magnetization is accompanied by copper loss in the production of that field and that loss is very often as large and sometimes larger than the other field losses. In some cases the total field losses are doubled by the introduction of interpoles. Now in contrast with this, another way of producing a suitable commutating zone is shown in Fig. 3, in which *B* indicates the interpoles supported from the magnet frame in any suitable way by non-magnetic material. Assuming again an armature magnetization of 10,000 ampere turns at the middle of the interpole, we can find a point indicated approximately at *X* in Fig. 3, at a convenient position mechanically and electrically which will have a strength of approximately 8,500 ampere turns, so that the winding on this type of interpole will require a magnetization of only the difference between the 10,000 point and the 8,500 point, equals 1500 ampere turns, plus that necessary for overcoming the air gap and magnetic circuit of the interpole in producing the desired commutating flux, or in the illustration approximately 3,000 ampere turns. It will be seen that with this type of interpole, with approximately 3,000 ampere turns, if properly designed, the same effect can be accomplished as with say—12,000

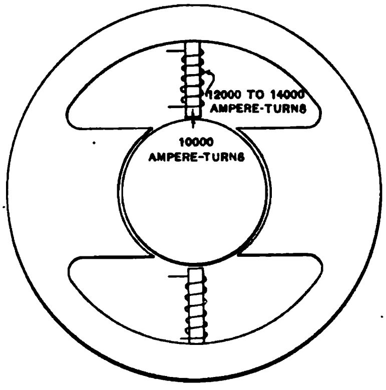


FIG. 2

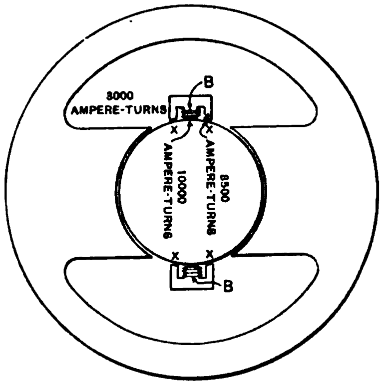


FIG. 3

ampere turns, in the type of interpole shown in Mr. Lamme's paper. This particular type is one that I have given considerable attention to in investigation and in tests, and believe it shows a greater promise than the old fashioned interpole that has been customary.

I note with interest Mr. Lamme's remarks on using a lesser number of interpoles than main poles. When we first started doing this some people thought that we had left something off the machine, particularly in two-pole machines where we used only one interpole, and in some six-pole machines with series connected armatures, where we used only two interpoles, but the success in the operation of the machines with a lesser number of interpoles than main poles when properly proportioned warranted this use, and I think Mr. Lamme confirms the correctness of using lesser interpoles than the total number of poles, provided you can get your other magnetic conditions correct.

One other point of great importance is, I think, not fully treated in the paper, and that is the effect of the number of coils per slot. We find in machines sometimes as many as five per slot and five commutator bars per slot. The difference in the effect between one-commutator bar per slot and many bars per slot is a matter of very serious consideration in the designing of interpole machines.

I regret that I did not have an opportunity to study this paper fully so as to be able to discuss it, and I will only conclude by expressing gratitude for the contribution which Mr. Lamme has made to the recorded knowledge on the subject which is available to those who are studying the designing of commutating machines.

R. B. Treat: Mr. Lamme's paper covers in a step by step manner several of the factors affecting commutation. The development can easily be followed, and each final formula has a real physical conception.

The last formula for B_i would seem to require a correction factor of $2/\pi$ due to the distributed magnetizing armature winding. One eminent author omits to apply this factor in his discussion of direct current machines, but does apply it when he discusses synchronous machines. Why he draws the distinction is a question to me.

Fig. 10, of the paper, shows a flux density diagram to the right of the slot picture. This shows a straight line increase of flux from the base of the conductor *d* to its top. The author appears to assume that the e.m.f., developed in the conductor of depth *d* by this flux is proportionately to one-half *d*. Extensive experiments have shown that the e.m.f. curve for the conductor of depth *d* will take the form of a parabola and the average e.m.f. is proportionately to $\frac{1}{3}$ *d*.

The formula for flux from tooth top..........contains the factors $0.54\sqrt{n}$ I have been unable to verify this part of the formula for experiments had indicated that it should be a logarithmic function of the armature core surface enclosed by the coil divided by the slot width. Again there seems to be no reference to the flux set up in the end connections by the commutating coil, this end flux being entirely independent of that referred to on a previous page which gives calculations for end flux set up by the armature magnetizing turns.

There is a chapter headed "Condensed approximate formula." The designer might well omit this chapter and adhere closely to the previously developed individual formula, for he may then easily note the effect of minor changes upon the design.

In the discussion of the effect of brush width given in the paper, I would ask if the factor B_i is not intended to be B_1 to conform to the value as given under the heading "Effective Armature Ampere Turns?"

The paper presents a theory of commutation which looks complicated and cumbersome, but if the designer will adhere to the elements which have been developed, he will find that the matter is of very easy application and it insures a little more accuracy than may have been obtained heretofore.

C. E. Wilson: During the preparation of the paper by Mr. Lamme I was engaged for a considerable time in checking over a large number of machines with the idea of verifying the formulae by tests. In other words, I have made practical application of the substance of the paper, not only in checking the tests on existing machines, but also in predicting the commutation on new designs. While engaged in this work I have been especially impressed by the importance of "over-compensation" and by the necessity for obtaining an approximately uniform distribution of current under the brushes.

A great deal of light can be thrown on these subjects by what are commonly called "brush curves". These curves are taken by reading the voltage between a point on the end of the brush near the commutator and a point on the commutator directly beneath. This voltage is a rough indication of the current density under the brush; but it is only an indication and not a measure of the current density, for, as is well known, the resistance between the brush and the commutator is exceedingly variable.

Fig. 1 shows the effect on the brush curves of changing the magnetizing force of the interpoles. These curves were taken at full load on a 1,500-kw., 500-rev. per min. machine, which has a high short circuit e.m.f. Curve *a* was taken with the machine under-compensated and sparking slightly; Curve *b* with it commutating perfectly, and Curve *c* with it over-compensated and sparking about the same as when Curve *a* was taken. These curves are characteristic. The current crowds to the leading tip when the machine is under-compensated and to the trailing tip when it is over-compensated.

In Fig. 2, Curve *a* was taken on a 15-h.p. motor at $\frac{3}{4}$ load. Curve *b* was taken on the same motor at the same load with brushes only half as wide, but with twice as many brushes per arm, so that the average current density was the same for both tests. Curve *a* shows flat compensation, while Curve *b* shows the machine to be under-compensated. This result is consistent with the theory advanced this evening. The difference would be more marked if this machine had a higher short circuit voltage due to the slot flux.

Changing the interpole gap has, of course, an effect on the brush curves similar to that produced by changing the interpole ampere turns. Fig. 3 shows tests at full load on a 125-kw. generator which has a low short circuit voltage. Curve *a* was taken with a ¼-in. (6.350 mm.) interpole gap and *b* with a 5/32-in. (3.969 mm.) gap, all other conditions being the same for both tests.

The same kind of brush curves can be taken on non-interpole

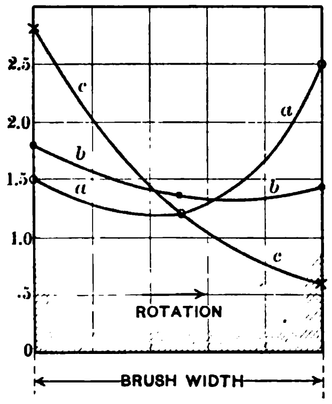


Fig. 1

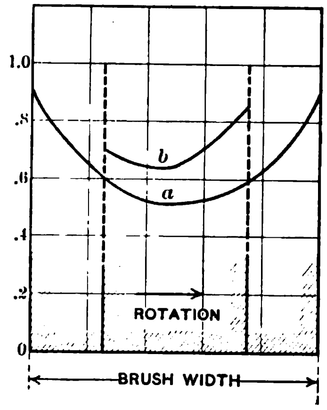


Fig. 2

machines. It is customary to shift the brushes of such machines to assist commutation. The short circuited coils, being in an active field at no load, have large local currents set up in them, which fact shows clearly in the brush curves. Fig. 4 shows the tests on a 100-h.p. non-interpole motor at no load and full load with the brushes given 2.7 bars lag. This machine commutated perfectly at no load and full load and only sparked slightly at 1½ load. The short circuit voltage per coil on this machine is 3.1 volts from test and 3.0 volts by calculation—a very close agreement. The test value of this short circuit voltage was obtained directly from the brush curves taken at no load and full load.

A good way to look at this problem is that, with the brushes on the no load neutral and the brush covering only one bar, the difference in voltage between the two edges of the brush is equal to the short circuit voltage per coil. For any other condition the approximate short circuit voltage can be found from the brush curves if due allowance is made for the number of bars covered by the brush and for any voltage assisting commutation due either to interpole flux or to the main flux in non-interpole machines if the brushes are shifted into an active field at no load.

In regard to saturation in the interpole circuit, Curves *a* and *b* shown in Fig. 5 are of special interest. They show the limits of sparkless commutation on a 62½-kw., 250 volt generator. This machine with flat compensation will begin to spark at 1½ load (point *A*), while with over-compensation it will not spark until 1¾ load (point *B*) is reached. This shows very clearly why an interpole machine should be over-compensated. Fig. 6 shows the brush curves taken on this same machine at

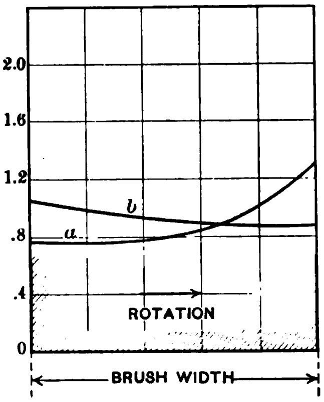


Fig. 3

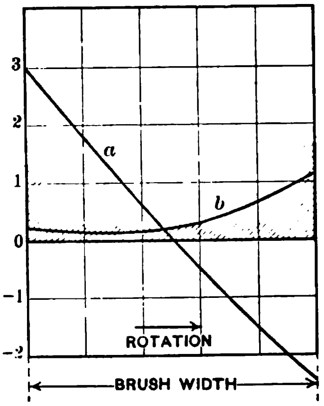


Fig. 4

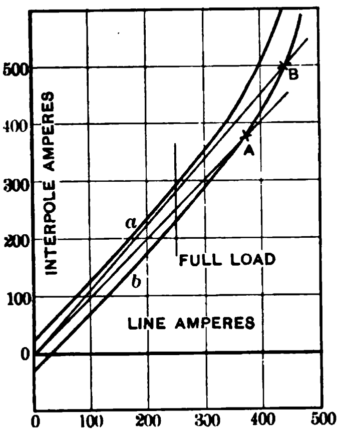


Fig. 5

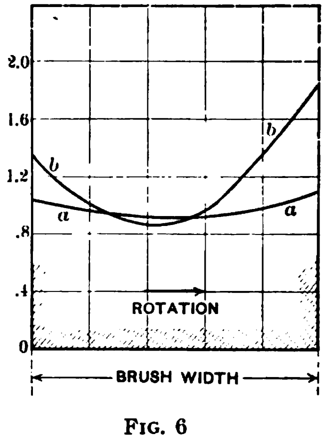


Fig. 6

full load, Curve *a*, and 1½ load, Curve *b*. Curve *b*, of course, shows under-compensation.

In a series or heavily compounded motor, saturation is even more serious than it is in a shunt machine, for that part of the magnetic circuit common to the main pole and interpole saturates as the load comes on. Fig. 7 shows the brush curves for a 5-h.p. heavily compounded motor, 1260 rev. per min., 5.25 amperes (Curve *a*), and at 813 rev. per min., 20 amperes, (Curve *b*).

Curve *a*, Fig. 8 shows another way of representing the saturation of the interpole circuit. It represents the resultant voltage between bars at the center of the brush for various armature currents in a series motor operating at constant speed. This 160-h.p. motor, on account of the conditions of the service required of it, is necessarily highly saturated. The dotted line *b* gives the over-compensation which would be obtained if there were no saturation in the interpole circuit, while the dotted line *c* gives the approximate short circuit voltage of the commutating coil. This curve shows clearly the special importance of over-compensation in the case of series motors which are required to operate at large overloads for short periods of time.

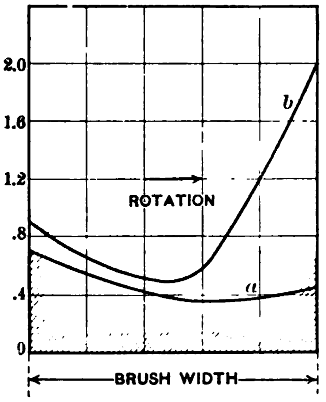


Fig. 7

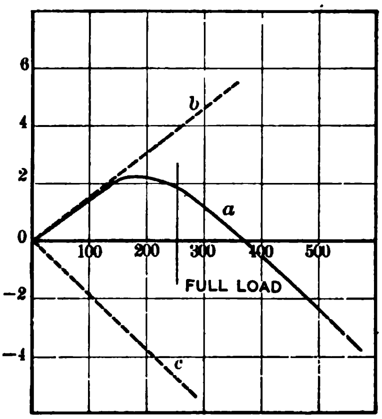


Fig. 8

The importance of properly shaping the interpole face, as explained by Mr. Lamme, is clearly shown by the brush curves in Fig. 2. Curve *a*, while it shows flat compensation, also shows current peaks at both sides of the brush. The effect is caused by too wide an interpole face and too small an interpole gap. When the interpole covers more than one slot pitch and the period of commutation does also, the increased flux density under the edges of the interpole will give a brush curve similar to Curve *a* in Fig. 2. Curve *b* shows this effect much reduced due to the shortening of the period of commutation, and, while it shows under-compensation, the current density at the leading brush tip is apparently less than in the case of Curve *a*. A machine with a brush curve like *a* is, of course, sensitive to interpole adjustment, brush setting, sudden changes of load, " etc ". It thus seems to be well worth while to so fashion the interpole face that good brush curves and approximately equal distribution of current under the brushes may be obtained.

B. G. Lamme: I have been much interested in the various points brought up in the discussion of the paper. There seems to be a general misunderstanding in regard to what is covered in this theory of commutation. This theory was intended to include only the internal actions involved in the operation of commutation and was not intended to cover extraneous conditions, such as undercutting of mica, quality or grade of brushes, type of brush holder, peripheral speed of commutator, etc. These features have largely to do with the mechanical conditions of commutation and, while they might be rightly classified under the broad subject of commutation, yet they do not belong to the theory as I have presented it. The undercutting of the mica, for instance, does not directly improve the commutating characteristics but it allows the brush to maintain more intimate contact with the copper surface of the commutator, at all times, than would be the case where the mica is not undercut. In the same way a good brush holder will maintain a more uniform contact and will prevent jumping and chattering of the brushes. The grade of brush will have a direct influence on the commutation, through its resistance, but the action of the brush is also mechanical, to a certain extent, as some brushes will give much smoother " riding " action on the commutator than others. The peripheral speed of the commutator face has considerable to do with the operation of the machine, through the operation of the brushes which ride on the commutator surface. However, these conditions do not modify the theory of the internal actions which take place in the armature during commutation.

A number of statements have been made this evening, to the effect that it should not be assumed that the method of analysis shown in this evening's paper will enable a beginner, or inexperienced person, to design good commutating machines. It was not the intention to create any such impression. A more careful perusal of the paper should call attention to the fact that, throughout, I have stated repeatedly that judgment and experience are required to use the method accurately. In fact, instead of making the problem appear simpler than formerly, in my mind, this method of analysis makes the problem appear more complex, for more conditions are included than in the usual methods of analysis, and at the same time these conditions have been shown to be extremely variable for different types of apparatus. In applying this method of analysis to non-interpole machines, for instance, the results obtained in many cases show that it is surprising that such machines have been made to work as well as they do. In fact, had this method been applied to any great extent in the past, I am inclined to think that many non-interpole machines now in existence would never have been undertaken, as their commutating characteristics would have appeared so questionable.

One direct consequence of the use of this method of analysis, in the author's own experience, has been to dispel a former notion that interpole machines were more sensitive, or more "delicate", in some ways than the non-interpole type. After long experience in the analysis of the various types of machines, the conclusion was reached that interpole machines work surprisingly well, considering some of the badly proportioned machines which have been put on the market in the past by various manufacturers.

Taking up in order some of the points brought up at the discussion this evening—Mr. Erban has evidently misread the paper when he intimates that I said that little has been done to determine the permissible overload, without flashing, at the time of short circuit. I did not intend to give any such impression, but did intend to give the idea that it is impracticable to build large, high speed machines, especially for the higher voltages, which could be dead-short-circuited without flashing, and I attempted to show in a simple way why certain limits could not be exceeded. I judge that his experience, as to the limiting load possible without flashing, coincides very closely with my own.

As to the presence of conducting gas or vapor assisting or causing flash-overs, I agree entirely with him, but it should be kept in mind that this vapor is not the initial cause of the flash but is a means for spreading it. The real cause lies in the excessive short circuit voltage under the brush due to the excessive armature current at the moment of short circuit, as explained in the paper.

Mr. Hoxie takes exception to my statement that field distortion has no effect on the commutation. His contention appears to be that, when there is saturation, the distortion of the main field flux has a direct influence on commutation. But this is the one exception I made to my broad statement, for I showed in Fig. 17, and the accompanying text, why a saturated main pole, with a consequent heavy series winding, tended to give, to some extent, the effect of an interpole.

Mr. Hobart brings up the point that, with a non-interpole machine, without lead at the brushes, this method of calculation might give very good results, but that when lead was considered it was hopelessly involved. I do not agree with him that the method is an impossible or impracticable one in this latter case, for it has been tried out, in many cases, with very enlightening results. In that method of direct current machine design in which the magnetic "field form" or field distribution is used as the basis of the calculations, this method comes in naturally and is simpler to apply and gives a better insight of the problem than taking the resultant of the magnetomotive forces instead of fluxes.

Mr. Hobart has also suggested that the maximum voltage between commutator bars should be given consideration as this may lead to commutation trouble. I do not see wherein either the average or the maximum voltage between bars has anything to do with the direct problem of commutation, but I do agree with him that it is the maximum voltage which we should consider in the design of the machine, instead of the average voltage. The sparking or flashing troubles caused by too high a maximum voltage between bars do not originate under the brush as a rule, but start between those commutator bars which are connected to conductors cutting the maximum field flux. I would define this as commutator, rather than commutation, trouble.

Professor MacLaren has called attention to the fact that while the e.m.f. due to the interpolar flux is increased due to chording, yet the increase is not very large. He is correct in this. The point which I intended to bring out in my paper was that chording was harmful rather than beneficial as far as the interpolar flux is concerned, and possibly the proportions shown in Fig. 4 in the paper, gave an exaggerated idea of this effect.

He also states that in working out certain large machines according to this method of calculation he found that the e.m.f. due to the interpolar flux in some cases was much greater than the e.m.fs. due to the slot and end fluxes. This appears to me to be rather unusual, in the ordinary type of machine, although I can see that if extra precautions were taken to reduce the e.m.fs. due to the slot and end fluxes, while none were taken to reduce the e.m.f. due to the interpolar flux, it might be possible to get the latter relatively high. Professor MacLaren also suggests that the effect of the chording on the slot e.m.f. as given in the paper, is greater than experience indicates. However, in many cases, especially where the end and interpolar fluxes are high, the effect of the chording would be relatively small, in the total result. In the example which he mentions, in which the e.m.f. due to the interpolar flux was 2.8 volts, that due to the end flux was 0.9 volts, while that due to the slot was 1.3 volts, it is obvious that a considerable variation in the latter value, due to the chording, would have a comparatively small effect on the total result. The influence of chording, as indicated in this paper, agrees very well with my own experience.

Professor Adams has called attention to the fact that the method of analysis given in the paper does not cover instantaneous conditions during the operation of commutation. I admit that there are small high frequency disturbances which occur periodically within the period of commutation, but these disturbances repeat themselves within a cycle of one tooth pitch, while in the method of calculation shown in the paper the results are averaged over one tooth pitch. The period of commutation is very seldom as short as that represented by one tooth pitch and is usually considerably longer. Therefore, in practice, the minor disturbances occur only in a relatively small part of the period of commutation. But even if we did calculate these disturbances with any accuracy, we could not use the results in our calculations for interpole machines, for it is only the average voltage which we can compensate, or it is only the average short circuit e.m.f. which the interpole neutralizes. While such minor disturbances may be interesting from a purely theoretical standpoint, yet in a practical design they need not be taken into account and, in fact, they may be, to a certain extent, suppressed by damping action in the interpole and elsewhere.

Professor Adams also calls attention to the possibility of secondary currents in the short circuited coils, set up by pulsations in the main field flux due to changing magnetic reluctance in the main circuit, as the armature rotates. This effect may be present, to a certain extent, and, if of materially large value, it would be harmful. However, this is an effect which the designer aims to eliminate, or to reduce to a low value for other reasons than commutation and, in well designed machines, it should have but very little influence on the commutation. In fact, I do not recall any instances in machines of recent design in which this effect has been sufficient to give any noticeable trouble.

Mr. Burke brings up a number of points which have been covered, to some extent, by what I have already said. He calls attention to the fact that certain machines operate in a satisfactory manner which, according to the ordinary calculations, should not be good operative machines. I have noted similar cases. As a rule, these cases have to do with machines for intermittent service, reversing machines, etc. In such machines, commutation characteristics appear to be allowable which are not at all permissible in ordinary continuous service. In fact, non-interpole railway motors without any lead at the brushes are, in many cases, operative at voltages across the brush, or between adjacent commutator bars, which would seem to be prohibitive. In fact, they would be prohibitive if it were not for the intermittent load and other special conditions of operation.

Mr. Burke has referred to a method of finding the true interpole commutating position, and he refers to adjustments which are departures from this position, in order to obtain more suitable conditions. I agree with him that it is very important to find the theoretically correct position of the brushes in interpole machines, but I think that it is wrong, especially in generators which are to operate in parallel, to permit any appreciable departure from this correct setting; for other troubles than commutation are liable to be encountered if the brushes are not set at the true neutral.

Mr. Burke has referred to a special arrangement of interpole which, in his opinion, may have some advantage over the usual type. The type of interpole he shows is not a new one to me, for an experimental machine of this type was designed by one of my associates several years ago. The arrangement is operative, but it seems to me that it is not a very practical scheme. Mr. Burke is somewhat in error in the figures which he gives for the advantages in this arrangement. For example, he assumes 10,000 ampere turns per pole in the armature and an excess of 2,000 to 4,000 ampere turns in the interpole, or say, 3,000 ampere turns excess. With the arrangement of interpole which he has shown he considers that the armature opposing ampere turns may be reduced to 1,500, instead of 10,000. The total ampere turns on the interpole then become 1,500 plus 3,000 =4,500. But he overlooks the return path required for the interpole flux and, assuming this to be of practically the same section as the interpole, 3,000 ampere turns additional are required for the gap in the return path. The total thus becomes 7,500 ampere turns, compared with 13,000 ampere turns with the usual type. This would be but little better than the interpole arrangement with half as many interpoles as main poles. However, a marked disadvantage in this scheme lies in the extra space required for the interpole. With the usual arrangement of interpole it is frequently difficult to find space for the interpole without reducing the effective circumferential width of the main poles. With this proposed arrangement the space for the special interpole would have to be increased very materially in most cases, and I believe that the decrease in output, due to such reduction in the width of the main poles would far more than balance the possible gain.

Mr. Treat has called attention to what he considers an apparent error in the derivation of the formulæ. He states that in connection with Fig. 10, " The author appears to assume that the e.m.f. developed in the conductor depth *d* by this flux is proportional to $\frac{1}{2}d$. Extensive experiments have shown that the e.m.f. curve for the conductor of depth *d* will take the form of a parabola and the average e.m.f. is proportional to $\frac{1}{3}d$."

Evidently Mr. Treat has not gone carefully into the derivation of the formulæ given in connection with this point, or he would have seen that the actual value used is in proportion to $\frac{1}{3}d$ instead of $\frac{1}{2}d$. The result obtained is therefore in accordance with his experience in this point.

He also states that he has not been able to verify the factor $0.54 \times \sqrt{n}$, which represents the flux from the tooth top and that, according to his calculations, the expression should be a logarithmic function of certain slot and surface dimensions. In reply to this I will say that, as worked out originally, this was a logarithmic function, but the expression was unduly complicated and was replaced by the simpler expression given in the paper, which is an equivalent within the practical range of design. Furthermore, a slight discrepancy in this part has such a small effect on the total result that it was considered advisable to use the simpler expression throughout.

I feel that the discussion of the paper has been hampered, to a certain extent, by the delay in its issue. A number of the statements made this evening have obviously been due to insufficient opportunity to go into the paper thoroughly, for a number of points which have been raised are, in reality, covered in the paper itself. In a number of cases I have not called attention to these, for I believe that more careful reading of the paper will furnish the answer to these points.

A paper presented at the 265th Meeting of the American Institute of Electrical Engineers, New York, November 10, 1911.

MAGNETIC PROPERTIES OF IRON AT FREQUENCIES UP TO 200,000 CYCLES

BY E. F. W. ALEXANDERSON

The measurements of magnetization, core loss and skin effect of iron at high frequencies described in the following were undertaken partly in order to get data for predetermining the core loss and magnetic densities in high frequency alternators and partly in order to investigate the possible usefulness of iron for transformer construction at high frequencies.

Description of Apparatus

The generator used for producing 200,000 cycles is substantially of the same type as the 100,000-cycle alternator described by the author in a paper before this Institute in June, 1909. The 200,000-cycle alternator was designed primarily for the purpose of wireless telegraph and telephone service, particularly in connection with small antennæ used for transmission over moderate distances, where the natural frequency is higher than it is in high antennæ intended for long distance work. The output that can be obtained from a generator of a certain physical size decreases with the frequency or, *vice versa*, increases with lower frequencies, which is a fortunate circumstance inasmuch as the power needed for short distance transmission in relatively small, whereas, long distances require a greater amount of power and a lower frequency.

In the design of the alternator for 200,000 cycles some new difficulties were encountered. The size of slot used in the 100,000-cycle alternator is almost as small as it is practicable to use, if an adequate insulation is to be provided for a normal operating potential of 110 volts and a high potential test of 750 volts. It was found possible to increase the number of slots from 600 to 800, retaining the samc insulation, but beyond that it seemed that any increase in frequency must be obtained by an increase in speed. It appeared that the factor of safety of the material would allow a certain increase of speed, but there was another consideration which made the increase of speed almost prohibitive; the air friction. From measurements made on the 100,000-cycle alternator the law for the variation of the air friction with the speed has been determined, and it was found by plotting the results on a logarithmic scale that all the measured points within the whole range of speed where measurements could be made, fall on a straight line, indicating that the air friction is an exact function of the 2.7th power of the speed. The air friction of a 12-in. (31 cm.) disk at 20,000 rev. per min. is 5 kw. and hence a general formula is found for the air friction.

On a 12-in. disk;

$$\text{Air friction in kw.} = 0.76 \left(\frac{\text{rev. per min.}}{10000}, \right)^{2.7}$$

Hence it was found that the speed needed for 200,000 cycles on the above assumption would require a power to be dissipated in air friction of 15 kw. which would be excessive particularly from the point of view of heating.

The 200,000-cycle alternator which has been developed has 800 slots but the machine runs at the same speed as the 100,000-cycle alternator, 20,000 rev. per min. The winding has one conductor per slot, insulated in the same way as the 100,000-cycle machine but the winding is of a special type so that 800 slots give the equivalent effect of 1200 poles. It generates 90 volts at no load.

SYSTEM FOR MEASUREMENT

Inasmuch as the principal object of this test was to ascertain the core loss in the iron, it was necessary to find a way of making wattmeter measurements. The ordinary hot wire instruments have been found very reliable for measuring amperes and volts at high frequencies but a wattmeter for such frequencies does not exist. However, it might be possible to construct such an instrument applying the a^2-b^2 principle to the hot wire meter. For these core loss measurements another method was adopted which required no special instruments. The principle of the method is the following:

A compound circuit is formed of the sample coil, a bank of condensers, and a variable inductance. The relation of the capacity to the inductance is varied until the volt-amperes input through the combination becomes a minimum for a constant value of current in the sample coil. The minimum input indicates the changing point from leading to lagging current of the combination and consequently the input represents the watts consumed by the group. The iron core is then removed and the same measurements of minimum input is made with the remaining air circuit. This second measurement represents the loss in all the auxiliary apparatus and wiring, and the difference between the two measurements is the watts core loss in the iron sample.

In following out this principle it must be observed that any presence of higher harmonics in the measured circuit might give a certain amount of wattless current even at the point of minimum input and any such disturbance must be eliminated. For this purpose the alternator was connected in series with a variable condenser which was in every case adjusted so as to give maximum current output for any given field excitation. In this way the harmonics were eliminated by tuning, so that a comparatively pure sine wave was delivered to the test circuit.

In arranging the test circuit there was a choice between connecting the condensers in multiple to the sample coil and tuning for minimum current input or connecting the condensers in series with the sample and tuning for minimum voltage input. Out of these two alternatives the second proved to be more convenient and was used throughout. The accuracy of the method was ascertained by substituting a known resistance in the place of the sample coil and checking the measured input to the same with the calculated $I^2 R$ loss. The results agreed within the errors of measurement. The arrangement of the test circuit employed is shown in Fig. 1. The sample coil is connected in series with a bank of condensers of one microfarad each, arranged in series, and a variable inductance consisting of a fine insulated wire about 1.5 meters along which was wound or unwound on a cylinder of wood about two cm. in diameter. This type of variable inductance was preferred to the one of the ordinary laboratory type where a blank wire is gradually rolled from an insulated to a conducting cylinder. It is of importance that no change be made in the losses in the auxiliary apparatus, because the core loss in the sample is determined by subtracting the losses in the apparatus from the total losses in the combination. The current was measured by a three-ampere hot wire meter with all the current flowing through the hot wire and the voltage was measured by a hot wire voltmeter with a full scale deflection corresponding to 0.23 amperes. Inasmuch as the series resistances of the voltmeter were originally made for ordinary frequencies it might be questioned whether they are sufficiently non-inductive for accurate readings on high frequencies. The tests made to ascertain this point indicate however that there is no measurable error even at 200,000 cycles. In the first place the readings of the voltmeter are proportional to the speed of the alternator over the whole range of speed in which the machine can be operated. For additional assurance it was tried to eliminate any possible inductance in the voltmeter by connecting it in series with a variable condenser, and it was found that the maximum reading was obtained without the condenser and that the readings decreased first slowly and then rapidly when the impedance of the condenser was increased.

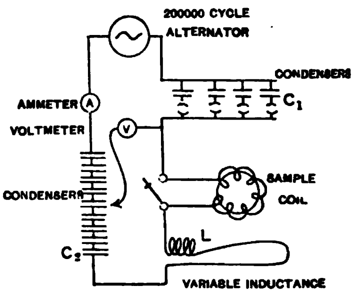


Fig. 1.—Arrangements for measurements

The sample coil consisted of a ring 2 in. (5.1 cm.) in diameter made up of 10 turns of a soft iron strip 0.003 in. thick and 0.75 in. wide (0.0076×1.9 cm.). The layers of the strip were separated by thin paper. The winding was made of 0.016 silk covered wire with two wires in multiple and 20 turns in series.

The test comprised two sets of measurements:

Skin effect measurements;

Core loss measurements.

This skin effect measurements consisted in determining the apparent permeability of the iron at various frequencies and densities by observing the volts and amperes at the terminals of the sample coil. The complete sets of observations at various frequencies are given in Table I.

TABLE I
SKIN EFFECT MEASUREMENTS

Frequency	Amperes	Volts	B_{max} lines per cm^2	Effective ampere turns per cm.
40,000	1.8	8.2	1550	2.25
	1.64	7.4	1440	2.05
	1.48	6.7	1300	1.85
	1.28	5.2	1010	1.6
60,000	2.02	12.8	1650	2.53
	1.88	11.3	1460	2.35
	1.74	10.2	1320	2.18
	1.33	7.2	930	1.66
80,000	2.37	18.4	1780	2.97
	2.14	15.3	1480	2.68
	1.82	12.3	1190	2.28
	1.48	9.2	890	1.85
100,000	2.46	20.4	1580	3.08
	2.06	16.2	1260	2.58
	1.64	11.5	890	2.05
	1.27	8.4	650	1.59
120,000	2.38	21.8	1410	2.98
	2.14	18.0	1160	2.68
	1.83	14.3	925	2.29
	1.38	10.2	660	1.73
	1.6	11.9	770	2.0
140,000	2.29	22.0	1220	2.86
	1.94	17.3	960	2.43
	1.69	13.7	760	2.11
160,000	2.26	22.6	1090	2.83
	2.00	19.0	920	2.5
	1.69	14.7	710	2.11
	1.37	11.3	550	1.71
180,000	2.32	24.8	1070	2.9
	2.20	22.8	885	2.75
	1.91	18.4	790	2.39
	1.50	13.5	580	1.88
	1.25	10.5	450	1.56
200,000	2.34	26.1	1010	2.93
	2.04	21.3	820	2.55
	1.67	16.4	635	2.09
	1.27	11.5	446	1.59

The method of procedure during the core loss measurements was as follows:

The alternator was held at constant speed and the outside condenser C_1 was tuned so as to give maximum current for that particular frequency. The voltmeter connection was then moved along the bank of condensers C_2 until the point was found which gave approximately minimum volt reading. The variable inductance was then wound or unwound until the exact minimum point was found. With this adjustment of the circuit, a continuous series of readings was taken by varying the field strength of the alternator and noting the volts and amperes input to the test circuit. The sample coil was then short circuited and another tuning of the auxiliary circuit undertaken leaving the voltmeter connection in the same place but increasing the variable inductance by winding up the wire until the minimum volt reading was found. The same procedure was repeated at each of the various frequencies from 40,000 to 200,000 cycles. The complete data as obtained from tests is given in Table II.

TABLE II

CORE LOSS MEASUREMENTS

Frequency	Amperes	Minimum volt total	Minimum volt auxiliaries	Minimum volt coil	Effective ampere turns per cm.
40,000	1.96	8.4	0.53	7.9	2.45
	1.87	7.4	0.48	6.9	2.34
	1.77	7.0	0.43	6.6	2.21
	1.52	5.1	0.32	4.8	1.9
	1.23	3.9	0.20	3.7	1.54
60,000	2.08	10.5	0.8	9.7	2.6
	1.92	9.2	0.68	8.5	2.4
	1.78	8.2	0.59	7.6	2.23
	1.63	7.2	0.49	6.7	2.04
	1.31	5.1	0.32	4.8	1.64
	1.06	4.1	0.21	3.9	1.33
80,000	2.32	15.2	1.2	14.0	2.9
	2.18	13.7	1.06	12.6	2.73
	2.05	12.1	0.94	11.2	2.56
	1.80	9.5	0.72	8.8	2.25
	1.60	8.2	0.57	7.6	2.0
	1.33	6.5	0.4	6.1	1.66
	1.08	4.9	0.26	4.3	1.35
100,000	2.29	16.5	1.68	14.8	2.88
	2.02	13.4	1.31	12.1	2.53
	1.76	11.0	1.0	10.0	2.20
	1.61	9.3	0.83	8.5	2.01
	1.33	7.4	0.57	6.8	1.66
	1.06	5.3	0.36	4.9	1.32
120,000	2.44	19.0	1.54	17.5	3.05
	2.26	17.1	1.3	15.8	2.82
	1.94	13.7	0.97	12.7	2.43
	1.65	10.5	0.70	9.8	2.06
	1.39	8.4	0.50	7.9	1.74
	1.12	6.6	0.32	6.3	1.4

TABLE II—CONTINUED

Frequency	Amperes	Minimum volt total	Minimum volt auxiliaries	Minimum volt coil	Effective ampere turns per cm.
140,000	2.24	17.8	1.46	16.3	2.8
	2.01	14.9	1.18	13.7	2.52
	1.67	11.4	0.81	10.6	2.09
	1.34	9.0	0.52	8.5	1.68
	1.08	7.0	0.34	6.7	1.35
	1.79	12.7	0.96	11.7	2.24
	2.20	17.6	1.42	16.2	2.75
160,000	2.75	26.9	3.1	23.8	3.44
	2.67	26.5	2.92	23.6	3.34
	2.40	22.5	2.37	20.1	3.0
	2.16	19.0	1.92	17.1	2.7
	1.89	15.3	1.46	13.8	2.36
	1.53	11.5	0.96	10.5	1.91
	1.26	9.0	0.65	8.4	1.58
	1.12	7.7	0.51	7.2	1.4
180,000	2.38	22.8	2.62	20.2	2.98
	2.50	24.8	2.9	21.9	3.13
	2.25	21.4	2.34	19.1	2.81
	1.95	17.4	1.76	15.6	2.44
	1.68	13.7	1.3	12.4	2.10
	1.32	10.1	0.81	9.3	1.65
	1.04	7.6	0.5	7.1	1.30
200,000	2.32	23.1	2.4	20.7	2.9
	1.93	17.6	1.66	16.0	2.41
	1.75	15.1	1.36	13.7	2.19
	1.52	12.6	1.03	11.6	1.9
	1.19	9.2	0.63	8.6	1.49
	0.98	7.4	0.44	7.0	1.22

For the sake of completing the data the same sample coil was tested at 1740 cycles and at 60 cycles in order to ascertain the permeability and core loss at lower frequencies. In these two sets of measurements a dynamometer was used for determining the core loss.

Although the apparatus described above for measurements at high frequencies was convenient to use and worked with all the accuracy that could be desired, it was found rather difficult in the beginning to get consistent measurements that could be reproduced. The quantity that was difficult to control was the actual properties of the iron due to the rapid changes in temperature. The skin effect or apparent permeability as well as the core loss is a function of the specific resistance, which on the other hand varies with the temperature. In order to maintain constant temperature the sample core was immersed in oil, however, the heat radiation was not as rapid as might be desired because the coil, in order to give a closed magnetic circuit was solidly wrapped in tape. At the highest densities and the highest frequencies that were used the heating of the iron was so rapid that the apparent permeability could be observed to increase about 20 per cent after the circuit was closed, the whole change taking place in about half a minute.

The following corrections of the measured results have been considered but it was concluded that none was of a magnitude that would be worth taking into account.

The inductance of the air circuit surrounding the winding of the coil.

Resistance of the winding.

Error due to current absorbed by the voltmeter.

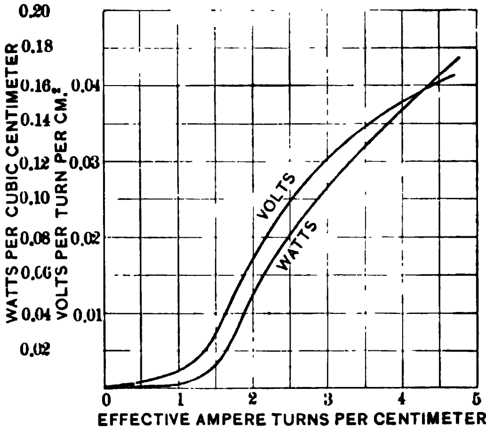


FIG. 2.—Saturation and core loss at 60 cycles

In order to ascertain the inductance of the air circuit of the coil winding an identical sample coil was made up with a paper core instead of the iron core. The inductance was, however, too low to be measured. Similarly it was found that the resistance could be neglected. The correction for the voltmeter current might be considered appreciable being about 4 per cent, however, considering the possible variations of several times this amount due to heating all efforts were bent to produce measurements which are consistent with each other rather than to correct the absolute quantities. The sets of readings given in Tables I and II are the results after several less successful attempts. The method finally adopted in taking the readings was to complete all the tuning, then let the coil cool and take the readings rapidly, closing the circuit only for a moment at a time.

The data represented by curves in Fig. 2 to Fig. 11, inclusive, are obtained from the test results given in Tables I and II in combination with some other tests made with a 100,000 cycle alternator in order to reach the intermediate frequencies.

Figs. 4 to 7 inclusive, give the data on magnetizing current and skin effect, whereas Figs. 8 to 11 inclusive, give the data on core loss. Fig. 5 gives the magnetization of the iron in the same form as ordinary alternating current saturation curves, one curve being drawn for each frequency. The curves in Fig. 6 show the decrease of the apparent magnetic densities with increasing frequency; in other words, the skin effect. Fig. 7 gives the magnetizing current in a form which is most useful for the purpose of design. Each curve represents a constant value of effective volts induced by a cross section of one square centimeter of the core. The curves show that, in spite of the skin effect, the magnetizing current needed to produce the same induced voltage decreases continually with increasing frequency. Fig. 11 gives the core loss in a similar form, each curve representing a constant value of volts produced per square centimeter of the core. The core loss, like the magnetizing current, decreases with increasing frequency.

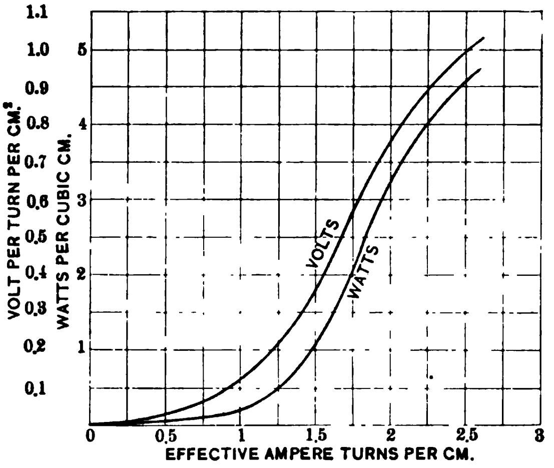


FIG. 3.—Saturation and core loss at 1,740 cycles

CONCLUSIONS

As a conclusion from these tests it may be stated that the opinion held quite extensively that iron does not respond to high frequencies is entirely without foundation. The iron not only responds but it seems to have the same permeability at 200,000 cycles as at 60 cycles. The apparent permeability of the iron in the sample which was tested is only about one-tenth as great at the high frequencies as at the low frequencies but the decrease of the apparent permeability is substantially in agreement with the change that would be expected from purely theoretical considerations due to the skin effect, which indicates that the actual permeability has remained unchanged.

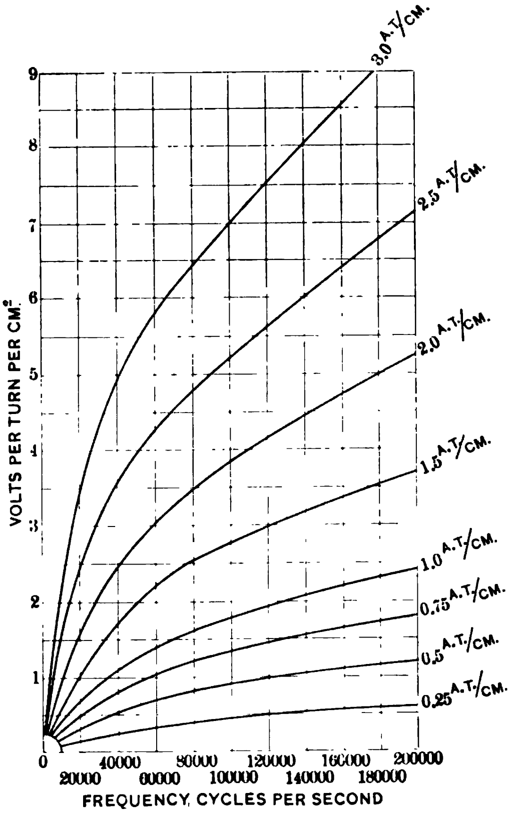


FIG 4.—Induced volts at constant ampere turns and varying frequency

In order to apply Steinmetz's formula for skin effect:

$$\text{Penetration, } l_p = \frac{3570}{\sqrt{\lambda \mu f}} \,^*$$

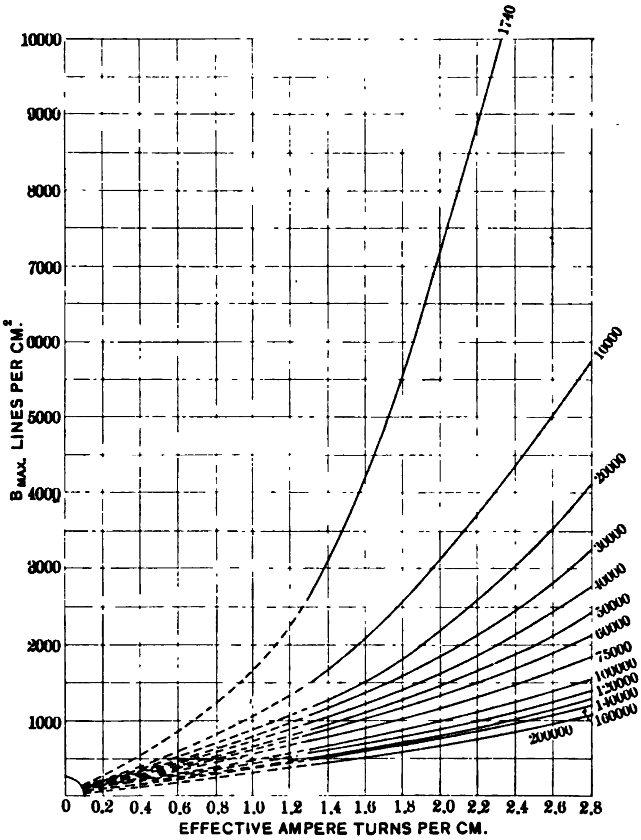


FIG. 5.—Alternating current saturation curves

The following constants may be used as average values for the sample

Permeability, $\mu = 2{,}250$

Conductivity, $\lambda = 0.9 \times 10^5$

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*Transient Electric Phenomena and Oscillations.

Hence penetration at 200,000 cycles should be

$$1_p = 0.00056 \text{ cm.}$$

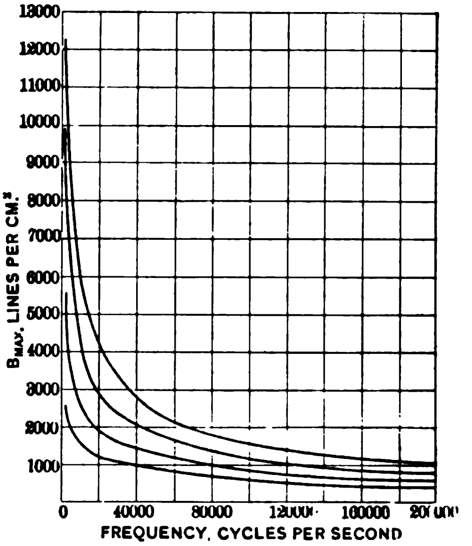

Fig. 6.—Apparent densities at constant ampere turns and varying frequency .

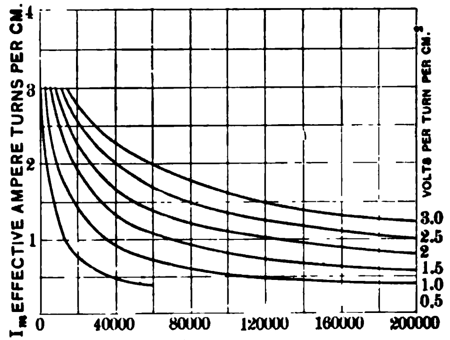

Fig. 7.—Magnetizing current at constant volts per turn and varying frequency

The data from the tests give

Average permeability at low frequency............ 2250

Average apparent permeability at 200,000 cycles.... 180

Hence $\text{penetration} = 0.0076 \times \frac{1}{2} \times \frac{180}{2250} = \ldots\ldots 0.0003$ cm.

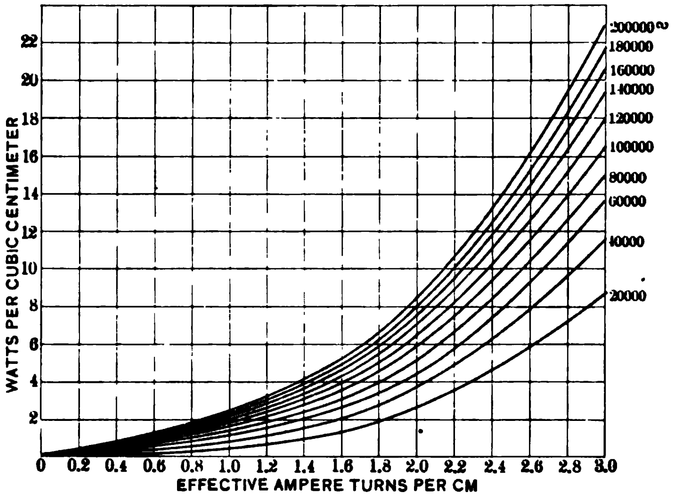

Fig 8.—Core loss at constant frequencies and varying ampere turns

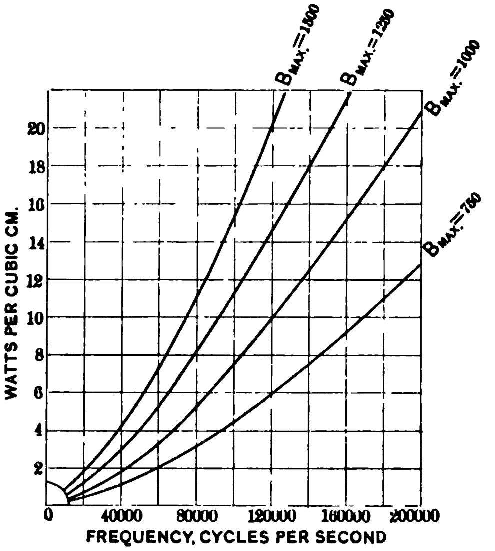

Fig. 9—Core loss at constant apparent densities and varying frequency

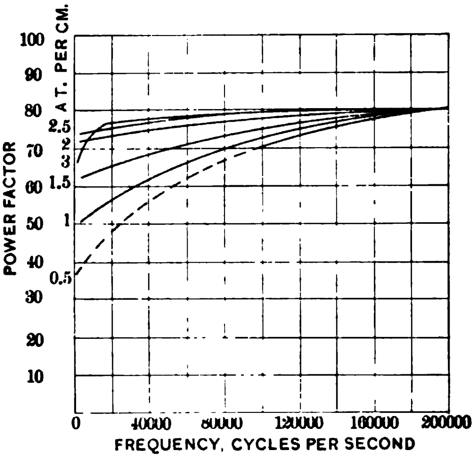

Fig. 10.—Power factor at varying frequency

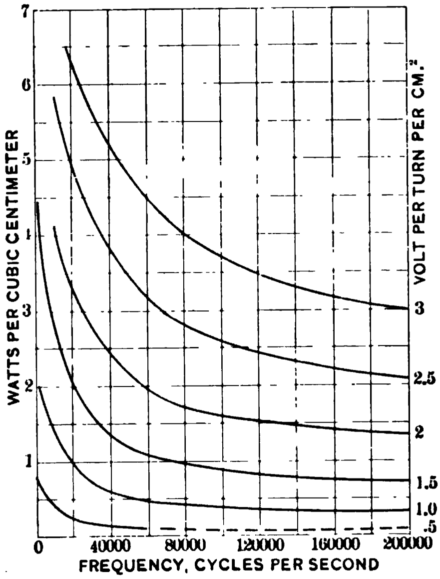

Fig. 11.—Core loss at constant volts per turn and varying frequency

The discrepancy between the measured and calculated values of the penetration is probably due to the fact that the coil has not a completely closed magnetic circuit, but depends upon the leakage from turn to turn of the laminations, whereby an additional skin effect is created.

The data on the magnetic properties of iron at high frequencies may be of more interest if it can be shown that iron may actually be useful in the construction of apparatus for such frequencies. Its usefulness may be claimed to be already demonstrated in as far as the construction of a 200,000 alternator would have been impossible without an iron armature. There is however, a generally accepted opinion that transformers for high frequencies cannot be made with iron core. On the other hand there is a well known rule that transformers for ordinary frequencies are lighter and more efficient at the higher frequencies. In order to examine the usefulness of iron for transformer design from the point of view of this rule a series of transformer designs have been made on the basis of the measurements of the skin effect and core loss. It might be expected that there would be found a turning point in the usefulness of iron, but actually it appears that the rule holds true at 200,000 cycles and probably a good deal higher.

TABLE III

DESIGNS OF FIVE-KW. TRANSFORMER AT VARIOUS FREQUENCIES

Frequency	60	1740	10000	50000	100000	200000
Dimension of core, cm	4.65	2.51	2.41	2.14	2.06	2.0
Weight, kg	19.5	3.05	2.7	1.9	1.7	1.5
Volts per turn Per cm^3 of iron	0.04	0.47	0.56	0.92	1.08	1.22
Core loss	120	156	92	56	54	52
$I^2 R$	450	70	62.4	44	39	35
Efficiency	90	96	97	98.1	98.2	98.3
Per cent magnetizing current	5	5.6	2.9	1.73	1.5	1.3

The series of transformer designs shown in Table III is made on a purely theoretical basis so as to illustrate the magnetic properties of iron on a uniform plan. Considerations that would be involved in a practical design such as insulation and capacity between turns have been left out entirely. All the designs have the same proportions of core and winding space and the heating is figured uniformly on the basis of one watt per square inch

(0.156 watt per cm.^{2}) for radiation of the core loss from the surface of the iron. The insulation between laminations has been assumed to be one-half of the thickness of the sheet. The current density in the winding is assumed to be 3000 amperes per square inch (470 amperes per cm^2) the copper occupying one-third of the winding space. The calculations show that the 200,000-cycle transformer has an astonishingly low weight and is very efficient, whereas, the 60 cycle transformer built on the same proportions is too inefficient to be acceptable and although it weighs twelve times as much as the 200,000-cycle transformer it should weigh a good deal more in order to reach a reasonable efficiency. It should also be noted that the 200,000-cycle transformer, in spite of its skin effect, has a magnetizing current of only 1.3 per cent, whereas the magnetizing current would be about 40 per cent if the core was removed. These figures are not given in order to advocate the use of iron transformers to the exclusion of air core transformers for special purposes where such apparatus is used but are simply intended to show that high frequency transformers can be built with iron and would compare very favorably in weight and efficiency with transformers for ordinary frequencies.

Discussion on Magnetic Properties of Iron at Frequencies up to 200,000 Cycles." New York, November 10, 1911.

Harold Pender: I should like to call attention to the fact that the results of this paper are confirmed by some experiments which were performed at the Institute of Technology last year under my direction by F. B. Silsbee. The method used was entirely different. For the high frequency generator we used an arc giving a frequency something like 200,000 cycles per second. This frequency was impressed upon a primary winding on a small iron core made up of very fine iron wire, and the electromotive force induced in a secondary winding on the same core was measured. This secondary induced electromotive force was also calculated on the assumption that the true permeability is independent of the frequency. In making this calculation the proper allowance was made for the non-uniform distribution of the flux in the core, usually referred to as the magnetic skin effect. The measured and calculated electromotive forces were found to agree quite closely.

B. A. Behrend: This paper is interesting in bringing again to our attention the question of high frequencies, a question which 20 years ago was perpetually under controversy. The paper of this evening is extremely instructive, although I regret that the able author does not strongly bring out the fact that, as far as fundamental laws and principles are concerned, the problem confronting the physicist or the electrical engineer, whether frequencies of 10 cycles, 25 cycles, 60 cycles, 120 cycles, or 100,000 cycles are involved, remains exactly where it was before; in other words, the permeability of iron as such remains unchanged independent of the frequency. This fact has, I believe, been known for some time. The high frequency induces vitiating factors which make it appear as though the permeability of iron were different for different frequencies, which it is not.

There is another point in the paper on which I wish to comment and that is the point of efficiency of high frequency apparatus. The apparatus may have a high efficiency, as the author has stated, but the apparatus is useless in its present form for practical applications other than wireless telegraphy, because of its want of regulative properties; in other words, a transformer or a generator of high frequency, though its efficiency may be entirely satisfactory, is an inferior piece of apparatus, in fact, an impossible piece of apparatus for the operating conditions required by the use of current-consuming devices of low power factor.

The statement which I have made is of a certain measure of importance and of interest to those of us older men, who argued in former years strongly in favor of lower frequencies in order to be able to design satisfactorily transformers, generators, and motors. The higher efficiency of the high-frequency transformer may be a great desideratum, yet its lack of regulative properties, owing to the high frequency, makes it an impracticable piece of apparatus for modern power plants.

Charles P. Steinmetz: The importance of this paper appears to me two-fold; first, in announcing the success of developing an alternator capable of generating at 200,000 cycles. It is very difficult, from the mere number, to realize what 200,000 cycles means. We were very proud when we succeeded in generating 10,000 cycles, one-twentieth of that frequency. We must realize that 200,000 cycles is 40 times as high a frequency as the highest frequency at which sound is audible; the pitch at which the ear fails is 1/40 of this frequency. The most powerful research instrument we have, the oscillograph, fails at 1/40 of this frequency, and even at 5,000 cycles it gives merely a single kick without shape or characteristic.

If we would send a current of 200,000 cycles into a line conductor, we would find that while the current flows into the conductor at one point, only 2,000 ft. away the current is still flowing in the opposite direction, is half a wave behind, and is zero 1,000 ft. away, at the center, and when the current stops at one point it is still maximum only 1,000 ft. away.

The second important feature of the paper is the information it gives us of the behavior of iron at these extremely high frequencies which are far beyond any machine frequency ever produced, but which are the frequencies of industrial importance in our wireless telegraph and telephone systems. The paper gives the quantitative values, showing that the permeability of iron at these frequencies is unchanged, and furthermore showing that with commercially practicable thickness of iron, while the eddy currents in the iron are formidable, still the effective permeability is 180 times that of air, that is, iron is still 180 times as good as air as the core for a 200,000 cycle transformer or reactive coil.

This may have an industrial value bearing in wireless apparatus. As you know, in wireless systems we transform the energy produced by the oscillating discharge from the voltage of the generator circuit to the much higher voltage impressed upon the antenna, by a transformer or auto-transformer, which is an air core apparatus. In these cases we deal not with alternating voltages, but with oscillating voltages; trains of waves which gradually, and usually quite rapidly, decrease in amplitude and then die out. The important problem, then, is to get the rate of dying out, the attenuation of the waves, as low as possible, to get as well sustained waves as possible, that is, to reduce the losses in transformation.

Iron has not been used in these transformers, not always because people did not believe iron would follow the frequency, but because the general impression is that the losses in the iron, even when as thin as commercially feasible, would be so formidable as to greatly increase the attenuation, increase the rapidity of dying out of the wireless wave.

As to the question of the magnetic action of the iron, the apparent permeability, I estimated theoretically, some years ago, the frequency at which the apparent permeability of iron, as affected by the eddy currents in the iron, would become inferior to air, that is, at what frequency the permeability would drop down to one, and I found this frequency to be of the magnitude of hundreds of millions of cycles, that is, up to a hundred million cycles iron still increases the magnetic flux, when using the thinnest feasible commercial sheet iron. But long before that, while the iron still increases the magnetic flux, it also increases the loss, and the increased loss, increased attenuation of the wave would make it useless.

Mr. Alexanderson's investigation gives for wireless frequencies, 200,000 cycles, definite numerical results of the apparent permeability, and shows this to be of the magnitude of 180. Thus iron is 180 times as good a conductor of magnetic flux as air, and in such transformers the losses in the iron are of the same magnitude as the $I^2 R$ loss. If we would redesign this iron core transformer tentatively designed by Alexanderson, to eliminate iron, using an air core, then the section of the air core to carry the same magnetic flux would have to be 180 times the size of the iron core used; that is to say, the length of the coil would have to be $\sqrt{180} = 13$ times as great; and to save half the loss, the core loss in the iron, we would have to increase the other half of the loss, the $I^2 R$ loss, 13 times.

So you see this investigation seems to show that by using an iron core of proper proportion, at wireless telegraph frequencies, we can materially improve the efficiency of the transformer or auto transformer, and so reduce the attenuation of the oscillating wave, and that is the problem of the power in wireless telegraphy.

Naturally, this design may be more or less, possibly materially modified, by the problem of the voltage that we have to generate, and the necessity of insulating for the voltage, which must be taken into consideration; but in general it seems from the conclusions of this paper that there should be a material advantage in wireless telegraphy in using iron core transformers, or auto transformers, instead of the air core apparatus used at the present time.

Reginald A. Fessenden (communicated after adjournment): For long it was supposed that iron lost its permeability at high frequencies, and was not able to respond quickly enough. This theory was a plausible one if Weber's theory that magnetization was due to currents circulating round the atoms, or if Kelvin and Maxwell's dictum that the electromagnetic rotation of light demonstrated conclusively that, to quote Maxwell, " some phenomenon of rotation is going on in a magnetic field ", be accepted.

But about 1890 or earlier J. J. Thomson had shown that iron had a permeability of at least 200 for frequencies of 1,000,000 per second, by measuring the velocity of propagation of an electric wave along an iron wire. In March 1895 the writer showed that the phenomena of magnetism could be explained both qualitatively and quantitatively by the theory that ionic charges, or electrons, circulated round the molecule with the velocity of light and that the maximum ferric induction so calculated agreed very closely with that found experimentally. Also, in March 1900 (*Phys. Rev.*) the writer showed that the mathematical work on which Kelvin and Maxwell based their conclusion was erroneous, in that the fundamental assumptions made virtually implied a theory of matter which was not justified, and that furthermore electromagnetic rotation could be mathematically deduced from the equations for an electric wave passing through an absorbing medium, without any assumptions as regards the nature of matter or of rotation, being a necessary consequence of the lag produced in the wave by the absorption and the presence of electrons.

Theory and J. J. Thomson's results were thus in agreement, but to what extent was unknown. Some experiments made with arc-produced high frequency currents in 1901 showed that the permeability of iron was of the same *order* as for low frequencies, but it might be a fifth as much or five times as much, for arc-produced high frequency currents are very unreliable, even when apparently continuous, having sometimes as much as 50 per cent of the waves missing, and also fluctuating so much in frequency as to make it absolutely impossible to get any accurate quantitative results. (*Lond. Electrician*, Feb. 15, 1907. Later confirmed by Fleming.)

Mr. Alexanderson's results form therefore the first definite and dependable knowledge in regard to this subject, and the paper is remarkable, both for its general interest and its scientific and practical importance. Workers with high frequency currents can now go ahead with confidence and use iron without fear. It will undoubtedly come into use as transformers, especially with the heterodyne system, where the electrostatic inductance between primary and secondary is of no importance and the shortening of the wave length along the wire winding does no harm, but may even in certain cases, be advantageous. I am able to confirm Mr. Alexanderson's statements as to the accuracy of the method used for measuring the power lost.

F. B. Silsbee (communicated after adjournment): In connection with Mr. Alexanderson's paper I would like to give a brief account of an investigation along similar lines which I carried on during the winter of 1910-11, at the Massachusetts Institute of Technology, under the direction of Professor Pender.

The object of the research was to decide whether or not iron can respond magnetically to a very rapidly alternating magnetomotive-force. Especial care was taken to arrange the conditions of the experiment so that they would be amenable to mathematical analysis, and all disturbing factors could be allowed for. The method used was quite different from Mr. Alexanderson's, as the specimens were magnetized by a primary winding, and the flux determined by measuring the voltage in a secondary winding. To allow for the magnetic " skin-effect " a careful mathematical analysis was made which took into account the effects of both eddy-currents and hysteresis. The assumptions it was necessary to make were: first, that the hysteresis loop, traversed by the iron during a complete cycle, was a simple symmetrical loop having the equation

$$\left(\frac{\mu H - B}{\nu B_0}\right)^2 + \left(\frac{H}{H_0}\right)^2 = 1 \qquad (1)$$

where H = m.m.f. per cm. at any point.
B = corresponding flux density.
H_0 = max. value of H during the cycle.
B_0 = value of B corresponding to $H = H_0$.

$$\mu = \frac{B_0}{H_0}$$

$$\nu = \frac{B \text{ at } H = O}{B \text{ at } H = H_0}$$

and second, that μ and ν were constant and independent of H_0. The iron used had been hardened in drawing so that the maximum permeability was only about 700. It was, however, very well suited for this investigation as μ was practically constant at 80 for values of H_0 between O and 3.5 c.g.s. units, ν also was practically constant over this range. From the above assumptions it can be proved that the secondary voltage is of the form

$$E_2 = -M_0\, \omega\, I\, [B + \nu A + j\,(A - \nu B)] \qquad (2)$$

where M_0 = mutual inductance between the two windings at low frequency.
ω = $2\,\pi$ times the frequency.
I = primary current.
$j = \sqrt{-1}$

A and B are functions of the radius of the core, the conductivity, permeability and hysteretic constant of the iron, and of the frequency; and are expressed as a rather complicated series.

The secondary voltage was measured by an alternating current modification of the potentiometer method, so commonly used in direct-current work. A variable inductance and resistance were connected in series with the primary winding, and the secondary winding was connected, in series with a suitable detector, so that its voltage was opposed to the $I\,Z$ drop in the impedance. The inductance and resistance were both adjusted until there was no current in the detector, thus eliminating any demagnetizing action from the secondary coil. Then from the known values of the resistance and inductance, and the primary current, as measured by a hot-wire ammeter, both the magnitude and phase relation of the secondary voltage could be computed.

The high-frequency current was obtained from a Poulsen arc formed between copper and carbon electrodes in an atmosphere of coal gas. The arc was supplied with 220 volts direct current and gave about two amperes at 350,000 cycles per second. The frequency was measured by the use of a resonating circuit, loosely coupled inductively to the primary circuit and containing inductance and capacity in series with a detector. The resonating circuit was adjusted for maximum current, and the frequency computed from its constants by the formula,

$$f = \frac{2\pi}{\sqrt{L\,C}} \tag{3}$$

The detector used in this and also in measuring the voltage of the specimen was a thermo-couple heated by the passage of the high-frequency current, and connected to a low resistance D'Arsonval galvanometer.

Four specimens were used, made up of fine iron wire 0.009, 0.005, 0.004, 0.002 in. (0.228, 0.127, 0.101, 0.050 mm.) in diam. respectively. Each size of wire was wound into a ring of circular cross-section about 8 cm. in diameter, and having a total sectional area of iron of about 0.2 sq. cm. Great care was taken in insulating between adjacent wires. The material was passed through thick shellac, and thoroughly dried by a blast of hot air before being wound into the ring. The electrical resistance between the ends of the iron wire after winding was practically that computed from its length and resistivity, thus showing that there could be no short circuit between turns.

It will be seen from equation (2) that when the potentiometer circuit is balanced

$$L = M_0\,(A - \nu\,B)$$

$$R = M_0\,\omega\,(B + \nu\,A)$$

This was found to be the case in most of the measurements to within 3 or 4 per cent which was the limit of precision of the measurements. This shows that the true permeability μ is unaffected by frequency up to 350,000 cycles, at least over the range of H used. In some cases there were quite wide discrepancies but it is probable that they were due, at least in part, to the rise in temperature as suggested by Mr. Alexanderson. The observed values of the energy component were uniformly low, which suggested that possibly the hysteresis loss per cycle might be less at the higher frequencies, when the molecules are subjected to such rapid agitation. Such a decrease in hysteresis has been suggested by Ewing, Lodge and others. It would be interesting to know if Mr. Alexanderson has separated his core losses, so that this fact could be determined.

A paper presented at the 266th meeting of the
American Institute of Electrical Engineers
New York, December 8, 1911.

METHODS OF VARYING THE SPEED OF ALTERNATING-CURRENT MOTORS*

BY GUS A. MAIER

Electrical developments of recent years have been toward the extension of the use of alternating currents in every field where practicable, due to the advantages of distributing systems and the simplicity of the induction motor. For constant speed work, the field is well covered by the squirrel cage and polar wound motors. Various methods have been proposed for varying and adjustable speed work and this paper is to discuss the methods used and proposed.

VARYING SPEED

For varying speed work the following types of motors have been used.

Induction.
- Squirrel cage (resistance control).
- Squirrel cage (compensator control).
- Double squirrel cage with one movable stator.
- Sliding armature.
- Polar wound armature.

Commutator.
- Repulsion (resistance).
- Repulsion (brush shifting).

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* A number of papers have been written on the individual methods of varying the speeds of alternating-current motors. No effort has been made to go into details of design or to discuss broadly those subjects which have recently been treated. Free use has been made of papers and all articles referring to this subject. This paper covers in a general way the methods of varying the speeds of alternating-current motors.

For adjustable speed work the following have been used:

Induction.

Multispeed
- Pole changing.
- Frequency changing.
- Cascade.

Commutator.

Shunt repulsion.
Induction in connection with shunt repulsion.

Induction Motors. The two principal methods of obtaining varying speed with induction motors are; first, primary control (varying the stator voltage of a motor having a high resistance rotor); and second, by secondary control (varying the rotor resistance). The first can be subdivided depending on whether the variation in the stator potential is obtained through a compensator or a resistance in the primary circuit of the motor.

The starting torque of an induction motor is equal to

$$K\left(\frac{E^2 r_1}{Z^2}\right)$$

Where K = a constant.
E = applied voltage.
r_1 = rotor resistance per phase.
Z = total impedance.

The starting current is equal to $\frac{E}{Z}$.

The running torque is equal to

$$K \frac{E^2 S r_1}{[(r_1+Sr)^2+S^2 X^2]}$$

Where X = total reactance.
r = stator resistance per phase.
s = slip.

It is thus directly dependent on the rotor resistance, the slip and the square of the voltage, and inversely on a function of resistance, slip and reactance.

Fig. 1 gives a comparison between the percentages of full-load torque and current at various percentages of synchronous speed for the two methods of speed variation. For full-load torque at starting, 2½ times full-load current is taken from the line with resistance control and 1.4 times full load current with compensator control.

A squirrel cage rotor designed for this service is necessarily very expensive, as the losses necessary to obtain the reduced speeds are all within the machine and, therefore, a large motor is necessary. Compensator control is complicated and expensive, owing to the fact that the compensator coils must be cut out and cannot be short circuited, as is the case with rheostatic control.

The sliding armature control, in which the rotor is displaced horizontally with respect to the stator and the double squirrel cage with one movable stator have the same limitations and characteristics, as noted above.

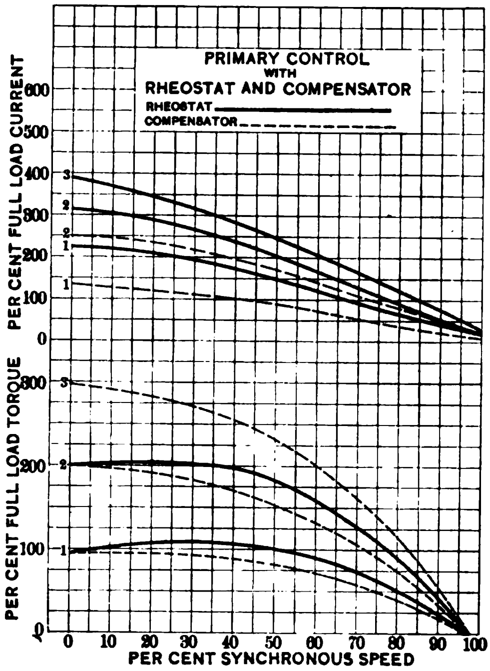

Fig. 1

The polar wound induction motor with variable rotor resistance is best adapted to variable speed, as the losses necessary to obtain reduced speeds are external to the motor itself. The primary current is approximately proportional to the torque, full load for full load torque. See Fig. 2.

The main objection to this type of motor even though the speed characteristics are satisfactory is the efficiency at reduced speed, the efficiency being reduced in direct proportion to the reduction in speed.

By comparing Fig. 1 and Fig. 2, you will note that in curve No. 3, the torque per ampere is the same for all three methods of control. This is because the total resistance in the polar wound rotor circuit, on this point of the controller, is equal to the resistance of the high resistance squirrel cage motor. It is of further importance to note the very much better inherent speed regulation of the polar wound motor over other methods of control.

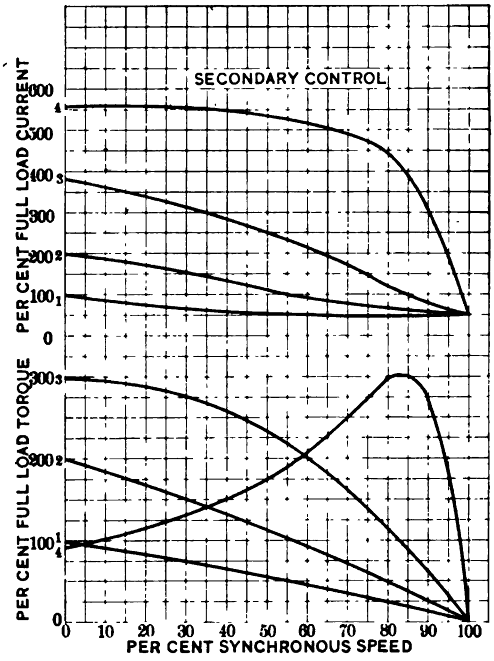

Fig. 2

Referring to Fig. 3 it is important to note the advantages of the polar wound equipment as compared with the other two methods. The current taken from the line is less, the power factor higher and the efficiency high throughout practically the entire range. Furthermore, the slip is less; the final slip shown by the resistance and compensator control is due to the use of high resistance squirrel cage windings.

Commutator Motors. The operation of single-phase alternating-current commutator motors can be explained in a simple manner by comparison with direct-current commutator machines. In each direct-current machine, the torque is developed by the resultant of the current flowing in the armature bars and the field in quadrature thereto. In case such a motor is supplied with alternating current, then also a torque will be exerted.

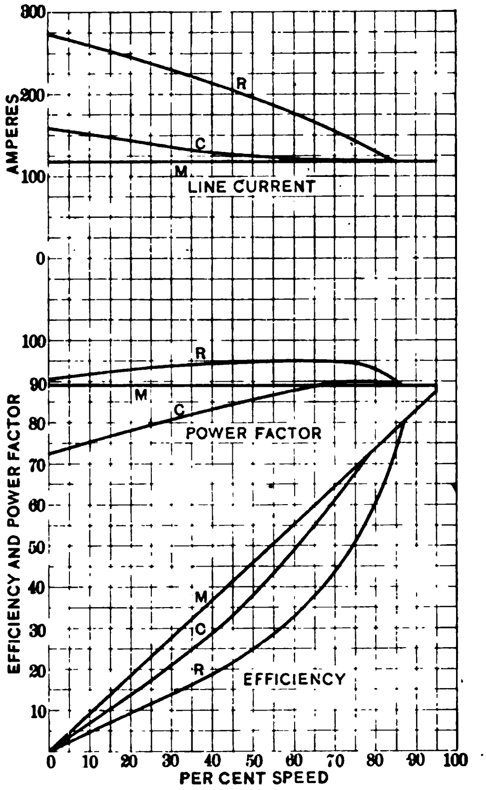

FIG. 3

Due to the alternating nature of the current and voltage, both field and armature current will fluctuate, passing through all intermediate values between maximum and minimum, and a pulsating instead of a constant torque will exist. In case field and armature circuit has been connected in series, the variation of the instantaneous values of field and armature current will be in complete time phase and the torque will never reverse;

for a reversal of torque, it being necessary that either the direction of the field or the direction of the armature current be changed, as is known from direct current practice. The possibility of driving a direct-current motor with alternating current was discovered by Dr. Louis Duncan.* By connecting the field and armature circuits in multiple, the torque will be considerably reduced, the current in both circuits differing in time phase, due to the difference of the self inductance of the two circuits. Hence, it even can happen that at the moment the armature current has just passed through its zero value, the torque is reversed. In a direct-current machine from the magnetic lines interlinked with the armature conductors, a flux will result, which is in quadrature to the field when the brushes are standing in the neutral. This flux decreases the perfection of the operation by distorting the shape of the main field and by inducing in the windings—short circuited under the brushes—and e.m.f. of rotation which can give rise to sparking. In modern direct current design, the armature flux usually is compensated by a special compensating winding or by interpoles. With alternating-current motors such a compensating winding is rarely omitted since the armature flux in this case also lowers the power factor by inducing a reactive voltage. This will be readily understood by considering that in an ordinary induction motor, the reactance is nearly exclusively caused by leakage lines inside the slots, from teeth to teeth and around the end connections, while in a commutator motor, the number of leakage lines is largely increased by the flux which finds a path of low magnetic reluctance in the field core. By using a cylindrical core instead of polar field, the influence of the armature flux is increased. The compensation of the armature flux can be obtained in the same way as the direct-current motors, by connecting the armature and compensating winding in series. (Compensation by conduction). See Fig. 4.

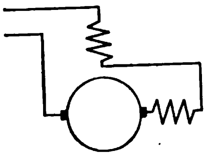
FIG. 4

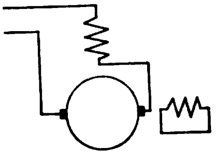
FIG. 5

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* See TRANSACTIONS A. I. E. E., 1888, Vol. V, p. 211.

The compensating winding can be closed upon itself in which case the compensation is obtained by induction as shown in Fig. 5. Compensation by induction is also possible by connecting field and compensating winding in series, in which case the armature current is induced in the short circuited armature, Fig. 6.

With the arrangement according to Fig. 5, the e.m.f. induced in the short circuited winding equals the product of current times impedance, while in the arrangement according to Fig. 6, in addition thereto, there has to be transferred from the compensating winding into the short circuited armature winding, an e.m.f. which is balanced by the e.m.f. generated by rotation of the armature conductors in the magnetic field.

Fig. 6 at the same time represents a type of repulsion motor, which, in its original form, as invented by Elihu Thomson in 1887, has but one stator winding, the axis of which is displaced over a certain angle to the axis of the brushes. (Fig. 7). In this case, in order to investigate the operation of the machine, we can always replace this single winding by two imaginary windings, so as to have the conditions outlined in Fig. 6.

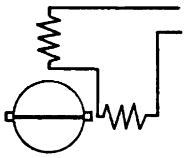

FIG. 6

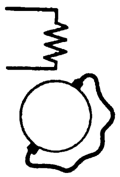

FIG. 7

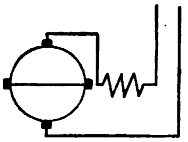

FIG. 8

Instead of providing on the stator both the field and the compensating winding, as outlined in Fig. 6, the field winding can be omitted if we place on the armature two brushes in quadrature to the short circuited brushes, and connect the compensating winding in series with the brushes, which may be called field brushes. (See Fig. 8.)

This arrangement covers the principle of the compensated repulsion motor, which was invented by Winter and Eichberg in Germany, and La Tour in France. In any motor, in which the armature flux is compensated by induction, the compensating winding and armature winding act as the primary and secondary of a transformer. Hence, as in any transformer, a flux lagging over 90 deg. in time phase behind the primary energy current will exist. This flux, which may be called " cross flux " can be made effective in improving the commutation of the armature current. In the compensated repulsion motor, apart from this effect, the cross flux generates an e.m.f. of rotation which compensates the self induction of the armature field circuit and also of the stator winding when connected in series to the armature field winding. When bearing in mind that between the brushes of a revolving armature, only an e.m.f. of rotation can be induced by a field winding, the axis of which is standing in quadrature thereto, and that this e.m.f. will always be in time phase with the field current in which the armature is rotating, it will be understood that between the exciting brushes only the cross flux interlinked with the compensating winding can induce an e.m.f. of rotation, and that between the short circuited energy brushes, only the flux interlinked with the armature field winding can induce an e.m.f. of rotation. Apart from the above outlined improvement, resulting from the introduction of field brushes on the commutator, the compensated repulsion motor does not differ from the other mentioned types of single-phase commutator machine.

To supply a motor with energy current, counter e.m.fs. have to be overcome. Of these, there are both useful and harmful. Of the useful there is chiefly the back e.m.f. induced in the rotor by rotation in the motor field, for the motor output is proportional to the product of this e.m.f. and the energy current. On the other hand, the counter e.m.f. of self induction of the rotor winding is injurious and can practically prevent any appreciable energy current flowing. It is, therefore, necessary to neutralize the inductance of the rotor winding along the brush axis and reduce the inductive pressure to a minimum. This is accomplished by means of a compensating winding on the stator, the axis of which is arranged along the brush axis. The winding is traversed by the rotor current or current proportional to this and serves to neutralize the armature field, the effective ampere turns being adjusted to secure this result. Thus we see that the compensating winding is an essential part of the commutator motor and the same can be either a separate winding or combined with the exciting winding.

Hence, in regard to speed control, the motors covered by Fig 4 to Fig. 8 can all be considered as direct current series, and the method used for speed regulation of the same can be applied.

Primary Voltage Control

Assuming constant torque, in which case the current has to be constant, the e.m.f. induced between the armature energy brushes will increase in the same ratio as the speed. Hence, since the requirement has to be fulfilled, that the voltage applied to the armature (either by conduction or induction), be equal to the resistance drop and the e.m.f. of rotation; the voltage applied to the terminals has to be raised in order to reach this condition. By connecting the terminals of the motor covered by Figs. 4 to 8 to a transformer with variable taps or to an induction regulator, the applied voltage can be changed and the speed regulation obtained. The principle has been demonstrated in Fig. 9, for compensated series motor according to Fig. 4.

Primary Resistance Control

Instead of using a transformer with variable ratio of transformation, the reduction of the voltage can also be obtained by inserting resistance between the line and the motor terminals. This method is less efficient than the former, however, due to simplicity of the required control apparatus, can be used for small units and control during short periods.

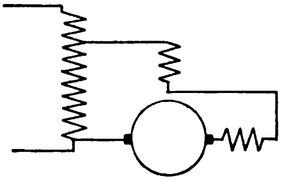

Fig. 9

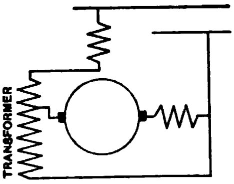

Fig. 10

Field Control

By connecting in parallel to the field variable resistance or by connecting the field to the secondary of the series transformer with variable taps, the ratio of field current to armature current can be changed and thus, the speed of the motor would be regulated.

Armature Voltage Control

This method is in principle the same as the methods described under the heading " Primary Voltage Control ", the only difference being that the armature is directly connected to the secondary of a transformer, which is laying in series with the field winding. (See Fig. 10.)

BRUSH SHIFTING

The brush shifting commutator motor is built both single and three-phase. On account of simplicity, the single-phase will be described.

The single-phase motor of this type is known as the Deri type or modification thereof, and is used for hoists, railway work, and in fact, such places where a motor with series characteristics is applicable.

The torque of a motor depends on the diameter and axial length of its rotor, on the number of conductors, the current and the field. The torque is commonly varied by variations in the current and field strength, but in the type of repulsion motor provided with brush displacement gear, it is also possible to vary the ratio of active field conductors to active rotor conductors.

By shifting the brushes out of the neutral, that is, by reducing the number of active conductors in which an e.m.f. is induced by rotation with the same field strength, the speed has to be increased in order to induce the same e.m.f. of rotation which again has to balance the energy voltage.

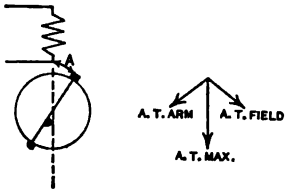

FIG. 11

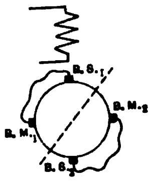

FIG. 12

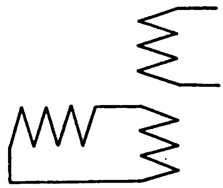

FIG. 13

In regard to brush shifting, special reference should be made to the motor according to Fig. 7, and which again has been represented in Fig. 11.

If the line of the brushes makes an angle A with the axis of the main winding and $A\ T_{max}$ represents the total magnetomotive force yielded by this winding, then we can resolve $A\ T_{max}$ into two components, one $A\ T_{arm}$ being along the line of the brushes and $A\ T_{field}$ at right angles to this line. By shifting the brushes, we increase one of the components and simultaneously decrease the other. There are two positions in which the torque vanishes, that is, for A equals zero, in which case the field is zero, and for A equals 90 degs., in which case the armature current is zero. In case A equals 20 to 30 degs., the torque will be maximum.

Since the motor, according to Figs. 7 and 11, is very sensitive in regard to the position of the brushes, an arrangement has been made for reducing the effect resulting from a certain displacement of the brushes.

Fig. 12 represents this arrangement for the Deri motor, which has four brushes for two poles, two of which are stationary (*Bs*1 and *Bs*2) and two movable (*Bm*1 and *Bm*2). Assuming that the position in which the line of the brushes *Bm*1, *Bm*2 falls along the line of the brushes, *Bs*1, *Bs*2 corresponds to a certain speed, then after the brushes have been shifted over an angle 2*A*, according to Fig. 12, the axis of the armature will have been shifted over an angle 1*A* only.

LaTour has proposed a scheme also with four brushes for two poles, which, however, in this case are moved simultaneously, keeping the same relative position to each other. In this case, a part of the armature, *i.e.*, the conductors between the brushes, *Bs*1 and *Bm*2 are always ineffective in regard to generation of e.m.f., whereas with the Deri motor, this part is ineffective at certain speeds only. The advantage of the arrangement according to LaTour is that the brushes are standing in such a position that less trouble is caused by induction of an e.m.f., which affects the commutation of the short circuited coils under the brushes.

* The importance of the brush shifting motor warrants further consideration and we may consider the theory of operation in a different manner. Let us suppose the rotor winding replaced by two equivalent windings, which are in space quadrature, and which, while independent of each other magnetically, must be regarded as electrically connected in series. (See Fig. 13.)

The magnetic axis of one winding is coincident with the axis of the stator poles, while the axis of the other winding is in space quadrature with the stator poles. The first winding alone is subject to the inductive action of the stator flux and plays the part of the secondary of a transformer, of which the stator winding forms the primary. This winding will be called the transformer winding—the secondary winding may be termed the quadrature winding. Any flux produced by the current in the quadrature winding will be in time phase with the current in the transformer winding and will produce accelerating torque at the rotor.

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* See the *Electrician*, London, January 3, 1908.

Figs. 14, 15 and 16 indicate three positions of the brushes and show the equivalent connection for the transformer and quadrature windings on the motor. Both when the axes are together and when they are separated by 180 space degrees, the torque is zero. In the former case, the transformer effect is zero, while in the latter case, there is no quadrature effect.

Under speed conditions, there are produced two e.m.fs., which affect the value of the secondary current, and therefore, alter the torque, which for each position of the brushes varies directly with the square of the current in the armature. One e.m.f. appears at the transformer axis and the other at the quadrature axis—the former is proportional to the product of the speed and the quadrature flux, and the latter varies directly with the product of the speed and the transformer flux, these two e.m.fs. have such values and time-place position as to tend to decrease the armature current with increase of speed.

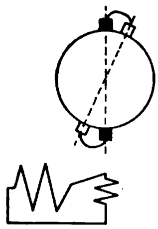

FIG. 14

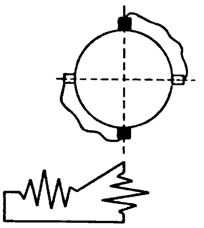

FIG. 15

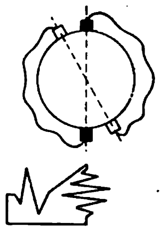

FIG. 16

With constant primary voltage at any given speed, the current and torque can be varied over a wide range by merely shifting the brushes, as explained above.

It will be seen from the above that the machine possesses the speed, current and speed torque characteristics of the direct-current series motor.

In general it can be said that a motor laid out for brush shift has to be built of larger size than one with stationary brushes. Considering that the ratio of the cost of the control apparatus to the cost of the motor is, with certain exceptions, smaller in proportion for large motors than for smaller ones, it would appear that the necessity of increasing the size of the motor—in case brush shifting be used, for small motor applications, less increase in the total cost of the equipment than by using controllers and transformers.

MULTISPEED*

The induction motor as usually spoken of is primarily a constant speed motor. Where variable speed is required a motor with collector rings is used, the speed variation being produced by varying the amount of resistance in the secondary circuit.

Many times it so happens that two or three speeds will be satisfactory for the operation of the machinery and these speeds must be independent of the load. Under such conditions, multispeed motors can frequently be used.

In these motors the different synchronous speeds are produced by changing the number of poles in the magnetic circuit. Each of these speeds is fixed, if no resistance is used in the secondary circuit. With multispeed motors, as with single speed motors, resistance may be used in the secondary circuit for varying the speed.

A change in the number of poles may be produced in any one of the following ways:

1. By the use of single magnetic and electric circuits, changing the number of poles by regrouping the coils.
2. By the use of single magnetic circuits and independent electric circuits.
3. By means of separate magnetic and electric circuits.

1. *Single Magnetic and Electric Circuits.* Where two speeds only are required and those of ratio 2:1, the motor with single magnetic and electric circuit is admirably adapted, due to economical use of material, all of which is active at both speeds and simplicity of pole changing, six leads only being required for two or three-phase motor. For any ratios other than 2:1, the switching and wiring becomes more or less complicated, the ordinary three-phase winding requiring 33 leads and two-phase requiring 16 leads.

2. *Single Magnetic and Independent Electric Circuits.* Where speed ratios other than 2:1 are required it is much more practicable to use independent electric circuits. These motors, having independent windings, have three leads for each speed in the stator for three-phase motors and four for two-phase motors.

Theoretically, it is possible to obtain a number of different speeds, but is usually impracticable to use more than two windings.

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* See TRANSACTIONS A. I. E. E., 1908. Articles by H. G. Reist and H. Maxwell and H. C. Specht.

Obviously, it is possible to combine 1 and 2, thereby obtaining four speeds requiring twelve leads for a two or three-phase motor.

In a motor with 2:1 speed ratio, using single magnetic and electric circuits, the material is used to the best advantage when the output at low speed is approximately one half of that at the high speed. It is possible, however, to vary the output at the low speed over a wide range depending upon the connections employed in the primary.

Speed	Connection	Approx. max. output
(1) 100	2 circuit delta	100
50	Y delta	11
(2) 100	2 circuit Y	100
50	1 circuit Y	22
(3) 100	2 circuit Y	100
50	1 circuit delta	66
(4) 100	1 circuit delta	100
50	2 circuit Y	117
(5) 100	1 circuit Y	100
50	2 circuit Y	350
(6) 100	Y delta	100
50	2 circuit delta	700

From this table it will be seen that there is a wide range of choice of outputs. These combinations are often valuable in order to maintain good efficiency and power factor at the desired loads.

Motors of this type are adapted for driving machine tools, printing presses (certain cases only), pumps, blowers, etc.

3. *Separate Magnetic and Electric Circuits*, (*Cascade*). Multispeed motors as referred to under (1) and (2) become objectionable in the larger sizes, especially where it is desirable to change speeds frequently, or it is objectionable to open the primary circuit.

Where two- or three-speed motors of very low speed are required, two motors operating in concatenation should be used since the ratio of the diameter to the length of the changeable pole motor becomes very great, resulting in excessively heavy construction to secure the necessary rigidity, and high costs. With the concatenated set, one motor operates alone to obtain either of the high speeds and the two motors operating in concatenation for the low speed. These sets are also designed for operation at variable speed in addition to the fixed speeds.

For the concatenated connection, they are generally of the polar wound type, although the last motor of the set may be of the squirrel cage construction. The rotors are usually mounted on the same shaft, making a two unit set. The primary of the first motor is connected to the source of supply at its secondary to the primary of the second motor, as shown in Fig. 17.

Considering two single-speed motors to comprise the set, the maximum number of speeds (not including resistance steps) obtainable is four, two of which consist of either motor with its primary connected to the line and secondary short circuited, the other two being obtained by direct and differential concatenation.

Two motors are connected in direct concatenation, if they have a tendency to start in the same direction and differential concatenation, if they tend to start in opposite directions.

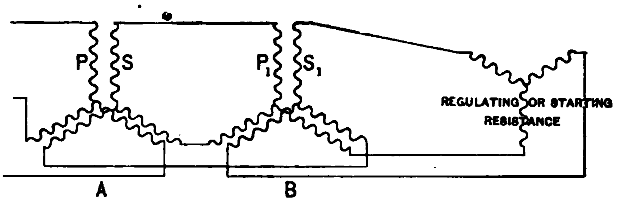

FIG. 17

Synchronous speed of motors in concatenation is as follows:

$$\text{Speed} = \frac{\text{Cycles} \times 120}{P_1 \pm P_2}$$

P_1 = poles motor *A*.
P_2 = poles motor *B*.

Plus sign to be used when motors are in direct concatenation and minus when in differential concatenation.

Where more speeds are required, it is possible to use three motors, but the cost becomes prohibitive and switching complicated. A much better and cheaper proposition is to use one or two, two-speed motors, depending on the number of speeds required.

The operation and control of the cascade set is simple, especially for direct concatenation.

With the multispeed (pole changing) motor in changing from one speed to the other, it is necessary to open the primary circuit; this may be obviated in the cascade sets by introducing resistance in the leads between motors *A* and *B*, this resistance to be cut in or out step by step, in changing from one speed to the other. Fig. 18.

In the concatenated motor, speed reduces somewhat as follows: With the polar wound single-speed motor, the reduction in speed is accomplished by reduction in efficiency, the power factor remaining constant. In the concatenated motor, the efficiency will remain approximately constant, but the power factor goes down as the speed is decreased. This low power factor is due to the fact that the motors are underloaded when run in concatenation, as compared with normal rating, as individual motors.

Recently there has been brought out in England, the Hunt or internal concatenated motor. The machine is a modified form of the "cascade" motor, having two magnetic field systems, superimposed upon one another in the same core body.

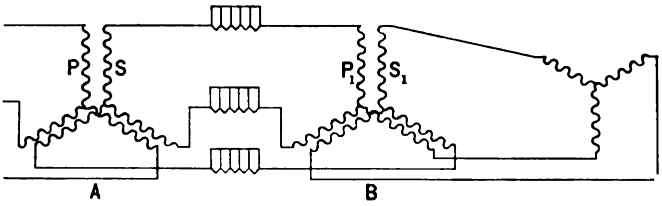

FIG. 18

The second field has its origin in the rotor and consequently induces secondary currents in the stator windings. (See Fig. 19).

The motor can be laid out for cascade speeds of 12, 18, 24 poles, *i.e.*, numbers divisible by six, the lowest number of poles being 12. A motor having a smaller number of poles is not satisfactory, as the resultant magnetic field is not symmetrical.

Considering an eight-pole stator winding with taps brought out in such places that the winding can be made responsive to a four pole field. The motor has a winding which gives the effect of an eight-pole and four-pole winding in series. The motor can operate either in cascade with a speed corresponding to 12 poles or as an eight-pole motor. In order to obtain three effective speeds, a change over switch is necessary in the stator circuit. This enables the connections to be changed so that the primary currents may produce either an eight-pole or a four-pole magnetic field.

The arrangement of the windings are such that if the slip rings are short circuited, the currents induced in the rotor give only one number of poles, and the machine under these conditions behaves exactly like an ordinary slip ring motor.

When the slip rings are open circuited, a small number of the rotor conductors become inoperative and the currents in the rotors then produce two number of poles, the second of which acts upon the stator and causes the motor to run at cascade speed.

Although no figures are available, it appears possible with Mr. Hunt's methods to design a cheaper two-speed motor than the ordinary collector ring type with two primary and two secondary windings. On account of the whole winding being used all the time, this type of motor ought to have a better efficiency and power factor than a double-wound motor.

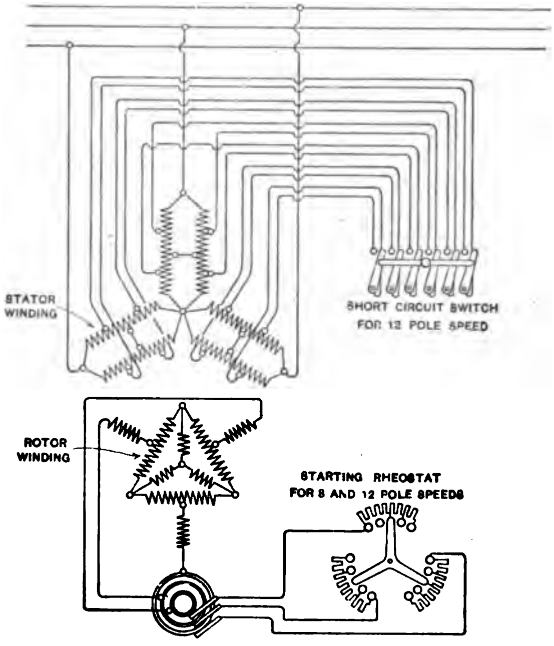

FIG. 19.—Hunts 2-speed motor for continuous service

With this construction, it seems possible to obtain a motor of smaller size and cost with better constants than a two-speed collector ring motor or cascade set. Continuous acceleration is possible by means of rheostats without opening the main circuit.

FREQUENCY CHANGERS

For adjustable speed work, where a large number of speeds are not required and frequent changes not necessary, a simple method may be used, *i.e.*, by using a frequency changer set, consisting of an alternating-current motor and required number of generators for the various frequencies.

This system is objectionable on account of high cost and large speed steps. It is also impracticable to change speeds while running. The efficiency of the combined outfit is comparatively low.

Where direct current is available, a simple method may be employed affording easy methods of control while running. Speed changes are obtained by changing the primary frequency applied to the driving motor operating as an ordinary induction motor.

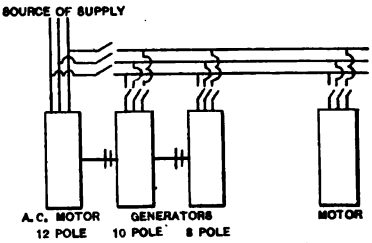

FIG. 20

By far the simplest method is to use a direct-current motor driving an alternator and by varying the speed, the frequency and voltage delivered by the alternator is in direct proportion to variations in speed.

In this case it is necessary for the units of the set to be of sufficient capacity to take care of driving motor *C*.

Another method of accomplishing the same result, is to use an induction motor in place of generator *B* and using the secondary of motor *B*, as the source of supply for motor *C*. (See Fig. 22.)

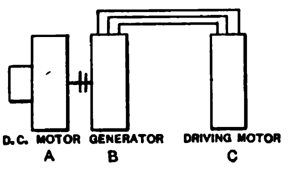

FIG. 21

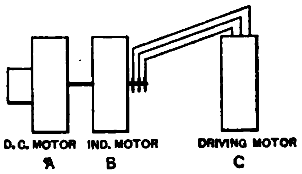

FIG. 22

The primary of the induction motor *B* is connected to the alternating current source of supply and the speed of the set is controlled by the direct-current motor. The secondary of motor *B* supplies power to the motor *C* operating at variable speeds.

When the frequency changer set is running at a speed corresponding to the synchronous speed of motor *B*, the frequency in the motor is zero. The frequency at any other speed may be determined as follows:

$$C = C_1 \left(\frac{N - N_1}{N} \right)$$

C = frequency in rotor B.
C_1 = frequency source of power motor B.
N = synchronous speed motor B.
N_1 = speed of frequency changer set.

The minus sign in the above formula becomes plus when the set is running in the opposite direction to the normal operation of motor B.

The voltage in the rotor B is the ratio of primary turns to secondary turns (neglecting losses in air gap), and varies directly as the frequency.

It is obvious that it can only be used for constant torque work with limited speed range, otherwise the costs which are already high become prohibitive. The direct-current motor A supplies only a fraction of the power supplied the motor C, depending upon the speed range.

Motor B must be of the same capacity as C. Motor A runs as a generator when the frequency set runs in the same direction as normal operation of motor B, as motor when running in the opposite direction.

A number of systems have been proposed involving more or less complicated and expensive features.

Another quite simple system has been proposed—differing from those previously referred to—in which the primary of the driving motor is connected to the power circuit of constant frequency, speed control being obtained by impressing different frequencies on the secondary winding of the motor. By supplying to the secondary of the driving motor, a variable frequency, and to the primary a constant frequency, the motor must run at speeds corresponding to the sum or difference of the primary and secondary frequencies. It is well known that the speed of an induction motor corresponds to the difference of primary and secondary frequencies. By supplying the secondary with a variable frequency, speed changes may be obtained corresponding to the resultant of two frequencies in the rotor, the one produced by the rotating field and the other from some outside source.

There are, however, a number of cases where a multispeed motor is not satisfactory since it does not give sufficient number of steps and there has been felt a real need for an alternating-current motor with shunt characteristics. In fact, the matter is very important, since the absence of such a motor often necessitates the application of direct-current apparatus, even if only a few adjustable speed motors are required and all other conditions would warrant the use of polyphase or single-phase alternating-current motors. Hence, the problem is not to obtain a motor which is better and more efficient than the direct-current adjustable speed motor, but the first step and at present, the most important, is to obtain an adjustable speed motor, no matter whether it be a little inferior in characteristics to a direct-current motor, in order to help us out in the above case and all schemes which are to be described in the following, have to be judged in this light.

Considerable work has already been done in this country for the adjustable speed, single-phase motor.* In order to outline the theory of this interesting type of motor, we may refer to the compensating repulsion motor as covered by Fig. 8, and see what is happening when this motor is running at synchronous speed. We assume the motor has a cylindrical core without salient poles, in which case the mutual inductive impedance is equal, in all directions. Neglecting the self inductive reactance of both circuits, it is apparent that at synchronism the current flowing between the exciting brushes has to be equal and 90 degs. displaced in time phase to the magnetizing current flowing in the compensating windings (reduced to the armature circuit). Otherwise the requirement would not be fulfilled that the e.m.f. of transformation be balanced by the e.m.f. of rotation.

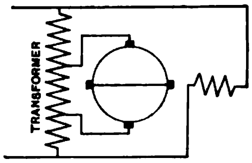

FIG. 23

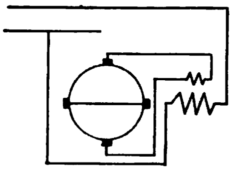

FIG. 24

In the circuit between the field brushes, a similar condition

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* See articles TRANSACTIONS A. I. E. E., 1909, Vol. XXVIII, page 475 by F. Creedy, 1909 by E. F. Alexanderson, Vol. XXVIII, page 511 and paper by W. A. Layman, May 1911, National Electric Light Association.

exists. The current flowing from brush to brush induces a reactive voltage, which at synchronous speed is completely balanced by the e.m.f. induced by a rotation through the cross flux interlinked with the compensating windings. Hence, at synchronism, in order to obtain a current which will be in time phase with the energy current. we have merely to apply a voltage to the brushes of the same time phase as the voltage applied to the energy circuit and which has only to compensate the *I R* drop in the exciting armature circuit.

This has been done in Fig. 23 by means of a. transformer. The stator proper can also be used as a transformer for supplying this voltage, if a second winding has been provided on the same, as shown in Fig. 24, or by equipping the compensating winding with suitable taps, so as to use it as an auto transformer. In this way we have obtained a shunt motor which will run at constant speed. It can be started as a series motor in the connection shown in Fig. 8.

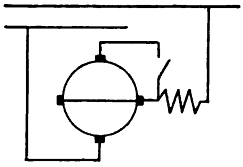

FIG. 25

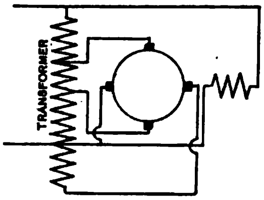

FIG 26

Fig. 25 represents a compound motor. If the switch is opened, then only half the armature works as field winding, and in this way the motor can be started as a series motor. When up to speed, the switch can be closed and the other half of the armature will receive shunt excitation.

In order to make out of the motor, according to Fig. 23, one which shall give equally good characteristics, at speeds differing from synchronous speed, we have to open the short circuit of the energy brushes and apply voltage to the same. This has been shown diagramatically in Fig. 26.

Suppose we apply to the energy brushes a voltage which is equal and in time phase with the voltage formerly induced between the short circuited brushes by transformation. If we further increase the current between the exciting brushes 41½ per cent and also the speed 41½ per cent, then it is clear that the e.m.f. of rotation induced between the energy brushes, which is proportional to both the field strength and the speed, will be increased 100 per cent, and will balance the voltage of the energy circuit, which is the sum of the applied and the voltage induced by transformation. But also the voltage in the exciting circuit will be balanced. The reactive voltage being increased over 41½ per cent, due to the increase of current and the voltage induced by rotation also being increased over 41½ per cent, since the speed has only been changed and not the field interlinked with the compensating winding, which induces the e.m.f. of rotation between these brushes. By thus properly choosing the voltage applied to the two different sets of brushes, it is possible to obtain different speeds, both above and below synchronism. Of course, it is again possible to omit the transformer by using the stator for this purpose.

Analytically, the conditions for equilibrium can be investigated in the following manner.

Let:

$\underset{.}{E}_{fa}$ = voltage applied to field brushes.
$\underset{.}{E}_{ft}$ = voltage induced by transformation between the field brushes.
$\underset{.}{E}_{fr}$ = voltage induced by rotation between the field brushes.
$\underset{.}{I}_{f}$ = current and field circuit.
$\underset{.}{E}_{ea} = x\,\underset{.}{E}_{et}$ = voltage applied to energy brushes.
$\underset{.}{E}_{et}$ = voltage induced by transformation between the energy brushes.
$\underset{.}{E}_{er}$ = voltage induced by rotation between energy brushes.
$\underset{.}{I}_{m}$ = magnetizing current in the compensating winding.
n = speed.
n_0 = synchronous speed.
Z = mutual inductive impedance.

Then at standstill we have in the field circuit, e.m.f. consumed by mutual inductive impedance:

$$\underset{.}{E}_{fa} = \underset{.}{I}_{f}\, Z$$

And for the energy circuit we have e.m.f. consumed by mutual inductive impedance:

$$\underset{.}{E}_{et} = \underset{.}{I}_{m}\, Z$$

At a speed n, we have in the energy circuit to meet the requirement that the voltage applied to the brushes has to be equal to the voltage consumed by mutual inductive impedance and by the e.m.f. of rotation. Hence,

$$\underset{\cdot}{E}_{ea}=\underset{\cdot}{I}_m Z+j\frac{n}{n_0}\underset{\cdot}{I}_f Z$$

If further

$$\underset{\cdot}{E}_{ea}=\underset{\cdot}{E}_{ei}$$

Then. since

$$\underset{\cdot}{E}_{ei}=\underset{\cdot}{I}_m Z,$$

we have

$$x\underset{\cdot}{I}_m Z=\underset{\cdot}{I}_m Z+j\frac{n}{n_0}\underset{\cdot}{I}_f Z$$

This is only possible if

$$\underset{\cdot}{I}_f=j\,y\,\underset{\cdot}{I}_m$$

or

$$x=1-y\frac{n}{n_0}$$

For the field circuit we have, that the applied voltage has to be equal to the sum of the voltages consumed by the e.m.fs. of transformation and rotation, *i.e.*:

$$\underset{\cdot}{E}_{fa}=\underset{\cdot}{I}_f Z-j\frac{n}{n_0}\underset{\cdot}{I}_m Z,$$

or since

$$\underset{\cdot}{I}_f=j\,y\,\underset{\cdot}{I}_m,$$

we have

$$\underset{\cdot}{E}_{fa}=\underset{\cdot}{I}_f Z\left(1-\frac{n}{n_0}y\right)$$

Neglecting the resistance of the field winding the requirements for the equilibrium of the e.m.f. in the field winding should be

$$\underset{\cdot}{E}_{fa}=0$$

or

$$y=\frac{n}{n_0}$$

Substituted in

$$x=1-y\,\frac{n}{n_0}$$

gives

$$x=1-\left(\frac{n}{n_0}\right)^2$$

or

$$\underset{\cdot}{E}_{ea}=\left\{1-\left(\frac{n}{n_0}\right)^2\right\}\underset{\cdot}{E}_{a}$$

For $n\ =1415$
$n_0=1000$
We would find

$$\underset{\cdot}{E}_{ea}=1-\left\{\left(\frac{1415}{1000}\right)^2\right\}\underset{\cdot}{E}_{at}$$

or

$$\underset{\cdot}{E}_{ea}=\underset{\cdot}{E}_{at},$$

and

$$I_f=j\,\frac{1415}{1000}\,I_m=j\,1.415\,I_m$$

We thus arrive at the same result as above.*

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* See *Electro Technische Zeitschrift* 1908, page 588 and page 857. [Eichberg.]

The above outlined method is a combination of armature and field control. With armature côntrol carried out in a different way, the same result can be obtained. Let us suppose we apply to the compensating winding, 70 per cent of the voltage originally applied to the same, and impress on the energy brushes this very same voltage. Then apart from magnetic saturation conditions, the cross flux interlinked with the exciting current of the compensating winding will be reduced to 70 per cent of its original value. If now again the speed increases 41½ per cent, then the conditions for equilibrium have been obtained. The current in the exciting windings will be unchanged since the e.m.f. of rotation is the same as before, the speed being increased 41½ per cent and the field strength being reduced to 70 per cent of its original value. Neither in this case do we need a transformer if we provide the stator with steps.

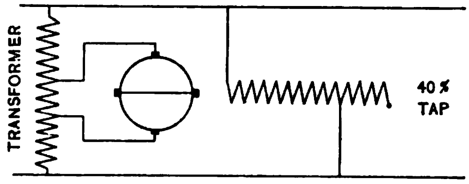


FIG. 27

In Fig. 27 a diagram of connections is demonstrated for a motor, which, in this connection will run at synchronous speed and 40 per cent above synchronous speed when connected according to Fig. 28. For matter of simplicity it has been assumed that both the armature and compensating winding have the same number of effective turns, in the connection of Fig. 27.

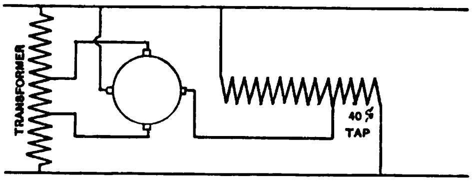


FIG. 28

In case we use armature control without providing special means for balancing the exciting circuit, also, speed regulation can be obtained. Then, however, the current in the exciting winding will be larger, due to a wattless component which does not contribute to the torque.

As long as this wattless component does not become excessive, so as to cause sparking, its presence is not objectionable.

Another interesting type of motor has been described by Mr. Alexanderson in the paper referred to above. With one of the two described motors, he uses field control and he has developed his method in such a way that it directly can be applied to the compensated repulsion motor. Instead of obtaining the equilibrium in the field and energy circuits by the methods described above, he makes use of the fact that the voltage appearing at the terminals of the exciting winding arranged on the stator in quadrature to the compensating winding is 90 degs. out of time phase with the line voltage and consequently in time phase with the e.m.f. induced between the field brushes by rotation through the cross flux. This voltage, after having been stepped up or down by means of a regulating transformer, is introduced between the field brushes and thus regulating the current in the field winding, the speed can be varied.

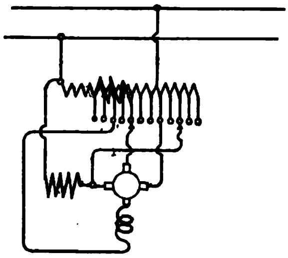


FIG. 29

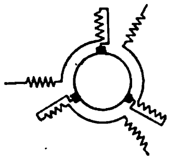
The variable speed motor described by Mr. Layman in his paper is in principle a combination of the different types which have been described above and for further reference we may refer to his paper.

Of course, in case of a motor with series characteristics it is also possible to impress voltage on the energy brushes and obtain, in this way, speed regulation. The methods follow directly from what has been explained in regard to the shunt motor.

POLYPHASE COMMUTATOR MACHINES

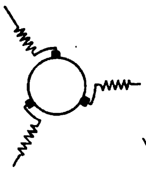
The polyphase commutator motor has a fixed winding consisting of a number of coil groups, corresponding with the number of phases, just like the stator of an ordinary induction motor, these groups of coils being displaced in space by an angle corresponding to the displacement of the several phases from one another; thus in a three-phase winding, they are separated by 120 deg. The revolving part of the motor is wound like a continuous current machine and provided with a commutator. The field of a polyphase commutator motor is produced by the exciting winding, which is fed by a current or pressure. If the energy current of a motor is used as exciting current, the rotary field produced will not be constant, but will depend on the output of the motor. Thus, if the exciting winding *E*, Fig. 30 is connected in series with the energy winding the motor will have series characteristics, the speed being a function of the torque. On the other hand, if the exciting winding *E* is fed from a constant voltage, Fig. 32, the field will be constant, independent of the load. This motor possesses a shunt characteristic and runs at approximately constant speed. In addition to these, combinations can also be made to give compound characteristics.

Figs. 30 to 33 show arrangements of winding of three-phase motor.



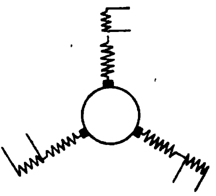
Series motor with separate exciting winding

FIG. 30



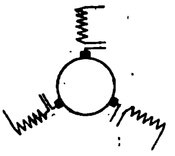
Series motor with combined exciting and compensating winding

FIG. 31



Shunt motor with separate exciting winding

FIG. 32



Shunt motor combined exciting and compensating winding

FIG. 33

Three-phase commutator motors

The difficulty of regulating the speed of induction motors hitherto was that no means were known whereby the low frequency of the rotor currents could be transformed to that of the mains. It was only the knowledge that the commutator always transformed the slip frequency to that of the motor field, thus to that of the supply—that made this possible, for, since the commutator can be directly connected to the network, energy can be given or taken, according to the conditions.

It has been shown that the three-phase commutator motor of moderate size and designed for moderate frequencies has been employed extensively in such cases where speed variation over a wide range and shunt characteristics are required. The design of the three-phase commutator motor meets, however, with difficulties if larger sizes or higher frequencies are required. The commutation becomes poor and costs high.

KRAEMER SYSTEM*

Previous to the introduction of the commutator motor, it was necessary to sacrifice efficiency for regulation, the imperfect method of inserting resistance in the rotor circuit being used.

It was a happy thought that led Kraemer to supply only that power to the commutator motor which would otherwise be consumed in the rotor resistances. (See Fig. 34.)

In this system the commutator machine is direct connected to the shaft of the main motor and the energy furnished by the slip rings of the main motor is returned as mechanical energy to the shaft, instead of being absorbed in resistance. This arrangement is of advantage in cases where a constant h.p. motor is required because the torque of the machine, which is proportional to the slip, is added to the torque of the main motor.

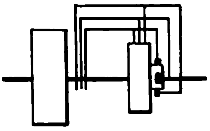


FIG. 34

The slip energy of the induction motor can also be recovered by the application of a rotary converter. This equipment has the disadvantage of greater complication and also special source of direct current. (See Fig. 35).

Referring to Fig. 35, the slip rings of the main motor are connected to a rotary converter, the direct current side of which feeds a direct-current shunt motor, the latter being connected mechanically to the main motor shaft. The speed is regulated by varying the field of the direct-current motor. When the resistance is cut out, the armature voltage and the direct current voltage of the converter will increase. Necessarily the alternating current voltage will increase also, thus the speed of the main motor will decrease. For wide ranges of regulation, this system is particularly applicable, as the design of the rotary

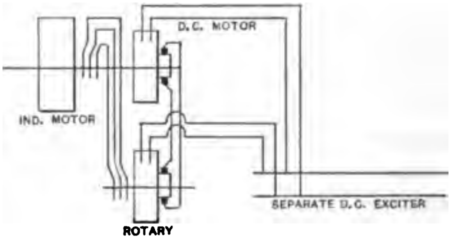


FIG. 35

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* See the *Electrician* (London), August, 1910.

converter is practically independent of the speed reduction required.

Scherbius

Speed regulation has been worked out in a slightly different manner by Scherbius, employing a polyphase commutator motor.

With reference to Fig. 36, a three-phase commutator motor (regulating motor) is connected to the slip rings of an induction motor. The regulating motor drives a generator which again delivers electrical energy to the line, or a regulating motor is direct connected or belted to the main load. Whether the slip energy of a main motor should be recovered as electrical or mechanical energy depends upon the nature and condition of the motor application. By adjusting the excitation of the regulating motors, speed variation of the main motor is obtained, as the slip of an induction motor is proportional to the secondary voltage. The speed of the regulating motor remains constant between no load and full load. The speed depends upon the excitation only, the armature voltage being practically the same as the rotor voltage of the induction motor. Ordinarily the commutator motor has shunt windings. Where a flywheel is used, series or compound windings are used; the speed will then drop over a certain adjustable range and use be made of the kinetic energy of the flywheel. It should be mentioned that by reversing the direction of the excitation, the regulation can be extended over synchronous speed (special exciter required) the regulating motor becoming a generator and delivering the slip energy instead of receiving it. The commutator motor can be designed so that it delivers wattless current and thus improves the power factor of the whole installation.

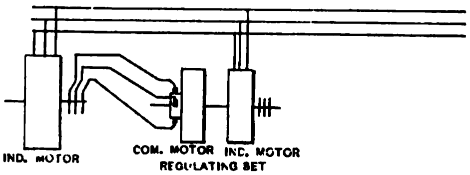


Fig. 36

The use of an independent regulating set has the advantage that the commutator machine can be designed for an economical speed and in this way becomes a cheaper proposition.

Very considerable progress has been made of the use of the alternating current commutator motor in Europe. The brush shifting motor as originally developed was a single-phase motor and is now being built three-phase. The simplest arrangement for three-phase operation is to use two single-phase motors using the T connection.

Various modifications of the Kraemer and Scherbius systems have been used showing excellent results as to regulation and economy of operation.

Discussion on "Methods of Varying the Speed of Alternating-Current Motors. (Maier) New York, December 8, 1911.

H. W. Buck: There is a popular idea that alternating-current motors can be used only where constant speed is allowable, and that where variable speed is required it is necessary to convert the alternating current into direct current and obtain in this way the flexible speed variation through direct-current motors.

Recent developments in this country and abroad in variable speed alternating-current motors, which have not as yet been extensively applied commercially, afford means for many industrial purposes of variable speed control almost as flexible as that obtainable with direct-current machinery.

The variable speed alternating-current motor is a very important field for development work and a paper like this of Mr. Maier which points out the direction in which the development is taking place, is of great value.

Mr. Maier's paper is an excellent resume of the art as it stands to-day and should be suggestive and instructive to those interested in this branch of electrical engineering.

R. N. Dickinson: I ask Mr. Buck if the developments he speaks of are in multiphase machines or single-phase machines?

H. W. Buck: In both.

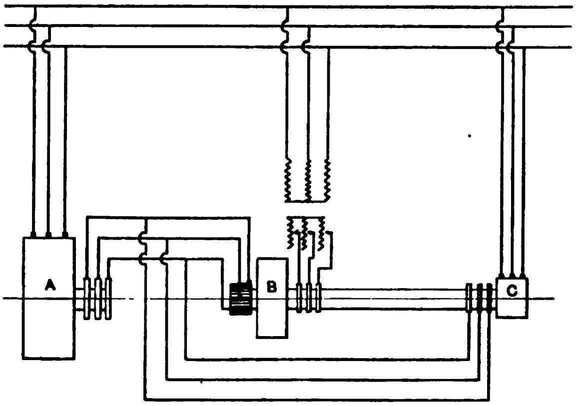
W. N. Smith: I will ask a question which perhaps Mr. Maier or some of the manufacturers represented here can answer. Take the case of a large outfit of motors to drive machine tools in shops such as you have in a locomotive repair shop, or in one of the larger industrial plants. Which of the systems mentioned in the paper this evening have actually been tried on a commercial scale at the present time, and found satisfactory? Possibly some information, showing what has been actually accomplished in practice, would be worth hearing.

G. A. Maier: At the present time in this country there is only one type of adjustable speed motor which is used, that is the multi-speed motor, which is used to a limited extent. In Europe they have used the adjustable speed single- and three-phase commutated motors but I do not think there has been much use made of those motors in this country up to the present time. The slip ring motor is used in some cases where the torque is constant and continuous, such as wheel laths or boring mills doing certain classes of work where the load is always on; the load must be constant and continuous. For general machine tool work, the alternating-current motor has not been used to any considerable extent in this country.

C. J. Fechheimer: An account of a system other than described by the author of this paper for obtaining adjustable speeds appeared in the *Electrotechnische Zeitschrift* of October 19, 1911. This is the system which has been brought out by A. Heyland and R. Ruedenberg. It is somewhat comparable to the Scherbius and Kraemer systems and the accompanying sketch shows the scheme of connections.

The large polyphase induction motor *A* is to have its speed adjusted to suit the requirements. The stator of this motor is connected to the line and its slip rings are connected to the commutator of a machine having an armature similar to that of a synchronous converter. The slip rings of this converter, or more properly, frequency changer, are connected to the secondary of the transformer, the primary of which is connected to the line.

On the same shaft with this machine is a small induction motor *C* with primaries connected to the line and secondary connected in parallel with the slip rings of the large induction motor. The speed is adjusted by changing the number of turns in the secondary of the transformer, which changes the voltage between slip rings of the frequency changer *B*, which in turn alters the pressure between the brushes on the commutator end of this machine. As is well known, a change in voltage between the slip rings of an induction motor will change its speed. The speed of the frequency changer is set by means of the small induction motor *C*.



It is very interesting to note that the frequency changer *B* has no windings on its stator except those which are used to neutralize the unbalanced higher harmonics of armature reaction and thus assist commutation.

Now when we consider whether it is advisable to use a polyphase commutator motor to secure adjustable or variable speed, we must remember that in America it is essential to design apparatus which is practically fool-proof. In other words, after the machine is installed, it is necessary for it to do the work intended for it and not require much attention.

Judging from the great length of time that has been required to successfully design direct current machines and secure satisfactory commutation, we may expect an equal or greater length of time to be able to predict satisfactorily the commutation in alternating-current commutator motors. It is in fact doubtful whether polyphase commutator motors can be designed in large sizes without sparking, so that the question arises: will any variable speed systems which involve the use of alternating-current commutator motors for large outputs be acceptable on the American market?

It has occurred to me that we might use a system which, while costly, would be absolutely certain of results. This system is outlined in Mr. Maier's paper and we suggest a modification of it—one which I have thought of employing a number of times. When there is a direct-current source of supply it consists of an inverted synchronous converter or motor-generator set. The speed is varied by changing the field strength of the converter or direct-current motor. The speed of the induction or synchronous motor changes with the speed of the converter or motor-generator. Or, if the source of supply be alternating, two synchronous converters may be used, one to change from alternating to direct and the other from direct to alternating. By using a split pole converter for the first machine and changing the field excitation of the second, or by having reactance coils in series with the first machine and varying the field strengths of both converters, the frequency may be altered as desired with corresponding changes in the speed of the main induction or synchronous motor.

These systems would be especially applicable to cases to which direct current cannot be directly applied, such as for large capacities and high speeds for which direct current motors cannot be built. We could then use synchronous or induction motors, and have their speeds varied as indicated above.

B. G. Lamme: The question was asked in this discussion, to what extent adjustable speed or variable speed alternating-current motors have been used in machine tool drive. Mr. Maier has replied to this question by stating that multi-speed motors without commutators have been used to a limited extent.

I can cite one case where commutator type alternating-current motors have been used for adjustable speed, not in machine tool work, but in print work where print rolls had to be driven at various speeds between zero and full speed. The motors used in this case were somewhat different from any described in the paper of the evening, although based upon the same principles.

In a single phase commutator type motor it is obvious that if the field winding be connected in shunt with the armature, the operation of the motor will be very poor, because the shunt field current, and therefore the field flux, will lag practically 90 deg. behind the armature current, whereas, for good operation the armature current and the field magnetism should be practically in phase. This condition can be obtained by connecting the field winding to a supply circuit which has practically a 90 deg. phase relation to the circuit which supplies the armature. This is illustrated in Fig. 1. With this arrangement the field current, or field flux, is still 90 deg. behind the exciting e.m.f., but as this latter is practically 90 deg. out of phase with the armature e.m.f., the field or exciting flux will thus be practically in phase with the armature current and the machine becomes the equivalent of an ordinary shunt machine. Such a motor can be connected either across a two-phase circuit or can be connected to a three-phase circuit by certain arrangements of the windings which give the equivalent of a two-phase condition.

With this arrangement the speed can be controlled just as with a shunt-wound direct current machine, either by field control or by control of the armature voltage. In the direct-current shunt machine, field control is obtained by insertion of resistance in the field circuit. In this alternating-current motor, field control is obtained by insertion of reactance in the field circuit, instead of resistance. As the field current already lags practically 90 deg. behind its supply e.m.f., the addition of reactance in series does not modify the phase relations materially. Added reactance thus causes weaker fields and increase in speed.

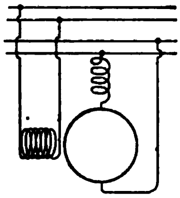


Fig. 1

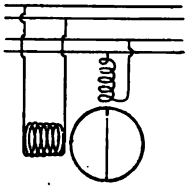


Fig. 2

The speed of such a motor can also be varied by means of resistance in the armature, the same as a direct-current machine, to give variable speed; or can be regulated by means of adjustable voltage supplied to the armature terminals from a transformer with taps. This will give practically constant speed characteristics for any given adjustment. The arrangement thus becomes the equivalent of a direct-current shunt motor with an adjustable supply voltage.

An equivalent arrangement to this was also built and tested, in which the armature was not connected directly across the line but was closed on itself and was supplied with current from a transformer winding on the primary or field core. This arrangement is shown in Fig. 2.

Several motors of the first described type were put into commercial operation. One of these motors was of 20 h.p. normal capacity and another of 30 h.p., both for 25 cycles. Both were controlled by adjustable armature voltage from transformers. The speed was adjustable from practically zero to 1000 rev. per min. The 20-h.p. motor was in operation for about 18 months and was apparently a very satisfactory machine. The 30-h.p. motor was operated for a much shorter period and was not nearly as satisfactory. Later both motors were taken out of service, partly because the system of operation of the plant was modified. The equipments were expensive, due partly to the cost of the regulating transformer, but they could be operated successfully in the hands of careful men. However, the apparatus was rather delicate in its characteristics compared with direct-current machines with equivalent characteristics.

In going over this paper hurriedly, I find several points which are open to discussion. On the last two pages of the paper where reference is made to the Kraemer and Scherbius systems, certain features of these systems are not brought out fully. Both of these are forms of the so-called cascade connection in which the secondary of an induction motor feeds current to another motor. Here the secondary motor is of the commutator type. The feature of this arrangement which makes the commutator motors operative in general, lies in the low frequency delivered by the secondary of the induction motor to the commutator motor. At synchronism of the induction motor, the frequency of the secondary is zero. If the speed is within 10 per cent of synchronism, for instance, the secondary frequency will be 10 per cent of the primary frequency; that is, with 60 cycles in the primary, the secondary frequency would be six cycles. At a speed of 30 per cent from synchronism, the secondary frequency will be 30 per cent, or 18 cycles, with a 60-cycle supply circuit. These low frequencies are particularly advantageous in the design of commutator type alternating current motors whether of the single-phase or three-phase type. However, the further the speed departs from synchronism the more difficult becomes the design and operating conditions. In consequence, such systems are usually built to operate within a certain limited per cent of synchronous speed, this per cent depending upon the frequency of the supply circuit. These conditions apply to the arrangement shown in Figs. 34 and 36. However, in the scheme shown in Fig. 35, the current from the secondary of the induction motor is fed to the collector rings of a synchronous converter and direct current from this synchronous converter is fed to a direct current motor, which thus absorbs the extra power of the secondary to the main motor.

A little consideration will show that the synchronous converter should be of the same capacity as the motor to be regulated, if operated at the primary frequency. It is therefore necessarily a large and expensive machine, relatively speaking. The lower frequency and the lower speed at which it is normally operated will be of indirect advantage in somewhat reducing the cost and size of this synchronous converter.

Furthermore, the direct-current motor operated from this synchronous converter cannot be of very economical proportions for it must be designed for operation over quite a wide range of voltage due to the fact that the synchronous converter will give different voltages with different speeds, and the direct-current motor must be adapted for operation at these voltages.

Therefore, considering this scheme as a whole, it appears to me that equivalent results would be obtained if the synchronous converter were connected directly to the supply circuit, so that its direct-current side supplies current to a direct-current motor which drives the load. The synchronous converter could be of the synchronous booster type for varying the direct-current voltage delivered by it. The size and capacity of the synchronous booster would be dependent upon the range of voltage, and speed variation desired, like the direct-current motor in Fig. 35. Therefore, in this arrangement the synchronous converter would correspond in capacity to the synchronous converter shown in Fig. 35, while the direct-current driving motor would correspond in capacity to the induction motor, and the synchronous booster would correspond in capacity to the direct-current motor in the same figure. The scheme, as a whole, appears to me to be simpler than that shown and no more expensive. With either of these schemes the permissible range of speed could be fairly wide.

C. P. Steinmetz: While the alternating-current motor has the reputation of being a constant speed motor, you see from the paper that in recent years a very large amount of work has been done in developing not only one but quite a number of types of variable speed alternating current motors, although the work is so recent that these motors have thus far come into industrial practice only to a limited extent, and there is no doubt that many further developments are still possible and desirable, There is one feature which impresses itself very forcibly, and that is, that in most of these variable speed alternating-current motors, the variable speed is connected with the use of the commutator. A commutator is not necessarily objectionable. We know that the direct current motor has a commutator and can be extremely successful. But, on the other hand, as the great advantage of the alternating-current motor has always been claimed its great simplicity, due to the absence of commutator or similar device. When we wish to get variable speed, in most cases we have to accept the commutator—the same complication which is inherent in the direct current motor whether variable speed or constant speed.

In order to see how far we have advanced, and what further work may still be done, it is desirable to make a classification of the problem. The Standardization Rules divide the motors in two types: constant speed and variable speed motors. Of the variable speed motors, three classes again are distinguished: multispeed motors, which have several steps of constant speed, hence, which can by adjustment be operated as constant speed motors not only at one but at two or more speeds; second, varying speed motors, in which the speed varies with the load as a rule decreasing with increase of load. Typical thereof is the direct-current series motor; third, adjustable speed motors, in which the speed can be adjusted to any desired value within a certain range, but when once adjusted the motor remains at this speed irrespective of the load, or approximately so. Typical of this class is the direct-current shunt motor with field control, where, by varying the field rheostat, you can set the speed at any desired value, and at that value it remains approximately from no load to over load.

Alternating-current motors of these three types can be produced as follows: Multi-speed motors are in industrial use to a considerable extent as induction motors without commutators—either by changing the number of poles, by some switchover device, or by concatenation of two motors, or by combining the two concatenated motors in one structure as an internally concatenated motor, or by a combination of several of these methods whereby we can then gain a considerable number of speed steps.

The varying speed motor also exists as a motor without commutator. It is the induction motor with slip rings and external rheostat. In this motor with increasing load the speed decreases. It has a characteristic in its speed torque relation similar to the direct current series motor, except that the speed is limited to a definite maximum speed, the speed of synchronism. But the limiting feature of this motor is that when the speed falls off, the efficiency also falls off. That is, at fractional speed you get correspondingly fractional efficiencies. The varying speed alternating current motor which is free from this dropping off of efficiency with dropping off of speed, is the commutator motor of the series type, or of the repulsion type, or of the combination type, the series-repulsion type, but, as stated, this is not as simple because it has a commutator.

The third type of variable speed motor is the adjustable speed motor, a motor which can be run at any constant value of speed, within a certain range. This motor does not yet exist in the commutatorless type, but it does exist in an almost infinite variety of commutator types of motor. Many of these are mentioned in the paper, but there exists no adjustable speed alternating current motor yet, which does not have a commutator, and this field is still open for development. It is this class of motor, which after all, would be the most useful, because very often in the industry we desire to be able to vary the motor speed over a moderate range as in cotton mills, to adjust it to the materials and the atmospheric conditions, so as to operate the mill at maximum efficiency. This means the ability of varying the speed, say, ten or twenty per cent, up or down, and still at any speed to have constancy independent of the load. In the absence of this motor there have been developed adjustable speed commutator motors which are in industrial use, are eminently satisfactory, but have not yet the simplicity of the plain induction motor.

In concluding, I may say, so far as my experience goes with the variable speed alternating-current motors, there are in use to a large extent: the induction motor with external rheostat as varying speed motor in hoisting work, in three-phase railroading, etc.; then there is in extensive use the multi-speed motor of the changed-pole type, and also to some extent, of the concatenated type; then there is in use the varying speed motor of the commutator type, as series and repulsion motor, and finally there is in use the adjustable speed motor of the so-called induction-repulsion type, especially in small units for general service.

C. O. Mailloux: It is remarkable what a transformation of feeling electrical engineers have undergone, in the last few years, in regard to the commutator. For a time, the commutator was charged with all the crimes in the electrical calendar; it was considered a great objection in connection with electrical machinery, in general. It was the fashion, among the partisans of the polyphase induction motor, to allege, as one of its most meritorious features, the entire *absence* of a commutator. It was very pleasing to me, at the last electrical Congress, at Turin, in the discussion on electric railways, before Section 5, where I had the honor to preside, to find evidence of a return to a spirit of greater toleration with respect to the commutator. The partisans of alternating-current systems of traction, at that Congress, were not as rabid as formerly, on the subject of " commutators ". The reason is that, in Europe, just now, the single-phase system of electric traction is in vogue; and that system, as we know, involves the use of a commutator-motor. That, of course, makes some difference. It is sufficient to change the opinion of many men who, some years ago, thought that a commutator-motor was not fit for traction purposes at all. Thus the " evil " of former days is fast becoming a " virtue." The old French proverb says: " *Chassez le naturel, et il revient au galop.*" (The nearest English equivalent is: " what is bred in the bone cannot be beaten out of the flesh "). Perhaps the commutator is much more " natural " than some people have been willing to admit. In dealing with the problems of speed-regulation, with alternating-current motors of any phase and type, we are obliged to come to the commutator, in order to increase the range of speeds obtainable, and to realize the fullest possibilities. It is the most " natural " thing to do, and it should have been done long ago. It is now being done because it cannot be put off any longer, presumably. Whatever may be the reason, the result will be to make the induction motor much more " fit " than it was or would ever be otherwise, for many applications, old as well as new, in competition with the direct-current motor.

Those of us who obtained their first practical experience with commutator machines, in the days when there were no others, were reconciled to commutator-motors. We were not afraid of them; and we knew that they could be made to do good work under any and all circumstances. For many purposes the commutator-motor did much better work than the commutatorless motor, even before the advent of the commutating pole, or the so-called " inter-pole " motor. Today, with the auxiliary or commutating pole, we are getting commutation which is absolutely satisfactory, if not absolutely faultless, in the case of direct-current motors of good design. The problem will not be quite as simple, however, with alternating-current motors. There are many difficulties here, due to electromagnetic phenomena and reactions which are not " in phase " with each other, and it is necessary to resort to compensating devices of various kinds to secure a line of commutation that is " stable "—which may not shift back and forth, describing an angle, during each period—and which " stays put " at least a portion of the time, same as in the direct-current motor. It is gratifying to find that considerable progress has been made in the adaptation of the commutator to polyphase motors, in the last few years. In that respect, the Europeans appear to be in the lead. During my recent trip abroad, I had occasion to learn something of the latest improvements made there in multi-speed, adjustable speed, and variable speed motors; and I was surprised at some of the results which are being obtained with alternating-current commutator motors. M. Latour, who is mentioned in the paper as the inventor of a method of speed-regulation involving the use of a commutator motor, has recently brought out some new methods of regulation which are of the greatest interest and merit. He kindly explained some of his methods to me and also gave me some results of practical tests and operation. I was astonished at the wonderful range of speed obtained, with motors of all sizes, with very satisfactory commutation, and with good power factor and efficiency. His results have not been excelled, and I doubt if they have been equalled, in this country. They are hard to match, even in Europe, and his methods have been adopted and are being used by several manufacturers, and they are being imitated by others there. M. Latour is one of the few alternanating-current specialists who never had *commutatorphobia.* It was he who first made a success of the single-phase commutator motor of the compensating type. (Even the German Patent Office had, ultimately, to give him the priority over all others). His recent work with polyphase commutator motors, evincing great originality, and producing valuable industrial results shows what can be done to make the commutator appear, even here, as a blessing in disguise. It is high time that the rest of the alternating-current designers and specialists should realize that the commutator not only need not, but that it should not, be banished from their "repertoire." With proper appreciation and treatment, its role may become as important and as useful here as in the case of the direct-current motors.

Gus A. Maier: It seems that in the discussion of the paper, the commutator of the motor is what practically everyone objects to. It is not considered desirable to go back to direct-current practice and use the commutator. We are agreed on that point, but there seems to be no way of getting away from it at the present time. The Europeans have done a large amount of work on the commutator motor and use it in many places. As far as I can learn, the commutation is fairly good, although I understand that the European standards of commutation are not as high as the American practise.

There is one point which has not been brought out which the advocates of the alternating-current motor can still hold on to, and that is, the voltage on the commutator is very low, compared to the line potential. It will probably run in the neighborhood of from 50 to 100 volts, and then with some sparking on the commutator, even if the commutator becomes black, it does not seem to affect the commutation or operation of the motor in any way. The Europeans are running these motors in various places, cotton mills, hoisting outfits, lathes, and using Scherbius sets in rolling mills, and Scherbius and Kraemer systems are giving very good results. I think it is up to the American engineers to follow closely the European developments and make improvements wherever possible, and I have no doubt the future will show some rapid progress in that direction.

A paper communicated to the American Institute of Electrical Engineers, by Dr. A. E. Kennelly.

TABLES OF HYPERBOLIC FUNCTIONS IN REFERENCE TO LONG ALTERNATING-CURRENT TRANSMISSION LINES

BY A. E. KENNELLY

The formulas relating to the behavior of long alternating-current lines of uniform resistance, inductance, capacitance and leakance, operated in the steady state, are very simply expressed in terms of hyperbolic functions; but are very long and cumbersome if expressed, with equal precision, without those functions. The absence of suitable tables* of hyperbolic functions of complex variables has, however, stood in the way of the general use of hyperbolic formulas. A few tables of such functions, in terms of real and imaginary components, have been published; but these are not so convenient for engineering purposes as tables of vector variables, using polar coördinates.

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* The following is a list of Tables of Complex Hyperbolic Functions thus far published:

1. " Hyperbolic Functions " by Dr. James McMahon, Chapter IV of " Higher Mathematics " by Merriman and Woodward, pp. 107–168. Sinh and cosh $(x+j\,y)$ to $x=1.5$, $y=1.5$. Wiley and Sons, New York, 1896.

2. " The Alternating-Current Theory of Transmission Speed over Submarine Telegraph Cables," by A. E. Kennelly. *Proc. Int. El. Congress. St. Louis*, Sec. A, Vol. I, pp. 68–105, 1904. Table of sinh, cosh, tanh, cosech, sech, coth $\rho\underline{/45°}$, up to $\rho=20$.

3. " The Distribution of Pressure and Current over Alternating-Current Circuits," by A. E. Kennelly, *Harvard Engineering Journal*, 1905–06. Tables of sinh, cosh, tanh, cosech, sech, coth $\rho\underline{/\delta}$ up to $\rho=1.5$ for five particular values of δ.

4. " Formulæ, Constants and Hyperbolic Functions for Transmission Line Problems," by W. E. Miller, *General Electric Review*, Schenectady, N. Y., May, 1910, Supplement. Sinh and cosh $(x+j\,y)$ up to $x=1$, $y=1$.

The accompanying tables have been computed by the writer for particular application to transmission-line problems. They extend up to 0.5 in modulus, by steps of 0.1, and from 60 deg. to 90 deg. inclusive, of argument, by steps of single degrees. The results are given to five significant digits of modulus, and to decimals of a degree, as far as the third decimal place, or to about 3.6 seconds of arc. The reasons for expressing circular angles in degrees and decimals of a degree, instead of in degrees, minutes, and seconds, were first—that the results are thereby more compactly expressed; second—that interpolations are simplified, and third—that reduction to French " grads " may be made if desired, merely by dividing with 0.9. The moduli results in the table, errors excepted, are correct up to the last digit, which is doubtful to the extent of unity. The circular angles resulting from the tables, errors excepted, are correct up to the last digit, which is doubtful to the extent of unity, representing 0.001 degree. Such a degree of precision in the tables is unnecessary for ordinary alternating-current engineering re-

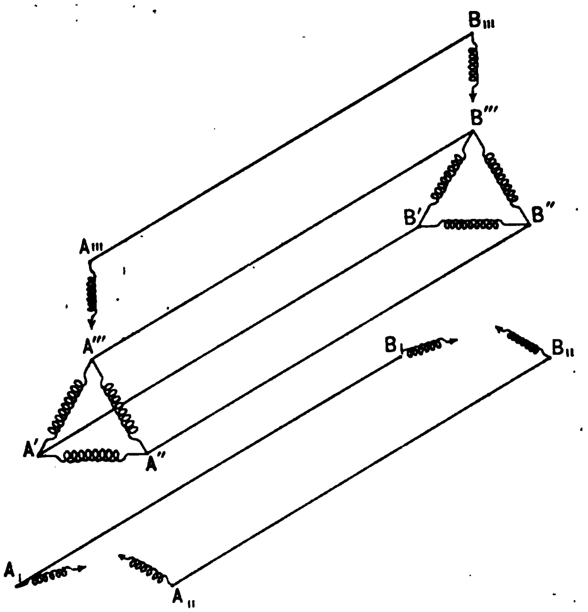

FIG. 1.—Analysis of a three-phase system into three component single-phase systems

quirements; but has been adopted because the tables are intended to form the nucleus of a subsequent and more extensive series, applicable to telephonic-circuit problems. The tables here offered suffice for the longest existing power-transmission lines, operated at fundamental frequencies. They do not purport to cover the behavior of such lines with higher harmonic frequencies.

DIRECTIONS FOR THE USE OF THE TABLES WITH POWER-TRANSMISSION THREE-PHASE LINES

Let the three-phase line considered be uniform throughout, and free from intermediate loads. If there be a load applied to the system at a substation along the line, the line must be divided at the substation into two distinct uniform sections, and each must be treated separately.

Analyze the three-phase line system $A' B'$, $A'' B''$, $A''' B'''$, into three separate single-phase lines, $A_, B_,$ $A_{,,} B_{,,}$ $A_{,,,} B_{,,,}$ Fig. 1, in the usual way, each such single line carrying one third of the load, and being operated, at star voltage, with respect to its enveloping surface of zero-potential. This well known step enables us to consider one line only, say $A B$, operated as though single-phase, with its independent linear constants.

Find the total resistance R in the line $A B$ (ohms).

Find the total reactance $j X = j \mathcal{L} \omega$ in the line $A B$ (ohms); where $\mathcal{L}$ is the wire inductance in henrys, and ω is the angular velocity $2 \pi f$, in radians per second, f being the frequency of operation, in cycles per second.

Find the total leakance G in the line $A B$, if any (mhos).

Find the total susceptance $j B = j C \omega$ in the line $A B$, (mhos); where C is the total wire capacitance to zero-potential surface, in farads.

Ordinarily, the leakance G is negligible, but it may be included for the sake of generality.

Proceed to find the total wire impedance $R + j X = Z \underline{/\beta_1}$ ohms, where Z is the vector impedance at the angle β_1. The numerical value of Z may be called the *modulus*, and the value of β_1 the *argument*, of the plane vector impedance.

Also find the total wire admittance $G + j B = Y \underline{/\beta_2}$ mhos, where Y is the vector admittance modulus, and β_2 the admittance argument.

Lay off a conductor, *A B*, Fig. 2, to represent the wire *A B* of the line by the well-known "split-condenser" method.[*] In the line *A B*, place an air-cored reactance coil of impedance *Z* ohms. At each end of the line, place a condenser of half the admittance *Y* mhos. Then the three-part artificial line *A B G G* will represent one wire of the line, except that it has its total dielectric admittance applied in two equal terminal lumps, instead of being applied in uniform distribution. The line *A B G G*, thus constituted, may be called the "***nominal** Π*" of one wire of the line; because the nominal values of the wire's impedance and admittance are ascribed to it. A nominal *Π* artificial line is unable correctly to imitate the behavior of an actual transmission line-wire, owing to the lumpiness error, just referred to; but it is readily possible so to modify the value of the impedance *Z* in the *architrave A B*, and also the value of the admittance *Y*/2 in each *pillar*, *A G* or *B G*, of the nominal *Π*, as will make the lumpiness error vanish, or will make the artificial line truly represent, at the terminals, the behavior of one line wire, taking distributed leakance and capacitance into account. Such a corrected artificial line may be called the "equivalent *Π*" of the line-wire[†]; because it then becomes externally equivalent

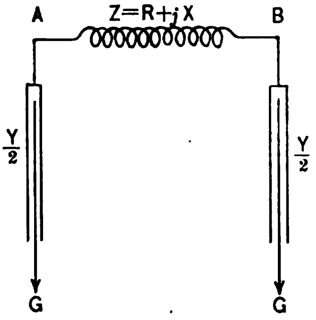

Fig. 2.—Nominal *Π* of one line wire *A B*

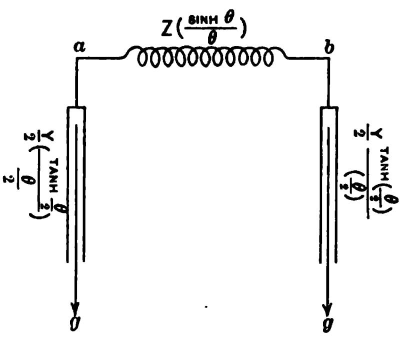

Fig. 3.—Equivalent *Π* of one line wire of angle *θ*

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[*] See a paper on "Calculation of the High-Tension Line," by P. H. Thomas, Transactions A. I. E. E., Part I, Vol. XXVIII, June, 1909. pp. 641–686.

[†] The theory and demonstration of this proposition are given in "The Application of Hyperbolic Functions to Electrical Engineering Problems," by A. E. Kennelly. The University of London Press. Chapter III, p. 33.

to the same in every respect, during steady operation. The algebraic factors that must be applied to Z and $Y/2$, in order to convert the nominal Π into the equivalent Π, may be called the *correcting-factors* of the nominal Π.

The correcting-factor of the architrave impedance $A\ B$ is $\frac{\sinh \theta}{\theta}$, and the corresponding correcting-factor of each pillar admittance, $A\ G$ or $B\ G$, is $\frac{\tanh \frac{\theta}{2}}{\frac{\theta}{2}}$; where θ is the hyperbolic angle subtended by the line, in hyperbolic radians,* defined by the relation:

$$\theta = \sqrt{Z\ Y} \qquad \text{hyperbolic radians } \underline{/}\ .$$

This hyperbolic angle will be a plane-vector quantity. Its modulus will not exceed 0.5, with transmission lines operated at ordinary frequencies, and its argument will lie between 60 deg. and 90 deg. The equivalent Π of a line-wire is represented diagrammatically at *a b g g*, Fig. 3.

Having, therefore, found the line-angle θ, enter Table IV, for the nearest given architrave correcting-factor, and Table V, for the nearest given pillar correcting-factor. In general, the corrected line-wire impedance, which becomes $Z\left(\frac{\sinh \theta}{\theta}\right)$, is slightly diminished in modulus, but slightly increased in argument. This ordinarily represents a slight reduction in conductor resistance, associated with a very small change in conductor reactance. That is, the $I^2\ R$ loss on a line wire is slightly reduced by correcting from the nominal Π to the equivalent Π; or, the distributed capacity in a line produces slightly less $I^2\ R$ loss, in actual operation, than if the line had its capacity applied in two equal terminal lumps. Again, Table V shows that the pillar correcting factor slightly increases the admittance modulus, but slightly diminishes the admittance argument; so that some $I^2\ R$ loss appears in the terminal condensers.

Having found the equivalent Π of each wire of the line as above, the behavior of the wire, under any assigned normal load, can be determined by known rules for simple alternating-current circuits.

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*See TRANSACTIONS American Institute of Electrical Engineers, Part I, Vol. XVII, pp. 698, 702, June 29, 1909. Discussion on Thomas' paper.

Harmonic Frequencies. It has been assumed in the foregoing discussion that the e.m.f. existing at each end of the line possesses a purely sinusoidal wave-shape; or, in other words, contains the fundamental frequency only. If harmonic frequencies in the impressed e.m.f. have to be taken into account, the process must be repeated for each harmonic frequency in turn. That is, the impedance Z, admittance Y, and line angle $\theta = \sqrt{Z\,Y}$, must be found for each harmonic frequency, independently. The nominal and equivalent Πs of the line are then found for each such frequency. Under a given condition of load, the distribution of pressure and current is thus obtained for each frequency component, as though it acted alone. The resulting total distribution of pressure and current is then derived through the superposition of the various component distributions, by the process of* " crab-addition ", or equivalent methods. In general, the accompanying tables are not sufficiently extended in modulus to provide for cases of harmonics higher than of triple-frequency on very long lines; although they may cover cases of higher harmonics on shorter lines.

Single-Phase Circuits. The analysis of a single-phase line, with two conductors, differs only from that of a three-phase line already discussed, in applying half the circuit load and impressed e.m.f., to each of the two wires, considered as operated to zero-potential surface.

Interpolation between Tabular Values. In using the Tables, interpolation is required both for intermediate values of the modulus, and for intermediate values of the associated argument. Thus, in finding $\frac{\sinh\theta}{\theta}$, from Table IV, for the particular value of $\theta = 0.25\underline{/70.5}$ deg., we require to find, first, the interpolated value $0.25\underline{/70}$ deg., which is $0.98174\underline{/0^\circ.4}$ (on the basis of simple proportion), and then to deduce the interpolated value for $0.25\underline{/70^\circ.5}$ by subtracting $0.00025\underline{/0.012}$, (the mean difference for 0.5 deg. of argument), obtaining, as the result, $0.98149\underline{/0^\circ.388}$. It should be observed, however, that owing to the rate at which the tabular values are changing, and the relatively large steps of 0.1 in modulus, simple proportional interpolation cannot be relied upon for precision in the fifth

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*" The Application of Hyperbolic Functions to Electrical Engineering Problems ", Chapter VII, p. 101.

digit. In other words, unless second differences are taken into account, which involves much labor, the interpolated values can only be relied on to four digits, with the last in some doubt. In general, a sufficiently good interpolation for most practical alternating-current engineering purposes can be made by direct inspection, at a corresponding sacrifice of attempted high precision.

TABLE I

HYPERBOLIC SINES

	0.1		0.2		0.3		0.4		0.5	
60°	0.099917	60°.082	0.19934	60°.331	0.29777	60°.746	0.39474	61°.330	0.48978	62.085
61°	0.099912	61.081	0.19929	61.324	0.29764	61.731	0.39441	62.302	0.48916	63.042
62°	0.099907	62.079	0.19925	62.317	0.29751	62.715	0.39409	63.273	0.48855	63.995
63°	0.099902	63.077	0.19921	63.309	0.29738	63.698	0.39379	64.243	0.48795	64.950
64°	0.099897	64.075	0.19918	64.301	0.29725	64.680	0.39349	65.212	0.48738	65.900
65°	0.099893	65.073	0.19914	65.293	0.29712	65.661	0.39320	66.179	0.48681	66.845
66°	0.099889	66.071	0.19911	66.284	0.29700	66.641	0.39293	67.144	0.48627	67.794
67°	0.099886	67.069	0.19908	67.275	0.29689	67.621	0.39266	68.108	0.48575	68.738
68°	0.099881	68.066	0.19904	68.265	0.29678	68.599	0.39240	69.070	0.48523	69.679
69°	0.099877	69.064	0.19901	69.255	0.29667	69.577	0.39214	70.030	0.48473	70.617
70°	0.099873	70.062	0.19898	70.245	0.29657	70.555	0.39190	70.990	0.48426	71.554
71°	0.099869	71.059	0.19895	71.235	0.29647	71.532	0.39167	71.948	0.48380	72.489
72°	0.099865	72.057	0.19892	72.225	0.29637	72.508	0.39145	72.905	0.48338	73.422
73°	0.099861	73.054	0.19889	73.214	0.29628	73.483	0.39123	73.861	0.48296	74.354
74°	0.099858	74.051	0.19887	74.203	0.29620	74.458	0.39104	74.817	0.48257	75.284
75°	0.099855	75.048	0.19885	75.192	0.29612	75.432	0.39084	75.771	0.48220	76.212
76°	0.099852	76.045	0.19883	76.180	0.29604	76.406	0.39066	76.724	0.48184	77.138
77°	0.099850	77.042	0.19881	77.168	0.29596	77.379	0.39050	77.676	0.48152	78.062
78°	0.099847	78.039	0.19878	78.156	0.29590	78.351	0.39034	78.628	0.48123	78.986
79°	0.099845	79.036	0.19876	79.144	0.29584	79.322	0.39018	79.578	0.48095	79.909
80°	0.099843	80.033	0.19875	80.181	0.29578	80.294	0.39004	80.528	0.48067	80.830
81°	0.099841	81.030	0.19873	81.118	0.29573	81.266	0.38993	81.477	0.48044	81.750
82°	0.099839	82.026	0.19872	82.106	0.29569	82.238	0.38983	82.425	0.48023	82.669
83°	0.099838	83.023	0.19871	83.093	0.29566	83.209	0.38973	83.373	0.48004	83.587
84°	0.099837	84.020	0.19870	84.080	0.29563	84.180	0.38965	84.321	0.47987	84.505
85°	0.099836	85.017	0.19869	85.067	0.29561	85.150	0.38958	85.268	0.47972	85.422
86°	0.099835	86.014	0.19868	86.054	0.29559	86.120	0.38952	86.215	0.47960	86.338
87°	0.099834	87.011	0.19868	87.041	0.29557	87.090	0.38948	87.162	0.47952	87.254
88°	0.099833	88.008	0.19867	88.028	0.29555	88.060	0.38946	88.108	0.47947	88.170
89°	0.099832	89.004	0.19867	89.014	0.29553	89.030	0.38944	89.054	0.47945	89.085
90°	0.099831	90.000	0.19867	90.000	0.29552	90.000	0.38942	90.000	0.47943	90.000

Example of the use of Table: sinh $(0.3\underline{/75^\circ}) = 0.29612\underline{/75.^\circ432} = 0.29612\underline{/75.^\circ25.'55''}$

NOTE.—sinh $\rho\underline{/\delta} = 0\underline{/\delta}$ when $\rho = 0$.

USE OF THE TABLES FOR HYPERBOLIC ANGLES BETWEEN 0.5 AND 1.0 IN MODULUS

Although the correcting factors $\frac{\sinh\theta}{\theta}$ and $\frac{\tanh(\theta/2)}{(\theta/2)}$, for reducing nominal to equivalent Πs, are only tabulated up to 0.5 in modulus of θ; yet, with a little extra effort, they can be used for lines whose hyperbolic angles do not exceed 1.0 in modulus. That is, they can be used for doubled range. It is evident that Table V is already sufficiently extended to apply to lines of

$\theta = 1$; since only the semi-angle $\theta/2$ in modulus has to be sought in it. It is only Table IV, therefore, which needs to be extended for the extra range. For this purpose, we may use the relation:

$$\frac{\sinh 2\theta}{2\theta} = \frac{\sinh\theta}{\theta}\cdot\cosh\theta \qquad \text{numeric } \underline{/}$$

TABLE II
HYPERBOLIC COSINES

	0.1		0.2		0.3		0.4		0.5	
60°	0.99751	0°.249	0.99012	0°.999	0.97810	2°.267	0.96191	4°.075	0.94219	6°.460
61°	0.99736	0.244	0.98952	0.979	0.97674	2.221	0.95948	3.998	0.93838	6.343
62°	0.99721	0.238	0.98893	0.957	0.97541	2.173	0.95711	3.914	0.93465	6.217
63°	0.99707	0.232	0.98835	0.934	0.97411	2.123	0.95478	3.826	0.93099	6.083
64°	0.99693	0.226	0.98780	0.910	0.97285	2.070	0.95251	3.733	0.92741	5.941
65°	0.99679	0.220	0.98725	0.885	0.97163	2.014	0.95031	3.635	0.92393	5.789
66°	0.99666	0.214	0.98672	0.859	0.97044	1.955	0.94816	3.532	0.92054	5.631
67°	0.99653	0.207	0.98621	0.832	0.96927	1.894	0.94608	3.423	0.91725	5.463
68°	0.99641	0.200	0.98571	0.804	0.96814	1.830	0.94407	3.310	0.91407	5.288
69°	0.99629	0.193	0.98523	0.775	0.96706	1.764	0.94213	3.193	0.91100	5.106
70°	0.99617	0.185	0.98477	0.744	0.96601	1.696	0.94026	3.072	0.90805	4.916
71°	0.99606	0.177	0.98433	0.713	0.96500	1.626	0.93846	2.946	0.90521	4.718
72°	0.99596	0.169	0.98391	0.681	0.96404	1.553	0.93674	2.816	0.90248	4.513
73°	0.99586	0.161	0.98350	0.648	0.96313	1.479	0.93510	2.682	0.89989	4.302
74°	0.99576	0.153	0.98312	0.614	0.96227	1.402	0.93354	2.544	0.89742	4.085
75°	0.99567	0.144	0.98276	0.580	0.96145	1.324	0.93207	2.403	0.89508	3.861
76°	0.99559	0.135	0.98242	0.545	0.96068	1.244	0.93069	2.258	0.89288	3.631
77°	0.99551	0.126	0.98210	0.509	0.95996	1.162	0.92938	2.111	0.89081	3.396
78°	0.99544	0.117	0.98181	0.472	0.95929	1.078	0.92816	1.960	0.88889	3.155
79°	0.99537	0.108	0.98154	0.435	0.95866	0.993	0.92705	1.807	0.88711	2.910
80°	0.99531	0.099	0.98128	0.397	0.95808	0.908	0.92603	1.652	0.88548	2.660
81°	0.99525	0.090	0.98105	0.359	0.95756	0.821	0.92509	1.494	0.88399	2.406
82°	0.99520	0.080	0.98085	0.320	0.95710	0.732	0.92425	1.333	0.88266	2.149
83°	0.99515	0.070	0.98067	0.281	0.95670	0.643	0.92351	1.170	0.88147	1.888
84°	0.99511	0.060	0.98051	0.242	0.95635	0.552	0.92287	1.006	0.88044	1.624
85°	0.99508	0.050	0.98037	0.203	0.95605	0.461	0.92233	0.841	0.87957	1.357
86°	0.99506	0.040	0.98026	0.163	0.95580	0.369	0.92189	0.675	0.87886	1.087
87°	0.99504	0.030	0.98018	0.123	0.95560	0.277	0.92154	0.508	0.87830	0.816
88°	0.99502	0.020	0.98012	0.082	0.95545	0.185	0.92128	0.340	0.87790	0.544
89°	0.99501	0.010	0.98009	0.041	0.95537	0.093	0.92112	0.171	0.87766	0.272
90°	0.99500	0.000	0.98007	0.000	0.95534	0.000	0.92106	0.000	0.87758	0.000

Example of the use of Table: $\cosh 0.5\underline{/81°} = 0.88399\underline{/2.°406} = 0.88399\underline{/2.°24.'22''}$.

Note.—$\cosh \rho\underline{/\delta} = 1.0\underline{/0°}$, when $\rho = 0$ whatever the value of δ.

Thus, suppose we require to find the value of the correcting-factor $\frac{\sinh\theta}{\theta}$ for the case of a line having a hyperbolic angle $\theta = 1.0\underline{/76°}$ which is beyond the direct range of Table IV. Find in that Table the value of $\frac{\sinh\theta}{\theta}$ for $0.5\underline{/76}$, the semi-line angle, which gives $0.96368\ \underline{/1°.138}$. Now find the cosine of

$0.5\underline{/76}$ deg. in Table II. It is $0.89288\underline{/3^\circ.631}$. The required value of $\frac{\sinh\theta}{\theta}$ for $1\underline{/76}$ deg. is then: $0.96368\underline{/1^\circ.138} \times 0.89288\underline{/3^\circ.631} = 0.86045\underline{/4^\circ.769}$.

Consequently, with the aid of this extra step, harmonic frequencies or lengths of line, can be dealt with extending up to 1.0 modulus in the line-angle θ.

TABLE III
HYPERBOLIC TANGENTS

	0.1		0.2		0.3		0.4		0.5	
60°	0.10017	59°.833	0.20133	59°.332	0.30444	58°.479	0.41037	57°.255	0.51983	55°.625
61°	0.10018	60.837	0.20140	60.345	0.30473	59.510	0.41107	58.305	0.52128	56.700
62°	0.10019	61.841	0.20148	61.360	0.30501	60.542	0.41176	59.359	0.52271	57.781
63°	0.10020	62.845	0.20156	62.375	0.30528	61.575	0.41244	60.417	0.52412	58.867
64°	0.10020	63.849	0.20164	63.391	0.30555	62.610	0.41311	61.479	0.52552	59.959
65°	0.10021	64.853	0.20171	64.408	0.30580	63.647	0.41376	62.544	0.52689	61.058
66°	0.10022	65.857	0.20179	65.425	0.30605	64.686	0.41441	63.612	0.52824	62.164
67°	0.10023	66.862	0.20186	66.443	0.30630	65.727	0.41504	64.685	0.52957	63.276
68°	0.10024	67.866	0.20193	67.461	0.30655	66.769	0.41565	65.760	0.53085	64.391
69°	0.10025	68.871	0.20199	68.480	0.30678	67.813	0.41623	66.837	0.53209	65.511
70°	0.10026	69.877	0.20205	69.501	0.30701	68.859	0.41680	67.918	0.53330	66.638
71°	0.10026	70.882	0.20212	70.522	0.30722	69.906	0.41735	69.002	0.53446	67.771
72°	0.10027	71.888	0.20217	71.544	0.30742	70.955	0.41788	70.089	0.53560	68.909
73°	0.10028	72.893	0.20223	72.566	0.30762	72.005	0.41838	71.179	0.53669	70.052
74°	0.10028	73.898	0.20228	73.589	0.30781	73.056	0.41886	72.272	0.53773	71.200
75°	0.10029	74.904	0.20234	74.612	0.30799	74.108	0.41932	73.368	0.53872	72.351
76°	0.10029	75.910	0.20239	75.635	0.30816	75.162	0.41975	74.466	0.53965	73.507
77°	0.10030	76.916	0.20243	76.659	0.30831	76.217	0.42016	75.566	0.54054	74.667
78°	0.10030	77.922	0.20246	77.684	0.30846	77.273	0.42054	76.668	0.54138	75.831
79°	0.10031	78.928	0.20250	78.709	0.30860	78.329	0.42088	77.771	0.54215	76.999
80°	0.10031	79.934	0.20254	79.734	0.30872	79.386	0.42120	78.876	0.54285	78.170
81°	0.10032	80.940	0.20257	80.759	0.30884	80.445	0.42150	79.983	0.54349	79.344
82°	0.10032	81.946	0.20260	81.785	0.30894	81.506	0.42177	81.092	0.54407	80.520
83°	0.10032	82.953	0.20263	82.812	0.30904	82.567	0.42201	82.203	0.54459	81.699
84°	0.10033	83.960	0.20265	83.838	0.30912	83.628	0.42222	83.315	0.54503	82.881
85°	0.10033	84.967	0.20267	84.864	0.30920	84.689	0.42239	84.427	0.54540	84.065
86°	0.10033	85.974	0.20268	85.891	0.30926	85.751	0.42252	85.540	0.54571	85.251
87°	0.10033	86.981	0.20270	86.918	0.30930	86.813	0.42264	86.654	0.54596	86.438
88°	0.10033	87.988	0.20270	87.946	0.30933	87.875	0.42274	87.768	0.54616	87.626
89°	0.10033	88.994	0.20271	88.973	0.30934	88.937	0.42279	88.883	0.54628	88.813
90°	0.10033	90.000	0.20271	90.000	0.30934	90.000	0.42280	90.000	0.54631	90.000

Example of the use of Table: tanh $0.2\underline{/70^\circ} = 0.20205\underline{/69.^\circ501} = 0.20205\underline{/69.^\circ30.'04''}$.
NOTE.—tanh $\rho\underline{/\delta} = 0\underline{/\delta}$ when $\rho = 0$.

Numerical Example. A three-phase transmission line has a length of 250 km. (155.34 statute miles), and consists of three No. 000 A. W. G. aerial copper wires, 0.41 inch (1.041 cm.) in diameter, supported symmetrically, on pole insulators, at a uniform interaxial distance of 72 in. (193 cm.). Required to find the equivalent Π of each wire, at an impressed sinusoidal frequency of 25 cycles per second. The following are the assumed linear constants of the line, referred to one wire:

Linear conductor resistance $r = 0.206$ ohm/km. $= 0.33$ ohm/mile.
Linear conductor inductance $l = 1.2223$ millihenry/km. $= 1.967$ millihenry/mile.
Linear dielectric leakance $g = 0$.
Linear dielectric capacitance $c = 0.0094828$ microfarad/km. $=$ 0.01526 microfarad/mile.

TABLE IV

CORRECTING FACTOR $\frac{\sinh \theta}{\theta}$

	0.1		0.2		0.3		0.4		0.5	
60°	0.99917	0°.082	0.99670	0°.331	0.99257	0°.746	0.98685	1°.330	0.97956	2°.085
61°	0.99912	0.081	0.99645	0.324	0.99213	0.731	0.98603	1.302	0.97832	2.042
62°	0.99907	0.079	0.99625	0.317	0.99170	0.715	0.98523	1.273	0.97710	1.998
63°	0.99902	0.077	0.99605	0.309	0.99127	0.698	0.98448	1.243	0.97590	1.950
64°	0.99897	0.075	0.99585	0.301	0.99083	0.680	0.98373	1.212	0.97476	1.900
65°	0.99893	0.073	0.99570	0.293	0.99040	0.661	0.98300	1.179	0.97364	1.848
66°	0.99889	0.071	0.99555	0.284	0.99000	0.641	0.98232	1.144	0.97254	1.794
67°	0.99885	0.069	0.99540	0.275	0.98963	0.621	0.98165	1.108	0.97150	1.738
68°	0.99881	0.066	0.99520	0.265	0.98927	0.599	0.98100	1.070	0.97046	1.679
69°	0.99877	0.064	0.99505	0.255	0.98891	0.577	0.98035	1.030	0.96946	1.617
70°	0.99873	0.062	0.99490	0.245	0.98857	0.555	0.97975	0.990	0.96852	1.554
71°	0.99869	0.059	0.99475	0.235	0.98823	0.532	0.97918	0.948	0.96760	1.489
72°	0.99865	0.057	0.99460	0.225	0.98790	0.508	0.97863	0.905	0.96676	1.422
73°	0.99861	0.054	0.99445	0.214	0.98760	0.483	0.97808	0.861	0.96592	1.354
74°	0.99858	0.051	0.99435	0.203	0.98733	0.458	0.97758	0.817	0.96514	1.284
75°	0.99855	0.048	0.99425	0.192	0.98707	0.432	0.97710	0.771	0.96440	1.212
76°	0.99852	0.045	0.99415	0.180	0.98680	0.406	0.97665	0.724	0.96368	1.138
77°	0.99850	0.042	0.99405	0.168	0.98653	0.379	0.97625	0.676	0.96304	1.062
78°	0.99847	0.039	0.99395	0.156	0.98633	0.351	0.97585	0.628	0.96246	0.986
79°	0.99845	0.036	0.99385	0.144	0.98613	0.322	0.97545	0.578	0.96190	0.909
80°	0.99843	0.033	0.99375	0.131	0.98593	0.294	0.97512	0.528	0.96134	0.830
81°	0.99841	0.030	0.99365	0.118	0.98577	0.266	0.97483	0.477	0.96088	0.750
82°	0.99839	0.026	0.99360	0.106	0.98563	0.238	0.97458	0.425	0.96046	0.669
83°	0.99838	0.023	0.99355	0.093	0.98553	0.209	0.97433	0.373	0.96008	0.587
84°	0.99837	0.020	0.99350	0.080	0.98543	0.180	0.97413	0.321	0.95974	0.505
85°	0.99836	0.017	0.99345	0.067	0.98537	0.150	0.97395	0.268	0.95944	0.422
86°	0.99835	0.014	0.99340	0.054	0.98530	0.120	0.97380	0.215	0.95920	0.338
87°	0.99834	0.011	0.99335	0.041	0.98523	0.090	0.97370	0.162	0.95904	0.254
88°	0.99833	0.008	0.99335	0.028	0.98517	0.060	0.97365	0.108	0.95894	0.170
89°	0.99832	0.004	0.99335	0.014	0.98510	0.030	0.97360	0.054	0.95890	0.085
90°	0.99831	0.000	0.99335	0.000	0.98507	0.000	0.97355	0.000	0.95886	0.000

Example of the use of Table: $\frac{\sinh 0.3/\underline{80^\circ}}{0.3/\underline{80^\circ}} = 0.98593/\underline{0.^\circ 294} = 0.95893/\underline{0.^\circ 17.' 38''}$

NOTE.—$\frac{\sinh \theta}{\theta} = 1.0/\underline{0^\circ}$ when $\theta = 0/\underline{\delta}$.

From these we obtain the following wire constants:

Total conductor resistance $R = 51.5$ ohms per wire.
Total conductor inductance $\mathfrak{L} = 0.30557$ henry per wire.
Total dielectric capacitance $C = 2.3707$ microfarads per wire.

At the frequency $f = 25$, the impressed angular velocity is $\omega = 157.08$ radians/sec. so that the total conductor reactance

$jX = j\mathcal{L}\,\omega = j\,47.999$ ohms per wire, and the total dielectric susceptance $jB = jC\,\omega = j\,3.72396 \times 10^{-4}$ mho per wire. The conductor impedance is $Z = 51.5 + j\,47.999 = 70.4\underline{/42°.985}$ ohms per wire. The dielectric admittance is $Y = 0 + j\,3.72396 \times 10^{-4} = 3.7239 \times 10^{-4}\underline{/90°}$ mho per wire. With these data, we obtain the nominal Π of each wire *A B G G*, Fig. 4.

TABLE V

CORRECTING FACTOR $\dfrac{\tanh\theta}{\theta}$

NOTE.—All the angles found in this table are negative

	0.1		0.2		0.3		0.4		0.5	
		°−		°−		°−		°−		°−
60°	1.0017	0.167	1.0067	0.668	1.0148	1.521	1.0259	2.745	1.0397	4.375
61°	1.0018	0.163	1.0070	0.655	1.0158	1.490	1.0278	2.695	1.0425	4.300
62°	1.0019	0.159	1.0074	0.640	1.0167	1.458	1.0294	2.641	1.0454	4.220
63°	1.0020	0.155	1.0078	0.625	1.0176	1.425	1.0311	2.583	1.0482	4.133
64°	1.0020	0.151	1.0082	0.609	1.0185	1.390	1.0328	2.521	1.0510	4.041
65°	1.0021	0.147	1.0086	0.592	1.0193	1.353	1.0344	2.456	1.0538	3.942
66°	1.0022	0.143	1.0090	0.575	1.0202	1.314	1.0360	2.388	1.0564	3.836
67°	1.0023	0.138	1.0093	0.557	1.0210	1.273	1.0376	2.315	1.0591	3.724
68°	1.0024	0.134	1.0097	0.539	1.0218	1.231	1.0391	2.240	1.0617	3.609
69°	1.0025	0.129	1.0100	0.520	1.0226	1.187	1.0406	2.163	1.0642	3.489
70°	1.0026	0.123	1.0103	0.499	1.0234	1.141	1.0420	2.082	1.0666	3.362
71°	1.0026	0.118	1.0106	0.478	1.0241	1.094	1.0434	1.998	1.0689	3.229
72°	1.0027	0.112	1.0109	0.456	1.0247	1.045	1.0447	1.911	1.0712	3.091
73°	1.0028	0.107	1.0112	0.434	1.0254	0.995	1.0460	1.821	1.0734	2.948
74°	1.0028	0.102	1.0114	0.411	1.0260	0.944	1.0472	1.728	1.0755	2.800
75°	1.0029	0.096	1.0117	0.388	1.0266	0.892	1.0483	1.632	1.0774	2.649
76°	1.0029	0.090	1.0120	0.365	1.0272	0.838	1.0494	1.534	1.0793	2.493
77°	1.0030	0.084	1.0122	0.341	1.0277	0.783	1.0504	1.434	1.0811	2.333
78°	1.0030	0.078	1.0123	0.316	1.0282	0.727	1.0513	1.332	1.0828	2.169
79°	1.0031	0.072	1.0125	0.291	1.0287	0.671	1.0522	1.229	1.0843	2.001
80°	1.0031	0.066	1.0127	0.266	1.0291	0.614	1.0530	1.124	1.0857	1.830
81°	1.0032	0.060	1.0129	0.241	1.0295	0.555	1.0538	1.017	1.0870	1.656
82°	1.0032	0.054	1.0130	0.215	1.0298	0.494	1.0544	0.908	1.0881	1.480
83°	1.0032	0.047	1.0132	0.188	1.0301	0.433	1.0550	0.797	1.0892	1.301
84°	1.0033	0.040	1.0133	0.162	1.0304	0.372	1.0555	0.685	1.0901	1.119
85°	1.0033	0.033	1.0134	0.136	1.0307	0.311	1.0560	0.573	1.0908	0.935
86°	1.0033	0.026	1.0134	0.109	1.0309	0.249	1.0563	0.460	1.0914	0.749
87°	1.0033	0.019	1.0135	0.082	1.0310	0.187	1.0566	0.346	1.0919	0.562
88°	1.0033	0.012	1.0135	0.054	1.0311	0.125	1.0568	0.232	1.0923	0.374
89°	1.0033	0.006	1.0135	0.027	1.0311	0.063	1.0570	0.117	1.0926	0.187
90°	1.0033	0.000	1.0136	0.000	1.0311	0.000	1.0570	0.000	1.0926	0.000

Example of the use of Table: $\dfrac{\tanh\ 0.4\underline{/71°}}{0.4\underline{/71°}} = 1.0434\,\overline{\diagdown 1.°998} = 1.0434\,\overline{\diagdown 1.°59.'53''}$.

NOTE. $\dfrac{\tanh\theta}{\theta} = 1.0\underline{/0°}$ when $\theta = 0\underline{/\delta}$.

We next proceed to find the line angle θ as follows:

$$\theta = \sqrt{70.4\underline{/42°.985} \times 3.7239 \times 10^{-4}\underline{/90°}}$$

$$= \sqrt{0.0262167\ \underline{/132°.985}}$$

$$= 0.161915\ \underline{/66°.492}\ \text{hyp.}$$

Entering Table IV with the line angle $0.162\underline{/69^\circ.5}$, we find the architrave correcting factor to be $0.997\underline{/0^\circ.18}$. Also, entering Table V with the semi-line angle $0.081\underline{/66^\circ.5}$, we find the pillar correcting factor $1.0016\overline{\diagdown 0^\circ.10^\circ}$. The corrected value of line impedance becomes therefore $70.4\underline{/42^\circ.985}\times 0.997\underline{/0^\circ.18}=70.19\underline{/43^\circ.165}$, and that of each pillar admittance $1.862\times 10^{-4}\underline{/90^\circ}\times 1.0016\overline{\diagdown 0^\circ.10}=1.865\underline{/89^\circ.9}\times 10^{-4}$ mho. These values produce the equivalent *Π* of each line-wire, as shown in Fig. 5. That is, the external behavior of the actual line at any load could not be distinguished from that of an artificial line of three equivalent *Π*s, each having, as architrave, an impedance coil of $70.19\underline{/43^\circ.165}$ ohms, and terminal pillar admittances, each of $1.865\times 10^{-4}\underline{/89^\circ.9}$ mhos, such as might be produced by a condenser of 2.375 microfarads in series with a resistance of 9.73 ohms. The voltage regulation and efficiency of the three-phase line in the steady state of operation can now be determined by known methods, since the actual line is replaced by definite groups of reactance coils and condensers.

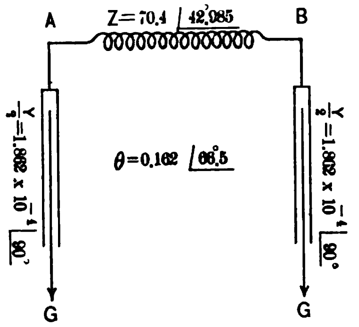

FIG. 4.—Nominal *Π* of line wire in numerical example

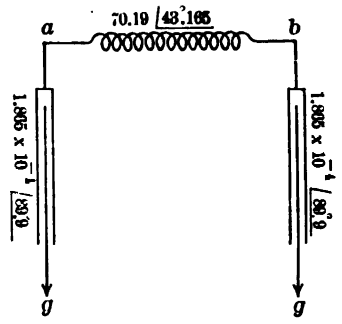

FIG. 5.—Equivalent *Π* of line wire in numerical example

A provisional report from the American Committee, subject to, and until such time as the official report of the Commission shall appear.

TURIN MEETING OF THE INTERNATIONAL ELECTROTECHNICAL COMMISSION SEPTEMBER 7-13, 1911*

Historical Outline

It was voted by the Chamber of Government Delegates at the International Electrical Congress of St. Louis in 1904, that an international electrotechnical commission should be established to carry on the work commenced at that Congress. The commission came into official existence in 1906, owing, in large measure, to the work of Colonel R. E. Crompton, C.B., who has served as its honorary secretary from its inception. The initiation of the organization was thus a sequel to the work of the St. Louis Congress. The first meeting of the commission, for organization, took place at London in 1906, and was attended on behalf of the American Institute of Electrical Engineers by Messrs. F. B. Crocker, C. O. Mailloux, and C. H. Sharp. Lord Kelvin was elected the first President of the Commission, an organization was formed, and statutes adopted M. C. leMaistre was appointed General Secretary, with an office in London, at 28 Victoria St.

A council meeting of the Commission was held in London in 1908, and was attended by Mr. C. O. Mailloux on the part of the United States Committee, appointed by the American Institute of Electrical Engineers. Professor Elihu Thomson was elected President on the demise of M. Mascart. Work was commenced at this meeting. Inasmuch as a principal difficulty in the deliberations of any internationally selected body lies in

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*A provisional report from the U. S. National Committee to the Board of Directors of the A. I. E. E., and subject to such official reports of the meeting as may be issued by the Commission.

diversities of language, two languages—French and English—were chosen by vote, as the official languages of the assembly; so that all reports, transactions and publications of the commission have to be prepared, delivered, and printed in these two languages simultaneously. French was voted for, instead of German, because the French language was more generally known to, and used by, the delegates, than the German. A single language, had it been possible, would have been much easier to deal with officially; because there is necessarily great difficulty in obtaining strictly equivalent renderings of any statement or resolution in two languages simultaneously. So great is this difficulty of bilingual renderings, that any attempt to introduce a third official language, and so to maintain three mutually equivalent renderings of all the proceedings, would probably break down in failure.

The meeting of 1908 saw work commenced on international nomenclature or lists of equivalent electrotechnical terms in one or more pairs of languages. Incidentally, it was decided as a matter of practical necessity, that all the quantitative resolutions of the commission should be stated and published in the international metric system of weights and measures, non-metric countries being allowed to employ their local equivalents of such values in parentheses.

In August, 1910, an unofficial meeting of the commission was held at Brussels, at the invitation of the Belgian committee, and under the acting presidency of Professor Eric Gerard. Messrs. A. E. Kennelly and Charles F. Scott attended this meeting on behalf of the United States Committee, appointed by the American Institute of Electrical Engineers. Active discussion took place, and good progress was made, in several technical directions, namely:

(1) International electrotechnical nomenclature and terminology.

(2) International symbols.

(3) International rating of machines.

The American Committee also reported the vote of the A. I. E. E., taken at the Jefferson convention, June, 1910, to refer to the commission the question of the direction of phase advance in alternating-current vector diagrams, for an international decision. The meeting accepted the proposition, and referred the question to all the national committees for consideration, and report at the next meeting.

It had been proposed to hold the next meeting at Berlin in 1911; but, in view of the proposal to hold an International Electrotechnical Congress at Turin in 1911, in connection with the Turin-Rome Exhibition, the German committee postponed its invitation, and the Italian committee's invitation was then accepted, to hold the next meeting of the Commission at Turin, in connection with the date of the Congress.

During the year 1910-1911, the central office in London circulated the questions raised at the Brussels meeting among the various national committees. The report of the Brussels meeting was published* in the Proceedings of the A. I. E. E., and the American committee reported the actions at Brussels to the Board of Directors of the A. I. E. E.

At the date of the Turin meeting of the Commission, the following countries had organized national committees of the Commission, sharing alike annually in the expenses of the Commission:

(1) Austria
(2) Belgium
(3) Brazil
(4) Canada
(5) Chile
(6) Denmark
(7) France
(8) Germany
(9) Great Britain
(10) Holland
(11) Hungary
(12) India (British)
(13) Italy
(14) Japan
(15) Mexico
(16) Spain
(17) Sweden
(18) Switzerland
(19) United States
(20) Uruguay

In some of these countries, such as British India, the committee is appointed by the government alone. In others, such as the United States, it is appointed by a single dominant electrotechnical Institution. In others, such as France, it is appointed by a number of electrotechnical societies, in coöperation. In all cases, however, the central office of the Commission recognizes and communicates solely with the local committee of each country, to the exclusion of any societies or institutions in that country. In other words, a national committee is organized in each country by the electrotechnical forces resident therein; but once the national committee is formed to represent that country, the central office deals exclusively with the committee, and communicates with that country through no other channel.

The Turin Meeting

The attendance at the Turin meeting by National Committees and their delegates was as follows:

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*Proceedings of the A. I. E. E., December 1910, pages 10-11.

President of the Commission, Dr. Elihu Thomson.

Belgium. MM. Armand Halleux, G. A. L'Hoest, E. Gevaert, O. De Bast (secretary).

British Indian Government. M. F. H. Meares, Government Delegate.

Canada. Professor L. W. Gill.

Denmark. M. S. A. Faber, Professor Absalon Larsen (secretary).

Ecuador. Sr. Richard Muller, Professor of Electrical Engineering at University of Quito.

France. MM. R. V. Picou (President), H. Armagnat, P. Boucherot, E. Brunswick, M. J. Blondin, Ch. David (secretary), Paul Janet, F. Laporte (Adjunct Secretary), R. Legouez, G. Roux.

Germany. Herr Professor Dr. E. Budde (President) President of the Verband Deutscher Elektrotechniker, Herr Georg Dettmar (Secretary), Herr Geh. Ober Postrat Professor Dr. D. K. Strecker, President of the Committee on Units and Symbols of the Verband Deutscher Electrotechniker.

Great Britain. MM. Alexander Siemens (President), British Government Delegate, W. Duddell, F. R. S., Major W. A. J. O'Meara, C.M.G., C.B., British Government Delegate, M. R. K. Gray, Dr. Silvanus P. Thompson F.R.S. British Government Delegate, Professor T. Mather, M. P. F. Rowell (Secretary)

Holland. Professor Clarence Feldmann, President of the Electrical Section of the Dutch Society of Engineers. M. L. M. Barnet Lyon.

Hungary. Professor Dr. Moritz de Hoor-Tempes.

Italy. Professor Luigi Lombardi (President), President of the Associazione Elettrotecnica Italiana, Signor C. Clerici, Professor Guido Grassi, Signor E. Jona, Signor C. Montu, Parliamentary Deputy, Signor G. Semenza (Secretary), Signor P. Verole.

Japan. Professor Dr. A. Oya.

Mexico. Senor. Alfonso Castello.

Panama. The Consul of Panama at Turin.

Portugal. Il Barone Nasi, the Portuguese Consul at Turin.

Spain. Senor Don Luis de la Peña.

Sweden. MM. C. A. Rossander (President), E. C. Ericson (Secretary).

Switzerland. M. le Professor J. Landry (Secretary), K. Tauber.

United States. MM. C. O. Mailloux (President) U. S. Government Delegate, Gano Dunn, President of the American Institute of Electrical Engineers and U. S. Gov. Delegate, Dr. A. E. Kennelly (Secretary), Dr. Clayton H. Sharp.

General Officers of the Commission

Colonel R. E. Crompton, C.B., (Honorary Secretary), M. C. leMaistre, General Secretary, M. E. Litton, Assistant.

Nineteen countries, and eleven languages, were thus represented by 51 delegates, including the Presidents of the American, Dutch, German and Italian Institutions of electrical engineers.

The meetings at Turin were held in the handsome council chamber of the Provincial Palace of the Prefecture.

OFFICIAL FIRST SESSION, SEPTEMBER 7TH

At the first meeting, on September 7, after a reception by President Elihu Thomson, an address of welcome was given by His Excellency Signor Calissano, Italian Minister of Posts and Telegraphs. President Thomson then read an address, in which he complimented the influence of Italian genius upon the philosophy science and art of the world, and offered the congratulations of the Commission to the Italian nation on the fiftieth anniversary of its political union. He called attention to the growth and development of the Commission, the importance of its work, and the desirability of appointing various international sub-committees to carry on the work during the interim periods between successive meetings. This suggestion was very cordially received by the assembly.

After the report of Honorary Secretary Colonel Crompton, indicating the recent progress of the Commission, an election was held for officers of the ensuing period. M. Picou, the President of the French Committee, indicated the importance of Dr. Budde's past services to electrotechnics and to the Commission. He moved that Dr. E. Budde, the President of the German Committee, and of the Verband Deutscher Elektrotechniker, should be elected as President of the Commission, to succeed President Elihu Thomson. This motion was carried unanimously with acclamation. The Honorary Secretary and General Secretary were reappointed also with acclamation. Professor Lombardi was also unanimously elected to preside over the unofficial meetings, on September 8 and 9. After an official photograph of the assembly had been taken, the meeting adjourned.

UNOFFICIAL SECOND SESSION, SEPTEMBER 8

At the unofficial meeting of the 8th, the first matter considered was international nomenclature. A subcommittee of one delegate from each of the countries—Belgium, France, Germany and Great Britain,—had been charged at the Brussels meeting, with the preparation of a report on this subject, and had met, earlier in the year, at Cologne. This sub-committee presented a list of 56 electrotechnical terms in general use, in connection with dynamo-electric machinery, drawn up in English and French, with their respective definitions in those official languages.

It may be said that it has hitherto been the policy of the

American Committee to leave nomenclature in the English language to their confrères in Great Britain, rather than to attempt a separate American nomenclature. In cases, however, where the American usage of a term differed appreciably from the British usage, notice of the fact was sent to the British Committee, which had then proceeded either to find a modification in terms suitable to the engineers of both countries; or to specify the distinction between the respective usages.

After some discussion, the report of the sub-committee was accepted, and provisionally adopted, by the meeting; with an order to print the same in both alphabetical and logical orders. A subcommittee on nomenclature consisting of one delegate from the British, French and German committees, was then voted, to continue the work, and to report at the next meeting of the Commission. The Danish and Spanish committees were especially invited to send delegates to follow the work of this subcommittee. Delegates from any or all national committees were held to be free to attend the meetings of this or other special subcommittees. The central office was, however, to arrange solely with the special subcommittee members for the dates of their meetings, the national committees being then informed of the dates selected.

The French committee submitted printed copies of its " Vocabulaire Electrotechnique " of 323 electrotechnical French terms, and their definitions in French, a task executed with a view to assisting in the work of the I. E. C.

The British committee also submitted printed copies of its latest work on nomenclature " Terms commencing with the letters F to M " being a list of 132 English terms in alphabetical order, with their English definitions, and also with unofficial definitions in French.

International Symbols. The report of the Brussels conference on symbols was then taken up. This report had been printed, and circulated among the various national committees, for nearly a year. The proposals were, after considerable discussion, adopted provisionally in the following form:

1. Instantaneous values of electrical quantities which vary with the time are to be represented by small letters.
2. Virtual or constant values of electrical quantities to be represented by capital letters.
3. Maximum values of periodic electrical quantities to be represented by capital letters followed by the subscript " *m* ".

4. Magnetic quantities, whether constant or variable, to be represented by capital letters of either script, Gothic, heavy-faced, or of any other special type.

5. Maximum values of magnetic quantities to be represented by capital letters of either script, Gothic, heavy-faced, or of any special type, followed by the subscript "m".

6. The following quantities to be represented by the following letters:

Electromotive force	E, e	
Electric quantity	Q, q	
Self-inductance[*]	$\mathcal{L}$, $\mathbf{L}$	[†]
Magnetic force	$\mathcal{H}$, $\mathbf{H}$	
Magnetic flux density	$\mathcal{B}$, $\mathbf{B}$	
Length	L, l	
Mass	M, m	
Time	T, t	

Dr. Budde, in the name of the German committee, and Mr. Alexander Siemens, in the name of the British Committee, proposed the definite adoption of the letters I, E and R to represent, respectively, the current, the electromotive force, and the resistance, in the simple algebraical expression for Ohm's law. This proposition was unanimously adopted.

M. Picou proposed, and Dr. Feldmann seconded, the proposition that in questions relating to alternating currents, the term "reactive power" be adopted to designate the quantity $U I \sin \phi$ where U is the virtual alternating potential difference, I the virtual current, and ϕ the difference of phase between them. This proposition was adopted.

A subcommittee consisting of one member from each of the following countries was appointed, to continue the study of the question of international symbols.

Belgium, France, Germany, Great Britain, Holland, Italy, Spain, Switzerland, United States.

Certain supplementary propositions of the French committee were referred to this subcommittee for consideration, and, at the suggestion of Mr. Feldmann, the question of special terms of a similar nature to "reactive power" was also referred to this subcommittee.

Summing up then the action of the assembly on symbols, the

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[*] Coefficient of self-induction.

[†] See Articles 4 and 5 as to font.

recommendations of the Brussels conference were not only provisionally adopted, with minor amendments; *i.e.* adopted subject to possible revision, in detail, at some future commission meeting, but, owing to the loyal international coöperation of Germany and Great Britain, an international symbolic statement was definitely adopted for Ohm's law.

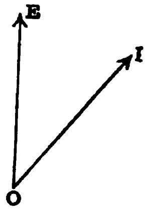
UNOFFICIAL THIRD SESSION, SEPTEMBER 9

Diagrams for Alternating-Current Quantities. The question as to the direction of phase advance in the graphic representation of alternating-current quantities was taken up, as proposed by the Brussels conference, and in pursuance of the action originally taken by the American Institute of Electrical Engineers at its Jefferson Convention in 1910.

After a brief discussion, in which unanimity of opinion was manifested, it was moved by Professor S. P. Thompson, that the direction of phase advance should be in the counter-clockwise direction, as originally taken by Dr. Fleming. The following proposition was then unanimously adopted.

In the graphical representation of alternating electric and magnetic quantities, advance in phase shall be represented in the counter-clockwise direction.

> NOTE.—The impedance of a reactive coil, of resistance R, and inductance L, is $R+\sqrt{-1}\,L\,\omega$, and that of a condenser of capacity C, is $\frac{1}{\sqrt{-1}\,C\,\omega}$, where ω is equal to $2\,\pi\times$frequency.
>
> It follows also that the diagram herewith represents the phase relations in a simple alternating-current circuit containing an impressed electromotive force $O\,E$ and a lagging current $O\,I$.



Summing up, then, the action of the assembly concerning alternating-current diagrams, or so-called vector diagrams, there was complete unanimity of opinion among all of the national committees, after a year of consideration for the subject, that the order of advance in phase in such diagrams should be counter-clockwise. All reference to the methods for deriving such diagrams was carefully avoided. That is to say, the question as to whether a "crank diagram", or a polar "time diagram", should be used, was not entered into, and only the relations of the final vector-diagram were discussed. This leaves to engineers entire freedom to arrive at the internationally standardized vector diagram in any desired manner.

Ratings of Electrical Machinery and Apparatus. The propositions of the Brussels conference in regard to rating were adopted without modification as follows:

1. The output of electric generators is defined as the electric power available at the terminals.
2. The output of electric motors is defined as the mechanical power available at the shaft.
3. Both the electric and mechanical powers to be expressed in international watts.

A subcommittee consisting of one member from each of the following national committees was appointed to study the subject of the international rating of electrical machinery and apparatus:

Belgium, France, Germany, Great Britain, Italy, Sweden, Switzerland, United States.

Professor Elihu Thomson drew attention to a printed document " Extract from the Rules of various countries in reference to the rating of electrical machinery," published by the central office. This work, the value of which he was glad to recognize, would be likely to prove of great utility to the subcommittee.

In the name of the Italian committee, Mr. Jona presented a report dealing with this subject and especially with the question of prime movers, when closely related to electrical machinery.

The report was referred to the special committee, with instructions to give most careful consideration to the proposals of the Italian committee.

The national committees were requested to put themselves into communication with the technical societies of their respective countries, in order to facilitate the work of the Commission.

Summing up then the actions of the assembly in regard to international rating, although but little has yet been accomplished in this direction; yet the special subcommittee is now in a fair way to accomplish much, that has hitherto been impossible owing to differences of language, of national customs, of construction, and of viewpoint.

FUTURE MEETINGS OF THE I. E. C.

The proposition of Signor Lombardi and Mr. Alexander Siemens, that the next official meeting of the I. E. C. be held at Berlin, in 1913, was adopted, the exact date to be fixed by the central office, after consultation with the national committees. An unofficial meeting may be arranged in the meantime, if necessary.

Mr. Gano Dunn, as President of the A. I. E. E., cordially invited the I. E. C. to hold an official meeting at San Francisco, in 1915, on the occasion of the Panama Pacific Exposition, to be held in celebration of the opening of the Panama Canal. He announced that the A. I. E. E. was desirous of holding, at the same time, an International Electrical Congress, that the Board of Directors of the A. I. E. E. had already taken official action, had passed resolutions authorizing the Congress, and had instructed him to appoint a Committee on Organization, provided that an expression of opinion favorable to the holding of such a Congress were obtained from the I. E. C. at its meeting in Turin.

The meeting thanked Mr. Gano Dunn for the very cordial invitation of the A. I. E. E., and on the proposition of Mr. Feldmann, seconded by Mr. Duddell, adopted the following resolution:

" The I. E. C. expresses its willingness to hold an official meeting at San Francisco in 1915, and instructs the Central Office, on the request of the A. I. E. E., to coöperate with it in the organization of the International Electrical Congress in San Francisco at the same time."

As will be noted later, in relation to the Council Meeting of September 13, the I. E. C. undertook, at the request of the International Electrical Congress of Turin, the task of assigning and appointing future International Electrotechnical Congresses, in so far as concerns their times and places of meeting, so that the invitation of the A. I. E. E., as extended through Mr. Gano Dunn of the American Committee, was accepted in its entirety, both as to the holding of a meeting of the I. E. C., and of an International Electrotechnical Congress, at San Francisco, in 1915.

ILLUMINATING ENGINEERING AND TECHNOLOGY

The Honorary Secretary of the Illuminating Engineering Society of London, Mr. Leon Gaster, personally invited to attend the meeting of the Commission, raised the question of the Commission studying the terms employed in matters of illumination, and requested that the national committees put themselves in communication with the Societies dealing with these questions in their respective countries.

Dr. Kennelly, speaking as President of the Illuminating Engineering Society of the United States, and Dr. Clayton H. Sharp, as a Past President, favored this idea, and the meeting expressed an opinion favorable to the suggestion.

OFFICIAL FOURTH SESSION, PLENARY MEETING, SEPTEMBER 11

At the plenary meeting of September 11, the various resolutions of the unofficial meetings on the 8th and 9th, were presented in writing, were read over in both of the official languages, and were formally voted without dissent. Many of these resolutions have been quoted at large, in this report.

COUNCIL MEETING OF THE I. E. C., SEPTEMBER 13

The council of the I. E. C. is a species of executive committee, to which, by statute, the affairs of the Commission are entrusted, when no plenary meeting is in session. It consists of the President of the Commission, the Honorary Secretary, the Presidents of the various National Committees, and one additional delegate from each national committee. At this meeting, the communication was received from the International Electrotechnical Congress of Turin, to the effect that in view of the authority, permanence, and international organization of the I. E. C., the latter body was especially qualified to determine the relations between succeeding international electrotechnical congresses. The said Commission was therefore requested to undertake the task of organizing such congresses in future, in so far as relates to their times and places of convention, the details of organization being referred in each case to the particular committee and country in which the congress is to be held.

The council expressed its thanks to the congress, and undertook to comply with the request.

The meeting then adjourned, after passing votes of thanks to the officers of the Commission, and to their hosts, Professor Lombardi and the Italian Committee.

SOCIAL FEATURES OF THE COMMISSION MEETING

Although the sessions of the I. E. C. occupied the working hours of the days on which they occurred; yet the official and technical duties of the delegates were delightfully relieved by the cordial and assiduous attentions of their Italian hosts, who spared no pains to make the visit to Turin memorable for social pleasure as well as for electrotechnical accomplishment. The Italian committee held a banquet in honor of the delegates on

September 9, at which mutual toasts and congratulations were offered on the success of the meetings. At this banquet, a souvenir of the convention, in the form of a Japanese work of Art, was presented to the retiring President Professor Elihu Thomson, by a number of the delegates, who desired to testify their appreciation of the valuable services he had rendered to the commission during his term of office. Colonel Crompton acted as Master of the Ceremonies during this presentation.

Over and above the official resolutions adopted by the Commission, the Turin meeting brought about very successful results in active and cordial coöperation, which are bound to manifest themselves in the actions of the standing subcommittees. The real requisite in any international undertaking is cordial coöperation, without which all endeavors may be nugatory, but with which success becomes assured. To the labors of the permanent staff of the Commission, its Honorary and General Secretaries, this spirit of cordiality is largely due, aided by President Thomson, and the Italian Committee. President Mailloux of the American Committee, moreover, gave most valuable service to this, as to past meetings, of the I. E. C., by his very unusual linguistic abilities. Although, theoretically, any delegate can share in the work of the meetings, who can speak either English or French; yet, practically, no delegate can take an active part in the proceedings, who is not familiar with both these languages. Mr. Mailloux being thoroughly well acquainted with the German, Spanish, and Italian languages, besides English and French, has on many occasions been able to bring about unity of thought and action among delegates of different countries, which might otherwise have been impossible.

In conclusion, electrical engineers all over the world, thanks to the work of the I. E. C., now possess an internationally ratified symbolic expression of Ohm's law, a list of other international symbols, a good start toward international rating of machinery, a good nucleus of official international electrotechnical nomenclature, and an international decision on the long debated question of phase rotation in alternating-current vector diagrams. Moreover, largely owing to the personal incentive of President Dunn of the American Institute of Electrical Engineers, the way has been officially paved for holding a commission meeting and a congress at San Francisco in 1915.

A. E. Kennelly,
Secretary of U. S. National Committee

Report to the American Institute of Electrical Engineers from its Delegation to the Congress, and subject to the official report of the Congress.

THE INTERNATIONAL ELECTROTECHNICAL CONGRESS OF TURIN*

HISTORICAL OUTLINE

Information was laid before the American Institute of Electrical Engineers in 1910, that an International Congress of the Applications of Electricity would be held, at Turin, from the 10th to the 17th of September, 1911, under the auspices of the Associazione Electrotecnica Italiana (A. E. I.) and of the Italian Electrotechnical Committee of the I. E. C., in connection with the Turin-Rome International Industrial Exhibition.

International Electrical Congresses since the year 1900, inclusive, have been held as follows:

In 1900, at Paris, in connection with the Paris Exposition Universelle.

In 1904, at St. Louis, in connection with the World's Fair of St. Louis.

In 1908, at Marseilles, in connection with the Marseilles Exhibition.

In connection with the Congresses of 1900 and 1904, there had been appointed a chamber of Government Delegates, charged with the work of international electrotechnical agreement, in addition to the Congress at large, in which papers were read and discussed. In pursuance of a vote taken in the chamber of Delegates at the St. Louis Congress of 1904, the duties of such Chambers were relegated to a special international commission. This International Electrotechnical Commission—the I. E. C.—was formed with a permanent organization in 1906, and no

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*A report to the Board of Directors of the A. I. E. E. from its Delegation to the Turin Congress, and subject to such official reports as the Congress may publish.

chamber of government delegates was formed at the Marseilles Congress.

In connection with the Turin Congress, it was arranged that a meeting of the I. E. C. should be held at Turin, immediately before (7th–12th Sept.) the convention of the Congress.

ORGANIZATION OF THE TURIN CONGRESS

The Turin Congress was organized under (1) A Committee of Honor. (2) An Organizing Committee. (3) An Executive Committee nominated by the General Council of the A. E. I.

(1) The Committee of Honor was headed by H. R. H. the Duke Degli Abruzzi, and included 28 members, among whom were Professor Elihu Thomson, President of the I. E. C., Colonel Crompton, Honorary Secretary of the I. E. C., and Signor Antonio Pacinotti, Honorary President of the A. E. I.

(2) The Committee of Organization comprised 30 members as follows:

President Professor Luigi Lombardi, President of the A. E. I.
Vice-President Professor Guido Grassi, President of the Turin Section of the A. E. I.
Secretaries, Signori C. A. Curti, and G. Semenza.

Members, Messrs. M. Ascoli, E. Jona, Q. M. Calatabiano, L. Amaduzzi, F. Fusco, G. Cesare, A. Vivarelli, A. Panzarasa, M. Pizzuti, G. Amati, S. Pagliani, C. Esterle, L. Pontiggia, L. A. Herdt, C. A. Rossander, A. Siemens, R. V. Picou, M. G. A. Hagemann, E. Budde, E. Gerard, O. T. Blathy, J. Alonso y Millan, F. Drexler, C. Feldmann, K. P. Tauber, C. Zipernowsky, H. Armagnat.

A sub-committee for the United States was formed as follows: President, J. W. Lieb, Jr.; Members, D. C. Jackson, A. E. Kennelly, C. O. Mailloux, T. C. Martin, H. G. Stott, S. W. Stratton.

(3) The Executive Committee comprised 23 members as follow:

President Professor Guido Grassi.
Secretaries Signori G. Lignana, and F. Nizza.
Treasurer, A. Luino.
Members, Messrs. E. Morelli, V. Tedeschi, C. Montu, O. Trossarelli, E. Soleri, P. Forster, A. Rostain, G. G. Ponti, R. Arnò, V. Treves, A. Miolati, L. Ferraris, T. Chiesa, P. Prat, G. Schultz, G. Bisazza, E. Lauchard, E. De-Benedetti, R. Pinna.

The technical work of the Congress was divided among eight sections. Thirty-one specific official topics were selected, and announced in advance, by the Committee of Organization, and papers or reports on these topics were secured, in advance, from as many authors in 10 countries: *viz.*, Austria, Belgium, Denmark, France, Germany, Great Britain, Italy, Sweden, Switzerland and the United States.

In addition to the officially prepared papers or reports on specified topics, about fifty independently offered papers on various subjects were accepted by the Organizing Committee.

The subscription for membership in the Congress was fixed at 25 Lire ($5), entitling the holder to all the publications and privileges of the organization. A reduced subscription of 10 Lire ($2), entitled a holder to attend the technical meetings, as a listener, but to the exclusion of all publications or privileges.

The technical meetings of the Congress were of two kinds—plenary and sectional. There were three plenary meetings, one of opening, the second, intermediate, and the third of closing. The sectional meetings were for the reading and discussion of papers. They took place between the first and third plenary meetings.

Four official languages were selected for the reception of papers and reports: namely, French, Italian, English and German. All papers in Italian, English, or German, were required to be accompanied by a summary in French. The official bulletins of the Congress were issued daily in Italian and French. Votes and resolutions in plenary meetings were presented in both Italian and French. When a single language was employed as a vehicle of discussion, it was most frequently the French; although discussions were admitted in any of the four official languages. All the technical meetings of the Congress were held at its headquarters—the Reale Politecnico, via Ospedale, Turin—a large technical college granting degrees, following five-year courses of instruction in civil engineering, mechanical engineering, (including electrical engineering), chemical engineering, and architecture.

The greater number of the reports and papers communicated to the Congress were printed, and circulated in advance of each day's sessions.

About 450 full-membership adhesions were published before the close of the Congress, and the total registration of members in both classes amounted to about 650.

The following is a list of the foreign government delegates and of the delegates from technical institutions.

REPRESENTATIVES OF FOREIGN GOVERNMENTS

Austria. Messrs. Alfredo Graf, Alfredo Deinlein.

Belgium. Messrs. Ernest Gerard, D. L'Hoest, l'Ing. Colson, Eugene Gevaert.

Denmark. Ing. Joh. Rasch.

France. Messrs. M. Barbillion, Swyngedauw.

Japan. Mr. Atsushi Oya.

British India. J. W. Meares.

Great Britain. Major W. A. J. O'Meara, Dr. R. T. Glazebrook, A. Siemens, Dr. S. P. Thompson.

Luxemburg. Mr. Charles Eydt.

Mexico. Ing. Alfonso Castello.

Portugal. Baron Nasi di Cossombrato.

Ecuador. Messrs. Richard Muller, Alfredo Gerard.

Russia. Messrs. Pierre Ossadtchy, Colonel de Mouromtzew.

Spain. Messrs. Alfredo Lasala, Jose Abbad Boned, Luis De La Pena, Manuel de Justo y Sanches Blanco.

United States. Messrs. Gano Dunn, C. O. Mailloux, H. B. Brooks.

Switzerland. Messrs. Gustavo Sulzberger, Christian Brunnschweiler, E. Vanoni.

Hungary. Frederic Koromzay.

DELEGATES OF TECHNICAL SOCIETIES

Austria.

Elektrotechnischer Verein, Vienna: Ing. Alfredo Grünhut.

Osterreichischer Ingenieur und Architekten Verein di Vienna: Ing. Alfredo Deinlein.

Belgium.

Association des Ingenieurs Electriciens sortis de l'Institut Montefiore de Liege: Omer de Bast.

Denmark.

Den Tekniske Ferening, Copenhagen: P. O. Pedersen.

Dansk Ingeniørferening, Copenhagen: P. O. Pedersen, Ing. Valdemar Poulsen.

France.

Société Internationale des Electriciens de Paris: Boucherot, Armagnat, Brunswick, Janet, Legouez, Blondin.

Syndicat Professionnel des Usines d'Electricité de Paris: Eschwège, Legouez, Brylinski, Berthelot.

Société des Ingénieurs Civils de Paris: Paul Lecler.

Société des Agriculteurs de France: Paul Lecler.

Germany.

Verband Deutscher Electrotechniker: Prof. Dr. E. Budde, G. Dettmar.

Great Britain.

The Institution of Electrical Engineers: S. Z. De Ferranti, Colonel R. E. Crompton, Dr. R. T. Grazebrook, R. Haye Gray, A. Siemens. S. P. Thompson, W. Duddell, W. A. J. O'Meara, R. Hammond, Æ. H. Patchell.

The Illuminating Engineering Society:Prof. S. P. Thompson, Leon Gaster, R. J. Wallis-Jones, Justuk Eck.

The Institution of Mechanical Engineers: W. H. Patchell.

Electrical Section, London Chamber of Commerce: Leon Gaster, Charles P. Sparks, A. Bruce Anderson.

Italy.

Accademia delle Scienze Fisiche e Matematiche di Napoli: Prof. Guido Grassi.

Collegio degli Ingegneri ed Architetti di Milano: Ing. Denti Eugenio.

Collegio Ingegneri ed Architetti di Napoli: Ing. G. D. Cangia.

Associazione Amichevole fra gli Ingegneri ex-Allievi della Scuola di Torino: Ing. Comm. Oreste Lattes.

Reale Istituto Lombardo di Scienze e Lettere di Milano: Prof. Riccardo Arnò, Prof. Francesco Grassi.

Societa Ingegneri ed Architetti Italiani di Roma: Ing. Prof. Giuseppe Revessi.

Collegio Veneto degli Ingegneri di Venezia: Ing. Cav. Filippo Danioni.

Collegio Nazionale degli Ingegneri Ferroviari Italiani di Roma: Comm. Lattes Oreste.

Collegio Ingegneri ed Architetti Pugliesi di Bari: Ing. Nicola Stea.

Associazione Elettrotecnica Italiana:

Sezione di Roma: Prof. Comm. Guglielmo Mengarini, Ing. Ulisse Del Buono.

Sezione di Genova: Cav. Vittorio Capellini.

Holland.

Institut van Ingenieurs d'Olanda: L. M. Barnet-Lyon Clarence Feldmann.

Argentine Republic.

Sociedad Cientifica Argentina di Buenos Ayres: Dr. Ing. Angel Gallardo, Ing. Domingo Selva.

Russia. Sezione elettrotecnica della Societa Imperiale Russa di Pietroburgo: Prof. M. de Chatelain.

Istituto elettrotecnico di Pietroburgo: Prof. Graftio.

Spain.

Sociedad Espanola de Fisica y Quimica di Madrid: A. Gabasso, Luis de la Pena, Enrique Hauser.

Instituto de Ingenieros Civiles di Madrid: Ing. Luis de la Pena, Ing. Don Antonio Gonzales Echorte.

Switzerland.

Physikalische Gesellschaft Zurich: Dr. H. Behn-Eschenburg, Dr. Ing. S. Guggenheim.

United States.

American Institute of Electrical Engineers: Prof. E.Thomson, C. O. Mailloux, Dr. A. E. Kennelly, Gano Dunn, Prof. C. A. Adams, H. B. Brooks, E. P. Burch, G. Faccioli, Etienne de Fodor, R. O. Heinrich, Francis Jehl, Hirayoshi Oshima, C. H. Sharp, J. F. Stevens, Philip Torchio.
Association of Edison Illuminating Companies: Philip Torchio.
American Electrochemical Society: C. O. Mailloux.
Franklin Institute: G. Faccioli, A. E. Kennelly.

PLENARY MEETINGS

(1) The opening plenary meeting took place at 10:30 a.m. on Sunday, September 10. An address of welcome was read by Senator P. Boselli, President of the Royal Academy of Sciences at Turin, in the presence of a number of official delegates. This was followed by an address by Professor Luigi Lombardi, who, in welcoming the members, set forth the hopes and purposes of the Congress, as well as the arrangements made for showing the most interesting electrotechnical plants, exhibits, and features of importance to visitors. A fine address was then delivered by the Minister of Posts and Telegraphs Signor Calissano, who was elected honorary president of the Congress. The following elections of officers were then made:

President—Prof. Luigi Lombardi, President of the A. E. I.

Vice-Presidents—Prof. Guido Grassi.
Ing. Emanuele Jona.

General Secretary—Ing. Guido Semenza.

Honorary Vice-Presidents—Antonio Pacinotti for Italy.
Silvanus P. Thompson and Alexander Siemens for England.
Gano Dunn for the United States.
Paul Janet for France.
Karl Strecker for Germany.
Alfred Graf for Austro-Hungary.
Pierre Ossactchy for Russia.
Gustave L'Hoest for Belgium.
Behn-Eschenburg for Switzerland.
De La Pena for Spain.
Poulsen for Denmark.

The following Section officers were then elected, after which the plenary meeting adjourned.

Section	Subject	President	Vice-Presidents	Secretary
I.	Electrical Machines and Transformers.	Boucherot (France)	Morelli (Italy) Feldmann (Holland)	C. Palestrino
II.	Electrical Installations and Networks	De Bast (Belgium)	Ferraris (Italy) Landry (Switzerland)	Del Buono Boccardo
III.	Instruments, Apparatus and Switching Devices.	Kennelly (U. S. A.)	Dina (Italy) Armagnat (France)	Emmanueli Barbagelata
IV.	Lighting and Heating by Electricity.	Rossander (Sweden)	Mengarini (Italy) Sharp (U. S. A.)	Danioni
V.	Electrical Traction and Propulsion.	Mailloux (U. S. A.)	Sartori (Italy) Barnet Lyon (Holland)	Fenzi Ponti
VI.	Telegraphy and Telephony.	O'Meara (Grt. Britain)	Larsen (Denmark) Di Pirro (Italy)	Bellini
VII.	Accumulators, Electro-Chemistry, Electro-metallurgy and other applications.	Beckmann (Germany)	Miolati (Italy) Duddell (Grt. Britain) Rumi (Italy)	
VIII.	Tariffs, Taxation and Legislation in regard to the distribution of electrical energy.	Arno (Italy)	Dettmar (Germany) Bonghi (Italy)	Botto Guiletti

(2) *The Intermediate Plenary Meeting of September* 13. After some discussion by President Lombardi on the subject of the organization of future electrotechnical congresses, the following resolution was put by M. Clarence Feldmann (Holland), and carried unanimously, after being seconded by Messrs. Bouchèrot (France), S. P. Thompson (Great Britain), Mailloux (U. S. A.), L'Hoest (Belgium), Strecker (Germany), De Chatelain (Russia), and Grassi (Italy).

Whereas, the provisional committee appointed by the International Electrotechnical Congress of Marseilles for the purpose of forming a permanent international committee of organization for electrotechnical congresses has made no report.

and *Whereas*, The International Electrotechnical Commission, permanently organized since 1906, is by its position, authority and statutes especially adapted to be the permanent organization for securing the connecting link between international electrotechnical congresses and to give effect to their labors.

The International Electrotechnical Commission is requested to accept the task of organizing future electrotechnical Congresses so far as concerns their dates, places of meeting, and objects, the details of the organization of each congress being confided to the electrotechnical committee of the country in which the congress is to be held, with the assistance of its technical societies, if necessary.

The meeting then adjourned.

The request was accepted by the Council of the I. E. C. on the 13th.

(3.) *The Closing Plenary Meeting of Saturday, September* 16. The following resolutions were presented to the plenary meeting after having been published in the official bulletin. All were adopted.

(1) From Section III.

The International Electrotechnical Congress of Turin compliments the American Institute of Electrical Engineers on the practice it has adopted of inserting in its publications the metric equivalent value, in parentheses, after each expression of values in English measures.

And since this procedure greatly facilitates the reading of these publications in all the countries using the metric system, while constituting a worthy example towards and in view of the much desired complete international unification of weights and measures.

Resolved, that the technical societies of all countries in which the metric system is not yet official, are invited to follow the above-mentioned example of the American Institute of Electrical Engineers.

This vote was carried with much acclamation.

(2) From Section IV.

The Congress favors the appointment of an International Commission for the general study of systems of illumination, and of all technical problems connected with illumination, proposing that the Illuminating Engineering Society of London be charged with the formation of this International Commission, placing itself for that purpose in communication with all other existing national and international photometric committees.

(3) From Section V.

The committee of the fifth section charged with the duty of examining the proposition of Mr. Mailloux concerning the definition and industrial determination of train acceleration reports as follows:

Whereas, in industrial traction, the speed of trains is always measured in kilometers per hour, (or in local equivalent values), and

Whereas, it is logical to start with such a definition of train velocity, in order to express the acceleration per second, and

Whereas, such definitions of train acceleration are invariably used in America, and are also largely used in other countries. *and*,

Whereas, a similar proposal has already been recommended favorably by the International Electrotechnical Congress of Marseilles (1908).

Resolved by this Congress that:

(1) Train accelerations be expressed in kilometers per hour per second.

(2) That the International Electrotechnical Commission be informed of this resolution.

(4) From Section VIII.

Two resolutions relating to political and fiscal questions of European rather than American interest.

(5) From certain individual members:

That the next International Electrotechnical Congress should form a section devoted to the subject of electrotechnical instruction.

Closing reports and addresses were then made by the President, the Section-Presidents and various government delegates, who expressed the general thanks of the assembly to the officers of the Congress, the President and Secretary, and the Italian committee for their attentions and courtesies during the exercises of the Congress.

The Congress than finally adjourned.

MEETINGS OF THE SECTIONS

A list of the reports and papers presented as to the different sections is appended herewith. The contributions numbered 80 in all, from writers in 14 different countries, 11 contributions being presented from America.

Meetings in Honor of Senator Antonio Pacinotti, and of Professor Galileo Ferraris. A meeting of the A. E. I., attended by a large number of members of the Congress, was held on September 13, to celebrate the semi-centennial anniversary of his invention of the Pacinotti-ring dynamo-electric machine, the original model of which was displayed in the electrical department of the Turin Exhibition. The President of the A. E. I., Professor Lombardi, with a very appropriate address, delivered to Signor Pacinotti a handsomely framed and illumined parchment, on which was reproduced, in miniature, the classic paper published by him on the dynamo-electric machine in " Il Nuovo Cimento " of 1865. An illuminated album accompanied the gift, containing the testimonial signatures of most of the members of the A. E. I., as well as of many Congress members.

Professors Mengarini and Silvanus P. Thomson expressed the admiration and respect of the electrotechnical world, both in Italy and abroad, for the work of the illustrious Italian scientist.

A brief and modest reply was made by Signor Pacinotti, which was enthusiastically received, after which the meeting adjourned.

At another hour, an unofficial gathering took place in the large public square, called the " Piazza Castello," at the foot of the monument erected to Galileo Ferraris, the well-known Turin scientist, teacher and electrician, who discovered and first applied the principal of the rotating magnetic field. Three wreaths were laid on the pedestal, one offered by the French delegation, another by the American delegation, and the third by the city of Frankfort on Maine.

The Mayor of Turin made an eloquent speech in tribute to the work of Ferraris, and responses were made by M. Ferdinand Meyer on behalf of the French electricians, Mr. Philip Torchio, on behalf of American electricians, and Mr. Hartmann of Hartmann & Brown, on behalf of the city of Frankfort.

Technical Visits. On Thursday September 14, an all-day trip was taken from Turin to Genoa and back, inspecting en route the " Giovi " lines and system. This is a three-phase electric railway, running from Busalla through Pontedecimo to Compasso. The locomotives are of European " Westinghouse " construction, supplied through overhead trolley lines with alternating currents at 3,000 volts pressure, at a frequency of 15 cycles per second. The locomotives are regenerative, and return power to the system when descending grades of 0.7 per cent or more.

Another visit was arranged for September 17 to Mont Cenis, where the Società delle Forze Idrauliche del Moncenisio has hydraulic plants that transmit three-phase power, at 30 kilovolts, to Turin and beyond, at a frequency of 50 cycles per second.

Entertainments. The social features of the Congress were prepared and carried out with great care by the Italian committee, each day being provided with some special social event. There was also a special committee of Italian ladies to meet the ladies of the guests, and arrange for their entertainment. The prevailing pleasant weather aided the entertainment committees on all occasions.

On the 10th, a dinner was given to the official delegations at the Superga, on the top of a hill 420 meters in height, about 10 kilometers north of Turin, with a rare and magnificent view of the Piedmontese Alps. On the 11th, an evening reception was given to the Congress by the Turin Municipality at the Circolo degli Artisti. On the 12th, a dinner was given by the City of Turin to the official delegates of the Congress, and another dinner, at the Exposition was given to the Congress members on the 13th. A lunch was also given by the Italian Committee, to the American delegation, which will be memorable to all who attended it, for the charm of the occasion and the sentiments it elicited.

There can be no doubt that not only the Congress guests as a whole; but also the American guests in particular, are under many obligations to their Italian hosts at Turin for attentions and courtesies during their very pleasant visit. The hope was expressed by President Dunn of the A. I. E. E., that many of the members of the Italian Society would be able to attend the Congress of 1915 at San Francisco, under the auspices of the American Institute. Mr. C. O. Mailloux, as the President of the American Delegation had frequent occasion to use his well known linguistic abilities, not only at the Congress sessions; but also at the social reunions, where his speeches in French and Italian were much applauded.

CONCLUSIONS

The permanent results of the Turin Congress may be summarized as follows:

(1) The record of some 80 reports and papers with discussions on the same, destined to be published in due season by the Congress, and distributed among the members.

(2) The resolutions of the plenary sessions of the Congress which bring its conclusions to a focus. As already detailed these particularly contain:

- (*a*) A provision for the regular convocation of future international electrotechnical congresses, by assigning them to the jurisdiction of the I. E. C.
- (*b*) The adoption of the kilometer-per-hour-per-second as the industrial unit of train acceleration.

(c) A formal vote of thanks to the A. I. E. E. for its help to foreign engineers, in printing parenthetical metric equivalents.

(d) That an international illuminating-engineering commission should be formed.

(3) Over and above the records of the Congress, there remains with all the members who attended it, the wealth of memories and experiences derived from meeting confrères of other countries, and from a sympathetic insight into the solutions of the problems which confront them.

A. E. Kennelly,
Secretary American Delegation.

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Dr. H. Behn-Eschenburg. Electrical and mechanical characteristics of modern generators.

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Dr. S. P. Thompson. Converters, rectifiers and motor-generators.

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Ing: C. Sarli. Variable speed three-phase motors, with special reference to rolling mills.

Papers.

Dr. Hallo. Motor-converters.

Boucherot. Electromagnetic phenomena resulting from the sudden short-circuit of an alternator.

R. Legouez. Commutator motors.

Section II

Installations, Central Stations, Switchboards, Conducting Lines

Reports.

Philip Torchio. The selection of voltage and the construction of switchboards and of substations in large electric systems.

J. Grosselin. High-tension underground networks in metallic connection with overhead lines.

E. Ragonot. The construction and use of automatic circuit breakers.

G. Semenza. The synchronous operation of several central stations feeding the same network.

Papers.

S. Q. Hayes. Commercial Electrical apparatus for 100,000 volt service.

Ing: E. Soleri. The extreme limits of high-tension employed in cables.

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*The titles are here given in English when presented in another language.

Dr. Ing. Leon Lichtenstein. Recent experiments and tests on high-tension cables.

Dr. Osuke Asano. Progress in electrical installations in Japan.

Etienne de Fodor. The present state of the problem of sewage disposal in connection with electric stations.

J. Routin. Automatic regulation.

Dr. Ing. W. Weicker. Protection against over voltage on aerial lines, especially with suspension insulators.

E. Hubet Stockar. Aluminum for electrical conductors.

Paul Lecler. Electrotechnical applications to reinforced concrete.

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INSTRUMENTS AND METHODS OF MEASUREMENT, PROTECTION OF INSTALLATIONS, MISCELLANEOUS

Reports.

G. Faccioli. High-tension switching phenomena.

Dr. C. H. Sharp. Electricity Meters, with special reference to different kinds of loads.

A. Durand. Electricity meters.

Papers.

Ing: Alberto Dina. The measurement of insulation-resistance in an alternating-current system during its operation.

Prof. Riccardo Arnò. Watt-volt ampere meters.

Ingg. A. Barbagelata and L. Emanueli. Zero methods of alternating-current measurement.

Prof. G. Revessi. A contribution to the study of stray currents.

A. E. Kennelly. The rotating electric current field.

Vladimir Karapetoff. Some practical calculations of electrostatic fields.

Jules Neher. Multiple-rate electricity meter.

C. O. Mailloux. Method of determination of the constant current producing the same heating effect as a variable current.

Dr. C. P. Steinmetz. The nature of electric transient phenomena.

Prof. E. E. F. Creighton. Protection of electric systems.

Ing. Alberto Dina. On methods of preventing internal over voltages.

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Ing. G. Martinez. Some new industrial alternating-current instruments.

J. L. Farny. On a new oscillographic process with the Braun tube.

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G. Neuhaus. Protective relays.

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C. A. Rossander. The present and future of electric heating.

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Ing. C. Herrgott. On the heating of textiles by electricity.

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Gustave L'Hoest. Overhead line construction on electric railroads.

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Dr. W. Kummer. On the development of single phase locomotives.

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C. O. Mailloux. The industrial definition and measurement of train-acceleration.

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Dr. Giovanni di Pirro. Long distance telephony.

Dr. Valdemar Poulsen. Wireless telephony.

H. Milon. Automatic and semi-automatic telephonic systems.

Prof. P. O. Pedersen. Secrecy in radiotelegraphic communications.

Major W. A. J. O'Meara. The different systems of multiple telegraphy.

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Prof. A. Majorana Calatabiano. Wireless telephone research.

J. Erksine Murray. A definition of the efficiency of a wireless telegraph system.

Ing. Ettore Bellini. The Hertzian azimuth compass.

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STORAGE-CELLS, ELECTROCHEMISTRY, ELECTROMETALLURGY AND OTHER APPLICATIONS

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Dr. H. Beckmann. The present state of the electric storage cell.

Ing. Remo Catani. The direct production of steel from its ores in the electric furnace.

Dr. Erlwein. The sterilization of water by electrical processes.

Paul Lecler. The distribution of electrical energy in farming.

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Dr. Sigm. Guggenheim. The electric results furnished by the chief induction furnaces of the steel industry.

Ing. Luigi De Andreis. The sterilization of water in the Rovigo aqueduct.

J. A. Montpellier. The technique of the electric storage-cell especially with reference to the iron-nickel cell.

Prof. Oscar Scarpa. On the sterilization of water by ultraviolet rays.

Dr. M. Recklinghausen. The sterilization of city water supplies by ultraviolet rays.

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Ing. Mario Bonghi. Comparative study of direct and indirect taxation of electricity in different countries.

Dr. H. Schreiber. Government regulations on the electric transmission of power.

E. C. Ericson. Government regulations on the electric transmission of power.

L. M. Barnet-Lyon. Government regulations on the electric transmission of power.

Papers.

Prof. Riccardo Arnò. A solution of the problem of a rational charge for electric energy.

Louis Prat. Mechanical problems in central stations.

STANDARDIZATION RULES

OF THE

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

HISTORY OF THE STANDARDIZATION RULES.

The first step taken by the Institute toward the standardization of electrical apparatus and methods was a topical discussion on " The Standardization of Generators, Motors and Transformers," which took place simultaneously in New York and Chicago on the evening of January 26, 1898. The discussion appears in the Institute Transactions, Vol. XV, pages 3 to 32. The opinions expressed were generally favorable to the scheme of standardization of electrical apparatus, although some members feared that difficulties might arise. As a result of this discussion, a Committee on Standardization was appointed by the Council of the Institute, consisting of the following members:

Francis B. Crocker, *Chairman.*

Cary T. Hutchinson
Arthur E. Kennelly
John W. Lieb, Jr.
Charles P. Steinmetz
Lewis B. Stillwell
Elihu Thomson

After a careful consideration of the matter and consultation with the members of the Institute and interested parties generally, a " Report of the Committee on Standardization," was presented and accepted by the Institute, June 26, 1899. Those original rules appeared in the Institute Transactions, Vol. XVI, pages 255 and 268.

As a result of changes and developments in the electric art, it was subsequently found necessary to revise the original report, this work being carried out by the following Committe on Standardization:

Francis B. Crocker, *Chairman.*

Arthur E. Kennelly
John W. Lieb, Jr.
C. O. Mailloux
Charles P. Steinmetz
Lewis B. Stillwell
Elihu Thomson

This revised report was adopted at the 19th Annual Convention at Great Barrington, Mass., on June 20, 1902, and appears in the Institute Transactions, Vol. XIX, pages 1075 to 1092.

In consequence of still further change and development in electrical apparatus and methods, it was decided in September, 1905 that a second revision was needed, and the following Committee was appointed to do this work.

Francis B. Crocker, *Chairman.*
Arthur E. Kennelly, *Secretary.*

Henry S. Carhart
John W. Lieb, Jr.
C. O. Mailloux
Robert B. Owens
Charles F. Scott
Charles P. Steinmetz
Henry G. Stott
S. W. Stratton

This Committee held monthly meetings and carried on extensive correspondence with manufacturers, consulting and operating engineers and other interested parties, and as a result, presented its report at the 23d Annual Convention, held at Milwaukee, May 28–30, 1906. After considerable discussion the report was accepted and referred back to the Committee for amendment and rearrangement in form. It was then to be submitted to the Board of Directors for final adoption. In September, 1906, the following Standardization Committee was appointed:

Francis B. Crocker, *Chairman.*
Arthur E. Kennelly, *Secretary.*
A. W. Berresford
Dugald C. Jackson
C. O. Mailloux
Robert B. Owens
Charles F. Scott
Charles P. Steinmetz
Henry G. Stott
S. W. Stratton
Elihu Thomson

This Committee held monthly meetings, also sub-committee meetings, and carefully referred the rules as a whole, and each part of them to the members of the Institute. The rules were also entirely rearranged as to form, and put in shape to facilitate ready reference to them and enable future revisions to be made without breaking up the logical arrangement. Thus amended the rules were submitted to the Board of Directors and approved by it on June 21, 1907. The Board also directed that the rules should be presented, as accepted by the Board, at the Annual Convention held at Niagara Falls, June 24 to 27, 1907, which action was taken by President Sheldon on June 26, 1907. By the Constitution which went into effect on June 10, 1907, this Committee has been made a standing Committee with the title " Standards Committee," consisting of nine members.

On August 12, 1910 the Board of Directors increased the size of the committee from nine to twelve members; on October 14 from twelve to fourteen, and on March 10, 1911 from fourteen to sixteen. The committee thus constituted is given below.

Comfort A. Adams, *Chairman.*
Arthur E. Kennelly, *Secretary.*
H. W. Buck
Gano Dunn
H. W. Fisher
H. B. Gear
J. P. Jackson
W. L. Merrill
Ralph D. Mershon
W. S. Moody
R. A. Philip
W. H. Powell
Charles Robbins
E. B. Rosa
Charles P. Steinmetz
Calvert Townley

This committee and several sub-committees held numerous meetings at which the general revision of the Standardization Rules of the Institute was considered. The complete Standardization Rules as revised by this committee, were presented to and approved by the Board of Directors on June 27th, 1911, at the Annual Convention held at Chicago, Ill.

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STANDARDIZATION RULES OF THE A. I. E. E.

As Approved June 27, 1911.

I. DEFINITIONS AND TECHNICAL DATA.

1 *Note*: The following definitions and classifications are intended to be practically descriptive and not scientifically rigid.

A. DEFINITIONS. CURRENTS AND E.M.F.'S.

2 A **Direct Current** is a unidirectional current.

3 A **Continuous Current** is a steady, or non-pulsating, direct current.

4 A **Pulsating Current** is a current equivalent to the superposition of an alternating current upon a continuous current.

5 An **Alternating Current or e.m.f.** is a current or e.m.f. which, when plotted against time in rectangular coördinates, consists of half-waves of equal area in successively opposite directions from the zero line.

5a **Cycle.** Two immediately succeeding half-waves constitute a cycle.

5b **Period.** The time required for the execution of a cycle is called a period

5c **Frequency.** The number of cycles per second is called the frequency.

5d **Wave-form.** The shape of the curve of e.m.f. or current plotted against time in rectangular coördinates, is ordinarily referred to as the wave-form or wave-shape. Two alternating quantities are said to have the same wave-shape if their corresponding phase ordinates bear a constant ratio. The wave-shape, as ordinarily understood, is thus independent of the scales to which the curve is plotted.

5e **Simple Alternating Wave.** Unless otherwise specified an alternating current or e.m.f. is assumed to be sinusoidal, and the wave a sinusoid, sine-wave or curve of sines. On this account a complete cycle is taken as 360 degrees, and any portion of a cycle may be expressed in degrees from any convenient reference point, such as the ascending zero-point.

5f A **Complex Alternating Wave** is a non-sinusoidal wave. A complex alternating wave is capable of being resolved into a single sine wave of fundamental frequency, with superposed odd-frequency harmonic waves, or ripples, of 3, 5, 7 . . . $(2n+1)$ times the fundamental frequency, each harmonic having constant amplitude, and a definite starting phase-relation to the fundamental sine-wave. It is customary when analyzing a complex wave, to neglect harmonics higher than the 11th; *i.e.*, of frequency higher than 11 times the fundamental. In special cases, however, frequencies still higher may have to be considered. In certain exceptional cases even harmonics are present.

5g **Root-Mean-Square Value** (sometimes called the Virtual or Effective Value). Unless otherwise specified, the rating of an alternating-current or e.m.f., in amperes or volts, is assumed to be the square root of the mean square value taken throughout one or more complete cycles. This is sometimes abbreviated to r.m.s. The term root-mean-square is to be preferred to the terms virtual or effective. The root-mean-square value is indicated by all properly calibrated alternating-current voltmeters and ammeters. In the case of a sine-wave, the ratio of the maximum to the r.m.s. value is $\sqrt{2}$.

5h FORM-FACTOR OF AN ALTERNATING WAVE. The ratio of the root-mean-square to the arithmetical mean ordinate of a wave, taken without regard to sign, is called its form-factor. The form-factor for a purely rectangular wave is the minimum, 1.0; for a sine wave it is 1.11, and for a wave more peaked than a sine wave it is greater than 1.11.

5i THE EQUIVALENT SINE WAVE is a sine wave having the same frequency and the same r.m.s value as the actual wave.

5j The DEVIATION of wave-form from the sinusoidal is determined by superposing upon the actual wave, (as determined by oscillograph), the equivalent sine wave of equal length, in such a manner as to give the least difference, and then dividing the maximum difference between corresponding ordinates by the maximum value of the equivalent sine wave.

5k PHASE DIFFERENCE. When corresponding cyclic values of two sinusoidal alternating quantities such as two alternating currents or e.m.fs. or of a current and an e.m.f., of the same frequency, occur at different instants, the two alternating quantities are said to differ in phase, their phase difference being the time interval, expressed in degrees or as a fraction of a cycle, between the occurrence of their corresponding values; *e.g.* their ascending zeros or their positive maxima.

5l EQUIVALENT PHASE DIFFERENCE. If two alternating quantities are non-sinusoidal, and of different wave shapes, the preceding definition of phase-difference is inapplicable, and phase-difference ceases to have exact significance. However, when the two complex alternating quantities are the voltage E and current I in a given circuit, the effective power P of which is known, it is customary to define the equivalent phase difference by the angle whose cosine is the power-factor, P/EI, of the circuit. See Sections 54 and 324.

5m SINGLE-PHASE. A term characterizing a simple alternating current circuit energized by a single alternating e.m.f. Such a circuit is usually supplied through two wires. The currents in these two wires counted positively outwards from the source, differ in phase by 180 degrees or half a cycle.

5n THREE-PHASE. A term characterizing the combination of three circuits energized by alternating e.m.fs. which differ in phase by one third of a cycle; *i.e.*, 120°.

5o QUARTER-PHASE, also called TWO-PHASE. A term characterizing the combination of two circuits energized by alternating e.m.fs. which differ in phase by a quarter of a cycle; *i.e.*, 90°.

5p SIX-PHASE. A term characterizing the combination of six circuits energized by alternating e.m.fs. which differ in phase by one sixth of a cycle; *i.e.*, 60°.

5q POLYPHASE is the general term applied to any alternating system with more than a single phase.

6 An OSCILLATING CURRENT is a current alternating in direction, and of decreasing amplitude.

B. DEFINITIONS. ROTATING MACHINES.

7 A GENERATOR transforms mechanical power into electrical power.

8 A DIRECT-CURRENT GENERATOR produces a direct current that may or may not be continuous.

9 An ALTERNATOR is an alternating-current generator, either single-phase or polyphase.

9a A SYNCHRONOUS ALTERNATOR comprises a constant magnetic field and an armature delivering either single-phase or polyphase current in synchronism with the rotation of the machine.

10 A POLYPHASE GENERATOR produces currents differing symmetrically in phase; such as quarter-phase currents, in which the terminal voltages of the two circuits differ in phase by 90 degrees; or three-phase currents, in which the terminal voltages of the three circuits differ in phase by 120 degrees.

11 A **Double-Current Generator** supplies both direct and alternating currents from the same armature-winding.

11a An **Inductor Alternator** is an alternator in which both field and armature windings are stationary.

11b An **Induction Generator** is a machine structurally identical with an induction motor, but driven above synchronous speed as an alternating-current generator.

12 A **Motor** transforms electrical power into mechanical power.

12a A **Direct-Current Motor** transforms direct-current power into mechanical power.

12b An **Alternating-Current Motop** transforms alternating-current power into mechanical power.

12c A **Synchronous Motor** is a machine structurally identical with a synchronous alternator, but operated as a motor.

12d A **Synchronous Phase Modifier**, sometimes called a Synchronous Condenser, is a synchronous motor, running either idle or under load, whose field excitation may be varied so as to modify the power-factor of the circuit, or through such modification to influence the voltage of the circuit.

12e An **Induction Motor** is an alternating-current motor, either single-phase or polyphase, comprising independent primary and secondary windings, one of which, usually the secondary, is on the rotating member. The secondary winding has no conductive connection with the supply circuit.

12f A **Repulsion Motor** is an induction motor, usually single phase, in which the magnetic axis of the secondary, (a closed coil winding mounted on the rotor), is maintained at a certain fixed angle with respect to the stationary primary coil by means of a multisegmental commutator and short-circuiting brushes.

12g A **Single-Phase Series Commutator Motor** is structurally similar to a series direct-current motor, except that it is usually provided in addition with a series compensating winding distributed around the outer air-gap periphery and supported in slots in the pole faces, for the purpose of diminishing the armature leakage reactance.

13 A **Booster** is a machine inserted in series in a circuit to change its voltage. It may be driven by an electric motor (in which case it is termed a motor-booster) or otherwise.

14 A **Motor-Generator** is a transforming device consisting of a motor mechanically connected to one or more generators.

15 A **Dynamotor** is a transforming device combining both motor and generator action in one magnetic field, either with two armatures, or with one armature having two separate windings and independent commutators.

16 A **Converter** is a machine employing mechanical rotation in changing electrical energy from one form into another. A converter may belong to either of several types, as follows:

17 *a.* A **Direct-Current Converter** converts from a direct current to a direct current, usually with a change of voltage.

18 *b.* A **Synchronous Converter** (commonly called a rotary converter) converts from an alternating to a direct current, or *vice versa.*

19 *c.* A **Motor-Converter** is a combination of an induction motor with a synchronous converter, the secondary of the former feeding the armature of the latter with current at some frequency other than the impressed frequency; *i.e.*, it is a synchronous converter concatenated with an induction motor.

20 *d.* A **Frequency Changer** converts the power of an alternating-current system from one frequency to another, with or without a change in the number of phases or in the voltage.

21 *e.* A **Rotary Phase Converter** converts from an alternating-current system of one or more phases to an alternating-current system of a different number of phases, but of the same frequency.

21a **Equalizing Connections** are low resistance connections between equipotential points of multiple-wound closed-coil armatures to equalize the induced voltage between brushes.

C. DEFINITIONS. STATIONARY INDUCTION APPARATUS.

22 STATIONARY INDUCTION APPARATUS changes electric energy to electric energy through the medium of magnetic energy. It comprises several forms, distinguished as follows:

23 *a.* TRANSFORMERS, in which the primary and secondary windings are insulated from one another.

23a A PRIMARY WINDING is that winding of a transformer, or of an induction motor, which receives power from an external source.

23b A SECONDARY WINDING is that winding of a transformer, or of an induction motor, which receives power from the primary by induction.

Note: The terms "High-voltage winding" and "Low-voltage winding" are suitable for distinguishing between the windings of a transformer, where the relations of the apparatus to the source of power are not involved.

24 *b.* AUTO-TRANSFORMERS, also called compensators, in which a part of the primary winding is used as a secondary winding, or conversely.

25 *c.* POTENTIAL REGULATORS, in which one coil is in shunt and one in series with the circuit, so arranged that the ratio of transformation between them is variable at will. They are of the following three classes:

26 1. CONTACT VOLTAGE REGULATORS, also called Compensator Regulators, in which the number of turns in use of one of the coils is adjustable.

27 (2) INDUCTION POTENTIAL REGULATORS in which the relative positions of the primary and secondary coils are adjustable.

28 (3) MAGNETO POTENTIAL REGULATORS in which the direction of the magnetic flux with respect to the coils is adjustable.

29 *d.* REACTORS or REACTANCE COILS, also called choke coils, are a form of stationary induction apparatus used to supply reactance or to produce phase displacement.

29a *e.* AN INDUCTION STARTER is a device used in starting induction motors, converters, etc., by voltage control, consisting of an auto-transformer combined with a suitable switching device.

29b A LEAKAGE REACTANCE or SERIES REACTANCE is a portion of the reactance of any induction apparatus which is due to stray or purely self-inductive flux.

D. GENERAL CLASSIFICATION OF APPARATUS.

30 COMMUTATING MACHINES. Under this head may be classed the following: Direct-current generators; direct-current motors; direct-current boosters; motor-generators; dynamotors; converters; compensators or balancers; closed-coil arc machines, and alternating-current commutating motors.

31 Commutating machines may be further classified as follows:

32 *a.* DIRECT-CURRENT COMMUTATING MACHINES, which comprise a magnetic field of constant polarity, a closed-coil armature, and a multisegmental commutator connected therewith.

33 *b.* ALTERNATING-CURRENT COMMUTATING MACHINES, which comprise a magnetic field of alternating polarity, a closed-coil armature, and a multisegmental commutator connected therewith.

34 *c.* SYNCHRONOUS COMMUTATING MACHINES, which comprise synchronous converters, motor-converters and double-current generators.

35 SYNCHRONOUS MACHINES, comprise a constant magnetic field, and an armature receiving or delivering alternating-currents in synchronism with the motion of the machine; *i.e.*, having a frequency equal to the product of the number of pairs of poles and the speed of the machine in revolutions per second.

36 STATIONARY INDUCTION APPARATUS, include transformers, auto-transformers, potential regulators, and reactors or reactance coils.

37 ROTARY INDUCTION APPARATUS, or INDUCTION MACHINES, include apparatus wherein the primary and secondary windings rotate with respect to each other; *i.e.*, induction motors, induction generators, frequency converters, and rotary phase converters.

38 UNIPOLAR or ACYCLIC MACHINES, direct-current machines, in which the voltage generated in the active conductors maintains the same direction with respect to those conductors.

39 RECTIFYING APPARATUS, PULSATING-CURRENT GENERATORS.

40 ELECTROSTATIC APPARATUS, such as condensers, etc.

41 ELECTROCHEMICAL APPARATUS, such as batteries, etc.

42 ELECTROTHERMAL APPARATUS, such as heaters, etc.

42a REGULATING APPARATUS, such as rheostats, etc.

42b SWITCHING APPARATUS.

43 PROTECTIVE APPARATUS, such as fuses, circuit-breakers, lightning arresters, etc.

44 LUMINOUS SOURCES.

E. MOTORS. SPEED CLASSIFICATION.

45 MOTORS may, for convenience, be classified with reference to their speed characteristics as follows:

46 *a.* CONSTANT-SPEED MOTORS, in which the speed is either constant or does not materially vary; such as synchronous motors, induction motors with small slip, and ordinary direct-current shunt motors.

47 *b.* MULTISPEED MOTORS (two-speed, three-speed, etc.), which can be operated at any one of several distinct speeds, these speeds being practically independent of the load, such as motors with two armature windings, or induction motors with controllers for changing the number of poles.

48 *c.* ADJUSTABLE-SPEED MOTORS, in which the speed can be varied gradually over a considerable range; but when once adjusted remains practically unaffected by the load, such as shunt motors designed for a considerable range of field variation.

49 *d.* VARYING-SPEED MOTORS, or motors in which the speed varies with the load, decreasing when the load increases; such as series motors.

F. DEFINITIONS. INSTRUMENTS.

49a AN AMMETER is a current-measuring instrument, indicating in amperes]

49b A VOLTMETER is a voltage-measuring instrument, indicating in volts.

49c A WATTMETER is an instrument for measuring electrical power, and indicating in watts.

49d RECORDING AMMETERS, VOLTMETERS, WATTMETERS, etc., are instruments which record graphically upon a time-chart the values of the quantities they measure.

49e A WATT-HOUR METER is an instrument for registering total watt-hours. This term is to be preferred to the term " integrating wattmeter ".

49f A VOLTMETER COMPENSATOR is a device in connection with a voltmeter, which causes the latter to indicate the voltage at some other point of the circuit.

49g A SYNCHROSCOPE is a synchronizing device which, in addition to indicating synchronism, shows whether the machine to be synchronized is fast or slow.

G. DEFINITION AND EXPLANATION OF TERMS.

(I) *LOAD FACTOR.*

50 The LOAD FACTOR of a machine, plant or system is the ratio of the average power to the maximum power during a certain period of time. The average power is taken over a certain period of time, such as a day or a year, and the maximum is taken over a short interval of the maximum load within that period.

51 In each case the interval of maximum load should be definitely specified. The proper interval is usually dependent upon local conditions and upon the purpose for which the load factor is to be determined.

(II) *DIVERSITY FACTOR.*

51a DIVERSITY FACTOR is the ratio of the sum of the maximum power demands of the subdivisions of any system or part of a system, to the maximum demand of the whole system or of the part of the system under consideration, measured at the point of supply.

(III) *DEMAND FACTOR.*

51b DEMAND FACTOR is the ratio of the maximum power demand of any system or part of a system to the total connected load of the system or of the part of system under consideration.

(*IV*) *NON-INDUCTIVE LOAD AND INDUCTIVE LOAD.*

52 A non-inductive load is a load in which the current is in phase with the voltage across the load.

53 An inductive load is a load in which the current lags behind the voltage across the load. A load in which the current leads the voltage across the load is sometimes called a condensive or anti-inductive load.

53a When voltage and current waves are sinusoidal but not in phase, the voltage may be resolved into two components one in phase with the current, and the other in quadrature therewith. The former is called the effective component (sometimes the energy component), and the latter the reactive component of the voltage. The current may be similarly subdivided with respect to the voltage, and the two components similarly named.

(*V*) *POWER-FACTOR AND REACTIVE FACTOR.*

54 The POWER-FACTOR in alternating-current circuits or apparatus is the ratio of the effective (*i.e.* the cyclic average) power in watts to the apparent power in volt-amperes. It may be expressed as follows:

$$\frac{\text{effective power}}{\text{apparent power}}=\frac{\text{effective watts}}{\text{total volt-amperes}}=\frac{\text{effective current}}{\text{total current}}=\frac{\text{effective voltage}}{\text{total voltage}}$$

55 The REACTIVE-FACTOR is the ratio of the reactive volt-amperes (*i.e.*, the product of the reactive component of current by voltage, or reactive component of voltage by current) to the total volt-amperes. It may be expressed as follows:

$$\frac{\text{reactive power}}{\text{apparent power}}=\frac{\text{reactive watts}}{\text{total volt-amperes}}=\frac{\text{reactive current}}{\text{total current}}=\frac{\text{reactive voltage}}{\text{total voltage}}$$

56 POWER-FACTOR and REACTIVE-FACTOR are related as follows:
If p = power-factor and q = reactive-factor, then with sine waves of voltage and current,

$$p^2+q^2=1$$

With distorted waves of voltage and current, q ceases to have definite significance.

(VI) *SATURATION-FACTOR.*

57 The SATURATION-FACTOR of a machine is the ratio of a small percentage increase in field excitation to the corresponding percentage increase in voltage thereby produced. The saturation factor is, therefore, a criterion of the degree of saturation attained in the magnetic circuit at any excitation selected. Unless otherwise specified, however, the saturation factor of a machine refers to the excitation existing at normal rated speed and voltage. It is determined from measurements of saturation made on open circuit at rated speed.

58 The PERCENTAGE OF SATURATION of a machine at any excitation may be found from its saturation curve of generated voltage as ordinates, against excitation as abscissas, by drawing a tangent to the curve at the ordinate corresponding to the assigned excitation, and extending the tangent to intercept the axis of ordinates drawn through the origin. The ratio of the intercept on this axis to the ordinate at the assigned excitation, when expressed in percentage, is the percentage of saturation and is independent of the scale selected for excitation and voltage. This ratio is equal to the reciprocal of the saturation-factor at the same excitation, deducted from unity. Thus, if f be the saturation-factor and p the percentage of saturation.

$$p = 1 - \frac{1}{f}$$

(VII) *VARIATION AND PULSATION.*

59 The Variation in Prime Movers which do not give an absolutely uniform rate of rotation or speed, as in reciprocating steam engines, is the maximum angular displacement in position of the revolving member expressed in degrees, from the position it would occupy with uniform rotation, and with one revolution taken as 360 degrees.

60 The Pulsation in Prime Movers is the ratio of the difference between the maximum and minimum velocities in an engine-cycle to the average velocity.

61 The Variation in Alternators or alternating-current circuits in general is the maximum difference in phase of the generated voltage wave from a wave of absolutely constant frequency of the same average value, expressed in electrical degrees (one cycle equals 360 degrees) and may be due to the variation of the prime mover.

62 The Pulsation in Alternators or alternating-current circuits, in general, is the ratio of the difference between maximum and minimum frequency during an engine cycle to the average frequency.

63 Relation of Variation in prime mover and alternator. If p = number of pairs of poles, the variation of an alternator is p times the variation of its prime mover, if direct-connected, and $p\ n$ times the variation of the prime mover if rigidly connected thereto in the velocity ratio n; so that the speed of the alternator is n times that of the prime mover.

II. PERFORMANCE SPECIFICATIONS AND TESTS.

A. RATING.

65 Rating by Output. All electrical apparatus should be rated by output and not by input. Generators, transformers, etc., should be rated by electrical output: motors by mechanical output, and preferably in kilowatts.

65a The following four classes of rating are recognized and recommended: they do not cover the rating of railway motors which is treated in Appendix B, and there are other large though less definitely definable classes of service in which each case must be treated by itself. Some of these may be later reduced to fairly simple terms and introduced into these Rules.

65b 1. Continuous Rating in which under load there is the attainment of approximately stationary temperature, and no other limit of capacity is exceeded.

65c 2. Intermittent Rating in which one minute periods of load and rest alternate until the attainment of approximately stationary temperature and no other limit of capacity is exceeded.

65d Note:—Since the temperature depends upon the losses and the capacity of the apparatus to emit them, a constant load may be substituted for the intermittent load in determining the temperature, provided the losses are equivalent.

65e 3. Minute Rating in which under load for one minute, no mechanical, thermal, magnetic, or electrical limit of capacity is exceeded and no permanent change is wrought in the apparatus.

65f 4. Variable Service Rating. It is desirable here to recognize this class of rating which is intended to cover the rating of motors for machine-tool and similar service, in which the thermal absorptive capacity plays a part. The specifications for this rating have not been fully determined at the time that this edition of the Rules goes to press.

66 Rating in Kilowatts. Electrical power should be expressed in kilowatts, except when otherwise specified.

67 Apparent Power, Kilovolt-Amperes. Apparent power in alternating-current circuits should be expressed in kilovolt-amperes as distinguished from effective power in kilowatts. When the power factor is 100 per cent, the apparent power in kilovolt-amperes is equal to the kilowatts.

68 The Rated (Full-Load) Current is that current which, with the rated terminal voltage, gives the rated kilowatts, or the rated kilovolt-amperes. In machines in which the rated voltage differs from the no-load voltage, the rated current should refer to the former.

69 Determination of Rated Current. The rated current may be determined as follows: If P = rating in watts, or volt-amperes if the power factor be other than 100 per cent, and E = full-load terminal voltage, the rated current per terminal is:

70 $I=\frac{P}{E}$ amperes, in a direct-current machine or single-phase alternator.

71 $I=\frac{1}{\sqrt{3}}\frac{P}{E}$ amperes, in a three-phase alternator.

72 $I=\frac{1}{2}\frac{P}{E}$ amperes, in a quarter-phase alternator.

73 Normal Conditions. The rating of machines or apparatus should be based upon certain normal conditions to be assumed as standard, or to be specified. These conditions include voltage, current, power-factor, frequency, wave shape and speed; or such of them as may apply in each particular case. Performance tests should be made under these standard conditions unless otherwise specified.

74 *a*. Power Factor. Since the inherent capacity of alternating current generators, synchronous motors, and transformers, depends upon their voltage and their current, they should be rated in kilovolt-amperes. If the apparatus is rated in kilowatts without specification as to the power factor, a power factor of 100 per cent shall be understood.

If rated in kilowatts and a power factor other than 100 per cent be specified, this should be understood as defining only the nature of the load, and not as implying an increase in the ampere rating of the apparatus, which should be based upon the kilowatt rating at 100 per cent power factor.

75 *b*. Wave Shape. In determining the rating of alternating-current machines or apparatus, a sine wave shape of alternating current and voltage is assumed, except where a distorted wave shape is inherent to the apparatus. See Secs. 79–80.

76 Fuses. The rating of a fuse should be the maximum current which it will continuously carry.

77 Circuit-Breakers. The rating of a circuit-breaker should be the maximum current which it is designed to carry continuously.

77a Note. In addition thereto, the maximum current and voltage at which a fuse or a circuit-breaker will open the circuit should be specified. It is to be noted that the behavior of fuses and of circuit-breakers is much influenced by the amount of electric power available on the circuit.

78 Indicating meters should be rated according to their full-scale reading of volts, amperes, or watts. In wattmeters the rated volts and rated amperes should also be included; *i.e.*, the volts and amperes which can be safely and continuously carried by the voltage and current coils respectively.

78a Watt-hour Meters should be rated in volts and amperes.

B. WAVE SHAPE.

79 The Sine Wave should be considered as standard, except where a deviation therefrom is inherent in the operation of the apparatus

80 A Maximum Deviation of the wave from sinusoidal shape not exceeding 10 per cent is permissible, except when otherwise specified. See Section 5j.

81, 82, 83. See Sections 5e to 5l.

C. EFFICIENCY.

(I) *DEFINITIONS.*

84 The **Efficiency** of an apparatus is the ratio of its output to its input. The output and input may be in terms of watt-hours, watts, volt-amperes, amperes, or any other quantity of interest, thus respectively defining energy-efficiency, power-efficiency, apparent-power-efficiency, current-efficiency, etc. Unless otherwise specified, however, the term is ordinarily assumed to refer to power-efficiency. An exception should be noted in the case of luminous sources, (see Sec. 346).

86 **Apparent Efficiency.** In apparatus in which a phase displacement is inherent to their operation, apparent efficiency should be understood as the ratio of net power output to volt-ampere input.

87 *a.* **Note.** Such apparatus comprises induction motors, synchronous phase modifiers, synchronous converters controlling the voltage of an alternating-current system, potential regulators, open magnetic circuit transformers, etc.

88 *b.* **Note.** Since the apparent efficiency of apparatus delivering electric power depends upon the power-factor of the load, the apparent efficiency, unless otherwise specified, should be referred to a load power-factor of unity.

(II) *MEASUREMENT OF EFFICIENCY.*

89 **Methods.** Efficiency may be determined by either of two methods, *viz.*: by measurement of input and output; or, by measurement of losses.

90 *a.* **Method of Input and Output.** The input and output may both be measured directly. The ratio of the latter to the former is the efficiency.

91 *b.* **Method by Losses.** The losses may be measured either collectively or individually. The total losses may be added to the output to derive the input, or subtracted from the input to derive the output.

92 **Comparison of Methods.** The output and input method is preferable with small machines. When, however, as in the case of large machines, it is impracticable to measure the output and input; or when the percentage of power loss is small and the efficiency is nearly unity, the method of determining efficiency by measuring the losses should be followed.

93 **Electric Power** should be measured at the terminals of the apparatus. In tests of polyphase machines, the measurement of power should not be confined to a single circuit but should be extended to all the circuits in order to avoid errors of unbalanced loading.

94 **Mechanical Power** in machines should be measured at the pulley, gearing, coupling, etc., thus excluding the loss of power in said pulley, gearing or coupling, but including the bearing friction and windage. The magnitude of bearing friction and windage may be considered, with constant speed, as independent of the load. The loss of power in the belt and the increase of bearing friction due to belt tension should be excluded. Where, however, a machine is mounted upon the shaft of a prime mover, in such a manner that it cannot be separated therefrom, the frictional losses in bearings and in windage, which ought, by definition, to be included in determining the efficiency, should be excluded, owing to the practical impossibility of separating them from those of the prime mover.

95 In **Auxiliary Apparatus**, such as an exciter, the power lost in the auxiliary apparatus should not be charged to the principal machine, but to the plant consisting of principal machine and auxiliary apparatus taken together. The plant efficiency in such cases should be distinguished from the machine efficiency.

96 **Normal Conditions.** Efficiency tests should be made under normal conditions herein set forth, which are to be assumed as standard. These conditions include voltage, current, power-factor, frequency, wave shape, speed, temperature and barometric pressure, or such of them as may apply in each particular case. Performance tests should be made under these standard conditions unless otherwise specified. See Secs. 73–75.

97 *a.* **Temperature.** The efficiency of all apparatus, except such as may be intended for intermittent service should be either measured at, or re-

duced to, the temperature which the apparatus assumes under continuous operation at rated load, referred to a room temperature of 25 deg. cent. See Secs. 267–292.

98 With apparatus intended for intermittent service, the efficiency should be determined at the temperature assumed under specified conditions.

99 *b*. POWER FACTOR. In determining the efficiency of alternating-current apparatus, the electric power should be measured when the current is in phase with the voltage, unless otherwise specified, except when a definite phase difference is inherent in the apparatus, as in induction motors, induction generators, frequency converters, etc.

100 *c*. WAVE SHAPE. In determining the efficiency of alternating-current apparatus, the sine wave should be considered as standard, except where a difference in the wave form from the sinusoidal is inherent in the operation of the apparatus. See Sec. 80.

(III) *MEASUREMENT OF LOSSES.*

101 LOSSES. The usual sources of losses in electrical apparatus and the methods of determining these losses are as follows:

102 (*A*) BEARING FRICTION AND WINDAGE.

The magnitude of bearing friction and windage (which may be considered as independent of the load) is conveniently measured by driving the machine from an independent motor, the output of which may be suitably determined. See Sec. 94.

(*B*) COMMUTATOR BRUSH FRICTION.

103 The magnitude of the commutator brush friction (which may be considered as independent of the load) is determined by measuring the difference in power required for driving the machine with brushes on and with brushes off (the field being unexcited).

(*C*) COLLECTOR-RING BRUSH FRICTION.

104 Collector-ring brush friction may be determined in the same manner as commutator brush friction. It is usually negligible.

(*D*) MOLECULAR MAGNETIC FRICTION AND EDDY CURRENTS.

105 These losses include those due to molecular magnetic friction and eddy currents in iron and copper and other metallic parts, also the losses due to currents in the cross-connections of cross-connected armatures.

106 In MACHINES these losses should be determined on open circuit and at a voltage equal to the rated voltage $+I\,r$ in a generator, and $-I\,r$ in a motor, where I denotes the current strength and r denotes the internal resistance of the machine. They should be measured at the correct speed and voltage, since they do not usually vary in any definite proportion to the speed or to the voltage.

107 NOTE. The TOTAL LOSSES in bearing friction and windage, brush friction, magnetic friction and eddy currents can, in general, be determined by a single measurement by driving the machine with the field excited, either as a motor, or by means of an independent motor.

108 RETARDATION METHOD. The no-load iron, friction, and windage losses may be segregated by the Retardation Method. The generator should be brought up to full speed (or, if possible, to about 10 per cent above full speed) as a motor, and, after cutting off the driving power and excitation, frequent readings should be taken of speed and time, as the machine slows down, from which a speed-time curve can be plotted. A second curve should be taken in the same manner, but with full field excitation; from the second curve the iron losses may be found by subtracting the losses found in the first curve.

109 The speed-time curves can be plotted automatically by belting a small separately excited generator (say 1/10 kw.) to the generator shaft and connecting it to a recording voltmeter.

(*E*) ARMATURE-RESISTANCE LOSS.

110 This loss may be expressed by $p\,I^2\,r$; where r = resistance of one armature circuit or branch, I = the current in such armature circuit or branch, and p = the number of armature circuits or branches.

(*F*) Commutator, Brush and Brush-Contact Resistance Loss.

111 It is desirable to point out that with carbon brushes these losses may be considerable in low-voltage machines.

(*G*) Collector-Ring and Brush-Contact Resistance Loss.

112 This loss is usually negligible, except in machines of extremely low voltage or in unipolar machines.

(*H*) Field-Excitation Loss.

113 With separately excited field, the loss of power in the resistance of the field coils alone should be considered. With either shunt- or series-field windings, however, the loss of power in the accompanying rheostat should also be included, the said rheostat being considered as an essential part of the machine, and not as separate auxiliary apparatus.

114 (*I*) Load Losses.

The load losses may be considered as the difference between the total losses under load and the sum of the losses as above specified and determined.

115 *a*. In Commutating Machines of small field distortion, the load losses are usually trivial and may, therefore, be neglected. When, however, the field distortion is large as in commutating-pole machines; or, as is shown, for instance, by the necessity for shifting the brushes between no load and full load on non-commutating pole machines; these load losses may be considerable, and should be taken into account. In this case the efficiency may be determined either by input and output measurements, or the load losses may be estimated by the method of Sec. 116.

116 *b*. Estimation of Load Losses. While the load losses cannot well be determined individually, they may be considerable and, therefore, their joint influence should be determined by observation. This can be done by operating the machine on short-circuit and at full-load current, that is, by determining what may be called the " short-circuit core loss." With the low field intensity and great lag of current existing in this case, the load losses are usually greatly exaggerated.

117 One-third of the short-circuit core loss may, as an approximation, and in the absence of more accurate information, be assumed as the load loss.

(IV) *EFFICIENCY OF DIFFERENT TYPES OF APPARATUS.*

(*A*) Direct-Current Commutating Machines.

118 In Direct-Current Commutating Machines the losses are:

119 *a*. Bearing Friction and Windage. See Measurement of Losses (*A*), Sec. 102.

120 *b*. Molecular Magnetic Friction and Eddy Currents. See Measurement of Losses (*D*), Sec. 105.

121 *c*. Armature Resistance Losses. See Measurement of Losses (*E*), Sec. 110.

122 *d*. Commutator Brush Friction. See Measurement of Losses (*B*), Sec. 103.

123 *e*. Commutator, Brush and Brush-Contact Resistance. See Measurement of Losses (*F*), Sec. 111.

124 *f*. Field Excitation Loss. See Measurement of Losses (*H*), Sec. 113.

125 *g*. Load Losses. See Measurement of Losses (*I*), Sec. 114.

126 Note. *b* and *c* are losses in the armature or " armature losses "; *d* and *e* " commutator losses "; *f* " field losses."

(*B*) Alternating-Current Commutating Machines.

127 In Alternating-Current Commutating Machines, the losses are:

128 *a*. Bearing Friction and Windage. See Measurement of Losses (*A*), Sec. 102.

129 *b*. Rotation Loss, measured with the machine at open circuit, the brushes on the commutator, and the field excited by alternating current when driving the machine by a motor.

130 This loss includes molecular magnetic friction and eddy currents, caused by rotation through the magnetic field, $I^2 r$ losses in cross-con-

nections of cross-connected armatures, I^2r and other losses in armature-coils and armature-leads which are short-circuited by the brushes as far as these losses are due to rotation.

131 *c.* Alternating or Transformer Loss: These losses are measured by wattmeter in the field circuit, under the conditions of test *b*. They include molecular magnetic friction and eddy-currents due to the alternation of the magnetic field, I^2r losses in cross-connections of cross-connected armatures, I^2r and other losses in armature coil and commutator leads which are short-circuited by the brushes, as far as these losses are due to the alternation of the magnetic flux.

132 The losses in armature-coils and commutator leads short-circuited by the brushes, can be separated in *b*, and *c*, from the other losses, by running the machine with and without brushes on the commutator.

133 *d.* I^2R Loss, other load losses in armature and compensating winding and I^2r loss of brushes, may be measured by a wattmeter connected across the armature and compensating winding.

134 *e.* Field Excitation Loss. See Measurement of Losses (*H*), Sec. 113.

135 *f.* Commutator Brush-Friction. See Measurement of Losses (*B*), Sec. 103.

(*C*) Synchronous Commutating Machines.

136 1. In Double-Current Generators, the efficiency of the machine should be determined as a direct-current generator, and also as an alternating-current generator. The two values of efficiency may be different, and should be clearly distinguished.

137 2. In Converters the losses should be determined when driving the machine by a motor. These losses are:

138 *a.* Bearing Friction and Windage. See Measurement of Losses (*A*), Sec. 102.

139 *b.* Molecular Magnetic Friction and Eddy Currents. See Measurement of losses (*D*) Sec. 105.

140 *c.* Armature-Resistance Loss. This loss in the armature is $q\, I^2r$, where I = direct current in armature, r = armature resistance and q, a factor which is equal to 1.47 in single-circuit single-phase, 1.15 in double-circuit single-phase, 0.59 in three-phase, 0.39 in two-phase, and 0.27 in six-phase converters.

141 *d.* Commutator-Brush Friction. See Measurement of Losses (*B*), Sec. 103.

142 *e.* Collector-Ring Brush Friction. See Measurement of Losses (*C*), Sec. 104.

143 *f.* Commutator, Brush and Brush-Contact Resistance Loss. See Measurement of Losses (*F*) Sec. 111.

144 *g.* Collector-Ring Brush-Contact Resistance Loss. See Measurement of Losses (*G*), Sec. 112.

145 *h.* Field-Excitation Loss. See Measurement of Losses (*H*), Sec. 109.

146 *i.* Load Losses. These can generally be neglected, owing to the absence of field distortion.

147 3. The Efficiency of Two Similar Converters may be determined by operating one machine as a converter from direct to alternating, and the other as a converter from alternating to direct, connecting the alternating sides together, and measuring the difference between the direct-current input and the direct-current output. This process may be modified by returning the output of the second machine through two boosters into the first machine and measuring the losses. Another modification is to supply the losses by an alternator between the two machines, using potential regulators.

(*D*) Synchronous Machines.

148 In Synchronous Machines the losses are:

149 *a.* Bearing Friction and Windage. See Measurement of Losses (*A*), Sec. 102.

150 *b.* Molecular Magnetic Friction and Eddy Currents. See Measurement of Losses (*D*), Sec. 105.

151 *c.* ARMATURE-RESISTANCE LOSS. See Measurement of Losses (*E*), Sec. 110.

152 *d.* COLLECTOR-RING BRUSH FRICTION. See Measurement of Losses (*C*), Sec. 104.

153 *e.* COLLECTOR-RING BRUSH-CONTACT RESISTANCE LOSS. See Measurement of Losses (*G*), Sec. 112.

154 *f.* FIELD-EXCITATION LOSS. See Measurement of Losses (*H*), Sec. 113.

155 *g.* LOAD LOSSES. See Measurement of Losses (*I*), Sec. 114.

(*E*) STATIONARY INDUCTION APPARATUS.

156 In STATIONARY INDUCTION APPARATUS, the losses are:

157 *a.* MOLECULAR MAGNETIC FRICTION AND EDDY CURRENTS measured at open secondary circuit, rated frequency, and at rated voltage $-I\,r$, where I = rated current, r = resistance of primary circuit.

158 *b.* RESISTANCE LOSSES, the sum of the $I^2 r$ losses in the primary and in the secondary windings of a transformer, or in the two sections of the coil in a compensator or auto-transformer, where I = rated current in the coil or section of coil, and r = resistance.

159 *c.* LOAD LOSSES, *i.e.*, eddy currents in the iron and especially in the copper conductors, caused by the current at rated load. For practical purposes they may be determined by short-circuiting the secondary of the transformer and impressing upon the primary a voltage sufficient to send rated load current through the transformer. The loss in the transformer under these conditions, measured by wattmeter, gives the load losses $+I^2 r$ losses in both primary and secondary coils.

160 In CLOSED MAGNETIC CIRCUIT TRANSFORMERS, either of the two circuits may be used as primary when determining the efficiency.

161 In POTENTIAL REGULATORS, the efficiency should be taken at the maximum voltage for which the apparatus is designed, and with non-inductive load, unless otherwise specified.

(*F*) ROTARY INDUCTION APPARATUS, OR INDUCTION MACHINES.

162 In ROTARY INDUCTION APPARATUS, the losses are:

163 *a.* BEARING FRICTION AND WINDAGE. See Measurement of Losses (*A*), Sec. 102.

164 *b.* MOLECULAR MAGNETIC FRICTION AND EDDY CURRENTS in iron, copper and other metallic parts; also $I^2 r$ losses which may exist in multiple-circuit windings. *a* and *b* together are determined by running the motor without load at rated voltage, and measuring the power input.

165 *c.* PRIMARY $I^2 R$ LOSS, which may be determined by measurement of the current and the resistance.

166 *d.* SECONDARY $I^2 R$ LOSS, which may be determined as in the primary. when feasible; otherwise, as in squirrel-cage secondaries, this loss is measured as part of *e*.

167 *e.* LOAD LOSSES; *i.e.*, molecular magnetic friction, and eddy currents in iron, copper, etc., caused by the stray field of primary and secondary currents, and secondary $I^2 R$ loss when undeterminable under (*d*). These losses may for practical purposes be determined by measuring the total power, with the rotor short-circuited at standstill and a current in the primary circuit equal to the primary energy current at full load. The loss in the motor under these conditions may be assumed to be equal to the load losses $+I^2 r$ losses in both primary and secondary coils.

(*G*) UNIPOLAR OR ACYCLIC MACHINES.

168 In UNIPOLAR MACHINES, the losses are:

169 (*a*) BEARING FRICTION AND WINDAGE. See Measurement of Losses (*A*), Sec. 102.

170 (*b*) MOLECULAR MAGNETIC FRICTION AND EDDY CURRENTS. See Measurement of Losses (*E*), Sec. 106.

171 (*c*) ARMATURE-RESISTANCE LOSSES. See Measurement of Losses (*E*), Sec. 110.

172 (*d*) COLLECTOR-BRUSH FRICTION. See Measurement of Losses (*C*), Sec. 104.

173 (*e*) COLLECTOR BRUSH-CONTACT RESISTANCE. See Measurement of Losses (*G*), Sec. 112.

174 (*f*) FIELD-EXCITATION. See Measurement of Losses (*H*), Sec. 113.

175 (*g*) LOAD LOSSES. See Measurement of Losses (*I*), Sec. 114.

(*H*) RECTIFYING APPARATUS, PULSATING-CURRENT GENERATORS.

176 This division includes: open-coil arc machines and mechanical and other rectifiers.

177 In RECTIFIERS the most satisfactory method of determining the efficiency is to measure both electric input and electric output by wattmeter. The input is usually inductive, owing to phase displacement and to wave distortion. For this reason the power factor and the apparent efficiency should also be considered, since the latter may be much lower than the true efficiency. The power consumed by auxiliary devices, such as the synchronous motor or cooling devices, should be included in the electric input.

178 In CONSTANT-CURRENT RECTIFIERS, transforming from constant potential alternating to constant direct current, by means of constant-current transforming devices and rectifying devices, the losses in the transforming devices are to be included in determining the efficiency and have to be measured when operating the rectifier, since in this case the losses may be greater than when feeding an alternating secondary circuit. In constant-current transforming devices, the load losses may be considerable, and therefore, should not be neglected.

179 In OPEN-COIL ARC MACHINES, the losses are essentially the same as in direct-current (closed coil) commutating machines. In this case, however, the load losses are usually greater, and the efficiency should preferably be measured by input- and output-test, using wattmeters for measuring the output.

179a In alternating-current rectifiers, the output should, in general, be measured by wattmeter and not by voltmeter and ammeter, since owing to pulsation of current and voltage, a considerable discrepancy may exist between watts and volt-amperes. If, however, a direct-current and an alternating-current meter in the rectified circuit (either a voltmeter or an ammeter) give the same reading, the output may be measured by direct-current voltmeter and ammeter. The type of alternating-current instrument here referred to should indicate the effective or root-of-mean-square value and the type of direct-current instrument the arithmetical mean value, which would be zero on an alternating-current circuit.

(*I*) TRANSMISSION LINES.

180 The EFFICIENCY of transmission lines should be measured with non-inductive load at the receiving end, with the rated receiving voltage and frequency, also with sinusoidal impressed wave form, except where expressly specified otherwise, and with the exclusion of transformers or other apparatus at the ends of the line.

(*J*) PHASE-DISPLACING APPARATUS.

183 In SYNCHRONOUS PHASE-MODIFIERS and exciters of induction generators, the determination of losses is the same as in other synchronous machines.

184 In REACTORS the losses are molecular magnetic friction, eddy losses and $I^2 r$ loss. They should be measured by wattmeter. The losses of reactors should be determined with a sine wave of impressed voltage except where expressly specified otherwise.

185 In CONDENSERS, the losses are due to dielectric hysteresis and leakage, and should be determined by wattmeter with a sine wave of voltage or by an alternating-current bridge method.

186 In POLARIZATION CELLS, the losses are those due to electric resistivity and a loss in the electrolyte of the nature of chemical hysteresis. These losses may be considerable. They depend upon the frequency, voltage and temperature, and should be determined with a sine wave of impressed voltage, except where expressly specified otherwise.

D. REGULATION.

(I) *DEFINITIONS.*

187 The Regulation of a machine or apparatus in regard to some characteristic quantity (such as terminal voltage, current or speed) is the ratio of the deviation of that quantity from its normal value at rated load to that normal value. The term "regulation," therefore, has the same meaning as the term "inherent regulation," occasionally used.

188 Constant Standard. If the characteristic quantity is intended to remain constant (*e.g.*, constant voltage, constant speed, etc.) between rated load and no load, the regulation is the ratio of the maximum variation from the rated-load value to the no-load value.

189 Varying Standard. If the characteristic quantity is intended to vary in a definite manner between rated load and no load, the regulation is the ratio of the maximum variation from the specified condition to the normal rated-load value.

190 (*a*) Note. If the law of the variation (in voltage, current, speed, etc.) between rated load and no load is not specified, it should be assumed to be a simple linear relation; *i.e.*, one undergoing uniform variation between rated load and no load.

191 (*b*) Note. The regulation of an apparatus may, therefore differ according to its qualification for use. Thus, the regulation of a compound-wound generator specified as a constant-potential generator, will be different from that which it possesses when specified as an over-compounded generator.

192 In Constant-Potential Machines, the regulation is the ratio of the maximum difference of terminal voltage from the rated-load value (occurring within the range from rated load to open circuit) to the rated load terminal voltage.

193 In Constant-Current Machines, the regulation is the ratio of the maximum difference of current from the rated-load value (occurring within the range from rated-load to short-circuit, or minimum limit of operation), to the rated-load current.

194 In Constant-Power Apparatus, the regulation is the ratio of maximum difference of power from the rated load value (occurring within the range of operation specified) to the rated power.

195 In Constant-Speed Direct-Current Motors and Induction Motors the regulation is the ratio of the maximum variation of speed from its rated load value (occurring within the range from rated load to no-load) to the rated load speed.

196 The regulation of an induction motor is, therefore, not identical with the slip of the motor, which is the ratio of the drop in speed from synchronism, to the synchronous speed.

197 In Constant-Potential Transformers, the regulation is the ratio of the rise of secondary terminal voltage from rated non-inductive load to no-load (at constant primary impressed terminal voltage) to the secondary terminal voltage at rated load.

198 In Over-Compounded Machines, the regulation is the ratio of the maximum difference in voltage from a straight line connecting the no-load and rated-load values of terminal voltage as function of the load current, to the rated-load terminal voltage.

199 In Converters, Dynamotors, Motor-Generators and Frequency Converters, the regulation is the ratio of the maximum difference of terminal voltage at the output side from the rated-load voltage, to the rated-load voltage on the output side.

200 In Transmission Lines, Feeders, etc., the regulation is the ratio of the maximum voltage difference at the receiving end, between rated non-inductive load and no load to the rated-load voltage at the receiving end (with constant voltage impressed upon the sending end).

201 In Steam Engines, the regulation is the ratio of the maximum variation of speed in passing slowly from rated-load to no-load (with constant steam pressure at the throttle) to the rated-load speed. For variation and pulsation see Secs. 59–64.

202 In a Hydraulic Turbine or Other Water-Motor, the regulation is the ratio of the maximum variation of speed in passing slowly from rated-load to no-load (at constant head of water; *i.e.*, at constant difference of level between tail race and head race), to the rated-load speed. For variation and pulsation see Secs. 59–64.

203 In a Generator-Unit, consisting of a generator united with a prime-mover, the regulation should be determined at constant conditions of the prime-mover; *i.e.*, constant steam pressure, head, etc. It includes the inherent speed variations of the prime-mover. For this reason the regulation of a generator-unit is to be distinguished from the regulation of either the prime-mover, or of the generator contained in it, when taken separately.

(II) *CONDITIONS FOR AND TESTS OF REGULATION.*

204 Speed. The Regulation of Generators is to be determined at constant speed, and of alternating apparatus at constant impressed frequency.

205 Non-Inductive Load. In apparatus generating, transforming or transmitting alternating currents, regulation should be understood to refer to non-inductive load, that is, to a load in which the current is in phase with the e.m.f. at the output side of the apparatus, except where expressly specified otherwise.

206 Wave Form. In alternating apparatus receiving electric power, regulation should refer to a sine wave of e.m.f., except where expressly specified otherwise.

207 Excitation. In commutating machines, rectifying machines, and synchronous machines, such as direct-current generators and motors, alternating-current and polyphase generators, the regulation is to be determined under the following conditions:

(1) At constant excitation in separately excited fields.
(2) With constant resistance in shunt-field circuits, and
(3) With constant resistance shunting series-field circuits; *i.e.*, the field adjustment should remain constant, and should be so chosen as to give the required rated-load voltage at rated-load current.

208 Impedance Ratio. In alternating-current apparatus, in addition to the non-inductive regulation, the impedance ratio of the apparatus should be specified; *i.e.*, the ratio of the voltage consumed by the total internal impedance of the apparatus at rated-load current, to its rated-load voltage. As far as possible, a sinusoidal current should be used.

209 Computation of Regulation. In synchronous machines the open-circuit exciting ampere-turns corresponding to terminal voltage plus armature-resistance-drop, and the exciting ampere-turns at short-circuit for rated-load current should be combined vectorially to obtain the resultant ampere-turns, and the corresponding internal e.m.f. should be taken from the saturation curve.

E. INSULATION.

(I) *INSULATION RESISTANCE.*

210 Insulation Resistance is the ohmic resistance offered by an insulating coating, cover, material or support to an impressed voltage, tending to produce a leakage of current through the same.

211 Ohmic Resistance and Dielectric Strength. The ohmic resistance of the insulation is of secondary importance only, as compared with the dielectric strength, or resistance to rupture by high voltage. Since the ohmic resistance of the insulation can be very greatly increased by baking, but the dielectric strength is liable to be weakened thereby, it is preferable to specify a high dielectric strength rather than a high insulation resistance. The high-voltage test for dielectric strength should always be applied.

212 Recommended Value of Resistance. The insulation resistance of completed apparatus should be such that the rated terminal voltage of the apparatus will not send more than $\frac{1}{1,000,000}$ of the rated-load current, through the insulation. Where the value found in this way exceeds one megohm, it is usually sufficient.

213 INSULATION RESISTANCE TESTS should, if possible, be made at the pressure for which the apparatus is designed.

(II) *DIELECTRIC STRENGTH.*

(*A*) TEST VOLTAGES.

214 DEFINITION. The dielectric strength of an insulating wall, coating, cover or path is measured by the voltage which must be applied to it in order to effect a disruptive discharge through the same.

215 BASIS FOR DETERMINING TEST VOLTAGES. The test voltage which should be applied to determine the suitability of insulation for commercial operation is dependent upon the kind and size of the apparatus and its normal operating voltage, upon the nature of the service in which it is to be used, and the severity of the mechanical and electrical stresses to which it may be subjected. The voltages and other conditions of test which are recommended have been determined as reasonable and proper for the great majority of cases and are proposed for general adoption, except when specific reasons make a modification desirable.

216 CONDITION OF APPARATUS TO BE TESTED. Commercial tests should, in general, be made with the completely assembled apparatus and not with individual parts. The apparatus should be in good condition and high-voltage tests, unless otherwise specified, should be applied before the machine is put into commercial service, and should not be applied when the insulation resistance is low owing to dirt or moisture. High-voltage tests should, in general, be made at the temperature assumed under normal operation. High-voltage tests considerably in excess of the normal voltages to determine whether specifications are fulfilled are admissable on new machines only. Unless otherwise agreed upon, high-voltage tests of a machine should be understood as being made at the factory.

217 POINTS OF APPLICATION OF VOLTAGE. The test voltage should be successively applied between each electric circuit and all other electric circuits including conducting material in the apparatus.

218 The FREQUENCY of the alternating-current test voltage is, in general, immaterial within commercial ranges. When, however, the frequency has an appreciable effect, as in alternating-current apparatus of high voltage and considerable capacity, the rated frequency of the apparatus should be used.

219 TABLE OF TESTING VOLTAGES. The following voltages are recommended for testing all apparatus, lines and cables, by a continued application for one minute. The test should be with alternating voltage having a virtual value (or root mean square referred to a sine wave of voltage) given in the table, and preferably for tests of alternating apparatus at the normal frequency of the apparatus.

	Rated Terminal Voltage of Circuit.	Rated Output.	Testing Voltage.
220	Not exceeding 400 volts	Under 10 kw	1,000 volts.
	" " "	10 kw. and over	1,500 "
	400 and over, but less than 800 volts	Under 10 kw	1,500 "
	" " " " "	10 kw. and over	2,000 "
	800 " " " 1,200 "	Any	3,500 "
	1,200 " " " 2,500 "	Any	5,000 "
	2,500 " "	Any	Double the normal rated voltages.

221 EXCEPTION.—TRANSFORMERS. Transformers having primary pressures of from 550 to 5,000 volts, the secondaries of which are directly connected to consumption circuits, should have a testing voltage of 10,000 volts, to be applied between the primary and secondary windings, and also between the primary winding and the core.

222 EXCEPTION.—FIELD WINDINGS. The tests for field windings should be based on the rated voltage of the exciter and the rated output of the machine of which the coils are a part. Field windings of synchronous motors and converters, which are to be started by applying alternating current to the armature when the field is not excited and when a high voltage is induced in the field windings, should be tested at 5,000 volts.

223 RATED TERMINAL VOLTAGE.—DEFINITION. The rated terminal voltage of circuit in the above table, means the voltage between the conductors of the circuit to which the apparatus to be tested is to be connected,—for instance, in three-phase circuits the delta voltage should be taken. In the following specific cases, the rated terminal voltage of the circuit is to be determined as specified in ascertaining the testing voltage:

224 (*a*) TRANSFORMERS. The test of the insulation between the primary and secondary windings of transformers, is to be the same as that between the high-voltage windings and core, and both tests should be made simultaneously by connecting the low-voltage winding and core together during the test. If a voltage equal to the specified testing voltage be induced in the high-voltage winding of a transformer it may be used for insulation tests instead of an independently induced voltage. These tests should be made first with one end and then with the other end of the high-tension winding connected to the low-tension winding and to the core.

225 (*b*) CONSTANT-CURRENT APPARATUS. The testing voltage is to be based upon a rated terminal voltage equal to the maximum voltage which may exist at open or closed circuit.

226 (*c*) APPARATUS IN SERIES. For tests of machines or apparatus to be operated in series, so as to employ the sum of their separate voltages the testing voltage is to be based upon a rated terminal voltage equal to the sum of the separate voltages except where the frames of the machines are separately insulated, both from the ground and from each other, in which case the test for insulation between machines should be based upon the voltage of one machine, and the test between each machine and ground to be based upon the total voltage of the series.

(*B*) METHODS OF TESTING.

227 CLASSES OF TESTS. Tests for dielectric strength cover such a wide range in voltage that the apparatus, methods and precautions which are essential in certain cases do not apply to others. For convenience, the tests will be separated into two classes:

228 CLASS 1. This class includes all apparatus for which the test voltage does not exceed 10 kilovolts, unless the apparatus is of very large static capacity, *e.g.*, a large cable system. This class also includes all apparatus of small static capacity, such as line insulators, switches and the like, for all test voltages.

229 METHOD OF TEST FOR CLASS 1. The test voltage is to be continuously applied for the prescribed interval,—(one minute unless otherwise specified). The test voltage may be taken from a constant-potential source and applied directly to the apparatus to be tested, or it may be raised gradually as specified for tests under Class 2.

230 CLASS 2. This class includes all apparatus not included in Class 1.

231 METHOD OF TEST FOR CLASS 2. The test voltage is to be raised to the required value smoothly and without sudden large increments and is then to be continuously applied for the prescribed interval,—(one minute, unless otherwise specified), and then gradually decreased.

232 CONDITIONS AND PRECAUTIONS FOR CLASS 1 and CLASS 2. The following apply to all tests:

233 The WAVE SHAPE should be approximately sinusoidal and the apparatus in the testing circuits should not materially distort this wave.

234 The SUPPLY CIRCUIT should have ample current-supply capacity so that the charging current which may be taken by the apparatus under test will not materially alter the wave form nor materially affect the test voltage. The circuit should be free from accidental interruptions.

235 RESISTANCE OR INDUCTANCE in series with the primary of a raising transformer for the purpose of controlling its voltage is liable seriously to affect the wave form, thereby causing the maximum value of the voltage to bear a different and unknown ratio to the root mean square value. This method of voltage adjustment is, therefore, in general, undesirable. It may be noted that if a resistance or inductance is employed to limit the current when burning out a fault, such resistance or inductance should be short circuited during the regular voltage test.

236 The INSULATION under test should be in normal condition as to dryness and the temperature should when possible be that reached in normal service.

237 ADDITIONAL CONDITIONS AND PRECAUTIONS FOR CLASS 2. The following conditions and precautions, in addition to the foregoing, apply to tests of apparatus included in Class 2.

238 SUDDEN INCREMENT OF TESTING VOLTAGE on the apparatus under test should be avoided, particularly at high voltages and with apparatus having considerable capacity, as a momentarily excessive rise in testing voltage will result.

239 SUDDEN VARIATIONS IN TESTING VOLTAGE of the circuit supplying the voltage during the test should be avoided as they are likely to set up injurious oscillation.

240 GOOD CONNECTIONS in the circuits supplying the test voltage are essential in order to prevent injurious high frequency disturbances from being set up. When a heavy current is carried by a small water rheostat, arcing may occur, causing high-frequency disturbances which should be carefully avoided.

241 TRANSFORMER COILS. In high-voltage transformers, the low-voltage coil should preferably be connected to the core and to the ground when the high-voltage test is being made, in order to avoid the stress from low-voltage coil to core, which would otherwise result through condenser action. The various terminals of each winding of the high-tension transformer under test should be connected together during the test in order to prevent undue stress on the insulation between turns or sections of the winding in case the high-voltage test causes a break-down.

(*C*) METHODS FOR MEASURING THE TEST VOLTAGE.

242 FOR MEASURING THE TEST VOLTAGE, two instruments are in common use, (1) the spark gap and (2) the voltmeter.

243 1. THE SPARK GAP is ordinarily adjusted so that it will break down with a certain predetermined voltage, and is connected in parallel with the insulation under test. It ensures that the voltage applied to the insulation is not greater than the break-down voltage of the spark gap. A given setting of the spark gap is a measure of one definite voltage, and, as its operation depends upon the maximum value of the voltage wave, it is independent of wave form and is a limit on the maximum stress to which the insulation is subjected. The spark gap is not conveniently adapted for comparatively low voltages.

244 In SPARK-GAP MEASUREMENTS, the spark gap may be set for the required voltage and the auxiliary apparatus adjusted to give a voltage at which this spark gap just breaks down. The spark gap should then be adjusted for, say, 10 per cent higher voltage, and the auxiliary apparatus again adjusted to give the voltage of the former breakdown, which is to be the assumed voltage for the test. This voltage is to be maintained for the required interval.

245 The SPARK POINTS should consist of new sewing needles, supported axially at the ends of linear conductors which are each at least twice the length of the gap. There should be no extraneous body near the gap within a radius of twice its length. A table of approximate striking distances is given in Appendix D. This table should be used in connection with tests made by the spark-gap methods.

246 A NON-INDUCTIVE RESISTANCE of about one-half ohm per volt should be inserted in series with each terminal of the gap so as to keep the discharge current between the limits of one-quarter ampere and 2 amperes. The purpose of the resistance is to limit the current in order to prevent the surges which might otherwise occur at the time of break-down.

247 2. The VOLTMETER gives a direct reading, and the different values of the voltage can be read during the application and duration of the test. It is suitable for all voltages, and does not introduce disturbances into the test circuit.

248 In VOLTMETER MEASUREMENTS, the voltmeter should, in general, derive its voltage from the high-tension testing circuit either directly or through an auxiliary ratio transformer. It is permissible, however, to measure the voltage at other places,—for example, on the primary of the transformer. provided the ratio of transformation does not materially vary during the test; or that proper account is taken thereof.

249 Spark Gap and Voltmeter. The spark gap may be employed as a check upon the voltmeter used in high-tension tests in order to determine the transformation ratio of the transformer, the variation from the sine wave form and the like. It is also useful in conjunction with voltmeter measurements to limit the stress applied to the insulating material.

(*D*) Apparatus for Supplying Test Voltage.

250 The Generator or Circuit supplying voltage for the test should have ample current carrying capacity, so that the current which may be taken for charging the apparatus to be tested will not materially alter the wave form nor otherwise materially change the voltage.

The Testing Transformer should be such that its ratio of transformation does not vary more than 10 per cent when delivering the charging current required by the apparatus under test. (This may be determined by short-circuiting the secondary or high voltage winding of the testing transformer and supplying 1/10 of the primary voltage to the primary under this condition. The primary current that flows under this condition is the maximum which should be permitted in regular dielectric test.)

251 The Voltage Control may be secured in either of several ways, which in order of preference, are as follows:

252 1. By generator field circuit.
253 2. By magnetic commutation.
254 3. By change in transformer ratio.
255 4. By resistance or choke coils.

256 . In Generator Voltage Control, the voltage of the generator should preferably be about its approximate normal rated load value when the full testing voltage is attained, which requires that the ratio of the raising transformer be such that the full testing voltage is reached when the generator voltage is normal. This avoids the instability in the generator which may occur if a considerable leading current is taken from it when it has low voltage and low field current.

257 In Magnetic Commutation, the control is effected by shunting the magmetic flux through a secondary coil so as to vary the induction through the coil and the voltage induced in it. The shunting should be effected smoothly, thus avoiding sudden changes in the induced voltage.

258 In Transformer Voltage Control, by change of ratio, it is necessary that the transition from one step to another be made without interruption of the test voltage, and by steps sufficiently small to prevent surges in the testing circuit. The necessity of this precaution is greater as the inductance or the static capacity of the apparatus in the testing circuit under test is greater.

259 When Resistance Coils or Reactors are used for voltage control, it is desirable that the testing voltage should be secured when the controlling resistance or reactance is very nearly or entirely out of circuit in order that the disturbing effect upon the wave form which results may be negligible at the highest voltage.

F. CONDUCTIVITY.

260 Copper. The conductivity of copper in annealed wires and in electric cables should not be less than 98 per cent of the Annealed Copper Standard, and the conductivity of hard-drawn copper wires should not be less than 95 per cent of the Annealed Copper Standard. The Annealed Copper Standard represents a mass-resistivity of 0.153022 ohm per meter-gram at 20 deg. cent. or 873.75 ohms per mile-pound at 20 deg. cent.; or using a density of 8.89, a volume-resistivity of 1.72128 microhm-cm., or microhms in a cm. cube, at 20 deg. cent. or 0.67767microhm-inch at 20 deg. cent.

G. RISE OF TEMPERATURE.

(I) *MEASUREMENT OF TEMPERATURE.*

(*A*) Methods.

261 There are two methods in common use for determining the rise in temperature, *viz.*: (1) by thermometer, and (2) by increase in resistance of an electric circuit.

262 1. By **Thermometer.** The following precautions should be observed in the use of thermometers:

263 *a*. **Protection.** The thermometers indicating the room temperature should be protected from thermal radiation emitted by heated bodies, or from draughts of air or from temporary fluctuations of temperature. Several room thermometers should be used. In using the thermometer by applying it to a heated part, care should be taken so to protect its bulb as to prevent radiation from it, and, at the same time, not to interfere seriously with the normal radiation from the part to which it is applied.

264 *b*. **Bulb.** When a thermometer is applied to the free surface of a machine, it is desirable that the bulb of the thermometer should be covered by a pad of definite area. A convenient pad may be formed of cotton waste in a shallow circular box about one and a half inches in diameter, through a slot in the side in which the thermometer bulb is inserted. An unduly large pad over the thermometer tends to interfere with the natural liberation of heat from the surface to which the thermometer is applied.

265 2. By **Increase in Resistance.** The resistance may be measured either by the Wheatstone bridge, the Thomson or Kelvin double bridge, the potentiometer method, or the ammeter and voltmeter method. If a temperature coefficient must be assumed, its value for copper may be taken to be 0.00394 per deg. cent. from and at 20 deg. cent., or 0.00428 per deg. cent. from and at 0 deg. cent. This value holds for average commercial *annealed* copper. If the copper wire is hard-drawn, or if the conductivity is known, a different value of temperature coefficient should be taken, according to the explanation and discussion of the temperature coefficient in Appendix E.

The temperature rise may be determined either (1) by dividing the per cent increase of initial resistance by the temperature coefficient for the initial temperature expressed in per cent; or (2) by multiplying the increase in per cent of the initial resistance by T plus the initial temperature in degrees cent., and then dividing the product by 100. ($-T$ is the " inferred absolute zero temperature of resistance " and is given in the last column of the table in Appendix E. For average commercial *annealed* copper it is 233.8).

266 3. **Comparison of Methods.** In electrical conductors, the rise of temperature should be determined by their increase of resistance where practicable. Temperature elevations measured in this way are usually in excess of temperature elevations measured by thermometers. In very low resistance circuits, thermometer measurements are frequently more reliable than measurements by the resistance method. Where a thermometer applied to a coil or winding, indicates a higher temperature elevation than that shown by resistance measurement, the thermometer indication should be accepted.

(*B*) Normal Conditions for Tests.

267 1. **Duration of Tests.** The temperature should be measured after a run of sufficient duration for the apparatus to reach a practically constant temperature. This is usually from 6 to 18 hours, according to the size and construction of the apparatus. It is permissible, however, to shorten the time of the test by running a lesser time on an overload in current and voltage, then reducing the load to normal, and maintaining it thus until the temperature has become constant.

268 2. **Room Temperature.** The rise of temperature should be referred to the standard condition of a room temperature of 25 deg. cent.

269 **Temperature Correction.** If the room temperature during the test

differs from 25 deg. cent., correction on account of difference in resistance should be made by changing the observed rise of temperature by one-half per cent for each degree centigrade. Thus with a room temperature of 35 deg. cent., the observed rise of temperature has to be decreased by 5 per cent, and with a room temperature of 15 deg. cent., the observed rise of temperature has to be increased by 5 per cent. In certain cases, such as shunt-field circuits without rheostat, the current strength will be changed by a change of room temperature. The heat-production and dissipation may be thereby affected. Correction for this should be made by changing the observed rise in temperature in proportion as the $I^2 R$ loss in the resistance of the apparatus is altered owing to the difference in room temperature.

270 3. Barometric Pressure. Ventilation. A barometric pressure of 760 mm. and normal conditions of ventilation should be considered as standard, and the apparatus under test should neither be exposed to draught nor enclosed, except where expressly specified. The barometric pressure needs to be considered only when differing greatly from 760 mm.

271 Barometric Pressure Correction. When the barometric pressure differs greatly from the standard pressure of 760 mm. of mercury, as at high altitudes, a correction should be applied. In the absence of more nearly accurate data, a correction of one per cent of the observed rise in temperature for each 10 mm. deviation from the 760-mm. standard is recommended. For example at a barometric pressure of 680 mm. the observed rise of temperature is to be reduced by $\frac{760-680}{10} = 8$per cent.

(II) *LIMITING TEMPERATURE RISE.*

272 General. The temperature of electrical machinery under regular service conditions, should never be allowed to remain at a point at which permanent deterioration of its insulating material takes place.

273 Limits Recommended. It is recommended that the following maximum values of temperature elevation, referred to a standard room temperature of 25 degrees centigrade, at rated load under normal conditions of ventilation or cooling, should not be exceeded.

(*A*) Machines in General.

274 In commutating machines, rectifying machines, pulsating-current generators, synchronous machines, synchronous commutating machines and unipolar machines, the temperature rise in the parts specified should not exceed the following:

275 Field and armature, 50 deg. cent.

276 Commutator and brushes, by thermometer, 55 deg. cent.

277 Collector rings, 65 deg. cent.

278 Bearings and other parts of machine, by thermometer, 40 deg. cent.

279 (*B*) Rotary Induction Apparatus. The temperature rise should not exceed the following:

280 Electric circuits, 50 deg. cent., by resistance.

281 Bearings and other parts of the machine 40 deg. cent., by thermometer.

282 In squirrel-cage or short-circuited armatures, 55 deg. cent., by thermometer, may be allowed.

(*C*) Stationary Induction Apparatus.

283 *a*. Transformers for Continuous Service. The temperature rise should not exceed 50 deg. cent. in electric circuits, by resistance; and in other parts, by thermometer.

284 *b*. Transformers for Intermittent Service. In the case of transformers intended for intermittent service, or not operating continuously at rated load, but continuously in circuit, as in the ordinary case of lighting transformers, the temperature elevation above the surrounding air-temperature should not exceed 50 deg. cent., by resistance in electric circuits and by thermometer in other parts, after the period corresponding to the term of rated load. In this instance, the test load should not be applied until the transformer has been in circuit for a sufficient time to attain the temperature elevation due to core loss. With transformers for commercial lighting, the duration of the rated-load test may be taken as three hours, unless otherwise specified.

285 *c.* Reactors, Induction- and Magneto-Regulators. Electric circuits by resistance and other parts by thermometer, 50 deg. cent.

286 *d.* Large Apparatus. Large generators, motors, transformers, or other apparatus in which reliability and reserve overload capacity are important, are frequently specified not to rise in temperature more than 40 deg. cent. under rated load and 55 deg. cent. at rated overload. It is, however, ordinarily undesirable to specify lower temperature elevations than 40 deg. cent. at rated load, measured as above.

(*D*) Rheostats.

287 In Rheostats, Heaters and other electrothermal apparatus, no combustible or inflammable part or material, or portion liable to come in contact with such material, should rise more than 50 deg. cent. above the surrounding air under the service conditions for which it is designed.

288 *a.* Parts of Rheostats. Parts of rheostats and similar apparatus rising in temperature, under the specified service conditions, more than 50 deg. cent., should not contain any combustible material, and should be arranged or installed in such a manner that neither they, nor the hot air issuing from them, can come in contact with combustible material.

(*E*) Limits Recommended in Special Cases.

289 *a.* Heat Resisting Insulation. With apparatus in which the insulating materials have special heat-resisting qualities, a higher temperature elevation is permissible.

290 *b.* High Air Temperature. In apparatus intended for service in places of abnormally high temperature, a lower temperature elevation should be specified.

291 *c.* Apparatus Subject to Overload. In apparatus which by the nature of its service may be exposed to overload, or is to be used in very high voltage circuits, a smaller rise of temperature is desirable than in apparatus not liable to overloads or in low-voltage apparatus. In apparatus built for conditions of limited space, as railway motors, a higher rise of temperature must be allowed.

292 *d.* Apparatus for Intermittent Service. In the case of apparatus intended for intermittent service, except railway motors, the temperature elevation which is attained at the end of the period corresponding to the term of rated load, should not exceed the values specified for machines in general. In such apparatus, including railway motors, the temperature elevation should be measured after operation, under as nearly as possible the conditions of service for which the apparatus is intended, and the conditions of the test should be specified.

H. OVERLOAD CAPACITIES.

293 Performance with Overload. All apparatus should be able to carry the overload hereinafter specified without serious injury by heating, sparking, mechanical weakness, etc., and with an additional temperature rise not exceeding 15 deg. cent., above those specified for rated loads, the overload being applied after the apparatus has acquired the temperature corresponding to rated load continuous operation. Rheostats to which no temperature rise limits are attached are naturally exempt from this additional temperature rise of 15 deg. cent. under overload specified in these rules.

294 Normal Conditions. Overload guarantees should refer to normal conditions of operation regarding speed, frequency, voltage, etc., and to non-inductive conditions in alternating apparatus, except where a phase displacement is inherent in the apparatus.

295 Overload Capacities Recommended. The following overload capacities are recommended:

296 *a*. Generators. Direct-current generators and alternating-current generators, 25 per cent for two hours.

297 *b*. Motors. Direct-current motors, induction motors and synchronous motors, not including railway and other motors intended for intermittent service, 25 per cent for two hours, and 50 per cent for one minute.

298 *c*. Converters. Synchronous converters, 25 per cent for two hours, 50 per cent for one-half hour.

299 *d*. Transformers and Rectifiers. Constant-potential transformers and rectifiers, 25 per cent for two hours; except in transformers connected to apparatus for which a different overload is guaranteed in which case the same guarantees shall apply for the transformers as for the apparatus connected thereto.

300 *e*. Exciters. Exciters of alternators and other synchronous machines, 10 per cent more overload than is required for the excitation of the synchronous machine at its guaranteed overload, and for the same period of time. All exciters of alternating-current, single-phase or polyphase generators, should be able to give at their rated speed, sufficient voltage and current to excite their alternators, at the rated speed, to the full-load terminal voltage, at the rated output in kilovolt-amperes and with 50 per cent power factor.

301 *f*. A Continuous-Service Rheostat, such as an armature- or field-regulating rheostat, should be capable of carrying without injury for two hours, a current 25 per cent greater than that at which it is rated. It should also be capable of carrying for one minute a current 50 per cent greater than its rated load current, without injury. This excess of capacity is intended for testing purposes only, and this margin of capacity should not be relied upon in the selection of the rheostat.

302 *g*. An Intermittent Service or Motor-starting Rheostat is used for starting a motor from rest and accelerating it to rated speed. Under ordinary conditions of service, and unless expressly stated otherwise, a motor is assumed to start in fifteen seconds and with 150 per cent of rated current strength. A motor-starter should be capable of starting the motor under these conditions once every four minutes for one hour.

303 (*a*) This Test may be carried out either by starting the motor at four-minute intervals, or by placing the starter at normal temperature across the maximum voltage for which it is marked, and moving the lever uniformly and gradually from the first to the last position during a period of fifteen seconds, the current being maintained substantially constant at said 50 per cent excess, by introducing resistance in series or by other suitable means.

304 (*b*) Other Rheostats for Intermittent-Service are employed under such special and varied conditions, that no general rules are applicable to them.

III. VOLTAGES AND FREQUENCIES.

A. VOLTAGES.

305 Direct-Current Generators. In direct-current, low-voltage generators, the following average terminal voltages are in general use and are recommended:

125 volts. 250 volts. 600 volts.

306 Low-Voltage Circuits. In direct-current low-voltage circuits, the following terminal voltages are in general use and are recommended:

115 volts. 230 volts. 550 volts.

In alternating-current low-voltage circuits, the following terminal voltages are in general use and are recommended.

110 volts. 220 volts. 440 volts. 550 volts.

307 Primary Distribution Circuits. In alternating-current, constant-potential, primary-distribution circuits, an average voltage of 2,200 volts, with step-down transformer ratios 1/10 and 1/20, is in general use, and is recommended.

308 Transmission Circuits. In alternating-current constant-potential transmission circuits, the following impressed voltages are recommended.

6,600 11,000 22,000 33,000 44,000 66,000 88,000 110,000

309 Transformer Ratio. It is recommended that the standard transformer ratios should be such as to transform between the standard voltages above named. The ratio will, therefore, usually be an exact multiple of 5 or 10, *e.g.*, 2,200 to 11,000; 2,200 to 44,000.

310 Range in Voltage. In alternating-current generators, or generating systems, a range of terminal voltage should be provided from rated voltage at no load to 10 per cent in excess thereof, to cover drop in transmission. If a greater range than ten per cent is specified, the generator should be considered as special.

B. FREQUENCIES.

311 In Alternating-Current Circuits, the following frequencies are, standard:

25 cycles 60 cycles

312 These frequencies are already in extensive use and it is deemed advisable to adhere to them as closely as possible.

IV. GENERAL RECOMMENDATIONS.

313 Name Plates. All electrical apparatus should be provided with a name plate giving the manufacturers' name, the voltage and the current in amperes for which it is designed. Where practicable, the kilowatt capacity, character of current, speed, frequency, type, designation and serial number should be added.

314 Diagrams of Connections. All electrical apparatus when leaving the factory should be accompanied by a diagram showing the electrical connections and the relation of the different parts in sufficient detail to give the necessary information for proper installation.

315 Rheostat Data. Every rheostat should be clearly and permanently marked with the voltage and amperes, or range of amperes, for which it is designed.

316 Colored Indicating Lights. When using colored indicating lights on switch-boards, red should denote danger such as " switch closed," or " circuit alive "; green should denote safety, such as " switch open," or " circuit dead."

317 When white lights are used a light turned on should denote danger, such as " switch closed " or " circuit alive "; while the light out should denote safety, such as " switch open," or " circuit dead." Low-efficiency lamps should be used on account of their lesser liability to accidental burn-out.

318 The use of colored lights is recommended, as safer than white lights.

319 Grounding Metal Work. It is desirable that all metal work near high potential circuits be grounded.

320 Circuit Opening Devices. The following definitions are recommended.

321 *a*. A Circuit-Breaker is an apparatus for breaking a circuit at the highest current which it may be called upon to carry.

322 *b*. A Disconnecting Switch is an apparatus designed to open a circuit only when carrying little or no current.

323 *c*. An Automatic Circuit-Breaker is an apparatus for breaking a circuit automatically under an excessive strength of current. It should be capable of breaking the circuit repeatedly at rated voltage and at the maximum current which it may be called upon to carry.

V. APPENDICES AND TABULAR DATA.

APPENDIX A. NOTATION.

The following notation is recommended:

Name of Quantity	Symbol	Unit
324 Voltage, e.m.f., potential difference	E, e,	volt
Current	I, i.	ampere
Resistance	R, r,	ohm
Reactance	X, x,	"
Impedance	Z, z,	"
Admittance	Y, y,	mho
Conductance	G, g,	"
Susceptance	B, b,	"
Power	P, p,	watt
Capacity	C, c,	farad
Inductance	$\mathcal{L}$, L,	henry
Magnetic flux	Φ	maxwell
Magnetic density	$\mathcal{B}$,	gauss
Magnetic force	H,	gilbert per cm.
Length	L, l,	cm. or inch
Mass	M, m,	gm. or lb.
Time	T, t,	second or hour

E_m, I_m and B_m should preferably be used for maximum cyclic values, e, i and p for instantaneous values, E and I for r.m.s. values (see Sec. 5g.) and P for the average value or effective power. These distinctions are not necessary in dealing with continuous-current circuits. Vector quantities are preferably represented by bold face capitals.

APPENDIX B.—RAILWAY MOTORS.

(I) *RATING.*

325 Introductory Note on Rating. Railway motors usually operate in a service in which both the speed and the torque developed by the motor are varying almost continually. The average requirements, however, during successive hours in a given class of service are fairly uniform. On account of the wide variation of the instantaneous loads, it is impracticable to assign any simple and definite rating to a motor which will indicate accurately the absolute capacity of a given motor or the relative capacity of different motors under service conditions. It is also impracticable to select a motor for a particular service without much fuller data with regard both to the motor and to the service than is required, for example, in the case of stationary motors which run at constant speeds.

326 Scope of Nominal Rating. It is common usage to give railway motors a nominal rating in horse power on the basis of a one-hour test. As above explained, a simple rating of this kind is not a proper measure of service capacity. This nominal rating, however, indicates approximately the maximum output which the motor should ordinarily be called upon to develop during acceleration. Methods of determining the continuous capacity of a railway motor for service requirements are given under a subsequent heading.

327 The Nominal Rating of a railway motor is the horse-power output at the car-axle, that is, including gear and other transmission losses, which gives a rise of temperature above the surrounding air (referred to a room temperature of 25 deg. cent.) not exceeding 90 deg. cent. at the commutator and 75 deg. cent. at any other part after one hour's continuous run at its rated voltage (and frequency, in the case of an alternating-current motor) on a stand, with the motor-covers removed, and with natural ventilation. The rise in temperature is to be determined by thermometer, but the resistance of no electrical circuit in the motor shall increase more than 40 per cent during the test.

(II) *SELECTION OF MOTOR FOR SPECIFIED SERVICE.*

328 GENERAL REQUIREMENTS. The suitability of a railway motor for a specified service depends upon the following considerations:

329 *a.* Mechanical ability to develop the requisite torque and speeds as given by its speed-torque curve.

330 *b.* Ability to commutate successfully the current demanded.

331 *c.* Ability to operate in service without occasioning a temperature rise in any part which will endanger the life of the insulation.

332 OPERATING CONDITIONS, TYPICAL RUN. The operating conditions which are important in the selection of a motor include the weight of load, the schedule speed, the distance between stops, the duration of stops, the rate of acceleration and of braking retardation, the grades and the curves, with these data at hand, the outputs which are required of the motor may be determined, provided the service requirements are within the limits of the speed-torque curve of the motor. These outputs may be expressed in the form of curves giving the instantaneous values of current and of voltage which must be applied to the motor. Such curves may be laid out for the entire line, but they are usually constructed only for a certain average or typical run, which is fairly representative of the conditions of service. To determine whether the motor has sufficient capacity to perform the service safely, further tests or investigations must be made.

333 CAPACITY TEST OF RAILWAY MOTOR IN SERVICE. The capacity of a railway motor to deliver the necessary output may be determined by measurement of its temperature after it has reached a maximum in service. If a running test cannot be made under the actual conditions of service, an equivalent test may be made in a typical run back and forth, under such conditions of schedule speed, length of run, rate of acceleration, etc., that the test cycle of motor losses and conditions of ventilation are essentially the same as would be obtained in the specified service.

334 METHODS OF COMPARING MOTOR CAPACITY WITH SERVICE REQUIREMENTS. Where it is not convenient to test motors under actual service conditions or in an equivalent typical run, recourse may be had to one of the two following methods of determining temperature rise now in general use:

335 1. METHOD BY LOSSES AND THERMAL CAPACITY CURVES. The heat developed in a railway motor is carried partly by conduction through the several parts and partly by convection through the air to the motor-frame whence it is distributed to the outside air. As the temperature of the several parts is thus dependent not only upon their own internal losses but also upon the temperature of neighboring parts, it becomes necessary to determine accurately the actual value and distribution of losses in a railway motor for a given service and reproduce them in an equivalent test-run. The results of a series of typical runs expressed in the form of thermal capacity curves will give the relation between degrees rise per watt loss in the armature and in the field for all ratios of losses between them met with in the commercial application of a given motor.

336 This method consists, therefore, in calculating the several internal motor losses in a specified service and determining the temperature rise with these losses from thermal capacity curves giving the degrees rise per watt loss as obtained in experimental track tests made under the same conditions of ventilation.

337 The following motor losses cause its heating and should be carefully determined for a given service: $I^2 R$ in the field; $I^2 R$ in the armature; $I^2 R$ in the brush contacts, core loss and brush friction.

338 The loss in the bearings (in the case of geared motors) also adds somewhat to the motor-heating, but owing to the variable nature of such losses they are generally neglected in making calculations.

339 2. METHOD BY CONTINUOUS CAPACITY OF MOTOR. The essential losses in the motor, as found in the typical run, are in most cases those in the motor windings and in the core. The mean service conditions may be expressed in terms of the current which would produce the same losses in the motor windings and the voltage which, with that current, would produce the same core losses as the average in service. The continuous capacity of the motor is given in terms of the amperes which it will carry when run on a testing stand—with covers on or off, as specified—at different voltages, say, 40, 60, 80 and 100 per cent of the rated voltage—with a temperature rise not exceeding 90 deg. cent at the commutator and 75 deg. cent at any other part, provided the resistance of no electric circuit in the motor increases more than 40 per cent. A comparison of the equivalent service conditions with the continuous capacity of the motor will determine whether the service requirements are within the safe capacity of the motor.

340 This method affords a ready means of determining whether a specified service is within the capacity of a given motor and it is also a convenient approximate method for comparing the service capacities of different motors.

APPENDIX C. PHOTOMETRY AND LAMPS.

341 Candle-Power. The luminous intensity of sources of light is expressed in candle-power. The unit of candle-power is the international candle maintained by the Bureau of Standards at Washington, D. C. The Hefner unit is 0.90 of the international candle.

342 Lumen. The total flux of light from a source is equal to its mean spherical intensity multiplied by 4π. The unit of flux is called the lumen. A lumen is the $\frac{1}{4\pi}$th part of the total flux of light emitted by a source having a mean spherical intensity of one candle-power.

344 Illumination. The fundamental physical unit of illumination is the centimeter-candle, or lumen per square centimeter of incident surface. This is a very intense illumination. It is, therefore, convenient to express illumination practically in thousandths of the fundamental unit; *i.e.*, in millilumens per sq. cm. In English-speaking countries, the unit of illumination commonly employed is the foot-candle or lumen per square foot. A foot-candle is nearly the same illumination as a millilumen per sq. cm. and is actually the more intense in the ratio 1.0764; so that n foot-candles $= 1.0764 \times n$ millilumens per sq. cm. A meter candle or lumen per square meter, is called a " lux ". A foot-candle is 10.764 lux, and a millilumen per sq. cm. is exactly 10 lux.

346 The Efficiency of Electric Lamps is properly stated in terms of lumens per watt at lamp terminals. This use of the term efficiency is to be considered as special, and not to be confused with the generally accepted definition of efficiency in Sec. 84.

347 *a.* Efficiency, Auxiliary Devices. In illuminants requiring auxiliary power-consuming devices outside of the luminous body, such as steadying resistances in constant potential arc lamps, a distinction should be made between the net efficiency and the gross efficiency of the lamp. This distinction should always be stated. The gross efficiency should include the power consumed in the auxiliary resistance, etc. The net efficiency should, however, include the power consumed in the controlling mechanism of the lamp itself. Comparison between such sources of light should be made on the basis of gross efficiency, since the power consumed in the auxiliary device is essential to the operation.

348 A Standard Circuit Voltage of 110 volts, or a multiple thereof may be assumed, except where expressly stated otherwise.

349 Watts per Candle. The specific consumption of an electric lamp is its watt consumption per mean spherical candle-power. " Watts per candle " is the term used commercially in connection with incandescent lamps, and denotes, watts per mean horizontal candle-power.

350 Photometric Tests in which the results are stated in candle-power should be made at such a distance from the source of light that the latter may be regarded as practically a point. Where tests are made at shorter distances, as for example in the measurement of lamps with reflectors, the results should always be given as " apparent candle-power " at the distance employed, which distance should always be specifically stated.

351 Basis for Comparison. Either the total flux of light in lumens, or the mean spherical candle-power, should always be used as the basis for comparing various luminous sources with each other, unless there is a clear understanding or statement to the contrary.

352 Incandescent Lamps, Rating. It is customary to rate incandescent lamps on the basis of their mean horizontal candle-power; but in comparing incandescent lamps in which the relative distribution of luminous intensity differs, the comparison should be based on their total flux of light measured in lumens, or on their mean spherical candle-power.

352a Life Tests. Similar filaments may be assumed to operate at the same temperature only when their lumens per watt consumed are the same. Life tests are comparable only when conducted under similar conditions as to filament temperatures.

353 The Spherical Reduction-Factor of a lamp

$$= \frac{\text{mean spherical candle-power}}{\text{mean horizontal candle-power}}$$

354 The Total Flux of light in lumens emitted by a lamp $= 4\pi \times$ mean horizontal candle-power × spherical reduction-factor.

355 The Spherical Reduction-Factor should only be used when properly determined for the particular type and characteristics of each lamp. The spherical reduction-factor permits of substantially accurate comparisons being made between the total lumens, or mean spherical candle-powers of different types of incandescent lamps, and may be used in the absence of proper facilities for direct measurement of the total lumens, or mean spherical candle-power.

356 " Reading Distance." Where standard photometric measurements are impracticable, approximate measurements of illuminants such as street lamps may be made by comparing their " reading distances;" *i.e.*, by determining alternately the distances at which an ordinary size of reading print can just be read, by the same person or persons, when all other light is screened. The angle below the horizontal at which the measurement is made should be specified when it exceeds 15 degrees. Reading-distance methods usually involve the comparison of very faint illuminations and hence the results may be seriously affected by the Purkinje effect.

357 In Comparing Different Luminous Sources not only should their candle-power be compared, but also their relative form, brightness, distribution of illumination and character of light.

357a The following symbols are recommended in connection with photometry:

Photometric magnitude	Symbol	Unit
Intensity of light.	I	International candle.
Luminous flux.	F	Lumen.
Illumination.	E	Lumen/cm.^{2}, foot-candle.
Specific radiation.	R	Foot-candle.
Brightness.	b	Candle/cm.^{2}
Quantity.	Q	Candle.
Lighting.	L	Lumen-second, lumen-hour.

APPENDIX D. SPARKING DISTANCES.

358 Table of Sparking Distances in Air between Opposed Sharp Needle-Points, for Various Root-Mean-Square Sinusoidal Voltages, in inches and in centimeters. The table applies to the conditions specified in Secs. 240–246.

359

Kilovolts R.M.S.	Distance. Inches	Distance. Cm.	Kilovolts R.M.S.	Distance. Inches	Distance. Cm.
5	0.225	0.57	140	13.95	35.4
10	0.47	1.19	150	15.0	38.1
15	0.725	1.84	160	16.05	40.7
20	1.0	2.54	170	17.10	43.4
25	1.3	3.3	180	18.15	46.1
30	1.625	4.1	190	19.20	48.8
35	2.0	5.1	200	20.25	51.4
40	2.45	6.2	210	21.30	54.1
45	2.95	7.5	220	22.35	56.8
50	3.55	9.0	230	23.40	59.4
60	4.65	11.8	240	24.45	62.1
70	5.85	14.9	250	25.50	64.7
80	7.1	18.0	260	26.50	67.3
90	8.35	21.2	270	27.50	69.8
100	9.6	24.4	280	28.50	72.4
110	10.75	27.3	290	29.50	74.9
120	11.85	30.1	300	30.50	77.4
130	12.90	32.8			

APPENDIX E. TEMPERATURE COEFFICIENT OF COPPER.

360 The fundamental relation between the rise of temperature and the increase of resistance of copper may be expressed thus:

$$R_t = R_{t_1} \; (1 + \alpha_{t_1} \; [t - t_1])$$

where R_t is the resistance at any temperature t deg. cent.; R_{t_1} is the resistance at any "initial temperature" (or "temperature of reference") t_1 deg. cent.; and α_{t_1} is the temperature coefficient from and at the initial temperature t_1 deg. cent. Obviously the temperature coefficient is different for different initial temperatures, and this variation is shown in the horizontal rows of the table below. Furthermore, it has been shown that the temperature coefficient is different for different conductivities, and that the temperature coefficient is substantially proportional to the conductivity. The results of this simple law are shown by the vertical columns of the table below.

TEMPERATURE COEFFICIENTS OF COPPER FOR DIFFERENT INITIAL TEMPERATURES AND DIFFERENT CONDUCTIVITIES

Ohms per meter-gram at 20 deg. cent	Per cent conductivity	α_0	α_{15}	α_{20}	α_{25}	α_{30}	α_{50}	$-T$ "Inferred absolute zero"
0.16108	95	0.00405	0.00381	0.00374	0.00367	0.00361	0.00336	-247.2
0.15940	96	0.00409	0.00386	0.00378	0.00371	0.00364	0.00340	-244.4
0.15776	97	0.00414	0.00390	0.00382	0.00375	0.00368	0.00343	-241.7
0.15727	97.3	0.00415	0.00391	0.00383	0.00376	0.00369	0.00344	-240.9
0 15614	98	0.00418	0.00394	0.00386	0.00379	0.00372	0.00346	-239.0
0.15457	99	0.00423	0.00398	0.00390	0.00383	0.00375	0.00349	-236.4
0.153022	100	0.00428	0.00402	0.00394	0.00386	0.00379	0.00352	-233.8
0.15151	101	0.00432	0.00406	0.00398	0.00390	0.00383	0.00355	-231.3

The quantity ($-T$) given in the last column of the above table is the calculated temperature on the centigrade scale at which copper of the particular conductivity concerned would have zero electrical resistance *provided* the temperature coefficient between 0 deg. cent and 100 deg. cent. applied continuously down to the absolute zero. The usefulness of this "inferred absolute zero temperature of resistance" in calculating temperature rise is evident from the following formula:

$$t - t_1 = \frac{R_t - R_{t_1}}{R_{t_1}} (T + t_1)$$

The presentation of the above table is intended to emphasize the desirability of determining the temperature coefficient rather than assuming it. Actual experimental determination is facilitated by the proportional relation between the temperature coefficient and the conductivity; a measurement of either quantity gives both. However, if a temperature coefficient *must* be assumed, the best value to take for average commercial *annealed* copper wire is that given in the table for 100 per cent conductivity, *viz.*,

$$\alpha_0 = 0.00428, \ \alpha_{20} = 0.00394, \ \alpha_{25} = 0.00386$$

This is the value recommended for wire wound on instruments and machines, since they are generally wound with annealed wire and experiments have shown that the distortions due to the winding of the wire do not appreciably affect the temperature coefficient.

If a value must be assumed for *hard-drawn* copper wire, the value recommended is that given in the table for 97.3 per cent conductivity, *viz.*,

$$\alpha_0 = 0.00415, \ \alpha_{20} = 0.00383, \ \alpha_{25} = 0.00376$$

The temperature coefficients in fahrenheit degrees are given by dividing any α above by 1.8. Thus, the 20 deg. cent. or 68 deg. fahr. temperature coefficient for copper of 100 per cent conductivity is 0.00394 per deg. cent., or 0.00219 per deg. fahr.

APPENDIX F. HORSEPOWER.

361 In view of the fact that a horsepower defined as 550 foot-pounds per second represents a power which varies slightly with the latitude and altitude (from 743.3 to 747.6 watts) and also in view of the fact that different authorities differ as to the precise value of the horsepower in watts, *the Standards Committee has adopted* 746 *watts as the value of the horsepower.* The number of foot-pounds per second to be taken as one horsepower is therefore such a value at any given place as is equivalent to 746 watts; the number varies from 552 to 549 foot-pounds per second, being 550 at 50 deg. latitude (London), and 550.5 at Washington. The Standards Committee, however, recommends that the kilowatt instead of the horsepower be used generally as the unit of power.

ADDENDA

TO THE

A. I. E. E. STANDARDIZATION RULES

G. Official Actions of the Turin Congress

AND

H. Rating of Electrical Machinery in Different Countries

Printed by order of the Board of Directors in Accordance with the Following Resolution Adopted October 13, 1911:

Resolved, that the Secretary be authorized to publish with the Standardization Rules of the Institute the decisions of the International Electrotechnical Commission at Turin in regard to standard symbology and the direction for indicating advancement of phase in graphic diagrams of alternating current quantities, and a resume of the principal features of the rating of electrical machinery of the leading foreign countries.

The following appendix, G, covers the principal official actions of the Turin Congress in 1911.

APPENDIX G.—INTERNATIONAL AGREEMENTS OF THE I.E.C.

The following is a brief résumé of the official actions of the International Electrotechnical Commission at Turin in 1911.

(a) *INTERNATIONAL SYMBOLS*

1. Instantaneous values of electrical quantities which vary with the time are to be represented by small letters.
2. Virtual or constant values of electrical quantities to be represented by capital letters.
3. Maximum values of periodic electrical quantities to be represented by capital letters followed by the subscript " *m* "
4. Magnetic quantities, whether constant or variable, to be represented by capital letters of either script, gothic, heavy-faced, or of any special type.
5. Maximum values of magnetic quantities to be represented by capital letters of either script, gothic, heavy-faced, or of any special type, followed by the subscript " *m* ".
6. The following quantities to be represented by the following letters:

Electromotive force	E, e	
Electric quantity	Q, q	
Self-inductance*	ℒ, **L**	†
Magnetic force	ℋ, **H**	
Magnetic flux density	ℬ, **B**	
Length	L, l	
Mass	M, m	
Time	T, t	

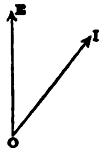
7. The letters I, E, and R were definitely adopted to represent, respectively, the current, the electromotive force, and the resistance, in the simple algebraical expression of Ohm's law.
8. In all questions relative to alternating currents, the expression " Reactive Power " is adopted to designate the quantity $UI \sin \phi$.

(b) *DIAGRAMS FOR ALTERNATING CURRENTS*

In the graphical representation of alternating electric and magnetic quantities, advance in phase shall be represented in the counter-clockwise direction.

NOTE. The impedance of a reactive coil, of resistance R, and inductance L, is $R+\sqrt{-1}\,L\,\omega$, and that of a condenser of capacity C, is

$\frac{1}{\sqrt{-1}\;C\;\omega}$ where ω is equal to $2\,\pi \times$ frequency.



It follows also that the diagram herewith represents the phase relations in a simple alternating-current circuit containing an impressed electromotive force $O\,E$ and a lagging current $O\,I$.

(c) *RATINGS OF ELECTRICAL MACHINERY AND APPARATUS*

The propositions of the Brussels conference in regard to rating were adopted without modification as follows:

1. The output of electric generators is defined as the electric power available at the terminals.
2. The output of electric motors is defined as the mechanical power available at the shaft.
3. Both the electric and mechanical powers to be expressed in international watts.

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*Coefficient of self-induction.

†As examples only.

Appendices H and I give comparisons of methods of rating electrical machinery, and particularly of D-C. machinery, in different countries, as compiled by the Secretary of the International Electrotechnical Commission. See Publication No. 9, "Rating of Electrical Machinery," of the I. E. C. August, 1911.

APPENDIX H.—RATING OF ELECTRICAL MACHINERY

The following are the comparative rules on the rating of direct current generators and motors as compiled by the General Secretary of the International Electrotechnical Commission from the National Rules of six countries as in force in 1911. These comparative rules have been appended to the Standardization Rules by order of the Board of Directors of the American Institute of Electrical Engineers, in order to present the extent of agreement or diversity existing on these rules, in 1911, among the leading electrical engineering societies of the world.

List of documents from which extracts have been made:

BELGIUM. "Prescriptions normales" for the reception of electrical machines and transformers issued by the "Chambre syndicale des Electriciens Belges" in 1908.

FRANCE. General instructions for the delivery and reception of electrical machines and transformers issued by the "Union des Syndicats de l'Electricité" in 1910.

Regulations for tenders, supply and testing of electrical machines and transformers issued by the "Association Alsacienne des Propriétaires d'Appareils à Vapeur," 1906.

GERMANY. Standard rules for the utilization and testing of electrical machines and transformers issued by the "Verband Deutscher Elektrotechniker" in 1910.

GREAT BRITAIN. Report of the British Engineering Standards Committee on "Electrical Machinery" issued in 1907.

SWEDEN. Rules for the testing and reception of electrical machines and transformers issued by the "Association of Swedish Engineers" in 1909.

UNITED STATES. Standardization Rules of the American Institute of Electrical Engineers as contained in its PROCEEDINGS, August, 1911.

ANALYSIS OF THE RULES

POWER

Method of Expressing the Power of Electrical Machines

BELGIUM.	
Generators.	Kw. at the machine terminals.
Motors.	Mechanical horse power at the shaft (75 kg-m. per sec.)
FRANCE.	
Generators.	Kw. at the machine terminals.
Motors.	Kw. or horse power at the shaft (75 kg-m. per sec.).
GERMANY.	
Generators.	Kw.
Motors.	Horse power (75 kg-m. per sec. =736 W.).
GREAT BRITAIN.	
Generators.	Kw.
Motors.	B.h.p. (1 Brake horse power=746 W.).
SWEDEN.	
Generators.	Kw.
Motors.	Horse power (75 kg-m. per sec.).
UNITED STATES.	
Generators.	Kw. at the machine terminals.
Motors.	B.h.p. (746 W.) Preferably in kilowatts.

NOTES.—BELGIUM. Motors usually have their power indicated as "H.P.," with consequent confusion as to which is intended; the English b.h.p. being 746 watts, the "cheval-vapeur" being equivalent to 736 watts.

FRANCE. The Association Alsacienne still allows the Poncelet of 100 kg-m. per sec.

RATING

Belgium.

(1) *Intermittent service.* In which the periods of work and rest alternate in minutes.

(2) *Momentary service.* In which periods of work of sufficiently short duration for a stationary temperature not to be reached are followed by periods of rest long enough for the temperature to fall to approximately that of the surrounding air.

(3) *Continuous service.* In which the periods of work are sufficiently prolonged to lead to the establishment of stationary conditions of temperature.

France.

(1) *Continuous service.*
(2) *Variable service.*
(3) *Intermittent service.* (Overhead traveler, crane, lift.)

Germany.

(1) *Intermittent service.* In which the periods of work and rest alternate in minutes. (Motors for cranes, lifts, tramways. and similar apparatus.)

(2) *Short period service.* In which the periods of work are not sufficiently long for the final (rated) temperature to be reached, whilst the periods of rest are long enough for the temperature to fall to approximately that of the surrounding air.

(3) *Continuous service.* In which the periods of work are sufficiently long for the final temperature (rated) to be attained.

Great Britain.

(1) *Continuous working.* The output of generators and motors for continuous working shall be the output at which they can work continuously for six hours and conform to the prescribed tests, and this output shall be defined as the Rated Load.

(2) *Intermittent working.* The output of motors for intermittent working shall be the output at which they can work for one hour and conform to the prescribed tests, and this output shall be defined as the Rated Intermittent Load.

Sweden.

(1) *Continuous service.* In which the temperature reaches stationary conditions.

(2) *Intermittent service.* In which the working periods do not exceed one hour or alternate with intervals of rest of a similar length.

United States.

(1) *Continuous rating.* In which, under load, there is the attainment of approximately stationary temperature.

(2) *Intermittent rating.* In which one minute periods of load and rest alternate until the attainment of approximately stationary conditions of temperature.

(3) *Minute rating.* In which, under load for one minute, no limit of capacity is exceeded, and no permanent change is wrought in the apparatus

(4) *Variable service rating.* Not yet defined.

Notes.—France. The Association Alsacienne does not recognize "variable service."

Great Britain. The reason for the " six hours " in the explanation of " continuous working " is due to the fact that the Committee considered standardization should not go beyond 1,000 kw., above that capacity being considered as special. One of the aims of standardization being to assist the manufacturer to carry stock, it would scarcely pay to do so above 1,000 kw.

" Continuous working " does not include machines running from week-end to week-end without a stop. The one-hour test under " Intermittent " is likely to be revised.

Sweden. The rated output under continuous service is defined as that output which can be obtained continuously for an unlimited period without the temperature limits specified being exceeded. The rated output under intermittent service is that output which can be obtained continuously for an hour without the temperature limits specified being exceeded.

NAME-PLATES

Information to be Stated on the Name-plates

Belgium. Service recognized (intermittent, momentary, for...... hours, or continuous). The rated values of the power, the pressure, the current and the angular speed.

France. The power for which the machine has been sold.

It is expressed thus: For generators, in kilowatts at the terminals; for motors, in kilowatts, or horse power of 75 kg-m. per second, available at the shaft.

Additional information to be stated: For continuous working, the rated values of the speed in revolutions per minute, the pressure, and the current; for variable working, the limits of pressure and current; for intermittent working (traveller, crane, lift, etc.) the power for a period of one hour, mentioning "intermittent working."

Germany. The power for generators to be stated in kilowatts (kw.). Mechanical power to be stated in kilowatts or in horse power (p.s.).

In addition, the name-plate on which the power is stamped, or a special name-plate, must indicate the rated value of the number of revolutions, the pressure and the current.

The name-plate must also indicate the rating: "Intermittent," or "For......hours," or "Continuous."

Great Britain. Unless otherwise stated, the output is to be considered to apply to continuous working. In the case of intermittent rating the word "Intermittent" is to be added.

The name-plate shall also indicate: For generators: Kw., volts, amperes, rev. per min. For motors: B.h.p., volts, amperes, rev. per min.; B.h.p (Intermittent), volts, amperes, rev. per min.

Sweden. Unless otherwise stated, the output shall be considered as applying to continuous working.

The values stated on the name-plate shall be in conformity with the rules, except it bear the indication "For special purposes."

The following information to be stated: For generators: the kilowatts, volts, amperes, and revs.per minute. For motors: the mechanical power, the volts, amperes, and revs. per minute.

N. B.—**The speed shall be the speed with the machine hot, and a tolerance of ± 7% shall be allowed.**

The service, adding the words: "For continuous service." "For intermittent service."

For variable service, the respective limits shall be given.

United States. Name of the maker, volts, amperes.

When practicable, the kilowatts, the revs. per minute, the type, the character of current, designation and serial number.

COMMUTATION

Belgium. Under all conditions not exceeding the rated load, the commutator of a machine shall not require to be cleaned or given other attention more than once a day, with the brushes remaining in the most favorable position.

France.

Union des Syndicats de L'Electricité. Under all loads included in running free up to the rated load, the brushes being ground and fixed in the most favorable position by means of a previous run, machines with a commutator must be capable of working for the period of the test (specified under *Heating: Duration of Test,*) without it being necessary to glass-paper the commutator or resort to any other method of cleaning.

Association Alsacienne. Unless otherwise specified, when once the brushes have been adjusted in the most favorable position, machines with commutators shall be capable of working, without appreciable sparking or adjustment of brushes, at all loads from no load to rated load, even with sudden changes of current.

Under continuous working, and at any load within the prescribed limits, the working of the commutator shall be such as to require attention (cleaning or lubricating) only at intervals of twelve hours. This condition also applies to collector rings.

GERMANY. Under all conditions of load from one quarter up to the rated load, and without alteration of the position of the brushes, the sparking produced must be so negligible as to permit the machine to work for at least 24 hours without the commutator requiring attention. The brushes to be ground in in the most favorable position.

GREAT BRITAIN. ————

SWEDEN. Unless otherwise specified, the commutation is to be without appreciable sparking from no load to 10 per cent overload, and without the position of the brushes being altered.

In the case of machines for variable working, the position of the brushes may be altered. The working position of the brushes is to be indicated.

UNITED STATES. The machine must carry the specified overload without injurious sparking or mechanical weakness.

OVERLOAD

The percentages given below imply an overload in excess of the rated load marked on the name-plate.

BELGIUM. For continuous service. 20 per cent of the rated load for a period equal to one-fifth of the duration of the heating test, with a maximum of one hour.

For intermittent service. 20 per cent of the rated load for one-fifth of an hour.

For momentary service. 20 per cent of the rated load for a period equal to one-fifth of the time marked on the name-plate. 40 per cent of the rated load for 3 minutes.

FRANCE. The test to be applied at the close of the rated load test. All machines must be capable of withstanding, without deterioration or appreciable sparking: 20 per cent of the rated load for one-tenth of the duration of the test for continuous working; 30 per cent of the rated load for 5 minutes.

Motors for intermittent service: 25 per cent of the rated load for 15 minutes; 30 per cent of the rated load for 5 minutes.

GERMANY. For generators and motors: 25 per cent of the rated load for half an hour.

For motors (constant potential): 40 per cent of the rated load for 3 minutes.

GREAT BRITAIN. (see *Heating at Conclusion of Overload Test*)

SWEDEN. 25 per cent in excess of the rated current without injurious sparking. Adjustment of brushes permitted.

UNITED STATES. For generators: 25 per cent of the rated load for 2 hours.

For motors (continuous working): 25 per cent of the rated load for 2 hours; 50 per cent of the rated load for 1 minute.

NOTES.—**BELGIUM.** The machine must be capable of sustaining the overload during the specified period without adjustment of the brushes. The overload test can be carried out at the close of the rated load test, in which case the rise in temperature must not exceed the limits prescribed for the rated load by more than 10 deg. cent.

FRANCE. The Association Alsacienne also specifies:

For motors intended to work for prolonged periods, an overload test of 40 per cent for three minutes.

All motors must be capable of withstanding an increase of 30 per cent in the rated speed for a period of five minutes.

GERMANY. The overload test is to be applied without reference to the temperature rise, and it must be commenced with the machine at such a temperature as to prevent the prescribed limits being exceeded.

At constant angular speed, generators must be capable of maintaining constant pressure with an overload of 15 per cent.

VARIATION OF PRESSURE AND SPEED

Belgium.

Generators. The pressure variation of a generator is measured by the greatest difference observed, at constant speed, between any two values of the pressure corresponding with loads, the current of which does not exceed that defined by the rated load, the pressure being regulated, at this current, to the rated load value. The brushes to remain adjusted in the working position.

Note.—The excitation of a generator, intended to work at constant pressure, must be capable of maintaining the rated value of the pressure under normal conditions of angular speed, at all loads from no load up to and including 1.15 times the rated load current.

Motors. The speed variation is defined as the difference in angular speed at rated load and at no load at normal working pressure, and without alteration of field rheostats adjusted for the rated load.

France.

Generators. The pressure variation shall be measured in passing from the rated load to no load, at constant speed, in the case of *self-exciting* machines, whilst maintaining constant the resistance in the field circuit; in the case of *separately excited* machines, whilst maintaining constant the exciting current. During this test, and unless otherwise specified, the brushes shall remain fixed in the rated load sparkless position.

Motors. The speed variation shall be measured in passing from the rated load to no load, with constant pressure at the terminals.

Germany. At constant rated load speed and excitation, the pressure to be observed at, at least, four points practically equidistant on the load curve. The greatest difference between the observed pressures is a measure of the variation. In regard to the adjustment of brushes, the conditions are to be governed according to the rating.

Great Britain. ————

Sweden. At constant rated load speed and excitation, the ratio of the increase in pressure at rated load and the pressure at no load, shall be taken:

(1) By observing the values of the pressure at no load and at rated load.

(2) By observing the values of the pressure with increasing values of the field current, both with no load on the armature and with the rated load current in the armature. Tolerance allowed ±2 per cent.

United States.

Generators. Ratio of maximum variation of terminal pressure from the normal (occurring between rated load and no load) to the rated load pressure.

Motors. Ratio of maximum variations in speed from the normal (occurring between rated load and no load) to the rated load speed.

HEATING

General Notes

Belgium. The test is to be carried out as nearly as possible under ordinary working conditions. An overload may be applied at the commencement of the test in order to attain more rapidly the final conditions of temperature.

The maximum temperature is to be compatible with the insulation employed. Special precautions are to be taken when employing thermometers. Unless otherwise specified, apparatus for variable speed and pressure shall be tested under the most severe conditions.

France. The maximum temperature is to be compatible with the insulation employed.

Germany. The tests are to be carried out under the conditions of service specified. Any ventilation which would naturally be produced under ordinary working conditions and which was provided for in the design may be imitated during the test. If a thermometer is employed, it is recommended that the bulb be covered with tinfoil and every precaution taken to prevent loss of heat.

Great Britain. The temperature allowed depends on the character of the insulation.

The temperature limits specified do not cover cases in which cotton, paper, and its preparations, linen, or similar materials are used solely as vehicles for insulation varnishes, or enamels, capable of continuously resisting temperatures above 125 deg. cent. Machines in which such materials are employed are dealt with separately.

A much higher temperature rise than that specified on page 44, may be permitted in machines in which the insulation is secured by means of special materials designed to resist high temperatures, but the amount of permissible temperature rise must depend on the properties of the insulating materials and the method of construction and must be settled specially for each class of machine.

Sweden. The test is to be carried out as nearly as possible under ordinary working conditions.

United States. The temperature allowed must not be such as to be detrimental to the insulation under ordinary working conditions.

An overload may be applied to shorten the duration of the test.

Methods

Belgium. By increase in resistance, if this method is practicable. Otherwise by means of thermometers.

France. Temperatures ascertained by means of thermometers, placed in the hottest part accessible; nevertheless, in the case of exciting circuits and all stationary windings, the temperature may be determined by increase in resistance.

Germany. Exciting coils and stationary windings by increase in resistance, other parts by thermometer.

Great Britain. Stationary windings by increase in resistance. Moving coils by increase in resistance, if possible, otherwise by thermometer or thermo-couple. Commutators, brushes, bearings by thermometer.

Sweden. Windings by increase in resistance, if possible, otherwise by thermometer. Other parts by thermometer.

United States. By thermometer. Conductors by rise in resistance, when possible.

Duration of Test

Belgium.

Continuous service. A period of sufficient duration to attain a practically constant temperature: ordinarily 5 hours up to 20 kilowatts, 8 hours above 20 kilowatts.

Intermittent service. One hour.

Momentary service. The number of hours indicated on the name-plate.

France.

Continuous working. A period sufficiently long to attain a practically constant temperature. (See Note.)

Intermittent service. One hour.

Note.—France. The periods are given, in a general manner, by the following table:

$$K = \frac{\text{Volts} \times \text{Amperes}}{\text{Rev.per.min.}}$$

K			
0– 10	2 hours	300– 500	7 hours
10– 30	3 "	500– 700	8 "
30–100	4 "	700–1,000	9 "
100–200	5 "	1,000–1,500	10 "

Above 1,500, according to the destination of the machine.

Germany.

Intermittent service. One hour.

Momentary service. The number of hours indicated on the name-plate.

Continuous service. Ten hours.

In the case of small machines the duration of test may be reduced.

Great Britain.

Continuous working. Six hours (up to 1,000 kw.).

Intermittent working. One hour (under revision).

Sweden.

Continuous service. A period sufficiently long to attain a practically constant temperature.

United States.

Continuous service. A period sufficiently long to attain a practically constant temperature.

Intermittent service. According to the conditions of service specified.

Limits of Temperature Rise in Deg. Cent.

Belgium.

(*i*) *Permissible limits.* (*ii*) *Recommended limits.*

	(*i*)	(*ii*)
Insulated with cotton	50	45
" " paper	60	45
" " mica, asbestos or similar preparations	80	65
Commutators, brushes	60	50
Bearings, terminals and connections	35	30

For permanently short-circuited windings, 5 degrees rise in excess of the above limits is permitted.

France.

Continuous service. For stationary windings, 10 deg. cent. in excess of the limits for moving coils.

Moving coils:

Insulated with cotton	50
" " paper	60
" " mica, asbestos or equivalent preparations	80
Iron and bare conductors	90
Commutators	60
Bearings	45

Intermittent service. In conformity with the regulations of the Milan Congress (see *Appendix I*).

Uninterrupted service. For commutating machines destined for an uninterrupted day and night service, the permissible temperature rise shall be in accordance with those for continuous service, reduced by 5 deg. cent.

Association Alsacienne.

(*a*) For field windings traversed by a continuous current:	
By resistance	55
By thermometer	45
(*b*) For all other windings and the iron in which the windings are embedded	45
(*c*) For permanently short-circuited windings	55
(*d*) For commutators, brushes	50
(*e*) For bearings, terminals and connections	30

N. B.—For enclosed machines, an extra rise of 5 degrees may be permitted.

For commutating machines destined to work day and night continuously the above limits of temperature shall be reduced by 5 degrees.

Germany. For stationary windings 10 deg. cent. above the limits for moving coils is permitted.

For moving coils as well as the iron in which the windings are embedded:

Insulated with cotton	50
" " " under oil and paper	60
" " enamel, mica, asbestos or equivalent preparations	80

Commutators	60
Bearings	50

Great Britain. For cotton, paper and its preparations, linen, micanite or similar materials:

Stationary coils (by increase in resistance)	60
Moving coils (by increase in resistance)	60
Moving coils (by thermo-couple)	50

Sweden. For stationary windings, 10 per cent above the limits for moving coils.

Moving coils:

Insulated with cotton	50
" " paper	60
" " mica, asbestos	80
Coreplates	60
Commutators	60
Bearings	40

United States.

Field and armature windings	50
Commutators and brushes	55
Bearings and other parts	40

N. B.—For large machines, a temperature limit of 40 deg. cent. under rated load conditions and 55 deg. cent. under overload is frequently specified.

Surrounding Air Temperature

(At the commencement of each paragraph the figure in deg. cent. indicates the standard adopted as the room temperature.)

Belgium.

35 deg. cent. The temperature to be adopted is the mean of the thermometer readings, taken at regular intervals during the last quarter of the test, in the currents of air passing to the apparatus, or, in default, at the middle height round the apparatus; in any case, as near as possible at a minimum distance to ensure safety from the effects of direct radiation.

France.

35 deg. cent. The thermometer indicating the surrounding air shall be placed in the line of air currents at about one metre from the machine and sheltered from all external influence. The mean value of the figures during the last quarter of the test shall be adopted.

Germany.

35 deg. cent. The temperature of the surrounding air shall be taken in the currents of air arriving, or, if there is no well-defined current of air, the mean value shall be taken of the air surrounding the machine at the middle height and at about one metre distant from the machine. The mean value of the figures during the last quarter of the test shall be adopted.

Great Britain.

25 deg. cent.

Sweden.

35 deg. cent.

United States.

25 deg. cent. The surrounding air of the test room is to be ascertained by means of several thermometers protected from direct radiation and air currents.

Heating at Conclusion of Overload Test

Belgium. After the application of an overload of 20 per cent for one-fifth of the specified test period and of 40 per cent for three minutes, the prescribed limits of temperature must not be exceeded by more than 10 deg. cent.

France. No conditions as to heating.

Germany. The temperature limits specified for the rated load must not be exceeded, therefore the overload must be applied for a sufficiently short period and at a time when the temperature of the machine is sufficiently low in order to insure that the permissible limits of temperature cannot be exceeded.

Great Britain. Under no conditions must the temperature exceed 85 deg. cent.

Sweden. ———

United States. When the overload test is applied directly after the rated load test under continuous service conditions, an allowance is permitted of 15 deg. cent. above the limits specified for the rated load.

Surrounding Air Temperature Corrections

Belgium. If the surrounding air is likely to exceed 35 deg. cent., the temperature limits must be reduced by an amount equivalent to the excess.

France. If the test is carried out with a surrounding temperature below 35 deg. cent., the temperature limits shall be reduced in the following ratio:

$$\frac{1}{1 + 0.005 \ (35-\theta)}$$

θ being the temperature of the surrounding air during the test.

If the machine is intended for a locality in which the surrounding air exceeds 35 deg. cent., the temperature limits shall be reduced in the following ratio:

$$\frac{1}{1 + 0.005 \ (\theta' - 35)}$$

θ' being the presumed temperature of the surrounding air.

Association Alsacienne. If the temperature of the surrounding air ordinarily exceeds 35 deg. cent., the temperature limits must be reduced by an equivalent amount.

Germany. ———

Great Britain. If the surrounding air in actual service is likely to exceed 25 deg. cent., the observed temperature rise shall be reduced by one degree for each degree of difference between the surrounding air and 25 deg. cent.

Sweden. ———

United States. The observed temperature shall be reduced or increased one-half per cent for every degree cent. difference between the surrounding air and 25 deg. cent.

RESISTIVITY OF COPPER

Variation with Temperature

Unless otherwise specified, the following values have been adopted:

Belgium. 0.004 per degree cent.

France. 0.004 per degree cent.

Germany. 0.0040 per degree cent.

Great Britain. 0.42 per cent per degree cent. from and at 0 deg. cent.

Sweden. 0.004 per degree cent.

United States. 0.428 per cent per degree cent. from and at 0 deg. cent. for annealed copper of 100 per cent standard conductivity (see *Appendix E*).

ATMOSPHERIC PRESSURE

United States. 760 mm. of mercury.

The observed temperature rise shall be reduced by 1 per cent for every 10 mm. deviation below 760. This correction is only to be applied when the atmospheric pressure differs greatly from the standard pressure of 760 mm. of mercury.

MECHANICAL TESTS

Belgium. Machines must be capable of withstanding for five minutes, at no load with and without excitation, a speed of 25 per cent in excess of the maximum speed specified for the particular service, unless the nature or method of using the machine in itself necessitates speeds exceeding this figure, in which case the excess must be 100 per cent.

France. Generators must be capable of withstanding a momentary increase in speed to be fixed, in each case, according to the particular method of driving. Continuous current motors must be capable of withstanding, during five minutes, a speed of 20 per cent in excess of the rated speed.

Association Alsacienne. Motors must be .capable of withstanding, during five minutes, a speed of 30 per cent in excess of the rated speed.

Germany. Machines intended to work at a practically constant speed must be capable of withstanding during five minutes, a speed of 15 per cent in excess of the rated speed, first without excitation and then with full excitation.

Great Britain. ————

Sweden. Machines must be capable of withstanding a speed of 20 per cent in excess of the rated speed. Generators driven by hydraulic turbines, unless otherwise specified, must be capable of withstanding a speed of 90 per cent in excess of the rated speed.

United States. ————

EFFICIENCY

General Conditions

Belgium. The method of determination is to be specified. Unless otherwise stated, the efficiency is to be ascertained at rated load and under the corresponding conditions as to temperature.

The power absorbed in the field rheostats of a machine is to be included as part of the power required to excite the machine. In the case of a separately excited machine, the efficiency of the machine and that of its exciter, shall be separately stated.

France. The efficiency tests should be made at, or reduced to, the temperature attained at the close of the working test. The efficiency shall be stated for the rated load, three-quarters and half load, and is to include the losses due to the auxiliary apparatus such as exciter, ventilation, circulating pumps, forming an integral portion of the plant.

Germany. The statement as to the efficiency applies to the rated load and the conditions of service to which it refers is to be mentioned. The power absorbed in excitation and in the rheostats must be considered as losses for the purpose of the calculations; unless otherwise stated the power absorbed in cooling the machine must also be considered as a loss.

For machines which are specially excited, the efficiency of the two machines must be stated separately.

Great Britain. ————

Sweden. The losses in excitation and in the rheostats are to be included.

Friction losses are only to be included in the case of machines with automatic lubrication.

Losses due to an independent flywheel are to be excluded.

The efficiency measurements are to be made at the temperature attained by the machine at the close of the rated load test and referred to a surrounding air temperature of 20 deg. cent.

United States. The test is to be carried out under ordinary working conditions and with the surrounding air at standard temperature.

In the case of belt-driven machines, the loss of power in belts and the increase of bearing friction, due to the increase of belt tension, is to be excluded.

In the case of a generator inseparable from its prime mover, bearing friction is to be excluded.

The losses in exciters or in auxiliary apparatus are to be considered separately, and charged to the plant consisting of the machine together with its auxiliary apparatus.

The plant efficiency is to be distinguished from the efficiency of the machine alone. The efficiency of a machine is to be measured at, or reduced to, the temperature which the machine assumes under rated load conditions, referred to a surrounding air temperature of 25 deg. cent.

Method

ALL COUNTRIES. Directly by input output, where possible. Indirectly by losses, if the direct method is not possible.

Enumeration of Losses

BELGIUM. ————

FRANCE.

Mechanical. (*a*) Bearing friction and ventilation.
(*b*) Friction of brushes on commutators and collecting rings.
Electrical. (*c*) Hysteresis and Foucault currents.
(*d*) Joule effects in the circuits.

GERMANY. The losses are not enumerated in tabular form.

GREAT BRITAIN. ————

SWEDEN.

(*a*) Bearing, brush and air friction.
(*b*) Hysteresis and Foucault currents.
(*c*) Ohmic losses in armature.
(*d*) Ohmic losses in brushes.
(*e*) Ohmic losses in exciting coils.

UNITED STATES.

(*a*) Bearing friction and windage losses.
(*b*) Molecular magnetic friction and Foucault current losses.
(*c*) Armature resistance losses.
(*d*) Commutator-brush friction loss.
(*e*) Brush and brush-contact resistance losses.
(*f*) Field excitation loss.
(*g*) Load losses.

NOTES.—FRANCE. *Association Alsacienne* includes under electrical losses brush contact resistance.

UNITED STATES. (*e*) Losses may be considerable with carbon brushes in low voltage machines. (*g*) Difference between total losses under load and sum of losses as here specified.

DIELECTRIC TESTS

General Conditions of Test

BELGIUM. Dielectric tests take the place of insulation tests unless the machine is intended for localities in which special conditions are imposed.

The test is to be carried out before the machine is put into actual service. The repetition of the test is to be avoided.

The test is to be carried out hot.

The windings of machines and transformers must be capable of withstanding, for a period of half an hour, a working pressure of 30 per cent in excess of the highest pressure of the service.

FRANCE. The test of the insulation is to be carried out hot, when possible, and shall only be made in the works of the manufacturer.

The test pressure is to be applied gradually. The circuits of machines and transformers shall be capable of withstanding, without undue strain, for a period of three minutes, a pressure 30 per cent in excess of the ordinary working pressure, provided no mechanical or electrical considerations are against it.

Germany. The tests, when possible, are to be carried out at the works of the manufacturer. They are not to be repeated except in very exceptional cases. For large machines the test is to be repeated, *in situ*, but previous to the machine being put into actual service.

The test is to be carried out hot. Machines and transformers must be capable of withstanding, during five minutes, a pressure 30 per cent in excess of the rated pressure.

Great Britain. ————

Sweden. The test to be carried out hot.

United States. In general, the test is to be carried out at the temperature corresponding to the rated load and when the machine is completely assembled, but before it is put into regular service.

The machine must be dry and free from dust.

Points of Application of the Pressure

All Countries. The various rules specify that the test pressure is to be applied between windings and the frame and between all electric circuits.

Nature of the Test Pressure

Belgium, France, Germany, Sweden. The values of the test pressure herein indicated are applicable to cases in which the testing current is similar to the ordinary working current.

If windings intended for continuous current work be tested with alternating current, the test pressure shall be seven-tenths of that specified.

Conversely, if windings intended for alternating current work be tested with continuous current, the test pressure shall be 1.4 times that specified.

The voltage curve of the alternator employed for the test shall be as nearly as possible a sine wave.

Great Britain. ————

United States. The test is to be carried out with alternating current at the normal frequency of the apparatus.

The values herein given are the root mean square values of the test pressure referred to a sine wave form.

The wave shape of the test pressure should be as nearly as possible sinusoidal, and should not be materially distorted by the testing circuit.

Test Pressure

Belgium.

Working Pressure.	Test Pressure.
Less than 300 v.	4 times the working pressure + 300 v.
300– 600 v.	3 " " " " + 600 v.
600– 1,200 v.	2,400 v.
1,200– 5,000 v.	Twice the working pressure.
5,000–10,000 v.	The working pressure + 5,000 v.
Above 10,000 v.	1.5 times the working pressure.

France.

Rated Pressure.		Test Pressure.
Up to 5,000 v.	Tested Hot.	Twice the rated pressure with minimum of 110 v.
From 5,000–10,000 v.		Rated pressure + 5,000 v.
Above 10,000 v.		1.5 times rated pressure.
Up to 5,000 v.	Tested Cold.	Three times the rated pressure with minimum of 500 v.
From 5,000–10,000 v.		Rated pressure + 10,000 v.
Above 10,000 v.		Twice the rated pressure.

GERMANY.

Rated Pressure.	Test Pressure.
Above 40 v.	At least 110 v.
From 5,000– 7,500 v.	The working pressure + 7,500 v.
Above 7,500 v.	Twice the working pressure.

GREAT BRITAIN. ————

SWEDEN.

Working Pressure	Test Pressure.
Up to 3,300 v.	3 times the working pressure with a minimum of 700 v.
Above 3,300 v.	1.5 times the working pressure.

UNITED STATES OF AMERICA.

Working Pressure.	Power.	Test Pressure.
Not exceeding 400 v.	Below 10 kw.	1,000 v.
	10 kw. and above.	1,500 v.
400 v. and above, but less than 800 v.	Under 10 kw.	1,500 v.
	10 kw. and over.	2,000 v.
800 v. and above, but less than 1,200 v.	Any power.	3,500 v.
1,200 v. and above, but less than 2,500 v.	"	5,000 v.
2,500 v. and over.	"	Double the normal rated voltage.

Length of Test

BELGIUM. 30 minutes.

FRANCE. 30 minutes (hot) } (*Association Alsacienne*, 5 minutes).
5 " (cold) }

GERMANY. 1 minute.

GREAT BRITAIN. ————

SWEDEN. At least 1 minute.

UNITED STATES. 1 minute.

APPENDIX I.

HEATING (see p. 43).

STIPULATION OF THE MILAN CONGRESS (September, 1906).

The heating of a motor is to be considered as excessive when, starting from a surrounding air temperature equivalent to 25 deg. cent., the motor, after 10 hours' working at its permanent power or after one hour working at its normal power, attains a final temperature exceeding that of the surrounding air by the following values:

(*a*)	For windings insulated with cotton...............	70 deg. cent.
	For windings insulated with paper................	80 " "
	For windings insulated with mica, asbestos or other substances presenting the same qualities of insulation and incombustibility.......	100 " "
(*b*)	For commutators.............................	80 " "
(*c*)	For metallic portions in which the windings are embedded, the value corresponding to that indicated for the windings, according to the insulation employed for the latter.	

When the windings are insulated with a combination of insulating materials, the lower limit will be taken. . . . By permanent power and normal power of a motor is to be understood that power which, the current being furnished at the normal pressure of supply, can be developed by the said motor during 10 consecutive hours, in the first case, and during an uninterrupted period of one hour, in the second case, without the heating being excessive in the sense indicated under the paragraph as to " Heating."

TOLERANCE AS REGARDS GUARANTEES

UNION DES SYNDICATS DE L'ELECTRICITÉ

The following table fixes:

(1) Tolerance allowed for errors in measurement, and beyond which the question as to reduction in price may arise.

(2) The limit beyond which the question as to the material not complying with the specification may arise.

Guarantees as to	Tolerance.	Limits.
Heating.	3 deg. cent.	10 deg. cent. above the limits fixed.
Auto-regulation.	20% of the guaranteed percentage	50% of the guaranteed percentage.
Efficiency.	20% of the sum of the total or measurable losses, as the case may be.	50% of the losses, total or measurable.

ASSOCIATION ALSACIENNE

As acceptance of the various guarantees given, it is usual to fix two limits, the first representing the permissible tolerance to allow for inexactitudes and errors of measurement, the second giving to the buyer the right to reject the material. Between these two limits it is usually a question of penalties in proportion to the deviation from the guarantee. The penalties for the different guarantees are cumulative.

The following values are to be recommended:

Guarantee as to	Tolerance *re* Measurements.	Limits of Rejection.	Penalties to be applied by the Buyer, between the Tolerance and Limits.
Heating.	4 deg. cent. above limits fixed (for resistance measurements only).	10 deg. cent. above limits fixed.	½% per degree.
Auto-regulation.	15% of the percentage guaranteed by the manufacturer.	40% of the guaranteed percentage.	1% for each 10% of the percentage guaranteed, applicable between 15% and 40%.
Efficiency.	15% of the sum of the losses, total or measurable, as the case may be.	40% of the losses, total or measurable.	2% for each 10% of the sum of the losses, total or measurable, applicable between 15% and 40%.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

REPORT OF THE BOARD OF DIRECTORS FOR THE FISCAL YEAR ENDING APRIL 30, 1911.

The Board of Directors of the American Institute of Electrical Engineers presents herewith for the information of the membership its annual report for the fiscal year ending April 30, 1911.

The report includes a brief summary of the more important work accomplished by the various standing and special committees, also detailed statements showing the condition of the various funds and finances of the Institute. It is highly gratifying to the Board of Directors to be able to report for this year, a surplus of nearly $17,000, and a net increase in the membership of 436 members. The Board also wishes to call attention to the investment last November, of $15,000, par value, in General Mortgage, four per cent, Chicago, Burlington and Quincy Railroad Company bonds.

The Board has held 10 regular monthly meetings during the year, and the Executive Committee two meetings.

The Annual Convention was held at Jefferson, N. H., June 26 to 30, 1910. The total registered attendance numbered 178. Twenty professional papers were presented and discussed.

Following the policy inaugurated last year of holding other Institute meetings in various portions of the country, partaking of the nature of conventions, there was held at Schenectady, N. Y., and Pittsfield, Mass., on February 14, 15 and 16, 1911, the Pittsfield-Schenectady Mid-Year Convention, at which 16 technical papers were presented and discussed. The attendance at this convention numbered 350 at Schenectady, and 150 at Pittsfield. The meeting was a most successful one in every way. An equally enthusiastic and successful meeting, known as the Pacific Coast Meeting, was held in Los Angeles, Cal., on April 25 to 29, 1911. All of the larger cities on the coast were represented at this meeting, and the total attendance numbered 189 members and 134 visitors. These meetings cannot fail to emphasize the national character of the Institute.

In addition to the work summarized and embodied in this report, a greal deal has been accomplished by temporary committees appointed by the Board from time to time throughout the year. Among these may be mentioned the special committee recently appointed to act in conjunction with the committees of other interested engineering organizations in opposing the legislation to license engineers. The report of this committee may be found in the April issue of the PROCEEDINGS. Much creditable work has also been performed by the representatives of the

Institute on various commissions and congresses, both in this country and abroad. Among the more important of these are: the Second National Conservation Congress at St. Paul, Minn., September 5 to 9, 1910, the American Mining Congress at Los Angeles, Cal., September 26–October 1, 1910, Annual Meeting, American Academy of Political and Social Science, Philadelphia, Pa., April 7 and 8, 1911, the International American Scientific Congress at Buenos Aires, Argentina, S. A., July 10 to 25, 1910, and the Reunion Amicale at Brussels, August 8 to 10, 1910.

The year has not only been one of the most prosperous in the history of the Institute, but an extremely busy and useful one.

Sections Committee.—The Sections Committee is able to report continued progress in Section and Branch affairs during the past year. One Section has been added to the roster—Detroit-Ann Arbor—and one discontinued—Norfolk, Va. Although the total attendance at the Section meetings does not add up to quite so large a figure as during the previous year, an analysis of the various Sections indicates that the small Sections have generally increased their attendance, while the decreases have taken place in one or two of the larger Sections.

The reports from the University Branches have been most encouraging. It is gratifying to note that a large percentage of the additions to the membership of the Institute comes from the ranks of the enrolled Students, and further to note that most of the enrolled Students have come into the Institute through the agency of the University Branches.

Considerable work has been done during the past year towards placing the matter of Sections expenditures on a uniform and logical basis. It is hoped that by-laws which incorporate this uniform basis of appropriation for this purpose will be in such shape that the proposed plan may be put into operation during the next administrative year, beginning August 1, 1911.

	For Year Ending			
	May 1 1908	May 1 1909	May 1 1910	May 1 1911
SECTIONS				
Number of Sections	21	24	25	25
Section Meetings held	141	169	187	208
Original papers and talks	120	167	178	181
Attendance	7,476	16,427	16,694	15,243
BRANCHES				
Number of Branches	22	26	31	36
Branch meetings held	143	198	237	255
Original papers and talks	84	158	147	147
Attendance	4,128	8,443	10,255	10,714

Meetings and Papers Committee.—This committee has arranged for nine regular Institute meetings during the year, at each of which one or more technical papers were presented. Seven of these meetings were held in New York, one in Boston, and one in Toronto. The committee also approved and coöperated in the Pittsfield-Schenectady Mid-Year Convention, and the Pacific Coast Meeting held in Los Angeles. Active preparations are now being made by the committee for the Annual Meeting and the Annual Convention, the latter to be held in Chicago from June 26 to 30 inclusive. The committee has received much assistance from the chairmen of the various technical committees, who have been active in securing papers and carrying out the policy of the Meetings and Papers Committee in handling the meetings held in New York.

Educational Committee.—The work of this committee for the year has consisted principally of an investigation into the advisability of offering prizes to students of electrical engineering for competitions of various sorts, such as thesis work or original designs. The committee unanimously decided against the establishment of such prizes. The remainder of the work of the committee has consisted in arranging a program for one of the sessions of the Annual Convention to be held in Chicago in June.

Industrial Power Committee.—This committee has been active in its field during the year, and as a result of its efforts a number of meetings have been held by the various Sections which were devoted to the presentation and discussion of industrial power subjects. A very successful meeting, at which the cost of industrial power was discussed, was held under the auspices of the committee in New York in March, with the coöperation of the American Society of Mechanical Engineers The committee has succeeded in obtaining from various sources a number of valuable papers on industrial power subjects which will be transmitted to the new committee at the close of the present administrative year.

Railway Committee.—A number of meetings have been held by this committee during the year. The advancement of railway electrification has been held as the principal object of the committee's existence, and it now appears that the committee is in a fair way to establish a precedent of unusual value to determine the practical results of electrification as carried out on a number of important lines. To this end, in addition to a paper read by Mr. W. S. Murray at an Institute meeting in Toronto giving considerable information concerning the New Haven installation, there will be a discussion of this paper at the Annual Convention in June. Among others, special details relating to the New York Central installation are expected. A most important paper giving construction, maintenance, and operating costs, is now being prepared for the Chicago Convention. Six or eight other valuable papers have been promised for the future.

Telegraphy and Telephony Committee.—The Telegraphy and Telephony Committee has held two meetings during the year. It has obtained several valuable papers in its field, for both the Pacific Coast Meeting, held at Los Angeles in April, and the Annual Convention, to be held in Chicago in June.

Electric Lighting Committee.—The Electric Lighting Committee secured five papers during the year, one of which was presented at the Institute meeting held in New York on February 10, 1911. The remaining four papers will all be presented at the Annual Convention in June.

Power Station Committee.—The Power Station Committee had assigned to it the May meeting, but this was changed in order to give an opportunity for the presentation ceremonies of the Edison Medal. The committee expects to secure papers for the Annual Convention.

Electrochemical Committee.—The efforts of this committee have been directed towards arranging for two or three papers on electrochemical subjects for a meeting scheduled tentatively for April, 1911. Owing to other engagements of those who were best qualified to deal with the subjects selected, and to the fact that practically all of the available material had been promised elsewhere, it was deemed inadvisable to hold an electrochemical meeting this year. Arrangements have virtually been completed, however, for one or two good papers for next fall or winter.

High Tension Transmission Committee.—The High Tension Transmission Committee has followed this year the custom of previous years. Up to the present time it has held five meetings, with an average attendance of six members, and presumably at least one more meeting will be held during the term of the present committee. At these meetings various questions coming before the committee were discussed and appropriately determined. The committee prepared the program for the regular Institute meeting held in New York on January 10, 1911. The committee also assisted in the preparation of the program for the Pacific Coast Meeting at Los Angeles. The committee expects to hold an " Extra High Tension Operation Meeting " at the Annual Convention in Chicago, giving data and discussion on the construction and operation of power systems utilizing 80,000 volts or higher. The most notable action of the committee during the present year was its participation, with the authorization of your Board, in the specifications for overhead crossings of electric light and power lines. These specifications were prepared with the idea of securing a nationally recognized crossing specification which could be uniformly used throughout the country by railway, telephone, telegraph, or whatever lines are crossed by power circuits. This specification is a joint report of committees of various engineering organizations, but it is believed, as a result of the coöperation of these various bodies through their representative committees, that this specification will be universally recognized and followed.

Editing Committee.—Since May 1, 1910, there have been edited and published 12 numbers of the PROCEEDINGS. The total number of pages contained in these PROCEEDINGS is 2,856. Of these, 360 pages have appeared in Section I, and 2,226 pages in Section II Of the 2,226 pages in Section II, 1,624 pages were devoted to technical papers, and 530 pages to discussions. Volume XXIX of the TRANSACTIONS, consisting of the papers and discussions presented during the calender year 1910 and the report of the Board of Directors for the fiscal year ending April 30, 1910, contains approximately 1,770 pages. The volume will be issued in two parts and is expected to be ready for delivery about the middle of June.

From May 1, 1910 to April 30, 1911 there have been published in full in the PROCEEDINGS seven papers read before various Sections and Branches, in addition to 11 abstracts of such papers, which appeared in Section I of the PROCEEDINGS.

The Editing Committee has gone carefully over the discussions which have been submitted by the Sections and Branches, and has supervised the editing of the discussions presented at the regular Institute meetings. The committee, in coöperation with the Meetings and Papers Committee, has revised and will have reprinted the pamphlet " Suggestions to

Authors ", bringing this up to present requirements as to style, illustrations, arrangement of matter for papers and discussions, with the end of assuring greater uniformity of style in the PROCEEDINGS and TRANSACTIONS and facilitating the actual handling of the papers and discussions.

Standards Committee.—The Standards Committee has held seven regular monthly meetings in New York since its appointment last August, and will hold one more in May. The additions and amendments to the Standardization Rules presented at the last Annual Convention have been completely revised, supplemented and incorporated into the rules. The rules thus revised will be presented to the Board of Directors at this meeting, and it is hoped will be ready for distribution in the early summer.

Last year, at the request of the Standards Committee, the U. S. Bureau of Standards undertook a thorough investigation of the resistivity and temperature coefficient of copper, to serve as a basis for a new Institute wire table. This work was completed during the summer, and the Bureau has now nearing completion the preparation of a very comprehensive set of tables under the direction of the Standards Committee.

In order to handle the various questions arising, 10 sub-committees were appointed during the year. The work of five of these sub-committees is still unfinished and will continue over until next year. The subjects under consideration by these five committees are: 1. *Definitions of electromotive force, potential difference, and voltage*; 2. *The standardization of the stranding of cables*; 3. *A definition of horse-power in terms of the watt*; 4. *The rating of electrical machinery, particularly intermittent rating*; 5. *Insulation testing and transformer regulation.*

International Electrotechnical Commission.—An unofficial conference was held by the International Electrotechnical Commission at Brussels August 8 to 10, 1910, at the invitation of the Belgian Electrotechnical Committee. The Conference was presided over by Professor Eric Gerard. Forty-seven delegates, representing 11 national committees, attended the conference. Messrs. A. E. Kennelly and Charles F. Scott represented the U. S. National Committee.

The resolution of the American Institute of Electrical Engineers adopted at the Jefferson Convention on June 29, 1910, referring the question of standard direction of alternating current vector-rotation to the Commission, (TRANSACTIONS A. I. E. E., 1910, pp. 1821–1822) was laid before the Conference by the U. S. delegates.

Substantial progress was made at the Conference in all of the four subjects taken up for discussion—nomenclature, symbols, vector-rotation, and rating. In the last three the United States National Committee had taken an especially active interest.

The official resumé of the actions at the Conference, issued in September by the General Secretary, was printed in the PROCEEDINGS of the American Institute of Electrical Engineers for December 1910, pages 10 and 11.

Six meetings have been held by the U S. National Committee in New York City during the year, with an average attendance of four members. At the meeting in October 1910 the actions of the Brussels Conference as printed in the Official Resumé were endorsed. Various documents have been received from the General Secretary and considered by the committee. Communications have been exchanged with the French

Committee on an inquiry received from them as to the nomenclature of reactive power in an alternating current circuit.

Local committees of the Commission have now been formed in the following countries: Austria, Belgium, Brazil, Canada, Denmark, France, Germany, Great Britain, Hungary, Italy, Japan, Mexico, Spain, Sweden, and the United States. A plenary meeting of the Commission is scheduled to be held in Turin from September 11 to 16, 1911, at the invitation of the Italian Committee, and in conjunction with the Turin International Electrical Congress.

The President of the Commission is Dr. Elihu Thomson, who succeeded Lord Kelvin in that office. The Honorary Secretary is Colonel R. E. Crompton, and the General Secretary is Mr. C. le Maistre, whose office is at 28 Victoria Street, London.

It is to be hoped that international standardization may be adopted at the forthcoming Turin meeting in some or all of the four subjects on which tentative progress was made at the Brussels Conference.

Code Committee.—The Code Committee, through its chairman, represented the Institute at the annual meeting of the National Board of Fire Underwriters, held in New York on March 20 and 21, 1911. The only matter of interest to the Institute, taken up at this meeting, was the grounding of secondaries, and the work of the Institute's representative resulted in the passing of a resolution by the Underwriters' Conference endorsing the practice of the grounding of secondaries and recommending that municipalities and lighting companies make such a rule mandatory, with the further resolution that the Institute use its efforts to bring about an agreement with the National Electric Light Association in the matter of grounding of secondaries up to 250 volts, instead of at 150 volts, the present adopted standard of the association.

Law Committee.—The Law Committee has considered several questions submitted to it by the Board of Directors, principally in reference to interpretation of the By-Laws and Constitution. Owing to the fact that this committee, under the Constitution, is merely an advisory committee, no constructive work has been done.

Conservation of Natural Resources Committee.—During the year the Conservation of Natural Resources Committee has corresponded, through its chairman, with various officials of the federal government relative to the regulations covering the development of water powers, dependent in whole or in part, upon the run-off from public lands.

On December 28 the Secretary of Agriculture issued a " Use Book " containing regulations and instructions for the use of the national forestry service, and a Manual of Procedure for forest officers, which, in respect of water powers, embodies substantially all of the suggestions presented in President Stillwell's presidential address at the Jefferson Convention, and approved by that convention in a resolution requesting the Board of Directors to take action looking to their adoption.

On February 10 the Board of Directors, by resolution, instructed the Conservation Committee to examine a bill introduced in the House of Representatives by Mr. Herbert Parsons, and to communicate to Mr. Parsons (and if deemed desirable, to other representatives, senators and officers of the administration) the views of the committee in respect thereto. In accordance with this instruction, the committee examined the bill and notified Mr. Parsons and Chairman Lundell, of the Committee on Public Lands, of its approval of the proposed legislation. No action upon the bill was taken by the Congress which adjourned in March.

Library Committee.—In accordance with Section 24 of the By-Laws of the Institute we beg leave to submit herewith our annual report for the year ending April 30, 1911, showing the state of the library and including the names of all donors to it.

The efforts of the committee have been directed towards the ultimate extension of the usefulness of the library to the members of the founder and associate societies and to non-members, whether resident or non-resident in New York City, and towards the harmonious, equitable and efficient administration of the joint libraries housed in the upper stories of the Engineers' Building.

The successful attainment of either of these ends predicates the assistance of a competent chief librarian. The former chief librarian having resigned on August 31, 1910, the administration was placed temporarily in the hands of Miss Alice J. Gates. On December 30, 1910 Dr. W. P. Cutter was appointed librarian of the libraries of the three Founder Societies and United Engineering Society and he assumed office about the first of February, 1911. To accept the appointment, Dr. Cutter, who had previously been connected with the Library of Congress and with the Library of the U. S. Dept. of Agriculture, resigned from the librarianship of the Forbes Library, Northampton, Mass.

The present status of joint occupancy and administration of the library is as follows: There are four separate libraries which are the respective properties of the three Founder Societies and the United Engineering Society. Many parts of the first three have been donated and are subject to limitations prescribed by the deeds of gift. At the suggestion of the United Engineering Society, the Founder Societies (minutes of Board of Directors A. I. E. E., June 12, 1908) agreed "That the administration of the library be placed in the hands of a chief librarian, all employes of the library to be subject to the direction of said chief librarian." As the result of another suggestion from the United Engineering Society, which was approved at the same time, by the Founder Societies, the salaries of all the library employes are now paid by the United Engineering Society and one third of the total amount is charged back to each of the Founder Societies. Any administrative act of the chief librarian, which involves an expenditure by any one of the four societies, must be subject to the constitutional or other limitations prescribed by that society. The Constitution of A. I. E. E. (§50) prescribes that the Library Committee "shall direct expenditures for books or other articles" for the library. The United Engineering Society allows the House Committee, which consists of the secretaries of the three Founder Societies, to direct expenditures which do not exceed $100. To facilitate the work of the chief librarian, therefore, the Library Conference Committee, consisting of one member from each of the Founder Societies, is of service. This committee, representation upon which was authorized by the Institute Board of Directors on October 26, 1906 and

March 29, 1907, for diplomatic reasons has assumed merely recommendatory functions and it has been agreed that all its actions must be unanimous. It meets once a month, at the close of the meeting of the House Committee, the members of the latter attending. To this committee the chief librarian makes recommendations, which, if approved, are ultimately referred to the body having power to effectuate the execution. The Institute's representative on this committee must be a member of the Library Committee, and has thus far been its chairman.

Statistical information concerning the library and its use during the year, including a list of donors, is given in the following tables:

DONORS

ACADEMIE ROYALE DES SCIENCES	1
ACHESON, E. G	1
ADAMS, E. D	7
AGENDA DE L'ELECTRO	1
AMERICAN ELECTROCHEMICAL SOCIETY	1
AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS	4
ARNOLD, BION J	1
ASSOCIATION OF RAILWAY TELEGRAPH SUPERINTENDENTS	1
BOSTON TRANSIT COMMISSION	1
CALDWELL, EDWARD	1
CHICAGO BUREAU OF STATISTICS	12
COLUMBIA UNIVERSITY, DEPARTMENT OF PHYSICS	2
CONGRESO CIENTIFICO (1° PAN AMERICANO)	2
CROCKER, F. B & ARENDT, M	1
GENERAL ELECTRIC COMPANY	6
HERING, C	1
INDIANA RAILROAD COMMISSIONER	2
INSTITUTION OF OPERATING ENGINEERS	1
INTERNATIONAL ELECTROCHEMICAL COMMISSION	1
INTERNATIONAL ELECTROTECHNIQUE COMMISSION	1
IOWA ENGINEERING SOCIETY	1
ITALY POSTE & TELEGRAFI	2
JANECKE, MAX	1
JOURNAL OF ELECTRICITY, POWER & GAS	1
KARAPETOFF, V	1
MAILLOUX, C. O	8
MARTIN, T. C	4
MASSACHUSETTS GAS AND ELECTRIC LIGHT COMPANY	1
MCCLURG, A. C. COMPANY	1
MCKAY, D	1
MCGRAW PUBLISHING COMPANY	6
MICHIGAN ELECTRIC ASSOCIATION	1
MILLS, JOHN	1
NACHOD, C. P	1
NATIONAL FIRE PROTECTION ASSOCIATION	1
NEW JERSEY PUBLIC UTILITY COMMISSIONER	2
PIERCE, A. L	1

POLYTECHNIC INSTITUTE OF BROOKLYN		1	
SEMENZA, G		1	
SOCIETE INTERNATIONALE DES ELECTRICIENS		1	
TELEFUNKEN WIRELESS TELEGRAPH COMPANY		14	
U. S. ELECTRICAL DEPARTMENT, WASHINGTON		1	
VAN NOSTRAND, D. COMPANY		6	
VERMONT PUBLIC SERVICE COMMISSIONER		1	
VILLARS, G.		3	
WARE, H. E		1	
WESTERN UNION TELEGRAPH COMPANY		1	
WESTINGHOUSE ELECTRIC & MANUFACTURING COMPANY		1	
DONOR UNKNOWN		6	
OLD MATERIAL		79	
			198
Exchanges	199		
Purchases	91		
			290
Total accessions			488

The following tabulation gives the state of the five accounts from which the Library committee is entitled to draw.

Donations (General Library Fund)

Dr.		Cr.	
Balance May 1, 1910	$258.04		
Interest May 1, 1911	6.48	Unexpended	$264.52
	$264.52		$264.52

Mailloux Endowment Fund ($1,000)

(Proceeds for the maintenance of certain sets of periodical publications.)

Balance May 1, 1910	$60.35	Expended	$34 00
Interest	15.00	Unexpended	41.35
	$75.35		$75.35

International Electrical Congress of St. Louis, 1904 Fund.)

(Proceeds available for the purchase of non-American International electrical literature)

Invested in New York City 4½% Bonds			$2268.00
Additions to the fund			49.65
Total fund			$2317.65
Balance on hand May 1, 1910	$129.12		
Interest to May 1, 1911	90.00	Unexpended	$219.12
	$219.12		$219.12

WEAVER DONATION

(Available for the purchase of early electrical literature.)

Balance on hand May 1, 1910	$65.44	Unexpended	$65.44
	$65.44		$65.44

INSTITUTE APPROPRIATION ACCOUNT

Dr.		Cr.	
Appropriation for the year ending October 1, 1911	$4,500.00	Librarian and assistants (to January 1, 1911)	$1,005.90
		Cataloguing (to Jan. 1, 1911)	554.00
		Desk attendant (to Jan. 1, 1911)	106.68
		Salary (one-third) of librarian, assistants, cataloguer and desk attendant (Jan. 1 to April 1, 1911)	528.34
		One-third running expenses of library for 1910	106.78
		Books	176.95
		Subscriptions	169.29
		Insurance	72.93
		Binding	322.85
		Miscellaneous	39.58
			$3,083.30
		Unexpended	1,416.70
	$4,500 00		$4,500.00

STATISTICS OF LIBRARY MAY 1, 1911

Source	Volumes	Pamphlets	Valuation
Report of May 1, 1910	14,865	1,279	$25,959.12
Purchases	91		315.84
Gifts and exchanges	288	30	583.50
Old material accessioned	49	34	66.50
	15,293	1,343	$26,924.96

In the following table are given the figures for the total valuation of the Library property:

Books	$26,924.96
Stacks	1,761.05
Furniture, Catalogues, cases, etc.	376.00
	$29,062.01

LIBRARY ATTENDANCE

	Day	Night	Total
May 1910	454	279	733
June "	471	239	710
July "	609	Closed	609
August "	560	"	560
September "	596	261	857
October "	665	328	993
November "	678	334	1,012
December "	699	306	1,005
January 1911	681	333	1,014
February "	617	334	951
March "	687	334	1,021
April "	756	293	1.049
Total May 1910–April 1911	7,473	3,041	10,514

Mr. Edward D. Adams has continued his contributions to the library as in previous years by the donation of the PROCEEDINGS and TRANSACTIONS of the Royal Society of London, and new volumes of the International Catalogue of Scientific Literature. He has also as heretofore had these volumes bound at his expense.

The income from the C. O. Mailloux Fund of $1000 has again been used to maintain the four important periodical sets which were originally presented to the library by Mr. Mailloux.

Respectfully submitted,

A. BEMENT.
C. F. BURGESS.
GANO DUNN.
J. P. MALLET.
C. E. SCRIBNER.
PHILIP TORCHIO.
SAMUEL SHELDON, *Chairman*.

LIBRARY COMMITTEE.

Edison Medal Committee.—At a meeting of the Edison Medal Committee held on November 26, 1910, the names of candidates submitted in accordance with the committee's by-laws were voted upon, and Mr. Frank J. Sprague was selected from the list to be voted on in December following. The voting in December was done in accordance with the provisions of the by-laws, and resulted in the award of the Edison Medal to Mr. Frank J. Sprague, for " Meritorious Achievement in Electrical Science, Engineering and Arts ", the result of the vote being transmitted to the Board of Directors under date of December 19, 1910. The presentation is to be made at the Annual Meeting of the Institute on May 16, 1911.

John Fritz Medal Board of Award.—The John Fritz Medal for 1910 was awarded to Alfred Noble, past-president, American Society of Civil Engineers, for " notable achievements as a civil engineer." The presentation was made on November 30, 1910, at the house of the American Society of Civil Engineers, New York City.

Board of Examiners.—The Board has held 11 meetings during the year. It has considered and reported to the Board of Directors a total of 1,748 applications for election to membership in the Institute, Student enrolment, and transfer to the grade of Member.

A summary of these applications is as follows:

Recommended for election as Associates	916
Not recommended for election as Associates	2
Recommended for transfer	54
Not recommended for transfer	27
Recommended for enrolment as Students	749
Total number of applications considered	1,748

This is an increase of 351 applications over last year.

Membership Committee.—On November 1, 1910 a letter was mailed by the committee to each member of the Institute requesting the names of desirable candidates for admission. The coöperation of the officers of the Institute Sections was also requested. In response to these communications over 1,200 names were suggested by the membership. All of these prospective candidates were communicated with promptly and supplied with printed matter relating to the Institute and its various activities.

The number of applications received from November 1, 1911, on which date the present committee began its active work, to April 30, 1911, is 661, and the total number received during the year ending April 30, 1911, is 937. The present total membership and the net increase during the past year are indicated in the following table:

	Hon. Mem.	Mem.	Assoc.	Total
Membership, April 30, 1910	1	640	6,040	6,681
Additions:				
New Associates			899	
Transferred		56		
Reinstated		3	43	
Deductions:				
Died		2	34	
Resigned		3	122	
Dropped		5	343	
Transferred			56	
Membership April 30, 1911	1	689	6,427	7,117

Net increase during the year in membership 436

Student Enrollment.—Since the enrollment of Students was authorized in 1902, the total number enrolled up to May 1, 1911 is 4,418. Of this number 1,348 are still enrolled as Students and 745 have become Associates, or their applications are pending. The remaining 2,325 are off the list by reason of expiration of the three year Student term, or through their failure to complete that term.

Resignations.—The following Members and Associates have resigned during the year in good standing.

Members.—T. L. Miller, D. W. Shea, William C. Woodward.

Associates.—L. Andrews, G. F. Atwater, H. Binney, G. W. Bissell, E. M. Blake, C. E. Boman, J. A. Britton, J. S. Brosius, H. B. Burley, K. O. Burrer, R. L. Cadwell, J. R. Carl, M. B. Carroll, F. J. Chisholm, Wm. Christensen, M. D. Church, W. R. Collier, C. A. Cornwall, A. G. Coursol, R. Dahlander, N. B. Davis, G. R. Davison, F. B. De Gress, R. J. Dunlop, J. J. Ehrenreich, F. W. Field. W. G. Fox, C. E. Frailey, D. H. Fry, W. Gale, Jr., F. H. Geer, S. D. Gilbert, G. B. Glassco, S. H. Goddard, J. R. Gordon, C. J. Graham, J. H. Granbery, E. W. T. Gray, L. H. Haight, F. G. Haldy, B. S. Harrison, H. H. Heaton, W. L. Hedenberg, A. S. Hegeman, C. J. Heilman, W. E. Hodge, H. Hollinger, J. C. Hunter, E. W. Jodrey, W. P. Judson, Grover Keeth, R. B. Kellogg, A. S. Kelly, C. G. R. Kemp, J. S. Kerine, John Langan, A. W. Lee, L. H. Lee, J. A. Leonard, D. R. Lovejoy, E. S. Lytch, R. T. MacKeen, H. E. De M. Malan, G. W. Martin, J. A. McCoy, S. A. Mendenhall, C. P. Merrill, H. C. Meyer, P. E. Mitchell, E. F. Morrill, F. C. Nelson, L. H. Newbert, E. C. Newton, E. W. Niles, Ray Oliphant, A. F. Ormsbee, W. H. Palmer, Jr., P. D. Parsons, J. E. Peavey, P. C. Petersen, W. P. Phillips, J. O. Plowden, J. H. Poole, F. H. Poor, G. L. Pratt, C. Rabello, L. C. Ralston, C. J. Ratterman, Arthur Rice, C. D. Richardson, G. B. Roberts, Raymond Roth, L. Searing, F. M. Shaw, F. B. Shuford, Mont Sleeth, C. H. Starkweather, L. Stocker, F. C. Sutter, Phillip Sweetser, W. M. Talbott, E. A. Taylor, E. L. Tessier, R. McK. Thomas, W. H. Thorpe, J. B. Tingley, H. C. Trow, R. T. Turnbull, W. E. Ver Planck, E. S. Vinten, W. E. Wardwell, K. Watson, W. F. Weber, W. C. Webster, S. F. Weston, H. B. P. Wicks, Carl Wiler, R. S. Willis, J. F. Wilson, H J. Wood, J. W. Wright, C. R. Wylie.

Total resignations, 125.

Deaths.—The following deaths have occurred during the year:

Members.—S. S. Dickenson, Joseph Wetzler.

Associates.—R. F. Adams, T. P. Bailey, S. M. Balls, C. K. Batchelder, E. A. Bessey, W. H. Browne, H. W. Deeds, R. Dickerson, J. D. E. Duncan, G. N. Eastman, E. R. French, A. Henderson, J. Heywood, F. F. Gardner, C. W. Hunt, E. J. Jenness, J. D. Keiley, W. C. Kerr, C. J. Larson, F. H. Lincoln, K. McCaskill, J. McKenzie, R. J. Nunn, T. G.Odell, J. F. Palecek C. E. Robles, A. Spies, Charles Talbott, O. Stephensen, D. A. Wilkes, E. B. Wintrobe, J. T. Wolfe, A. V. Woodard, S. Yoshisaki.

Total deaths, 36.

Delinquent.—Dropped as delinquent during the year, 348.

Intermediate Grade of Membership Committee.—The work of this committee has consisted chiefly in the gathering of data relating to the establishment of a third grade of membership. The committee being so widely scattered, it has been deemed wise to call a meeting to be held sometime during the Annual Convention. It is also planned to have a discussion of the subject by the Section delegates at the convention.

Indexing Transactions Committee.—The work of this committee has been largely devoted to the perfection of a plan providing for a suitable index to the Institute TRANSACTIONS. The actual work of indexing the papers and discussions, however, is now well under way, although the work must necessarily be slow at the present time.

It is the intention to make this index complete in every detail, and yet not encumber it with the minutæ which fill ordinary indexes. It is planned to have the index so arranged that one in search of information on any given subject will first find all papers bearing directly on the subject, and then all references which may be parts of other papers or discussions.

Building Fund.—The amount collected from subscribers during the year was $1,672.00. The interest on the bank balance amounted to $119.54, making a total of $1,791.54 to the credit of the Building Fund during the year.

LAND, BUILDING AND ENDOWMENT FUND.

RECEIPTS.		DISBURSEMENTS.	
Before appointment of Committee	$ 6,100.00	Paid United Engineering Society, acct. of contract	$ 8,000.00
Collected by Committee	147,553.05	Paid United Engineering Society, acct. of mortgage	126,000.00
Interest on balances	6,086.55	Paid United Engineering Society, acct. of interest	19,529.45
Reimbursement by Institute	9,221.95	Expenses of Committee	10,440.73
		Balance in bank, May 1, 1911	4,991.37
Total	$168,961.55	Total	$168,961.55

Finance Committee.—The following correspondence and financial statements form a complete summary of the work of the Finance Committee for the year.

BOARD OF DIRECTORS, May 16, 1911.
American Institute of Electrical Engineers.

Dear Sirs: The Finance Committee respectfully submits herewith the following report for the year ending April 30, 1911.

The committee has held regular monthly meetings throughout the year. It has examined and approved the expenditures of the Institute for various purposes, and has otherwise performed the duties prescribed for it in the Constitution and By-Laws. Messrs. Peirce, Struss and Company, chartered accountants, have audited the Institute books, and their certification of the Institute finances follows.

Your Secretary, a representative of the firm of chartered accountants, and your committee have examined the securities owned by the Institute and find them to be in accordance with the accountants' report. In this connection attention may be directed to the purchase by the Institute during the past year, of $15,000 par value of Chicago, Burlington & Quincy 4% Bonds, the selection being made from the list of securities available for legal investment by savings banks in New York and Massachusetts.

In general it will be seen from the reports submitted that the finances of the Institute are in good condition and that the increase in expenditures resultant from increased activities is being successfully met by the increase in income.

Respectfully submitted,
A. W. BERRESFORD,
Chairman Finance Committee.

MR. A. W. BERRESFORD, NEW YORK, May 8, 1911.
Chairman Finance Committee.

Dear Sir: In accordance with your instructions, we have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30th,1911.

The results of this examination are presented in four exhibits, attached hereto: as follows:—

Exhibit " *A* " Balance Sheet, April 30, 1911.
Exhibit " *B* " Receipts and disbursements for general purposes for year ended April 30, 1911.
Exhibit " *C* " Receipts and Donations for designated purposes, also expenditures for year ended April 30, 1911.
Exhibit " *D* " Condensed Cash Statement.

We beg to present attached hereto our certificate to the aforesaid exhibits.

Yours very truly.
(Signed) PEIRCE, STRUSS & Co.,
Certified Public Accountants.

MR. A. W. BERRESFORD, NEW YORK, May 8, 1911.
Chairman Finance Committee:

Dear Sir: Having audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1911, we hereby certify that the accompanying Balance Sheet is a true exhibit of its financial condition as of April 30, 1911, and that the accompanying statements of Cash Receipts and Disbursements are correct.

(Signed) PEIRCE, STRUSS & Co.,
Certified Public Accountants.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Balance Sheet, April 30, 1911.

Exhibit A.

Assets.

Cash:		
Land, Building and Endowment Funds	4,991.37	
General Library fund	264.52	
Compounded Membership fund.....	4,742.99	9,998.88
Farmers' Loan and Trust ctf. of dep..		1,000.00
General cash in bank	18,866.49	
Mailloux fund, interest..............	41.35	
Weaver donation...	65.44	
International Elec. Congress of St. Louis Library fund interest..........	268.77	
Total cash deposit..	19,242.05	
Secretary's petty cash on hand....	750.00	19,992.05
Land, Building and Endowment fund, accrued interest..	43.02	
General Library fund accrued interest..	2.75	
Mailloux Fund accrued interest....	22.50	
International Electrical Congress of St. Louis, 1904, Library Fund accrued interest....	45.00	113.27
Mailloux fund, principal Bond.......		1,000.00
International Electrical Congress of St. Louis 1904, Library Fund, N. Y. City 4½ % Bonds, due 1917..	2,268.00	
N.Y. City 4½% Gold Bonds, due 1957..	30,000.00	
Premium on bonds.	1,952.50	
Westinghouse Electric & Mfg. Co's. stock............	50.00	
C., B.& Q. 4% Bonds (15M.), due 1958, cost.............	14,606.25	46,608.75
Equity in Engineering Societies Building (25 to 33 West 39th St.)........	353,346.61	
One-third cost of land (25 to 33 West 39th St.)...	180,000.00	533,346.61
Library Volumes and Fixtures.........	28,096.17	
Transactions.......	8,123.50	
Office Furniture and Fixtures.........	7,084.10	
Works of Art, Paintings, etc.........	2,543.60	
Badges............	339.95	46,187.32
Accounts Receivable:		
Members for current dues.............	470.00	
Members for past dues, suspense account............	7,216.50	
Members for entrance fees.......	305.00	
Miscellaneous......	367.93	
For Advertising....	1,803.50	
Accrued interest on Bonds...........	675.00	
Accrued interest on bank balance.....	212.38	11,050.31
Total Assets.......		$671,565.19

Liabilities and Surplus.

Funds:		
Land, Building and Endowment Fund	5,034.39	
General Library fund	267.27	
Compounded Membership Fund.....	4,742.99	
Mailloux Fund.....	1,063.85	
International Electrical Congress of St. Louis 1904, Library Fund:		
Bonds...........	2,268.00	
Cash, on deposit...	268.77	
Accrued interest....	45.00	13,690.27
Reserve, for Furniture and Fixtures.		2,237.98
Accounts payable, Subject to approval by the Finance Committee..............		3,973.40
United Engineering Society (for cost of land).........		54,000.00
Total Liabilities....		73,901.65
Surplus:		
In Cash...........	18,866.49	
New York City bonds...........	31,952.50	
Chicago, Burlington & Quincy Bonds.	14,606.25	
In property and accounts receivable.	532,238.30	597,663.5
Total Liabilities and surplus..		671,565.19

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Receipts and Disbursements for General Purposes for Year Ended April 30, 1911.

Exhibit B.

Receipts.		
Entrance Fees	4,430.00	
Current Dues	63,550.87	
Past Dues	5,410.00	
Advance Dues	262.00	
Students Dues	4,185.00	
Transfer Fees	530.00	
Badges	1,845.00	
		80,212.87
Sales, Transactions, etc.	1,271.86	
Subscriptions, Proceedings	1,596.77	
Advertising	9,350.66	
Binding	149.50	
Exchange	20.26	
		12,389.05
Interest:		
Bonds	1,650.00	
Bank Balance	795.30	
		2,445.30
Royalty		100.00
Total		$95,147.22

Disbursements.		
Stationery and Printing	2,987.01	
Postage	2,730.05	
General Expenses	2,568.01	
Meeting Expenses	4,613.03	
Section Meetings	6,831.85	
Badges purchased	1,541.14	
Salaries	11,380.00	
Interest on Mtge	2,160.00	
Office Furniture	613.74	
Advertising Expense	3,491.41	
Year Book and Catalogue	2,645.79	
Express	232.43	
		$41,794.46
Proceedings:		
Printing	$7,777.88	
Paper and Envelopes	5,731.13	
Engraving	2,224.77	
Binding and Mailing	3,860.32	
Salaries	3,372.00	
		22,966.10
Transactions:		
Vol. 28	5,474.50	
Vol. 29	1,084.82	
		6,559.32
Library (including salaries)		3,083.30
United Engineering Society.. Assessments for office space		4,000.00
Total		$78,403.18
Excess Receipts over Disbursements		16,744.04
		$95,147.22

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Receipts and Donations for Designated Purposes, also Expenditures for Year Ended April 30, 1911.

Exhibit C.

Receipts.	
Land, Building and Endowment Fund, Donations, Interest, etc	$1,791.54
General Library Fund, Interest	6.48
Compounded Membership Fund, Interest	231.62
International Electrical Congress of St. Louis 1904, Library Fund, Donations and interest	113.85
Special Library account	127.00
Total	2,270.49

Expenditures.	
Mailloux Fund	32.25
Compounded Membership Fund	506.87
Certificate of Deposit F. L. & T. Co	1,000.00
N. Y. Telephone Bond, due 1939	986.75
C. B. & Q. Bonds purchased	14,729.58
Special Library account (to be reimbursed)	77.95
Total	17,333.40

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

CONDENSED CASH STATEMENT.

EXHIBIT D.

Cash on deposit April 30, 1910	27,559.80	
Secretary's Petty Cash, April 30, 1910	750.00	
		28,309.80
Receipts for general purposes, Exhibit "B"	95,147.22	
Receipts for designated purposes, Exhibit "C"	2,270.49	
		97,417.71
		125,727.51
Disbursements for general purposes Exhibit "B"	78,403.18	
Expenditures for designated purposes, Exhibit "C"	17,333.40	
		95,736.58
Balance on hand April 30, 1911		29,990 93
On deposit for designated purposes, Exhibit "A"	9,998.88	
*On deposit in General cash, Exhibit "A"	19,242.05	
Secretary's Petty Cash, Exhibit "A"	750.00	
		29,990.93
Property acquired during the year, Office Furniture and Fixtures		613.74
*This includes the following unexpended balances:		
Mailloux Fund	41.35	
Weaver Donation	65.44	
Int. Elec. Congress of St. Louis Library Fund	268.77	
		375.56

RECEIPTS AND DISBURSEMENTS PER YEAR PER MEMBER.

During each fiscal year for the past seven years.

Year	**1905**	**1906**	**1907**	**1908**	**1908**	**1910**	**1911**
Membership, April 30, each year	3460	3870	4521	5674	6400	6681	7117
Receipts per Member	$12.32	$12.77	$12.21	$13.01	$13.21	$13.35	$13.37
Disbursements per Member:	$10.72	$10.48	$11.62	$11.73	$10.49	$12.03	$11.03
Credit Balance per Member	$1.60	$2.29	.59	$1.28	$2.72	$1.32	$2.34

Respectfully submitted for the Board of Directors,

RALPH W. POPE, Secretary.

New York, May 16, 1911.

SYNOPTICAL AND TOPICAL

INDEX

OF

A. I. E. E. TRANSACTIONS

Vol. XXX, Parts I, II and III

Prepared under the supervision of the Committee on Indexing the Transactions

The main headings under which these synopses are classified were arrived at by a careful study of all the papers contributed since the organization of the Institute.

The method of making this classification may be called the automatic method, since it is created by sorting the papers themselves into groups and then naming the groups.

Many papers fall naturally into several different groups and in such cases they are inserted under as many different heads as it is thought they rightfully belong.

The classified synopses are designed for those searching for comprehensive information on any given topic, while the subject index is intended for those looking up specific and definite data or information.

MAIN SECTIONS OF SYNOPTICAL INDEX

1. EDUCATION

TENTATIVE SCHEME OF ORGANIZATION AND ADMINISTRATION, FOR A STATE UNIVERSITY

Ralph D. Mershon **Vol. xxx—1911, pp. 2337-2344**

Endeavor to improve the efficiency of university organization system.

Discussion, pp, 2346-2358 by Messrs. A. H. Ford, C. F. Harding, B. B. Brackett, A. S. Langsdorf, George D. Shepardson, M. C. Beebe, W. L. Upson and Ralph D. Mershon.

Criticisms of author's plan. Other plans of organization.

2. GENERAL THEORY

MECHANICAL FORCES IN MAGNETIC FIELDS

Charles P. Steinmetz **Vol. xxx—1911, pp. 367-385**

Elementary theory of the energy transformation in plunger magnets, giving equations for the mechanical work and the efficiency. Brief mathematical consideration of the mechanical forces exerted by short-circuit currents in transformers, cables and general circuits.

Discussion, pp. 386-413, by Messrs. A. S. McAllister, G. Faccioli, Cassius M. Davis, Mr. Harper, F. C. Green, C. J. Barrow, Henry Pikler, John J. Frank, Charles F. Scott, H. C. Cox, V. Karapetoff, K. Faye-Hansen, J. Murray Weed, H. B. Dwight, A. C. Zelewsky, E. Jasse and C. P. Steinmetz.

General discussion of the mechanical forces exerted on short circuits in transformers,—equations, tests and experience. Energy transformations in plunger magnets.

THE TEMPERATURE GRADIENT IN OIL-IMMERSED TRANSFORMERS

James Murray Weed **Vol. xxx—1911, pp. 427-446**

General discussion of the complex temperature gradient from the hottest part of the transformer coil to the external air. Thermal properties of various substances. Laws of heat dissipation.

Discussion incorporated with that of paper by F. C. Green on " Problems in the Operation of Transformers."

THE DIRECTION OF ROTATION IN ALTERNATING-CURRENT VECTOR DIAGRAMS

Ernst J. Berg **Vol. xxx—1911, pp. 575-579**

Advantages of rectangular and polar coordinate systems of representing alternating quantities.

Discussion incorporated with that of paper by W. S. Franklin on " Conventions in Clock-Diagram Representation."

CONVENTIONS IN CLOCK-DIAGRAM REPRESENTATION

W. S. Franklin **Vol. xxx—1911, pp. 581-585**

Presentation of clock-diagram with both methods of representing time-phase displacement.

Discussion, (including that of paper by Ernst J. Berg on " The Direction of Rotation in Alternating-Current Vector Diagrams "), pp. 586-596, by Messrs. G. L. Hoxie, Charles P. Steinmetz, Louis F. Blume, O. J. Ferguson, and C. A. Adams.

General discussion of the relative advantages of the polar coordinate system and the crank diagram in representing alternating quantities.

CISOIDAL OSCILLATIONS

George A. Campbell **Vol. xxx—1911 pp. 873-909**

Development of system of equations for general electric circuit. Application in the calculation of quantities in networks of conductors; free oscillations of eddy currents and skin effect.

Discussion, pp. 910-913, by Mr. C. L. Cory.

Physical meaning of author's equations.

THE ELECTRIC STRENGTH OF AIR—II

John B. Whitehead **Vol. xxx—1911, pp. 1857-1887**

Account of experimental investigation of various factors which affect corona or critical e.m.f. and critical surface intensity at which corona forms. Discussion of results with reference to their bearing upon ionization theory.

No discussion.

THE LAW OF CORONA AND THE DIELECTRIC STRENGTH OF AIR

F. W. Peek, Jr. **Vol. xxx—1911, pp. 1889-1965**

Account of extensive investigations of corona and dielectric phenomena on a 230,000-volt transmission line. Study of the effects of various atmospheric conditions; of wire diameter, spacing and nature of surface; and of frequency, etc., on dielectric phenomena. Methods of measuring corona losses. Development of formulas for expressing the relations between the various factors. Results tabulated and plotted as curves.

Discussion, pp. 1966-1988, by Messrs. C. P. Steinmetz, Cassius M. Davis, A. B. Hendricks, Jr., Charles F. Scott, Harris J. Ryan, John B. Whitehead and F. W. Peek, Jr.

Results of other corona tests. Relation between critical e.m.f. and diameter giving critical diameter. Comparison of results of Dr. Whitehead and Mr. Peek with earlier experimental work. Terminology of dielectric phenomena. Dielectric strength of oil under pressure.

A THEORY OF COMMUTATION AND ITS APPLICATION TO INTERPOLE MACHINES

B. G. Lamme **Vol. xxx—1911, pp. 2359-2404**

Development of theory of commutation with method of calculation based on fundamental law of electromagnetic induction. Analysis of various fluxes and their effect on commutation. Design of interpoles.

Discussion, pp. 2405-2431, by Messrs. H. F. T. Erben, Geo. L. Hoxie, H. M. Hobart, Malcolm MacLaren, C. A. Adams, C. F. Scott, James Burke, R. B Treat, C. E. Wilson, and B. G. Lamme.

Theoretical and experimental methods of studying commutation.

TABLES OF HYPERBOLIC FUNCTIONS IN REFERENCE TO LONG ALTERNATING-CURRENT TRANSMISSION LINES

A. E. Kennelly **Vol. xxx—1911, pp. 2495-2506**

Formula for use with hyperbolic functions in calculation of long transmission lines. Tables calculated for this purpose. Examples of the use of the formulas and tables in calculation of different types of lines. Bibliography of tables of complex hyperbolic functions.

No discussion.

3. MEASUREMENTS AND INSTRUMENTS

HIGH-TENSION TESTING OF INSULATING MATERIALS

A. B. Hendricks, Jr. **Vol. xxx—1911, pp. 167-218**

Outline of methods of testing the dielectric properties of materials, covering briefly general conditions, choice of apparatus, factors which affect the results and properties of commercial insulating materials. Description of the method used by the author for standard tests and results of numerous tests on commercial materials given in the form of curves. Design of new type of spark gap for oil testing.

Discussion, incorporated with that of paper by H. R. Wilson on " Commercial Problems of Transformer Design."

COMMERCIAL PROBLEMS OF TRANSFORMER DESIGN

H. R. Wilson **Vol. xxx—1911, pp. 219-223**

Brief mention of some of the commercial factors which enter into the design of transformers.

Discussion, (including that of paper by A. B. Hendricks, Jr., on "High-Tension Testing of Insulating Materials," and W. J. Wooldridge on " Hysteresis and Eddy Current Exponents for Silicon Steel), pp. 224-243, by Messrs. Charles P. Steinmetz, Henry Pikler, Ralph D. Mershon, William L. Puffer, A. S. McAllister, L. T. Robinson, George F. Seaver, J. R. Craighead, J. L. R. Hayden, F. M. Farmer, H. L. Schermerhorn, E. M. Hewlett, W. J. Foster, Charles F. Scott, C. A. Adams, and William A. Del Mar.

Relative merits of various methods measuring high e.m.f. Outline of proper method of insulating electrical machinery. Analytical study of stress distribution in cable insulation with proposed explanation of early breakdown in inner layers of insulation.

TEST OF LOSSES ON HIGH-TENSION LINES

G. Faccioli **Vol. xxx—1911, pp. 337-355**

Description of corona loss tests on lines of Central Colorado Power Company. Methods of measurement. Results in form of curves and in tables. Experimental law of corona loss. Equation of actual corona e.m.f. as function of spacing and diameter.

Discussion, (including that of paper by J. H. Cunningham on " Design, Construction and Test of an Artificial Transmission Line;" paper by E. E. F. Creighton on " Protection of Electrical Transmission Lines" and paper by C. I. Burkholder and R. H. Marvin on " Tests of Arcing-Ground Suppressor on the 40,000-Volt System of the Southern Power Company "), pp. 356-365, by Messrs. Ralph D. Mershon, Charles S. Ruffner, L. C. Nicholson, Taylor Reed, J. L. R. Hayden, F. W. Peek, Jr., L. T. Robinson, Charles F. Scott, G. Faccioli and E. E. F. Creighton.

Discussion of corona loss and critical e.m.f. laws. Experimental determinations. Actual distribution of insulator flash-overs among the three conductors of a three-phase line. Protection of lines from insulator spill-overs.

COMMERCIAL TESTING OF SHEET IRON FOR HYSTERESIS LOSS

L. T. Robinson **Vol. xxx—1911, pp. 741-760**

Outline of the requirements of a shop method of testing iron. Description of testing outfits developed by the author, with results of tests of commercial iron. Relative accuracy and cost of testing compared with Epstein method.

Discussion, incorporated with that of paper by Malcolm MacLaren on " The Effect of Temperature upon the Hysteresis Loss in Sheet Steel."

THE EFFECT OF TEMPERATURE UPON THE HYSTERESIS LOSS IN SHEET STEEL

Malcolm MacLaren **Vol. xxx—1911, pp. 761-775**

Description of iron-loss tests on samples of iron at temperatures up to non-magnetic point. Eddy-current and hysteresis coefficients and value of hysteresis loss plotted with temperature for different densities. Also a description of method of measuring hysteresis loss directly.

Discussion, (including that of paper by L. T. Robinson on " Commercial Testing of Sheet Iron for Hysteresis Loss "), pp. 776-802, by Messrs. C. H. Sharp. Edwin F. Northrup, J. A. Capp, L. W. Chubb, R. B. Treat, W. J. Wooldridge, W. R. Whitney, C. J. Fechheimer, C. A. Adams, W. S. Franklin, Malcolm MacLaren, L. T. Robinson, Henry Pikler, M. G. Lloyd and J. D. Ball.

General remarks on commercial methods of testing magnetic properties of iron. Relation between flux density and hysteresis exponent. Method of calculating the hysteresis exponent from observed iron losses.

A POWER DIAGRAM INDICATOR FOR HIGH-TENSION CIRCUITS

Harris J. Ryan **Vol. xxx—1911, pp. 1069-1113**

Cathode ray power indicator—its theory of operation and its mode of construction. Account of tests made with power indicator, demonstrating its accuracy and usefulness. Measurement of very low power at high e.m.f. Photographs of apparatus and connection diagrams.

Discussion, pp. 1114-1119 by Messrs. E. F. Scattergood, C. L. Cory, J. J. Frank, Ralph D. Mershon, H. J. Ryan and R. W. Sorensen.

General remarks on the uses and advantages of the cathode-ray power indicator.

ELECTRIC LINE OSCILLATIONS

G. Faccioli **Vol. xxx—1911, pp. 1803-1850**

Account of switching tests on 100,000-volt transmission system of the Great Western Power Company. Detailed discussion of large number of oscillograms. Recommendation for switching on high-tension systems. Appendix by W. W. Lewis on construction and operation of the photographic attachment to the oscillograph.

Discussion, pp. 1851-1856, by Messrs. C. P. Steinmetz, Max H. Collbohm, D. B. Rushmore and Percy H. Thomas.

THE LAW OF CORONA AND THE DIELECTRIC STRENGTH OF AIR

F. W. Peek, Jr. **Vol. xxx—1911, pp. 1889-1965**

Accounts of extensive investigations of corona and dielectric phenomena on a 230,000-volt transmission line. Study of the effects of various atmospheric conditions of wire diameter, spacing and nature of surface, and of frequency, etc., on dielectric phenomena. Methods of measuring corona losses. Development of formulas for expressing the relations between the various factors. Results tabulated and plotted as curves.

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Results of other corona tests. Relation between critical e.m.f. and diameter giving critical diameter. Comparison of results of Dr. Whitehead and Mr. Peek with earlier experimental work. Terminology of dielectric phenomena. Dielectric strength of oil under pressure.

THE APPLICATION OF CURRENT TRANSFORMERS TO THREE-PHASE CIRCUITS

J. R. Craighead **Vol. xxx—1911, pp. 2167-2179**

Analysis of the performance of series transformers with secondaries interconnected in various ways, the primaries being included in polyphase circuits.

Discussion, incorporated with that of paper by Messrs. E. C. Stone and R. W. Atkinson on " Cost of Transformer Losses."

MAGNETIC PROPERTIES OF IRON AT FREQUENCIES UP TO 200,000 CYCLES

E. F. W. Alexanderson **Vol. xxx—1911, pp. 2433-2448**

Measurements of magnetizing core loss and skin effect at high frequencies. Method of measuring power at high frequency. Relation between the various factors and frequency plotted as curves.

Discussion, pp. 2449-2454, by Messrs. Harold Pender, B. A. Behrend, Charles P. Steinmetz, Reginald A. Fessenden and F. B. Silsbee.

Behavior of iron at high frequency.

4. DIELECTRIC PHENOMENA

OPEN ATMOSPHERE AND DRY TRANSFORMER OIL AS HIGH VOLTAGE INSULATORS

Harris J. Ryan **Vol. xxx—1911, pp. 1-76**

Analytical study of corona formation and the factors which influence it, covering practically all the work that has thus far been done by the author and other investigators. Complete references to all original work. Brief explanation of the ionization theory, which is applied to explain the phenomena that take place on high-tension lines. Laws formulated for calculation of critical e.m.f. under all conditions.

Discussion, incorporated with that of paper by E. L. West on " High Voltage Line Loss Tests made on the 100-Kilovolt, 60-Cycle, 180-Mile Transmission Line of the Central Colorado Power Company."

HIGH-VOLTAGE LINE LOSS TESTS MADE ON THE 100-KILOVOLT, 60-CYCLE, 180-MILE TRANSMISSION LINE OF THE CENTRAL COLORADO POWER COMPANY

E. L. West **Vol. xxx—1911, pp. 77-85**

Communication to Professor Ryan by the author, giving detailed results of tests on extensive transmission system with and without load.

Discussion, (including that of paper by Harris J. Ryan on " Open Atmosphere and Dry Transformer Oil as High-Voltage Insulators ") pp. 86-130, by Messrs. M. I. Pupin, G. Faccioli, C. P. Steinmetz, William S. Stanley, J. B. Whitehead, Erich Hausmann, Samuel Sheldon, H. W. Fisher, J. E. Noeggerath, A. E. Kennelly, F. W. Peek, Jr., George L. Hoxie, A. B. Hendricks, Jr., J. A. Koontz, C. E. Bennett, and Harris J. Ryan. General discussion of Professor Ryan's conclusions. Criticism and defense of electron theory. Formula for practical corona calculations on long-distance lines. Method of constructing minute needle points. Relation between conductivity of gases and vapors and corona phenomena. Results of tests on high-tension lines and also with short-spark discharge apparatus.

HIGH-TENSION TESTING OF INSULATING MATERIALS

A. B. Hendricks, Jr. **Vol. xxx—1911, pp. 187-218**

Outline of methods of testing the dielectric properties of materials, covering briefly general conditions, choice of apparatus, factors which affect the results and properties of commercial insulating materials. Description of the method used by the author for standard tests and results of numerous tests on commercial materials given in the form of curves. Design of new type of spark gap for oil testing.

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G. Faccioli **Vol. xxx—1911, pp. 337-355**

Description of corona loss tests on lines of Central Colorado Power Company. Methods of measurement. Results in form of curves and in tables. Experimental law of corona loss. Equation of actual corona e.m.f. as function of spacing and diameter.

Discussion, (including that of paper by J. H. Cunningham on " Design, Construction and Test of an Artificial Transmission Line;" paper by E. E. F. Creighton on " Protection of Electrical Transmission Lines" and paper by C. I. Burkholder and R. H. Marvin on " Tests of Arcing Ground Suppressor on the 40,000-Volt System of the Southern Power Company,") pp. 356-365, by Messrs. Ralph D. Mershon, Charles S. Ruffner, L. C. Nicholson, Taylor Reed, J. L. R. Hayden, F. W. Peek, Jr., L. T. Robinson, Charles F. Scott, G. Faccioli and E. E. F. Creighton.

Discussion of corona loss and critical e.m.f. laws. Experimental determinations. Actual distribution of insulator flash-overs among the three conductors of a three-phase line. Protection of lines from insulator spill-overs.

THE ELECTRIC STRENGTH OF AIR—II

John B. Whitehead **Vol. xxx—1911, pp. 1837-1857**

Account of experimental investigation of various factors which affect corona or critical e.m.f. and critical surface intensity at which corona forms. Discussion of results with reference to their bearing upon ionization theory.

No discussion.

THE LAW OF CORONA AND THE DIELECTRIC STRENGTH OF AIR

F. W. Peek, Jr. **Vol. xxx—1911, pp. 1859-1965**

Account of extensive investigations of corona and dielectric phenomena on a 230,000-volt transmission line. Study of the effects of various atmospheric conditions of wire diameter, spacing and nature of surface, and of frequency, etc., on dielectric phenomena. Methods of measuring corona losses. Development of formulas for expressing the relations between the various factors. Results tabulated and plotted as curves.

Discussion, pp. 1966-1988, by Messrs C. P. Steinmetz, Cassius M. Davis, A. B. Hendricks, Jr., Charles F. Scott, Harris J. Ryan, John B. Whitehead and F. W. Peek, Jr.

Results of other corona tests. Relations between critical e.m.f. and diameter giving critical diameter. Comparison of results of Dr. Whitehead and Mr. Peek with earlier experimental work. Terminology of dielectric phenomena. Dielectric strength of oil under pressure.

THE HIGH-EFFICIENCY SUSPENSION INSULATOR

A. O. Austin **Vol. xxx—1911, pp. 2303-2328**

Mathematical study of the efficiency of different shapes and types of suspension insulator. Judging an insulator by flash-over tests. Design of insulators for high tension.

Discussion, pp. 2329-2335, by Messrs. E. E. F. Creighton, Paul M. Lincoln, P. H. Thomas, E. M. Hewlett, N. J. Neall and A. O. Austin.

Behavior of insulators under high-tension stresses. Distribution of stresses between several parts of link insulators.

5. ELECTRIC CONDUCTORS

ELECTROLYTIC CORROSION IN REINFORCED CONCRETE

C. Edward Magnusson and C. H. Smith **Vol. xxx—1911, pp. 2055-2079**

Account of experimental investigation of damage done by stray currents in reinforced concrete. Study of various methods of protecting reinforcement from corrosion.

Discussion, pp. 2080-2101, by Messrs. Burton McCollum, E. W. Stevenson, G. G. Shepardson, George A. Hoadley, A. S. Langsdorf, Maxmilian Toch, Guy F. Shaffer, Harry Barker and W. L. Upson.

Description of similar investigations by the Bureau of Standards and others. Criticism of the authors' methods, especially of the high e.m.fs. used. Account of actual example of corrosion in a building.

6. MAGNETIC PROPERTIES AND TESTING OF IRON

HYSTERESIS AND EDDY-CURRENT EXPONENTS FOR SILICON STEEL

W. J. Wooldridge **Vol. xxx—1911, pp. 215-217**

Results of tests plotted in the form of curves.

Discussion, incorporated with that of paper by H. R. Wilson on " Commercial Problems of Transformer Design."

COMMERCIAL TESTING OF SHEET IRON FOR HYSTERESIS LOSS

L. T. Robinson **Vol. xxx 1911 pp. 741-760**

Outline of the requirements of a shop method of testing iron. Description of testing outfits developed by the author, with results of tests of commercial iron. Relative accuracy and cost of testing compared with Epstein method.

Discussion, incorporated with that of paper by Malcolm MacLaren on " The Effect of Temperature upon the Hysteresis Loss in Sheet Steel."

THE EFFECT OF TEMPERATURE UPON THE HYSTERESIS LOSS IN SHEET STEEL

Malcolm MacLaren **Vol. xxx—1911, pp. 761-775**

Description of iron loss tests on samples of iron at temperatures up to non-magnetic point. Eddy current and hysteresis coefficients and value of hysteresis loss plotted with temperature for different densities. Also a description of method of measuring hysteresis loss directly.

Discussion, (including that of paper by L. T. Robinson on " Commercial Testing of Sheet Iron for Hysteresis Loss ") pp. 776-802, by Messrs. C. H. Sharp, Edwin F. Northrup, J. A. Capp, L. W. Chubb, R. B. Treat, W. J. Wooldridge, W. R. Whitney, C. J. Fechheimer, C. A. Adams, W. S. Franklin, Malcolm MacLaren, L. T. Robinson, Henry Pikler, M. G. Lloyd and J. D. Ball.

General remarks on commercial methods of testing magnetic properties of iron. Relation between flux density and hysteresis exponent. Method of calculating the hysteresis exponent from observed iron losses.

MAGNETIC PROPERTIES OF IRON AT FREQUENCIES UP TO 200,000 CYCLES

E. F. W. Alexanderson **Vol. xxx—1911, pp 2423-2448**

Measurements of magnetizing core loss and skin effect at high frequencies. Methods of measuring power at high frequency. Relation between the various factors and frequency plotted as curves.

Discussion, pp. 2449-2454, by Messrs. Harold Pender, A. B. Behrend Charles P. Steinmetz, Reginald A. Fessenden, F. B. Silsbee.

Behavior of iron at high frequency.

8. TRANSFORMERS

OPEN ATMOSPHERE AND DRY TRANSFORMER OIL AS HIGH-VOLTAGE INSULATORS

Harris J. Ryan **Vol. xxx—1911, pp. 1-76**

Analytical study of corona formation and the factors which influence it, covering practically all the work that has thus far been done by the author and other investigators. Complete references to all original work. Brief explanation of the ionization theory, which is applied to explain the phenomena that take place on high tension lines. Laws formulated for calculation of critical e.m.f. under all conditions.

Discussion, incorporated with that of paper by E. L. West on " High Voltage Line Loss Tests made on the 100-Kilovolt, 60-Cycle, 180-Mile Transmission Line of the Central Colorado Power Company."

COMMERCIAL PROBLEMS OF TRANSFORMER DESIGN

H. R. Wilson **Vol. xxx—1911, pp. 219-223**

Brief mention of some of the commercial factors which enter into the design of transformers.

Discussion, (including that of paper by A. B. Hendricks, Jr., on " High Tension Testing of Insulating Materials " and W. J. Wooldridge on " Hysteresis and Eddy Current Exponents for Silicon Steel "), pp. 224-243, by Messrs. Charles P. Steinmetz, Henry Pikler, Ralph D. Mershon, William L. Puffer, A. S. McAllister, L. T. Robinson, George F. Seaver, J. R. Craighead, J. L. R. Hayden, F. M. Farmer, H. L. Schermerhorn, E. M. Hewlett, W. J. Foster, Charles F. Scott, C. A. Adams, and William A. Del Mar.

Relative merits of various methods of measuring of high e.m.f. Outline of proper method of insulating electrical machinery. Analytical study of stress distribution in cable insulation with proposed explanation of early breakdown in inner layers of insulation.

MECHANICAL FORCES IN MAGNETIC FIELDS

Charles P. Steinmetz **Vol. xxx—1911, pp. 367-385**

Elementary theory of the energy transformation in plunger magnets, giving equations for the mechanical work and the efficiency. Brief mathematical consideration of the mechanical forces exerted by short-circuit currents in transformers, cables and general circuits.

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General discussion of the mechanical forces exerted on short circuits in transformers, equations, tests and experience. Energy transformations in plunger magnets.

THE REGULATION OF DISTRIBUTING TRANSFORMERS

C. E. Allen **Vol. xxx—1911, pp. 415-426**

Comparison of a number of the most important regulation formulas. Effect of different electric and magnetic constants upon regulation characteristics with varying power-factor. Modern winding construction for good regulation.

Discussion, incorporated with that of paper by F. C. Green on " Problems in the Operation of Transformers."

THE TEMPERATURE GRADIENT IN OIL IMMERSED TRANSFORMERS

James Murray Weed **Vol. xxx—1911, pp. 427-446**

General discussion of the complex temperature gradient from the hottest part of the transformer coil to the external air. Thermal properties of various substances. Laws of heat dissipation.

Discussion, incorporated with that of paper by F. C. Green on " Problems in the Operation of Transformers."

DISSIPATION OF HEAT FROM SELF-COOLED, OIL-FILLED TRANSFORMER TANKS

J. J. Frank and H. O. Stephens **Vol. xxx—1911, pp. 447-456**

Description of various modern types of transformer tanks, followed by results of tests bearing on the relative merits of each type in the dissipation of heat.

Discussion, incorporated with that of paper by F. C. Green on " Problems in the Operation of Transformers."

PROBLEMS IN THE OPERATION OF TRANSFORMERS

F. C. Green **Vol. xxx—1911, pp. 457-475**

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Discussion, (including that of paper by James Murray Weed on " The Temperature Gradient in Oil-Immersed Transformers " and paper by J. J. Frank and H. O. Stephens on " Dissipation of Heat from Self-Cooled, Oil-Filled Transformer Tanks "), pp. 476-494, by Messrs. W. S. Moody, Henry Pikler, E. G. Reed, Louis F. Blume, E. A. Wagner, C. A. Adams, A. E. Walden, Ralph D. Mershon, C. W. Stone, E. F. Alexanderson, C. P. Steinmetz, J. M. Weed, R. W. Atkinson and C. E. Allen.

General discussion of the regulation, cooling and protection of transformers from short-circuits.

COMMERCIAL TESTING OF SHEET IRON FOR HYSTERESIS LOSS

L. T. Robinson **Vol. xxx—1911, pp. 741-760**

Outline of the requirements of a shop method of testing iron. Description of testing outfits developed by the author, with results of tests of commercial iron. Relative accuracy and cost of testing compared with Epstein method.

Discussion, incorporated with that of paper by Malcolm MacLaren on "The Effect of Temperature upon the Hysteresis Loss in Sheet Steel."

THE APPLICATION OF CURRENT TRANSFORMERS TO THREE-PHASE CIRCUITS

J. R. Craighead **Vol. xxx—1911, pp. 2167-2179**

Analysis of the performance of series transformers with secondaries interconnected in various ways, the primaries being included in polyphase circuits.

Discussion, incorporated with that of paper by Messrs. E. C. Stone and R. W. Atkinson on "Cost of Transformer Losses."

COST OF TRANSFORMER LOSSES

E. C. Stone and R. W. Atkinson **Vol. xxx—1911 pp. 2181-2199**

Classification of transformer losses and complete analytical discussion of the cost of various losses under various operating conditions.

Discussion, (including that of paper by J. R. Craighead on "The Application of Current Transformers to Three-Phase Circuits"), pp. 2200-2207, by Messrs. A. H. Pikler, W. C. Smith, E. A. Wagner, H. B. Gear, and R. W. Atkinson.

Progress in transformer design in a period of 12 years. Importance of long life as compared with energy efficiency.

MAGNETIC PROPERTIES OF IRON AT FREQUENCIES UP TO 200,000 CYCLES

E. F. W. Alexanderson **Vol. xxx—1911, pp. 2433-2448**

Measurements of magnetizing core loss and skin effect at high frequencies. Method of measuring power at high frequency. Relation between the various factors and frequency plotted as curves.

Discussion, pp. 2449-2454, by Messrs. Harold Pender, B. A. Behrend, Charles P. Steinmetz, Reginald A. Fessenden and F. B. Silsbee.

Behavior of iron at high frequency.

9. ELECTRICAL MACHINERY AND APPARATUS

MECHANICAL FORCES IN MAGNETIC FIELDS

Charles P. Steinmetz **Vol. xxx—1911, pp. 367-385**

Elementary theory of the energy transformation in plunger magnets, giving equations for the mechanical work and the efficiency. Brief mathematical consideration of the mechanical forces exerted by short-circuit currents in transformers, cables and general circuits.

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General discussion of the mechanical forces exerted on short circuits in transformers, equations, tests and experience. Energy transformations in plunger magnets.

VOLTAGE REGULATION OF GENERATORS

H. A. Laycock **Vol. xxx—1911, pp. 555-571**

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Desirability of poor inherent regulation of large generators.

FLYWHEEL LOAD EQUALIZER

W. N. Motter and L. L. Tatum **Vol. xxx—1911, pp. 729-739**

Description of a fly-wheel-generator installation designed to take the sudden peak loads encountered in the operation of ore bridges. Results of tests of power consumption and actual saving in the cost of handling the ore.

No discussion.

THE USE OF POWER-LIMITING REACTANCES WITH LARGE TURBO-ALTERNATORS

R. F. Schuchardt and E. O. Schweitzer **Vol. xxx—1911, pp. 1143-1194**

Detailed account of exhaustive tests made to determine the action of reactors in the leads of turbo-generators under various conditions, with respect to the effect on the generators, on the secondary apparatus and on the stability of the system. Oscillograms of currents and e.m.fs. are given for each test. Bibliography.

Discussion, incorporated with that of paper by Charles P. Steinmetz on " Development of the Modern Central Station."

INDUCTION MACHINES FOR HEAVY SINGLE-PHASE MOTOR SERVICE

E. F. W. Alexanderson **Vol. xxx—1911, pp. 1357-1369.**

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Arthur C. Eastwood **Vol. xxx—1911, pp. 1519-1537**

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R. Wikander **Vol. xxx—1911, pp. 2019-2051**

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WAVE SHAPE OF CURRENTS IN AN INDIVIDUAL ROTOR CONDUCTOR OF A SINGLE-PHASE INDUCTION MOTOR

H. Weichsel **Vol. xxx—1911, pp. 2103-2124**

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CHOICE OF ROTOR DIAMETER AND PERFORMANCE OF POLYPHASE INDUCTION MOTORS

Theodore Hoock **Vol. xxx—1911, pp. 2127-2158**

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B. G. Lamme **Vol. xxx—1911, pp. 2359-2404**

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MAGNETIC PROPERTIES OF IRON AT FREQUENCIES UP TO 200,000 CYCLES

E. F. W. Alexanderson **Vol. xxx—1911, pp. 2433-2448**

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Behavior of iron at high frequency.

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Gus A. Maier **Vol. xxx—1911, pp. 2455-2484**

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Description of additional methods of speed variation.

11. POWER PLANTS

COMMENTS ON FIXED COSTS IN INDUSTRIAL POWER PLANTS

John C. Parker **Vol. xxx—1911, pp. 637-652**

Analysis of fixed charges in isolated plants, covering investment, insurance, interest, depreciation, obsolescence, supervision, profit, real estate and space. Definition of marginal principle of figuring fixed cost.s Development of equation for amortization. Estimated and actual cost of energy production in isolated plants compared with central station service.

Discussion, incorporated with that of paper by Aldis E. Hibner on " The Cost of Industrial Power."

THE COST OF INDUSTRIAL POWER

Aldis E. Hibner **Vol. xxx—1911, pp. 653-664**

Brief consideration of the various factors that enter into the cost of energy production in isolated steam and gas producer plants. Effect of factory heating on cost of energy production. Investment, fixed charges and operating charges for heating plant, steam power plant and gas producer plant for a given industrial installation.

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TRANSMISSION APPLIED TO IRRIGATION

O. H. Ensign and James M. Gaylord **Vol. xxx—1911, pp. 803-834**

Detailed description of the Minidoka and Salt River Projects of the Reclamation Service covering power houses, transmission lines and pumping stations.

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THE USE OF POWER-LIMITING REACTANCES WITH LARGE TURBO-ALTERNATORS

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DEVELOPMENT OF THE MODERN CENTRAL STATION

Charles P. Steinmetz **Vol. xxx—1911, pp. 1213-1225**

Short sketch of the development of modern alternating-current station. Uses to which reactors can be put in localizing disturbance in different parts of the system and improving parallel operation of units. Electromechanical synchronizing of distant stations.

Discussion, (including that of paper by R. F. Schuchardt and E. O. Schweitzer, " The Use of Power-Limiting Reactances with Large Turbo-Alternators " and paper by E. B. Merriam on " Some Recent Tests of Oil Circuit Breakers "), pp. 1226-1249, by Messrs. John W. Lieb, Jr., M. H. Collbohm, D. B. Rushmore, C. W. Stone, B. G. Lamme, W. L. Waters, J. J. Frank, Louis A. Ferguson, R. B. Williamson, Clarence P. Fowler, P. Junkersfeld and C. P. Steinmetz.

General remarks on problems in operation of very large central stations, with special reference to the use of reactance in the circuits.

12. PARALLEL OPERATION

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13. TRANSMISSION LINES

OPEN ATMOSPHERE AND DRY TRANSFORMER OIL AS HIGH-VOLTAGE INSULATORS

Harris J. Ryan **Vol. xxx—1911, pp. 1-76**

Analytical study of corona formation and the factors which influence it, covering practically all the work that has thus far been done by the author and other investigators. Complete references to all original work. Brief explanation of the ionization theory, which is applied to explain the phenomena that take place on high-tension lines. Laws formulated for calculation of critical e.m.f. under all conditions.

Discussion, incorporated with that of paper by E. L. West on " High-Voltage Line Loss Tests made on the 100-Kilovolt, 60-Cycle, 180-Mile Transmission Line of the Central Colorado Power Company."

HIGH-VOLTAGE LINE LOSS TESTS MADE ON THE 100-KILOVOLT, 60-CYCLE, 180-MILE TRANSMISSION LINE OF THE CENTRAL COLORADO POWER COMPANY

E. L. West **Vol. xxx—1911, pp. 77-85**

Communication to Professor Ryan by the author, giving detailed results of tests on extensive transmission system with and without load.

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DESIGN, CONSTRUCTION AND TEST OF AN ARTIFICIAL TRANSMISSION LINE

J. H. Cunningham **Vol. xxx—1911, pp. 243-256**

Detailed account of the mode of construction. Oscillograms of current and e.m.f. when the line was opened and closed.

Discussion, incorporated with that of paper by G. Faccioli on " Tests of Losses on High-Tension Lines."

PROTECTION OF ELECTRICAL TRANSMISSION LINES

E. E. F. Creighton **Vol. xxx—1911, pp. 257-326**

Comprehensive discussion of line disturbances due to arcing ground on insulated, dead-grounded and resistance-grounded systems, with description of arcing-ground suppressor and tests to which it has been submitted. Detailed treatment of line capacity under various conditions of grounding. General discussion of lightning disturbances. Theory of lightning induction. Elements in the design of transmission line that affect lightning disturbances. Arcing-over tests on insulators under various conditions. Profusely illustrated with photographs of arcs between lines, in horn gaps and over insulators, with oscillograms and circuit diagrams of the tests and of the arc suppressors.

Discussion, incorporated with that of paper by G. Faccioli on " Tests of Losses on High Tension Lines."

TESTS OF ARCING-GROUND SUPPRESSOR ON THE 40,000-VOLT SYSTEM OF THE SOUTHERN POWER COMPANY

C. I. Burkholder and R. H. Marvin **Vol. xxx—1911, pp. 327-335**

Description of tests of automatic insulator protector with high-power arcs for insulator. Circuit diagrams, photographs of arcs and oscillograms of potential rise and current.

Discussion, incorporated with that of paper by G. Faccioli on " Tests of Losses on High-Tension Lines."

TESTS OF LOSSES ON HIGH-TENSION LINES

G. Faccioli **Vol. xxx—1911, pp. 337-356**

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Discussion of corona loss and critical e.m.f. laws. Experimental determinations. Actual distribution of insulator flash-overs among the three conductors of a three-phase line. Protection of lines from insulator spill-overs.

ECONOMIC LIMITATIONS TO AGGREGATION OF POWER SYSTEMS

Robert A. Philip **Vol. xxx—1911, pp. 597-630**

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O. H. Ensign and James M. Gaylord **Vol. xxx—1911, pp. 803-834**

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TRANSMISSION SYSTEMS FROM THE OPERATING STANDPOINT

R. J. C. Wood **Vol. xxx—1911, pp. 835-841**

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Magnus T. Crawford **Vol. xxx—1911, pp. 1049-1071**

Brief description of Snoqualmie Falls transmission system, with outline of the general method of manipulation in the maintenance of service. Log of service interruptions covering period of four years and giving date, character, extent and cause of each disturbance. General discussion of causes of interruptions, and methods of avoiding them.

Discussion, pp. 1072-1079, by Messrs. R. J. C. Wood, P. M. Downing, E. F. Scattergood, G. H. Stockbridge, R. W. Van Norden, C. O. Poole, Ralph Bennett, D. D. Morgan, W. B. Gump and M. T. Crawford.

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ELECTRIC LINE OSCILLATIONS

G. Faccioli **Vol. xxx—1911, pp. 1803-1850**

Account of switching tests on 100,000-volt transmission system of the Great Western Power Company. Detailed discussion of large number of oscillograms. Recommendation for switching on high-tension systems. Appendix by W. W. Lewis on construction and operation of the photographic attachment to the oscillograph.

Discussion, pp. 1851-1856, by Messrs. C. P. Steinmetz, Max H. Collbohm, D. B. Rushmore and Percy H. Thomas.

HIGH-TENSION TRANSMISSION

Percy H. Thomas **Vol. xxx—1911, pp. 1989-1993**

General discussion of answers to a list of questions submitted by the High-Tension Committee to companies operating transmission systems at 80,000 volts or more.

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P. T. Hanscom **Vol. xxx—1911, pp. 2007-2014**

Answers to the list of questions submitted by the High-Tension Committee covering line construction, protection and operation at 100,000 volts. Detailed log of lightning disturbances.

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General remarks on the protection of high-tension systems.

SOLUTION TO PROBLEMS IN SAGS AND SPANS

Wm. Le Roy Robertson **Vol. xxx—1911, pp. 2209-2228**

Comparison of the different types of equations for solution of sag problems. System of charts and tables for facilitating the calculation of sag in transmission wires.

Discussion, incorporated with that of paper by Pender and Thomson on " The Mechanical and Electrical Characteristics of Transmission Lines."

SAG CALCULATIONS FOR SUSPENDED WIRES

Percy H. Thomas **Vol. xxx—1911, pp. 2229-2239**

Short method of calculating sag. Description of use of method. Mathematical development of formulas.

Discussion, incorporated with that of paper by Pender and Thomson on " The Mechanical and Electrical Characteristics of Transmission Lines."

THE MECHANICAL AND ELECTRIC CHARACTERISTICS OF TRANSMISSION LINES

Harold Pender and H. F. Thomson **Vol. xxx—1911, pp. 2241-2279**

Equations and charts for calculation of the mechanical design of wire spans under various conditions of climate and topography. Equations, charts and tables for the calculation of the electrical performance of various types of transmission lines.

Discussion, (including " Solution to Problems in Sags and Spans " and " Sag Calculations for Suspended Wires "), pp. 2280-2302, by Messrs. Paul M. Lincoln, L. C. Nicholson, N. J. Neall, Jean Bart Balcomb, W. L. R. Robertson, H. F. Thomson, H. V. Carpenter, Hugh Pastoriza, R. S. Brown, Frank F. Fowle, R. C. Darrow, P. H. Thomas, Harold Pender and H. F. Thomson.

Calculation and design of transmission lines. Experience as to effects of wind, sleet and ice upon lines.

THE HIGH-EFFICIENCY SUSPENSION INSULATOR

A. O. Austin **Vol. xxx—1911, pp. 2303-2328**

Mathematical study of the efficiency of different shapes and types of suspension insulator. Judging an insulator by flash-over tests. Design of insulators for high tension.

Discussion, pp. 2329–2335, by Messrs. E. E. F. Creighton, Paul M. Lincoln, P. H. Thomas, E. M. Hewlett, N. J. Neall, A. O. Austin.

Behavior of insulators under high-tension stresses. Distribution of stresses between several parts of link insulators.

TABLES OF HYPERBOLIC FUNCTIONS IN REFERENCE TO LONG ALTERNATING-CURRENT TRANSMISSION LINES

A. E. Kennelly **Vol. xxx—1911, pp. 2495-2506**

Formula for use with hyperbolic functions in calculation of long transmission lines. Tables calculated for this purpose. Examples of the use of the formulas and tables in calculation of different types of lines. Bibliography of tables of complex hyperbolic functions.

No discussion.

14. ELECTRIC SERVICE DISTURBANCES AND PROTECTION

PROTECTION OF ELECTRICAL TRANSMISSION LINES

E. E. F. Creighton **Vol. xxx—1911, pp. 257-326**

Comprehensive discussion of line disturbances due to arcing ground on insulated, dead grounded and resistance grounded system, with description of arcing ground suppressor and tests to which it has been submitted. Detailed treatment of line capacity under various conditions of grounding. General discussion of lightning disturbances. Theory of lightning induction. Elements in the design of transmission line that affect lightning disturbances. Arcing-over tests on insulators under various conditions. Profusely illustrated with photographs of arcs between lines, in horn gaps and over insulators, with oscillograms and circuit diagram of the tests and of the arc suppressors.

Discussion, incorporated with that of paper by G. Faccioli on " Tests of Losses on High-Tension Lines."

TESTS OF ARCING GROUND SUPPRESSOR ON THE 40,000-VOLT SYSTEM OF THE SOUTHERN POWER COMPANY

C. I. Burkholder and R. H. Marvin **Vol. xxx 1911, pp. 327-336**

Description of tests of automatic insulator protector with high-power arcs for insulator. Circuit diagrams, photographs of arcs and oscillograms of potential rise and current.

Discussion, incorporated with that of paper by G. Faccioli on " Tests of Losses on High-Tension Lines."

CONTINUITY OF SERVICE IN TRANSMISSION SYSTEMS

Magnus T. Crawford **Vol. xxx—1911, pp. 1049-1071**

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Discussion, pp. 1072-1079, by Messrs. R. J. C. Wood, P. M. Downing, E. F. Scattergood, G. H. Stockbridge, R. W. Van Norden, C. O. Poole, Ralph Bennett, D. D. Morgan, W. B. Gump and M. T. Crawford.

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Discussion, (including that of paper by J. P. Jollyman on " Transmission System of the Great Western Power Company," paper by W. S. Lee on " Transmission System of the Southern Power Company " and paper by M. Hebgen on " Transmission System of the Great Falls Power Company "), pp. 2015-2018, by Messrs. M. H. Collbohm, Paul M. Lincoln, L. C. Nicholson, N. J. Neall, J. F. Vaughan and Hugh Pastoriza.

General remarks on the protection of high-tension systems.

15. DISTRIBUTION SYSTEMS

ADVANTAGES OF UNIFIED ELECTRIC SYSTEMS COVERING LARGE TERRITORIES

William B. Jackson **Vol. xxx—1911, pp. 131-151**

Statement of factors that make it economically possible to serve a large territory from one comprehensive transmission system. Maps and brief abstracts of a large number of hydroelectric and steam-electric distribution systems which cover extensive areas. Discussion of the advantages that a comprehensive system has over a large number of isolated small plants from the standpoint of economy and good service'

Discussion, pp. 152-165, by Messrs. P. Junkersfeld, W. L. Robb, F. Darlington, George H. Lukes, Norman T. Wilcox, Philip Torchio, L. L. Elden, Charles P. Steinmetz and William B. Jackson. General discussion of the advantages of a comprehensive distribution system over large territory and some of the factors that bear upon the success of such an undertaking. Actual comparison of the operation of the New York Edison plant with the London, Municipal and Private installations. Analysis of actual results obtained by the Boston Edison Company in the operation of a suburban system.

ECONOMIC LIMITATIONS TO AGGREGATION OF POWER SYSTEMS

Robert A. Philip **Vol. xxx—1911, pp. 597-630**

General discussion of the principles underlying the success of electric distribution of energy. Power diagram developed for inter-connected distribution systems in which the e.m.f. at all generating receiving stations is maintained constant by varying the reactive power in each receiving station. Numerical examples showing application of the line performance equations and vector diagrams.

Discussion, pp. 631-636, by Messrs. A. E. Kennelly, N. T. Wilcox, E. A. Ekern, Edw. N. Lake and R. P. Jackson.

General remarks on practicability of maintaining constant potential in all parts of a large system.

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ELECTROLYTIC CORROSION IN REINFORCED CONCRETE

C. Edward Magnusson and C. H. Smith **Vol. xxx—1911, pp. 2055-2079**

Account of experimental investigation of damage done by stray currents in reinforced concrete. Study of various methods of protecting reinforcement from corrosion.

Discussion, pp. 2080-2101, by Messrs. Burton McCollum, E. W. Stevenson, G. D. Shepardson, George A. Hoadley, A. S. Langsdorf, Maxmilian Toch, Guy F. Shaffer, Harry Barker and W. L. Upson.

Description of similar investigations by the Bureau of Standards and others. Criticism of the authors' methods, especially of the high e.m.fs. used. Account of actual example of corrosion in a building.

THE APPLICATION OF CURRENT TRANSFORMERS TO THREE-PHASE CIRCUITS

J. R. Craighead **Vol. xxx—1911, pp. 2167-2179**

Analysis of the performance of series transformers with secondaries interconnected in various ways, the primaries being included in polyphase circuits.

Discussion, incorporated with that of paper by Messrs. E. C. Stone and R. W. Atkinson on " Cost of Transformer Losses."

COST OF TRANSFORMER LOSSES

E. C. Stone and R. W. Atkinson **Vol. xxx—1911, pp. 2181-2199**

Classification of transformer losses and complete analytical discussion of the cost of various losses under various operating conditions.

Discussion, (including that of paper by J. R. Craighead on " The Application of Current Transformers to Three-Phase Circuits ", pp. 2200-2207, by Messrs. A. H. Pikler, W. C. Smith, E. A. Wagner, H. B. Gear and R. W. Atkinson.

Progress in transformer design in a period of 12 years. Importance of long life as compared with energy efficiency.

16. CONTROL, REGULATION AND SWITCHING

HIGH-VOLTAGE LINE LOSS TESTS MADE ON THE 100-KILOVOLT, 60-Cycle, 180-MILE TRANSMISSION LINE OF THE CENTRAL COLORADO POWER COMPANY

E. L. West **Vol. xxx—1911, pp. 77-85**

Communication to Professor Ryan by the author, giving detailed results of tests on extensive transmission system with and without load.

Discussion, (including that of paper by Harris J. Ryan on " Open Atmosphere and Dry Transformer Oil as High Voltage Insulators "), pp. 86-130, by Messrs. M. I. Pupin, G. Faccioli, C. P. Steinmetz, William S. Stanley, J. B. Whitehead, Erich Hausmann, Samuel Sheldon, H. W. Fisher, J. E. Noeggerath, A. E. Kennelly, F. W. Peek, Jr., George L. Hoxie, A. B. Hendricks, Jr., J. A. Koontz, C. E. Bennett and Harris J. Ryan. General discussion of Professor Ryan's conclusions. Criticism and defense of electron theory. Formula for practical corona calculations on long distance lines. Method of constructing minute needle points. Relation between conductivity of gases and vapors and corona phenomena. Results of tests on high-tension lines and also with short spark discharge apparatus.

THE REGULATION OF DISTRIBUTING TRANSFORMERS

C. E. Allen **Vol. xxx—1911, pp. 415-426**

Comparison of a number of the most important regulation formulas. Effect of different electric and magnetic constants upon regulation characteristics with varying power-factor. Modern winding construction for good regulation.

Discussion, incorporated with that of paper by F. C. Green on " Problems in the Operation of Transformers."

OIL CIRCUIT BREAKERS

E. B. Merriam **Vol. xxx —1911 pp. 495-502**

Brief discussion of the functions and operative characteristics of automatic oil switches, and of factors which influence their performance.

Discussion, pp. 503-527, by Messrs. B. C. Jamieson, E. M. Hewlett, L. C. Nicholson, W. L. R. Emmet, R. D. Mershon, C. J. Barrow, H. W. Cheney, A. S. McAllister, Charles F. Scott, C. P. Steinmetz, H. L. Smith, E. W. Rice, Jr., E. B. Merriam, Chester Lichtenberg, A. G. Collis and E. Zachrisson.

Experience and tests with oil circuit breakers opening short-circuits. Theoretical analysis of the influence of different kinds. of load upon rupturing capacity of oil switches.

VOLTAGE REGULATION OF GENERATORS

H. A. Laycock **Vol. xxx—1911, pp. 555-571**

Outline of requirements of generator and exciters essential to successful application of automatic regulation. Description of Tirrill regulator applied to alternating-current and direct-current systems. Examples of very large installations.

Discussion, pp. 572-573, by Messrs. H. G. Reist, Carl J. Fechheimer and E. F. Alexanderson.

Desirability of poor inherent regulation of large generators.

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General discussion of the principles underlying the success of electric distribution of energy. Power diagram developed for inter-connected distribution systems in which the e.m.f. at all generating receiving stations is maintained constant by varying the reactive power in each receiving station. Numerical examples showing application of the line performance equations and vector diagrams.

Discussion, pp. 631-636, by Messrs. A. E. Kennelly, N. T. Wilcox, E. A. Ekern, Edw. N. Lake, R. P. Jackson.

General remarks on practicability of maintaining constant potential in all parts of a large system.

TRANSMISSION SYSTEMS FROM THE OPERATING STANDPOINT

R. J. C. Wood **Vol. xxx—1911, pp. 835-841**

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Capital charges against irrigated land. Reliability of transmission systems.

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THE USE OF POWER-LIMITING REACTANCES WITH LARGE TURBO-ALTERNATORS

R. F. Schuchardt and E. O. Schweitzer **Vol. xxx—1911, pp. 1143-1194**

Detailed account of exhaustive tests made to determine the action of reactors in the leads of turbo-generators under various conditions, with respect to the effect on the generators, on the secondary apparatus and on the stability of the system. Oscillograms of currents and e.m.fs. are given for each test. Bibliography.

Discussion, incorporated with that of paper by Charles P. Steinmetz on " Development of the Modern Central Station."

SOME RECENT TESTS OF OIL CIRCUIT BREAKERS

E. B. Merriam **Vol. xxx—1911 pp. 1195-1212**

Account of short-circuit rupturing tests with oil switch fed from a 12,000-kw. turbo-generator. Records of oil pressure generated, also e.m.f. and current oscillograms. Construction of oil switch to utilize the explosion energy.

Discussion, incorporated with that of paper by Dr. Charles P. Steinmetz on " Development of the Modern Central Station."

DEVELOPMENT OF THE MODERN CENTRAL STATION

Charles P. Steinmetz **Vol. xxx—1911, pp. 1213-1225**

Short sketch of the development of modern alternating-current station. Uses to which reactors can be put in localizing disturbance in different parts of the system and improving parallel operation of units. Electromechanical synchronizing of distant stations.

Discussion. (including that of paper by R. F. Schuchardt and E. O. Schweitzer " The Use of Power-Limiting Reactances with Large Turbo-Alternators and paper by E. B. Merriam on " Some Recent Tests of Oil Circuit Breakers "), pp. 1226-1249, by Messrs. John W. Lieb, Jr., M. H. Collbohm, D. B. Rushmore, C. W. Stone, B. G. Lamme, W. L. Waters, J. J. Frank, Louis A. Ferguson, R. B. Williamson, Clarence P. Fowler, P. Junkersfeld and C. P. Steinmetz.

General remarks on problems in operation of very large central stations, with special reference to the use of reactance in the circuits.

AUTOMATIC MOTOR CONTROL FOR DIRECT-CURRENT MOTORS

Arthur C. Eastwood **Vol. xxx—1911, pp. 1519-1537**

Advantages of automatic control over manual and description of various types of series current-limit controllers, with curves and data on their actual performance in operation.

Discussion, pp. 1538-1545, by Messrs. E. J. Murphy, Arthur C. Eastwood, Ragner Wikander and Theodore Varney.

Experience in the design of series current-control contactors.

SOME LIMITATIONS OF RHEOSTATIC CONTROL

G. R. Radley and L. L. Tatum **Vol. xxx—1911, pp. 1547-1561**

Discussion of the factors that influence the design of resistors for different classes of service. Data on heating and limiting temperatures for resistance terminals and contacts.

No discussion.

ELEVATOR CONTROL

T. E. Barnum **Vol. xxx—1911, pp. 1563-1583**

Brief description of the construction and performance of typical passenger elevator electric motive power equipment. Photographs, connections diagrams and current-time performance curves.

Discussion, pp. 1584-1585, by Messrs. Fred J. Newman, S. N. Clarkson, Theodore Varney and T. E. Barnum.

Efficiency of worm gears.

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Account of switching tests on 100,000-volt transmission system of the Great Western Power Company. Detailed discussion of large numbers of oscillograms. Recommendation for switching on high-tension systems. Appendix by W. W. Lewis on construction and operation of the photographic attachment to the oscillograph.

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Calculation and design of transmission lines. Experience as to effects of wind, sleet and ice upon lines.

METHODS OF VARYING THE SPEED OF ALTERNATING-CURRENT MOTORS

Gus A. Maier **Vol. xxx—1911, pp. 2455-2484**

Classification of types of varying and adjustable speed alternating-current motors. Comprehensive description of various types of speed control for induction and commutator motors, including rheostatic, compensator, brush shifting, pole changing, concatenation, frequency changer and regulating motors.

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Description of additional methods of speed variation.

17. TRACTION

SOME RECENT DEVELOPMENTS IN RAILWAY TELEPHONY

Gregory Brown **Vol. xxx—1911, 1007-1036**

Outline of requirements of train dispatching service, messenger service and block service. Description of various telephone systems in use on railroads. Photographs of apparatus and circuit diagrams.

Discussion, pp. 1037-1048, by Messrs. Kempster B. Miller, L. B. Cramer, Ralph Bennett, Ralph W. Pope, C. F. Elwell, S. G. McMeen, J. A. Lighthipe, S. J. Lisberger and Gregory Brown.

Experience with telephony in operation of railway and transmission systems.

INDUCTION MACHINES FOR HEAVY SINGLE PHASE MOTOR SERVICE

E. F. W. Alexanderson **Vol. xxx—1911, pp. 1357-1369**

Description, theory and performance characteristics of phase converter for changing single-phase to polyphase energy.

Discussion, incorporated with that of paper by William S. Murray on "Electrification Analyzed, and Its Practical Application to Trunk Line Roads, Inclusive of Freight and Passenger Operation."

ELECTRICAL OPERATION OF THE WEST JERSEY AND SEASHORE RAILROAD

B. F. Wood **Vol. xxx—1911, pp. 1371-1390**

Brief description of electric system and equipment. Cost of construction, maintenance and operation charges in detailed tabular form, and logs of repairs and train detentions.

Discussion, incorporated with that of paper by William S. Murray on "Electrification Analyzed, and Its Practical Application to Trunk Line Roads, Inclusive of Freight and Passenger Operation."

ELECTRIFICATION ANALYZED AND ITS PRACTICAL APPLICATION TO TRUNK LINE ROADS, INCLUSIVE OF FREIGHT AND PASSENGER OPERATION

William S. Murray **Vol. xxx—1911, pp. 1391-1456**

Analysis of passenger, freight and switching service of steam locomotives. Performance and economy tests. Characteristics and advantages of electric locomotives for the same classes of service. Statistical record of operation of New Haven Road, covering train delays, power house, distribution systems and locomotives. Wiring diagrams and constructional drawings for locomotives and distribution system. Cost data for electrification of railroads.

Discussion, (including that of paper by E. F. W. Alexanderson on "Induction Machines for Heavy Single-Phase Motor Service" and paper by B. F. Wood on "Electrical Operation of the West Jersey and Seashore Railroads"), pp. 1457-1518, by Messrs. Frank J. Sprague, Edwin B. Katte, L. C. Fritch, J. L. Woodbridge, Dugald C. Jackson, N. W. Storer, John W. Lieb, Jr., Charles F. Scott, E. F. W. Alexanderson, Philip Dawson, W. N. Smith, B. F. Wood, John B. Taylor, R. E. Hellmund, C. P. Steinmetz, H. Graftio, Gisbert Kapp and W. S. Murray.

General discussion of the problem of equipping trunk lines with electric motive power. Experience with direct-current and single-phase systems. Standardization of electric apparatus.

ELECTROLYTIC CORROSION IN REINFORCED CONCRETE

C. Edward Magnusson and C. H. Smith **Vol. xxx—1911, pp. 2055-2079**

Account of experimental investigation of damage done by stray currents in reinforced concrete. Study of various methods of protecting reinforcement from corrosion.

Discussion, pp. 2080-2101, by Messrs. Burton McCollum, E. W. Stevenson, G. D. Shepardson, George A. Hoadley, A. S. Langsdorf, Maxmilian Toch, Guy F. Shaffer, Harry Barker and W. L. Upson.

Description of similar investigations by the Bureau of Standards and others. Criticism of the author's methods, especially of the high e.m.fs. used. Account of actual example of corrosion in a building.

18. LIGHT, LIGHTING AND LAMPS

THE COST OF ARC LIGHTING AND GENERAL SERVICE FROM MEDIUM AND SMALL SIZE MUNICIPAL OR PRIVATE PLANTS

W. Edgar Reed **Vol. xxx—1911, pp. 1121-1133**

Analysis of cost of lighting small communities. Character of various items that enter into the construction and conditions which modify them. Actual cost of lighting different cities.

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19. ELECTRICITY IN THE ARMY AND NAVY

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